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THE KURTOSIS STATISTICAL CHARACTERISTIC OF OCEAN AMBIENT NOISE AND ITS SIMULATION

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The kurtosis is the most sensitive parameter to judge how different a distribution is from the Gaussian. This paper makes a statistical analysis of the kurtosis of the ocean ambient noise and generates random numbers with a specified kurtosis and power spectral density (PSD), the results show that when the desired kurtosis is bigger than three, the simulation can always be done, but if it is more than 1.5 and less than three, the filter required to meet some certain conditions, the smaller the kurtosis is, the greater the pass-band cutoff frequency of the filter should be.

Keywords: Kurtosis; power spectral density; non-Gaussian ocean ambient noise

1. Introduction

In underwater acoustics, knowledge of the statistical characteristics of the ocean ambient noise is of paramount importance. In general, it is assumed that the ocean ambient noise is stationary and Gaussian, the reason is that the noise is a random combination of an infinite number of independent sources. However, some measurements indicate to the contrary. The degree of deviating from Gaussian distribution can be described using kurtosis.

In 1988, a statistical analysis was given of ambient noise from several ocean acoustic environment by F.W.Machell *et al*, and found that the kurtosis values lying typically between 2.30 and 3.67, also his work showed that the seismic data always is leptokurtic while the merchant data is platykurtic.[1-2] In 1993, R.J.Webster proposed a model under the consideration of kurtosis and studied on how to generate noise sequence with specific kurtosis in 1994.[3-4]

This paper begins with a statement of the definition of the kurtosis and the method to generate the white noise with a certain kurtosis. Then, some numerical simulations are made and discussed in two different cases to generate noise with certain kurtosis. Finally, the statistical characteristic of kurtosis of the ocean ambient noise obtained from an experiment is analyzed, and also random sequences with a similar kurtosis and power spectral density (PSD) with the actual noise is simulated.

2. Statistical Parameter and Theoretical Foundation

2.1. Kurtosis

The ocean ambient noise always takes on apparent non-Gaussianity, and kurtosis is a classical measure of non-Gaussianity of random variable. It is the fourth central moment divided by the fourth power of the standard deviation, [5]which is defined as follows:

$$\beta_2 = \frac{E\left[(X-\mu)^4\right]}{\left\{E\left[(X-\mu)^2\right]\right\}^2} \quad , \tag{1}$$

where X is a random variable and μ is the expected value of the random variable.

Kurtosis is a measure of how outlier-prone or how ‘peaked’ a PDF is. For a normal distribution, the kurtosis value is three. Distributions that have kurtosis greater than three are called leptokurtic, which is associated with the impulsive source, means the tail of the PDF of the variable is thicker than the standard normal distribution. At the same time, a smaller kurtosis indicates a flat distribution, which is connected with sinusoids wave (the kurtosis value is 1.5).

Ref. 4 shows that if x_n are independent identically distribution zero-mean random variables with kurtosis $\beta_2\{x_n\} = \beta_2\{x\}$ and scalars a_n , then

$$\beta_2\{y\} = 3 + (\beta_2\{x\} - 3)\frac{\sum_{n=1}^{N} a_n^{\ 4}}{(\sum_{n=1}^{N} a_n^{\ 2})^2} \quad , \tag{2}$$

$$y = \sum_{n=1}^{N} a_n x_n \quad . \tag{3}$$

If we account a_n as the coefficients of a filter, then according to the expectation kurtosis value and a PSD shaping filter, the theoretical kurtosis values of the input noise can be obtained.

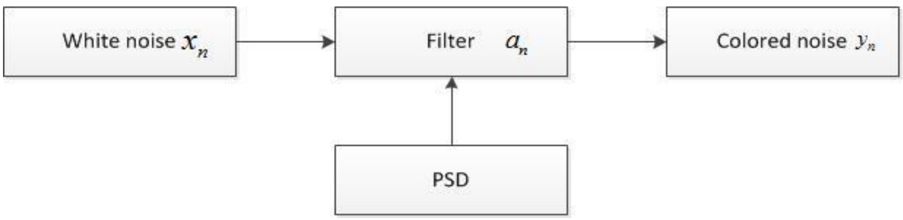

Fig. 1. Non-Gaussian noise simulation flow chart

2.2 White noise generation

In the theory of stochastic processes, white noise sequences can be generated by mathematical and physical methods, including multiplicative congruential and mixed congruential and so on. In this article, because the white noise need to have a specific kurtosis, so the means to produce the noise request to have a certain relationship with the kurtosis in mathematics.

