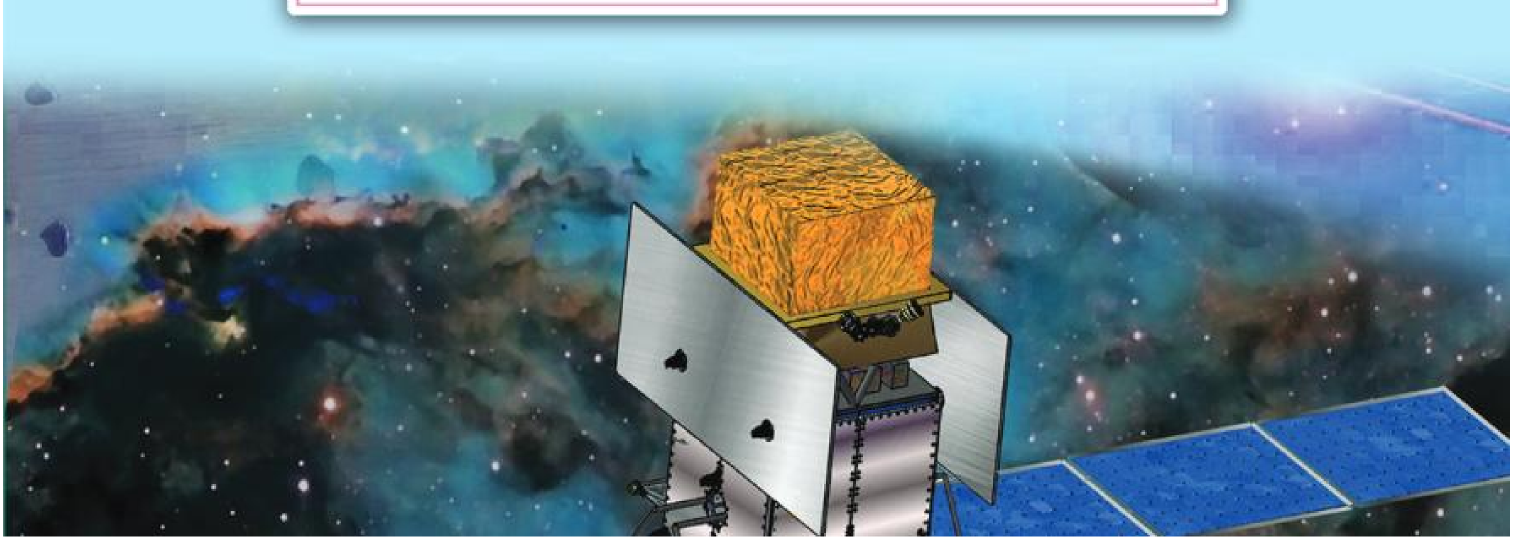
Chapter - 4

CURRENT ELECTRICITY

- Electric current, Drift speed, Ohm's law, Resistance
- Combination of resistance, Electric power
- Kirchhoff's laws, Wheatstone Bridge, Cells
- Emf, Combination of cells, Metre Bridge
- Potentiometer, CR circuits



4.1 INTRODUCTION

In our daily life we listen music on a tape recorder or radio receiver, see different programs on television, enjoy cool breeze from electric fan or cooler. Do you know, what makes these appliances work? It is electric current. Electricity is a unique gift of science to mankind. Our every day life is governed directly or indirectly by many applications of electricity. We cannot imagine to live without electricity in the modern world.

When a potential difference is applied across a conductor an electric field sets in the conductor and electrons drift in the direction opposite to the field. Due to electron drift, charge flows through the conductor and we say that electric current is flowing through it. The direction of current is taken opposite to the direction of flow of electrons (negative charge) and in the direction of flow of positive charge. Let a charge ΔQ be passing across a conducting wire in short time interval Δt then, we define electric current as $I = \dfrac{\Delta Q}{\Delta t}$.

If a net charge 'Q' passes through any cross section of the conductor in time 't' then the average current 'I' is given by

$$\text{Average current } I = \frac{Q}{t}$$

The electric current through any conductor is the rate of transfer of charge from one side of any cross section of conductor to the other side.

The SI unit of current is ampere (symbol A). If one coulomb of charge passes through a cross-section of the conductor per second then the current is one ampere.

$$\text{Ampere(A)} = \frac{\text{coulomb(C)}}{\text{second(s)}}$$

Commonly used sources of current are electric cells, electric generators etc.

If the rate of flow of charge with time is not constant, the instantaneous current is defined by the equation,

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad \text{.... (4.1)}$$

Current is a scalar quantity. It is a macroscopic quantity like the mass of a body or volume of a container. In SI Units, current is one of the fundamental physical quantity. It is dimensionally denoted by [I] or [A].

Example-4.1

The number of electrons striking the screen of CRT is 7.5×10^{15} in 10 s. Calculate the electric current.

Solution :

$$i = \frac{q}{t} = \frac{ne}{t} = \frac{7.5 \times 10^{15} \times 1.6 \times 10^{-19}}{10}$$
$$= 120\ \mu A$$

Example-4.2

In a hydrogen atom, electron moves in an orbit of radius 5×10^{-11} m with a speed of 2.2×10^{6} m/s. Calculate the equivalent current.

Solution :

Number of rotations made by the electron/sec.

$$n = \frac{v}{2\pi r}$$

$$\text{Current } i = n.e = \frac{v}{2\pi r}.e = \frac{2.2 \times 10^{6}}{2\pi \times 5 \times 10^{-11}} \times 1.6 \times 10^{-19}$$
$$= 1.12 \times 10^{-3} \text{ amp} = 1.12 \text{ mA}$$

4.2 CURRENT CARRIERS

The charged particles whose flow in a definite direction constitutes the electric current are called current carriers.

(a) **Current carriers in solid conductors :** In solid conductors like metals, the valence electrons of the atoms do not remain attached to individual atoms but are free to move throughout the volume of the conductor. Under the effect of an external electric field, the valence electrons move in a definite direction causing electric current in the conductors. Thus valence electrons are the current carriers in solid conductors.

(b) **Current carriers in liquids** : In an electrolyte like NaCl etc., there are positively and negatively charged ions (like Na^+, Cl^-). These are forced to move in definite directions under the effect of an external electric field, causing electric current. Thus in liquids, the current carriers are positively and negatively charged ions.

(c) **Current carriers in gases :** In general all gases are insulators of electricity. But they can be ionized by applying a high potential difference at low pressures or by their exposure to X rays etc. The ionized gas contains positive ions and electrons. Thus positive ions and electrons are the current carriers in gases.

4.3 ELECTRIC CURRENTS

Atoms and molecules are made of negatively charged electrons and positively charged nuclei, which are bound to each other and are thus not free to move. In a material the molecules are so closely packed that some of the electrons in it are no longer attached to individual nuclei, but free to move inside the matter like molecules in a gas. Such electrons move under the effect of electric field, in a definite direction, resulting in a current in the material. The materials which have large number of free electrons and develop strong electric currents in them, when an electric field is applied are called Conductors.

There are some other materials in which the electrons will be bound and they will not be accelerated, even if the electric field is applied, i.e. no current on applying electric field. Such materials are called insulators. For example, wood, plastic, rubber etc.

In case of a solid conductor (i.e., Cu, Fe, Ag etc.) atoms are tightly bound to each other. There are large number of free electrons in them, which will be responsible for the strong current in them when electric field is applied on them.

When no electric field is applied on a solid conductor, the free electrons move like molecules in a gas due to their thermal velocities. There is no preferential direction for the velocities of the electrons. Therefore, at a cross-section of the conductor, the number of free electrons travelling in one direction will be equal to the number of free electrons travelling in the opposite direction. Due to it, there is no net flow of electric charge in a direction inside the conductor and hence no current in it.

When an electric field is applied on a solid conductor in the shape of cylinder of circular cross - section by attaching positively and negatively charged circular discs of a dielectric of the same radius as that of the solid conductor, at the two ends as shown in Fig. 4.1. An electric field is set up in the conductor from positive charged disc towards negative charged disc. Due to this electric field, the free electrons in the conductor will be accelerated towards the positive disc side, in order to neutralise the charges of discs. The motion of the electrons will be there till the electric field inside the conductor exists. In this situation, there will be a current for a very short time called transient current in the conductor.

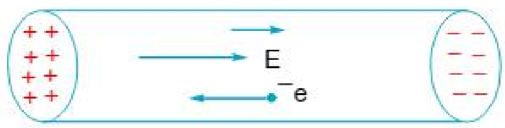


Fig 4.1 Electric currents in conductors

If we apply a steady electric field in the body of conductor by connecting a cell or battery across the two ends of a conductor, then there will be a steady current in the conductor rather than a transient current.

4.4 OHM'S LAW

In 1826 German Scientist George Simon Ohm studied the relation between current flowing in a conductor and potential difference applied across it. He expressed this relation in the form of a law known as Ohm's law, "the electric current through a conductor is directly proportional to the potential difference across it provided the physical conditions such as temperature are unchanged".

Let V be the potential difference applied across the conductor and I be the current flowing through it. According to Ohm's law,

$$V \alpha I \text{ or } V = RI \text{ or } \frac{V}{I} = R \qquad .... (4.2)$$

Where constant R is the electrical resistance applied by the conductor for the flow of electric current.

Resistance is the property of the material of the conductor which opposes the flow of current through it. The V-I graph for a metallic conductor is a straight line (Fig 4.2)

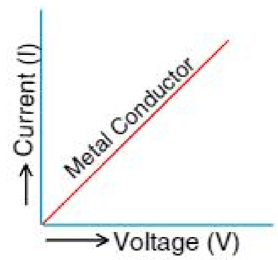


Fig 4.2 Graph of V - I

Unit of resistance is Ohm. It is expressed by symbol Ω (Omega)

1 Ohm = 1 Volt / 1 ampere

The factors which effect the resistance of a conductor are :

1. The resistance of the conductor is directly proportional to the length (l) of the conductor i.e., $R \alpha l$

2. The resistance in a conductor is inversely proportional to the area of cross-section (A) i.e, $R \alpha \frac{1}{A}$.

Thus on combining above two relations,

$$R \alpha \frac{\ell}{A} \text{ or } R = \rho \frac{\ell}{A} \qquad .... (4.3)$$

If $\ell = 1$ m, A = 1 m^2, then ρ = (R Ohm) (1 m^2)/1 m = R Ohm × m. Thus, the value of resistivity of a material is equal to resistance of a wire of the material of 1m length and 1 m^2 area of cross- section and is expressed in $\Omega - $m.

Inverse of resistivity is called conductivity or specific conductance and is represented by σ.

$$\sigma = \frac{1}{\rho} \qquad .... (4.4)$$

Unit of conductivity is Ohm^{-1} metre^{-1}($\Omega^{-1}m^{-1}$) ($\Omega^{-1}m^{-1}$) or mho meter $^{-1}$

$$V = I \times R = \frac{I\rho\ell}{A} \qquad .... (4.5)$$

Current per unit area (taken normal to the current), I/A is called current density and is denoted by j. The SI units of the current density are A/m^2. Further, if E is the magnitude of uniform electric field in the conductor whose length is l, then the potential difference V across its ends is El. Using these, the last equation can be written as

$$El = j\rho l \text{ or } E = j\rho \qquad .... (4.6)$$

The above relation for magnitudes E and j can indeed be cast in a vector form. The current density, (which we have defined as the current through unit area normal to the current) is also directed along E, and is also a vector $\bar{j}$ $(= j\bar{E}/E)$. Thus, the last equation can be written as, $\bar{E} = \bar{j}\rho$ or $\bar{j} = \sigma\bar{E}$ (Ohm's law is often stated in an equivalent form) where $\sigma = 1/\rho$ is called the conductivity.

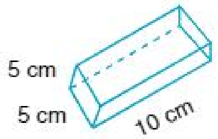
Example-4.3

A rectangular block has dimensions 5cm × 5cm × 10cm. Calculate the resistance measured between (a) two square ends and (b) the opposite rectangular ends. Specific resistance of the material is $3.5 \times 10^{-5}\,\Omega$m.

Solution :

a) Resistance between two square ends $R_1 = \frac{\rho\ell}{A}$

$$R_1=\frac{3.5\times10^{-5}\times10\times10^{-2}}{5\times5\times10^{-4}}=1.4\times10^{-3}\,\Omega$$



b) Resistance between the opposite rectangular ends

$$R_2=\frac{\rho\ell}{A}$$

$$R_2=\frac{3.5\times10^{-5}\times5\times10^{-2}}{5\times10\times10^{-4}}=3.5\times10^{-4}\,\Omega$$

Example-4.4

Two wires of same material have their lengths in the ratio of 2 : 3 and radii 8 : 9. Equal value of p.d is applied between their ends (separately). Calculate the ratio of current through those two.

Solution :

We know that $R=\frac{\rho\ell}{A}=\frac{\rho\ell}{\pi r^2}$

$$R=\frac{\rho\ell}{\pi r^2}$$

Substitute $R=\frac{V}{I}$

$$\frac{V}{I}=\frac{\rho\ell}{\pi r^2}\Rightarrow I=\frac{V\pi r^2}{\rho\ell}$$

$$I\propto\frac{r^2}{\ell}$$

$$\frac{I_1}{I_2}=\frac{r_1^2}{\ell_1}\times\frac{\ell_2}{r_2^2}$$

$$\frac{I_1}{I_2}=\frac{r_1^2}{r_2^2}\times\left(\frac{\ell_2}{\ell_1}\right)=\frac{8^2\times3}{9^2\times2}=\frac{64\times3}{81\times2}=\frac{32}{27}$$

$I_1 : I_2 = 32 : 27$

Example-4.5

Two wires of equal diameters of resistivities ρ_1 and ρ_2 and length x_1 and x_2 respectively are joined in series. Find the equivalent resistivity of the combination.

Solution :

Resistance, $R_1=\frac{\rho_1\ell_1}{A_1};R_2=\frac{\rho_2\ell_2}{A_2}$

$\ell_1=x_1,\ell_2=x_2$

As the wires are of equal diameters $A_1=A_2=A$.

$$R_1=\frac{\rho_1x_1}{A},R_2=\frac{\rho_2x_2}{A}$$

$$R=\frac{\rho x}{A}$$

where $x=x_1+x_2$

$$R=R_1+R_2$$

$$\frac{\rho x}{A}=\frac{\rho_1x_1}{A}+\frac{\rho_2x_2}{A}$$

$$\rho x=\rho_1x_1+\rho_2x_2$$

$$\rho(x_1+x_2)=\rho_1x_1+\rho_2x_2\quad[\because x=x_1+x_2]$$

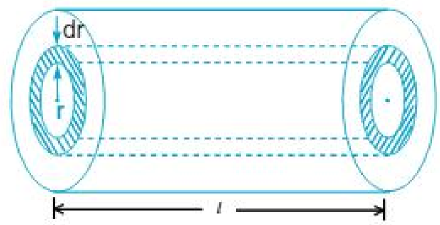
$$\therefore\rho=\frac{\rho_1x_1+\rho_2x_2}{x_1+x_2}$$

Example-4.6

A long round conductor of cross-sectional area S is made of material whose resistivity depends only on distance r from the axis of the conductor as $\rho=\frac{\alpha}{r^2}$, where α is a constant. Find : (a) the resistance per unit length of such a conductor (b) the electric field strength in the conductor due to which a current i flows through it

Solution :

Consider a cylindrical element of radii between r and $(r+dr)$. Its resistance



$$dR=\frac{\rho l}{2\pi rdr}\text{ (or) }\frac{1}{dR}=\frac{2\pi rdr}{\rho l}\qquad\text{..... (i)}$$

$$\therefore\frac{1}{R}=\int_0^a\frac{1}{dR}=\int_0^a\frac{2\pi}{\rho l}rdr$$

(where a is the radius of the conductor)

$$=\int_0^a\frac{2\pi rdr}{\left(\frac{\alpha}{r^2}\right)l}=\frac{2\pi}{\alpha l}\int_0^a r^3dr=\frac{2\pi}{\alpha l}\left(\frac{a^4}{4}\right)=\frac{(\pi a^2)^2}{2\pi\alpha l}=\frac{S^2}{2\pi\alpha l}$$

$$R=\frac{2\pi\alpha l}{S^2}\qquad\text{..... (ii)}$$

The resistance per unit length of wire $R=\frac{2\pi\alpha}{S^2}$

(b) Equation (ii) can be written as $R=\left(\frac{2\pi\alpha}{S}\right)\left(\frac{l}{S}\right)$

compare with $R=\frac{\rho l}{S}$, we get $\rho=\frac{2\pi\alpha}{S}$.

By Ohm's law $E=j\rho=\frac{i}{S}\times\frac{2\pi\alpha}{S}=\frac{2\pi\alpha i}{S^2}$

Example-4.7

A wire of silver has a resistance of 1 ohm. Specific resistance of constantan is 30 times the specific resistance of silver. Find the resistance of a constantan wire whose length is one third length of the silver wire and radius half the radius of the silver wire.

Solution :

$R_s = 1\Omega$, $\rho_C = 30\,\rho_S$

Resistance of constantan $R_C = ?$

Length $\ell_C = \frac{\ell_S}{3}$

radius $r_s = 2r_c$

We know that $R = \frac{\rho\ell}{a} = \frac{\rho\ell}{\pi r^2}$

$$\frac{R_c}{R_s} = \frac{\rho_c}{\rho_s}\frac{\ell_c}{\ell_s}\left(\frac{r_s}{r_c}\right)^2 = \frac{30}{3}\times 4 = 40$$

Resistance of constantan $R_c = 40 \times 1 = 40\Omega$

4.5 DRIFT VELOCITY

A metal conductor has large number of free electrons or conduction electrons, whose number density (i.e., no. of electrons per unit volume) is about $10^{29}\ m^{-3}$. These electrons at room temperature move at random within the body of the conductor, like the molecules of a gas. The average thermal speed of the free electrons in random motion at room temperature is of the order of $10^5\ ms^{-1}$. The directions of motion of these free electrons are so randomly distributed that the average thermal velocity of the electrons is zero. If $\overline{u}_1, \overline{u}_2, \overline{u}_3, ...\overline{u}_n$ are random thermal velocities of n free electrons in the metal conductor, then the average thermal velocity of electrons is

$$\frac{\overline{u}_1 + \overline{u}_2 + \overline{u}_3 + ... + \overline{u}_n}{n} = \overline{0}$$

As a result of which there will be no net flow of electrons or charge in one particular direction in a metal conductor, hence no current. When some potential difference is applied across the two ends of a metal conductor, say a copper wire; an electric field is set up inside the conductor. As a result of which, the free electrons in the conductor experience a force in a direction opposite to that of electric field and are accelerated from negative end to positive end of the conductor. On their way, the accelerated free electrons suffer frequent collisions against the copper ions/atoms and lose their gained kinetic energy. After each collision, the free electrons are again accelerated due to electric field towards the positive end of conductor and lose their gained kinetic energy in the next collision with the ions/atoms of the conductor. It means, the extra velocity gained by free electron is lost in subsequent collision. The process continues till the electrons reach the positive end of the conductor. Thus under the effect of electric field applied, the free electrons are having random thermal velocities and acquired small velocities by virtue of acceleration due to electric field, with which they move towards the positive end of the conductor.

These velocities are in different directions. Due to it, each free electron describes a curved path between two successive collisions, Fig 4.3 As a result of it, the free electrons drift towards the positive end of the conductor with some average velocity called drift velocity. Thus drift velocity is defined as the average velocity with which the free electrons get drifted towards the positive end of the conductor under the influence of an external electric field applied : The drift velocity of electrons is of the order of $10^{-4} ms^{-1}$.

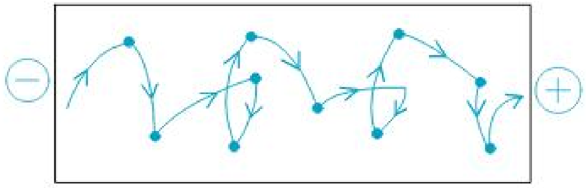


Fig 4.3 path of free electron motion between successive collisions

If V is the potential difference applied across the ends of the conductor of length *l*, the magnitude of electric field set up is $E = V/l$.

The direction of this electric field is from positive end to negative end of conductor. Since the charge on electron is –e, each free electron in the conductor experiences of force, $\overline{F} = -e\overline{E}$.

Here the negative sign shows that the direction of force is opposite to that of electric filed applied. If m is the mass of an electron, the acceleration of each electron is $\overline{a} = \frac{-e\overline{E}}{m}$.

Due to this acceleration, the free electron, apart from its thermal velocity, acquires additional velocity component in a direction opposite to the direction of electric field.

However, the gain in velocity of electron due to electric field is very small and is lost in the next collision with ion/atom of the conductor. As a result the acceleration of an electron is not proportional to the external electric field. The positive ions also experience a force due to electric field but they cannot move as they are heavy and tightly bound in the metal.

At any instant of time, the velocity acquired by electron having thermal conductor will be $\overline{v}_1 = \overline{u}_1 + \overline{a}\tau_1$ where τ_1 is the time elapsed since it has suffered its last collision with ion/ atom of the conductor. Similarly, the velocities acquired by other electrons in the conductor will be

$$\overline{v}_2 = \overline{u}_2 + \overline{a}\tau_2,\ \overline{v}_3 = \overline{u}_3 + \overline{a}\tau_3, \ldots\ldots\ldots\ldots$$

$$\overline{v}_n = \overline{u}_n + \overline{a}\tau_n,$$

The average velocity of all the free electrons in the conductor is the drift velocity $\overline{v}_d$ of the free electrons.

$$\text{Thus } \overline{v}_d = \frac{\overline{v}_1 + \overline{v}_2 + \ldots\ldots\ldots\ldots\ldots\overline{v}_n}{n}$$

$$= \frac{(\overline{u}_1 + \overline{a}\tau_1) + (\overline{u}_2 + \overline{a}\tau_2) + \ldots\ldots\ldots(\overline{u}_n + \overline{a}\tau_n)}{n}$$

$$= \left(\frac{\overline{u}_1 + \overline{u}_2 + \ldots\ldots + \overline{u}_n}{n}\right) + \overline{a}\,\frac{\tau_1 + \tau_2 + \ldots\ldots.\tau_n}{n}$$

$$= 0 + \overline{a}\tau = \overline{a}\tau$$

where $\tau = \frac{\tau_1 + \tau_2 + \ldots + \tau_n}{n}$ = average time that has elapsed since each electron suffered its last collision with ion/atom of conductor and is called relaxation time. Its value is of the order of 10^{-14} second.

Putting the value of $\overline{a}$ in the above relation, we have

$$\overline{v}_d = \frac{-e\overline{E}\tau}{m} \qquad \ldots.\ (4.7)$$

$\therefore$ Average drift speed, $v_d = \frac{eE}{m}\tau$.

This formula tells us that the electrons move with an average velocity which is independent of time.

Because of this drift, there will be net transport of charge carriers across any area normal to the field direction.

4.6 RELATION BETWEEN CURRENT AND DRIFT VELOCITY

Consider a conductor (say a copper wire) of length l and of uniform area of cross - section A. Therefore volume of the conductor = Al.

