1. INDEFINITE INTEGRATION

SYNOPSIS

GENERAL FORMULAE

1. If $K \in R$, $\int K dx = Kx + C$
2. If $n \neq -1$, $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
3. $\int \frac{1}{x} dx = \log|x| + C$
4. $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$
5. $\int e^x .dx = e^x + C$
6. $\int a^x dx = \frac{a^x}{\log a} + C$ (for $a > 0$, $a \neq 1$)
7. $\int \sqrt{x} dx = \frac{2}{3} x\sqrt{x} + C$
8. $\int \text{Sin}x\, dx = -\text{Cos}x + C$
9. $\int \text{Cos}x\, dx = \text{Sin}x + C$
10. $\int \text{Tan}x\, dx = \log|\text{Sec}x| + C = -\log|\text{Cos}x| + C$
11. $\int \text{Cot}x\, dx = \log|\text{Sin}x| + C = -\log|\text{Cosec}x| + C$
12. $\int \text{Sec}x\, dx = \log|\text{Sec}x + \text{Tan}x| + C = \log\left|\text{Tan}\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$
13. $\int \text{Cosec}x\, dx = \log|\text{Cosec}x - \text{Cot}x| + C = \log\left|\text{Tan}\frac{x}{2}\right| + C = -\log|\text{Cosec}x + \text{Cot}x| + C$
14. $\int \text{Sec}x\, \text{Tan}x\, dx = \text{Sec}x + C$
15. $\int \text{Cosec}x\, \text{Cot}x\, dx = -\text{Cosec}x + C$
16. $\int \text{Sec}^2 x\, dx = \text{Tan}x + C$
17. $\int \text{Cosec}^2 x\, dx = -\text{Cot}x + C$
18. $\int \text{Sinh}\, x\, dx = \text{Cosh}\, x + C$
19. $\int \text{Cosh}\, x\, dx = \text{Sinh}\, x + C$
20. $\int \text{Tanh}\, x\, dx = \log|\text{Cosh}\, x| + C$
21. $\int \text{Coth}\, x\, dx = \log|\text{Sinh}\, x| + C$
22. $\int \text{Sech}\, x\, dx = 2\text{Tan}^{-1}(e^x) + C$
23. $\int \text{Cosech}\, x\, dx = \log\left|\text{Tan h}\frac{x}{2}\right| + C$

24. $\int \text{Sech}^2 x\,dx = \text{Tanh } x + C$

25. $\int \text{Cosech}^2 x\,dx = -\text{Coth } x + C$

26. $\int \text{Sech } x \tanh x\,dx = -\text{Sech } x + C$

27. $\int \text{Cosech } x \text{ Coth } x\,dx = -\text{Cosech } x + C$

28. $\int \frac{dx}{\sqrt{1-x^2}} = \text{Sin}^{-1}x + C$ (or) $-\text{Cos}^{-1}x + C$

29. $\int \frac{dx}{1+x^2} = \text{Tan}^{-1}x + C$ (or) $-\text{Cot}^{-1}x + C$

30. $\int \frac{dx}{x\sqrt{x^2-1}} = \text{Sec}^{-1}x + C$ (or) $-\text{Cosec}^{-1}x + C = -\text{Sec}^{-1}x + C$ (or) $\text{Cosec}^{-1}x + C$. If $x < -1$

31. $\int \frac{1}{\sqrt{1+x^2}}dx = \text{Sinh}^{-1}x + C$

32. $\int \frac{1}{\sqrt{x^2-1}}dx = \text{Cosh}^{-1}x + C$

Important Results :

1. $\int f^1(ax+b)dx = \frac{f(ax+b)}{a} + C$

2. $\int \frac{f^1(x)}{f(x)}dx = \log|f(x)| + C$

3. $\int \frac{f^1(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$

4. If $\int f(x)dx = F(x)$ and $g(x)$ is a differentiable function then $\int (f0g)(x)g^I(x)dx = F\left[g(x)\right] + c$

5. $\int \{f(x)\}^n . f^1(x)dx = \frac{\{f(x)\}^{n+1}}{n+1} + C\ (n \neq -1)$

6. $\int f^1\{g(x)\}.g^1(x)\,dx = f\{g(x)\} + C$

7. $\frac{d}{dx}\left(\int f(x)\,dx\right) = f(x)$

8. $\int \left[\frac{d}{dx}\ f(x)\right]dx = f(x) + C$

Standard Results :

1. $\int \frac{1}{\sqrt{a^2-x^2}}dx = \text{Sin}^{-1}\left(\frac{x}{a}\right) + c$

2. $\int \frac{1}{\sqrt{x^2-a^2}}dx = \text{Cosh}^{-1}\left(\frac{x}{a}\right) + c$ (or) $= \log\left|x + \sqrt{x^2-a^2}\right| + c$

3. $\int \frac{1}{\sqrt{x^2+a^2}}dx = \text{Sinh}^{-1}\left(\frac{x}{a}\right) + c$ (or) $= \log\left|x + \sqrt{x^2+a^2}\right| + c$

4. $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\text{Tan}^{-1}\left(\frac{x}{a}\right)+c$

5. $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

6. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$

7. $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\text{Sin}^{-1}\left(\frac{x}{a}\right)+c$

8. $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\text{Cosh}^{-1}\left(\frac{x}{a}\right)+c = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$

9. $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\text{Sinh}^{-1}\left(\frac{x}{a}\right)+c = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c$

Points to remember

1. $\int \frac{f^1(x)}{\sqrt{a^2-[f(x)]^2}}dx = \text{Sin}^{-1}\left[\frac{f(x)}{a}\right]+c$

2. $\int \frac{f^1(x)}{\sqrt{[f(x)]^2-a^2}}dx = \text{Cosh}^{-1}\left[\frac{f(x)}{a}\right]+c = \log\left|f(x)+\sqrt{[f(x)]^2-a^2}\right|+c$

3. $\int \frac{f^1(x)}{\sqrt{[f(x)]^2+a^2}}dx = \text{Sinh}^{-1}\left[\frac{f(x)}{a}\right]+c = \log\left|f(x)+\sqrt{[f(x)]^2+a^2}\right|+c$

Integration of Partial Fractions :

1. $\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}\log\left|\frac{x+a}{x+b}\right|+c$

2. $\int \frac{1}{(ax+b)(cx+d)}dx = \frac{1}{ad-bc}\log\left|\frac{ax+b}{cx+d}\right|+K$

3. $\int \frac{1}{(x^2+a^2)(x^2+b^2)}dx = \frac{1}{b^2-a^2}\left[\frac{1}{a}\tan^{-1}\frac{x}{a}-\frac{1}{b}\tan^{-1}\frac{x}{b}\right]+c$

4. $\int \frac{x}{(x^2+a^2)(x^2+b^2)}dx = \frac{1}{2(b^2-a^2)}\log\left|\frac{x^2+b^2}{x^2+a^2}\right|+c$

5. $\int \frac{1}{x(x^n+1)}dx = \frac{1}{n}\log\left|\frac{x^n}{1+x^n}\right|+c$

6. $\int \frac{1}{x(x^n-1)}dx = \frac{1}{n}\log\left|\frac{x^n-1}{x^n}\right|+c$

7. $\int \frac{1}{x(1-x^n)}dx = \frac{1}{n}\log\left|\frac{x^n}{1-x^n}\right|+c$

Special Types of Integration :

1. If the given integral is of the form $\int\frac{px+q}{ax^2+bx+c}dx$, $\int\frac{px+q}{\sqrt{ax^2+bx+c}}dx$, $\int(px+q)\sqrt{ax^2+bx+c}\,dx$ then put $px+q = A\frac{d}{dx}(ax^2+bx+c)+B$, and $A=\frac{p}{2a}$, $B=q-bA$

2. If the given integral is of the form $\int\frac{1}{(px+q)\sqrt{ax^2+bx+c}}dx$ then put $px+q=\frac{1}{t}$.

3. If the given integral is of the form $\int\frac{1}{(ax^2+b)\sqrt{cx^2+d}}dx$ then put $x=\frac{1}{t}$.

4. If the given integral is of the form $\int\frac{px+q}{\sqrt{ax+b}}dx$ (or) $\int\frac{\sqrt{ax+b}}{px+q}dx$ (or) $\int(px+q)\sqrt{ax+b}\,dx$ (or) $\int\frac{1}{(px+q)\sqrt{ax+b}}dx$ then put $ax + b = t^2$

5. If the integral is of the form $\int\frac{dx}{a^2\cos^2x+b^2\sin^2x}$ (or) $\int\frac{dx}{a\cos^2x+b\sin x\cos x+c\sin^2x}$ then multiply both numerator and denominator with $\sec^2x$ and put $\tan x = t$.

6. $\int\frac{dx}{a^2\sin^2x+b^2\cos^2x}=\frac{1}{ab}\tan^{-1}\left(\frac{a\sin x}{b\cos x}\right)+c$

7. If the integral is of the form $\int\frac{a\text{Cos}x+b\text{Sin}x+c}{d\text{Cos}x+e\text{Sin}x+f}dx$ then take

   $a\text{Cos}x+b\text{Sin}x+c = A\frac{d}{dx}(d\text{Cos}x+e\text{Sin}x+f)+B(d\text{Cos}x+e\text{Sin}x+f)+C$

8. $\int\frac{a\text{Cos}x+b\text{Sin}x}{c\text{Cos}x+d\text{Sin}x}dx=\frac{ac+bd}{c^2+d^2}.x+\frac{ad-bc}{c^2+d^2}\log|c\cos x+d\sin x|+K$

9. If the integral is of the form $\int\frac{dx}{a+b\cos x}$ (or) $\int\frac{dx}{a+b\sin x}$ (or) $\int\frac{dx}{a\cos x+b\sin x+c}$ put $\tan\frac{x}{2}=t$.

   Then $dx=\frac{2dt}{1+t^2}$. $\sin x=\frac{2t}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$.

Integration by parts :

1. If u and v are function of x, then $\int udv = uv-\int vdu$.

2. $\int f(x).g(x)\,dx = f(x)\int g(x)dx-\int\left[f^1(x)\int g(x)dx\right].dx$ where $f(x)$ = First function, $g(x)$ = Second function.

3. **Proper choice of first and second function :**

   a) The first function is the function which comes first in the word ILATE.

   b) If one of the two functions is not directly integrable, then take this function as the first function.

   c) If one of the function is not directly integrable, and there is no other function, then unity is taken as the second function.

Point to remember

If u and v are easily derivable, integrable functions of x, [u, u^1, u^{II}, u^{III},... and v, v_1, v_2, ...] then

$\int uv\,dx = uv_1 - u^1v_2 + u^{11}v_3 - u^{111}v_4 + \ldots$

4. $\int x^n.\log x dx = \frac{x^{n+1}}{n+1}\left[\log x - \frac{1}{n+1}\right] + c$ (for $n \neq -1$)

5. $\int e^x\left[f(x) + f'(x)\right]dx = e^x f(x) + c$

6. $\int e^{-x}\left[f(x) - f^I(x)\right]dx = -e^{-x}f(x) + c$

7. $\int e^{ax}\left[f(x) + \frac{f'(x)}{a}\right]dx = \frac{e^{ax}f(x)}{a} + c$

8. $\int\left[xf'(x) + f(x)\right]dx = x\,f(x) + c$

9. $\int e^{ax}\text{Sin}(bx+c)dx = \frac{e^{ax}}{a^2+b^2}[a\text{Sin}(bx+c) - b\text{Cos}(bx+c)] + K$

10. $\int e^{ax}\text{Cos}(bx+c)dx = \frac{e^{ax}}{a^2+b^2}[a\text{Cos}(bx+c) + b\text{Sin}(bx+c)] + K$

11. $\int a^x.\text{Sin}(bx+c)dx = \frac{a^x}{(\log a)^2+b^2}[(\log a).\text{Sin}(bx+c) - b\text{Cos}(bx+c)] + K$

12. $\int a^x\text{Cos}(bx+c)dx = \frac{a^x}{(\log a)^2+b^2}[(\log a).\text{Cos}(bx+c) + b\text{Sin}(bx+c)] + K$

Integral of Inverse Trigonometic functions :

1. $\int \text{Sin}^{-1}x.dx = x\text{Sin}^{-1}x + \sqrt{1-x^2} + c$

2. $\int \text{Cos}^{-1}x.dx = x\text{Cos}^{-1}x - \sqrt{1-x^2} + c$

3. $\int \text{Tan}^{-1}x.dx = x\text{Tan}^{-1}x - \frac{1}{2}\log(1+x^2) + c$

4. $\int \text{Cot}^{-1}x.dx = x\text{Cot}^{-1}x + \frac{1}{2}\log(1+x^2) + c$

5. $\int \text{Sec}^{-1}x.dx = x\text{Sec}^{-1}x - \text{Cosh}^{-1}x + c$

6. $\int \text{Cosec}^{-1}x.dx = x\text{Cosec}^{-1}x + \text{Cosh}^{-1}x + c$

7. $\int \text{Sinh}^{-1}x.dx = x\text{Sinh}^{-1}x - \sqrt{x^2+1} + c$

8. $\int \text{Cosh}^{-1}x.dx = x\text{Cosh}^{-1}x - \sqrt{x^2-1} + c$

9. $\int \text{Sinh}^{-1}\left(\frac{x}{a}\right).dx = x\text{Sinh}^{-1}\left(\frac{x}{a}\right) - \sqrt{x^2+a^2} + c$

10. $\int \text{Cosh}^{-1}\left(\frac{x}{a}\right).dx = x\text{Cosh}^{-1}\left(\frac{x}{a}\right) - \sqrt{x^2-a^2} + c$

Reduction Formulae :

1. For $n \in N$, if $I_n = \int \text{Sin}^n x\,dx$ then $I_n = \frac{-\text{Sin}^{n-1}x\text{Cos}x}{n} + \frac{n-1}{n} I_{n-2}$

2. For $n \in N$, if $I_n = \int \text{Cos}^n x\,dx$ then $I_n = \frac{\text{Cos}^{n-1}x\text{Sin}x}{n} + \frac{n-1}{n} I_{n-2}$

3. For $n \in N$, if $I_n = \int \text{Tan}^n x\,dx$ then $I_n = \frac{\text{Tan}^{n-1}x}{n-1} - I_{n-2}$

4. For $n \in N$, if $I_n = \int \text{Cot}^n x\,dx$ then $I_n = -\frac{\text{Cot}^{n-1}x}{n-1} - I_{n-2}$

5. For $n \in N$, if $I_n = \int \text{Sec}^n x\,dx$ then $I_n = \frac{\text{Sec}^{n-2}x\text{Tan}x}{n-1} + \frac{n-2}{n-1}.I_{n-2}$

6. For $n \in N$, if $I_n = \int \text{Cosec}^n x\,dx$ then $I_n = \frac{-\text{Cosec}^{n-2}x\text{Cot}x}{n-1} + \frac{n-2}{n-1}.I_{n-2}$

7. For $n \in N$, if $I_{m,n} = \int \text{Sin}^m x\text{Cos}^n x\,dx$ then $I_{m,n} = \frac{\text{Sin}^{m+1}x\text{Cos}^{n-1}x}{m+n} + \frac{n-1}{m+n}.I_{m,n-2}$

$= -\frac{\text{Sin}^{m-1}x\text{Cos}^{n+1}x}{m+n} + \frac{m-1}{m+n}.I_{m-2,n}$

8. For $n \in N$, if $I_n = \int x^n e^{ax} dx$ then $I_n = \frac{e^{ax}}{a}.x^n - \frac{n}{a} I_{n-1}$

9. For $n \in N$, if $I_n = \int (\log x)^n dx$ then $I_n = x(\log x)^n - n.I_{n-1}$

10. If $I_{m,n} = \int x^m (\log x)^n dx$ then $I_{m,n} = \frac{x^{m+1}}{m+1}(\log x)^n - \frac{n}{m+1} I_{m,n-1}$

LECTURE SHEET

EXERCISE-I

(Formula based Problems, Integration by Substitution & Integration by using partial Fractions)

LEVEL-I (MAIN)

Single answer type questions

1. $\int \frac{x^4+1}{x^6+1}\,dx =$

1) $\tan^{-1}x - \tan^{-1}x^3 + c$ 2) $\tan^{-1}x - \frac{1}{3}\tan^{-1}(x^3) + c$

3) $\tan^{-1}x + \tan^{-1}(x^3) + c$ 4) $\tan^{-1}x + \frac{1}{3}\tan^{-1}(x^3) + c$

2. If $x \in \left(\pi, \frac{3\pi}{2}\right)$ then $\int \left(\sqrt{1+\sin x} - \sqrt{1-\sin x}\right) dx =$

1) $4\cos\frac{x}{2} + c$ 2) $-4\cos\frac{x}{2} + c$ 3) $4\sin\frac{x}{2} + c$ 4) $-4\sin\frac{x}{2} + c$

3. If $0 < x < \frac{\pi}{2}$ then $\int [1 + 2\cot x(\cot x + \operatorname{cosec} x)]^{1/2}\, dx =$

1) $\sqrt{\cot x + \operatorname{cosec} x} + C$ 2) $2 \log |\sin x/2| + C$ 3) $\log |\operatorname{cosec} x + \cot x| + C$ 4) $2 \log |\cos x/2| + C$

4. $\int \left(\sum_{r=0}^{\infty} \frac{x^r 3^r}{r!} \right) dx =$

1) $e^x + c$ 2) $\frac{e^{3x}}{3} + c$ 3) $\frac{-3}{1-3x} + c$ 4) $3e^{3x} + c$

5. $\int \frac{\sec^2 x}{\log(\tan x)^{\tan x}} dx =$

1) $\log|\tan x| + C$ 2) $\log|\log(\tan x)| + C$ 3) $\frac{1}{\log(\tan x)} + C$ 4) $\frac{\tan x}{\log|\tan x|} + C$

6. If $\int \frac{e^x - 1}{e^x + 1} dx = f(x) + c$ then $f(x) =$

1) $2 \log(e^x + 1)$ 2) $\log(e^{2x} - 1)$ 3) $2 \log(e^x + 1) - x$ 4) $\log(e^{2x} + 1)$

7. If $\ell^r(x)$ means log log log----(x), the 'log' being repeated 'r' times, then

$\int \frac{1}{x\, \ell(x)\, \ell^2(x)\, \ell^3(x) - - - \ell^r(x)} dx =$

1) $\ell^r(x) + c$ 2) $\ell^{r+1}(x) + c$ 3) $\frac{\ell^{r+1}(x)}{r+1}$ 4) $\frac{\ell^r(x)}{r} + c$

8. If $\int (\sin 2x - \cos 2x) dx = \frac{1}{\sqrt{2}} \sin(2x - a) + b$ then

1) $a = \frac{\pi}{4}, b \in R$ 2) $a = \frac{-\pi}{4}, b \in R$ 3) $a = \frac{5\pi}{4}, b \in R$ 4) $a = \frac{-5\pi}{4}, b \in R$

9. $\int \cos x \cos 2x \cos 3x\, dx =$

1) $\frac{1}{4}\left[\frac{1}{6}\sin 6x + \frac{1}{2}\sin 2x - \frac{1}{4}\sin 4x + x\right] + c$ 2) $\frac{1}{4}\left[\frac{1}{6}\sin 6x - \frac{1}{2}\sin 2x - \frac{1}{4}\sin 4x - x\right] + c$

3) $\frac{1}{4}\left[\frac{1}{6}\sin 6x + \frac{1}{2}\sin 2x + \frac{1}{4}\sin 4x + x\right] + c$ 4) $\frac{1}{6}\sin 6x + c$

10. If $\int \frac{\sin(x - a)}{\cos(x - b)} dx = A.\log|\sec(x - b)| + Bx + C$ then $B =$

1) $\cos(b - a)$ 2) $\sin(b - a)$ 3) $\cos(b + a)$ 4) $\sin(b + a)$

11. $\int \sqrt{x - 3} \left\{ \sin^{-1}(\log x) + \cos^{-1}(\log x) \right\} dx =$

1) 0 2) does not exist 3) 1 4) –1

12. $\int x^x (1 + x + x \log x) dx =$

1) $x^{x+1} + c$ 2) $x^{x+2} + c$ 3) $x^x + x + c$ 4) $x^{x+1} + x + c$

13. $\int\sqrt{\frac{\cos x-\cos^3 x}{1-\cos^3 x}}\,dx=$

1) $\frac{-2}{3}\sin^{-1}[(\cos x)^{3/2}]+c$ 2) $\frac{2}{3}\sin^{-1}[(\cos x)^{3/2}]+c$ 3) $-\frac{1}{3}\sin^{-1}[(\cos x)^{3/2}]+c$ 4) $\frac{1}{3}\sin^{-1}[(\sin x)^{3/2}]+c$

14. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx=$

1) $\sqrt{2}\text{Sin}^{-1}\left(\sin x+\cos x\right)+c$ 2) $\sqrt{2}\text{Cos}^{-1}\left(\sin x+\cos x\right)+c$

3) $\sqrt{2}\text{Cos}^{-1}\left(\sin x-\cos x\right)+c$ 4) $\sqrt{2}\text{Sin}^{-1}\left(\sin x-\cos x\right)+c$

15. $\int\sin^{5/2}x\cos^3 x\,dx=$

1) $2\sin^{7/2}x\left(\frac{1}{7}+\frac{1}{11}\sin^2 x\right)+c$ 2) $2\sin^{7/2}x\left(\frac{1}{7}-\frac{1}{11}\sin^2 x\right)+c$

