Chapter - 2

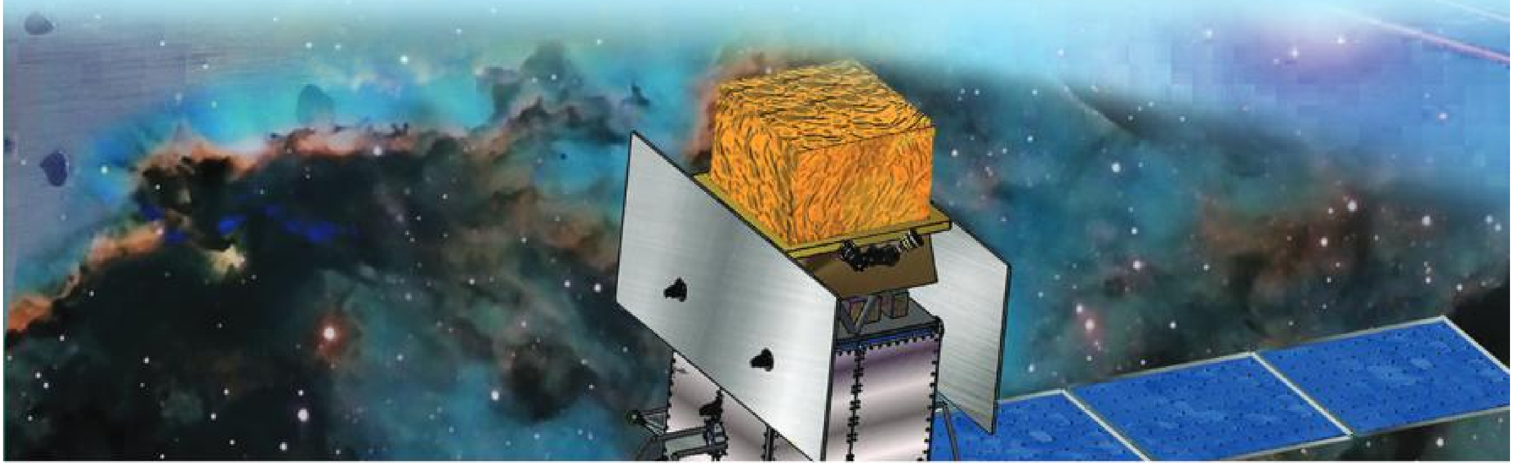
GAUSS'S LAW

✤ Electric flux - Gauss's law ✤

✤ Applications of Gauss's law ✤

✤ Metal Conductors in Electric Field ✤

✤ Van De Graff Generator ✤



2.1 ELECTRIC FLUX

The word 'flux' means to flow in Latin. Any group of electric lines of force or electric field lines passing through a given surface is known as electric flux. It is denoted by ϕ. (We have already discussed about magnetic flux in Magnetism).

Consider flow of a liquid with velocity 'v' through a small flat surface ds, in a direction normal to the surface. The rate of flow of liquid is given by the volume of the liquid crossing the area per second which is given as Vds. This is known as flux of liquid flowing across the plane. Here $\overline{V}$ is flow velocity of the liquid and ds is the area of a small flat surface in a direction normal to the surface. If the normal to the surface is not parallel to the direction of flow of liquid, but makes an angle θ with $\overline{V}$, liquid flux is given by $Vds\cos\theta$. In vector form we can write the flux as $\overline{V}.\hat{n}\,ds$

The electric flux through a small element of surface $d\overline{s}$ is defined as $d\phi = \overline{E}.d\overline{s}$ $\Rightarrow d\phi = E\cos\theta\, ds = Eds\cos\theta$ or $E \perp ds$

The total flux through the surface is

$$\int d\phi = \int \overline{E}.d\overline{s}$$

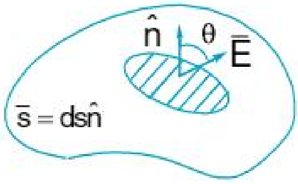


Fig 2.1

We treat area of a surface as vector quantity and its direction is along the outward normal to the surface at any point. i.e., $d\overline{s} = ds\hat{n}$ where $\hat{n}$ is unit vector along outward normal.

We can associate a vector to the area of a curved surface also. For this the given surface must be divided into a large number of very small area elements. Each small area element may to treated as planar and a vector associated with it may be along outward normal to that surface.

- Electric flux is a scalar quantity and its SI unit is Vm.

2.2 $\overline{E}$ IN TERMS OF ϕ

We know that the density of electric field lines gives the magnitude of electric field strength. Actually the electric field strength at a point in the region of electric field can be given as the electric flux passing through a unit normal area at that point.

If the electric field is uniform, and if ϕ be the electric flux passing through an area S which is normal to the field lines, the value of electric field strength on this surface can be given as

$$E = \frac{\phi}{s} \quad \text{or} \quad \phi = Es$$

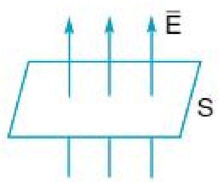


Fig 2.2(a)

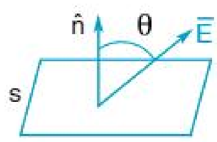


Fig 2.2 (b)

If the surface is not normal to the field direction, then $\phi = Es\cos\theta$ or $E = \dfrac{\phi}{s\cos\theta}$

- In a non uniform electric field, the electric flux through a given surface can be obtained from the formula $\int d\phi = \int E\ ds\ \cos\theta$.
- Electric flux may be positive, negative or even zero as shown below. In ward flux is taken as positive, outward flux as negative.

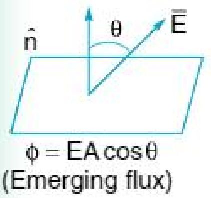


Fig.2.3(a)

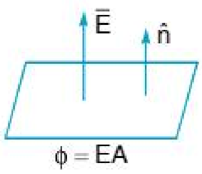


Fig. 2.3(b)

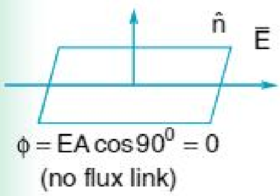


Fig. 2.3(c)

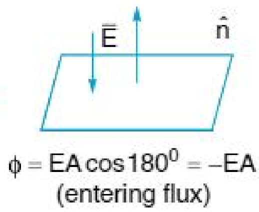


Fig. 2.3(d)

2.3 SOLID ANGLE

Solid angle is the three dimensional angle subtended by the lateral surface of a cone at its vertex

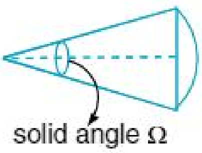


Fig 2.4(a)

Let us calculate the solid angle subtended by a surface X at a point O. Join all the points of the periphery of the surface X to the point O by straight lines as shown. It gives a cone with vertex at O.

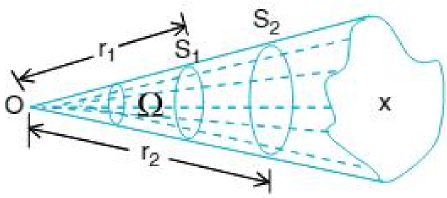


Fig 2.4(b)

By taking centre at O, we draw several spherical sections on this cone of different radii as shown. Let the area of spherical section which is of radius r_1 be s_1 and the area of section of radius r_2 be s_2. The ratio of area of any surface intersected by cone to the square of radius of that sphere is a constant and it gives actually the solid angle Ω From the figure, solid angle subtended by surface X at the point O is given by $\Omega = \frac{S_1}{r_1^2} = \frac{S_2}{r_2^2}$.

- SI unit of solid angle is steradian and it is a dimensionless quantity.
- One steradian is the solid angle subtended at the centre of sphere by the surface of the sphere having area equal to square of the radius of the sphere.

  The surface subtending solid angle need not be normal to axis of cone. For example consider a surface X of area $d\bar{s}$ as shown. The axis of cone formed by the surface at O is not normal to the surface. In this cone solid angle Ω subtended at point O can be given as $\Omega = \frac{ds\cos\theta}{r^2}$

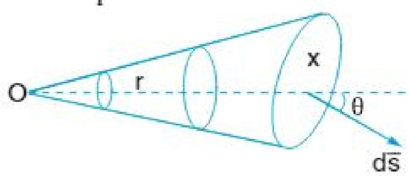


Fig 2.4(c)

Here θ is the angle between $d\bar{s}$ and axis of the cone.

2.4 RELATION BETWEEN SEMI-VERTEX ANGLE OF A CONE AND SOLID ANGLE SUBTENDED

Consider a spherical surface of radius R. Let X be a surface on that sphere which subtends a half angle θ (in radian) at the centre of the sphere. Now consider an elemental strip of this section of radius $r = R\sin\alpha$ and angular width $d\alpha$ as shown. Then surface area of this strip is given by

$$ds = (2\pi R\sin\alpha)Rd\alpha.$$

The total area of spherical section can be obtained by integrating this elemental area from 0 to θ.

Total area of spherical section is

$$S = \int ds = \int_0^{\theta} 2\pi R^2 \sin\alpha \, d\alpha$$

$$= 2\pi R^2 (-\cos\alpha)_0^{\theta} = 2\pi R^2 (1-\cos\theta)$$

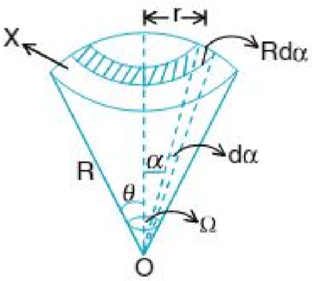


Fig 2.5

If Ω is solid angle subtended by this section at the centre O, then its area δ is given by $S = \Omega R^2$ (as discussed earlier) So, we can write

$$\Omega R^2 = 2\pi R^2 (1-\cos\theta) \text{ and } \Omega = 2\pi(1-\cos\theta)$$

- The solid angle subtended by a hemispherical surface at its centre is given by $\Omega = 2\pi(1-\cos 90^0) = 2\pi$ steradians

If $\theta = 180^0$ in the previous curve, we get the solid angle substended by a closed surface on

$$\Omega = 2\pi(1-\cos 180^0) = 4\pi \text{ steradians}$$

The total solid angle subtended by a closed surface is always 4π steradians, irrespective of the size and shape of the closed surface.

2.5 ELECTRIC FLUX DUE TO A CHARGE AND CHARGES

Case (i) Charge inside the closed surface

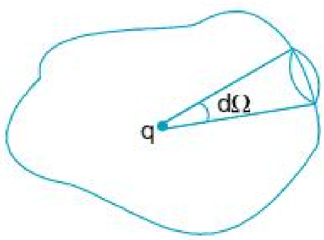


Fig 2.6(a)

Consider a point charge q inside a closed surface as shown. A small element of that closed surface subtends a solid angle dΩ at the position of the charge q. Now flux through that elemental surface is $d\phi = E\, ds \cos\theta$

i.e., $d\phi = \dfrac{q}{4\pi\epsilon_0}\dfrac{ds}{r^2}\cos 0^0 = \dfrac{q}{4\pi\epsilon_0}d\Omega$

(Refer to the explanation about solid angle)

So, total flux is $\phi = \int d\phi = \dfrac{q}{4\pi\epsilon_0}\int d\Omega$

But we know that $\int d\Omega = 4\pi$ in this case ie closed surface

$$\Rightarrow \phi = \left(\frac{q}{4\pi\epsilon_0}\right)4\pi = \frac{q}{\epsilon_0}$$

The total flux through a closed surface is always $\dfrac{1}{\epsilon_0}$ times charge enclosed irrespective of shape and size of the closed surface and position of charge.

Case(ii): Charge outside the closed surface

Consider a point charge q kept outside the closed surface as shown.

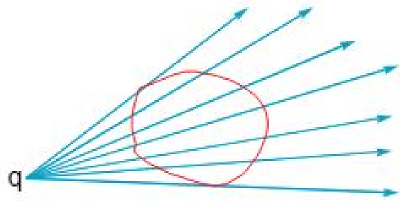


Fig 2.6(b)

The direction of arrow head gives the direction of field $\overline{E}$ due to q at each point on that surface. Here we can observe that total flux entering the surface (ϕ_{in}) and the total flux emerging out from the surface (ϕ_{out}) are the same. We know that ϕ_{in} is negative and ϕ_{out} is positive by convention. So total flux associated with the closed surface of any shape due to charge outside the surface is zero as

$$\phi_{total} = \phi_{in} + \phi_{out} = -\phi + \phi = 0$$

- If negative charge is enclosed by a closed surface then the total flux through that surface is $\phi = \frac{-q}{\epsilon_0}$, i.e., the flux is entering into that surface. But if the same negative charge is outside a closed surface then total flux is again zero.

