3. PARTIAL FRACTIONS

SYNOPSIS

1. **Polynomial :** An expression $a_0 + a_1x + a_2x^2 + ... + a_n x^n$ is called a polynomial in x with real coefficients. Polynomials are generally denoted by $f(x)$, $g(x)$ $R(x)$ ..... etc.
   ie $f(x) = a_0 + a_1x + a_2x^2 + ... + a_n x^n$ where $(a_n \neq 0) \Rightarrow \deg f(x) = n$
2. Remainder obtained when $f(x)$ is divided by $x-a$ is $f(a)$. If $f(a) = 0$ then $x - a$ is a factor of $f(x)$
   If degree of divisor is 'n', then the degree of remainder is $(n - 1)$.
3. **Rational fraction :** If $f(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$ then $\frac{f(x)}{g(x)}$ is called a rational fraction.
4. **Proper & improper fraction :** A rational fraction $\frac{f(x)}{g(x)}$ is called
   i) Proper fraction if **deg** $f(x)$ < **deg** $g(x)$
   ii) Improper fraction if **deg** $f(x)$ $\geq$ **deg** $g(x)$

Note : 1) Proper fraction is expressed as the sum of two or more proper fractions.
2) If it is Improper use **division Algorithm**

Division Algorithm : $f(x)$, $g(x)$ are two polynomials. If $g(x) \neq 0$, then $\exists$ two polynomials $q(x)$, $r(x)$ such that $\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$ if the degree of $f(x)$ is $\geq$ that of $g(x)$ is > that of $r(x)$

Method of resolving proper fraction $\frac{f(x)}{g(x)}$ into partial fractions

Type 1 : When $g(x)$ contains non-repeated linear factors i.e. $g(x) = (x - a)(x - b)(x - c)$

$$\frac{f(x)}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} \quad \text{where } A = \frac{f(a)}{(a-b)(a-c)}, B = \frac{f(b)}{(b-a)(b-c)}, C = \frac{f(c)}{(c-a)(c-b)}$$

Type 2 : When $g(x)$ contains repeated and non-repeated linear factors i.e. $g(x) = (x - a)^2 (x - b)$,

$$\frac{f(x)}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b} \quad \text{where A} = \frac{f(a)}{a-b} - \frac{f(a)}{(a-b)^2}, B = \frac{f(a)}{a-b}, C = \frac{f(b)}{(a-b)^2}$$

Note : Polynomial of the form $ax^2 + bx + c$, where $a,b,c \in R$ and $b^2 < 4ac$ is the irreducible polynomial over real.

Type 3 : When $g(x)$ contains non repeated irreducible quadratic factors.

i.e. $g(x) = (ax^2 + bx + c)(x - d)$

$$\frac{f(x)}{(ax^2+bx+c)(x-d)} = \frac{Ax+B}{(ax^2+bx+c)} + \frac{C}{x-d} \quad \text{where } C = \frac{f(d)}{ad^2+bd+c}$$

$f(x) = (x - d)(Ax + B) + C(ax^2 + bx + c)$ and by equating the coefficients, we get A and B.

Type 4 : When $g(x)$ contains repeated irreducible quadratic factors i.e. $g(x) = (ax^2 + bx + c)^2 (x - d)$

$$\frac{f(x)}{(ax^2+bx+c)^2(x-d)} = \frac{Ax+B}{(ax^2+bx+c)} + \frac{Cx+D}{(ax^2+bx+c)^2} + \frac{E}{x-d} \text{ where } E = \frac{f(d)}{(ad^2+bd+c)^2}$$

We write $f(x) = (Ax + B)(ax^2 + bx + c)(x - d) + (Cx + D)(x - d) + E(ax^2 + bx + c)^2$ and by equating the coefficients, we get A, B, C and D.