3) $3\sin^{7/2}x\left(\frac{1}{5}-\frac{1}{9}\sin^2 x\right)+c$ 4) $3\sin^{7/2}x\left(\frac{1}{7}-\frac{1}{11}\sin^2 x\right)+c$

16. $\int\frac{dx}{\cos x\sqrt{\cos 2x}}=$

1) $\cos^{-1}(\tan x)+C$ 2) $\cos(\tan^{-1}x)+C$ 3) $\sin^{-1}(\tan x)+C$ 4) $\tan(\sin^{-1}x)+C$

17. If $x\in(0,1)$ then $\int\cos\left\{2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx=$

1) $\cot^{-1}x+c$ 2) $-\cot^{-1}x+c$ 3) $\frac{x^2}{2}+c$ 4) $\frac{-x^2}{2}+c$

18. If $\int\frac{1}{x\sqrt{1-x^3}}dx=a\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|+b$ then $a=$

1) $\frac{2}{3}$ 2) $\frac{1}{3}$ 3) $-\frac{2}{3}$ 4) $-\frac{1}{3}$

19. $\int\frac{(x-x^3)^{1/3}}{x^4}dx=$

1) $\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+C$ 2) $\frac{-3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+C$ 3) $\frac{3}{8}\left(1+\frac{1}{x^2}\right)^{4/3}+C$ 4) $\frac{-3}{8}\left(1+\frac{1}{x^2}\right)^{4/3}+C$

20. $\int\frac{1}{x^4\sqrt[4]{x^4+1}}dx=$

1) $\frac{1}{3}\left(1+\frac{1}{x^4}\right)^{3/4}+C$ 2) $\frac{-1}{3}\left(1+\frac{1}{x^4}\right)^{3/4}+C$ 3) $\frac{2}{3}\left(1+\frac{1}{x^4}\right)^{3/4}+C$ 4) $\frac{-2}{3}\left(1+\frac{1}{x^4}\right)^{3/4}+C$

21. $\int\frac{(x+\sqrt{a^2+x^2})^n}{\sqrt{a^2+x^2}}dx\ (n\neq 0)=$

1) $\frac{1}{n+1}(x+\sqrt{a^2+x^2})^{n+1}+C$ 2) $\frac{1}{n(x+\sqrt{a^2+x^2})}+C$

3) $(x+\sqrt{a^2+x^2})^n+C$ 4) $\frac{1}{n}(x+\sqrt{a^2+x^2})^n+C$

22. $\int \frac{1}{(2x+1)^{5/6}(3x+5)^{7/6}}dx =$

1) $\frac{6}{7}\left(\frac{2x+1}{3x+5}\right)^{1/6}+C$ 2) $-\frac{6}{7}\left(\frac{2x+1}{3x+5}\right)^{1/6}+C$ 3) $\frac{6}{7}\left(\frac{3x+5}{2x+1}\right)^{1/6}+C$ 4) $-\frac{6}{7}\left(\frac{3x+5}{2x+1}\right)^{1/6}+C$

23. If $\alpha>1$, then $\int \frac{dx}{x^2+2\alpha x+1} =$

1) $\frac{1}{\sqrt{1-\alpha^2}}\tan^{-1}\left(\frac{x+\alpha}{\sqrt{1-\alpha^2}}\right)+c$ 2) $\frac{1}{\sqrt{1-\alpha^2}}\cot^{-1}\left(\frac{x+\alpha}{\sqrt{1-\alpha^2}}\right)+c$

3) $\frac{1}{2\sqrt{\alpha^2-1}}\log\left(\frac{x+\alpha-\sqrt{\alpha^2-1}}{x+\alpha+\sqrt{\alpha^2-1}}\right)+c$ 4) $\frac{1}{2\sqrt{\alpha^2-1}}\log\left(\frac{x+\alpha+\sqrt{\alpha^2-1}}{x+\alpha-\sqrt{\alpha^2-1}}\right)+c$

24. $\int \frac{e^{3x}+e^x}{e^{4x}-e^{2x}+1}dx =$

1) $\frac{1}{4}\log(e^{4x}-e^{2x}+1)+C$ 2) $\tan^{-1}(e^x-e^{-x})+C$

3) $\tan^{-1}(e^x+e^{-x})+C$ 4) $\tan^{-1}(e^{-x}-e^x)+C$

25. If $\int \frac{1}{x(x^5-1)(x^5+1)}dx = A\log x + B\log(x^5-1)+C\log(x^5+1)$ then$(A,B,C)=$

1) $\left(-1,\frac{1}{10},\frac{1}{10}\right)$ 2) $\left(-\frac{5}{4},\frac{1}{8},-\frac{1}{8}\right)$ 3) $\left(-\frac{5}{4},-\frac{1}{8},\frac{1}{8}\right)$ 4) $\left(1,\frac{1}{10},-\frac{1}{10}\right)$

26. If $\int \frac{1-x^7}{x(1+x^7)}dx = a\ln|x|+b\ln|x^7+1|+C$ then$(a,\ b) =$

1) $\left(1,\frac{2}{7}\right)$ 2) $\left(1,-\frac{7}{2}\right)$ 3) $\left(1,-\frac{2}{7}\right)$ 4) $\left(2,-\frac{2}{7}\right)$

27. If $\int \frac{1}{a^4-x^4}dx = A\tan^{-1}\left(\frac{x}{a}\right)+B\log\left|\frac{a+x}{a-x}\right|+C$ then $(A,B) =$

1) $\left(-\frac{1}{2a^3},\frac{1}{4a^3}\right)$ 2) $\left(\frac{1}{2a^3},\frac{1}{4a^3}\right)$ 3) $\left(\frac{1}{2a^3},\frac{1}{4a}\right)$ 4) $\left(\frac{1}{a},\frac{1}{2a}\right)$

28. Is the statement $\frac{d}{dx}\left[\int f(x)dx\right]=\int\left[\frac{d}{dx}f(x)\right]dx$ is true ?

1) yes 2) no 3) cannot be determined 4) none

29. The anti derivative of $f(x) = 1 + 2^x \log 2$ is $g(x)$ and the curve $y = g(x)$ passes through $\left(-1,\frac{1}{2}\right)$ then the curve $y = g(x)$ meets the y-axis at

1) (0, –2) 2) (0, –1) 3) (0, 2) 4) (1, 0)

30. $\int \frac{\log\left(1+\frac{1}{x}\right)}{x(1+x)}dx =$

1) $-\frac{1}{2}[\log(x+1)]^2 - \frac{1}{2}[\log x]^2 + \log x.\log(x+1) + C$ 2) $\frac{1}{2}[\log(x+1)]^2 + \frac{1}{2}[\log x]^2 - \log x.\log(x+1) + C$

3) $-\frac{1}{2}[\log(x+1)]^2 - \frac{1}{2}[\log x]^2 + \log[x(x+1)] + C$ 4) $\frac{1}{2}[\log(x+1)]^2 + \frac{1}{2}[\log x]^2 - \log[x(x+1)] + C$

31. $\int\left(\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x\right)\log_e^x dx =$

1) $\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x + c$ 2) $\left(\frac{x}{e}\right)^x - \left(\frac{e}{x}\right)^x + c$ 3) $-\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x + c$ 4) $\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x + \log x$

Numerical value type Questions

32. $\int \frac{\sec^2 x}{(\sec x+\tan x)^5}dx = \frac{-1}{4(\sec x+\tan x)^4}\left[A + \frac{1}{3(\sec x+\tan x)^2}\right] + C$ then A= .....

33. $\int \frac{\cos x - \sin x}{\sqrt{8-\sin 2x}}dx =$ Sin^{-1} (A(sinx + cosx)) +c then A = ...

34. If $\int f(x)dx = f(x)$, and $\int\{f(x)\}^2 dx = t\{f(x)\}^2$ then t = ...

LEVEL-II (ADVANCED)

Single answer type questions

1. If $\frac{1}{x^4+1}dx = A\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) - B\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right| + C$ then

a) $A = 2B$ b) $A = -2B$ c) $2A = B$ d) $2A = -B$

2. $\int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx =$

a) $\frac{x^7}{2x^7+x^2+1}+C$ b) $\frac{x^5}{x^2+1+2x^2}+C$

c) $\frac{-1}{2x^7+x^2+1}+C$ d) $\frac{p(x)}{q(x)}, \deg p(x) = \deg q(x) = 7$

3. $\int(x - 11C_1x^2 + 11C_2x^3 - 11C_3x^4 + ...11C_{11}x^{12})dx =$

a) $\frac{(1-x)^{13}}{13} - \frac{(1-x)^{12}}{12} + C$ b) $\frac{(1-x)^{13}}{13} + \frac{(1-x)^{12}}{12} + C$

c) $\frac{(1-x)^{13}}{13} - \frac{(1+x)^{12}}{14} + C$ d) $\frac{(1+x)^{13}}{13} - \frac{(1-x)^{12}}{12} + C$

4. A curve $g(x)=\int x^{27}(1+x+x^2)^6(6x^2+5x+4)dx$ is passing through the origin, then

a) $g(-1)=\frac{1}{8}$ b) $g(-1)=\frac{1}{7}$ c) $g(1)=\frac{1}{7}$ d) $g(-1)=\frac{2}{7}$

5. If $0<x<1$ then $\int\sqrt{\underset{n\to\infty}{Lt}(1+x^2)(1+x^4)(1+x^8)....(1+x^{2^n})}\,dx=$

a) $\frac{1}{1-x^2}+c$ b) $\frac{1}{\sqrt{1-x^2}}+c$ c) $\sin^{-1}x+c$ d) $\sin^{-1}(x^2)+c$

6. If $x f(x)=3f^2(x)+2$ then $\int\frac{2x^2-12xf(x)+f(x)}{(6f(x)-x)(x^2-f(x))^2}dx=$

a) $\frac{1}{x^2-f(x)}+c$ b) $\frac{1}{x^2+f(x)}+c$ c) $\frac{1}{2x^2-f(x)}+c$ d) $\frac{2}{x^2-f(x)}+c$

7. $\int\tan x\tan 2x\,dx=$

a) $\frac{1}{2}\log\tan\left(\frac{\pi}{4}+x\right)-x+C$ b) $\log\tan\left(\frac{\pi}{4}+x\right)-x+C$

c) $\log\tan\left(\frac{\pi}{4}-x\right)+x+C$ d) $\frac{1}{2}\log\tan\left(\frac{\pi}{4}+x\right)+x+C$

8. Let $f(x)$ be a polynomial of degree three satisfying $f(0)=-1$ and $f(1)=0$ also, 0 is a stationary point of $f(x)$ does not have an extremum at $x=0$, then the valueof the integral $\int\frac{f(x)}{x^3-1}dx$ is

a) $\frac{x^2}{2}+c$ b) $x+c$ c) $\frac{x^3}{b}+c$ d) $-\frac{x^2}{2}+c$

9. $\int\frac{1}{x}\left\{\log_{ex}e.\log_{e^2x}e.\log_{e^3x}e\right\}dx=$

a) $\frac{1}{2}\log\left[\log_e ex\right]-\log\left[\log_e(e^2x)\right]+\frac{1}{2}\log\left[\log_e e^3x\right]+C$

b) $\frac{1}{2}\log\left[\log_e ex\right]-\log\left[\log_e(e^2x)\right]+\frac{2}{3}\log\left[\log_e e^3x\right]+C$

c) $\frac{1}{2}\log\left[\log_e ex\right]-\log\left[\log_e(e^2x)\right]+\frac{4}{3}\log\left[\log_e e^3x\right]+C$

d) $\frac{1}{2}\log\left[\log_e ex\right]-\log\left[\log_e(e^2x)\right]+\frac{1}{3}\log\left[\log_e e^3x\right]+C$

10. $\int\sqrt{\frac{e^x-1}{e^x+1}}dx=$

a) $\log\left|e^x+\sqrt{e^{2x}-3}\right|-\sec^{-1}(e^x)+c$ b) $\log\left|e^x+\sqrt{e^{2x}-1}\right|-\sec^{-1}(e^x)+c$

c) $\log\left|e^x+\sqrt{e^{2x}+2}\right|-\sec^{-1}(e^x)+c$ d) $\log\left|e^x+\sqrt{e^{2x}-1}\right|-\operatorname{cosec}^{-1}(e^x)+c$

11. If m is a non zero number and $\int \frac{\left(x^{5m-1}+2x^{4m-1}\right)}{\left(x^{2m}+x^{m}+1\right)^{3}}dx = f(x) + c$ then $f(x) =$

a) $\frac{x^{5m}}{2m(x^{2m}+x^{m}+1)^2}$ b) $\frac{x^{4m}}{2m(x^{2m}+x^{m}+1)^2}$ c) $\frac{x^{2m}(x^{5m}+x^{4m})}{(x^{2m}+x^{m}+1)^2}$ d) $\frac{x^{5m}-x^{4m})}{2m(x^{2m}+x^{m}+1)^2}$

12. For any natural number m, evaluate, $\int (x^{3m}+x^{2m}+x^{m})(2x^{m}+3x^{m}+6)^{\frac{1}{m}}\,dx : x > 0$

a) $\frac{1}{6}\frac{(2x^{3m}+3x^{2m}+6x^{m})^{\frac{m+1}{m}}}{m+1}+c$ b) $\frac{1}{6}\frac{(2x^{3m}+x^{2m}+6x^{m})^{\frac{m+1}{m}}}{m+1}+c$

c) $\frac{1}{6}\left(\frac{(x^{3m}+x^{2m}+x^{m})^{\frac{m+1}{m}}}{m+1}\right)+c$ d) $\frac{1}{6}\left(\frac{(2x^{3m}+3x^{2m}+6x^{m})^{\frac{m}{m+1}}}{m+1}\right)+c$

More than one correct answer type questions

13. If the primitive of $\sin(\ln x)$ is $f(x)\{\sin g(x)-\cos K(x)\}+C$ then

a) $\lim_{x\to 2} f(x)=1$ b) $\lim_{x\to 2}\frac{g(x)}{K(x)}=1$ c) $g(e^3) = 3$ d) $g(e) = 2$

14. If $\int\frac{x^2-8x+7}{(x^2-3x-10)^2}dx = P\log|x-5|+Q.\frac{1}{x-5}-R.\log|x+2|-s.\frac{1}{x+2}+c.$ Then

a) $P=-\frac{45}{98}$ b) $Q=\frac{8}{49}$ c) $R=-\frac{15}{49}$ d) $S=\frac{27}{49}$

15. If $f\left(\frac{3x-4}{3x+4}\right)=x+2$ and $\int f(x)dx = Ax + B\log|x-1|+C$ then

a) $A=\frac{2}{3}$ b) $A=\frac{4}{3}$ c) $B=\frac{8}{3}$ d) $B=-\frac{8}{3}$

Linked comprehension type questions

Passage - I :

Using the properties of differentation and Integration simplify the following

16. Let $g(x)$ be an antiderivative for $f(x)$. Then $\log(1+(g(x)^2)$ is an antiderivative for

a) $\frac{f(x)g(x)}{1+g^2(x)}$ b) $\frac{-f(x)g(x)}{1+g^2(x)}$ c) $\frac{-2f(x)g(x)}{1+g^2(x)}$ d) $\frac{2f(x)g(x)}{1+g^2(x)}$

17. A function f satisfying $f^1(\sin x) = \cos^2 x,\ \forall x$ and $f(1) = 1$ is

a) x b) x^2 c) $f(x) = x-\frac{x^3}{3}+\frac{1}{3}$ d) $2 - x^2$

18. If $x = f^{11}(t)\cos t + f^{1}(t)\sin t;\ y = -f^{11}(t)\sin t + f^{1}(t)\cos t$ then $\int\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]^{1/2}dt =$

a) $f(t)+f^{11}(t)$ b) $f(t)+f^{1}(t)$ c) $f(t)-f^{1}(t)$ d) $f^{1}(t)+f^{11}(t)$

Matrix matching type questions

19. **COLUMN -I** **COLUMN -II**

A) $\int \frac{5x^4+4x^5}{(x^5+x+1)^2}dx =$ p) $\frac{2}{\alpha-\beta}\sqrt{\frac{x-\alpha}{\beta-x}}+c$

B) $\int \frac{x^2(x\sec^2 x+\tan x)dx}{(x\tan x+1)^2} =$ q) $\sqrt{2}\log\left|\operatorname{cosec}\left(\frac{x}{2}+\frac{\pi}{4}\right)-\cot\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+c$

C) $\int \frac{dx}{(x-\beta)\sqrt{(x-\alpha)(\beta-x)}} =$ r) $\frac{-x^2}{x\tan x+1}+2\log|x\sin x+\cos x|+c$ *

D) $\int \frac{dx}{\sqrt{1+\sin x}} =$ s) $\frac{x^5}{x^5+x+1}+C$

Integer answer type questions

20. If $\int\left(\frac{x^3+x}{x^4-9}\right)dx = A\log|x^4-9|+B\log\left|\frac{x^2-3}{x^2+3}\right|+c$ then $A/B =$

21. If $\int\frac{\sqrt{x^2+1}}{x^4}dx = -\frac{1}{A}\frac{(x^2+1)^{3/2}}{x^3}+c$ then $A =$

22. If $f(x) = \underset{n\to\infty}{Lim}\left[2x+4x^3+.....+2nx^{2n-1}\right]$ $(0<x<1)$ and $\int f(x)dx = \frac{p}{Q+Rx^2}+C$ then $P+Q-R =$

EXERCISE-II

Special Types of integration, Rational functions of sinx and cosx

LEVEL-I (MAIN)

Single answer type questions

1. $\int\sqrt{\frac{x-1}{x-2}}dx =$

1) $\sqrt{x^2-3x+2}+\frac{1}{2}\cosh^{-1}(2x-3)+C$ 2) $\sqrt{x^2-3x+2}-\frac{1}{2}\cosh^{-1}(2x-3)+C$

3) $\sqrt{x^2-3x+2}+\frac{1}{2}\cosh^{-1}(2x+3)+C$ 4) $\sqrt{x^2-3x+2}-\frac{1}{2}\cosh^{-1}(2x+3)+C$

2. $\int\frac{1}{(2x+1)\sqrt{x^2-x-2}}dx =$

1) $-\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{4x+7}{6x+3}\right)+c$ 2) $-\frac{1}{\sqrt{5}}\sin^{-1}\left(\frac{4x-7}{6x-3}\right)+c$

3) $-\frac{1}{\sqrt{5}}\cos^{-1}\left(\frac{4x+7}{6x+3}\right)+c$ 4) $-\frac{1}{\sqrt{5}}\sin^{-1}\left(\frac{4x+7}{6x+3}\right)+c$

3. $\int \frac{x^2-1}{x^4+1}dx =$

1) $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+C$
2) $\frac{1}{\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+C$
3) $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right|+C$
4) $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x-1}{x^2-\sqrt{2}x+1}\right|+C$

4. If $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = Ax + B\log(9e^x-4e^{-x})+C$ then

1) $A=\frac{-19}{36}, B=\frac{35}{36}, C=0$
2) $A=\frac{35}{36}, B=\frac{-19}{36}, C=0$
3) $A=\frac{-19}{36}, B=\frac{35}{36}, C\in R$
4) $A=\frac{35}{36}, B=\frac{-19}{36}, C\in R$

5. $\int\left(\frac{x^2-x}{x^3-x^2+x-1}\right)dx$

1) $\frac{1}{2}\log(x^2+1)+c$
2) $\frac{1}{2}\log\left|x^2-1\right|+c$
3) $\log(x^2+1)+c$
4) $\log\left|x^2-1\right|+c$

6. If $\int \frac{5\tan x}{\tan x-2}dx = x + a\log\left|\sin x-2\cos x\right|+k$ then a = ?

1) -1
2) -2
3) 1
4) 2

7. $\int\sqrt[3]{\frac{\sin^n x}{\cos^{n+6} x}}dx =$

1) $\frac{4}{3+n}(\tan x)^{\frac{n}{3}+1}+c$
2) $\frac{3}{3+n}(\tan x)^{\frac{n}{3}+1}+c$
3) $\frac{3}{3-n}(\tan x)^{\frac{n}{3}+1}+c$
4) $\frac{3}{2+n}(\tan x)^{\frac{n}{3}+1}+c$

8. $\int\frac{xdx}{(2-x^2)+\sqrt{2-x^2}}$

1) $\log\left|1+\sqrt{2+x^2}\right|+c$
2) $-\log\left|1+\sqrt{2-x^2}\right|+c$
3) $-x\log\left|1-\sqrt{2-x^2}\right|+c$
4) $x\log\left|1-\sqrt{2+x^2}\right|+c$

9. $\int\frac{\sin x\cos^3 x}{1+\cos^2 x}dx =$

1) $\frac{-1}{2}(1+\cos^2 x)+\frac{1}{2}\log(1+\cos^2 x)+c$
2) $\frac{1}{2}(1+\cos^2 x)-\frac{1}{2}\log(1+\cos^2 x)+c$
3) $\frac{1}{2}(1+\cos^2 x)+\frac{1}{2}\log(1+\cos^2 x)+c$
4) $-\frac{1}{2}(1+\cos^2 x)-\frac{1}{2}\log(1+\cos^2 x)+c$

