SIDDHARTH NILAKHE

MA - 641 Assignment 4

Q3] Given :- Suppose that $\{Y_t\}$ is generated according to $Y_t = e_t + c(e_{t-1} + e_{t-2} + e_{t-3} + \dots + e_0)$ for $t > 0$ & $c > 0$

a] To Find → Mean & covariance fns for $\{Y_t\}$

$E(Y_t) = E(e_t + c(e_{t-1} + e_{t-2} + e_{t-3} + \dots e_0))$

$= E(e_t) + c\, E(e_{t-1}) + c\, E(e_{t-2}) \dots + c\, E(e_0)$

giving us $\Rightarrow 0$

for $e_t \sim WN(0, \sigma_e^2)$

$Var(Y_t) = Var(e_t) + c^2 \sum_{i=1}^{t} Var(e_{t-i}) = \sigma_e^2 + \sigma_e^2 (t c^2)$

depends on t, $c \neq 0$

Now finding covariance

$\therefore Cov(Y_t, Y_{t-k}) = Cov(e_t + c \sum_{i=1}^{t} e_{t-i}, e_{t-k} + c \sum_{i=1}^{t-k} e_{t_i - k})$

giving us $\Rightarrow c(1 + ct)\sigma_e^2$

Here we can clearly see it is dependent on t. Hence it is not stationary.

b] $\nabla Y_t = Y_t - Y_{t-1}$

To Find - mean & covariance for ∇Y_t

$\therefore E(\nabla Y_t) = E(e_t) + c - 1\, E(e_t - 1)$

giving us $\Rightarrow 0$

Now finding covariance

$Cov(Y_t, Y_{t-k}) = Cov(e_t + (c-1)e_{t-1}, e_{t-k} + c(c-1)e_{t-k})$

Solving for the above we get

$\Rightarrow \sigma_e^2 (c^2 - 2c + 2)$ for $k = 0$

$\Rightarrow \sigma_e^2 (c - 1)$ for $k = 1$

& 0 for $k = 2, 3$ i.e $k > 1$

As we can see it is not dependent i.e independent of t we can say that ∇Y_t is infact stationary

c] We can identify $\{Y_t\}$ as a ARIMA (0, 1, 1) process.

Q 1] a] $Y_t = Y_{t-1} - 0.25 Y_{t-2} + e_t - 0.1 e_{t-1}$

To find - p, d & q & param values

→ From the given we can figure out that

i. Order of moving average (MA) → q → = 1

ii. Differentiating order → d = 1

iii. & the order of autoregressive i.e AR → p = 2

iv. $\phi_1 = 1$ v. $\phi_2 = -0.25$ $\theta_1 = -0.1$

Checking the stationarity condition

$\phi_1 + \phi_2 = 0.75 < 1$ $|\phi_2| = 0.25 < 1$

$\phi_2 - \phi_1 = -1.25 < 1$

Hence we were able to identify the process as ARIMA (2, 1, 1) with $\phi_1 = 1$ $\phi_2 = -0.25$ $\theta_1 = -0.1$ as parameters

b] Similarly for (b)

$Y_t = 0.5 Y_{t-1} - 0.5 Y_{t-2} + e_t - 0.5 e_{t-1} + 0.25 e_{t-2}$

→ p = 2 → q = 2 → d = 1

$\phi_1 = 0.5$ $\phi_2 = -0.5$

$\theta_1 = -0.5$ $\theta_2 = 0.25$

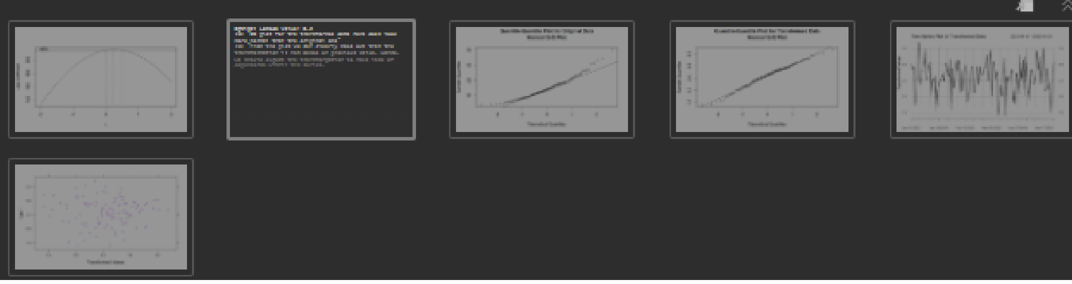
$\phi_1 + \phi_2 < 1$ $|\phi_2| < 1$ $\phi_2 - \phi_1 < 1$

Hence we get ARIMA (2, 1, 2) with $\phi_1 = 0.5$ $\phi_2 = -0.5$ $\theta_1 = -0.5$ $\theta_2 = 0.25$ as params

