Chapter - 4

ALTERNATING CURRENT

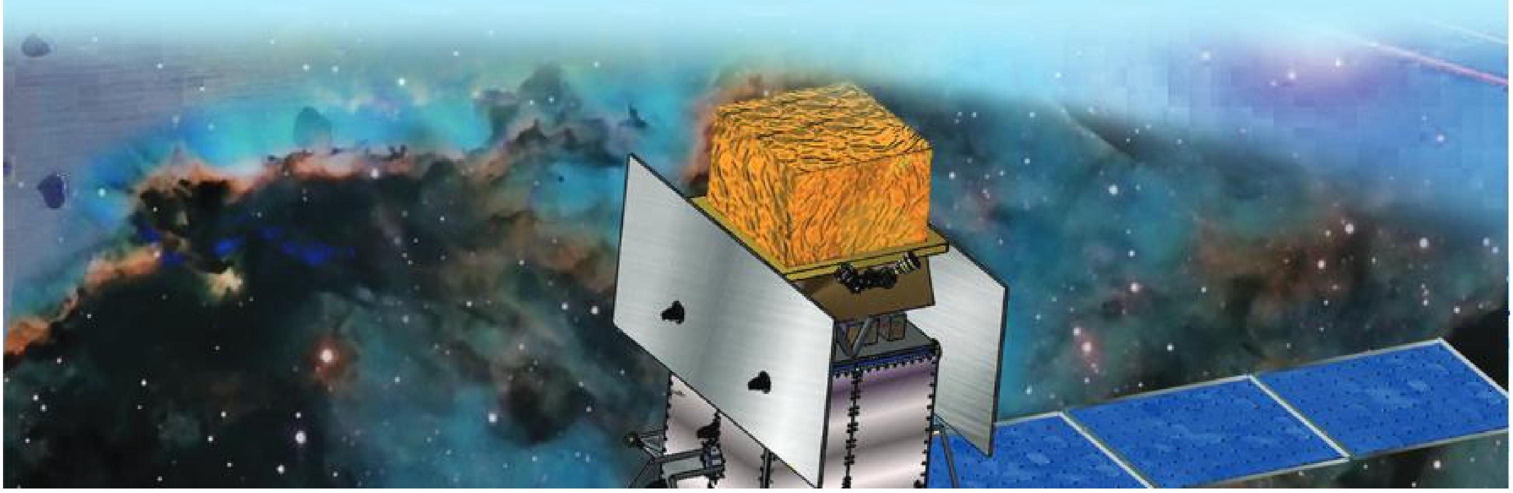
✤ Alternating current and voltage ✤

✤ RMS value ✤ Phasor notation ✤

✤ AC through resistor ✤ Capacitor and inductor ✤

✤ Power in AC circuit ✤ Series resonance ✤

✤ LC oscillations ✤



4.1.1 ALTERNATING CURRENT (A.C)

Alternating current is that current which continuously varies in magnitude and periodically reverses its direction.

The alternating current (a.c) is generally expressed as (sinusoidal form)

$$I = I_0 \sin(\omega t + \phi) \qquad ....(4.1)$$

I = Instantaneous value of current

I_0 = Maximum (or) peak value

$(\omega t + \phi)$ = Phase

ω = driving frequency

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

The variation of a.c. with time is shown in Fig 4.1.

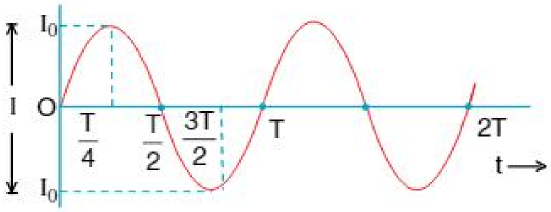


Fig 4.1

e.m.f of an a.c source is generally given by

$$E = E_0 \sin(\omega t + \phi) \qquad .... (4.2)$$

where E_0 is the maximum value or peak value or amplitude of e.m.f. Its angular frequency is called the driving frequency and is given by

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

where T is the time period and ν is called the frequency of alternating e.m.f.

The pictorial symbol used to represent the a.c. source in a given circuit is shown in Fig 4.2.

Fig 4.2

1. Direction of current is not marked on the symbol of A.C source
2. Alternating voltage can be of many forms, some of them are shown below: (non sinusoidal)

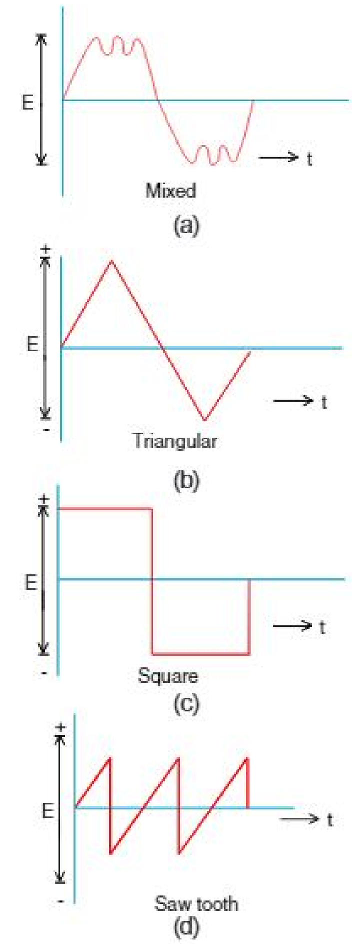


Fig 4.3

Example-4.1

Write the general equation for the instantaneous e.m.f of a 50 Hz generator whose peak voltage is 250 V.

Solution :

Here,

$E_0 = 250V$ and $\nu = 50Hz$

$\omega = 2\pi\nu = 2\pi \times 50 = 100\pi$

and $E = E_0 \sin\omega t = 250 \sin 100\pi t$

4.1.2 MEAN OR AVERAGE VALUE OF ALTERNATING CURRENT

The value of steady current which sends the same amount of charge through a circuit in a certain time interval as is sent by an alternating current through the same circuit in half the time period (or half cycle) is known as mean or average value of alternating current over half cycle. (consider either positive half cycle or negative half cycle)

Expression for average value

Let an alternating current be represented by

$I = I_0 \sin\omega t$

The charge sent by the current I in time dt is given by

$$dq = Idt = I_0 \sin\omega t\, dt \qquad \left(\because I = \frac{dq}{dt}\right)$$

Therefore, the charge sent by a.c. in the first half cycle (i.e. t = 0 to t = T/2) is

$$\int_0^q dq = \int_0^{T/2} I_0 \sin\omega t\, dt$$

$$\text{or } q = I_0 \int_0^{T/2} \sin\omega t\, dt = I_0\left[\frac{-\cos\omega t}{\omega}\right]_0^{T/2}$$

$$q = -\frac{I_0}{\omega}\left[\cos\omega t\right]_0^{T/2} = \frac{-I_0}{(2\pi/T)}\left[\cos\frac{2\pi}{T}t\right]_0^{T/2}$$

$$q = \frac{-I_0 T}{2\pi}\left[\cos\frac{2\pi}{T}\times\frac{T}{2} - \cos 0\right]\left[\because \omega = \frac{2\pi}{T}\right]$$

$$q = \frac{-I_0 T}{2\pi}\left[\cos\pi - \cos 0\right]$$

$$\Rightarrow q = \frac{-I_0 T}{2\pi}(-1-1) \quad q = \frac{I_0 T}{\pi} \quad \text{...... (i)}$$

Let I_{av} be the mean value of a.c. over positive half cycle, then the charge sent by it in T/2 is given by

$$q = I_{av} \times \frac{T}{2} \quad \text{.... (ii)}$$

From (i) and (ii), we get $I_{av} \times \frac{T}{2} = \frac{I_0 T}{\pi}$

$$\text{or } I_{av} = \frac{2I_0}{\pi} = 0.637 I_0 \quad \text{.... (4.3)}$$

$$\text{or } I_{av} = 63.7\% I_0$$

Thus, mean value of an a.c. over any positive half cycle of it is 63.7% of its peak value.

The mean or average value of a.c. over a complete cycle is zero. Mean value of a.c over positive half of a.c. is 0.637 I_0. Similarly mean value of a.c. over negative half is –0.637 I_0. Therefore

mean value a.c. over a full cycle

$= 0.637\ I_0 - 0.637 I_0 = 0$

Mean value of a.c. over a half cycle may be zero (if it is neither positive nor negative half cycle)

4.1.3 ROOT MEAN SQUARE (R.M.S) VALUE OF ALTERNATING CURRENT

Root mean square value of a.c is defined as that steady current which produces the same amount of heat in a conductor in a certain time interval as is produced by the a.c. in the same conductor during the time period T (i.e., full cycle)

It is represented by I_{rms}.

Root mean square value of a.c. is also known as efffective value (I_{eff}) or virtual value (I_v).

Expression : Let an alternating current $I = I_0 \sin\omega t$ flow through a conductor of resistance R for time dt. Then, heat produced in the conductor is given by

$$dH = I^2 R dt = (I_0^2 \sin^2\omega t) R\ dt$$

$$dH = I_0^2 R \sin^2\omega t\ dt$$

Now heat produced in the conductor when current flows for time period T is

$$\int dH = \int_0^T I_0^2 R \sin^2 \omega t \, dt$$

$$H = I_0^2 R \int_0^T \frac{1-\cos 2\omega t}{2} dt$$

$$H = \frac{I_0^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$H = \frac{I_0^2 R}{2}[T-0], \; H = \frac{I_0^2 R}{2}T \qquad \text{...... (i)}$$

$$(\because \sin 4\pi = \sin 0 = 0) \qquad \text{...... (i)}$$

Let I_{rms} be the r.m.s value of a.c which flows through the conductor of resistance R for time T.

$\therefore$ Heat produced in the conductor,

$$H = I^2_{rms} RT \qquad \text{...... (ii)}$$

According to definition of r.m.s value of a.c.; equation (i) = equation (ii)

$$I^2_{rms} RT = \frac{I_0^2 RT}{2}$$

$$\text{or } I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0 \qquad \text{.... (4.4)}$$

Clearly r.m.s. value of an alternating current is 70.7% of its peak value. Similarly r.m.s or virtual or effective value of alternating e.m.f is given as

$$E_{r.m.s} = \frac{E_0}{\sqrt{2}}$$

✤ A.C. ammeter and voltmeter read the r.m.s value i.e., effective value of alternating current and voltage respectively. Therefore they are known as hotwire meters. If an a.c. voltmeter plugged into a household electric outlet reads 150V, then 150V is the r.m.s value of voltage and the peak value of voltage

$$E_0 = \sqrt{2} E_{r.m.s}$$

i.e. $1.414 \times 150 = 212.1$ V

Example-4.2

The equation of an alternating current is $I = 20 \sin 300\pi t$. Calculate the frequency and r.m.s value of current.

Solution :

Here $I = 20 \sin 300\pi t$

i) Compare it with standard equation

$I = I_0 \sin \omega t = I_0 \sin 2\pi \nu t$

we get $I_0 = 20A$ and $2\pi\nu t = 300\pi t$ or $\nu = 150$ Hz

ii) We know, $I_{rms} = \frac{I_0}{\sqrt{2}}$

$\therefore I_{rms} = \frac{20}{\sqrt{2}} = 14.14A.$

Example-4.3

(a) The peak voltage of a.c. supply is 600 V. What is its rms voltage?

(b) The rms value of current in an a.c. circuit is 20A. What is its peak current?

Solution :

a) Here $E_0 = 600V$

$\therefore E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{600}{\sqrt{2}} = 424.3$

b) Here $I_{rms} = 20$ A.

We know $I_{rms} = \frac{I_0}{\sqrt{2}}$

$\therefore I_0 = \sqrt{2} I_{rms}$ or $I_0 = \sqrt{2} \times 20 = 1.414 \times 20$
$= 28.28A$

4.2 AC INSTRUMENTS

If ordinary DC measuring instruments (ammeter and voltmeter) are used for measuring alternating current and alternating voltage, they will show zero reading. This is because the average of positive and negative half cycles in AC is zero. On the other hand, hot wire instruments depend, for their functioning, on the heating effect caused by electric current. Whether the current is flowing in one direction or in the reverse direction, heating effect will be there. This is because the heat produced is independent of the direction of flow of current. Moreover, heat produced is proportional to square of current and is always positive. Hence hot-wire instruments are largely employed in measuring AC voltages and currents. These instruments measure the virtual voltage or virtual current as the case may be. The alternating current is measured in 'virtual ampere' and the alternating voltage is measured in 'virtual volt'.

Ac ammeter and AC voltmeter read the rms values of alternating current and voltage respectively. Their common name is 'hot wire meters.'

One virtual ampere is that value of alternating current which when flowing through a resistance for a certain time generates the same quantity of heat as is done by a steady (direct) current of an ampere through the same resistance for the same time.

The given values of alternating current and voltage are virtual values unless otherwise specified.

Illustration : 220 V AC means $E_v = 220V$

One virtual volt is that value of alternating e.m.f. which when applied across a resistance for a certain time generates the same amount of heat as is generated by a steady (direct) potential difference of one volt applied across the same resistance for same time.

Note : While the scale of DC instruments is linearly graduated, the scales of AC instruments are not evenly graduated.

4.3 PHASOR AND PHASOR DIAGRAMS

(i) In A.C. circuits, although the frequency of alternating current is the same as that of alternating e.m.f but it is not always essential that they are in same phase. Generally, when the current in the circuit is maximum, the e.m.f. is not maximum and vice versa. The phase difference between current and e.m.f. depends upon the nature of circuit elements (i.e., circuit consists of a resistor or an inductor or a capacitor or any combination of these circuit elements). In certain circuits, the current reaches maximum value after the e.m.f becomes maximum. Then the currents is said to lag behind the emf. In certain other circuits, current attains the maximum value before the e.m.f has become maximum. In such cases, the current is said to lead the e.m.f.