10. If $\int \frac{1}{\cos^6 x+\sin^6 x}dx = \tan^{-1} f(x)+C$ then $f(x)$ =

1) $\tan x - \cot x$ 2) $\tan x + \cot x$ 3) $\sec x + \tan x$ 4) $\sec x - \tan x$

11. $\int \sqrt{1+\sec x}\, dx$

1) $2\tan^{-1}(\sqrt{\sec x - 1})+c$ 2) $2\cosh^{-1}(\sqrt{2}\cos\frac{x}{2})+c$

3) $2\tanh^{-1}(\sqrt{\sec x - 1})+c$ 4) $-2\cos^{-1}(\sqrt{2}\cos x/2)+c$

12. $\int \frac{\text{Tan}x}{1+\text{Tan}x+\text{Tan}^2 x}dx = x - \frac{K}{\sqrt{A}}\text{Tan}^{-1}\left(\frac{K\text{Tan}x+1}{\sqrt{A}}\right)+3$ then the ordered pair (K, A) = ____

1) (2, 3) 2) (–2, 3) 3) (–2, –3) 4) (2, –3)

13. $\int \cos(\log_e^x)dx$ =

1) $\frac{x}{2}[\cos(\log_e^x)+\sin(\log_e^x)]+c$ 2) $x[\cos(\log_e^x)+\sin(\log_e^x)]+c$

3) $\frac{x}{2}[\cos(\log_e^x)-\sin(\log_e^x)]+c$ 4) $x[\cos(\log_e^x)-\sin(\log_e^x)]+c$

14. $\int x\sqrt{\frac{1-x^2}{1+x^2}}dx$ =

1) $\frac{1}{2}\sin^{-1}(x^2)+\sqrt{1-x^2}+C$ 2) $\frac{1}{2}\left[\sin^{-1}(x^2)+\sqrt{1-x^4}\right]+C$

3) $\frac{1}{2}\sin^{-1}(x^2)-\sqrt{1-x^4}+C$ 4) $\frac{1}{2}\left[\sin^{-1}(x^2)-\sqrt{1-x^4}\right]+C$

15. $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ =

1) $\frac{2(\sqrt{x}-1)}{\sqrt{1-x}}+c$ 2) $\frac{\sqrt{x}-1}{\sqrt{1-x}}+c$ 3) $\frac{2(\sqrt{x}-1)}{\sqrt{1+x}}+c$ 4) $\frac{2(\sqrt{x}+1)}{\sqrt{1-x}}+c$

16. If $f(x)=\int \frac{x^2\,dx}{(1+x^2)(1+\sqrt{1+x^2})}$ and $f(0)=0$ then $f(1)$ =

1) $\log(1+\sqrt{2})$ 2) $\log(1+\sqrt{2})-\frac{\pi}{4}$ 3) $\log(1+\sqrt{2})+\frac{\pi}{4}$ 4) $\log(1+\sqrt{2})+\frac{\pi}{3}$

17. $\int \frac{x-1}{x+1}\cdot\frac{1}{\sqrt{x(x^2+x+1)}}dx$ =

1) $\tan^{-1}\sqrt{x+\frac{1}{x}+1}+C$ 2) $\sqrt{2}\tan^{-1}\sqrt{x+\frac{1}{x}+1}+C$

3) $2\tan^{-1}\sqrt{x+\frac{1}{x}+1}+C$ 4) $\frac{1}{\sqrt{2}}\tan^{-1}\sqrt{x+\frac{1}{x}+1}+C$

18. $\int \frac{(x^2-1)}{(x^4+3x^2+1)\text{Tan}^{-1}\left(\frac{x^2+1}{x}\right)}dx =$

1) $\log \sin^{-1}\left(x-\frac{1}{x}\right)+c$ 2) $\log \text{Tan}^{-1}\left(x-\frac{1}{x}\right)+c$ 3) $\log \sin^{-1}\left(x+\frac{1}{x}\right)+c$ 4) $\log \text{Tan}^{-1}\left(x+\frac{1}{x}\right)+c$

19. If $\int \frac{dx}{x+x^7} = p(x)$ then $\int \frac{x^6}{1+x^7}dx$ =

1) $\log|x| - p(x)+c$ 2) $\log|x| + p(x)+c$ 3) $x - p(x) + c$ 4) $x + p(x) + c$

20. $\int \frac{\sin^2 x\sec^2 x+2\tan x\sin^{-1}x\sqrt{1-x^2}}{(\sqrt{1-x^2})(1+\tan^2 x)}dx =$

1) $\sin^2 x\sin^{-1}x+c$ 2) $\sin^2 x\cos^{-1}x+c$ 3) $\cos^2 x\sin^{-1}x+c$ 4) $2\sin^2 x\sin^{-1}x+c$

21. If $\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = A\sqrt{7-6x-x^2}+B\sin^{-1}\left(\frac{x+3}{4}\right)+c$ then $(A, B) =$

1) $(-2, -1)$ 2) $(2, -1)$ 3) $(-2, 1)$ 4) $(2, 1)$

22. $\int \frac{1}{1+\cos^2 x+2\cos x\sin x}dx =$

1) $\tan^{-1}(\tan x+1)+C$ 2) $\frac{1}{2}\tan^{-1}\{\tan x+1\}+C$ 3) $\frac{1}{2}\tan^{-1}\left[\frac{1}{2}(\tan x+1)\right]+C$ 4) $\tan^{-1}\left[\frac{1}{2}(\tan x+1)\right]+C$

23. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^A + C(Tanx)^B + k$ where k is integration constant then $A + B + C$ equals

1) $\frac{16}{5}$ 2) $\frac{27}{10}$ 3) $\frac{7}{10}$ 4) $\frac{21}{5}$

24. $\int \frac{(\sin^2 x\cos^2 x)\ dx}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}$

1) $\frac{1}{3(1+Tan^3 x)}+c$ 2) $\frac{1}{1+\cos^3 x}+c$ 3) $\frac{-1}{1+\cos^3 x}+c$ 4) $\frac{-1}{3(1+Tan^3 x)}+c$

25. $\int \frac{2+\cos x/2}{x+\sin x/2}dx =$

1) $\log\left|x+\sin\frac{x}{2}\right|+C$ 2) $\frac{1}{2}\log\left|x+\sin\frac{x}{2}\right|+C$ 3) $2\log\left|x+\sin\frac{x}{2}\right|+C$ 4) $-\frac{1}{2}\log\left|x+\sin\frac{x}{2}\right|+C$

Numerical value type questions

26. If $\int \frac{1}{5+4\cos 2\theta}\ d\theta = A\tan^{-1}(B\tan\theta)+c$ then A =

27. $\int \frac{dx}{(x-2)^{7/8}(x+3)^{9/8}} = A\left(\frac{x-2}{x+3}\right)^{1/8}+c$ then A =

28. $\int \sec^2 x\,\text{cosec}^4 x\,dx = A(-\cot^3 x)+k\tan x-2\cot x+c \Rightarrow A =$

LEVEL-II (ADVANCED)

Single answer type questions

1. If $\int \frac{dx}{\sin^2 x+\tan^2 x} = A\cot x + B\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)+c$ then $\frac{A}{B}=$

a) 2 b) $\sqrt{2}$ c) $-\sqrt{2}$ d) -2

2. $\int \sqrt{\frac{\cos^3 x}{\sin^{11} x}}dx =$

a) $-2\left[\frac{1}{9}\tan^{-9/2} x+\frac{1}{5}\tan^{-5/2} x\right]+c$ b) $-2\left[\frac{1}{5}\tan^{-9/2} x+\frac{1}{5}\tan^{-5/2} x\right]+c$

c) $2\left[\frac{1}{9}\tan^{-9/2} x+\frac{1}{5}\tan^{-5/2} x\right]+c$ d) $3\left[\frac{1}{9}\tan^{-9/2} x+\frac{1}{5}\tan^{-5/2} x\right]+c$

3. $\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}dx =$

a) $\log\left|\sec\left(\frac{x^2+1}{2}\right)\right|+c$ b) $\log\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$ c) $2\log\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$ d) $2\log\left|\sec\left(\frac{x^2+1}{2}\right)\right|+c$

4. Let $n\geq 2$ be a natural number and $0<\theta<\frac{\pi}{2}$. Then $\int \frac{(\sin^n\theta-\sin\theta)^{1/n}.\cos\theta}{\sin^{n+1}\theta}d\theta =$

a) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+c$ b) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+c$

c) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+c$ d) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+c$

5. $\int \frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx =$

a) $\frac{1}{2}\left[x+\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+\sin 2x}{\sqrt{2}-\sin 2x}\right|\right]+c$ b) $\frac{1}{2}\left[x-\frac{1}{2}\log\left|\frac{\sqrt{2}-\sin 2x}{\sqrt{2}+\sin 2x}\right|\right]+c$

c) $\frac{1}{2}\left[x-\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+\sin 2x}{\sqrt{2}-\sin 2x}\right|\right]+c$ d) $\frac{1}{2}\left[x+\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}-\sin 2x}{\sqrt{2}+\sin 2x}\right|\right]+c$

6. $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} =$

a) $2\sqrt{x}+3\sqrt[3]{x}-6\sqrt[6]{x}+6\log(\sqrt[6]{x}+1)+c$ b) $2\sqrt{x}-3\sqrt[3]{x}-6\sqrt[6]{x}+6\log(\sqrt[6]{x}+1)+c$

c) $2\sqrt{x}+3\sqrt[3]{x}-6\sqrt[6]{x}-6\log(\sqrt[6]{x}+1)+c$ d) $2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log(\sqrt[6]{x}+1)+c$

7. If $\int \frac{(1+x)\sin x dx}{(x^2+2x)\cos^2 x-(1+x)\sin 2x} = \frac{1}{2}\ln\left|\frac{\sin x-(1+x)\cos x-A}{\sin x-(1+x)\cos x+B}\right|+C$ then $A+B=$

a) 0 b) 1 c) 2 d) –2

8. $\int \frac{1}{1-\sin^4 x}dx =$

a) $\frac{1}{2}\tan x+\frac{1}{2}\tan^{-1}\left(\sqrt{2}\tan x\right)+c$ b) $\frac{1}{2\sqrt{2}}\tan x+\frac{1}{2}\tan^{-1}\left(\sqrt{2}\tan x\right)+c$

c) $\frac{1}{2}\tan x+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right)+c$ d) $\frac{1}{2}\tan^{-1}x+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right)+c$ 3

9. If $\int\left(\frac{bx\cos 4x-a\sin 4x}{x^2}\right)dx=\frac{a\sin 4x}{x}$ then

a) $(a, b) = (1, 5)$ and $(1/4, 1)$ b) $(a, b) = (1, 4)$ and $(1/5, 1)$

c) $(a, b) = (1, 4)$ and $(1/4, 1)$ d) $(a, b) = (1, 3)$ and $(1/4, 1)$

10. $\int \frac{dx}{\sec x+\operatorname{cosece} x} =$

a) $\frac{1}{2}\left[\sin x-\cos x-\frac{1}{\sqrt{2}}\log\left|\operatorname{cosec}\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right|\right]+c$

b) $\left[\sin x-\cos x-\frac{1}{\sqrt{2}}\log\left|\operatorname{cosec}\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right|\right]+c$

c) $\frac{1}{2}\left[\sin x-\cos x-\frac{1}{2}\log\left|\operatorname{cosec}\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right|\right]+c$

d) $\frac{1}{2}\left[\sin x-\cos x-\log\left|\operatorname{cosec}\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right|\right]+c$

11. If $\int \frac{dx}{x^{2010}(1+x^{2010})^{\frac{1}{2010}}} = \frac{-1}{a}(1+x^{-b})^{\frac{c-2}{b}}+k$. Then

a) a,b,c are in A.P. b) a,b,c are in G.P. c) a,b,c are in H.P. d) a,b,c are in A.G.P.

12. If $\int x^5(1+x^3)^{2/3}dx = A(1+x^3)^{8/3}+B(1+x^3)^{5/3}+C$ then $A-B=$

a) $\frac{13}{40}$ b) $\frac{-3}{40}$ c) $\frac{-3}{40}$ d) $\frac{-13}{40}$

13. $\int \frac{\sin 12\theta-\sin 9\theta}{2\cos 7\theta-1}d\theta =$

a) $\frac{\cos 2\theta}{2}+\frac{\cos 5\theta}{5}+C$ b) $\frac{\cos 5\theta}{2}-\frac{\cos 2\theta}{5}+C$ c) $\frac{\cos 2\theta}{5}-\frac{\cos 5\theta}{2}+C$ d) $\frac{\cos 2\theta}{2}-\frac{\cos 5\theta}{5}+C$

14. $\int \frac{\cos^4 x}{\sin^3 x[\sin^5 x+\cos^5 x]^{3/5}}dx =$

a) $\frac{1}{2}(1+\cot^5 x)^{2/5}+c$ b) $-\frac{1}{2}(1-\cot^5 x)^{2/5}+c$ c) $-\frac{1}{2}(1+\cot^5 x)^{2/5}+c$ d) $-\frac{1}{3}(1+\cot^5 x)^{2/5}+c$

15. $\int \frac{x^2}{(x\sin x+\cos x)^2}dx =$

a) $\frac{x}{\cos x(x\sin x+\cos x)}-\text{Tan}x+c$ b) $\frac{x}{\cos x(x\sin x+\cos x)}+\text{Tan}x+c$

c) $\frac{-x}{\cos x(x\sin x+\cos x)}+\text{Tan}x+c$ d) $\frac{-x}{\cos x(x\sin x+\cos x)}-\text{Tan}x+c$

16. $\int \frac{1}{\tan^2 x+\sec^2 x}dx =$

a) $\frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x)-\tan^{-1}(\tan x)+c$ b) $\frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x)-\tan^{-1}(\tan x)+c$

c) $\sqrt{2}\tan^{-1}(\sqrt{2}\tan x)-\tan^{-1}(\tan x)+c$ d) $\sqrt{2}\tan^{-1}(\tan x)-\tan^{-1}(\sqrt{2}\tan x)+c$

More than one correct answer type questions

17. If $\int \frac{1}{x^4+8x^2+9}dx=\frac{1}{6}\left[\frac{1}{\sqrt{14}}f(x)-\frac{1}{\sqrt{2}}g(x)\right]+C$ then

a) $f(x)=\tan^{-1}\left(\frac{x^2-3}{x\sqrt{14}}\right)$ b) $f(x)=\tan^{-1}\left(\frac{x^2+3}{x\sqrt{14}}\right)$ c) $g(x)=\tan^{-1}\left(\frac{x^2+3}{x\sqrt{2}}\right)$ d) $g(x)=\tan^{-1}\left(\frac{x^2-3}{x\sqrt{2}}\right)$

18. If $\int \frac{dx}{(\sec x+\text{cosec}\,x+\tan x+\cot x)^2}=\frac{x}{P}+\frac{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}{Q}+\frac{\cos 2x}{R}+K$ then

a) $P=2$ b) $\frac{P}{Q}=1$ c) $R=-8$ d) $P+Q+R=-4$

19. If $I=\int \frac{\sin x+\sin^3 x}{\cos 2x}dx=A\cos x+B\log|f(x)|+C$, then

a) $A=\frac{1}{4}, B=-\frac{1}{\sqrt{2}}, f(x)=\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$ b) $A=\frac{1}{2}, B=-\frac{3}{4\sqrt{2}}; f(x)=\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$

c) $A=-\frac{1}{2}, B=\frac{3}{\sqrt{2}}, f(x)=\frac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$ d) $A=\frac{1}{2}, B=\frac{3}{4\sqrt{2}}, f(x)=\frac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$

20. If $\int f(x)\sin x\cos x dx=\frac{1}{2A}\log f(x)+c$ then

a) $A=(b^2+a^2)$ b) $f(x)=\frac{1}{a^2\sin^2 x+b^2\cos^2 x}$

c) $f(x)=a^2\sin^2 x+b^2\cos^2 x$ d) $A=b^2-a^2$

21. If $\int \frac{\sec x(2+\sec x)}{(1+2\sec x)^2}dx = \frac{f(x)}{2+g(x)}+C$ then

a) $f^2(x) - g^2(x) = 1$ b) $f^2(x) + g^2(x) = 1$

c) Domain of $f(x)$, $g(x)$ is R d) Range of $f(x)$ and $g(x)$ is R

22. If $\int \frac{(3+2\cos x)}{(2+3\cos x)^2}dx = \frac{P(x)}{A+BQ(x)}+C$ then

a) $P^2(x)+Q^2(x)=1$ b) $P^2(x)-Q^2(x)=1$ c) $AB = 6$ d) $AB = 1$

23. $\int \frac{dx}{2\sin x+\sec x} = A\log\left|\operatorname{cosec}\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right| - \frac{1}{B(\sin x+\cos x)}+C$ then

a) $B=\frac{1}{\sqrt{2}}$ b) $A=\frac{1}{2\sqrt{2}}$ c) $A=\frac{1}{2\sqrt{3}}$ d) B = 2

24. If $\int \frac{dx}{3\sin x+\sin^3 x} = A\log f(x)+B\log\left|\frac{1-\cos x}{1+\cos x}\right|+C$ then

a) $A+B=\frac{1}{4}$ b) $A+B=\frac{1}{5}$ c) $f(x)=\left|\frac{2+\cos x}{2-\cos x}\right|$ d) $f(x)=\left|\frac{1+\cos x}{1-\cos x}\right|$

25. $\int \frac{\operatorname{Tan}x}{\sqrt{a+b\operatorname{Tan}^2 x}}dx =$

a) $\frac{1}{\sqrt{b-a}}\operatorname{Tan}^{-1}\left(\frac{\sqrt{a+b\operatorname{Tan}^2 x}}{b-a}\right)+C$ b) $\frac{1}{\sqrt{a-b}}\operatorname{Tan}^{-1}\left(\frac{\sqrt{a+b\operatorname{Tan}^2 x}}{b-a}\right)+C$

c) $\frac{1}{\sqrt{b-a}}\operatorname{Tan}^{-1}\left(\frac{\sqrt{a+b\operatorname{Tan}^2 x}}{a-b}\right)+C$ d) $\frac{1}{\sqrt{b-a}}\operatorname{Tan}^{-1}\left(\sqrt{\frac{b-a}{b}}\cos x\right)+C$

26. $\int \sin(\sqrt[3]{x})dx = a\left\{(2-x^{2/3})\cos(\sqrt[3]{x})+b\sqrt[3]{x}.\sin\sqrt[3]{x}\right\}$ than

a) $a = 3$ b) $b = 2$ c) $a = 1$ d) $b = 1$

27. $f(x)=\int \frac{\cos 4x+1}{\cot x-\tan x}dx$ and $f\left(\frac{\pi}{8}\right)=0$ then

a) $f(x)$ is an odd function b) $|f(x)|$ has fundamental period

c) The Range of $f(x)$ is $\left[\frac{-1}{8},\frac{1}{8}\right]$ d) $f(x)$ is bounded function

Linked comprehension type questions

Passage - I :

Let $f(x,n)=\int \frac{x^2+n(n-1)}{(x\sin x+n\cos x)^2}dx$ and $f(0, n) = 0$

28. If $f\left(\frac{\pi}{4},3\right)=\frac{\lambda-\pi}{\lambda+\pi}$ then the value λ is

a) $\frac{1}{12}$ b) 12 c) $\frac{1}{6}$ d) 6

29. If $f\left(\frac{\pi}{3},7\right)=Tan(A-B)$ then the value of (TanA)=?, When $0<B<\frac{\pi}{4}$

a) $\sqrt{3}$ b) $2\sqrt{3}$ c) $3\sqrt{3}$ d) $4\sqrt{3}$

30. If $f\left(\frac{\pi}{6},2\right)=\frac{\lambda(4\lambda-\pi)}{(4\lambda^3+\pi)}$ then the value of $(1+\lambda+\lambda^2)(1-\lambda+\lambda^2)$ is

a) 3 b) 7 c) 13 d) 21

Matrix matching type questions

31. $I=\int\frac{dx}{a+b\cos x}$, where $a, b>0$ and $a+b=u$, $a-b=v$

COLUMN - I	COLUMN - II
A) $v=0$	p) $\frac{1}{\sqrt{u}}\log\left(\frac{\sqrt{u}+\tan x/2}{\sqrt{u}-\tan x/2}\right)+c$
B) $v>0$	q) $\frac{2}{\sqrt{uv}}\tan^{-1}\left(\tan x/2\sqrt{v/u}\right)+c$
C) $v<0$	r) $\frac{1}{\sqrt{-uv}}\log\left\|\frac{\sqrt{u}+\sqrt{-v}\tan x/2}{\sqrt{u}-\sqrt{-v}\tan x/2}\right\|+c$
D) $v=-1$	s) $\frac{2}{u}\tan x/2+c$

32.

