Chapter - 1

WAVE MOTION AND SOUND

✤ Wave function and wave equation ✤ Transverse waves in strings ✤
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✤ Interference of waves ✤ Beats ✤
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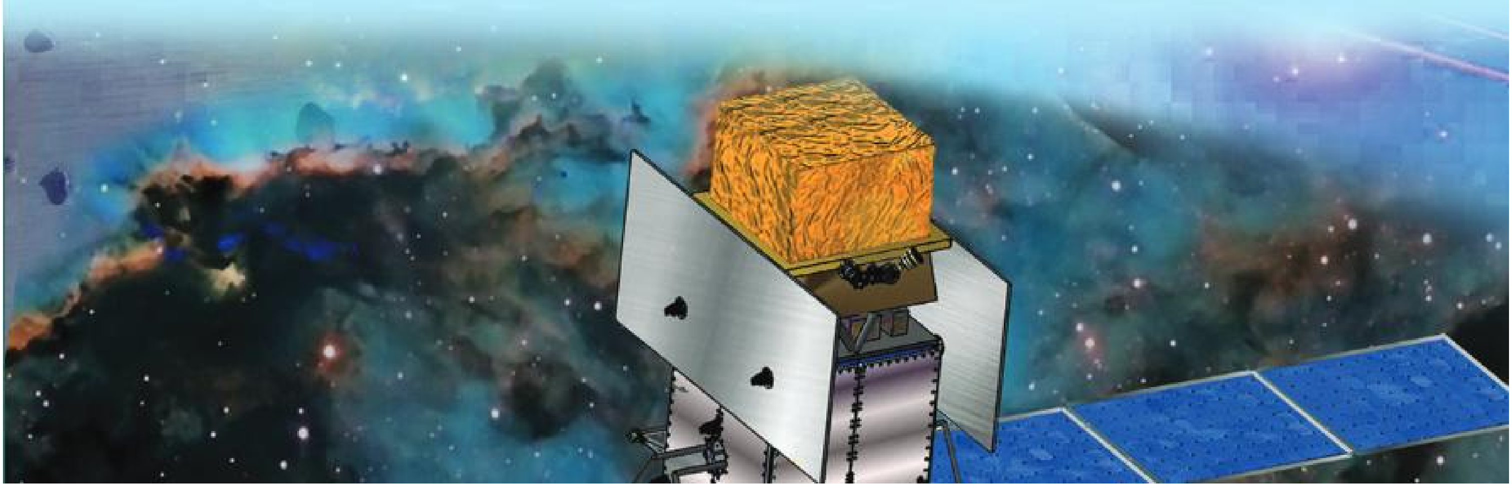


1.1 INTRODUCTION

Most of the information comes to us in the form of waves. It is through wave motion that sounds come to our ears, light to our eyes and electromagnetic signals to our radios and television sets. Sound waves can travel only in a material medium and they are mechanical waves. Light and other electromagnetic waves can propagate even in the absence of a medium. In this chapter we concentrate on mechanical waves.

When a stone is dropped into water in a pond, waves are produced at the point where the stone strikes the water. The waves (ripples) travel outward and particles of the water vibrate up and down about their mean positions. This can be clearly seen when the leaves floating on the surface of the pond move up and down as the ripples pass on. They do not travel along the waves. Similarly, when a tuning fork is set into vibrations, waves are produced in the surrounding air making the particles of the air oscillate about their mean positions. Hence a wave motion can be defined as a form of disturbance which travels through the medium due to repeated periodic motion of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next. In this process the energy and momentum of the particles are successively transmitted through the medium in the wave form known as a progressive wave or a travelling wave. Such a wave travels with a constant speed in the medium.

1.2 LONGITUDINAL AND TRANSVERSE WAVES

The waves are classified into two types depending on the direction of propagation of waves relative to the direction of vibration of the particles of the medium.

1) longitudinal waves and

2) transverse waves.

Longitudinal waves

If the particles of the medium vibrate parallel to the direction of propagation of the waves, the waves are called 'Longitudinal waves'.

The propagation of longitudinal waves can be demonstrated as shown in the Fig. 1.1(a).

If a light spring is held horizontally and given a push along its length, the spring gets compressed sending a pulse of pressure along the spring. Soon this compression tends to release the pressure in the region by pushing the neighbouring (particles) layers of the spring. This is known as rarefaction. Thus the compression is transmitted horizontally along the length of the spring. If the pushing at the end of the spring is repeated at regular intervals of time a periodic longitudinal progressive wave takes place along the length of the spring. In a compression, particles move in the direction in which the wave advances where as in a rarefaction, particles move in the opposite direction in which the wave advances.

Fig.1.1 (a) Longitudinal waves in a spring

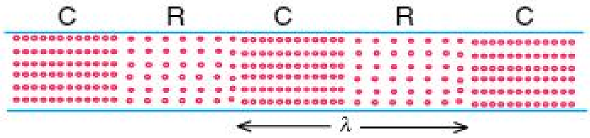


Fig 1.1(b)

Fig 1.1(c)

Sound waves are longitudinal waves. Longitudinal waves can travel in solids, liquids and in gases as well.

Transverse waves

If the direction of propagation of the waves is perpendicular to the direction of vibration of the particles, the waves are said to be 'Transverse waves'.

Transverse waves can be demonstrated as shown in the fig 1.2(a).

If a light spring held horizontally at one end is moved vertically up and down, periodically, the length of the spring gets into the shape of the curve as shown in the Fig. 1.2 (b) with "ups" (c) and "down" (T).

The 'ups' (c) are the locations of maximum displacements upward and are known as crests. The 'downs' (T) are the maximum displacements in the downward direction and are known as troughs.

Thus the displacements crests and troughs produced at one end vertically, are handed over to the successive particles (coils) of the spring propagating a transverse progressive wave.

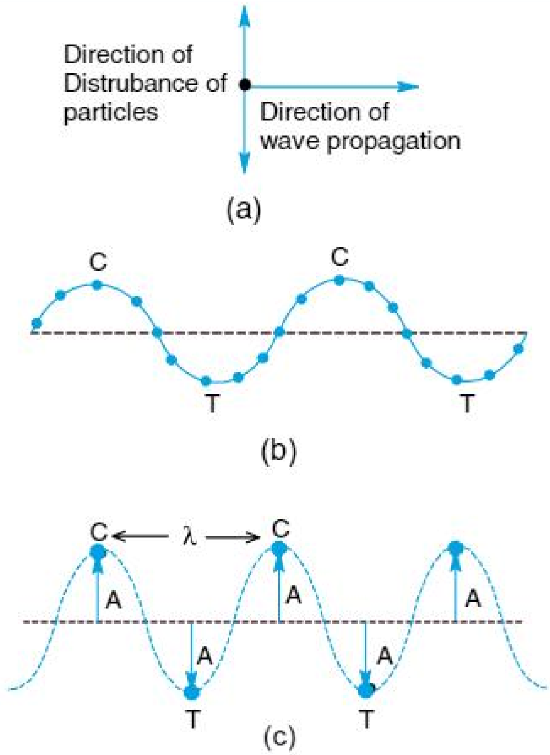


Fig 1.2

Transverse waves can propagate only in solids and on the surfaces of liquids.

Parameters of progressive wave motion :

The following are the parameters of a progressive wave motion (see fig 1.3)

(i) Amplitude (A) : The maximum displacement of any particle on the wave from its mean position in either direction is known as the amplitude of the wave. The SI unit of amplitude is 'metre'.

(ii) Phase (ϕ) : The phase of the wave at any instant at a point is the state of vibration of the particle at that point, with regard to its position and direction. The SI unit of phase is "radian", as it is represented as phase angle. For example, if the phase of a particle at the crest (or at the compression) is $\frac{\pi}{2}$ radian, the phase at the next trough (or at the next rarefaction) is $\frac{3\pi}{2}$ radian. Therefore the phase difference between a crest and the next trough, or between a compression and the next rarefaction is π radian. The phase difference between two consecutive crests or troughs (compressions or rarefactions) is 2π.

(iii) Wavelength (λ) : The distance between two successive particles which are in the same phase of vibration on the wave is known as the wavelength. The SI unit of wave length is "metre". On a transverse wave it can be identified as the linear separation between two consecutive crests or troughs. Similarly the separation between two consecutive compressions or rarefactions is equal to the wavelength of a longitudinal wave. Actually the phase difference between the particles separated by λ is 2π and they are said to be "in phase".

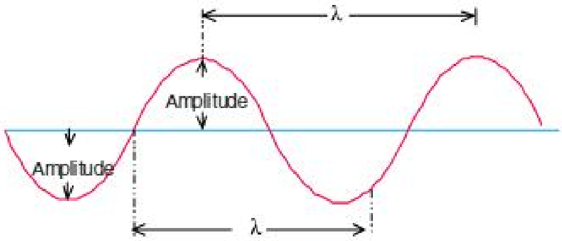


Fig 1.3

(iv) Time period (T): It is the time taken by the disturbance to advance through a distance of one wavelength in the direction of propagation. It is equal to the period of oscillation of any particle on the wave. Its SI unit is 'second.'

(v) Frequency (n) : It is equal to the number of vibrations per second completed by any particle on the wave. In other words it is the number of wavelengths that take place in unit time (second) along the direction of propagation of the wave. Its SI unit is 'hertz'. If 'T' is the period of the wave we can write $n = \frac{1}{T}$. It is independent of medium and depends on type of source.

(vi) Velocity (v) : It is the distance, through which the disturbance is propagated per unit time along the wave. The SI unit of speed of the wave is "metre per second". It depends on the nature of the medium in which the wave travels. Crests and troughs or compressions and rarefactions advance through a distance of one wavelength (λ) in a time period (T).

$\therefore$ The velocity of the wave $v = \frac{\lambda}{T}$ or $n\lambda$

$\therefore v = n\lambda$.

This velocity is not the velocity of the oscillating particle on the wave. The particle velocity changes every moment during the oscillation, but the wave velocity is constant for a progressive wave and the energy is transmitted in the medium with this velocity.

1.3 WAVE FUNCTION

It is a mathematical description of the disturbance or wave created in the medium. In transverse mechanical wave this function is displacement of the particle (y), whereas in longitudinal wave it may be displacement of particle (y) or pressure fluctuation (Δp) or density fluctuation (Δd). Any function which depends on two variables, space and time,

i.e., F = f(x,t) and satisfies the equation $\frac{d^2F}{dx^2} = \frac{1}{v^2}\frac{d^2F}{dt^2}$ represents a wave. Here v is the ratio of coefficient of t to coefficient of x.

e.g : $y = A\sin kx \cos\omega t$, $y = A\sin(\omega t - kx)$,

$y = Ae^{-(ax+bt)^2}$ or $y = \frac{A}{B + (x - vt)^2}$ etc.

1.4 TRAVELLING WAVE

If in the wave function space and time variables appear in the combination of $(ax \pm bt)$, the wave function should be finite every where and at all times then the function represents a travelling wave i.e.,the equation of travelling wave form is $F = f(ax \pm bt)$.

e.g : $y = A\sin(\omega t - kx)$, $y = a\cos^2(bt - ax)$,

$y = (ax + bt)^2$, $y = \frac{A}{B + (ax - bt)^2}$, $y = Ae^{-(ax-bt)^2}$

etc, represent travelling waves. The other combination of x and t does not represent travelling wave eg : $y = A\sin kx \cos\omega t$, $y = A\sin(ax^2 + bt)$ etc.

The quantity $(ax \pm bt)$ is called phase of the travelling wave function. This quantity is constant for any point on the pulse if the shape of the wave does not change as the wave propagates.

i.e., $ax \pm bt = \text{constant}$

$$\therefore a\frac{dx}{dt} \pm b = 0 \qquad \therefore \frac{dx}{dt} = \mp\frac{b}{a} = \mp v$$

where 'v' is the wave velocity or phase velocity.

i.e., The speed of the travelling wave is the ratio of co-efficient of t to co-efficient of x.

The negative sign between x and t implies the wave travelling in positive x-direction and positive sign between x and t implies wave travelling in negative x-direction.

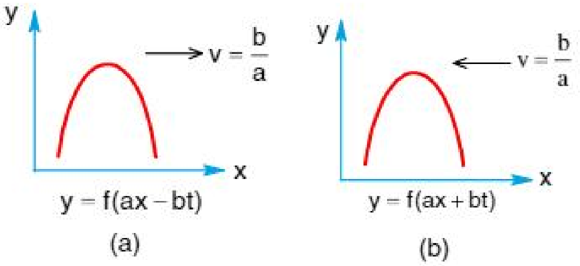


Fig 1.4

eg : $y = A\sin(ax - bt)$ represents a travelling wave in positive x-direction, $y = \frac{A}{B + (ax + bt)^2}$ represents travelling pulse in negative x-direction.

> ***Note:*** The travelling wave function can also be of the form, $y = f\left(t \pm \frac{x}{v}\right)$. To understand the travelling wave form easily we consider a string stretched along the x-axis whose vibrations are in the y-direction.

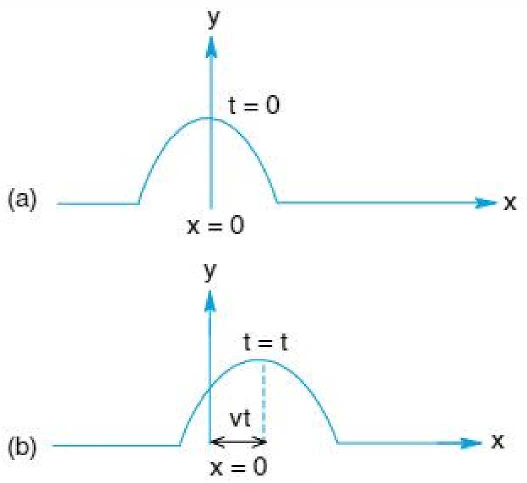


Fig 1.5

If there is a pulse in the string, we choose $t = 0$ when the displacement y of the string at $x = 0$ is maximum (peak). If the pulse starts to travel down in the string with speed (v) in the positive x-direction, the wave travels a distance $x = vt$ in time t, so the time interval between the formation of wave (peak) at $x = 0$ and its arrival at the point x is $\frac{x}{v}$.

