7. LOGARITHMS

SYNOPSIS

1. If $a \neq 1$, N are two positive real numbers and $a^x = N$, then x is called logarithm of N to the base a and we write $x = \log_a N$
2. Logarithms are defined only for positive numbers.
3. $\log_a a = 1$
4. $\log_a 1 = 0$
5. $\log_a(xy) = \log_a x + \log_a y$
6. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
7. $\log_a(x^n) = n \log_a x$
8. $\log_{a^n} x = \frac{1}{n}\log_a x$
9. $a^{\log a^m} = M$
10. $\log_b a \log_c b = \log_c a$
11. $\log_b a \log_a b = 1$
12. $\log_a x = \frac{\log_b x}{\log_b a}$
13. $\log_b a = \frac{1}{\log_a b}$
14. $a^{\log c^b} = b^{\log c^a}$
15. $a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$
16. If x, y, z are in G.P., then $\log_a x$, $\log_a y$, $\log_a z$ are in A.P.
17. $\log_a x$ is a decreasing function of x if $0 < a < 1$. i.e., $x > y \Leftrightarrow \log_a x < \log_a y$
18. $\log_a x$ is an increasing function of x if $a > 1$. i.e., $x > y \Leftrightarrow \log_a x > \log_a y$
19. $\log_b a$ is positive if $a > 1$ & $b > 1$ or $0 < a < 1$ & $0 < b < 1$
20. $\log_b a$ is negative if $0 < a < 1$ & $b > 1$ or $a > 1$ & $0 < b < 1$

LECTURE SHEET

LEVEL-I (MAIN)

Single answer type questions

1. $\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi}$

   1) is equalt o 2 2) is less than 2
   3) is greater than 2 4) cannot be determined

2. If $\frac{1}{\log_2 a} + \frac{1}{\log_4 a} + \frac{1}{\log_8 a} + \frac{1}{\log_{16} a} + \ldots\ldots$ to n terms $= \frac{n(n+1)}{2k}$, then $k =$

   1) $\log_a 2$ 2) $\log_a 4$ 3) $\log_2 a$ 4) $\log_4 a$

3. If $\log(x+y) = \log\left(\frac{3x-3y}{2}\right)$, then $\log x - \log y =$

   1) log 2 2) log 3 3) log 4 4) log 5

4. If x,y,z are three consecutive natural numbers then $\log(1 + xz) =$

   1) 0 2) y 3) log y 4) 2 log y

5. If $a^x = b^y = c^z$ and x,y,z are in G.P., then

1) $\log_c b = \log_a c$ 2) $\log_a c = \log_b a$ 3) $\log_a b = \log_c b$ 4) $\log_b a = \log_c b$

6. If $x = \log_a bc, y = \log_b ca, z = \log_c ab$ then $xyz =$

1) $x + y + z$ 2) $x + y + z + 1$ 3) $x + y + z + 2$ 4) $x + y + z + 3$

7. If $\log_a abc = x+1$, $\log_b abc = y+1$, $\log_c abc = z+1$, then $x + y + z + 2 =$

1) 0 2) xyz 3) $2yz$ 4) $3yz$

8. If $\log_{30} 3 = a$, $\log_{30} 5 = b$, then $\log_{30} 8 =$

1) $3(1+ a + b)$ 2) $3(1- a + b)$ 3) $3(1-a-b)$ 4) $3(1+a-b)$

9. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then $a^{b+c} b^{c+a} c^{a+b} =$

1) 0 2) 2 3) 1 4) 4

10. If $\frac{\log a}{y-z} = \frac{\log b}{z-x} = \frac{\log c}{x-y}$, then $a^{y^2+yz+z^2}\, b^{z^2+zx+x^2}\, c^{x^2+xy+y^2} =$

1) 0 2) 1 3) -1 4) 2

11. If $\frac{xy\log(xy)}{x+y} = \frac{yz\log(yz)}{y+z} = \frac{zx\log(zx)}{z+x}$ then

1) $x^x = y^y = z^z$ 2) $x = y = z$ 3) $x + y + z = 0$ 4) none

12. If $\log_{12} 81 = x$, then $\log_6 3 =$

1) $\frac{x}{x+4}$ 2) $\frac{x+4}{x}$ 3) $\frac{2x}{x+4}$ 4) $\frac{x+4}{2x}$

13. If $(4.2)^x = (0.42)^y = 100$, then $\frac{1}{x} - \frac{1}{y} =$

1) 1/2 2) 1/3 3) 1/4 4) 1/5

14. If $\frac{\log_a n}{\log_c n} = \frac{\log_a n - \log_b n}{\log_b n - \log_c n}$ then a,b,c are in

1) A.P. 2) H.P. 3) G.P. 4) A.G.P.