Case (iii) : Charges inside and outside a closed surface

Consider a system of point changes as shown.

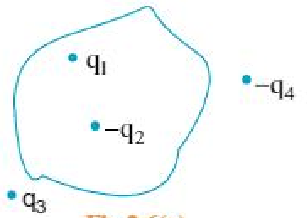


Fig 2.6(c)

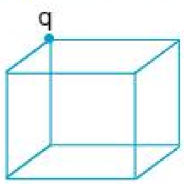
In this case flux associated with charges inside the closed surface is $\phi_1 = \frac{q_1}{\epsilon_0} + \frac{-q_2}{\epsilon_0}$.

Flux associated due to charges outside the closed surface is $\phi_2 = 0$

$$\Rightarrow \text{Total flux } = \phi_1 + \phi_2 = \frac{(q_1 - q_2)}{\epsilon_0}$$

Example-2.1

A point charge q is placed at one corner of a cubical box. Find the total flux associated with that box.



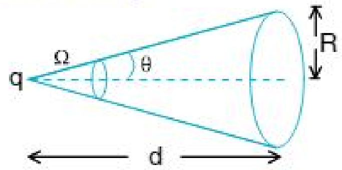
Solution :

If q is placed as shown, seven more identical cubes are required to enclose, that charge completely. Then the total flux through all faces of 8 cubes is $\frac{q}{\epsilon_0}$. Now contribution of this flux through each cube (including the given one) is $\frac{q}{8\epsilon_0}$.

Here it is interesting to note that the faces which have common edge on which the given charge is kept have no flux and the flux $\frac{q}{8\epsilon_0}$ emerges through the remaining three faces of the cube.

Example-2.2

A point charge q is placed at a distance d from the centre of a circular disc of radius R. Find electric flux through the disc due to that charge



Solution :

We know that total flux originated from a point charge q in all directions is $\frac{q}{\epsilon_0}$. This flux is originated in a solid angle 4π. In the given case solid angle subtended by the cone formed by the disc at the point charge is

$\Omega = 2\pi(1 - \cos\theta)$

So, the flux of q which is passing through the surface of the disc is

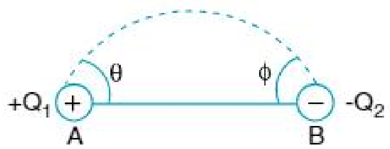
$$\phi = \frac{q}{\epsilon_0}\frac{\Omega}{4\pi} = \frac{q}{2\epsilon_0}(1 - \cos\theta)$$

From the figure, $\cos\theta = \frac{d}{\sqrt{d^2 + R^2}}$

$$\text{so } \phi = \frac{q}{2\epsilon_0}\left\{1 - \frac{d}{\sqrt{d^2 + R^2}}\right\}$$

Example-2.3

Two point charges $+Q_1$ and $-Q_2$ are placed at A and B respectively. A line of force emanates from Q_1 at an angle θ with the line joining A and B. At what angle will it terminate at B ?



Solution :

We know that number of emerging lines of force is proportional to magnitude of the charge. The field lines emanating from Q_1 are spread out equally in all directions. The number of field lines or flux through cone of half angle θ is $\frac{Q_1}{4\pi}2\pi(1 - \cos\theta)$.

Similarly the number of lines of force terminating on $-Q_2$ at an angle ϕ is $\frac{Q_2}{4\pi}2\pi(1 - \cos\phi)$.

The total lines of force emanating from Q_1 is equal to the total lines of force terminating on Q_2

$$\Rightarrow \frac{Q_1}{4\pi}2\pi(1-\cos\theta) = \frac{Q_2}{4\pi}2\pi(1-\cos\phi)$$

or $\frac{Q_1}{2}(1-\cos\theta) = \frac{Q_2}{2}(1-\cos\phi)$

$$Q_1 \sin^2\theta/2 = Q_2 \sin^2\phi/2$$

$$\sin\phi/2 = \sqrt{\frac{Q_1}{Q_2}}\sin\theta/2$$

$$\Rightarrow \phi = 2\sin^{-1}\left\{\sqrt{\frac{Q_1}{Q_2}}\sin\theta/2\right\}$$

2.6 GAUSS'S LAW

In electrostatics Gauss's law is a powerful tool which is useful in simplifying electric field calculations where there is symmetry in charge distribution. This law can be used to find total flux associated with a closed surface. It also gives us insight into how electric charge distributed itself over conducting bodies. The statement of Gassuss's law is as given below

"The total electric flux through a closed surface is equal to the net charge enclosed by the surface divided by ϵ_0".

Here ϵ_0 is permitivity of free space. Mathematically the Gauss's law or Gauss's theorem can be stated as $\oint_s \bar{E}.d\bar{s} = \frac{q}{\epsilon_0}$.

Here q is the total charge enclosed by the surface s and $\oint_s$ denotes that the surface should be closed, enclosing entire volume inside which the total charge is q. Here the hypothetical surface assumed is known as Gaussian surface

2.7 PROOF OF GAUSS'S LAW

Consider charges q_1, q_2, q_3,........q_n inside a closed surface and charges Q_1, Q_2,......Q_n outside that surface.

Consider a point P on the surface. Let $\bar{E}_1, \bar{E}_2, ...\bar{E}_n$ be the fields produced by q_1, q_2, q_3,.......q_n at P and $\bar{E}_1^{'}, \bar{E}_2^{'}, ...\bar{E}_n^{'}$ be the fields produced by the charges Q_1, Q_2, ...Q_n at P.

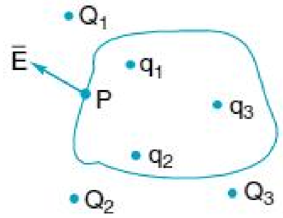


Fig 2.7

Now the resultant electric field at P is given by $\bar{E} = \left(\bar{E}_1 + \bar{E}_2 + ... + \bar{E}_n\right) + \left(\bar{E}_1^{'} + \bar{E}_2^{'} + ... + \bar{E}_n^{'}\right)$

The flux of resultant electric field through the closed surface is

$$\phi = \oint \bar{E}.d\bar{s} = \left\{\oint \bar{E}_1.d\bar{s} + \oint \bar{E}_2.d\bar{s} + ... + \oint \bar{E}_n.d\bar{s}\right\}$$
$$+\left\{\oint \bar{E}_1^{'}.d\bar{s} + \oint \bar{E}_2^{'}.d\bar{s} + ... + \oint \bar{E}_n^{'}.d\bar{s}\right\}$$

Here $\oint \bar{E}_1.d\bar{s}$ is the flux due to q_1 which is q_1/ϵ_0 and $\oint \bar{E}_1^{'}.d\bar{s}$ due to Q_1 is zero. As it is not enclosed by the Gaussian surface.

Similarly we can get the flux due to the other charges. Now we can write

$$\oint \bar{E}.d\bar{s}$$
$$= \left\{\frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + ... + \frac{q_n}{\epsilon_0}\right\} + \{0 + 0.....\}$$
$$= \left(\frac{q_1 + q_2 + ... + q_n}{\epsilon_0}\right)$$
$$\Rightarrow \oint \bar{E}.d\bar{s} = \frac{\sum q_{enclosed}}{\epsilon_0}$$

- Here $\sum q_{enclosed}$ is the sum of all enclosed charges which can be positive, negative or zero.
- The electric field in this case is net field due to all charges present inside as well as outside the closed surface.
- By using coulomb's law and the principle of superposition, we can find electric field strength at a given point. But in case of more complex configuration of the charge, Gauss's law is useful to calculate field or flux.

2.8 COULOMB'S LAW FROM GAUSS'S LAW

Consider a point charge q_1 at O. Let us construct a Gaussian surface in the form of a closed sphere, having its centre at O and with a radius OP = r. Here P is a point on the surface of that sphere.

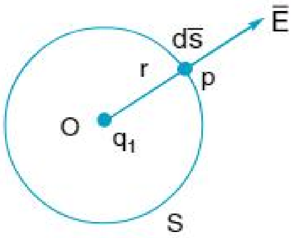


Fig 2.8

The electric field strength at P due to charge q_1 will be $\bar{E}$ which will be radially outwards. Let us consider a small area element ds at P on the surface of the sphere. $d\bar{s}$ will be along outward normal to the surface and is parallel to $\bar{E}$. So we have $\bar{E}.d\bar{s} = Eds\cos 0 = Eds$. This condition is applicable at every point on the surface of the sphere.

Now $\oint_s \bar{E}.d\bar{s} = \oint E.ds = Es = E4\pi r^2$

But from Gauss's law $\oint_s \bar{E}.d\bar{s} = \frac{q_{enclosed}}{\epsilon_0}$

$\Rightarrow E4\pi r^2 = \frac{q_1}{\epsilon_0}$ or $E = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}$

This is the field intensity due to point charge q_1 at a distance r from that charge.

If we keep a charge q_2 at P, the force acting between q_1 and q_2 is $F = Eq_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$ which is Coulomb's law in electrostatics.

2.9 IMPORTANT POINTS REGARDING GAUSS'S LAW

a) The surface to which Gauss's law is applied need not be a real physical surface. In most applications of this law, the imaginary surface considered may be in empty space or embedded in a solid body or partly in space and partly within a body.

b) Gauss's law is true for any closed surface irrespective of its shape or size.

c) Gauss's law is often useful for easier calculation of the electrostatic field when the system has some symmetry.

d) Gauss's law is based on the inverse square dependence on distance contained in the Coulomb's law.

e) While applying Gauss's law one can choose any Gaussian surface. But let the Gaussian surface should not pass through any discrete charge. The reason is electric field due to a system of discrete charges is not well defined at the location of every charge whereas the Gaussian surface can pass through a continuous charge distribution.

f) The electric field at every point on the Gausian surface is either perpendicular or tangential.

g) Magnitude of electric field at every point where it is perpendicular to the surface has constant value.

h) If coulomb's law involves $1/r^3$ dependence instead of $1/r^2$, Gauss's law will not be true in the form mentioned.

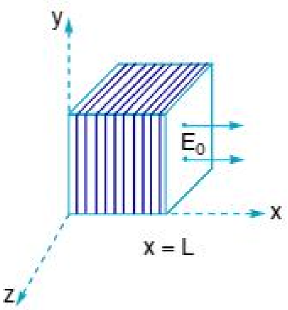
Example-2.4

The electric field in a region is given by $\vec{E} = E_0\frac{x}{L}\hat{i}$. Find the charge contained inside a cubical volume bounded by the surface x = 0, x = L, y = 0, y = L, z = 0 and z = L.

Solution :

At x = 0, E = 0 and at x = L, $\vec{E} = E_0\hat{i}$

The direction of the field is along the X-axis, so it will cross the yz-face of the cube. The flux of this field

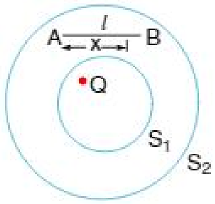


$\phi = \phi_{left\ face} + \phi_{right\ face} = 0 + E_0L^2 = E_0L^2$

By Gauss's law, $\phi = \frac{q}{\epsilon_0}$ $\quad \therefore q = \epsilon_0\phi = \epsilon_0 E_0L^2$

Example-2.5

Calclate the total flux of the electrostatic field through the spheres S_1 and S_2. The wire AB shown here has a linear charge density $\lambda = kx$, where x is the distance measured along the wire from end A.



Solution :

Charge on a small element of length dx of the wire is $\lambda dx = kxdx$

Total charge on the wire $= \int_0^l kxdx = \frac{kl^2}{2}$

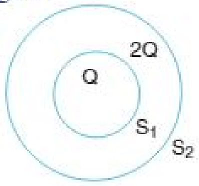
Flux through $S_1 = \frac{Q}{\epsilon_0}$

Flux through $S_2 = \frac{Q + kl^2/2}{\epsilon_0}$

Example-2.6

S_1 and S_2 are two hollow concentric spheres enclosing charges Q and 2Q respectively. (a) What is the ratio of electric flux through S_1 and S_2 (b) Instend of air if a medium of dielectric construct 5 is introduced inside S_1, how will the electric flux through it?

