2. THEORY OF EQUATIONS

SYNOPSIS

Definition : An equation of the form $f(x) = p_0x^n + p_1x^{n-1} + p_2x^{n-2} + \text{..................} + p_n = 0 \;\; (p_0 \neq 0)$ is called an n^{th} degree polynomial equation, where $p_0, \; p_1, \text{...............} \; p_n$ are complex numbers

Relation between roots and coefficients :

1. If $\alpha_1, \alpha_2, \alpha_3, \text{.......}, \alpha_n$ are the roots of the equation $f(x) = p_0x^n + p_1x^{n-1} + p_2x^{n-2} + ... + p_n = 0$ then

i) Sum of the roots $= \sum \alpha_1 = s_1 = -\dfrac{p_1}{p_0}$

ii) Sum of the product of roots taken two at a time $= s_2 = \dfrac{p_2}{p_0}$

iii) Sum of the product of roots taken three at a time $= s_3 = -\dfrac{p_3}{p_0}$

iv) Product of the roots $= \alpha_1\alpha_2\alpha_3....\alpha_n = s_n = (-1)^n \dfrac{p_n}{p_0}$

➤ If α, β, γ are the roots of $ax^3+bx^2+cx+d=0$ then

i) $s_1 = \alpha+\beta+\gamma = -\dfrac{b}{a}$ ii) $s_2 = \alpha\beta+\beta\gamma+\gamma\alpha = \dfrac{c}{a}$ iii) $s_3 = \alpha\beta\gamma = -\dfrac{d}{a}$

i.e. the equation having roots α, β, γ is

$\Rightarrow x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma = 0$

$\Rightarrow x^3 - s_1x^2 + s_2x - s_3 = 0$

➤ If $\alpha, \beta, \gamma, \delta$ are the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$ then

i) $s_1 = \alpha+\beta+\gamma+\delta = -\dfrac{b}{a}$

ii) $s_2 = \alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = \dfrac{c}{a}$

iii) $s_3 = \alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta = -\dfrac{d}{a}$

iv) $s_4 = \alpha\beta\gamma\delta = \dfrac{e}{a}$

i.e. the equation having roots $\alpha, \beta, \gamma, \delta$ is

$\Rightarrow x^4 - (\alpha+\beta+\gamma+\delta)x^3 + (\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)x^2 - (\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta)x + \alpha\beta\gamma\delta = 0$

$\Rightarrow x^4 - s_1x^3 + s_2x^2 - s_3x + s_4 = 0$

➤ If α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0$

i) $\sum\alpha^2 = s_1^2 - 2s_2$ ii) $\sum\alpha^3 = s_1^3 - 3s_1s_2 + 3s_3$

iii) $\sum\alpha^4 = s_1^4 - 4s_1^2s_2 + 4s_1s_3 + 2s_2^2$ iv) $\sum\alpha^2(\beta+\gamma) = s_1s_2 - 3s_3 = \sum\alpha^2\beta$

2. To solve a cubic equation, when the roots are

i) in A.P., they are taken as $\alpha-\beta, \alpha, \alpha+\beta$ ii) in H.P., they are taken as $\dfrac{1}{\alpha-\beta}, \dfrac{1}{\alpha}, \dfrac{1}{\alpha+\beta}$

iii) in G.P. they are taken as $\dfrac{\alpha}{\beta}, \; \alpha, \alpha\beta$.