15. If $\frac{1}{2}\log_{10} 2 + \log_{10}\sqrt{x^2+4x+5} - \frac{1}{2}\log_{10}(x^2-4x+5) = \frac{1}{2}$, then $x =$

1) 0 or 1 2) 1 or 5 3) 0 or 5 4) 0 or 1 or 5

16. If $\log 2 = a$, $\log 3 = b$, $\log 7 = c$ and $6^x = 7^{x+4}$ then $x =$

1) $\frac{4b}{c+ab}$ 2) $\frac{4c}{a+b-c}$ 3) $\frac{4b}{c-a-b}$ 4) $\frac{4b}{a+b-c}$

17. $0.0001 < n < 0.001 \Rightarrow$

1) $-4 < \log n < -3$ 2) $-3 < \log n < -2$ 3) $-2 < \log n < -1$ 4) $-5 < \log n < -4$

18. $\frac{\log_a(\log_b a)}{\log_b(\log_a b)} =$

1) $\log_b a$ 2) $\log_a b$ 3) $-\log_a b$ 4) $-\log_b a$

19. If $\log 2 = a$, $\log 3 = b$, $\log 7 = c$ and $2^x.3^{x+4} = 7^x$, then the value of x in terms of a, b, c is

1) $\frac{4b}{(c-a-b)}$ 2) $\frac{4b}{(c+a+b)}$ 3) $\frac{4}{(c+a+b)}$ 4) $\frac{4}{(c-a-b)}$

20. The solution set of $\log_{1/2}(x^2-1)>0$ is:

1) $(1,\sqrt{2})$ 2) $(-\sqrt{2},-1)$ 3) $(-\sqrt{2},\sqrt{2})$ 4) $(-\sqrt{2},-1)\cup(1,\sqrt{2})$

21. The set of all x satisfying the equation, $x^{\log_3 x^2+(\log_3 x)^2-10}=\frac{1}{x^2}$ is :

1) {1, 9} 2) $\left\{1,9,\frac{1}{81}\right\}$ 3) $\left\{1,4,\frac{1}{81}\right\}$ 4) $\left\{9,\frac{1}{81}\right\}$

22. The set of all the solutions of the inequality, $\log_{1-x}(x-2)\geq -1$ is:

1) $(-\infty,0)$ 2) $(2,\infty)$ 3) $(-\infty,1)$ 4) ϕ

Numerical value type questions

23. $\log_2\left(1+\frac{1}{2}\right)+\log_2\left(1+\frac{1}{3}\right)+.....+\log_2\left(1+\frac{1}{31}\right)=$

24. If $\log_{10}\left[\frac{1}{2^x+x-1}\right]=x\left[\log_{10}5-1\right]$ then $x=$

25. If $\log_{10}3=0.4771$, then the number of digits in 3^{40} is

26. Number of solution to the equation $[x] = \log x$.

27. The number of solution to (Real solutions) $x^{\frac{3}{4}(\log_2 x)^2+\log_x x-\frac{5}{4}}=\sqrt{2}$

28. If $x=\log_{2a}^{a}:y=\log_{3a}^{2a}:z=\log_{4a}^{3a}$ then $(x-2)yz=$

29. $a>0(a\neq 1),b>0(b\neq 1)$ such that $a^{(\log_a^b)^x}=b^{(\log_b^a)^x}$ then $x=$

30. There exist positive integers A, B and C with no common factors greater than 1, such that $A\log_{200}5 + B\log_{200}2 = C$. The sum $A + B + C$ equals

LEVEL-II (ADVANCED)

Single answer type questions

1. If $c^2=(ca)^{\log_a b}$ conditions $a,b,c>0, b>a \Rightarrow \ln a,\ln b,\ln c$ are in (natural logarithms)

a) A.P b) G.P c) H.P d) None

2. Given real numbers a, b, $c > 0$ (^{1} 1) such that $\log_{\log_c a} e$, $\log_{(a^{c/2})} e$, $\log_{(\log_b c)} e$ ar e in H.P. then c equal to

a) $\log_a(\log_a b)$ b) $\log_a(\log_b a)$ c) $\log_b(\log_b a)$ d) $\log_b(\log_a b)$