Solution :



(a) Flux through $S_1 = \frac{Q}{\epsilon_0}$;

Flux through $S_2 = \frac{Q+2Q}{\epsilon_0} = \frac{3Q}{\epsilon_0}$

Ratio of the flux is $= \frac{1}{3}$

(b) $E_m = E_{air}/5$

New flux through $S_1 = \oint \bar{E}.d\bar{s} = \frac{1}{5}\int \bar{E}_{air}.d\bar{s}$

$$= \frac{1}{5}\frac{Q}{\epsilon_0} = \frac{Q}{5\epsilon_0}$$

2.10 APPLICATIONS OF GAUSS'S LAW

a) Electric field due to a point charge

Consider a point charge q at point O, Consider a spherical Gaussian surface of radius r around the charge with O as the centre.

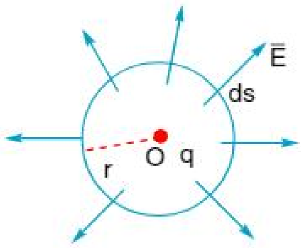


Fig 2.9 (a)

At every point on this sphere the electric field E has same magnitude and everywhere it is radial. If we consider an elemental area ds on the sphere,

$\bar{E}.d\bar{s} = Eds\cos 0^0 = Eds$

From Gauss's law $\oint_s \bar{E}.d\bar{s} = \frac{q_{enclosed}}{\epsilon_0}$

$$\oint_s \bar{E}.d\bar{s} = E4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

b) Electric field due to a linear charge distribution

Consider an infinitely long thin straight wire with uniform linear charge density λ.

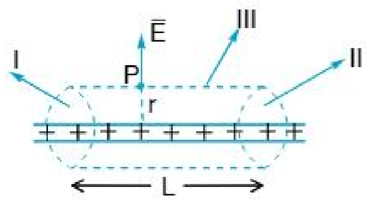


Fig 2.9 (b)

We have to calculate the electric field at a point P which is at a distance r from the line charge. Consider a Gaussian surface in the form of a cylinder of length L and radius r such that point P is on the curved surface. Here electric field E will be everywhere radial. The flat surfaces I and II have their area vectors perpendicular to $\bar{E}$. So $\bar{E}.d\bar{s}$ will be zero through I and II. Along the curved surface III, field $\bar{E}$ is radial and $d\bar{s}$ is also radial everywhere so $\bar{E}.d\bar{s} = Eds$ for III . So

$$\oint_s \bar{E}.d\bar{s} = \oint_I \bar{E}.d\bar{s} + \oint_{II} \bar{E}.d\bar{s} + \oint_{III} \bar{E}.d\bar{s}$$

$$= \oint_{III} Eds = E2\pi rL \quad \left(as \int ds = 2\pi rL\right)$$

From Gauss's law

$$\oint_s \bar{E}.d\bar{s} = \frac{q_{encl}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0} \qquad (as\, q = \lambda L)$$

$$\text{or } E2\pi rL = \frac{\lambda L}{\epsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r} \text{ and } \bar{E} = \left(\frac{\lambda}{2\pi\epsilon_0 r^2}\right)\hat{r}$$

If λ is positive $\bar{E}$ is radially outwards and if λ is negative $\bar{E}$ is radially inwards.

We know that $V(r) = -\int \bar{E}.d\bar{r}$

Here $E = \frac{\lambda}{2\pi\epsilon_0 r}$ and $\bar{E}.d\bar{r} = Edr$

$$\text{So } V(r) = -\int Edr = -\int \frac{\lambda}{2\pi\epsilon_0 r} dr$$

$$= \left(\frac{-\lambda}{2\pi\epsilon_0} \log_e r\right) + c$$

where C is constant of integration V(r) gives electric potential at a distance r from the line of charge.

c) Electric field due to infinite nonconducting plane sheet of charge

Consider an infinite plane sheet of charge with uniform surface charge density σ. Let us find the electric field $\bar{E}$ at a distance r in front of that sheet. By symmetry the field must have the same magnitude and opposite directions at two points equidistant from the sheet on opposite sides of it. The field lines are directed away from the sheet and perpendicular to it.

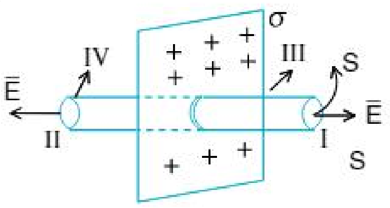


Fig 2.9 (c)

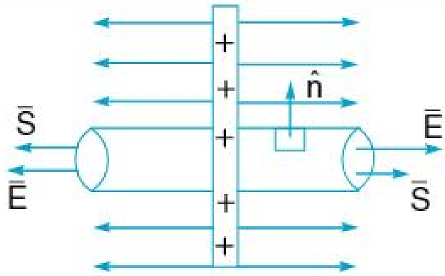


Fig 2.9 (d)

Let us consider a Gaussian surface in the form of a closed cylinder with end caps of area S arranged to penetrate the sheet perpendicular as shown.

$$\text{Now } \oint_s \bar{E}.d\bar{s} = \oint_I \bar{E}.d\bar{s} + \oint_{II} \bar{E}.d\bar{s} + \oint_{III} \bar{E}.d\bar{s} + \oint_{IV} \bar{E}.d\bar{s}$$

But at surfaces III and IV, $\bar{E}$ is perpendicular to the area vector. So $\oint_{III} \bar{E}.d\bar{s} = 0$ and $\oint_{IV} \bar{E}.d\bar{s} = 0$

$$\Rightarrow \oint_s \bar{E}.d\bar{s} = \oint_I \bar{E}.d\bar{s} + \oint_{II} \bar{E}.d\bar{s}$$

$$= E(S + S) = 2ES$$

where S is area of each flat surface of the cylinder. From Gauss's law $\oint_s \bar{E}.d\bar{s} = \frac{q_{enclosed}}{\epsilon_0}$

$$\Rightarrow 2ES = \frac{\sigma S}{\epsilon_0} \text{ or } E = \frac{\sigma}{2\epsilon_0}$$

- The magnitude of electric field is independent of the distance from the sheet. This is true as long as the sheet is large when compared to the distance of the point from the sheet.
- The above result holds good even for finite sheet of charge when the point is not nearer to the edge and the distance of the point from the sheet is small compared of the dimensions of the sheet.
- If σ is negative in the above case, $\bar{E}$ will be along the inward normal
- Electrostatic potential due to an infinite plane sheet of charge at a perpendicular distance r from the sheet is given by

$$V(r) = -\int \bar{E}.d\bar{r} = -\int Edr$$

As $E = \frac{\sigma}{2\epsilon_0}$, we can write

$$V(r) = -\int \frac{\sigma}{2\epsilon_0} dr = \left(\frac{-\sigma}{2\epsilon_0} r\right) + C$$

where C is constant of integration

- The magnitude of electric field of an infinite plane sheet of charge is independent of distance r from the sheet whereas $E \propto \frac{1}{r^2}$ in the case of point charge. The reason is charge is not localised at a point but distributed on a sheet.

d) Electric field due to infinite conducting sheet or near a charged conducting surface

Consider an infinite conducting sheet as shown. When charge is given to it, it distributes itself over the outer surface of the sheet.

For a thin sheet, the charge distributes on both of its faces. So, conducting sheet is equivalent to the combination of two non-conducting sheets, with the same charge density σ.

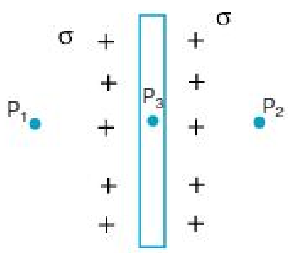


Fig 2.9 (e)

The electric field at any points is the superposition of the fields of two conducting charged sheets.

Now resultant field at P_1 is

$$E_1 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Now resultant field at P_2 is

$$E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Now resultant field at P_3 is

$$E_3 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

so, $E_1 = E_2 = \frac{\sigma}{\epsilon_0}$.

Similarly, we can show that electric field near a charged conducting surface is $\frac{\sigma}{\epsilon_0}$ and is normal to the surface as explained below.

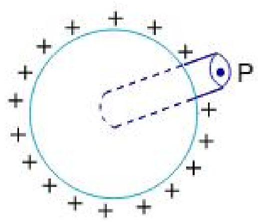


Fig 2.9 (f)

Consider a point P near a charged conducting surface. Let us consider a Gaussian surface in the form of a pill box which partially lies inside the conductor. As the field is normal to the surface at P, $\oint_s \overline{E}.d\overline{s} = Eds$ where ds is the elemental surface of the flat cap at P. Here $\overline{E}.d\overline{s} = 0$ for the curved surface and flat surface inside the conductor. Now from Gauss's law, $\oint \overline{E}.d\overline{s} = \frac{q_{encl}}{\epsilon_0}$

$$\Rightarrow Eds = \frac{\sigma ds}{\epsilon_0} \quad \text{or} \quad E = \frac{\sigma}{\epsilon_0}$$

e) Electric field due to a charged spherical shell (conducting)

Consider a uniformly charged thin spherical shell of radius R and total charge q on it. The charge spreads uniformly on its outer surface and so charge inside the shell will be zero.

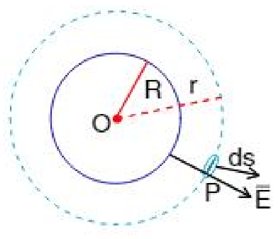


Fig 2.9 (g)

The electric field at any point on the shell or outside the shell will be radial (outwards for positive charge and inwards for negative charge)

i) For the point P at a distance r from the centre of the shell (r > R), consider a Gaussian surface with point P on it. This surface is concentric with the charged spherical shell. At all points on the Gaussian surface, E is same in magnitude and radial so $\overline{E}.d\overline{s} = Eds$ everywhere

$$\oint_S \overline{E}.d\overline{s} = \oint_S Eds = E4\pi r^2$$

From Gauss's law $\oint_S \overline{E}.d\overline{s} = \frac{q}{\epsilon_0}$

$$\Rightarrow E4\pi r^2 = \frac{q}{\epsilon_0} \text{ or } E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

If σ is surface charge density, $q = 4\pi R^2\sigma$

$$\text{so } \vec{E} = \frac{\sigma}{\epsilon_0}\left(\frac{R^2}{r^3}\right)\vec{r}$$

- For points outside a uniformly charged spherical shell, it behaves as if the entire charge on it were concentrated at the centre of the shell.

Now potential at any point outside the shell is

$$V(r) = -\int \overline{E}.d\overline{r} = -\int Edr$$

$$= -\int \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr = \frac{1}{4\pi\epsilon_0}\frac{q}{r} + C$$

where C is constant of integration

If $r \to \infty$, $V(\infty) \Rightarrow 0$ and $C = 0$

$$\text{Now } V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad (r > R)$$

ii) If the point P lies on the surface of the charged shell, we take r = R and we get $E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$ and $q = 4\pi R^2\sigma$

$$\text{So } E = \frac{\sigma}{\epsilon_0} \quad \text{and} \quad \overline{E} = \frac{\sigma}{\epsilon_0}\hat{n}$$

Now electrostatic potential at P such that r = R is given by $V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

iii) If the point P is inside the charged spherical shell, we consider a concentric Gaussian surface with radius r < R as shown. We have

$$\oint_S \overline{E}.d\overline{s} = \oint_S Eds = E4\pi r^2$$

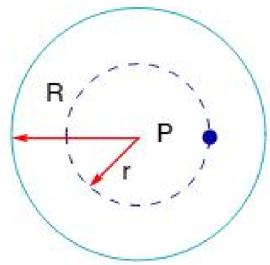


Fig 2.9 (h)

As P is inside the shell, there is no charge enclosed by the Gaussian surface as charge resides only on the outer surface of shell.

From Gauss's law $\oint_S \overline{E}.d\overline{s} = \frac{q_{emf}}{\epsilon_0} = 0$

$\Rightarrow E = 0$ inside the charged shell.