3. To solve a biquadratic equation, take the roots as
   i) $\alpha-3\beta, \alpha-\beta, \alpha+\beta, \alpha+3\beta$ when they are in A.P.
   ii) $\frac{1}{\alpha-3\beta}, \frac{1}{\alpha-\beta}, \frac{1}{\alpha+\beta}, \frac{1}{\alpha+3\beta}$ when they are in H.P.
   iii) $\frac{\alpha}{\beta^3}, \frac{\alpha}{\beta}, \alpha\beta, \alpha\beta^3$ when they are in G.P.
4. The condition that the roots of $ax^3+bx^2+cx+d=0$ are in A.P. is $2b^3 + 27a^2d = 9abc$.
5. The condition that the roots of $ax^3+bx^2+cx+d = 0$ are in G.P. is $ac^3 = b^3d$.
6. The condition that the roots of $ax^3+bx^2+cx+d=0$ are in H.P. is $2c^3+ 27ad^2 = 9bcd$.
7. The condition that one root of $ax^3+bx^2+cx + d = 0$ is sum of the other two roots, is $8a^2d + b^3 = 4abc$.
8. The condition that the products of two of the roots of $ax^3 + bx^2 + cx + d = 0$ is -1, is $a(a + c) + d(b + d) = 0$
9. If $\alpha+\sqrt{\beta}$ is a root of $f(x) = 0$, whose coeffcients are rational then $\alpha-\sqrt{\beta}$ is also a root, where $\sqrt{\beta}$ is irrational.
10. If $a + ib$ is a root of $f(x) = 0$ whose coefficients are real then $a - ib$ is also a root, where $i = \sqrt{-1}$.
11. In an equation, if all the coefficients are of the same sign then the equation has no positive root.
12. In an equation, if all the coefficients of even powers of x are all positive (or all negative) and the coefficients of odd powers of x are of opposite sign, then the equation has no negative root.
13. The equation of lowest degree with rational coefficients, having a root $\sqrt{a}+\sqrt{b}$ is $x^4 - 2(a + b)x^2 + (a - b)^2 = 0$
14. The equation of lowest degree with rational coefficients, having a root $\sqrt{a}+i\sqrt{b}$ is $x^4 - 2(a - b)x^2 + (a + b)^2 = 0$
15. If $f(x) = (x-\alpha)^2\phi(x)$ when $\phi(x)$ is not divisible by $x-\alpha$ then α is called second order multiple root of $f(x) = 0$.
16. A multiple root α of order n of $f(x) = 0$ is multiple root of order $n - 1$ of $f'(x) = 0$.
17. If α,β,γ are the roots of $f(x) = x^3+px^2+qx+r=0$ then the equation having roots
    i) $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ is $f(-p - x) = 0$
    ii) $\alpha\beta, \beta\gamma, \gamma\alpha$ is $f\left(-\frac{r}{x}\right)=0$

Transformed equations :

1. The second term of $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_n = 0$ can be removed by diminishing the roots by h where $h = -\frac{a_1}{na_0}$ and a_0 is the coeffcient of the first term a_1 is the coefficient of the second term n is the degree of $f(x) = 0$
2. If $f(x) = 0$ is an equation of degree n then to eliminate r^{th} term in $f(x) = 0$ can be transformed to $f(y + h) = 0$ where h is a constant such that $f^{(n-r+1)}(h) = 0$.

Reciprocal equation (R.E.) :

1. If an equation is unaltered by changing x as $\frac{1}{x}$, then it is a reciprocal equation.
2. If $x^n + p_1x^{n-1} + p_2x^{n-2} + ... + p_{n-1}x + p_n = 0$ is a polynomial equation such that $p_n = 1$, $p_1 = p_{n-1}, p_2 = p_{n-2}$ etc ; then the equation is R.E. of the 1st type.
3. If $p_n = -1$, $p_1 = -p_{n-1}, p_2 = -p_{n-2}$ etc; then the equation is R.E. of the 2nd type.
4. A reciprocal equation of the 1st type and even degree is called a standard R.E.
5. $x + 1$ is a factor of R.E. of the 1st type and odd degree.
6. x^2-1 is a factor of R.E. of the 2nd type and even degree.

LECTURE SHEET

EXERCISE-I

Quotient, Remainders :

1. The Quotient obtained when $x^4 + 11x^3 - 44x^2 + 76x + 48$ is divided by $x^2 - 2x + 1$ is
   1) $x^2 - 13x + 5$ 2) $x^2 + 13x - 19$ 3) $x^2 - 13x + 19$ 4) $x^2 + 13x + 25$
2. The value of k so that $x^4-3x^3+ 5x^2 - 33x + k$ is divisible by $x^2 - 5x + 6$ is
   1) 45 2) 48 3) 51 4) 54

Formation of equations :