<u>QUESTION 7</u>

Question 7

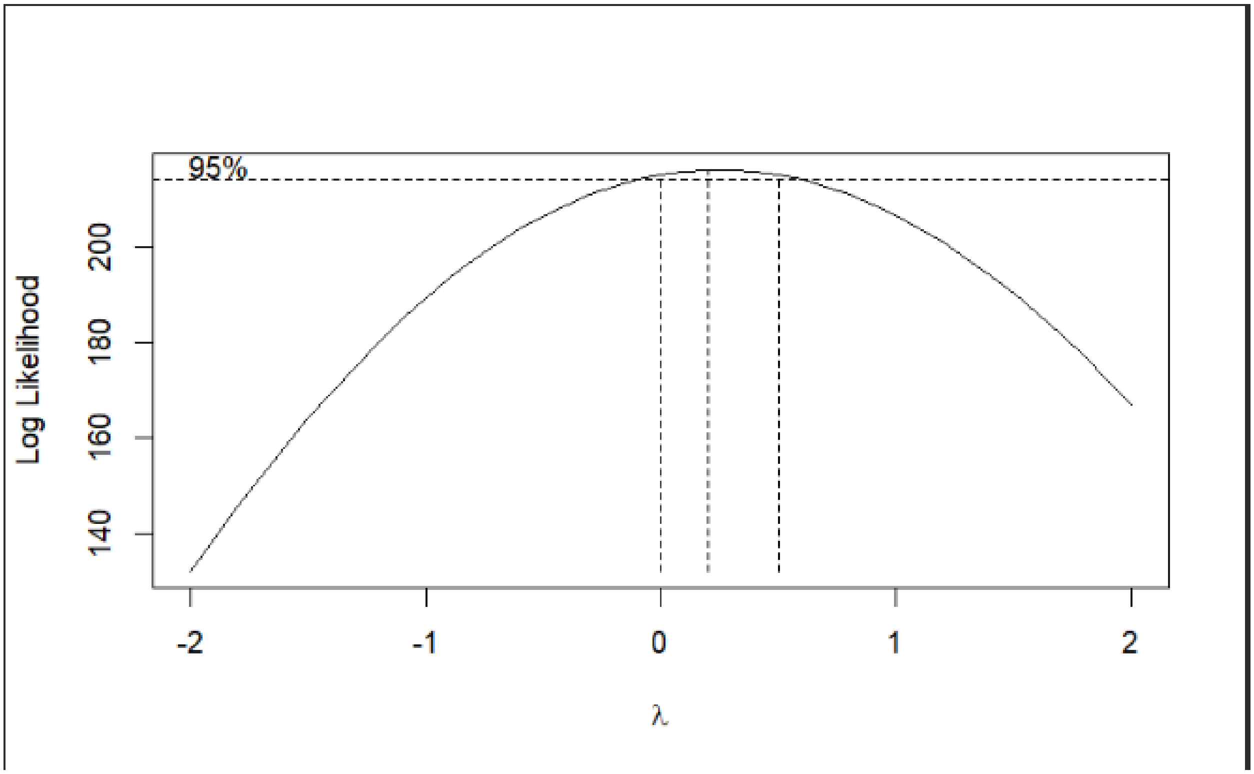
```r
{r q7}

library(lattice)
library(xts)
data(larain)
larain_data <- as.xts(larain)

boxcox_obj <- BoxCox.ar(larain_data)
larain_data_transformed <- log(larain_data)
best_lambda <- which.max(boxcox_obj$loglike)
cat("Optimal Lambda value:", boxcox_obj$lambda[best_lambda], "\n")
larain_data_dates <- as.Date("2023-01-01") + 0:(length(larain_data_transformed) - 1
)
larain_data_transformed_xts <- xts(larain_data_transformed, order.by =
larain_data_dates)

qqnorm(larain_data)
qqline(larain_data)
title("Quantile-Quantile Plot for Original Data", line = 3)

qqnorm(larain_data_transformed)
qqline(larain_data_transformed)
title("Quantile-Quantile Plot for Transformed Data", line = 3)

print("QQ plot for the transformed data here does look more normal than the
original one")

plot(larain_data_transformed_xts, main = "Time Series Plot of Transformed Data",
ylab = "Transformed Values")

xyplot(zlag(larain_data_transformed) ~ larain_data_transformed, ylab = "Lag 1",
xlab = "Transformed Values", col = "purple")

print("From the plot we can clearly make out that the transformation is not based
on previous value. Hence, we should expect the transformation to have lack of
dependence within the series.")
```