COLUMN - I $f(x)$	COLUMN - II $\int f(x)dx$
A) $\frac{1}{(a^2+b^2)-(a^2-b^2)\cos x}$	p) $\frac{1}{ab}\tan^{-1}\left(\frac{a}{b}\tan\frac{x}{2}\right)+c$
B) $\frac{1}{a^2\sin^2 x+b^2\cos^2 x}$	q) $\frac{1}{a^2\sin\alpha}\tan^{-1}\left(\frac{\tan x}{\sin\alpha}\right)+c,\alpha=\cos^{-1}\frac{b}{a}$
C) $\frac{1}{a\cos x+b\sin x}$	r) $\frac{1}{ab}\tan^{-1}\left(\frac{a}{b}\tan x\right)+c$
D) $\frac{1}{a^2-b^2\cos^2 x}$	s) $\frac{1}{\sqrt{a^2+b^2}}\log\left\|\tan\frac{1}{2}\left(x+\tan^{-1}\frac{a}{b}\right)\right\|+c$

33. **COLUMN - I** **COLUMN - II**

A) $\int \frac{x^2+a^2}{x^4-a^2x^2+a^4}dx =$ p) $\frac{1}{3a}\left(\frac{x^2}{a+bx^2}\right)^{3/2}+C$

B) $\int \frac{x^2}{(a+bx^2)^{5/2}}dx =$ q) $\frac{-16}{105}$

C) If $\int \frac{dx}{(2\sin x+\sec x)^4} = \frac{A}{t^5}+\frac{B}{t^6}+\frac{C}{t^7}+K$ r) $\frac{-4\sqrt{2}}{5}$

where t = 1+tan$x$ then $A+B+C$ =

D) If $\int \frac{\sqrt{\sin^3 2x}}{\sin^5 x}dx = \lambda\cot^{5/2}x+C$ then λ = s) $\frac{1}{a}\text{Tan}^{-1}\left(\frac{x^2-a^2}{ax}\right)+C$

Integer answer type questions

34. If $\int \frac{dx}{\sqrt{(5+2x+x^2)^3}} = K\left(\frac{x+1}{\sqrt{5+2x+x^2}}\right)+c$ then $4K$ =

35. If $I=\int \frac{(x+x^{2/3}+x^{1/6})}{x(1+x^{1/3})}dx = \frac{K}{L}x^{2/3}+M\tan^{-1}(x^{1/6})+c$ then KL/M =

36. If $\int \sqrt{x^3+\sqrt{x^{11}}}\,dx = \frac{P}{Q}(1+x^{5/2})^{3/2}+C$ then $Q-2P$ =

37. If $\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx = A\sin x+B\cos ec\,x+C\tan^{-1}(\sin x)+K$ then $A+B-C$ =

38. If $\int \frac{dx}{x^{22}(x^7-6)} = \frac{1}{A}\left\{\ln|t|-\frac{t^3}{3}-3t+\frac{3t^2}{2}\right\}+C\left(\text{where } t=1-\frac{6}{x^7}\right)$ then $A/1008$ =

39. If $\int \frac{x^2}{(1+x^2)^{3/2}}dx = A\log\left[x+\sqrt{1+x^2}\right]+B\frac{x}{\sqrt{1+x^2}}+c$ then $A-B$=

40. If $\int \frac{dx}{(2ax+x^2)^{3/2}} = \frac{P}{a^Q}\cdot\frac{(x+a)}{\sqrt{2ax+x^2}}+c$ then $Q+P$=

41. If $\int \frac{x\,dx}{1+2x+2x^2+2x^3+x^4} = \frac{1}{P}\left[\tan^{-1}x+\frac{Q}{R+x}\right]+C$ then $P-Q+R$ =

42. If $\int \frac{1}{(\sin x+2\cos x)(\cos x+2\sin x)}dx = \frac{1}{P}\log\left|\frac{1+2\tan x}{2+\tan x}\right|+c$ then P =

EXERCISE-III

Integration By parts and Reduction formulae

LEVEL-I (MAIN)

Single answer type questions

1. If $\int \log_{10} x\,dx = K.x\log f(x)+c$ then K, $f(x)$ =

1) $\log_e 10, \frac{x}{e}$ 2) $\log_{10} e, \frac{x}{e}$ 3) $\log_2 10, \frac{x}{e}$ 4) $\log_{10} e, \frac{x}{e}$

2. $\int \log\sqrt{x+1}\,dx =$

1) $\frac{1}{2}[(x-1)\log(x+1)-x] + c$ 2) $\frac{1}{2}[(x+1)\log(x+1)-x] + c$

3) $\frac{1}{2}\left[x\log(x+1)-\frac{x^2}{2}\right]+c$ 4) $\frac{1}{2}\left[(x-1)\log(x+1)-\frac{x}{2}\right]+c$

3. $\int (\log x)^3\,dx =$

1) $x[(\log x)^3-(\log x)^2+(\log x)-6]+c$ 2) $x[(\log x)^3+(\log x)^2-(\log x)-6]+c$

3) $x[(\log x)^3+(\log x)^2+(\log x)-6]+c$ 4) $x[(\log x)^3-3(\log x)^2+6(\log x)-6]+c$

4. $\int \text{Sin}\sqrt{x}\,dx =$

1) $2\left[\sqrt{x}\text{Cos}\sqrt{x}-\text{Sin}\sqrt{x}\right]+c$ 2) $2\left[-\sqrt{x}\text{Cos}\sqrt{x}-\text{Sin}\sqrt{x}\right]+c$

3) $2\left[\sqrt{x}\text{Cos}\sqrt{x}+\text{Sin}\sqrt{x}\right]+c$ 4) $2\left[-\sqrt{x}\text{Cos}\sqrt{x}+\text{Sin}\sqrt{x}\right]+c$

5. $\int \text{Tan}^{-1}\sqrt{\frac{1-x}{1+x}}dx =$

1) $\frac{1}{2}(x\ \cos^{-1}x-\sqrt{1-x^2}\ )+c$ 2) $\frac{1}{2}(x\ \cos^{-1}x+\sqrt{1-x^2}\ )+c$

3) $\frac{1}{2}(x\ \sin^{-1}x-\sqrt{1-x^2}\ )+c$ 4) $\frac{1}{2}(x\ \sin^{-1}x+\sqrt{1-x^2}\ )+c$

6. $\int \frac{\text{Sin}^{-1}x-\text{Cos}^{-1}x}{\text{Sin}^{-1}x+\text{Cos}^{-1}x}\,dx =$

1) $\frac{4}{\pi}\left[x\text{Sin}^{-1}x+\sqrt{1-x^2}\right]-x+c$ 2) $\frac{1}{\pi}\left[x\text{Sin}^{-1}x+\sqrt{1-x^2}\right]-x+c$

3) $\frac{2}{\pi}\left[x\text{Sin}^{-1}x+\sqrt{1-x^2}\right]-x+c$ 4) $\frac{2}{\pi}\left[x\text{Sin}^{-1}x-\sqrt{1-x^2}\right]-x+c$

7. If $\int \frac{x^2\text{Tan}^{-1}x}{1+x^2}\,dx = x\text{Tan}^{-1}x-\frac{1}{2}\log(1+x^2) + f(x) + c$ then $f(x) =$

1) $-\frac{\text{Tan}^{-1}x}{2}$ 2) $-\frac{1}{2}(\text{Tan}^{-1}x)^2$ 3) $\frac{\text{Tan}^{-1}x}{2}$ 4) $\frac{(\text{Tan}^{-1}x)^2}{2}$

8. $\int \frac{\text{Sin}^{-1}\sqrt{x}}{\sqrt{1-x}}\,dx =$

1) $2\left\{\sqrt{x}-\sqrt{1-x}\ \text{Sin}^{-1}\sqrt{x}\right\}+c$ 2) $2\left\{\sqrt{x}+\sqrt{1-x}\ \text{Sin}^{-1}\sqrt{x}\right\}+c$

3) $2\left\{\sqrt{x}-\sqrt{1-x}\ \text{Cos}^{-1}\sqrt{x}\right\}+c$ 4) $\sqrt{x}\ \text{Sin}^{-1}\sqrt{x}+c$

9. If $\int e^x.(2\text{Sin}3x+5\text{Cos}3x)dx = \frac{e^x}{a}[b\text{Sin}3x + c\text{Cos}3x] + K$ then $(a,b,c)=$

1) (10, 17, 1) 2) $\frac{1}{10}$(, 17, –1) 3) $(\frac{1}{10}$, –17, 1) 4) (10, 17, –1)

10. $\int e^x \frac{(1-x)^2}{(1+x^2)^2}\,dx =$

1) $\frac{e^x}{(1-x)^2}+c$ 2) $\frac{e^x}{(1+x^2)}+c$ 3) $e^x\left(\frac{1-x^2}{1+x^2}\right)+c$ 4) $\frac{e^x}{(1+x^2)^2}+c$

11. $\int\left(\frac{2-\sin 2x}{1-\cos 2x}\right) e^x dx =$

1) $-2e^x \cot x + c$ 2) $e^x \cot x + c$ 3) $2e^x \cot x + c$ 4) $-e^x \cot x + c$

12. $\int e^x . \frac{x^3+x+1}{(1+x^2)^{3/2}}.dx =$

1) $\frac{e^x}{\sqrt{1+x^2}}+c$ 2) $\frac{e^x\sqrt{1+x^2}}{x}+c$ 3) $e^x.\frac{x}{\sqrt{1+x^2}}+c$ 4) $\frac{e^x.x}{(1+x^2)^{3/2}}+c$

13. $\int\left[\log(\log x)+(\log x)^{-2}\right]dx =$

1) $x\left\{\log(\log x)-\frac{1}{\log x}\right\}+c$ 2) $x\log(\log x)-\frac{1}{\log x}+c$

3) $\log(\log x)-\frac{x}{\log x}+c$ 4) $x\left[\log(\log x)-\log x\right]+c$

14. $\int e^x(x\cos x+\sin x)dx =$

1) $\frac{1}{3}e^x(x\sin x+x\cos x-\cos x)+c$ 2) $\frac{1}{2}e^x(x\sin x+x\cos x-\cos x)+c$

3) $\frac{1}{2}e^x(x\cos x+x\sin x-\cos x)+c$ 4) $\frac{1}{2}e^x(x\sin x-x\cos x+\cos x)+c$

15. $\int e^{2x}\left(\frac{\text{Cot}2x-1}{\text{Cos}x\ \text{Sin}x}\right)dx =$

1) $e^{2x}\text{Cosec}2x+c$ 2) $e^{2x}\text{Cot}2x+c$ 3) $-e^{2x}\text{Cosec}2x+c$ 4) $e^x\text{Cosec}2x+c$

16. $\int\frac{x+\text{Sin}x}{1+\text{Cos}x}dx=$

1) $x\tan\frac{x}{2}+c$ 2) $x\,\text{Cot}\frac{x}{2}+c$ 3) $\log|1+\text{Cos}x|+c$ 4) $x\ \log|\text{Cos}x|+c$

17. If $I_n=\frac{t^n}{1+t^2}dt$ then $I_6+I_4=$

1) $\frac{t^5}{5}$ 2) $\frac{t^7}{7}$ 3) $\frac{t^6}{6}$ 4) $\frac{t^3}{3}$

18. If $I_n=\int\frac{\text{Cos}nx}{\text{Cos}x}dx$ then $I_n=$

1) $-\frac{2}{(n-1)}\text{Cos}(n-1)x+I_{n-2}$ 2) $\frac{2}{(n-1)}\text{Cos}(n-1)x+I_{n-2}$

3) $\frac{2}{(n-1)}\text{Sin}(n-1)x-I_{n-2}$ 4) $-\frac{2}{(n-1)}\text{Sin}(n-1)x+I_{n-2}$

19. If $I_n = \int \frac{\sin^2 nx}{\sin^2 x}\,dx$ then $I_{n+1} - 2I_n + I_{n-1} =$

1) $\frac{1}{n}\sin 2nx + c$ 2) $-\frac{1}{n}\sin 2nx + c$ 3) $\frac{1}{n}\cos 2nx + c$ 4) $-\frac{1}{n}\cos 2nx + c$

20. If $I_n = \int \cot^n x\,dx$ then $I_0 + I_1 + 2(I_2 + I_3) + I_4 + I_5 =$

1) $-\left[\text{Cot}x + \frac{\text{Cot}^2 x}{2} + \frac{\text{Cot}^3 x}{3} + \frac{\text{Cot}^4 x}{4}\right] + c$ 2) $-\left[\text{Cot}x + \frac{\text{Cot}^3 x}{3}\right] + c$

3) $-\left[\frac{\text{Cot}^2 x}{2} + \frac{\text{Cot}^4 x}{4}\right] + c$ 4) $-\left[\text{Cot}x + \frac{\text{Cot}^3 x}{3} + \frac{\text{Cot}^5 x}{5}\right] + c$

21. If $I_{m,n} = \int \frac{x^m}{(\log x)^n}dx$ then $(m+1)I_{m,n} - n.I_{m,n+1} =$

1) $\frac{x^m}{(\log x)^n} + c$ 2) $-\frac{x^m}{(\log x)^n} + c$ 3) $-\frac{x^{m+1}}{(\log x)^n} + c$ 4) $\frac{x^{m+1}}{(\log x)^n} + c$

22. $\int \left\{\frac{\log x - 1}{1 + (\log x)^2}\right\}^2 dx =$

1) $\frac{\log x}{(\log x)^2 + 1} + c$ 2) $\frac{x}{x^2 + 1} + c$ 3) $\frac{xe^x}{1 + x^2} + c$ 4) $\frac{x}{(\log x)^2 + 1} + c$

23. $\int e^x \left(\frac{2 + \sin 2x}{1 + \cos 2x}\right)dx =$

1) $e^x \cot x + c$ 2) $2e^x \sec^2 x + c$ 3) $e^x \cos 2x + c$ 4) $e^x \tan x + c$

24. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to

1) $\frac{1}{3}[x^3\psi(x^3) - \int x^2\psi(x^3)dx] + C$ 2) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

3) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$ 4) $\frac{1}{3}[x^3\psi(x^3) - \int x^3\psi(x^3)dx + C]$

25. If $\int f(x)dx = g(x) + c$, then $\int f^{-1}(x)dx =$

1) $xf^{-1}(x) + c$ 2) $f(g^{-1}(x)) + c$ 3) $xf^{-1}(x) - (g(f^{-1}(x)) + c$ 4) $g^{-1}(x) + c$

Numerical value type questions

26. If $I_n = \int \text{Cos}^n x\,dx$ and $I_7 - \frac{\text{Cos}^6 x\text{Sin}x}{7} = t\,I_5$ then $t =$

27. If $I_n = \int \frac{\sin nx}{\sin x}dx$ where $n(>1) \in N$ and $I_n - I_{n-2} = \frac{1}{A(n-1)}\sin(n-1)x + c$ then A =

28. $\int \cos^{-1}(2x^2 - 1)dx = \frac{1}{t}\left(x\cos^{-1}x - \sqrt{1 - x^2}\right) + c$ then t =

LEVEL-II (ADVANCED)

Single answer type questions

1. If $\int \sin^{-1}x\cos^{-1}x\,dx = f^{-1}(x)\left[Ax - xf^{-1}(x) - 2\sqrt{1-x^2}\right] + \frac{\pi}{2}\sqrt{1-x^2} + 2x + c$ then $f(x)=$ and $A=$

a) $\sin x, \frac{\pi}{2}$ b) $\sin x, \frac{\pi}{4}$ c) $\cos x, \frac{\pi}{2}$ d) $\cos x, \frac{\pi}{4}$

2. $\frac{\int (e^x + e^{-x}) + (e^x - e^{-x})\sin x}{1+\cos x}dx =$

a) $(e^x - e^{-x})\tan\left(\frac{x}{2}\right) + c$ b) $(e^x + e^{-x})\cot\frac{x}{2} + c$ c) $(e^x + e^{-x})\tan\frac{x}{2} + c$ d) $(e^x - e^{-x})\cot\frac{x}{2} + c$

3. $\int \tan^{-1}\left\{\sqrt{\sqrt{x}-1}\right\}dx =$

a) $x\tan^{-1}\left(\sqrt{x-1}\right) - \sqrt{x-1} - \frac{1}{3}\left(\sqrt{x}-1\right)^{3/2} + c$ b) $x\tan^{-1}\left(\sqrt{x-1}\right) - \sqrt{x-1} - \frac{1}{3}\left(\sqrt{x}-1\right)^{3/2} + c$

c) $x\tan^{-1}\left(\sqrt{\sqrt{x}-1}\right) - \frac{1}{3}\left(\sqrt{x}-1\right)^{3/2} + c$ d) $x\tan^{-1}\left(\sqrt{\sqrt{x}-1}\right) - \sqrt{\sqrt{x}-1} - \frac{1}{3}\left(\sqrt{x}-1\right)^{3/2} + c$

4. Let $P = \int e^{ax}\cos bx\,dx, Q = \int e^{ax}\sin bx\,dx$. If $P + iQ = Z + C$ where Z is a complex function and C is constant then $|Z|=$

a) e^{ax} b) $\frac{e^{ax}}{a+b}$ c) $\frac{e^{ax}}{\sqrt{a^2+b^2}}$ d) $\frac{e^{bx}}{\sqrt{a^2+b^2}}$

5. $\int e^{x\sin x+\cos x}\left(\frac{x^4\cos^3 x - x\sin x + \cos x}{x^2\cos^2 x}\right)dx$

a) $e^{x\sin x+\cos x}\left(x - \frac{\sec x}{x}\right) + c$ b) $e^{x\sin x+\cos x}\left(x\sin x - \frac{\cos x}{x}\right) + c$

c) $e^{x\sin x+\cos x}\left(\frac{x}{\tan x} - \frac{\sec x}{x}\right) + c$ d) $xe^{x\sin x+\cos x} - \int e^{x\sin x+\cos x}\left(1 - \frac{\cos x - x\sin x}{x^2\cos^2 x}\right)dx$

6. If $\int \frac{\tan^{-1}x}{x^4}dx = A\frac{\tan^{-1}x}{x^3} + B\log\left|\frac{x^2+1}{x^2}\right| + \frac{c}{x^2} + k$ then $A + B + C =$

a) $-2/3$ b) $-1/2$ c) $1/3$ d) $1/2$

7. $\int e^{\sec x}.\sec^3 x\,(\sin^2 x + \cos x + \sin x + \sin x\cos x)dx =$

a) $e^{\sec x}(\sec x - \tan x) + c$ b) $e^{\sec x}(\sec x + \tan x) + c$

c) $e^{\sec x}(\sec x + \cot x) + c$ d) $e^{\sec x}(\operatorname{cosec} x - \tan x) + c$

8. $\int e^{\tan x}(\sin x - \sec x)dx =$ ____

a) $-e^{\tan x}.\cos x + c$ b) $e^{\tan x}.\sin x + c$ c) $e^{\tan x}.\sec x + c$ d) $e^{\tan x}.\cos x + c$

9. Let $g(x)$ be a differentiable function satisfying $\frac{d}{dx}\{g(x)\} = g(x)$ and $g(0) = 1$, then $\int g(x)\left(\frac{2-\sin 2x}{1-\cos 2x}\right)dx$ is equal to

a) $g(x)\cot x + c$ b) $-g(x)\cot x + c$ c) $\frac{g(x)}{1-\cos 2x} + c$ d) $g(x) + \cot x + c$

10. If $\int \text{Tan}^{-1}\left(\frac{2\cos^2\theta}{2-\sin 2\theta}\right)\sec^2\theta d\theta = \text{Tan}\theta + \log\cos\theta - A + \frac{1}{2}\log(2 - 2\text{Tan}\theta + \text{Tan}^2\theta) + C$ then $A =$

a) $\left(1-\text{Tan}^{-1}\theta\right)(1-\text{Tan}\theta)$ b) $(1-\text{Tan}\theta)\text{Tan}^{-1}(1-\text{Tan}\theta)$

c) $\text{Tan}^{-1}(1-\text{Tan}\theta)$ d) $\frac{1}{2}\text{Tan}^{-1}(1-\text{Tan}\theta)$

11. $\int Tan^{-1}\left(\frac{8x-4}{3-4x^2+4x}\right)dx =$

a) $(2x-1)Tan^{-1}\left(\frac{2x-1}{2}\right) - \log(4x^2-4x+5) + C$ b) $(2x-1)Tan^{-1}\left(\frac{2x-1}{2}\right) - \log(4x^2+4x+5)C$

c) $(2x-1)Tan^{-1}\left(\frac{2x-1}{2}\right) - \frac{1}{2}\log(4x^2-4x-5) + C$ d) $(2x+1)Tan^{-1}\left(\frac{2x-1}{2}\right) - \log\sqrt{(2x+1)^2+1} + C$

12. $\int Sin^{-1}\left(\sqrt{\frac{x}{a+x}}\right)dx$

a) $a\left[\sqrt{\frac{x}{a}}Tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + Tan^{-1}\frac{x}{a}\right] + C$ b) $a\left[Tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + Tan^{-1}\sqrt{\frac{x}{a}}\right] + C$

c) $a\left[\frac{x}{a}Tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + Tan^{-1}\sqrt{\frac{x}{a}}\right] + C$ d) $a\left[\frac{x}{a}Tan^{-1}\sqrt{\frac{x}{a}} + \sqrt{\frac{x}{a}} + Tan^{-1}\sqrt{\frac{x}{a}}\right] + C$

More than one correct answer type questions

13. If $\int e^x\left(\frac{x^2-x+1}{(x^2+1)^{3/2}}\right)dx = e^x f(x) + c$ then

a) $f(x)$ is an even function b) $f(x)$ is an odd function

c) $f(x)$ is a bounded function d) The range of $f(x)$ is $(0, 1]$

14. $\int e^{-x}\log(e^x+1)dx = x - Q(x)\log h(x) - \log(h(x)) + C$ than

a) $h(x) = e^x + 1$ b) $Q(x) = e^x$ c) $h(x) = e^{-x} + 1$ d) $Q(x) = e^x$

15. $\int \log(\sqrt{x-a} + \sqrt{x-b})dx$

a) $\left[2x - (a+b)\log\left(\sqrt{x-a} + \sqrt{x-b}\right)\right] + C$

b) $\frac{1}{2}[2x-(a+b)]\left(\log(b-a) - \log(\sqrt{x-a} - \sqrt{x-b}\right) - \frac{1}{2}\sqrt{(x-a(x-b)} + C$

c) $\frac{1}{2}[2x-(a+b)]\log(\sqrt{x-a} - \sqrt{x-b}) - \frac{1}{2}\sqrt{(x-a(x-b)} + C$ d) None

Linked comprehension type questions

Passage - I :

Repeated application of integration by parts gives us, the reducation formula if the inte grand is dependent of n, $n \in N$. On the basis of above information, answer the following question.