3. The equation whose roots are $\sqrt{2}, -\sqrt{2}, 3i, -3i$ is
   1) $x^4+7x^2-18=0$ 2) $x^4-7x^2+18=0$ 3) $x^4+7x^2+18=0$ 4) $x^4-7x^2-18=0$
4. The equation of lowest degree with rational coefficients having a root $\sqrt{3}+\sqrt{2}$ is
   1) $x^4 + 10x^2 - 1 = 0$ 2) $x^4 - 10x^2 + 1 = 0$ 3) $x^4 + 10x^2 + 1 = 0$ 4) $x^4 - 10x^2 - 1 = 0$

Models on roots of the equation :

5. If $\alpha, \beta, 1$ are roots of $x^3 - 2x^2 - 5x + 6 = 0$ $(\alpha > 1)$ then $3\alpha+\beta=$
   1) 7 2) 5 3) 14 4) 10
6. The condition that the product of two of the roots $x^3 + px^2 + qx + r = 0$ is -1, is
   1) $r^2 + pr + q + 1=0$ 2) $q^2 + pq + q + 1 = 0$ 3) $p^2 + pq + p + 1=0$ 4) $r^2 - pr - q + 1 = 0$
7. If one root of $24x^3 - 14x^2 - 63x + 45 = 0$ is double the other then the roots are
   1) $-1, \frac{1}{2}, 2$ 2) $2, 2, -1$ 3) $\frac{3}{4}, \frac{3}{2}, -\frac{5}{3}$ 4) $-\frac{3}{2}, -\frac{3}{4}, -\frac{1}{3}$
8. If $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$ has a pair of equal roots then the roots are
   1) $-1, -1, 2, 2$ 2) $1, 1, -2, -2$ 3) $-1, -1, 3, 3$ 4) $1, 1, -3, -3$
9. If $\sqrt{5}+\sqrt{2}$ is a root of $3x^5-4x^4-42x^3+56x^2+ 27x-36=0$ then the rational root is
   1) $\frac{4}{3}$ 2) $\frac{3}{4}$ 3) $-\frac{3}{4}$ 4) $-\frac{4}{3}$

10. If $-1+i$ is a root of $x^4 + 4x^3 + 5x^2 + 2x + k = 0$ then the other roots are

1) $-1, -1$ 2) $-\frac{1}{2}, -\frac{3}{2}$ 3) $-1 \pm \sqrt{2}$ 4) $1 \pm \sqrt{2}$

11. If the roots of $x^3 - 9x^2 + kx + l = 0$ are in A.P with common difference 2 then $(k, l) =$

1) $(15, -15)$ 2) $(23, -15)$ 3) $(15, -23)$ 4) $(-15, 23)$

12. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$ which are in A.P. where $\alpha = 1$ then common difference of that A.P is $(\alpha < \beta < \gamma < \delta)$

1) 1 2) 2 3) 3 4) 4

13. If the roots of $x^3 - 13x^2 + kx - 27 = 0$ are in G.P then $k =$

1) -30 2) 30 3) 39 4) -39

14. If the roots of the equation $x^4 - 10x^3 + 50x^2 - 130x + 169 = 0$ are of the form $a \pm ib$ and $b \pm ia$ then $(a, b) =$

1) $(3, 2)$ 2) $(2, 1)$ 3) $(-3, 2)$ 4) $(-3, -2)$

Symmetric Functions of the roots :

15. If α, β, γ are the roots of $x^4 - px^2 + qx - r = 0$ then $\alpha^2 + \beta^2 + \gamma^2 =$

1) $p^2 - 2q$ 2) $p^3 - 3pq + 3r$ 3) $p^4 - 3p^2q + 3pr + 2q^2$ 4) $2q$

16. If α, β, γ are the roots of $x^3 - px^2 + qx - r = 0$ then $\alpha^3 + \beta^3 + \gamma^3 =$

1) $p^2 - 2q$ 2) $p^3 - 3pq + 3r$ 3) $p^4 - 3p^2q + 3pr + 2q^2$ 4) $2q$

17. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ then $\Sigma\alpha^2(\beta + \gamma) =$

1) $p^2 - 2q$ 2) $-p^3 + 3pq - 3r$ 3) $p^4 - 4p^2q + 4pr + 2q^2$ 4) $3r - pq$

18. If α, β, γ are the roots of $x^3 + 2x^2 - 3x - 1 = 0$ then $\alpha^{-2} + \beta^{-2} + \gamma^{-2} =$

1) 12 2) 13 3) 14 4) 15

19. If α, β, γ are the roots of the equation $px^3 + qx^2 + rx + s = 0$ then $\Sigma\alpha^2\beta^2 =$

1) $\frac{r^2 + 2qs}{p^2}$ 2) $\frac{r^2 - 2qs}{p^2}$ 3) $\frac{ps + r^2}{p^2}$ 4) $\frac{ps - r^2}{p^2}$