(ii) In order to simplify the study of A.C circuits, it is preferred to treat alternating current and alternating e.m.f as vectors with the angle between the vector equal to the phase difference between the current and the e.m.f. The current and e.m.f are more appropriately called **Phasors.**

(iii) A diagram representing alternating current and alternating voltage (of same frequency) as vectors (phasors) with the phase angle between them is called a **Phasor Diagram.**

Let us consider a circuit in which emf and current vary sinusoidally with time and are mathematically expressed as

$$E = E_0 \sin \omega t \text{ and } I = I_0 \sin(\omega t + \phi)$$

Where ϕ is the phase angle between alternating emf and current, and E_o and I_0 are the peak values of alternating emf and current.

The instantaneous values E and I may be regarded as projections of E_0 and I_0 respectively.

When a quantity varies sinusoidally with time and can be represented as projection of a rotating vector, then it is called as phasor.

A diagram, representing alternating emf and current (of same frequency) as rotating vectors (Phasors) with phase angle between them is called as phasor diagram.

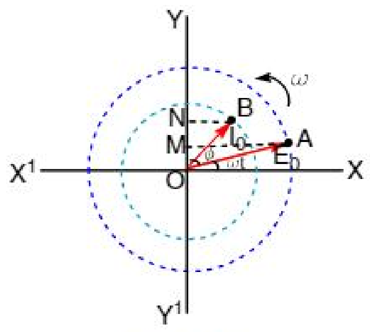


Fig 4.4 (a)

In Fig 4.4 (a), $\overline{OA}$ and $\overline{OB}$ represent two rotating vectors having magnitudes E_0 and I_0 in anti clockwise direction with same angular velocity 'ω'. OM and ON are the projections of $\overline{OA}$ and $\overline{OB}$ on Y-axis respectively. So, OM = E and ON = I, represent the instantaneous values of alternating emf and current. $\underline{|BOA} = \phi$ represents the phase angle by which current I_0 leads the alternating emf E_0. Figure (a) also represents the phasor diagram.

The phasor diagram, in a simple representation is shown in fig (b). Moreover, when we are interested only in phase relationship, the phasor diagram may also be represented as shown in fig (c).

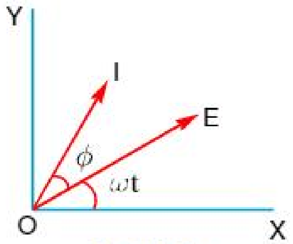


Fig 4.4 (b)

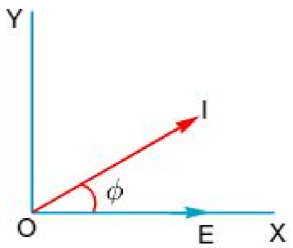


Fig 4.4 (c)

Knowledge Plus 4.1

☺ **Skin Effect :** When d.c. is allowed through a wire it is distributed uniformly over the whole cross-section of the wire, But when an a.c. of high frequency flow through the wire, it is not distributed uniformly over the outer layers. In case of very high frequencies, the current is almost wholly confined to the surface layer of the wire. The phenomenon is called skin effect. Since a.c. of high frequency do not pass through the entire cross-section of the wire, the effective cross-section of the wire, the effective resistance of a wire for a.c. is much greater than that for d.c. Hence the conductor required to carry high frequency a.c. consists of a number of stands of fine wire connected in parallel at their ends, and insulated throughout their length from each other. This increases the surface area and thus decreases the resistance.

4.4 ALTERNATING VOLTAGE APPLIED TO A RESISTOR

An A.C source applied to a resistor of resistance R is shown in Fig 4.5(a). Such a circuit is known as a resistive circuit.

The alternating e.m.f . applied is given by

$$E = E_0 \sin \omega t \qquad .....(i)$$

Let I be the current in the circuit at any instant t.

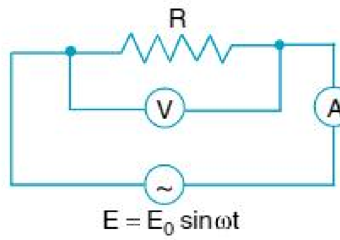


Fig 4.5(a)

The potential difference across Resistor = IR.

So, $E = IR$ or $I = \frac{E}{R}$ Using equation (i)

we get $I = \frac{E_0}{R} \sin \omega t$ or $I = I_0 \sin \omega t$ .....(ii)

where $I_0 = \frac{E_0}{R}$ is the peak value of alternating current.

Comparison of eqn.(i) and (ii) shows that the current and e.m.f. across the resistor are in phase (Fig (4.5(b)), phasor diagram for pure resistance is shown in Fig 4.5(c).

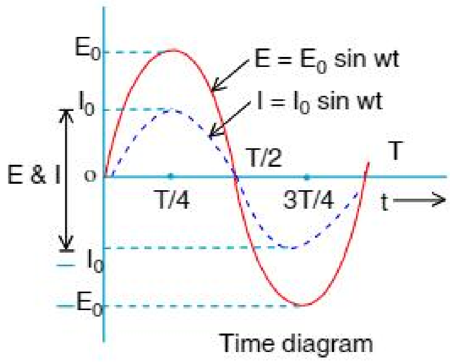


Fig 4.5 (b)

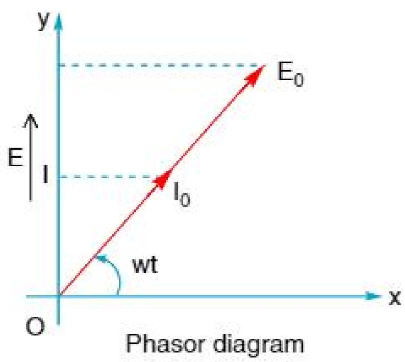


Fig 4.5 (c)

It is clear from eqn. (i) and eqn. (ii) that in a purely resistive circuit, there is no phase difference between voltage and current.

1. There is no phase difference between the voltage and current in a pure ohmic resistor.

2. R-f curve of purely resistive circuit is shown in figure below. Clearly value of resistance R does not change with change in frequency.

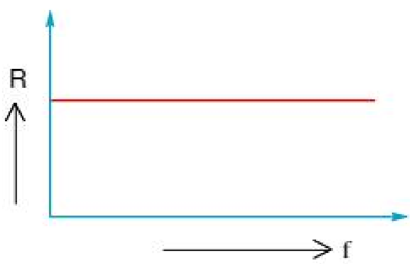


Fig 4.5(d)

4.5.1 ALTERNATING VOLTAGE APPLIED TO AN INDUCTOR

An alternating voltage applied to an inductor of inductance L is shown in figure below. Such a circuit is known as purely inductive circuit.

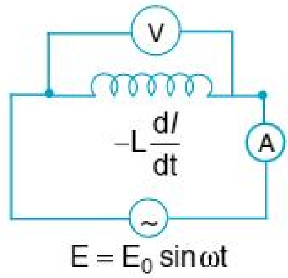


Fig 4.6(a)

The alternating e.m.f. applied is given by

$E = E_0 \sin \omega t$ .....(i)

The induced e.m.f. across the inductor $= -L.\frac{dI}{dt}$ which opposes the growth of current in the circuit. As there is no potential drop across the circuit, so

$$E + \left(-L.\frac{dI}{dt}\right) = 0 \text{ or } L.\frac{dI}{dt} = E \text{ or } \frac{dI}{dt} = \frac{E}{L}$$

Using equation (i) we, get

$$\frac{dI}{dt} = \frac{E_0}{L}\sin \omega t \quad \text{or} \quad dI = \frac{E_0}{L}\sin \omega t \, dt \text{ ...(ii)}$$

Integrating both sides, we get

$$\int dI = \int \frac{E_0}{L}\sin \omega t \, dt = \frac{E_0}{L}\int \sin \omega t \, dt$$

$$\text{or } I = \frac{E_0}{L}\left(\frac{-\cos \omega t}{\omega}\right) = \frac{E_0}{L\omega}(-\cos \omega t) \quad \text{....(1)}$$

Since $(-\cos \omega t) = \sin\left(\omega t - \frac{\pi}{2}\right)$ and peak value,

$$I_0 = \frac{E_0}{L\omega};\ I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{.... (2)}$$

Comparison of eqn. (1) and (2) shows that current lags behind the e.m.f. by an angle of $\frac{\pi}{2}$.

In other words phase difference between alternating voltage and alternating current is $\frac{\pi}{2}$ in an a.c. circuit containing only an inductor Fig 4.6(b).

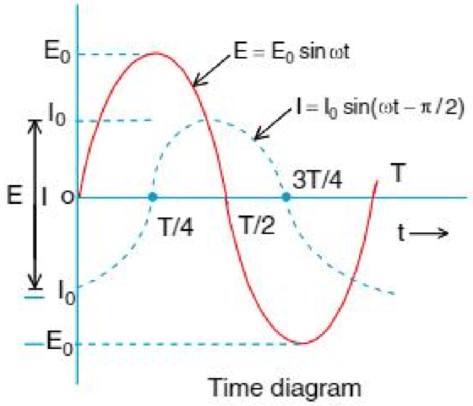


Fig 4.6(b)

The phasor diagram for purely inductive circuit is shown in Fig 4.6(c).

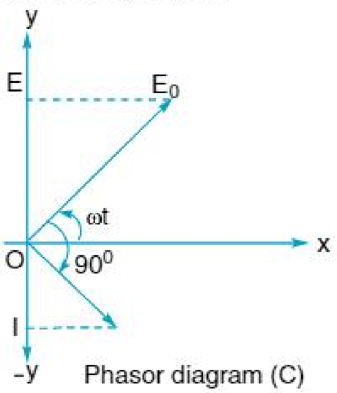


Fig 4.6(c)

4.5.2 INDUCTIVE REACTANCE $[X_L]$

Here $I_0 = \frac{E_0}{\omega L}$. Comparing it with $I_0 = \frac{E_0}{R}$, we conclude that, (ωL) has the dimensions of resistance. The term (ωL) is known as inductive reactance.

The inductive reactance is the effective opposition offered by the inductor to the flow of alternating current in the circuit.

The unit of inductive reactance is ohm.

Inductive reactance is given by relation.

$X_L = \omega L = L \times 2\pi f$

For d.c.; $f = 0$ $\qquad X_L = 0$

Thus, inductor offers no resistance to the flow of d.c. Hence d.c. can flow easily through the inductor.

For a.c f is finite

X_L = finite value

Thus, inductor offers finite resistance to the flow of a.c.

1. $X_L = L(2\pi f)$, So X_L increases with the increase in the frequency of a.c., $X_L \to \infty$ inductor behaves as good as an open circuit (i.e., $I = 0$) for high frequency of a.c.

2. $X_L = L(2\pi f) = \text{henry} \times (\text{second})^{-1}$

$$= \frac{\text{volt}}{\text{ampere / second}} \times (\text{second})^{-1}$$

$$= \frac{\text{volt}}{\text{ampere}} = \text{ohm}$$

3. Variation of voltage and current with time in purely inductive circuit can also be represented as shown in figure.

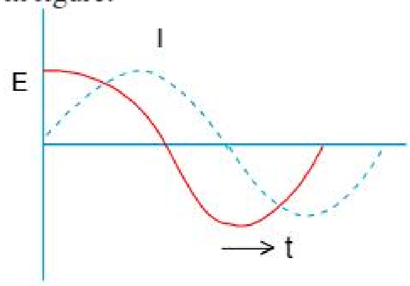


Fig 4.6(d)

4. $X_L - f$ diagram of a purely inductive circuit is shown in figure.

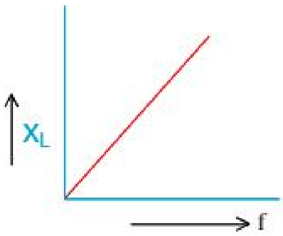


Fig 4.6(e)

5. Practically an inductor always possesses some resistance because of the wire used to make its coil. It is, therefore, only theoretical to consider a pure inductor.

4.5.3 POWER IN A PURELY INDUCTIVE CIRCUIT

Instantaneous power

$$= E_0 I_0 \sin\omega t \sin\left(\omega t - \pi/2\right)$$

$$= -E_0 I_0 \sin\omega t \cos\omega t = -\frac{E_0 I_0}{2}\sin 2\omega t$$

Average power over one cycle of alternating current

$$= \frac{\int_0^T -\frac{E_0 I_0}{2}\sin\omega t\, dt}{\int_0^T dt}$$

$$= -\frac{E_0 I_0}{2T}\int_0^T \sin 2\omega t\, dt = 0$$

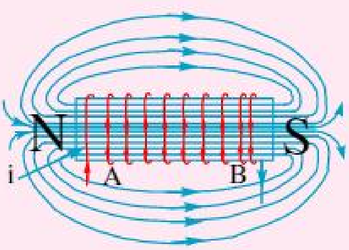
$\therefore$ Average power = 0

So, there is no power consumption in a purely inductive circuit. If power is positive during a certain half - cycle, it is negative during the next half cycle.

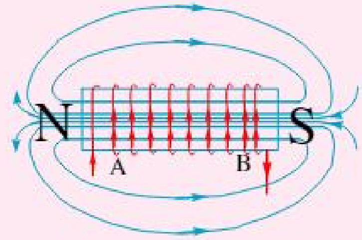
So, the average power consumed over the full cycle is zero. Positive power implies the supply of energy to magnetic field. Negative power implies that the energy is supplied back by the magnetic field.

Note : The average power supplied to an inductor over one complete cycle is zero.

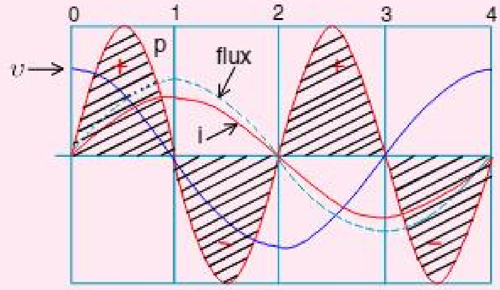
Figure below explain the magnetisation and the demagnetisation of an inductor.