3. For $x\in R; x>1$ and $n\in I>1$. Let $f_n(x)=\frac{n}{\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+...+\frac{1}{\log_n x}}$, then $(5!)^{f_5(x)}$

a) 5^x b) 5^5 c) x^5 d) 5^{5x}

4. Define a real valued function of a real variable x given by $f(x)=a\sin x+b\sqrt[3]{x}+6$ where a and b are real numbers. If $f(\log_{30}(\log_{15}30))=7$, the the value of $f\left(\log_{30}(\log_{30}15)\right)$ is.

a) 5 b) 7 c) –5 d) –7

5. If $a_n = \sum_{r=1}^{n} \log_e a^r, a>0$ and $S_n = \sum_{r=1}^{n} a_r$, then minimum number of terms of S_n to be taken so that the sum is greater than $1335\log_e a^n$ is

a) 88 b) 89 c) 91 d) 92

6. If the equation $\frac{\log_{12}(\log_8(\log_4 x))}{\log_5(\log_4(\log_y(\log_2 x)))} = 0$ has a solution for 'x' when $c<y<b, y\neq a$, where 'b' is as large as possible and 'c' is as small as possible, then the value of $(a+b+c)$ is equal to

a) 18 b) 19 c) 20 d) 21

7. If $x>1$, $y>1$, $z>1$ are in G.P., then $\log_{ex} e$, $\log_{ey} e$, $\log_{ez} e$ are in

a) A.P b) H.P c) G.P d) A.G.P

8. Which of the following is/are true

a) $\log_2^3 < \log_5^{17}$ b) $\log_2^{24}(\log_{96}^2)^{-1} - \log_2^{192}(\log_{12}^2)^{-1} = 3$

c) $(\log_2^5)^2 > \log_2^{20}$ d) $\log_{10}^5 . \log_{10}^{20} + (\log_{10}^2)^2 = 1$

More than one answer type questions

9. If $\frac{\log_2 x}{4} = \frac{\log_2 y}{6} = \frac{\log_2 z}{3k}$ and $x^3y^2z = 1$, then k is equal to

a) -8 b) -4 c) 0 d) $\log_2\left(\frac{1}{256}\right)$

10. A line $x=k$ intersects the graph of $y=\log_5 x$ and $y=\log_5(x+4)$. The distance between the points of intersection is 0.5 given that $k=a+\sqrt{b}$, where $a,b\in N$, then

a) $a=2, b=4$ b) $a=1, b=5$ c) $a+b=6$ d) $a=b=3$

11. A line $x=k$ intersects the graph of $y=\log_5 x$ and the graph of $y=\log_5(x+4)$. The distance between the point of intersection is 0.5. Given that $k=a+\sqrt{b}$ where $a,b\in N$ then

a) $a=2, b=4$ b) $a=1, b=5$ c) $a+b=6$ d) $a=b=3$

12. Given $x^{2\log x} = 10x^2$ (log x is to base 10) solve for x

a) $10^{\sqrt{3}+1}$ b) $10^{-\sqrt{3}-1}$ c) $10^{-\sqrt{3}+1}$ d) $10^{\sqrt{3}-1}$

Linked comprehension type questions

Passage - I :

An equation of the form $2m\log_a f(x) = \log_a g(x), a>0, a\neq 1, m\in N$ is equivalent to the system

$$\begin{cases} f(x)>0 \\ f^{2m}(x) = g(x) \end{cases}$$

13. The number of solutions of $2\log_e 2x = \log_e(7x-2-2x^2)$ is

a) 1 b) 2 c) 3 d) infinite

14. The number of solutions of $\ln 2x = 2\ln(4x-15)$ is

a) 0 b) 1 c) 2 d) infinite

15. The number of solutions of $\log(3x^2+x-2) = 3\log(3x-2)$ is

a) 1 b) 2 c) 3 d) 0

Passage - II :

Equations of the form (i) $f(\log_a x)=0, a>0, a\neq 1$ and (ii) $g(\log_x A)=0, A>0$, then eq. (i) is equivalent to $f(t) = 0$, where $t = \log_a x$

If $t_1, t_2, t_3,....,t_k$ are the roots of $f(t) = 0$, then $\log_a x=t_1, \log_a x=t_2,....,\log_a x=t_k$ and eq. (ii) is equivalent to $f(y) = 0$, where $y = \log_x A$. If $y_1, y_2, y_3,.....,y_k$ are the roots of $f(y) = 0$, then $\log_x A=y_1, \log_x A=y_2,.....\log_x A=y_k$.