Transformed Equations :

20. If $f(x) = 5x^3 + 4x^2 - 13x - 25$ and $f(x-3) = 5x^3 - 41x^2 + 98x + k$ then $k =$

1) 85 2) -85 3) 105 4) -105

21. The equation whose roots are multiplied by 3 of those of $2x^3 - 3x^2 + 4x - 5 = 0$ is

1) $2x^3 - 9x^2 + 36x - 135 = 0$ 2) $2x^3 - 9x^2 - 36x + 135 = 0$
3) $x^3 - 9x^2 + 36x + 135 = 0$ 4) $2x^3 - 9x^2 + 36x + 135 = 0$

22. If α, β, γ are the roots of $x^3 + 2x^2 - 4x - 3 = 0$ then the equation whose roots are $\frac{\alpha}{3}, \frac{\beta}{3}, \frac{\gamma}{3}$ is

1) $x^3 + 6x^2 - 36x - 81 = 0$ 2) $9x^3 + 6x^2 - 4x - 1 = 0$
3) $9x^3 + 6x^2 + 4x + 1 = 0$ 4) $x^3 - 6x^2 + 36x + 81 = 0$

23. The equation whose roots are squares of the roots of $x^4 + x^3 + 2x^2 + x + 1 = 0$ is
1) $x^4 - 3x^3 + 4x^2 + 3x + 1 = 0$ 2) $x^4 + 3x^3 + 4x^2 + 3x + 1 = 0$
3) $x^4 - 3x^3 - 4x^2 + 3x + 1 = 0$ 4) $x^4 - 3x^3 - 4x^2 - 3x + 1 = 0$

24. The equation whose roots are cubes of the roots of $x^3 + 3x^2 + 2 = 0$ is
1) $x^3 + 33x^2 + 12x + 8 = 0$ 2) $x^3 + 33x^2 - 12x - 8 = 0$
3) $x^3 - 33x^2 + 12x - 8 = 0$ 4) $x^3 + 33x^2 + 12x - 8 = 0$

25. If α, β, γ are the roots of $x^3-3x+1=0$ then the equation whose roots are $\alpha - \frac{1}{\beta\gamma}, \beta - \frac{1}{\gamma\alpha}, \gamma - \frac{1}{\alpha\beta}$ is
1) $x^3 - 3x + 8 = 0$ 2) $x^3 - 6x + 8 = 0$ 3) $x^3 - 9x + 8 = 0$ 4) $x^3 - 12x + 8 = 0$

EXERCISE-II

Multiple Roots :

1. The repeated root of the equation $4x^3 - 12x^2 - 15x - 4 = 0$ is
1) $\frac{5}{2}$ 2) $-\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $-\frac{1}{3}$

2. If $f(x)= 0$ has a repeated root α, then another equation having α as root is
1) $f(2x) = 0$ 2) $f(3x) = 0$ 3) $f'(x) = 0$ 4) $f(4x) = 0$

3. The equation $x^3 - 3qx + 2r = 0$ has a repeated root then
1) $q^2 = r^3$ 2) $q = r^2$ 3) $q^3 = r$ 4) $q^3 = r^2$

4. The polynomial $x^3 - 3x^2 - 9x + C$ can be written in the form $(x-a)^2 (x-b)$ then $C =$
1) 5 2) 3 3) 25 4) 27

Conditional roots :

5. If the roots of $ax^3 + bx^2 + cx + d = 0$ are in G.P. then the roots of $ay^3 + bky^2 + ck^2y + dk^3 = 0$ are in
1) A.P 2) G.P 3) H.P 4) A.G.P

6. If 2,5,7,–4 are the roots of $ax^4+bx^3+cx^2+dx+e=0$ then the roots of $ax^4 - bx^3 + cx^2 - dx + e = 0$ are
1) 2, 5, 7, –4 2) –2, –5, –7, 4 3) 2, 5, 7, 4 4) 2, –5, 7, –4