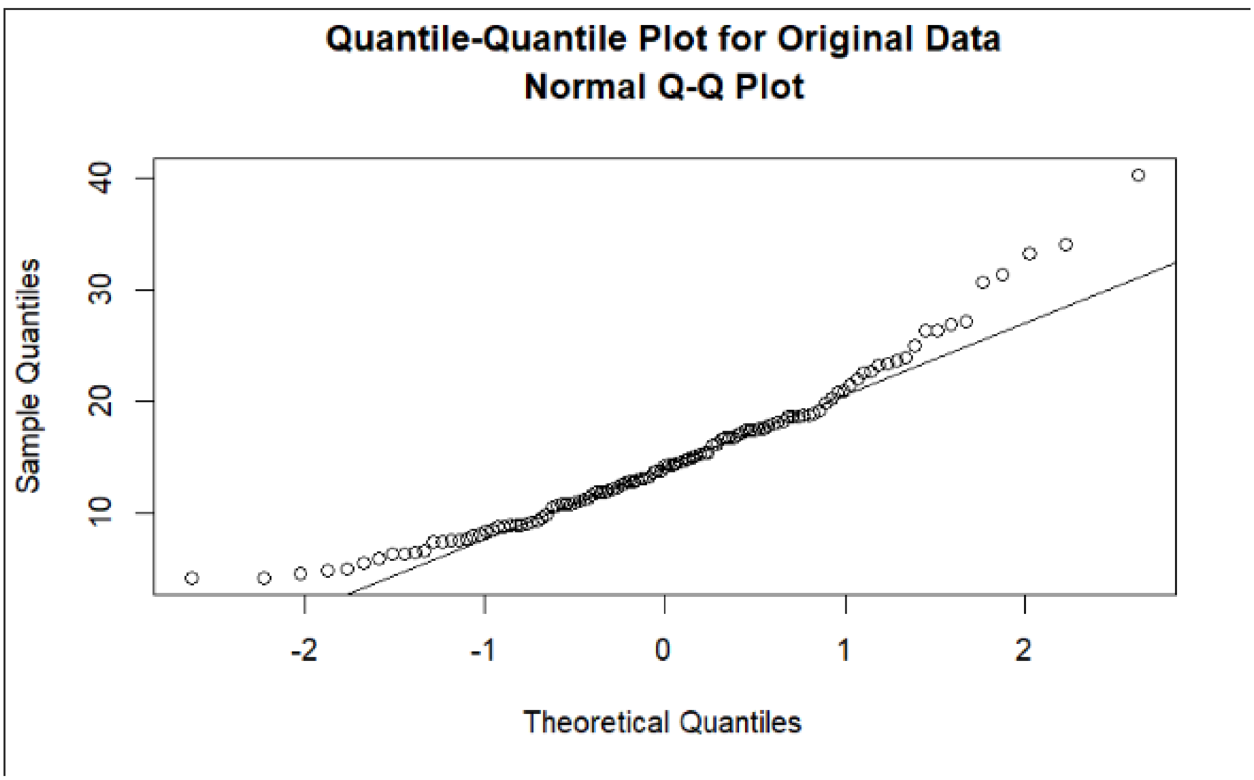


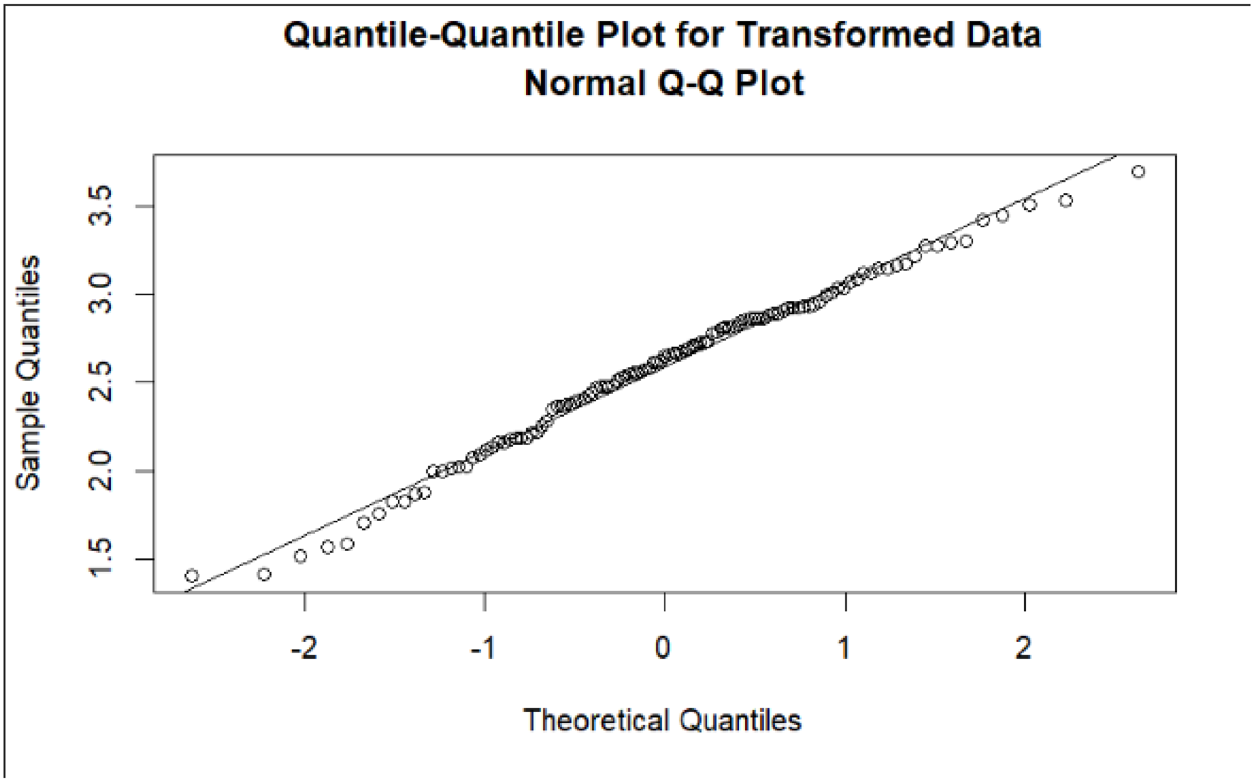
<u>CODE IN TEXT FORM</u>

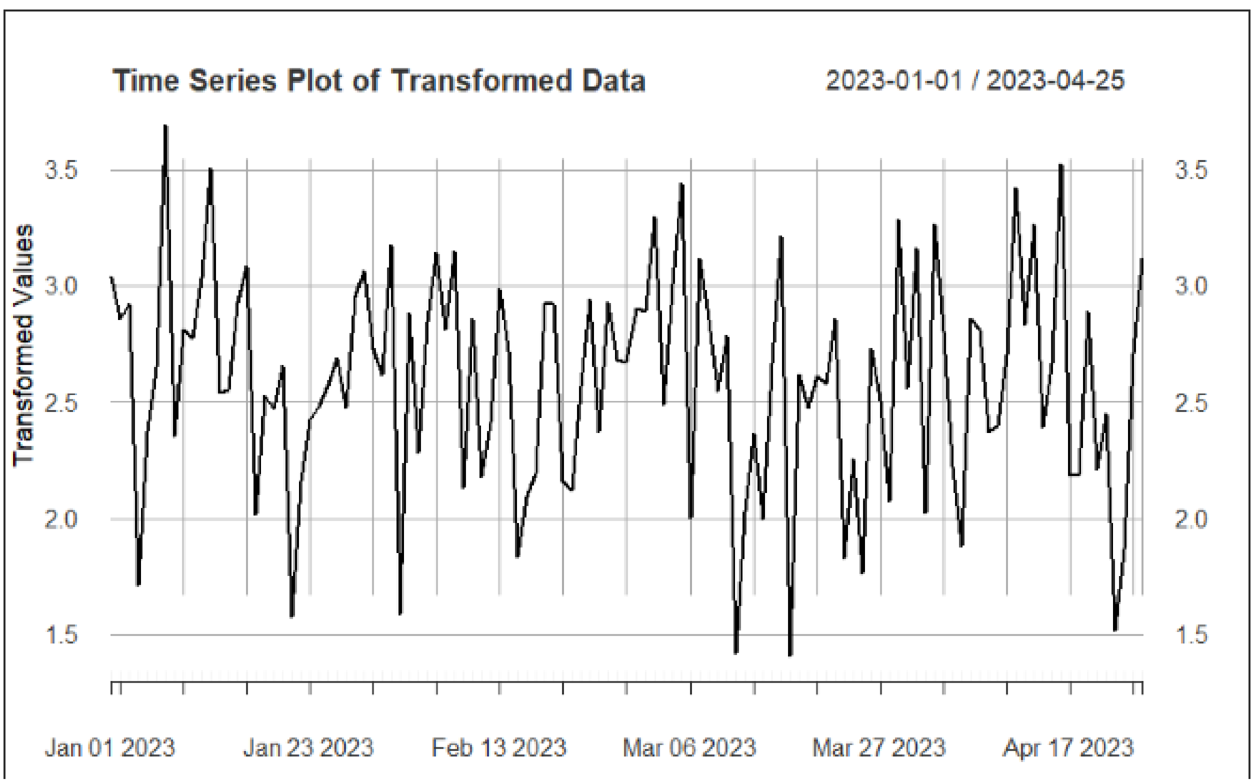
```
library(lattice)
library(xts)
data(larain)
larain_data <- as.xts(larain)

boxcox_obj <- BoxCox.ar(larain_data)
larain_data_transformed <- log(larain_data)
best_lambda <- which.max(boxcox_obj$loglike)
cat("Optimal Lambda value:", boxcox_obj$lambda[best_lambda], "\n")
larain_data_dates <- as.Date("2023-01-01") + 0:(length(larain_data_transformed) - 1)
larain_data_transformed_xts <- xts(larain_data_transformed, order.by = larain_data_dates)

qqnorm(larain_data)
qqline(larain_data)
title("Quantile-Quantile Plot for Original Data", line = 3)

qqnorm(larain_data_transformed)
qqline(larain_data_transformed)
title("Quantile-Quantile Plot for Transformed Data", line = 3)

print("QQ plot for the transformed data here does look more normal than the original one")

plot(larain_data_transformed_xts, main = "Time Series Plot of Transformed Data", ylab =
"Transformed Values")

xyplot(zlag(larain_data_transformed) ~ larain_data_transformed, ylab = "Lag 1", xlab =
"Transformed Values", col = "purple")

print("From the plot we can clearly make out that the transformation is not based on previous
value. Hence, we should expect the transformation to have lack of dependence within the
series.")
```