5. $\dfrac{Px+q}{x^2(x-a)} = \dfrac{-q}{ax^2} - \dfrac{Pa+q}{a^2x} + \dfrac{Pa+q}{a^2(x-a)}$

6. $\dfrac{1}{x^3(x+a)} = \dfrac{1}{a^3x} - \dfrac{1}{a^2x^2} + \dfrac{1}{ax^3} - \dfrac{1}{a^3(x+a)}$

7. $\dfrac{1}{(x-a)(x^2+b)} = \dfrac{1}{a^2+b}\left[\dfrac{1}{x-a} - \dfrac{x+a}{x^2+b}\right]$

8. The partial fractions of $\dfrac{1}{(x^2+a^2)(x^2+b^2)}$ are

i) $\dfrac{1}{b^2-a^2}\left[\dfrac{1}{x^2+a^2} - \dfrac{1}{x^2+b^2}\right]$ ii) $\dfrac{1}{a^2-b^2}\left[\dfrac{1}{x^2+b^2} - \dfrac{1}{x^2+a^2}\right]$

LECTURE SHEET

EXERCISE

1. The remainders of the polynomial $f(x)$ when divided by $x + 1$, $x + 2$, $x - 2$ are 6, 15, 3 the remainder of $f(x)$ when divided by $(x + 1)(x + 2)(x - 2)$ is

1) $2x^2 - 3x + 1$ 2) $3x^2 - 2x + 1$ 3) $2x^2 - x - 3$ 4) $3x^2 - 2x + 1$

2. If $\dfrac{x+1}{(x-a)(x-3)} = \dfrac{2}{x-a} + \dfrac{b}{x-3}$ then $(a, b) =$

1) $(7, -1)$ 2) $(-4, 1)$ 3) $(4, 1)$ 4) $(-4, -1)$

3. If $\dfrac{x^2-10x+13}{(x-1)(x^2-5x+6)} = \dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{K}{x-3}$ then K =

1) -1 2) -2 3) -3 4) -4

4. If $\dfrac{x^2+5x+1}{(x+1)(x+2)(x+3)} = \dfrac{A}{x+1} + \dfrac{B}{(x+1)(x+2)} + \dfrac{C}{(x+1)(x+2)(x+3)}$ then $B =$

1) 1 2) -5 3) 0 4) 10

5. The partial fractions of $\dfrac{x^3-5}{x^2-3x+2}$ are

1) $x + 3 - \dfrac{4}{x-1} + \dfrac{3}{x-2}$ 2) $x + 3 + \dfrac{4}{x-1} - \dfrac{3}{x-2}$

3) $x + 3 - \dfrac{4}{x-1} - \dfrac{2}{x-2}$ 4) $x + 3 + \dfrac{4}{x-1} + \dfrac{3}{x-2}$

6. If $\dfrac{x^4}{(x-a)(x-b)(x-c)} = P(x) + \dfrac{A}{x-a} + \dfrac{B}{x-b} + \dfrac{C}{x-c}$ then $P(x) =$

1) $x - a$
2) $x - a - b$
3) $x - a - b - c$
4) $x + a + b + c$

7. If $\dfrac{1}{x^3(x+3)} = \dfrac{1}{Ax} - \dfrac{1}{Bx^2} + \dfrac{1}{Cx^3} - \dfrac{1}{D(x+3)}$ then $A + B + C + D =$

1) 56
2) 6
3) 66
4) 76

8. If $\dfrac{3x^2+x+1}{(x-1)^4} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3} + \dfrac{D}{(x-1)^4}$ then $A + B - C + D =$

1) 0
2) 15
3) 1
4) 10

9. The no.of partial fractions of $\dfrac{5x-6}{x^3(x^2-1)^3}$ is

1) 6
2) 7
3) 8
4) 9

10. The partial fractions of $\dfrac{x^2}{(x^2+a^2)(x^2+b^2)}$ are

1) $\dfrac{1}{a^2+b^2}\left[\dfrac{a^2}{x^2+a^2} - \dfrac{b^2}{x^2+b^2}\right]$
2) $\dfrac{1}{b^2-a^2}\left[\dfrac{a^2}{x^2+a^2} - \dfrac{b^2}{x^2+b^2}\right]$
3) $\dfrac{1}{a^2-b^2}\left[\dfrac{a^2}{x^2+a^2} - \dfrac{b^2}{x^2+b^2}\right]$
4) $\dfrac{1}{a^2-b^2}\left[\dfrac{1}{x^2+a^2} - \dfrac{1}{x^2+b^2}\right]$

11. The partial fractions of $\dfrac{1}{(x^2+9)(x^2+16)}$ are

1) $\dfrac{1}{7}\left[\dfrac{1}{x^2+9} - \dfrac{1}{x^2+16}\right]$
2) $\dfrac{1}{9}\left[\dfrac{1}{x^2+9} - \dfrac{1}{x^2+16}\right]$
3) $\dfrac{1}{7}\left[\dfrac{1}{x^2+16} - \dfrac{1}{x^2+9}\right]$
4) $\dfrac{1}{25}\left[\dfrac{1}{x^2+9} - \dfrac{1}{x^2+16}\right]$