7. If 1, 2, 3 are the roots of $ax^3+bx^2+cx+d=0$ then the roots of $ax\sqrt{x} + bx + c\sqrt{x} + d = 0$ are
1) 2, 3, 4 2) 1, 4, 9 3) 2, 4, 6 4) 1, $\sqrt{2}, \sqrt{3}$

Removal of 2nd, 3rd and fractional Coefficients :

8. The transformed equation of $x^3-6x^2+5x+8=0$ by eliminating second term is
1) $x^3 + 7x + 2 = 0$ 2) $x^3 - 7x + 2 = 0$ 3) $x^3 + 5x + 2 = 0$ 4) $x^3 - 5x + 2 = 0$

9. Number of transformed equations of $x^3+2x^2+x+1=0$ by eliminating third term is
1) 4 2) 3 3) 2 4) 1

10. The transformed equation $x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$ by removing fractional coefficients, is
1) $x^3 - 3x^2 - x + 12 = 0$ 2) $x^3 - 3x^2 - x + 6 = 0$
3) $x^3 - 3x^2 - 24x - 216 = 0$ 4) $x^3 - 15x^2 - 14x + 2 = 0$

11. The transformed equation of $2x^3-\frac{3x^2}{2}-\frac{x}{8}+\frac{3}{16}=0$ by eliminating fractional coefficients and having unity for the coefficient of the first term is

1) $x^3-3x^2-x+6=0$ 2) $x^3-3x^2-x+3=0$
3) $x^3-3x^2-x+12=0$ 4) $x^3-24x^2-3x+3=0$

Reciprocal Equation :

12. The reciprocal equation among the following is

1) $2x^3+4x^2+2x+2=0$ 2) $2x^3+4x^2+4x+2=0$
3) $2x^3+4x^2+2x+4=0$ 4) $2x^3+2x^2+4x-4=0$

13. The equation $6x^6-25x^5+31x^4-31x^2+25x-6=0$ is reciprocal equation of

1) class one and $x=1$ is a root 2) class one and $x=-1$ is a root
3) class two and $x=\pm1$ are roots 4) class two and $x=1$ is a root but not $x=-1$

14. If the equation whose roots are p times the roots of $x^4+2x^3+46x^2+8x+16=0$ is a reciprocal equation then p =

1) 2 2) 3 3) $\pm\frac{1}{2}$ 4) $\pm\frac{1}{3}$

15. If $3x^5-7x^4+4x^3+ax^2+bx+c=0$ is a reciprocal equation of second type then (a, b, c) =

1) (3, -7, 4) 2) (-3, 7, -4) 3) (-4, 7, -3) 4) (4, -7, 3)

16. Theequations $ax^2+bx+a=0$, $x^3-2x^2+2x-1=0$ have two roots in common then $a+b$ must be equal to

1) 1 2) –1 3) 0 4) 2

17. Roots of the equation $ax^3+bx^2+cx+d=0$ remain unchanged by increasing each coefficient by one, then

1) $a=b=c=d\neq0$ 2) $a=b\neq c=d\neq0$ 3) $a\neq b=c=d\neq0$ 4) $a=b=c\neq d\neq0$

18. If α,β,γ are the roots of $x^3+qx+r=0$ then the equation whose roots are $\alpha^2+\alpha\beta+\beta^2$, $\beta^2+\gamma\beta+\gamma^2$, $\gamma^2+\gamma\alpha+\alpha^2$ is

1) $(x+q)^3=0$ 2) $(x-q)^3=0$ 3) $(x+2q)^3=0$ 4) $(x+3q)^3=0$

KEY SHEET (LECTURE SHEET)

EXERCISE-I

1) **2**	2) **4**	3) **1**	4) **2**	5) **1**	6) **1**	7) **3**	8) **4**	9) **1**	10) **3**
11) **2**	12) **1**	13) **3**	14) **1**	15) **1**	16) **2**	17) **4**	18) **2**	19) **2**	20) **2**
21) **1**	22) **2**	23) **2**	24) **1**	25) **4**					

EXERCISE-II

1) **2**	2) **3**	3) **4**	4) **4**	5) **2**	6) **2**	7) **2**	8) **2**	9) **3**	10) **4**
11) **1**	12) **2**	13) **3**	14) **3**	15) **3**	16) **3**	17) **1**	18) **1**		