We know that $E = -\frac{dV}{dr}$

Here for r < R in the case of charged spherical shell, E = 0 and so we can write $-\frac{dV}{dr} = 0$

$\Rightarrow$ V is constant.

But on the surface (r = R), $V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

which is constant for $r \leq R$

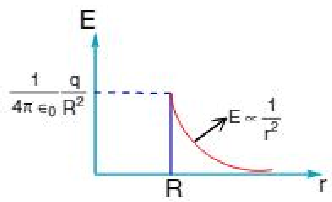


Fig 2.9 (i)

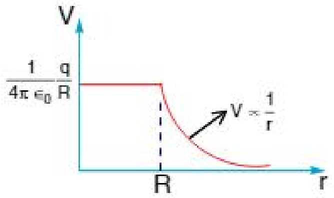


Fig 2.9 (j)

f) Electric field and potential due to a uniformly charged non-conducting solid sphere

Consider a charged sphere of radius R with total charge q uniformly distributed on it. Here volume charge density $\rho = \frac{q}{V}$ where V is $\frac{4}{3}\pi R^3$. If we consider a Gaussian surface which is concentric sphere (around the charged sphere) with radius $r > R$, from Gauss's law $\oint_S \bar{E}.d\bar{s} = \frac{q}{\epsilon_0}$

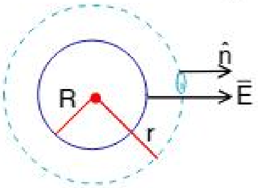


Fig 2.9 (k)

where $\oint_S \bar{E}.d\bar{s} = E4\pi r^2$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \text{ for } r > R$$

and $\bar{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^3}\right)\bar{r}$.

Let us consider a concentric Gaussian surface of radius $r < R$. Here also $\bar{E}$ will be radial everywhere but charge enclosed by the Gaussian surface is

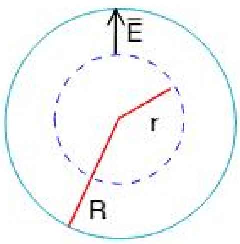


Fig 2.9 (l)

$$q_{en} = \frac{q}{V}\times\frac{4}{3}\pi r^3 = \frac{qr^3}{R^3}$$

From Gauss's law $\oint_S \bar{E}.d\bar{s} = \frac{q_{en}}{\epsilon_0}$

$$\Rightarrow E4\pi r^2 = \frac{qr^3}{\epsilon_0 R^3}$$

and $E = \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3}$

But $q = \rho\frac{4}{3}\pi R^3$

$$\Rightarrow E = \frac{\rho r}{3\epsilon_0} \quad \text{and} \quad \bar{E} = \left(\frac{\rho}{3\epsilon_0}\right)\bar{r}$$

At the centre of the sphere $r = 0 \Rightarrow E = 0$

On the surface of the sphere $r = R$ and $E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$

- E is a continuous function of r in the case of uniform spherical charge distribution. In the case of charged conducting sphere, it is discontinuous at $r = R$ as it jumps from $E = 0$ to $E = \frac{\sigma}{\epsilon_0}$

We can find the electrostatic potential due to spherical charge distribution as given below. At any point for $r > R$, we have $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

Now we use $E = \frac{-dV}{dr}$

or $dV = -\bar{E}.d\bar{r} = -Edr$

$$\int_\infty^V dV = -\int_\infty^V \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr$$

$$\Rightarrow V - 0 = \frac{1}{4\pi\epsilon_0}\frac{q}{r}\Big]_\infty^r$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad (\text{as } V = 0 \text{ at } r = \infty)$$

At $r = R$; $V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

i.e., at the surface $V = V_S = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

Now let us take a point at r < R for which

$$E = \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3}$$

again $dV = -\overline{E}.d\overline{r} = -Edr$

$$\int_{V_s}^{V} dV = -\int_R^r E\,dr = -\int_R^r \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3}\,dr$$

$$V - V_s = -\frac{1}{4\pi\epsilon_0}\frac{q}{R^3}\left(\frac{r^2}{2}\right)_R^r$$

$$V - \frac{1}{4\pi\epsilon_0}\frac{q}{R} = -\frac{1}{4\pi\epsilon_0}\frac{q}{R^3}\left[\frac{r^2}{2}-\frac{R^2}{2}\right]$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}\times\left[\frac{3}{2}-\frac{r^2}{2R^2}\right]$$

At the centre r = 0 and

$$V_C = \frac{1}{4\pi\epsilon_0}\frac{3q}{2R} = \frac{3}{2}V_s$$

So in the case of a uniformly charged spherical distribution,

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \text{ and } V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad (\text{for } r > R)$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2} \text{ and } V = \frac{1}{4\pi\epsilon_0}\frac{q}{R} \quad (\text{for } r = R)$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3} \text{ and}$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}\left(\frac{3}{2}-\frac{r^2}{2R^2}\right)(\text{for } r < R)$$

$$E = 0 \text{ and } V = \frac{3}{2}\frac{1}{4\pi\epsilon_0}\frac{q}{R}(\text{for } r = 0)$$

The variation of E and V with r is as shown in the graph

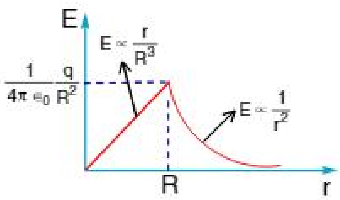


Fig 2.9 (m)

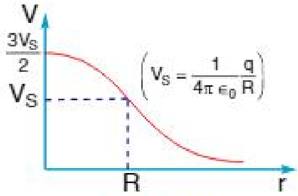


Fig 2.9 (n)

g) Electric field strength due to long uniformly charged cylinder

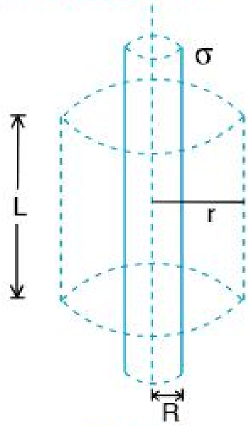


Fig 2.10

Consider a long cylinder of radius R which is uniformly charged with surface charge density σ. We know that at the interior points of a metal body electric field strength is zero. Let us find the electric field at a distance r from the axis of the cylinder. Consider a cylindrical Gaussian surface of radius r and length L as shown in the figure.

From Gauss's law, we can write

$$\oint \overline{E}.d\overline{s} = \frac{1}{\epsilon_0}(q_{en})$$

Here $q_{enclosed} = \sigma 2\pi RL$

Here electric flux through the circular faces is zero. So, from Gauss's law

$$\oint \overline{E}.d\overline{s} = \frac{\sigma 2\pi RL}{\epsilon_0} \text{ or } E2\pi rL = \frac{\sigma 2\pi RL}{\epsilon_0}$$

$$\Rightarrow E = \frac{\sigma R}{2\epsilon_0 r}$$

h) Field due to uniformly charged non-conducting cylinder

Consider a long cylinder of radius R charged with volume charge density ρ uniformly. Let us find electric field at a distance r from the axis of the cylinder. Consider a cylindrical Gaussian surface of length L and radius r as shown,

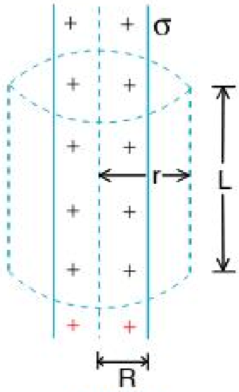


Fig 2.11

$$\oint \bar{E}.d\bar{s} = \frac{q_{encl}}{\epsilon_0}; \text{ where } q_{encl} = \rho\pi R^2 L$$

Here electric flux through the circular faces is zero. So, from Gauss's law

$$\oint \bar{E}.d\bar{s} = \frac{\rho\pi R^2 L}{\epsilon_0}$$

$$\Rightarrow E2\pi rL = \frac{\rho\pi R^2 L}{\epsilon_0} \text{ or } E = \frac{\rho R^2}{2\epsilon_0 r}$$

If $r < R$, $q_{encl} = \rho\pi r^2 L$

from Gauss's law

$$\oint \bar{E}.d\bar{s} = \frac{q_{encl}}{\epsilon_0}$$

$$E2\pi rL = \frac{\rho\pi r^2 L}{\epsilon_0}; \ E = \frac{\rho r}{2\epsilon_0}$$

✤ In vector form $\bar{E} = \frac{\rho\vec{r}}{2\epsilon_0}$

2.11 MECHANICAL FORCE ON THE CHARGED CONDUCTOR

We know that like charges repel each other. So, when a conductor is charged, the charge at any point of the conductor is repelled by the charge on its remaining part. It means surface of a charged conductor experiences mechanical force.

Consider a charged conductor as shown. Let ds be the surface area of a small element on the conductor. The electric field at point P_1 near the conductor surface can be considered as the superposition of fields $\bar{E}_1$ and $\bar{E}_2$. Here $\bar{E}_1$ is the field produced by that elemental surface and $\bar{E}_2$ is the field due to the remaining surface of the conductor.

$$\bar{E} = \bar{E}_1 + \bar{E}_2$$

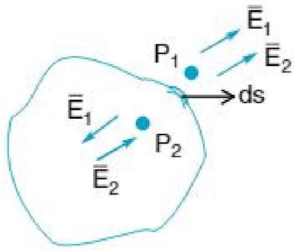


Fig 2.12

But we know that $E = \frac{\sigma}{\epsilon_0}$ at P_1 which is just outside the conductor and is zero at P_2 which is just inside the conductor

So at P_1 we have $E_1 + E_2 = \frac{\sigma}{\epsilon_0}$

and at P_2 we have $E_1 - E_2 = 0$

$$\Rightarrow E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$$

Now the force experienced by small surface ds due to the charge on the rest of the surface is

$$F = (dq)E_2 = (\sigma ds)E_2 = \frac{\sigma^2 ds}{2\epsilon_0}$$

$$\text{and } \frac{\text{Force}}{\text{Area}} = \frac{F}{ds} = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2}\epsilon_0 E^2$$

2.12 ELECTRIC PRESSURE ON A CHARGED SURFACE

From the above derivation we observed that a small surface of a charged conductor will experience a force by the remaining surface. The force per unit area of the surface is $\frac{1}{2}\epsilon_0 E^2$ or $\frac{\sigma^2}{2\epsilon_0}$

This is known as electric pressure on the charged metal surface.

$$\Rightarrow P_e = \frac{1}{2}\epsilon_0 E^2$$

Suppose a charged body is in an external electric field. Let us find out the electric pressure on the surface of that charged body.

Consider a surface uniformly charged with charge density σ. On that surface 'ds' is the surface area of a small element. The charge on that element is $dq = \sigma ds$

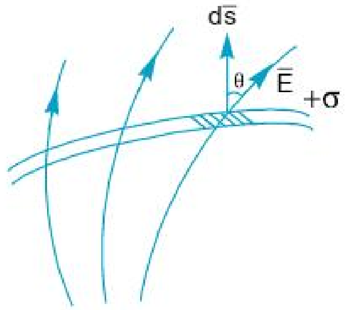


Fig 2.13

The given surface is in an external electric field represented by the field lines as shown.

Let E be the intensity of electric field on the elemental surface. Here angle between $\bar{E}$ and $d\bar{s}$ is θ. In this case $\bar{E}$ has two components.

Component parallel to the surface is

$E_{\parallel} = E \sin\theta$

and component normal to the surface is

$E_{\perp} = E \cos\theta$

Here force due to $E_{\parallel}$ on the surface is tangential which tries to stretch the surface. Where as the force due to $E_{\perp}$ applies outward pressure on the surface. Now outward force on the elemental surface is

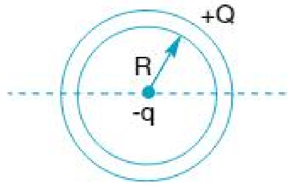
$$dF = (dq)E_{\perp} = \sigma\, ds\, E_{\perp}$$

So, the outward electric pressure on the surface is

$$P_e = \frac{dF}{ds} = \sigma E_{\perp} \Rightarrow P_e = \sigma E \cos\theta$$

Example-2.7

A thin spherical shell radius of r has a charge Q uniformly distributed on it. At the centre of the shell, a negative point charge -q is placed. If the shell is cut into two identical hemispheres, still equilibrium is maintained. Then find the relation between Q and q?



