SIDDHARTH PARAAG NILAKHE

ML HW-4

Q1] Bayes Theorem

Proof:

We know that

$$P(E_i|A) = \frac{P(E_i \cap A)}{P(A)} \quad \text{—— (i)}$$

$$P(E_i \cap A) = P(E_i|A).P(A) = P(A|E_i)P(E_i) \quad \text{—— (ii)}$$

Then we know that,

$$P(A) = \sum_{k=1}^{n} P(A|E_k).P(E_k) \quad \text{—— (iii)}$$

Replacing (ii) and (iii) in (i)

$$P(E_i|A) = \frac{P(A|E_i).P(E_i)}{\sum_{k=1}^{n} P(A|E_k).P(E_k)}$$

Hence, Prooved

Before understanding why it is useful, let us understand what Bayes Theorem is? It is a method of calculating conditional probability which is the probability that an event occurs given that another event occurred. Conditional probabilities are vital to get accurate prediction in machine learning problem. ~~It~~ ~~its~~ The main use is that it helps in building more accurate models.

Q2] $P(\text{cancer}) = 0.008$ $\quad P(\neg\text{cancer}) = 0.992$

$P(+|\text{cancer}) = 0.98$ $\quad P(+|\neg\text{cancer}) = 0.03$

$$P(\text{cancer}|+) = \frac{P(+|\text{cancer}) \cdot P(\text{cancer})}{P(+|\text{cancer}) \cdot P(\text{canor}) + P(+|\neg\text{cancer}) P(\neg\text{cancer})}$$

$$= \frac{0.98 \times 0.008}{(0.98 \times 0.008) + (0.003 \times 0.992)}$$

$$= 0.208$$

$$P(\neg\text{cancer}|+) = 1 - P(\text{cancer}|+) = 1 - 0.208$$

$$= 0.792$$

$$P(\text{cancer}|++) = \frac{P(+|\text{cancer}) \times P(\text{cancer}|+)}{P(+|\text{cancer}) \times P(\text{cancer}|+) + P(+|\neg\text{caner})\, P(\neg\text{cancer}|+)}$$

$$= \frac{0.98 \times 0.208}{(0.98 \times 0.208) + (0.03 \times 0.792)}$$

$$P(\text{cancer}|++) = 0.895$$

$$P(\neg\text{cancer}|++) = 1 - P(\text{cancer}|++)$$

$$= 1 - 0.895$$

$$= 0.105$$

Q3] From the table given we get,

$P(Yes) = 8/12$ $\quad$ $P(No) = 4/12$

Outlook	Yes	No
Sunny	2/8	3/4
Rain	3/8	1/4
Overcast	3/8	0/4

Humidity	Yes	No
High	3/8	3/4
Normal	5/8	1/4

Temperature	Yes	No
Hot	1/8	2/4
Mild	4/8	1/4
Cool	3/8	1/4

Wind	Yes	No
Weak	5/8	2/4
Strong	3/8	2/4

Instance to Find <Outlook = Sun, Temp = Cool, Humidity = High, Wind = Strong>

For Yes,

$P(Yes) \cdot P(Sunny|Yes) \cdot P(Cool|Yes)\, P(High|Yes) \cdot P(Strong|Yes)$

$= \frac{8}{12} \cdot \frac{2}{8} \cdot \frac{3}{8} \cdot \frac{3}{8} \cdot \frac{3}{8} = 0.0087$

For No,

$P(No) \cdot P(Sunny|No)\; P(Cool|No)\; P(High|No)\; P(Strong|No)$

$= \frac{4}{12} \times \frac{3}{4} \times \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} = 0.023$

~~Hence~~ $P(No) > P(Yes)$

Hence, classified as No.

Q4] For the first iteration:

$net\_c = w_{c0} + a \times w_{ca} + b \times w_{cb}$

$= 0.1 + 0.1 + 0 = 0.2$

$O_c = \frac{1}{1+e^{-netc}} = \frac{1}{1+e^{-0.2}} = 0.55$

$net\_d = w_{d0} + O_c \times w_{d0}$

$= 0.1 + (0.55 \times 0.1) = 0.155$

$O_d = \frac{1}{1+e^{-net\,d}} = \frac{1}{1+e^{-0.155}} = 0.539$

By using backpropagation we get,

$S_d = O_d \cdot (1 - O_d) \cdot (t_a - O_a)$

$= 0.539 (1 - 0.539)(1 - 0.539)$

$= 0.1215$

$\Delta w_{d0} = 0.034$

$w_{dc} = w_{dc} + \Delta w_{dc} = 0.1 + 0.019 = 0.119$

$w_{d0} = w_{d0} + \Delta w_{d0} = 0.1 + 0.034 = 0.134$

$S_c = O_c \; (1 - O_c) \; (w_c \times S_d)$

$= 0.55 \; (1 - 0.55) \; (0.1 \times 0.115)$

$= 0.003$

Now,

$\Delta w_{ca} = \eta\, \delta_c x_a + \alpha \cdot 0$

$= 0.3 \times 0.003 \times 1 = 0.001$

$\Delta w_{co} = 0.001$

$\Delta w_{cb} = 0$

$w_{co} = w_{co} + \Delta w_{co} = 0.1 + 0.001 = 0.101$

$w_{ca} = w_{ca} + \Delta w_{ca} = 0.1 + 0.001 = 0.101$

$w_{cb} = w_{cb} + \Delta w_{cb} = 0.1 + 0 = 0.1$

For 2nd iteration we have

$net\_c = 0.101 + 0 + 0.100 = 0.201$

$O_c = \dfrac{1}{1+e^{-0.201}} = 0.53$

$net\_d = 0.134 + 0.88 \times 0.119 = 0.1994$

$O_d = \dfrac{1}{1+e^{-0.1999}} = 0.5496$

By using back propagation we get,

$\delta_d = 0.5496\,(1-0.5496)\,(0-0.5496)$

$= -0.136$

$\Delta w_{dc} = 0.3\,(-0.136) \cdot 0.88 + (0.9 \times 0.019)$

$= -0.0053$

$\Delta w_{do} = -0.01$

$w_{dc} = 0.119 - 0.0053 = 0.113$

Page No.
Date

$W_{do} = 0.134 - 0.01 = 0.124$

$\delta_c = 0.55 \times (1 - 0.55)(0.113)^-(0.136)$
$= -0.003$

Now, as done before

$\Delta W_{ca} = 0.0009$
$\Delta W_{co} = 0$
$\Delta W_{cb} = -0.001$

As done before we get

$W_{co} = 0.101$
$W_{ca} = 0.1042$
$W_{cb} = 0.009$

$W_{do} = 0.124$
$W_{dc} = 0.113$

~~Answer~~