16. If $I_n = \int \tan^n x\,dx$ and $I_n = \frac{\tan^{n-1} x}{(n-1)} + \lambda I_{n-2}$ then λ is equal to

a) $\frac{1}{(n-1)}$ b) $\frac{1}{(n-2)}$ c) $\frac{1}{n}$ d) -1

17. If $I_{m-2,n+2} = \int \sin^{m-2} x \cos^{n+2} x\,dx$ and $I_{m,n} = -\frac{\sin^{m-1} x \cos^{n+1} x}{(n+1)} + f(m,n) I_{m-2,n+2}$, then $f(2, 3)$ is equal to

a) 1/2 b) 1/3 c) 1/4 d) 1/5

18. If $I_{m,p} = \int x^m (a+bx^n)^p dx$ & $I_{m,p} = \frac{x^{m+1}(a+bx^n)^p}{(m+1)} - f(m,n,p,b) I_{m+n,p-1}$ then $f(1, 2, 3, 4) =$

a) 8 b) 10 c) 12 d) 6

Passage - II :

If A is square matrix and e^A is defined as $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + ... = \frac{1}{2}\begin{bmatrix} f(x) & g(x) \\ g(x) & f(x) \end{bmatrix}$, where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$ and $0<x<1$. I is an identity matrix.

19. $\int \frac{g(x)}{f(x)} dx =$

a) $\log(e^x + e^{-x}) + c$ b) $\log(e^x - e^{-x}) + c$ c) $\log(e^{2x} - 1) + c$ d) $\log(e^{2x} + 1) + c$

20. $\int (g(x) + 1)\sin x\,dx =$

a) $\frac{e^x}{2}(\sin x - \cos x) + c$ b) $\frac{e^{2x}}{5}(2\sin x - \cos x) + c$ c) $\frac{e^x}{5}(\sin 2x - \cos 2x) + c$ d) $\frac{e^{2x}}{5}(2\sin x + \cos x) + c$

21. $\int \frac{f(x)}{\sqrt{g(x)}} dx =$

a) $\frac{1}{2\sqrt{e^x - 1}} - \text{cosec}^{-1}(e^x) + c$ b) $\frac{2}{\sqrt{e^x - e^{-x}}} - \sec^{-1}(e^x) + c$

c) $\sqrt{e^{2x} - 1} + \sec^{-1}(e^x) + c$ d) $\frac{2}{\sqrt{e^x - e^{-x}}} + \sec^{-1}(e^x) + c$

Integer answer type questions

22. Let $f(x)$ be a function such that $f(0) = f^1(0) = 0$, $f^{11}(x) = \sec^4 x + 4$ then the function is $\frac{A}{B}\log|\sec x| + \frac{1}{C}\tan^2 x + Dx^2$ then $A+B-C+D =$

23. If $\int \cot^{-1}(x^2 + x + 1)dx = (x+1)\tan^{-1}(1+x) - x\tan^{-1} x + A\log\left|1 + (1+x)^2\right| + B\log\left|1 + x^2\right| + C$ then $A+B =$

24. If $I_{m,n} = \int \cos^m x \sin nx\,dx$ and $7I_{4,3} - 4I_{3,2} = -\cos Ax\,\cos^B x + c$ then $A + B =$

KEY SHEET (LECTURE SHEET)

EXERCISE-I

LEVEL-I

1) **4** 2) **3** 3) **2** 4) **2** 5) **2** 6) **3** 7) **2** 8) **4**
9) **3** 10) **2** 11) **2** 12) **1** 13) **1** 14) **4** 15) **2** 16) **3**
17) **4** 18) **2** 19) **2** 20) **2** 21) **4** 22) **1** 23) **3** 24) **2**
25) **1** 26) **3** 27) **2** 28) **2** 29) **3** 30) **1** 31) **2** 32) **0.5**
33) **0.33** 34) **0.5**

LEVEL-II

1) **a** 2) **a** 3) **a** 4) **b** 5) **c** 6) **a** 7) **a** 8) **b**
9) **a** 10) **b** 11) **b** 12) **a** 13) **abc** 14) **bd** 15) **ad** 16) **d**
17) **c** 18) **a** 19) **A-s;B-r;C-p;D-q** 20) **3** 21) **3** 22) **3**

EXERCISE-II

LEVEL-I

1) **1** 2) **4** 3) **1** 4) **3** 5) **1** 6) **4** 7) **2** 8) **2**
9) **1** 10) **1** 11) **1** 12) **1** 13) **1** 14) **2** 15) **1** 16) **2**
17) **3** 18) **4** 19) **1** 20) **1** 21) **1** 22) **1** 23) **1** 24) **4**
25) **3** 26) **0.5** 27) **0.33** 28) **0.5**

LEVEL-II

1) **b** 2) **a** 3) **b** 4) **c** 5) **c** 6) **d** 7) **c** 8) **c**
9) **c** 10) **a** 11) **a** 12) **a** 13) **d** 14) **d** 15) **c** 16) **c**
17) **ac** 18) **abcd** 19) **bd** 20) **bd** 21) **bc** 22) **ac** 23) **bd** 24) **ac**
25) **ad** 26) **ab** 27) **bcd** 28) **b** 29) **a** 30) **c**
31) **A-s;B-q;C-r;D-p** 32) **A-p;B-r;C-s;D-q** 33) **A-s;B-r;C-q;D-p**
34) **A-s;B-p;C-q;D-q** 35) **1** 36) **1** 37) **7** 38) **5** 39) **9**
40) **1** 41) **2** 42) **3**

EXERCISE-III

LEVEL-I

1) **4** 2) **2** 3) **4** 4) **4** 5) **1** 6) **1** 7) **2** 8) **1**
9) **4** 10) **2** 11) **4** 12) **3** 13) **1** 14) **2** 15) **3** 16) **1**
17) **1** 18) **3** 19) **1** 20) **1** 21) **4** 22) **4** 23) **4** 24) **3**
25) **3** 26) **0.85** 27) **0.5** 28) **0.5**

LEVEL-II

1) **a** 2) **c** 3) **d** 4) **c** 5) **a** 6) **a** 7) **b** 8) **a**
9) **b** 10) **b** 11) **a** 12) **c** 13) **acd** 14) **ab** 15) **bc** 16) **d**
17) **c** 18) **c** 19) **a** 20) **b** 21) **c** 22) **1** 23) **0** 24) **7**

PRACTICE SHEET

EXERCISE-I

Formula based Problems, Integration by Substitution & Integration by using partial Fractions

LEVEL-I (MAIN)

Single answer type questions

1. $\int \frac{x^4+x^2+1}{x^2-x+1}\,dx =$

1) $\frac{x^5}{5}+\frac{x^3}{3}+x+c$ 2) $\frac{x^3}{3}-\frac{x^2}{2}+x+c$ 3) $\frac{x^4}{4}-\frac{x^2}{3}+c$ 4) $\frac{x^3}{3}+\frac{x^2}{2}+x+c$

2. $\int \frac{x^5}{x^2+1}\,dx =$

1) $\frac{x^4}{4}+\frac{x^3}{3}-\tan^{-1}x+c$ 2) $\frac{x^4}{4}-\frac{x^2}{2}+\frac{1}{2}\log(x^2+1)+c$

3) $\frac{x^4}{4}+\frac{x^3}{3}+\tan^{-1}x+c$ 4) $\frac{x^4}{4}+\frac{x^3}{5}-\tan^{-1}x+c$

3. If $\int \frac{\sin^2\alpha-\sin^2 x}{\cos x-\cos\alpha}\,dx = f(x)+Ax+B$ and $B\in R$ then

1) $f(x) = 2\sin x,\ A = \cos\alpha$ 2) $f(x) = 2\sin x,\ A = 2\cos\alpha$

3) $f(x) = \sin x,\ A = \cos\alpha$ 4) $f(x) = \sin x,\ A = 2\cos\alpha$

4. $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x.\cos^2 x}\,dx =$

1) $\sin x+\cos x+c$ 2) $\tan x+\cot x+c$ 3) $\sec x-\operatorname{cosec} x+c$ 4) $\sin n\ x-\cos x+c$

5. $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}\,dx =$

1) $\frac{1}{2}\cos 2x+C$ 2) $\frac{-1}{2}\cos 2x+C$ 3) $\frac{1}{2}\sin 2x+C$ 4) $\frac{-1}{2}\sin 2x+C$

6. If $\int \frac{1}{\sin(x-a)\sin(x-b)}\,dx = A\log\left[\frac{\sin(x-b)}{\sin(x-a)}\right]+c$ then $A =$

1) $\sin(a-b)$ 2) $\sin(b-a)$ 3) $\operatorname{cosec}(b-a)$ 4) $\operatorname{cosec}(a-b)$

7. $\int \frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}\,dx =$

1) $\sin^{-1}\left(\frac{\sin x+\cos x}{2}\right)+c$ 2) $\sin^{-1}\left(\frac{\sin x+\cos x}{3}\right)+c$ 3) $\sin^{-1}\left(\frac{\sin x+\cos x}{4}\right)+c$ 4) $\sin^{-1}(\sin x+\cos x)+c$

8. $\int \frac{\cos x + x\sin x}{x(x+\cos x)}\,dx =$

1) $\log\left|\frac{x+\cos x}{x}\right|+c$ 2) $\log|x\,(x+\cos x)|+c$ 3) $\log\left|\frac{x}{x+\cos x}\right|+c$ 4) $\log|x(\cos x+x\sin x)|+c$

9. If $\int \frac{dx}{x^2\sqrt{1+x^2}} = k\,\frac{\sqrt{1+x^2}}{x}+c$ then $k =$

1) 1 2) 1/2 3) −1/2 4) −1

10. $\int \frac{7x^8+8x^7}{(1+x+x^8)^2}dx = f(x)+c \Rightarrow f(x) =$

1) $\frac{x^8}{1+x+x^8}$ 2) $28\log(1+x+x^8)$ 3) $\frac{1}{1+x+x^8}$ 4) $\frac{-1}{1+x+x^8}$

11. $\int x^2\sqrt{x^6-1}\,dx =$

1) $\frac{1}{6}\left\{x^3\sqrt{x^6-1}-\log\left(x^3+\sqrt{x^6-1}\right)\right\}+c$ 2) $\frac{1}{6}\left\{x^3\sqrt{x^6-1}+\log\left(x^3+\sqrt{x^6-1}\right)\right\}+c$

3) $\frac{1}{6}\left\{x^3\sqrt{x^6-1}-\sin^{-1}(x^3)\right\}+c$ 4) $\frac{1}{6}\left\{x^3\sqrt{x^6-1}+\sinh^{-1}(x^3)\right\}+c$

12. $\int \frac{\cot x}{1+\sin^2 x}\,dx =$

1) $\log\left|\frac{\sin x}{\sqrt{1+\sin^2 x}}\right|+C$ 2) $\log\left|\frac{\sqrt{1+\sin^2 x}}{\sin x}\right|+C$ 3) $\frac{1}{2}\log\left|\frac{\sin x}{\sqrt{1+\sin^2 x}}\right|+C$ 4) $\frac{1}{2}\log\left|\frac{\sqrt{1+\sin^2 x}}{\sin x}\right|+C$

13. The indefinite integral of 2 sin$x$ w.r.t. cos$x$ is

1) cos$x - x + c$ 2) sin$x + x + c$ 3) $\frac{1}{2}\sin 2x - x + c$ 4) $\frac{1}{2}\cos 2x + x + c$

LEVEL-II (ADVANCED)

Single answer type questions

1. $\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx =$

a) $\frac{x^4}{6(2x^4+3x^2+1)^4}+c$ b) $\frac{x^4}{(2x^4+3x^2+1)^4}+c$ c) $\frac{x^{12}}{6(2x^4+3x^2+1)^4}+c$ d) $\frac{x^{12}}{(2x^4+3x^2+1)^4}+c$

2. $\int \frac{\{f(x).\phi^1(x)-f^1(x).\phi(x)\}}{f(x).\phi(x)}\{\log\phi(x)-\log f(x)\}\,dx =$

a) $\log\frac{\phi(x)}{f(x)}+k$ b) $\frac{1}{2}\left\{\log\frac{\phi(x)}{f(x)}\right\}^2+k$ c) $\frac{\phi(x)}{f(x)}\log\frac{\phi(x)}{f(x)}+k$ d) $\left\{\log\frac{\phi(x)}{f(x)}\right\}^2+k$

3. If $f(0) = f^1(0) = 0$ and $f^{11}(x) = \tan^2 x$ then $f(x)$ is

a) $\log\sec x-\frac{1}{2}x^2$ b) $\log\cos x+\frac{1}{2}x^2$ c) $\log\sec x+\frac{1}{2}x^2$ d) $-\log\sec x+\frac{1}{2}x^2$

4. If $\int f(x)dx = f(x)$, then $\int \{f(x)\}^2 dx =$

a) $\frac{1}{2}\{f(x)\}^2$ b) $\{f(x)\}^3$ c) $\frac{\{f(x)\}^3}{3}$ d) $\{f(x)\}^2$

5. $\int \frac{f(x)}{x^3-1}dx =$ (where $f(x)$ is a polynomial of degree 2 in x such that $f(0) = f(1) = 3f(2) = -3$)

a) $-\log|x-1| + \log|x^2+x+1| + \frac{2}{\sqrt{3}}\text{Tan}^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

b) $-\log|x-1| + \log|x^2+x+1| + \frac{4}{\sqrt{3}}\text{Tan}^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

c) $-\log|x-1| + \log|x^2+x+1| + \frac{6}{\sqrt{3}}\text{Tan}^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

d) $-\log|x-1| + \log|x^2+x+1| + \frac{1}{\sqrt{3}}\text{Tan}^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

More than one correct answer type questions

6. If $\int \text{cosec}2x dx = f(g(x)) + C$, then

a) range $g(x) = (-\infty, \infty)$ b) dom $f(x) = (-\infty, \infty) \sim \{0\}$ c) $g^1(x) = \sec^2 x$ d) $f^1(x) = \frac{1}{x} \forall x \in (0, \infty)$

7. If $f^1(x) = \frac{1}{-x+\sqrt{x^2+1}}$ and $f(0) = -\frac{1+\sqrt{2}}{2}$ then $f(1)$ is equal to

a) $-\frac{1}{2}\log(\sqrt{2}-1)$ b) $-\log(\sqrt{2}-1)$ c) $\log(1+\sqrt{2})$ d) $\frac{1}{2}\log(1+\sqrt{2})$

8. If $f(x) = \int \frac{x^8+4}{x^4-2x^2+2}dx$ and $f(0) = 0$ then

a) $f(x)$ is an odd function b) $f(x)$ has range R

c) $f(x)$ has atleast one real root d) $f(x)$ R monotonic function

Matrix matching type questions

9.

COLUMN - I	COLUMN - II
A) $\int \frac{dx}{\sqrt{2}(2+x)}$	p) $\frac{1}{\sqrt{2}}Tan^{-1}\left(\frac{x}{2}\right)+C$
B) $\int \frac{dx}{\sqrt{x}(2+\sqrt{x})}$	q) $\sqrt{2}Tan^{-1}\left(\frac{\sqrt{x}}{2}\right)+C$
C) $\int \frac{dx}{x(2+x)}$	r) $2\log_e(2+\sqrt{x})+C$
D) $\int \frac{1}{2+x^2}dx$	s) $\frac{1}{2}\log_e\left(\frac{x}{x+2}\right)+C$

EXERCISE-II

Special Types of integration: Rational functions of sinx and cosx

LEVEL-I (MAIN)

Single answer type questions

1. $\int \frac{dx}{(x+1)\sqrt{4x+3}} =$

1) $\tan^{-1}\sqrt{4x+3}+c$ 2) $3\tan^{-1}\sqrt{4x+3}+c$ 3) $2\tan^{-1}\sqrt{4x+3}+c$ 4) $4\tan^{-1}\sqrt{4x+3}+c$

2. If $\int \frac{1}{5+4\cos 2\theta}\, d\theta = A\tan^{-1}(B\tan\theta)+c$ then $(A, B) =$

1) $\left(\frac{1}{2},\frac{1}{2}\right)$ 2) $\left(\frac{1}{3},\frac{1}{3}\right)$ 3) $\left(\frac{1}{2},3\right)$ 4) $\left(\frac{1}{3},2\right)$

3. $\int \frac{1}{4\cos^2 x+9\sin^2 x}dx =$

1) $\frac{1}{6}\tan^{-1}\left(\frac{2}{3}\tan x\right)+C$ 2) $\tan^{-1}\left(\frac{1}{3}\tan x\right)+C$ 3) $\frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right)+C$ 4) $\frac{1}{6}\tan^{-1}\left(\frac{1}{2}\tan x\right)+C$

4. $\int \frac{\sin x + 8\cos x}{4\sin x + 6\cos x}dx =$

1) $x+\frac{1}{2}\log(4\sin x+6\cos x)+c$ 2) $2x+\log(2\sin x+3\cos x)+c$

3) $x+2\log(2\sin x+3\cos x)+c$ 4) $\frac{1}{2}\log(4\sin x+6\cos x)+c$

5. $\int \frac{1}{\sqrt{x}+x}dx =$

1) $\log(x+\sqrt{x})+c$ 2) $\log(1+\sqrt{x})+c$ 3) $2\log(x+\sqrt{x})+c$ 4) $2\log(1+\sqrt{x})+c$

6. $\int \frac{dx}{(x-2)^{7/8}(x+3)^{9/8}} =$

1) $\frac{8}{5}\left(\frac{x-2}{x+3}\right)^{1/8}+c$ 2) $\frac{8}{5}\left(\frac{x+2}{x+3}\right)^{1/8}+c$ 3) $\frac{4}{5}\left(\frac{x-2}{x+3}\right)^{1/8}+c$ 4) $\frac{5}{8}\left(\frac{x-2}{x+3}\right)^{1/8}+c$

7. $\int \frac{x^2+1}{x^4+1}dx =$

1) $\frac{1}{\sqrt{2}}\text{Tan}^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$ 2) $\sqrt{2}\text{Sin}^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$ 3) $\frac{1}{\sqrt{2}}\text{Sinh}^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$ 4) $\sqrt{2}\text{Cosh}^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$

8. If $\int \frac{5e^x+3e^{-x}}{7e^x+4e^{-x}}dx = Ax+B\ln(7e^{2x}+4)+c$ then $4A + 14B =$

1) 1 2) $\frac{11}{4}$ 3) 18 4) $\frac{13}{4}$

9. $\int \sec^2 x \operatorname{cosec}^4 x\, dx = -\frac{1}{3}\cot^3 x + k\tan x - 2\cot x + c \Rightarrow k =$

1) 4 2) 3 3) 2 4) 1

10. $\int\left(\sqrt{\frac{a+x}{a-x}} + \sqrt{\frac{a-x}{a+x}}\right)dx =$

1) $2\sin^{-1}\left(\frac{x}{a}\right)+c$ 2) $2a\sin^{-1}\left(\frac{x}{a}\right)+c$ 3) $2\cos^{-1}\left(\frac{x}{a}\right)+c$ 4) $2a\cos^{-1}\left(\frac{x}{a}\right)+c$

11. $\int \frac{dx}{x^2\sqrt{4+x^2}} =$

1) $\frac{1}{4}\sqrt{4+x^2}+c$ 2) $\frac{-1}{4}\sqrt{4+x^2}+c$ 3) $\frac{-1}{4x}\sqrt{4+x^2}+c$ 4) $\frac{9}{4x}\sqrt{4+x^2}+c$

12. $\int \frac{dx}{\sqrt{x-x^2}} =$

1) $2\sin^{-1}\sqrt{x}+c$ 2) $2\sin^{-1}x+c$ 3) $2x\sin^{-1}x+c$ 4) $\sin^{-1}\sqrt{x}+c$

LEVEL-II (ADVANCED)

Single answer type questions

1. If $\int \frac{3\cot 3x - \cot x}{\tan x - 3\tan 3x}dx = ax + b\ln\left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right| + c$ then $(a, b) =$

a) $\left(1, -\frac{1}{\sqrt{2}}\right)$ b) $\left(1, -\frac{1}{\sqrt{3}}\right)$ c) $\left(1, \frac{1}{\sqrt{3}}\right)$ d) $\left(1, -\frac{1}{\sqrt{5}}\right)$