Solution :

Here the outward electric pressure at every point on the shell due to its own charge is

$$P_1 = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2\epsilon_0}\left(\frac{Q}{4\pi r^2}\right)^2$$

$$P_1 = \frac{Q^2}{32\pi^2 \epsilon_0 r^4}$$

Due to –q, the electric field on the surface of the shell is $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

This electric field pulls every point of the shell in inward direction. The inward pressure on the surface of the shell due to the negative charge is $P_2 = \sigma E$

$$= \left(\frac{Q}{4\pi r^2}\right)\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right) = \frac{Qq}{16\pi^2 \epsilon_0 r^4}$$

For equilibrium of the hemispherical shells $P_2 \geq P_1$

or $\frac{Qq}{16\pi^2 \epsilon_0 r^4} \geq \frac{Q^2}{32\pi^2 \epsilon_0 r^4}$

$$q \geq \frac{Q}{2}$$

2.13 ELECTROSTATIC FIELD ENERGY

Consider a charged particle of mass m and charge q placed in an electric field E. If that particle is released from rest, it starts moving due to the force applied by electric field on it.

As a result that particle gains some kinetic energy. Here the electric field is doing work to increase the kinetic energy of that particle. In other words the electric field has some energy which enables it to do that work.

So, whenever electric field exists, field energy also must exist. We can find energy density of the electric field as explained below.

Consider a charged conductor as shown. We know that electric field just outside the surface of that conductor at any point is given by $E = \frac{\sigma}{\epsilon_0}$

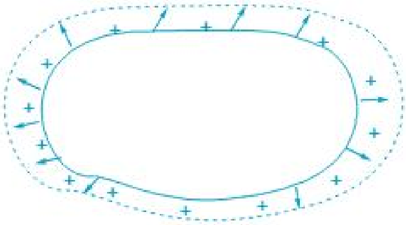


Fig 2.14

We have already proved that the charged body experiences an outward electric pressure given by $P = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2}\epsilon_0 E^2$

If we treat the surface of the charged body flexible, due to the outward pressure it expands as shown. Inside the body there will be no field. Earlier before expansion there was field in the region with volume dv which corresponds to increase in volume of the charged body.

During expansion the energy with in the volume dv vanished. What happened to this vanished field?.

The answer is very simple. We know that to expand the charged body work is done by electric forces in the body. This work done in increasing the volume is equal to loss of field energy in the volume dv.

Now we can write dW = PdV

The field energy stored in the volume dv is given by $dU = dW = PdV$ and $\frac{dU}{dV} = P$

$\frac{dU}{dV} = u$, which is known as field energy density in the electric field.

$$u = P = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2}\epsilon_0 E^2$$

If the electric field is uniform in a region, total field energy stored in a given volume V is given by $U = \frac{1}{2}\epsilon_0 E^2 V$

If the electric field is non uniform in a given region, total field energy stored is given by $\int dU$ where $dU = \frac{1}{2}\epsilon_0 E^2 dV$

2.14 SELF ENERGY

In the previous topic we have discussed about interaction potential energy of a system of point charges. Now let us discuss about self energy of a charged body. When a body is charged, all the charge on it must be brought from infinity (the reference taken by us) onto that body. In doing so, work has to be done against the electric field of that body. This work will be stored in that body in the form of potential energy which is known as self energy "Self energy of a charged body is the total field energy associated with the electric field due to this body in its surrounding". Let us consider two important cases for which we are going to find self energy.

a) Self energy of a charged conducting sphere

Consider a conducting sphere of radius R charged with charge Q on it. In the process of charging, we have to bring charge to the sphere from infinity in steps of elemental charge each dq. While bringing the elemental charge the field produced by the charge on the sphere already accumulated opposes the charge element. At an instant sphere has a charge 'q' on it. Due to this charge, potential of the sphere is $V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

As we brought charge dq to its surface from infinity, work done by the external agency is

$$dW = V\,dq = \frac{1}{4\pi\epsilon_0}\frac{q}{R}dq$$

Total work done in charging the sphere to have final charge Q on it is given by

$$W = \int dW = \int_0^R \frac{1}{4\pi\epsilon_0}\frac{q}{R}dq$$

$$\Rightarrow W = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{2R}$$

So self energy of charged conducting sphere is $U = \frac{Q^2}{8\pi\epsilon_0 R}$

We can find the self energy from energy density also as explained below.

We know that energy per unit volume in an electric field is $u = \frac{1}{2}\epsilon_0 E^2$.

When the sphere has no charge, there was no electric field in its surroundings. But when the sphere is charged, there exists an electric field in its surroundings from its surface to infinity.

Electric field due to the charged sphere at outer point is given by $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$.

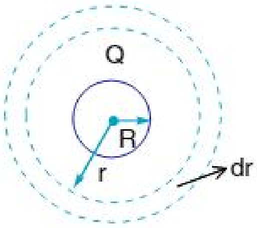


Fig 2.15

Consider an elemental spherical shell of radius r and width dr as shown. The volume enclosed in this shell is

$$dV = 4\pi r^2\, dr$$

The field energy stored in this volume is

$$dU = \frac{1}{2}\epsilon_0 E^2 dV = \frac{Q^2}{8\pi\epsilon_0 r^2} dV$$

Total field energy associated with the sphere can be calculated by integrating the above expression from $r = R$ to $r = \infty$ (no electric field inside the sphere). So, total field energy in the surrounding of the sphere is

$$U = \int dU = \int_R^\infty \frac{Q^2}{8\pi\epsilon_0 r^4} dV$$

$$\Rightarrow U = \frac{Q^2}{8\pi\epsilon_0 R}$$

b) Self energy of a uniformly charged non conducting sphere

Consider a non conducting sphere charged uniformly with a charge Q on it. We know that outside region of that sphere, every point is same as that of conducting sphere of radius R only. So, the field energy in the surroundings of this sphere from its surface to infinity can be given as

$$U_{out} = \frac{Q^2}{8\pi\epsilon_0 R}$$

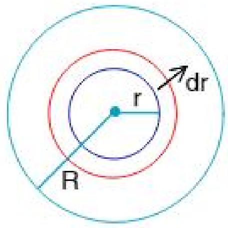


Fig 2.16

We know that at interior point of this sphere, $E \neq 0$. So, field energy exists in the interior region. Consider an elemental shell of radius r and thickness dr as shown.

Field energy in the volume of this shell is given as $dU_{in} = \frac{1}{2}\epsilon_0 E^2 dV$

$$= \frac{1}{2}\epsilon_0\left(\frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}\right)^2 4\pi r^2 dr = \frac{Q^2 r^4}{8\epsilon_0 R^6}\, dr$$

$$\left(\begin{array}{l} \because E = \frac{1}{4\pi\epsilon_0}\frac{Q\,r}{R^3} \quad \text{for} \quad r < R \\ \text{and} \quad dV = 4\pi r^2\, dr \end{array}\right)$$

Now total field energy inside the sphere will be given as

$$U_{in} = \int dU_{in} = \int_0^R \frac{Q^2 r^4}{8\epsilon_0 R^6} dr = \frac{Q^2}{40\pi\epsilon_0 R}$$

So, total self energy of this sphere is given by

$$U = U_{out} + U_{in} = \frac{3Q^2}{20\pi\epsilon_0 R}$$

- Total electrostatic energy of a system of charges is the sum of self energy of all charged bodies and interaction energy of all possible pairs of charged bodies.

$$U_{total} = \sum U_{self} + \sum U_{interaction}$$

- A point charge does not have any self energy. So for a pair of charges there will be interaction energy only.

  If q_1 and q_2 are two point charges separated by a distance r, interaction energy of that system is $U_i = q_1V_2 = q_2V_1$.

  Here V_2 is potential at q_1 due to charge q_2 and V_1 is potential at q_2 due to charge q_1.

Example-2.8

Two uniformly charged conducting spheres of radii R_1 and R_2 having charges Q_1 and Q_2 respectively are seperated by distance r. Find total electrostatic energy of this system?

Solution :

Here $U = U_{self} + U_{interaction}$

$$U_{self} = \frac{Q_1^2}{8\pi \epsilon_0 R_1} + \frac{Q_2^2}{8\pi \epsilon_0 R_2}$$

$$U_{interaction} = \frac{1}{4\pi \epsilon_0}\frac{Q_1Q_2}{r}$$

$$\Rightarrow U = \frac{1}{8\pi \epsilon_0}\left\{\frac{Q_1^2}{R_1} + \frac{Q_2^2}{R_2} + \frac{2Q_1Q_2}{r}\right\}$$

If the two spheres are non conducting, in this case,

$$U_{self} = \frac{3Q_1^2}{20\pi \epsilon_0 R_1} + \frac{3Q_2^2}{20\pi \epsilon_0 R_2}$$

$$\text{So, } U = \frac{1}{4\pi \epsilon_0}\left\{\frac{3Q_1^2}{5R_1} + \frac{3Q_2^2}{5R_2} + \frac{Q_1Q_2}{r}\right\}$$

Example-2.9

Two concentric shells of radii R_1 and R_2 are charged uniformly with charges Q_1 and Q_2 respectively. Find the total electrostatic energy of the system.

Solution :

$$U = U_{self} + U_{interaction}$$

$$U_{self} = \frac{Q_1^2}{8\pi \epsilon_0 R_1} + \frac{Q_2^2}{8\pi \epsilon_0 R_2}$$

$$U_{interaction} = \frac{1}{4\pi \epsilon_0}\frac{Q_1Q_2}{R_2}$$

$$\text{So, } U = \frac{1}{8\pi \epsilon_0}\left\{\frac{Q_1^2}{R_1} + \frac{Q_2^2}{R_2} + \frac{2Q_1Q_2}{R_2}\right\}$$

Example-2.10

A shell of radius R has a charge Q uniformly distributed over it. A point charge q is placed at the centre of the shell. Find the work done to increase the radius of the shell from R to 2R?

Solution :

$$U_i = (U_s + U_{int}) \text{ initially} = \frac{Q^2}{8\pi \epsilon_0 R} + \frac{1}{4\pi \epsilon_0}\frac{Qq}{R}$$

$$U_f = (U_s + U_{int}) \text{ finally} = \frac{Q^2}{16\pi \epsilon_0 R} + \frac{1}{8\pi \epsilon_0}\frac{Qq}{R}$$

Work done by the electric field = $U_i - U_f$

$$= \frac{Q}{8\pi \epsilon_0 R}\left(\frac{Q}{2} + q\right)$$

2.15 METAL CONDUCTORS IN ELECTRIC FIELD

When metal conductor is kept in an electric field, there will be momentary flow of free electrons. After this flow stops, as the conductor will be in electrostatic equilibrium. At such condition, conductor will have the following main properties.

a) "Net electric field inside the conductor is zero". Let us consider a metal block kept in an external uniform electric field $\overline{E}_0$. Due to this field each electron experiences force eE_0 in a direction opposite to $\overline{E}_0$. This makes the electrons move to one face at which there will be net negative charge. As a result on the opposite face there will be an equal positive charge. These charges are called induced charges (we have discussed about this in the previous topic). These induced charges establish an electric field $\overline{E}_i$ within the metal which opposes the external field $\overline{E}_0$. This applies a force on each free electron equal to eE_i in the direction opposite to $\overline{E}_i$.