The displacement 'y' of the string at x at any time t is exactly the same as the value of y at x = 0 at the earlier time $\left(t - \frac{x}{v}\right)$. Hence the desired form of 'y' in terms of x and t is $y = f\left(t - \frac{x}{v}\right)$ for the wave travelling in positive x-direction. Similarly for the wave travelling in the negative x-direction the wave form is $y = f\left(t + \frac{x}{v}\right)$.

1.5 HARMONIC AND PLANE PROGRESSIVE WAVE

A simplest progressive wave is the one resulting from the simple harmonic oscillations of the particles of the medium. Such a wave is known as a simple harmonic wave. All the phases of vibration of any particle can be found on the consecutive particles in order, along the direction of propagation of the wave and the wave advances with constant velocity. Therefore the progressive wave equation should simultaneously represent the displacements of all the particles at any given instant and the phases of vibration of a particle at a given point along the direction of propagation. Hence the required wave equation should be periodic in time as well as in position.

If the source of the wave is a simple harmonic oscillator, then the travelling wave is a sin or cos function of $(ax \pm bt)$, then the wave is said to be harmonic or plane progressive wave. During harmonic wave propagation the particles in the medium execute S.H.M on the same plane.

The displacement form of one dimensional harmonic plane progressive wave is

$y = A\sin(kx \mp \omega t + \phi_0)$ (or)
$y = A\cos(kx \mp \omega t + \phi_0)$

The pressure form of 1-D plane progressive wave is

$\Delta P = \Delta P_0 \cos(kx \mp \omega t + \phi_0)$ (or)
$\Delta P = \Delta P_0 \sin(kx \mp \omega t + \phi_0)$

(i) In displacement wave y represents the displacement of the particle from its mean position, at position x and time 't'. In pressure wave form ΔP represents the pressure fluctuation in the medium at position x and time t.

(ii) The maximum displacement of vibrating particle is called displacement amplitude (A) and the maximum pressure fluctuation in the medium is called pressure amplitude (ΔP_0).

(iii) The function $(kx \mp \omega t + \phi_0)$ is called phase (ϕ) of the wave, which gives the position and direction of motion of the particle at time 't'.

(iv) ' ϕ_0 ' is called initial phase or epoch, which gives the direction and motion and position and particle at x = 0 from where time is considered i.e., at t = 0.

(v) k is called wave number or propagation constant or wave vector and has unit (rad/m). This is related with wavelength (λ) as $k = \frac{2\pi}{\lambda}$.

The propagation constant k or wavelength λ depend on the nature of the medium and on the source velocity producing the waves.

(vi) ω is called angular frequency. Its S.I unit is rad/sec.

$\therefore \omega = 2\pi n = \frac{2\pi}{T}$

n, is called frequency of the wave.

(vii) 'x' represents the position of disturbance in time 't' (or) displacement of the wave in time t from the source.

(viii) The phase or wave velocity with respect to medium will be given by

$$v = \frac{dx}{dt} = \frac{\text{Coefficient of t}}{\text{Coefficient of x}} = \frac{\omega}{k}$$

and depends only on the nature of the medium in which the wave propagates and is independent of the source producing the waves. If the same wave passes through two different media, the medium in which the speed is less relative to the other medium is called denser and other one is said to be rarer.

(ix) If in a medium waves of different wavelength have different velocity, the medium is said to be dispersive medium, in which the energy is transmitted with less speed than wave speed. However if in a medium waves of different wavelength have same velocity, the medium is said to be non-dispersive, in which the energy is transmitted with speed as that of wave speed.

(x) A plane progressive wave either mechanical transverse or longitudinal or non mechanical can be written in many forms such as

(a) $y = A\sin(\omega t \mp kx)$

(b) $y = A\sin 2\pi\left(nt \mp \frac{x}{\lambda}\right)$

(c) $y = A\sin 2\pi\left(\frac{t}{T} \mp \frac{x}{\lambda}\right)$

(d) $y = A\sin \omega\left(t \mp \frac{x}{\upsilon}\right)$

Example-1.1

The equation for the displacement of a stretched string is given by $y = 4\sin 2\pi\left[\frac{t}{0.02} - \frac{x}{100}\right]$ where y and x are in cm and t in sec. Determine the (a) direction in which wave is propagating (b) amplitude (c) time period (d) frequency (e) angular frequency (f) wavelength (g) propagation constant (h) velocity of wave (i) phase constant and (j) the maximum particle velocity.

Solution :

Comparing the given equation with the general wave equation :

$y = A\sin(\omega t - kx + \phi)$,

i.e., $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda} + \phi\right]$

we find that :

(a) As there is negative sign between t and x terms, the wave is propagating along positive x-axis.

(b) The amplitude of the wave A= 4 cm = 0.04m.

(c) The time period of the wave T= 0.02 s = (1/50) s.

(d) The frequency of the wave f = (1/T) = 50 Hz.

(e) Angular frequency of the wave $\omega = 2\pi f = 100\pi\ \text{rad/s}$

(f) The wavelength of the wave $\lambda = 100\,\text{cm} = 1\,\text{m}$.

(g) The propagation constant.
(= wave vector = angular wave number
$= (2\pi/\lambda) = 2\pi$ rad/m.

(h) The velocity of wave $v = f\lambda = 50 \times 1 = 50\,\text{m/s}$.

(i) The phase constant. i.e., initial phase $\phi = 0$.

(j) The maximum particle velocity
$(v_{Pa})_{max} = A\omega = 0.04 \times 100\pi = 4\pi\,\text{m/s}$

1.6 DEPENDENCE OF WAVE FUNCTION ON FRAME OF REFERENCE

Consider a plane progressive wave $y = \sin(\omega t - kx)$ propagating in medium S with speed $v = \frac{\omega}{k}$. The phase or wave velocity $\frac{\omega}{k}$ is true with respect to the medium in which it propagates.

Consider a reference frame S^1 moving in the same positive x-direction with constant velocity V relative to medium S. The velocity of wave with respect to frame S^1 is $\vec{V}_{ws^1} = \vec{v}_w - \vec{v}_{s^1} = \vec{v} - \vec{V}$

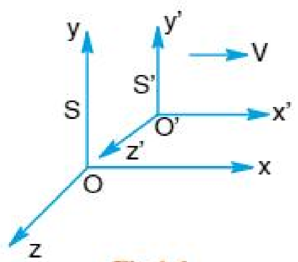


Fig 1.6

Therefore, the angular frequency of wave in reference frame S^1 is

$$\omega^1 = v_{ws^1} k = (v - V)k \quad \text{where } k = \frac{\omega}{v}$$

Hence $\omega^1 = \omega\left(1 - \frac{V}{v}\right)$. It should be noted that the wavelength of the wave remains the same.

Hence the equation of the wave in reference frame S^1 is $y = A\sin\left[\omega\left(1 - \frac{V}{v}\right)t - kx^1\right]$, where x^1 is the position of wave with reference to frame S^1.

1.7 GRAPHICAL REPRESENTATION OF HARMONIC WAVE

Two kinds of graphs may be plotted to represent a wave.

a) Displacement-position graph

It shows the displacements y of the vibrating particles of the medium at different positions 'x' from the source at a certain instant. This graph explains what happens to all the particles of the medium at an instant.

The position of the particles at a particular time and their corresponding displacement position graph is as shown in Fig 1.7(a) for transverse wave and 1.7(b) for longitudinal wave.

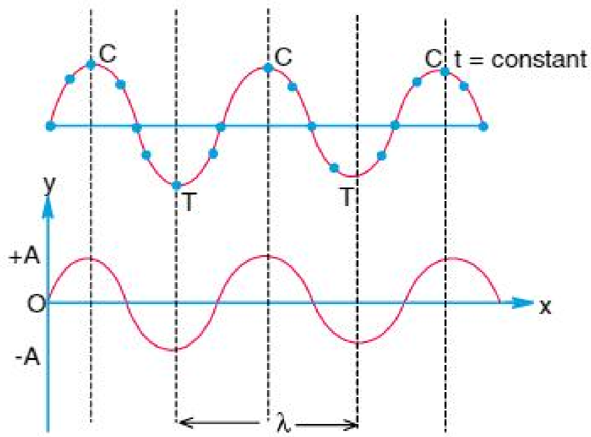


Fig 1.7(a)

The y-x graph of a transverse waves at an instant.

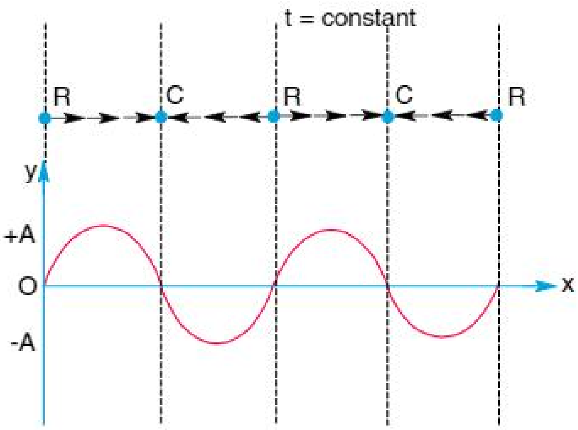


Fig 1.7(b)

The y-x graph of a longitudinal wave at an instant.

b) Displacement - time graph

It shows how the displacement of one particle at a particular position from the source varies with time. This graph explains what happens at a particular position w.r.t. time.

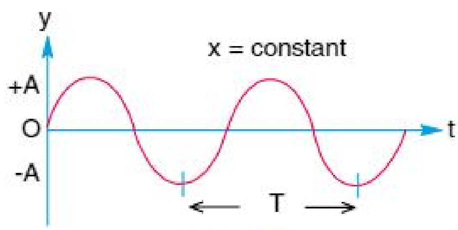


Fig 1.7(c)

1.8 RELATION BETWEEN WAVE VELOCITY (V) AND PARTICLE VELOCITY (V_P)

The velocity with which a disturbance propagates through a medium is called the wave velocity (v).

It depends only on the properties of the medium and is independent of time and position.

Particle velocity (v_p) is the rate at which displacement of a particle vary as a function of time i.e., particle velocity depend on time. Consider the wave function, $y = A\sin(kx - \omega t + \phi_0)$, then the particle velocity.

$$v_p = \frac{dy}{dt} = -\omega A\cos(kx - \omega t + \phi_0) \text{ (or)}$$

$$v_p = \omega\sqrt{A^2 - y^2}$$

The maximum particle velocity, $A\omega$ is called velocity amplitude. Particle possesses this maximum speed at mean position. At extreme position particle velocity is zero.

Now, the particle velocity function can be written as $v_p = \frac{dy}{dt} = -\left(\frac{\omega}{k}\right)\frac{dy}{dx} = -v\left(\frac{dy}{dx}\right)$

$\therefore v_p = -$(wave velocity) (slope of $y - x$ curve)

i.e. $v_p = -v$ (slope of $y - x$ curve)

In a transverse wave particle velocity is perpendicular to the line of direction of propagation. In a longitudinal wave, particle velocity is along the line of direction of propagation.

Case (i)

If the wave travels along positive x- direction, then v is positive.

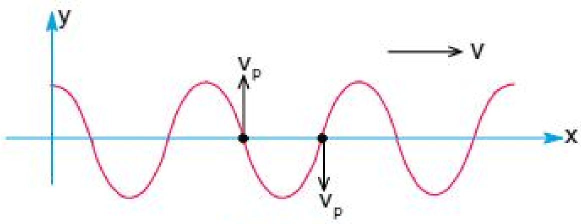


Fig 1.8(a)

(a) If $\frac{dy}{dx}$ is positive, then v_p is negative and it is downward in transverse wave or negative in longitudinal wave.

(b) If $\frac{dy}{dx}$ is negative, then v_p is positve and it is upward in transverse wave or positive in longitudinal wave.

Case (ii)

If the wave travels along negative x- direction then v is considered as negative.

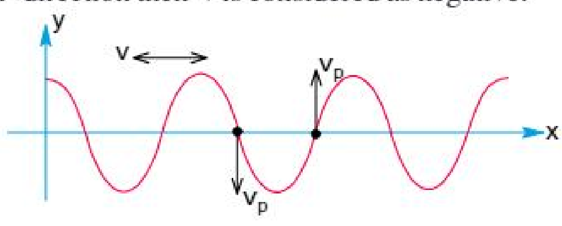


Fig 1.8(b)

a) If $\frac{dy}{dx}$ is positive, then v_p is upward in transverse wave or positive in longitudinal wave.

b) If $\frac{dy}{dx}$ is negative, then v_p is downward in transverse wave or negative in longitudinal wave.

1.9 EQUATION OF MOTION OF THE PARTICLE OF THE MEDIUM

The acceleration of the particle

$$a_p = \frac{dV_P}{dt} = \frac{d^2y}{dt^2} = -\omega^2 A\sin(kx - \omega t + \phi_0) = -\omega^2 y$$

$$[\therefore y = A\sin(kx - \omega t + \phi_0)]$$

The maximum value of acceleration $\omega^2 A$ is called acceleration amplitude. The above equation gives that acceleration of a particle is directly proportional to negative of its displacement, thus the particle in the medium execute simple harmonic motion during the harmonic wave propagation. Hence the particle has maximum acceleration at extreme position and zero at mean position.