If n is the number of free electrons per unit volume of the conductor, then total number of free electrons in the conductor = Aln. If e is the charge of each electron, then total charge of all the free electrons in the conductor, q = A$l$ne.

Let a constant potential difference V be applied across the ends of the conductor with the help of battery.

The electric field set up across the conductor is given by E = V/l (magnitude). Due to this field, the free electrons present in the conductor will begin to move with a drift velocity v_d towards the left hand side as shown in Fig 4.4.

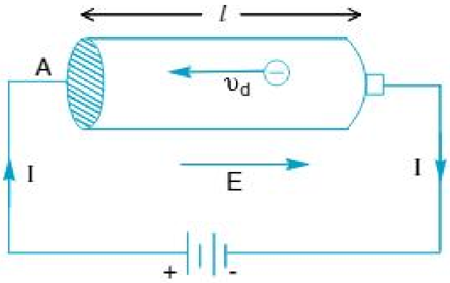


Fig 4.4 Current in a metallic conductor

$\therefore$ Time taken by the free electrons to cross the conductor, $t = l / V_d$

Hence current, $I = \frac{q}{t} = \frac{A\,l ne}{l / V_d}$

$$I = A\,neV_d$$

The equation gives the relation between the current flowing through the conductor and velocity of the electrons.

Putting the value of $V_d = \left|\frac{eE\tau}{m}\right|$ in the above equation, we have $I = \frac{Ane^2\,\tau E}{m}$ .... (4.8)

In a conductor, there are large number of free electrons or conduction electrons. If there is one free electron per atom, the number of free electrons per cubic meter of the conductor will be of the order of 10^{29}.

Knowledge Plus 4.1

☺ Even though the drift velocity is of the order of 10^{-4}m/s, an electric bulb at home glows immediately after it is switched on. How is it possible ?

☛ When we close the circuit, the electric field is set up in the entire closed circuit instantly with the speed of electromagnetic wave which causes electron drift at every portion of the circuit due to which the current is set up in the entire circuit instantly. The current so set up does not wait for the electrons to flow from one end of the conductor to other end. It is due to this reason, the electric bulb glows immediately when switched on.

Example-4.8

A current of 5A is passing through a metallic wire of cross sectional area 14×10^{-6}m^2. If the density of the charge carries in the wire is 5×10^{26}/m^3, find the drift speed of the electrons (charge carries).

Solution :

$I = 5A,\ A = 14\times10^{-6}\ m^2$

density of electron $= 5\times10^{26} / m^3$

charge of electron $e = 1.6\times10^{-19}C$.

drift speed $v_d = \frac{I}{neA}$.

$$= \frac{5}{5\times10^{26}\times1.6\times10^{-19}\times14\times10^{-6}}$$

$$= \frac{10^{-1}}{1.6\times14} = 4.46\times10^{-3}\ m/s$$

4.7 RELATION BETWEEN DRIFT VELOCITY AND CURRENT DENSITY

Let I be the current distributed uniformly across a conductor of cross-sectional area A. The magnitude of the current density for all points on that cross-section of the conductor is

$J = \frac{I}{A}$ (J is known as current density)

$\because\ I = A\,ne\,\upsilon_d$

Therefore, $J = \frac{I}{A} = neV_d$

$\overline{J}$ is a vector in the direction of $i = \overline{J}.\overline{A}$

❖ The unit of current density is [Am^{-2}]

As $I = \frac{Ane^2\tau E}{m}$, we can write $|\overline{J}| = \frac{ne^2\tau E}{m}$

Here vector $\overline{J}$ is parallel to $\overline{E}$ and so

$$\overline{J} = \frac{ne^2\tau\overline{E}}{m}$$

from ohm's law, we have $\overline{J} = \frac{\overline{E}}{\rho} = \sigma\overline{E}$ on comparison, we can write $\sigma = \frac{ne^2\tau}{m}$

So, conductivity, of a material depends on τ, n and m.

Example-4.9

Potential difference of 100 V is applied to the ends of a copper wire one metre long. Calculate the average drift velocity of the electrons. Compare it with thermal velocity at 27°C. Assume that there is one conduction electron per atom. The density of copper is $9.0\times10^3 kg/m^3$; Atomic mass of copper is 63.5g. Avogadro's number = 6.0×10^{23} per gram-mole. Conductivity of copper is $5.81\times10^7 \Omega^{-1}$. Boltzmann constant = $1.38\times10^{-23} JK^{-1}$.

Solution :

Here $V = 100V$, $l = 1$ m ; $M = 63.5g = 63.5\times10^{-3}kg$;
$\rho = 9.0\times10^3 kg/m^3$; $N = 6.0\times10^{23}$ per gram-mole;
$\sigma = 5.81\times10^7\Omega^{-1}$.

Since 6×10^{23} copper atoms have a mass of 63.5g, and there is one conduction electron per atom, number of electrons per unit volume is

$$n = \frac{6.0\times10^{23}}{63.5\times10^{-3}}\times9.0\times10^3 kg/m^3 = 8.5\times10^{28} m^{-3}$$

Electric field $E = \frac{V}{l} = \frac{100}{1} = 100 Vm^{-1}$

As $J = \sigma E = nev_d$

$$\therefore V_d = \frac{\sigma E}{ne} = \frac{(5.81\times10^7)\times(100)}{(8.5\times10^{28})\times1.6\times10^{-19}} = 0.43 ms^{-1}$$

Thermal velocity $v_{rms} = \sqrt{\frac{3k_BT}{m_e}}$

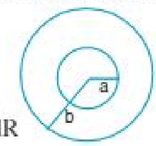
$$= \sqrt{\frac{3\times1.38\times10^{-23}\times300}{9.1\times10^{-31}}} = 1.17\times10^5 m/s$$

$$\frac{V_d}{V_{rms}} = \frac{0.43}{1.17\times10^5} = 3.67\times10^{-6}.$$

Example-4.10

The region between two concentric conducting spheres with radii a and b is filled with a conducting material of resistivity ρ. (a) Find the radial resistance between the spheres. (b) Dirive the expression for current density as a function of radius i.e., J(r) if V is potential difference between the spheres.

Solution :



(a) Consider a shell of thickness dr and radius r. Resistance of such shell is dR

$$dR = \frac{\rho dr}{4\pi r^2} \Rightarrow R = \int_a^b dR \Rightarrow R = \int_a^b \frac{\rho dr}{4\pi r^2} = \frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$$

(b) $I = \frac{V}{R} = \frac{V4\pi ab}{(b-a)}$

$$J(r) = \frac{I}{4\pi r^2} = \frac{V4\pi ab}{(b-a)4\pi r^2} \Rightarrow J(r) = \frac{Vab}{\rho(b-a)r^2}$$

4.8 MOBILITY

Mobility (μ), is defined as the magnitude of drift velocity per unit electric field applied. i.e.

$$\mu = \frac{\text{drift velocity}}{\text{electric field}} = \frac{|V_d|}{E} = \frac{eE\tau/m}{E} = \frac{e\tau}{m} \quad ...(4.9a)$$

Mobility of holes (i.e. positive charge carriers present in semiconductors) will be given by

$$\mu_h = \frac{e\tau_h}{m_h} \quad ...(4.9b)$$

where, τ_h is average relaxation time for holes, m_h refer to mass of hole and Charge is e.

Mobility is positive for both positive current carriers and negative current carriers, although their drift velocities are opposite to each other.

SI unit of mobility is $m^2s^{-1}V^{-1}$ or $ms^{-1}N^{-1}C$

Mobility of some materials, at room temperature are given in Table 4.1.

Table 4.1 Mobility of some materials

Materials	Electrons Mobilities ($cm^2 V^{-1} s^{-1}$)	Holes mobilities ($cm^2 V^{-1} s^{-1}$)
Diamond	1800	1200
Silicon	1350	480
Germanium	3600	1800
InSb	800	450
GaAs	8000	300

4.9 RELATION BETWEEN RESISTIVITY, FREE ELECTRON CONCENTRATION AND RELAXATION TIME

Resistivity depends on the nature of the material but it is independent of its dimensions, whereas the resistance of a conductor depends on its dimensions as well as on the material of which it is made up of.

In the terms of free electron concentration (n) and relaxation time (τ) the expression for resistivity is,

$$\rho = \frac{m}{ne^2\tau} \quad ....(4.10)$$

where e and m respectively are the charge and mass of an electron. The resistivity at a temperature is inversely proportional to free electron concentration. Thus, for a conductor having large number of free electrons resistivity and hence resistance is small. Silver is the best conductor. Copper and aluminium are also good conductors. Due to very low resistance copper and aluminium wires are used as connecting wires for joining various components of electrical circuits and house hold fitting. Several resistance wires of high resistivity are made of materials obtained by alloying some metals.

Some of the important alloys are manganin (84% Cu, 4% Ni and 12% Mn), constanton (60% Cu, 40% Zn) and nichrome (80 % Ni, 20% Cr). Resistance wires for electric heater, electric iron etc, are made of these alloys.

Due to extremely high resistivity ebonite, mica, china clay, fused quartz etc are used as insulators. In household wiring, copper and aluminium conductors are covered with a layer of some insulating materials like P.V.C (polyvinyl chloride), rubber, cotton etc.

We have materials like germanium (Ge) and silicon (Si) which have resistivity much smaller than that of insulators but much greater than that of metals.

They are called semi-conductors. Semi-conductors are used to make electronic devices such as diode, transistor etc.

4.10 TEMPERATURE DEPENDENCE OF RESISTANCE

The relation for resistivity and relaxation time is $\rho = \frac{m}{ne^2\tau}$ according to which the resistivity is inversely proportional to relaxation time τ. The resistance of a wire of length l and area of cross - section A is given by,

$$R = \rho\frac{l}{A} = \frac{ml}{ne^2\tau A}$$

For a given wire l, A and n are constants, therefore resistance $R \alpha \frac{1}{\tau}$

Relaxation time is the average time between successive collisions of electrons with the lattice ions (positive ions of metal). If mean free path (the average distance between two successive collisions) of electrons is λ and root mean square speed is v_{rms}, then

$$\tau = \frac{\lambda}{v_{rms}} \qquad \text{Therefore } R \alpha \frac{1}{\tau} \alpha \frac{v_{rms}}{\lambda}$$

With increase of temperature, root mean square speed increases ($v_{rms} \ \alpha \ T$) and mean free path decreases (because amplitude of vibration of lattice increases) so that collisions of electrons with the lattice take place more frequently. As a result resistivity and hence, resistance of the wire increases with increase of temperature. In other words conductivity of conductor decreases with increase of temperature.

If R_1 and R_2 are the values of resistance of a wire at t_1^0C and t_2^0C respectively then R_2 may be obtained by relation.

$$R_2 = R_1[1+\alpha(t_2 - t_1)] \qquad .... (4.11)$$

where α is a constant, called temperature coefficient of resistance (or resistivity). w.r.t. temp t_1

At $t_2 = t\ ^0C; t_1 = 0\ ^0C; \alpha$ w.r.t temp 0^0C, then $R_t = R_0(1+\alpha t)$

$$\text{Then } \alpha = \frac{R_t - R_0}{R_0 t} = \frac{R_t - R_0}{R_0} \text{ (per } 1^0C)$$

If $R_0 = 1$ and $t = 1^0$ C, then; $\alpha = R_t - R_0$, the increase in resistance, thus temperature coefficient of resistance is equal to changes in resistance of a wire of resistance one ohm at 0^0C when temperature changes by 1^oC.

If the resistance of a wire at temperature t_1^0C is R_1 and at t_2^0C is R_2, then

$$R_1 = R_0\ (1+\alpha t_1) \text{ and } R_2 = R_0(1+\alpha t_2)$$

$$\text{On dividing } \frac{R_1}{R_2} = \frac{1+\alpha t_1}{1+\alpha t_2}$$

$$\text{So that } \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1} (\text{per}^0C) \qquad .... (4.12)$$

Example-4.11

The temperature coefficient of resistance of a wire is $0.00125^0\ C^{-1}$ w.r.t temperature at 300 K at which the resistance of the wire in one ohm. Find the temperature at which the resistance of the wire will be 2 ohm.

Solution :

$\alpha = 0.00125^0 C,\ t_1 = 300K, R_1 = 1\Omega$

$R_2 = 2\Omega,\ t_2 = ?$

Temperature coefficient resistance

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}\ ;\ 0.00125 = \frac{2-1}{1(t_2 - 300)}$$

$$t_2 - 300 = \frac{1}{0.00125}$$

$$t_2 = 300 + \frac{10^3}{1.25} = 300 + 800 = 1100K$$

Final temperature $t_2 = 1100K$

Example-4.12

A wire has a resistance of 2.5Ω at 100^0 C. Temperature coefficient of resistance of the material $\alpha = 3.6 \times 10^{-3} K^{-1}$ at 25^0C. Find its resistance at 25^0C.

Solution :

$R = 2.5\Omega,\ t_2 = 100^0 C$

$\alpha = 3.6 \times 10^{-3} K^{-1},\ R_1 = ?$

$t_2 = 100 + 273 = 373$ K. $t_1 = 25^0C = 273 + 25 = 298K$

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$$

$$3.6 \times 10^{-3} = \frac{2.5 - R_1}{R_1(373 - 298)} = \frac{2.5 - R_1}{R_1 \times 75}.$$

$$3.6 \times 10^{-3} \times 75 \times R_1 = 2.5 - R_1$$

$$0.270 \times R_1 = 2.5 - R_1$$

$$0.270 \times R_1 + R_1 = 2.5$$

$$1.27\ R_1 = 2.5 \Rightarrow R_1 = 1.96\Omega$$

Resistance at 25^0C, $R_1 = 1.97\ \Omega$

Example-4.13

At room temperature, 27^0C, the resistance of a heating element is $100\,\Omega$. At temperature t^0C, its ressitance is found to be $117\,\Omega$. If $\alpha = 1.7 \times 10^{-4} / {}^0C$, find *t*.

Solution :

$R_2 = R_1\{1 + \alpha(t_2 - t_1)\}$

$R_1 = 100\Omega$ and $R_2 = 117\Omega$, $t_1 = 27^0C$, $t_2 = 1$

$$(t_2 - t_1) = \frac{R_2 - R_1}{R_1 \alpha} = \frac{117 - 100}{100 \times 1.7 \times 10^{-4}} = 1000^0 C$$

$\Rightarrow t_2 = t_1 = 1027^0C$

4.11 TEMPERATURE DEPENDANCE OF RESISTIVITY

The resistivity of alloys also increases with increase of temperature but increase is very small in comparison to that for metals. For alloys such as manganin, constanton and nichrome temperature coefficient of resistance is negligibly small and resistivity is high, hence these are used to make resistance wires or standard resistances.

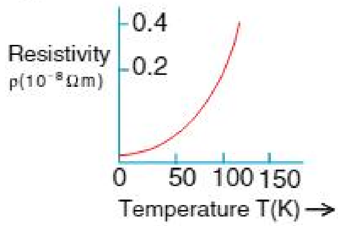
The resistivity of some materials like carbon, silicon, germanium etc. decrease with increase of temperature that is temperature coefficient of resistivity for these material is negative.

Over a limited range of temperatures, the resistivity of a metallic conductor is approximately given by

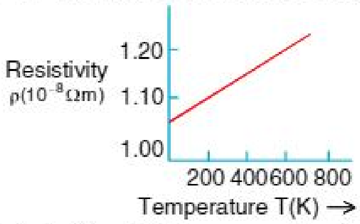
$$\rho_T = \rho_0[1 + \alpha(T - T_0)] \qquad .... (4.13)$$

where ρ_T is the resistivity at a temperature T; ρ_0 is the same at a reference temperature T_0 and α is the temperature coefficient of resistivity.

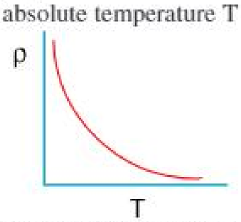
Resistivity versus temperature graphs are shown in Fig 4.5.



Resistivity ρ_T of copper as a function of temperature T



Resistivity ρ_T of nichrome as a function of absolute temperature T



Temperature dependence of resistivity for a typical semiconductor

Fig 4.5 Graph of ρ versus T

Example-4.14

The temperature coefficient of resistance of platinum is $\alpha = 3.92\times10^{-3}\,K^{-1}$ at 0^0 C. Find the temperature at which the increase in the resistance of platinum wire is 10% of its value at 0^0C.

Solution :

Let resistance of the wire be R_1 at $t_1^0C(=0^0C)$

At t_2^0C resistance of the wire R_2

$$R_2 = \frac{110R_1}{100} = 1.1R_1, \quad \alpha = 3.92\times10^{-3}K^{-1}$$

$$\Delta t = \frac{R_2 - R_1}{R_1\alpha} \Rightarrow = \frac{1.1R_1 - R_1}{R_1\alpha}$$

$$= \frac{R_1(1.1-1)}{R_1\alpha} = \frac{0.1R_1}{R_1\alpha} = \frac{0.1}{3.92\times10^{-3}}\,;\ \Delta t = 25.51^0C$$

$$t_2 = 25.51 + 0 = 25.51^0C$$

Example-4.15

The temperature coefficient of resistivity of a material is 0.00004 K^{-1}. When the temperature is increased by 50 ^{0}C., the resistivity increases by $2\times10^{-8}\,\Omega - m$. Find the initial resistivity of the material.

Solution :

$t_2 - t_1 = 50^0C,\ \rho_2 - \rho_1 = 2\times10^{-8}\,\Omega - m$

$\alpha = 0.00004\,K^{-1}$. Initial Resistivity $\rho_1 = ?$

$$\alpha = \frac{(\rho_2 - \rho_1)/\rho_1}{(t_2 - t_1)}$$

$$\therefore \rho = \frac{(\rho_2 - \rho_1)}{\alpha(t_2 - t_1)} = \frac{2\times10^{-8}}{0.00004\times50}$$

$$= 1000\times10^{-8}\,\Omega - m$$

4.12 ATOMIC EXPLANATION OF OHM'S LAW

Let us consider a cylindrical conductor of cross -sectional area A, in which an electric field E exists. In time Δt charge drifts a distance $v_d\Delta t$, where v_d is the drift velocity. Consider a length $V_d\Delta t$ of the conductor (Fig 4.6).

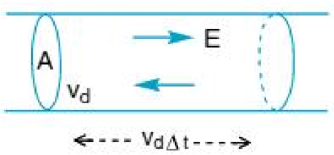


Fig 4.6 Cross - section of conductor

The volume of this portion is $Av_d\ \Delta t$. Let there be n free electrons per unit volume of conductor. The number of electrons in this portion is $nAv_d\ \Delta t$. All these electrons cross the area A in time Δt.

Therefore, charge crossing this area in time Δt is, $\Delta Q = nAV_d(\Delta t)e$ or current

$$I = \frac{\Delta Q}{\Delta t} = Ane\,v_d = \frac{Ane^2\tau E}{m} = \frac{Ane^2V}{ml}$$

Thus, $I \propto V$

This is Ohm's law.

It is clear from this expression that the current is directly proportional to the drift velocity of free electrons in a conductor.

Example-4.16

A total of 6.0×10^{16} electrons pass through any cross-section of a conducting wire per second Find the current.

Solution : The total charge crossing the cross section in one second is, $\Delta Q = ne = 6.0\times10^{16}\times1.6\times10^{-19}C$

$$= 9.6\times10^{-3}C$$

The value of current

$$I = \frac{\Delta Q}{\Delta t} = \frac{9.6\times10^{-3}C}{1s} = 9.6\times10^{-3}A$$

Example-4.17

Calculate the drift speed of electrons when 1A of current flows in a copper wire of cross section 2 mm^2. The number of free electrons in 1 cm^3 of copper is 8.5×10^{22}.

Solution :

From the current and drift velocity relation $I = Ane\,v_d$,

$$v_d = \frac{I}{Ane} = \frac{1}{2\times10^{-6}\times8.5\times10^{28}\times1.6\times10^{-19}}$$

$$= 3.7\times10^{-5}\ m/s$$

4.13 SUPER CONDUCTORS

We see that for some materials below a certain temperature resistivity suddenly becomes zero. This temperature is called critical temperature for this transition. The material in this state is called superconductor and the phenomenon is called superconductivity. It was observed for mercury in 1911 by Kamerleigh Onnes. Critical temperature for mercury is 4.2 K.

If an electric current is set up in a superconductor, it can persist for long time even for months and years after removing the applied potential difference. Superconductivity exists at very low temperatures which are difficult to obtain. Scientists are trying to prepare compounds and alloys which would be superconducting at room temperatures (300K).

Superconductivity at around 125 K has already been achieved and efforts are on to improve upon this. Superconductors are used to construct very strong magnets. Possible applications of superconductors are ultra fast computer switches and transmission of electric power through superconducting power lines.

Example-4.18

Two wires A and B of same mass and material are taken. Diameter of wire A is half of wire B. If resistance of wire A is 24 Ω find the resistance of wire B.

Solution :

Let r_A and r_B be radii and l_A, l_B be lengths of wire A and B respectively. As mass and density of the wires is the same, we have $\pi r_A^2 l_A d = \pi r_B^2 l_B d$

Therefore $\frac{l_A}{l_B} = \frac{r_B^2}{r_A^2}$

Resistances of wires A and B are

$$R_A = \rho \frac{l_A}{\pi r_A^2} \text{ and } R_B = \rho \frac{l_B}{\pi r_B^2}$$

$$\therefore \frac{R_A}{R_B} = \frac{\ell_A}{\ell_B} \times \frac{r_B^2}{r_A^2} = \frac{r_B^2}{r_A^2} \times \frac{r_B^2}{r_A^2} = \left(\frac{r_B}{r_A}\right)^4$$

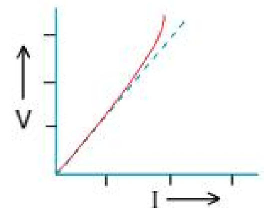
$$\Rightarrow r_A = \frac{r_B}{2} \quad \Rightarrow \quad \frac{R_A}{R_B} = (2)^4 = 16$$

$$\therefore R_B = \frac{R_A}{16} = \frac{24}{16} = 1.5 \text{ ohm.}$$

4.14 OHMIC AND NON-OHMIC RESISTANCES

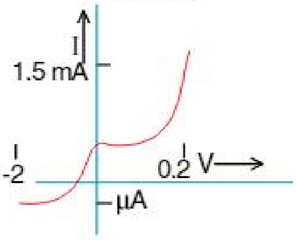
By drawing graph between voltage and current across conductors we observe that many conductors obey Ohm's law. Their resistance is called Ohmic resistance or linear resistance. But Ohm's law does not always hold good. If we replace the resistance wire by a torch bulb in an electrical circuit and note down values of current (I) for different voltages (V) then we see that the entire V–I graph drawn is not straight line (Fig 4.7(a)). For low values of V, it remains straight line and then becomes curved.

For high voltage, current through the filament of the bulb becomes large so that the temperature of the filament of bulb becomes higher and higher as current increases in the filament. Ratio V/I for low value of I gives resistance of the filament. Ohm's law holds in metallic wires for low values of current only.