OUTPUT

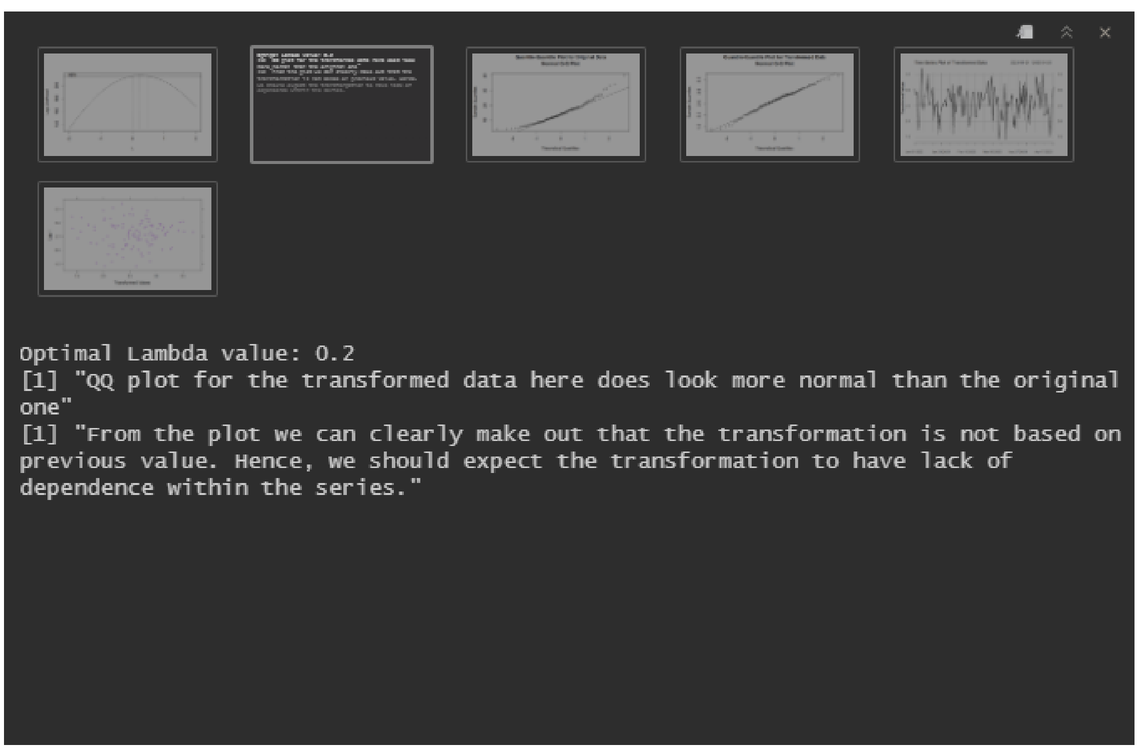

95%
Log Likelihood
140
160
180
200
-2
-1
0
1
2
λ


Quantile-Quantile Plot for Original Data
Normal Q-Q Plot
Sample Quantiles
10
20
30
40
-2
-1
0
1
2
Theoretical Quantiles