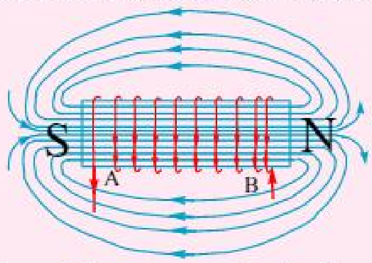
(0-1) : Current i through the coil entering at A increase from zero to a maximum value. Flux lines are set up i.e., the core gets magnetised. With the polarity shown voltage and current are both positive. So their product p is positive. Energy is absorbed from the source.



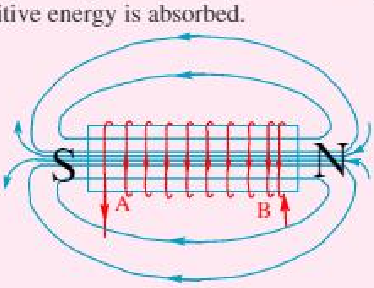
(1-2) : Current in the coil is still positive but is decreasing. The core gets demagnetised and the net flux becomes zero at the end of a half cycle. The voltage v is negative (since *di/dt* is negative). The product of voltage and current is negative, and energy is being returned to source.



One complete cycle of voltage/current.
Note that the current lags the voltage.



(2-3) : Current *i* becomes negative i.e., it enters at B and comes out of A. Since the direction of current has changed the polarity of the magnet changes. The current and voltage are both negative. So their product p is positive energy is absorbed.



(3-4) : Current *i* decreases and reaches its zero value at 4 when core is demagnetised and flux is zero. The voltage is positive but the current is negative. The power is, therefore, negative. Energy absorbed during the 1/4 cycle 2-3 is returned to the source.

Example-4.4

A 100 Hz a.c. is flowing in a coil of inductance 10mH. What is the reactance of the coil?

Solution :

Here $L = 10\text{ mH} = 10\times10^{-3}\text{H} = 10^{-2}\text{H}$

$\nu = 100\text{ Hz}$

Using $X_L = L\omega = L\times2\pi\nu$, we get

$X_L = 10^{-2}\times2\times3.14\times100 = 6.28\ \Omega$.

Example-4.5

A 44mH inductor is connected to 220V, 50Hz a.c supply. Determine the rms value of the current in the circuit.

Solution :

Here $L = 44\text{m H} = 44\text{x}10^{-3}\text{H}$.

$\nu = 50\text{Hz}$

$E_{rms} = 220\text{ V}$

Reactance of inductor.

$$X_L = L\omega = L\times2\pi\nu$$
$$= 44\times10^{-3}\times2\times3.14\times50$$
$$= 13.82\,\Omega$$

Current (rms) through the circuit

$$I_{rms} = \frac{E_{rms}}{X_L} = \frac{220}{13.82} = 15.9\text{A}$$

Example-4.6

Find the maximum value of current when an inductance of 2H is connected to 150 V, 50 cycle supply.

Solution :

Here $L = 2\text{H},\ E_{rms} = 150\text{ V},\ \nu = 50\text{ Hz}$

$X_L = L\omega = L\times2\pi\nu$

$2\times2\times3.14\times50 = 628\text{ ohm}$

RMS value of current through the inductor ,

$$I_{rms} = \frac{E_{rms}}{X_L} = \frac{150}{628} = 0.24$$

Maximum value (or peak value) of current is given by

$$I_{rms} = \frac{I_0}{\sqrt{2}} \text{ or } I_0 = \sqrt{2}\,I_{rms}$$
$$= 1.414\times0.24 = 0.339\text{A}$$

4.6.1 ALTERNATING VOLTAGE APPLIED TO A CAPACITOR

Alternating voltage applied to a capacitor is shown in Figure. Such a circuit is known as purely Capacitive circuit

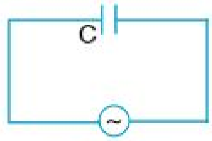


Fig 4.7(a)

The alternating e.m.f applied across the capacitor is given by

$$E = E_0 \sin\omega t \qquad .....(a)$$

Let q be the charge on the capacitor at any instant.

$\therefore$ Potential difference across the capacitor,

$$V_C = \frac{q}{C}$$

But $V_C = E$ or $\frac{q}{C} = E = E_0.\sin\omega t$

$$q = E_0 C \sin\omega t$$

Now, $I = \frac{dq}{dt} = \frac{d}{dt}(E_0\ C\sin\omega t)$

$$= E_0C(\cos\omega t)\omega = \frac{E_0}{(1/C\omega)}\cos\omega t$$

since $\cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$

$$I = I_0\sin\left(\omega t + \frac{\pi}{2}\right) \qquad .....(b)$$

where $I_0 = \frac{E_0}{\left(\frac{1}{C\omega}\right)}$ is the peak value of a.c.

Comparison of eqn (a) and (b) shows that current leads the e.m.f by an angle $\frac{\pi}{2}$ in an a.c. circuit containing a capacitor only. [Fig 4.7(b)]. The phasor diagram for capacitor is shown in [Fig 4.7(c)]

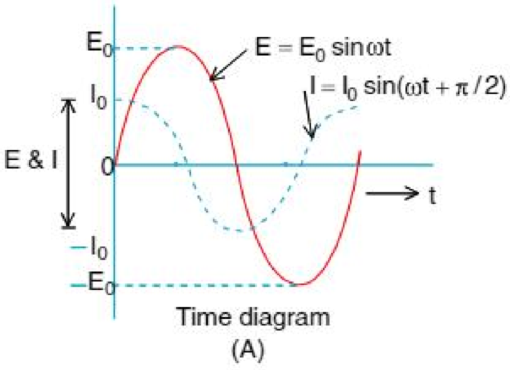


Fig 4.7(b)

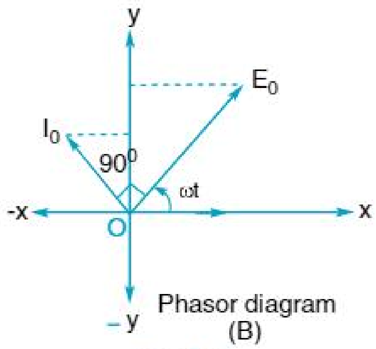


Fig 4.7(c)

Capacitive Reactance (X_C)

Here $I_0 = \dfrac{E_0}{\left(\dfrac{1}{\omega C}\right)}$, Comparing it with $I_0 = \dfrac{E_0}{R}$, we conclude that $\left(\dfrac{1}{\omega C}\right)$ has the dimensions of resistance. The term $\left(\dfrac{1}{\omega C}\right)$ is known as capacitive reactance.

4.6.2 CAPACITIVE REACTANCE (X_C)

The Capacitive reactance is the effective opposition offered by the capacitor to the flow of current in the circuit. The unit of capacitive reactance is ohm.

capacitive reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{C \times 2\pi f}$

For D.C., f = 0 $\quad \therefore X_C = \dfrac{1}{0} = \infty$

Thus, capacitor offers infinite resistance to the flow of d.c., so d.c. cannot pass through the capacitor, however small the capacitance of the capacitor is

For A.C. , f = finite

$$\therefore X_C = \frac{1}{\text{finite value}} = \text{very small}$$

Thus, capacitor offers small opposition to the flow of a.c., so a.c. can be considered to pass through it easily.

i) $X_C = \dfrac{1}{C \times 2\pi f}$ i.e., $X_C \propto \dfrac{1}{f}$.

For very high frequency of a.c. $X_C \to 0$. Thus capacitor behaves as a conductor for high frequency of a.c.

ii) Variation of X_C with v is shown in figure

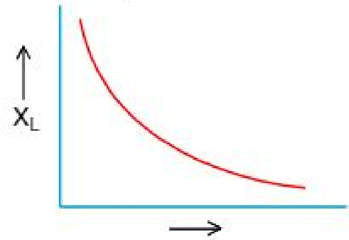


Fig 4.7 (d)

4.6.3 POWER IN A PURELY CAPACITIVE CIRCUIT

Instantaneous power

$$= EI = E_0 I_0 \sin\omega t \sin(\omega t + \pi/2)$$

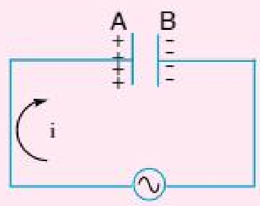
$$= \frac{E_0 I_0}{2} \sin 2\omega t$$

Average power over one cycle

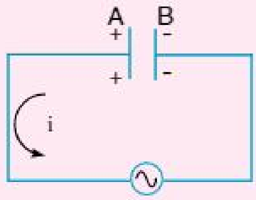
$$= \frac{\int_0^T \frac{E_0 I_0}{2} \sin 2\omega t\, dt}{\int_0^T dt} = \frac{E_0 I_0}{2T} \int_0^T \sin 2\omega t\, dt = 0$$

Conclusion : No power is consumed over one cycle.

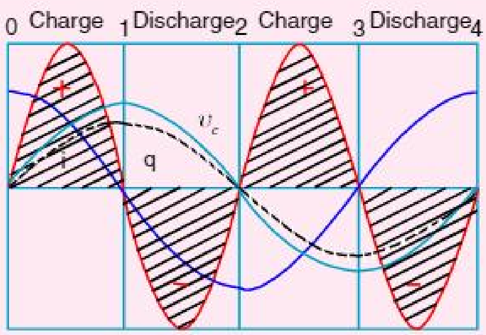
Note: Average power supplied to a capacitor over one complete cycle is zero.



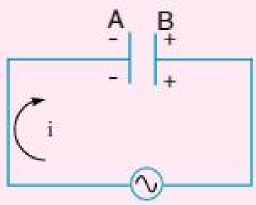
(0-1) : The current i flows as shown and from the maximum at 0, reaches a zero value at 1. The plate A is charged to positive polarity while negative charge q builds up in B reaching a maximum at 1 until the current becomes zero. The voltage $v_c = q/C$ is in phase with q and reaches maximum value at 1. Current and voltage are both positive. So $p = v_c i$ is positive. Energy is absorbed from the source during this quarter cycle as the capacitor is charged.



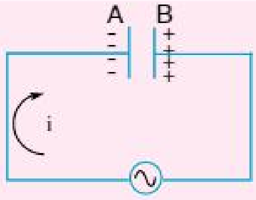
(1-2) : The current i reverse its direction. The accumulated charge is depleted i.e., the capacitor is discharged during this quarter cycle. The voltage gets reduced but is still positive. The current is negative. Their product, the power is negative. The energy is absorbed during the 1/4 cycle (i) is returned during this quarter.



One complete cycle of voltage/current.
Note that thecurrent leads the voltage.



(2-3) : As *i* continues to flow from A to B, the capacitor is charged to reversed polarity i.e., the plate B acquires positive and A acquires negative charge. Both the current and the voltage are negative. Their product p is positive. The capacitor absorbs energy during this 1/4 cycle.



(3-4) : The current *i* reverse its direction at 3 and flows from B to A. The accumulated charge is depleted and the magnitude of the voltage υ_c is reduced. υ_c becomes zero at 4 when the capacitor is fully discharged. The power is negative. Energy absorbed during (iii) is returned to the source. Net energy absorbed is zero.

Example-4.7

A50μF capacitor is connected to a 100 V, 50Hz a.c. supply. Determine the rms value of the current in the circuit.

Solution :

Here $C = 50\mu F = 50\times10^{-6}F = 5\times10^{-5}F$

$E_{rms} = 100V.$

$\nu = 50$ Hz.

Step 1 : Capacitive reactance,

$$X_C = \frac{1}{C\omega} = \frac{1}{C\times 2\pi\nu}$$

$$X_C = \frac{1}{5\times10^{-5}\times2\times3.14\times50} = 63.69\ \Omega$$

Step 2 : $I_{rms} = \dfrac{E_{rms}}{X_C} = \dfrac{100}{63.69} = 1.57$ A

4.7 ALTERNATING VOLTAGE APPLIED TO L R SERIES CIRCUIT

Consider an a.c. source of e.m.f E (r.m.s. value) connected to a series combination of an inductor of pure inductance L and a resistor of resistance R as shown in Figure A. Let I be the r.m.s. value of current flowing through the circuit. The potential difference across the inductor is given by, $V_L = IX_L$ .....(a)

Voltage V_L leads the current I by an angle of $\frac{\pi}{2}$ when a.c. flows through the inductor. In Fig 4.8(b) V_L is represented by OB along Y-axis and current I along X-axis. The potential difference across the resistor,

$V_R = IR$ .....(b)

Since the voltage and current are in phase when a.c. flows through the resistor, so V_R is represented along X-axis (the direction along which I is represented).

Therefore, the resultant of V_L and V_R is represented by OC and is given by

$$OC = \sqrt{OA^2 + OB^2} \text{ or } E = \sqrt{V_R^{\,2} + V_L^{\,2}}$$

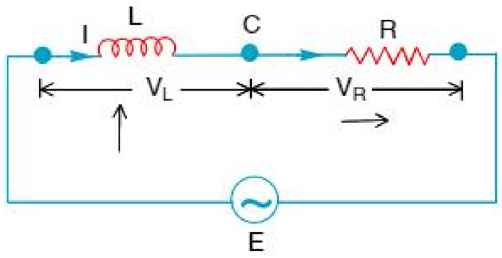


Fig 4.8 (a)

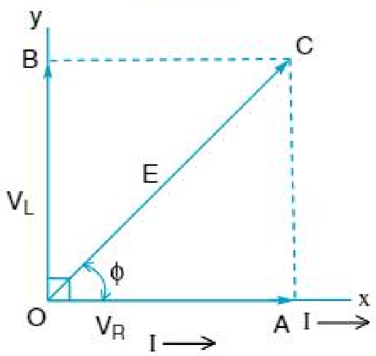


Fig 4.8 (b)

Using eqn. (i) and (ii)

$$E = \sqrt{I^2R^2 + I^2X_L^{\,2}} = I\sqrt{R^2 + X_L^{\,2}}$$

where $X_L = \omega L$ is the inductive reactance.

$$\text{or } I = \frac{E}{\sqrt{R^2 + X_L^{\,2}}} \quad .....(i)$$

If Z_{LR} is effective opposition offered by the LR circuit to a.c., then

$$I = \frac{E}{Z_{LR}} \quad ....(ii)$$

From eqn. (i) and (ii) we get

$$Z_{LR} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + L^2\omega^2} \quad .....(1)$$

The effective opposition offered by LR circuit to a.c. is called the impedence of LR circuit. Let ϕ be the angle made by the resultant of V_L and V_R with the X-axis, then from figure, we get

$$\tan\phi = \frac{AC}{OA} = \frac{OB}{OA} = \frac{V_L}{V_R} = \frac{IX_L}{IR}$$

$$\text{or } \tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} \quad ....(2)$$

In series LR circuit, voltage leads the current or in other words the current is said to lag behind the voltage by an angle ϕ given by eqn.(2)

1. $Z_{LR} = \sqrt{R^2 + L^2\omega^2} = \sqrt{R^2 + L^2 \times 4\pi^2 f^2}$.