PRACTICE SHEET

EXERCISE-I

Quotient, Remainders :

1. The remainder when $2x^5 - 3x^4 + 5x^3 - 3x^2 + 7x - 9$ is divisible by $x^2 - x - 3$ is :

   1) $33x + 4$ 2) $41x + 3$ 3) $41x + 44$ 4) $33x - 4$

Formation of equations :

2. The polynomial equation of the lowest degree having roots 1, $\sqrt{3}i$ is

   1) $x^3 + x^2 + 3x + 3 = 0$ 2) $x^3 - x^2 + 3x - 3 = 0$ 3) $x^3 + x^2 - 3x - 3 = 0$ 4) $x^3 - x^2 - 3x + 3 = 0$

Models on roots of the equation :

3. If the sum of two of the roots of $x^3 + ax + b = 0$ is zero then $b =$

   1) a 2) 1 3) -1 4) 0

4. The condition that one root of $x^3 + px^2 + qx + r = 0$ is sum of the other two roots, is

   1) $q^3 = 4(pq - 2r)$ 2) $p^3 = 4(pq - 2r)$
   3) $r^3 = 4(pr - 2q)$ 4) $p^3 = 4(pq - r)$

5. If one root of $x^3 - 12x^2 + kx - 18 = 0$ is thrice the sum of remaining two roots then $k =$

   1) 29 2) -29 3) 19 4) 15

6. If two of the roots of $2x^3 - 3x^2 - 3x + 2 = 0$ are differ by 3 then roots are

   1) $-1, \frac{1}{3}, 2$ 2) $-\frac{3}{2}, -\frac{4}{3}, -\frac{5}{3}$ 3) $-1, \frac{1}{2}, 2$ 4) $2, 2, -1$

7. If two roots of $x^3 - 9x^2 + 14x + 24 = 0$ are in the ratio 3 : 2 then the roots are

   1) 6, 4, -1 2) 3, 2, 4 3) $\frac{1}{2}, \frac{1}{3}, \frac{49}{6}$ 4) 6, 4, 2

8. If the sum of two of the roots of $x^4 - 2x^3 - 3x^2 + 10x - 10 = 0$ is zero then the roots are

   1) $\pm\sqrt{5}, 1 \pm i$ 2) $\pm\sqrt{5}, 1, -1$ 3) $\frac{1}{2}, -\frac{1}{5}, \pm 1$ 4) $\sqrt{2}, \sqrt{5}, \pm 2$

9. If the product of two of the roots of $x^4 - 5x^3 + 5x^2 + 5x - 6 = 0$ is 3 then the roots are

   1) 1, -2, 4, -8 2) $\pm 1, 2, 3$ 3) $\pm 2i, 2, 3$ 4) $-\frac{3}{2}, -\frac{1}{3}, 2 \pm \sqrt{3}$

10. If the roots of $x^4 + 5x^3 - 30x^2 - 40x + 64 = 0$ are in G.P. then the roots are

    1) 1, -2, 4, -8 2) $\pm$ 1, 2, 3 3) $\pm 2i$, 2, 3 4) $\frac{3}{2}, -\frac{1}{3}, 2 \pm \sqrt{3}$

Symmetric Functions of the roots :

11. If α, β, γ are the roots of $x^3 - x^2 + 8x - 6 = 0$ then $(1+\alpha)(1+\beta)(1+\gamma) =$

    1) 8 2) 12 3) 16 4) 24

12. If α, β, γ are the roots of the equation $x^3 - px^2 + qx + r = 0$ then $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha) =$

1) $2(p^2 - 3q)$ 2) $r - pq$ 3) $qp + r$ 4) $\frac{p^2 - 2q}{r^2}$

13. If α, β, γ are the roots of $x^3+ax^2+bx+c=0$ then $(1+\alpha^2)(1+\beta^2)(1+\gamma^2) =$

1) $(c - a)^2 + (b - 1)^2$ 2) $(c + a)^2 + (b + 1)^2$

3) $(c - a)^2 - (b - 1)^2$ 4) $(c + a)^2 - (b + 1)^2$

14. If α, β, γ are the roots of $x^3 + qx + r = 0$ then $\sum(\beta+\gamma)^{-1} =$

1) $\frac{q}{r}$ 2) $\frac{r}{q}$ 3) $-\frac{q}{r}$ 4) $-\frac{r}{q}$

15. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$ then $\Sigma\frac{1}{\alpha^2\beta^2} =$

