4. COMPLEX NUMBERS

SYNOPSIS

INFORMATION :

* The quantity $\sqrt{-1}$ denoted by i, is called Imginary unit.

$i^2 = \sqrt{-1} \times \sqrt{-1} \neq 1$, but equals to -1.

$$i^n = \begin{cases} (-1)^{n/2} \text{ if n is even integer} \\ (-1)^{\frac{n-1}{2}} \text{ if n id odd integer} \end{cases}$$

$\sqrt{a}\sqrt{b} = \sqrt{ab}$ holds only when alteast one of a, b is non-negative

* If $a, b \in R$, $a+ib$ is called complex number usually denoted by Z

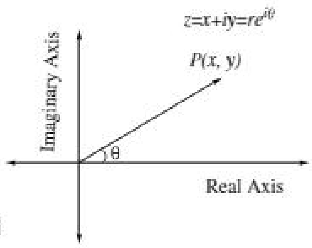
$$Z \text{ is } \begin{cases} \text{purely real if } \operatorname{Im}(Z) = 0 \\ \text{purely imaginary if } \operatorname{Re}(Z) = 0 \\ \text{imaginary if } \operatorname{Im}(Z) \neq 0 \end{cases}$$

If Z_1, Z_2 are two complex numbers (which are not purely real) then either $Z_1 = Z_2$ or $Z_1 \neq Z_2$ only holds.

$Z_1 > Z_2$, $Z_1 < Z_2$ are meaningless i.e. **there is no order among complex numbers.**

* Every Complex number $Z = a+ib$ can be represented by an ordered pair (a,b) and hence can be represented by a point with co-ordinates (a,b) in the complex plane (Argand diagram).

* **Modulus & Argument of 'Z'**



Let $Z = x+iy$, then

$|Z|$ = Distance of Z from origin = $\sqrt{x^2 + y^2}$

$|Z| \geq 0$ but $|Z| = \begin{cases} z & for\ z > 0 \\ -z & for\ z < 0 \end{cases}$ is not correct.

Let $Z = x+iy$ be represented by P, then the length of $\overline{OP}$ is $|Z|$

If OP makes angle 'θ' with positive direction of real axis, then 'θ' is argument or amplitude of Z

$$\theta = Tan^{-1}\left(\frac{y}{x}\right)$$

* **Conjugate of Z is denoted by $\overline{Z}$.**

If $Z = a+ib$, $\overline{Z} = a - ib$

Note that $Z + \overline{Z} = 2\operatorname{Re}(Z)$

$Z - \overline{Z} = 2i\operatorname{Im}(Z)$

$Z\overline{Z} = (\operatorname{Re}(Z))^2 + (\operatorname{Im}(Z))^2 = |Z|^2$

if $Z_1 = x_1+iy_1$, $Z_2 = x_2+iy_2$, then $\operatorname{Re}(Z_1\overline{Z_2}) = \operatorname{Re}(\overline{Z_1}Z_2) = x_1x_2 + y_1y_2$

Z is purely real $\Leftrightarrow Z = \overline{Z}$

Z is purely Imaginary $\Leftrightarrow Z = -\overline{Z}$

Geometrically, '$\overline{Z}$' is reflection of 'Z' in real axis. $Z, \overline{Z}, -Z, -\overline{Z}$ in order form rectangle.

* **Properties of Conjugate of Z**

$$Z_1+Z_2+.....+Z_n=\overline{Z_1}+\overline{Z_2}+.....+\overline{Z_n}$$

$$Z_1\,Z_2.......Z_n=\overline{Z_1}\,\overline{Z_2}.....\overline{Z_n}$$

$$\overline{\left(\frac{Z_1}{Z_2}\right)}=\frac{\overline{Z_1}}{\overline{Z_2}}$$

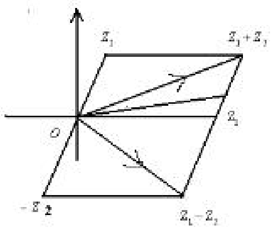
If $f(x+iy)=a+ib$, then $f(x-iy)=a-ib$

$f(z)=g(z)$ then $f(\overline{Z})=g(\overline{Z})$

* **Geometrical meaning of addition & subtraction of complex numbers**

Given the points Z_1 and Z_2, complete the parallelogram with OZ_1 and OZ_2 as adjacent sides, where O is the origin. Then the 4th vertex of this paralleogram is Z_1+Z_2

Like wise Z_1-Z_2 is the 4th vertex of parallelogram having O, Z_1-Z_2 as three of its vertices



We can see that Z_1+Z_2 & Z_1-Z_2 represent diagonals of the parallelogram having Z_1, Z_2 as adjacent sides like $\overline{a}+\overline{b},\overline{a}-\overline{b}$ represent diagonals of the parallelogram having $\overline{a},\overline{b}$ as adjacent sides

* **Complex number as a vector**

If $z=x+iy$ is represented by 'P' then $\overrightarrow{OP}$ ie position vector of P w.r.t.'O' represents 'Z'. Magnitude of vector $\overrightarrow{OP}$ is given by $|Z|$, direction of vector $\overrightarrow{OP}$ is given by argZ.

If P,Q reperesent the complex numbers Z_1,Z_2 then the vector $\overrightarrow{PQ}$ represzents Z_2-Z_1, since $\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=Z_2-Z_1$.

* **Properties of Moduli**

  * $-|Z|\le \mathrm{Re}(Z)\le |Z|$
  * $-|Z|\le \mathrm{Im}(Z)\le |Z|$
  * $|Z|\le |\mathrm{Re}(Z)|+|\mathrm{Im}(Z)|\le \sqrt{2}|Z|$
  * $|Z|=|\mathrm{Re}(Z)|+|\mathrm{Im}(Z)|$ if Z is purely real or purely imaginary
  * $\sqrt{2}|Z|=\mathrm{Re}(Z)+\mathrm{Im}(Z)$ if $|\mathrm{Re}(Z)|=|\mathrm{Im}(Z)|$
  * $|Z_1Z_2Z_3.......Z_n|=|Z_1||Z_2|.........|Z_n|$
  * $\left|\frac{Z_1}{Z_2}\right|=\frac{|Z_1|}{|Z_2|}$ if $Z_2\ne 0$

* $\left||Z_1|-|Z_2|\right| \le |Z_1+Z_2| \le |Z_1|+|Z_2|$ (Triangle inequility)

  $|Z_1+Z_2| = |Z_1|+|Z_2|$ holds if 0, Z_1, Z_2 are collinear and Z_1, Z_2 lie on same side of origin

  i.e when $\arg Z_1 - \arg Z_2 = 0$ i.e $\dfrac{Z_1}{Z_2} > 0$

  $|Z_1-Z_2| = |Z_1|+|Z_2|$ holds if 0, Z_1, Z_2 are collinear and Z_1, Z_2 lie on opposite side of origin

  i.e when $\arg Z_1 - \arg Z_2 = \pi$ i.e $\dfrac{Z_1}{Z_2} < 0$

  $|Z_1-Z_2| = \left||Z_1|-|Z_2|\right|$ holds if 0, Z_1, Z_2 are collinear and Z_1, Z_2 lie on same side of origin

  $|Z_1+Z_2| = \left||Z_1|-|Z_2|\right|$ holds if 0, Z_1, Z_2 are collinear and Z_1, Z_2 lie on opposite side of origin

* In general, $|Z_1+Z_2+......+Z_n| \le |Z_1|+|Z_2|+........+|Z_n|$

  Equality holds when 0, Z_1, Z_2.......Z_n are collinear and Z_1, Z_2, Z_3, Z_n lie on same side of origin

* $|Z_1+Z_2|^2 = (Z_1+Z_2)(\overline{Z_1}+\overline{Z_2}) = |Z_1|^2+|Z_2|^2+2\,\mathrm{Re}(Z_1\overline{Z_2})$

* $|Z_1-Z_2|^2 = (Z_1-Z_2)(\overline{Z_1}-\overline{Z_2}) = |Z_1|^2+|Z_2|^2-2\,\mathrm{Re}(Z_1\overline{Z_2})$

* $|Z_1+Z_2|^2+|Z_1-Z_2|^2 = 2(|Z_1|^2+|Z_2|^2)$ (Parallelogram law)

* If a, $b \in R$ and Z_1, Z_2 are complex numbers then

  $$|aZ_1+bZ_2|^2+|bZ_1-aZ_2|^2 = (a^2+b^2)(|Z_1|^2+|Z_2|^2)$$

* If θ is an argument of Z, then $2n\pi+\theta, n \in Z$ is general argument of Z

  The principal argument of Z lies in the interval $(-\pi, \pi]$

  Let $Z = x + iy$

  Step - I : Find $\theta = Tan^{-1}\left(\left|\dfrac{y}{x}\right|\right)$ then $\theta \in \left[0, \dfrac{\pi}{2}\right)$

  Step - II : Find the quadrant in which Z lies from the signs of x & y coordinates

  Then arg(Z) will be θ or $\pi-\theta$ or $\theta-\pi$ or $-\theta$ according as Z lies in I or II or III or IV quadrants

  If $Z = 0$, arg$Z$ is not defined

* Properties of arg(Z)

$ArgZ = \theta \Rightarrow Arg\overline{Z} = -\theta, Arg\left(\dfrac{1}{Z}\right) = -\theta, Arg\left(\dfrac{1}{\overline{Z}}\right) = \theta$ if Z is not purely real.

$Arg(Z_1Z_2) = ArgZ_1 + ArgZ_2 + 2k\pi \;\; k \in \{0, 1, -1\}$

$Arg\left(\dfrac{Z_1}{Z_2}\right) = ArgZ_1 - ArgZ_2 + 2k\pi \;\; k \in I$

$Arg(Z^n) = n\arg(Z) + 2k\pi \;\; k \in \{0, 1, -1\}$

Proper value of K must be choosen so that RHS also lies in $(-\pi, \pi]$

For eg:-

If $Z_1 = cis\frac{2\pi}{5}, Z_2 = cis\frac{4\pi}{5}$ then $Z_1Z_2 = cis\left(\frac{6\pi}{5}\right)\pi$

$Arg(Z_1Z_2) = \frac{-4\pi}{5}$ since $Z_1Z_2 \in Q_3$ but $ArgZ_1 + ArgZ_2 = \frac{6\pi}{5}$ so if K is choosen as '–1'

then RHS = $\frac{6\pi}{5} - 2\pi$ hence both sides will be same

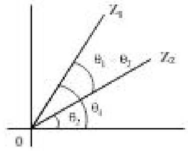
If |Z| = *r*, Arg (Z) = θ then $Z = r\,(\cos\theta + i\sin\theta)$ is called trigometric form or polar form of complex number Z. Here we should take Principal value of arg(*Z*)

If |Z| = *r*, Arg (Z) = θ, then $Z = re^{i\theta}$ is called exponential form or Euler's form of Z

* **Rotation Theorem**

If Z_1, Z_2 are two complex numbers then $\arg\left(\frac{Z_1}{Z_2}\right)$ is the angle through which OZ_2 must be turned in order that it coincides with OZ_1

$$\frac{Z_1}{Z_2} = \frac{|Z_1|e^{i\theta_1}}{|Z_2|e^{i\theta_2}} = \frac{|Z_1|}{|Z_2|}e^{i(\theta_1-\theta_2)}$$



In general, if Z_1, Z_2, Z_3 be three vertices of ΔABC described in the Counter-Clock wise sence and $\underline{|CAB} = \alpha$

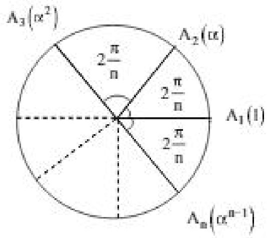
* **Demoiver's Theorem:** If $n \in N$, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

* If $n \in Q$ (set of rationals) then $\cos n\theta + i\sin n\theta$ is one of the values of $(\cos\theta + i\sin\theta)^n$. This theorem is useful in finding the roots of any complex number

* nth root of unity are Solutions of $Z^n = 1$. They are given by $Cis\left(\frac{2r\pi}{n}\right)$, $r = 0, 1, 2, ...n-1$

* If $Cis\left(\frac{2\pi}{n}\right)$ is defined by α then $1, \alpha, \alpha^2, ...\alpha^{n-1}$ are n^{th} roots of unity. The points representing these n^{th} roots are located at the vertices of a regular polygon of '*n*' sides inscrbed in a unit circle having centre at the origin. One vertex being on +ve real axis

* $1, \alpha, \alpha^2, ...\alpha^{n-1}$ are in G.P with Common ratio $e^{i\frac{2\pi}{n}}$



* $1 + \alpha + \alpha^2 + ... + \alpha^{n-1} = \left(\frac{1-\alpha^n}{1-\alpha}\right) = 0$

$$\Rightarrow \sum_{r=0}^{n-1}\cos\left(\frac{2r\pi}{n}\right) = 0, \sum_{r=0}^{n-1}\sin\left(\frac{2r\pi}{n}\right) = 0$$

* $1.\alpha.\alpha^2.......\alpha^{n-1} = \alpha^{\frac{n(n-1)}{2}} = \left(cis\frac{2\pi}{n}\right)^{\frac{n(n-1)}{2}} = (cis\pi)^{n-1}$

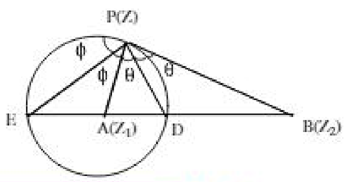
Thus product of nth root of unity $= (-1)^{n-1}\begin{cases} -1, \text{ if n is even} \\ 1, \text{ if n is odd}\end{cases}$

* If n^{th} roots of unity are denoted by $1, \alpha_1, \alpha_2, ...\alpha_{n-1}$ then

$$1^m + \alpha_1^m + \alpha_2^m + ... + \alpha_{n-1}^m = \begin{cases} 0 \text{ if } m \text{ is not integral multiple of } n \\ n \text{ if } m \text{ is integral multiple of } n\end{cases} \qquad \Rightarrow |\alpha_i - \alpha_j| = 2\sin\left|\frac{(i-j)\pi}{n}\right|$$

* If $1, \alpha_1, \alpha_2, ...\alpha_{n-1}$ are roots of $Z^n - 1 = 0$ then $\alpha_1, \alpha_2, ...\alpha_{n-1}$ are roots of $Z^{n-1} + Z^{n-2} + ... + Z + 1 = 0$
  Hence we have
  $(Z-\alpha_1)(Z-\alpha_2).....(Z-\alpha_{n-1}) = Z^{n-1} + Z^{n-2} + .. + Z + 1$ and hence $(1-\alpha_1)(1-\alpha_2).....(1-\alpha_{n-1}) = n$
  $$(1+\alpha_1)(1+\alpha_2).....(1+\alpha_{n-1}) = \begin{cases} 0 \text{ if } n \text{ is even} \\ 1 \text{ if } n \text{ is odd} \end{cases}$$
* $Z^n = P^n$ has roots $P, P\alpha_1, P\alpha_2, ...P\alpha_{n-1}$ where α_i's are n^{th} roots of unity
* Product of all values of $(z_1)^{1/n}$ is $(-1)^{n-1} z_1$
* **Cube roots of Unity :**
  $1, \dfrac{-1+i\sqrt{3}}{2}, \dfrac{-1-i\sqrt{3}}{2}$ are roots of $Z^3 = 1$
  If $\dfrac{-1+i\sqrt{3}}{2} = w$, then $\dfrac{-1-i\sqrt{3}}{2} = w^2$ or $\bar{w}$ or $\dfrac{1}{w}$
  Also $w^3 = 1$, $1+w+w^2 = 0$

  Twelth roots of unity are
  $\pm 1, \pm W, \pm W^2$
  $\pm i, \pm iW, \pm iW^2$
* In general, $1 + w^m + w^{2m} = \begin{cases} 0 \text{ if } m \in I, \text{ but not integral multiple of 3} \\ 3 \text{ if } m \text{ is integral nultiple of 3} \end{cases}$
* $1, w, w^2$ are the vertices of an equilateral triangle inscribed in the circle $|Z|=1$
* If a,b,c are non zero numbers such that $a+b+c = a^2+b^2+c^2$ then $a:b:c = 1:w:w^2$
  The linear factors of
  i) x^3+y^3 are $(x+y)$, $(x+wy)$, $(x+w^2y)$ ii) x^3-y^3 are $(x-y)$, $(x-wy)$, $(x-w^2y)$
  iii) $x^2+y^2+z^2-xy-yx-zx$ are $(x+wy+w^2z)$, $(x+w^2y+wz)$
  iv) $x^3+y^3+z^3-3xyz$ are $(x+y+z)$, $(x+wy+w^2z)$, $(x+w^2y+wz$
* **Different Loci :**
* $|Z_1 - Z_2|$ represents distance between the points Z_1, Z_2
* Locus of Z satisfying $|Z - Z_0| = r$ is circle with centre at Z_0 and of radius r
  $|Z - Z_0| < r, |Z - Z_0| > r$ represents the regions inside and outside the circle $|Z - Z_0| = r$ respectively
* $r_1 < |Z - Z_0| < r_2$ represent the region of argnad plane lying between the two concentric circles $|Z - Z_0| = r_1$, $|Z - Z_0| = r_2$
* Locus of Z satisfying $|Z - Z_1| = |Z - Z_2|$ is perpendicular bisector of line segmnet joining Z_1, Z_2
  If Z is such that $|Z - Z_1| = |Z - Z_2| = |Z - Z_3|$ then Z is circumcentre of triangle formed by the vertices Z_1, Z_2, Z_3
* If $\left|\dfrac{Z-Z_1}{Z-Z_2}\right| = K(K \neq 1, > 0)$, the locus of Z is Applonius circle with $\dfrac{KZ_2+Z_1}{K+1}, \dfrac{KZ_2-Z_1}{K-1}$ as diametical ends and with radius $\dfrac{K|Z_1-Z_2|}{|K^2-1|}$



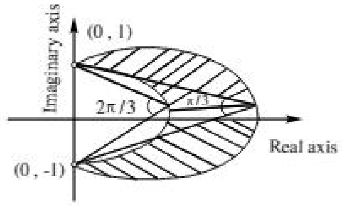
* Locus of Z satisfying $\operatorname{Re}\left(\frac{Z-Z_1}{Z-Z_2}\right)=0$ *or* $\frac{Z-Z_1}{Z-Z_2}=\lambda i$ *or* $Arg\left(\frac{Z-Z_1}{Z-Z_2}\right)=\pm\frac{\pi}{2}$ is a circle with Z_1, Z_2 as diametrical end points

  $Arg\left(\frac{Z-Z_1}{Z-Z_2}\right)=\frac{\pi}{2}\Rightarrow$ Locus of 'Z' is semi-circle with Z_1, Z_2 is extremities (Locus excludes the points Z_1, Z_2)

  If $\left|Arg\left(\frac{Z-Z_1}{Z-Z_2}\right)\right|=\frac{\pi}{2}$, Locus of Z is a circle with Z_1&Z_2 as ends of diameter.

* $Arg\left(\frac{Z-Z_1}{Z-Z_2}\right)=\theta\Rightarrow$ Locus of Z is major arc or minor arc of circle with Z_1, Z_2 as end points of a chord of cricle (Locus excludes the points Z_1, Z_2) according as $\theta\in\left(0,\frac{\pi}{2}\right)$ or $\theta\in\left(\frac{\pi}{2},\pi\right)$

* Suppose Z is such that $\frac{\pi}{3}\le\arg\left(\frac{Z+i}{Z-i}\right)\le\frac{2\pi}{3}$. Then locus of Z is shaded region in adjacent fig (excluding the points i,-i)



* Locus of Z satisfying $\operatorname{Im}\left(\frac{Z-Z_1}{Z-Z_2}\right)=0, Arg\left(\frac{Z-Z_1}{Z-Z_2}\right)=0$ or π is the line passing through Z_1, Z_2

  $$Arg\left(\frac{Z-Z_1}{Z-Z_2}\right)=\begin{cases}0 \Rightarrow \text{Locus of } Z \text{ is the line joining} \\ \quad Z_1, Z_2 \text{excluding the line segment } \overline{Z_1Z_2} \\ \pi \Rightarrow \text{Locus of } Z \text{ is the segment joining } Z_1, Z_2\end{cases}$$

* Locus of Z satisfying $\operatorname{Arg}(Z-Z_1)=\alpha$ is a ray with vertex at Z_1 and making angle 'α' with +ve direction of real axis (Locus excludes the point Z_1)

* Locus of Z satisfying $|Z-Z_1|+|Z-Z_2|=2a$ when $2a>|Z-Z_2|$ is an ellipse with Z_1, Z_2 as foci and '$2a$' as length of major axis and eccentricity $e=\frac{|Z_1-Z_2|}{2a}$. When $2a>|Z-Z_2|$ Locus of Z is the line segment joining Z_1, Z_2. There can not be any point in the argand plane such that $2a>|Z_1-Z_2|$

* Locus of Z satisfying $\left||Z-Z_1|-|Z-Z_2|\right|=2a$ is a hyperbola if $2a<|Z_1-Z_2|$, its foci are Z_1,Z_2 & $e=\frac{|Z_1-Z_2|}{2a}$ In case, $|Z_1-Z_2|=2a$, locus of Z is the line $\overleftrightarrow{Z_1Z_2}$ excluding the line segment joining Z_1 and Z_2

* Locus of Z satisfying $|Z-Z_1|^2+|Z-Z_2|^2=K$ is a circle if $k\ge\frac{1}{2}|Z_1-Z_2|^2$

* **Geometrical Applications :**

* If l,m,n are three real numbers such that $lZ_1 + mZ_2 + nZ_3 = 0$ and $l+m+n=0$, l,m,n are not all zeros simultaneously, then the points Z_1, Z_2, Z_3 are collinear

* The points Z_1, Z_2, Z_3 are collinear if $\dfrac{Z_3 - Z_1}{Z_2 - Z_1} = \dfrac{\overline{Z_3} - \overline{Z_1}}{\overline{Z_2} - \overline{Z_1}}$ *or* $\sum \overline{Z_1}(Z_2 - Z_3) = 0$

or $\begin{vmatrix} Z_1 & \overline{Z_1} & 1 \\ Z_2 & \overline{Z_2} & 1 \\ Z_3 & \overline{Z_3} & 1 \end{vmatrix} = 0$

* The equation of line joining Z_1, Z_2 in different form

1) $Z = Z_1 + t(Z_1 - Z_2)$ 't' being parameter (parametric form)

2) $\begin{vmatrix} Z & \overline{Z} & 1 \\ Z_1 & \overline{Z_1} & 1 \\ Z_2 & \overline{Z_2} & 1 \end{vmatrix} = 0$ (determinant form)

3) $Z(\overline{Z_1} - \overline{Z_2}) - \overline{Z}(Z_1 - Z_2) = \overline{Z_1}Z_2 - Z_1\overline{Z_2}$

This equation can put in the form $\bar{\alpha}Z + \alpha\overline{Z} + \beta = 0$ where β is real and α is a non zero complex constant $\bar{\alpha}Z + \alpha\overline{Z} + \beta = 0$ is the general equation of line in complex plane

Equation of line parallel to the line $\bar{\alpha}Z + \alpha\overline{Z} + \beta = 0$ is $\bar{\alpha}Z + \alpha\overline{Z} + \gamma = 0$, γ is a real number

Equation of line perpendicular to the line $\bar{\alpha}Z + \alpha\overline{Z} + \beta = 0$ is $\bar{\alpha}Z - \alpha\overline{Z} + i\gamma = 0$ where γ is a real number

* Perpendicular bisector of the line segment joining Z_1 and Z_2 is

$$(\overline{Z_1} - \overline{Z_2})Z + (Z_1 - Z_2)\overline{Z} = |Z_1|^2 - |Z_2|^2$$

* The equation of circle with centre at Z_0 and with radius r is

$|Z - Z_0| = r \Rightarrow (Z - Z_0)(\overline{Z} - \overline{Z_0}) = r^2 \Rightarrow Z\overline{Z} - Z_0\overline{Z} - \overline{Z_0}Z + Z_0\overline{Z_0} - r^2 = 0$

which is of the form $Z\overline{Z} + \bar{\alpha}Z + \alpha\overline{Z} + \beta = 0$ (called general equation of circle)

The centre of this circle is '$-\alpha$', radius is $\sqrt{\alpha\bar{\alpha} - \beta}\,(\beta \in IR)$

* The equation of circle described on the line segment joining Z_1 and Z_2 as diameter is

$$(Z - Z_1)(\overline{Z} - \overline{Z_2}) + (Z - Z_2)(\overline{Z} - \overline{Z_1}) = 0$$

* The condition for four points Z_1, Z_2, Z_3 & Z_4 to be concyclic is, the number $\dfrac{Z_3 - Z_1}{Z_3 - Z_2} \cdot \dfrac{Z_4 - Z_2}{Z_4 - Z_1}$ is real

Hence the equation of a circle through 3 non collinear points Z_1, Z_2 & Z_3 can be taken as

$\left(\dfrac{Z - Z_2}{Z - Z_1}\right)\left(\dfrac{Z_3 - Z_1}{Z_3 - Z_2}\right)$ is real $\Rightarrow \dfrac{(Z - Z_2)(Z_3 - Z_1)}{(Z - Z_1)(Z_3 - Z_2)} = \dfrac{(\overline{Z} - \overline{Z}_2)(\overline{Z_3} - \overline{Z}_1)}{(\overline{Z} - \overline{Z}_1)(\overline{Z_3} - \overline{Z}_2)}$

