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MA 641 HW 5

Q4] Given → $\{X_t\} \Rightarrow$ stationary AR(1)

$Y_t = X_t + N_t$

To Find → a] Autocorrelation fn

b] ARIMA model for $\{Y_t\}$

a] $Var(Y_t) = Var(X_t) + Var(N_t)$

$= \sigma_x^2 + \sigma_N^2$

Now cov

$cov(Y_t, Y_{t-k}) = cov(X_t + N_t, X_{t-k} + N_{t-k})$

$\gamma_k = \phi^k \sigma_e^2$

$\therefore$ Corr (Y_t, Y_{t-k}) will be $\dfrac{\phi^k}{1+\sigma_x^2/\sigma_N^2}$

b] Now we know $\theta = \dfrac{1}{1+\sigma_x^2/\sigma_N^2}$

Here we can see that the given autocorrelation of Y_t is in the form $\theta\phi^k$

$\therefore$ we know it can be ARMA(1,1) model

Q2] Given - $n = 100$ $\quad \gamma_2 = 0.31$ $\quad \gamma_4 = 0.11$

$\gamma_1 = -0.99$ $\quad \gamma_3 = -0.21$ $\quad |\gamma_k| < 0.09$ for $k > 4$

To find :- ARIMA models

Sol^n - $\dfrac{2}{\sqrt{n}} \Rightarrow \dfrac{2}{10} = 0.2$

$\therefore |\gamma_i| > 0.2$ when $i = 1, 2, 3$

$|\gamma_i| < 0.2$ $i > 3$

Hence we can see it can be either MA (2) or MA(3)

For MA(2) we can see that

$$Var(\gamma_3) = \frac{(1+2(-0.49)^2 + (2(0.31)^2)}{100} \cong 0.0167$$

$$\frac{\gamma_3}{\sqrt{Var}} = -1.62$$

$\therefore$ we can say MA(2) is not rejected

Q3) Length = 121 $\hat{\phi}_{11} = 0.8$ $\hat{\phi}_{22} = -0.6$ $\hat{\phi}_{33} = 0.08$ $\phi_{44} = 0.0$

Threshold $\Rightarrow \frac{2}{\sqrt{n}} = 0.181$

$|\hat{\phi}_{ii}| > 0.181$ 2 for $i \leq 2$

$|\hat{\phi}_{ii}| < 0.181$ $i > 2$

Hence we can say it is AR(2) model

Q1] Threshold $\Rightarrow \frac{2}{\sqrt{n}} = 0.2$

From the given we see only lag 1 of ∇Y_t above the threshold we got

Hence for IMA(1,1) model

$$Var(\gamma_2) = \frac{(1+2(-0.42)^2)}{100} = 0.0135$$

$$\gamma_2 / \sqrt{Var} = 1.55$$

Hence unable to reject MA 1 for ∇Y_t series

Q5) $Y_t = X_t + e_t$

↳ Y_t here follows a random walk with noise

where, $X_t = X_{t-1} + N_t$ is the random walk

e_t is the observational noise

Here we consider N_t & e_t are independent gaussian white noise series with a zero mean & var = 1.

Params ⇒ Mean of $N_t = 0$, mean of $e_t = 0$

Variance of both 1