1) $\frac{q^2 - 2pr}{r^2}$ 2) $q^3 - 3pqr + 3r^2$ 3) $\frac{p^2 - 2q}{r^2}$ 4) $\frac{pq}{r-3}$

Transformed Equations :

16. If $f(x) = x^4 + 3x^2 - 6x - 2$ then the coefficient of x^3 in $f(x + 1)$ is

1) 24 2) –24 3) 4 4) –4

17. The equation whose roots are those of equation $x^4 - 3x^3 + 5x^2 - 2 = 0$ with contrary sign is

1) $x^4 + 3x^3 + 5x^2 - 2 = 0$ 2) $x^4 + 3x^3 + x^2 + 7x - 2 = 0$

3) $x^4 - 3x^3 + 8x^2 + 4 = 0$ 4) $10x^4 - 13x^2 + 40 = 0$

18. The equation whose roots exceed by 2 than the roots of $4x^4 + 32x^3 + 83x^2 + 76x + 21 = 0$ is

1) $4x^4 + 13x^2 + 9 = 0$ 2) $4x^4 - 13x^2 + 9 = 0$

3) $4x^4 + 12x^2 - 9 = 0$ 4) $4x^4 - 13x^2 - 9 = 0$

19. If α, β, γ are the roots of $x^3+x^2+2x+3 = 0$ then the equation whose roots $\beta+\gamma,\ \gamma+\alpha, \alpha+\beta$ is

1) $x^3 + 2x^2 + 3x - 1 = 0$ 2) $x^3 + 2x^2 + 3x + 1 = 0$

3) $x^3 + 2x^2 - 3x - 1 = 0$ 4) $x^3 - 2x^2 + 3x - 1 = 0$

20. If α, β, γ are the roots of $x^3 + 3x^2 - 4x + 2 = 0$ then the equation whose roots are $\frac{1}{\alpha\beta}, \frac{1}{\beta\gamma}, \frac{1}{\gamma\alpha}$ is

1) $4x^3 - 6x^2 + 4x + 1 = 0$ 2) $4x^3 + 6x^2 - 4x - 1 = 0$

3) $4x^3 + 6x^2 - 4x + 1 = 0$ 4) $4x^3 - 6x^2 - 4x - 1 = 0$

21. If α, β, γ are the roots of $x^3 - 6x - 4 = 0$ then the equation whose roots are $\beta\gamma + \frac{1}{\alpha}, \gamma\alpha + \frac{1}{\beta}, \alpha\beta + \frac{1}{\gamma}$ is

1) $4x^3+30x^2+125=0$ 2) $x^3+15x^2-120=0$

3) $4x^3+30x^2-125=0$ 4) $4x^3-30x^2-125=0$

EXERCISE-II

Multiple Roots :

1. The non-repeated root of $x^3 + 4x^2 + 5x + 2 = 0$ is

   1) $-\frac{5}{3}$ 2) -2 3) -1 4) 1

2. If $x^4 - 5x^3 + 9x^2 - 7x + 2 = 0$ has a multiple root of order 3 then the roots are

   1) 1, 1, 1, 2 2) 1, 2, 2, 2 3) –1, –1, –1, 2 4) 1, –2, –2, –2

3. The multiple roots of $x^4 - 2x^3 - 11x^2 + 12x + 36 = 0$ are

   1) 1, 2 2) –1, 2 3) 2, 3 4) –2, 3

Conditional roots :

4. If 1, 2, 3 are the roots of $ax^3 + bx^2 + cx + d = 0$ then the roots of $ax^3 + 2bx^2 + 4cx + 8d = 0$ are

   1) 2, 4, 6 2) 3, 4, 5 3) $\frac{1}{2}, 1, \frac{3}{2}$ 4) –1,0,1

Removal of 2nd, 3rd and fractional Coefficients :