* Let Z_1, Z_2, Z_3 be vertices of a ΔABC, then area of $\Delta ABC = \frac{1}{4}\left|\sum \overline{Z_1}(Z_2 - Z_3)\right|$

Centroid $= \frac{Z_1 + Z_2 + Z_3}{3}$; Circumcentre $= \frac{\sum |Z_1|^2 (Z_2 - Z_3)}{\sum \overline{Z_1}(Z_2 - Z_3)}$

Orthocentre $= \frac{\sum \overline{Z_1}(Z_2 - Z_3)(Z_2 + Z_3 - Z_1)}{\sum \overline{Z_1}(Z_2 - Z_3)}$

* If Z_1, Z_2, Z_3, Z_4 are vertices of a quadrilatearl then

$$|Z_1 - Z_3||Z_2 - Z_4| \le |Z_1 - Z_2||Z_3 - Z_4| + |Z_1 - Z_4||Z_2 - Z_3|$$

Equality holds if and only if it is cyclic quadrilateral

* If $Z_1, Z_2, \ldots\ldots Z_n$ are vertices of n sided regular polygon and Z_0 is its centroid then

i) $\sum_{i=1}^{n} Z_i = nZ_0$ ii) $\sum_{i=1}^{n} Z_i^2 = nZ_0^2$

* If Z_1, Z_2, Z_3 are vertices of equilateral triangle then

i) $(Z_1 - Z_2)^2 + (Z_2 - Z_3)^2 + (Z_3 - Z_1)^2 = 0$ ii) $\sum \frac{1}{Z_1 - Z_2} = 0$

iii) $\sum Z_i^2 = \sum Z_1 Z_2$ iv) $(Z_1 - Z_2)^2 = (Z_2 - Z_3)(Z_3 - Z_1)$

Logarithm of a complex number : $\log_e^Z = \log_e^{|Z|} + i(2n\pi + \arg Z)$ where $n \in I$

DeMoivers theorem, n_{th} roots of unity etc.,

1. i) $(cis\alpha)(cis\beta) = cis(\alpha + \beta)$ ii) $(cis\alpha)/(cis\beta) = cis(\alpha - \beta)$

2. ***DeMoivre's theorem for integral index :***

If n is an integer, then

i) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ ii) $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$

iii) $(\cos\theta - i\sin\theta)^n = \cos n\theta - i\sin n\theta$ iv) $(\cos\theta - i\sin\theta)^{-n} = \cos n\theta + i\sin n\theta$

v) $\cos n\theta + i\sin n\theta = \frac{1}{\cos n\theta - i\sin n\theta} \Rightarrow \cos n\theta - i\sin n\theta = \frac{1}{\cos n\theta + i\sin n\theta}$

vi) $(\sin\theta + i\cos\theta)^n = i^n(\cos\theta - i\sin\theta)^n = i^n(\cos n\theta - i\sin n\theta)$

vii) $\sin\theta - i\cos\theta = -i\, cis\theta$ viii) $(\sin\theta - i\cos\theta)^n = (-i)^n\, cis n\theta$

ix) $1 + cis\theta = 2\cos\frac{\theta}{2} cis\frac{\theta}{2}$ x) $1 + cis(-\theta) = 2\cos\frac{\theta}{2} cis\left(\frac{-\theta}{2}\right)$

xi) $\frac{1 + cis\theta}{1 + cis(-\theta)} = cis\theta$ xii) $cis\theta - 1 = 2i\sin\frac{\theta}{2} cis\frac{\theta}{2}$

xiii) $\frac{cis\theta - 1}{cis\theta + 1} = i\tan\frac{\theta}{2}$

3. ***n^{th} root of a complex number :*** The n^{th} roots of a complex number $z = rcis\theta$ are $r^{1/n} cis\left(\frac{2k\pi + \theta}{n}\right)$ where $k = 0, 1, 2, \ldots\ldots n-1$

4. ***nth roots of unity :*** The nth roots of unity are $1,\alpha,\alpha^2,......,\alpha^{n-1}$ where $\alpha = cis\frac{2\pi}{n}$

i) The n^{th} roots of unity form a G.P with common ratio $\text{cis}\frac{2\pi}{n}$

ii) The sum of n^{th} roots of unity = 0 if $n > 2$

iii) The product of n^{th} roots of unity = $(-1)^{n-1}$

iv) Sum of n^{th} roots of any complex number z is 0 if $n > 2$

v) Product of n^{th} roots of any complex number z is $z(-1)^{n-1}$

vi) n^{th} roots of any unity are the vertices of a regular polygon of n sides which is inscribed in the unit circle with centre at origin and length of each side $2\sin\frac{\pi}{n}$ and Area $n\sin\frac{\pi}{n}\cos\frac{\pi}{n}$

vii) If $1,\alpha_1,\alpha_2,.....\alpha_{n-1}$ are n^{th} roots of unity then $\left|\alpha_i - \alpha_j\right| = 2\sin\left|\frac{(i-j)\pi}{n}\right|$

viii) Cube roots of unity are $1,\text{cis}\frac{2\pi}{3},\text{cis}\frac{4\pi}{3}$, there are generally denoted by $1,w,w^2$.

$1,w,w^2$ are the vertices of an equilateral triangle which is inscribed in a unit circle with centre at origin length of its side is $\sqrt{3}$ and area is $\frac{3\sqrt{3}}{4}$ sq.units

ix) Fourth roots of unity are $1,i,-1,-i$. These are denoted by $\text{cis}0,\text{cis}\frac{\pi}{2},\text{cis}\pi$ and $\text{cis}\left(\frac{-\pi}{2}\right)$ these are the vertices of a square inscribed in a unit circle with centre at origin and length of each side is $\sqrt{2}$ units and Area 2 square units

5. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then

i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$

ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$

iii) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$

iv) $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$

v) $\cos 2^n\alpha + \cos 2^n\beta + \cos 2^n\gamma = 0$

vi) $\sin 2^n\alpha + \sin 2^n\beta + \sin 2^n\gamma = 0$

vii) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 3/2$

viii) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3/2$

ix) $\cos(2\alpha-\beta-\gamma) + \cos(2\beta-\gamma-\alpha) + \cos(2\gamma-\alpha-\beta) = 3$

x) $\sin(2\alpha-\beta-\gamma) + \sin(2\beta-\gamma-\alpha) + \sin(2\gamma-\alpha-\beta) = 0$

6. ***Demoivre's Theorem for rational index:***

If $\frac{p}{q}$ $(q > 1)$ is a rational number then $\cos\frac{p\theta}{q} + i\sin\frac{p\theta}{q}$ is one root of the q^{th} roots of $(\cos\theta + i\sin\theta)^p$

Amp. mod–Amp forms & Euler forms :

$1 = \text{cis}0 = e^{i0}$ $\quad i = \text{cis}\frac{\pi}{2} = e^{\frac{i\pi}{2}}$ $\quad -1 = \text{cis}\,\pi = e^{i\pi}$ $\quad -i = \text{cis}\left(-\frac{\pi}{2}\right) = e^{\frac{-i\pi}{2}}$

$1+i = \sqrt{2}\text{cis}\frac{\pi}{4} = \sqrt{2}e^{\frac{\pi}{4}i}$ $\quad 1-i = \sqrt{2}\text{cis}\left(-\frac{\pi}{4}\right) = \sqrt{2}e^{-\frac{\pi}{4}i}$

$-1+i = \sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right) = \sqrt{2}e^{\frac{3\pi}{4}i}$ $\quad -1-i = \sqrt{2}\text{cis}\left(\frac{-3\pi}{4}\right) = \sqrt{2}e^{-\frac{3\pi}{4}i}$

LECTURE SHEET

EXERCISE-I

Modulus, Argument, Conjugate of Complex Numbers SQRT, Cube Root; Problesm on modulus; Problems on Argument

LEVEL-I (MAIN)

Single answer type questions

1. If $\frac{3}{2+\cos\theta+i\sin\theta}=x+iy$ then $(x-1)(x-3)=$

   1) y^2 2) $-y^2$ 3) 0 4) 1

2. If $(x+iy)^{\frac{1}{3}}=a+ib$ then $\left(\frac{x}{a}\right)+\left(\frac{y}{b}\right)=$

   1) a^2-b^2 2) $4(a^2-b^2)$ 3) $2(a^2-b^2)$ 4) $2(a^2+b^2)$

3. If $\frac{\sin(x/2)+\cos(x/2)+i\tan x}{1+2i\sin(x/2)}$ is real then $x=$

   1) $n\pi$ or $n\pi+\frac{\pi}{4}$ 2) $2n\pi$ or $n\pi+\frac{\pi}{4}$

   3) $n\pi$ or $n\pi+\frac{\pi}{2}$ 4) $n\pi$ or $n\pi+\frac{\pi}{6}$

4. If $(1-i)(1-2i)$.........$(1-ni)=x-iy$, then 2. 5. 10.... $(1+n^2)=$

   1) x^2+y^2 2) $x+y$ 3) x^2-y^2 4) $x-y$

5. If $(1+\cos\theta-i\sin\theta)(1+\cos 2\theta+i\sin 2\theta)=x+iy$ then $x^2+y^2=$

   1) $16\cos^2\theta\sin^2\left(\frac{\theta}{2}\right)$ 2) $16\sin^2\theta\cos^2\left(\frac{\theta}{2}\right)$

   3) $16\sin^2\theta\sin^2\left(\frac{\theta}{2}\right)$ 4) $16\cos^2\theta\cos^2\left(\frac{\theta}{2}\right)$

6. If $|z_1|=|z_2|=|z_3|=.......=|z_n|=1$, then $|z_1+z_2+z_3+.......+z_n|=$

   1) $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}+....+\frac{1}{z_n}\right|$ 2) $\left|\frac{1}{z_1}-\frac{1}{z_2}+\frac{1}{z_3}-....+\frac{1}{z_n}\right|$

   3) $\left|\frac{1}{z_1^2}+\frac{1}{z_2^2}+\frac{1}{z_3^2}+....+\frac{1}{z_n^2}\right|$ 4) $\left|\frac{1}{z_1^2}-\frac{1}{z_2^2}+\frac{1}{z_3^2}-....+\frac{1}{z_n^2}\right|$

7. If z_1, z_2 are two complex numbers satisfying $\left|\frac{z_1-3z_2}{3-z_1\bar{z}_2}\right|=1$, $|z_1|\neq 3$, then $|z_2|=$

   1) 1 2) 2 3) 3 4) 4

8. $\sqrt{4ab-2(a^2-b^2)i}=$

1) $\pm\left\{\dfrac{(a-b)\ \ +i(a-b)}{2}\right\}$ 2) $\pm\{(i+5b)+(a-4b)\}$

3) $\pm\{(a+b)-i(a-b)\}$ 4) $\pm\{(i+b)-i(a-b)\}$

9. $\sqrt{x+i\sqrt{x^4+x^2+1}}=$

1) $\dfrac{\sqrt{x^2+x+1}+i\sqrt{x^2-x+1}}{\sqrt{2}}$ 2) $\dfrac{\sqrt{x^2+x+1}-i\sqrt{x^2-x+1}}{2}$

3) $\sqrt{x+1}+i\sqrt{x-1}$ 4) $\sqrt{x-1}+i\sqrt{x+1}$

10. If $\sqrt{x+iy}=\pm(a+ib)$ then $\sqrt{-x-iy}=$

1) $\pm(b+ia)$ 2) $\pm(a-ib)$ 3) $\pm(a+ib)$ 4) $\pm(b-ia)$

11. If z_1 and z_2 are two complex numbers satisfying the equation $\left|\dfrac{z_1+z_2}{z_1-z_2}\right|=1$, then $\dfrac{z_1}{z_2}$ is a number which is

1) Positive real 2) Negative real

3) Zero or purely imaginary 4) None of these

12. If $|z_1-1|<1, |z_2-2|<2$ and $|z_3-3|<3$, then $|z_1+z_2+z_3|$

1) is less than 6 2) is greater than 6 3) is less than 12 4) lies between 6 and 12

13. If z is a complex number such that $-\dfrac{\pi}{2}\le \arg Z\le\dfrac{\pi}{2}$ then which of the following in equality is true.

1) $|z-\vec{z}|\le|z|\,|\arg z-\arg\vec{z}|$ 2) $|z-\vec{z}|\le|\arg z-\arg\vec{z}|$

3) $|z-\vec{z}|>|z|\,|\arg z-\arg\vec{z}|$ 4) $|z-\vec{z}|=1$

14. If $Z=\begin{vmatrix}1 & 1-2i & 3+5i\\ 1+2i & -5 & 10i\\ 3-5i & -10i & 11\end{vmatrix}$ then

1) Z is purely imaginary 2) Z is purely real

3) $Z=0$ 4) $\arg Z=\dfrac{-\pi}{4}$

15. If $z=i^{i^i}$ where $i^2=-1$ then $|z|=$

1) 1 2) $e^{-\pi/2}$ 3) $e^{-\pi}$ 4) $e^{\pi/2}$

Numerical value type questions

16. The minimum value of $|z-1|+|z-2|+|z-3|+|z-4|+|z-5|$ is

17. If $Z = i\,Log(2-\sqrt{3})$ then $\cos Z =$

18. If Complex Number Z satisfies the condition $|Z-2+i| \le 1$, than maximum distance of origin from $A(4+i(2-z))$is

19. If $Z=(3+7i)(\lambda+\mu i)$ where $\lambda, \mu \in I-\{\sigma\}$ and $i=\sqrt{-1}$ is purely imagainary then minimum value of $|z|^2$ is

20. If $\left|Z-\frac{6}{2}\right|=5$ and maximum and minimum values of $|z|$ are λ and μ respectively. then $\frac{\lambda^{\mu}+\mu^{\lambda}}{\lambda^{\mu}-\mu^{\lambda}}=$

21. Number of ordered pairs of (a,b) of real mumbers such that $(a+ib)^{2020}=a-ib$ holds good is

22. If z_1, z_2, z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=1$ then $|z_1-z_2|^2+|z_2-z_3|^2+|z_3-z_1|^2$ can not exceed.

23. If a, b, c a_1, b_1 and c_1, are non-zero complex numbers satisfying $\frac{a}{a_1}+\frac{b}{b_1}+\frac{c}{c_1}=1+i$ and $\frac{a_1}{a}+\frac{b_1}{b}+\frac{c_1}{c}=0$ where $i=\sqrt{-i}$, the value of $\frac{\left|\frac{a^2}{a_1^2}+\frac{b^2}{b_1^2}+\frac{c^2}{c_1^2}\right|}{1000}=$

LEVEL-II (ADVANCED)

Single answer type questions

1. If the complex number Z satisfying $Z+|Z|=2+8i$ then value of $|Z|=$

   a) 8 b) 17 c) 15 d) 24

2. If $|Z+2-i|=5$ then maximum value of $|3Z+9-7i|=$

   a) 20 b) 15 c) 5 d) 16

3. If z and w are two complex numbers such that $\overline{z}+i\overline{w}=0$ and $\arg(zw)=\pi$ then $\arg(z)=$

   a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{4}$ d) $\frac{5\pi}{4}$

4. If Z_1 and Z_2 are two complex numbers such that $Z_1^2+Z_2^2 \in R$ and $Z_1(Z_1^2-3Z_2^2)=2$; $Z_2(3Z_1^2-Z_2^2)=11$ then $Z_1^2+Z_2^2=$

   a) 5 b) 125 c) 25 d) 15

5. Let $z_r (1 \le r \le 4)$ be complex numbers such that $|z_r|=\sqrt{r+1}$ and $|30z_1+20z_2+15z_3+12z_4|=k|z_1z_2z_3+z_2z_3z_4+z_3z_4z_1+z_4z_1z_2|$ Then the value of k equals

   a) $|z_1z_2z_3|$ b) $|z_2z_3z_4|$ c) $|z_4z_1z_2|$ d) None of these

6. If $|z_1|=2$, $|z_2|=3$, $|z_3|=4$ and $|2z_1+3z_2+4z_3|=4$, then absolute value of $8z_2z_3+27z_3z_1+64z_1z_2$ equals

   a) 24 b) 48 c) 72 d) 96

7. If the ratio $\frac{1-z}{1+z}$ is purely imaginary, then

a) $0 < |z| < 1$
b) $|z| = 1$
c) $|z| > 1$
d) bounds for $|z|$ can not be decided

8. If P and Q are represented by the numbers z_1 and z_2 such that $\left|\frac{1}{z_2}+\frac{1}{z_1}\right| = \left|\frac{1}{z_2}-\frac{1}{z_1}\right|$, then the circumcentre of ΔOPQ, (where O is the origin) is

a) $\frac{z_1 - z_2}{2}$
b) $\frac{z_1 + z_2}{2}$
c) $\frac{z_1 + z_2}{3}$
d) $z_1 + z_2$

9. If z_1, z_2 are two complex numbers satisfying the equation $\left|\frac{z_1+z_2}{z_1-z_2}\right| = 1$, then $\frac{z_1}{z_2}$ is a number which is

a) Positive real
b) Negative real
c) Zero
d) Lying on imaginary axis

10. Let z be a complex number satisfying $|z+16| = 4|z+1|$ then

a) $|z| = 4$
b) $|z| = 5$
c) $|z| = 6$
d) $3 < |z| < 6$

11. If $c^2 + s^2 = 1$, then $\frac{1+c+is}{1+c-is}$ is equal to

a) $c - is$
b) $c + is$
c) $s + ic$
d) $s - ic$

12. If $\log_{\tan 30°}\left(\frac{2|z|^2 + 2|z| - 3}{|z|+1}\right) < -2$ then

a) $|z| < \frac{3}{2}$
b) $|z| > \frac{3}{2}$
c) $|z| > 2$
d) $|z| < 2$

13. If $|z| = \min\{|z-1|, |z+1|\}$ then $|z+\bar{z}| =$

a) 1
b) 2
c) 3
d) 4

14. If z_1 is a root of the equation $a_0 z^n + a_1 z^{n-1} + ... + a_{n-1} z + a_n = 3$, where $|a_i| < 2$ for $i = 0, 1, ...n$. Then

a) $|z_1| > \frac{1}{3}$
b) $|z_1| < \frac{1}{4}$
c) $|z_1| > \frac{1}{4}$
d) $|z| < \frac{1}{3}$

15. If $(a+ib)^5 = \alpha + i\beta$ then $(b+ia)^5$ is equal to

a) $\beta - i\alpha$
b) $\beta + i\alpha$
c) $\alpha - \beta$
d) $-\alpha - i$

16. If $(1 + i)(1 + 2i)(1 + 3i) .....(1 + ni) = \alpha + i\beta$, then $2 \cdot 5 \cdot 10.....(1+n^2)$ is equal to (where $\alpha, \beta, n \in R$)

a) $\alpha - i\beta$
b) $\alpha^2 - \beta^2$
c) $\alpha^2 + \beta^2$
d) none of these

17. If $z = \left(1+\sqrt{3}i\right)^{10} + \left(1-\sqrt{3}i\right)^{10}$, then $\arg z$ is

a) $\frac{\pi}{2}$
b) π
c) $\frac{\pi}{4}$
d) none of these

18. If Z_1, Z_2 are two complex numbers such that $|Z_1| = 1$, $|Z_2| = 1$ then the maximum value of $|Z_1 + Z_2| + |Z_1 - Z_2|$ is

a) 2
b) $2\sqrt{2}$
c) 4
d) none of these

19. Let z be a complex number satisfying $|z+16|=4|z+1|$ then

a) $|z|=4$ b) $|z|=5$ c) $|z|=6$ d) $3<|z|<6$

20. If (a, b) lies on $x^2 + y^2 = 100$ and z_1, z_2 lies on curve $z^2 + z^{-2} = 12$ then $\dfrac{|az_1 - bz_2|^2 + |bz_1 + az_2|^2}{10\left(|z_1|^2 + |z_2|^2\right)} =$

a) 1 b) 10 c) 100 d) $\sqrt{10}$

21. If $|z|=1$ and $w=\dfrac{z-1}{z+1},(z \neq -1)$ then Re(w) is

a) 0 b) $\dfrac{1}{|z+1|^2}$ c) $\dfrac{z}{|z+1|}\dfrac{1}{|z+1|^2}$ d) $\dfrac{\sqrt{2}}{|z+1|^2}$

22. The least value of $n \in N$ such that $(1+i)^n \pi = (1-i)^n 2 \text{ Sin}^{-1}\left(\dfrac{1+x^2}{2x}\right)$

a) 2 b) 4 c) 8 d) 16

More than one correct answer type questions

23. If a and b are complex numbers, then $|a+\sqrt{a^2-b^2}|+|a-\sqrt{a^2-b^2}|$ is

a) Equal to $|a+b|+|a-b|$

b) Equal to $|a+b|-|a-b|$

c) $\leq \dfrac{1}{2}\left(|a|^2+|b|^2\right)$

d) $\leq 2(|a|+|b|)$

24. If $z=\sqrt{20i-21}+\sqrt{21-20i}$, then the principal value of arg 'z' can be

a) $\dfrac{\pi}{4}$ b) $\dfrac{3\pi}{4}$ c) $-\dfrac{3\pi}{4}$ d) $-\dfrac{\pi}{4}$

25. If $\arg(z^{3/8})=(1/2)\arg(z^2+\bar{z}z^{1/2})$ then which of the following is (are) true ?

a) $|z|=1$ b) z is real c) z is pure imaginary d) $z^{1/2}=1$

26. For any complex number $z = x + iy$, define $(z)=|x|+|y|$. If z_1 and z_2 are any complex numbers, then

a) $(z_1+z_2)\leq(z_1)+(z_2)$

b) $(z_1+z_2)=(z_1)+(z_2)$

c) $(z_1+z_2)\geq(z_1)+(z_2)$

d) $|(z_1+z_2)|\leq|(z_1)|+|(z_2)|$

27. If for any 2 complex nos. z_1 and z_2, ($|z_2| \neq 1$) $\sqrt{z_1}-i\sqrt{z_2}=|z_2|\sqrt{z_1}+i|z_1|\sqrt{z_2}$, then which of the following options are correct

a) $z_1\bar{z}_2+1=0$ b) $z_1+z_2=0$ c) $\arg\left(\dfrac{z_1}{z_2}\right)=\pi$ d) $\arg\left(\dfrac{z_1}{z_2}\right)=\dfrac{\pi}{2}$

28. Let z satisfies arg $(z-3-4i)$ = arg $(z-1+2i)$ + arg (iz) + arg $(\bar{z})$ and z_1 is a fixed complex number such that $|z-z_1|$ is constant then

a) z lies on a semi-circle

b) $|z_1^8|$ is equal to 625

c) argument of z_1 is $\dfrac{\pi}{4}$

d) arg of (z_1-2) is $\dfrac{\pi}{2}$

29. For points z_1, z_2, z_3, z_4 in complex plane such that $|z_1|<1, |z_2|=1$ and $|z_3|\le 1$ and $z_3=\frac{z_2(z_1-z_4)}{\bar{z}_1 z_4-1}$, then $|z_4|$ can be

a) 2 b) $\frac{2}{5}$ c) $\frac{1}{3}$ d) $\frac{5}{2}$

30. If z_1 lies on $|z| = 1$ and z_2 lies on $|z| = 2$, then

a) $3\le|z_1-2z_2|\le 5$ b) $1\le|z_1+z_2|\le 3$ c) $|z_1-3z_2|\ge 5$ d) $|z_1-z_2|\ge 1$

31. If $z_1=a+ib$; $z_2=c+id$ are complex numbers such that $|z_1|=|z_2|=1$ and $\mathrm{Re}(z_1\bar{z}_2)=0$, then the pair of complex numbers $\omega_1=a+ic$ and $\omega_2=b+id$ satisfies

a) $|\omega_1|=1$ b) $|\omega_2|=1$ c) $\mathrm{Re}(\omega_1\bar{\omega}_2)=0$ d) $\mathrm{Re}(\omega_1\bar{\omega}_2)=1$

Linked comprehension type questions

Passage - I :

Let α,β,γ be three real numbers such that $\alpha^2+\beta^2+\gamma^2-\gamma=0$ and $z=\frac{\alpha+i\beta}{1-\gamma}$

32. $|z|^2$ equals

a) γ b) $1-\gamma$ c) $\frac{\gamma}{1-\gamma}$ d) $\frac{1-\gamma}{\gamma}$

33. α equals

a) $\frac{z+\bar{z}}{2(1+|z|^2)}$ b) $\frac{(\bar{z}-z)i}{2(1+|z|^2)}$ c) $\frac{(z-\bar{z})i}{2(1+|z|^2)}$ d) $\frac{(z-\bar{z})i}{2(1+|z|^2)}$

34. β equals

a) $\frac{z-\bar{z}}{2(1+|z|^2)}$ b) $\frac{(\bar{z}-z)i}{2(1+|z|^2)}$ c) $\frac{z+\bar{z}}{2(1+|z|^2)}$ d) $\frac{(z-\bar{z})i}{2(1+|z|^2)}$

Passage-II :

Let Z is any complex number satisfying the equation $z^2+pz+q=0$ $(q\in C)$ and both the roots of the equation have unit modulus

35. Modulus of q is

a) $\frac{1}{2}$ b) 2 c) 1 d) 3

36. Which of following is true

a) $|p|\le 2$ b) $|p|\le 1$ c) $|p|\ge 1$ d) $|p|\ge 2$

37. If $\arg(p)=\alpha$ and $\arg(q)=\beta$, then relation between α and β is

a) $\alpha=2\beta$ b) $2\alpha=\beta$ c) $\alpha=\frac{1}{\beta}$ d) $\alpha=-\beta$

Passage-III :

Given $z^{-1}=(a+ib)^{-1}+(a+ic)^{-1}$, $z=x+iy, a,b,c$ being real with $a+ib$, $a+ic$ not zero.