At equilibrium there will be no movement of free electrons and $eE_0 = eE_i \Rightarrow E_0 = E_i$

But $\overline{E}_i$ and $\overline{E}_0$ are opposite in direction. So net electric field inside the metallic conductor is zero. $(\overline{E}_0 + \overline{E}_i = 0)$

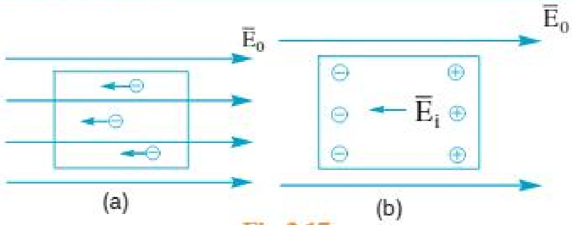


Fig 2.17

Instead of conductor, if

we put a dielectric slab in the field $\overline{E}_0$, net electric field in that slab will be $\overline{E} = \frac{\overline{E}_0}{K}$

$$\frac{\overline{E}_0}{K} = \overline{E}_0 + \overline{E}_i \Rightarrow \overline{E}_i = -\overline{E}_0\left(1-\frac{1}{K}\right)$$

So, induced field has magnitude

$E_0\left(1-\frac{1}{K}\right)$ and it opposes the external field.

b) "The magnitude of electric field just out side the charged conductor is σ/ϵ_0." where σ is surface charge density. This result is valid to conductor of any shape but preferably for large electric fields where the charge density on the conductor is high.

c) "The net charge inside a conductor is zero". We have studied about this in the previous topic according to which charge resides on the outer surface of the conductor. This is verified from Gauss's law.

When a conductor is charged positively or negatively, like charges repel each other. So, the charges try to get as far away from each other as they can. As a result, charges move to the surface of the conductor.

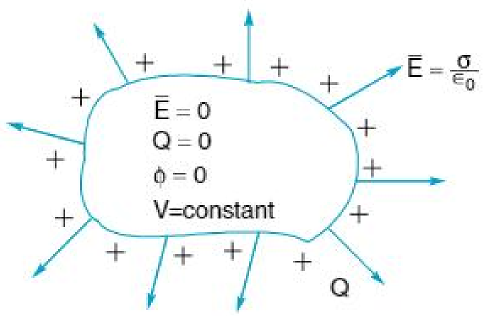


Fig 2.18

d) "The electric field on the surface or just outside the charged conductor is normal to the surface of the conductor at every point". This means that component of electric field along the tangent to the surface is zero.

e) "The electric potential at the surface and inside the charged conductor is the same or constant".

Inside a charged conductor $\overline{E} = 0$

$$E = \frac{-dv}{dr} = 0 \Rightarrow V = \text{constant}$$

So, conductor is an equipotential surface.

f) "Electric flux inside the charged conductor is zero". As charge enclosed is zero, flux inside the conductor is also zero.

2.16 CAVITY IN THE CONDUCTOR

We have discussed that there will be no electric field inside a charged conductor and all the charge resides on its outer surface only. Suppose that charged conductor has a cavity or cavities and there are no charges within the cavity or cavities, even then charge resides on the outer surface of the conductor. There will be no charge on the walls of the cavity or cavities. This can be verified very easily using Gauss's law by enclosing the cavity with a Gaussian surface.

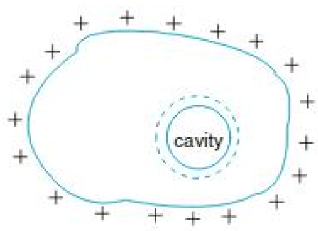


Fig 2.19

$$\oint \overline{E}.d\overline{s} = 0$$

For the dotted surface. $\Rightarrow q = 0$ inside cavity.

The field in a cavity inside a conductor is zero resulting in electrostatic shielding.

Consider a conductor with spherical cavity inside it. There is no charge on the conductor. Now a point charge +q is kept at the centre of the cavity. Due to this charge, a charge –q is induced on the inner surface of cavity. The total flux originated by +q will terminate on the cavity walls and no field lines enter into the conductor body.

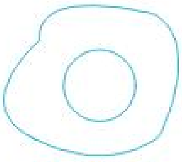
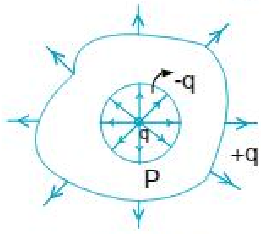


Fig 2.20(a)



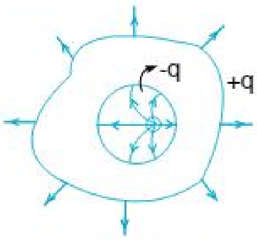


Fig 2.20(b) Fig 2.20(c)

We can consider a Gaussian surface around the cavity and prove that induced charge on the cavity walls is –q. The reason is $\overline{E} = 0$ inside the material of the conductor $\Rightarrow$ Total enclosed charge within the Gaussian surface = 0. Here the conductor is initially uncharged. From conservation of charge, we can say that on the outer surface of the conductor a charge +q will be induced. At any point inside the material of conductor, say at P, the electric field produced by +q in the cavity is cancelled by the field produced by charges induced on the walls of cavity and on the outer surface of the conductor. If the point charge is not at the centre of the spherical cavity, even then induced charges on the cavity walls and on the outer surface of the conductor are –q and +q only.

But the distribution of induced charges will change in such a way that at any point P in the material of the conductor resultant electric field is zero.

Suppose the conductor has charge q_0 on it initially. This charge resides on the outer surface of the conductor. If point charge q is kept inside the cavity, induced charges on the walls of cavity and on the outer surface of the conductor are the same as before. i.e., – q and + q. But the total charge on the outer surface of the conductor is $(q_0 + q)$ now.

If the charge inside the cavity is displaced, the induced charge distribution on inner surface of the body changes such that at any point inside the material of the conductor resultant field is zero. In this case the charge distribution on outer surface of the conductor does not change and only the charge distribution on the cavity walls will change.

Now the charge inside the cavity is fixed. If another charge is brought towards the conductor from outside, it will not affect the charge distribution inside the cavity and only the distribution of charge on the outer surface will be affected.

2.17 ELECTROSTATIC SHIELDING

When a conductor of any shape is charged, we know that electrostatic field is zero at all points inside that conductor. If that conductor has a cavity in it, with no charge inside the cavity, we know that electric field at all points inside that cavity is also zero. These conditions are valid irrespective of the size and shape of the cavity, charge on the conductor and the external fields in which the conductor was placed. From these observations, it is very clear that any cavity inside a conductor remains shielded from outside electric influence. This shielding effect is unaffected what ever may be charge and field configuration outside that conductor. Always field inside the cavity in a conductor is zero provided there are no charges in side it. This is known as electrostatic shielding.

2.18 PRINCIPLE OF GENERATOR

An instrument used for producing high voltages (million volt range) is known as generator. If a charged conductor is brought into contact with a hollow conductor, (former inside the latter) all of its charge transfers to the hollow conductor irrespective of the high potential of later.

Consider a spherical conductor 1 of radius r_1 holding charge q_1 uniformly distributed on it. It is kept inside a hollow conductor 2 of radius r_2 which is uncharged.

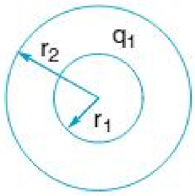


Fig 2.21(a)

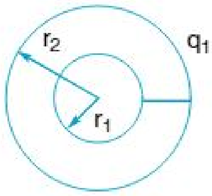


Fig 2.21(b)

Now electric potential of inner sphere is

$$V_1 = \frac{1}{4\pi\epsilon_0}\left\{\frac{q_1}{r_1}+\frac{0}{r_2}\right\} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1}$$

Electric potential of outer shell is

$$V_2 = \frac{1}{4\pi\epsilon_0}\left\{\frac{q_1}{r_2}+\frac{0}{r_2}\right\} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_2}$$

Now potential difference between the two conductors is $V_1 - V_2 = \frac{q_1}{4\pi\epsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$

From this we can conclude that potential difference in this case depends on q_1 only. It does not depend on any charge on the outer shell. Let us check the same as given below. If q_2 is the charge on the outer shell,

$$V_1 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right) \text{and } V_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2}{r_2}\right)$$

Now $V_1 - V_2 = \frac{q_1}{4\pi\epsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$

Here the potential difference $(V_1 - V_2)$ will remain the same for any value of q_2.

What happens if the two spheres are connected as shown in figure (b)? In this case charge flows from higher potential to lower potential. Ultimately all the charge resides on the surface of outer sphere. This is possible for all cases i.e., $q_1 > q_2$ or $q_2 > q_1$ and $V_1 > V_2$ or $V_2 > V_1$.

Example-2.11

Two spherical conductors of radii r_1 and r_2 have charges q_1 and q_2 on them. If they are connected by a metal wire, discuss their final charge distribution?

Solution :

We know that charge flows from higher potential to lower potential till the potentials of the two spheres are the same. Let q_1^1 and q_2^1 be the final charges on the spheres

$$\frac{1}{4\pi\epsilon_0}\frac{q_1^1}{r_1} = \frac{1}{4\pi\epsilon_0}\frac{q_2^1}{r_2}$$

Let σ_1 and σ_2 be the final charge densities on the two spheres.

Then we have $q_1^1 = \sigma_1 4\pi r_1^2$ and $q_2^1 = \sigma_2 4\pi r_2^2$

$$\sigma_1 r_1 = \sigma_2 r_2 \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$$

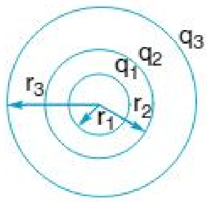
We can say $\sigma \propto \frac{1}{r}$

This explains that for charged metal bodies at same potential, the surface charge density is inversely proportional to the radius of curvature of the body.

So, when a metal body of irregular shape is charged, the charge resides on the outer surface such that surface charge density is more at sharp edges where radius of curvature is less.

Example-2.12

Three concentric spherical shells of radii r_1, r_2 and r_3 have charges q_1, q_2 and q_3 respectively. If the central sphere is now connected to the earth by a conducting wire, find the final potentials of the three shells



Solution :

When the middle shell is earthed, charge flows between the shell and the earth and its potential will be zero. Let q be the final charge on the middle shell.

$$V_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_2}+\frac{q}{r_2}+\frac{q_3}{r_3}\right) = 0$$

$$\frac{q_1}{r_2}+\frac{q}{r_2}+\frac{q_3}{r_3} = 0$$

$$\frac{q}{r_2} = -\left(\frac{q_1}{r_2}+\frac{q_3}{r_3}\right) \Rightarrow q = -\left(q_1+\frac{q_3 r_2}{r_3}\right)$$

Final potential of innermost shell is

$$V_1 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1}+\frac{q}{r_2}+\frac{q_3}{r_3}\right)$$

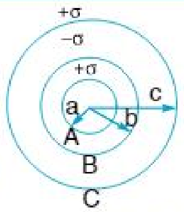
Final potential of middle shell is $V_2 = 0$

Final potential of outermost shell

$$V_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q+q_3}{r_3}\right)$$

Example-2.13

Three concentric spherical metal shells A, B, C of radii a, b, c (c>b>a) have surface charge densities $+\sigma, -\sigma$ and $+\sigma$ respectively.



a) Find the potentials of the three shells?

b) If the shells A and C are at the same potential, find the relation between a,b and c.

Solution :

a) Charges on the three shells are

$$q_A = +\sigma 4\pi a^2 \, ; \; q_B = -\sigma 4\pi b^2 \; ; \; q_C = +\sigma 4\pi c^2$$

$$V_A = \frac{1}{4\pi \in_0}\left\{\frac{q_A}{a}+\frac{q_B}{b}+\frac{q_C}{c}\right\} = \frac{\sigma}{\in_0}(a-b+c)$$

$$V_B = \frac{1}{4\pi \in_0}\left\{\frac{q_A}{b}+\frac{q_B}{b}+\frac{q_C}{c}\right\} = \frac{\sigma}{\in_0}\left(\frac{a^2-b^2}{b}+c\right)$$

$$V_C = \frac{1}{4\pi \in_0}\left\{\frac{q_A}{c}+\frac{q_B}{c}+\frac{q_C}{c}\right\} = \frac{\sigma}{\in_0}\left(\frac{a^2-b^2+c^2}{c}\right)$$

b) If $V_A = V_C$, on substitution,

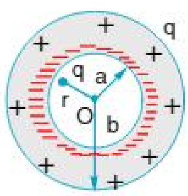
we get $a + b = c$

Example-2.14

A point charge q is at a distance r from the centre O of an uncharged spherical conducting layer, whose inner and outer radii are equal to a and b respectively. Find the potential at the point O if r<a.

Solution :

Equal amount of charge will be induced on the inner and outer surface of the conductor. Then each negative charge is at an equal distance a from O, and so potential due to this charge at O is $V_- = \frac{1}{4\pi \in_0}\frac{(-q)}{a}$.