2. $\int \frac{(1-\cos\theta)^{2/7}}{(1+\cos\theta)^{9/7}}d\theta =$

a) $\frac{7}{11}\left(\tan\frac{\theta}{3}\right)^{11/7}+c$ b) $\frac{7}{11}\left(\tan\frac{\theta}{2}\right)^{11/7}+c$ c) $\frac{11}{7}\left(\tan\frac{\theta}{2}\right)^{11/7}+c$ d) $\frac{7}{11}\left(\cot\frac{\theta}{2}\right)^{11/7}+c$

3. $\int \frac{1}{1-\cos^4 x}dx =$

a) $\frac{-1}{2}\tan x - \frac{1}{2\sqrt{2}}\cot^{-1}(\sqrt{2}\cot x)+c$ b) $\frac{-1}{2}\cot x + \frac{1}{2\sqrt{2}}\tan^{-1}(\sqrt{2}\cot x)+c$

c) $\frac{-1}{3}\cot x - \frac{1}{4\sqrt{2}}\tan^{-1}(\sqrt{2}\cot x)+c$ d) $\frac{-1}{2}\cot x - \frac{1}{2\sqrt{2}}\tan^{-1}(\sqrt{2}\cot x)+c$

4. $\int \frac{dx}{x^4\sqrt{a^2+x^2}} =$

a) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{3x^3}\sqrt{a^2+x^2}\right]+c$ b) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{3x^3}(a^2+x^2)^{3/2}\right]+c$

c) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{2\sqrt{x}}(a^2+x^2)^{3/2}\right]+c$ d) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{x^3}(a^2+x^2)^{3/2}\right]+c$

5. $\int\left(\frac{2\sin 2x-\cos x}{6-\cos^2 x-4\sin x}\right)dx =$

a) $2\log\left|\sin^2 x-3\sin x+5\right|+7\tan^{-1}(\sin x-2)+c$

b) $2\log\left|\sin^2 x-4\sin x+5\right|+8\tan^{-1}(\sin x-2)+c$

c) $2\log\left|\sin^2 x-4\sin x+5\right|+7\tan^{-1}(\sin x-2)+c$

d) $\log\left|\sin^2 x-\sin x+5\right|+7\tan^{-1}(\sin x-2)+c$

More than one correct answer type questions

6. If $\int[1+\tan x\tan(x+\alpha)]dx = A\log\left|\frac{\sec(x+\alpha)}{f(x)}\right|+c$ then

a) $A=\cot\alpha$ b) $A=\tan\alpha$ c) $f(x)=\sec x$ d) $f(x)=\cos x$

7. $\int\frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}=(\beta>\alpha)$

a) $2\text{Cos}^{-1}\sqrt{\left(\frac{\beta-x}{\beta-\alpha}\right)}+c$ b) $2\text{Sin}^{-1}\sqrt{\frac{x-\alpha}{\beta-\alpha}}+c$

c) $2\tan^{-1}\sqrt{\frac{x-\alpha}{\beta-x}}+c$ d) $2\cot^{-1}\sqrt{\frac{\beta-\alpha}{x-\alpha}}+c$

8. If $\int\frac{dx}{\tan x+\cot x+\sec x+\text{cosec}x}=\frac{1}{2}[f(x)-g(x)-x]+c$ then

a) $f(x)=\sin x$ b) $f(x)=-\sin x$ c) $g(x)=\cos x$ d) $g(x)=-\cos x$

9. If $\int\sec^2 x\,\text{cosec}^4 x\,dx=\frac{-1}{3}[f(x)]^3+g(x)-2f(x)+c$

a) $f(x)=\cot x$ b) $g(x)f(x)=1$ c) $g(x)f(x)=-1$ d) $f(x)=\tan x$

10. If $\int\frac{dx}{x^n\left(1+x^n\right)^{1/n}}=A(f(x))^{1-\frac{1}{n}}+C$ then

a) A = 1- n b) $A=\frac{1}{1-n}$ c) f(x) = $1+x^n$ d) $f(x)=1+\frac{1}{x^n}$

Linked comprehension type questions

Passage - I :

If the integrande is a rational function of x and fractional power of a linear fractional function of the form $\left(\frac{ax+b}{cx+d}\right)$ ie, $\int f\left(x,\left(\frac{ax+b}{cx+d}\right)^{m/n},.....,\left(\frac{ax+b}{cx+d}\right)^{r/s}\right)dx$ in this form substitute $\frac{ax+b}{cx+d}=t^m$ where m is the LCM of the denominatiors of fractional powers of $\left(\frac{ax+b}{cx+d}\right)$

11. The value of $\int \frac{dx}{(1+x)^{1/2}-(1+x)^{1/3}}$ is

a) $2\lambda^{1/2}+3\lambda^{1/3}+6\lambda^{1/6}+6In\left|\lambda^{1/6}-1\right|+c$ b) $2\lambda^{1/2}-3\lambda^{1/3}+6\lambda^{1/6}+6In\left|\lambda^{1/6}-1\right|+c$

c) $2\lambda^{1/2}+3\lambda^{1/3}-6\lambda^{1/6}+6In\left|\lambda^{1/6}-1\right|+c$

d) $2\lambda^{1/2}+3\lambda^{1/3}+6\lambda^{1/6}-6In\left|\lambda^{1/6}-1\right|+c$ Where $\lambda=1+x$

12. The value of $\int \frac{1+x^{1/2}-x^{2/3}}{1+x^{1/3}}dx$ is

a) $\frac{3}{4}x^{4/3}+\frac{6}{7}x^{7/6}+x+\frac{6}{5}x^{5/6}+2x^{1/2}-6x^{1/6}+6\tan^{-1}(x^{1/6})+c$

b) $-\frac{3}{4}x^{4/3}+\frac{6}{7}x^{7/6}+x+\frac{6}{5}x^{5/6}-2x^{1/2}+6x^{1/6}-6\tan^{-1}(x^{1/6})+c$

c) $\frac{3}{4}x^{4/3}-\frac{6}{7}x^{7/6}+x-\frac{6}{5}x^{5/6}+2x^{1/2}-6x^{1/6}+6\tan^{-1}(x^{1/6})+c$

d) $-\frac{3}{4}x^{4/3}+\frac{6}{7}x^{7/6}+x-\frac{6}{5}x^{5/6}+2x^{1/2}-6x^{1/6}+6\tan^{-1}(x^{1/6})+c$

13. $\int \frac{\sqrt{x}}{\sqrt{x}-\sqrt[3]{x}}dx=$

a) $6\left\{\frac{x}{6}+\frac{x^{5/6}}{5}+\frac{x^{1/2}}{2}+\frac{x^{1/3}}{3}+\log(x^{1/6}-1)\right\}+c$ b) $6\left\{\frac{x}{6}+\frac{x^{6/5}}{5}+\frac{x^{1/2}}{3}+\frac{x^{1/3}}{2}+\log\left(x^{1/6}-1\right)\right\}+c$

c) $6\left\{\frac{x}{6}+\frac{x^{5/6}}{5}+\frac{x^{2/3}}{4}+\frac{x^{1/2}}{3}+\frac{x^{1/3}}{2}+x^{1/6}+\log(x^{1/6}-1)\right\}+c$

d) $3\left\{\frac{x}{6}+\frac{x^{5/6}}{5}+\frac{x^{1/2}}{2}+\frac{x^{1/3}}{3}+\log(x^{1/6}-1)\right\}+c$

Matrix matching type questions

14.

COLUMN - I	COLUMN - II
A) $\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}dx=$	p) $\frac{-\cot(x+\alpha)}{a^2+b^2}+c,\ \alpha=\tan^{-1}\left(\frac{b}{a}\right)$
B) $\int \frac{dx}{(a\sin x+b\cos x)^2}=$	q) $\sin^{-1}x+\sqrt{1-x^2}-\sqrt{\frac{1-x}{1+x}}+c$
C) $\int \frac{1+x\cos x}{x\left(1-x^2e^{2\sin x}\right)}dx=$	r) $\frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right)+c$
D) $\int \frac{2-x^2}{(1+x)\sqrt{1-x^2}}dx=$	s) $\frac{1}{2}\log\left\|\frac{x^2e^{2\sin x}}{1-x^2e^{2\sin x}}\right\|+c$

15. **COLUMN - I** **COLUMN - II**

A) $\int \frac{dx}{x^4+x^6} =$ p) $\left(\frac{\sqrt{2}}{9}, \sin x - \cos x\right)$

B) $\int \frac{\sin 2x}{a^2\sin^4 + b^2\cos^4 x}dx =$ q) $\text{Tan}^{-1}x - \frac{1}{3x^3} + \frac{1}{x} + C$

C) $\int \frac{dx}{\sin 2x.Tan^9 x} =$ r) $\frac{-Cot^9 x}{18} + C$

D) If $\int \frac{dx}{\sin^3 x + \cos^3 x} = A\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right| + B\text{Tan}^{-1}(t) + C$, s) $\frac{1}{ab}\text{Tan}^{-1}\left(\frac{a}{b}\text{Tan}^2 x\right) + C$

then $(AB, t) =$

Integer answer type questions

16. If $\int \sqrt{1+\text{cosec}x}dx = \sin^{-1}(A\sin x + B) + c$ then $A + B =$

17. If $\int \frac{dx}{1+\sqrt{x^2+2x+2}} = A\log\left|x+1+\sqrt{x^2+2x+2}\right| + B\frac{\sqrt{x^2+2x+2}-1}{x+1} + c$ then $A + B =$

18. If $\int \frac{(x^4-x)^{1/4}}{x^5}dx = \frac{A}{B}\left(1-\frac{1}{x^3}\right)^{5/4} + c$ then $B-2A=$

19. If $\int \frac{\cos^5 x}{\sin^2 x}dx = A\sin^3 x + B\sin x + C\text{cosec}x + D$ then $3A - 2B - C =$

20. If $\int \frac{dx}{\cos^2 x + \cot^2 x} = A\tan x + B\tan^{-1}\left(\frac{\cot x}{\sqrt{2}}\right) + c$ then $\frac{A}{B^2} =$

21. If $\int \frac{(2x+1)}{(x^2+4x+1)^{3/2}}dx = \frac{Ax}{(x^2+4x+1)^{B/C}} + C$ then $A+B+C =$

22. If $\int \frac{\sqrt{5+x^{10}}}{x^{16}}dx = \frac{-1}{15K}\left[1+\frac{5}{x^{10}}\right]^{3/2} + C$ then $K =$

23. If $\int \frac{dx}{x^3\sqrt{x^2-1}} = \frac{1}{p}\left[\frac{\sqrt{x^2-1}}{x^2} + Q\text{Tan}^{-1}\sqrt{x^2-1}\right] + C$ then $P/Q =$

24. If $\int \sqrt{\frac{3-x}{3+x}}Sin^{-1}\left(\frac{1}{\sqrt{6}}\sqrt{3-x}\right)dx$

$= \frac{1}{A}\left\{2x + 2\sqrt{9-x^2}Cos^{-1}\left(\frac{x}{3}\right) + B\left(\cos^{-1}\frac{x}{3}\right)^2\right\} + C$ then the value of A – B is

25. If $\int \frac{\cos^2 x\sin x}{\sin x - \cos x}dx = \frac{1}{A}\log|\sin x - \cos x| + \frac{\sin 2x + \cos 2x}{B} + C$ then $B/A =$

26. If $\int \frac{x+(\cos^{-1}3x)^2}{\sqrt{1-9x^2}}dx = A\sqrt{1-9x^2}+B(\cos^{-1}3x)^3+C$ then the value of $A-B=$

27. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx = \frac{\sqrt{2x^4-2x^2+1}}{Ax^B}+C$ then $A+B=$

28. If $\int \sec(x-\frac{\pi}{3})\operatorname{cosec}\left(x-\frac{\pi}{6}\right)dx = \frac{A}{\sqrt{B}}\log\left|\frac{\sin(x-\frac{\pi}{6})}{\cos(x-\frac{\pi}{3})}\right|+C$ then $AB=$

EXERCISE-III

Integration by parts and Reduction formulae

LEVEL-I (MAIN)

Single answer type questions

1. $\int \cos^{-1}(2x^2-1)dx=$

1) $2\left(x\sin^{-1}x+\sqrt{1-x^2}\right)+c$ 2) $2\left(x\cos^{-1}x+\sqrt{1-x^2}\right)+c$

3) $2\left(x\cos^{-1}x-\sqrt{1-x^2}\right)+c$ 4) $2\left(x\sin^{-1}x-\sqrt{1-x^2}\right)+c$

2. If $I_1=\int \text{Sin}^{-1}\left(\frac{2x}{1+x^2}\right)dx$, $I_2=\int \text{Cos}^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$, $I_3=\int \text{Tan}^{-1}\left(\frac{2x}{1-x^2}\right)dx$, then $I_1+I_2-I_3=$

1) $2x\text{Tan}^{-1}x-\log(1+x^2)+c$ 2) $2[x\text{Tan}^{-1}x-\log(1+x^2)]+c$

3) $2x\text{Tan}^{-1}x+\log(1+x^2)+c$ 4) 0

3. $\int[\text{Sin}(\log x)+\text{Cos}(\log x)]\,dx=$

1) $x\ \text{Cos}(\log x)+c$ 2) $-x\ \text{Cos}(\log x)+c$ 3) $x\ \log(\text{Sin}x)+c$ 4) $x\ \text{Sin}(\log x)+c$

4. $\int(1+x-x^{-1})e^{x+x^{-1}}dx=$

1) $(x+1)e^{x+x^{-1}}+c$ 2) $(x-1)e^{x+x^{-1}}+c$ 3) $\frac{1}{2}xe^{x+x^{-1}}+c$ 4) $x.e^{x+x^{-1}}+c$

5. If $I_n=\int \text{Cos}^n x\,dx$ then $I_7-\frac{\text{Cos}^6 x\text{Sin}x}{7}=$

1) $\frac{6}{7}I_5$ 2) $-\frac{6}{7}I_5$ 3) $\frac{5}{8}I_5$ 4) $-\frac{5}{8}I_5$

6. If $I_{m,n}=\int x^m(\log x)^n dx$ then $I_{m,n}-\frac{x^{m+1}}{(m+1)}(\log x)^n=$

1) $\frac{n}{m+1}\cdot I_{m,n-1}$ 2) $\frac{m}{n+1}\cdot I_{m,n-1}$ 3) $-\frac{n}{m+1}\cdot I_{m,n-1}$ 4) $\frac{n}{m+1}\cdot I_{m-1,n-1}$

7. If $I_n = \int \frac{\sin nx}{\sin x} dx$ where $n(>1) \in N$ then $I_n - I_{n-2} =$

1) $\frac{2}{n-1}\cos(n-1)x + c$ 2) $\frac{2}{n-1}\sin(n-1)x + c$ 3) $\frac{-2}{n-1}\sin(n-1)x + c$ 4) $\frac{-2}{n-1}\cos(n-1)x + c$

8. If $\phi(x) = \int \cot^4 x\, dx + \frac{1}{3}\cot^3 x - \cot x$ and $\phi\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$ then $\phi(x) =$

1) $\pi - x$ 2) $x - \pi$ 3) $\frac{\pi}{2} - x$ 4) x

LEVEL-II (ADVANCED)

Single answer type questions

1. If $\int e^x \left[\frac{x^3 + 3x^2 + 4}{(x+1)^3}\right] dx = e^x \left[\frac{x^2 + Ax + B}{(x+1)^2}\right] + C$ then A+B =

a) 4 b) 0 c) 2 d) –4

2. Let $\int e^x \{f(x) - f^1(x)\} dx = \phi(x)$. Then $\int e^x f(x) dx$ is

a) $\phi(x) + e^x f(x)$ b) $\phi(x) - e^x f(x)$ c) $\frac{1}{2}\{\phi(x) + e^x f(x)\}$ d) $2\{\phi(x) + e^x f(x)\}$

3. $\int e^x \left(\frac{x+2}{x+4}\right)^2 dx =$

a) $e^x\left(\frac{x}{x+1}\right) + c$ b) $e^x\left(\frac{x}{x+2}\right) + c$ c) $e^x\left(\frac{x}{x+3}\right) + c$ d) $e^x\left(\frac{x}{x+4}\right) + c$

4. $\int \text{Sec}^{-1}\sqrt{x}\, dx =$

a) $x\sec^{-1}\sqrt{x} + \sqrt{x-1} + C$ b) $x\sec^{-1}\sqrt{x} - \sqrt{x-2} + C$

c) $x\sec^{-1}\sqrt{x} - \sqrt{x+1} + C$ d) $x\sec^{-1}\sqrt{x} - \sqrt{x-1} + C$

More than one correct answer type questions

5. If $\int x\log(1+x^2) dx = \phi(x).\log(1+x^2) + \Psi(x) + c$ then

a) $\phi(x) = \frac{1+x^2}{2}$ b) $\Psi(x) = \frac{1+x^2}{2}$ c) $\Psi(x) = -\frac{1+x^2}{2}$ d) $\phi(x) = -\frac{1+x^2}{2}$

6. If $\int e^{2ax}\left[\frac{1-\cos 2ax}{1+\sin 2ax}\right] dx = \frac{-1}{2a} e^{f(x)} g(x) + c$ then

a) $f(x) = ax$ b) $f(x) = 2ax$ c) $g(x) = \cot\left(\frac{\pi}{4} + ax\right)$ d) $g(x) = \tan\left(\frac{\pi}{4} - ax\right)$

7. If $\int x\log\left(1 + \frac{1}{x}\right) dx = \frac{x^2}{2}\log f(x) + \frac{1}{2} g(x) + C$ than

a) $f(x) = \frac{x+1}{x}$ b) $f(x) = \frac{x}{x+1}$ c) $g(x) = x + \log(1+x)$ d) $g(x) = x - \log(1+x)$

Linked comprehension type questions

Passage - I :

If $f(x)$ is a differentiable function, then $\int e^x[f(x)+f'(x)]dx = e^x f(x)+c$ Using the above information. Simplify the following.

8. $\int e^x \frac{2-x^2}{(1-x)\sqrt{1-x^2}}dx =$

a) $e^x\sqrt{\frac{1-x}{1+x}}\frac{1}{(1-x)^2}+c$ b) $e^x\sqrt{\frac{1+x}{1-x}}+c$ c) $e^x\sqrt{\frac{1-x}{1+x}}+c$ d) $-e^x\sqrt{\frac{1-x}{1+x}}+c$

9. $\int \frac{e^{\tan^{-1}x}}{(1+x^2)}\left[(\sec^{-1}\sqrt{1+x^2})^2+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]dx =$

a) $e^{\tan^{-1}x}(\tan^{-1}x)^2+c$ b) $e^{\tan^{-1}x}(\cot^{-1}x)^2+c$ c) $e^{\tan^{-1}x}(\sin^{-1}x)^2+c$ d) $e^{\tan^{-1}x}(\cos^{-1}x)^2+c$

10. *If* $x\in\left(0,\frac{\pi}{2}\right)$, then $\int e^{-x/2}.\frac{\sqrt{1-\sin x}}{1+\cos x}dx =$

a) $e^{-x/2}\sec\frac{x}{2}+c$ b) $-e^{-x/2}\sec\frac{x}{2}+c$ c) $e^{x/2}.\sec\frac{x}{2}+c$ d) $-e^{x/2}.\sec\frac{x}{2}+c$

KEY SHEET (PRACTICE SHEET)

EXERCISE-I

LEVEL-I

1) 4 2) 2 3) 3 4) 3 5) 4 6) 3 7) 2 8) 3
9) 4 10) 1 11) 1 12) 1 13) 3

LEVEL-II

1) c 2) b 3) a 4) a 5) a 6) abc 7) ad 8) abcd
9) A-q;B-r;C-s-D-p

EXERCISE-II

LEVEL-I

1) 3 2) 2 3) 3 4) 1 5) 4 6) 1 7) 1 8) 2
9) 4 10) 2 11) 3 12) 1

LEVEL-II

1) b 2) b 3) d 4) b 5) c 6) ac 7) abc 8) ac
9) ab 10) bd 11) a 12) d 13) c 14) A-r;B-p;C-s;D-q
15) A-q;B-s;C-r;D-p 16) 1 17) 0 18) 7 19) 6 20) 4
21) 4 22) 5 23) 2 24) 7 25) 2 26) 0 27) 4 28) 6

EXERCISE-III

LEVEL-I

1) 3 2) 1 3) 4 4) 4 5) 1 6) 3 7) 2 8) 4

LEVEL-II

1) b 2) c 3) d 4) d 5) ac 6) bcd 7) ad 8) b
9) a 10) b

ADDITIONAL EXERCISE

LEVEL-I (MAIN)

Single answer type questions

1. $\int\{1+\tan x.\tan(x+\alpha)\}dx =$

1) $\cot\alpha\log|\cot(x+\alpha)|+c$ 2) $\cot\alpha\log\left|\frac{\cot(x+\alpha)}{\cos x}\right|+c$

3) $\cot\alpha\log\left|\frac{\cos x}{\cos(x+\alpha)}\right|+c$ 4) $\cot\alpha\log\left|\frac{\sin x}{\cot(x+\alpha)}\right|+c$

2. The equation of a curve passing through origin is given by $y=\int x^3\cos x^4dx$. If the equation of the curve is written in the form $x = g(y)$, then

1) $g(y)=\sqrt[3]{\sin^{-1}(4y)}$ 2) $g(y)=\sqrt{\sin^{-1}(4y)}$ 3) $g(y)=\sqrt[4]{\sin^{-1}(4y)}$ 4) $g(y)=\sqrt[5]{\sin^{-1}(4y)}$