38. The value of $x^2 + y^2$ equals

a) $\dfrac{(a^2-b^2)(a^2-c^2)}{4a^2+(b+c)^2}$ b) $\dfrac{(a^2+b^2)(a^2+c^2)}{4a^2+(b+c)^2}$ c) $\dfrac{(a^2+c^2)(a+b)}{a^2+b^2}$ d) $a^2+b^2+c^2$

39. The value of $(x-a)^2+y^2$ equals

a) $\dfrac{(a^2-bc)^2}{4a^2+(b+c)^2}$ b) $\dfrac{a^2}{4a^2+c^2}$ c) $\dfrac{a^2+b^2+c^2}{a^2+b^2+4c^2}$ d) $\dfrac{(a^2+bc)^2}{4a^2+(b+c)^2}$

40. The value of Re(z) i.e., 'x' equals

a) $\dfrac{2a^2+b^2+c^2}{4a^2+(b+c)^2}$ b) $\dfrac{a(2a^2+b^2+c^2)}{4a^2+b^2}$ c) $\dfrac{2(2a^2+b^2+c^2)}{a^2+b^2+c^2}$ d) $\dfrac{a(2a^2+b^2+c^2)}{4a^2+(b+c)^2}$

Passage-IV :

If P is the point (a, b) on the Argand plane corresponding to the complex number $Z = a + ib$, then $\vec{OP} = a\hat{i} + b\hat{j}$. $\therefore \left|\vec{OP}\right| = \sqrt{a^2+b^2} = |Z|$ and arg(Z) = direction of the vector $\vec{OP} = \tan^{-1}\left(\dfrac{b}{a}\right)$. Therefore, complex number Z can also be represented by $\vec{OP}$.

41. Let (r,θ) denote the point $r(\cos\theta + i\sin\theta)$ in the Argand plane. If $a = (1,\alpha)$, $b = (1,\beta)$, $c = (1,\gamma)$ and $a + b + c = 0$ then $a^{-1}+b^{-1}+c^{-1} =$

a) 1 b) -1 c) 0 d) None of these

42. Let Z_1, Z_2 be two complex numbers such that $|Z_1+Z_2| = |Z_1|+|Z_2|$, then

a) $\arg(Z_1) = \arg(Z_2)$ b) $\arg(Z_1) + \arg(Z_2) = 0$

c) $\arg(Z_2/Z_1) = 1$ d) None of these

43. If Z_1, Z_2 and Z_3, Z_4 are two pairs of conjugate complex numbers, then $\arg\left(\dfrac{Z_1}{Z_4}\right)+\arg\left(\dfrac{Z_2}{Z_3}\right) =$

a) 0 b) $\dfrac{\pi}{2}$ c) $\dfrac{3\pi}{2}$ d) π

Matrix matching type questions

44. If $z = \cos\alpha + i\sin\alpha, 0 < \alpha < \pi/6$, then the principal argument of:

COLUMN - I	COLUMN - II
A) $1+z^3$	p) $2\alpha - \dfrac{\pi}{2}$
B) $1-z^4$	q) $\dfrac{\pi}{2}+\dfrac{\alpha}{2}$
C) $\dfrac{1+z^3}{1-z^4}$	r) $\dfrac{3\alpha}{2}$
D) $\dfrac{z^4-1}{z^3+1}$	s) $\dfrac{\pi}{2}-\dfrac{\alpha}{2}$

45. **COLUMN - I** **COLUMN - II**

COLUMN - I	COLUMN - II
A) If $Z_1 = 1+i, Z_2 = \sqrt{3}+i$, then arg $\left(\frac{Z_1}{Z_2}\right)^4 =$	p) $\frac{3\pi}{4}$
B) If $\lvert Z-i\rvert = \sqrt{2}$. Then $\arg\left(\frac{Z-1}{Z+1}\right) =$	q) $\pm\frac{\pi}{2}$
C) If $\overline{Z} + i\overline{W} = 0$, arg(Z.W) = π, then arg(Z) =	r) $\frac{\pi}{3}$
D) If $\lvert Z^2-1\rvert = \lvert Z^2\rvert + 1$, then arg(Z) =	s) $\frac{\pi}{4}$

Integer answer type questions

46. If $z \neq 0$ and $2+\cos\theta + i\sin\theta = \frac{3}{z}$, then the value of $2(z+\overline{z}) - |z|^2$ is

47. The number of complex numbers z satisfying the conditions $\left|(z/\overline{z}) + (\overline{z}/z)\right| = 1, |z| = 1$ and $\arg z \in (0, 2\pi)$ is

48. If $a, b \in R$, then the number of complex numbers $a + ib$ for which $(a+ib)^2 = (a-ib)^3$ is

49. Number of solution of the equation $z^3 = \overline{z}\, i|z|$ are _____

50. If α, β, γ and a, b, c are complex numbers such that $\frac{\alpha}{a} + \frac{\beta}{b} + \frac{\gamma}{c} = 1+i$ and $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 0$ then the value of $\left|\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} + \frac{\gamma^2}{c^2}\right|$ is equal to

51. The quadratic equation $z^2 + (a+ib)z + (c+id) = 0$ (a, b, c, d are real and $bd \neq 0$) has equal roots. Then the value of ab/d is ________

EXERCISE-II

Demoivers theorem, n_{th} roots of unity etc.,

LEVEL-I (MAIN)

Single answer type questions

1. $[(\cos\alpha - \cos\beta) + i(\sin\alpha - \sin\beta)]^n + [(\cos\alpha - \cos\beta) - i(\sin\alpha - \sin\beta)]^n =$

1) $2^{n+1}.\sin^n\left(\frac{\alpha-\beta}{2}\right)\cos\left[n\left(\frac{\pi}{2} + \frac{\alpha+\beta}{2}\right)\right]$ 2) $2^{n-1}.\sin^n\left(\frac{\alpha-\beta}{2}\right)\cos\left[n\left(\frac{\pi}{2} + \frac{\alpha+\beta}{2}\right)\right]$

3) $2^{n+1}.\sin^n\left(\frac{\alpha+\beta}{2}\right)\cos\left[n\left(\frac{\pi}{2} + \frac{\alpha-\beta}{2}\right)\right]$ 4) $2^{n-1}.\sin^n\left(\frac{\alpha+\beta}{2}\right)\cos\left[n\left(\frac{\pi}{2} + \frac{\alpha-\beta}{2}\right)\right]$

2. If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$, then $\frac{x}{y} - \frac{y}{x} =$

1) $2i\sin(\alpha+\beta)$ 2) $2i\sin(\alpha-\beta)$ 3) $2i\cos(\alpha+\beta)$ 4) $2i\cos(\alpha-\beta)$

3. If n is a positive integer and $(1+i\sqrt{3})^n+(1-i\sqrt{3})^n=2^{n+1}\cos\theta$, then the value of θ is

1) $n\pi/3$ 2) $n\pi/2$ 3) $n\pi/4$ 4) $n\pi/6$

4. If n is a positive integer, then the $(a+ib)^{m/n}+(a-ib)^{m/n}=$

1) $2(a^2+b^2)^{m/2n}\cos\left[\frac{m}{n}\mathrm{Tan}^{-1}\frac{b}{a}\right]$ 2) $2(a^2+b^2)^{m/2n}\sin\left[\frac{m}{n}\mathrm{Tan}^{-1}\frac{b}{a}\right]$

3) $2(a^2+b^2)^{m/2n}\cot\left[\frac{m}{n}\mathrm{Tan}^{-1}\frac{b}{a}\right]$ 4) $2(a^2+b^2)^{m/2n}\tan\left[\frac{m}{n}\mathrm{Tan}^{-1}\frac{b}{a}\right]$

5. The product of the four values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{3/4}$ is

1) -1 2) 1 3) i 4) $-i$

6. If $(\sqrt{3}+i)^n=2^n$, $n\in I$, the set of integers, then n is a multiple of

1) 6 2) 10 3) 9 4) 12

7. If $x_n=\cos\frac{\pi}{2^n}+i\sin\frac{\pi}{2^n}$, then $\prod_{n=1}^{\infty}x_n=$

1) -1 2) 1 3) $1/\sqrt{2}$ 4) $i/\sqrt{2}$

8. The value of $1+\sum_{k=0}^{14}\left\{\cos\frac{(2k+1)\pi}{15}+i\sin\frac{(2k+1)\pi}{15}\right\}$ is

1) 0 2) 1 3) -1 4) i

9. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ then $x^{\sin(2\alpha-\beta-\gamma)}.x^{\sin(2\beta-\gamma-\alpha)}.x^{\sin(2\gamma-\alpha-\beta)}=$

1) 0 2) 1 3) x 4) x^3

10. If $A+B+C=\pi$ and $\cos A+\cos B+\cos C=0=\sin A+\sin B+\sin C$ then $\cos 3A+\cos 3B+\cos 3C=$

1) 3 2) -3 3) 0 4) 1

11. The common roots of the equation $x^{12}-1=0$, $x^4+x^2+1=0$ are

1) $\omega,\ \omega^2$ 2) $-\omega,\ -\omega^2$ 3) $\pm\omega,\ \pm\omega^2$ 4) $\pm\omega^2,\pm\omega^5$

12. The product of all n^{th} roots of unity ($n>1$) is

1) 0 2) $(-1)^{n-1}$ 3) -1 4) 1

13. If $1,\alpha_1,\alpha_2,.....\alpha_{n-1}$ are the n^{th} roots of unity then $(1-\alpha_1)(1-\alpha_2).....(1-\alpha_{n-1})=$

1) $n-1$ 2) n 3) -1 4) 1

14. If $1,\alpha_1,\alpha_2,.....\alpha_{n-1}$ are the n^{th} roots of unity and n is an odd natural number then $(1+\alpha_1)(1+\alpha_2).....(1+\alpha_{n-1})=$

1) 1 2) -1 3) 0 4) -2

15. If α is an n^{th} root of unity, the $1+2\alpha+3\alpha^2+.....+n\alpha^{n-1}=$

1) $\frac{n}{1-\alpha}$ 2) $\frac{-n}{1-\alpha}$ 3) $\frac{-n}{(1-\alpha)^2}$ 4) $\frac{n}{(1-\alpha)^2}$

16. If $x^2+x+1=0$, then the value of $\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+....+\left(x^{27}+\frac{1}{x^{27}}\right)^2$ is

1) 27 2) 72 3) 45 4) 54

17. If ω is a complex cube root of unity, then the value of $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}$ is

1) 0 2) –1 3) 1 4) 2

18. If ω is an imaginary cube root of unity, then the value of the expression $1(2-\omega)(2-\omega^2)+2(3-\omega)(3-\omega^2)+...+(n-1)(n-\omega)(n-\omega^2)$ is

1) $\frac{1}{4}n^2(n+1)^2-n$ 2) $\frac{1}{4}n^2(n+1)^2+n$ 3) $\frac{1}{4}n^2(n+1)-n$ 4) $\frac{1}{4}n(n+1)^2-n$

19. If $\omega\neq 1$ is a cube root of unity satisfying $\frac{1}{a+w}+\frac{1}{b+w}+\frac{1}{c+w}=2w^2$ and $\frac{1}{a+w^2}+\frac{1}{b+w^2}+\frac{1}{c+w^2}=2w$ then the value of $\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}$ is

1) 2 2) –2 3) ω^2 4) ω

20. If z is a complex number satisfying $z^4+z^3+2z^2+z+1=0$, then the set of possible values of $|z|$ is

1) {1, 2} 2) {1} 3) {1, 2, 3} 4) {1, 2, 3, 4}

21. If $|z|=1$ then $\left(\frac{1+z}{1+\bar{z}}\right)^n+\left(\frac{1+\bar{z}}{1+z}\right)^n$ is equal to

1) $2\cos(n(\arg z))$ 2) $2\sin(\arg z)$ 3) $2\cos\left(n\left(\arg\left(\frac{z}{2}\right)\right)\right)$ 4) $2\sin\left(n\left(\arg\left(\frac{z}{2}\right)\right)\right)$

22. If $\omega\neq 1$ be a cube root of unity and $(1+\omega)^7=l+m\omega$, then the value of $l+m=$

1) 0 2) 1 3) 2 4) –1

23. If $2w+1=\sqrt{3}i$, then the determinant $\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & -1-w^2 & w^2\\ 1 & w^2 & w^4\end{vmatrix}$ is

1) $3w$ 2) $3w(w-1)$ 3) $3w^2$ 4) $3w(1-w)$

24. If w is complex cube root of 1, then $\cos\left(\left((1-w)(1-w^2)+(2-w)(2-w^2)+....+(10-w)(10-w^2)\right)\frac{\pi}{900}\right)=$

1) –1 2) 0 3) 1 4) $\frac{\sqrt{3}}{2}$

25. $S_1 \equiv$ If $z^2+z+1=0$ and $n \in N$ then $\sum_{k=1}^{n} z^k + z^{-k} = -n+3\left[\frac{n}{3}\right]$ where [ ] gif

$S_2 \equiv$ If $w \neq 1$ is a cube root of 1 then $w^k + w^{-k} = \begin{cases} -1 \text{ if } k \text{ is not a multiple of 3} \\ 2 \text{ if } k \text{ is a multiple of 3} \end{cases}$ then

1) S_1, S_2 both true and S_2 explains S_1

2) S_1, S_2 both true but S_2 is not a correct explanation of S_1

3) S_1 true , S_2 False

4) S_1 false , S_2 true

26. The area of the figure formed by the roots of $z^5 = (z-1)^5$ in argand plane is

1) $32\cos\frac{2\pi}{5}$ 2) $1-\cos^2\frac{2\pi}{5}-\sin^2\frac{2\pi}{5}$ 3) $\cot\frac{\pi}{5}$ 4) $2\cot\frac{\pi}{5}$

27. If α is non real fifth root of 1 then $\log_2\left|1+\alpha+\alpha^2+\alpha^3-\frac{1}{\alpha}\right|$

1) 0 2) $\frac{1}{2}$ 3) 1 4) 2

28. If 1, $\alpha_1, \alpha_2, ...\alpha_{99}$ are roots of $Z^{100}=1$ then $\sum_{1\le i<j\le 99}\alpha_i\alpha_j$ equal to

1) 0 2) 1 3) 99 4) 100

29. If $\cos x+2\cos y+3\cos z=\sin x+2\sin y+3\sin z=0$ then the value of $\sin 3x+8\sin 3y+27\sin 3z$ is

1) $\sin(x+y+z)$ 2) $3\sin(x+y+z)$ 3) $18\sin(x+y+z)$ 4) $\sin(x+2y+z)$

Numerical value type questions

30. If $Z+\frac{1}{2}=2\cos 6^\circ$ and $z^{1000}+\frac{1}{z^{1000}}+1$ is

31. If $(\sqrt{3}+i)^{100}=2^{99}(a+ib)$ then $\frac{a^2+b^2}{100}=$

32. If n is not a multiple of 3, and if $\lambda=\sin\left\{(\omega^n+\omega^{2n})\pi-\frac{\pi}{4}\right\}$ then $\frac{\lambda}{\sqrt{2}}=$

33. If $a=1+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 6}+\frac{1}{\lfloor 9}+....$, $b=1+\frac{1}{\lfloor 4}+\frac{1}{\lfloor 7}+....$, $c=\frac{1}{\lfloor 2}+\frac{1}{\lfloor 5}+\frac{1}{\lfloor 8}+....$, then $a^3+b^3+c^3-3abc=$

34. If m and n are the smallest positive integers satisfying the relation $\left(2cis\frac{\pi}{6}\right)^m=\left(4cis\frac{\pi}{4}\right)^n$ where $i=\sqrt{-1}$ $\left(\frac{n}{m}\right)^2$ equals to

LEVEL-II (ADVANCED)

Single answer type questions

1. α,β,γ are the roots of $x^3-3x^2+3x+7=0$ (ω is cube root of unity) then $\left(\frac{\alpha-1}{\beta-1}+\frac{\beta-1}{\gamma-1}+\frac{\gamma-1}{\alpha-1}\right)$ is

a) $\frac{3}{\omega}$ b) ω^2 c) $2\omega^2$ d) 3ω

2. If α,β,γ are the cube roots of $p(<0)$, then $\dfrac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}$ for any x, y, z is equal to (where ω is complex cube root of unity)

   a) 1 b) 0 c) ω^2 d) 3

3. The number of common roots of the equations $x^3+2x^2+2x+1=0$ and $x^{2012}+x^{2014}+1=0$ is

   a) 1 b) 2 c) 3 d) 4

4. If $\alpha=-1+i\sqrt{3}$ and n is a positive integer which is not a multiple of 3, then $\alpha^{2n}+2^n\alpha^n+2^{2n}=$

   a) 1 b) –1 c) 0 d) α^2

5. If z is a non real root of $\sqrt[7]{-1}$ then $z^{86}+z^{175}+z^{289}$ is equal to

   a) 0 b) –1 c) 3 d) 1

6. Number of ordered pairs of (a, b) of real numbers such that $(a+ib)^{2008}=a-ib$ holds good, is

   a) 2008 b) 2009 c) 2010 d) 1

7. Let $f(x)=x^4+ax^3+bx^2+cx+d$, $a, b, c, d\in R$. Suppose all roots of $f(x)=0$ are real and $|f(i)|=1$, then $a+b^2+c^3+d^4$ equals

   a) –1 b) 1 c) 4 d) None of the above

8. All the roots of the equation $11z^{10}+10iz^9+10iz-11=0$ lie

   a) inside $|z|=1$ b) on $|z|=1$ c) outside $|z|=1$ d) can't say

9. Let $z=e^{i\theta}$ then the value of $\sum_{m=1}^{15}\mathrm{Im}(z^{2m-1})$, when $\theta=2^0$ is

   a) $\dfrac{1}{\sin 2^0}$ b) $\dfrac{1}{3\sin 2^0}$ c) $\dfrac{1}{2\sin 2^0}$ d) $\dfrac{1}{4\sin 2^0}$

10. If $az^2+bz+1=0$, $a,b,z\in C$ and $|a|=\dfrac{1}{2}$, have a root α such that $|\alpha|=1$ then $|a\bar{b}-b|=$

    a) 1/4 b) 1/2 c) 5/4 d) 3/4

11. Let $C=\cos\dfrac{2\pi}{7}+\cos\dfrac{4\pi}{7}+\cos\dfrac{8\pi}{7}$ and $S=\sin\dfrac{2\pi}{7}+\sin\dfrac{4\pi}{7}+\sin\dfrac{8\pi}{7}$, then

    a) $C=\dfrac{1}{2}$ b) $S=\dfrac{1}{2}$ c) $C=\dfrac{\sqrt{7}}{2}$ d) $S=\dfrac{\sqrt{7}}{2}$

More than one correct answer type questions

12. If $S=\sum_{k=1}^{10}\left(\sin\dfrac{2\pi k}{11}-i\cos\dfrac{2\pi k}{11}\right)$ then

    a) $S+\bar{S}=0$ b) $S\bar{S}=1$ c) $\sqrt{S}=\pm\dfrac{1}{\sqrt{2}}(1+i)$ d) $S-\bar{S}=0$

13. Let $P(x)$ and $Q(x)$ be two real polynomials. Suppose that $f(x)=P(x^3)+xQ(x^3)$ is divisible by x^2+x+1, then

a) $P(x)$ is divisible by $(x-1)$ but $Q(x)$ is not divisible by $x-1$

b) $Q(x)$ is divisible by $(x-1)$ but $P(x)$ is not divisible by $x-1$

c) Both $P(x)$ and $Q(x)$ are divisible by $x-1$ d) $f(x)$ is divisible by $x-1$

14. Let a complex number $\alpha,(\alpha \neq 1)$ to be root of the equation $Z^{p+q}-Z^p-Z^q+1=0$, where p, q are distinct primes. Then

a) $1+\alpha+\alpha^2+.........\alpha^{p-1}=0$ and $1+\alpha+\alpha^2+........\alpha^{q-1}\neq 0$

b) $1+\alpha+\alpha^2+.........\alpha^{p-1}\neq 0$ and $1+\alpha+\alpha^2+........\alpha^{q-1}=0$

c) $1+\alpha+\alpha^2+.........\alpha^{p-1}=0$ and $1+\alpha+\alpha^2+........\alpha^{q-1}=0$

d) $1+\alpha+\alpha^2+.........\alpha^{p-1}\neq 0$ and $1+\alpha+\alpha^2+........\alpha^{q-1}\neq 0$

15. If $(1+x)^n=C_0+C_1x+........C_nx^n$, where n is a positive integer, then

a) $C_0-C_2+C_4.......=2^{n/2}\cos\left(\frac{n\pi}{4}\right)$ b) $C_1-C_3+C_5.......=2^{n/2}\sin\left(\frac{n\pi}{4}\right)$

c) $C_0+C_4+C_8+.......=2^{n-2}+2^{\left(\frac{n-2}{2}\right)}\cos\left(\frac{n\pi}{4}\right)$ d) $C_0+C_3+C_6+.......=\frac{1}{3}\left(2^n+2\cos\left(\frac{n\pi}{3}\right)\right)$

Linked comprehension type questions

Passage-I :

If n is a natural number, define polynomial $P_n(x)$ of degree n as follows:

$\cos n\theta=P_n(\cos\theta)$

For example, $P_2(x)=2x^2-1$ and $P_3(x)=4x^3-3x;\ x\in R$.

16. $\frac{1}{2x}[P_{n+1}(x)+P_{n-1}(x)]$ equals

a) $P_{n+2}(x)$ b) $P_{n-1}(x)+P_n(x)$ c) $P_n(x)$ d) $P_{n+1}(x)-P_n(x)$

17. $\left(x+\sqrt{x^2-1}\right)^n+\left(x-\sqrt{x^2-1}\right)^n$ equals

a) $P_n(x)$ b) $P_{n+1}(x)+P_{n-1}(x)$ c) $2P_n(x)$ d) $P_{n+1}(x)-P_{n-1}(x)$

Passage-II :

The equations $z^n-1=0$ has 'n' roots which are called the n^{th} roots of unity. The n^{th} roots of unity are $1,\alpha,\alpha^2,.........\alpha^{n-1}$ which are in GP, where $\alpha=\cos\left(\frac{2\pi}{n}\right)+i\sin\left(\frac{2\pi}{n}\right)$; $i=\sqrt{-1}$, Then we have following results.

i) $\sum_{r=0}^{n-1}\alpha^r=0$ (or) $\sum_{r=0}^{n-1}\cos\left(\frac{2\pi r}{n}\right)=0$ and $\sum_{r=0}^{n-1}\sin\left(\frac{2\pi r}{n}\right)=0$ ii) $z^n-1=\sum_{r=0}^{n-1}(z-\alpha^r)$

18. The value of $\sum_{r=1}^{n-1}\frac{1}{(2-\alpha^r)}$ is equal to

a) $(n-2)2^n$ b) $\frac{(n-2)2^{n-1}+1}{2^n-1}$ c) $\frac{(n-2)2^{n-1}}{2^n-1}$ d) $\frac{(n-1)2^{n-1}}{2^n-1}$

19. The algebraic sum of perpendicular distance from the points $1,\alpha,\alpha^2,.........\alpha^{n-1}$ to the line $\bar{a}z+a\bar{z}+b=0$ (where α is complex number and 'b' is real) is equal to

a) $\frac{n}{2|a|}$ b) $\frac{n|b|}{2a}$ c) $\frac{nb}{|a|}$ d) $\frac{nb}{2|a|}$

20. Given $Z=\cos\left(\frac{2\pi}{2n+1}\right)+i\sin\left(\frac{2\pi}{2n+1}\right)$, n is a positive integer, then the equation whose roots are $\alpha=Z+Z^3+Z^5+.....Z^{2n-1}$ and $\beta=Z^2+Z^4+.....Z^{2n}$ is

a) $x^2+x+1=0$ b) $x^2+x-1=0$

c) $x^2+x+\frac{1}{4\cos^2\frac{\pi}{2n+1}}=0$ d) $x^2-x+\frac{1}{4\cos^2\frac{\pi}{2n+1}}=0$

Passage-III :

Let $A_1, A_2, A_3,....A_n$ be vertices of a regular polygon of n sides whose centre 'O' is z_0 let $A_1, A_2, A_3,......A_n$ be $z_1, z_2,.......z_n$ respectively. Let $OA_1 = OA_2 = ... OA_n = 1$

21. The value of $\left|z_1^2+z_2^2+z_3^2+...........+z_n^2\right|=k\left|z_0\right|^2$ then k

a) 1 b) 3 c) n d) n^2

22. If $Z_0=(0,0)$, $|A_1A_2|^2+|A_1A_3|^2+..........|A_1A_n|^2=k|A_1A_2||A_1A_3|......|A_1A_n|$ then $k=$

a) 0 b) 1 c) 2 d) $2n$

Matrix matching type questions

23. **COLUMN - I** **COLUMN - II**

A) If $|a_i|<1;\lambda_i\ge 0$ for i = 1,2,3,....n and $\lambda_1+\lambda_2+\lambda_3+.......\lambda_n=1$ and ω is a complex cube root of unity, then $\left|\lambda_1a_1\omega+\lambda_2a_2\omega^2+.......\lambda_na_n\omega^n\right|$ cannot exceed — p) $|z|^n+\frac{1}{|z|}$

B) If Re(z) < 0, then the value of $(1+z+z^2+.......+z^n)$ cannot exceed — q) 2

C) If $\omega(\ne 1)$ is a cube root of unity, then $\frac{1}{\sqrt{3}}\left|1+2\omega+3\omega^2+.....+3n\omega^{3n-1}\right|$ $(n\in N)$ cannot exceed — r) n

D) If $1,\omega,\omega^2,.......\omega^{n-1}$ are the n, n^{th} roots of unity, then $(2-\omega)(2-\omega^2).....(2-\omega^{n-1})=$ — s) 2^n-1

t) $|a_1|+|a_2|+....+|a_n|$

a) A - q ; B - t ; C - s ; D - r b) A - q ; B - r ; C - s ; D - q

c) A - t ; B - p ; C - r ; D - s d) A - s ; B - qr ; C - qt ; D - t

24. If z_1, z_2, z_3, z_4 are the roots of the equation $z^4 + z^3 + z^2 + z + 1 = 0$, then

COLUMN - I	COLUMN - II
A) $\left\|\sum_{i=1}^{4} z_i^4\right\|$ is equal to	p) 0
B) $\sum_{i=1}^{4} z_i^5$ is equal to	q) 4
C) $\prod_{i=1}^{4}(z_i+2)$ is equal to	r) 1
D) Least value of $\left[\|z_1+z_2\|\right]$ is	s) 11
where [ ] represents greatest integer function	t) $\left\|4\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right\|$

25.