5. Number of transformed equations of $x^4+2x^3-12x^2+2x-1= 0$ by eliminating third term is

   1) 0 2) 1 3) 2 4) 3

6. To remove the 2nd term of the equation $x^4 -10x^3 +35x^2 -50x+24=0$, diminish the roots by

   1) $\frac{2}{5}$ 2) $-\frac{2}{5}$ 3) $\frac{5}{2}$ 4) $-\frac{5}{2}$

7. The transformed equation with integer coefficients whose roots are multiplied by some constant of those of $x^4 - \frac{1}{2}x^3 + \frac{3}{4}x^2 - \frac{5}{4}x + \frac{1}{16} = 0$ is

   1) $y^4 - y^3 + 3y^2 - 10y + 1 = 0$ 2) $y^4 - 24y^2 + 9y - 24 = 0$

   3) $y^4 - 2y^3 + 6y - 6 = 0$ 4) $y^4 - 5y^3 + 3y^2 - 9y + 27 = 0$

Reciprocal Equation :

8. The equation $x^4 + 3x^3 - 3x - 1 = 0$ is a reciprocal equation of

   1) class one and odd order 2) class two and even order

   3) class one and even order 4) class two and odd order

9. If 2, 3 are two roots of the reciprocal equation $6x^5-41x^4+97x^3-97x^2+41x-6 = 0$ then the other roots are

   1) 1, –2, 3 2) –1, –2, –3 3) $-1, \frac{1}{2}, \frac{1}{3}$ 4) $1, \frac{1}{2}, \frac{1}{3}$

10. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4-2x^3-8x^2+18x-9=0$ such that $\alpha + \beta = 0$ then $3\gamma + 4\delta =$

    1) 7 2) 10 3) 25 4) 15

11. The roots of the equation $x^4+px^3+qx^2+rx+s=0$ are such that sum of the two roots is equal to sum of the other two then $p^3 - 4pq =$

    1) r 2) $8r$ 3) $-8r$ 4) $-r$

12. If the roots of $x^4 - 8x^3 + 23x^2 + kx + 12 = 0$ are such that the difference of the two roots is equal to the difference of other two then $k =$

1) 28 2) –28 3) 30 4) –30

13. If α, β, γ are the roots of $x^3 - 7x + 6 = 0$ then the equation whose roots are $(\alpha+\beta)^2, (\beta+\gamma)^2, (\gamma+\alpha)^2$ is

1) $x(x^2 + 7) = 36$ 2) $x(x + 7)^2 = 36$ 3) $x(x^2 - 7) = 36$ 4) $x(x - 7)^2 = 36$

14. If α, β, γ are the roots of $x^3 + x + 1 = 0$ then the equation whose roots are $(\alpha-\beta)^2, (\beta-\gamma)^2, (\gamma-\alpha)^2$ is

1) $(x - 2)^3 + 3(x - 2) + 27 = 0$ 2) $(x + 1)^3 + 3(x + 1)^2 + 27 = 0$

3) $(x + 2)^3 + 3(x + 2) + 27 = 0$ 4) $(x - 1)^2 + 3(x - 1) + 27 = 0$

15. If α, β, γ are the roots of $x^3 + x - 2 = 0$ then the equation whose roots are $\frac{\beta}{\gamma}+\frac{\gamma}{\beta}, \frac{\gamma}{\alpha}+\frac{\alpha}{\gamma}, \frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ is

1) $4x^3 + 12x^2 + 13x + 6 = 0$ 2) $4x^3 - 12x^2 - 13x + 6 = 0$

3) $4x^3 + 12x^2 - 13x - 6 = 0$ 4) $4x^3 - 12x^2 + 13x - 6 = 0$

KEY SHEET (PRACTICE SHEET)

EXERCISE-I

1) **2**	2) **2**	3) **4**	4) **2**	5) **1**	6) **3**	7) **1**	8) **1**	9) **2**	10) **1**
11) **3**	12) **3**	13) **1**	14) **1**	15) **3**	16) **3**	17) **1**	18) **2**	19) **1**	20) **4**
21) **3**									

EXERCISE-II

1) **2**	2) **1**	3) **4**	4) **1**	5) **3**	6) **3**	7) **1**	8) **2**	9) **3**	10) **1**
11) **3**	12) **2**	13) **4**	14) **2**	15) **1**					