Thus Z_{LR} increases with the frequency of a.c., so Z_{LR} is low for low frequency of a.c. and high for higher frequency of a.c.

2. The phase angle between voltage and current $\left[\phi = \tan^{-1}\left(\frac{2\pi fL}{R}\right)\right]$ increases with the increase in the frequency of a.c.

Example-4.8

A series combination of a coil of inductance L and a resistor of resistance $12\,\Omega$ is connected across a 12 V, 50 Hz supply. Calculate L if the circuit current is 0.5 A.

Solution :

Impedance, $Z = \frac{E}{I} = \frac{12}{0.5} = 24\Omega$ and $Z = \sqrt{R^2 + \omega^2 L^2}$

or $Z^2 = R^2 + \omega^2 L^2$ or $L^2 = \frac{Z^2 - R^2}{\omega^2}$

Here $\omega = 2\pi\nu = 2 \times \frac{22}{7} \times 50$

$= 314 \text{ rads}^{-1}$ and $R = 12\Omega$

$$L^2 = \frac{(V)^2 - (12)^2}{(12)^2} \text{ or } L = \frac{12\sqrt{3}}{314} = 0.066\text{H}$$

Example-4.9

A 50V, 10W lamp is run on 100V, 50Hz a.c. mains Calculate the inductance of the choke coil required.

Solution :

Voltage marked on lamp, V = 50 V

Power, P = 10W

$\therefore$ Resistance of lamp, $R = \dfrac{V^2}{P} = \dfrac{50 \times 50}{10} = 250\,\Omega$.

Current rating of lamp, $I_{rms} = \dfrac{P}{V} = \dfrac{10}{50} = \dfrac{1}{5}\,A$

The given circuit is equivalent to LR circuit.

Here $E_{rms} = 100V$, $\nu = 50Hz$

When the lamp is worked on a.c., the impedance of the circuit is

$$Z_L = \frac{E_{rms}}{I_{rms}} = \frac{100}{\frac{1}{5}} = 500 \text{ ohm.}$$

$$Z_L^2 = R^2 + \omega^2 L^2;$$

$$Z_L^2 = R^2 + 4\pi^2 \nu^2 L^2$$

$$\Rightarrow 4\pi^2 \times (50)^2 L^2 = 500^2 - 250^2 = 3 \times 250^2$$

$$\Rightarrow L = \frac{250\sqrt{3}}{2\pi \times 50} = \frac{5\sqrt{3}}{2\pi} = 1.38H$$

Example-4.10

A coil of inductance 0.50 H and resistance 100 ohm is connected to 240 V, 50 Hz a.c. supply.

a) What is the peak current in the coil?

b) What is the time lag between the peak voltage and the peak current?

Solution :

Here $L = 0.50H$, $R = 100$ ohm.

$E_{rms} = 240V$, $\nu = 50\,Hz$

Step 1 :

We know

$E_0 = \sqrt{2}\,E_{rms} = 1.414 \times 240 = 339.4V$

Impedance of LR circuit is

$$Z_L = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2}$$

$$= \sqrt{(100)^2 + 4 \times (3.14)^2 \times (50)^2 \times (0.50)^2}$$

$$= 186.1 \text{ ohm}$$

Peak current, $I_0 = \dfrac{E_0}{Z_L} = \dfrac{339.4}{186.1} = 1.82A$

In LR circuit the phase difference between current and voltage ϕ is given by

$$\tan\phi = \frac{L\omega}{R} = \frac{L \times 2\pi f}{R};$$

$$\tan\phi = \frac{0.50 \times 2 \times 3.14 \times 50}{100} = 1.57$$

$$\phi = \tan^{-1}(1.57) = 57^0 30^1 = 57.5^0$$

$$= \frac{57.5 \times \pi}{180} = 0.3194\,\pi \text{ radians}$$

Here $E = E_0 \sin\omega t$; $E_0 = E_{rms}\sqrt{2}$

$\therefore$ Time lag between E_0 and I_0 is given by

$$t = \frac{\phi}{\omega} = \frac{\phi}{2\pi\nu} = \frac{0.3194\pi}{2\pi \times 50}$$

$$= 0.003194\,s = 3.194 \times 10^{-3}\,s.$$

4.8 CR SERIES CIRCUIT WITH ALTERNATING VOLTAGE

Let an alternating source of e.m.f. E (r.m.s value) be connected to a series combination of a capacitor of pure capacitance (C) and a resistor of resistance (R) as shown in Fig 4.9(a).

Let I be the r.m.s value of current flowing through the circuit. The potential difference across the capacitor.

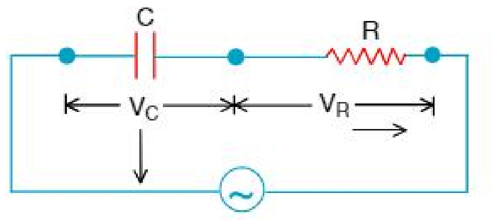


Fig 4.9 (a)

$V_C = IX_C$ .....(i)

The potential difference across the resistor,

$V_R = IR$ .....(ii)

The Voltage lags behind the current by an angle of $\pi/2$ when a.c. flows through the capacitor, as in Fig 4.9(b). V_C is represented by OB along negative Y-axis and the current I is represented along X-axis.

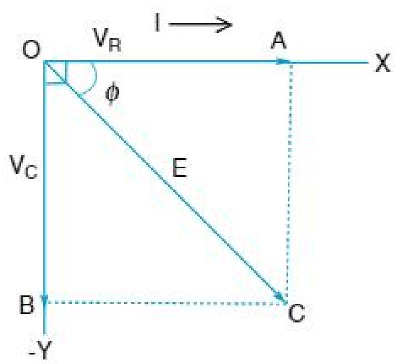


Fig 4.9 (b)

Also, the voltage and current are in phase when a.c. flows through the resistor, so V_R is represented by OA along X-axis.

Therefore, the resultant potential difference of V_C and V_R is represented by OC and is given by

$$OC = \sqrt{OA^2 + OB^2} \quad \text{or } E = \sqrt{V_R^{\ 2} + V_C^{\ 2}}$$

Using eqn. (i) and (ii)

$$E = \sqrt{I^2R^2 + I^2X_C^{\ 2}} = I\sqrt{R^2 + X_C^{\ 2}}$$

where $X_C = \dfrac{1}{C\omega}$ is the capacitive reactance.

or $$I = \frac{E}{\sqrt{R^2 + X_C^{\ 2}}} \quad .....(iii)$$

If Z_{CR} is the effective opposition offered by the CR circuit to a.c., then

$$I = \frac{E}{Z_{CR}} \quad .....(iv)$$

From eqn. (iii) and (iv), we get

$$Z_{CR} = \sqrt{R^2 + X_C^2} \quad .....(1)$$

which is the impedance of CR circuit. Let ϕ be the angle made by E with X-axis, then from Figure (B)

$$\tan\phi = \frac{AC}{OA} = \frac{V_C}{V_R} = \frac{IX_C}{IR}$$

or $$\tan\phi = \frac{X_C}{R} = \frac{I}{C\omega R} \quad .....(2)$$

In series CR circuit, voltage lags behind the current or in other words, the current is said to lead the voltage by an angle ϕ given by eqn. (2).

$$Z_{CR} = \sqrt{R^2 + X_C^{\ 2}} = \sqrt{R^2 + \frac{1}{C^2\omega^2}}$$

$$= \sqrt{R^2 + \frac{1}{4\pi^2 f^2 C^2}}$$

Thus $Z \propto \dfrac{1}{f}$ for very high frequency (f) of a.c. $Z \to R$ and for very low frequency of a.c. $Z \to \infty$ since phase angle between voltage and current is given by

$$\tan\phi = \frac{1}{C\omega R} = \frac{1}{2\pi fCR}$$

As ν increases phase angle ϕ decreases.

Example-4.11

A series circuit contains a resistor of 20Ω, a capacitor and an ammeter of negligible resistance. It is connected to a source of 200 V, 50 Hz. If the reading of ammeter is 2.5 A, calculate the reactance of the capacitor.

Solution :

Here $R = 20\Omega$, $E_{rms} = 200V$, $\nu = 50Hz$, $I_{rms} = 2.5A$

The circuit is CR circuit.

Impedance of circuit.

$$Z_{CR} = \frac{E_{rms}}{I_{rms}} = \frac{200}{2.5} = 80\,\Omega$$

But, $Z_{CR} = \sqrt{R^2 + X_C^{\ 2}}$

or $Z_{CR}^{\ 2} = R^2 + X_C^{\ 2}$

or $X_C^{\ 2} = Z_{CR}^{\ 2} - R^2$

or $X_C = \sqrt{Z_{CR}^{\ 2} - R^2} = \sqrt{(80)^2 - (20)^2} = 77.46$ ohm

Example-4.12

A $10\mu F$ capacitor is in series with a 50Ω resistance and the combination is connected to a 220V, 50 Hz line. Calculate (i) the capacitive reactance, (ii) the impedance of the circuit and (iii) the current in the circuit.

Solution :

Here, $C = 10\mu F = 10\times10^{-6} = 10^{-5}F$

$R = 50$ ohm, $E_{rms} = 220V, f = 50Hz$,

(i) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\times3.14\times50\times10^{-5}} = 318.5\ \Omega$$

(ii) Impedance of CR circuit.

$$Z_{CR} = \sqrt{R^2 + X_C^{\ 2}} = \sqrt{(50)^2 + (318.5)^2}$$

$$= 322.4\Omega$$

(iii) Current, $I_{rms} = \dfrac{E_{rms}}{Z_{CR}} = \dfrac{220}{322.4} = 0.68A$

4.9 L, C AND R SERIES CIRCUIT WITH ALTERNATING VOLTAGE

A circuit containing inductor of pure inductance (L) capacitor of pure capacitance (C) and resistor of resistance (R), all joined in series, is shown in figure. Let E be the r.m.s value of the applied alternating e.m.f. to the LCR circuit.

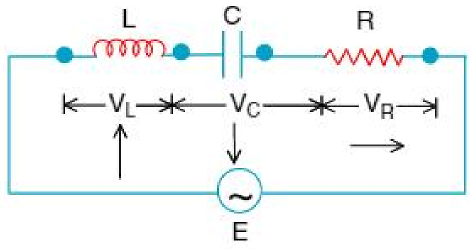


Fig 4.10 (a)

Let I be the r.m.s value of current flowing through all the circuit elements. The potential difference across L,

$V_L = IX_L$ .....(i)

(leads current I by an angle of $\frac{\pi}{2}$)

The potential difference across C,

$V_C = IX_C$ .....(ii)

(lags behind the current I by an angle of $\frac{\pi}{2}$. The potential difference across R,

$V_R = IR$ .....(iii)

(in phase with the current)

Since V_R and I are in phase, so V_R is represented by OA in the direction of I .

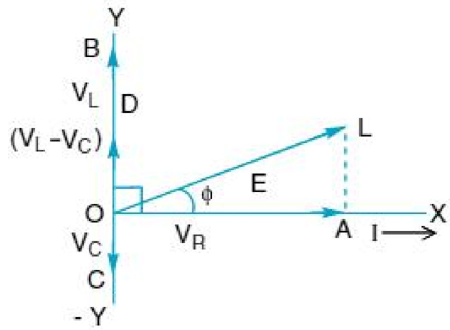


Fig 4.10 (b)

The current lags behind the potential difference V_L by an angle of $\frac{\pi}{2}$, so V_L is represented by OB perpendicular to the direction of I. The current leads the potential difference V_C by an angle of $\frac{\pi}{2}$ so V_C is represented by OC perpendicular to the direction of I.

Since V_L and V_C are in opposite phase, so their resultant $(V_L - V_C)$ is represented by OD (Here $V_L > V_C$)

The resultant of V_R and $V_L - V_C$) is given by OL. The magnitude of OL is given by

$$OL = \sqrt{(OA)^2 + (OD)^2}$$

$$= \sqrt{{V_R}^2 + (V_L - V_C)^2}$$

$$\text{or } E = \sqrt{{V_R}^2 + (V_L - V_C)^2}$$

Using eqns. (i), (ii) and (iii), we get

$$E = \sqrt{I^2R^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{or } \frac{E}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

Now $\frac{E}{I} = Z$, the effective opposition of LCR circuit to A.C. called impedance (Z) of the circuit.

$\therefore$ Impedance (Z) of LCR circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad .....(iv)$$

Again, $I = \frac{E}{Z}$

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

$$\left(\because X_L = L\omega \text{ and } X_C = \frac{1}{C\omega}\right)$$

Let ϕ be the phase angle between E and I, then from above Figure.

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

Putting $X_L = L\omega$ and $X_C = \frac{1}{C\omega}$

$$\tan\phi = \frac{\left(L\omega - \dfrac{1}{C\omega}\right)}{R}$$

The relation (iv) is known to be the general relation for impedance. Clearly, if X_L and X_C both are equal to zero then Z = R i.e., expression for pure resistance circuit. If $X_L = 0$ then $Z = \sqrt{R^2 + X_C{}^2}$ i.e., expression for series RC circuit. Similarly if $X_C = 0$ then $Z = \sqrt{R^2 + X_L{}^2}$ i.e. expression for series RL circuit.