COLUMN - I	COLUMN - II
A) Let $z_k (k=0,1,2,3,4,5,6)$ be the roots of the equation $(z+1)^7+(z)^7=0$ then $\sum_{k=0}^{6}\text{Re}(z_k)$	p) -1
B) If α,β,γ and a, b, c are complex numbers such that $\frac{\alpha}{a},\frac{\beta}{b},\frac{\gamma}{c}=1+i$ and $\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=0$, then the value of $\frac{\alpha^2}{a^2},\frac{\beta^2}{b^2},\frac{\gamma^2}{c^2}$	q) $-\frac{7}{2}$
C) If $z_1, z_2 ....z_6$ are six roots of the equations $z^6-z^5+z^4-z^3+z^2-z+1=0$, then the value of $\prod_{i=1}^{6}(z_1+1)$	r) 1
D) If $\|z\| = \min(\|z-1\|,\|z+1\|)$, then the value of $\|z+\bar{z}\|$	s) 4

Integer answer type questions

26. If $a_1, a_2, ..., a_n$ are real numbers and $\cos\alpha + i\sin\alpha$ is a root of $x^n + a_1x^{n-1} + a_2x^{n-2} + ...$ $+a_{n-1}x + a_n = 0$ then the value of $|a_1\cos\alpha + a_2\cos 2\alpha + ... + a_n\cos(n\alpha)|$ is

27. If $z = \frac{1}{2}(\sqrt{3}-i)$ and the smallest value of the positive integer n for which $(z^{89}+i^{97})^{94} = z^n$ is λ then $\frac{\lambda}{2} =$

28. If $1, x_1, x_2, x_3$ are the roots of $x^4 - 1 = 0$ and ω is a complex cube root of unity, the value of $\frac{(\omega^2-x_1)(\omega^2-x_2)(\omega^2-x_3)}{(\omega-x_1)(\omega-x_2)(\omega-x_3)}$ is

29. The number of common roots of the equations $x^5 - x^3 + x^2 - 1 = 0$ and $x^4 - 1 = 0$ is

30. If $z_n = \left(1+i\sqrt{3}\right)^n$ and the value of $\sqrt{3}\text{Im}(z_5\bar{z}_4) = \lambda$ then the integral part of $\frac{\lambda}{100}$ is

31. If Z_1 and $\overline{Z_1}$ represents adjacent vertices of a regular polygon of n sides and if $\frac{\text{Im}(Z_1)}{\text{Re}(Z_1)} = \sqrt{2}-1$ then n is equal to

EXERCISE-III

Geometrical Applications

LEVEL-I (MAIN)

Single answer type questions

1. Let A, B and C represents the complex numbers z_1, z_2 and z_3 in the Argand plane. If circumcentre of the triangle ABC is at the origin, then the complex number corresponding to orthocentre is

   1) $\frac{1}{4}(z_1+z_2+z_3)$ 2) $\frac{1}{3}(z_1+z_2+z_3)$ 3) $\frac{1}{2}(z_1+z_2+z_3)$ 4) $z_1+z_2+z_3$

2. If $z = x + iy$ then the equation $\left|\frac{2z-i}{z+1}\right| = m$ does not represent a circle when

   1) $m=\frac{1}{2}$ 2) $m = 1$ 3) $m = 2$ 4) $m = 3$

3. If $|z|=\sqrt{2}$ then the point given by "$3+4z$" lies on a circle whose radius is

   1) $2\sqrt{2}$ 2) $3\sqrt{2}$ 3) $4\sqrt{2}$ 4) $5\sqrt{2}$

4. $z = x+iy$ and $w=\frac{1-iz}{z-i}$, then $|w| = 1 \Rightarrow$ in the complex plane

   1) z lies on the imaginary axis 2) z lies onthe real axis
   3) z lies on the unit circle 4) z lies on the parabola

5. If $|z+\bar{z}|+|z-\bar{z}|=2$ then z lies on

   1) a straight line 2) a square 3) a circle 4) parallelogram

6. If $z(\bar{z}+3)=2$ then the locus of $z = x+iy$ is

   1) $x^2+y^2+3x-2=0,\ y=0$ 2) $x=0$ such that $y>2/3$
   3) $x^2+y^2+2x-4y=0$ such that $y<0,\ x^2+y^2>1$ 4) $x^2+y^2+2x-4y=0$ such that $2x-y+4>0$

7. The locus of z satisfying the inequality $\log_{\frac{1}{3}}|z+1| > \log_{\frac{1}{3}}|z-1|$ is

   1) $\mathrm{Re}(z)>0$ 2) $\mathrm{Re}(z)<0$ 3) $\mathrm{Im}(z)>0$ 4) $\mathrm{Im}(z)<0$

8. If $z_1 = 8+4i$, $z_2 = 6+4i$ and z be a complex number suchthat $\mathrm{Arg}\left(\frac{z-z_1}{z-z_2}\right)=\frac{\pi}{4}$, then the locus of z is

   1) $(x-7)^2+(y-5)^2=2$ 2) $(x-7)^2+(y+5)^2=2$
   3) $(x-7)^2+(y-5)^2=4$ 4) $(x+7)^2+(y+5)^2=4$

9. If $|z| = 3$, the area of the triangle whose sides are z, $z\omega$ and $z+\omega z$ (where ω is a complex cube root of unity) is

1) $\frac{9\sqrt{3}}{4}$ 2) $\frac{8\sqrt{3}}{2}$ 3) $\frac{5}{2}$ 4) $\frac{8\sqrt{3}}{3}$

10. Let z_1 and z_2 be two non-zero complex numbers such that $\frac{z_1}{z_2}+\frac{z_2}{z_1}=1$, then the origin and points represented by z_1 and z_2

1) Lie on a straight line 2) Form a right triangle

3) Form an equilateral triangle 4) form an isosceles triangle

11. If a and b are real numbers between 0 and 1 such that the points $z_1 = a+i$, $z_2 = 1+bi$ and $z_2 = 0$ form an equilateral triangle then a, b are

1) $2-\sqrt{3},\ 2-\sqrt{3}$ 2) $2-\sqrt{3},\ 2+\sqrt{3}$ 3) $2+\sqrt{3},\ 2-\sqrt{3}$ 4) $2-\sqrt{2},\ 2+\sqrt{2}$

12. The roots of the cubic equation $(z+\alpha\beta)^3=\alpha^3, \alpha\neq 0$ represent the vertices of a triangle of sides of length

1) $\frac{1}{\sqrt{3}}|\alpha\beta|$ 2) $\sqrt{3}\ |\alpha|$ 3) $\sqrt{3}\ |\beta|$ 4) $\frac{1}{\sqrt{3}}\ |\alpha|$

13. The equation $\left||z+i|-|z-i|\right|=k$ represents a hyperbola if

1) $-3<k<2$ 2) $k>2$ 3) $0<k<2$ 4) $k>4$

14. Let $z=1-t+i\sqrt{t^2+t+2}$, where t is a real parameter. The locus of z in the Argand plane is

1) a hyperbola 2) an ellipse 3) a straight line 4) a circle

Numerical value type questions

15. One vertex of an equilateral triangle is at the origin and the other two vertices are, roots of $2z^2+2z+k=0$, then k is

16. Let $Z=1-t+i\sqrt{t^2+t+2}$ where t is a real parameter. Locus of Z is a conic then eccentricity of conic is

17. If $az^2+bz+1=0$ where $a,b,c\in Z$ $|a|=\frac{1}{2}$ and have a roof α such that $|\alpha|=1$ and $|a\overline{b}-b|=$

18. Let z and ω be complex numbers. If Re(z) = |z–2|, Re(ω) = |ω–2| and Arg(z–ω) $=\frac{\pi}{3}$, the value of $\frac{\text{Im}(z+\omega)}{\sqrt{3}}$ is

LEVEL-II (ADVANCED)

Single answer type questions

1. If z_1, z_2, z_3 and z_4 be the consecutive vertices of a square, then $z_1^2+z_2^2+z_3^2+z_4^2$ equals

a) $z_1z_2+z_2z_3+z_3z_4+z_4z_1$ b) $z_1z_2+z_1z_3+z_1z_4+z_2z_3+z_2z_4+z_3z_4$

c) 0 d) None of the above

2. If z_1, z_2 and z_3 are the vertices of an isosceles right angled triangle, right angled at the vertex z_2, then $(z_1 - z_2)^2 + (z_3 - z_2)^2$ equals

a) 0 b) $(z_1 - z_3)^2$ c) $\left(\frac{z_1 + z_3}{2}\right)^2$ d) None of these

3. The point of intersection of the curves $\arg(z-3i) = \frac{3\pi}{4}$ and $\arg(2z+1-2i) = \frac{\pi}{4}$ is

a) $\frac{1}{4}(3+9i)$ b) $\frac{1}{4}(3-9i)$ c) $\frac{1}{2}(3+2i)$ d) None of these.

4. If z_1, z_2, z_3 are non-zero complex numbers representing the points A, B, C such that $\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}$. Then

a) A, B, C are collinear.

b) Circle passes through points A, B, C has centre at origin O

c) Circle passes through A, B, C passes through origin.

d) None of these.

5. If $|2z-4-2i| = |z|\sin\left(\frac{\pi}{4} - \arg z\right)$, then locus of z is/an

a) Ellipse b) Circle c) Parabola d) Pair of straight line

6. If $|z-3i| = 3$, (where $i = \sqrt{-1}$) and $\arg z \in (0, \pi/2)$, then $\cot(\arg(z)) - \frac{6}{z}$ is equal to

a) 0 b) $-i$ c) i d) none of these

7. The complex number $3 + 4i$ is rotated about origin by an angle of $\frac{\pi}{4}$ and then stretched 2 times. The complex number corresponding to new position is

a) $\sqrt{2}(-3+4i)$ b) $\sqrt{2}(-1+7i)$ c) $\sqrt{2}(3-4i)$ d) $\sqrt{2}(-1-7i)$

8. If the complex numbers z_1, z_2, z_3 satisfying $3z_1 = 5z_2 - 2z_3$, then z_1, z_2 and z_3 lie in a

a) circle b) parabola c) line d) hyperbola

9. The value of $\sin\left[\log_e\left\{\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^z\right\}\right]$ is, where z satisfies the equation $|z-2i| = 1$ and has least modulus

a) 1 b) 0 c) -1 d) $\frac{1}{2}$

10. Let a, b, c be three points lying on the circle $|z| = 1$ and suppose $\alpha \in (0, \pi/2)$ be such that $a + b\cos\alpha + c\sin\alpha = 0$, then

a) $b = \pm ic$ b) $2a^2 + b^2 + c^2 = 0$ c) $a^2 + b^2 + c^2 = 0$ d) none of these

11. Let $z_1, z_2, z_3 \in C$ such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $|z_1^2 + 2z_2^2 + z_3^2| =$

a) 1 b) 2 c) 3 d) 4

12. If $w \neq 1$ is a cube root of 1 and $|z-1|^2 + 2|z-w|^2 = 3|z-w^2|^2$ then z lies on

a) Straight line b) a parabola
c) An ellipse d) a rectangular hyperbola

13. If $z \neq 1$, and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

a) on a circle with centre at origin
b) either on real axis or on a circle not passing through the origin
c) on imaginary axis
d) either on the real axis or on a circle passing through origin

14. Locus of z if $\arg(z-(1+i)) = \begin{cases} \frac{3\pi}{4} & \text{if } |z| \leq |z-2| \\ -\frac{\pi}{4} & \text{if } |z| > |z-4| \end{cases}$ is

a) straight lines passing through (2,0) b) straight lines passing through (2,0),(1,1)
c) a line segment d) a set of two rays

15. Let S_1 and S_2 are concentric circles with radius 1 and 8/3 respectively, having centre at (3,0), on the argand plane. If z satisfies the in equality $\log_{\frac{1}{3}}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right) > 1$ then

a) z lies outside S_1 but inside S_2 b) z lies inside of both S_1 and S_2
c) z lies outside both of S_1 and S_2 d) z lies on S_1

16. $P = \{z : \operatorname{Im} z \geq 1\}$; $Q = \{z : |z-2-i| = 3\}$; $R = \{z : \operatorname{Re}(1-i)z = \sqrt{2}\}$. If $Z \in P \cap Q \cap R$ then $|z+1-i|^2 + |z-5-i|^2 =$

a) 23 b) 36 c) 72 d) 34

17. If $z_1, z_2, z_3, \ldots\ldots z_{10}$ denote vertices of regular polygon of 10 sides with z_0 as centre of polygon then $\frac{z_1^2 + z_2^2 + z_3^2 + \ldots + z_{10}^2}{z_0^2}$ equals to

a) 1 b) 10 c) 100 d) 2

18. The length of the curve which is the locus represented by $\arg\left(\frac{z+i}{z-i}\right) = \frac{\pi}{4}$ is equal to

a) $\frac{3\pi}{2}$ b) $\frac{3\pi}{\sqrt{2}}$ c) $\frac{\pi}{\sqrt{2}}$ d) $\pi\sqrt{2}$

19. The image of complex number z satisfying $\arg\left(\frac{z-i}{z-3}\right) = \frac{\pi}{6}$, about real axis is

a) $\arg\left(\frac{z+i}{z-3}\right) = \frac{\pi}{6}$ b) $\arg\left(\frac{z-i}{z+3}\right) = \frac{\pi}{6}$ c) $\arg\left(\frac{z+i}{z-3}\right) = \frac{-\pi}{6}$ d) $\arg\left(\frac{z-i}{z+3}\right) = \frac{-\pi}{6}$

20. If ω is the complex cube root of unity, then the arg $(i\omega)$ + arg $(i\omega^2)$ is

a) 0 b) $\frac{\pi}{2}$ c) π d) $\frac{\pi}{3}$

21. The locus of z such that $\arg\left(\frac{z-z_1}{z-z_2}\right)=\frac{\pi}{4}$ where $z_1=10+6i, z_2=4+6i$ is part of a circle whose radius is

a) $\sqrt{2}$ b) $2\sqrt{2}$ c) $3\sqrt{2}$ d) 1

More than one correct answer type questions

22. Let z_1, z_2, z_3 be the complex numbers representing the vertices A, B, C of a triangle described in counterclock sense respectively. Consider the following statements.

I) ΔABC is equilateral

II) $z_3-z_1=(z_2-z_1)\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

III) $z_2-z_1=(z_3-z_1)\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$

IV) $z_1+z_2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)+z_3\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)=0$

Then which one is correct:

a) $\text{I}\Rightarrow\text{II}$ b) $\text{II}\Rightarrow\text{III}$ c) $\text{III}\Rightarrow\text{IV}$ d) $\text{IV}\Rightarrow\text{I}$

23. If from a point P representing the complex numbers z_1 on the curve $|z|=2$, pair of tangents are drawn to the curve $|z|=1$, meeting at points $Q(z_2)$ and $R(z_3)$, then

a) complex number $\frac{z_1+z_2+z_3}{3}$ will lie on the curve $|z|=1$

b) $\left(\frac{4}{\bar{z}_1}+\frac{1}{\bar{z}_2}+\frac{1}{\bar{z}_3}\right)\left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)=9$

c) $\arg\left(\frac{z_2}{z_3}\right)=\pm\frac{2\pi}{3}$

d) orthocentre and circumcentre of ΔPQR will coincide

24. The equations of two lines making an angle 45^0 with a given line $\bar{a}z+a\bar{z}+b=0$ (where 'a' is a complex number and b is real) and passing through a given point C (c) (c is a complex number), is/are

a) $\frac{z+c}{a}+i\frac{\bar{z}-\bar{c}}{\bar{a}}=0$

b) $\frac{z-c}{a}+i\frac{\bar{z}-\bar{c}}{\bar{a}}=0$

c) $\frac{z-c}{a}-i\frac{\bar{z}-\bar{c}}{\bar{a}}=0$

d) $\frac{z+c}{a}-i\frac{\bar{z}-\bar{c}}{\bar{a}}=0$

25. Let z_1 and z_2 are non-zero (given) complex numbers and k be any positive real number. Consider the system of equations $|3z-z_1-2z_2|=|z_1-z_2|$ and $\arg\left(\frac{z_1-z_2}{z-kz_1-(1-k)z_2}\right)=\pm\frac{\pi}{2}$. Then

a) The system of equations has no solution if $k\in\left(\frac{2}{3},\infty\right)$

b) The system of equations have more than one solutions if $k\in\left(0,\frac{2}{3}\right)$

c) The system of equations have no solution if $k\in\left(0,\frac{2}{3}\right)$

d) The system of equations have more than one solution if $k\in\left(\frac{2}{3},\infty\right)$

26. If α is a variable complex number such that $|\alpha|>1$ ($|\alpha|$ is a constant) and $z=\alpha+\frac{1}{\alpha}$ lies on a conic then

a) Eccentricity of the conic is $\frac{2|\alpha|}{1+|\alpha|^2}$

b) Distance between foci is 4

c) Length of latusrectum is $\frac{2\left(|\alpha|^2-1\right)}{|\alpha|^2+1}$

d) Distance between directrices is $\left(|\alpha|+\frac{1}{|\alpha|}\right)^2$

27. Let a,b,c be distinct complex numbers with $|a|=|b|=|c|=1$ and z_1, z_2 be the roots of the equation $az^2+bz+c=0$ with $|z_1|=1$. Let P and Q represent the complex numbers z_1 and z_2 in the argand plane with $\angle POQ=\theta$, $0^0<\theta<180^0$ (where O being the origin) then

a) $b^2=ac; \theta=\frac{2\pi}{3}$

b) $\theta=\frac{2\pi}{3}; PQ=\sqrt{3}$

c) $PQ=2\sqrt{3}; b^2=ac$

d) $\theta=\frac{\pi}{3}; b^2=ac$

Linked comprehension type questions

Passage-I :

In acute angle ΔABC altitude are drawn from vertices A, B, C which meets sides BC, AC, AB at $D(z_1)$, $E(z_2)$, $F(z_3)$ such that arg $\left(\frac{z_3-z_2}{z_2-z_1}\right)$ purely img. If BC = 4 units

28. Circum radius of ΔABC is

a) $3\sqrt{2}$

b) $2\sqrt{2}$

c) 2

d) None of these

29. The length of EF

a) $2\sqrt{2}$

b) $\sqrt{2}$

c) 2

d) None of these

30. Circum radius of ΔDEF

a) 4

b) 2

c) $2\sqrt{2}$

d) None

Passage-II :

Tangents are drawn to a point P to circle $|z-(1+i)|=4$; such that angle between them is $\frac{\pi}{3}$, intersect the circle at A and B. If I is incentre of ΔPAB.

31. Area of ΔIAB

a) $3\sqrt{3}$

b) $4\sqrt{3}$

c) 4

d) None

32. Circum radius of ΔIAB

a) 2

b) 4

c) $2\sqrt{2}$

d) None

33. Area of ΔPAB

a) $12\sqrt{3}$

b) $4\sqrt{3}$

c) $8\sqrt{3}$

d) None

Passage-III :

If Z_1, Z_2 be the complex numbers representing two points A and B then we define complex slope of line AB as $\mu = \frac{Z_1 - Z_2}{\bar{Z}_1 - \bar{Z}_2}$. It can be noted that $|\mu| = 1$ and μ remains same for any two points on the line AB. Now answer the following questions.

34. The complex slope of line $a\bar{Z} + \bar{a}Z + b = 0$ where a is complex and b is real is

a) $\frac{a}{\bar{a}}$ b) $-\frac{a}{\bar{a}}$ c) $\frac{\bar{a}}{a}$ d) $-\frac{\bar{a}}{a}$

35. If a line in Argand plane passes through two points which are represented by Z_1, Z_2 and the line has complex slope μ then the equation of line must be of the form

a) $Z - Z_1 = \mu(Z - Z_2)$ b) $Z - Z_1 = \mu(\bar{Z} - \bar{Z}_1)$ c) $Z - Z_1 = \frac{1}{\mu}(\bar{Z} - \bar{Z}_1)$ d) $\frac{Z - Z_1}{Z - Z_2} = \mu + \frac{Z - Z_2}{Z - Z_1}$

36. If the complex slope of a line passing through origin which is not parallel to y-axis is $\cos\phi + i\sin\phi$ then the line makes angle θ with x-axis such that

a) $\theta = 2\phi$ b) $\theta = 90^0 - \phi$ c) $\theta = \phi/2$ d) $\theta = \phi$

Passage-IV :

A person walks $2\sqrt{2}$ units away from origin in south west direction to reach A, then walks $\sqrt{2}$ units in south east ($S45^0E$) direction to reach B. From B he travels 4 units horizontally towards east to reach C, then he travels along a circular path with centre at origin through an angle of $\frac{2\pi}{3}$ in anti clockwise direction to reach his destination D.

37. Position of B in argand plane is

a) $\sqrt{2}e^{-i\frac{3\pi}{4}}$ b) $\sqrt{2}(2+i)e^{-i\frac{3\pi}{4}}$ c) $\sqrt{2}(1+2i)e^{-i\frac{3\pi}{4}}$ d) $-3 + i$

38. Let the complex number Z represents C in argand plane then $\arg(Z)$ =

a) $-\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $-\frac{\pi}{4}$ d) $\frac{\pi}{3}$

Matrix matching type questions

39.

COLUMN - I	COLUMN - II
A) Locus of the point 'z' satisfying the equation $\text{Re}(z^2) = \text{Re}(z + \bar{z})$	p) A parabola
B) Locus of the point 'z' satisfying the $\lvert z - z_1\rvert + \lvert z - z_2\rvert = \lambda, \lambda \in R^+$ and $\lambda\lvert z_1 - z_2\rvert$	q) A straight line
C) Locus of the point 'z' satisfying the $\left\lvert\frac{2z - i}{z+1}\right\rvert = m$, Where $i = \sqrt{-1}$ and $m \in R^+$	r) An ellipse
D) If $\lvert z\rvert = 25$, then the points representing the complex number $-1 + 75\bar{z}$ will be on 'a'	s) A rectangular hyperbola
	t) A circle

40. Which of the following condition/s in column-II are satisfied by the quadrilateral formed by Z_1, Z_2, Z_3, Z_4 in order given in column-I.

COLUMN - I	COLUMN - II
A) Parallelogram	p) $Z_1 - Z_4 = Z_2 - Z_3$
B) Rectangle	q) $\lvert Z_1 - Z_3 \rvert = \lvert Z_2 - Z_4 \rvert$
C) Rhombus	r) $\dfrac{Z_1 - Z_2}{Z_3 - Z_4}$ is purely real
D) Square	s) $\dfrac{Z_1 - Z_3}{Z_2 - Z_4}$ is purely imaginary
	t) $\dfrac{Z_1 - Z_2}{Z_3 - Z_2}$ is purely imaginary

Integer answer type questions

41. If the points $1 + 2i$ and $-1 + 4i$ are reflections of each other in the line $z(1+i)+\overline{z}(1-i)+K=0$, then the value of K is ______

42. If the area of a triangle with vertices Z_1, Z_2 and Z_3 is the absolute value of the number $\lambda i\begin{vmatrix} Z_1 & \overline{Z}_1 & 1 \\ Z_2 & \overline{Z}_2 & 1 \\ Z_3 & \overline{Z}_3 & 1 \end{vmatrix}$ then the value of $1/\lambda$ is equal to ______.

43. If the complex number z is such that $|z-1| \le 1$ and $|z-2| = 1$, then the maximum possible value $|z|^2$ is

44. If $0 \le \arg(z) \le \pi/4$, then the least value of $\sqrt{2}|z-i|$ is

45. The number of points z in the complex plane satisfying both the equations $|z-4-8i| = \sqrt{10}$ and $|z-3-5i|+|z-5-11i| = 4\sqrt{5}$ is _________.