Example-1.2

For a wave described by, $y = A\sin(\omega t - kx)$, consider the following points (i) x = 0, (ii) $x = \frac{\pi}{4k}$

For a particle at each of these points at t = 0, describe whether the particle is moving or not and in what direction and describe whether the particle is speeding up, slowing down or instantaneously not accelerating.

Solution :

$y = A\sin(\omega t - kx)$, Particle velocity

$$v_P(x,t) = \frac{\delta y}{\delta t} = \omega A\cos(\omega t - kx)$$

and particle acceleration

$$a_P(x,t) = \frac{\delta^2 y}{\delta t^2} = -\omega^2 A\sin(\omega t - kx)$$

i) t = 0, x = 0: $v_P = +\omega A$ and $a_p = 0$ i.e., particle is moving upwards but its acceleration is zero.

ii) At $t = 0$, $x = \frac{\pi}{4k}$, $V_P = \frac{A\omega}{\sqrt{2}}$, upward and

$a_p = \frac{\omega^2 A}{\sqrt{2}}$ upward

Example-1.3

A transverse wave is travelling along a string from left to right. Fig. below represents the shape of the string at a given instant. At this instant

(a) which points have an upward velocity?

(b) which points will have downward velocity?

(c) which points have zero velocity?

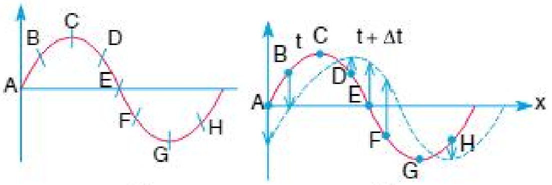
(d) which points have maximum magnitude of velocity?

Solution :

For a wave $\upsilon_{P_a} = -\upsilon \times (\text{slope})$ i.e., particle velocity is proportional to the negative of the slope of y/x curve; so

(a) For upward velocity υ_{P_a} = positive, so slope must be negative which is at points D, E and F.

(b) For downward velocity υ_{P_a} = negative, so slope must be positive which is at A, B and H.



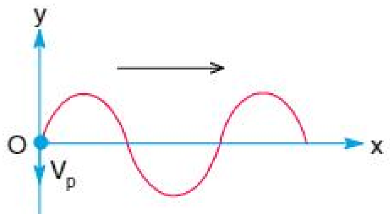
(a) (b)

(c) For zero velocity slope must be zero which is at C and G.

(d) For maximum magnitude of velocity $|\text{slope}|$ = maximum which is at A and E.

1.10 INITIAL PHASE (ϕ_0) OF A WAVE

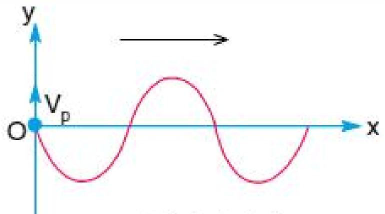
In general, the equation of a harmonic wave travelling along the positive x axis is expressed as $y = A\sin(kx - \omega t \pm \phi_0)$, where ϕ_0 is called the initial phase. ϕ_0 determines the displacement and direction of motion of the particle at x = 0, when t = 0.



$y = A\sin(kx - \omega t)$

$\frac{dy}{dx}$ is + ve, at $x = 0, t = 0$, $v_p(\downarrow)$

(a)



$y = A\sin(\omega t - kx)$

$\frac{dy}{dx}$ is – ve, at $x = 0, t = 0$, $v_p(\uparrow)$

(b)

Fig 1.9

Case (i)

With $\phi_0 = 0$, the particle is at mean position at x = 0 and the wave form starts from origin O. The wave form of $y = A\sin(kx - \omega t)$ and $y = A\sin(\omega t - kx)$ as shown in Fig 1.9(a) and 1.9(b) respectively.

These two waves represent the same type (but not same) of travelling waves with wave velocity $\frac{\omega}{k}$, but they differ by phase π as shown in figure 1.9.

In $y = A\sin(kx - \omega t)$, at x=0, t=0, the particle is at mean position, and the particle velocity is downward (in transverse wave) or left ward (in longitudinal).

In $y = A\sin(\omega t - kx)$, at x = 0, t = 0, the particle is at mean position and the particle velocity is upward (in transverse wave) or right ward (in longitudinal).

Case (ii)

With $\phi_0 \neq 0$ the particle with non zero displacement at x = 0 and t = 0. To express the travell-ing wave in $y = A\sin(kx - \omega t \pm \phi_0)$ or $y = A\sin(\omega t - kx \mp \phi_0^1)$ forms, upper signs are considered when the sine curve (in y-x graph) is assumed to start at A from left of the origin as shown in Fig 1.10(a) and lower signs are considered when sine curve is assumed to start at A from right of the origin as shown in Fig 1.10(b). The value of ϕ_0 or ϕ_0^1 is phase difference between the points O and A. It is same as the phase difference between moving particle at t = 0 and its mean position.

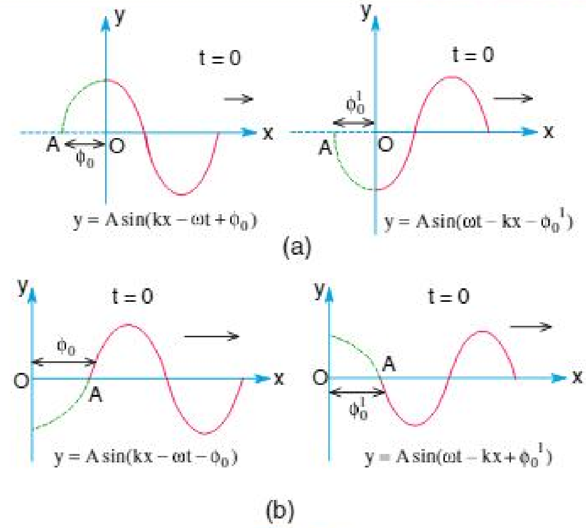


Fig 1.10

For example consider the wave travelling in positive x direction as shown in the Fig 1.11. The equation of that travelling wave is

$$y = A\sin\left(kx - \omega t + \frac{\pi}{3}\right)$$

$$\text{(or) } y = A\sin\left(\omega t - kx + \frac{2\pi}{3}\right)$$

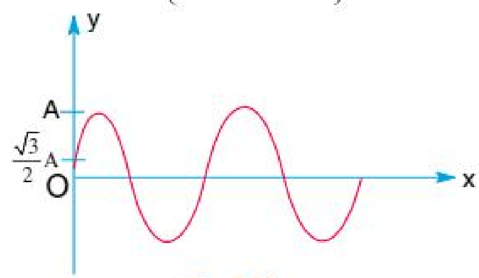


Fig 1.11

The form $y = A\sin(\omega t - kx)$ is used when our interest is to study what happens at a particular point in space with the passage of time. The form $y = A\sin(kx - \omega t)$ is used when our interest is to study what happens to all the particles at a particular instant.

Example-1.4

A sinusoidal wave travelling in the positive direction on a stretched string has amplitude 2.0 cm, wavelength 1.0 m and wave velocity 5.0 m/s. At x = 0 and t = 0, it is given that y = 0 and $\frac{\delta y}{\delta t} < 0$. Find the wave function y(x, t).

Solution :

We start with a general form for a rightward moving wave. $y(x,t) = A\sin(kx - \omega t + \phi)$.

The amplitude given is A = 2.0 cm = 0.02 m.

The wavelength is given as, $\lambda = 1.0\text{m}$

$\therefore$ Wave number $k = \frac{2\pi}{\lambda} = 2\pi \text{m}^{-1}$

Angular frequency $\omega = vk = 10\pi \text{ rad/s}$

$\therefore y(x,t) = (0.02)\sin[2\pi(x - 5.0t) + \phi]$

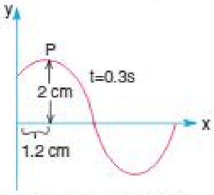
We are told that for $x = 0, t = 0, y = 0$ and $\frac{\delta y}{\delta t} < 0$

From these conditions, we may conclude that $\phi = 2n\pi$ where $n = 0, 2, 4, 6, ...$ Therefore,

$y(x,t) = (0.02\text{m})\sin[(2\pi \text{m}^{-1})x - (10\pi \text{s}^{-1})t]\,\text{m}$

Example-1.5

Figure shows a snapshot of a sinusoidal travelling wave taken at t = 0.3s. The wavelength is 7.5 cm and the amplitude is 2 cm. If the crest P was at x = 0 at t = 0, write the equation of travelling wave.



Solution :

Given, $A = 2\text{cm},\ \lambda = 7.5\text{cm}$

$\therefore k = \frac{2\pi}{\lambda} = 0.84\,\text{cm}^{-1}$

The wave has travelled a distance of 1.2 cm in 0.3s.

Hence, speed of the wave, $v = \frac{1.2}{0.3} = 4\,\text{cm/s}$

$\therefore$ Angular frequency $\omega = (v)(k) = 3.36$ rad/s

Since the wave is travelling along positive x-direction and crest (maximum displacement) is at $x = 0$ at $t = 0$, we can write the wave equation as,

$y = A\sin(kx - \omega t + \frac{\pi}{2})$ (or) $y(x,t) = A\cos(kx - \omega t)$

$y(x,t) = (2\text{cm})\cos[0.84\text{cm}^{-1})x - (3.36\text{rad/s})t]\text{cm}$

1.11 PHASE DIFFERENCE BETWEEN TWO PARTICLES IN THE MEDIUM

The argument of harmonic function $(\omega t - kx + \phi_0)$ is called phase (ϕ) of the wave.

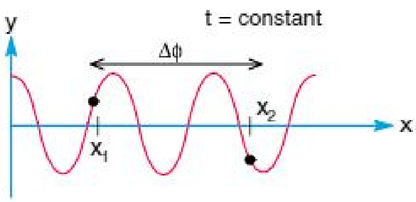


Fig 1.12

If we consider two points (particles) at positions x_1 and x_2 on a wave at a given instant t, as shown Fig 1.12.

Then $\phi_1 = \omega t - kx_1 + \phi_o$ and $\phi_2 = \omega t - kx_2 + \phi_o$

$\therefore$ Phase difference between them

$$\phi_1 - \phi_2 = k(x_2 - x_1)$$

i.e., $\Delta\phi = \frac{2\pi}{\lambda}.\Delta x$

where $\Delta x = (x_2 - x_1)$ is said to be path difference or particle separation. $\Delta x = n\lambda$, where $n = 0, 1, 2, 3,$ .... the particles are said to be in phase, i.e, their properties are similar at any instant.

(ii) If the two - particle separation is $\Delta x = (2n-1)\frac{\lambda}{2}$, where n = 1, 2, 3, ..., the particles are said to be out of phase, i.e., their properties are same in magnitude but opposite in sign.

(iii) If the two - particle separation is $\Delta x < \lambda$, they always differ by a constant phase. The phase difference between them is in between 0 and 2π. i.e., within one wave cycle, no two particles are under same phase.

1.12 VARIATION OF PHASE OF A PARTICLE WITH TIME

If we consider a particle at a given position x_0 on a wave, then $\phi = \omega t - kx_0 + \phi_0$. At $t = t_1$, $\phi_1 = \omega t_1 - kx_0 + \phi_0$ and at $t = t_2$, $\phi_2 = \omega t_2 - kx_0 + \phi_0$

Then the phase change of the particle in the given time interval $(t_2 - t_1)$ is $\phi_2 - \phi_1 = \omega(t_2 - t_1)$

i.e., $\Delta\phi = \frac{2\pi}{T}\Delta t$,

where T is the time period of oscillation. It is clear that

(i) If $\Delta t = nT$, where n = 1,2,3 ......, the particle repeats the motion. This implies after one time period the phase of oscillating particle becomes same as in the beginning.

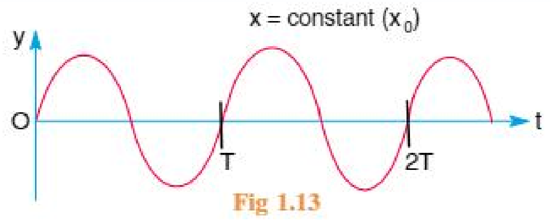


Fig 1.13

(ii) During the harmonic wave motion, all the particles in the medium change their phase by same amount in the same interval of time.

1.13 SPEED OF TRANSVERSE WAVE IN A STRING

On a stretched string as shown Fig 1.14 (a), if a transverse jerk is given, a pulse is created as shown in Fig 1.14 (b) which travels toward right with a wave speed v as shown in Fig 1.14(c). We start our analysis by looking at the pulse carefully as shown in enlarged view of Fig 1.14(d). For convenience of our analysis we choose a reference frame in which the pulse remains stationary or we assume that our frame is moving along with the pulse at speed v.

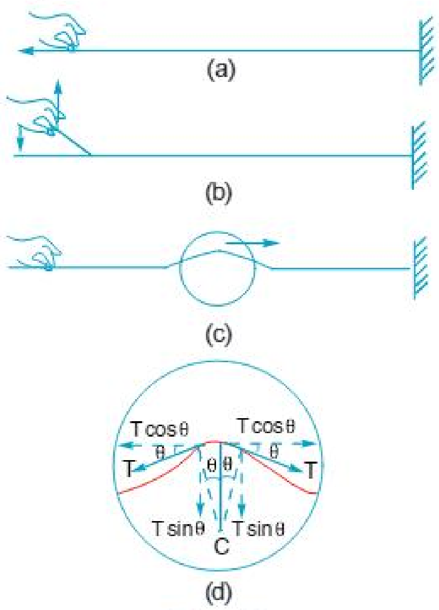


Fig 1.14

Now consider a small element of length dl on this pulse as shown. This element is forming an arc, say of radius R with centre at C and subtending an angle 2θ at C. We can see that two tensions T are acting on the edges of dl along tangential directions as shown in Fig 1.14(d).

