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MA 641 HW 1

Q6] If X and Y are dependent but $V(X) = V(Y)$

T.F $\Rightarrow$ $Cov(X+Y, X-Y)$

$\therefore Cov(X+Y, X-Y) \Rightarrow E[(X+Y)(X-Y)] - E(X+Y) E(X-Y)$

which gives us

$\Rightarrow E(X^2 - Y^2) - [E(X^2) - E(Y^2)]$

but as we know

$E(X) = E(Y)$

therefore we get

$\Rightarrow E(X^2 - Y^2) - 0 \Rightarrow 0$

$\because E(X^2) = E(Y^2)$

$V(X) = E(X^2) - [E(X)]^2$

$V(Y) = E(Y^2) - [E(Y)]^2$

$\therefore Cov(X+Y, X-Y) = E[X^2 - Y^2] - [E(X^2) - E(Y^2)]$

$= 0$

Q4] To show that $\{Y_t\}$ is stationary we need to show $\mu_t = E(Y_t) = \mu$ is constant for all t and $\gamma_{t,t-k} = Cov(Y_t, Y_{t-k})$

a] To Find $\rightarrow E(Y_t)$

$E(Y_t) = E\left(\sum_{s=-\infty}^{\infty} a_s e_{t-s}\right)$

$= \sum_{s=-\infty}^{\infty} a_s E(e_{t-s})$

but from the given in the Q we get

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$$E(Y_t) = \sum_{s=-\infty}^{\infty} a_s \times 0 = 0$$

which tells us $E(Y_t) = 0$ for all t and is stationary.

6] $Cov(Y_t, Y_{t-k}) = \sigma^2 \sum_{i=-\infty}^{\infty} a_{k+i} a_i$

To Find :- Is the process $\{Y_t\}$ stationary

$\Rightarrow Cov(Y_t, Y_{t-k}) = Cov\left(\sum_{s=-\infty}^{\infty} a_s e_{t-s}, \sum_{s=-\infty}^{\infty} a_s e_{t-k-s}\right)$

i) for $t \neq s$ $E(e_t) = 0$
$\therefore$ we get cov between e_t and e_s will be 0.

ii) For $t = s$
$Cov(Y_t, Y_{t-k}) = Cov(a_0 e_t, a_k e_{t-k})$
$\Rightarrow a_0 a_k Cov(e_t, e_{t-k})$

as we know e_t and e_{t-k} are iid
which means that cov depends on ~~lag~~ lag k
$Cov(e_t, e_{t-k}) = \sigma^2 \gamma_k$
simply giving us

$$Cov(Y_t, Y_{t-k}) = \begin{cases} a_0 a_k \sigma^2 & k \neq 0 \\ 0 & k = 0 \end{cases}$$

$$\therefore Cov(Y_t, Y_{t-k}) = \sigma^2 \sum_{i=-\infty}^{\infty} a_{k+i} a_i$$

Hence, we get to know that ~~at~~ it clearly depends on the lag k and not on time t.
$\therefore \{Y_t\}$ is Stationary
Cov of Y_t and $E(Y_t)$ is not dependent on t.

Q7] a] $Y_t = X$

$E(X) = 4$ and $Var(X) = \sigma^2$

Let us assume that X does not depend on t

then $E(Y_t) = E(X) = 4$

$Var(Y_t) = Var(X) = \sigma^2$

$Cov(Y_t, Y_{t-k}) = Cov(X, X)$

giving us $Var(X)$

which is nothing but σ^2

As we can understand from above that none of the above are dependent on t, the process $\{Y_t\}$ is stationary