46. Let $\dfrac{1}{a_1-2i}, \dfrac{1}{a_2-2i}, \dfrac{1}{a_3-2i}, \dfrac{1}{a_4-2i}, \dfrac{1}{a_5-2i}, \dfrac{1}{a_6-2i}, \dfrac{1}{a_7-2i}, \dfrac{1}{a_8-2i}$ are vertices of regular octagon. If the area of octagon is A, then the value of $8\sqrt{2}A$ is (where $a_j \in R$ for j = 1, 2, 3, 4, 5, 6, 7, 8 and $i = \sqrt{-1}$) _____

47. Let 1, w, w^2 be the cube roots of unity. The least possible degree of a polynomial with real coefficients having roots $2w$, $(2 + 3w)$, $(2 + 3w^2)$, $(2 - w - w^2)$, is

KEY SHEET (LECTURE SHEET)

EXERCISE-I

LEVEL-I

1) 2	2) 2	3) 2	4) 1	5) 4	6) 1	7) 1	8) 3
9) 1	10) 4	11) 3	12) 3	13) 1	14) 2	15) 1	16) 6
17) 2	18) 4	19) 3364	20) 1.4	21) 2022	22) 9	23) 0.2	

LEVEL-II

1) b	2) a	3) c	4) a	5) c	6) d	7) b	8) b
9) d	10) a	11) b	12) c	13) a	14) a	15) b	16) c
17) b	18) b	19) a	20) b	21) a	22) b	23) ad	24) abcd
25) ab	26) ad	27) abc	28) abd	29) bc	30) abcd	31) abc	32) c
33) a	34) b	35) c	36) a	37) a	38) b	39) d	40) d
41) c	42) b	43) a	44) A-r;B-p;C-s;D-q			45) A-r;B-p;C-p;D-q	
46) 3	47) 8	48) 6	49) 4	50) 2	51) 2		

EXERCISE-II

LEVEL-I

1) 1	2) 2	3) 1	4) 1	5) 2	6) 4	7) 1	8) 2
9) 2	10) 2	11) 3	12) 2	13) 1	14) 1	15) 2	16) 4
17) 2	18) 1	19) 1	20) 2	21) 1	22) 3	23) 2	24) 2
25) 1	26) 2	27) 3	28) 2	29) 3	30) 0	31) 0.04	32) 0.5
33) 1	34) 0.25						

LEVEL-II

1) a	2) c	3) b	4) c	5) b	6) c	7) d	8) b
9) d	10) d	11) d	12) abc	13) cd	14) ab	15) abcd	
16) c	17) c	18) b	19) d	20) c	21) c	22) c	
23) c	24) A-r;B-qt;C-s;D-p			25) A-q;B-p;C-s;D-r			26) 1
27) 5	28) 1	29) 2	30) 7	31) 8			

EXERCISE-III

LEVEL-I

1) 4	2) 3	3) 3	4) 2	5) 2	6) 1	7) 2	8) 1
9) 1	10) 3	11) 1	12) 2	13) 3	14) 1	15) 0.67	16) 1.41
17) 0.75	18) 1.33						

LEVEL-II

1) a	2) a	3) d	4) c	5) a	6) c	7) b	8) c
9) c	10) a	11) a	12) a	13) d	14) d	15) a	16) b
17) b	18) d	19) c	20) c	21) c	22) abcd	23) abcd	24) bc
25) ab	26) abd	27) ab	28) c	29) a	30) c	31) a	32) b
33) a	34) b	35) b	36) c	37) c	38) c		
39) A-p;B-q;C-r;D-st			40) A-pr;B-pqrt;C-prs;D-pqrst				
41) 6	42) 4	43) 3	44) 1	45) 2	46) 2	47) 5	

PRACTICE SHEET

EXERCISE-I

Modulus, Argument, Conjugate of Complex Numbers SQRT, Cube Root; Problesm on modulus; Problems on Argument

LEVEL-I (MAIN)

Single answer type questions

1. The real value of θ for which the expression $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely imaginary is

   1) $n\pi,\ n\in Z$ 2) $n\pi \pm\frac{\pi}{2},\ n\in Z$ 3) $n\pi \pm\frac{\pi}{3},\ n\in Z$ 4) $n\pi \pm\frac{\pi}{6},\ n\in Z$

2. If $z = 3+i$, then $(z+1)(\overline{z}+1)=$

   1) 17 2) -17 3) $\sqrt{17}$ 4) $-\sqrt{17}$

3. If z_1, z_2 are conjugate complex numbers and z_3, z_4 are also conjugate, then $\operatorname{Arg}\left(\frac{z_3}{z_2}\right)=$

   1) $\operatorname{Arg}\left(\frac{z_1}{z_4}\right)$ 2) $\operatorname{Arg}\left(\frac{z_4}{z_1}\right)$ 3) $\operatorname{Arg}\left(\frac{z_2}{z_4}\right)$ 4) $\operatorname{Arg}\left(\frac{z_1}{z_3}\right)$

4. If $|z_1+z_2| = |z_1-z_2|$, then the difference in the arguments of z_1 and z_2 is

   1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) π

5. Let z,ω be complex numbers such that $\overline{z}+i\overline{\omega}=0$ and $\operatorname{Arg}(z\omega)=\pi$ then Arg(z) =

   1) $\frac{\pi}{4}$ 2) $\frac{5\pi}{4}$ 3) $\frac{3\pi}{4}$ 4) $\frac{\pi}{2}$

6. If $(a_1+ib_1)(a_2+ib_2).....(a_n+ib_n)=A+iB$ then $\operatorname{Tan}^{-1}\left(\frac{b_1}{a_1}\right)+\operatorname{Tan}^{-1}\left(\frac{b_2}{a_2}\right)+.......+\operatorname{Tan}^{-1}\left(\frac{b_n}{a_n}\right)=$

   1) $\operatorname{Tan}^{-1}\left(\frac{A}{B}\right)$ 2) $2n\pi+\operatorname{Tan}^{-1}\left(\frac{B}{A}\right)$ 3) $\operatorname{Tan}^{-1}\left(\frac{2A}{B}\right)$ 4) $n\pi+\operatorname{Tan}^{-1}\left(\frac{B}{A}\right)$

7. If z and ω are two non-zero complex numbers such that $|z\omega|=1$ and $\arg(z)-\arg(\omega)=\frac{\pi}{2}$, then $\overline{z}\omega$ is equal to

   1) 1 2) –1 3) i 4) $-i$

Logarithm of a complex number :

8. Log(log$i$) =

   1) $\log\left[\left(\frac{\pi}{2}\right)+i\left(\frac{\pi}{2}\right)\right]$ 2) $\log\left[\left(\frac{\pi}{2}\right)-i\left(\frac{\pi}{2}\right)\right]$ 3) $\log\left(\frac{\pi}{2}\right)+i\left(\frac{\pi}{2}\right)$ 4) $\log\left(\frac{\pi}{2}\right)-i\left(\frac{\pi}{2}\right)$

9. The value of $i^{i^{i.....\infty}}=x+iy$ then $x^2+y^2=$

   1) $e^{-\pi y}$ 2) $e^{+\pi y}$ 3) πy 4) $-\pi y$

10. The real and imaginary parts of log(1+i)=

1) $\left(\frac{1}{2}, \frac{\pi}{4}\right)$ 2) $\left(\log 2, \frac{\pi}{4}\right)$ 3) $\left(\log\sqrt{2}, \frac{\pi}{4}\right)$ 4) $\left(\log\frac{1}{2}, \frac{\pi}{4}\right)$

11. $\tan\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\}=$

1) ab 2) $\frac{2ab}{a^2-b^2}$ 3) $\frac{a^2-b^2}{2ab}$ 4) $\frac{2ab}{a^2+b^2}$

12. A : If $z=i\log(2-\sqrt{3})$ then $\cos z=2$

R : $\cosh\theta=\frac{e^{i\theta}+e^{i\theta}}{2}$

1) Both A and R are true R is correct explanation to A
2) Both A and R are true but R is not correct explanation to A
3) A is true R is false
4) A is false R is true

Numerical value type questions

13. If $\frac{5z_2}{7z_1}$ is purely imaginary, then $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|=$

14. If $|z+2-i|=5$ then the maximum value of $|3z+9-7i|$ is

15. If $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, then $\sin^2\theta$ is

16. Number of integral solutions satisfying the inequidity $|\text{Re}(z)|+|\text{Im}(z)|<21$ is

17. Let Z be a complex number such that the imaginary part of Z is non-zero and $a=z^2+z+1$ is real. Then a cannot take the value

LEVEL-II (ADVANCED)

Single answer type questions

1. Let z_1 and z_2 be any two complex numbers then $\left|z_1+\sqrt{z_1^2-z_2^2}\right|+\left|z_1-\sqrt{z_1^2-z_2^2}\right|$ is equal to

a) $\left|z_1^2-z_2^2\right|+\left|z_1^2+z_2^2\right|$ b) $\left|z_1-z_2\right|+\left|z_1^2+z_2^2\right|$ c) $\left|z_1+z_2\right|+\left|z_1^2+z_2^2\right|$ d) $\left|z_1+z_2\right|+\left|z_1-z_2\right|$

2. If z_1, z_2, z_3 are any three complex numbers on Argand plane then $z_1\left(\text{Im}\left(\bar{z}_2z_3\right)\right)+z_2\left(\text{Im}\left(\bar{z}_3z_1\right)\right)+z_3\left(\text{Im}\left(\bar{z}_1z_2\right)\right)$ is equal to

a) 0 b) $z_1+z_2+z_3$ c) $z_1z_2z_3$ d) $\left(\frac{z_1+z_2+z_3}{z_1z_2z_3}\right)$

3. Suppose two complex numbers $z=a+ib; w=c+id$ satisfy the equation $\frac{z+w}{z}=\frac{w}{z+w}$ then

a) both a & c are zeros b) both b & d are zeros
c) both b & d must be non zeros d) at least one of b & d is non-zero

4. If Z and W are two complex numbers such that $\bar{z}+i\overline{w}=0$ and arg $(Zw)=\pi$ then arg (Z) = _____

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{4}$ d) $\frac{5\pi}{4}$

5. If $|z| = 1$ and $z \neq \pm 1$ then one of the possible values of $\arg(z) - \arg(z+1) - \arg(z-1)$, is

a) $-\pi/6$ b) $\pi/3$ c) $-\pi/2$ d) $\pi/4$

6. If z_1, z_2, z_3 are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|z_2-z_3|}=\frac{b}{|z_3-z_1|}=\frac{c}{|z_1-z_2|}$ then $\frac{a^2}{z_2-z_3}+\frac{b^2}{z_3-z_1}+\frac{c^2}{z_1-z_2}$ is

a) $3abc$ b) $(abc)^3$ c) $a+b+c$ d) 0

7. If z_1, z_2 are complex numbers such that $\text{Re}(z_1)=|z_1-2|, \text{Re}(z_2)=|z_2-2|$ and $\arg(z_1-z_2)=\pi/3$, then $\text{Im}(z_1+z_2)=$

a) $2/\sqrt{3}$ b) $4/\sqrt{3}$ c) $2\sqrt{3}$ d) $\sqrt{3}$

8. If $z=x+iy$ is such that $|z-4|<|z-2|$, then

a) $x>0.\ y>0$ b) $x<0.\ y>0$

c) $x>2, y>3$ d) $x>3$ and y is any real number

9. If a is a positive real number, $z = a + 2i$ and $z|z| - az + 1 = 0$, then

a) z is pure imaginary b) $a^2 = 2$

c) $a^2 = 4$ d) No such complex number exists

10. If $|z-3+2i|=4$, then the absolute difference between the maximum and minimum values of $|z|$ is

a) 8 b) 4 c) $2\sqrt{13}$ d) $4\sqrt{13}$

11. Let $z_1=10+6i$ and $z_2=4+6i$. If z is any complex number such that the argument of $(z-z_1)/(z-z_2)$ is $\pi/4$, then $|z-7-9i|$ is equal to

a) $2\sqrt{3}$ b) $3\sqrt{2}$ c) 3 d) 2

12. If the complex number z satisfies the condition $|z| \geq 3$, then the least value of $\left|z+\frac{1}{z}\right|$ is equal to

a) 5/3 b) 8/3 c) 11/3 d) none of these

13. For $Z_1=\sqrt[6]{\frac{1-i}{1+i\sqrt{3}}}$; $Z_2=\sqrt[6]{\frac{1-i}{\sqrt{3}+i}}$; $Z_3=\sqrt[6]{\frac{1+i}{\sqrt{3}-i}}$ which of the following holds good?

a) $\sum|Z_1|^2=\frac{3}{2}$ b) $|Z_1|^4+|Z_2|^4=|Z_3|^{-8}$

c) $\sum|Z_1|^3+|Z_2|^3=|Z_3|^{-6}$ d) $Z_1|^4+|Z_2|^4=|Z_3|^8$

14. Let z be a complex number. Then the region represented by $|z+2|<|z+4|$ is given by

a) $R_e(z)>-3$ b) $I_m(z)<-3$ c) $R_e(z)>-3, I_m(z)<-3$ d) None of these

15. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1|=|z_2|$. If z_1 has positive real part and z_2 has imaginary part, then which of the statements are correct for the value of $\frac{z_1+z_2}{z_1-z_2}$

a) zero b) real and positive c) real and negative d) purely imaginary

16. If Z is a non-real complex number, then the minimum value of $\frac{\operatorname{Im} Z^5}{\operatorname{Im}^5 Z}$ is

a) –1 b) –2 c) –4 d) –5

More than one correct answer type questions

17. If z_1 lies on $|z| = 1$ and z_2 lies on $|z| = 2$, then

a) $3 \le |z_1 - 2z_2| \le 5$ b) $1 \le |z_1 + z_2| \le 3$ c) $|z_1 - 3z_2| \ge 5$ d) $|z_1 - z_2| \ge 1$

18. If Z_1, Z_2, Z_3 be three unimodular complex numbers such that $\frac{Z_1^2}{Z_2Z_3} + \frac{Z_2^2}{Z_1Z_3} + \frac{Z_3^2}{Z_1Z_2} + 1 = 0$ then $|Z_1 + Z_2 + Z_3|$ can take the value equal to.

a) 4 b) 3 c) 2 d) 1

19. If $z_1 = 5 + 12i$ and $|z_2| = 4$ then

a) maximum $(|z_1 + iz_2|) = 17$

b) minimum $(|Z_1 + (1+i)z_2|) = 13 - 4\sqrt{2}$

c) minimum $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{4}$

d) maximum $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{3}$

20. Let a, b, c be distinct complex numbers with $|a| = |b| = |c| = 1$ and z_1, z_2 be the roots of the equation $az^2 + bz + c = 0$ with $|z_1| = 1$. Also let P and Q represents the complex numbers z_1 and z_2 is the complex plane with $\angle POQ = \theta$ where O being the origin then

a) $b^2 = ac, \theta = \frac{2\pi}{3}$ b) $\theta = \frac{2\pi}{3}, PQ = \sqrt{3}$ c) $PQ = 2\sqrt{3}, b^2 = ac$ d) $\theta = \frac{\pi}{3}, b^2 = ac$

21. If $z_1, z_2 \in C$ are such that $\left|\frac{z_1 - z_2}{1 - \bar{z}_1 z_2}\right| = 1$ then

a) $|z_1| = 1$ b) $|z_2| = 1$ c) $z_1 = e^{i\theta}, \theta \in R$ d) $z_2 = e^{i\phi}, \phi \in R$

22. If $|z| = 2\sqrt{2}$ *and* $\arg\left(\frac{z-(5+i)}{z-(1+i)}\right) = \frac{\pi}{4}$. Then Z is

a) $\left(\frac{3+\sqrt{7}}{2}\right) + i\left(\frac{3-\sqrt{7}}{2}\right)$

b) $\left(\frac{3-\sqrt{7}}{2}\right) + i\left(\frac{3+\sqrt{7}}{2}\right)$

c) $\left(\frac{3+\sqrt{7}}{2}\right) + i\left(\frac{3+\sqrt{7}}{2}\right)$

d) $2 - 2i$

23. For any complex numbers Z and ω such that $|z|^2 \omega - |\omega|^2 z = z - \omega$ then

a) $z = \omega$ b) $z\bar{\omega} = \bar{z}\omega$ c) $z\bar{\omega} = 1$ d) $\frac{z}{\omega} =$ purely imaginary

Linked comprehension type questions

Passage-I :

Let z_1, z_2 be two complex numbers such that

$$|z_1| = |z_2| = r > 0 \text{ and } X = \left(\frac{r(z_1+z_2)}{r^2+z_1z_2}\right)^2 + \left(\frac{r(z_1-z_2)}{r^2-z_1z_2}\right)^2, \text{ then}$$

24. Which of the following is true always ?

a) $|X| = 2$ b) X can be equal to zero

c) X is purely real d) X is purely imaginary

25. Which of the following is correct

a) $|X+1| \geq 5$ b) $|X+1| \geq 3$ c) $|X+1| \geq 2$ d) none of these

26. If $z_1 = re^{i2x}$, $z_2 = re^{i2y}$, then X is equal to

a) $\frac{\cos^2(x+y)}{\sin^2(x-y)} + i\frac{\sin^2(x+y)}{\cos^2(x-y)}$ b) $\frac{\cos^2(x-y)}{\sin^2(x+y)} + i\frac{\sin^2(x-y)}{\cos^2(x+y)}$

c) $\tan(x-y)+i\tan(x-y)$ d) none of these

Passage-II :

Let z_1 be a complex number of magnitude unity and z_2 be a complex number given by $z_2 = z_1^2 - z_1$. Answer the following questions.

27. If $\arg z_1 = \theta$ then $|z_2|$ is equal to

a) $2\left|\sin\frac{\theta}{2}\right|$ b) $2\left|\cos\frac{\theta}{2}\right|$ c) $\sqrt{2}\left|\sin\frac{\theta}{2}\right|$ d) $\sqrt{2}\left|\cos\frac{\theta}{2}\right|$

28. If $\arg z_1 = \theta$ and $4n\pi < \theta < (4n+2)\pi$, ($n$ is an integer), then $\arg z_2$ is equal to

a) $\frac{3\theta}{2}$ b) $\frac{\pi - 3\theta}{2}$ c) $\frac{\pi + 3\theta}{2}$ d) $\frac{\pi + \theta}{2}$

29. If $\arg z_1 = \theta$ and $(4n+2)\pi < \theta < (4n+4)\pi$, (n is an integer), then $\arg z_2$ is equal to

a) $\frac{\pi}{2}+3\theta$ b) $\frac{3\pi}{2}+\frac{3\theta}{2}$ c) $\frac{3\pi}{2}+3\theta$ d) $\frac{\pi}{2}+\frac{3\theta}{2}$

Passage-III :

Suppose z and w are two complex numbers such that $|z| \leq 1$ and $|w| \leq 1$ and $|z + iw| = |z - i\overline{w}| = 2$ we know that $|z|^2 = z\overline{z}$ and $|z + w| \leq |z| + |w|$ then, answer the following questions.

30. Which of the following is correct

a) $|z|=|w|=\frac{1}{2}$ b) $|z|=\frac{1}{2}$ $|w|=\frac{3}{4}$ c) $|z|=\frac{1}{4}$ $|w|=\frac{3}{4}$ d) $|z| = |w| = 1$

31. Number of complex number z satisfying above conditions are

a) 1 b) 2 c) 4 d) infinite

32. Which of the following is true for z and w

a) Re(z) = Re(w) b) Im(z) = Im(w) c) Re(z) = Im(w) d) Im(z) = Re(w)

Matrix matching type questions

33. Number solution of

COLUMN - I	COLUMN - II
A) $z^2+\|z\|=0$	p) 1
B) $z^2+\bar{z}^{2}=0$	q) 3
C) $z^2+8\bar{z}=0$	r) 4
D) $\|z-2\|=1$ and $\|z-1\|=2$	s) Infinite

EXERCISE-II

Demoivers rule, n_{th} roots of unity etc.,

LEVEL-I (MAIN)

Single answer type questions

1. If $z=\cos\theta+i\sin\theta$ then $\dfrac{z^{2n}-1}{z^{2n}+1}=$

   1) $\cos n\theta$ 2) $\sin n\theta$ 3) $-i\sin n\theta$ 4) $i\tan n\theta$

2. If α and β are the roots of $x^2-2x+4=0$ then $\alpha^n+\beta^n=$

   1) $2^n.\cos(n\pi/2)$ 2) $2^{n+1}.\cos(n\pi/3)$ 3) $2^{n-1}.\sin(n\pi/6)$ 4) $2^{n+1}.\sin(n\pi/3)$

3. If $p=\cos2\alpha+i\sin2\alpha$, $q=\cos2\beta+i\sin2\beta$ then $\sqrt{p/q}-\sqrt{q/p}=$

   1) $2i\sin(\alpha-\beta)$ 2) $2\sin(\alpha-\beta)$ 3) $2i\cos(\alpha-\beta)$ 4) $2\cos(\alpha-\beta)$

4. One value of $(1+i)^{1/2}$ is $2^{1/4}e^{i\pi/8}$. The other value is

   1) $2^{1/4}e^{-i(\pi/8)}$ 2) $2^{1/4}e^{i(5\pi/8)}$ 3) $2^{1/4}e^{-i(5\pi/8)}$ 4) $2^{1/4}.e^{i(9\pi/8)}$

5. The value of $\prod_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$ is

   1) -1 2) 0 3) $-i$ 4) i

6. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=$

   1) 0 2) 1/2 3) 3/2 4) 2

7. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ then $\cos(2\alpha-\beta-\gamma)+\cos(2\beta-\gamma-\alpha)+\cos(2\gamma-\alpha-\beta)=$

   1) 0 2) 1 3) 2 4) 3

8. The product of the distinct $(2n)^{th}$ roots $1+i\sqrt{3}$ is

   1) 0 2) $-1-i\sqrt{3}$ 3) $1+i\sqrt{3}$ 4) $-1+i\sqrt{3}$

9. $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)$ to 2n factors =

1) 2 2) 2^{2n} 3) $2n$ 4) 2^n

10. If $1,\omega,\omega^2$ are the cube roots of unity, then $(x+y+z)(x+y\omega+z\omega^2)\ (x+y\omega^2+z\omega)=$

1) $x^3+y^3+z^3$ 2) $x^3+y^3+z^3-3xyz$

3) $x^3+y^3+z^3+3xyz$ 4) $x^3+y^3+z^3-6xyz$

11. Least positive argument of the 4th root of the complex number $2-i\sqrt{12}$ is

1) $\pi/6$ 2) $5\pi/12$ 3) $7\pi/12$ 4) $11\pi/12$

Numerical value type questions

12. If $\left(\dfrac{1+\cos\theta+i\sin\theta}{\sin\theta+i+i\cos\theta}\right)^4=\cos n\theta+i\sin n\theta$, then $n=$

13. If $z_k=\cos\left(\dfrac{k\pi}{10}\right)+i\sin\left(\dfrac{k\pi}{10}\right)$, then $z_1z_2z_3z_4$ is equal to

14. The value of $e\left(Cis(-i)-Cis(i)\right)$ is equal to

LEVEL-II (ADVANCED)

Single answer type questions

1. Sum of the common roots of the equations $z^3+2z^2+2z+1=0$ and $z^{97}+z^{29}+1=0$ is equal to

a) 1 b) –1 c) 0 d) none of these

2. If $1,\alpha_1,\alpha_2,......,\alpha_8$ are nine, ninth roots of unity (taken in counter-clockwise direction) then $|(2-\alpha_1)(2-\alpha_3)(2-\alpha_5)(2-\alpha_7)|$ is equal to

a) $\sqrt{255}$ b) $\sqrt{1023}$ c) $\sqrt{511}$ d) 15

3. If $a=1+\dfrac{x^3}{\lfloor 3}+\dfrac{x^6}{\lfloor 6}+\dfrac{x^9}{\lfloor 9}+.....,b=x+\dfrac{x^4}{\lfloor 4}+\dfrac{x^7}{\lfloor 7}+.....$ and $c=\dfrac{x^2}{\lfloor 2}+\dfrac{x^5}{\lfloor 5}+\dfrac{x^8}{\lfloor 8}+.....$, then $a^3+b^3+c^3-3abc=$

a) 0 b) 1 c) e^x d) –1

4. Given α, β respectively the fifth and the fourth non-real roots of unity, then the value of $(1+\alpha)(1+\beta)(1+\alpha^2)(1+\beta^2)(1+\beta^3)(1+\alpha^4)$ is

a) 0 b) $(1+\alpha+\alpha^2)(1-\beta^2)$

c) $(1+\alpha)(1+\beta+\beta^2)$ d) None of these

5. $\sqrt{-1-\sqrt{-1-\sqrt{-1\,.......\infty}}}$ is equal to :

a) ω or ω^2 b) $-\omega$ or $-\omega^2$ c) $1+i$ or $1-i$ d) $-1+i$ or $-1-i$

where ω is the imaginary cube root of unity and $i=\sqrt{-1}$

6. The solution of the $z^4+4z^3i-6z^2-4zi=0$ are the vertices of a convex polygon in the complex palane. The area of the polygon is

a) $2^{3/4}$ b) $2^{3/2}$ c) $2^{5/4}$ d) 2

7 If $\alpha = Cos\frac{8\pi}{11}+i\ Sin\frac{8\pi}{11}$ then real part of $\sum_{K-1}^{5}\alpha^k$ is .....

a) $\frac{1}{2}$ b) 0 c) $\frac{-1}{2}$ d) 1

8. $Sin\frac{\pi}{3}+\frac{1}{2}\sin\frac{2\pi}{3}+\frac{1}{3}\sin\frac{3\pi}{3}+....\infty=$

a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$

9. If α is non real root of $x^7 = 1$, then $1+3\alpha+5\alpha^2+7\alpha^3+.....+13\alpha^6$ α^6 is equal to

a) 0 b) $\frac{14}{1-\alpha}$ c) $\frac{14}{\alpha-1}$ d) none of these

10. Let z_1 and z_2 be n^{th} roots of unity which subtend a right angle at the origin. Then n must be of the form

a) $4k + 1$ b) $4k + 2$ c) $4k + 3$ d) $4k$

11. 8^{th} roots of unity and 6^{th} roots of unity in the Argand plane formed two regular polygons. Then the ratio of their Areas is

a) 4 : 3 B) $8:3\sqrt{3}$ c) $8:3\sqrt{6}$ d) $4:3\sqrt{3}$

12. The equation $z^n-1 = 0$ and $z^m-1 = 0$ have only one common root where m, n $m,n \in N$, then

a) n and m are primes
b) atleast one of m and n is prime
c) m and n are coprimes
d) one of m and n is even and the other is odd