In this essay, a method of sampling damped sinusoids uniformly is adopted. Assume that the damping function is like $[\log 1/t]^n$, $0 < t \le 1$, so we could use two uniform variables to generate the noise

$$x_m = \left[\log \frac{1}{t_{2m-1}} \right]^n \sin\left(2\pi t_{2m}\right) \quad , \tag{4}$$

Here, the kurtosis can be represented by the Gamma function[6]

$$\beta_2 \{x\} = \frac{3}{2} \frac{\Gamma(4n+1)}{\left[\Gamma(2n+1)\right]^2} \quad . \tag{5}$$

3. Non-Gaussian Ocean Ambient Noise Simulation Analysis

In order to verify the feasibility of the theory, some numerical simulations are made. During the simulation procedure, the empirical formula proposed by Knudsen and Wenz is used to describe the anticipant PSD .[7]

$$NL\left(f , U \right) = 25 - 10 \cdot \log\left[f^{5/3} \right] + 10 \cdot \log\left[\left(U/5 \right)^{5/3} \right] \quad . \tag{6}$$

where f is frequency ($0.5 \le f \le 5.0\text{kHz}$) and U is wind speed ($2.5 \le U \le 4.0\text{knots}$). In the simulation, assuming that $U = 3.0\text{knots}, f_s = 20\text{kHz}$ and the spectral level below 0.5kHz is calculated according to the value of 0.5kHz.

The shape of the noise level (design objective) is shown in Fig. 2, and what shown in Figs. 3 and 4 are the amplitude and phase responses of the filter get from Fig.2. It can be seen that the magnitude of the filter has the same shape as the ideal noise level, and at the same time the phase is linear.

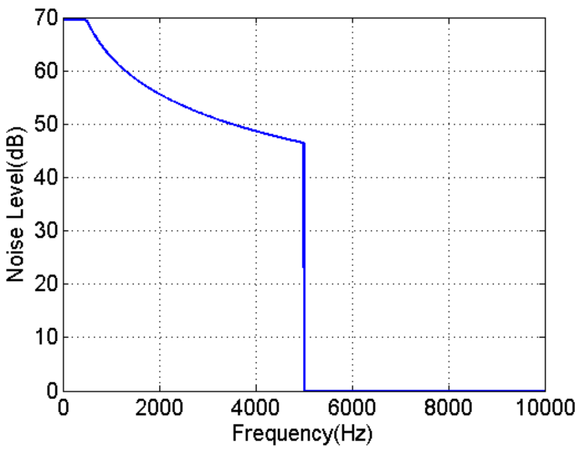

Fig. 2. The ideal noise level

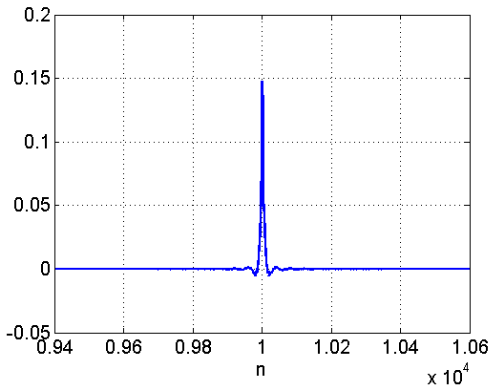

Fig. 3. Unit impulse response

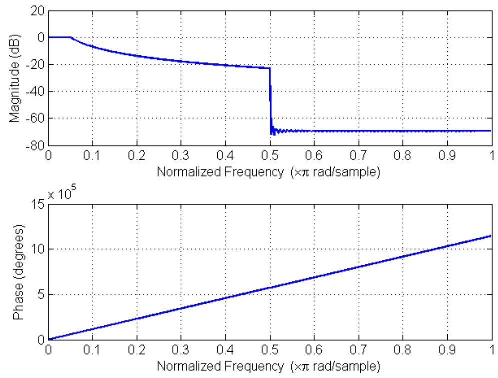

Fig. 4. Amplitude and phase responses

To verify the universality of the method mentioned above, the simulations are accomplished in two cases separately: the kurtosis is greater than three and less than three.

(1) The desired kurtosis $\beta_2\{y\} > 3.0$.

Assuming the expected value is four, then the parameter n can be calculated according to Eq. (2) and (5), and so we can get the form of the damped sinusoidal signal given in Fig. 5. After sampled the sine uniformly and pass through the filter shown in Figs. 3 and 4, the normalized power spectral density (NPSD) is accessible, presented in Fig. 6. And Table 1 shows the kurtosis comparison result of the numerical simulation.