12. $\dfrac{3x^3-2x^2-1}{x^4+x^2+1} =$

1) $\dfrac{2x+1}{x^2+x+1} + \dfrac{x-2}{x^2-x+1}$
2) $\dfrac{2x+1}{x^2+x+1} - \dfrac{x-2}{x^2-x+1}$
3) $\dfrac{2x-1}{x^2+x+1} + \dfrac{x+2}{x^2-x+1}$
4) $\dfrac{3x+4}{x^2+x+1} - \dfrac{x+3}{x^2-x+1}$

13. $\frac{1}{x^4+1}=$

1) $\frac{x+\sqrt{2}}{2\sqrt{2}(x^2+\sqrt{2}x-1)}+\frac{\sqrt{2}-x}{2\sqrt{2}(x^2+\sqrt{2}\,x-1)}$

2) $\frac{x+\sqrt{2}}{2\sqrt{2}(x^2+\sqrt{2}x+1)}+\frac{\sqrt{2}-x}{2\sqrt{2}(x^2-\sqrt{2}\,x+1)}$

3) $\frac{x+\sqrt{2}}{2\sqrt{2}(x^2+\sqrt{2}x-1)}+\frac{\sqrt{2}-x}{2\sqrt{2}(x^2-\sqrt{2}\,x+1)}$

4) $\frac{x+\sqrt{2}}{2\sqrt{2}(x^2-\sqrt{2}x+1)}+\frac{\sqrt{2}-x}{2\sqrt{2}(x^2-\sqrt{2}\,x+1)}$

14. The no. of partial fractions of $\frac{5x^2+9}{(x^2+1)^5}$ is

1) 10 2) 3 3) 2 4) 7

15. If $\frac{1}{(x-a)(x^2+b)}=\frac{A}{x-a}+\frac{Bx+C}{x^2+b}$ then $\frac{1}{(x-a)(x^2+b)^2}=$

1) $\frac{A^2}{x-a}+\frac{A(Bx+C)}{x^2+b}+\frac{Bx+C}{(x^2+b)^2}$ 2) $\frac{A^2}{x+a}+\frac{A(Bx-C)}{x^2+b}+\frac{Bx+C}{(x^2-b)^2}$

3) $\frac{A^2}{x+a}+\frac{A(Bx+C)}{x^2+b}+\frac{Bx-C}{(x^2+b)^2}$ 4) $\frac{A^2}{x-a}+\frac{A(Bx-C)}{x^2-b}+\frac{Bx+C}{(x^2-b)^2}$

16. The coefficient of x^5 in $\frac{x^2+1}{(x^2+4)(x-2)}$ is

1) $-\frac{1}{256}$ 2) $-\frac{1}{199}$ 3) $\frac{1}{256}$ 4) $\frac{1}{199}$

17. The coefficient of x^n in $\frac{x-4}{x^2-5x+6}$ is

1) $\frac{1}{3^{n+1}}-\frac{1}{2^n}$ 2) $\frac{1}{3^{n+1}}+\frac{1}{2^n}$ 3) $\frac{1}{5^{n+1}}+\frac{1}{2^n}$ 4) $\frac{1}{5^n}+\frac{1}{2^n}$

18. The coefficient of x^n in $\frac{x+1}{(x-1)^2(x-2)}$ is

1) $1-2n-\frac{3}{2^{n+1}}$ 2) $1-2n-\frac{3}{2^{n-1}}$ 3) $1+2n+\frac{3}{2^{n+1}}$ 4) $1+2n-\frac{3}{2^{n-1}}$

19. If $\frac{1}{x(x+1)(x+2).....(x+n)}=\frac{A_0}{x}+\frac{A_1}{x+1}+\frac{A_2}{x+2}+......+\frac{A_n}{x+n}$ then $A_r=$

1) $\frac{(-1)^r r!}{(n-r)!}$ 2) $\frac{(-1)^r}{r!(n-r)!}$ 3) $\frac{1}{r!(n-r)!}$ 4) $\frac{1}{r!(n+r)!}$