Also, $\cos\phi = \frac{R}{Z}$

1. When $X_L = X_C$ or $L\omega = \frac{1}{C\omega}$.

Then $\tan\phi = 0$. or $\phi = 0°$. Thus, there is no phase difference between current and potential difference. Therefore, the given LCR circuit is equivalent to a pure resistive circuit. The impedance of such LCR circuit is given by Z = R. It is independent of the frequency of a.c.

2. When $X_L > X_C$ or $L\omega > \frac{1}{C\omega}$,

$\tan\phi = +ve$ or $\phi = +ve$

This means, the potential difference leads the current by angle of ϕ such that LCR circuit is known as inductance dominated circuit.

3. When $X_L < X_C$ or $L\omega < \frac{1}{C\omega}$,

$\tan\phi = -ve$ or $\phi = -ve$

Thus, the potential difference lags behind the current by an angle of ϕ. Such LCR circuit is called capacitance dominated circuit.

4. If $V_C > V_L$ in LCR circuit, then phasor diagram shown in Figure becomes as shown in Figure and impedance of the LCR circuit can be written as

$$Z = \sqrt{R^2 + (X_C - X_L)^2} \text{ and}$$

$$\tan\phi = \frac{X_C - X_L}{R}$$

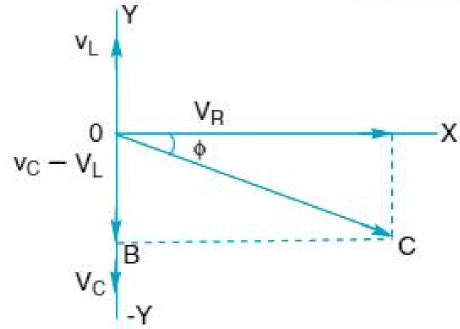


Fig 4.10 (c)

Example-4.13

When a capacitor of small capacitance is connected in series with series L-R circuit. the alternating current in the circuit increases. Explain why?

Solution :

Addition of capacitor in the given circuit decreases the impedance Z of the circuit and hence increases current I in the circuit as $I = \frac{V}{Z}$

where $Z = \sqrt{R^2 + X_L{}^2}$ without capacitor

and new $Z = \sqrt{R^2 + (X_L - X_C)^2}$ with capacitor

4.10 IMPEDANCE AND ADMITTANCE

The total effective opposition offered by LCR circuit to alternating current is known as Impedance.

In general, impedance (Z) comprises of three parts i.e., resistance (R), inductive reactance (X_L) and capacitive reactance (X_C) when X_L and X_C are opposite to each other and total reactance is taken as $\pm(X_L - X_C)$. Reciprocal of reactance is known as susceptance.

Impedance (Z) of LCR circuit can be represented diagramatically by Impedance triangle as shown in Figure (A).

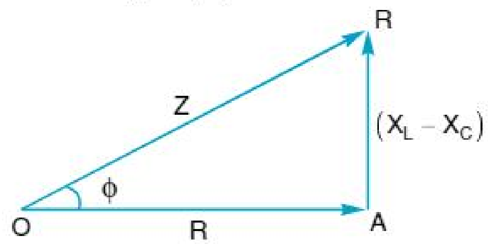


Fig 4.10 (d)

$$Z=\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}=\sqrt{R^2+(X_L-X_C)^2}$$

Impedance of LR circuit is given by

$$Z=\sqrt{R^2+(L\omega)^2}=\sqrt{R^2+X_L^2}$$

and is represented by Figure (B)

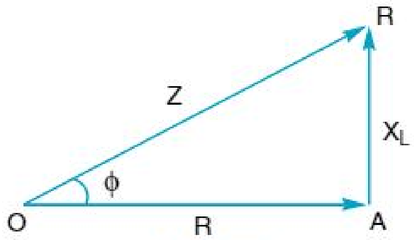


Fig 4.10 (e)

The phase difference between current and voltage is $\tan\phi=\frac{L\omega}{R}=\frac{X_L}{R}$ .This shows that current leads the voltage by an angle of ϕ. Impedance of CR circuit is given by

$$Z=\sqrt{R^2+\left(-\frac{1}{C\omega}\right)^2}=\sqrt{R^2+X_C{}^2}$$

and is represented by Figure (C)

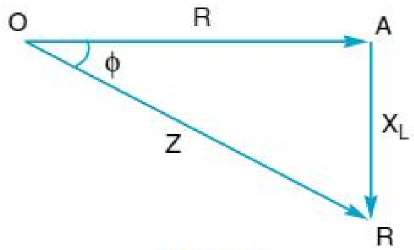


Fig 4.10 (f)

The voltage lags behind the current by an angle

$$\tan\phi=\frac{X_C}{R}=\frac{1}{\omega CR}=\frac{1}{\omega CR}$$

ADMITTANCE

Reciprocal of impedance of a circuit is called Admittance of the circuit.

i.e., Admittance (Y) = $\frac{1}{Z}$

Unit of Impedance (Z) of the circuit is ohm.

Unit of Admittance of the circuit is ohm^{-1} i.e., mho or siemen.

4.11 ANALYTICAL SOLUTION

The voltage equation for the circuit is

$$L\frac{di}{dt}+Ri+\frac{q}{C}=\upsilon\ =E_0\sin\omega t$$

we know that i = dq/dt. Therefore, di/dt=d^2q/dt^2. Thus, in terms of q, the voltage equation becomes

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=E_0\sin\omega t$$

This is like the equation for a forced, damped oscillator,

Let us assume a solution $q=q_m\sin(\omega t+\theta)$

so that $\frac{dq}{dt}=-q_m\cos(\omega t+\theta)$

and $\frac{d^2q}{dt^2}=-q_m\omega^2\sin(\omega t+\theta)$

Substituting these values in Eq., we get $q_m\omega\left[R\cos(\omega t+\theta)+(X_c-X_L)\sin(\omega t+\theta)\right]=E_0\sin\omega t$

where we have used the relation $Xc=1/\omega C, X_L=\omega L$. multiplying and dividing Eq. by $Z=\sqrt{R^2+(X_C-X_L)^2}$, we have

$$q_m\omega Z\left[\frac{R}{Z}\cos(\omega t+\theta)+\frac{(X_C-X_L)}{Z}\sin(\omega t+\theta)\right]$$
$$=E_0\sin\omega t$$

Now, let $\frac{R}{Z}=\cos\phi$ and $\frac{(X_C-X_L)}{Z}=\sin\phi$

so that $\phi=\tan^{-1}\frac{X_C-X_L}{R}$

substituting this in Eq. and simplifying, we get: $q_m\omega Z\cos(\omega t+\theta-\phi)=E_0\sin\omega t$

Comparing the two sides of this equation, we see that $E_0=q_m\omega Z=I_0Z$

where $I_0=q_m\omega$ and $\theta-\phi=-\frac{\pi}{2}$ or $\theta=-\frac{\pi}{2}+\phi$

Therefore, the current in the circuit is

$$i=\frac{dq}{dt}=q_m\omega\cos(\omega t+\theta)$$

$$I = I_0 \cos(\omega t + \phi) \text{ or } I = I_0 \sin(\omega t + \phi)$$

where $I_0 = \dfrac{E_0}{Z} = \dfrac{E_0}{\sqrt{R^2 + (X_C - X_L)^2}}$ and

$$\phi = \tan^{-1} \frac{X_C - X_L}{R}$$

Thus, the analytical solution of the amplitude and phase of the current in the circuit agrees with that obtained by the technique of phasors.

4.12 ELECTRICAL RESONANCE – LCR SERIES CIRCUIT

Electrical Resonance is said to take place in a series LCR circuit when the circuit allows maximum current for a given frequency of alternating supply at which capacitive reactance becomes equal to the inductive reactance.

The current (I) in a series LCR circuit is given by

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \quad .....(i)$$

From the above eqn., it is clear that current I will be maximum if the impedance (Z) of the circuit is minimum .

At low frequencies, $L\omega = L \times 2\pi f$ is very small and $\dfrac{1}{C\omega} = \dfrac{1}{C \times 2\pi f}$ is very large.

At high frequencies, $L\omega$ is very large and $\dfrac{1}{C\omega}$ is very small. For a particular frequency (f_0), $L\omega = \dfrac{1}{C\omega}$ i.e. $X_L = X_C$ and the impedance (Z) of LCR circuit is minimum and is given by $Z = R$. Therefore, at the particular frequency (f_0), the current in LCR circuit becomes maximum. The frequency (f_0) is known as the resonant frequency and the phenomenon is called electrical resonance. Again, for electrical resonance $(X_L - X_C) = 0$.

i.e. $X_L = X_C$

or $L\omega = \dfrac{1}{C\omega}$ or $\omega^2 = \dfrac{1}{LC}$

or $\omega = \dfrac{1}{\sqrt{LC}}$ or $(2\pi f_0) = \dfrac{1}{\sqrt{LC}}$

or $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

This is the value of resonant frequency.

The resonant frequency is independent of the resistance R in the circuit. However, the sharpness of resonance decreases with the increase in R.

Series LCR circuit is more selective when resistance of this circuit is small.

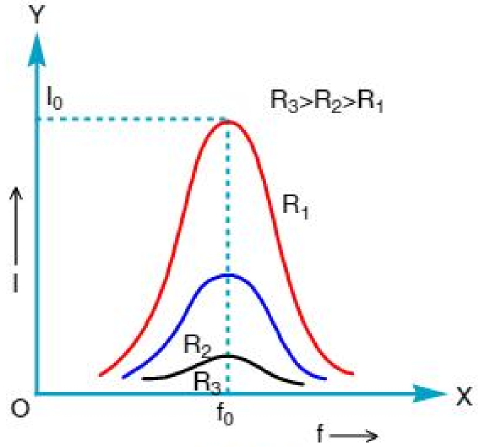


Fig 4.11

Applications : Series LCR circuit at resonance admit maximum current at particular frequencies so they can be use to tune the desired frequency or filter unwanted frequencies. They are used in transmitters and receivers of radio, television and telephone carrier equipment etc.

4.13 RESONANCE IN L-C-R CIRCUIT

At resonance,

a) Net reactance $X = 0$

b) $X_L = X_C$

c) Impedance $Z = R$ (minimum)

d) peak value of current $I_0 = \dfrac{E_0}{Z} = \dfrac{E_0}{R}$

(maximum but not infinity)

e) Resonant frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

f) Voltage and current will be in phase

g) power factor $\cos\phi = 1$

h) Resonant frequency is independent of value of R.

i) **A series L - C - R circuit behaves like a pure resistive circuit at resonance.**

Note :

i) Resonant circuits have a variety of applications, for example, in the tuning mechanism of a radio or a TV set. The antenna of a radio accepts signals from many broadcasting stations. The signals picked up in the antenna acts as a source in the tuning circuit of the radio, so the circuit can be driven at many frequencies. But to hear one particular radio station, we tune the radio. In tuning, we vary the capacitance of a capacitor in the tuning circuit such that the resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal received. When this happens, the amplitude of the current with the frequency of the signal of the particular radio station in the circuit is maximum.

ii) It is important to note that resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit. Only then do the voltages across L and C cancel each other (both being out of phase) and the current amplitude is E_0/R. The total source voltage appearing across R. This means that we cannot have resonance in a RL or RC circuit.

Sharpness of resonance

The amplitude of the current in the series LCR circuit is given by

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

and is maximum when $\omega = \omega_0 = 1/\sqrt{LC}$. The maximum value is

$$I_{0max} = E_0/R$$

For values of ω other than ω_0, the amplitude of the current is less than the maximum value. Suppose we choose a value of ω for which the current amplitude is $1/\sqrt{2}$ times its maximum value. At this value, the power dissipated by the circuit becomes half. From the curve in Fig. we see that there are two such values of ω, say, ω_1 and ω_2, one greater and the other smaller than ω_0 and symmetrical about ω_0. We may write

$$\omega_1 = \omega_0 + \Delta\omega$$

$$\omega_2 = \omega_0 - \Delta\omega$$

The difference $\omega_1 - \omega_2 = 2\Delta\omega$ is often called the bandwidth of the circuit. The quantity $(\omega_0/2\Delta\omega)$ is regarded as a measure of the sharpness of resonance. The smaller the $\Delta\omega$, we note that the current amplitude i_0 is

$(1/\sqrt{2})\, I_{0max}$ for $\omega_1 = \omega_0 + \Delta\omega$. Therefore,

$$\text{at } \omega_1, I_0 = \frac{E_0}{\sqrt{R^2\left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}}$$

$$= \frac{I_0^{max}}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}}$$

$$\text{or} \quad \sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} = R\sqrt{2}$$

$$R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = 2R^2$$

$$\omega_1 L - \frac{1}{\omega_1 C} = R$$

which may be written as,

$$(\omega_0 + \Delta\omega)L - \frac{1}{(\omega_0 + \Delta\omega)C} = R$$

$$\omega_0 L\left(1 + \frac{\Delta\omega}{\omega_0}\right) - \frac{1}{\omega_0 C + \left(1 + \frac{\Delta\omega}{\omega_0}\right)C} = R$$

Using $\omega_0^2 = \frac{1}{LC}$ in the second term on the left hand side, we get

$$\omega_0 L\left(1 + \frac{\Delta\omega}{\omega_0}\right) - \frac{\omega_0 L}{\left(1 + \frac{\Delta\omega}{\omega_0}\right)} = R$$

we can approximate

$\left(1 + \frac{\Delta\omega}{\omega_0}\right)^{-1}$ as $\left(1 - \frac{\Delta\omega}{\omega_0}\right)$ since $\frac{\Delta\omega}{\omega_0} << 1$.