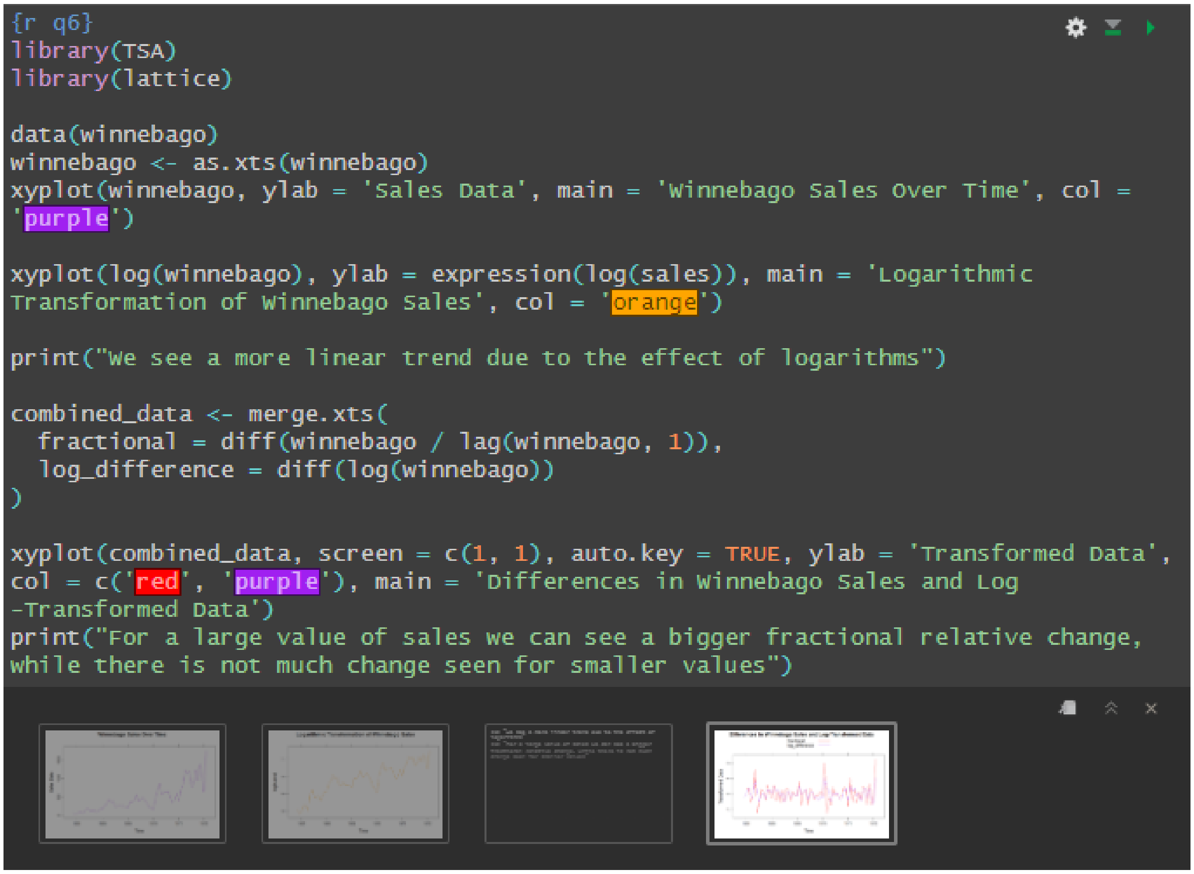

Quantile-Quantile Plot for Transformed Data
Normal Q-Q Plot
Sample Quantiles
1.5
2.0
2.5
3.0
3.5
-2
-1
0
1
2
Theoretical Quantiles


Time Series Plot of Transformed Data
2023-01-01 / 2023-04-25
Transformed Values
3.5
3.0
2.5
2.0
1.5
Jan 01 2023
Jan 23 2023
Feb 13 2023
Mar 06 2023
Mar 27 2023
Apr 17 2023

ALL OUTPUTS ALONG WITH CONCLUSIONS BELOW



QUESTION 6



```
{r q6}
library(TSA)
library(lattice)

data(winnebago)
winnebago <- as.xts(winnebago)
xyplot(winnebago, ylab = 'Sales Data', main = 'Winnebago Sales Over Time', col =
'purple')

xyplot(log(winnebago), ylab = expression(log(sales)), main = 'Logarithmic
Transformation of Winnebago Sales', col = 'orange')

print("We see a more linear trend due to the effect of logarithms")

combined_data <- merge.xts(
  fractional = diff(winnebago / lag(winnebago, 1)),
  log_difference = diff(log(winnebago))
)

xyplot(combined_data, screen = c(1, 1), auto.key = TRUE, ylab = 'Transformed Data',
col = c('red', 'purple'), main = 'Differences in Winnebago Sales and Log
-Transformed Data')
print("For a large value of sales we can see a bigger fractional relative change,
while there is not much change seen for smaller values")
```

CODE IN TEXT FORM

library(TSA)
library(lattice)

data(winnebago)
winnebago <- as.xts(winnebago)
xyplot(winnebago, ylab = 'Sales Data', main = 'Winnebago Sales Over Time', col = 'purple')