PRACTICE SHEET

EXERCISE

1. The remainder obtained when the polynomial $x^4 - 3x^3 + 9x^2 - 27x + 81$ is divided by $x - 3$ is
   1) 81 2) 243 3) 405 4) 18

2. $(a - 1)$ is a factor of $a^5 - a^4 - 4a^3 + 4a^2 + 4a + k$ then $k =$
   1) 4 2) –4 3) 2 4) -2

3. If the remainders of the polynomial $f(x)$ when divided by $x - 1, x - 2$ are 2, 5 then the remainder of $f(x)$ when divided by $(x - 1)(x - 2)$ is
   1) 0 2) $1 - x$ 3) $2x–1$ 4) $3x–1$

4. If the remainders of the polynomial $f(x)$ when divided by $x+1$ and $x–1$ are 7, 3 then the remainder of $f(x)$ when divided by $x^2–1$ is
   1) $3x+5$ 2) $2x+7$ 3) $–2x+5$ 4) $3x+7$

5. If $\frac{2x-1}{(x-1)(2x+3)} = \frac{1}{5(x-1)} - \frac{k}{5(2x+3)}$ then $k =$
   1) 0 2) 1 3) 2 4) –8

6. If $\frac{1}{x^2-25} = \frac{1}{k}\left[\frac{1}{x-5} - \frac{1}{x+5}\right]$ then $k =$
   1) 10 2) -10 3) $\frac{1}{10}$ 4) $-\frac{1}{10}$

7. If $\frac{x^3}{(x-a)(x-b)(x-c)} = 1 + \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$ then $A =$
   1) $\frac{a^3}{(c-b)(c-a)}$ 2) $\frac{a^3}{(b-c)(b-a)}$ 3) $\frac{a^3}{(a-b)(a-c)}$ 4) $\frac{1}{(a-b)(a-c)}$

8. If $\frac{x^4}{(x-1)(x+2)} = \frac{1}{3(x-1)} - \frac{16}{3(x+2)} + x^2 - x + k$ then $k =$
   1) 0 2) 1 3) 2 4) 3

9. If $\frac{(1+x)(1+2x)(1+3x)}{(1-x)(1-5x+6x^2)} = k + \frac{A}{1-x} + \frac{B}{1-2x} + \frac{C}{1-3x}$ then
   1) $k + A = 13$ 2) $k + B = 29$ 3) $k + C = 19$ 4) $k + A + B + C = 0$

10. The partial fractions of $\frac{x^2+13x+15}{(2x+3)(x+3)^2}$ are
    1) $\frac{1}{2x+3} - \frac{1}{x+3} + \frac{5}{(x+3)^2}$ 2) $\frac{-1}{2x+3} + \frac{1}{x+3} + \frac{5}{(x+3)^2}$
    3) $\frac{-1}{2x+3} - \frac{1}{x+3} + \frac{5}{(x+3)^2}$ 4) $\frac{1}{2x+3} + \frac{1}{x+3} + \frac{5}{(x+3)^2}$

11. The No. of partial fractions of $\frac{x^2+5x+7}{x^3-x}$ is

1) 2 2) 3 3) 4 4) 1

12. If $f(x)$ is a function of x such that $\frac{1}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{f(x)}{(1+x^2)}$ for all $x \in R$ then $f(x)$ is

1) $\frac{1-x}{2}$ 2) $\frac{1+x}{2}$ 3) $1-x$ 4) $1+x$

13. If $\frac{x}{(1+x^2)(3-2x)} = \frac{A}{3-2x} + \frac{Bx+C}{1+x^2}$ then $C =$

1) $\frac{-1}{13}$ 2) $\frac{2}{13}$ 3) $\frac{-2}{13}$ 4) $\frac{1}{13}$

14. The partial fractions of $\frac{(x+1)^2}{x(x^2+1)}$ are

1) $\frac{1}{2x}+\frac{x}{x^2+1}$ 2) $\frac{1}{x}-\frac{2}{x^2+1}$ 3) $\frac{1}{x}+\frac{1}{x^2+1}$ 4) $\frac{1}{x}+\frac{2}{x^2+1}$