3. $\int\frac{\sin^2 x.\sec^2 x+2\tan x.\sin^{-1}x.\sqrt{1-x^2}}{\sqrt{1-x^2}(1+\tan^2 x)}dx =$

1) $(\cos^2x).(\sin^{-1}x)+c$ 2) $(\sin^2x).(\sin^{-1}x)+c$ 3) $(\sec^2x).(\cos^{-1}x)+c$ 4) $(\sec^2x).(\tan^{-1}x)+c$

4. If $f(x)$ is a polynomial of nth degree then $\int e^x f(x)dx =$ where $f^n(x)$ denotes nth order derivative of $f(x)$ w.r.t.x

1) $e^x[f(x)-f^1(x)+f^{11}(x)+f^{111}(x)+.....+(-1)^{-1}f^n(x)]$

2) $e^x[f(x)+f^1(x)+f^{11}(x)+f^{111}(x)+.....+(-1)^{-1}f^n(x)]$

3) $e^x[f(x)+f^1(x)+f^{11}(x)+f^{111}(x)+.....+(-1)^{-1}f^{2n}(x)]$

4) $e^x[f(x)+f^1(x)+f^{11}(x)+f^{111}(x)+.....+(-1)^{-1}f^{3n}(x)]$

5. $\int x\{f(x^2)g''(x^2)-f''(x^2)g(x^2)\}dx$

1) $f(x^2)g'(x^2)-g(x^2)f'(x^2)+c$ 2) $\frac{1}{2}\{f(x^2)g(x^2)f'(x^2)\}+c$

3) $\frac{1}{2}\{f(x^2)g'(x^2)-g(x^2)f'(x^2)\}+c$ 4) $f(x^2)g(x^2)+c$

6. $\int e^{x/2}\sec 2x(1+4\tan 2x)dx$ is equal to

1) $4e^{x/2}\sec 2x+c$ 2) $2e^{x/2}\sec 2x+c$ 3) $e^{x/2}\sec 2x+c$ 4) $\frac{1}{2}e^{x/2}\sec 2x+c$

7. $\int\frac{x^2}{(a+bx)^2}dx =$

1) $\frac{1}{b^3}\left[(a+bx)-2a\log(a+bx)-\frac{a^2}{a+bx}\right]+c$ 2) $\frac{1}{b^3}\left[2a\log(a+bx)+\frac{a^2}{a+bx}\right]+c$

3) $\frac{1}{b^3}[2a-(a+bx)^2+a^2]+c$ 4) $\frac{1}{b^3}\left[2a\log(a+bx)-\frac{a^2}{a+bx}\right]+c$

8. $\int \frac{\left(\sqrt{x}\right)^3}{\left(\sqrt{x}\right)^5 + x^4} dx = A\log\left(\frac{x^k}{x^k+1}\right) + c$ then the values of A & K respectively are

1) $\frac{3}{2}, \frac{2}{3}$ 2) $\frac{3}{2}$, 2 3) 2, $\frac{3}{2}$ 4) $\frac{2}{3}, \frac{3}{2}$

9. $\int |x+y|\,dx$ where $\frac{dy}{dx} = 0$ is equal to

1) 0 2) $\frac{(x+y)^2}{2} + c$ 3) $\frac{-(x+y)^2}{2} + c$ 4) $\frac{(x+y)^2|x+y|}{2} + c$

10. $\int \frac{mx^{m+2n-1} - nx^{n-1}}{x^{2m+2n} + 2x^{m+n} + 1} dx$ is equal to

1) $\frac{x^m}{x^{m+n}+1} + c$ 2) $\frac{x^n}{x^{m+n}+1} + c$ 3) $\frac{x^{m+n}-1}{x^{m+n}+1} + c$ 4) $-\frac{x^n - 1}{x^{m+n}+1} + c$

11. $\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1+\sqrt[3]{x})} dx$ is equal to

1) $\frac{3}{2}x^{2/3} + 6\tan^{-1}x^{1/6} + c$ 2) $\frac{3}{2}x^{2/3} - 6\tan^{-1}x^{1/6} + c$

3) $-\frac{3}{2}x^{2/3} + 6\tan^{-1}x^{1/6} + c$ 4) $-\frac{3}{2}x^{2/3} - 6\tan^{-1}x^{1/6} + c$

12. $\int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2} dx = \frac{-x^2}{x\tan x + 1} + f(x) + c$ then $f(x) =$

1) $\log|x\sin x + \cos x| + c$ 2) $\log|x\cos x + \sin x| + c$ 3) $2\log|x\sin x + \cos x| + c$ 4) $2\log|x\cos x + \sin x| + c$

13. If [.] denotes greatest integer function and $x \in \left(\frac{2}{3}, 1\right)$ and $\int \frac{[5-3x]}{x-2} dx = k\log|x-2| + c$ then K =

1) 1 2) 2 3) 3 4) 4

14. $\int \frac{1}{(x+1)^2\sqrt{x^2+2x+2}} dx =$

1) $-\frac{\sqrt{x^2+2x+2}}{x+1} + c$ 2) $\frac{\sqrt{x^2+2x+2}}{x+1} + c$ 3) $2\frac{\sqrt{x^2+2x+2}}{x+1} + c$ 4) $\frac{1}{2}\frac{\sqrt{x^2+2x+2}}{x+1} + c$

15. If $f(x) = \lim_{n\to\infty} n^2(x^{1/n} - x^{1/(n+1)}), x > 0$ then $\int x f(x) dx$ is equal to

1) $\frac{x^2}{2} + c$ 2) 0 3) $x^2\log x - \frac{1}{2}x^2 + c$ 4) $\frac{x^2}{2}\log x - \frac{x^2}{4} + c$

16. The value of $\int (x^2+x)(x^{-8} + 2x^{-9})^{\frac{1}{10}} dx$ is

1) $\frac{5}{11}(x^2+2x)^{\frac{11}{10}} + c$ 2) $\frac{5}{6}(x+1)^{\frac{11}{10}} + c$ 3) $\frac{6}{7}(x+1)^{\frac{11}{10}} + c$ 4) $\frac{3}{5}(x+1)^{\frac{11}{10}} + c$

17. If $\int \frac{\sqrt[3]{(1+2x^3)^2}}{x^6}dx = -\frac{1}{K}\left(L+\frac{1}{x^3}\right)^{\frac{M}{N}}+c$ (HCF of M and N is 1). The value of (K+L+M+N)

1) 5 2) 10 3) 15 4) 20

18. Let $f(x)=\frac{x}{(1+x^n)^{1/n}}$ for $n>2$ and $g(x)=\underbrace{fofo.....of}_{f \text{ occurs } n \text{ times}}(x)$. then $\int x^{n-2}g(x)dx$ equal to

1) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$ 2) $\frac{1}{(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$

3) $\frac{1}{n(n-1)}(1+nx^n)^{1+\frac{1}{n}}+K$ 4) $\frac{1}{(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$

19. $\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}}dx =$

1) $2[\sqrt{x}-\sqrt{1-x}\arcsin\sqrt{x}]+c$ 2) $2[\sqrt{x}+\sqrt{1-x}\arcsin\sqrt{x}]+c$

3) $2[\sqrt{x}+\sqrt{1-x}\arccos\sqrt{x}]+c$ 4) $2[\sqrt{x}-\sqrt{1-x}\arccos\sqrt{x}]+c$

20. $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}dx$ is equal to

1) $12\left[\frac{(1+\sqrt[4]{x})^{7/3}}{7}+\frac{(1+\sqrt[4]{x})^{4/3}}{4}\right]+c$ 2) $12\left[\frac{(1+\sqrt[4]{x})^{7/3}}{7}-\frac{(1+\sqrt[4]{x})^{4/3}}{4}\right]+c$

3) $\frac{1}{12}\left[\left(\frac{1+x^{1/4}}{4}\right)^{4/3}-x\right]+c$ 4) $\frac{1}{12}\left[\left(\frac{1+x^{1/4}}{4}\right)^{4/3}+x\right]+c$

21. If $\frac{dI}{dy}=3^{\cos y}.\sin y$ then I is equal to

1) $3^{\cos y}+c$ 2) $-\frac{3^{\cos y}}{\log 3}+c$ 3) siny+c 4) $3^{\text{siny}}+c$

22. If $\int \sqrt{x}(1-x^3)^{-1/2}dx=\frac{2}{3}g(f(x))+c$ then

1) $f(x)=\sqrt{x}, g(x)=\sin^{-1}x$ 2) $f(x)=x^{3/2}, g(x)=\sin^{-1}x$

3) $f(x)=x^{2/3}, g(x)=\cos^{-1}x$ 4) $f(x)=\sqrt{x}, g(x)=\cos^{-1}x$

23. If $\int \frac{dx}{ae^{mx}+be^{-mx}}=K\tan^{-1}(Pe^{mx})+C$, then $K, P =$

1) $K=\frac{1}{\sqrt{ab}}, P=\sqrt{\frac{a}{b}}$ 2) $K=\frac{1}{m\sqrt{ab}}, P=\sqrt{\frac{a}{b}}$ 3) $K=m\sqrt{ab}, P=\sqrt{\frac{b}{a}}$ 4) $K=\frac{1}{m\sqrt{ab}}, P=\sqrt{\frac{b}{a}}$

LEVEL-II

LECTURE SHEET (ADVANCED)

Single answer type questions

1. If y implicit differentiable function of x such that $y(x+y)^2 = x$ then $\int \frac{dx}{x+3y} =$

a) $\ln((x+y)^2+1)+c$ b) $\frac{1}{2}\ln((x-y)^2+1)+c$ c) $\ln((x-y)^2+1)+c$ d) $\frac{1}{2}\ln((x+y)^2+1)+c$

2. If $x = f''(t)\cos t + f'(t)\sin t$, $y = -f''(t)\sin t + f'(t)\cos t$ then $\int \left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right)^{\frac{1}{2}} dt$

a) $f'(t)+f(t)+C$ b) $f''(t)+f(t)+C$ c) $f'''(t)+f'(t)+C$ d) $f''(t)+f'(t)+C$

3. The value of $\int \frac{1+(\sin x)^{2/3}}{1+(\sin x)^{4/3}} . d(\sqrt[3]{\sin x})$

a) $\sin^{-1}(1+\sqrt[3]{\sin x})+c$ b) $\sin^{-1}(\sqrt[3]{\sin x})+c$ c) $\frac{1}{2}\tan^{-1}\left(\frac{\sqrt[3]{\sin^2 x}-1}{\sqrt{2}\sqrt[3]{\sin x}}\right)+c$ d) $\frac{1}{2}\tan^{-1}\left(\frac{\sqrt[3]{\sin x}}{\sqrt{2}}\right)+c$

4. The integral $\int \frac{\sin 2x\left(1-\frac{3}{2}\cos x\right)}{e^{\sin^2 x+\cos^3 x}} =$

a) $e^{\sin^2 x+\cos^3 x}+c$ b) $-e^{-(\sin^2 x+\cos^3 x)}+c$ c) $e^{(\sin^2 x+\cos^3 x)^2}+c$ d) $e^{\sin^2 x-\cos^3 x}+c$

5. $\int e^{\log x^2} . \log_{e^x} x\,dx =$

a) $e^{(2\log_e x)^2}+C$ b) $\frac{1}{2}e^{(2\log_e x)^2}+C$ c) $\frac{e^{|2\log_e x|}}{4}(2\log_e x-1)+C$ d) $\frac{1}{4}e^{2\log_e x}+C$

6. $\int \frac{\sin 2x+2\tan x}{(\cos^6 x+6\cos^2 x+4)} dx$

a) $2\sqrt{\frac{1+\cos^2 x}{\cos^7 x}}$

b) $\tan^{-1}\frac{1}{\sqrt{2}}\left(\frac{1+\cos^2 x}{\cos^7 x}\right)+c$

c) $\frac{1}{12}\log_e\left(\frac{1+\cos^2 x}{\cos^7 x}\right)+c$

d) $\frac{1}{12}\log\left(1+\frac{6}{\cos^4 x}+\frac{4}{\cos^6 x}\right)+c$

7. The value of integral $\int e^x\left(\frac{1}{\sqrt{1+x^2}}-\frac{1-2x^2}{\sqrt{(1+x^2)^5}}\right)dx$ is equal to

a) $e^x\left(\frac{1}{\sqrt{1+x^2}}+\frac{x}{\sqrt{(1+x^2)^3}}\right)+c$

b) $e^x\left(\frac{1}{\sqrt{1+x^2}}-\frac{x}{\sqrt{(1+x^2)^3}}\right)+c$

c) $e^x\left(\frac{1}{\sqrt{1+x^2}}+\frac{x}{\sqrt{(1+x^2)^5}}\right)+c$

d) $\frac{e^x}{\sqrt{1+x^2}}+c$

8. The value of integral $\int \tan^{-1}\left(\frac{x-3}{\sqrt{6x-x^2}}\right)dx$ is equal to

a) $\sin^{-1}\left(\frac{x-3}{3}\right)+\sqrt{6x-x^2}+c$

b) $\cos^{-1}\left(\frac{x-3}{3}\right)+\sqrt{6x-x^2}+c$

c) $(x-3)\sin^{-1}\left(\frac{x-3}{3}\right)+\sqrt{6x-x^2}+c$

d) $(x-3)\cos^{-1}\left(\frac{x-3}{3}\right)+\sqrt{6x-x^2}+c$

9. $\int \frac{x^2-1}{x\sqrt{(x^2+\alpha x+1)(x^2+\beta x+1)}}dx$ is equal to

a) $\log_e\left[\frac{\sqrt{x^2+\alpha x+1}+\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right]+c$

b) $2\log_e\left[\frac{\sqrt{x^2+\alpha x+1}+\sqrt{x^2+\beta x+1}}{\sqrt{x}}\right]+c$

c) $\log_e\left[\sqrt{x^2+\alpha x+1}-\sqrt{x^2+\beta x+1}\right]+c$

d) $\log_e\left[\sqrt{x^2+\alpha x+1}+\sqrt{x^2+\beta x+1}\right]+c$

10. $\int \frac{\tan x\,dx}{\sqrt{\sin^4 x+\cos^4 x}}=$

a) $\log\left|\tan^2 x+\sqrt{1+\tan^4 x}\right|+C$

b) $\frac{1}{2}\log\left|\tan^2 x+\sqrt{1+\tan^4 x}\right|+C$

c) $\frac{1}{4}\log\left|\tan^2 x+\sqrt{1+\tan^4 x}\right|+C$

d) $\log\left|\tan^2 x-\sqrt{1+\tan^4 x}\right|+C$

11. If $(x-y+3)^2 = x+y$ then $\int \frac{dx}{x+y+6}$ is equal to

a) $\frac{1}{2}\log(x-y+3)+\frac{1}{2\sqrt{6}}\tan^{-1}\left(\frac{x+y+6}{\sqrt{6}}\right)+c$

b) $\frac{1}{2}\tan^{-1}(x-y+3)+\frac{1}{2\sqrt{6}}\log\left(\frac{x+y+6}{\sqrt{6}}\right)+c$

c) $\frac{1}{2}\log(x-y+6)+\frac{1}{2\sqrt{6}}\tan^{-1}\left(\frac{x+y+3}{\sqrt{6}}\right)+c$

d) $\frac{1}{2}\tan^{-1}(x-y+6)+\frac{1}{2\sqrt{6}}\log\left(\frac{x+y+3}{\sqrt{6}}\right)+c$

12. $\int \frac{(\sin^3\theta-\cos^3\theta-\cos^2\theta)(\sin\theta+\cos\theta+\cos^2\theta)^{2007}}{(\sin\theta)^{2009}(\cos\theta)^{2009}}d\theta=$

a) $\frac{1}{2008}(\sec\theta+\operatorname{cosec}\theta+\cot\theta)^{2008}+c$

b) $\frac{1}{2008}(\tan\theta.\sec\theta+\operatorname{cosec}\theta.\cot\theta-\operatorname{cosec}^2\theta)^{2008}+c$

c) $\frac{1}{2009}(\sin\theta+\cos\theta+\tan\theta)^{2009}+c$

d) $\frac{1}{2009}(\cot\theta.\operatorname{cosec}\theta+\tan\theta\sec\theta-\sec^2\theta)^{2009}+c$

13. If $I=\int\sqrt{\tan x}\,dx$, then I equal

a) $\frac{1}{\sqrt{2}}[\sin^{-1}(\sin x-\cos x)+\log|\sin x+\cos x+\sqrt{\sin 2x}|+C$

b) $\sqrt{2}[\sin^{-1}(\sin x+\cos x)+\log|\sin x-\cos x+\sqrt{\sin 2x}|+C$

c) $\sqrt{2}[\sin^{-1}(\cos x-\sin x)+\log\sin 2x]+C$ d) none of these

14. If $I=\int\sqrt{e^{2x}+ae^{x}}\,dx$, then I equal

a) $\sqrt{e^{2x}+ae^{x}}+a\log(e^{x}+ae^{2x})+C$ b) $\sqrt{e^{2x}+ae^{x}}+a\log(e^{x/2}+a^{ex})+C$

c) $\sqrt{e^{2x}+ae^{x}}+a\log\left(e^{x/2}+\sqrt{e^{x}+a}\right)+C$ d) none of these

15. If $I=\int\frac{dx}{\sin^3 x+\cos^3 x}$, then I equals

a) $\frac{1}{3\sqrt{2}}\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|+\tan^{-1}(t)+C$ b) $\frac{1}{3\sqrt{2}}\log\left|\frac{\sqrt{2}-t}{\sqrt{2}+t}\right|+\frac{1}{3}\tan^{-1}(t)+C$

c) $\frac{1}{3\sqrt{2}}\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|+\frac{2}{3}\tan^{-1}(t)+C$ d) $\frac{1}{3\sqrt{2}}\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|-\frac{2}{3}\tan^{-1}(t)+C$

where $t=\sin x+\cos x$

More than one correct answer type questions

16. $\int\sin\left(\sqrt[3]{x}\right)dx=a\left\{(2-x^{2/3})\cos\left(\sqrt[3]{x}\right)+b^3\sqrt[3]{x}.\sin\left(\sqrt[3]{x}\right)\right\}+c$

a) $a=3$ b) $b=2$ c) $a=1$ d) $b=1$

17. If $\int\frac{\cos x+\sin 2x}{(2-\cos^2 x)(\sin x)}dx=\int\frac{A}{\sin x}dx+B\int\frac{\sin x}{1+\sin^2 x}dx+C\int\frac{dx}{1+\sin^2 x}$

a) $A+B+C=4$ b) $A+B+C=2$ c) $A+B+C=-1$ d) $A+B+C=5$

18. If $\int\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}=\frac{A}{\pi}\left[\sqrt{x-x^{B}}-(C-Dx)\sin^{-1}\sqrt{x}\right]-x+c$ then

a) A+B+C+D = 7 b) $(A+B).(C+D)=12$ c) $(A+B).C=4$ d) $A+B+C=5$

19. $\int\frac{dx}{(1+x)^5\sqrt{x^2+2x}}=\frac{1}{8}\left(3\theta-2\sin 2\theta+\frac{\sin 4\theta}{4}\right)+C$, then

a) $\theta=\sin^{-1}(x+1)$ b) $\theta=\sin^{-1}\left(\frac{1}{x+1}\right)$ c) $\theta=\frac{\pi}{2}-\cos^{-1}(x+1)$ d) $\theta=\frac{\pi}{2}-\cos^{-1}\left(\frac{1}{x+1}\right)$

20. If $\int\operatorname{cosec}2x\,dx=f(g(x))+C$, then

a) range $g(x)=(-\infty,\infty)$ b) dom $f(x)=(-\infty,\infty)\sim\{0\}$

c) $g'(c)=\sec^2 x$ d) $f'(x)=1/x$ for all $x\in(0,\infty)$

21. If $I=\int\frac{\sin x+\sin^3 x}{\cos 2x}dx=A\cos x+B\log|f(x)|+C$, then

a) $A=1/4, B=-1/\sqrt{2}, f(x)=\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$ b) $A=1/2, B=-3/4\sqrt{2}$

c) $A=-1/2, B=3/4\sqrt{2}, f(x)=\frac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$ d) $A=1/2, B=3/4\sqrt{2}, f(x)=\frac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$

22. If $I=\int e^{x\sin x+\cos x}\left(\frac{x^4\cos^3 x-x\sin x+\cos x}{x^2\cos^2 x}\right)dx$ then I equals

a) $e^{x\sin x+\cos x}\left(x-\frac{\sec x}{x}\right)+C$ b) $e^{x\sin x+\cos x}\left(x\sin x-\frac{\cos x}{x}\right)$

c) $e^{x\sin x+\cos x}\left(\frac{x}{\tan x}-\frac{\sec x}{x}\right)+C$ d) $xe^{x\sin x+\cos x}-\int e^{x\sin x+\cos x}\left(1-\frac{\cos x-x\sin x}{x^2\cos^2 x}\right)dx$

23. If $\int\frac{3x+4}{x^3-2x-4}dx=\log|x-2|+K\log f(x)+C$, then

a) $K=-1/2$ b) $f(x)=x^2+2x+2$ c) $f(x)=|x^2+2x+2|$ d) $K=1/4$

24. If $I=\int(\tan x)^{1/3}dx=A\log\frac{t^4-t^2+1}{(t^2+1)^2+B}\tan^{-1}\frac{2t^2-1}{2\sqrt{3}}+C$ where $t=\tan^{1/3}x$ then

a) $A=1/4$ b) $B=\sqrt{3}/2$ c) $A=1/4$ d) $B=2\sqrt{3}$

25. If the primitive of $\frac{1}{x^6+x^4}$ is $f(x)+\frac{1}{x}+g(x)x^{-3}+C$ then

a) $f(x)=\tan^{-1}x$ b) $f(x)=2\tan^{-1}x$ c) $g(x)$ is a constant d) $g(x)=-1/3$ function

26. If $\int f(x)dx=\frac{3}{55}\sqrt[3]{\tan^5 x}(5\tan^2 x+11)+C$ then $f(x)$ is equal to

a) $\sqrt[3]{\sin^2\cos^{-14}x}$ b) $\sqrt[3]{\tan^2 x(1+\tan^2 x)^6}$ c) $\sqrt[3]{\cos^2 x\sin^{-14}x}$ d) $\frac{7}{3}\sqrt[3]{\sin^2 x\cos^{-14}x}$

Linked comprehension type questions

Passage - I :

If the integrand is a rational function of x and fractional powers of a linear fractional function of the form $\frac{ax+b}{cx+d}$, then rationalization of the integral is affected by the substitution $\frac{ax+b}{cx+d}=t^m$, where m is 1 c.m. of fractional powers of $\frac{ax+b}{cx+d}$.