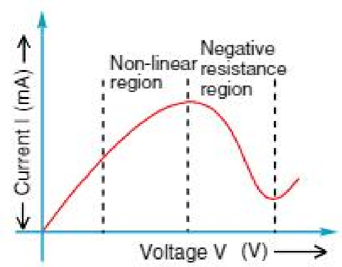
(a) The dashed line represents the linear Ohm's law.

The solid line is the voltage V versus current I for a good conductor.



(b) Characteristic curve of a diode.

Note the different scales for negative and positive values of the voltage and current.



(c) Variation of current versus voltage for Gas

Fig 4.7 V-I Graph for ohmic and non–ohmic conductors

Other examples of non-ohmic resistances are vacuum diode, semiconductor diode, transistor liquid electrolytes etc. In Vacuum diode ohm's law does not hold even for low values of current. Its V–I curve is shown in Fig 4.7 b.

The relation between V and I is not unique, i.e., there is more than one value of V for the same current I. A material exhibiting such behaviour is GaAs (Fig 4.7(c)).

4.15 THERMISTOR

It is heat-sensitive resistor (semiconducting material) such that its resistance varies appreciably with temperature. The temperature coefficient of resistance of a thermistor is very high.

Thermistor type 1 has a negative temperature coefficient of resistance i.e., resistance decreases with increase of temperature.

Thermistor type 2 has a high positive temperature coefficient of resistance.

A mixture of oxides of manganese and nickel is used in the fabrication of thermistors.

Important applications of thermistor are

1. The thermistor with a high negative temperature coefficient of resistance is made use in resistance thermometer in very low temperature measurement.
2. Thermistors can be used to detect even very small change in temperature.
3. Thermistor with negative temperature coefficient are employed to safeguard against current surges in circuit where this could be harmful.
4. Thermistors are used in temperature-control units.
5. Thermistors can be used for the detection of excessive temperature in an industrial equipment.
6. Thermistors are used to protect the windows of transformers, motors etc.
7. Thermistors are used for voltage stabilisation.
8. Thermistors are used for remote sensing.
9. Thermistors are used to protect the filament (heater unit) of the picture tube of a television set against the variation of current.

4.16 RESISTORS

For different electrical and electronic circuits we require resistors of different values. Resistors may be divided into two groups; wire wound resistors and carbon resistors. In a wire wound resistor a resistance wire (of magnanin, constanton or nichrome) of definite length according to value of resistance is wound two fold over insulating cylinder to make it non inductive.

To make carbon resistor, carbon with a suitable binding agent is molded into a cylinder. Wire leads are attached to this cylinder for connecting it to an electrical circuit. The value of resistance is indicated by four coloured bands marked on the cylinder (Fig 4.8) and meaning of different colours are given in table 4.2 the colours and their orders may be remembered by the statement given.

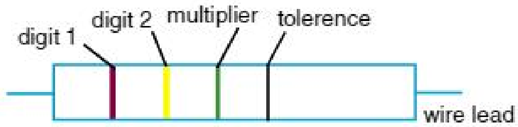


Fig 4.8 Colour bands on a resistor

Table 4.2 Resistance codes (resistance given in ohm)

Colour	Digit	Multiplier	Tolerance
Black	0	1	
Brown	1	10	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		0.1	5%
Silver		0.01	10%

B	B	R	O	Y
Black	Brown	Red	Orange	Yellow
Great	Britain	Very	Good	Wife
Green	Blue	Violet	Gray	White

For example suppose the colours on the resistor shown in Fig 4.8 are brown, yellow, green and gold as read from left to right. Using the table the resistance is

Brown	Yellow	Green	Gold
1	4 $\longrightarrow$	$\times\ 10^5 \longrightarrow$	$+5\% \longrightarrow$

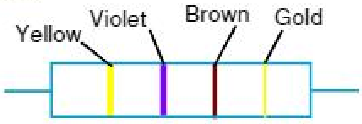
$$= 14 \times 10^5 \left(1 \pm \frac{5}{100}\right)\Omega$$

$$= (1.4 \pm 0.07)10^6\,\Omega = (1.4 \pm 0.07)\text{M}\Omega$$

Some times tolerance is missing from the code and there are only three bands. Then the tolerance is 20%.

Example-4.19

A carbon resistor has coloured stripes as shown. What is its resistance?



Solution :

For yellow - 4 is significant

violet - 7 is significant

As for brown - 1 is significant, number of zeros to be attached = 1

For Gold stripe tolerence = 5%

So value of resistance = $(470 \pm 5\%)\ \Omega$

4.17 COMBINATIONS OF RESISTORS

4.17.1 SERIES COMBINATION

Connect more resistors in series by joining them end to end such that same current passes through all the resistors. In Fig 4.9 three resistors of resistances R_1, R_2 and R_3 are shown connected in series. The combination can be connected to a battery or other circuit at ends A and D. Let a current I flow through the series combination when it is connected to a battery of voltage V. Potential difference V1, V_2 and V_3 be developed across R_1, R_2 and R_3 respectively due to this current I then $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$. But sum of V_1, V_2 and V_3 is equal to V i.e.,

$$V = V_1 + V_2 + V_3 \Rightarrow V = IR_1 + IR_2 + IR_3$$

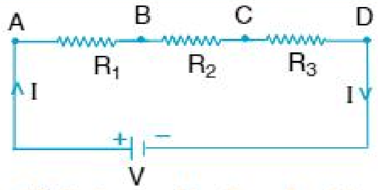


Fig 4.9 Series combination of resistance

If equivalent resistance of this series combination is R, then

$$V = IR = I(R_1 + R_2 + R_3) \quad \text{or} \quad R = R_1 + R_2 + R_3$$

This arrangement may be extended for any number of resistors.

$$R = R_1 + R_2 + R_3 + R_4 + \ldots.. \qquad \ldots.\ (4.15)$$

Thus, equivalent resistance of a series combination of resistors is equal to sum of resistances of all resistors.

If we require to apply a voltage across a resistor (say electric bulb) less than the constant voltage of supply source, we connect another resistor in series to it.

4.17.2 PARALLEL COMBINATION

Connect the resistors in parallel by joining their one end at one point and other end at another point. In parallel combination same potential difference exists across all resistors.

Fig.4.10 shows the parallel combination of three resistors of resistances R_1, R_2 and R_3. Let the combination be connected to a battery of voltage V and draw a current I from the source.

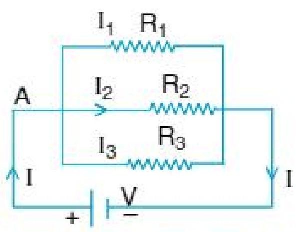


Fig 4.10 Parallel Combination of Resistance

The main current is divides into three parts. Let I_1, I_2, I_3 be the currents flowing through resistors R_1, R_2, R_3 respectively, then $I_1 = V/R_1$, $I_2 = V/R_2$ and $I_3 = V/R_3$. But sum of V_1, V_2 and V_3 is equal to V. i.e.,

The main current is the sum of I_1, I_2 and I_3

i.e. $I = I_1 + I_2 + I_3$ or $I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

If the equivalent resistance of combination is R, then $V = IR$ or $I = V/R$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}\ ;\ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The process may be extended for any number of resistors so that,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \ldots\ldots \quad \ldots. (4.16)$$

From this we infer that inverse of equivalent of resistance of parallel combination is equal to sum of inverses of individual resistances.

For two resistors in parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1R_2}{R_1+R_2}$$

Knowledge Plus 4.2

☺ In our homes all the electrical appliances bulbs, fans, heaters etc are connected in parallel and each has separate switch (Fig. 4.11). Why ?

☛ Potential difference across each appliance remains same so that current flowing in any of them does not depend upon the other. In other words each appliance can be operated independent of the presence of the others. As we go on switching bulbs, and fans, the resistance of the electrical circuit of the house goes on decreasing and current drawn from mains goes on increasing.

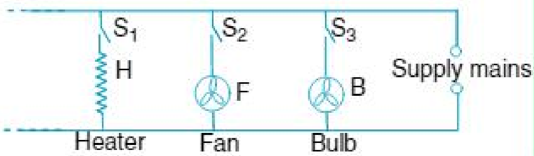


Fig 4.11 Electrical circuit at our homes

- The equivalent resistance of parallel combination is smaller than the smallest individual resistance. Consider a simple electrical circuit having a resistor of 2Ω resistance connected across a battery of voltage 2 volt. It will draw a current of 1 ampere.

- When another resistor of 2 ohm resistance is connected in parallel, then it will also draw a current of 1 ampere. That is total current drawn from battery is 2 ampere, hence resistance of the circuit is halved. If we go on increasing the number of resistors in parallel the resistance of circuit goes on decreasing and the current drawn from battery goes on increasing.

4.17.3 DIVISION OF CURRENT IN RESISTORS CONNECTED IN PARALLEL

Let the two resistors of resistances R_1 and R_2 be connected in parallel between points A and B (Fig 4.12). The main current I be divided into two parts I_1 and I_2 flowing through R_1 and R_2 respectively. The main current I is sum of I_1 and I_2 i.e $I_1 + I_2 = I$.

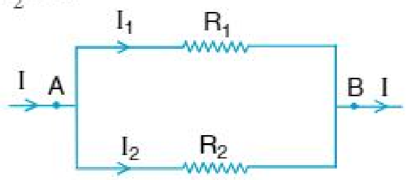


Fig 4.12 Division of current in resistance

According to Ohm's law $V_A - V_B = I_1R_1$ and also $V_A - V_B = I_2R_2$

Therefore $I_1R_1 = I_2R_2$

$$I_1R_1 = (I - I_1)R_2$$

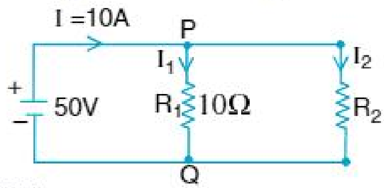
$$I_1(R_1 + R_2) = IR_2$$

or $I_1 = \frac{R_2}{R_1+R_2} I$,

Similarly, $I_2 = \frac{R_1}{R_1+R_2} I$.

Example-4.20

For a circuit shown in Fig find the value of resistance R_2 and current I_2 flowing through R_2



Solution :

If equivalent resistance of parallel combination of R_1 and R_2 is R, then

$$R=\frac{R_1R_2}{R_1+R_2}=\frac{10R_2}{10+R_2}$$

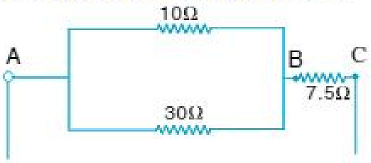
According to Ohm's law,

$$R=\frac{50}{10}=5\Omega \Rightarrow \frac{10R_2}{10+R_2}=5 \Rightarrow R_2=10\Omega.$$

The current is equally divided into R_1 and R_2. Hence $I_2=5A$.

Example-4.21

Find equivalent resistance of the network in Fig. between points (i) A and B and (ii) A and C.



Solution :

(i) The 10Ω and 30Ω resistors are connected in parallel between points A and B. The equivalent resistance between A and B is

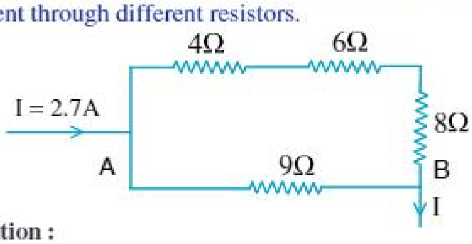
$$R_1=\frac{10\times 30}{10+30}\text{ohm}=7.5\Omega$$

(ii) The resistance R_1 is connected in series with resistor of 7.5Ω, hence the equivalent resistance between points A and C is,

$R_2=(R_1+7.5)$ ohm $=(7.5+7.5)$ ohm $=15\Omega$.

Example-4.22

Find potential difference between points A and B of the network shown in Fig. and distribution of given main current through different resistors.



Solution :

Between points A and B resistors of 4Ω, 6Ω and 8Ω resistances are in series and these are in parallel to 9Ω resistor.

Equivalent resistance of series combination is

$R_1=(4+6+8)$ ohm $=18\,\Omega$

Equivalent resistance between A and B is

$R=9\times 18/(9+18)$ ohm $=6\,\Omega$

Potential difference between A and B is

$V=IR=2.7\times 6V=16.2V$

Current through $9\,\Omega$ resistor = 16.2/9=1.8A

Current through $4\,\Omega$, $6\,\Omega$ and $8\,\Omega$ resistors = $2.7-1.8=0.9A$.

4.18 ELECTRICAL ENERGY AND POWER

Consider a conductor with end points A and B, in which a current I is flowing from A to B. The electric potential at A and B are denoted by V(A) and V(B) respectively. Since current is flowing from A to B, V(A)>V(B) and the potential difference across AB is $V=V(A)-V(B)>0$.

In a time interval Δt, an amount of charge $\Delta Q=I\Delta t$ travels from A to B. The potential energy of the charge at A, by definition was, Q V(A) and similarly at B, it is Q V(B). Thus, change in its potential energy ΔU_{pot} is

ΔU_{pot} = Final potential energy - Initial potential energy

$$=\Delta Q[V(B)-V(A)]=-(\Delta Q)V=-IV\Delta t<0.$$

If charges moved without collisions through the conductor, their kinetic energy would also change so that the total energy is unchanged. Conservation of total energy would then imply that,

$\Delta K=-U_{pot}$ that is, $\Delta K=IV\Delta t>0$

Thus, in case charges were moving freely through the conductor under the action of electric field, their kinetic energy would increase as they move. During collisions, the energy gained by the charges thus is shared with the atoms. The atoms vibrate more vigorously, i.e., the conductor heats up.

Thus, in an actual conductor, an amount of energy dissipated as heat in the conductor during the time interval Δt is,

$\Delta W=IV\Delta t$

The energy dissipated per unit time is the power dissipated $P=\Delta W/\Delta t$ and we have, $P=IV$

Using Ohm's law $V=IR$, we get

$$P=I^2R=V^2/R \quad \text{.... (4.14)}$$

as the power loss ("ohmic loss") in a conductor of resistance R carrying a current I. It is this power which heats up.

for example, the coil of an electric bulb to incandescence, radiating out heat and light.

The connecting wires from the power station to the device has a finite resistance R_c. The power dissipated in the connecting wires, which is wasted is P_c with $P_c = I^2R_c = \frac{P^2R_c}{V^2}$

Thus, to drive a device of power P, the power wasted in the connecting wires is inversely proportional to V^2. The transmission cables from power stations are hundreds of miles long and their resistance R_c is considerable.

To reduce P_c, these wires carry current at enormous values of V and this is the reason for the high voltage danger signs on transmission lines - a common sight as we move away from populated areas.

Using electricity at such voltage is not safe and hence at the other end, a device called a transformer lowers the voltage to a value suitable for use.

Knowledge Plus 4.3

☺ Electrical power is transmitted from power stations to home and factories, which may be hundreds of miles away, via transmission cables. One obviously wants to minimise the power loss in the transmission cables connecting the power stations to homes and factories. How this can be achieved ?

☛Consider a device R, to which a power P is to be delivered via transmission cables having a resistance R_c to be dissipated by it finally. If V is the voltage across R and I the current through it, then P = VI.

Example-4.23

Two electric lamps of 40 W each are connected in parallel across the mains supply. Find the total power consumed by the two bulbs together.

Solution :

Power of each bulb P = 40W

Total consumption of power by the two bulbs

$$P = P_1 + P_2 = 40 + 40 = 80W.$$

Example-4.24

Two electric bulbs have their resistances in the ratio 2 : 3. They are connected (a) first in series and then (b) in parallel across the same voltage. Find the ratio of powers consumed by each of the two bulbs in the two combinations.

Solution : $\frac{R_1}{R_2} = \frac{2}{3}$

a) Series combination :

power $P = i^2R$. $\quad \therefore \frac{p_1}{p_2} = \frac{i^2R_1}{i^2R_2}$

$\because$ current is same in series combination

$$\therefore \frac{p_1}{p_2} = \frac{R_1}{R_2} = \frac{2}{3} \qquad \therefore p_1 : p_2 = 2:3$$

b) Parallel combination:

$$\frac{p_1}{p_2} = \frac{i_1^2R_1}{i_2^2R_2} = \frac{\left(\frac{V^2}{R_1^2}\right)R_1}{\left(\frac{V^2}{R_2^2}\right)R_2} \qquad [\because V = iR]$$

$$= \frac{1/R_1}{1/R_2} = \frac{R_2}{R_1} = \frac{3}{2} \qquad \therefore p_1 : p_2 = 3:2.$$

Example-4.25

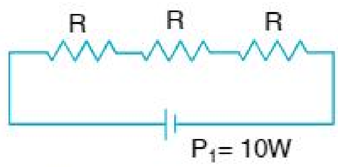
Three equal resistors connected in series across a source of e.m.f. together dissipate 10W power. Find the power dissipated if the same resistors are connected in parallel.

Solution :

$P_1 = 10W$

In series combination, total resistance = 3R

$$\text{power} = P_1 = \frac{V^2}{3R}$$



In parallel combination, total resistance =R/3

$$P_2 = \frac{V^2}{R/3}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{R/3}{3R} \Rightarrow \frac{10}{P_2} = \frac{R/3}{3R}$$

$$\frac{10}{P_2} = \frac{R}{3\times 3R} = \frac{1}{9} \Rightarrow P_2 = 90 \text{ watts}$$

Example-4.26

Two heater coils separately take 10 minutes and 5 minutes to boil a certain amount of water. Find the time taken by both the coils connected in series to boil the same amount of water.

Solution :

$t_1 = 10$ min, $t_2 = 5$ min.

$$\Rightarrow \frac{t}{R} = \text{constant}\ ;\ \frac{t_1}{t_2} = \frac{R_1}{R_2} \Rightarrow \frac{10}{5} = \frac{R_1}{R_2}$$

$\therefore R_1 : R_2 = 2:1$

Resultant resistance $R = R_1 + R_2 = 2R + R = 3R$. If 2R heater takes 10 minutes then 3R heater takes 15 minutes for heating purpose.

Example-4.27

Ten 50W bulbs are operated on an average for 10 hours a day. Find the energy consumed in kWh in one month of 30 days.

Solution :

Electrical energy consumed by Ten 50 W bulbs at the rate of 10 hours a day for 30 days = total wattage × hours of use × 30

$$= (10\times 50)\times 10\times 30 \text{ watt} - \text{hours}$$

$$= \frac{10\times 50\times 10\times 30}{1000} \text{ kWh} = 150\text{kWh}$$

Example-4.28

A bulb rated 100 W, 220 V is connected across 110V mainsline. Find the power consumed.

Solution :

Resistance of bulb $R = \frac{V^2}{P} = \frac{(220)^2}{100} = 22\times 22 = 484\,\Omega$

Power consumed by the bulb when connected to mainsline

$$P = \frac{V_1^2}{R} = \frac{110\times 110}{484} = 25 \text{ watts}$$

4.19 KIRCHOFF'S LAW

Ohm's law gives current - voltage relation in simple electrical circuits. But when the circuit is complicated, it will be difficult to find current distribution by Ohm's law.

Kirchhoff in 1842 formulated the following two laws which enable us to find the distribution of current in complicated electrical circuits or electrical networks.

(i) Kirchoff's First Law (Junction Law)

It states that the sum of all the currents directed towards a junction (point) in an electrical network is equal to the sum of all the currents directed away from the junction. Thus, in Fig 4.13.

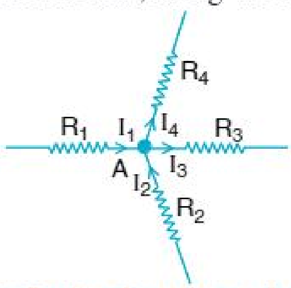


Fig 4.13 Distribution of current at a junction in the circuit

$$I_1 + I_2 = I_3 + I_4 \text{ or } I_1 + I_2 - I_3 - I_4 = 0$$

If we take currents approaching point A in Fig as positive and that leaving the point as negative, then the above relation may be written as

$$I_1 + I_2 + (-I_3) + (-I_4) = 0$$

Hence the first law may be stated in other words that the algebraic sum of currents at a junction is zero. Kirchoff's first law tells us that there is no accumulation of charge at any point if steady current flows in it. The net charge coming towards the point should be equal to that going away from it in the same time.

$$\Sigma I = 0 \qquad .... (4.17)$$

(ii) Kirchoff's Second Law (Loop Law)

This law is generalization of Ohm's law. It tells that the algebraic sum of the products of the currents and resistances in any closed loop (or mesh) in an electrical network is equal to the algebraic sum of electromotive forces acting in the loop.

While using this law we start from a point on the loop and go along the loop either clock wise or anti clock - wise to reach the same point again. The product of current and resistance is taken as positive when we traverse in the direction of current and e.m.f. is taken positive when we traverse from negative to positive electrode through the cell. Mathematically we can write the law as

$$\sum IR = \sum E \qquad .... (4.18)$$

Let us take an electrical network shown in Fig. 4.14 For closed mesh ADCBA,

$I_1R_1 - I_2R_2 = E_1 - E_2$.

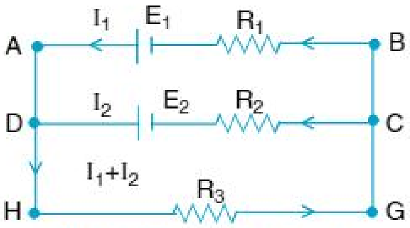


Fig 4.14 Electrical net work

For mesh DHGCD

$I_2R_2 + (I_1 + I_2)R_3 = E_2$ and for mesh AHGBA

$I_1R_1 + (I_1 + I_2)R_3 = E_1$

In more general form Kirchhoff's second law is stated as: The algebraic sum of all the potential difference along a closed loop in a circuit is zero.

Example-4.29

Consider the network as shown in Fig. Current is supplied to the network by two batteries as shown. Find the values of currents I_1, I_2, I_3. The direction of the currents are as indicated by the arrows.

Solution :

Applying Kirchhoff's 1st law to junction C, we get

$I_1 + I_2 - I_3 = 0$

Applying Kirchhoff's IInd law to the closed meshes ACDA and BCDB, we get

$5I_1 + 2I_3 = 12$ ..... (2)

$3I_2 + 2I_3 = 6$ ..... (3)

Subtracting eq (3) from eq (2) we get

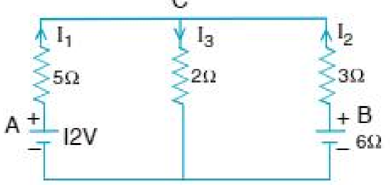
$5I_1 - 3I_2 = 6$ ..... (4)

Multiplying eq. (1) by 2 and adding with eq. (2) we get

$7I_1 + 2I_2 = 12$ ...... (5)

Multiplying eq (4) by 2 and eq. (5) by 3 and adding them we get

$31I_1 = 48 \Rightarrow I_1 = 1.548$ A.



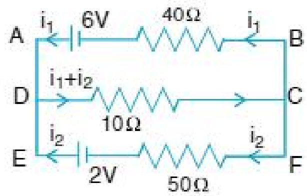
Putting value of I_1 in eq. (5) we get $I_2 = 0.58$A and from eq. 1 we get $I_3 = I_1 + I_2 = 2.128$ A.