Therefore $\omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right)-\omega_0 L\left(1-\frac{\Delta\omega}{\omega_0}\right)=R$

or $\omega_0 L\frac{2\Delta\omega}{\omega_0}=R \qquad \Delta\omega=\frac{R}{2L}$

The sharpness of resonance is given by.

$$\frac{\omega_0}{2\Delta\omega}=\frac{\omega_0 L}{R}$$

The ratio $\frac{\omega_0 L}{R}$ is also called the quality factor, Q of the circuit.

$$Q=\frac{\omega_0 L}{R}$$

From the above Eq, we see that $2\Delta\omega=\frac{\omega_0}{Q}$

So, larger the value of Q, the smaller is the value of $2\Delta\omega$ or the bandwidth and sharper is the resonance. Using $\omega_0^2=1/LC$ we can write $Q=1/\omega_0 CR$

We see from Fig. that if the resonance is less sharp, not only is the maximum current less, the circuit is close to resonance for a larger range $\Delta\omega$ of frequencies and the tuning of the circuit will not be good. So, less sharp the resonance, less is the selectivity of the circuit or vice versa. From this we see that if quality factor is large. i.e., R is low or L is large, the circuit is more selective.

4.14 QUALITY FACTOR AND SHARPNESS OF RESONANCE

The sharpness or selectivity of a resonance circuit is measured by Q-factor, called quality factor. The Q-factor of series resonance circuit is defined as the ratio of voltage developed across the inductance or capacitance at resonance to the impressed voltage (which is the voltage across R).

$$Q=\frac{\text{voltage across L or C}}{\text{Applied Voltage(= voltage acrossR)}}$$

$$Q=\frac{IX_L}{IR}=\frac{X_L}{R}=\frac{\omega L}{R}=\frac{1}{\sqrt{LC}}\frac{1}{R}$$

$$Q=\frac{1}{R}\sqrt{\frac{L}{C}}$$

Note :

i) Q is just a number

ii) Q factor is also called voltage amplification factor.

iii) Q factor will be large i.e., the circuit will have more sharpness if R is low or L is large or C is low.

4.15.1 POWER IN LCR CIRCUIT

Instantaneous power = EI

$$=E_0\sin(\omega t+\phi)I_0\sin\omega t$$

$$=E_0 I_0\sin\omega t\sin(\omega t+\phi)$$

$$=E_0 I_0\sin\omega t(\sin\omega t\cos\phi+\cos\omega t\sin\phi)$$

$$=E_0 I_0\cos\phi\sin^2\omega t+E_0 I_0\sin\phi\sin\omega t\cos\omega t$$

If the instantaneous power is assumed to remain constant for a small time dt, then work done over a complete cycle is given by

$$W=\int_0^T\left[E_0 I_0\cos\phi\sin^2\omega t+\frac{E_0 I_0}{2}\sin\phi(2\sin\omega t\cos\omega t)\right]dt$$

$$W=E_0 I_0\cos\phi\int_0^T\sin^2\omega t+\frac{E_0 I_0}{2}\sin\phi\int_0^T\sin 2\omega t\,dt$$

$$\therefore W=E_0 I_0\cos\phi\times\frac{T}{2}$$

Average power over complete cycle,

$$P_{av}=\frac{W}{T}=\frac{E_0 I_0}{2}\cos\phi$$

$$=\frac{E_0}{\sqrt{2}}\times\frac{I_0}{\sqrt{2}}\cos\phi=E_v I_v\cos\phi$$

Also, $P_{av}=E_v I_v\frac{R}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}$ or

$$P_{av}=E_v\times\frac{E_v}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}\times\frac{R}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}$$

$$=\frac{E_v^2 R}{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}$$

Average power is also known as true power. The quantity $E_v I_v$ is called the apparent power or virtual power. It is customary to express true power in kW and apparent power in kVA.

$\cos\phi$ is called the power factor of LCR circuit. Its value varies from zero to 1.

Power factor is defined as the ratio of true power to apparent power.

$$\text{Power factor} = \cos\phi = \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

Special Cases :

Case (i) : If the AC circuit contains pure resistance, then $\phi = 0^0$.

$$P_{av} = E_v I_v \cos 0^0 = E_v I_v = \frac{E_v^2}{R}$$

Case (ii) : If the AC circuit contains pure inductance, then $\phi = 90^0 . \therefore P_{av} = 0$

Case (iii) : If the AC circuit contains pure capacitance, then $\phi = 90^0 . \therefore P_{av} = 0$

Case (iv) : If the AC circuit contains L and R, then

$$\cos\phi = \frac{R}{\sqrt{R^2 + L^2\omega^2}}$$

$$\therefore P_{av} = E_v I_v \frac{R}{\sqrt{R^2 + L^2\omega^2}}$$

$$= \frac{E_v^2 R}{R^2 + L^2\omega^2}$$

Case (v) : If the AC circuit contains C and R, then

$$\cos\phi = \frac{R}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}.$$

$$\therefore P_{av} = \frac{E_v^2 R}{R^2 + \frac{1}{C^2\omega^2}}.$$

4.15.2 WATTLESS CURRENT OR IDLE CURRENT

If the voltage and current differ in phase by $\pi/2$, then Power factor, $\cos\phi = \cos 90^0 = 0$.

In this case, the current has no power. Such a current is, therefore, called wattless current. Since this current does not perform any work, therefore, this current may also be called idle current. Such a current flows only in purely inductive or in purely capacitive circuits.

4.16.1 ADVANTAGES OF ALTERNATING CURRENT OVER DIRECT CURRENT

1. The cost of generation of AC is less than that of DC.
2. AC can be conveniently converted into DC with the help of rectifiers.
3. AC is available in a wide range of voltages. These voltages can be easily stepped up or stepped down with the help of transformers.
4. AC appliances are simple, robust and require less care as compared to DC devices.
5. By supplying AC at high voltages, we can minimise line losses.

4.16.2 DISADVANTAGES OF ALTERNATING CURRENT OVER DIRECT CURRENT

1. AC is more dangerous than DC.
2. AC is transmitted more by the surface of the conductor. This is called skin effect. It is for this reason that several strands of thin insulated wire, instead of a simple thick wire, need to be used.
3. For electroplating, electrorefining, electrotyping, only DC can be used and not AC.

Example-4.14

A 100 mH inductor, a 20μF capacitor and a 10 Ω resistor are connected in series to a 100 V a.c. source. Calculate

i) impedance of circuit at resonance

ii) current at resonance

iii) resonant frequency

Solution :

(i) Impedance of circuit at resonance $Z = R = 10\Omega$

(ii) Current at resonance $= \frac{E}{R} = \frac{100}{10} = 10A$

(iii) Resonant frequency, $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$$f_0 = \frac{1}{2\pi\sqrt{(100\times10^{-3})(20\times10^{-6})}} = 112.5Hz$$

4.17(A) CHOKE COIL

Choke coil (or ballast) is a device having high inductance and negligible resistance. It is used to control current in **ac** circuits and is used in fluorescent tubes. The power loss in a circuit containing choke coil is least.

If a.c. source is directly connected to a mercury tube, the tube will be damaged. To avoid this a choke coil is connected in series with the tube. This is a simple L.R circuit with impedance $Z = \sqrt{R^2 + \omega^2L^2}$. If the applied voltage is $E = E_0 \sin\omega t$, the peak current through the circuit is

$$I_0 = \frac{E_0}{\sqrt{R^2 + \omega^2L^2}}.$$

The rms current is given by

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_0/\sqrt{2}}{\sqrt{R^2 + \omega^2L^2}} = \frac{E_{rms}}{\sqrt{R^2 + \omega^2L^2}}$$

The rms voltage across the resistor is

$$V_{R_{rms}} = R.I_{rms} = \frac{R.E_{rms}}{\sqrt{R^2 + \omega^2L^2}}$$

If choke coil were not used, the voltage across the resistor would be the same as the applied voltage. Thus, by using the choke coil, the voltage across the resistor is reduced by a factor.

$$V_{R_{rms}} = R.I_{rms} = \frac{R.E_{rms}}{\sqrt{R^2 + \omega^2L^2}}.$$

The advantage of using a choke coil to reduce the voltage is that an inductor does not consume power. Hence, we do not lose electrical energy in the form of heat.

If we connect an additional resistor in series with the tube, to reduce the voltage instead of a choke coil, power will be lost due to Joule's heating ($H = i^2Rt$).

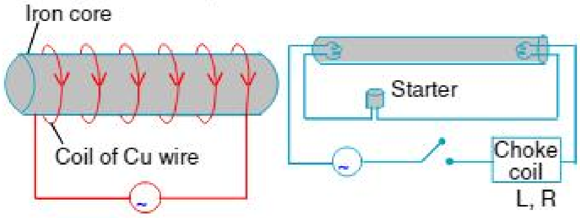


Fig 4.12

1) It consist of a Cu coil wound over a soft iron laminated core.
2) Thick Cu wire is used to reduce the resistance (R) of the circuit.
3) Soft iron is used to improve inductance (L) of the circuit.
4) The inductive reactance or effective opposition of the choke coil is given by $X_L = \omega L = 2\pi fL$
5) For an ideal choke coil r = 0, no electric energy is wasted i.e., average power P = 0.
6) In actual practice, choke coil is equivalent to a R - L circuit.
7) Choke coil for different frequencies are made by using different substances in their core.

For low frequency, L should be large thus iron core choke coil is used. For high frequency **ac** circuit, L should be small, so air cored choke coil is used.

4.17(B) SKIN EFFECT

A direct current flows uniformly throughout the cross-section of the conductor. An alternating current, on the other hand, flows mainly along the surface of the conductor. This effect is known as skin effect. The reason is that when alternating current flows through a conductor, the flux changes in the inner part of the conductor are higher. Therefore the inductance of the inner part is higher than that of the outer part. Higher the frequency of alternating current, more is the skin effect.

The depth upto which **ac** flows through a wire is called skin depth.

Since **ac** of high frequency do not pass through the entire cross-section of the wire, the effective resistance of the wire for **ac** is much greater than that for **dc**. Hence the conductor required to carry high frequency **ac** consists of a number of strands of fine wire connected in parallel at their ends, and insulated throughout their length from each other. This increases the surface area and thus decreases the resistance.

4.17(C) THE WORKING PRINCIPLE OF METAL DETECTOR

Metal detector works on the principle of resonance in **ac** circuits. When a person walks through a metal detector, in fact he walks through a coil of many turns connected to a capacitor tuned suitably. If he carries some metal, impedance of the circuit changes, bringing significant changes in the current. This change in current is detected and the electronic circuit causes the alarm.

Example-4.15

A 100 mH inductor, a 20μF capacitor and a 10 Ω resistor are connected in series to a 100V ac source. Calculate (i) impedance of circuit at resonance (ii) current at resonance (iii) resonant frequency.

Solution :

(i) Impedance of circuit at resonance $Z = R = 10\Omega$

(ii) Current at resonance $= \dfrac{E}{R} = \dfrac{100}{10} = 10\,A$

(iii) Resonant frequency, $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$f_0 = \frac{1}{2\pi\sqrt{(100\times10^{-3})(20\times10^{-6})}}$$

$= 112.5\,Hz$

4.18 TRANSFORMER

The transformer is a device which is used to change the voltage of an alternating current. As such they are of two types. When it changes the low alternating current at high voltage into high alternating current at low voltage, then it is know as step-down transformer. On the other hand, if it changes high alternating current at low voltage into low alternating current at high voltage then it is called step-up transformer i.e. a transformer is called as step-up or step-down type depending upon it increases or decreases the voltage of alternating current respectively.

Principle

It works on the principle of Mutual Induction i.e. when current flowing through a coil or magnetic flux linked with a coil changes, an induced e.m.f. is produced in the other coil.

Construction

It consists of two coils of copper wire wound separately over a rectangular and laminated soft iron core [The core is made by placing soft-iron strips one above the other. These strips are insulated from each other to reduce the eddy currents and hence the loss of energy in the core]. These coils are kept insulated from each other as well as from the iron core. The two coils are known as primary coil P and secondary coil S. The a.c. source is connected across the primary coil while transformed voltage or transformer output is obtained across the secondary coil S.

In case of a step-up transformer, the primary coil consists of smaller number of turns of thick copper wire while the secondary coil consists of larger number of turns of thin copper wire. In step down transformer, the order is just reversed.

Theory

When alternating current flows through the primary coil, a magnetic field is produced inside the coil which also varies both in magnitude and direction as per nature of input A.C source. Due to this, core is first magnetised in one direction and then in opposite direction in each cycle of current. Hence the electrical energy of the primary coil is transformed into the magnetic energy of the core. Since the secondary coil is also wrapped on the same core and the core is made of soft iron, the magnetic flux passing through it also changes continuously due to repeated magnetisation and demagnetisation of the core. As no leakage of magnetic flux occurs in the process of transformation, an alternating e.m.f. of the same frequency is induced in the secondary coil. Thus magnetic energy of iron-core is now transformed into the electrical energy of the secondary coil. The magnitude of induced e.m.f. produced in the secondary coil depends upon the ratio of number of turns in the two coils.

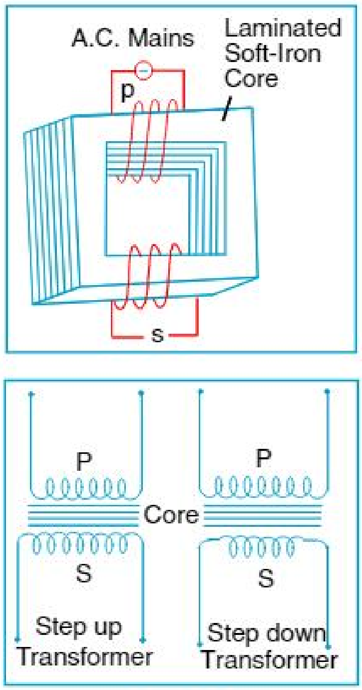


Fig 4.13

Let ϕ be the magnetic flux linked with each turn and N_P and N_S be the number of turns in the primary and secondary coils respectively. Because there is no leakage of magnetic flux in the process of transformation, same magnetic flux ϕ will be linked with each turn of primary and secondary coils at every instant. Therefore, according to Faraday's law of electromagnetic induction, induced e.m.f's produced in primary and secondary coils are given by

$$e_p = -N_p\left(\frac{\Delta\phi}{\Delta t}\right) \text{ and } e_s = -N_s\left(\frac{\Delta\phi}{\Delta t}\right),$$

$$\text{Hence } \frac{e_s}{e_p} = \frac{N_s}{N_p} \quad ..... (1)$$

If there is no loss of energy in the primary coil and its resistance is assumed negligible then induced emf produced in the primary coil will be nearly equal to the applied potential difference V_p between its ends. Similarly, because the secondary coil is open hence potential difference across its ends will be equal to emf induced in it. i.e. under these ideal conditions,

$$\frac{e_s}{e_p} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = r \quad ..... (2)$$

Here, r is called Transformation ratio. For step-up transformer $r > 1$; and for step down transformer $r < 1$.