The horizontal components of these tensions cancel each other, but the vertical components add to form a radial restoring force in downward direction, which is given as

$$F_R = 2T\sin\theta \simeq 2T\theta \qquad [\text{As } \sin\theta \simeq \theta]$$

$$= T\frac{dl}{R} \qquad \left[2\theta = \frac{dl}{R}\right] \quad .....(1)$$

If μ be the mass per unit length of the string, the mass of this element is given as $dm = \mu dl$ In our reference frame if we look at this element, it appears to be moving toward left with speed v then we can say that the acceleration of this element in our reference frame is

$$a = \frac{v^2}{R} \qquad .....(2)$$

Now from equations (1) and (2) we have

$$F_R = \frac{dmv^2}{R} \text{ or } T\frac{dl}{R} = \frac{(\mu dl)v^2}{R}$$

$$\text{or } v = \sqrt{\frac{T}{\mu}} \qquad ....(3)$$

If 'S' is the cross-section and ρ is the volumetric density of the wire, $\mu = \rho S$ so,

$$V = \sqrt{\frac{T}{\rho S}} = \sqrt{\frac{\text{stress}}{\rho}} \quad \left(\because \frac{T}{S} = \text{stress}\right).$$

Thus the speed of a transverse wave along a stretched string depends only on the tension and the linear mass density of the string and not on the wave characteristics. It must be remembered that we derive the expression for the velocity of wave always with respect to medium only unless otherwise stated.

Example-1.6

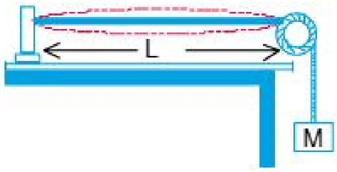
A wire of uniform cross-section is stretched between two points 1 m apart. The wire is fixed at one end and a weight of 9 kg hung over a pulley at the other end produces fundamental frequency of 750 Hz.

(a) What is the velocity of transverse waves propagating in the wire?

(b) If now the suspended weight is submerged in a liquid of density (5/9) that of the weight, what will be the velocity and frequency of the waves propagating along the wire?

Solution :

a) In case of fundamental vibrations of string $(\lambda/2) = L$, i.e., $\lambda = 2 \times 1 = 2m$.



Now as $v = f\lambda$ and $f = 750$ Hz, $V_T = 2 \times 750 = 1500$ m/s

i.e.., $\lambda = 2m$, $f = 750$ Hz and $V_T = 1500$ m/s

(b) Now as in case of a wire under tension $v = \sqrt{\frac{T}{m}}$

so $\frac{V_A}{V_B} = \sqrt{\frac{T_A}{T_B}}$, i.e., $V_B = 1500\sqrt{\frac{T_B}{T_A}}$ (or)

$$V_B = 1500\sqrt{\frac{Mg'}{Mg}} = 1500\sqrt{\frac{g[1-\sigma/\rho]}{g}}\left[\text{as } g' = g\left(1-\frac{\sigma}{\rho}\right)\right]$$

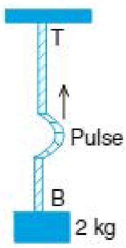
or $V_B = 1500\sqrt{1-\frac{5}{9}} = 1000 m/s$

so from $v = f\lambda$, $f_B = \frac{V_B}{\lambda_B} = \frac{1000}{2} = 500Hz$ i.e., in this situation, $\lambda = 2$ m, f = 500 Hz and v = 1000 m/s.

Example-1.7

A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope ?

Solution



As the rope has a mass and a mass is also suspended from the lower end, the tension in the rope will be different at different points. Now as $v = \sqrt{(T/m)}$

or $\frac{v_T}{v_B} = \sqrt{\frac{T_T}{T_B}} = \sqrt{\frac{(6+2)g}{2g}} = 2$ (or) $\left[\frac{f_T\lambda_T}{f_B\lambda_B}\right] = 2$

[as $v = f\lambda$]

Here $f_T = f_B$ as frequency is the characteristic of the source producing the waves.

So $\lambda_T = 2\lambda_B = 2 \times 0.06 = 0.12m$

Example-1.8

A wave pulse starts propagating in +x-direction along a non-uniform wire of length L m with mass per unit length given by $m = m_0 + \alpha x$ and under a tension of T N. Find the time taken by the pulse to travel from the lighter end (x=0) to the heavier end.

Solution

Velocity of transverse wave in a string.

$$\upsilon = \frac{dx}{dt} = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{m_0 + \alpha x}}$$

$$\therefore \sqrt{m_0 + \alpha x}\, dx = \sqrt{T}\, dt$$

Integrating within proper limits,

$$\int_0^L \sqrt{m_0 + \alpha x}\, dx = \sqrt{T}\int_0^t dt$$

$$\Rightarrow \left[\frac{2(m_0 + \alpha x)^{3/2}}{3\alpha}\right]_0^L = \sqrt{T}t$$

$$\therefore t = \frac{2}{3\alpha\sqrt{T}}[(m_0 + \alpha L)^{3/2} - m_0^{3/2}]$$

1.14 ENERGY, POWER AND INTENSITY TRANSPORTED BY A HARMONIC WAVE

As mentioned earlier, a wave is an energy transporting phenomenon which transport energy along a medium without transporting matter. To produce a wave, we have to apply a force to a portion of the wave medium. This force do work on the system. As the wave propagates each portion of the medium exerts and does work on the adjoining portion. In this way a wave can transport energy from one region of space to another.

When a transverse wave propagates along a string, it transports energy in two forms, kinetic energy and potential energy. Kinetic energy due to motion of particle (elements) and potential energy due to the amount to which it is stretched.

Suppose the wave propagating along the string is represented by $y = A\sin(kx - \omega t)$.

If μ is the mass per unit length of the rope, then the kinetic energy of a small element of length dx is

$$dK = \frac{1}{2}(dm)v_p^{\ 2} \Rightarrow dK = \frac{1}{2}(\mu dx)\left(\frac{dy}{dt}\right)^2$$

$$\text{(or)}\ dK = \frac{1}{2}\mu\omega^2 A^2[\cos^2(kx - \omega t)]dx \ ...\ (1)$$

If T is the tension in the string then the potential energy stored in the element of length dx is

$$dU = \frac{1}{2}(Tdx)\left(\frac{dy}{dx}\right)^2 \quad \text{(or)}$$

$$dU = \frac{1}{2}Tk^2A^2[\cos^2(kx - \omega t)]dx \quad .....\ (2)$$

$[\because y = A\sin(kx - \omega t)]$

Since $v = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$, therefore $Tk^2 = \mu\omega^2$ ..... (3)

From (2) and (3)

$$dU = \frac{1}{2}\mu\omega^2A^2[\cos^2(kx - \omega t)]dx \quad .....(4)$$

Total mechanical energy is $dE = dK + dU$

(or) $dE = \mu\omega^2A^2\cos^2(kx - \omega t)dx$ .....(5)

The linear mechanical energy density is

$$\frac{dE}{dx} = \mu\omega^2A^2\cos^2(kx - \omega t) \quad .....(6)$$

The equation (6) explains that the energy density is maximum at mean position

($y = 0,\ \phi = 0, \pi, 2\pi, 3\pi......$) and minimum at the extreme position

($y = \pm A,\ \phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}..........$).

This is because for a string segment, the potential energy depends on the slope of the string and is maximum when the slope is maximum, which is at the equilibrium position (B) of the segment, the same position for which the kinetic energy is maximum. At the extreme position (C) slope is zero, hence potential energy is zero and kinetic energy is zero. At the extreme position, the particle transfers entire energy to next particle.

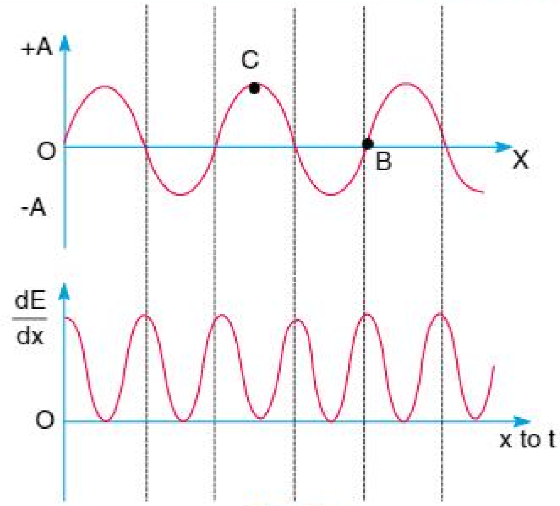


Fig 1.15

We see that $\frac{dE}{dx}$ varies with time (or) position in figure. The frequency of the energy density is double that of displacement wave. Since the average value of $\cos^2(kx - \omega t)$ at any instant is $\frac{1}{2}$.

The average linear mechanical energy density

$$< \frac{dE}{dx} > = \frac{1}{2}\mu\omega^2A^2 \quad ....(7)$$

If 'S' is the area of cross section and ρ is the density of rope, then the average mechanical energy per unit volume $< \frac{dE}{dv} > = u = \frac{1}{2}\frac{\mu\omega^2A^2}{S}$ ....(8)

$\therefore$ The volumetric energy density of travelling wave $u = \frac{1}{2}\rho\omega^2A^2$ ....(9)

$\therefore$ the total energy associated over a length l, and area of cross section S is

$$E = \frac{1}{2}\rho\omega^2A^2Sl \quad ....(10)$$

Thus amount of energy carried by a wave is related to the amplitude of the wave. A high energy wave is characterized by a high amplitude, a lower energy wave is characterized by a low amplitude.

Putting a lot of energy in a medium into a transverse pulse will not effect the wavelength, the frequency or the speed of the pulse i.e., the energy imparted to a pulse will only effect the amplitude of the wave.

The rate of transmission of energy is called power. The average power transmitted by the wave through area of cross section S is

$$<p> = \frac{<dE>}{dt} = \frac{1}{2}\mu\omega^2 A^2\left(\frac{dx}{dt}\right)$$

$$<P> = \frac{1}{2}\mu\omega^2 A^2 v \qquad .....(11)$$

$$\text{(or)} <p> = \frac{1}{2}\rho\omega^2 A^2 S v \quad (\because \mu = \rho S) \qquad .....(12)$$

The average intensity is defined as average power transmitted per unit area.

$$\text{So } <I> = \frac{<p>}{S} = \frac{1}{2}\frac{\mu\omega^2 A^2 v}{S} \text{ (or) } <I> = \frac{1}{2}\rho\omega^2 A^2 v$$

Example-1.9

A thin string is held at one end and oscillates vertically so that, $y(x=0,t) = 8\sin 4t(\text{cm})$ Neglect the gravitational force. The string's linear mass density is 0.2 kg/m and its tension is 1 N. The string passes through a bath filled with 1 kg water. Due to friction heat is transferred to the bath. The heat transfer efficiency is 50%. Calculate how much time passes before the temperature of the bath rises by one degree kelvin.

Solution :

Comparing the given equation with equation of a travelling wave, $y = A\sin(kx \pm \omega t)$ at $x = 0$ we find,

$A = 8$ cm $= 8 \times 10^{-2}$ m $\omega = 4$ rad/s Speed of travelling wave,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1}{0.2}} = 2.236 \text{m/s}$$

Further, $\rho S = \mu = 0.2 \text{kg/m}$ The average power over a period is, $P = \frac{1}{2}(\rho S)\omega^2 A^2 v$

Substituting the values, we have

$$P = \frac{1}{2}(0.2)(4)^2(8\times10^{-2})^2(2.236) = 2.29\times10^{-2} \text{J/s}$$

Now let it take *t* second to raise the temperature of 1 kg water by one degree kelvin. Then $Pt = ms\Delta t$

Here, s = specific heat of water = $4.2\times10^3 \text{J/kg}-\text{K}$

$$\therefore \quad t = \frac{ms\Delta t}{P\times\frac{1}{2}} = \frac{(1)(4.2\times10^3)(1)}{1.145\times10^{-2}} \simeq 4.2 \text{ day}$$

1.15 SUPERPOSITION OF WAVES

When two or more waves travel simultaneously in a medium the resultant displacement at a point is the vector sum of the individual displacements due to each wave at that point. This is called the principle of superposition.

i.e., $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ... + \vec{y}_n$

$$\text{(or) } y(x,t) = \sum_{n=1}^{n=n} y_n(x,t).$$

Here y_1, y_2 are the individual wave functions and y is their sum wave function. This principle of superposition holds for all types of waves (mechanical and electromagnetic waves) and valid only when the amplitude of the wave is much less than the wavelength, $A << \lambda$, and velocity of the wave is much larger than the particle velocity, $v >> \frac{dy}{dt}$.

Case (i)

Consider two wave pulses travelling simultaneously in opposite directions.

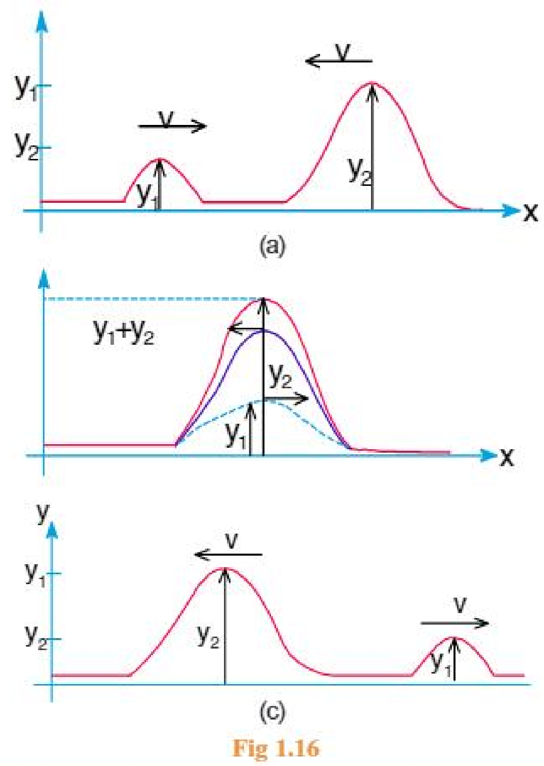


Fig 1.16

When they overlap each other, the displacement of particle on string is the sum of the two displacements, as the displacements of the pulses are in same direction as shown in Fig 1.16 (b).