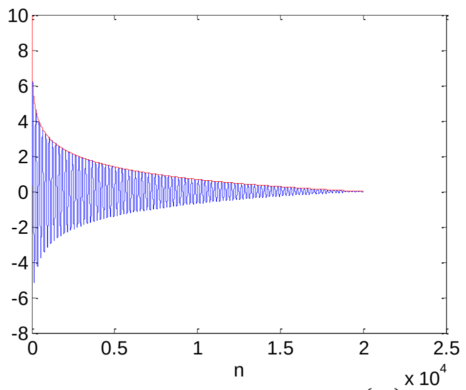

Fig. 5. The damped sine signal ($\beta_2\{y\} > 3$)

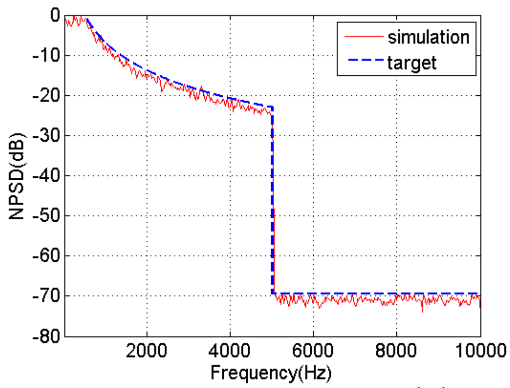

Fig. 6. NPSD simulation result ($\beta_2\{y\} > 3$)

Table 1. Kurtosis simulation result ($\beta_2\{y\} > 3$)

	$\beta_2\{x\}$	$\beta_2\{y\}$
Theory	9.72	4.00
Simulation	9.25	3.97

As shown in Fig. 6, the shape of the simulated NPSD is very similar to the target one, except for some fluctuation in the frequency domain, and this is always called “truncation effect” in digital signal processing. The simulated kurtosis is 3.97 as shown in Table 1, and this is also very close to the target value four. If the kurtosis is changed to other value, a random sequence with a specific PSD and kurtosis can always be simulated.

(2) The desired kurtosis $1.5 \le \beta_2\{y\} \le 3.0$

Assuming the expected kurtosis is 2.80, the results are displayed in Figs. 7 and 8. In this case, it worked out fine. But while the expected kurtosis is 2.70, the simulation can’t be carried out, because now $\beta_2\{x\} < 1.5$, which is contradicted with Eq. (5), meaning the kurtosis should be no less than 1.5.

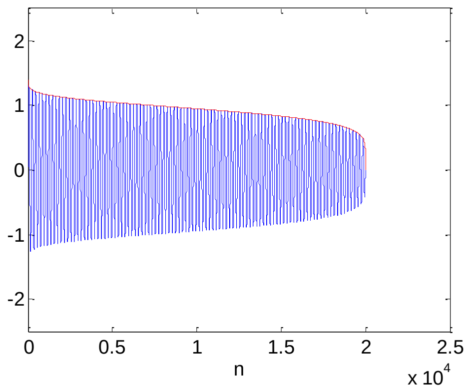

Fig. 7. The damped sine signal

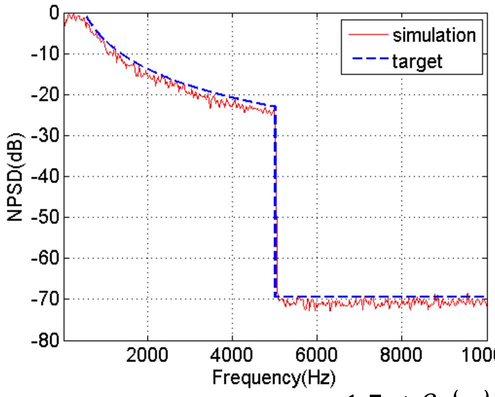

Fig. 8. NPSD simulation result ($1.5 \le \beta_2\{y\} \le 3$)

Table 2. Kurtosis simulation result ($1.5 \le \beta_2\{y\} \le 3$)

	$\beta_2\{x\}$	$\beta_2\{y\}$
Theory	1.66	2.80
Simulation	1.65	2.81

Considering Eq. (2) and (5), some conclusions can be drawn (to denote $\frac{\sum_{n=1}^{N} a_n^{\ 4}}{(\sum_{n=1}^{N} a_n^{\ 2})^2} = \eta$):

(i) If the expectation kurtosis $\beta_2\{y\} > 3$, the kurtosis of white noise $\beta_2\{x\} > 3$; if $\beta_2\{y\}$ is a certain value, $\beta_2\{x\}$ decreases as η increases.