Similarly due to positive charge, potential at O

$$V_+ = \frac{1}{4\pi \in_0}\frac{q}{b}$$

Potential at O due to charge q ; $V_0 = \frac{1}{4\pi \in_0}\frac{q}{r}$

Thus total potential $V = V_0 + V_- + V_+$

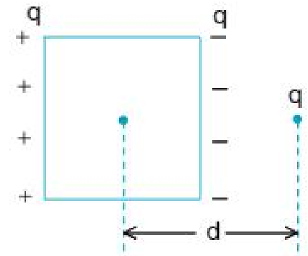
$$= \frac{q}{4\pi \in_0}\left[\frac{1}{r}-\frac{1}{a}+\frac{1}{b}\right].$$

Example-2.15

A positive charge q is placed in front of a conducting solid cube at a distance d from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

Solution :

Charges will be induced on the surface of the cube due to the charge q. The net electric field at the centre of the cube due to all the charges must be zero. Let E_1 be the electric field due to the charges appearing on the surface of the cube. If E_2 is the electric field due to charge q, then



$\vec{E}_1 + \vec{E}_2 = 0$ or $\vec{E}_1 = -\vec{E}_2$ or $E_1 = E_2$

The electric field at the centre of the cube, due to charge q

$$E_2 = \left(\frac{1}{4\pi \in_0}\right)\frac{q}{d^2} \Rightarrow E_1 = \left(\frac{1}{4\pi \in_0}\right)\frac{q}{d^2}$$

2.19 ELECTRIC FIELD DUE TO TWO PARALLEL NON-CONDUCTING SHEETS OF CHARGE

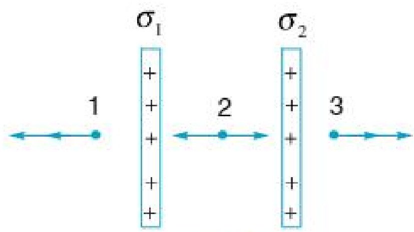


Fig 2.22

Consider two infinite non conducting sheets of charge with surface charge densities σ_1 and σ_2 as shown.

Fields produced by the sheets at a point near them are given by $\frac{\sigma_1}{2\in_0}$ and $\frac{\sigma_2}{2\in_0}$ directed normally outwards.

Now at point 1 field $E_1 = \frac{\sigma_1}{2\in_0} + \frac{\sigma_2}{2\in_0} = \frac{\sigma_1 + \sigma_2}{2\in_0}$

at point 2 field $E_2 = \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} = \frac{\sigma_1 - \sigma_2}{2\epsilon_0}$

at point 3 field $E_3 = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} = \frac{\sigma_1 + \sigma_2}{2\epsilon_0}$

Suppose, $\sigma_1 = \sigma_2 = \sigma$ then

$E_1 = E_3 = \frac{\sigma}{\epsilon_0}$ and $E_2 = 0$

Similarly try for $\sigma_1 = \sigma$ and $\sigma_2 = -\sigma$

2.20 ELECTRIC FIELD AT THE SURFACE OF A CHARGED CONDUCTOR

Consider a charged conductor of any shape let σ be the surface charge density of that conductor. Let $\hat{n}$ be a unit vector along outward normal to the surface considered on that conductor.

Consider a pill box in the form of a short cylinder as the Gaussian surface about any point P on the surface of the conductor. Such pill box will be partly inside and partly outside the surface of the conductor. Area of cross section of that pill box is ds which is very small.

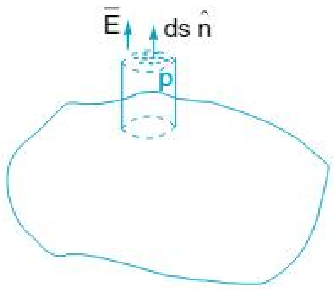


Fig 2.23

Inside the surface of the changed conductor electrostatic field is zero. Just outside the conductor, the electric field is normal to the surface. The total contribution of the electric flux through the pill box will be due to the outer surface only. If $\bar{E}$ is the electric field at P, the total flux through the Gaussian surface considered will be Eds (Here $\sigma > O$). Here $\bar{E}$ is constant and $\bar{E}$ is parallel to $d\bar{s}$. Charge enclosed by the pill box is σds.

From Gauss's law, $Eds = \frac{\sigma ds}{\epsilon_0}$

$$\Rightarrow E = \frac{\sigma}{\epsilon_0}$$

If σ is positive, $\bar{E}$ will be along outward normal to the surface and it σ is negative, $\bar{E}$ will be along inward normal to the surface.

Knowledge Plus 2.1

☺ We have a very sensitive electronic instrument and we want to protect it from external electric fields. How?

☛ We can surround that instrument with a conducting box or we can keep that inside the cavity of a conductor. By doing so the charge on the box or conductor with cavity distributes such that net electric field inside the box or cavity is zero. In this way instrument can be protected from the external fields. Do you know ? This is called shielding.

2.21 VAN DE GRAFF GENERATOR

Whenever a charge is given to a metal body it will spread on the outer surface of it. If we put a charged metal body inside the hollow metal body and the two are connected by a wire, whole of the charge of the inner body will flow to the outer surface of the hollow body. No matter how large the charge is on the inner body, it completely transfers to the outer body. This principle is used in Van de Graaff generator. It is used to develop very high charges and intense electric field or very high voltages.

The figure shows various parts of Van de Graaff generator. It consists of a large spherical metal shell S mounted on two insulating supports. A belt made of insulating material is run at high speed over pulleys P_1 and P_2. The pulley P_1 is in the base of the machine which is run by a motor. The pulley P_2 is at the centre of the spherical shell.

The metal combs C_1 and C_2 with sharp ends are mounted on the generator as shown. Comb C_1 is called emitter comb and it is held near the lower end of the belt. This is given a high positive potential about 10^4V with respect to the ground. The other comb C_2 is called collector comb and it is placed near the upper end of the belt such that its pointed ends touch the belt. The other end is in contact with the inner surface of the metal sphere S.

As C_1 is given high positive potential with respect to the earth, its positive ends set up positive ions. These positive charges are sprayed on the belt which carries them to the top. As the positive charges reach C_2, they are transferred to the metal sphere. This charge immediately moves to the outer surface of the sphere. Now the uncharged belt runs down to take positive charge from C_1. This process continues and more and more charge builds up on the outer surface of the spherical shell. As the charge on the shell increases, its potential also increases. When the shell is charged to a sufficient potential, the leakage of charge due to ionisation of surrounding air begins. So there is a limiting value of potential to which the shell can be charged.

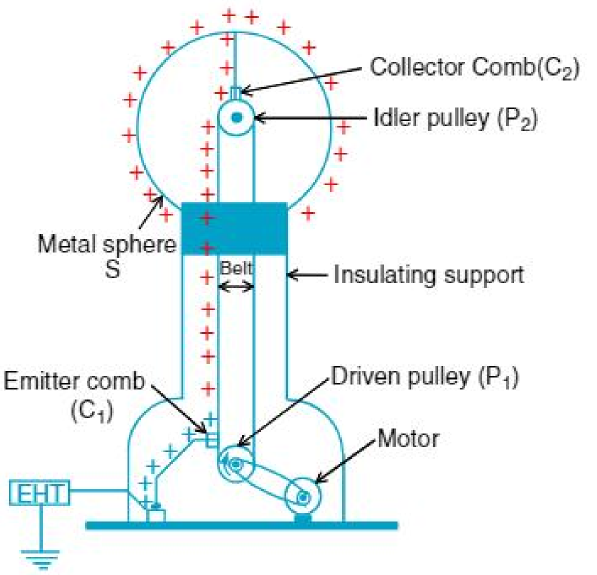


Fig 2.22

At a Glance

1. The flux of electric field over a given surface $\overline{s}$ is given by $\int_s \overline{E}.d\overline{s}$.
2. Gauss's law is useful in finding the flux linked with a closed surface or electric field intensity at a point. According to this $\oint \overline{E}.d\overline{s} = \frac{q_{encl}}{\epsilon_0}$
3. There can be electric field on the gaussian surface even if charge enclosed by it is zero.
4. Coulomb's inverse square law and Gauss's law are equivalent.
5. Gauss's law is only true if inverse square law of electric force is true.
6. Net charge inside a closed Gaussian surface drawn in any conducting shell is zero.
7. Potential of the earthed conductor is zero.
8. Charge remains constant in any isolated conductor if it is not earthed.
9. Equal and opposite charges appear on opposite faces of a body due to induction.
10. If two conductors are connected by a conducting wire, they are at the same potential.

EXERCISE

LONG ANSWER QUESTIONS

1. Define electric flux. State Gauss's law in electrostatics. Using Gauss's law, derive coulomb's inverse square law.
2. Applying Gauss's law derive the expression for electric intensity due to an infinite long straight charged wire.
3. Applying Gauss's law derive the expression for electric intensity due to an infinite plane sheet of charge.
4. Applying Gauss's law derive the expression for electric intensity due to a charged conducting spherical shell at (a) a point outside the shell (b) a point on the surface of the shell (c) a point inside the shell.
5. Show that electric field near a charged conductor is $\frac{\sigma}{\epsilon_0}$ where σ is surface charge density.

SHORT ANSWER QUESTIONS

1. What do you mean by electric flux? What is its SI unit?
2. State and prove Gauss's law.
3. Derive Coulomb's law from Gauss's law.
4. Derive Gauss's law from Coulomb's law.
5. State Gauss's law and explain its importance in electrostatics.
6. Applying Gauss's law show that electric field inside a charged conductor is zero every where.

VERY SHORT ANSWER QUESTIONS

1. A Gaussian surface does not enclose a charge. Does it mean that E = 0 on its surface ?

A. No. There can be electric field on the Gaussian surface even if the charge enclosed by it is zero. However, the net flux through the surface will be zero. For instance, the Gaussian surface may have a point charge outside it.

2. If electric force between point charged varies inversely as the cube of the distance, will Gauss's law be valid?

A. No. Gauss's law is only true if inverse square law of electric force is true.

3. Can you give any situation where Gauss's law cannot be helpful?

A. Consider an electric dipole. The field does have axial symmetry about the dipole axis but there is no simple surface over which normal component of $\bar{E}$ is constant. So, practically it is difficult to apply Gauss's law to such a system even though the law is valid in this situation.

4. If charge distribution within a Gaussian surface changes inside it, will electric field strength change inside and outside the Gaussian surface?

A. As the total charge inside the Gaussian surface remains unchanged, the same electric flux will pass through the Gaussian surface. However, due to the change in charge distribution, the value of E will change inside as well as outside the Gaussian surface.

5. Even though electric flux is a scalar quantity, we consider the flux flowing out of a surface as positive and flux entering into the surface as negative. Keeping this fact in mind answer the following question :
The total electric flux through a Gaussian surface is zero when there is a charge outside the surface. Why is it so when the charge produces electric field?

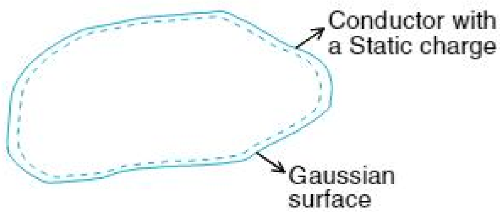
A. The electric field will always be along the direction away from the charge (assumed positive). But the flux will have both positive and negative values with equal magnitudes. Hence total flux will be zero for charge outside the closed surface.

6. We have derived the formula $E = \dfrac{\lambda}{2\pi \varepsilon_0 r}$ considering only a finite length ℓ of the wire. How are we justified in taking the equation valid for the entire wire of infinite length?

A. This is because, the total charge enclosed will be λL and total surface area will be $(2\pi r)$ L and L cancels out in Eqn. $E(2\pi r)\,L = \dfrac{\lambda L}{\varepsilon_0}$, even when ℓ is infinity.

7. In conductors, the outer electrons of each atom or molecule are weakly bound to the atom or molecule. So, these electrons are almost free to move throughout the conductor. Hence, these are called free electrons or conduction electrons. When such a conductor is in an electric field, the free electrons inside redistribute themselves on the surface of the conductor in such a way that the electric field at every point inside the conductor is zero.
Applying Gauss's law, try to prove that any excess static charge given to an insulated conductor resides entirely on its outer surface.