16. The solution set of $(\log_5 x)^2+\log_5 x+1=\frac{7}{\log_5 x-1}$ contains

a) (1, 3) b) {1} c) {25} d) {1, 25}

17. The set of all x satisfying the equation $x^{\log_3 x^3+(\log_3 x)^2-10}=\frac{1}{x^2}$ is

a) {1, 9} b) $\left\{9,\frac{1}{81}\right\}$ c) $\left\{1,4,\frac{1}{81}\right\}$ d) $\left\{1,9,\frac{1}{81}\right\}$

18. If $\frac{(\ln x)^2-3\ln x+3}{\ln x-1}<1$, then x belongs to

a) $(0, e)$ b) $(1, e)$ c) $(1, 2e)$ d) $(0, 3e)$

Passage - III :

If $\frac{\log x}{b-c}=\frac{\log y}{c-a}=\frac{\log z}{a-b}$, $l = \log_{2k}k$, $m = \log_{3k}2k$, $n = \log_{4k}3k$ and $\log_{12}27 = p$ then

19. $xyz =$

a) 1 b) abc c) 0 d) –1

20. $x^a.y^b.z^c =$

a) 0 b) 2 c) 3 d) 1

21. $x^{b^2+bc+c^2}.y^{c^2+ca+a^2}.z^{a^2+ab+b^2} =$

a) 0 b) 1 c) -1 d) 2

Passage - IV :

If $f(x) = \log x$, $g(x) = x^3$, $h(x) = \cos x$; $w(x)=\frac{1-x}{1+x}$,

$T(x) = (fow)(x)$, $u(x)=\frac{1+x}{1-x}$ and

$V(x) = (fou)(x)$

22. $V(x) + V(y) =$

a) $V(x + y)$ b) $V(x)+\frac{V(y)}{1+xy}$ c) $(x+y)V\left(\frac{1}{1+xy}\right)$ d) $V\left(\frac{x+y}{1+xy}\right)$

23. $u\left(\frac{3x+x^3}{1+3x^2}\right)=$ -----

a) $3v(x)$ b) $v(x)$ c) $2v(x)$ d) none of 1,2,3

Matrix matching type questions

24.

COLUMN - I	COLUMN - II
A) If $\log_{34}5$ lies in the interval (a, b), then	p) $[10a+10b]=8$, where [.] denotes the greatest integer function
B) If $\log_{300}4$ lies in the interaval (a, b), then	q) (10a + 10b) = 5 where (.) denotes the least integer function
C) If $\log_{400}4$ lies in the interval (a, b), then	r) [6b–3a] = 2 where [.] denotes the greatest integer function
	s) [10a + 10b] = 3 where [.] denotes the greatest integer function
	t) (6b – 3a)= 1 where (.) denotes the least integer function

25.

COLUMN - I	COLUMN - II
A) The solution set of $\log_{100}\lvert x+y\rvert=\frac{1}{2}$, $\log_{10}y-\log_{10}\lvert x\rvert=\log_{100}4$ is	p) $\{\sqrt{2},2\}$
B) The solution set of $4\log_2^2 x+1=2\log_2 y$ and $\log_2 x^2\geq\log_2 y$	q) {1, 1}
C) The solution set of $\log_4 x-\log_2 y=0$ and $x^2-5y^2+4=0$	r) {–10, 20}
	s) {4, 2}
	t) $\left\{\frac{10}{3},\frac{20}{3}\right\}$

26.

LIST - I (equation)	LIST - II (solution)
1) $\log_8 x+\log_x 8=\frac{10}{3}$	a) 8, ½
2) $5^{(\log_{10}x-2)}=3^{(\log_{10}x-2)}$	b) 512, 2
3) $2\log_{\_}\log_2 x+\log_2\log_2(2\sqrt{2}x)$	c) 100
4) $x^{\log_2(x+4)}=6$	d) 1, $\frac{1}{3}$
	e) 1

	1	2	3	4		1	2	3	4
a)	b	c	a	e	b)	a	e	c	d
c)	a	b	d	c	d)	d	a	c	b

Integer answer type questions

27. $\log(a+c)$, $\log(a+b)$, $\log(b+c)$ are in A.P and a, c, b are in H.P where $a, b, c > 0$. If $a+b=k\,c/4$ then the value of 'k' is

28. If $x,y\in R^+$ satisfy $\log_8 x+\log_4 y^2=5$ and $\log_8 y+\log_4 x^2=7$ then the value of $\frac{(x^2+y^2)}{2080}=$ .....