(ii) If the expectation kurtosis $1.5 \le \beta_2\{y\} \le 3$, the kurtosis of white noise $1.5 \le \beta_2\{x\} \le 3$; if $\beta_2\{y\}$ is a certain value, the changes of $\beta_2\{x\}$ and η are synchronous.

(iii) If the expectation kurtosis $1 \le \beta_2\{y\} < 1.5$, the simulation is unachievable.

Therefore, the value of $\beta_2\{x\}$ can be adjusted to more than 1.5 by increasing the value of η.

For a filter, the value η is connected with its cutoff frequency and order. Their relationship is shown in Figs. 9 and 10. It can be concluded that while the filter order is constant, η increases with the increase of cutoff frequency. And while the cutoff frequency is constant, η is reduced with the growth of filter order.

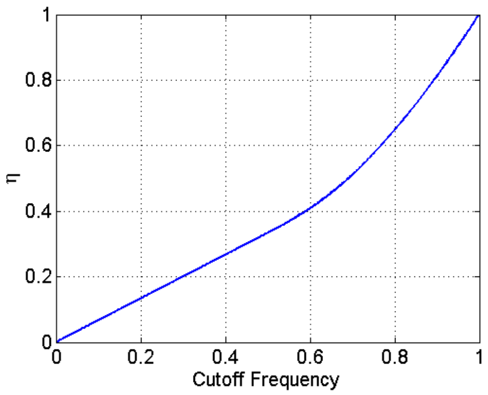

Fig. 9. η and cutoff frequency

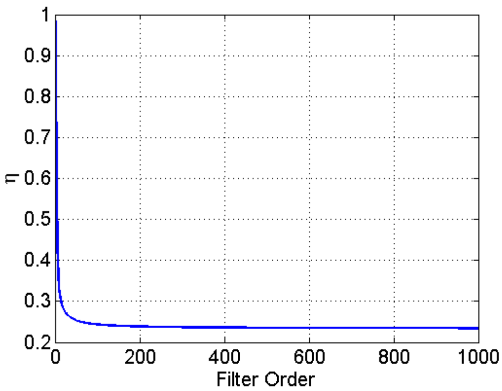

Fig. 10. η and filter order

In consequence, the value of η can be increased by advancing the cutoff frequency and reducing the filter order. That means when the expected kurtosis value is between 1.5 and three, the filter required to meet some certain conditions: the smaller the kurtosis is, the greater the pass-band cutoff frequency should be.

The pass-band cutoff frequency for a filter can be expressed as follows:

$$w_p = 2\pi f_p / f_s \quad . \tag{7}$$

where f_p is the highest frequency of the signal in the pass-band (Hz) and f_s is sampling frequency (Hz). So the cutoff frequency goes up as the sampling frequency goes down and when the anticipant kurtosis $\beta_2\{y\}$ is less than three, the noise simulation can be completed by means of downsampling or using a lower filter order.

So, when the sampling frequency reduced from $f_s = 20\mathrm{kHz}$ to $f_s = 14\mathrm{kHz}$, the results are present in Figs.11, 12 and Table 3. At this moment, the NPSD and the kurtosis are both consistent with the target.

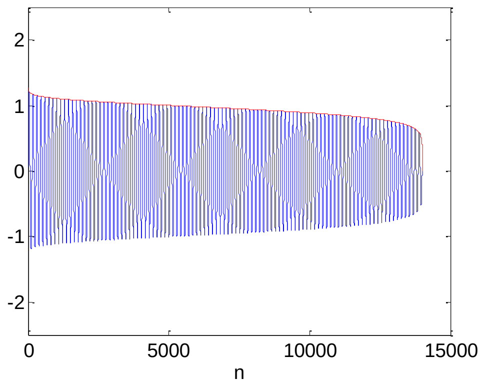

Fig. 11. The damped sine signal

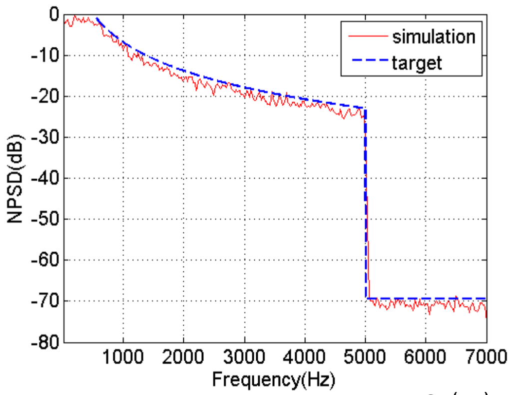

Fig. 12. NPSD simulation result ($1.5 \le \beta_2\{y\} \le 3$)