A. As shown in Fig, consider a conductor of any arbitrary shape having a static charge. At an infinitesimal distance from the surface of the conductor construct a Gaussian surface lying inside the conductor. The flux through this Gaussian surface must be zero since E is zero everywhere inside the conductor and on all points on the Gaussian surface. In accordance with Gauss's law there can be no net charge inside the Gaussian surface.



This shows that all the net charges must be on the surface of the conductor since the Gaussian surface is within an infinitesimal distance of the surface. Thus under static conditions, there can be no net charge inside a conducting body and all the charge must reside on its surface.

The fact that the field inside a conductor is zero holds good for a hollow conductor also. This phenomenon is used in electrostatic shielding to protect electrical instruments.

8. **The magnitude of the electric field of an infinite plane sheet of charge (as given by $E = \frac{\sigma}{\epsilon_0}$ is independent of the distance r from the sheet. $E \propto \frac{1}{r^2}$ is not followed here. Can you guess why?**

A. This is because the entire charge is on the infinite plane and does not depend on the distanced r from the plane.

9. **Let us consider an infinite plane charged conducting plate. Now the charge distributes on both sides of the conducting plate. The field of such charged plate arises from the superposition of the fields of two sheets of charge. Can you guess the value of E for points outside the plate? What will be the field inside the plate, and why?**

A. $E = \frac{\sigma}{\varepsilon_0}$ and inside the plate E = 0.

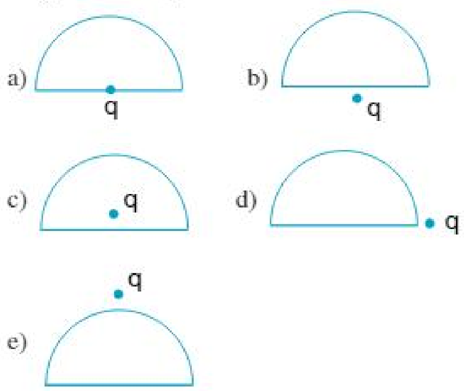
10. **Inside a hollow conducting sphere of charge Q, the electric intensity (E) is zero. But, the potential (V) is not zero. How should be the potential inside the charged conducting sphere to satisfy the condition that E is zero inside?**

A. The potential should be the same everywhere inside the sphere so that $\frac{dV}{dr} = 0$.

PROBLEMS

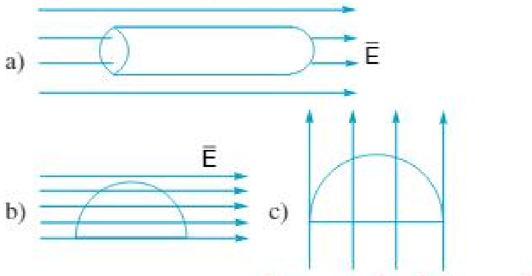
LEVEL - I

1. Find the total flux due to charge q associated with the given hemispherical surface.



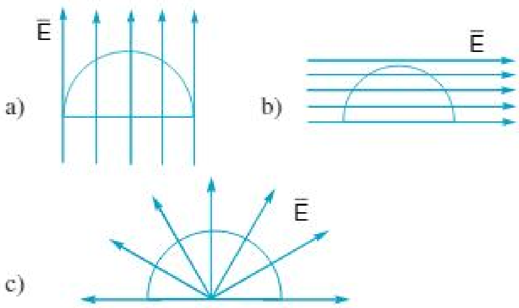
[**Ans:** (a) $\frac{q}{2\epsilon_0}$; (b) zero ; (c) $\frac{q}{\epsilon_0}$; (d) zero; (e) zero]

2. In a uniform electric field, find the total flux associated with the given surfaces.



[**Ans:** zero in all the cases]

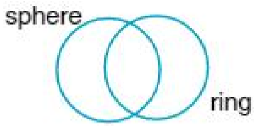
3. Find the flux due to the electric field through the curved surface (R is radius of curvature)



(Radial field with same intensity)

[**Ans:** (a) $\pi R^2 E$; (b) zero ; (c) $2\pi R^2 E$]

4. A charge q is distributed uniformly on a ring of radius R. A sphere of equal radius R is constructed with its centre at the periphery of the ring, Find the flux of the electric field through the surface of the sphere

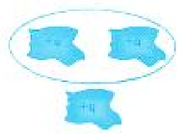


[**Ans:** $\frac{q}{3\epsilon_0}$]

5. A point charge +Q is located at a distance of r from the centre of an uncharged conducting sphere of radius R. Find the electric potential of that sphere Find the electric field and electric potential at the centre of the sphere due to induced charges on the sphere?

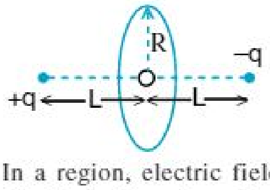
[**Ans:** $\frac{1}{4\pi\epsilon_0}\frac{Q}{r}, \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$ (towards +Q), zero]

6. Shown below is a distribution of charges. Find the flux of electric field due to these charges through the surface S



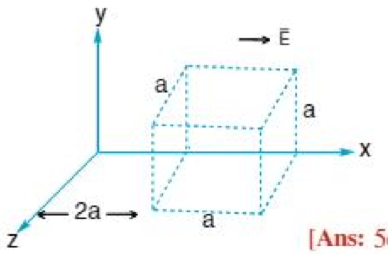
[**Ans:** $2q/\varepsilon_0$]

7. Two point charges q and –q are seperated by a distance 2L. Find the flux of the electric field vector across the circle of radius R as shown.



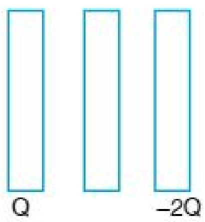
[Ans: zero]

8. In a region, electric field depends on X-axis as $E = E_0 x^2$. There is a cube of edge a as shown. Then find the charge enclosed in that cube.



[Ans: $5\epsilon_0 a^4 E_0$]

9. Three identical metal plates with large surface areas are kept parallel to each other as shown in figure. The left most plate is given a charge Q, the right most charge –2Q and the middle one remains neutral. Find the charge appearing on the outer surface of the right most plate.

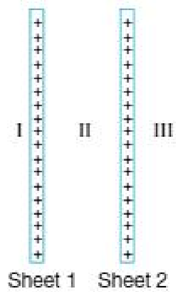


[Ans: –Q/2]

LEVEL - II

1. A charge q is placed at the centre of the open end of a cylindrical vessel. Find the flux of the electric field through the surface of the vessel. [Ans: $\frac{q}{2\epsilon_0}$]

2. Two parallel plane sheets 1 and 2 carry uniform charge densities σ_1 and σ_2 as in fig. Find the electric field in the region marked II $(\sigma_1 > \sigma_2)$.



[Ans: $\frac{(\sigma_1 - \sigma_2)}{2\varepsilon_0}$]

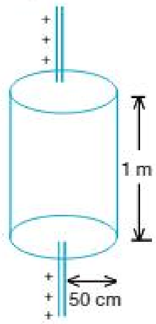
3. The length of each side of a cubical closed surface is L. If charge 48C is situated at one of the corners of the cube, find the flux passing through the cube. [Ans: $\frac{6}{\epsilon_0}$]

4. A cylinder of radius R and length L is placed in a uniform electric field E parallel to the cylinder axis. Find the total flux from the surface of the cylinder [Ans: zero]

5. Eight dipoles of charges of magnitude e are placed inside a cube. Find the total electric flux coming out of the cube. [Ans: zero]

6. A cube is arranged such that its length, breadth and height are along X,Y and Z directions. One of its corners is situated at the origin. Length of each side of the cube is 25cm. The components of electric field are $E_x = 400\sqrt{2}$ N/C, $E_y = 0$ and $E_z = 0$ respectively. Find the flux coming out of the cube at the right end. [Ans: $25\sqrt{2}\ \text{Nm}^2/\text{C}$]

7. Electric charge is uniformly distributed along a long straight wire of radius 1 mm. The charge per cm length of the wire is Q coulomb. Another cylindrical surface of radius 50 cm and length 1 m symmetrically encloses the wire as shown in the figure. Find the total electric flux passing through the cylindrical surface.



[Ans: $\frac{100Q}{\varepsilon_0}$]

8. A point charge q is placed at the centre of a cubical box. Find the (a) total flux through the box. (b) through each face of the box

[Ans: (a) $\frac{q}{\epsilon_0}$; (b) $\frac{q}{6\epsilon_0}$]

9. In the previous problem if q is placed at the centre of one face find total flux through the box. If q is placed at one corner of the cube, find total flux through the box and flux through the faces.

[Ans: $\frac{q}{2\epsilon_0}, \frac{q}{8\epsilon_0}$, no flux is linked with the three faces at the junction of which q is kept, flux emerges equally through the other three front faces of the cube and equal to $\frac{q}{24\epsilon_0}$]

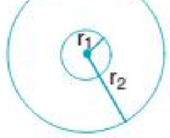
10. A point charge q is placed at a depth d below the centre of mouth of a conical vessel, whose open end is circular having a radius r. Find the electric flux through the lateral surface of the vessel ?

[Ans : $\frac{q}{2\in_0}\left\{1+\frac{d}{\sqrt{d^2+r^2}}\right\}$]

11. A non conducting sphere of radius a is charged uniformly with volume charge density ρ . There is a spherical cavity in the sphere with radius b. The position vector of centre of the cavity with respect to centre of the original sphere is $\vec{C}$. Find the electric field at any point inside the cavity whose position vector is $\overline{r}$ with respect to centre of the original sphere.

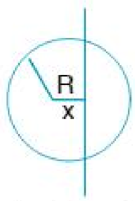
[Ans : $\frac{\rho\vec{C}}{3\in_0}$]

12. In a spherical charge distribution, volume charge density varies as $\rho = Cr^{-1}$ for $r_1 < r < r_2$ as shown. Here C is a positive constant. When a point charge q is kept at the centre of the sphere, the electric field strength in the region $r_1 < r < r_2$ will be a constant. For this find the value of C.



[Ans : $C = \frac{q}{2\pi r_1^2}$]

13. An infinite uniformly charged sheet with surface charge density σ cuts through a spherical Gaussion surface of radius R at a distance x from its centre as shown.



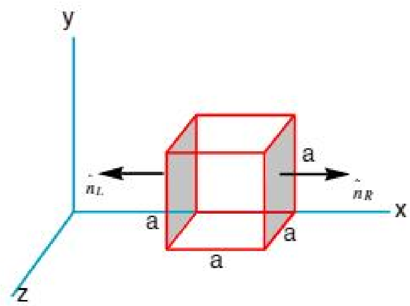
Find the electric flux through the Gaussian surface.

[Ans: $\frac{\pi(R^2-x^2)\sigma}{\in_0}$]

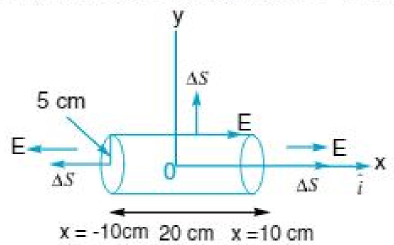
14. A point charge is kept at the centre of circular face of a cylinder of radius r and length r. Find the electric flux through remaining curved surface (i.e., other than circular face)

[Ans : $\frac{q}{2\in_0}\frac{1}{\sqrt{2}}$]

15. The electric field components in Fig. are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$, in which α = 800 N/C $m^{1/2}$. Calculate (a) the flux through the cube, and (b) the charge within the cube. Assume that a = 0.1 m.



16. An electric field is uniform, and in the positive x direction for positive x, and uniform with the same magnitude but in the negative x direction for negative x. It is given that E = 200 $\hat{i}$ N/C for x> 0 and E = –200 $\hat{i}$ N/C for x < 0. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin an its axis along the x-axis so that one face is x = + 10 com and the other is at x = –10 cm (Fig.).



(a) What is the net outward flux through each flat face?

(b) What is the flux through the side of the cylinder?

(c) What is the net outward flux through the cylinder?

(d) What is the net charge inside the cylinder?

17. An early model for an atom considered it to have a positively charged point nucleus of charge Ze, surrounded by a uniform density of negative charge up to a radius R. The atom as a whole is neutral. For this model, what is the electric field at a distance r from the nucleus?