b] To find - γ_k for $\{Y_t\}$

$\gamma_k = Cov(Y_t, Y_{t-k})$

$= Cov(X, X)$

giving us $Var(X) = \sigma^2$

$\therefore \gamma_k = \sigma^2$

Q2] Given - First difference $W_t = Y_t - Y_{t-1}$

Second difference $Z_t = W_t - W_{t-1}$

To write the model in the form

$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t$

Now, $Z_t = W_t - W_{t-1}$

$= Y_t - Y_{t-1} - (Y_{t-1} - Y_{t-2})$

$= Y_t - 2Y_{t-1} - Y_{t-2}$

$Y_t = Z_t + 2Y_{t-1} - Y_{t-2}$

$Y_t = e_t + 2Y_{t-1} - Y_{t-2}$

~~where~~ $\therefore \phi_1 = 2, \quad \phi_2 = -1 \quad e_t = 2t$

$\therefore$ Model, $Y_t = 2Y_{t-1} - Y_{t-2} + e_t$

Q5] a] $Y_t = \sum_{i=1}^{t} e_t$ $\quad E(e_t) = \mu > 0$

Now we get

$E(Y_t) = E\left(\sum_{i=1}^{t} e_t\right) \Rightarrow \sum_{i=1}^{t} E(e_t)$

which gives us nothing but

$E(Y_t) = t\mu$

which tells us the process $\{Y_t\}$ is not stationary because $E(Y_t)$ depends on t

b] $E(Y_t) = \sum_{i=1}^{t} E(e_t) = 0$

$\therefore \quad Var(Y_t) = Var\left(\sum_{i=1}^{t} e_t\right)$

$= \sum_{i=1}^{t} Var(e_t)$

from the given information as we know $Var(e_t) = \sigma^2$

Hence $\Rightarrow \sum_{i=1}^{t} Var(e_t) = t\sigma^2$

which tells us the process $\{Y_t\}$ is not stationary because $V(Y_t)$ depends on t.

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Q1) Cauchy Schwarz Inequality says that for any RV X & Y
$[E(XY)]^2 \leq E(X^2)\, E(Y^2)$
If $E(X^2) = 0$ then $P(X=0) = 1$
and $E(XY) = 0$

a] Now, $0 < E(X^2) < \infty$ & $0 < E(Y^2) < \infty$
For a & b, we get
① $0 \leq E[(aX + bY)^2] = a^2 E(X^2) + b^2 E(Y^2) + 2ab\, E(XY)$
② $\Rightarrow 0 \leq E[(aX - bY)^2] = a^2 E(X^2) + b^2 E(Y^2) - 2ab\,E(XY)$

For ①
$0 \leq E[(\sqrt{E(Y^2)}\,X + \sqrt{E(X^2)}\,Y)^2] = E(Y^2)E(X^2) + E(X^2)E(Y^2) + 2\sqrt{E(Y^2)E(X^2)}\,E(XY)$

giving us
$0 \leq E(X^2)E(Y^2) + \sqrt{E(Y^2)E(X^2)}\,E(XY)$
↳ ③

In the same way,
$0 \leq E[(aX - bY)^2]$ will give
$0 \leq E(X^2)E(Y^2) - \sqrt{E(Y^2)E(X^2)}\,E(XY)$
↳ ④

Solving ③
$-\sqrt{E(Y^2)E(X^2)}\,E(XY) \leq E(X^2)E(Y^2)$
$E(XY) \leq \sqrt{E(X^2)E(Y^2)}$
Similarly, $E(XY) \geq -\sqrt{E(X^2)E(Y^2)}$
Hence we get
$-\sqrt{E(X^2)E(Y^2)} \leq E(XY) \leq \sqrt{E(X^2)E(Y^2)}$
Simplifying $[E(XY)]^2 \leq E(X^2)\,E(Y^2)$
telling us that the Cauchy Schwarz holds

6] $[E(UW)]^2 \le E(U^2) E(W^2)$

$-\sqrt{E(U^2) E(W^2)} \le E(UW) \le \sqrt{E(U^2) E(W^2)}$

$U = X - E(X)$

$U^2 = (X - E(X))^2$

$E(U^2) = E[X - E(X)]^2 \Rightarrow Var(X)$

$E(W^2) = Var\ Y$

$-\sqrt{Var(X)\, Var(Y)} \le cov(XY) \le \sqrt{Var(X)\, Var(Y)}$

$-1 \le \dfrac{cov(XY)}{\sqrt{var(X)\, Var(Y)}} \le 1$

giving us $-1 \le corr(XY) \le 1$