29. Number of values of x satisfy the equation $\log_{2x+3}(6x^2+23x+21)=4-\log_{3x+7}(4x^2+12x+9)$ (is/are)

30. If $\log_{10}x+\frac{1}{2}log_{10}x+\frac{1}{4}\log_{10}x+....=y$ and $\frac{1+3+5+....+(2y-1)}{4+7+10+....+(3y+1)}=\frac{20}{7\log_{10}x}\forall x,y\in N$, then the value of $\log_y^x$ must be

PRACTICE SHEET

LEVEL-I (MAIN)

Single answer type questions

1. $3^{\sqrt{\log_3 7}} - 7^{\sqrt{\log_7 3}} =$

   1) 0 2) 1 3) -1 4) 2

2. $\log_2 17 \log_{1/5} 2 \log_3 (1/5)$

   1) is equal to 2 2) is less than 2 3) is greater than 2 4) cannot be determined

3. If $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots\ldots + \frac{1}{\log_{43} n} = \frac{1}{\log_x n}$, then $x =$

   1) 43 2) 43^n 3) 43! 4) n^{43}

4. If $y = \frac{10^{\log x}}{x^2}$ and $x = y^A$ then $A =$

   1) $y = \frac{10^{\log x}}{x^2}$ 2) $\frac{1}{\log 10 + 2}$ 3) $\frac{1}{\log 10 - 3}$ 4) $\frac{1}{\log 10 + 3}$

5. If $x^3 + y^3 = 0$, $x + y \neq 0$ then $\log(x + y) =$

   1) $\frac{1}{2}(\log x + \log y - \log 2)$ 2) $\frac{1}{2}(\log x + \log y + \log 3)$ 3) $\log x + \log y + \log 2$ 4) $\log x + \log y - \log 2$

6. If $x = \log_{0.1} 0.001$, $y = \log_9 81$ then $\sqrt{x - 2\sqrt{y}} =$

   1) $3 - \sqrt{2}$ 2) $\sqrt{3} - 2$ 3) $\sqrt{2} - 1$ 4) $\sqrt{2} - 2$

7. If $f(a) = \log \frac{2+a}{2-a}$, then $\frac{1}{2} f\left(\frac{8a}{4+a^2}\right) =$

   1) $f(a)$ 2) $2f(a)$ 3) $\frac{1}{2} f(a)$ 4) $-f(a)$

8. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$ then $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} =$

   1) 1 2) 2 3) 3 4) 4

9. If $a = 1 + \log_x yz$, $b = 1 + \log_y zx$, $c = 1 + \log_z xy$ then $ab + bc + ca =$

   1) 0 2) $2abc$ 3) $a^2 + b^2 + c^2$ 4) abc

10. If $a^{1/(1-\log_a x)} = y$, $a^{1/(1-\log_a y)} = z$ then $a^{1/(1-\log_a z)} =$

    1) x 2) $2x$ 3) x^2 4) $x/2$

11. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then $a^{b+c} b^{c+a} c^{a+b} =$

    1) 0 2) 2 3) 1 4) 4

12. If $(x - y)\log_a 2 = (y - z)\log_b 2 = (z - x)\log_c 2$ then $5^{abc} =$

    1) 0 2) 1 3) 2 4) 5

13. If $b = \frac{a^x + a^{-x}}{2}$, then $x =$

    1) $\log_a(b + \sqrt{b^2 + 1})$ 2) $\log_b(a + \sqrt{a^2 + 1})$ 3) $\log_a(b + \sqrt{b^2 - 1})$ 4) none

14. If $5^x = (0.5)^y = 1000$ then $\frac{1}{x} - \frac{1}{y} =$

    1) 1 2) 1/2 3) 1/3 4) 1/4

15. If $a^x = b^y = c^z = d^w$, then $\log_a(bcd) =$

1) $x\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$ 2) $\frac{1}{x}\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$ 3) $x\left(\frac{1}{y}+\frac{1}{z}-\frac{1}{w}\right)$ 4) $\frac{1}{x}\left(\frac{1}{y}+\frac{1}{z}-\frac{1}{w}\right)$

16. If a,b,c are distnict positive real numbers different from 1 such that $(\log_b a \log_c a - \log_a a)+(\log_a b \log_c b - \log_b b)+(\log_a c \log_b c - \log_c c) = 0$ then $abc =$

1) 0 2) 1 3) –1 4) none

17. If $\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$ and $x \neq y$, then $x + y =$

1) 2 2) 65/8 3) 37/6 4) $8+\sqrt[3]{2}$

18. If $\log_l x, \log_m x, \log_n x$ are in A.P. and $x \neq 1$ then $(\ln)^{\log_l m} =$

1) n 2) n^2 3) n^3 4) n^4

19. If x, y, z are positive and the p^{th}, q^{th}, r^{th} terms respectively of a G.P., then $(q \quad r)\log x + (r \quad p)\log y + (p \quad q)\log z =$