If we assume that there are no energy losses in the process of transformation, then instantaneous output power = instantaneous input power

$$\text{or } V_s I_s = V_p . I_p \quad \text{or} \quad \frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = r$$

Thus a step-up transformer increases the voltage by decreasing the current and step-down transformer decreases the voltage by increasing the current. This happens according to law of conservation of energy. In other words, a transformer simply transforms the voltages and currents but does not generate electricity.

In real practice, the energy obtained from the secondary coil is always little bit less than the energy given to primary coil. This is because of the fact that some energy of the primary coil is spent in heating its wire & some is spent in the process of magnetisation and demagnetisation of core.

The ratio of output power to the input power is called efficiency (η) of transformer.

$$\eta = \frac{P_{0/p}}{P_{i/p}} = \frac{E_s I_s}{E_p I_p}$$

4.19 LC OSCILLATIONS

A capacitor (C) and an inductor (L) are connected as shown in the figure. Initially the charge on the capacitor is Q

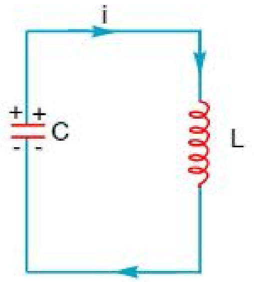


Fig 4.14(a)

$\therefore$ Energy stored in the capacitor

$$U_E = \frac{Q^2}{2C} \qquad \text{.... (1)}$$

The energy stored in the inductor = 0.

The capacitor now begins to discharge through the inductor and current begins to flow in the circuit. As the charge on the capacitor decreases, U_E decreases but the energy $U_B = \frac{1}{2}LI^2$ in the magnetic field of the inductor increases. Energy is thus transferred from capacitor to inductor. When the whole of the charge on the capacitor disappears, the total energy stored in the electric field in the capacitor gets converted to magnetic field energy in the inductor. At this stage there is maximum current in the inductor.

Energy now flows from inductor to the capacitor except that the capacitor is charged oppositely. This process of energy transfer continues at a definite frequency (ϑ). Energy is continuously shuttled back and forth between the electric field in the capacitor and the magnetic field in the inductor.

If no resistance is present in the LC circuit the LC oscillation will continue infinitely as shown.

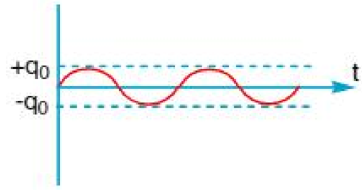


Fig 4.14(b)

However in an actual LC circuit, some resistance is always present due to which energy is dissipated in the form of heat. So LC oscillation will not continue infinitely.

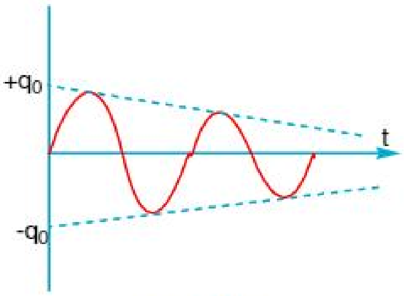


Fig 4.14(c)

Let q be the charge on the capacitor at any time t and $\frac{di}{dt}$ be the rate of change of current. Since no battery is connected in the circuit,

$$\frac{q}{c} - L.\frac{di}{dt} = 0 \qquad \text{..... (1)}$$

$$\text{but } i = -\frac{dq}{dt} \qquad \text{.... (2)}$$

from equation (1) and (2) we have

$$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0 \qquad \text{.... (3)}$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \qquad \text{.... (4)}$$

This equation (2) is analogus to

$$\frac{d^2r}{dt^2}+\omega^2 r=0 \qquad \text{.... (5)}$$

comparing (4) and (5) we have

$$\omega^2=\frac{1}{LC}; \qquad \omega=\frac{1}{\sqrt{LC}}$$

$$2\pi f=\frac{1}{\sqrt{LC}}; \quad f=\frac{1}{2\pi\sqrt{LC}}$$

The charge therefore oscillates with a frequency $f=\frac{1}{2\pi\sqrt{LC}}$ and varies sinusoidally with time.

Example-4.16

A step up transformer operates on a 230 V line and a load current of 2 ampere. The ratio of the primary and secondary windings is 1 : 25. What is the current in the primary?

Solution :

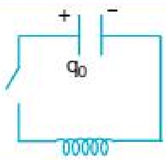
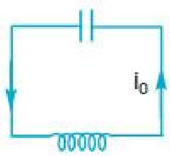
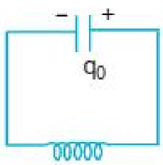
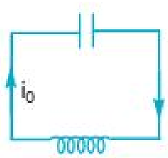
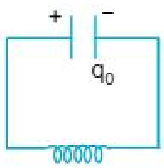
Using the relation $\frac{N_P}{N_S}=\frac{I_S}{I_P}; I_P=\frac{N_S I_S}{N_P}$

Here $N_p/N_s=1/25$ or

$N_s/N_p=25/1=25$ and $I_S=2A$

Current in primary, $I_P=25\times 2=50A$

Comparison between LC Oscillations with Spring Mass Oscillations

t = 0		$U_E=\frac{q_0^2}{2C}; U_B=0$	 at extreme position	$PE=\frac{1}{2}kA^2;\quad KE=0$
t = T/4		$U_E=0;\ U_B=\frac{1}{2}Li_0^2$	 at mean position	$PE=0;\ KE=\frac{1}{2}mv_0^2$
t = t/2		$U_E=\frac{q_0^2}{2C};\ U_B=0$	 at extreme position	$PE=\frac{1}{2}kA^2;\ KE=0$
t = 3T/4		$U_E=0; U_B=\frac{1}{2}Li_0^2$	 at mean position	$PE=0; KE=\frac{1}{2}mv_0^2$
t = T		$U_E=\frac{q_0^2}{2C};\ U_B=0$	 at extreme position	$PE=\frac{1}{2}kA^2;\ KE=0$

Example-4.17

A step down transformer converts a voltage of 2200 V into 220 V in the transmission line. Number of turns in primary coil is 5000. Efficiency of transformer is 90% and its output power is 8kW. Calculate (i) number of turns in secondary coil (ii) input power.

Solution :

$$\text{Efficiency } \eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\text{or Input power} = \frac{\text{Output power}}{\eta}$$

$$= \frac{8}{90/100} = 8.9\text{kW}$$

$$\text{Again } \frac{N_s}{N_p} = \frac{E_s}{E_p} \text{ or } N_s = \frac{E_s}{E_p} \times N_p$$

$$\text{Here } N_p = 5000, E_s = 220\text{V}$$

$$E_p = 2200\text{V} \Rightarrow N_S = \frac{220}{2200} \times 5000 = 500$$

Example-4.18

A transfer mer having efficiency 90% is working on 100 V and at 2.0 kW power. If the current in the secondary coil is 5A, calculate

i) the current in the primary coil and

ii) voltage across the secondary coil.

Solution :

$$\text{Here } \eta = 90\% = \frac{9}{10}, I_s = 5\text{A}$$

$$E_p = 100\text{V},$$

$$E_p I_p = 2\text{kW} = 2000\text{W}$$

$$\text{i) } I_p = \frac{2000}{E_p} \text{ or } I_p = \frac{2000}{100} = 20\text{A}$$

$$\text{ii) } \eta = \frac{\text{Output power}}{\text{Input power}} = \frac{E_s I_s}{E_p I_p} \text{ or } E_s I_s = \eta \times E_p I_p$$

$$= \frac{9}{10} \times 2000 = 1800\text{W}$$

$$\therefore E_s = \frac{1800}{I_s} = \frac{1800}{5} = 360 \text{ volt}$$

Example-4.19

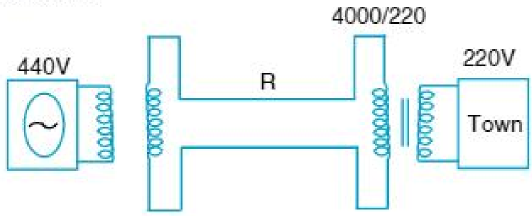
A small town with a demand of 800 kW of electric power at 220V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two line wires carrying power is Ω 0.5/km. The town gets power from the lines through a 4000-220 V step down transformer at a substation in the town.

a) Estimate the line power loss in the form of heat

b) How much power must the plant supply, assuming there is negligible power loss due to leakage ?

c) Characterize the step up transformer at the plant

Solution :



Power requirement of the town is given by EI, where E is the voltage at the receiving end of the line and I is the line current

Here

$$P = 800 \times 1000 \text{ W}$$

$$E = 4000 \text{ V then}$$

$$I = \frac{P}{E} = \frac{800 \times 1000}{4000} = 200\text{A}$$

(a) Line power loss

$$= I^2R = (200)^2 \times (2 \times 15 \times 0.5)$$

$$= 600,000 \text{ W} = 600\text{kW}$$

(b) Total power delivered by power plant

$$= 800 + 600 = 1400\text{kW}$$

(c) Voltage at sending end of line

= Receiving end line voltage+Voltage drop in line

$$= 4000 + (IR) = 4000 + 200(2 \times 15 \times 0.5)$$

$$= 4000 + 3000 = 7000\text{V}$$

$$\therefore \text{ Stepup transformer of } \frac{7000\text{V}}{440\text{V}} \text{ is required}$$

Example-4.20

An LC circuit contains a 20mH inductor and a 50 μF capacitor with an initial charge of 10mC. The resistance of the circuit is negligible. If the circuit is closed at t = 0.

(a) What is the total energy stored initially?

(b) What is the natural frequency of the circuit?

(c) At what time is the energy stored completely electrical?

(d) At what time is the total energy stored equally between the inductor and the capacitor?

Solution :

$$\text{(a) Total energy stored} = \frac{Q_0^2}{2C} = 1\text{J}$$

$$\text{(b) Natural frequency } f = \frac{1}{2m\sqrt{LC}} = 159\text{Hz}$$

(c) Charge on the capacitor varies simple harmonically such that at t = 0 charge $Q = Q_0$.

$$\Rightarrow Q = Q_0 \cos\omega t = Q_0 \cos\left(\frac{2\pi}{T}\right)t$$

where $T = \frac{1}{f} = 6.3$ ms

Energy stored in the circuit will be completely, electrical when $\cos\left(\frac{2\pi}{T}t\right) = \pm 1$

at $t = 0, \frac{T}{2}, T, \frac{3T}{2}, ...$

(d) Energy stored will be completely magnetic if = Q= 0

$$\cos\left(\frac{2\pi}{T}t\right) = \cos(2n+1)\frac{\pi}{2}$$

at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$

(e) $\frac{Q^2}{2C} = \frac{1}{2}\left(\frac{Q_0^2}{2e}\right) \Rightarrow Q = \pm\frac{Q_0}{\sqrt{2}}$

From $Q = Q_0 \cos\left(\frac{2\pi}{T}t\right)$

For $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, ...$ Total energy is stored equally between the inductor and capacitor.

Example-4.21

A series LCR circuit with L = 0.12 H, C = 480 nF, R = 23 Ω is connected to a 230 V variable frequency supply (a) What is the source frequency for which current amplitude is maximum? Obtain the maximum value?

(b) What is the source frequency for which average power absorbed by the circuit is maximum? Find the value of maximum power?

(c) For what frequencies is the power transferred to the circuit half the power at resonant frequency?

(d) What is the Q-factor of the given circuit?

Solution :

(a) Current amplitude is maximum at resonant frequency given by $f_0 = \frac{1}{2\pi\sqrt{LC}} = 663\text{Hz}$

$$\text{Maximum current amplitude} = \frac{V_0}{R} = \frac{\sqrt{2}V_{rms}}{R}$$

$$= \frac{\sqrt{2}\times 230}{23} = 14.14\text{A}$$

(b) Average power $<P> = \frac{1}{2}I_0^2R$

$$= \frac{1}{2}(14.14)^2(23) = 2300\text{W}$$

(c) $\Delta\omega = \frac{R}{2L} = \frac{23}{2\times 0.12} = 958\,\text{rad}\,\text{s}^{-1}$

$\Delta f = \frac{\Delta\omega}{2\pi} = 15.2\text{Hz}$

Power absorbed is half the peak power at $f = f_0 \pm \Delta f$

i.e., f = 678 Hz and 648 Hz

Curent amplitude at these frequencies is $\frac{I_0}{\sqrt{2}} = 10\text{A}$

(or) Q-factor $= \frac{\omega_0 L}{R} = 21.7$

At a Glance

1. A current that changes its direction periodically is called alternating current
2. Alternating current varies sinusoidally with time as $i = i_0 \sin(\omega t + \phi)$.
3. Alternating emf varies sinusoidally with time as $e = e_0\sin(\omega t+\phi)$.
4. The factor $(\omega t + \phi)$ is called phase.
5. Average value of a function from t_1 to t_2 is defined as $\langle f\rangle = \frac{\int_{t_1}^{t_2} f dt}{t_2 - t_1}$. We can also find the value of $\int_{t_1}^{t_2} f dt$ graphically. Average value is the area of f - t graph from t_1 to t_2.
6. $i_{average} = \frac{\int_{t_1}^{t_2} i\,dt}{t_1 - t_2}$
7. $i_{rms} = \sqrt{\frac{\int_{t_1}^{t_2} i^2\,dt}{t_2 - t_1}}$
8. $i_{average} = \frac{2i_0}{\pi} = 0.6370\,i_0$
9. $i_{rms} = \frac{i_0}{\sqrt{2}} = 0.707\,i_0$
10. ac voltmeter and ac ammeters are used to measure alternating voltage and current.
11. In a pure ohmic resistor there is no phase difference between current and voltage.
12. In a pure inductor voltage leads current by $\frac{\pi}{2}$

13. In a pure capacitor voltage lags behind current by $\frac{\pi}{2}$
14. The effective opposition offered by the inductor to the flow of ac is called inductive reactance (X_L)
15. $X_L = \omega L = 2\pi\nu L$
16. Inductor offers no resistance to the flow of d.c.
17. The effective opposition offered by the capacitor to the flow of current in the circuit is called capacitive reactance (x_C).
18. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$.
19. Capacitor offers infinite resistance to d.c.
20. Power consumed in an A.C. circuit $= V_{rms}\, i_{rms} \cos\phi$.
21. Impedance $(z) = \frac{V_{rms}}{i_{rms}}$. Admitance $= \frac{1}{Z}$
22. In a LCR series circuit $\tan\phi = \frac{\pm(X_L - X_C)}{R}$.
23. In a LCR series circuit if
    i) $X_L > X_C$, the circuit is called inductive dominated circuit.
    ii) $X_C > X_L$, the circuit is called capacitive dominated circuit.
    iii) $X_C = X_L$, then resonance sets in.
24. Resonant frequency $V_0 = \frac{1}{2\pi\sqrt{LC}}$.
25. Power $P = V_{rms}\, i_{rms} \cos\phi$.