27. If $I=\int\frac{(2x-3)^{1/2}}{(2x-3)^{1/3}+1}dx=3\left[\frac{1}{7}(2x-3)^{7/6}-\frac{1}{5}(2x-3)^{5/6}+\frac{1}{3}(2x-3)^{1/2}-(2x-3)^{1/6}+g(x)\right]+C$ then $g(x)$ is equal to

a) $\tan^{-1}(2x-3)^{1/6}$ b) $(2x-3)^{1/6}$ c) $3\tan^{-1}(2x-3)^{1/6}$ d) $4(2x-3)^{1/6}$

28. If $I=\int\frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}}=A\left(\frac{x-1}{x+2}\right)^{1/4}+C$ then A is equal to

a) 1/3 b) 2/3 c) 3/4 d) 4/3

29. If $I=\int\frac{dx}{\sqrt[3]{(x+1)^2(x-1)^4}}=K\sqrt[3]{\frac{1+x}{1-x}}+C$ then K is equal to

a) 2/3 b) –3/2 c) 1/3 d) 1/2

30. If $I=\int\frac{dx}{(1-x)\sqrt{1-x^2}}=K\sqrt{\frac{x+1}{1-x}}+C$ then K is equal to

a) 1/2 b) 1 c) 1/3 d) 2/3

Integer answer type questions

31. $\int(\log_e x)\sin^{-1}x\,dx=P(x)\sqrt{1-x^2}(\log_e^x-1)+Q\log_e\left|\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right|$ then $Q+\frac{3}{2}=$

32. $\int\sin\left(2\tan^{-1}\sqrt{\frac{3-x}{3+x}}\right)dx=P\cos^{-1}\frac{x}{3}+Q\sin\left(2\cos^{-1}\frac{x}{3}\right)+C$ then $P+2Q$ is

33. Evaluate $\int\frac{\left(\sin^{3/2}x+\cos^{3/2}x\right)dx}{(\sin^3x\cos^3x\sin(x+a))^{1/2}}=2\left(\frac{1}{P}\sqrt{\cos a-\sin a\cos x}+\frac{1}{Q}\sqrt{\cos a\tan x-\sin x}+C\right)$ then

$P^2+Q^2=$

34. $\int\frac{\cos x(1+4\cos 2x)}{\sin x+4\sin x\cos^2 x}dx=P\log|\sin x|+Q\log|\sin^2 x+R\cos^2 x|+c$, then find $P+Q+R$

35. $\int\frac{(\tan\theta)^{7/6}-(\tan\theta)^{17/6}\,d\theta}{(\tan\theta)^{1/3}(\sec^2\theta+\tan^2\theta)^{1/2}+(\tan\theta)^{1/2}(\sec^2\theta+\tan\theta)^{1/3}}=P\left(\frac{y^3}{3}-\frac{y^2}{2}+y\right)+Q\log|1+y|+c$ where

$y=\left(\tan\theta+\frac{1}{\tan\theta}+1\right)^{1/6}$, then find $P+Q$.

36. $\int\frac{dy}{y^2(1+y^2)}=\frac{P}{y}+Q\tan^{-1}y+R\sin(S\tan^{-1}y)+T\sin(4\tan^{-1}y)+c$, then find $P-\frac{8}{3}Q-2R+S-32T$.

37. If $\int(x^{2010}+5x^{804}+x^{402})(2x^{1608}+5x^{402}+10)dx=\frac{1}{4030}(2x^{2010}+5x^{804}+10x^{402})^{\frac{a}{402}}$, then the sum of the digit in 'a'.

38. If $\left(\frac{\sin x+\sin 3x+\sin 5x+\sin 7x+\sin 9x+\sin 11x}{\cos x+\cos 3x+\cos 5x+\cos 7x+\cos 9x+\cos 11x}\right)dx$ is equals $\frac{1}{6}\ln(\sec mx)$, where $m\in N$, find

'$m-1$'

PRACTICE SHEET (ADVANCED)

Single answer type questions

1. Let $f(x)=\frac{x}{(1+x^n)^{1/n}}$ for $n>2$ and $g(x)=fofo...of(x)$ (n times). Then $\int x^{n-2}g(x)dx$ equals

a) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$ b) $\frac{1}{(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$

c) $\frac{1}{n(n-1)}(1+nx^n)^{1+\frac{1}{n}}+K$ d) $\frac{1}{(n-1)}(1+nx^n)^{1+\frac{1}{n}}+K$

2. If $I=\int\frac{x^2+20}{(x\sin x+5\cos x)^2}dx$, then I equals

a) $-\frac{x}{\cos x(x\sin x+5\cos x)}+\tan x+C$ b) $\frac{x}{\sin x(x\sin x+5\cos x)}+\cot x+C$

c) $(x\sin x-5\cos x)^{-1}\sin x+7x+C$ d) none of these

3. If $I=\int\frac{dx}{\sqrt{2x+3}+\sqrt{x+2}}$, then I equals

a) $2(u-v)+\log\left|\frac{u-1}{u+1}\right|+\log\left|\frac{v-1}{v+1}\right|+C$; $u=\sqrt{2x+3}$, $v=\sqrt{x+2}$

b) $\log\left|\frac{\sqrt{x+2}+\sqrt{2x+3}}{\sqrt{x+2}-\sqrt{2x+3}}\right|+C$ c) $\log\left(\sqrt{x+2}+\sqrt{2x+3}\right)+C$

d) is transcedental function in u and v, $u=\sqrt{2x+3}$, $v=\sqrt{x+2}$

4. If $I=\int\frac{dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)}$, then I equals

a) $\frac{1}{bc}\tan^{-1}(a)+\frac{1}{ca}\tan^{-1}(b)+\frac{1}{ab}\tan^{-1}(c)+k$

b) $\frac{1}{b^2-c^2}\tan^{-1}(a)+\frac{1}{c^2-a^2}\tan^{-1}(b)+\frac{1}{a^2-b^2}\tan^{-1}(c)+k$

c) $\frac{\tan^{-1}a+\tan^{-1}b+\tan^{-1}c}{a^2+b^2+c^2}+k$ d) none of these

5. If $I=\int\frac{\sec^2 x-7}{\sin^7 x}dx$, then I equals

a) $\frac{\tan x}{(\sin x)^7}+C$ b) $\frac{\cos x}{(\sin x)^7}+C$ c) $\frac{\sin x}{\cos^7 x}+C$ d) none of these

6. If $I=\int \frac{e^{3x}}{1+e^{x}}dx$, then I equal

a) $(1/2)(1+e^x)(e^x-3)+\log(1+e^x)+C$ b) $(1/2)(1+e^x)^2-(1/3)(1+e^x)+\log(1+e^x)+C$

c) $(1/2)(1+e^x)(e^x+3)+\log(1+e^x)+C$ d) $(1/2)(1+e^x)^2-2\log(1+e^x)+C$

7. If $I=\int(1+\cot(x-\alpha)\cot(x+\alpha)dx$, then I equals

a) $\log\left|\frac{\cot x-\cot\alpha}{\cot x+\cot\alpha}\right|+C$ b) $\cot 2\alpha\log\left|\frac{1-\cot x\tan\alpha}{1+\cot x\tan\alpha}\right|+C$

c) $\operatorname{cosec}2\alpha\log\left|\frac{\tan x-\cot\alpha}{\tan x+\cot\alpha}\right|+C$ d) $\log|\tan x|-x\log|\tan\alpha|+C$

8. If $I=\int\frac{dx}{x^4\sqrt{1+x^2}}$, then I equals

a) $\frac{\sqrt{x^2+1}}{x}-\frac{1}{2x^2}+C$ b) $\frac{\sqrt{1+x^2}}{x}-\frac{1}{2x^3}+C$ c) $-\frac{\sqrt{1+x^2}}{x}+\frac{2x}{\sqrt{1+x^2}}+C$ d) none of these

9. If $I=\int\frac{x^2}{(x-a)(x-b)}dx$, then I equals

a) $x+\frac{1}{a-b}\log\left|\frac{x-a}{x-b}\right|+C$ b) $x+\frac{1}{a-b}\log\left|\frac{x-a}{x-b}\right|^{a^2+b^2}+C$

c) $x+\frac{1}{a-b}\{a^2\log|x-a|-b^2\log|x-b|\}+C$ d) none of these

10. If $I=\int\cot^{-1}\left(\frac{a^2-ax+x^2}{a^2}\right)dx$, then I equals

a) $x\tan^{-1}\left(\frac{x}{a}\right)-(x-a)\tan^{-1}\left(\frac{x-a}{a}\right)+C$ b) $\frac{a}{2}\log(2a^2-2ax+x^2)-\frac{a}{2}\log(x^2+a^2)+C$

c) $x\tan^{-1}\left(\frac{x}{a}\right)+(x-a)\tan^{-1}\left(\frac{x-a}{a}\right)+\frac{a}{2}\log(2a^2-2ax+x^2)+C$ d) none of these

11. If $I=\int\tan^{-1}(1+\sqrt{x})dx$, then I equals

a) $x^2\tan^{-1}(1+\sqrt{x})-\sqrt{x}+\log(x+2\sqrt{x}+2)+C$

b) $(2\sqrt{2}+1)\tan^{-1}(1+\sqrt{x})+\sqrt{x}-\log(x+2\sqrt{x}+2)+C$

c) $(\sqrt{x}-1)^2\tan^{-1}(1+\sqrt{x})-\sqrt{x}+\log(\sqrt{x}+1)+C$ d) none of these

12. If $I=\int \tan x \tan 2x \tan 3x$, then I equals

a) $\log|\cos x|+(1/2)\log|\cos 2x|+(1/3)\log|\cos 3x|+C$ b) $\log|\cos x|-(1/2)\log|\cos 2x|-(1/3)\log|\cos 3x|+C$

c) $\log|\cos x|+(1/2)\log|\cos 2x|-(1/3)\log|\cos 3x|+C$ d) none of these

More than one correct answer type questions

13. $\int \frac{xdx}{x^4+x^2+1}$ equals to

a) $\frac{2}{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C$ b) $\frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right]+C$

c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C$ d) $\frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right]+C$

14. $\int \sqrt{1+\cos x}\,dx$ equals to

a) $2\sin^{-1}\sqrt{\sin x}+c$ b) $\sqrt{2}\cos^{-1}\sqrt{\cos x}+c$ c) $c-2\sin^{-1}(1-2\sin x)$ d) $\cos^{-1}(1-2\sin x)+c$

15. If $\int \sqrt[3]{(\operatorname{cosec}x-\cot x)}\left(1+\sqrt[3]{(\operatorname{cosec}x-\cot x)^2}\right)dx = A\log_e|f(x)|+Bg(x)+c$, then

a) $A=\frac{3}{2}$ b) $B=-\frac{3}{4}$ c) $2A=B^2$ d) $A^2=4B$

16. $f(x)=\int \frac{x}{2}d\left(\frac{x^2-1}{x^2}\right)$ and $f(2)=\frac{1}{2}$ & $g(x)=fff.....r$ times $f(x)$

$g_r(x)=f(x)$. $g_2=ff(x)$ and so on, then identify the correct statement(s)

a) $\frac{d}{dx}\left(g_{3n-2}(x)\right)=1$ whenever exist, $n\in Z$ b) $\frac{d}{dx}\left(g_{3n}(x)\right)=1$ whenever exist, $n\in Z$

c) $\lim\limits_{x\to 1}\frac{\sum\limits_{r=1}^{100}(g_{3r}(x))^r+x-101}{x-1}=5050$

d) Slope of the tangent to the graph of the function $y=g_{80}(x)$ at $x=\frac{1}{2}$ is 4.

17. The integral $\int \frac{(2x+5)}{(x+1)(x+2)(x+3)(x+4)}dx$ is equal to

a) $\frac{1}{2}\ln\left|\frac{3x+4}{5x+6}\right|+c$ b) $\frac{1}{2}\ln\left|\frac{x^2+3x+4}{x^2+5x+6}\right|+c$ c) $\frac{1}{2}\ln\left|\frac{x^2+5x+4}{x^2+5x+6}\right|+c$ d) $\frac{1}{2}\ln\left|\frac{(x+1)(x+4)}{(x+2)(x+3)}\right|+c$

18. If $\int \frac{3x+4}{x^3-2x-4}dx=\log|x-2|+K\log f(x)+C$, then

a) $K=-1/2$ b) $f(x)=x^2+2x+2$ c) $f(x)=|x^2+2x+2|$ d) K = 1/4

19. If $\int \frac{dx}{5+4\cos x} = K\tan^{-1}\left(M\tan\frac{x}{2}\right)+C$, then

a) $K = 1$ b) $K = 2/3$ c) $M = 1/3$ d) $M = 2/3$

20. If $I = \int (\tan x)^{1/3} dx = A\log\frac{t^4 - t^2 + 1}{(t^2+1)^2} + B\tan^{-1}\frac{2t^2-1}{2\sqrt{3}} + C$ where $t = \tan^{1/3}x$ then

a) A = 1/4 b) B = $\sqrt{3}/2$ c) A = 1/4 d) B = $2\sqrt{3}$

21. If $I = \int \frac{\sin x + \sin^3 x}{\cos 2x} dx = A\cos x + B\log|f(x)| + C$, then

a) $A = 1/4, B = -1/2, f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$ b) $A = 1/2, B = -3/4\sqrt{2}$

c) $A = -1/2, B = 3/\sqrt{5}, f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$ d) $A = 1/2, B = 3/4\sqrt{2}, f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$

22. The value of the integral $\int \frac{\log(x+1) - \log x}{x(x+1)} dx$ is

a) $-(1/2)(\log(x+1))^2 - (1/2)(\log x)^2 + \log(x+1)\log x + C$

b) $-[(\log(x+1)^2 - (\log x)^2] + \log(x+1)\log x + C$

c) $C - (1/2)(\log(1+1/x)^2$ d) none of these

Linked comprehension type questions

Passage - I :

we can derive reduction formulas for the integral of the form $\int \sin^n x\,dx$, $\int \cos^n x\,dx$, $\int \tan^n x\,dx$, $\int \cot^n x\,dx$, and other integrals of these form using integration by parts. In turn these reduction formulas can be used to compute integrals of higher power of sinx, cosx etc.

23. If $\int \sin^5 x dx = -\frac{1}{5}\sin^4 x\cos x + A\sin^2 x\cos x - \frac{8}{15}\cos x + C$ then A is equal to

a) –2/15 b) –3/15 c) –4/15 d) –1/15

24. If $\int \tan^6 x dx = \frac{1}{5}\tan^5 x + A\tan^3 x + \tan x - x + C$ then A is equal to

a) 1/3 b) 2/3 c) –2/3 d) –1/3

25. If $I = \int \sec^6 x\,dx = \frac{1}{5}\tan^3 x + A\tan^3 x + C$ then A is equal to

a) 1/3 b) 2/3 c) –2/3 d) –1/3

26. If $\int \text{cosec}^n x\,dx = \frac{-\text{cosec}^{n-2}x\cot x}{n-1} + A\int \text{cosec}^{n-2}x\,dx$ then A is equal to

a) $\frac{1}{n-2}$ b) $\frac{n}{n-2}$ c) $\frac{n-1}{n-2}$ d) $\frac{n-2}{n-1}$

Matrix matching type questions

27. The antiderivative of

COLUMN - I	COLUMN - II
A) $f(x)=\frac{1}{x^{1/2}+x^{1/3}}$	p) $2x^{1/2}-3x^{1/3}+6x^{1/6}-6\log(x^{1/6}+1)+C$
B) $f(x)=\frac{1}{x+x^{1/3}}$	q) $\log\frac{x}{\left(1+\sqrt[10]{x}\right)^{10}}+\frac{10}{\sqrt[10]{x}}-\frac{5}{\sqrt[5]{x}}+\frac{10}{3\sqrt[10]{x^3}}-\frac{5}{2\sqrt[5]{x^2}}+C$
C) $f(x)=\frac{1}{x\left(\sqrt{x}+\sqrt[5]{x^2}\right)}$	r) $2\sqrt{x}-3\sqrt[3]{x}-8\sqrt[4]{x}+6\sqrt[6]{x}+48\sqrt[12]{x}+3\log(1+\sqrt[12]{x})+\frac{33}{2}\log\left(\sqrt[6]{x}-\sqrt[12]{x}+2\right)-\frac{171}{\sqrt{7}}\tan^{-1}\frac{2\sqrt[12]{x}+1}{\sqrt{7}}+C$
D) $f(x)=\frac{1}{\sqrt{x}+\sqrt[3]{x}+2\sqrt[4]{x}}$	s) $\frac{3}{2}\log(1+x^{2/3})+C$

Integer answer type questions

28. Let $F(x)$ be the antiderivative of $f(x) = 3\cos x - 2\sin x$ whose graph passes through the point $(\pi/2, 1)$. Then F(0) is equal to

29. Let $F(x)$ be the antiderivative of $f(x) = 1/(3+5\sin x+3\cos x)$ whose graph passes through the point (0,0). The $F(\pi/2)-\frac{1}{10}\log\frac{8}{3}+1982$ is equal to

30. Let f be a function satisfying $f''(x) = x^{-3/2}$, $f'(x) = 2$ and $f(0) = 0$. Then $f(784)$ is equal to

31. $f\int\frac{1}{e^{2x}+e^{-2x}}dx=\frac{A}{842}\tan^{-1}e^{2x}+C$ then A is equal to

32. If $\int\frac{dx}{(x^2+1)^2}=\frac{A}{148}\tan^{-1}x+\frac{1}{2}\frac{x}{x^2+1}+C$ then A is equal to

33. If $\int\frac{f(x)}{1-x^3}dx=\log\left|\frac{x^2+x+1}{x-1}\right|\frac{A}{948\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}}+C$ then A = _____, where $f(x)$ is a polynomial of second degree in x such that $f(0) = f(1) = 3f(2) = 3$.

34. If $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx=\frac{A}{748\pi}[-(\sin^{-1}\sqrt{x})(1-2x)+\sqrt{x}\sqrt{1-x}]-x+C$ then A is equal to

35. If $\int \frac{\cos^7 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx=-\frac{A}{248}\sin^3 x+5\sin x-\frac{2}{\sin x}-12\tan^{-1}(\sin x)+C$ then A is equal to

KEY SHEET (ADDITIONAL EXERCISE)

LEVEL-I (MAIN)

1) **3**	2) **3**	3) **2**	4) **1**	5) **3**	6) **2**	7) **1**	8) **4**	9) **4**	10) **4**
11) **1**	12) **3**	13) **2**	14) **1**	15) **4**	16) **1**	17) **3**	18) **1**	19) **1**	20) **2**
21) **2**	22) **2**	23) **2**							

LEVEL-II

LECTURE SHEET (ADVANCED)

1) **d**	2) **b**	3) **c**	4) **b**	5) **c**	6) **d**	7) **a**	8) **c**	9) **b**	10) **b**
11) **c**	12) **a**	13) **a**	14) **c**	15) **c**	16) **ab**	17) **bc**	18) **abcd**	19) **bd**	20) **abc**
21) **bd**	22) **ad**	23) **abc**	24) **ab**	25) **acd**	26) **ab**	27) **a**	28) **d**	29) **b**	30) **b**
31) **2**	32) **0**	33) **1**	34) **6**	35) **0**	36) **8**	37) **7**	38) **5**		

PRACTICE SHEET (ADVANCED)

1) **a**	2) **a**	3) **a**	4) **d**	5) **a**	6) **a**	7) **b**	8) **a**	9) **c**	10) **d**
11) **a**	12) **c**	13) **bc**	14) **ad**	15) **ac**	16) **bd**	17) **cb**	18) **abc**	19) **bc**	20) **ab**
21) **bd**	22) **ac**	23) **c**	24) **d**	25) **b**	26) **d**	27) **A-p;B-s;C-q;D-r**			
28) **0**	29) **1982**	30) **2240**	31) **421**	32) **74**	33) **1896**	34) **1496**		35) **744**	