1) 0 2) 1 3) pqr 4) xyz

20. If $\log_3 2, \log_3(2^x - 5), \log_3(2^x - 7/2)$ are in A.P., then $x =$

1) 1 2) 2 3) 3 4) 4

21. If $1 < \log_2 x^2 < 2$, then $x \in$

1) $(-2,\sqrt{2})\cup(\sqrt{2},2)$ 2) $(-2,-\sqrt{2})\cup(\sqrt{2},2)$ 3) $(2,\sqrt{2})\cup(-\sqrt{2},2)$ 4) $(-2,\sqrt{2})\cup(-\sqrt{2},2)$

22. If $\log_{0.1}(x-3) < \log_{0.01}(x-3)$, then x lies in

1) (3,4) 2) [3, 4] 3) $(4,\infty)$ 4) none

23. If $\log_{1/3}(\log_4(x^2-5) > 0$, then

1) $x \in (-3,-\sqrt{6})\cup(\sqrt{6},3)$ 2) $x \in (3,-\sqrt{6})\cup(\sqrt{6},3)$ 3) $x \notin (-3,-\sqrt{6})\cup(\sqrt{6},-3)$ 4) $x \notin (-3,-\sqrt{6})\cup(\sqrt{6},3)$

24. If $x = \log_2 3$, $y = \log_3 11$, then

1) $x = y$ 2) $x < y$ 3) $x > y$ 4) none

25. $\frac{\log_a x + \log_b x}{\log_a x - \log_b x} =$

1) $2\log_{a/b} a$ 2) $2\log_{a/b} b$ 3) $\log_{a/b} ab$ 4) $\log_{b/a} ab$

26. The set $\{x : |1 - \log_{1/5} x| + 2 = |3 - \log_{1/5} x|\}$ is equal to:

1) $(0,\infty)$ 2) $\left[\frac{1}{5},\infty\right)$ 3) $\left[\frac{1}{5},5\right]$ 4) $\left(0,\frac{1}{5}\right]$

27. If $|x-2|^{\log_2 x^3 - 3\log_x 4} = (x-2)^3$, then a value of x is :

1) –4, –3 2) 4 , 3 3) 3, –4 4) 4

28. If $\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$ and $x \neq y$, then $x + y$ is equal to:

1) $\frac{25}{3}$ 2) $\frac{65}{8}$ 3) $\frac{37}{6}$ 4) $8 + 2^{1/3}$

29. The solution set of the equation, $\frac{(x+3)\log_{10} x}{(x-4)(x^2+9)} > 0$:

1) $(0,1)\cup[4,\infty)$ 2) $(0,2)\cup[4,\infty)$ 3) $(0,1]\cup[4,\infty)$ 4) $(0,1)\cup(1,\infty)$

Numerical value type questions

30. $\log_{\sqrt{2}}[\log_{\sqrt{2}}\{\log_{\sqrt[4]{3}}(\log_3 27)\}] =$

31. If $\log_{10} 343 = 2.5353$ then the least positive integer n such that $7^n > 10^5$ is

32. If $\log_a\left[1+\log_b\{1+\log_c(1+\log_a x\}\right]=0$ then $x =$

33. If $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, then the number of digits in $(2^{10}3^5)^{20}$ is

34. If $\log_{10}3 = 0.4771$ then the position of first significant figure in 81^{-25} is

LEVEL-II (ADVANCED)

Single answer type questions

1. The position of the first significant figure in 3^{-65} is (given that $\log_{10}^3 = 0.4771$)

a) 30 b) 31 c) 32 d) 29

2. If a, b, c are unequal positive number and $\Sigma[\log_b^a \log_c^a - 1]=0$ then $abc =$

a) 0 b) $\cos 0^0$ c) $\cos 30^0$ d) $\cos\pi$

3. a, b are the lengths of sides and c is the length of hypotenuse of a right angled triangle then $\frac{\log_{c+b}^a + \log_{c-b}^a}{\log_{c+b}^a . \log_{c-b}^a} =$

a) 0 b) 1 c) –1 d) 2

4. Solution set of $\log_3(x^2-2) < \log_3\left(\frac{3}{2}|x|-1\right)$ is:

a) $(-\sqrt{2},-1)$ b) $(-2,+\sqrt{2})$ c) $(-2,-\sqrt{2})\cup(\sqrt{2},2)$ d) Null set

5. If $\log_{105}^7 = a, \log_7^5 = b$ then $\log_{35}^{105} =$

a) ab b) $(b+1)a$ c) $\frac{1}{ab}$ d) $\frac{1}{a(b+1)}$

More than one answer type questions

6. $\log_{\cos x}\sin x + \log_{\sin x}\cos x = 2 \Rightarrow$ solve for $x =$ .....