EXERCISE

LONG ANSWER QUESTIONS

1. Derive expression for impedance and current of an a.c. circuit containing inductance and resistance in series and (ii) Capcitance and resistance in series.
2. Obtain an expression for impedance and current in series LCR circuit. Deduce an expression for the resonating frequency of an LCR series resonating circuit.

SHORT ANSWER QUESTIONS

1. Explain instantaneous, maximum, and rms values of a current.
2. Discuss the flow of a.c. through a pure resistor.
3. Obtain an expression for the current through an inductor when a.c. e.m.f. is applied.
4. Obtain an expression for the current in a capacitor when an a.c. e.m.f. is applied.

VERY SHORT ANSWER QUESTIONS

1. Where is the power dissipation in an alternating current? In resistance? In inductance? in capacitance?

A. Power is dissipated in an a.c. circuit in resistance only.

2. What is average value of a.c. over a complete cycle and why?

A. Average value of a.c. over a complete cycle is zero, because a.c. is positive during one half cycle and equally negative during the other half cycle.

3. What is peak value of 220V a.c?

A. $E_0 = \sqrt{2}E_v = \sqrt{2} \times 220\,\text{volt} = 311\,\text{volt}$

4. What is meant by admittance of an a.c. circuit?

A. Admittance of an a.c. circuit is the reciprocal of impedence (Z) of the circuit.

5. In an inductor, current rises to a steady value at a constant rate. Comment.

A. No, current rises only exponentially.

6. A larger value of Q implies sharper resonance. Comment.

A. Yes, it is correct

7. What are the dimensions of R/L?

A. $M^0 L^0 T^{-1}$

8. What is the significance of time constant of R - L circuit?

A. Time constant of R - L circuit tells us how fast or how slow is the growth/decay of current in the R - L circuit. Low value of time constant indicates that the growth and decay are fast. Large values of time constant indicate that growth and decay of current in the circuit are slow.

9. Can we use 25 c/s. a.c for lighting purposes?

A. Yes, we can use 25 c/s. a.c. for lighting purposes. The fluctuations in current will be so rapid (50 times/sec) that the bulb will appear glowing continuously due to persistence of vision.

10. A bulb connected in series with a solenoid is lit by a.c. source. If a soft iron core is introduced in the solenoid, will bulb glow brighter?

A. No, the bulb will glow dimmer. This is because, on introducing soft iron core in the solenoid, its inductance L increases, the inductive reactance $X_L = \omega L$ increases and hence the current through the bulb decreases.

11. An electric lamp connected in series with capacitor and an a.c. source is glowing with certain brightness. How does the brightness of the lamp change on reducing the capacitance?

A. Brightness of the lamp decreases. This is because on reducing C ; X_C increases Z increases and I decreases.

12. What is the power dissipation in an a.c. circuit in which voltage and current are given by $V = 300\sin(\omega t + \pi/2)$ and $I = 5\sin\omega t$?

A. As phase difference between voltage and current is $\pi/2$, power dissipation is zero.

PROBLEMS

LEVEL - I

1. The equation of alternating current for a circuit is given by $I = 50\cos 100\pi t$. Find (i) frequency of a.c applied (ii) mean value of current during positive half of the cycle (iii) virtual value of current and (iv) the value of current 1/300 sec after it was zero
[Ans: (i) 50Hz, (ii) 31.8 A, (iii) 35.35 A, (iv) 43.3 A]

2. Find the virtual value of current through a capacitor of capacitance 10μF, when connected to a source of 110 volt at 50 cycles supply .What is its reactance?
[Ans: 0.346 A, 318.2Ω]

3. A coil of indutance $4/\pi$ H is joined in series with a resistance of 30Ω .Calculate the current flowing in the circuit when connected to a.c mains of 200 V and frequency 50 Hz. **[Ans:** 0.499 A]

4. A circuit consists of a resistance of 10Ω and a capacitance of 0.1μF. If an alternating e.m.f of 100 V, 50 Hz is applied, calculate the current in the circuit.
[Ans: 3.14×10^{-3}A]

5. The current through a 1.0 H inductor varies sinusoidally with an amplitude of 0.5 A and a frequency of 50 Hz. Calculate the potential difference across the terminals of the inductor.
[Ans: 111V]

6. What is the inductive reactance of a coil if the current through it is 80 m A and voltage across it is 40 V?
[Ans: 500 Ω]

7. Calculate the frequency at which the inductive reactance of 0.7 H inductor is 220 Ω. **[Ans:** 50 Hz)]

8 What is the capacitive reactance of a 5μF capacitor when it is part of a circuit whose frequency is (i) 50 Hz (ii) 10^6 Hz ?
[Ans: (i) 637 Ω, (ii) 3.18×10^{-2} Ω]

9. A coil of inductance 0.50H and resistance 100 Ω is connected to a 240V-50Hz a.c supply. What is the maximum current in the coil and the time lag between voltage maximum and current maximum?
[Ans: 1.82A, 3.2×10^{-3}s]

10. A resistor of 50 Ω, an inductor of (20/π)H and a capacitor of (5/π)μF are connected in series to a voltage source 230V, 50Hz. Find the impedance of the circuit. **[Ans:** 50 Ω]

11. A 1μF capacitor is connected to 220V-50Hz a.c. source Find the virtual value of current through the circuit. What is the peak voltage across the capacitor?
[Ans: 0.07A, 311 V]

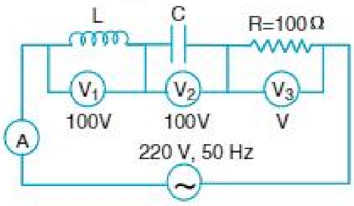
12. An alternating current of 1.5 mA rms and angular frequency ω=100rad/s flows through a 10k Ω resistor and a 0.50μF capacitor in series. Calculate the rms voltage across the capacitor and the impedance of the circuit. **[Ans:** 30 V, 2.236×10^4 Ω]

13. A circuit contains a resistance of 40 Ω and an inductance of 0.68 H, and an alternating effective e.m.f of 500 V at a frequency of 120Hz is applied to it. Find the value of the effective current in the circuit and power factor. **[Ans:** 0.99 A,1/128]

14. An alternating voltage of 100 virtual volt is applied to a circuit of resistance 0.5 Ω and inductance 0.01H, the frequency being 50 Hz .What is the current and lag in time between voltage and current?
[Ans: 31 A,1/222 sec]

15. A resistor of 100 ohm is connected in series with an inductor of 10H and a capacitor of 0.1μF All these elements are connected to a 220 volt, 50 Hz a.c. supply. Calculate the total impedance of the circuit.
[Ans: 28707.2 Ω]

16. In the circuit shown, what will be the reading of the voltmeter V_3 and ammeter A ?



[Ans: 220V, 2.2 A]

LEVEL - II

1. An a.c circuit has a choke coil L and resistance R. The potential difference across the choke is $v_L = 160V$ and that across the resistance $v_R = 120V$. Find the virtual value of the applied voltage. If the virtual current in the circuit be 1.0 A , then calculate the total impedance of the circuit. If a direct current be passed in the circuit, then what will be the potential difference in the circuit?

[Ans: $E = 200$ V, $Z=200\,\Omega$,V_R=120 V]

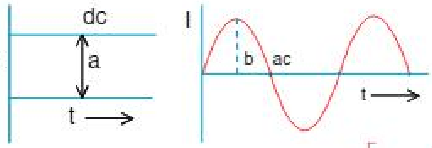
2. A $12\,\Omega$ resistance and an inductance of $0.05/\pi$ Hz with negligible resistance are connected in series. Across the ends of this circuit is connected a 130V alternating voltage of frequency 50 Hz. Calculate the alternating current in the circuit and the potential difference across the resistance and that across the inductance. [Ans: 10 A,120 V and 50 V]

3. A 1.9 H inductor, a 100μF capacitor and $25\,\Omega$ resistor are connected in series to an a.c source whose e.m.f (in volt) varies with time t (in seconds) according to the expression E = 282 sin 100t. Determine (i) the reactance, (ii) the impedance (iii) the r.m.s. value of the current and (iv) the rate of dissipation of heat

[Ans: (i) $90\,\Omega$; (ii) $93.4\,\Omega$; (iii) 2.135A and (iv) 113.95 W]

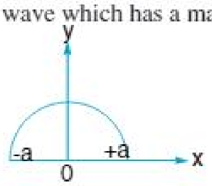
4. A resistor of $12\,\Omega$, a capacitor of reactance $14\,\Omega$ and a pure inductor of inductance 0.1Hz are joined in series and placed across a 200V, 50Hz a.c. supply. Calculate (i) The current in the circuit and (ii) The phase angle between the current and the voltage. Take $\pi = 3$. [Ans: (i) 10 A, (ii) $\tan^{-1}(4/3)$]

5. If a direct current of value a ampere is superimposed on an alternative current $I = b \sin \omega t$ flowing through a wire, what is the effective value of the resulting current in the circuit?



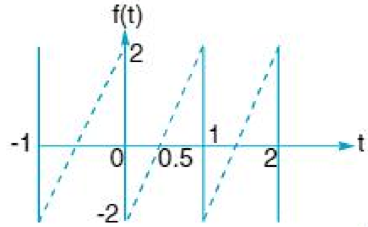
[Ans: $\left[a^2 + \frac{b^2}{2}\right]^{1/2}$]

6. Determine the rms value of a semi-circular current wave which has a maximum value of a.



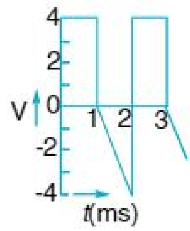
[Ans: $\sqrt{\frac{2}{3}}a$]

7. Find the rms and the average values of the saw tooth wave form shown in fig.



[Ans: $\sqrt{\frac{4}{3}}$, zero]

8. Calculate the rms and the average values of the voltage wave shown Fig.



[Ans: $\sqrt{\frac{32}{3}}$V; 1V]

9. The Voltage applied to a purely inductive coil of self inductance 15.9m H is given by the equation V = 100 sin 314t + 75sin942t + 50sin1570t. Find the equation of the resulting current.

[Ans : $20\sin\left(314t - \frac{\pi}{2}\right) + 5\sin\left(942t - \frac{\pi}{2}\right) + 2\sin\left(1570t - \frac{\pi}{2}\right)$]

10. For a sinusoidally varying alternating current, what is the ratio of the average value and rms value ?

[Ans : $\frac{2\sqrt{2}}{\pi}$]

ADDITIONAL EXERCISE

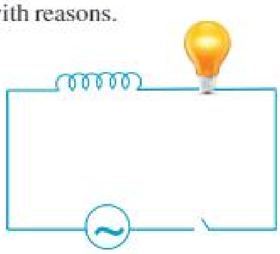
1. A light bulb is rated at 100W for a 220 V supply. Find (a) the resistance of the bulb; (b) the peak voltage of the source; and (c) the rms current through the bulb.

2. A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

3. A lamp is connected in series with a capacitor. Predict your observations for dc and ac connections. What happens in each case if the capacitance of the capacitor is reduced?

4. A 15.0 μ F capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

5. A light bulb and an open coil inductor are connected to an ac source through a key as shown in Fig. The switch is closed and after sometime, an iron rod is inserted into the interior of the inductor. The glow of the light bulb (a) increases; (b) decreases; (c) is unchanged, as the iron rod is inserted. Give your answer with reasons.



6. A resistor of $200\,\Omega$ and a capacitor of $15.0\,\mu F$ are connected in series to a 220 V, 50 Hz ac source. (a) Calculate the current in the circuit; (b) Calculate the voltage (rms) across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

7. (a) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

   (b) Power factor can often be improved by the use of a capacitor of appropriate capacitance in the circuit. Explain.

8. A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series *LCR* circuit in which $R = 3\,\Omega$, $L = 25.48$ mH, and $C = 796\,\mu F$. Find

   (a) the impedance of the circuit;

   (b) the phase difference between the voltage across the source and the current;

   (c) the power dissipated in the circuit; and

   (d) the power factor.

9. Suppose the frequency of the source in the previous example can be varied.

   (a) What is the frequency of the source at which resonance occurs?

   (b) Calculate the impedance, the current, and the power dissipated at the resonant condition.

10. At an airport, a person is made to walk through the doorway of a metal detector, for security reasons. If she/he is carrying anything made of metal, the metal detector emits a sound. On what principle does this detector work?

11. Show that in the free oscillations of an *LC* circuit, the sum of energies stored in the capacitor and the inductor is constant in time.