Example-4.30

Find the current in each cell considering circuit given in fig.

Solution :

Applying Kirchhoff's law for the loop ABCDA



$6 = 40i_1 + (i_1 + i_2)10$

$6 = 40i_1 + 10i_1 + 10i_2$

$6 = 50i_1 + 10i_2$ .... (1)

For the loop DCFED

$2 = 10(i_1 + i_2) + 50i_2$

$2 = 10i_1 + 10i_2 + 50i_2$

$2 = 10i_1 + 60i_2$ ...... (2)

From (1) and (2)

$290\,i_1 = 34$

$i_1 = 0.1172\,A$ .... (3)

Substituting (3) in (2)

$2 = 10 \times 0.1172 + 60i_2$; $2 = 1.172 + 60i_2$

$60i_2 = 2 - 1.172$; $i_2 = \dfrac{2 - 1.172}{60} = 0.0138A$

Current in the 6V cell = 0.1172A

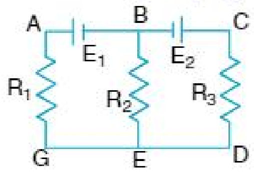
Current in the 2V cell = 0.0138A.

Example-4.31

In the given circuit values are as follows

$\varepsilon_1 = 2V, \varepsilon_2 = 4V, R_1 = 1\Omega$ and $R_2 = R_3 = 1\Omega$.

Calculate the Currents through R_1, R_2 and R_3.

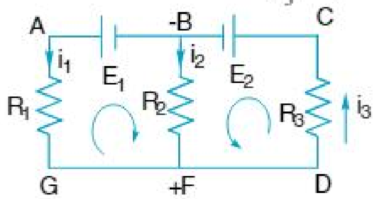


Solution :

Potential difference across BE is equal to the e.m.f. of E_1.

i) The current through R_1 is zero because the resultant p.d across AG = 0. The p.d. across BE due to $E_2 = -2\Omega$ volts and $E_1 = +2$ volts, hence , p.d. across AG = + 2 − 2 = 0.

ii) p.d. across R_3 = −2 volts.; $i_3 = \dfrac{-2}{R_3} = \dfrac{-2}{1} = -2A$

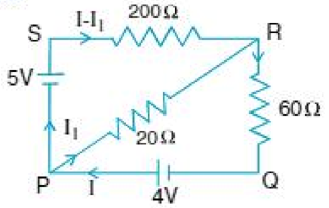


iii) p.d. across BE = −2 volts. $i_2 = \dfrac{-2}{R_2} = \dfrac{-2}{1} = -2A$

Note : You may solve this using Kirchoff's Law as usual.

Example-4.32

In the given circuit, the two cells have no internal resistance. Calculate the potential difference across the $20\,\Omega$ resistor.



Solution :

Applying Kirchoff's law to loop PRQP :

$60I + 20I_1 = 4$ .... (1)

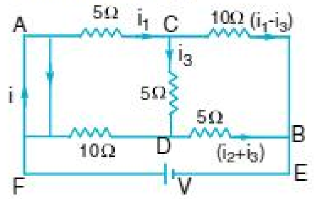
For Loop PSRP : $200(I-I_1)-20I_1 = 5$ .... (2)

Solving (1) and (2), $I_1 = \frac{5}{172}A$

$\Rightarrow V_{20} = \frac{5}{172}\times 20 = \frac{25}{43}$ or 0.594volt

Example-4.33

Find the equivalent resistance between the points A and B of the circuit shown in the fig.



Solution :

Supposing a source is connected between the terminals A and B. The current distribution is shown in fig.

At junction A, $i = i_1 + i_2$

Resistance between A and B, $R_{AB} = \frac{V}{i} = \frac{V}{i_1 + i_2}$

In close loop A C D A, $-5i_1 - 5i_3 + 10i_2 = 0$

or $-i_1 - i_3 + 2i_2 = 0$ .... (i)

In close loop C B D E

$-10(i_1 - i_3) + 5(i_2 + i_3) + 5i_3 = 0$

or $-2i_1 + i_2 + 4i_3 = 0$ .... (ii)

Now in close loop A C B E F A

$-5i_1 - 10(i_1 - i_3) + V = 0$ or $-3i_1 + 2i_3 = -\frac{V}{5}$ .... (iii)

From equations (i) and (iii), we get

$-5i_1 + 4i_2 = -\frac{V}{5}$ ..... (iv)

From equations (ii) and (iii),

we get $4i_1 + i_2 = \frac{2V}{5}$ .... (v)

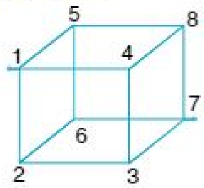
Solving equations (iv) and (v), we get

$i_1 = \frac{9V}{105}$; $i_2 = \frac{6V}{105}$; $i_3 = \frac{V}{35}$

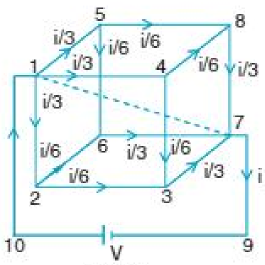
$$R_{AB} = \frac{V}{i_1 + i_2} = \frac{V}{\left(\frac{9V}{105} + \frac{6V}{105}\right)} = 7\Omega$$

Example-4.34

Twelve equal wires, each of resistance r ohm are connected so as to form a skeleton cube. Find the equivalent resistance between the diagonally opposite points 1 and 7.



Fig(a)



Fig(b)

Solution :

Connect a source between points 1 and 7

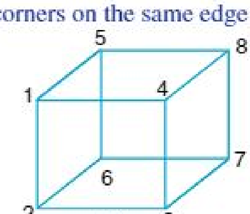
The network is symmetrical about the diagonal 1-7. Therefore current in resistors are distributed symmetrically about the diagonal. The current distribution is shown in fig(b).

Choose a close loop 1-2-3-7-9-10-1, we have

$-r\frac{i}{3} - r\frac{i}{6} - r\frac{i}{3} + V = 0$ (or) $\frac{V}{i} = \frac{5}{6}r$ (or) $R_{17} = \frac{V}{i} = \frac{5}{6}r$

Example-4.35

Twelve equal wires each of resistance r are joined to form a skeleton cube. Find the equivalent resistance between two corners on the same edge of the cube.



(a)

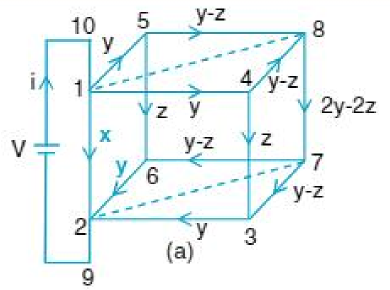
Solution :

Method - I

Connect a source between points 1 and 2. Let current i enter through point 1 into the network. The network is symmetrical about dotted lines. The currents above and below dotted line are symmetrically distributed as shown in fig.

By junction rule at 1, we have $i = x + 2y$

$\therefore R_{12} = \frac{V}{i} = \frac{V}{x + 2y}$ .... (i)



In close loop 1-2-9-10-1, we have

$$-rx + V = 0 \text{ or } x = \frac{V}{r} \quad \text{.... (ii)}$$

In close loop 1-4-3-2-1

$$-rz - rz - ry + rx = 0 \quad (\text{or}) \quad x - 2y - z = 0 \quad \text{.... (iii)}$$

In close loop 4-8-7-3-4

$$-r(y-z) - r \times 2(y-z) - r(y-z) + rz = 0$$

$$\text{or} \quad -4(y-z) + z = 0 \text{ or } 4y + 5z = 0 \quad \text{... (iv)}$$

From equations (iii) and (iv), we get

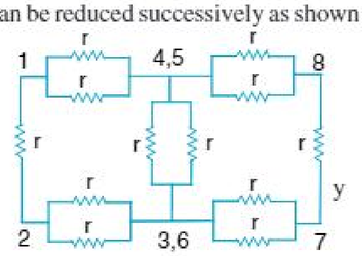
$$-4y + 5(x-2y) = 0 \text{ or } 5x = 14y$$

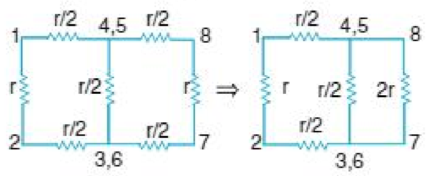
$$\text{Since } x = \frac{V}{r} \quad \therefore y = \frac{5}{14} \times \frac{V}{r}$$

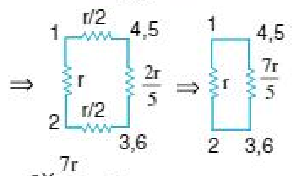
$$\text{Now } R_{12} = \frac{V}{x+2y} = \frac{V}{\frac{V}{r} + 2 \times \frac{5V}{14r}} = \frac{7r}{12}$$

Method II :

Our previous knowledge reveals that points 3 and 6 must be at the same potential. So must be 4 and 5. If points of equal potential are joint by a wire, the currents in the circuit do not change. The given network of resistors can be reduced successively as shown in figure.



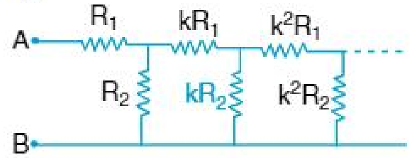




$$R_{12} = \frac{r \times \frac{7r}{5}}{r + \frac{7r}{5}} = \frac{7}{12}r$$

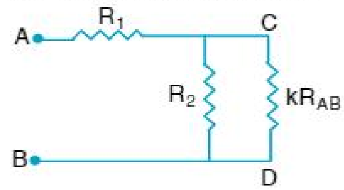
Example-4.36

The circuit diagram shown in fig consists of a very large (infinite) number of elements. The resistances of the resistors in each subsequent element differ by a factor k from the resistances of the resistors in the previous element. Determine the resistance R_{AB} between points A and B if the resistances of the first element are R_1 and R_2.



Solution :

From symmetry considerations, we can remove the first element from the circuit ; the resistance of the remaining circuit between points C and D will be $R_{CD} = kR_{AB}$. Therefore, the equivalent circuit of the infinite chain will have the form shown in figure.



$$\text{Thus } R_{AB} = R_1 + \frac{R_2(kR_{AB})}{R_2 + (kR_{AB})}$$

$$R_{AB}[R_2 + (kR_{AB})] = R_1[R_2 + kR_{AB}] + kR_2 R_{AB}$$

$$R_2 R_{AB} + kR_{AB}^2 = R_1 R_2 + kR_1 R_{AB} + kR_2 R_{AB}$$

$$\text{or } kR_{AB}^2 + (R_2 - kR_1 - kR_2)R_{AB} - R_1 R_2 = 0$$

$$\therefore R_{AB} = \frac{-(R_2 - kR_1 - kR_2) \pm \sqrt{(R_2 - kR_1 - kR_2)^2 + 4kR_1R_2}}{2k}$$

As resistance cannot be negative

$$R_{AB} = \frac{(kR_1 + kR_2 - R_2) + \sqrt{(R_2 - kR_1 - kR_2)^2 + 4kR_1R_2}}{2k}$$

4.20 WHEATSTONE BRIDGE

A resistance can be measured by Ohm`s law using a voltmeter and an ammeter in an electrical circuit. But this measurement is not accurate. To measure it more accurately Kristie devised and Wheatstone popularized a special network design called Wheatstone Bridge. It is an arrangement of four resistances which can be used to measure one of them in terms of the rest.

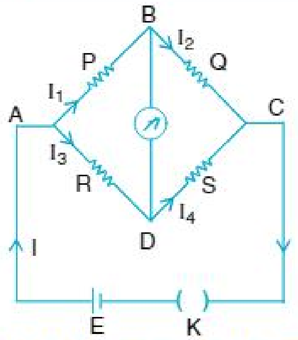


Fig 4.15 A Wheatstone bridge circuit

Consider the circuit as shown in Fig 4.15 where:

(i) S is a unknown resistance to be measured. Arm CD of the bridge is called unknown arm.

(ii) P and Q are two adjustable resistances connected in two ratio arms AB and BC of the bridge

(iii) R is adjustable known resistance. Arm AD is called known arm.

(iv) A sensitive galvanometer G is connected in one of the cross arm BD of the bridge.

(iv) A battery E along with a key K is connected in other cross arm AC. Arm AC and BD are called conjugate arms.

On closing the key, in general there will be some current I flowing through the galvanometer and we will get some deflection in the galvanometer. It indicates that there is some potential difference between points B & D.

Now consider the following three cases

i) Point B is at higher potential than point D: Current will flow from B towards D and galvanometer will show deflection in one direction.

ii) Point B is at lower potential than point D: Current will flow from point D towards B and galvanometer will show deflection in opposite direction.

iii) Both points B and D are at same potential : In this case no current flows through the galvanometer which will show no deflection i.e., the galvanometer is in condition. In this condition the Wheat stone bridge is said to be in the State of Balance.

The points B and D will be at the same potential only when the potential drop across P is equal to that across R. Thus at the null state.

$$I_1P = I_3R \qquad .... (4.19)$$

Applying Kirchoff's first law at junction B and D we get $I_1 - I_2 - I_G = 0$; $I_3 + I_G - I_4 = 0$

At the null state $I_G = 0$

Therefore $I_1 = I_2$ .... (4.20)

and $I_3 = I_4$ .... (4.21)

Also potential drop across Q will be equal to that across S. So that

$$I_2Q = I_4S \qquad .... (4.22)$$

Dividing equation (4.19) by equation (4.22)

$$\frac{I_1P}{I_2Q} = \frac{I_3R}{I_4S}$$

Using eqns (4.20) and (4.21) we get

$$\frac{P}{Q} = \frac{R}{S} \qquad ..... (4.23)$$

This is the condition for which a Wheat stone Bridge is balanced From eq. (v), unknown resistance S is $S = \frac{QR}{P}$

Measurement of resistance by Wheatstone`s Bridge method has the following merits.

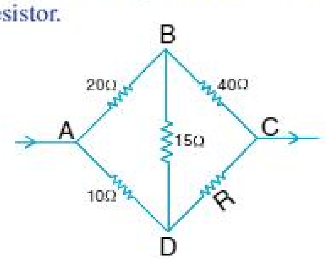
i) The balance condition given by eq (4.23) at null position is independent of the applied voltage E. In other words if we change the e.m.f. of the cell, the balance will not change.

ii) The measurement of resistance does not depend on the accuracy of calibration of the galvanometer. Galvanometer is used only as a null indicator.

The main factor affecting the accuracy of measurement by Wheatstone Bridge is its sensitivity with which the changes in the null condition can be detected. It has been found that the bridge has the greatest sensitivity when the resistances are as nearly equal as possible.

Example-4.37

Find the value of R in Fig. so that there is no current in the 15Ω resistor.



Solution :

This is the Wheatstone bridge with the galvanometer replaced by 15Ω resistor. The bridge is balanced because there is no current in 15Ω resistor, hence,

$20 / 10 = 40 / R$

$$\therefore R = \frac{40 \times 10\ \Omega}{20} = 20\ \Omega.$$

4.21 METER BRIDGE

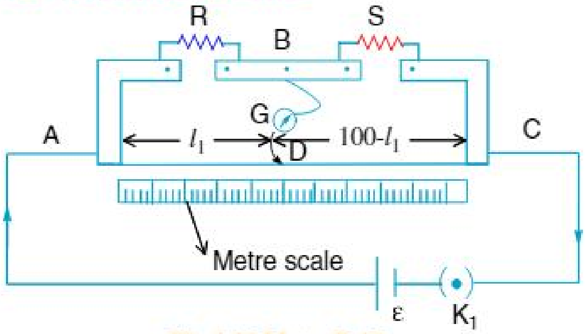


Fig 4.16 Metre Bridge

In the Fig 4.16 meter bridge is shown. Meter bridge consists of a wire of length 1 m and of uniform cross sectional area stretched taut and clamped between two thick metallic strips bent at right angles. As shown in Fig 4.16.

The metallic strips has two gaps. In between the gaps, the resistor can be connected. The wire is clamped at the end circuits and these circuit are connected to a cell through a key. One end of a galvanometer is connected to the metallic strip midway between the two gaps.

The other end of the galvanometer is connected to a jockey' which is essentially a metallic rod whose one end has a knife-edge and can slide over the wire to make electrical connection.

R is an unknown resistance, whose value we want to determine which is connected across one of the gaps. Across the other gap, we connect a standard known resistance S.

The jockey is connected to some point D on the wire, a distance l cm from the end A. The jockey can be moved along the wire. The portion AD of the wire has a resistance $R_{cm} l_1$ where R_{cm} is the resistance of the wire per unit centimetre. The portion DC of the wire similarly has a resistance $R_{cm}(100\text{-}l)$.

The four arms AB, BC, DA and CD [with resistances R, S, $R_{cm} l$ and $R_{cm}(100-l)$] obviously form a Wheatstone bridge with AC as the battery arm and BD the galvanometer arm. The galvanometer shows no current at one position, if we move the jockey along the wire.

Let the distance of the jockey from the end A at the balance point be $l = l_1$. The four resistances of the bridge at the balance point then are R, S, $R_{cm}\, l_1$ and $R_{cm}(100-l_1)$. The balance condition, gives

$$\frac{R}{S} = \frac{R_{cm} l_1}{R_{cm}\left(100 - l_1\right)} = \frac{l_1}{100 - l_1}$$

If we find l_1 then the unknown resistance R is known in terms of the standard known resistance S by

$$R = S\frac{l_1}{100 - l_1} \quad \text{.... (4.24)}$$

We would get various values of l_1 by choosing various values of S and calculate R each time.

If D there is error in measurement of l_1, then we will get an error in R.

The percentage error in R can be minimized by adjusting the balance point near the middle of the bridge.

Example-4.38

In a metre bridge, the null point is found at a distance of 33.7 cm from A. If now a resistance of 12Ω is connected in parallel with S, the null point occurs at 51.9 cm. determine the values of R and S.

Solution :

From the first balance point, $\frac{R}{S}=\frac{33.7}{66.3}$ ..... (i)

After S is connected in parallel with a resistance of 12Ω, the resistance across the gap changes from S to S_{eq}, where

$$S_{eq}=\frac{12S}{S+12}$$

and hence the new balance condition gives

$$\frac{51.9}{48.1}=\frac{R}{S_{eq}}=\frac{R(S+12)}{12S}$$

Substituting the value of R/S from equation (i), we get

$$\frac{51.9}{48.1}=\frac{S+12}{12}\cdot\frac{33.7}{66.3}$$

which gives $S=13.5\Omega$.

Using the value of R/S above

we get $R=6.86\Omega$

Example-4.39

A balance point in a meter bridge experiment is obtained at 30 cm from the left. If right gap contains 3.5Ω, what is the resistance in the left gap ?

Solution :

The balancing length from left l = 30 cm

Resistance in right gap $R=3.5\Omega$

$$R=S\times\frac{\ell}{(100-\ell)}\ ;\ R=3.5\times\frac{30}{70}\Rightarrow R=1.5\Omega$$

4.22 POST OFFICE BOX

Initially this apparatus was used for measuring the resistance of the telephone or the telegraph wires, or for finding faults in these wires.

Post office box is a compact form of Wheatstone Bridge.

Description :

The post office box is a resistance box in which two arms AB and BC are connected in series. Each of these arms contains resistances $10\Omega, 100\Omega$ and 1000Ω. In the third known arm AD, there are resistance from 1Ω to 5000Ω arranged in a U - shape.

The unknown resistance S to be determined is connected in the arm CD. The galvanometer G is connected between B and D through the key K_2 and the cell is connected between A and C through the key K_1.

If the bridge is balanced (zero deflection of galvanometer) by Wheatstone's principle $\frac{P}{Q}=\frac{R}{S}$. However this balanced condition cannot be obtained in general, because resistances in R can take only integral values.

Suppose galvanometer deflects to the left for $R = R_1$ and to the right for $R = R_1 + 1$. Then for the case $P = Q = 10\Omega$, S lies between R_1 and $R_1 + 1$.

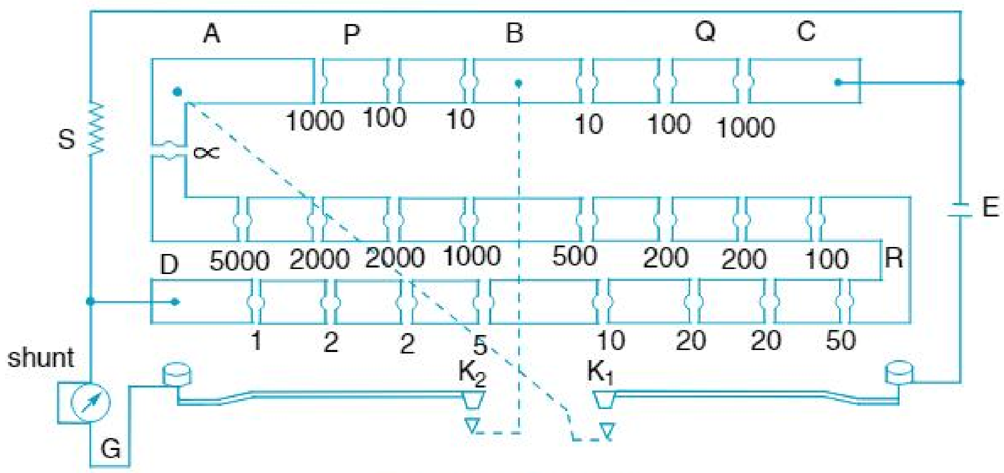


Fig 4.17 Post office Box

Repeat the experiment for $P = 100\,\Omega$ and $Q = 10\,\Omega$ $\left(\text{i.e}\frac{P}{Q}=10\right)$ and then for $P = 1000\,\Omega$ and $Q = 10\,\Omega$ $\left(\text{i.e}\frac{P}{Q}=100\right)$ to obtain a more accurate value of S.

4.23 ELECTROMOTIVE FORCE (E.M.F.) AND POTENTIAL DIFFERENCE

EMF is the short form of electromotive force. EMF of a cell or battery is equal to the potential difference between its terminals when the terminals are not connected externally.

To understand the difference between e.m.f. and potential difference of a cell, connect a cell in a circuit having a resistor R and key K. A voltmeter of very high resistance is connected in parallel to the cell as shown in Fig. 4.18 When key is closed voltmeter reading will decrease.

Actually when key K is opened no significant current flows through the loop having cell and voltmeter due to e.m.f. E of the cell which is the potential difference between terminals of the cell when no current is drawn from it. When key K is closed current flows outside and inside the cell. The cell introduces a resistance r, called internal resistance to the circuit.

Let current I be flowing in the circuit. Potential drop Ir across internal resistance r due to current flow acts opposite to the e.m.f. of the cell. Hence, voltmeter reading will be E-Ir and is equal to V. But V = IR.

Therefore $E - Ir = IR = V$ or $E = V + Ir$

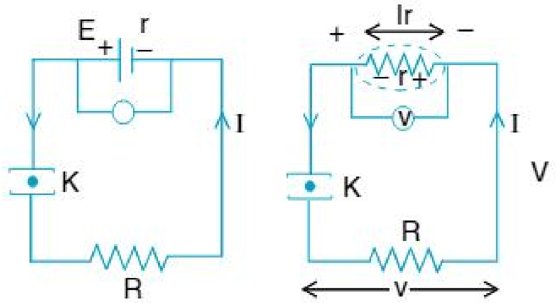


Fig 4.18 e.m.f. of a cell

Thus, e.m.f of a cell depends on

i) the liquid used in the cell

ii) the material of the plates, and

iii) temperature of the liquid.