13. Suppose two complex numbers $z=a+ib; w=c+id$ satisfy the equation $\frac{z+w}{z}=\frac{w}{z+w}$ then

a) both a & c are zeros
b) both b & d are zeros
c) both b & d must be non zeros
d) at least one of b & d is non-zero

More than one correct answer type questions

14. $\alpha \neq 1$ is a 5^{th} root of 1 and $\beta \neq -1$ is a 5^{th} root of –1. Which of the following assertions is (are) true?

a) $\alpha\beta$ is a 5^{th} root of –1
b) $\alpha^2\beta$ is a 5^{th} root of 1
c) $|\alpha\beta|=1$
d) $|\alpha+\beta|<2$

Linked comprehension type questions

Passage - I :

Let $w=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}\alpha=w+w^2+w^3\beta=w^3+w^5+w^6$ then

15. α,β are the roots of the equation

a) $x^2+x+1=0$ b) $x^2+x+2=0$ c) $x^2+2x+5=0$ d) $x^2+3x+5=0$

16. $2\alpha=$

a) $1+\sqrt{7}i$ b) $-1-\sqrt{7}i$ c) $1+\sqrt{7}i$ d) $-1+\sqrt{7}i$

17. $\sum_{K=0}^{6} w^{k^2}$

a) $\sqrt{7}i$ b) i c) $-i$ d) $-\sqrt{7}i$

Passage - II :

The roots of $x = 1^{1/3}$ are called the cube roots of unity they are usually denoted by $1,\omega,\omega^2$ it follows that $1+\omega+\omega^2=0$ and $\omega^3=1$, ω and ω^2 are called complex cube roots of unity

18. If $\frac{1}{a+\omega}+\frac{1}{b+\omega}+\frac{1}{c+\omega}=2\omega^2$ and $\frac{1}{a+\omega^2}+\frac{1}{b+\omega^2}+\frac{1}{c+\omega^2}=2\omega$ then $\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}$ is

a) 0 b) 1 c) 2 d) 1/2

19. If $x+y+z\neq 0$ and $\begin{vmatrix} \frac{x}{1+\omega} & \frac{y}{\omega+\omega^2} & \frac{z}{\omega^2+1} \\ \frac{y}{\omega+\omega^2} & \frac{z}{\omega^2+1} & \frac{x}{1+\omega} \\ \frac{z}{\omega^2+1} & \frac{x}{1+\omega} & \frac{y}{\omega+\omega^2} \end{vmatrix}=0$ then

a) $x^2+y^2+z^2=0$ b) $x+y\omega+z\omega^2=0$ or $x=y=z$

c) $x\neq y\neq z\neq 0$ d) None of these

20. $\frac{1}{a+\omega}+\frac{1}{b+\omega}+\frac{1}{c+\omega}=\frac{1}{\omega}$ where a, b, c are real then $\sum\frac{1}{a^2-a+1}$ is

a) 1 b) 0 c) -1 d) 2

Passage - III :

If α is any of 7th roots of unity , then $\alpha=\frac{cis2k\pi}{7},(k=0,1,.....6)$ and $\sum_{i=1}^{6}\alpha^i=-1,(\alpha\neq 1)$ and $\alpha^7=1$ Answer the following

21. The equation whose roots are $\alpha+\alpha^2+\alpha^4$ and $\alpha^3+\alpha^5+\alpha^6$ is

a) $x^2+x-2=0$ b) $x^2+x+2=0$

c) $x^2-x+2=0$ d) $x^2-x-2=0$

22. If $f(x)=1+2x+3x^2+4x^3+5x^4+6x^5+7x^6$ then for

$\alpha \neq 1, f(x)+f(\alpha x)+f(\alpha^2 x)+f(\alpha^3 x)+........+f(\alpha^6 x)=$

a) 42 b) 21 c) 14 d) 7

23. $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}=\sin\frac{4\pi}{7}\sin\frac{5\pi}{7}\sin\frac{6\pi}{7}=$

a) $\frac{\sqrt{7}}{16}$ b) $\frac{\sqrt{7}}{8}$ c) $\frac{\sqrt{7}}{32}$ d) $\frac{\sqrt{7}}{64}$

Passage-IV :

If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ and $\delta=\frac{\alpha+\beta+\gamma}{3}$. The difference of any two of the angles α,β,γ is equivalent to $\frac{2\pi}{3}$. Then

24. The value of $\sin 6\alpha+\sin 6\beta+\sin 6\gamma$ interms of δ is

a) $3\sin 2\delta$ b) $\sin 6\delta$ c) $3\sin 3\delta$ d) $3\sin 6\delta$

25. If one of the angles α,β,γ is $\frac{\pi}{36}$ then the

a) 0 b) 3 c) $\frac{3}{2}$ d) $\frac{2049}{2048}$

26. If $\alpha=10^0$ then

a) $\frac{113}{128}$ b) $\frac{117}{128}$ c) $\frac{51}{128}$ d) none of these

Passage-V :

If $z=r(\cos\theta+i\sin\theta)$ and n is a positive integer, then

$$z^{1/n}=r^{1/n}\left[\cos\frac{(2k\pi+\theta)}{n}+i\sin\frac{(2k\pi+\theta)}{n}\right]$$

Where k = 0, 1, 2, 3, …….. (n -1), given n, nth roots of the complex number z.

27. If $1,\omega,\omega^2 .....\ \omega^{n-1}$ are the n, nth roots of unity and z_1 and z_2 are any two complex numbers, then $\sum_{k=0}^{n-1}|z_1+\omega^k z_2|^2$

a) $n\left[|z_1|^2+|z_2|^2\right]$ b) $(n-1)\left[|z_1|^2+|z_2|^2\right]$

c) $(n+1)\left[|z_1|^2+|z_2|^2\right]$ d) $(n+2)\left[|z_1|^2+|z_2|^2\right]$

28. If $1,\alpha,\alpha^2$, ... α^{n-1} are the nth roots of unity then $\sum_{i=1}^{n-1}\frac{1}{2-\alpha^i}$ is equal to

a) $\frac{(n-2)2^{n-1}+1}{2^n-1}$ b) $(n-2).2^n$ c) $\frac{(n-2).2^{n-1}}{2^n-1}$ d) $\frac{(n+2).2^{n-1}}{2^n-1}$

22. If $f(x)=1+2x+3x^2+4x^3+5x^4+6x^5+7x^6$ then for $\alpha\neq 1, f(x)+f(\alpha x)+f(\alpha^2 x)+f(\alpha^3 x)+........+f(\alpha^6 x)=$

a) 42 b) 21 c) 14 d) 7

23. $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}=\sin\frac{4\pi}{7}\sin\frac{5\pi}{7}\sin\frac{6\pi}{7}=$

a) $\frac{\sqrt{7}}{16}$ b) $\frac{\sqrt{7}}{8}$ c) $\frac{\sqrt{7}}{32}$ d) $\frac{\sqrt{7}}{64}$

Passage-IV :

If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ and $\delta=\frac{\alpha+\beta+\gamma}{3}$. The difference of any two of the angles α,β,γ is equivalent to $\frac{2\pi}{3}$. Then

24. The value of $\sin 6\alpha+\sin 6\beta+\sin 6\gamma$ interms of δ is

a) $3\sin 2\delta$ b) $\sin 6\delta$ c) $3\sin 3\delta$ d) $3\sin 6\delta$

25. If one of the angles α,β,γ is $\frac{\pi}{36}$ then the

a) 0 b) 3 c) $\frac{3}{2}$ d) $\frac{2049}{2048}$

26. If $\alpha=10^0$ then

a) $\frac{113}{128}$ b) $\frac{117}{128}$ c) $\frac{51}{128}$ d) none of these

Passage-V :

If $z=r(\cos\theta+i\sin\theta)$ and n is a positive integer, then

$$z^{1/n}=r^{1/n}\left[\cos\frac{(2k\pi+\theta)}{n}+i\sin\frac{(2k\pi+\theta)}{n}\right]$$

Where k = 0, 1, 2, 3, …….. (n -1), given n, nth roots of the complex number z.

27. If $1,\omega,\omega^2 .....\ \omega^{n-1}$ are the n, nth roots of unity and z_1 and z_2 are any two complex numbers, then $\sum_{k=0}^{n-1}|z_1+\omega^k z_2|^2$

a) $n\left[|z_1|^2+|z_2|^2\right]$ b) $(n-1)\left[|z_1|^2+|z_2|^2\right]$

c) $(n+1)\left[|z_1|^2+|z_2|^2\right]$ d) $(n+2)\ [|z_1|^2+|z_2|^2]$

28. If $1,\alpha,\alpha^2$, …… α^{n-1} are the nth roots of unity then $\sum_{i=1}^{n-1}\frac{1}{2-\alpha^i}$ is equal to

a) $\frac{(n-2)2^{n-1}+1}{2^n-1}$ b) $(n-2).2^n$ c) $\frac{(n-2).2^{n-1}}{2^n-1}$ d) $\frac{(n+2).2^{n-1}}{2^n-1}$

29. If $1, \omega, \omega^2, ...\omega^{n-1}$ are the nth roots of unity then $(1-\omega)(1-\omega^2)....(1-\omega^{n-1})$ is equal to

a) 0 b) 1 c) n d) n^2

Matrix matching type questions

30. **COLUMN - I** / **COLUMN - II**

COLUMN - I	COLUMN - II
A) Let w be a non real cube root of unity then the number of distinct elements in the set $\{(1+w+w^2+.....+w^n)^m \mid m,n \in N\}$ is	p) 0
B) Let 1, w, w^2 be the cube root of unity. The least possible degree of a polynomial with real cofficient having roots $2w, (2+3w), (2+3w^2), (2-w-w^2), (2-w-w^2)$, is	q) 5
C) If w is a complex root of $z^n - 1 = 0$ and p is not a multiple of n, then $1 + w^p + w^{2p} + ... + (w^{n-1})^p =$	r) 6
D) If the roots of the equation $z^4 + z^2 + 1 = 0$ are z_1, z_2, z_3, z_4 then $(w - z_1)(w - z_2)(w - z_3)(w - z_4)$ (where w is a complex cube root of unity) =	s) 8

31. If α, β are the roots of the equation $z^2\sin^2\theta - z\sin 2\theta + 1 = 0, 0 < \theta < \frac{\pi}{2}$, and n is an integer, then the value of

COLUMN - I	COLUMN - II
A) $(\alpha\beta)^n$	p) $2^n \cot^n \theta$
B) $(\alpha+\beta)^n$	q) $\operatorname{cosec}^{2n}\theta$
C) $\alpha^n + \beta^n$	r) $2\operatorname{cosec}^n\theta\cos n\theta$
D) $\lvert\alpha\rvert^n + \lvert\beta\rvert^n$	s) $2\operatorname{cosec}^n\theta$

32. Match the following

COLUMN - I **COLUMN - II**

(ω is complex cube roots of unity, i = $\sqrt{-1}$)

Equation	**roots**
A) $x^6 + 1 = 0$	p) ± 1
B) $x^8 + x^4 + 1 = 0$	q) $\pm i$
C) $x^{12} - 1 = 0$	r) $\pm\omega, \pm\omega^2$
D) $x^6 - 1 = 0$	s) $\pm i\omega, \pm i\omega^2$

Integer answer type questions

33. If $n \geq 3$ and $1, \alpha_1, \alpha_2, \alpha_3......\alpha_{n-1}$ are n roots of unity, then value of $\sum_{1 \le i < j \le n-1} \alpha_i \alpha_j$ is

EXERCISE-III

Geometrical applications

LEVEL-I (MAIN)

Single answer type questions

1. The complex equation $|z+1-i| = |z+i-1|$ represents a

1) straight line 2) circle 3) parabola 4) hyperbola

2. If $\text{Im}(z^2) = 2$ then the locus of z is

1) $xy = 1$ 2) $xy = 0$ 3) $xy = -1$ 4) $xy = 2$

3. If $\alpha + i\beta = \tan^{-1}(z)$, $z = x + iy$ and α is constant then the locus of z is

1) $x^2 + y^2 + 2x\cot 2\alpha = 1$ 2) $\cot 2\alpha.(x^2 + y^2) = 1 + x$

3) $x^2 + y^2 + 2y\tan 2\alpha = 1$ 4) $x^2 + y^2 + 2y\tan 2\alpha = 1$

4. If $\log_{\tan 30^o}\left(\dfrac{2|z|^2 + 2|z| - 3}{|z| + 1}\right) < -2$, then

1) $|z| < 3/2$ 2) $|z| > 3/2$ 3) $|z| > 2$ 4) $|z| < 2$

5. In the Argand Diagram,all the complex numbers z satisfying $|z-4i| + |z+4i| = 10$ lie on a

1) straight line 2) circle 3) ellipse 4) parabola

6. The region of the argand plane defined by $|z-i| + |z+i| \le 4$ is

1) interior of an ellipse 2) exterior of a circle

3) interior and boundary of an ellipse 4) parabola

7. In Argand diagram, O, P, Q represent the origin, z and $z + iz$ respectively. then $\angle OPQ =$

1) $\dfrac{\pi}{4}$ 2) $\dfrac{\pi}{6}$ 3) $\dfrac{\pi}{2}$ 4) $\dfrac{\pi}{3}$

8. Let the complex number z_1 z_2, z_3 be the vertices of an equilateral triangle. Let z_0 be the circumcentre of the triangle. Then $z_1^2 + z_2^2 + z_3^2 =$

1) $2z_0^2$ 2) $3z_0^2$ 3) $4z_0^2$ 4) $4z_0^3$

9. The equation $|z-i| + |z+i| = k$, $k > 0$, can represent an ellipse if k^2 is

1) < 1 2) < 2 3) > 4 4) < 3

10. Two points P and Q in the Argand diagram represent z and $2z + 3 + i$. If P moves on a circle with centre at the origin and radius 4, then the locus of Q is a circle with centre

1) (3, 1) 2) (0, 0) 3) (2, –3) 4) (–3, 1)

11. Image of point $\frac{2-i}{3+i}$ in the line $(1+i)z+(1-i)\overline{z}=0$ is the point

1) $\frac{1+i}{2}$ 2) $\frac{1-i}{2}$ 3) $\frac{-1+i}{2}$ 4) $-\frac{i+1}{2}$

12. If 'z' is a complex number then the locus of 'z' satisfying the condition $|2z-1|=|z-2|$ is

1) parabola 2) a pair of lines 3) a circle

4) perpendicular bisector of the segment joining $z=\frac{1}{2}$ and $z=2$

13. If $A(Z_1)$, $B(Z_2)$ and $C(Z_3)$ are the vertices of triangle ABC, such that $|Z_1|=|Z_2|=|Z_3|$ and $|Z_1+Z_2|=|Z_1-Z_2|$ then $\underline{|C} =$

1) $\pi/2$ 2) $\pi/4$ 3) $\pi/3$ 4) $\pi/6$

Numerical value type questions

14. If the roots of $z^3+iz^2+2i=0$ represent the vertices of a ΔABC in the Argand plane, then the area of the triangle is

15. In the Argand plane the non-zero solutions of the equation $z^3=\overline{z}$ form polygon whose area is

16. In the argand plane, the point A,B and C represent the Complex Numbers z, ωz, and $z+\omega z$ where $z\neq 0$ and $\omega = cis\frac{2\pi}{3}$. If Area of $(\Delta ABC)=\sqrt{3}$ then $|z|$ is

17. If $|z+3|=2\ |z-3|$ represents equation of Circle, then its radius is

LEVEL-II (ADVANCED)

Single answer type questions

1. If $|z_1-1|<1, |z_2-2|<2$ and $|z_3-3|<3$, then $|z_1+z_2+z_3|$

a) is less than 6 b) is greater than 6

c) is less than 12 d) lies between 6 and 12

2. If $\arg\left(\frac{z_1-\frac{z}{|z|}}{\frac{z}{|z|}}\right)=\frac{\pi}{2}$ and $\left|\frac{z}{|z|}-z_1\right|=3$ then $|z_1|$ equals to

a) $\sqrt{26}$ b) $\sqrt{10}$ c) $\sqrt{3}$ d) $2\sqrt{2}$

3. If P lies on $|z-1-i|=1$ and Q lies on $|z-i|=4$ maximum and minimum values of $\overline{PQ}$ are 'l' and 'm' respectively then l + m equals

a) 2 b) 4 c) 6 d) 8

4. If a complex number 'z' satisfies $|2z + 10 + 10i| \le 5\sqrt{3} - 5$, then the least principal argument of 'z' is

a) $-\frac{5\pi}{6}$ b) $-\frac{\pi}{6}$ c) $-\frac{2\pi}{3}$ d) $-\frac{11\pi}{12}$

5. All complex numbers 'z' which satisfy the relation $\left|z-|z+1|\right| = \left|z+|z-1|\right|$ on the complex plane lie on the

a) Line $y = 0$ b) Line $x = 0$ c) circle $x^2 + y^2 = 1$

d) Line $x = 0$ or on a line segment joining $(-1, 0)$ to $(1, 0)$

6. If $|Z-1| = 1$, then $\frac{Z-2}{Z} =$

a) iTan(arg Z) b) iCot(arg Z) c) Tan (Arg Z) d) Cot(arg Z)

7. If the complex no.z satisfies $|z-6i| = I_m(z)$ then range of arg z–arg $\bar{z}$ is ____________

a) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ b) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ c) $\left[\frac{2\pi}{3}, \frac{4\pi}{3}\right]$ d) $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$

8. In the Argand plane, the points A, B and C represent the complex numbers z, ωz and $z+\omega z$, where $z \ne 0$ and $\omega = cis\frac{2\pi}{3}$. If area $(\Delta ABC) = \sqrt{3}$, then $|z|$ equals

a) 2 b) 1 c) $\sqrt{3}$ d) $\frac{\sqrt{3}}{2}$

9. If $|z-25i| \le 15$, then |maximum arg (z) - minimum arg (z)| equals

a) $2\cos^{-1}(3/5)$ b) $2\cos^{-1}(4/5)$

c) $\pi/2 + \cos^{-1}(3/5)$ d) $\sin^{-1}(3/5) - \cos^{-1}(3/5)$

10. If the two circles $z\bar{z} + \bar{a}_1 z + a_1\bar{z} + b_1 = 0$ and $z\bar{z} + \bar{a}_2 z + a_2\bar{z} + b_2 = 0$ (where b_1, b_2 are real) intersect orthogonally , then

a) $a_1a_2 + \overline{a_1}\,\overline{a_2} = b_1 + b_2$ b) $a_1a_2 - \overline{a_1}\,\overline{a_2} = b_1 - b_2$

c) $a_1\overline{a_2} + \overline{a_1}\,a_2 = b_1 + b_2$ d) $a_1\overline{a_2} - \overline{a_1}\,a_2 = b_1 - b_2$

11. If the points A1, A2, ..., A8 be the affixes of the roots of the equation $z^8 - 1 = 0$ in the argand plane, then the area of the triangle $A_3A_6A_7$ is equal to

a) $\sqrt{2}$ b) $\sqrt{3}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

12. Locus of z if $\arg(z-(1+i)) = \begin{cases} \frac{3\pi}{4}, & when\ |z| \le |z-4| \\ -\frac{\pi}{4} & when\ |z| > |z-4| \end{cases}$ is

a) A set of 4 rays
b) A set of 2 rays
c) circle
d) a line segment only

More than one correct answer type questions

13. Equation of tangent drawn to circle $|z| = r$ at the point $A(z_0)$ is

a) $\mathrm{Re}\left(\frac{z}{z_0}\right) = 1$
b) $z\overline{z_0} + z_0\overline{z} = 2r^2$
c) $\mathrm{Im}\left(\frac{z}{z_0}\right) = 1$
d) $\mathrm{Im}\left(\frac{z_0}{z}\right) = 1$

14. If $|Z| = 1$, then the locus of Z_1 which moves such that $Z_1 = \frac{5-4Z}{4Z-2}$ in the complex plane, is

a) circle centred at $\frac{1}{2}$
b) circle of radius $4\frac{1}{2}$
c) circle centred at $-\frac{1}{2}$ and of radius 1
d) circle passing through all quadrants

15. If $A(z_1), B(z_2)$ & $C(z_3)$ are three points in argand plane where

$|z_1 + z_2| = |z_1| - |z_2|$ & $|(1-i)z_1 + iz_3| = |z_1| + |z_3 - z_1|$ then

a) A, B, C lie on a fixed circle with centre $\frac{z_2 + z_3}{2}$
b) A, B, C form right triangle
c) A, B, C form equilateral triangle
d) A, B, C form an obtuse triangle

16. Let z be a complex number such that $\left|z + \frac{1}{z}\right| = 2$. If $|z| = r_1$ & r_2 for $\arg z = \frac{\pi}{4}$

a) $|r_1 - r_2| = \sqrt{2}$ for arg $z = \frac{\pi}{4}$
b) Range of $|r_1 - r_2| = [0, 2]$ as arg z varies
c) $|r_1 - r_2| = 3$ if arg $z = \frac{\pi}{3}$
d) $|r_1 - r_2| = 2$ if arg z = $\frac{\pi}{2}$

17. Let A and B two distance points denoting the complex number α and β respectively. A complex number z lies btween A and B where $z \ne \alpha, z \ne \beta$. Which of the following relation(s) hold good

a) $|\alpha - z| + |z - \beta| = |\alpha - \beta|$
b) $\exists$ a positive real number 't' such tha $tz = (1-t)\alpha + t\beta$
c) $\begin{vmatrix} z-\alpha & \overline{z}-\overline{\alpha} \\ \beta-\alpha & \overline{\beta}-\overline{\alpha} \end{vmatrix} = 0$
d) $\begin{vmatrix} z & \overline{z} & 1 \\ \alpha & \overline{\alpha} & 1 \\ \beta & \overline{\beta} & 1 \end{vmatrix} = 0$

18. If z is a complex number such that z, iz and $z + iz$ are the vertices of a triangle then

a) area of the triangle is $\frac{1}{2}\ |z|^2$ b) orthocentre of the triangle is $z + iz$

c) circumcentre of the triangle is $\frac{1}{2}(z+iz)$ d) the triangle is equilateral

19. Equation of the straight line making an angle 45° with the line $\bar{a}z + a\bar{z} + b = 0$ (where a is complex and b is real) and passing through C (C is complex) are

a) $\frac{z-c}{a} + i\frac{\bar{z}-\bar{c}}{\bar{a}} = 0$ b) $\frac{\bar{z}-\bar{c}}{\bar{a}} + i\frac{z-c}{a} = 0$ c) $\frac{z-c}{a} - i\frac{\bar{z}-\bar{c}}{\bar{a}} = 0$ d) $\frac{\bar{z}-\bar{c}}{\bar{a}} - i\frac{z-c}{a} = 0$

Linked comprehension type questions

Passage-I :

In argand plane $|z|$ represent the distance of a point z from the origin. In general $|z_1 - z_2|$ represent the distance between two points z_1 and z_2. Also for a general moving point z in argand plane, if $\arg(z) = \theta$, then $z = |z|e^{i\theta}$, where $e^{i\theta} = \cos\theta + i\sin\theta$

20. The equation $|z - z_1| + |z - z_2| = 10$ if $z_1 = 3 + 4i$ and $z_2 = -3 - 4i$ represents

a) point circle b) ordered pair (0,0) c) ellipse d) line segment

21. If $|z - (3+2i)| = \left|z\cos\left(\frac{\pi}{4} - \arg z\right)\right|$, then locus of z is

a) circle b) parabola c) ellipse d) hyperbola

Passage-II :

Let Z be a complex number and K be a real number.consider the sets

$A:\{Z : |\mathrm{Im}(Z)| = K - |\mathrm{Re}(Z) - K|\}$, $B:\{Z : |Z - K| > |Z - 2K|\}$

C : Circle inscribed in the geometrical figure formed by A

22. Area bounded by A is

a) $\frac{3K^2}{2}$ b) K^2 c) $2K^2$ d) $\frac{K^2}{2}$

23. Radius of C is

a) $\frac{K}{2}$ b) $\frac{K}{\sqrt{2}}$ c) K d) $\frac{3K}{2}$

24. Number of points of contact of C with A that belong to B is

a) 0 b) 2 c) 3 d) 4

Passage-III :

Consider the region R in the argand plane described by the complex no.s Z satisfying the inequalities $|Z-2| \le |Z-4|$, $|Z-3| \le |Z+3|$, $|Z-i| \le |Z-3i|$, $|Z+i| \le |Z+3i|$.