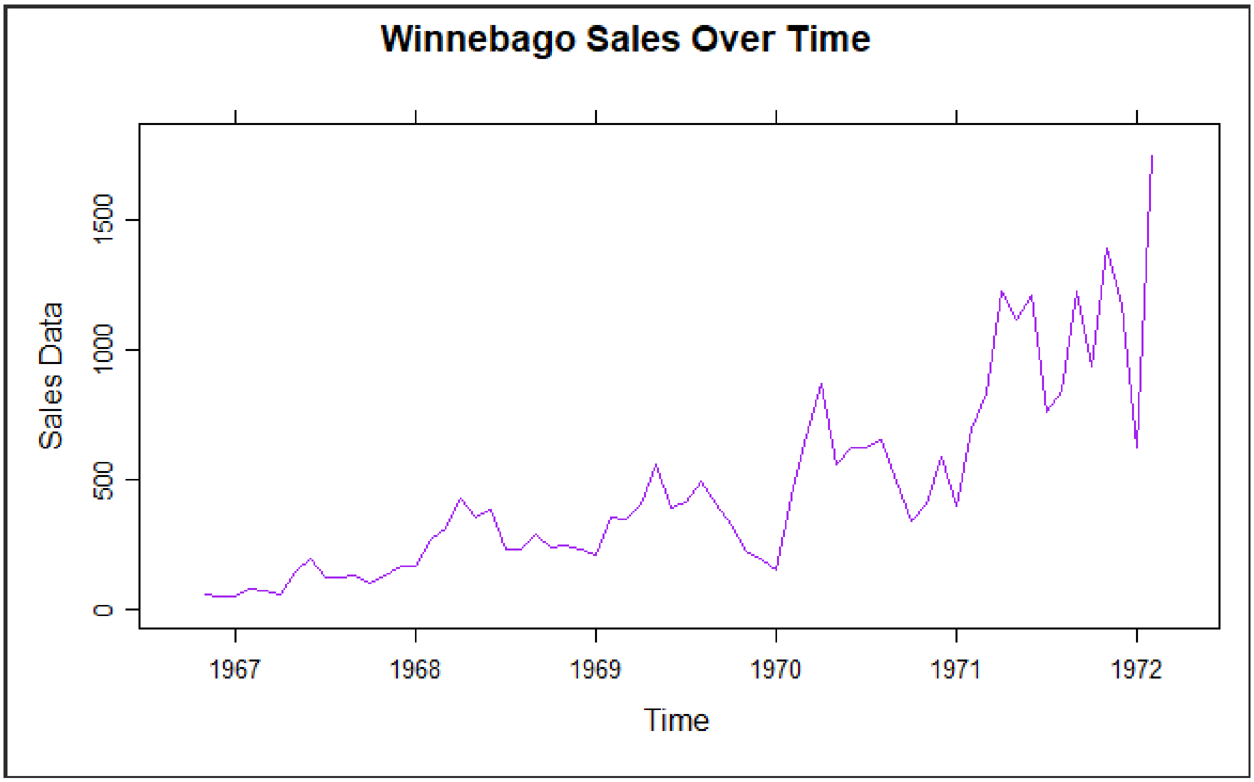
xyplot(log(winnebago), ylab = expression(log(sales)), main = 'Logarithmic Transformation of Winnebago Sales', col = 'orange')

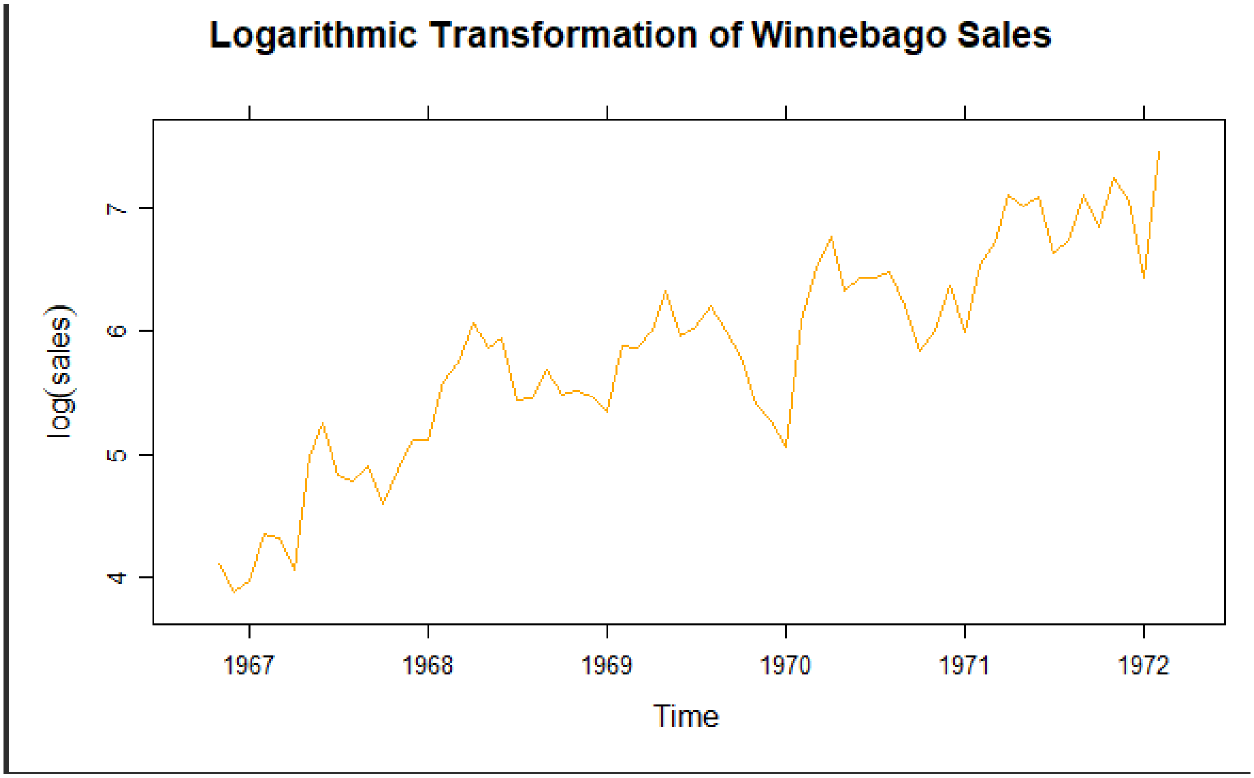
print("We see a more linear trend due to the effect of logarithms")

