

ML HW-2

$$\begin{aligned}
 ① E(w) &= \sum_{i=1}^m (w^T x^i - y^i)^2 + \lambda \sum_{i=1}^m w_i^2 \\
 &= (x^T w - y)^T (x^T w - y) + \lambda w^T w \\
 &= (x^T w)^T (x^T w) - (x^T w)^T y - y^T (x^T w) + y^T y + \lambda w^T w \\
 &= w^T x^T x w - w^T x^T y - y^T x^T w + y^T y + \lambda w^T w
 \end{aligned}$$

$$\begin{aligned}
 ② E(w) &= 2x^T x w - 2x^T y + 2\lambda w = 0 \\
 &\therefore 0
 \end{aligned}$$

Thus $(x^T x + \lambda I) w = x^T y$

We then get,

$$w = (x^T x + \lambda I)^{-1} x^T y$$

② ① How many parameters? \rightarrow We need $(n+1)k$ parameters

What are these? $\rightarrow \theta_0, \theta_1, \theta_2, \dots, \theta_n$ for each class

② We know

$$J(\theta) = -\frac{1}{m} \sum_{i=1}^m \sum_{k=1}^K y_k^{(i)} \log \left(\hat{p}_k^{(i)} \right)$$

$$\text{So, } \hat{p}_k^{(i)} = c(s_k(x))_k$$

$k \rightarrow$ number of classes

$$= \frac{\exp(s_k(x))}{\sum_{i=1}^K \exp(s_k(x))}$$

Here $s_k = x^T (\theta^{(k)})$

$$\frac{\partial J(\theta)}{\partial \theta_k} = \frac{\partial J(\theta)}{\partial \hat{p}_k} \cdot \frac{\partial \hat{p}_k}{\partial s_k} \cdot \frac{\partial s_k}{\partial x}$$

$$\frac{\partial \hat{p}_k}{\partial s_k} = -\frac{1}{m} \sum_{i=1}^m \sum_{k=1}^K y_k^{(i)} \frac{1}{\hat{p}_k^{(i)}}$$

Then,

$$\frac{\partial \hat{p}_k^{(i)}}{\partial s_k} = \frac{\exp(s_k(x))}{\sum_{i=1}^K \exp(s_k(x))} \left[\sum_{i=1}^K \exp(s_k(x)) - \exp(s_k(x)) \right]^2$$

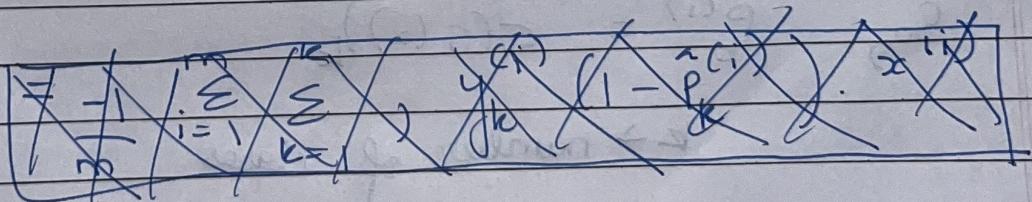
which gives

$$= \hat{p}_k^{(i)} \left(1 - \hat{p}_k^{(i)} \right)$$

$$\text{As } \frac{\exp(s_k(x))}{\sum_{i=1}^K \exp(s_k(x))} = \hat{p}_k^{(i)}$$

From the equation earlier,

$$\frac{\partial J(\theta)}{\partial \theta_k} = -\frac{1}{m} \sum_{i=1}^m \sum_{k=1}^K y_k^{(i)} \cdot \frac{1}{\hat{p}_k^{(i)}} \cdot \hat{p}_k^{(i)} \left(1 - \hat{p}_k^{(i)} \right) \cdot \frac{\partial \hat{p}_k^{(i)}}{\partial s_k}$$



As $y_k^{(i)} = 1$ when it belongs to k^{th} class

$$\frac{\partial J(\theta)}{\partial \theta_k} = \frac{1}{m} \sum_{i=1}^m \sum_{k=1}^n (p_k^{(i)} - y_k^{(i)}) x^{(i)}$$

Hence, Proved