Answer the following questions.

25. The maximum value of $|Z|$ for any Z in R is

a) 5 b) 3 c) 1 d) $\sqrt{13}$

26. Area of largest circle that can be inscribed in R is

a) 5π b) $\frac{9\pi}{4}$ c) 9π d) 25π

27. Maximum of $|Z_1 - Z_2|$ given that Z_1, Z_2 are any two complex no.s lying in the region R is

a) 5 b) 14 c) $\sqrt{13}$ d) 12

Passage-IV :

Points $D(z_1)$, $E(z_2)$, $F(z_3)$ lie on a circle centred at the origin O. The tangents to the circle at D, E, F intersect at A, B and C. A, B, C are opposite D, E, F respectively. Based on this solve the following problems.

28. If A is represented by z, then arg $\frac{z_3 - z}{z_3}$ equals to

a) $-\frac{\pi}{2}$ b) $-\frac{\pi}{3}$ c) $-\pi$ d) $-\frac{2\pi}{3}$

29 The complex number z is given by

a) $\frac{2z_1z_3}{z_1+z_3}$ b) $\frac{2z_2z_3}{z_2+z_3}$ c) $\frac{2z_1z_2}{z_1+z_2}$ d) none of these

30. The complex representation of M (point of intersection of AO and DE) is given by

a) $\left(\frac{z_1+z_3}{z_1+z_2}\right)z_1$ b) $\left(\frac{z_1-z_3}{z_1-z_2}\right)z_1$ c) $\left(\frac{z_1+z_2}{z_1+z_3}\right)z_3$ d) $\frac{z_1z_3}{z_1+z_3}$

Passage-V :

Consider a square OABC in the Argand plane, where O is origin and A = A (z_0)

31. Equation to circum circle of OABC is

a) $|z - z_0(1+i)| = |z_0|$ b) $\left|z - z_0\left(\frac{1+i}{2}\right)\right| = \frac{|z_0|}{\sqrt{2}}$

c) $\left|z - z_0\left(\frac{1+i}{2}\right)\right| = \sqrt{2}\,|z_0|$ d) $|z - z_0(1+i)| = \sqrt{2}\,|z_0|$

32. Area of the square OABC

a) $|z_0|^2$ b) $\frac{1}{2}|z_0|^2$ c) $2|z_0|^2$ d) $4|z_0|^2$

Matrix matching type questions

33. Match the following :

COLUMN - I (Equation of circle)	COLUMN - II (Radius)
A) $\lvert z-1\rvert^2+\lvert z+1\rvert^2=4$	p) 3
B) $\text{Amp}\left(\frac{z-2i}{z+2i}\right)=\frac{\pi}{2}$	q) 4
C) $z\bar{z}-z(1-2i)-\bar{z}(1+2i)=4$	r) 2
D) $\lvert z+3\rvert=2\lvert z-3\rvert$	s) 1

34. Match the following :

COLUMN - I	COLUMN - II
A) If z_1, z_2, z_3 are vertices of an equilateral triangle then	p) $(z_1-z_2)^2+3(z_3-z_2)^2=0$
B) If z_1, z_2, z_3 are vertices of A, B, C respectively right angle at C, then of an isosceles right angled triangle with	q) $(z_2-z_3)^2=3(z_1-z_2)(z_3-z_1)$
C) If z_1, z_2, z_3 are vertices of A, B, C respectively each equal to $\frac{\pi}{6}$ then of an isosceles triangle and angles B & C are	r) $(z_1-z_2)^2=2(z_1-z_3)(z_3-z_2)$
D) If z_1, z_2, z_3 are vertices of A, B, C respectively of a triangle ABC in which	s) $(z_1-z_2)^2+(z_2-z_3)^2+(z_3-z_1)^2=0$

35. Match the following Column – I with Column – II

COLUMN - I	COLUMN - II
A) Locus of the point 'z' satisfying the equation	p) a straight line $\text{Re}(z^2)=\text{Re}(z+\bar{z})$ is
B) Locus of the point 'z' satisfying the equation fixed complex numbers such that $\lambda>\lvert z_1-z_2\rvert$ is	q) a circle $\lvert z-z_1\rvert+\lvert z-z_2\rvert=\lambda,\ \lambda\in R^+$ and z_1,z_2
C) Locus of the point 'z' satisfying the equation $\left\lvert\frac{2z-i}{z+1}\right\rvert=m$, where $i=\sqrt{-1},\ m\in R^+$ is	r) an aretwo ellipse
D) If $\lvert\bar{z}\rvert=25$ then the locus of the points represented by $-1+75\bar{z}$ is	s) a hyperbola

Integer answer type questions

36. Let z be a complex number then the minimum possible value of $\frac{\lvert z\rvert^2+\lvert z-3\rvert^2+\lvert z-6i\rvert^2}{10}$ is

KEY SHEET (PRACTICE SHEET)

EXERCISE-I

LEVEL-I

1) 3	2) 1	3) 1	4) 3	5) 3	6) 2	7) 4	8) 3
9) 1	10) 3	11) 2	12) 2	13) 1	14) 20	15) 0.33	16) 840
17) 0.75							

LEVEL-II

1) d	2) a	3) d	4) c	5) c	6) d	7) b	8) d
9) d	10) c	11) b	12) b	13) b	14) a	15) d	16) c
17) abcd	18) cd	19) abd	20) ab	21) abcd	22) ab	23) abcd	24) c
25) c	26) d	27) a	28) c	29) b	30) d	31) b	32) d
33) A-q;B-s;C-r;D-p							

EXERCISE-II

LEVEL-I

1) 4	2) 2	3) 1	4) 4	5) 1	6) 3	7) 4	8) 2
9) 2	10) 2	11) 2	12) 4	13) –1	14) 6.38		

LEVEL-II

1) b	2) c	3) b	4) a	5) a	6) c	7) c	8) d
9) c	10) d	11) c	12) c	13) d	14) ac	15) b	16) d
17) a	18) c	19) b	20) a	21) b	22) d	23) b	24) d
25) c	26) b	27) a	28) a	29) c	30) A-r;B-q;C-p;D-p		
31) A-q;B-p;C-r;D-s			32) A-qs;B-rs;C-pqrs;D-pr				
33) 1							

EXERCISE-III

LEVEL-I

1) 1	2) 1	3) 1	4) 3	5) 3	6) 3	7) 3	8) 2
9) 3	10) 1	11) 3	12) 3	13) 2	14) 2	15) 2	16) 2
17) 4							

LEVEL-II

1) c	2) b	3) d	4) a	5) d	6) a	7) b	8) a
9) b	10) c	11) c	12) b	13) ab	14) cd	15) ab	16) abd
17) abcd	18) ac	19) abc	20) d	21) b	22) c	23) b	24) a
25) d	26) b	27) a	28) a	29) b	30) c	31) b	32) a
33) A-t;B-r;C-p;D-q			34) A-s;B-r;C-q;D-p				
35) A-s;B-r;C-pq;D-q			36) 3				

ADDITIONAL EXERCISE

LECTURE SHEET (ADVANCED)

Single answer type questions

1. If z_1 and z_2 are two complex numbers satisfying $\frac{z_1}{2z_2}+\frac{2z_2}{z_1}=i$ and if 0, z_1, z_2 form two non similar triangles and if α, β are the least angles in the two triangles then $\cot\alpha+\cot\beta$ equals

a) $\sqrt{5}$ b) $2\sqrt{5}$ c) 1 d) 2

2. If the roots of the equation $z^3-(2\cos\alpha+1)z^2+(2\cos\alpha+1)z-1=0$ are the affixes of the vertices of a triangle in the argand plane ($0<\alpha<\pi$) of maximum area then the perimeter of the triangle is

a) $4\sqrt{3}$ b) $3\sqrt{3}$ c) $2\sqrt{3}$ d) 6

3. If $|z_2+iz_1|=|z_1|+|z_2|$ and $|z_1|=3$ & $|z_2|=4$ then area of ΔABC, if affix of A, B & C are (z_1), (z_2) & $\left(\frac{z_2-iz_1}{1-i}\right)$

a) 5/2 b) 0 c) 25/2 d) 25/4

4. The polynomial $(\cos\theta+x\sin\theta)^n-\cos n\theta-x\sin n\theta$ is divisible by

a) x^3+1 b) x^2+1 c) x^4+1 d) x^2-1

5. If $m\neq n$ are two natural numbers and c is a non-zero complex number then the number of solutions of the equation $z^m=c(\overline{z})^n$ is__

a) m – n b) m + n c) m d) m + n + 1

6. Area of the region bounded by the set of all complex numbers in the Argand plane which satisfy $|Z|\le 5$ and $|Z-6|\le 5$ is

a) $25\sin^{-1}\left(\frac{4}{5}\right)-6$ b) $25\sin^{-1}\left(\frac{4}{5}\right)-12$ c) $50\sin^{-1}\left(\frac{4}{5}\right)-24$ d) $50\sin^{-1}\left(\frac{4}{5}\right)-12$

7. If z_1, z_2, z_3 are the vertices of a triangle such that $\arg\left(\frac{z_3-z_2}{2z_1-z_2-z_3}\right)=\frac{\pi}{3}$ and $2|z_1-z_3|=|z_2-z_3|=5$ then the value of $\begin{vmatrix} z_1 & \overline{z}_1 & 1 \\ z_2 & \overline{z}_2 & 1 \\ z_3 & \overline{z}_3 & 1 \end{vmatrix}$ equals.

a) $\frac{25}{4}\sqrt{3}$ b) $\frac{25}{8}\sqrt{3}$ c) $\frac{25}{2}\sqrt{3}$ d) $25\sqrt{3}$

8. A particle starts to travel from a point P on the curve $C_1:|z-3-4i|=5$, where $|z|$ is maximum. From P, the particle moves through an angle $\tan^{-1}\frac{3}{4}$ in anticlock wise direction on $|z-3-4i|=5$ and reaches at point Q. From Q, it comes down parallel to imaginary axis by 2 units and reaches at point R. Complex number corresponding to point R in the Argand plane is

a) (3+5i) b) (3+7i) c) (3+8i) d) (3+9i)

9. A,B,C are vertices of a triangle inscribed in the circle $|z|=1$. Altitude from A meets the circumcircle again at D. If D,B,C represents the complex number z_1, z_2, z_3 respectively then the complex number representing the reflection of D in the line BC, is

a) $\frac{z_1z_2+z_1z_3+z_2z_3}{z_1}$ b) $\frac{z_1z_2+z_2z_3+z_1z_3}{z_1z_2z_3}$ c) $\frac{z_1z_2+z_1z_3-z_2z_3}{z_1}$ d) $\frac{z_1z_2+z_1z_3-z_2z_3}{z_1z_2z_3}$

10. A point P representing the complex number z moves in the Argand plane so that it lies always in the region defined by $|z-1| \le |z-i|$ and $|z-2-2i| \le 1$. If P describes the boundary of this region then the value of $|z|$ when the arg(z) has least value, is

a) $\sqrt{5}$ b) 7 c) $\sqrt{7}$ d) 5

11. Let P(z) be a variable point in the complex plane such that $|z| = \min\{|z-1|, |z+1|\}$ then the value of $(z+\bar{z})$ is

a) 1 if Re(z)>0 b) 1 if Re z<0 c) 0 if Re z>0 d) 0 if Re z <0

12. If z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=1$ then area of region common to given triangle and another triangle having vertices $-z_1, -z_2, -z_3$, is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{3}}{4}$ c) $\frac{7\sqrt{3}}{4}$ d) $\frac{5\sqrt{3}}{4}$

13 If $az^2+bz+1=0$, $a,b,z \in a,b,z \in C$ and $|a|=\frac{1}{2}$, have a root α such that $|\alpha|=1$ then $|a\bar{b}-b|=$

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{5}{4}$ d) $\frac{3}{4}$

14. If $|z-1| \le 2$ & $|wz-1-w^2| = a$ (where 'w' is a cube root of unity) then complete set of values of a is

a) $0 \le a \le 2$ b) $\frac{1}{2} \le a \le \frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{3}}{2}-\frac{1}{2} \le a \le \frac{1}{2}+\frac{\sqrt{3}}{2}$ d) $0 \le a \le 4$

15. For distinct complex numbers $z_1, z_2, \ldots, z_n$, the value of $\frac{|z_2-z_1|^2+|z_3-z_2|^2+\ldots\ldots+|z_n-z_{n-1}|^2}{|z_n-z_1|^2}$ can not be less than

a) $\frac{1}{n-1}$ b) $\frac{1}{n}$ c) $\frac{1}{n+1}$ d) $\frac{1}{n-2}$

16. If z is a complex number satisfying $|z|^2+2(z+\bar{z})+3i(z-\bar{z})+4=0$, $i=\sqrt{-1}$, then the complex number $z+3+2i$ will lie on a circle with

a) centre $1-5i$, radius 4 b) centre $1+5i$, radius 4

c) centre $1+5i$, radius 3 d) centre $1-5i$, radius 3

17. The value of $i\log_e(x-i)+i^2\pi+i^3\log_e(x+i)+i^4(2\tan^{-1}x)$, $x>0$, $i=\sqrt{-1}$ is

a) 0 b) 1 c) 2 d) 3

18. If $\left|\frac{z_1}{z_2}\right|=1$ and arg $(z_1 z_2) = 0$, then

a) $z_1 = z_2$ b) $|z_2|^2 = z_1 z_2$ c) $z_1 z_2 = 1$ d) $z_1 z_2 = 2$

19. Let z be a complex number and a_k, $b_k (k = 1,2,3)$ are real numbers then the value of

$$\begin{vmatrix} a_1z+b_1\overline{z} & a_2z+b_2\overline{z} & a_3z+b_3\overline{z} \\ b_1z+a_1\overline{z} & b_2z+a_2\overline{z} & b_3z+a_3\overline{z} \\ b_1z+a_1 & b_2z+a_2 & b_3z+a_3 \end{vmatrix} =$$

a) $(a_1a_2a_3+b_1b_2b_3)|z|^2$ b) $(a_1a_2a_3-b_1b_2b_3)|z|^2$

c) $a_1^{\ 2}-a_1^{\ 2}$ d) $|z|^2$

20. If $z = x = 3i$, then the value of $\int_2^4\left[\arg\left|\frac{z-i}{z+i}\right|\right]dx$, where [.] denotes the greatest integer function and $i=\sqrt{-1}$ is

a) $\sqrt{6}$ b) $6\sqrt{3}$ c) $3\sqrt{2}$ d) 0

21. The greatest positive argument of complex number satisfying $|z-4|=\mathrm{Re}(z)$ is

a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

22. In a ΔMAP, on sides MA and AP squares are drawn. If P and D are on the same side of AM and M, E lies on opposite side of AP where D and E are the centres of the squares on MA and AP respectively then angle between MP and DE is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

23. If $A=\{z\in C: z=x+xi-1\,\forall x\in R\}$ and $|z|\le|\omega|\,\forall z,\omega\in A$ then z is

a) $\frac{1}{2}(1+i)$ b) $-\frac{1}{2}(1-i)$ c) $-\frac{1}{2}(1+i)$ d) $\frac{1}{3}(1-2i)$

24. If the points in the complex plane which satisfy the equations $\log_5(|z|+3)-\log_{\sqrt{5}}\left||z|-1\right|=1$ and $\arg(z-1-i)=\frac{\pi}{4}$ is of the form $(A_1 + iB_1)$, then the value of $(A_1 + B_1)$ is equal to

a) $2\sqrt{2}$ b) $\sqrt{2}$ c) $4\sqrt{2}$ d) 0

25. A complex number z with Im(z) = 4 and a positive integer n be such that $\frac{z}{z+n}=4i$, then value of n is

a) 4 b) 16 c) 17 d) 32

26. If z_1 and z_2 satisfy $2|z+3| = |\operatorname{Re} z|$ and $\arg\frac{(z+3)}{(i+1)} = \frac{\pi}{2}$, then $\arg\left(\frac{z_1+3}{z_2+3}\right)$ is equal to

a) 0 b) $\pm\frac{\pi}{2}$ c) $\pm\pi$ d) None of these

27. If $\arg\left(\frac{z_1 - \frac{z}{|z|}}{\frac{z}{|z|}}\right) = \frac{\pi}{2}$ and $\left|\frac{z}{|z|} - z_1\right| = 3$, then $|z_1|$ is equal to

a) $\sqrt{26}$ b) $\sqrt{10}$ c) $\sqrt{3}$ d) $2\sqrt{2}$

28. $\frac{|z-z_1|}{|z-z_2|} = r$ $(r > 1)$ represents

a) a circle for which z_1 and z_2 are interior points

b) a circle for which z_1 & z_2 lie outside the circle

c) a circle for which z_1 lies inside and z_2 lies outside the circle

d) a circle for which z_1 lies outside z_2 lies inside the circle

More than one correct answer type questions

29. If $z_1 = 5 + 12i$ and $|z_2| = 4$ then

a) maximum $(|z_1 + iz_2|) = 17$ b) minimum $(|z_1 + (1+i)z_2|) = 13 - 9\sqrt{2}$

c) minimum $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{4}$ d) maximum $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{3}$

30. Consider two curves represented by $\arg(Z - Z_1) = \frac{3\pi}{4}$ and $\arg(2Z + 1 - 2i) = \frac{\pi}{4}$

a) Two curves do not intersect if $Z_1 = 3i$ b) Two curves do not intersect if $Z_1 = 2 + i$

c) Two curves intersect if $Z_1 = 3 + i$ d) Two curves intersecting if $Z_1 = 3$

31. Consider the complex numbers satisfying $|z - 25i| \le 15$ then

a) Complex number with least argument has argument $\tan^{-1}\left(\frac{4}{3}\right)$

b) Complex number with maximum argument has amplitude $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

c) Complex number with least magnitude is 10

d) Complex number with maximum magnitude is 40

32. If $|z_1| = |z_2| = |z_3| = 1$ and $S = |z_2 - z_3|^2 + |z_3 - z_1|^2 + |z_1 - z_2|^2$. Let α = minimum value of S and β = maximum value of S as z_1, z_2, z_3 vary on $|z| = 1$, then

a) $\alpha = 0$ b) $\beta = 9$ c) $\beta = 3$ d) $\alpha\alpha = 2$

33. If the lines $a\bar{z}+\bar{a}z+b=0$ and $c\bar{z}+\bar{c}z+d=0$ are mutually perpendicular, where a and c are non-zero complex numbers and b and d are real numbers, then

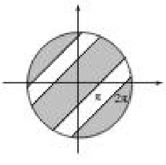
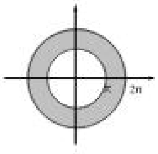
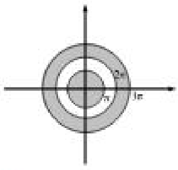
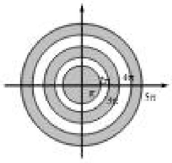
a) $a\bar{a}+c\bar{c}=0$ b) $a\bar{c}$ is purely imaginary

c) $\arg\left(\frac{a}{c}\right)=\pm\frac{\pi}{2}$ d) $\frac{a}{\bar{a}}=\frac{c}{\bar{c}}$

34. For points z_1, z_2, z_3, z_4 in complex plane such that $|z_1|<1, |z_2|=1$ and $|z_3|\le 1$ and $z_3=\frac{z_2(z_1-z_4)}{\bar{z}_1 z_4-1}$, then $|z_4|$ can be

a) 2 b) 2/5 c) 1/3 d) 5/2

35. The region described by the complex number z satisfying $\sin|z|>0$ is

a)  b)  c)  d) 

36. If $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$ then

a) $\cos 2^n(\alpha+\beta)+\cos 2^n(\beta+\gamma)+\cos 2^n(\gamma+\alpha)=0,\ n\in N$

b) $2^{\cos(2\alpha-\beta-\gamma)}\times 2^{\cos(2\beta-\gamma-\alpha)}\times 2^{\cos(2\gamma-\alpha-\beta)}=8$

c) If $\cos 3\alpha=\frac{3}{4}$, then $\cos^8\alpha\cos^8\beta+\cos^8\gamma=\frac{27}{32}$ d) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{3}{2}$

37. If conjugate of a complex number $z=a+ib$ is defined as $\bar{z}=b+ai$ then which of the following is/are true

a) $\overline{z_1 z_2}=-i\bar{z}_1\bar{z}_2$ b) $\overline{z+\bar{z}}=\bar{z}+z$ c) $I\left(\frac{\overline{z_1+z_2}}{z_3}\right)=\frac{\bar{z}_1\bar{z}_2}{\bar{z}_3}$ d) $\left(\frac{\bar{z}_1}{z_2}\right)=i\frac{\bar{z}_1}{\bar{z}_2}$

38. z_1 and z_2 are the roots of the equation $z^2-az+b=0$ where $|z_1|=|z_2|=1$ and a, b are non zero complex numbers then

a) $|a|+|b|\le 3$ b) $|a|\le 2$ c) $arg(a^2)=arg(b)$ d) $arg(a)=arg(b^2)$

Linked comprehension type questions

Passage-1:

Let $A(z_1)$, $B(z_2)$, $C(z_3)$ and $D(z_4)$ be the vertices of a trapezium in an argand plane. Let $|z_1-z_2|=4$, $|z_3-z_4|=10$ and the diagonals AC and BD intersect at P. it is given that $\arg\left(\frac{Z_4-Z_2}{Z_3-Z_1}\right)=\frac{\pi}{2}$ and $\arg\left(\frac{Z_3-Z_2}{Z_4-Z_1}\right)=\frac{\pi}{4}$.

39. Area of the trapezium ABCD is equal to

a) $\frac{130}{3}$ b) $\frac{160}{3}$ c) $\frac{190}{3}$ d) $\frac{140}{3}$

40. Area of triangle PCB is equal to

a) $\frac{100}{21}$ b) $\frac{200}{21}$ c) $\frac{100}{7}$ d) $\frac{400}{21}$

41. $|CP - DP|$ is equal to

a) $\frac{10}{\sqrt{21}}$ b) $\frac{16}{\sqrt{21}}$ c) $\frac{17}{\sqrt{21}}$ d) $\frac{19}{\sqrt{21}}$

Passage-II :

Suppose z and w be two complex numbers such that $|z| \le 1$, $|w| \le 1$ and $|z + iw| = |z - i\bar{w}| = 2$. Use the result $|z|^2 = z\bar{z}$ and $|z + w| \le |z| + |w|$, answer the following

42. Which of the following is true about $|z|$ and $|\omega|$

a) $|z| = |w| = \frac{1}{2}$ b) $|z| = \frac{1}{2}, |w| = \frac{3}{4}$ c) $|z| = |w| = \frac{3}{4}$ d) $|z| = |w| = 1$

43. Which of the following is true for z and ω

a) $\text{Re}(z) = \text{Re}(w)$ b) $I_m(z) = I_m(w)$ c) $\text{Re}(z) = I_m(w)$ d) $I_m(z) = \text{Re}(w)$

44. Number of complex numbers satisfying the above conditions is

a) 1 b) 2 c) 4 d) indeterminate

Passage-III :

Let z_1, z_2, z be three points A,B,P respectively in the Argand plane. Let P moves in the plane such that $\left|\frac{z - z_1}{z - z_2}\right| = \lambda \neq 1$, then locus of P is a circle. Let $z_1 = 2 + i, z_2 = -4 - 7i$.

45. If $\lambda = 2$, then the maximum area of the triangle ABP is

a) $\frac{100}{3}$ *sq.units* b) $\frac{200}{3}$ *sq.units* c) 34sq units d) 36 sq units

46. If $\lambda = 3$, then the number of points P in the Argand plane such that area of $\Delta ABP = 19$ *sq.units*

a) 1 b) 2 c) 3 d) 0

47. If $\lambda = \frac{2}{3}$, then the maximum distance of P from the line AB, is

a) 6 b) 12 c) 30 d) 10

Passage-IV :

In argand plane consider $A(z_1)$, $B(z_2)$ and $C(z_3)$ as distinct complex numbers lying on the curve $|z| = \sqrt{3}$

48. If a root of $z_1 z^2 + z_2 z + z_3 = 0$ has modulus unity then z_1, z_2, z_3 are in

a) A.P b) G.P c) H.P d) All of above

49. If ΔABC is equilateral and $P(z_p)$ be any point such that $|z_1| = |z_P|$ then the values of $|z_P - z_1|^2 + |z_P - z_2|^2 + |z_P - z_3|^2$ is

a) 9 b) 6 c) 18 d) 3

50. If $z_1z^2 + z_2z + z_3 = 0$ and $z_2z^2 + z_3z + z_1 = 0$ each have a common root having modulus unity then ΔABC is

a) isosceles b) equilateral c) isosceles right angled d) scalene

Matrix matching type questions

51. Let the complex numbers z_1, z_2 and z_3 represent the vertices A, B and C of a triangle ABC respectively, which is inscribed in the circle of radius unity and centre at origin. The internal bisector of the angle A meets the circumcircle again at the point D, which is represented by the complex number z_4 and altitude from A to BC meets the circumcircle at E, given by z_5. Now match the entries from the following two columns.