Unlike particles, when waves meet they do not alter one another and each propagates through the medium as if the other wave is not there. After crossing, each pulse travels along string as if nothing had happened and it has its original shape as shown in Fig 1.16(c).

Case (ii)

Consider two wave pulses travelling simultaneously in opposite directions as shown in Fig 1.17 overlapping each other. The displacement of particle on string is the difference of the two displacements as the displacements of the pulses are in opposite directions as shown in Fig1.17. After crossing, each pulse travels along string as if nothing had happened and it has its original shape as shown in Fig 1.17.

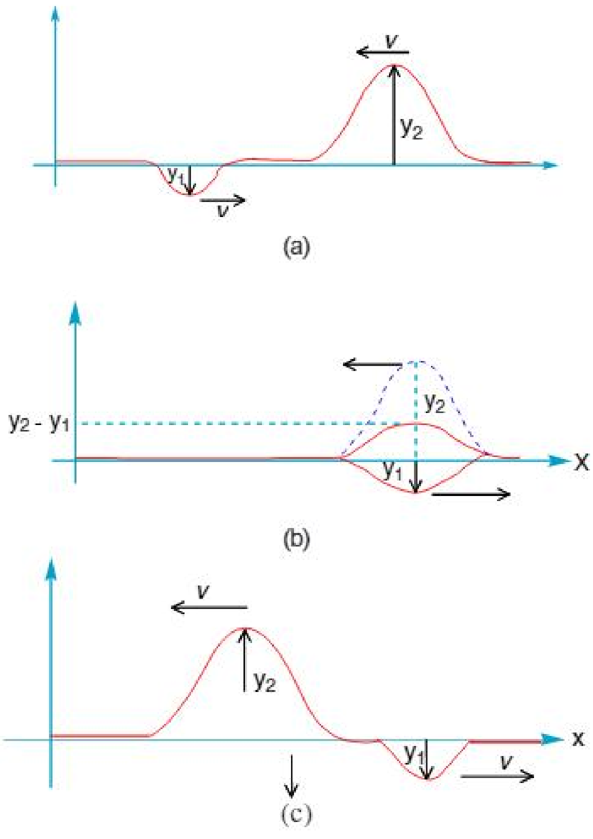


Fig 1.17

There are three phenomena that can be explained by the superposition of waves. These are

1) Interference : Superposition of same frequency waves that differ in phase or not, but travelling in same direction.

2) Standing waves : Superposition of waves that differ in direction.

3) Beats : Superposition of waves that differ in frequency travelling is same direction.

1.16 INTERACTION OF WAVES WITH BOUNDARIES

As a wave travels through a medium, it will often reach the end of the medium and encounter an obstacle or overlaps another medium through which if could travel. When one medium ends another medium begins, the interface of the two media is referred to as the boundary and the behavior of a wave at that boundary is described as its boundary behavior.

1.16.1 REFLECTION OF WAVES

a) Reflection of transverse wave at fixed end

First consider an elastic rope stretched from end to end whose one end is securely attached to a wall while other end is held in hand in order to produce pulses in to the medium.

Because the right end of the rope is attached to the wall, the last particle of the rope will be unable to move when a disturbance reaches it. This end of the rope is referred to as a fixed end. If a pulse is introduced at the left end of the rope it will travel towards right end. This pulse is called the incident pulse. When the incident pulse reaches the boundary, two things occur.

(i) A portion of the energy carried by the pulse is reflected and returns towards left end of the rope. The disturbance which returns from the wall is known as the reflected pulse.

(ii) A portion of the energy carried by the pulse is transmitted to the wall, causing wall to vibrate. The vibration of the wall is not visibly obvious, the energy transmitted to it is not typically discussed.

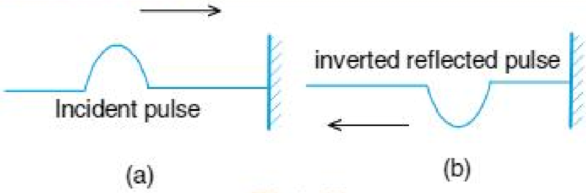


Fig 1.18

Here the reflected pulse is inverted. i.e., if a crest is incident towards a fixed end boundary, it will reflect and return as a trough. Similarly, if a trough is incident towards a fixed end boundary, it will reflect and return as a crest. This is shown in Fig 1.18.

When a crest reaches the end of a medium (say medium A), the last particle of the medium A receives an upward displacement. This particle is attached to the first particle of the other medium (say medium B) on the other side of the boundary. As the last particle of medium A pulls upwards on the first particle of medium B, the first particle of medium B pulls downwards on the last particle of medium A. This is due to Newton's third law of action -reaction. The effect of the downward pull on the last particle of medium A results in causing the upward displacement to become a down ward displacement. i.e., the crest becomes a trough. Thus a phase change of π takes place in the displacement when a transverse wave is reflected at the rigid support. The other characteristics of the reflected pulse include.

(i) The frequency of the reflected wave is same as these of incident wave.

(ii) The wavelength and speed of the reflected pulse are the same as these of the incident pulse. This is because both pulses are in the same medium.

(iii) The amplitude and hence energy of the reflected pulse is slightly less than the amplitude and the energy of the incident pulse, since some of the energy of the pulse was transmitted into wall at the boundary. The reflection of a pulse at a fixed end can be treated as superposition of incident pulse and an imaginary pulse which is moving opposite to incident pulse with same dimension but with inversion from the rigid medium.

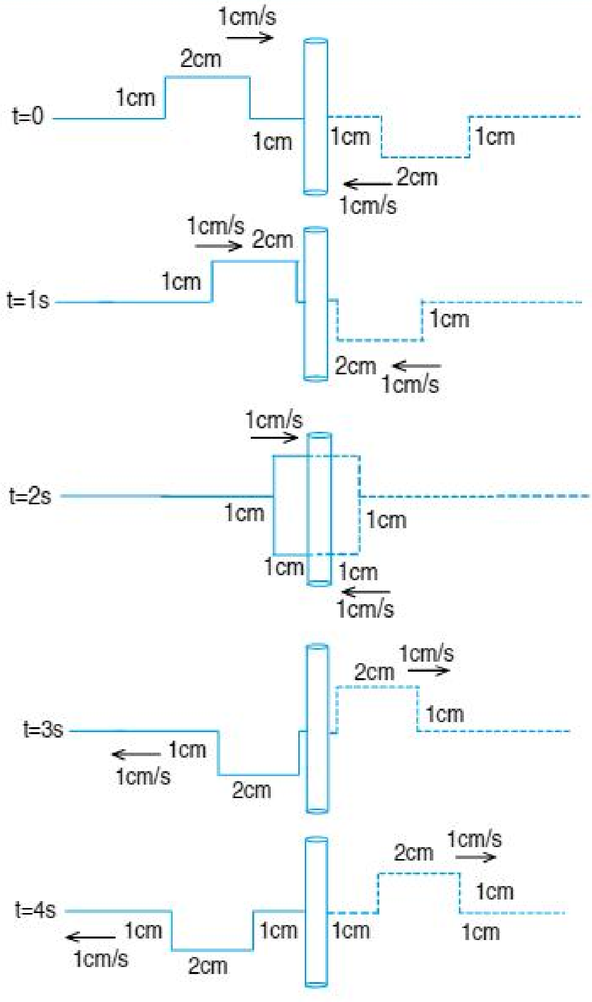


Fig 1.19

b) Reflection of transverse wave at free end

Consider the end of the rope free to move like attached to a wall, suppose it is attached to a ring which can slide freely on the pole as shown in Fig 1.20.

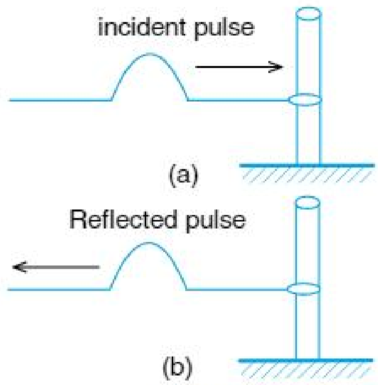


Fig 1.20

The last particle of the rope will be able to move when a disturbance reaches it. This end of the rope is referred to as a free end. If a pulse is introduced at the left end of the rope, it will travel through the rope towards right end of the medium. When the incident pulse reaches the end of the medium, the last particle of the rope can no longer interact with the first particle of the pole. When a crest reaches the end of the rope the last particle of the rope receives the same upward displacement only because there is no adjoining particle to pull it downward it to cause it to be inverted.

The result is that the reflected pulse is not inverted. When a crest is incident upon a free end, it returns as a crest after reflection and when a trough is incident upon a free end, it returns as a trough after reflection. Inversion is not observed on free end reflection. Thus no phase change takes place in the displacement, when a transverse wave is reflected at free end. This is shown in Fig 1.21.

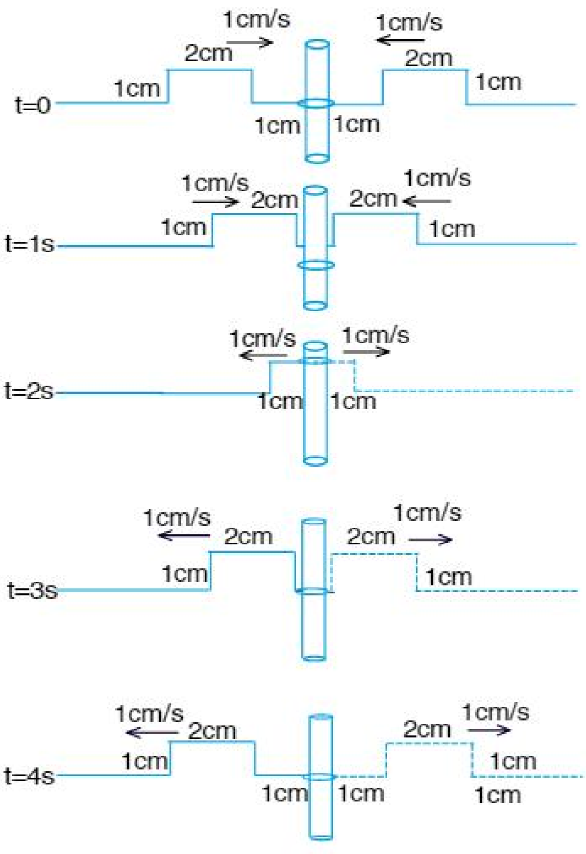


Fig 1.21

The reflection of a pulse at a free end can be treated as superposition of incident pulse and an imaginary pulse which is moving opposite to incident pulse with same dimension without inversion from the rarer medium.

1.16.2 REFRACTION OF WAVES

a) When the incident wave travels from rarer to denser medium

Let us consider a thin rope (less dense or rarer) attached to a thick rope (high dense or denser), with each rope held at opposite ends by supports as shown Fig 1.22.

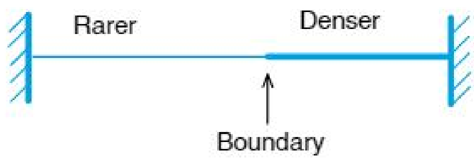


Fig 1.22

Suppose that a pulse is introduced at the supporting end of the thin rope. There will be an incident pulse travelling in the less dense medium (thin rope) towards the boundary with a more dense medium (thick rope). Upon reaching the boundary.

(i) A portion of the energy carried by the incident pulse is reflected and returns toward left end of the thin rope. The disturbance which returns after bouncing off the boundary is known as the reflected pulse.

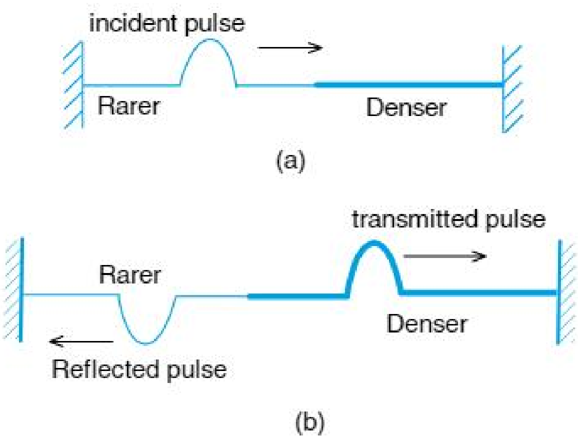


Fig 1.23

(ii) A portion of the energy carried by the incident pulse is transmitted into the thick rope (denser medium). The disturbance which continues moving to the right is known as the transmitted pulse.

In this case the reflected pulse will be found to be inverted and the transmitted pulse is not inverted. During the interaction between the two media at the boundary, the first particle of the more dense medium overpowers the smaller mass of the last particle of the less dense medium. This causes the crest to become a trough and vice versa in the less dense medium upon reflection. The first particle of high dense medium receives an upward pull when the incident pulse reaches the boundary. Since the more dense medium was originally at rest, any change in their state of motion would be in the same direction as the displacement of the particles of the incident pulse. For this reason, the transmitted pulses can never be inverted.

a) The transmitted pulse (in denser) is travelling slower than the reflected pulse (in rarer)

b) The transmitted pulse (in denser) has a smaller wavelength than the reflected pulse (in rarer).

c) Since reflected pulse and incident pulse are in the same medium the speed and wavelength of the reflected pulse and same as these is the incident pulse.

d) The frequency of the wave depends on the source of disturbance only. Hence the frequency of incident pulse, reflected pulse and transmitted pulse will be same.