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{5\pi}{4}$ d) $\frac{7\pi}{4}$

Linked comprehension type questions

Passage - I :

An equation of the form $2m\log_a f(x) = \log_a g(x), a>0, a\neq 1, m\in N$ is equivalent to the system

$$\begin{cases} f(x)>0 \\ f^{2m}(x)=g(x) \end{cases}$$

7. Solution set of the equation $\log_{(x^3+6)}(x^2-1)=\log_{(2x^2+5x)}(x^2-1)$ is

a) {–2} b) {1} c) {3} d) {–2, 1, 3}

8. Solution set of the equation $\log(x-9)+2\log\sqrt{(2x-1)}=2$ is

a) $\{\phi\}$ b) {1} c) {2} d) {13}

Passage - II :

Equations of the form (i) $f(\log_a x)=0, a>0, a\neq 1$ and (ii) $g(\log_x A)=0, A>0$,

then eq. (i) is equivalent to $f(t) = 0$, where $t = \log_a x$.

If t_1, t_2, t_3 ..... , t_k are the roots of $f(t) = 0$, then $\log_a x=t_1, \log_a x=t_2, ..., \log_a x=t_k$ and eq. (ii) is equivalent to $f(y) = 0$, where $y = \log_x A$. If y_1, y_2, y_3, ... , y_k are the roots of $f(y) = 0$, then $\log_x A=y_1, \log_x A=y_2, ...\log_x A=y_k$.

9. The number of solutions of the equation $\frac{1-2(\log x^2)^2}{\log x-2(\log x)^2}=1$ is

a) 0 b) 1 c) 2 d) infinite

10. The number of solutions of the equation $\log_x^3 10-6\log_x^2 10+11\log_x 10-6=0$ is

a) 0 b) 1 c) 2 d) 3

Passage-III :

If $\frac{\log x}{b-c}=\frac{\log y}{c-a}=\frac{\log z}{a-b}$, $l = \log_{2k}k$, $m = \log_{3k}2k$, $n = \log_{4k}3k$ and $\log_{12}27 = p$ then

11. $(l-1)mn+1=$

a) 2mn b) 0 c) *l*mn d) mn

12. $4\left(\frac{3-p}{3+p}\right)=$

a) $\log_6 16$ b) $\log_6 12$ c) $\log_6 18$ d) $\log_6 21$

Passage-IV :

If $f(x) = \log x$, $g(x) = x^3$, $h(x) = \cos x$; $w(x)=\frac{1-x}{1+x}$, $T(x) = (\text{fow})(x)$, $u(x)=\frac{1+x}{1-x}$ and $V(x) = (\text{fou})(x)$

13. $f\{g(a)\}+f\{g(b)\}=$

a) $f\{g(a)+g(b)\}$ b) $f\{g(ab)\}$ c) $g\{f(ab)\}$ d) $g\{f(a)+f(b)\}$

14. $(hof)\left(\frac{1}{x}\right).(hof)\left(\frac{1}{y}\right)-\frac{1}{2}\left[(hof)(xy)+(hof)\left(\frac{x}{y}\right)\right]=$

a) 0 b) $f(x)+f(y)$ c) $f(x).f(y)$ d) $f(x)-f(y)$

15. $T\left(\frac{2x}{1+x^2}\right)=$

a) ½ (fow) (*x*) b) –(fow) (*x*) c) (fow) (*x*) d) 2(fow) (*x*)

Matrix matching type questions

16. Observe the following lists :

LIST - I	LIST - II
1) $\log_a bc = x$, $\log_b ca = y$, $\log_c ab = z$	a) $xyz = xy + yz + zx$
2) $x = 1+\log_a bc$, $y = 1+\log_b ca$, $z = 1+\log_c ab$	b) $xyz = x + y + z + 2$
3) $\log_{2a} a = x$, $\log_{3a} 2a = y$, $\log_{4a} 3a = z$	c) $abc = 1$
4) $\frac{\log x}{a+b-2c} = \frac{\log y}{b+c-2a} = \frac{\log z}{c+a-2b}$	d) $1 + xyz = 2yz$
	e) $xyz = 1$

	1	2	3	4		1	2	3	4
a)	b	a	d	e	b)	c	a	b	d
c)	a	b	d	c	d)	d	a	b	c

17.