This means that if you have two cells of different size one big and one small, can the e.m.f.s be the same? Then what is the difference ?

Ans : Yes the e.m.f will be the same if the cells are made up of same materials and liquid. If the cell is of large size it will offer less resistance to the passage of current through it.

Note that the e.m.f of a cell does not depend at all on the size of the cell i.e. on the area of plates and distance between them.

4.24 DISTINCTION BETWEEN E.M.F. OF A CELL AND POTENTIAL DIFFERENCE

EMF of a cell	Potential difference
1. The emf is the maximum potential difference between the two electrodes of a cell when the cell is in the open circuit	1. The P.D. between the two points is the difference of potential between those two points in a closed circuit
2. The e.m.f is independent of the resistance of the circuit.It depends upon the nature of electrodes and the nature of electrolyte of the cell.	2. The P.D. depends upon the resistance between the two points of the circuit and current flowing through the circuit
3. It is used as the source of electric current	3. It is measured between any two points of the electric circuit.
4. E.M.F. is a cause	4. Potential difference is an effect.

Example-4.40

When a current drawn from a battery is 0.5A, its terminal potential difference is 20V. And when current drawn from it is 2.0A, the terminal voltage reduces to 16V. Find out e.m.f and internal resistance of the battery.

Solution :

Let E and r be the e.m.f and internal resistance of battery. When a current I ampere is drawn from it, then potential drop across internal resistance or inside the cell is = Ir. Then

$V = E - Ir$; $I = 0.5\,A$, $V = 20$ Volt, we have

$20 = E - 0.5r$ ..... (i)

$I = 2\,A$, $V = 16$ Volt, we have $16 = E - 2r$ .... (ii)

From eqs (i) and (ii)

$2E - r = 40$ and $E - 2r = 16$

On Solving we get $E = 21.3V$, $r = 2.67\,\Omega$.

Example-4.41

When a battery is connected to the resistance of 10Ω the current in the circuit is 0.12A. The same battery gives 0.07A current with 20Ω. Calculate e.m.f. and internal resistance of the battery.

Solution :

We know that $E = Ir + IR$

$$I_1r + I_1R_1 = I_2r + I_2R_2 \Rightarrow I_1r - I_2r = I_2R_2 - I_1R_1$$

$$r(I_1 - I_2) = I_2R_2 - I_1R_1 \Rightarrow r = \frac{I_2R_2 - I_1R_1}{I_1 - I_2}$$

$$r = \frac{0.07\times 20 - 0.12\times 10}{0.12 - 0.07} = \frac{1.4 - 1.2}{0.05} = \frac{0.2}{0.05} = 4\Omega$$

Internal resistance $r = 4\Omega$,

e. m. f $E = I_1r_1 + I_1R_1$

$0.12\times 4 + 0.12\times 10 = 0.48 + 1.2 \Rightarrow E = 1.68$ volt.

Example-4.42

A cell on open circuit has e.m.f. 2.0V and in closed circuit having current of 0.05A, the p.d is 1.5V. Calculate internal resistance of the cell.

Solution :

e. m. f of cell $E = 2V$

Current in circuit $i = 0.05A$

p.d across resistance $V = 1.5\,V$

Let 'r' be the internal resistance of the cell

$E = (Ir + V)$

$$r = \frac{E - V}{I} = \frac{2 - 1.5}{0.05} = \frac{0.5}{0.05} = 10\,\Omega$$

Internal resistance of the cell is $10\,\Omega$

Example-4.43

A battery of e.m.f 6V and internal resistance 1Ω gives a p.d of 5.8V when connected to a resistance. Find the external resistance.

Solution :

e. m. f of battery $E = 6V$

Internal resistance of battery $r = 1\Omega$

p.d across the resistance $V = 5.8V$

Let the external resistance be 'R'

$E = V + Ir$

$$E = V + \frac{V}{R}r \Rightarrow \frac{r}{R} = \frac{E - V}{V}$$

$$R = \frac{Vr}{E - V} = \frac{5.8\times 1}{6 - 5.8} = \frac{5.8}{0.2} = 29\Omega.$$

Example-4.44

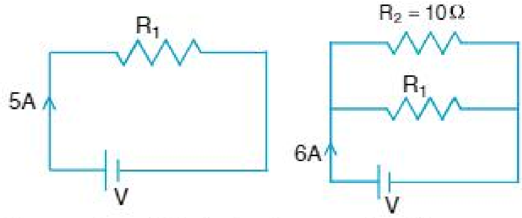
An ideal battery passes a current of 5A through a resistor. When it is connected to another resistance of 10Ω in parallel, the current is 6A. Find the resistance of the first resistor.

Solution :

For ideal battery internal resistance $r = 0$

Resistance of the first wire $= R_1$

Resistance of the second wire $R_2 = 10\Omega$



Current through R_1 in the first case $i_1 = 5A$

Current in the second case $i_2 = 6A$

Effective resistance in the second case $R = \dfrac{R_1R_2}{R_1 + R_2}$

$$V = I_1R_1 \text{ and } V = I_2\frac{R_1R_2}{R_1 + R_2}$$

$$I_1R_1 = I_2\frac{R_1R_2}{R_1 + R_2} \Rightarrow I_1 = I_2\frac{R_2}{R_1 + R_2}$$

$$5 = 6\times\frac{10}{R_1 + 10} \Rightarrow 5(R_1 + 10) = 60$$

$5R_1 + 50 = 60$, $5R_1 = 10$

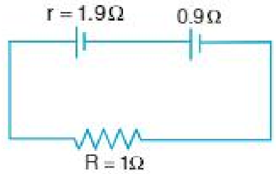
$$R_1 = \frac{10}{5} = 2\Omega \Rightarrow R_1 = 2\Omega.$$

Example-4.45

Two cells A and B each of 2 V are connected in series to an external resistance R = 1 ohm. The internal resistance of A is r_A = 1.9 ohm and that of B is r_B = 0.9 ohm. Find the potential difference between the terminals of A.

Solution :

Total current through the circuit $i = \dfrac{\text{voltage}}{\text{Total resistance}}$



$= \dfrac{4}{(1+1.9+0.9)} = \dfrac{4}{3.8}\text{A}$.

Potential difference of A, $V_A = \varepsilon - ir$,

$$= 2 - \frac{4}{3.8} \times 1.9 = 2 - 2 = 0.$$

Example-4.46

When a resistant of 11 Ω is connected in series with an electric cell, 0.5A current flows through it. If the 11 Ω resistor is replaced by 5 Ω resistor the current flowing through it is 0.9 A. Find the internal resistance of the cell.

Solution :

When the 11Ω resistance is connected in series

$i = 0.5\text{A}, R = 11\Omega$

$V = i(R + r)$ = where r is the internal resistance

$= 0.5\,(11 + r)$ ..... (1)

When the 5Ω resistance is connected in series $i = 0.9\text{ A}$

$V = 0.9\,(5 + r)$ ..... (2)

From (1) & (2) ; $0.5(11 + r) = 0.9(5 + r)$

$0.5 \times 11 + 0.5r = 0.9 \times 5 + 0.9r$

$5.5 + 0.5r = 4.5 + 0.9r \Rightarrow 0.4r = 1 \Rightarrow r = 2.5\Omega$.

Example-4.47

Two cells A and B with same e.m.f of 2 V each and with internal resistances $r_A = 3.5\Omega$ and $r_B = 0.5\Omega$ are connected in series with an external resistance $R = 3\Omega$. Find the terminal voltages across the two cells.

Solution :

We know $i = \dfrac{V}{R} = \dfrac{\varepsilon}{R + r}$

Current through the circuit

$$i = \frac{\varepsilon}{(R + r)} = \frac{2+2}{(3+3.5+0.5)} = \frac{4}{7}$$

i) $R = 3\Omega, r_A = 3.5\Omega, E = 2V$

Terminal voltage across A, $V_A = E - ir_A$

$$= 2 - \frac{4}{7} \times 3.5 = 0 \text{ volt}$$

ii) $r_B = 0.5\Omega, R = 3\Omega, E = 2V$

Terminal voltage across B, $V_B = E - ir_B$

$$= 2 - \frac{4}{7} \times 0.5 = 1.714 \text{ volts.}$$

4.25 COMBINATION OF TWO CELLS IN SERIES AND PARALLEL

(a) Cells in series

The two cells are said to be connected in series between two points A and C if one terminal of each cell is joined together and the other terminals of each cell is free,

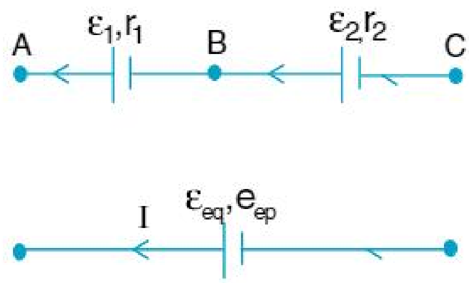


Fig 4.19 (Two cells in series)

Let $\varepsilon_1, \varepsilon_2$ be the e.m.f.s of the two cells. Let r_1, r_2 be their internal resistances respectively. Let the cells be sending the current in a circuit not shown in Fig. Let V_A, V_B and V_C be the potential at points A, B and C, and I be the current flowing through them. Potential difference between positive and negative terminals of the first cell,

$$V_{AB} = V_A - V_B = \varepsilon_1 - Ir_1$$

Potential difference between positive and negative terminals of the second cell.

$$V_{BC} = V_B - V_C = \varepsilon_2 - Ir_2$$

P.D. between A & C of the two cells,

$$V_{AC} = V_A - V_C = (V_A - V_B) + (V_B - V_C)$$

$$= (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2)$$

$$= (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2) \quad ..... \text{(i)}$$

Let the series combination of two cells be replaced by a single cell between A and C of emf ε_{eq} and internal resistance r_{eq} then

$$V_{AC} = \varepsilon_{eq} - Ir_{eq} \quad .... (ii)$$

Comparing (i) and (ii), we get $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ and $r_{eq} = r_1 + r_2$

If n cells of emfs $\varepsilon_1, \varepsilon_2 ....\varepsilon_n$ and of internal resistances $r_1, r_2 ....r_n$ respectively, are connected in series between A and C then equivalent emf, $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_n$ .... (4.25)

equivalent internal resistance of the cell is $r_{eq} = r_1 + r_2 + ... + r_n$. .... (4.26)

Rules for series combination of cells are as follows:

(i) The equivalent emf of a series combination of cells is equal to the sum of their individual emfs.

(ii) The equivalent internal resistance of a series combination of cells is equal to sum of their individual internal resistances.

✤ In the series combination of two cells, if negative terminal of first cell is connected to the negative terminal of the second cell between points A and C, as shown in Fig 4.20 then

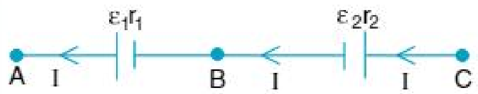


Fig 4.20 Two cells in parallel

$$V_{BC} = V_B - V_C = -\varepsilon_2 - Ir_2$$

Then equivalent emf of the two cells is

$$\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2$$

But equivalent internal resistance is

$$r_{eq} = r_1 + r_2$$

(b) Cells in parallel

The two cells are connected in parallel between two points A and D, if positive terminal of each cell is connected to one point and negative terminal of each cell to the other point, as shown in Fig. Let $\varepsilon_1, \varepsilon_2$ be the emfs of the two cells and r_1, r_2 be their internal resistance respectively.

Let I_1, I_2 be the currents from the two cells flowing towards point B and I be the current flowing out of B. As charge entering in is equal to charge leaving out, $I_1 + I_2 = I$.

Let $V_B.V_C$ be the potential at B and C respectively and V be the potential difference between B and C. Here, the potential difference across the terminals of first cell is equal to the potential difference across the terminals of the second cell. So for the first cell.

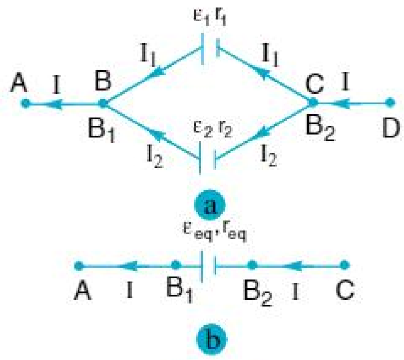


Fig 4.21

$$V = V_B - V_C = \varepsilon_1 - I_1 r_1 \text{ or } I_1 = \frac{\varepsilon_1 - V}{r_1}$$

For the second cell

Putting values in (i), we have

$$I = \left(\frac{\varepsilon_1 - V}{r_1}\right) + \left(\frac{\varepsilon_2 - V}{r_2}\right)$$

$$= \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$= \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - V\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

$$\text{or } V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - \frac{I r_1 r_2}{r_1 + r_2}$$

If the parallel combination of cells is replaced by a single cell between B and C of emf ε_{eq} and internal resistance r_{eq} then

$$V = \varepsilon_{eq} - Ir_{eq} \quad .... (iii)$$

Comparing (ii) and (iii), we have

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \quad .... (iv)$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \quad \text{.... (v)}$$

or $$\frac{1}{r_{eq}} = \frac{r_1 + r_2}{r_1 r_2} = \frac{1}{r_1} + \frac{1}{r_2} \quad \text{.... (vi)}$$

Dividing (iv) by (v), we have

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

If n cells of emfs e_1, e_2....e_n and internal resistance r_1, r_2......r_n are connected in parallel, whose equivalent emf is ε_{eq} and equivalent internal resistance is r_{eq}, then

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_n} \quad \text{.... (4.27)}$$

and $$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + ... \frac{\varepsilon_n}{r_n}. \quad \text{.... (4.28)}$$

4.26 COMBINATION OF A NUMBER OF IDENTICAL CELLS

There are three types of grouping of identical cells.

a) Cells in Series,

b) Cells in Parallel,

c) Cells in Mixed Grouping

(a) Cells in series

When n identical cells each of emf *e* and internal resistance *r* are connected to the external resistor of resistance R as shown in Fig 4.22, then the cells are connected in series.

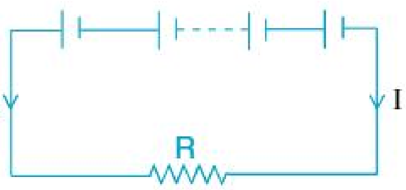


Fig 4.22

Equivalent emf of n cells in series,

$e_{eq} = e + e + ...$ upto n terms = ne

Equivalent internal resistance of n cells in series,

$r_{eq} = r + r + ...$ upto n terms = nr

Total resistance of the circuit = nr + R

Current in the resistance R is given by

$$I = \frac{n\varepsilon}{nr + R} \quad \text{..... (4.29)}$$

Special cases

Case (i)

If R < < nr, then R can be neglected in comparison to nr. Then $I = \frac{n\varepsilon}{nr} = \frac{\varepsilon}{r}$

Thus, the current in the external resistor is the same as due to single cell.

Case (ii)

If R >> nr, then nr can be neglected as compared to R, then $I = \frac{n\varepsilon}{R}$

Thus, the current in the external resistor is n time the current due to a single cell.

From above, we conclude that the maximum current can be drawn from the series combination of cells if the value of external resistance is very high as compared to the net internal resistance of the cells.

(b) Cells in parallel

When 'n' identical cells each of emf ε and internal resistance r are connected to the external resistor of resistance R as shown in Fig 4.23, then the cells are connected in parallel.

As the cells are connected in parallel, their equivalent internal resistance r_{eq} is given by

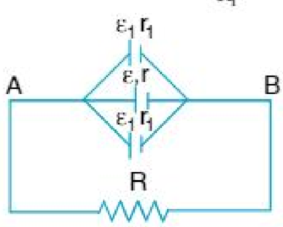


Fig 4.23

$$\frac{1}{r_p} = \frac{1}{r} + \frac{1}{r} + ... \text{ upto n terms} = \frac{n}{r}$$

or $r_{eq} = r/n$.

As R and r_{eq} are in series, total resistance in the circuit = R + r/n.

In parallel combination of identical cells, the effective emf in the circuit is equal to the emf due to a single cell.

Current in the resistance *R* is given by

$$I = \frac{\varepsilon}{R + r/n} = \frac{n\varepsilon}{nR + r} \qquad .... (4.30)$$

Special Cases

Case (i) : If R< < r, then R can be neglected compared to r.

$$\therefore \quad I = \frac{n\varepsilon}{r}$$

Thus the current in the external resistance is n times the current due to a single cell.

Case (ii) : If r < < R, then r can be neglected as compared to n R

$$\therefore \quad I = \frac{n\,\varepsilon}{n\,R} = \frac{\varepsilon}{R}$$

The current in the external resistance is same as due to a single cell.

Thus the maximum current can be drawn from the parallel combination of cells if the external resistance is very low as compared to the internal resistance of the cells.

(c) Cells in Mixed Combination

If the cells are connected as shown in Fig 4.24, they are said to be connected in mixed grouping.

Let there be n cells in series in one row and m rows of cells in parallel. Suppose all the cells are identical. Let each cell be of e.m.f. ε and internal resistance r.

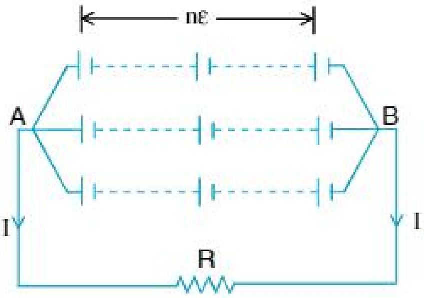


Fig 4.24

In each row, there are n cells in series, therefore their total internal resistance = nr

Their total e.m.f = $n\varepsilon$

Since there are m rows of cells in parallel, therefore, total internal resistance (r_p) of all the cells given by $\frac{1}{r_p} = \frac{1}{nr} + \frac{1}{nr} + ...$ up to m terms $= \frac{m}{nr}$

or $r_p = nr/m$.

Total resistance in the circuit = R + nr / m

Effective e.m.f. of the cells = $n\varepsilon$

The current in the external resistance R is given by $I = \frac{n\varepsilon}{R + nr/m} = \frac{mn\varepsilon}{mR + nr}$ .... (4.31)

The current I will be maximum if (mR + nr) is minimum, i.e.,

$$\left[\left(\sqrt{mR}\right)^2 + \left(\sqrt{nr}\right)^2 - 2\sqrt{mnRr}\right] + 2\sqrt{mnRr} = \text{minimum}$$

or $\left(\sqrt{mR} - \sqrt{nr}\right)^2 + 2\sqrt{mnRr} =$ minimum

It will be so if $\sqrt{mR} - \sqrt{nr} = 0$

$$\sqrt{mR} = \sqrt{nr}$$

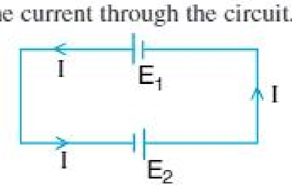
or $\quad mR = nr$ or $R = \frac{nr}{m}$.

i.e., external resistance = total internal resistance of all the cells.

We get the maximum current in mixed grouping of cells if the value of external resistance is equal to the total internal resistance of all the cells.

Example-4.48

Two cells of e.m.f.'s E_1 and E_2 are connected in series in a circuit. Let r_1 and r_2 be the internal resistance of the cells. Find the current through the circuit.



Solution :

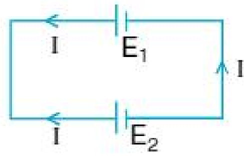
The cells are in series. $E = E_1 + E_2$

r_1 and r_2 are the internal resistances

current $i = \frac{E_1 + E_2}{(r_1 + r_2)}$.

Example-4.49

Let two cells of e.m.f.'s E_1 and E_2 be connected in parallel in a circuit. Let r_1 and r_2 be the internal resistances of the cells. Find the value of the current.



Solution :

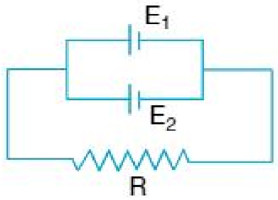
Resultant p.d.due to E_1 and $E_2 = E_1 - E_2$

Sum of internal resistance $= r_1 + r_2$

$\therefore$ current $i = \dfrac{E_1 - E_2}{(r_1 + r_2)}$.

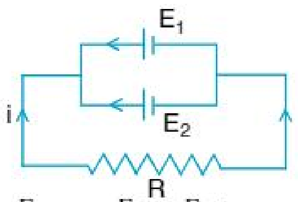
Example-4.50

Let two cells of e.m.f's E_1 and E_2 and internal resistances r_1 and r_2 be connected in parallel to a circuit with an external resistance R. Find the value of the current I through the given resistor.



Solution :

Electric cells connected in parallel to an external resistor. The current through the given resistor



$$I = \frac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2}}{1 + R\left[\dfrac{1}{r_1} + \dfrac{1}{r_2}\right]} = \frac{\dfrac{E_1 r_2 + E_2 r_1}{r_1 r_2}}{1 + \dfrac{R\left[r_1 + r_2\right]}{r_1 r_2}}$$

$$I = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2 + R\left[r_1 + r_2\right]}.$$

Example-4.51

An electric current is passed through a circuit containing two wires of the same material, connected in parallel. The lengths of the wires are in the ratio of 4:3 and radii of the wires are in the ratio of 2:3. Find the ratio of the currents passing through the wires.

Solution :

$l_1 : l_2 = 4 : 3,\ r_1 : r_2 = 2 : 3$

We know that $R \propto \dfrac{\ell}{r^2}$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{r_2^2}{r_1^2} = \frac{4}{3} \times \frac{9}{4} = \frac{3}{1}$$

Since connection is parallel, current is inversely proportional to resistance

$$i \propto \frac{1}{R} \Rightarrow \frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{1}{3}$$

Example-4.52

Twelve cells each having the same emf are connected in series and are kept in a closed box. Some of the cells are wrongly connected. This battery is connected in series with an ammeter and two cells identical with each other. The current is 3A when the cells and battery aid each other and 2A when the cells and battery oppose each other. How many cells are wrongly connected ?

Solution :

Let *m* cells wrongly connected in the battery and $\xi_{net} = (12 - 2m)\xi$

When two cells aid the battery, then current

$$3 = \frac{(12 - 2m)\xi + 2\xi}{R} \quad \text{..... (i)}$$

where *R* is the total resistance of the circuit. When two cells oppose the battery, then

$$2 = \frac{(12 - 2m)\xi - 2\xi}{R} \quad \text{..... (ii)}$$

Solving above equations, we get $m = 1$

Hence one cell is wrongly connected in the battery.

4.27 POTENTIOMETER

To measure e.m.f of a source or potential difference across a circuit element we use voltmeter. An ideal voltmeter should have infinite resistance so that it does not draw any current when connected across a source of e.m.f. Practically it is not possible to make a voltmeter which will not draw any current. To overcome this difficulty a circuit known as potentiometer is used for measuring the e.m.f. of a source or the potential difference across a circuit element without drawing any current from it. It employs a null method. The potentiometer can also be used for the measurement of the internal resistance of a cell, the current flowing in a circuit and comparison of resistances.