Example:

Let A_i, A_t and A_r be the amplitude of the incident, transmitted and reflected pulse. V_i and V_t are the speeds of the incident and transmitted pulses, ω is the angular frequency of incident pulses. If the equation of incident wave is

$y_i = A_i \sin\omega\left(t - \frac{x}{v_i}\right)$, then the equation of reflected wave

$y_r = A_r \sin\omega\left(t + \frac{x}{v_i} + \pi\right)$ and equation of transmitted wave

$$y_t = A_t \sin\omega\left(t - \frac{x}{v_t}\right)$$

b) When the incident wave travels from denser to rarer medium

Let us consider a thick rope attached to a thin rope, with the incident pulse originating in the thick rope. There will be an incident pulse travelling from denser medium to towards the boundary of rarer medium. There will be partial reflection and partial transmission at the boundary.

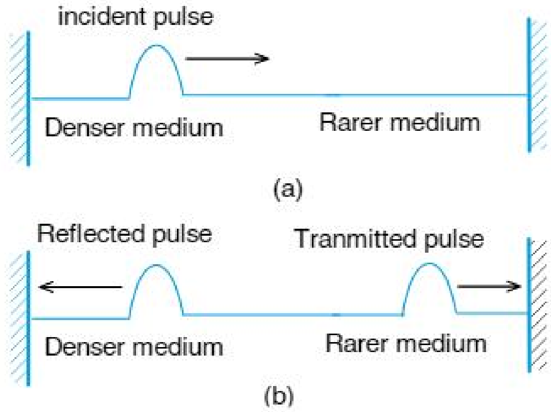


Fig 1.24

The reflected pulse in this situation will not be inverted. Similarly, the transmitted pulse is not inverted. When the incident pulse in denser medium, reaches the boundary, the first particle of the rarer medium does not have sufficient mass to overpower the last particle of the denser medium. Crest incident towards the boundary will reflect as a crest and vice versa. The transmitted pulse can never be inverted. Since the particles in this medium are originally at rest, any change in their state of motion would be in the same direction as the displacement of the particles of the incident pulse.

a) The transmitted pulse (in rarer) travels faster than the reflected pulse

b) The transmitted pulse has a larger wavelength than the reflected pulse

c) Since the reflected pulse and incident pulse are in the same medium, the speed and the wavelength of the reflected pulse are the same as the speed and wavelength of the incident pulse.

d) The frequency of the incident pulse, reflected pulse and transmitted pulse will be the same.

Example

Let A_i, A_t and A_r be the amplitude of the incident, transmitted and reflected pulses, V_i and V_t be the speeds of the incident and transmitted pulses, ω be the angular frequency of the incident pulse. If the equation of the incident pulse is $y_i = A_i \sin\omega\left(t - \frac{x}{v_i}\right)$, the equation of the reflected pulse $y_r = A_r \sin\omega\left(t + \frac{x}{v_i}\right)$ and the equation of the transmitted pulse $y_t = A_t \sin\omega\left(t - \frac{x}{v_t}\right)$

1.17 ANALYTICAL CONCEPT AT THE BOUNDARY BETWEEN A LIGHT AND HEAVY STRING

When a wave pulse encounters a boundary between a light and heavy string it undergoes partial reflection and transmission. Here

a) Incident, reflected and transmitted pulses have the same frequency i.e., $n_i = n_t = n_r$

b) By the principle of superposition the sum of displacement of the incident pulse (y_i) and the reflected pulse (y_r) is equal to displacement of transmitted pulse (y_t).i.e., $y_i + y_r = y_t$ and further $A_i + A_r = A_t$. Also at the boundary the slope of the wave pulse will be continuous

i.e., $(\text{slope})_1 = (\text{slope})_2$. i.e., $\frac{dy_i}{dx} + \frac{dy_r}{dx} = \frac{dy_t}{dx}$.

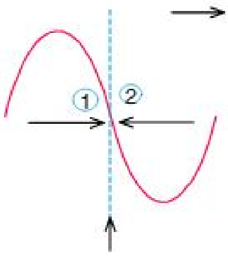


Fig 1.25

c) The incident pulse, reflected pulse and transmitted pulse are always in the same plane.

d) The average power of incident pulse (P_i) is equal to the sum of the power of reflected pulse (P_r) and transmitted pulse (P_t)

i.e. $P_i = P_r + P_t$

If A_i is the amplitude of the incident pulse in medium 1, A_r is the amplitude of the reflected pulse in medium 1, A_t is the amplitude of the transmitted pulse in medium 2, V_i and V_t are the pulse velocities in incident medium and transmission medium, μ_i and μ_t are the corresponding linear densities of the strings, then by conservation of energy

$$P_i = P_r + P_t \quad \text{...... (1)}$$

$$\frac{1}{2}\mu_i\omega^2 A_i^2 v_i = \frac{1}{2}\mu_i\omega^2 A_r^2 v_i + \frac{1}{2}\mu_t\omega^2 A_t^2 v_t$$

$$\text{(or)} \left(\frac{T}{v_i^2}\right)A_i^2 v_i = \left(\frac{T}{v_i^2}\right)A_r^2 v_i + \left(\frac{T}{v_t^2}\right)A_t^2 v_t$$

$$\left(\therefore v = \sqrt{\frac{T}{\mu}}\right) \text{(or)} \frac{A_i^2}{v_i} = \frac{A_r^2}{v_i} + \frac{A_t^2}{v_t} \quad \text{..... (2)}$$

Further $A_i + A_r = A_t$ ..... (3)

Solving above equations (2) and (3), we get

$$A_r = \left(\frac{V_t - V_i}{V_i + V_t}\right)A_i \text{ and } A_t = \left(\frac{2V_t}{V_i + V_t}\right)A_i$$

If k_i and k_t are the corresponding propagation constants in the strings.

$$\text{Then } A_r = \left(\frac{k_i - k_t}{k_i + k_t}\right)A_i \text{ and } A_t = \frac{2k_i}{k_i + k_t}$$

Case (i)

If $V_i > V_t$, i.e., incident pulse medium is rarer and transmitted pulse medium is denser, then A_r is negative indicating reflected pulse suffers a phase change of π and A_t is positive indicates transmitted pulse suffers no phase change.

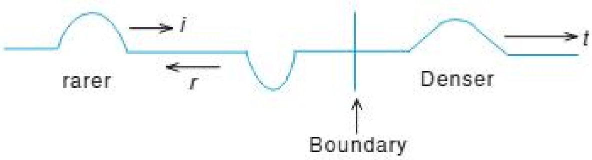


Fig 1.26(a)

Case (ii)

If $V_i < V_t$ i.e., incident pulse medium is denser and transmitted pulse medium is rarer, both A_r and A_t are positive.

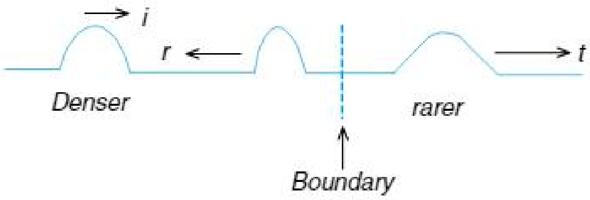


Fig 1.26 (b)

Case (iii)

If $V_t <<< V_i$, i.e., transmission medium is much denser, then $A_t = 0$ and $A_r = -A_i$.

This is the case similar to wave reflection at fixed end. i.e., the entire energy reflected.

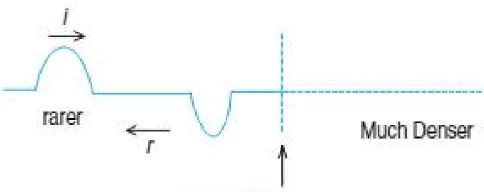


Fig 1.26 (c)

Case (iv)

If $V_t >>> V_i$ i.e., transmission medium is much rarer, then $A_r = A_i$ and $A_t = A_i$. The above result seems to be violation of conservation of energy, but it is not true, since $V_t >>> V_i$, then $P_t <<< P_i$ $(\because\ P \propto V)$

i.e., there is no transmitted wave hence $P_r = P_i$

i.e., $A_r = A_i$

This is similar to wave reflection at free end.

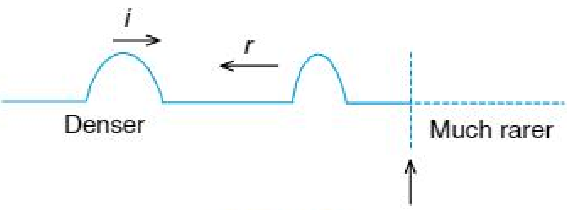
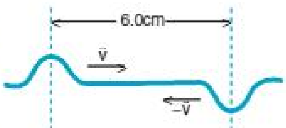


Fig 1.26 (d)

Example-1.10

In Figure. two pulses travel along a string in opposite directions. The wave speed v is 2.0 m/s and the pulses are 6.0 cm apart at t = 0.

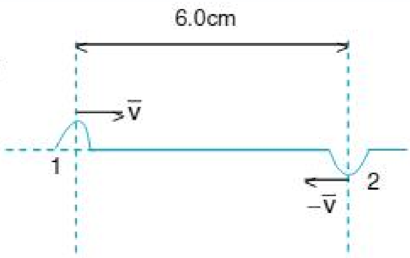


(a) Sketch the wave patterns when t is equal to 20 ms.

(b) In what form (or type) is the energy of the pulse at t = 15 ms?

Solution :

At t = 0 sec



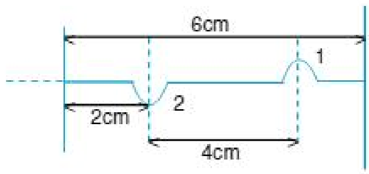
(a) At t = 20 ms.

Distance travelled by the first pulse in 20 ms is

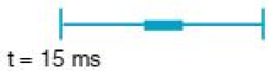
$$= Vt = 2 \times 20 \times 10^{-3} = 4\,cm.$$

Distance travelled by the second pulse in 20 ms is also

$$= Vt = 2 \times 20 \times 10^{-3} = 4\,cm$$



(b) At t = 15 ms, by the principle of superposition resultant wave displacement is zero. Hence, the shape of the string is straight at t = 15 ms. Hence, total energy is purely kinetic.



1.18 STANDING WAVES (OR) STATIONARY WAVES

When two travelling waves of same amplitude, frequency and velocity but moving in opposite directions are superposed, the phenomenon of standing waves is observed.

Consider two waves with same amplitude, velocity and frequency but travelling in opposite directions as in Fig 1.27 (a) and (b).

$y_1 = A\sin(kx - \omega t)$ and $y_2 = A\sin(kx + \omega t + \phi_0)$

To understand these waves easily, let us discuss the special case when $\phi_0 = 0$.

Using the principle of superposition, their resultant is

$y = y_1 + y_2 \quad y = A\sin(kx - \omega t) + A\sin(kx + \omega t)$

(or) $y = 2A\sin kx \cos \omega t$

The function $y = 2A\sin kx \cos \omega t$ does not have the form of $f(ax \pm bt)$ and it does not describe a travelling wave.

Hence it is known as standing wave. The wavelength and frequency of this resultant wave are equal to that of the individual waves which are superposed.

Since $y = 2A\sin kx\cos\omega t$ (or) $y = A_s\cos\omega t$, where A_s =2A sin kx, known as amplitude of the wave, is not constant but varies with position.

The equation $y = A_s\cos\omega t$ explains that particles of the medium executes simple harmonic motion.

All the particles vibrate with same frequency but their amplitudes are not equal. The amplitude of oscillation of a particle depends on its position as $A_s = 2A\sin kx$.

The given Fig 1.27(c) represents the stationary wave form of $y = A\sin kx\cos\omega t$, at t = 0.

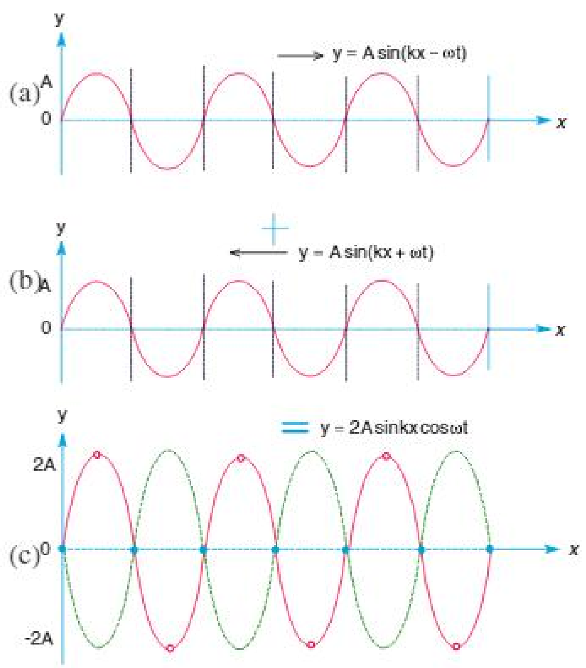


Fig 1.27

Position of nodes

In particular, there are points where the amplitude |2A sin kx| = 0. This will be the case when sin kx = 0

$\therefore\ kx = n\pi$, where n = 0, 1, 2, 3, ...

i.e., $kx = 0, \pi, 2\pi$....... (or)

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}...... \quad \left(\because\ k = \frac{2\pi}{\lambda}\right)$$

i.e., $x = n\frac{\lambda}{2}$, where n = 0, 1, 2, 3, ...

These are the points at which particles never displace from their mean position as the two waves pass them simultaneously although these points are not physically clamped. All these are called as nodes.Whose displacements are zero at all times. The distance between two successive nodes is $\frac{\lambda}{2}$.