15. If $\frac{ax-1}{(1-x+x^2)(2+x)} = \frac{x}{1-x+x^2} - \frac{1}{2+x}$ then $a =$

1) 3 2) –3 3) 2 4) –2

16. If $\frac{1}{x(x^2+a^2)} = \frac{A}{x} + \frac{Bx+C}{x^2+a^2}$ then $\tan^{-1}\left(\frac{A}{B}\right) =$

1) $\frac{3\pi}{4}$ 2) $\frac{\pi}{4}$ 3) $-\frac{\pi}{4}$ 4) $\frac{\pi}{3}$

17. If $\frac{(x+1)^2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ then $\cos^{-1}\left(\frac{A}{C}\right) =$

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

18. The number of partial fractions of $\frac{2}{x^4+x^2+1}$ is

1) 2 2) 3 3) 4 4) 5

19. $\frac{x^2+1}{(x^2+x+1)^2} =$

1) $\frac{1}{x^2+x+1} - \frac{x}{(x^2+x+1)^2}$ 2) $\frac{1}{x^2+x+1} + \frac{x}{(x^2+x+1)^2}$

3) $\frac{1}{x^2+x+1} + \frac{2x+3}{(x^2+x+1)^2}$ 4) $\frac{2x+1}{x^2+x+1} - \frac{x}{(x^2+x+1)^2}$

20. If $\frac{1}{(ax+b)(cx+d)}=\frac{A}{ax+b}+\frac{B}{cx+d}$ then $\frac{1}{(ax+b)^2(cx+d)}=$

1) $\frac{A}{(ax-b)^2}+\frac{AB}{ax-b}+\frac{B^2}{cx+d}$ 2) $\frac{A}{(ax+b)^2}+\frac{AB}{ax-b}+\frac{B^2}{cx+d}$

3) $\frac{A}{(ax+b)^2}+\frac{AB}{ax+b}+\frac{B^2}{cx+d}$ 4) $\frac{A}{(ax+b)^2}+\frac{B}{ax+b}+\frac{B^2}{cx+d}$

21. If $\frac{x+1}{x^2-px+q}=\frac{A}{x-\alpha}+\frac{B}{x-\beta}$, α and β are the roots of $x^2-px+q=0$ then $\frac{A-B}{A+B}=$

1) $\frac{p+2}{\sqrt{p^2-4q}}$ 2) $\frac{p}{\sqrt{p^2-4q}}$ 3) $\frac{p-2}{\sqrt{p^2+4q}}$ 4) $\frac{p+2}{\sqrt{p^2+4q}}$

22. The coefficient of x^4 in $\frac{3x^2+2x}{(x^2+2)(x-3)}$ is

1) $\frac{11}{27}$ 2) $\frac{77}{324}$ 3) $-\frac{77}{324}$ 4) $-\frac{11}{27}$

23. The coefficient of x^n in $\frac{x}{(x-a)(x-b)}$ is

1) $\frac{a^n+b^n}{a-b}\frac{1}{a^nb^n}$ 2) $\frac{a^n-b^n}{a-b}\frac{1}{a^nb^n}$ 3) $\frac{a^n-b^n}{a+b}\frac{1}{a^nb^n}$ 4) $\frac{a^n-b^n}{a-b}a^nb^n$

24. The coefficient of x^n in $\frac{(1+x)(1+2x)(1+3x)}{(1-x)(1-2x)(1-3x)}$ is

1) $12-30.2^n+20.3^n$ 2) $12+30.2^n+20.3^n$ 3) $12+30.2^n-20.3^n$ 4) $12-30.2^n-20.3^n$

25. If $\frac{x^2}{(x-1)^2(x+1)}=\frac{A}{x-1}+\frac{1}{2(x-1)^2}+\frac{B}{x+1}$ the length of the vector $4A\bar{i}+4B\bar{j}$ is

1) 10 2) 8 3) $\sqrt{10}$ 4) $\frac{\sqrt{10}}{4}$

KEY SHEET

LECTURE SHEET

1) **1**	2) **1**	3) **4**	4) **3**	5) **4**	6) **4**	7) **3**	8) **3**	9) **4**	10) **3**
11) **1**	12) **1**	13) **2**	14) **3**	15) **1**	16) **1**	17) **1**	18) **1**	19) **2**	

PRACTICE SHEET

1) **1**	2) **2**	3) **4**	4) **3**	5) **4**	6) **1**	7) **3**	8) **4**	9) **3**	10) **2**
11) **2**	12) **1**	13) **3**	14) **4**	15) **1**	16) **3**	17) **3**	18) **1**	19) **1**	20) **3**
21) **1**	22) **2**	23) **2**	24) **1**	25) **3**					