4.27.1 DESCRIPTION OF THE POTENTIOMETER

The potentiometer consists of a wooden board on which a number of resistance wires (usually ten) of uniform cross - sectional area are stretched parallel to each other. The wire is of magnanin or nichrome. These wires together act as a single wire of length equal to the sum of the length of all the wires joined in series by thick copper strips. The end terminals of the wires are provided with connecting screws.

A meter scale is fixed on the wooden board parallel to wires. A jockey (a sliding contact maker) is provided with the arrangement. It makes a knife edge contact at any desired point on the wire. Jockey has pointer which moves over the scale. It determines the position of the knife edge contact. In Fig 4.25 a ten wire potentiometer is shown. A and B are ends of the wire. K is a jockey and S is a scale. Jockey slides over a rod CD.

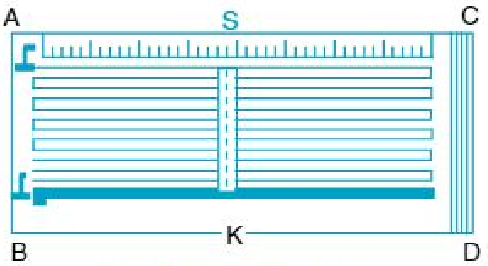


Fig 4.25 (Potentiometer)

Theory of Measurement by Potentiometer

Let us consider that a steady source of e.m.f E (say an accumulator) be connected across the uniform resistance wire AB of length *l*. Positive terminal of accumulator is connected at end A (Fig 4.26). A steady current I flows through the wire.

Potential difference across AB is given by

$V_{AB} = RI$

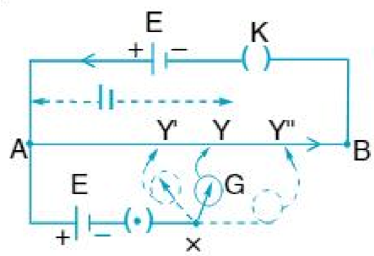


Fig 4.26 Circuit diagram of potentiometer

If ρ is the resistance per unit length of the wire, and k is the potential fall across unit length of the wire,

Then,

$$\therefore V_{AB} = k\ell = E \text{ or } k = \frac{E}{\ell}$$

For a length ℓ_1 of the wire, potential fall

$$V_1 = k\ell_1 = \frac{E}{\ell}\ell_1$$

Thus, potential falls linearly with the distance along the wire from A to B.

Let us measure an unknown voltage say a cell of e.m.f. V. The positive terminal of the cell is connected to end A of the wire and negative terminal through a galvanometer to jockey having variable contact Y. Note that V must be less than E.

Let us start jockey moving from A towards B. Suppose at position Y^{I} potential fall across the length AY^{1} of the wire be less than voltage V. Then current in the loop AY^{1} XA due to voltage V exceeds the current due to potential difference across 'AY'. Hence galvanometer shows some deflection in one direction.

Then jockey is moved away say at Y^{II} such that potential fall across AY^{II} is greater than the voltage V, then galvanometer shows deflection in other direction. Now in between $Y^{I}$and Y^{II} the jockey is moved slowly. The stage is reached say point Y such that potential fall across AY is equal to voltage V. Then point X and Y will be at same voltage and hence the galvanometer will not show any deflection i.e. null point is achieved. If ℓ_1 is the length between A and Y, then

$$V = k\ell_1 = \frac{E\ell_1}{\ell}$$

Thus, the unknown voltage V is measured when no current is drawn from it. The potentiometer has certain advantage. They are as follows;

(i) When the potentiometer is balanced, no current is drawn from the circuit on which the measurement is being made.

(ii) It produces no change in condition in any circuit to which it is connected

(iii) It makes use of null method for the measurement, the galvanometer used need not be calibrated.

4.27.2 COMPARISON OF THE E.M.F.S OF TWO CELLS BY POTENTIOMETER

We shall now use the same technique for comparison of e.m.f.s of two cells. Let us take, for example, a Daniel cell and a Leclanche cell and let E_1 and E_2 be their e.m.f.s.

Potentiometer connections are made as shown in Fig 4.27. One cell say of e.m.f E_1, is connected in the circuit by connecting terminals of 1 and 3 of key K_1. The balance point is obtained by moving the jockey on the potentiometer wire as explained earlier. Let the balance point on potentiometer be at point Y and let the length $AY_1 = \ell_1$. Then other cell of e.m.f. E_2 is connected in the circuit by connecting terminals 2 and 3 of the key K. Again balance is obtained at point Y_2 and let length $AY_2 = \ell_2$.

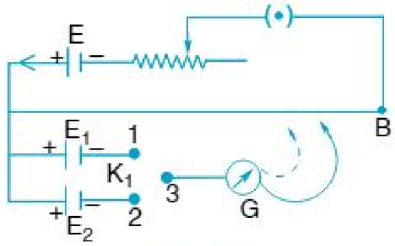


Fig 4.27

Applying potentiometer principle,

$E_1 = k\ell_1$ and $E_2 = k\ell_2$

where k is the potential gradient along the wire AB

$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$$

If e.m.f. of one cell is known, say E_2, the e.m.f. of other cell can be determined

$$E_1 = \frac{\ell_1}{\ell_2} E_2$$

4.27.3 DETERMINATION OF INTERNAL RESISTANCE OF THE CELL

Cells always offer resistance to the flow of current through them, which is often very small. This resistance is called the internal resistance of the cell and depends on the size of the cell i.e. the area of the plates immersed in the liquid, the distance of the plates and the strength of the electrolyte used in the cell.

Let us now learn how to measure the internal resistance of the cell using a potentiometer. Connections are made as shown in Fig 4.28. There is a cell of emf E_1 and internal resistance r.

A resistance box R with a key K_1 is connected in parallel with the cell. Rest of the circuit is similar to that in previous section. First of all key K is closed and a current I flows through wire AB. The key E_1 is kept open and on moving jockey balance is obtained with the cell at point say Y_1. Let $AY_1 = \ell$ then

$$E_1 = kl_1 \qquad \text{..... (i)}$$

Now key K_1 is closed. This introduces a resistance across the cell. A current say I_1 will flow in loop $E_1RK_1E_1$ due to cell E_1. This current I_1 is given by Ohm's law as,

$$I_1 = \frac{E_1}{R+r}$$

where r is the internal resistance of the cell. Now, terminal potential difference V_1 of the cell will be less than E_1 by an amount I_1r.

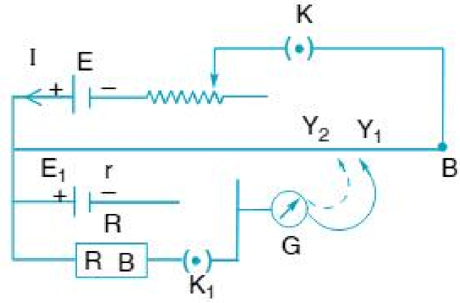


Fig 4.28

The value of V_1 is

$$V_1 = I_1 R = \frac{E_1}{R+r} R \qquad .... (ii)$$

Then potential difference V_1 is balanced on the potentiometer wire without change in current I. Let the balance point be at point Y_2 such that $AY_2 = l_2$ then,

$$V_1 = k\ell_2 \qquad .... (iii)$$

From eqs (i) and (iii)

$$\frac{E_1}{V_1} = \frac{\ell_1}{\ell_2}$$

From eq (ii)

$$\frac{E_1}{V_1} = \frac{R+r}{R}$$

$$\Rightarrow \frac{R+r}{R} = \frac{\ell_1}{\ell_1}$$

$$\text{or } r = R\left(\frac{\ell_1}{\ell_2} - 1\right) \qquad ..... (4.32)$$

Thus, by knowing R, *l1 and* l_2 the value of r is calculated.

Example-4.53

A cell of e.m.f 2V and internal resistance 1Ω is connected to a potentiometer of length 1m and resistance 4Ω. Calculate the potential drop per cm.

Solution :

e.m.f of cell E = 2V

Internal resistance of potentiometer wire $R = 4\Omega$

Potential drop $V = E\left(\frac{R}{R+r}\right) = \frac{2\times 4}{4+1} = 1.6V$

Potential drop per cm

$$= \frac{V}{\ell} = \frac{1.6}{100} = 1.6\times 10^{-2} \text{ volt/cm.}$$

Example-4.54

In a potentiometer experiment the balancing length with a cell is 560 cm. When an external resistance of 10Ω is connected in parallel to the cell, the balancing length changes by 60 cm. Find the internal resistance of the cell.

Solution :

$R = 10\Omega$,

Balancing length $\ell_1 = 560\,\text{cm}$

Change in balancing length $(\ell_1 - \ell_2) = 60\,\text{cm}$

internal resistance = r = ?

$\ell_1 - \ell_2 = 60$, $560 - \ell_2 = 60$

$\therefore \ell_2 = 500\,\text{cm}$

$$r = R\left(\frac{\ell_1 - \ell_2}{\ell_2}\right) = 10\times\frac{60}{500} = \frac{6}{5} = 1.2\,\Omega$$

Example-4.55

Length of a potentiometer wire is 5m. It is connected with a battery of fixed e.m.f. Null point is obtained for Daniel cell at 100cm on it. If the length of the wire is made 7m, then what will be the position of null point?

Solution :

Let the e.m.f of battery be E volt, the potential gradient is

$$k_1 = \frac{E}{5}\,V/m$$

When the length of potentiometer wire is 7m potential gradient is $k_2 = \frac{E}{7}\,V/m$

Now, if null point is obtained at length ℓ_2, then

$$E_1 = k_2\ell_2 = \frac{E}{7}\ell_2 V$$

Here same cell is balanced in two arrangements, hence

$$\frac{E}{5}\ell_1 = \frac{E}{7}\ell_2$$

$\ell_1 = 1\text{m}$; $\ell_2 = 7/5 = 1.4$ m.

Example-4.56

In a potentiometer experiment when a battery of e.m.f. 2V is included in the secondary circuit, the balance point is at 500cm. Find the balancing length from the same end when a cadmium cell of e.m.f. 1.018V is connected to the secondary circuit.

Solution :

e.m.f. of first cell $E_1 = 2V$

e.m.f. of second cell $E_2 = 1.018V$

First balancing length $\ell_1 = 500\,\text{cm}$

Let second balancing length = l_2

$E \propto \ell$

$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$$

$$\ell_2 = \frac{E_2}{E_1}\times\ell_1 = \frac{1.018}{2}\times 500 = 254.5\,\text{cm}$$

Example-4.57

In the secondary circuit of a potentiometer, a cell of internal resistance 1.5Ω gives a balancing length of 52 cm. To get a balancing length of 40 cm, how much resistance is to be connected across the cell?

Solution :

$r = 1.5\Omega$; $l_1 = 52$ cm, $l_2 = 40$ cm

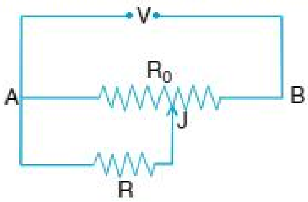
$$\frac{r}{R} = \frac{\ell_1 - \ell_2}{\ell_2}$$

$$\text{Resistance } R = \frac{r.\ell_2}{(\ell_1 - \ell_2)} = \frac{1.5\times 40}{(52-40)}$$

$$= \frac{1.5\times 40}{12} = 5\Omega$$

Example-4.57

A resistance R draws a current from a potentiometer R_0 is resistance of potentiometer. V is voltage supplied to potentiometer. Find voltage across R when the sliding contact is in the middle of the potentiometer.



Solution :

$R_{AJ} = R_{JB} = R_0/2$

Resistance between A and J is $R' = \dfrac{R_0 R}{R_0 + 2R}$

Total resistance between A and B is $R'' = R' + R_0/2$

Current in the circuit $= I = \dfrac{V}{R'} = \dfrac{2V}{R_0 + 2R'}$

Voltage across R is given by $V = IR' = \dfrac{2VR}{R_0 + 4R}$

4.28.1 GROWTH OF CHARGE IN C-R CIRCUIT

Consider a circuit containing a capacitor of capacitance C and a resistance "R" connected to a battery of constant emf E and negligible internal resistance, through a two-way switch S, as shown in fig.

When the switch S is closed between the points a and b, there occurs a flow of charge (i.e.current) through the capacitor leads but not through the dielectric of the capacitor and the capacitor is gradually charged.

The charging current becomes zero when the potential difference across the plates of capacitor becomes equal to applied emf E.

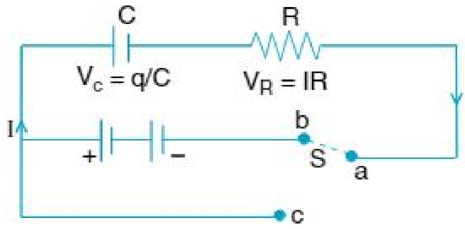


Fig 4.29

The potential difference V_C gradually developed across the plate of the capacitor acts in a direction opposite to applied emf i.e. it plays the role of opposing back emf in the circuit.

As a result of this, charge on the plate of capacitor does not reach the final steady value CE instantly, but grows at a certain rate depending upon the values of resistance and capacitance in the circuit.

Suppose at any instant during the growth of charge on the capacitor, I be the charging current (rate of growth of charge i.e. (dq/dt) and q be the charge on the plates of capacitor.

Then, instantaneous potential difference across the plates of capacitor, $V_C = \dfrac{q}{C}$ and that across the resistance $R, V_R = IR$

Since V_C is opposite to applied emf E, the resultant potential difference across the resistance $R = E - \dfrac{q}{C}$

According to Ohm's law, this must be equal to IR.

$$\text{Hence } E - \frac{q}{C} = IR \qquad \text{..... (1)}$$

$$E - \frac{q}{C} = \frac{dq}{dt}R \Rightarrow \frac{CE - q}{C} = \frac{dq}{dt}R$$

$$dt = \left(\frac{CR}{CE - q}\right)dq$$

Integrating above equation on both sides, we get :

$$t = -CR \log_e(CE - q) + K \quad .... (2)$$

Now, initially at t = 0, there is no charge on the plates of capacitor i.e., at t = 0, q = 0

Hence $0 = -CR \log_e CE + K$

or $K = CR \log_e CE \quad .... (3)$

Putting the value of K in equation 2, we get:

$$t = -CR \log_e(CE - q) + CR \log_e CE$$

$$t = -CR \log_e\left(\frac{CE - q}{CE}\right)$$

$$\log_e\left(\frac{CE - q}{CE}\right) = \frac{-t}{CR}$$

$$\frac{CE - q}{CE} = e^{\frac{-t}{CR}}$$

$$CE - q = CE\, e^{\frac{-t}{CR}}$$

$$q = CE\left(1 - e^{\frac{-t}{CR}}\right) \quad .... (4)$$

When the capacitor is fully charged, the current in the circuit becomes zero i.e, when $q = q_0$, $I = 0$. In this situation there is no potential difference across R and the whole e.m.f is found across capacitor.

Putting the condition q = q0 when I = 0 in eq (1), we get

$$E - \frac{q_0}{C} = 0 \text{ or } CE = q_0 \quad ....(5)$$

Hence eq. (4) can be written as :

$$q = q_0(1 - e^{-t/CR}) \quad ..... (4.33)$$

Above equation gives the value of charge on the capacitor at any time t during its charging. It shows that charge on the capacitor grows exponentially with time

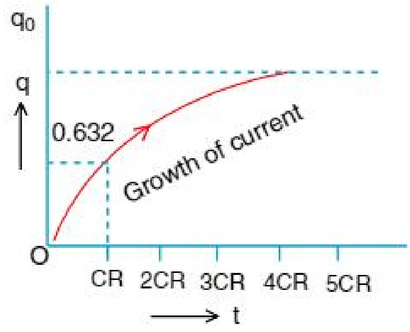


Fig 4.30

Rate of growth of charge or the charging current : It can be calculated easily by just differentiating eq. (4.33) with respect to time. That is

$$I = \frac{dq}{dt} = \frac{d}{dt}\left(q_0\left(1 - e^{\frac{-t}{CR}}\right)\right)$$

$$I = \frac{q_0}{CR}\, e^{\frac{-t}{CR}} = I_0 e^{\frac{-t}{CR}}$$

$$I = \frac{q_0}{CR}\left(1 - \frac{q}{q_0}\right) \quad ...... (4.34)$$

$$I = \frac{1}{CR}(q_0 - q) \quad ..... (4.35)$$

Above equation shows that smaller the product CR, the more quickly the charge grows on the plates of capacitor The product CR is known as capacitive time constant of the C – R circuit. It is represented by τ_C i.e. $\tau_C = CR$. If capacitance C is given in farad and resistance in ohm, the time constant is in second.

Consider $q = q_0\left(1 - e^{\frac{-t}{CR}}\right)$

When t = CR

$$q = q_0\left(1 - e^{-1}\right) = q_0\left(1 - \frac{1}{e}\right)$$

$q = 0.632\ q_0$

Thus, the capacitive time constant of C-R circuit is the time in which the charge on the plates of capacitor grows from zero to 0.632 (or 63.2%) of its maximum value.

It can be easily shown mathematically that when $t = 2$ CR, 3CR,4CR 5CR, the charge on the capacitor will be $0.865\ q_0$, $0.950 q_0$, $0.982\ q_0$ and $0.993\ q_0$ respectively. Finally when $t = \infty$, $q = q_0$ i.e., charge on the plates of a capacitor in a C-R circuit will attain its maximum value after infinite time.

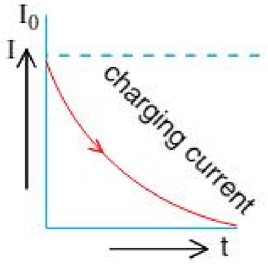


Fig 4.31

It follows from eq. (4.35) that charge on the plate of a capacitor in a C-R circuit will attain its maximum value after infinite time.

Charging current at the initial instant (i.e, t = 0) of charging process. It decreases exponentially from its maximum value I_0 to zero, as shown in fig.

4.28.2 DECAY OF CHARGE IN CR CIRCUIT

i) Let the switch S of the circuit shifted from position ab to position ac at the instant (counted as t = 0) when the capacitor C is fully charged ($q = q_0$). As the switch S is closed between the points a and c, the battery gets out of the circuit, the C-R circuit is again closed and there is again a flow of charge (current) while the capacitor is gradually discharged. The discharging current vanishes when the potential difference across the capacitor drops to zero.

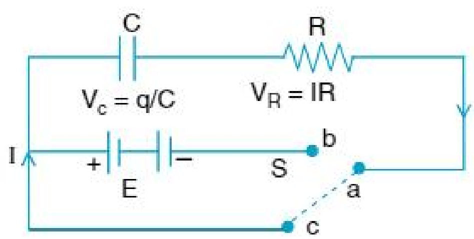


Fig 4.32

Suppose at any instant, during the decay of charge, q be charge on the capacitor and I, the value of current in the circuit. As battery is no more present in the circuit, the equation for decay of charge is

$$-\frac{q}{C} = IR = \left(\frac{dq}{dt}\right)R \text{ or, } dt = -CR\frac{dq}{q}$$

Integrating on both sides of above equation, we get :

$$t = -CR\log_e q + k \qquad .... (1)$$

where K is some constant of integration

Now, initially at t = 0 (i.e., at the moment of removing battery from the circuit), $q = q_0$.

Hence from above equation

$$0 = CR\ \log_e q_o + K$$

$$K = CR\ \log_e q_0 \qquad .... (2)$$

$$K = CR\log_e q_0$$

putting the value of K in eq. (1), we get :

$$t = -CR\ \log_e q + CR\ \log_e q_0$$

$$t = -CR\left[\log_e q - \log_e q_0\right]$$

$$t = -CR\ \log_e\left[\frac{q}{q_0}\right]$$

$$\frac{-t}{CR} = \log_e\left(\frac{q}{q_0}\right)$$

$$\frac{q}{q_0} = e^{\frac{-t}{CR}}$$

$$q = q_0\, e^{\frac{-t}{CR}} \qquad .... (4.36)$$

Above equation gives the value of charge q at any instant during the discharging of the capacitor through the resistance R. This equation shows that charge on the capacitor decays exponentially, with time. A graph showing the decay of charge with respect to time is shown in fig.

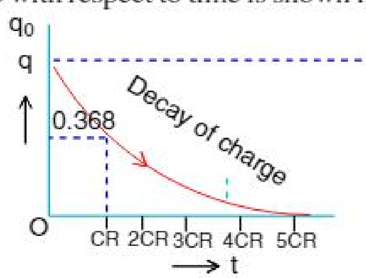


Fig 4.33

4.28.3 RATE OF DECAY OF CHARGE OR DISCHARGING CURRENT

This can be calculated by differentiating eq (12) i.e

$$I = \frac{dq}{dt} \qquad \Rightarrow I = \frac{d}{dt}\left(q_0 e^{\frac{-t}{CR}}\right)$$

$$I = \frac{-q_0}{CR} e^{\frac{-t}{CR}} \Rightarrow I = \frac{-q_0}{CR} \cdot \frac{q}{q_0}$$

$$I = \frac{-q}{CR} \qquad .... (4.37)$$

Above equation again shows that smaller the capacitive time-constant CR, more rapid is the discharge of the capacitor.

$$q = q_0 e^{\frac{-t}{CR}}$$

When t = CR

$$q = q_0 e^{-1} = \frac{q_0}{e}$$

$$q = \frac{q_0}{2.718} = 0.368 q_0$$

Therefore capacitive time constant of C-R circuit may also be defined as the time in which the charge on the capacitor decays from maximum to 0.368 (or 36%) of the minimum value. It can be again show mathematically, that at t = 2 CR, 3 CR, 4 CR, 5 CR, the charge on the capacitor in CR circuit falls to the values $0.135q_0$, $0.050q_0$, $0.018q_0$ and $0.017q_0$ respectively and charge will become zero after infinite time.

Discharging current as a function of time : From eq (13), we find

$$I = -\frac{q_0}{CR} e^{-t/CR} = -I_0 e^{-t/CR}.$$

4.29 ATMOSPHERIC ELECTRICITY

Lightning phenomenon can be explained by simple principles of physics. In the higher atmosphere or magnetosphere solar terrestrial interaction produces strong electric currents. In the lower atmospheric layers, weak currents will be produced and which will be maintained by thundrstorm.

Clouds of the earth's atmosphere majorly contain ice flakes or particles. These ice particles grow, collide, break aparts continuously. Due to this fractures, smaller ice particles get positive charge and larger particles get negative charge. Gravity and updrifts in the clouds seperate these charged particles subsequently.

Due to this separation, upper parts of the clouds will be positively charged and lower parts negatively charged. So, the clouds get dipole like structures. At the time of thunderstorm, ground will be positively charged as very weak positive charge is found near the base of the cloud.

Besides these radioactive radiation and cosmic lays ionise air into positive and negative ions. As a result air becomes a weak conductor. Charge separation results in huge potential within the cloud as well as between cloud and ground. This grows to millions of volts and as a consequence, electrical resistance of air breaks down and thousands of amperes of current flows as lighting flash.