Position of Antinodes

The points where the amplitude is 2A, i.e., the points with maximum amplitude are called antinodes. The position of antinodes are $\sin kx = \pm 1$.

$$\therefore\ kx = (2n-1)\frac{\pi}{2},\ \text{where } n = 1, 2, 3 .....$$

$$\text{i.e } kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}..... \quad \text{(or)}$$

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}....... \quad \left(\because\ k = \frac{2\pi}{\lambda}\right)$$

$$\text{i.e., } x = (2n-1)\frac{\lambda}{4},\ n = 1, 2, 3, ...$$

There are certain points in the diagram marked as empty dots (o), whose displacements periodically vary between zero and maximum in opposite directions. Such points are the antinodes.

The distance between two adjacent anti-nodes is also $\frac{\lambda}{2}$, while that between a node and an antinode is $\frac{\lambda}{4}$. The maximum amplitude of the standing wave is double the amplitude of the individual wave

i.e., $A_{max} = \pm 2A$

Standing waves can be transverse or longitudinal e.g. in strings (under tension) if reflected wave exists, the waves formed are transverse stationary, while in organ pipes waves are longitudinal stationary.

1.19 PROPERTIES OF STATIONARY WAVES

a) The nodes divide the medium into segments (or loops), All the particles in a segment vibrate in same phase, but in opposite phase (differ by phase π) with the particles in the adjacent segment. i.e., Two particles in consecutive loops always move in opposite direction

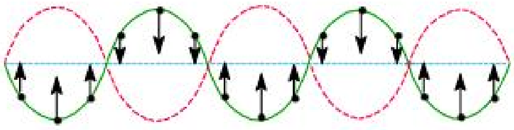


Fig 1.28

Hence in a stationary wave two particles differ in phase either by 0 or π

b) Within a segment (or loop) all the particles pass through their mean position simultaneously in same direction with their own maximum velocity ($A_s\omega$). In one time period particles cross their mean position twice. The particle velocity in a stationary wave

$$V_p = \frac{dy}{dt} = \frac{d}{dt}(A_s \cos\omega t) = A_s\omega \sin\omega t$$

where A_s is amplitude of the particle.

c) The energy density in stationary wave is twice that of the progressive wave.

d) As in stationary waves nodes are permanently at rest, no energy can be transmitted across them. i.e., energy of one region is confined in that region. This energy oscillates between elastic potential energy and kinetic energy of the particles of the medium. When all the particles are at their extreme position kinetic energy is minimum, elastic potential energy is maximum and when all the particles pass through their mean position kinetic energy will be maximum. While elastic potential energy is minimum. Then the total energy confined in a segment always remains the same. At any given position (except nodes), the distribution of kinetic energy and potential energy changes with time. At a given instant, the ratio of kinetic to potential energy is same for all the particles.

e) In a stationary wave if the amplitudes of the component waves are not equal, then nodes will not be permanently at rest ($A_{min} \neq 0$) and so some energy will pass across the node and that energy in transfered to another medium at boundary as a transmitting wave. Hence the wave will be partially standing.

The extent to which the resultant wave translates energy is proportional to $\frac{A_i - A_r}{A_i + A_r}$.

Less this value lesser energy propagates at node. For $A_i = A_r$, the percentage of energy crossing the loop is zero. For $A = A_t$, $A_r = 0$ the percentage of energy transfer is 100, which is nothing but a travelling wave.

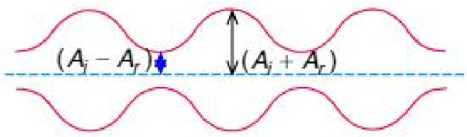


Fig 1.29

f) The equation of standing wave basically depends on the component waves. The different forms of standing waves are

(i) $y = 2A \sin kx \cos \omega t$

(ii) $y = 2A \sin kx \sin \omega t$

(iii) $y = 2A \cos kx \sin \omega t$

(iv) $y = 2A \cos kx \cos \omega t$

(i) In **$y = 2A \sin kx \cos \omega t$,** standing waves are formed on reflection of waves from fixed end. If x = 0 is fixed end, then that end acts as node and at t = 0 the particles are at extreme position.

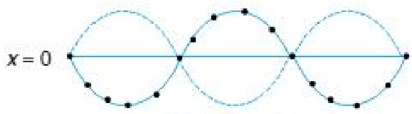


Fig 1.30 (a)

(ii) In **$y = 2A \sin kx \sin \omega t$,** standing waves are formed on reflection of waves from fixed end. If x = 0 is fixed end, then that end is node and at t = 0 the particles are crossing their mean position with different speeds.

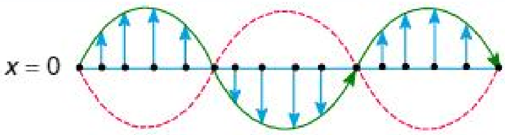


Fig 1.30 (b)

(iii) In **y = 2A cos kx sin ω t,** standing waves are formed on reflection of waves from free end. If x = 0 is that free end then that end is antinode and at t = 0, the particles are crossing their mean position with different speeds.

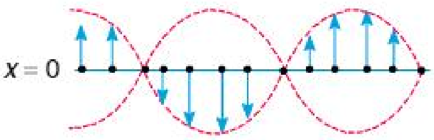


Fig 1.30 (c)

(iv) In **y = 2A coskx cos ω t, standing** waves are formed on reflection of waves from free end. If x = 0 is that free end, then that end is antinode and at t = 0, the particles are at extreme position.

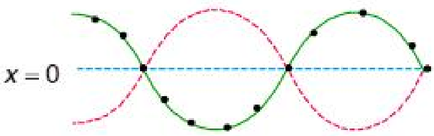


Fig 1.30 (d)

Example-1.11

The vibrations of a string of length 60 cm at both ends are represented by the equation.

$$y = 4\sin\left[\frac{\pi x}{15}\right]\cos(96\pi t)$$

where *x* and *y* are in cm and *t* is in sec.

a) What is the maximum displacement at x = 5 cm?

b) Where are the nodes located along the string?

c) What is the velocity of the particle at x = 7.5 cm and t = 0.25 s?

d) Write down the equations of component waves whose superposition gives the above wave.

Solution :

a) For x = 5, $y = 4\sin(5\pi/15)\cos(96\pi t)$

or $y = 2\sqrt{3}\cos(96\pi t)$

So y will be maximum when

$\cos(96\pi t) = \max = 1$, i.e., $(y_{max})_{x=5} = 2\sqrt{3}$cm

b) At nodes amplitude of wave is zero,

$$4\sin\left[\frac{\pi x}{15}\right] = 0 \text{ or } \frac{\pi x}{15} = 0, \pi, 2\pi, 3\pi, ...$$

So x = 0, 15, 30, 45, 60 cm

[as length of string = 60 cm].

c) As $y = 4\sin(\pi x/15)\cos(96\pi t)$

$$\frac{dy}{dt} = -4\sin\left[\frac{\pi x}{15}\right]\sin(96\pi t)\times(96\pi)$$

So the velocity of the particle at x=7.5 cm and t=0.25 s

$$v_{pa} = -384\pi\sin(7.5\pi/15)\sin(96\pi\times 0.25)$$

$$v_{pa} = -384\pi\times 1\times 0 = 0$$

d) As 2 sin A cos B = sin (A + B) + sin (A - B)

$$\text{So } y = 4\sin\left[\frac{\pi x}{15}\right]\cos(96\pi t)$$

$$= 2\left[\sin\left(\frac{\pi x}{15} + 96\pi t\right) + \sin\left(\frac{\pi x}{15} - 96\pi t\right)\right]$$

$$y = 2\sin\left[96\pi t + \frac{\pi x}{15}\right] - 2\sin\left[96\pi t - \frac{\pi x}{15}\right]$$

$[\text{as}\sin(-\theta) = -\sin\theta]$

$$y = y_1 + y_2 \quad \text{with } y_1 = 2\sin\left[96\pi t + \frac{\pi x}{15}\right]$$

$$y_2 = -2\sin\left[96\pi t - \frac{\pi x}{15}\right]$$

Example-1.12

A standing wave is formed by two harmonic waves, $y_1 = A\sin(kx - \omega t)$ and $y_2 = A\sin(kx + \omega t)$ travelling on a string in opposite directions. Mass density of the string is ρ and area of cross-section is s. Find the total mechanical energy between two adjacent nodes on the string.

Solution :

The distance between two adjacent nodes is $\frac{\lambda}{2}$ or $\frac{\pi}{k}$.

∴ Volume of string between two nodes will be

V = (area of cross-section) (distance between two nodes)

$= (s)\left(\frac{\pi}{k}\right)$. Energy density (energy per unit volume) of a travelling wave is given by $u = \frac{1}{2}\rho A^2\omega^2$.

A standing wave is formed by two idential waves travelling in oposite directions. Therefore, the energy stored between two nodes in a standing wave

E = 2[energy stored in a distance of $\frac{\pi}{k}$ of travelling wave] = 2 (energy density) (volume)

$$= 2\left(\frac{1}{2}\rho A^2\omega^2\right)\left(\frac{\pi s}{k}\right) \text{ or } E = \frac{\rho A^2\omega^2\pi s}{k}$$

1.20 FORMATION OF WAVES ON A STRETCHED STRING

A stretched string fixed at both the ends vibrates transversely, when plucked perpendicular to its length. The transverse pulses travel either way along the length of the string and get reflected at the fixed ends. These reflected waves, travelling in opposite directions with the same amplitude and frequency overlap along the length of the string. The resultant wave is known as the standing wave or stationary wave. It can be analytically obtained from the principle of superposition of waves.

Let incident and reflected transverse progressive waves with same amplitude (A), wavelength (λ) and frequency (n) travel in opposite direction along the stretched string.

Let $y_i = A\sin(kx - \omega t)$ and
$y_r = A\sin(kx + \omega t + \pi)$,

where k is known as propagation constant.

According to the principle of super position.

$y = y_i + y_r$

$\therefore\ y = 2A\sin kx\cos\omega t$ (or) $y = A_s\cos\omega t$

where $A_s = 2A\sin kx$ represents amplitude of particles in stationary wave. The string will vibrate in such a way that fixed or clamped points of the string are nodes, as the string at these points is not free to move. The point of plucking or free end is an antinode as here displacement will be maximum as shown in Fig 1.31 (a) and (b).

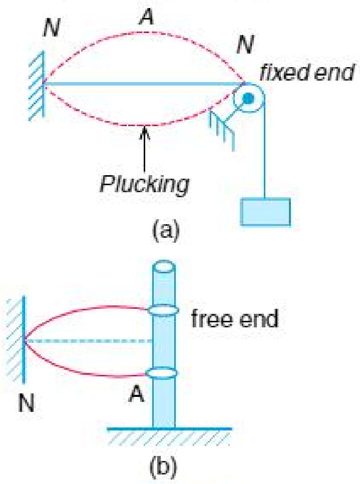


Fig 1.31

Table 1.1 Comparison between travelling wave and stationary wave

S.No	Travelling waves	Stationary waves
1.	These waves advance in a medium with definite velocity.	These waves remain stationary between two boundaries in the medium.
2.	In these waves, all particles of the medium oscillate with same frequency and amplitude	In these waves, all particles except nodes oscillate with same frequency but different amplitudes. Amplitude is zero at nodes and maximum at antinodes.
3.	At any instant phase of vibration varies continuously from one particle to the other i.e, phase difference between two particles can have any value between 0 and 2π.	At any instant the phase of all particles between two successive nodes is the same, but phase of particles on one side of a node is opposite to the phase of particles on the other side of the node i.e., phase difference between any two particles can be either zero or π
4.	In these waves, at no instant all the particles of the medium pass through their mean positions simultaneously.	In these waves all particles of the medium pass through their mean positions simultaneously twice in each time period.
5.	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.

a) String fixed at both ends

A stretched string of length l is clamped between two points. It may vibrate in the form of one or more number of segments (or loops) which are called normal modes. These modes of vibration are known as harmonics. The wavelength associated with the standing waves can take on many different values and it depends on number of harmonics. The distance between adjacent nodes is $\lambda/2$, so that in a string fixed at both ends there must be exactly an integral number 'n' of half wavelengths ($\lambda/2$), as shown in Fig 1.32.

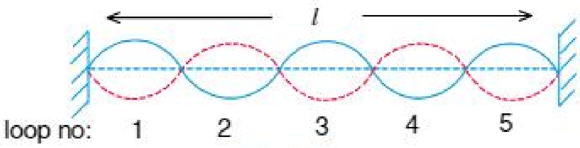


Fig 1.32

i.e., $n(\lambda/2) = l$

(or) $\lambda = \dfrac{2l}{n}$, where $n = 1, 2, 3$ ....

But $v = f\lambda$ and $v = \sqrt{\dfrac{T}{\mu}}$, so that the natural frequencies of oscillation of the string are

$$f = n\left(\frac{v}{2l}\right) \text{ (or) } f = \frac{n}{2l}\sqrt{\frac{T}{\mu}},\ n = 1, 2, 3, \ldots\ldots,$$

where μ is the linear density of the string and T is the tension in it.

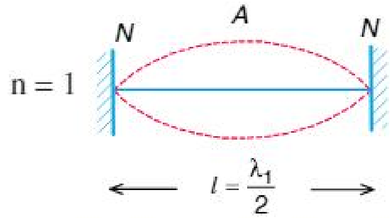
If the string vibrate as one segment $(n = 1)$ as shown in Fig 1.33(a), there is smallest frequency f_1 corresponding to the largest wavelength $\lambda_1 = 2l$

$$\therefore f_1 = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

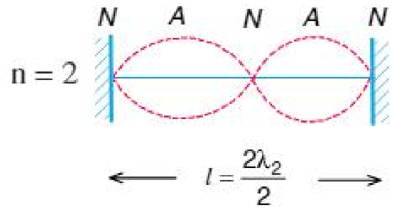
This is known as fundamental frequency (or) first harmonic. In this mode an antinode is formed at the middle of the string and two nodes are formed at the two ends.