LIST - I (In equility)	LIST - II (solution set)
1) $\log_{0.5}(x - 2) < \log_{0.25}(x - 2)$	a) (1, 2)
2) $\log_3(x - 5) < \log_9(x - 5)$	b) (5, 6)
3) $\log_4(x - 3) > \log_{16}(x - 3)$	c) $(-1, 3) \cup (3, \infty)$
4) $\log_{0.2}(x - 1) > \log_{0.04}(x - 1)$	d) $(3, \infty)$
	e) $(4, \infty)$

	1	2	3	4		1	2	3	4
a)	d	b	c	a	b)	a	c	b	d
c)	e	a	d	b	d)	d	b	e	a

18.

LIST - I	LIST - II
1) $\log_{10}\tan 40^\circ + \log_{10}\tan 41^\circ + ... + \log_{10}\tan 50^\circ$	a) 3
2) $\frac{\log 49\sqrt{7} + \log 25\sqrt{5} - \log 4\sqrt{2}}{\log 17.5}$	b) 857
3) $\log_3\left(1+\frac{1}{2}\right) + \log_3\left(1+\frac{1}{3}\right) + ... + \log_3\left(1+\frac{1}{53}\right)$	c) 0
4) $3^{\frac{4}{\log_4 9}} + (27)^{\frac{1}{\log_{36} 9}} + (81)^{\frac{1}{\log_5 3}}$	d) $\frac{3}{2}$
	e) 870

	1	2	3	4		1	2	3	4
a)	c	d	a	b	b)	d	a	c	b
c)	a	c	b	d	d)	a	d	c	b

Integer answer type questions

19. If (x_1, y_1) and (x_2, y_2) are solutions of the system of simultaneous equation $\log_8(xy) = 3\log_8 x.\log_8 y$, $4\log_8 \left(\frac{x}{y}\right) = \frac{\log_8 x}{\log_8 y}$ then $x_1x_2 + y_1y_2$ equals to

20. The absolute value of x satisfying the equation $\log_{10}(98+\sqrt{x^3-x^2-12x+36}) = 2$ is

21. No.of solutions to (Real solutions) $|x-1|^{(\log x)^2-\log x^2} = |x-1|^3$

22. If $\log\left(\frac{x^2}{y^3}\right) = 1$ and $\log(x^2y^3) = 7$ then $\log(|xy|)$ is equal to

KEY SHEET

LECTURE SHEET

LEVEL-I

1) **3**	2) **3**	3) **4**	4) **4**	5) **4**	6) **3**	7) **2**	8) **3**
9) **3**	10) **2**	11) **1**	12) **3**	13) **1**	14) **3**	15) **2**	16) **2**
17) **1**	18) **3**	19) **1**	20) **4**	21) **2**	22) **4**	23) **4**	24) **1**
25) **20**	26) **0**	27) **3**	28) **1**	29) **1**	30) **7**		

LEVEL-II

1) **c**	2) **b**	3) **b**	4) **c**	5) **a**	6) **b**	7) **b**	8) **b**
9) **abcd**	10) **ad**	11) **bc**	12) **bc**	13) **b**	14) **b**	15) **d**	16) **c**
17) **d**	18) **a**	19) **a**	20) **d**	21) **b**	22) **d**	23) **a**	
24) **A-pr, B-qt, C-st**			25) **A-rt, B-p, C-qs**			26) **a**	27) **8**
28) **2**	29) **1**	30) **5**					

PRACTICE SHEET

LEVEL-I

1) **1**	2) **3**	3) **3**	4) **1**	5) **2**	6) **3**	7) **1**	8) **1**
9) **4**	10) **1**	11) **3**	12) **4**	13) **3**	14) **3**	15) **1**	16) **2**
17) **4**	18) **2**	19) **1**	20) **3**	21) **2**	22) **3**	23) **1**	24) **2**
25) **4**	26) **2**	27) **2**	28) **4**	29) **4**	30) **4**	31) **6**	32) **1**
33) **108**	34) **48**						

LEVEL-II

1) **a**	2) **b**	3) **d**	4) **c**	5) **d**	6) **a**	7) **c**	8) **d**
9) **c**	10) **d**	11) **d**	12) **a**	13) **b**	14) **a**	15) **d**	16) **a**
17) **d**	18) **a**	19) **1**	20) **4**	21) **3**	22) **1**		