The lightning flash consists of series of strokes with each flash about for 30 seconds. Average peak power per stroke is 10^{12} W and electric field is about $10^5 Vm^{-1}$.

Even when the weather is fair, electric field originates due to surface charge density at ground and atmospheric conductivity as well as due to the flow of current from the ionosphere to the earth's surface.

Surface charge density at ground is negative and total charge will be about 600 KC (negative). So, in the atmosphere an equal positive charge generates. We may not identify the electric field in such case as everything virtually is conductor compared to air.

Example-4.58

A circuit having resistor of 2 mega-ohm and capacitor of 1μF is placed in series with a battery of 2 volt. Find the time after which the charge reaches 86.4% of its maximum value.

Solution :

$C = 1\mu F = 10^{-6}$ Farad.

$R = 2$ mega ohm $= 2\times10^{6}$ ohm,

$E = 2$ Volt

Now, $CR = 10^{-6}\times2\times10^{6} = 2s$

and $\frac{q}{q_0} = \frac{86.4}{100} = 0.864$

According to the relation $q = q_0\left[1 - e^{-t/CR}\right]$

we get : $\frac{q}{q_0} = 1 - e^{-t/CR}$

or , $0.864 = 1 - e^{\frac{-t}{2}}$

$e^{\frac{-t}{2}} = 1 - 0.864 = .136 \Rightarrow e^{\frac{t}{2}} = \frac{1}{0.136}$

$t = 2\ln(7.352) = 4\,\text{sec}$.

Example-4.59

A capacitor is being charged through a resistance of 3 mega ohm. If it reaches 75% of its final potential in 0.5 sec, find its capacitance.

Solution :

At any moment during the charging process, the potential difference across the plates of a capacitor is given by

$V = V_0(1 - e^{-t/CR})$

where V_0 is the final potential

Give that $\frac{V}{V_0} = 75\% = 0.75, t = 0.5$ sec

and $R = 3\times10^{6}\,\Omega$

Hence, $0.75 = 1 - e^{-0.5/(3\times10^{6})C}$

or, $e^{-0.5/(3\times10^{6})C} = 1 - 0.75 = 0.25$

or, $e^{-0.5/(3\times10^{6})C} = \frac{1}{0.25} = 4$

or, $\frac{0.5}{3\times10^{6}C} = \log_e 4 = 2.3026\times\log_{10}^{4}$

or, $C = \frac{0.5}{(3\times10^{6})\times2.3026\times0.6021}$

$= 0.12\times10^{-6}F = 0.12\mu F$

Example-4.60

A capacitor, charged to 10V, is being discharged through a resistance R. At the end of 1s the voltage across the capacitor is 5V. What will be the voltage after 2s?

Solution :

The equation for discharging of a capacitor through a resistance R is given by

$q = q_0 e^{-t/CR}$

or, $\frac{q_0}{q} = e^{t/CR}$

or, $\log_e \frac{q_0}{q} = \frac{t}{CR}$

or, $t = CR\log_e\left(\frac{q_0}{q}\right)$

As charge is proportional to voltage in case of a capacitor

$t = CR\log_e\left(\frac{V_0}{V}\right)$

Given that $V_0 = 10$ Volt,

At $t = 1$ sec, $V = 5$ Volt.

Hence $1 = CR\log_e\left(\frac{10}{5}\right) = CR\log_e(2)$ ....(1)

At $t = 2$ sec, the voltage will be given by :

$2 = CR\log_e\left(\frac{10}{V}\right)$ ....(2)

Dividing eq.(2) by eq. (1), we get:

$2 = \frac{\log_e(10/V)}{\log_e(2)} = \frac{\log_e 10 - \log_e V}{\log_e 2}$

or, $\log_e V = \log_e 10 - 2\log_e 2$

$= \log_e 10 - \log_e 4$

or, $V = \left(\frac{10}{4}\right) = (2.5)$ Volts

Example-4.61

A capacitor of capacitance C farad is being charged from a.d.c. supply of E volts through a resistance of R ohms. i) Show that most of the voltage across the capacitor builds up during the first time constant. ii) Show that capacitor is almost fully charged after time equal to 5 time constants.

Solution :

The voltage developed across the capacitor during charging is :

$$V = V_0(1 - e^{-t/RC})$$

i) During the first time constant i.e.,

at t = RC seconds,

$$V = V_0(1 - e^{-RC/RC})$$

$$= V_0(1 - e^{-1}) = 0.632V_0 \text{ Volts}$$

Hence, most of the voltage (i.e.,63.2%) builds up across the capacitor during first time constant :

ii) After time equal to 5 time constants i.e., at t=5 RC,

$$V = V_0(1 - e^{-5RC/RC})$$

$$= V_0(1 - e^{-5}) = 0.993V_0$$

Example-4.62

A 2μF Capacitor is connected to a.d.c. source of 100 volt through 1 MΩ series resistance. Calculate i) time constant ii) initial charging current iii) initial rate of rise of p.d across the capacitor iv) voltage across the capacitor 6 seconds after the application of voltage v) time taken for the capacitor to be fully charged.

Solution :

i) Time constant, $\tau_C = RC = 10^6 \times (2\times10^{-6}) = 2s$

ii) Initial charging current,

$$I_0 = \frac{V_0}{R} = \frac{100}{10^6} = 100\mu A$$

iii) Now, $V_0 = V + CR\dfrac{dV}{dt}$

At the instant, the switch is closed, V = 0

so, $V_0 = CR\left(\dfrac{dV}{dt}\right)$

or $\dfrac{dV}{dt} = \dfrac{V_0}{CR} = \dfrac{100}{2} = 50V/s$

iv) $V = V_0(1 - e^{-t/RC})$

$$= 100(1 - e^{-6/2}) = 100(1 - e^{-3}) = 95.1\text{Volt}$$

v) Time taken for the capacitor to be fully charged

$= 5RC = 5 \times 2 = 10$ sec

Example-4.63

A resistor R and a 4μF capacitor are connected in series across a 200 V d.c. supply. Across the capacitor is connected a neon lamp that strikes at 120 V in 5 seconds. Find R.

Solution :

$V = V_0(1 - e^{-t/\tau_C})$ where $\tau_C = RC$

or, $120 = 200\ (1 - e^{-5/\tau_C})$

or, $(e^{-5/\tau_C}) = 1 - \dfrac{120}{200} = 0.4$

or, $(e^{5/\tau_C}) = \dfrac{1}{0.4} = 2.5$

or, $(5/\tau_C) = \log_e 2.5$

$\therefore \tau_C = \dfrac{5}{\log_e 2.5} = 5.46\,\text{sec}$

or, RC = 5.46

or, $R = \dfrac{5.46}{C} = \dfrac{5.46}{4\times10^{-6}} = 1.364\times10^6\,\text{ohm} = 1.364M\Omega$

Example-4.64

A capacitor of 1μF and resistance 82kΩ are connected in series with a d.c.source of 100 Volt. Calculate the magnitude of energy and the time in which energy stored in the capacitor will reach half of its maximum value.

Solution :

i) Maximum energy stored $= \dfrac{CV_0^2}{2}$

Energy stored αV_0^2

Half of maximum will be stored when voltage across capacitor is

$V = \left(100/\sqrt{2}\right)\text{Volt} = 70.7\text{Volt}$

Energy stored

$$= \frac{1}{2}CV^2 = \frac{1}{2}\times(1\times10^{-6})\times(70.7)^2 = 0.0025J$$

ii) $\dfrac{V_0}{\sqrt{2}} = V_0\left(1 - e^{-t/RC}\right)$

or, $70.7 = 100\left[1 - e^{-t/(82\times10^3\times10^{-6})}\right]$

or, $70.7 = 100\left[1 - e^{-t/0.082}\right]$

or, $e^{-t/0.082} = 1 - \dfrac{70.7}{100} = 0.293$

or, $e^{t/0.082} = 3.413$

$\therefore t = 0.082\log_e 3.413 = 0.1\,\text{sec}$

Example-4.65

A battery is charged at a potential of 15V for 8 hours when the current flowing is 10A. The battery on discharge supplies a current of 5A for 15 hours. The mean terminal voltage during discharge is 14V. Find the watt hour efficiency of the battery?

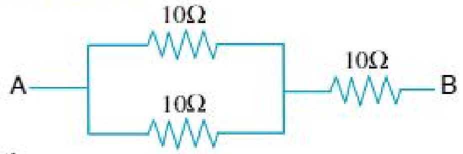
Solution :

$$\text{Efficiency of the battery} = \frac{\text{Output energy}}{\text{Input energy}}$$

$$= \frac{14\times4\times15}{15\times10\times8} = 0.875 = 87.5\%$$

Example-4.66

Three equal resistances each of 10Ω are connected as shown. The maximum power consumed by each resistor is 20W. Then find the maximum power consumed by the combination?

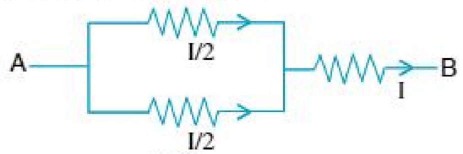


Solution :

If I is rated current in each resistor, from $P = I^2R$,

$$I = \sqrt{\frac{20}{10}} = \sqrt{2}\text{A}$$

The current distribution for maximum power consumption will be as shown



$$\text{Total power} = 2\left(\frac{I}{2}\right)^2 R + I^2R = 30\text{W}$$

4.30 JOULE'S LAW

Whenever electric current is passed through a conductor, it becomes hot after some time. This indicates that the electrical energy is being converted into heat energy. This effect is known as heating effect of current or joule heating effect.

If we reverse the direction of current, heat is still produced in the conductor. **Joule heating is an irreversible phenomenon** and it cannot be used to convert heat into electrical energy.

According to **Joule's law**, the current passing through a conductor produces heat.

The quantity of heat produced 'Q' is equal to the work done by the electric field on the free electrons. If constant current 'i' passes through a conductor for time 't' under a potential difference 'V', then work done (W) in the time 't' is

$W = Vit$

But according to Ohm's law, $V = iR$ where 'R' is the resistance of the conductor.

Now, work done, $W = (iR)\,i\,t$

$$= i^2Rt = \frac{V^2}{R}t = Vit$$

This work is converted into energy in the conductor. It means the electric current through a conductor produces thermal energy in the conductor and the conductor gets heated.

$\therefore$ Thermal energy produced, $Q = i^2Rt$

From the above equation, we can say that

i) The heat produced in a given conductor in a given time is proportional to the square of the current passing through it.

i.e., $Q \propto i^2$

ii) The heat produced in a given conductor and current is proportional to the time for which the current passing through it.

i.e., $Q \propto t$

iii) The heat produced in a given conductor by the current in a given time is proportional to the resistance of the conductor.

i.e., $Q \propto R$

These laws are known as Joule's laws.

Electric heater, electric iron, electric bulb, electric stove etc., are some of the instruments which work on the Joule's law and convert electrical energy into heat energy.

4.30.1 BULBS IN SERIES AND PARALLEL

Every electrical appliance will have rated or design values like wattage, voltage printed on it.

These values give information about resistance and allowable current etc., Let P and V be the power and voltage ratings on a bulb.

Resistance of filament of the bulb $R = \frac{V^2}{P}$

If V is constant, $R \alpha \frac{1}{P}$

So, if we compare (100 W – 230 V) and (60W – 230 V) bulbs, filament of 100 W bulb will have less resistance compared to that of 60 W bulb.

Allowable current in the bulb $i = \frac{P}{V}$

So, $i \alpha P$ if V is constant.

So, in the above case, 100 W bulb draws more current.

If two bulbs with power ratings P_1 and P_2 are connected in series with a voltage source V.

If P_1' and P_2' are consumed powers of the two bulbs,

$$\frac{P_1'}{P_2'} = \frac{i^2 R_1}{i^2 R_2} = \frac{R_1}{R_2}$$

If $P_1 > P_2$ then $R_1 < R_2 \Rightarrow P_1' < P_2'$

So, in series bulb with low wattage rating glows brighter than high wattage rated bulb.

Now total power consumed is

$$P = \frac{V^2}{R} = \frac{V^2}{R_1 + R_2}$$

But $R_1 = \frac{V^2}{P_1}$ and $R_2 = \frac{V^2}{P_2}$

$$\Rightarrow \frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

If the two bulbs are connected in parallel, voltage across them is same.

So, $\frac{P_1'}{P_2'} = \frac{R_2}{R_1}$ (as $P \alpha \frac{1}{R}$)

If $P_1 > P_2$ then $R_1 < R_2 \Rightarrow P_1' > P_2'$

So, in parallel high wattage bulb glows brighter than less wattage bulb.

Now total power consumed is

$$P = \frac{V^2}{R} \text{ and } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$P = P_1 + P_2$$

Application:

Maximum power transfer theorem

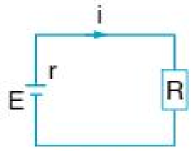


Fig 4.34

Consider a device of resistance R connected to a source of e.m.f E and internal resistance r as shown.

Current in the circuit is $i = \left(\frac{E}{R + r}\right)$.

Power dissipated in the device is $P = i^2 R$

$$\Rightarrow P = \frac{E^2 R}{(R + r)^2}$$

For maximum power dissipated in the device

$$\frac{dP}{dR} = 0 \Rightarrow \frac{d}{dR}\left[\frac{E^2 R}{(R + r)^2}\right] = 0$$

On simplification, we can get $R = r$

So, **the power dissipated in an external resistance is maximum if that resistance is equal to internal resistance of the source supplying the current to that device.**

4.30.2 FUSE WIRE

A fuse wire is generally prepared from tin - lead alloy (63% tin + 37% lead). **It should have high resistivity, low melting point.**

The fuse wire is used in series with the electrical installations and protects them from the high currents. All of sudden, if high current flows, the fuse wire melts away, causing the breakage in the circuit, thereby saving the main installations from being damaged.

Let R be the resistance of fuse wire.

We know that $R = \frac{\rho L}{\pi r^2}$

(L and r denote length and radius)

The heat produced in the fuse wire is

$$H = i^2 R = \frac{i^2 \rho L}{\pi r^2}$$

If H_0 is heat loss per unit surface area of the fuse wire, then heat radiated per second is = $H_0 2\pi r L$

At thermal equilibrium,

$$\frac{i^2 \rho L}{\pi r^2} = H_0 2\pi r L \text{ (or)}$$

$$H_0 = \frac{i^2 \rho}{2\pi^2 r^3}$$

According to Newton's law of cooling.

$$H_0 = K\theta$$

Where θ is the increase in temperature of fuse wire and K is a constant.

$$\theta = \frac{i^2 \rho}{2\pi^2 r^3 K}$$

Here θ is independent of length L of the fuse wire provided i remains constant.

For a given material of fuse wire $i^2 \alpha\, r^3$.

Application :

If radiation losses are neglected, due to heating effect of current the temperature of fuse wire will increase continuously, and it will melt in time 't', such that

$$H = mc\ \Delta\theta;\ \frac{I^2 Rt}{J} = mc\left(\theta_{mp} - \theta_r\right)$$

$$I^2\left(\frac{\rho L}{\pi r^2}\right)\frac{t}{J} = \pi r^2 Ld\ \ c\left(\theta_{mp} - \theta_r\right)$$

$$t = \frac{\pi^2 r^4 d\ c\left(\theta_{mp} - \theta_r\right)J}{I^2 \rho};\ t \propto r^4$$

i.e, in absence of radiation losses, the time in which fuse will melt is also independent on length and varies with radius as r^4.

Example-4.67

A copper electric kettle weighing 1000g, contains 900 g of water at 20^0C. It takes 12 minutes to raise the temperature to 100^0C. If the electric energy is supplied at 210V, calculate the strength of electric current assuming 10% of the heat is wasted. (specific heat of copper = 0.1 $\text{cal g}^{-1}\,{}^0\text{C}^{-1}$)

Solution :

Amount of heat required =

$$\{(1000 \times 0.1 \times 80) + (900 \times 1 \times 80)\}\ \text{cal}$$

$$= 80{,}000\ \text{cal}$$

As 10% is wasted, amount of heat produced

$$= \frac{100}{90} \times 80000\ \text{cal} = \frac{8}{9} \times 10^5 \times 4.2\ \text{J}$$

$$\text{From } Q = VIt,\ I = \frac{\left(\frac{8}{9}\right) \times 10^5 \times 4.2}{210 \times 12 \times 60} = 2.5\text{A}$$

At a Glance

1. Drift velocity is the average velocity component with which electrons move opposite to the field when an electric field exists in a conductor.
2. Electric current through any cross - sectional area is the rate of transfer of charge from one side to other side of the area. Unit of current is ampere denoted by A.
3. Ohm's law states that the current flowing through a conductor is proportional to the potential difference when physical conditions, temperature etc. remain unchanged.
4. Resistivity (or specific resistance) of a material equals the resistance of a wire of the material of 1 m length and 1m^2 area of cross section. Unit of resistivity is ohm meter.
5. Ratio V/I is called resistance and is denoted by R. Unit of resistance is ohm (denoted by)
6. Resistance of a conductor for which V/I ratio is not constant but depends on the value of voltage applied, is called non-ohmic resistance.
7. For a series combination of resistors the equivalent resistance is sum of resistances of all resistors.
8. For parallel combination of resistors inverse of equivalent resistance is sum of inverse of all the resistances.

9. Kirchhoff's laws to study systematically the complicated electrical circuits are:

   Law I: The sum of all the currents directed towards a point in an electric network is equal to the sum of all currents directed away from the point.

   Law II: The algebraic sum of all potential difference along a closed loop in an electrical network is zero.

10. The Wheatstone Bridge circuit is used to measure accurately an unknown resistance (S) by comparing it with known resistances (P,Q and R). In the balanced condition P/Q=R/S.

11. The emf of a cell is equal to potential difference between its terminals when a circuit is not connected to it. i.e., it is in open circuit.

12. A potentiometer measures voltages without drawing a current. Therefore, it can be used to measure e.m.f of source that has appreciable internal resistance.

EXERCISE

LONG ANSWER QUESTIONS

1. What is drift velocity of free electrons in a metallic conductor? For a current carrying conductor establish relation between current, drift velocity v_d concentration of conduction electrons and electronic charge.
2. Define electric current and discuss Ohm's law.
3. Three resistors R_1, R_2 and R_3 , and are connected (i) in series (ii) in parallel. Find out equivalent resistance of combination in each case.
4. What is the difference in emf and potential difference between electrodes of a cell. Derive relation between the two.
5. What are Kirchhoff's laws governing the currents and electromotive forces in an electrical network?
6. How will you potentiometer measure unknown potential difference measuring resistances with the help of a potentiometer?
7. Describe potentiometer method of comparing emfs of two cells.
8. How will you find the internal resistance of a cell with the help of a potentiometer?
9. Derive expressions for currents through a load resistance in series and parallel combinations of cells.
10. Derive expressions for the equivalent emf of (a) series combination and (b) parallel combination of electric cell.

SHORT ANSWER QUESTIONS

1. Define resistivity of a conductor. How does the resistance of a wire depend upon the resistivity of its material, its length and area of cross- section?
2. Define electrical conductivity. Write its unit. How does electrical conductivity depend. upon free electron concentration of the conductor?
3. Explain the difference between ohmic and non-ohmic resistances. Give some examples of non-ohmic resistances.
4. What is the effect of temperature on the resistivity of a material? Why does the electrical conductivity of a conductor decrease with increase in temperature?
5. State and explain Ohm's law. Define ohm.
6. On what factors does the resistance of a conductor depend? Explain.
7. Define conductance. What are its units? How does it change with temperature?
8. State and explain Kirchhoff's laws.
9. Apply Kirchhoff's laws to Wheatstone bridge.
10. Explain the principal of potentiometer
11. Derive the balancing condition of a Wheatstone bridge.
12. Explain parallel combination of cells.

VERY SHORT ANSWER QUESTIONS

1. An uncharged conductor is electrically neutral. It is only a charged conductor that produces an electric field outside. Now think of a current carrying conductor (for simplicity let us assume a long straight wire). Will it give rise to an electric field? (It gives rise to a magnetic field as discussed in the chapter of electromagnetism)

A. No

2. Does the random motion of free electrons in a conductor contribute to the drift of the electrons?

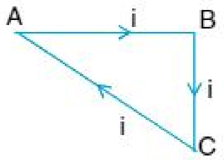
A. No.

3. **How can you ascertain that the drift is steady and not accelerated?**

A. An accelerated charge gives out radiation. Here there is no radiation)

4. **We put an arrow mark ($\rightarrow$) to show the direction of current flow. Why is the current not a vector quantity?**

A. The arrow of current only denotes the sense of current in which direction the charge flows. But current does not obey the laws of vector addition and hence current is **not a vector.**



It is a scalar only. Consider a current flowing along ABCA. If current is a vector, the resultant should be zero. However, the current remains the same i. Thus, current is **not a vector.**

5. **(a) Is the drift speed a characteristic of the material of the conductor?**

(b) A given conductor has got different areas of cross section. From the formula for drift speed can we say the current i will be different at different cross sections?

A. (a) No. $V_d = \frac{i}{neA}$ For a given material n and e are constants. But v_d depends on the amount of current i flowing through the conductor and also on the area of cross section A.

That is why we find $v_d \sim 10^{-3}\ ms^{-1}$ etc. with different orders of magnitude for copper.

(b) No. Current will be the same at all cross sections.

6. **When no external electric field is applied (that is when no current is passing through the conductor) the average velocity of a conduction electron over a large time will be zero and at any given time the average velocity of all the free electrons will also be zero. Will the situation be same when current is passing (electric field is applied)?**

A. No

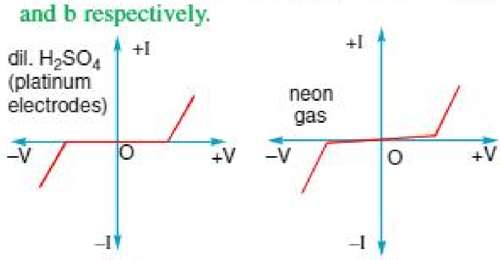
7. **The electron gets its drift speed under the influence of external force (due to applied electric field). Why the force does not give rise to acceleration of the electron?**

A. Due to collisions with positive ions inside the conductor the electron acquires only a drift speed.

8. **When a conductor is heated due to the passage of current through it, the resistivity (or resistance) of the conductor increases. What happens to the drift speed V_d of conduction electrons in this case?**

A. The drift speed decreases. This is due to increase in energy of free electrons, which results in more number of collisions with lattice ions over a given time and consequent loss of more energy.

9. **V - i characteristics of dilute sulphuric acid with tungsten electrodes and neon gas are shown in fig a and b respectively.**



(a) (b)

What can you say about the nature of these conductors ?

A. They are non - ohmic.

10. **A non - ohmic resistor does not obey Ohm's law. Can such non - ohmic resistors be of any use?**

A. Actually many useful components in the electrical industry and in electronics are non - ohmic in nature. Diode, transistor, light emitting diodes (LED), thermistor are all non-ohmic resistors (conductors). With out the non - ohmic resistors (conductors) there would have been no progress in electrons.