Table 3. Kurtosis simulation result ($f_s = 14\mathrm{kHz}, 1.5 \le \beta_2\{y\} \le 3$)

	$\beta_2\{x\}$	$\beta_2\{y\}$
Theory	1.59	2.70
Simulation	1.59	2.65

4. Experiment Data Analysis

4.1. Kurtosis statistics

To get the ocean ambient noise, an experiment was carried out in a certain sea. During the experiment, the hydrophone ($f_s = 22\mathrm{kHz}$) is fixed to the bottom of the sea, and the data were measured ten minutes per half hour for more than 50 hours totally.

For every ten minutes data, it is divided into 10 blocks of length 1320000, corresponding to 60 seconds, and for each block the value of kurtosis is calculated separately. Fig.13 shows the statistics of the whole 50h, 100blocks in total, by boxplot graph. Then the data are regrouped into 20 blocks, 30 blocks, 40 blocks, 100 blocks, 600 blocks and 1200 blocks sequentially. For different block numbers, the mean, the median, the 95% and 5% of the kurtosis are calculated, and the outcomes are shown in Table 4 and Fig. 14. The results show that with larger segment lengths, the data have a larger kurtosis and the values lie typically between 2.56 and 3.51, which is agree with F.W.Machell’s observation 2.30~3.67 in different environments. Although the amount of data is not large enough for statistical research, yet it is a reference for further study.

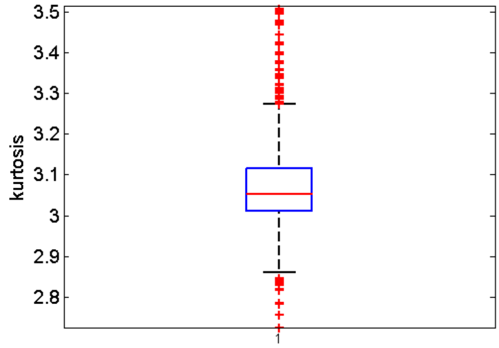

Fig. 13. Boxplot graph of kurtosis (block length is 60 seconds)

Table 4. The spread of kurtosis of different block length (95%-5%)

block length(s)	60	30	20	15	6	1	0.5
95%	**3.51**	3.36	3.31	3.27	3.25	3.33	3.40
5%	2.93	2.89	2.88	2.87	2.82	2.66	**2.56**

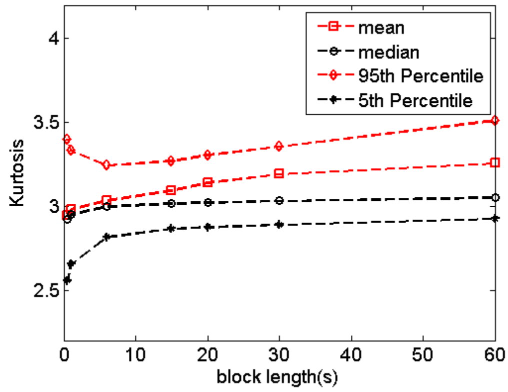

Fig. 14 Spread of data kurtosis (mean-median, 95%-5%)

4.2 Kurtosis simulation

According to the method mentioned above, generating the random sequence having the similar kurtosis and NPSD with the real ambient noise.

Firstly, the kurtosis of target noise is 3.1, the duration is 30 seconds, the sample rate is 22 kHz and the result is shown in Figs. 15 and 16.