COLUMN - I	COLUMN - II
A) $\arg\left(\frac{z_2z_3}{z_4^2}\right)$ is equal to	p) π
B) $\arg\left(\frac{z_4}{z_2 - z_3}\right)$ is equal to	q) $\frac{\pi}{2}$
C) $\arg\left(\frac{z_2z_3}{z_1z_5}\right)$ is equal to	r) $\frac{\pi}{4}$
D) $\arg\left(\frac{z_4^2}{z_1z_5}\right)$ is equal to	s) 0
	t) $-\frac{\pi}{2}$

52. Match the following:

COLUMN - I	COLUMN - II
A) The roots of cubic equation $(z+\alpha\beta)^3 = \alpha^3, (\alpha = 0)$ represent the vertices of a triangle of area equal to	p) $\lvert\tan\alpha\rvert$
B) If α is a complex number then the radius of the circle $\left\lvert\frac{z-\alpha}{z-\bar{\alpha}}\right\rvert = 2$ is equal to	q) $\frac{3\sqrt{3}}{4}\lvert\alpha\rvert^2$
C) If $\arg z = \alpha$ and $\lvert z-1\rvert = 1$ then $\left\lvert\frac{z-2}{z}\right\rvert$ is equal to	r) $\frac{2}{3}\lvert\alpha-\bar{\alpha}\rvert$
D) Let A and B represent complex numbers z_1 and z_2, which are roots of the equation $z^2 + pz + q = 0$. If $\angle AOB = \alpha \neq 0$ and OA = OB, where O is the origin then $\frac{p^2}{q}$ is equal to	s) $4\cos^2\frac{\alpha}{2}$
	t) $\lvert\alpha+\bar{\alpha}\rvert^2$

53. Match the statements/expressions in **Column I** with the open intervals in **Column II**

COLUMN - I	COLUMN - II
A) $\sin\frac{\pi}{900}\left\{\sum_{r=1}^{10}(r-\omega)(r-\omega^2)\right\}=$	p) 0
B) If roots of $t^2+t+1=0$ be α,β then $\alpha^4+\beta^4+a^1\beta^{-1}=$	q) 4
C) If $\left[\frac{1+\cos\theta+i\sin\theta}{\sin\theta+i(1+\cos\theta)}\right]^4=\cos n\theta+i\sin n\theta$, then $n=$	r) i
D) If $z_r=\cos\frac{\pi}{3^r}+i\sin\frac{\pi}{3^r}$, $r=1,2,3,\ldots,$ then value of $z_1z_2z_3\ \ldots\ \infty=$	s) 1

54. Number of solutions of

COLUMN - I	COLUMN - II
A) $z^2+\lvert z\rvert=0$	p) 1
B) $z^2+\bar{z}^2=0$	q) 3
C) $z^2+8\bar{z}=0$	r) 4
D) $\lvert z-2\rvert=1$ and $\lvert z-1\rvert=2$	s) Infinite

55. Match the following:

COLUMN - I	COLUMN - II
A) If $\frac{(x-1)^2}{x}=-1$ and $p=\frac{x^{16000}+1}{x^{8000}}$ and q be the digit at unit place in the number $2^{2^n}+1, n\in N, n>1$, then $p+q=$	p) 3
B) The possible integral values of α for which $\lvert z-\alpha^2+7\alpha-11-i\rvert=1$ and arg $z>\frac{\pi}{2}$ is satisfied for at least one z, is/are	q) 4
C) If z lies on the curve $\arg(z+1)=\frac{\pi}{4}$ then the minimum value of $\lvert z-w\rvert+\lvert z+w\rvert$, where w is complex cube root of unity, is	r) 5
D) If z be a complex number such that $\lvert z+5\rvert\le 7$, then possible integral values of $\lvert z-3\rvert$ are	s) 2
	t) 6

56. Match the following equation in z column-I with corresponding values arg Z in column II

COLUMN - I	COLUMN - II
A) $z^2 - z + 1 = 0$	p) $-\frac{2\pi}{3}$
B) $z^2 + z + 1 = 0$	q) $-\frac{\pi}{3}$
C) $2z^2 + 1 + i\sqrt{3} = 0$	r) $\frac{\pi}{3}$
D) $2z^2 + 1 - i\sqrt{3} = 0$	s) $\frac{2\pi}{3}$
	t) $\frac{\pi}{6}$

Integer answer type questions

57. Let A, B, C three set of complex number defined below

$A = \{z : |z+1| \le 2 + \mathrm{Re}(z)\}$; $B = \{z : |z-1| \ge 1\}$; $C\left\{z : \left|\frac{z-1}{z+1} \ge 1\right|\right\}$

The number of points having integral coordinate in region $A \cap B \cap C$ is

58. If $x + \frac{1}{x} = 1$; and $p = x^{4000} + \frac{1}{x^{4000}}$ and q be the digit at unit place in the number 2^{2^n}, $n \in N$ and $n > 1$, then the value of $p + q =$

59. The number of complex numbers Z satisfying the conditions $\left|\frac{Z}{\overline{Z}} + \frac{\overline{Z}}{Z}\right| = 1, |Z| = 1$ and $\arg(Z) \in (0, 2\pi)$ is

60. Let $Z = \frac{\sqrt{3}}{2} - \frac{i}{2}$ Then the smallest positive integer "n" such that $(Z^{95} + i^{67})^{94} = Z^n$ then the value of $\frac{n}{2}$ is

61. The number of solutions of the equation $z^2 - z - |z|^2 + \frac{64}{|z|^5} = 0$ is (where $z = x + iy$ $x, y \in R$ and $x \ne \frac{1}{2}$)

62. If magnitude of a complex number 4 – 3i is tripled and rotated by an angle π anticlockwise about origin then resulting complex number would – 12 + λi then λ must be equal to

63. If the complex numbers z for which $\arg\left[\frac{3z - 6 - 3i}{2z - 8 - 6i}\right] = \frac{\pi}{4}$ and $|z - 3 + i| = 3$, are $\left(k - \frac{4}{\sqrt{5}}\right) + i\left(1 + \frac{2}{\sqrt{5}}\right)$ and $\left(k + \frac{4}{\sqrt{5}}\right) + i\left(1 - \frac{2}{\sqrt{5}}\right)$ then k must be equal to

64. If $x_1 + iy_1$, $x_2 + iy_2$ are the solutions of the equations $\left|\frac{Z - 12}{Z - 8i}\right| = \frac{5}{3}$ and $\left|\frac{Z - 4}{Z - 8}\right| = 1$, then $|y_1 - y_2| =$

65. If $z_1, z_2, z_3, \ldots\ldots\ldots z_n$ are in G.P with first term as unity such that $z_1 + z_2 + z_3 + \ldots\ldots + z_n = 0$. Now if $z_1, z_2, z_3, \ldots\ldots\ldots z_n$ represents the vertices of n-polygon, then the distance between incentre and circumcentre of the polygon is represented by $4k$. Find k.

PRACTICE SHEET (ADVANCED)

Single answer type questions

1. $f(z)$ = the real part of z. If $a \in N$, $n \in N$ then the value of $\sum_{n=1}^{6a} \log_2 \left| f[1+i\sqrt{3})^n] \right|$ has the value equal to

a) $18a^2 + 3a$ b) $18a^2 - 3a$ c) $18a^2 - a$ d) $18a^2 + a$

2. If $|z-3i| = 3$, (where i = $\sqrt{-1}$) and arg $z \in (0, p/2)$, then cot (arg (z)) $-\frac{6}{z}$ is equal to

a) 0 b) –i c) i d) p

3. If $\alpha_0, \alpha_1, \alpha_2, ...\alpha_{n-1}$ be the n^{th} roots of unity, then value of $\sum_{i=0}^{n-1} \frac{\alpha_i}{(3-\alpha_i)} =$

a) $\frac{n}{3^n - 1}$ b) $\frac{n-1}{3^n - 1}$ c) $\frac{n+1}{3^n - 1}$ d) $\frac{n+2}{3^n - 1}$

4. The least value of P for which the two curves arg $z = \frac{\pi}{6}$ and $|z - 2\sqrt{3}i| = P$ have a solution is

a) $\sqrt{3}$ b) 3 c) $\frac{1}{\sqrt{3}}$ d) $\frac{1}{3}$

5. If a complex number Z lies on a circle of radius $\frac{1}{2}$ then the complex number –1+4Z lies on a circle of radius

a) $\frac{1}{2}$ b) 1 c) 2 d) 4

6. If arg $\left(\frac{z-1}{z+1}\right) = \pm\frac{\pi}{4}$ then length of the path traced by the points in the locus is

a) $3\sqrt{2}\pi$ b) 3π c) $\frac{\pi}{\sqrt{2}}$ d) $\sqrt{2}\pi$

7. If $|z-i| \le 2$ and $z_0 = 5 + 3i$, the maximum value of $|iz + z_0|$ is

a) $2+\sqrt{31}$ b) $\sqrt{31}-2$ c)7 4) 3

8. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is

a) 0 b) 7 c) 2 d) 17

9. If $\beta \ne 1$ be any n[th] root of unity then $1 + 3\beta + 5\beta^2 + ...n$ terms equals

a) $\frac{2n}{1-\beta}$ b) $\frac{-2n}{1-\beta}$ c) $-\frac{2n}{(1-\beta)^2}$ d) $\frac{2n}{(1-\beta)^2}$

10. $\sin\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3} + \frac{1}{3}\sin\frac{3\pi}{3} + ...\infty =$

A) 0 B) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$

11. If $|z - 1| + |z + 3| \leq 8$, then the range of values of $|z - 4|$ is,

a) (0, 8) b) [1, 9] c) [0, 8] d) [5, 9]

12. If w is a complex number such that $|w| = r¹ 1$ then $z = \omega + \frac{1}{\omega}$ describes a conic. The distance between the foci is

a) 1 b) $2(\sqrt{2} - 1)$ c) 3 d) 4

13. If $|z| = 1$ and $z ¹ \pm 1$, then all the values of $\frac{z}{1 - z^2}$ lie on

a) a line not passing through the origin
b) $|z| = \sqrt{2}$
c) the x-axis
d) the y-axis

14. let a be a complex number such that $|a| = 1$ If the equation $az^2 + z + 1 = 0$ has a pure imaginary root, then tan(arg a) =

a) $\frac{\sqrt{5} - 1}{2}$ b) $\frac{\sqrt{5} + 1}{2}$ c) $\sqrt{\frac{\sqrt{5} - 1}{2}}$ d) $\sqrt{\frac{\sqrt{5} + 1}{2}}$

15. Let z_1 and z_2 be two non real complex cube roots of unity and $|z - z_1|^2 + |z - z_2|^2 = \lambda$ be the equation of a circle with z_1, z_2 as ends of a diameter, then the value of λ is

a) 4 b) 3 c) 2 d) $\sqrt{2}$

16. Let $z \in C$ and if $A = \left\{z : \arg(z) = \frac{\pi}{4}\right\}$ and $B = \left\{z : \arg(z - 3 - 3i) = \frac{2\pi}{3}\right\}$ then $n(A \cap B)$ is equal to

a) 1 b) 2 c) 3 d) 0

17. The common roots of the equations $z^3 = 2z^2 + 2z + 1 = 0$ and $z^{1985} + z^{100} + 1 = 0$ are

a) $-1, \omega$ b) $-1, \omega^2$ c) ω, ω^2 d) 1, -1

18. The range of real number α for which the equation $z - \alpha|z - 1| + 2i = 0$ has a solution is

a) $\left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$ b) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{13}}{2}\right]$ c) $\left[0, \frac{\sqrt{15}}{2}\right]$ d) $\left[-\infty, \frac{\sqrt{5}}{2}\right] \cup \left[\frac{\sqrt{5}}{2}, \infty\right]$

19. Let z_1, z_2, z_3 are the roots of $z^3 + 3az^2 + 3bz + c = 0$ (where a, b and c are complex no.s) correspond to the points A, B, C on Gaussion plane. If ΔABC is equilateral then

a) $a = b$ b) $a^2 = b^3$ c) $a^2 = b$ d) $a = b^2$

20. A point P is representing the complex numbers z moves in the argand diagram so that it lies always in the region defined by $|z-1| \leq |z-i|$ and $|z-2-2i| \leq 1$. If P describes the boundary of this region the value of $|z|^2$ when arg(z) has ths smallest value

a) 3 b) 5 c) 7 d) 9

More than one correct answer type questions

21. The locus of the centre of the circle which touches the circles $|z-z_1|=a$ & $|z-z_2|=b$ externally then which of the following statement is/are true ?

a) Is hyperbola if $|a-b|<|z_2-z_1|$

b) right bisector of line joining $A(Z_1)$ & $B(Z_2)$ if a = b

c) an empty set if $|a-b|>|z_2-z_1|$

d) set of points on the line AB & out side the line AB segment

22. Which of the following statement(s) is/are true ?

a) The condition that $z^2+az+b=0$ has a purely imaginary root where $a,b\in C$ is__ $(a+\bar{a})(a\bar{b}+\bar{a}b)+(b-\bar{b})^2=0$

b) The condition that the equation $az^2+bz+c=0$ has both purely imaginary roots $a,b,c\in C$ is $\frac{a}{\bar{a}}=\frac{-b}{\bar{b}}=\frac{c}{\bar{c}}$

c) The condition that the equation $az^2+bz+c=0$ has both real roots where a, b, c are complex constants is $\frac{a}{\bar{a}}=\frac{b}{\bar{b}}=\frac{c}{\bar{c}}$

d) The condition that the equation $z^2+az+b=0$ has a purely real root where a & b are complex constants is $(a\bar{b}-\bar{a}b)(\bar{a}-a)=(b-\bar{b})^2$

23. If $1,\alpha_i's$ (i = 1, 2, 3,.....6) are roots of $z^7+1=0$ then

a) $\alpha_1,\alpha_2.........\alpha_6$ are roots of $x^6-x^5+x^4-x^3+x^2-x+1=0$

b) The equation where roots are $\alpha_1^9,\alpha_2^9......\alpha_6^9$ is $x^6+x^5+x^4+x^3+x^2+x+1=0$

c) The equation where roots are $\alpha_1^{10},\alpha_2^{10}......\alpha_6^{10}$ is $x^6+x^5+x^4+x^3+x^2+x+1=0$

d) $\prod_{i=1}^{6}\alpha_i^3=-1$

Linked comprehension type questions

Passage-I :

The complex slopes of a line passing through two points represented by complex numbers z_1 and z_2 is defined by $\frac{z_2-z_1}{\bar{z}_2-\bar{z}_1}$ and we shall denote by ω If z_0 is complex number and c is a real number, then $\bar{z}_0z+z_0\bar{z}+c=0$ represents a straight line. Its complex slope is $-\frac{z_0}{\bar{z}_0}$.

Now consider two lines $\alpha\bar{z}+\bar{\alpha}z+i\beta=0$ ....(i) and $a\bar{z}+\bar{a}z+b=0$ ....(ii)

where α, β and a, b are complex constants and let their complex slopes be denotes by ω_1 and ω_2 respectively.

24. If the lines are inclined at an angle of 120° to each other, then

a) $\omega_2\bar{\omega}_1 = \omega_1\bar{\omega}_1$ b) $\omega_2\bar{\omega}_1^{\,2} = \omega_1\bar{\omega}_2^{\,2}$ c) $\omega_1^2 = \omega_2^2$ d) $\omega_1 + 2\omega_2 = 0$

25. Which of the following must be true

a) α must be pure imaginary b) β must be pure imaginary

c) α must be real d) β must be imaginary

26. If line (i) makes an angle of 45° with real axis, then $(1+i)\left(-\frac{2\alpha}{\bar{\alpha}}\right)$ is

a) $2\sqrt{2}$ b) $2\sqrt{2}i$ c) $2(1-i)$ d) $-2(1+i)$

Passage-II :

Consider a triangle having vertices at the points $A\left(2e^{\frac{i\pi}{4}}\right), B\left(2e^{\frac{11i\pi}{12}}\right), C\left(2e^{\frac{-5i\pi}{12}}\right)$. Let the in-circle of ΔABC touches the sides BC, CA and AB at D,E and F respectively. Which are respresented by complex number Z_d, Z_e, Z_f in order. If P(z) be any point on the in circle, then

27. $AP^2 + BP^2 + CP^2$ is equal to

a) 12 b) 15 c) 16 d) $\frac{27}{2}$

28. $\text{Re}\left(\frac{1}{z_d}+\frac{1}{z_e}+\frac{1}{z_f}\right)$ is equal to

a) $\sqrt{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{-1}{\sqrt{2}}$ d) 0

29. If the altitude through vertex 'A' cuts the circum circle of ΔABC at Q, then the complex number representing 'Q' is

a) $-\sqrt{2}(1+i)$ b) $\sqrt{2}(1+i)$ c) $\frac{-(1+i)}{\sqrt{2}}$ d) $\frac{-1}{2}(1+i)$

Passage-III :

Let z be a complex number satisfying $z^2 + 2z\lambda + 1 = 0$; where λ is a parameter which can take any real value.

30. The roots of this equation lie on a certain circle if

a) $-1 < \lambda < 1$ b) $\lambda > 1$ c) $\lambda < 1$ d) None of these

31. One root lies inside the unit circle and one outside if

a) $-1 < \lambda < 1$ b) $\lambda > 1$ c) $\lambda < 1$ d) None of these

Passage-IV :

Consider the locus of the complex number z in the Argand plane given by $\text{Re}(z) - 2 = |z - 7 + 2i|$. Let $P(z_1)$, $Q(z_2)$ be two complex numbers satisfying the given locus and also satisfying

$$\arg\left(\frac{z_1 - (2-\alpha i)}{z_2 - (2+\alpha i)}\right) = \frac{\pi}{2} (\alpha \in R)$$

32. The minimum value of PQ is

a) 4 b) 12 c) 10 d) 8

33. The minimum value of PR.QR where R represents the point (7,-2) is

a) 25 b) 12 c) 10 d) $\sqrt{50}$

Passage-V :

A (a), B (b), C (c) be complex numbers lie on |z| = 1.

34. If Altitude through A meet the circumcircle of ΔABC at D then the complex number representing reflection of D in the line BC is

a) $-\frac{bc}{a}$ b) $\frac{ab+ac-bc}{a}$

c) $a+b+c$ d) $-(a+b+c)$

35. If internal angular bisectors of $\angle A, \angle B, \angle C$ intersect circumference at P, Q, R respectively then centroid of triangle PQR is

a) $\frac{\sqrt{ab}+\sqrt{bc}+\sqrt{ca}}{3}$ b) $\frac{(a+b+c)}{3}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

c) $\frac{ab+bc+ca}{3}$ d) $\frac{a+b+c}{3}$

36. The point that divides segment joining nine point centre and orthocentre of ΔABC in the ratio 2:3 internally is

a) $\frac{4(a+b+c)}{5}$ b) $\frac{7(a+b+c)}{10}$

c) $\frac{3(a-2b+c)}{5}$ d) 0

Matrix matching type questions

37.

COLUMN - I	COLUMN - II
A) If $Z_1 = 1+i, Z_2 = \sqrt{3}+i$, then $\arg\left(\frac{Z_1}{Z_2}\right)^4 =$	p) $\frac{3\pi}{4}$
B) If $\lvert Z-i\rvert = \sqrt{2}$. Then $\arg\left(\frac{Z-1}{Z+1}\right) =$	q) $\pm\frac{\pi}{2}$
C) If $\overline{Z} + i\overline{W} = 0$, arg(Z.W) = π, then arg(Z) =	r) $\frac{\pi}{3}$
D) If $\lvert Z^2-1\rvert = \lvert Z^2\rvert + 1$, then arg(Z) =	s) $\frac{\pi}{4}$

38. **COLUMN - I** | **COLUMN - II**

A) Let z_1 & z_2 be two non-real complex cube roots of unity & $|z-z_1|^2+|z-z_2|^2=\lambda$ be the equation of a circle with z_1, z_2 as ends of diameter then 'λ'=___ — p) 0

B) The equation on $|z-1+i|^2+|z-4|^2=k$, represents a circle for $k \geq l$ then 'l' may be — q) 4

C) $|z-i|+|z+i|=k,\ k>0$ represents ellipse of $k^2>$ — r) 5

D) $|z+3i|-|z-3i|=k$ represents a hyperbola then 'k' may be equal to — s) 3

39. **COLUMN - I** | **COLUMN - II**

A) The possible value of 'a' so that the line arg(z) = $\frac{\pi}{6}$ intersect the circle $|z-2\sqrt{3}i|=a$ is — p) 3

B) If $z_1=1+i, z_2=1-i$, z & origin are four concyclic points then $|z|$ cannot be equal to — q) 4

C) If z_1, z_2, z_3, z_4, z_5 are 5th roots of unity then $\frac{1}{3-z_1}+\frac{1}{3-z_2}+\frac{1}{3-z_3}+\frac{1}{3-z_4}=k$ then $[k]$ where [ ] is g.i.f — r) 0

D) In the complex plane A & B are at z_1 = 5 – 2i & z_2 = 1 + i. If $p(z)$ moves such that $|z-z_1|=2|z-z_2|$ then Area(PAB) may be equal to — s) 5

40. Match the following loci

COLUMN - I	COLUMN - II
A) $\arg\frac{z+1}{z-1}=\frac{\pi}{4}$	p) Parabola
B) $z=\frac{3i-t}{2+it}(t\in R)$	q) Part of a circle
C) $\arg\ z=\frac{\pi}{4}$	r) Full circle
D) $z=t+it^2(t\in R)$	s) Ray

Integer answer type questions

41. The complex number z satisfying $|z+2+i|+|z-2+i|=4$, $0\le\arg(z+2+2i)\le\frac{\pi}{4}$ and $3\frac{\pi}{4}\le\arg(z-2+2i)\le\pi$ will lie on a line segment of the length k. Find k.

42. If the argument of $(z-a)(\overline{z}-b)$ is equal to that of $\frac{(\sqrt{3}+i)(1+\sqrt{3}i)}{1+i}$, where a,b are real numbers. If locus of z is a circle with centre $\frac{3+i}{2}$ then find $(a+b)$.

43. If ω is the imaginary cube roots of unity, then find the number of pairs of integers (a,b) such that $|a\omega+b|=1$

44. $f(z)$ is a complex valued function $f(z)=(a+ib)z$, where $a,b\in R$ and $|a+ib|=\frac{1}{\sqrt{2}}$. If has the property that $f(z)$ is always equidistant from 0 and z, then $a-b=$ ______

45. A is the region of the complex plane $\{z : z/4$ and $4/\overline{z}$ have real and imaginary part in $(0,1)$ $\}$, then $[p]$ (where p is the area of the region A and [.] denotes the greatest integer function) is _____

46. $|z| = 1$ is a curve in argand plane. $z_1 = -1 + i$ represents a pt. A in the argand plane. A variable line through A intersects the curve $|z| = 1$ at $B(z_2)$ and $C(z_3)$. P is a point on the line ABC such that AB, AP, AC are in G.P. the locus of P is a circle whose radius is

47. For the equation $z^6 - z^3 - 2450 = 0$ where z is a complex number the number of roots having positive real part is α, negative real part is β, positive imaginary part is γ, negative imaginary part is δ then $(\alpha+\beta)-(\gamma+\delta)$ equals

KEY SHEET (ADDITIONAL EXERCISE)

LECTURE SHEET (ADVANCED)

1) d	2) b	3) d	4) b	5) d	6) c	7) a	8) b	9) c	10) c
11) a	12) a	13) d	14) d	15) a	16) c	17) a	18) b	19) c	20) d
21) d	22) b	23) b	24) a	25) c	26) c	27) b	28) d	29) ad	30) abcd
31) cd	32) ab	33) c	34) bc	35) cd	36) abcd	37) abd	38) acd	39) d	40) b
41) a	42) d	43) b	44) d	45) a	46) d	47) b	48) b	49) c	50) b

51) **A-s;B-qt;C-p;D-p** 52) **A-q;B-r;C-p;D-s** 53) **A-s;B-p;C-q;D-r**

54) **A-q;B-s;C-r;D-p** 55) **A-t;B-pq;C-s;D-pqrst** 56) **A-qr;B-ps;C-qs;D-pr**

57) **6** 58) **5** 59) **8** 60) **5** 61) **0** 62) **9** 63) **4** 64) **9** 65) **0**

PRACTICE SHEET (ADVANCED)

1) **c** 2) **c** 3) **a** 4) **b** 5) **c** 6) **a** 7) **c** 8) **c** 9) **b** 10) **d**

11) **b** 12) **d** 13) **d** 14) **d** 15) **b** 16) **d** 17) **c** 18) **a** 19) **c**

20) **c** 21) **abd** 22) **abcd** 23) **abcd** 24) **b** 25) **b** 26) **c** 27) **b** 28) **d** 29) **a**

30) **a** 31) **b** 32) **c** 33) **c** 34) **c** 35) **a** 36) **b** 37) **A-r;B-s;C-p;D-p**

38) **A-s;B-r;C-rs;D-qrs** 39) **A-p;B-pq;C-r;D-pqs** 40) **A-q;B-q;C-s;D-p** 41) **2**

42) **3** 43) **6** 44) **0** 45) **0** 46) **1** 47) **2**