11. **A battery B of emf 1.5V and internal resistance $r = 5\,\Omega$ is first connected across a resistor of resistance $25\,\Omega$ as in fig (a). Next, the same battery is connected across a resistor of resistance $5\,\Omega$ as in fig (b). In which case, the voltmeter reading will be higher?**

(a) (b)

A. In the first case $i = \frac{1.5}{5+25} = \frac{1.5}{30}$ and

$V = 1.5 - \left(\frac{1.5}{30}\right)5 = 1.5 - 0.25 = 1.25V.$ ....(1)

In the second case

$i = \frac{1.5}{5+5} = \frac{1.5}{10}$ and $V = 1.5 - \left(\frac{1.5}{10}\right)5 = 0.75V$ ....(2)

The voltmetre reading is higher in the first case.

12. Out of the two Kirchhoffs laws which one explicitly shows that electrostatic force is a conservative force?

A. Kirchhoff's second law.

13. With a metre bridge, you are advised to preferably obtain a balancing length ℓ in the middle one third of the wire. Why ?

A. The bridge is most sensitive when $\frac{P}{Q} = \frac{R}{S} = 1$. That is when $\frac{\ell}{100-\ell} = 1$ or $\ell = 50$cm. More than 15 cm deviation is not good. So, it is preferable to obtain a balancing length in the middle one third of the wire.

14. You are to compare two resistance that are in the ratio 1 : 2. The wire of metre bridge given is of length 99 cm only. With full calculations explain how the error is minimized by taking additional readings with resistance interchanged.

A. $\frac{R}{S} = \frac{1}{2} = \frac{\ell}{99-\ell}$ gives $\ell = 33$ cm.

But we read the lengths as $\ell = 33$ cm and

100 -33 = 67 cm and write $\frac{R}{S} = \frac{33}{67} = 0.4925.$

Now, then we interchange the resistances, we should have $\frac{R}{S} = \frac{2}{1} = \frac{\ell}{99-\ell}$ which gives $\ell = 66$ cm - But we read lengths as $\ell = 66$ cm and 100 - 66 = 32 cm and write $\frac{R}{S} = \frac{66}{34}$ or $\frac{R}{S} = \frac{34}{66} = 0.5152.$

The average of 0.4925 and 0.5152 is 0.5038 and is very close to $\frac{1}{2}$

15. Define drift velocity of charge carriers in a conductor. Mention the units of drift velocity.

A. **Drift velocity :** The speed with which an electron gets drifted in a metallic conductor under the application of an external electric field is called the drift velocity (v_d).

16. Define mobility of charge carriers in a conductor. Mention the units of mobility.

A. **Mobility :** The mobility of a charge carrier is the average drift velocity resulting from the application of unit electric field strength.

It is represented by μ.

17. A conductor has got different areas of cross sections. Are the currents same at different cross sections? What conservation law helps you to decide the answer?

A. Conductor may be having different cross sections at different points along its length. But the current i will be the same for all cross sections of the conductor. This is a consequence of the law of conservation of charge.

18. Explain Ohm's law.

A. At constant temperature the current passing through a conductor is proportional to the potential difference between its ends.

If I is the strength of current flow through the conductor when the p.d. across its ends is V volt.

Then $I \propto V$

$V = IR$

R is a constant called electrical resistance. Its value depends on the nature of the material of conductor, dimensions and temperature of the conductor.

Ohm's law gives the relation between the current and voltage. It is not a universal law. It is not applicable for all substances. It is applicable only at constant temperature.

19. What are ohmic and non ohmic devices? Give examples.

A. The resistor which obeys ohm's law is ohmic resistor. Eg : Metals

The resistor which does not obey ohm's law is non-ohmic resistor.

Eg : Semiconductors (Silicon and Germanium), vacuum tubes, thermistors etc.

20. Define ohm.

A. The resistance of a conductor is said to be one ohm, if one volt of p.d. across its ends causes one ampere of current in it.

$R=\frac{V}{I}$; $\text{one ohm}=\frac{\text{one volt}}{\text{one ampere}}$

21. What is conductance: Give its unit.

A. The reciprocal of resistance is called conductance. It's S.I. unit is ohm^{-1}.

22. On what factors does the resistance of a conductor depend?

A. a) Nature of the substance.

b) Length and area of cross-section of the conductor.

c) Temperature.

23. The manganin wire is used in the preparation of standard resistance. Why?

A. For manganin temperature coefficient of resistance (α) is very very small. So change in resistance of the manganin wire with temperature is almost negligible. So it is used to prepare standard resistance.

24. What is temperature coefficient of resistivity? What are its units?

A. The ratio of the change in resistivity per 1^0C rise in temperature to the resistivity at 0^0C is called the temperature coefficient of resistivity.

Its units are $(C^0)^{-1}$ or K^{-1}.

25. Is Ohm's law a fundamental law?

A. Ohm's law is not a fundamental law.

Ohm's law does not hold good in case of gases, crystal rectifier, thermionic valves, transistor etc., and it is applicable only at constant temperature.

26. What is an ideal cell?

A. A cell or battery whose internal resistance is zero, is called ideal cell.

For ideal cell p.d., is equal to e.m.f.

27. What is the terminal voltage of a cell ? When it will be equal to the emf of the cell?

A. The potential difference across the terminals of a cell in an open circuit is called its emf ε .

28. When a cell is charged by sending current into the cell, what will be the terminal potential difference of the cell.

Ans Electric cell converts electrical energy into chemical energy, when current passes through it. While charging the cell, e.m.f. is less than the terminal voltage $(\in < v)$ and the direction of current inside the cell is form +ve terminal to negative terminal. $v=\varepsilon+ir$.

29. What are good conductors? Give examples.

A. Materials which have low values of resistivity are called good conductors.

Eg : Silver, Copper, Aluminium.

30. Which of the two, namely voltmeter and potentiometer is preferable to measure the emf of a battery? Why?

A. Potentiometer is preferable to measure the e.m.f of a battery. While measuring e.m.f. voltmeter draws current from battery. Whereas potentiometer draws no current when it is balanced. Hence the reading of p.d. between two points is more accurate.

31. Define kilowatt - hour unit of energy.

A. It is defined as the electrical energy consumed at the rate of one kilowatt for one hour.

32. On what conservation principles the first and the second laws of Kirchhoff are based?

A. First law is a consequence of the law of conservation of charge.

Second law is a consequence of the law of conservation of energy for electric circuits.

33. What is the advantage of a meter bridge over a Wheatstone bridge?

A. Meter bridge is an improvement on Wheatstone's bridge in the sense that we can change resistance continuously.

34. What is the principle of potentiometer?

A. The potential difference across any length of the potentiometer wire is directly proportional to its length. This is the principle of the potentiometer.

35. When is the series combination of cells advantageous and why?

A. Series combination is advantageous when high e.m.f (terminal potential difference at low current is required. In series combination of cells,

$i=\frac{n\varepsilon}{nr+R}$

where r <<R, then $i=\frac{n\varepsilon}{R}$

$\therefore$ ir = nε

$\therefore$ Potential difference across the external resistance is n times e.m.f of each cell.

36. **When is the parallel combination of cells advantageous and why ?**

A. Parallel combination is advantageous when more current is to be drawn at low potential from a circuit.

In parallel combination of cells, $i = \frac{\varepsilon}{R + (r/n)}$

If R << r, then $i = (n\varepsilon / R)$ that means, if external resistance is very small, current drawn is 'n' times that of a single cell.

37. **What are the advantages of using a potentiometer?**

A. Potentiometer can be used to measure e.m.f. of a cell, when the cell connected in the secondary circuit does not draw any current.

Hence, its e.m.f is measured against, balanced wire. No other instrument measure e.m.f of a cell except ammeters and voltmeters.

38.. **"Electrons alone are the current carriers in conductors". Explain whether this statement is correct or not?**

A. Yes, In a metallic conductor, on an average there will be one free electron available per atom.

39. **What conclusion can you draw from the following observation on a resistor made of alloy manganin ?**

Current A	Voltage V	Current A	Voltage V
0.2	3.94	3.0	59.2
0.4	7.87	4.0	78.8
0.6	11.8	5.0	98.6
0.8	15.7	6.0	18.5
1.0	19.7	7.0	38.2
2.0	39.4	8.0	58.0

Ans Ohm's law is valid to a high accuracy ; the resistivity of the alloy manganin is nearly independent of temperature.

40. **Answer the following questions :**

a) A steady current flows in a metallic conductor of non-uniform cross-section. Which of these quantities is constant along the conductor : current, current density, electric field, drift speed?

b) Is Ohm's law universally applicable for all conducting elements? If not, give examples of elements which do not obey Ohm's law.

c) A low voltage supply from which one needs high currents must have very low internal resistance. Why?

d) A high tension (HT) supply of, say, 6kV must have a very large internal resistance. Why ?

A. a) Only current (because it is given to be steady). The rest depends on the area of cross-section inversely.

b) No. Examples of non-ohmic elements; vacuum diode, semiconductor diode.

c) Because the maximum current drawn from a source $= \varepsilon / r$.

d) Because, if the circuit is shorted (accidentally), the current drawn will exceed safety limits, if internal resistance is not large.

41. **Choose the correct alternative :**

a) Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.

b) Alloys usually have much (lower/higher) temperature coefficients of resistance than pure metals.

c) The resistivity of the alloy manganin is nearly independent of temperature.

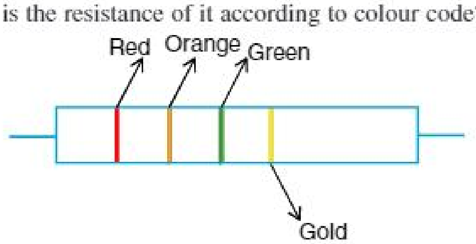
d) The resistivity of a typical insulator (e.g., amber) is greater than that of a metal by a factor of the order of (10^{22} / 10^3).

A. (a) greater, (b) lower, (c) nearly independent of, (d) 10^{22}

PROBLEMS

LEVEL - I

1. The colours on the resistor shown in Fig., are red orange, green and gold as read from left to right. What is the resistance of it according to colour code?



[Ans : $(2.3 \pm 0.115)\,M\Omega$]

2. A wire of length 0.1 m and radius 0.1mm has a resistance of $100\,\Omega$. Find the resistivity of the material. [Ans : $\pi \times 10^{-5}\,\Omega\,m$]

3. Consider a wire of length 4m and cross - sectional area 1 mm^2 carrying a current of 2A. If each cubic meter of the material contains 10^{29} free electrons find the average time taken by an electron to cross the length of the wire. [Ans : 8.9 hours]

4. Suppose you have three resistors each of value $30\,\Omega$. List all the different resistances you can obtain. [Ans : 10Ω, 20Ω, 45Ω, 90Ω]

5. The potential difference between the terminals of a battery of e.m.f 6.0V and internal resistance $1\,\Omega$ drops to 5.8V when connected across an external resistor. Find the resistance of the external resistor. [Ans : $29\,\Omega$]

6. Four resistors P,Q, R and X whose values are 2,2,2 and 3 ohms respectively are joined to form a Wheatstone Bridge. Calculate the value of resistance with which the resistance X must be shunted in order that the bridge may be balanced. [Ans : $6\,\Omega$]

7. The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4Ω, what is the maximum current that can be drawn from the battery? [Ans : 30 A]

8. (a) Three resistors $1\Omega, 2\Omega$ and 3Ω are combined in series. What is the total resistance of the combination?

   (b) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.

   [Ans : (a) 6Ω, (b) 2V, 4V, 6V]

9. (a) Three resistors $2\Omega, 4\Omega$ and 5Ω are combined in parallel. What is the total resistance of the combination ?

   (b) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.

   [Ans : (a) $(20/19)\Omega$; (b) 10A, 5A, 4A, 19A]

10. A negligibly small current is passed through a wire of length 15m and uniform cross-section $6.0\times10^{-7}\,m^2$, and its resistance is measured to be 5.0Ω. What is the resistivity of the material at the temperature of the experiment ? [Ans : $2.0\times10^{-7}\,\Omega m$]

11. A silver wire has a resistance of 2.1Ω at 27.5^0C, and a resistance of 2.7Ω at 100^0C. Determine the temperature coefficient of resistivity of silver at 27.5°C. [Ans : $0.0039\ ^0C^{-1}$]

12. In a potentiometer arrangement a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell ? [Ans : 2.25 V]

13. (a) In a metre bridge, the balance point is found to be at 39.5 cm from the end A, when the resistor Y is of 12.5Ω. Determine the resistance of X. Why are the connections between resistors in a Wheatstone or meter bridge made of thick copper strips ?

    (b) Determine the balance point of the above bridge if X and Y are interchanged.

    (c) What happens if the galvanometer and cell are interchanged at the balance point of the bridge ? Would the galvanometer show any current ?

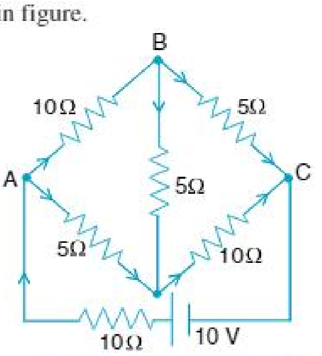
    [Ans : a) $X = 8.2\Omega$; to minimise resistance of the connections which are not accounted for in the bridge formula. b) 60.5 cm from A. c) The galvanometer will show no current.]

LEVEL - II

1. A battery of emf 10 V and internal resistance 3Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor ? What is the terminal voltage of the battery when the circuit is closed? [Ans : $17\Omega, 8.5\Omega$]

2. At room temperature (27.0^0C) the resistance of a heating element is 100Ω. What is the temperature of the element if the resistance is found to be 117Ω, given that the temperature coefficient of the material of the resistor is $1.70\times10^{-4}\ {}^0C^{-1}$ at 27°C. [Ans : $1027\ ^0C$]

3. Determine the current in each branch of the network shown in figure.



[Ans : Current in branch AB = (4/17)A. in BC = (6/17)A, in CD = (–4/17)A, in AD = (6/17)A, in BD = (–2/17)A, total current = (10/17)A]

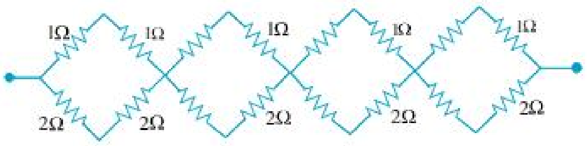
4. A storage battery of emf 80V and internal resistance 0.5Ω is being charged by a 120 V dc supply using a series resistor of 15.5Ω. What is the terminal voltage of the battery during charging ? What is the purpose of having a series resistor in the charging circuit ?

[Ans : 81.25 V; the series resistor limits the current drawn from the external source. In its absence, the current will be dangerously high]

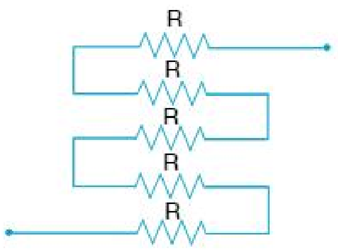
5. Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter ? Hence explain why aluminium wires are preferred for overhead power cables. ($\rho_{Al} = 2.63\times10^{-8}\Omega$m), $\rho_{Cu} = 1.72\times10^{-8}\Omega$m, relative density of Al= 2.7, Cu = 8.9.)

[Ans : The mass ratio of copper to aluminium wire is $(1.72/63)\times(8.9/2.7) = 2.2$. Since aluminium is lighter, it is preferred for long suspensions of cables]

6. (a) Given 'n' resistors each of resistance R, how will you combine them to get the (i) maximum (ii) minimum effective resistance / What is the ratio of the maximum to minimum resistance ?



(a)



(b)

(b) Determine the equivalent resistance of networks shown in figures above

(c) Given the resistances $1\Omega, 2\Omega, 3\Omega$ how will you combine them to get an equivalent resistance of

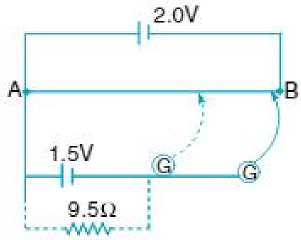
(i) $(11/3)\,\Omega$ (ii) $(11/5)\,\Omega$ (iii) $6\,\Omega$ (iv) $(6/11)\,\Omega$?

[Ans (a) (i) in series (ii) all in parallel; n^2.

(b) (i) $(16/3)\,\Omega$ (ii) 5R.

(c) (i) Join $1\,\Omega$, $2\,\Omega$ in parallel and the combination in series with $3\,\Omega$. (ii) parallel combination of $2\,\Omega$ and $3\,\Omega$ in series with $1\,\Omega$, (iii) all in series, (iv) all in parallel.]

7. Figure shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of $9.5\,\Omega$ is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.



[Ans 1.7 Ω]

8. A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8A. What is the steady temperature of the heating element if the room temperature is 27.0^{0}C. Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $1.70 \times 10^{-4}\ {}^0C^{-1}$.

[Ans : 867 ^{0}C]

9. The earth's surface has a negative surface charge density of 10^{-9} C m^{-2}. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to the low conductivity of the lower atmosphere) in a current of only 1800 A over the entire globe. If there were no mechanism of sustaining atmospheric electric field, how much time (roughly) would be required to neutralise the earth's surface ? (This never happens in practice because there is a mechanism to replenish electric charges, namely the continual thunderstorms and lightning in different parts of the globe.) (Radius of earth = 6.37×10^6m).

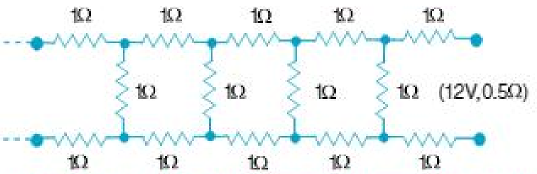
[Ans : Using the radius of earth, obtain total charge of the globe. Divide it by current to obtain time = 283 s. Still this method gives you only an estimate; it is not strictly correct.]

10. (a) Six lead-acid secondary cells each of emf 2.0 V and internal resistance 0.015Ω are joined in series to provide a supply to a resistance 8.5Ω. What are the current drawn from the supply and its terminal voltage?

(b) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380Ω. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car ?

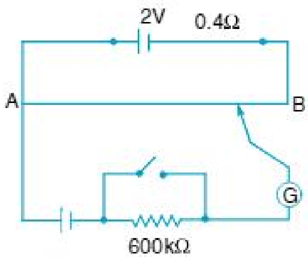
[Ans : (a) 1.4A, 11.9 V; (b) 0.005 A; impossible because a starter motor requires large current (~100A) for a few seconds]

11. Determine the current drawn from a 12V supply with internal resistance $0.5\,\Omega$ by the infinite network shown in figure. Each resistor has $1\,\Omega$ resistance.



[**Ans :** Hint : Let X be the equivalent resistance of the infinite network. Clearly, $2+X/(X+1)=X$ which gives $X=(1+\sqrt{3})\Omega$; therefore the current is 3.7A]

12. Figure shows a potentiometer with a cell of 2.0 V and internal resistance $0.40\,\Omega$ maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant emf of 1.02 V (for every moderate currents upto a few mA) gives a balance point at 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of $600\,\Omega$ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown emf ε and the balance point found similarly, turns out to be at 82.3 cm length of the wire.



(a) What is the value ε ?

(b) What purpose does the high resistance of $600\text{k}\,\Omega$ have?

(c) Is the balance point affected by this high resistance?

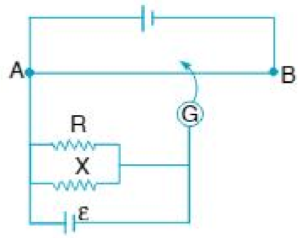
(d) Is the balance point affected by the internal resistance of the driver cell?

(e) Would the method work in the above situation if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V?

(f) Would the circuit work well for determining an extremely small emf, say of the order of a few mV (such as the typical emf of a thermo-couple)? If not, how will you modify the circuit?

[**Ans :** (a) ε = 1.25V ; (b) to reduce current through the galvanometer when the movable contact is far from the balance point. (c) No; (d) No ; (e) No. If ε is greater than the emf of the driver cell of the potentiometer, there will be no balance point on wire AB. (f) The circuit, as it is would be unsuitable, because the balance point (for ε of the order of a few mV) will be very close to the end A and the percentage error in measurement will be very large. The circuit is modified by putting a suitable resistor R in series with the wire AB so that potential drop across AB is only slightly greater than the emf to be measured. Then, the balance point will be at larger length of the wire and the percentage error will be much smaller]

13. Figure shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor $R=10.0\,\Omega$ is found to be 72.6 cm, while that with the unknown resistance X is 68.5 cm. Determine the value of X. What might you do if you failed to find a balance point with the given cell of emf ε ? (Take internal resistance $r=1\Omega$)



[**Ans :** $X=6.0\,\Omega$. If there is no balance point, it means potential drop across R or X is greater than the potential drop across the potentiometer wire AB. The obvious thing to do is to reduce current in the outside circuit (and hence potential drops across R and X) suitably by putting a series resistor.]

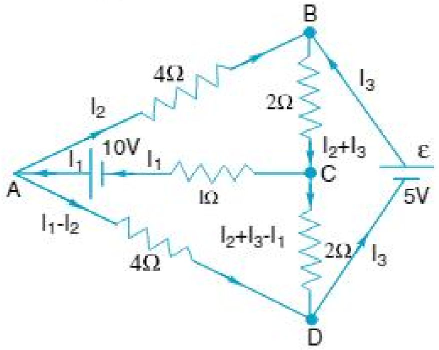
14. Two resistors with temperature coefficients of resistance α_1 and α_2 have resistances R_1 and R_2 at 0°C. Find the temperature coefficient of the compound resistor consisting of the two resisters connected in (a) series (b) parallel.

[**Ans:** (a) $\dfrac{R_1\alpha_1+R_2\alpha_2}{R_1+R_2}$ (b) $\dfrac{R_1\alpha_2+R_2\alpha_1}{R_1+R_2}$]

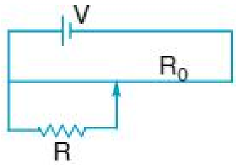
15. It is desired to make a $20\,\Omega$ coil of wire with zero thermal coefficient of resistance. To do this a carbon resistor of resistance R_1 is placed in series with an iron resistor of resistance R_2. The proportions of iron and carbon are so chosen that $R_1+R_2=20\,\Omega$ at all temperatures near 20°C. How large are R_1 and R_2 ? [$\alpha_C=-7.5\times10^{-2}\,{}^0C^{-1}$ and $\alpha_{Fe}=5.0\times10^{3}\,{}^0C^{-1}$

[**Ans:** $R_1=1.25\,\Omega$ and $R_2=18.75\,\Omega$]

17. Determine the current in each branch of the network shown in Fig.



16. A voltage V is supplied to a potentiometer wire of resistance R_0. A resistance R is connected as shown. Find voltage across R when the sliding contact is at the middle of potentiometer wire



[Ans: $\left(\frac{2VR}{R_0 + 4R}\right)$]

18. The four arms of a Wheatstone bridge (Fig.) have the following resistances : AB = 100 Ω, BC = 10 Ω, CD = 5 Ω, and DA = 60.