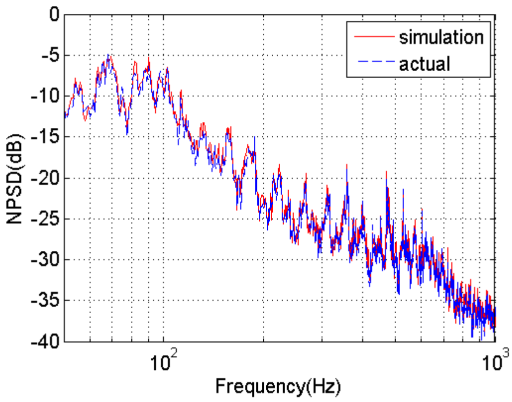

Fig. 15 Comparison of NPSD (kurtosis=3.144)

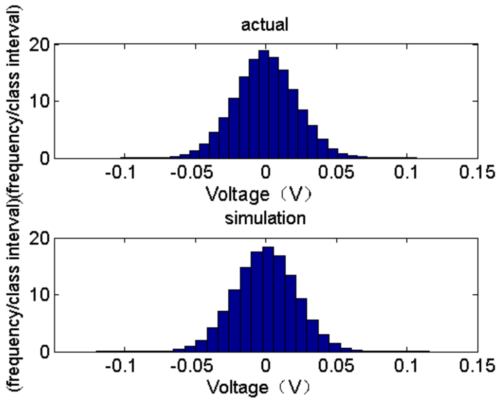

Fig. 16 Comparison of frequency histogram (kurtosis=3.144)

Secondly, the kurtosis of target noise is 2.9, and the duration time is also 30 seconds but the sample rate is decreased to 4.8 kHz .Figs. 17 and 18 show the comparison of the NPSD and frequency histogram.

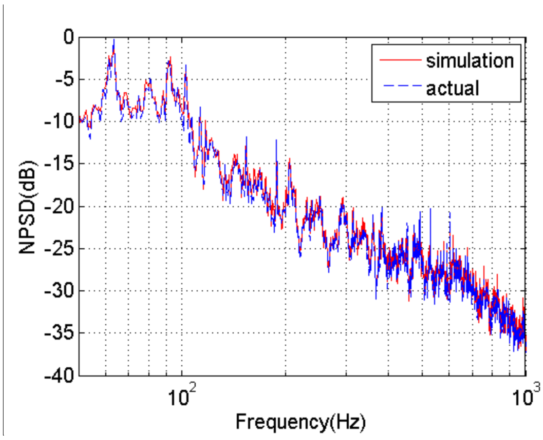

Fig. 17 Comparison of NPSD (kurtosis=2.918)

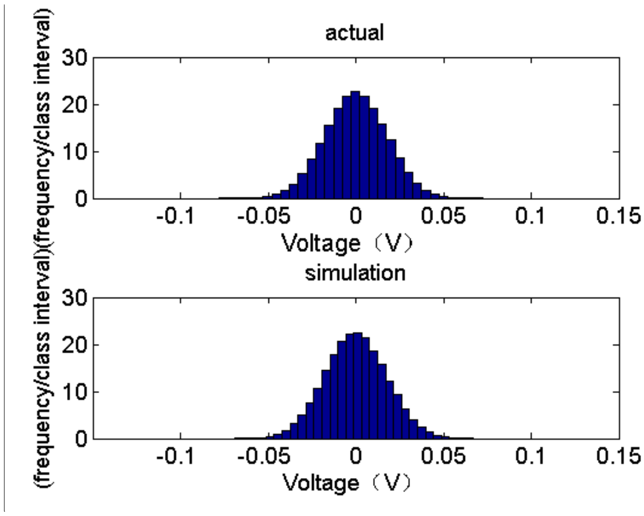

Fig. 18 Comparison of frequency histogram (kurtosis=2.918)

Table 5. Comparison of the kurtosis value

Actual	3.144	2.918
Simulation	3.214	2.921

From the comparison of NPSD between simulation noise series and actual ambient noise (Figs.15 and 17), it can be seen that they are approximately equal, and there are 1~2dB differences in individual frequencies.

Contrast of frequency histogram (shown in Figs.16 and 18) show that the kurtosis of actual noise is 3.144 and 2.918 while the simulation data's are 3.214 and 2.921, very close to the truth. It should be pointed out that due to the randomness of the white noise, results of each simulation may be a bit different, and the deviation is ± 0.02.

5. Conclusions

Kurtosis is a classical measure of non-Gaussianity of random variable. This study analyzed the statistical characteristics of ocean ambient noise, and produced non-Gaussian noise having almost the same PSD and kurtosis with the real one. The results show that when the expected kurtosis is bigger than three, it can be achieved with any arbitrary PSD; at the same time, when it is smaller than 1.5, the simulation unable to be achieved, and in this case, the kurtosis have no significance; when the kurtosis values falls in between 1.5 and three, only the PSD shape filter satisfy some certain conditions can the simulation be carried out. The bigger the anticipant kurtosis is, the bigger the cutoff frequency (the smaller the sampling frequency) should be, or the order of the filter should be reduced.

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