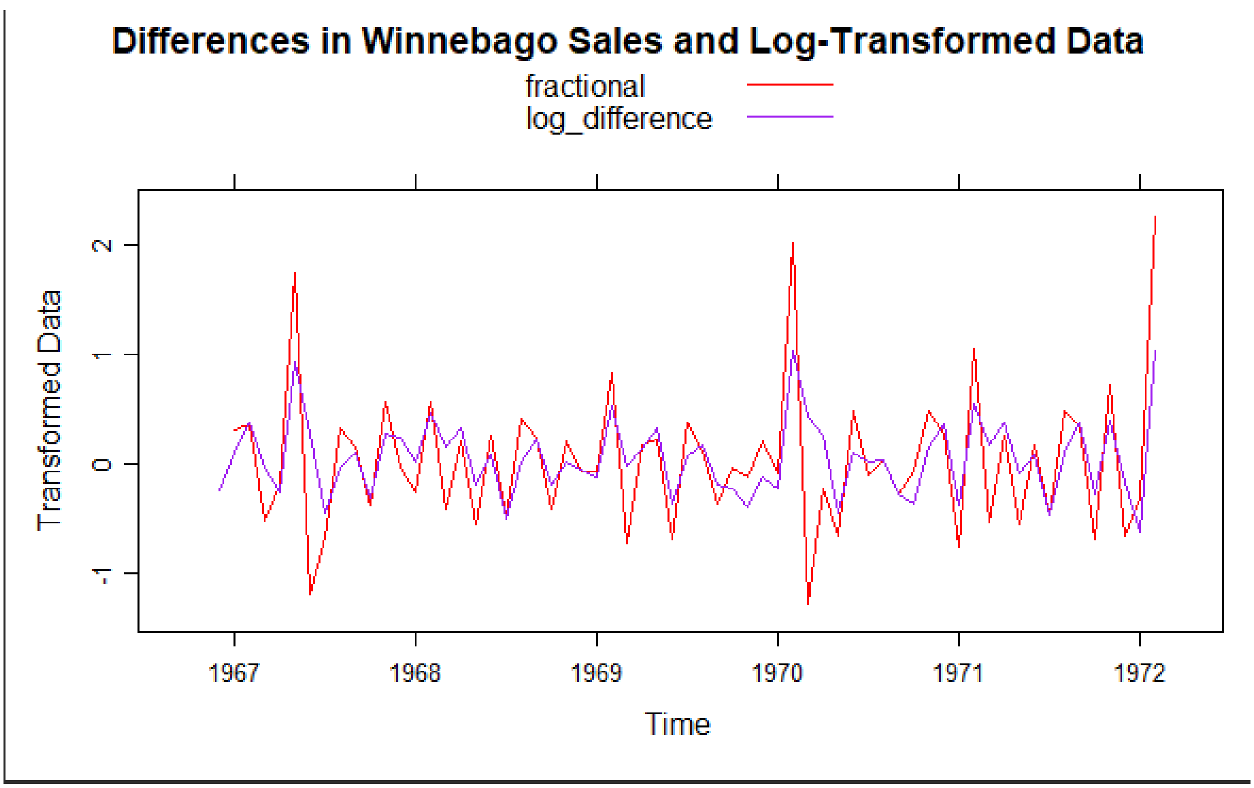
combined_data <- merge.xts(

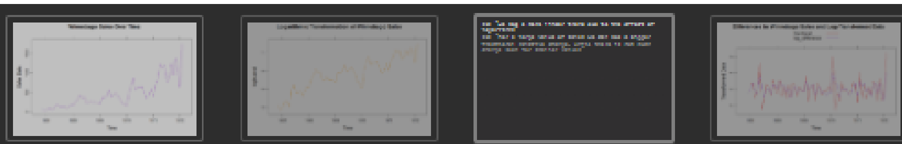
```
  fractional = diff(winnebago / lag(winnebago, 1)),
  log_difference = diff(log(winnebago))
)

xyplot(combined_data, screen = c(1, 1), auto.key = TRUE, ylab = 'Transformed Data', col =
c('red', 'purple'), main = 'Differences in Winnebago Sales and Log-Transformed Data')
print("For a large value of sales we can see a bigger fractional relative change, while there is not
much change seen for smaller values")
```

OUTPUT




Logarithmic Transformation of Winnebago Sales
log(sales)
4
5
6
7
1967
1968
1969
1970
1971
1972
Time


Differences in Winnebago Sales and Log-Transformed Data
fractional
log_difference
Transformed Data
-1
0
1
2
1967
1968
1969
1970
1971
1972
Time



```
[1] "We see a more linear trend due to the effect of logarithms"
[1] "For a large value of sales we can see a bigger fractional relative change,
while there is not much change seen for smaller values"
```