If the string vibrate as two segments $(n = 2)$ as shown Fig 1.33(b) the resonant frequency.

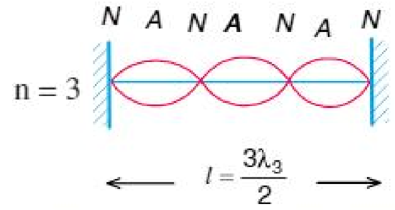
$$f_2 = 2\left(\frac{v}{2l}\right) = \frac{2}{2l}\sqrt{\frac{T}{\mu}} \qquad \because\ f_2 = 2f_1$$



(a) First harmonic (or) Fundamental mode



(b) Second harmonic (or) First over tone



(c) Third harmonic (or) Second overtone

Fig 1.33

This frequency is called as second harmonic or first overtone. The other standing wave frequencies are

$$f_3 = 3\left(\frac{v}{2l}\right) = \frac{3}{2l}\sqrt{\frac{T}{\mu}}$$

$f_3 = 3\ f_1$ and so on. The frequencies which are integral multiples of the fundamental frequency are called harmonics. All possible higher frequencies other than fundamental are called overtones.

If the string vibrates in n^{th} harmonic, its frequency will be n f_1, the number of loops or antinodes will be n, while total number of nodes (n+1). The mode of vibration is taken to be fundamental unless stated otherwise. In the string, the position of plucking is antinode. Depending on that position, mode of vibration changes.

b) Vibration of a string fixed at one end

Standing waves can be produced on a string which is fixed at one end and whose other end is free to move in a transverse direction. Such a free end can be nearly achieved by connecting the string to a very light thread. At fixed end there is node and at free end antinode always. The wavelength associated with the standing wave can take on many different values.

The distance between node and adjacent antinode is $\frac{\lambda}{4}$, so that in a string of length 'l' fixed at one end and the other is free, there must be exactly an odd integral number 'n' of quarterly wavelengths as shown in Fig 1.34.

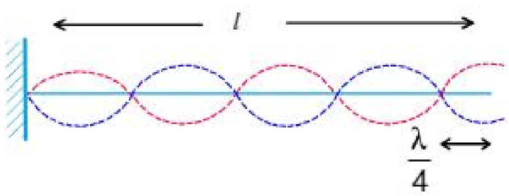


Fig 1.34

i.e., $n.\left(\frac{\lambda}{4}\right) = l$ (or) $\lambda = \frac{4l}{n}$, where n=1,3,5,7, ...

But $v = f\lambda$ and $v = \sqrt{\frac{T}{\mu}}$,

So that the natural frequencies of oscillations of the string are $f = n.\left(\frac{v}{4l}\right)$ (or) $f = \frac{n}{4l}\sqrt{\frac{T}{\mu}}$, n = 1, 3, 5, 7, ... where μ is linear density of the string and 'T' is the tension in the string.

If the string vibrates as one half loop (n = 1), there is smallest frequency f_1 corresponding to the largest wavelength $\lambda_1 = 4l$

$$\therefore f_1 = \frac{1}{4l}\sqrt{\frac{T}{\mu}}$$

This is known as fundamental frequency (or) first harmonic.

If the string vibrate as three half loops, then frequency $f_2 = 3f_1$

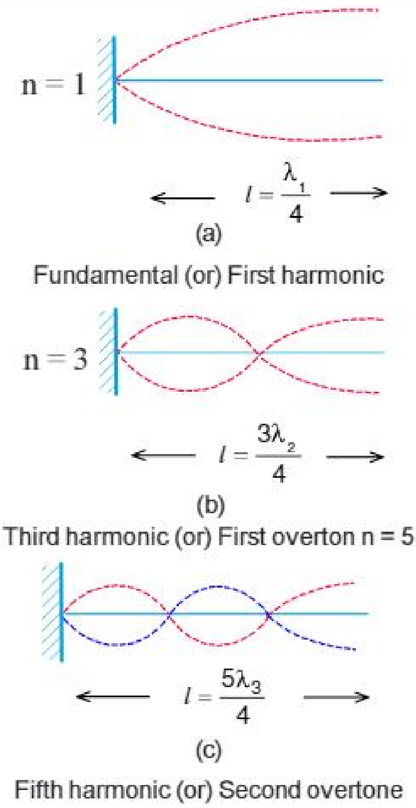


Fig 1.35

This frequency is called Third harmonic (or) first overtone frequency. The other standing wave frequencies are $f_3 = 5\,f_1$ and so on.

c) Vibration of composite string

Under vibrations of composite string (string made up by joining two strings of different lengths, cross sections and densities) having same tension through out, the joint is a node or antinode while lowest common fundamental frequency of the string will be $f_c = n_1f_1 = n_2f_2$.

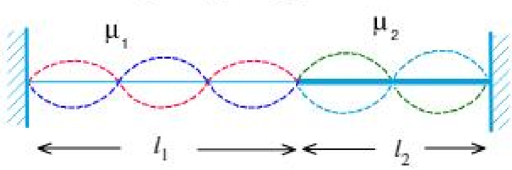


Fig 1.36

where f_1 and f_2 are the individual fundamental frequencies of strings 1 and 2 respectively under the same tension as that of composite string. The higher harmonic frequencies will be integral multiple of common frequency f_c.

1.21 LAWS OF TRANSVERSE VIBRATIONS IN A STRETCHED STRING

When a string is fixed at both ends, its fundamental frequency is $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$, We can state three laws of transverse vibrations as follow:

(i) First law or law of lengths

The fundamental frequency of a vibrating string is inversely proportional to the length (l) of the string, when the tension (T) in the string and its linear density (μ) are constant.

i.e., $f \propto \frac{1}{l}$, if T and μ are constant

(or) $f\,l$ = constant (or) $\frac{f_1}{f_2} = \frac{l_2}{l_1}$

(ii) Second law (or) Law of tensions

The fundamental frequency of a vibrating string is directly proportional to the square root of the tension (T), when the length of the string (l) and its linear density are constant.

i.e., $f \propto \sqrt{T}$, if l and μ are constant (or)

$\frac{f}{\sqrt{T}}$ = constant (or) $\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$

(iii) Third law or Law of linear density

The fundamental frequency of vibrating string is inversely proportional to square root of linear density, when the length of the string (l) and tension (T) are constant.

i.e., $f \propto \frac{1}{\sqrt{\mu}}$, if l and T are constant

(or) $f\sqrt{\mu}$ = constant (or) $\frac{f_1}{f_2} = \sqrt{\frac{\mu_2}{\mu_1}}$

According to these the frequency of a string can be changed by changing its length, tension or linear mass density. These three laws can be verified by sonometer experiment.

Example-1.13

A sonometer wire has a length of 114 cm between two fixed ends. Where should two bridges be placed to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 3 : 4 ?

Solution :

In case of a given wire under specific tension, fundamental frequency of vibration $f \propto (1/L)$. So for having fundamental frequencies in the ratio of 1 : 3: 4, the vibrating length should be in the ratio 1 : (1/3) : (1/4),

i.e., $L_1 : L_2 : L_3 :: 12 : 4 : 3$common factor is x, $12x + 4x + 3x = 114$ = length of the string $19x = 114$ or $x = 6$

$\Rightarrow L_1 = 12\times6 = 72\text{ cm}, L_2 = 4\times6 = 24\text{ cm}, L_3 = 3\times6 = 18\text{ cm}$

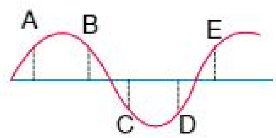
Example-1.14

A string 120 cm in length sustains a standing wave, with the points of string at which the displacement amplitude is equal to $\sqrt{2}$ mm being separated by 15.0 cm, Find the maximum displacement amplitude. Also find the harmonic corresponding to this wave.

Solution :

From figure. points A, B, C, D and E are having equal displacement amplitude.

Further, $x_E - x_A = \lambda = 4\times15 = 60\text{cm}$



As $\lambda = \frac{2l}{n} = \frac{2\times120}{n} = 60$

$\therefore\ n = \frac{2\times120}{60} = 4$

So, it corresponds to 4th harmonic.

Also, distance of node from A is 7.5 cm and no node is between them. Taking node at origin, the amplitude of stationary wave can be written as, $a = A\sin kx$

Here $a = \sqrt{2}\text{mm}; k = \frac{2\pi}{\lambda} = \frac{2\pi}{60}$ and $x = 7.5$ cm

$\therefore\ \sqrt{2} = A\sin\left(\frac{2\pi}{60}\times7.5\right) = A\sin\frac{\pi}{4}$

Hence, A = 2 mm

Example-1.15

An aluminium wire of cross-sectional area 10^{-6} m^2 is joined to a copper wire of the same cross-section. This compound wire is stretched on a sonometer, pulled by a load of 10 kg. The total length of the compound wire between two bridges is 1.5 m of which the aluminium wire is 0.6 m and the rest is the copper wire. Transverse vibrations are set up in the wire in the lowest frequency of excitation for which standing waves are formed such that the joint in the wire is a node. What is the total number of nodes observed at this frequncy excluding the two at the ends of the wire ? The density of aluminium is 2.6×10^4 kg/m^3.

Solution :

As the total length of the wire is 1.5 m and out of which $L_A = 0.6$ m, so the length of copper wire

$L_c = 1.5 - 0.6 = 0.9$ m. The tension in the whole wire is same (=Mg = 10g N) and as fundamental frequency of vibration of string is given by

$$f = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{T}{\rho A}} \quad [\text{as } m = \rho A\ ]$$

$$\text{So } f_A = \frac{1}{2L_A}\sqrt{\frac{T}{\rho_A A}} \text{ and } f_c = \frac{1}{2L_c}\sqrt{\frac{T}{\rho_c A}} \quad \text{...... (1)}$$

Now as in case of composite wire, the whole wire will vibrate with fundamental frequency

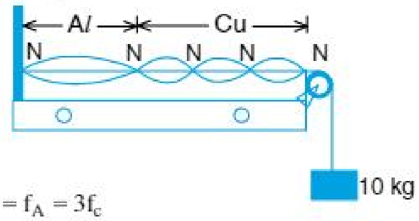
$$f = n_A f_A = n_c f_c \quad \text{.... (2)}$$

Substituting the values of f_A and f_c from Eqn.(1) in (2)

$$\frac{n_A}{2\times 0.6}\sqrt{\frac{T}{A\times 2.6\times 10^3}} = \frac{n_c}{2\times 0.9}\sqrt{\frac{T}{A\times 1.0401\times 10^4}}$$

$$\text{i.e., } \frac{n_A}{n_c} = \frac{2}{3}\sqrt{\frac{2.6}{10.4}} = \frac{2}{3}\times\frac{1}{2} = \frac{1}{3}$$

So that for fundamental frequency of composite string, $n_A = 1$ and $n_c = 3$, i.e., aluminium string will vibrate in first harmonic and copper wire at second, overtone as shown in figure.



$$\therefore\ f = f_A = 3f_c$$

This in turn implies that total number of nodes in the string will be 5 and so number of nodes excluding the nodes at the ends = 5 – 2 = 3, and

$$f = f_A = \frac{2}{2\times 0.6}\sqrt{\frac{10\times 9.8}{10^{-6}\times 2.6\times 10^3}} = 161.8\,\text{Hz}(= 3f_c)$$

1.22 TUNING FORK

It is a U shaped metal bar made of steel or an alloy with a handle attached at the bend. When it is struck against a hard rubber pad, its prongs begin to vibrate as shown in Fig 1.37(a).

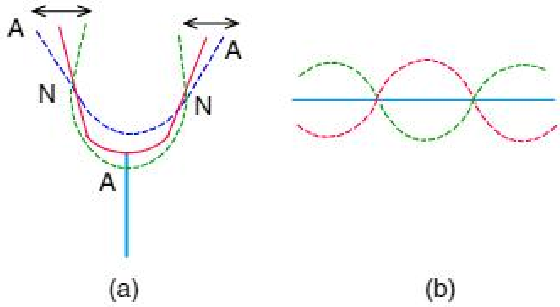


Fig 1.37

A tuning fork emits a single frequency note, that is, a fundamental with no overtones.

A tuning fork may be considered as a vibrating free bar as shown Fig 1.37(b) that has been bent into U-shape. Two antinodes are formed one at each free end of the bar which are in phase.

The frequency of a tuning fork of arm length 'l' and thickness 'd' in the direction of vibration is given by

$$f = \frac{d}{l^2}v = \frac{d}{l^2}\sqrt{\frac{Y}{\rho}},\ \left[\text{as } v = \sqrt{\frac{Y}{\rho}}\right]$$

where Y is the Young's modulus and ρ is the density of the material of the turning fork. Using the tuning fork we can produce transverse waves in solids and longitudinal waves in solids, liquids and gases. Transverse vibrations are present in the prongs. Longitudinal vibrations are present in the shank. Further more loading or waxing a tuning fork increases its inertia and so decrease its frequency, while filing a tuning fork decreases its inertia and so increases its frequency. When tuning fork is heated its frequency decreases due to decrease in elasticity.

1.23 TYPES OF VIBRATION

a) Free Vibrations

If an elastic body is excited and left to it self, it vibtates with definite constant amplitude and definite single frequency known as its natural frequency. Such vibrations are known as free or natural vibrations.

Analytically $F_{net} = -Ky$, with solution $y = A\sin(\omega_0 t + \phi)$ where ω_0 is the natural angular frequency

e.g. pendulum and tuning fork oscillation in vacuum

The natural frequency of a body depends on the dimensions of the body, its elastic properties and mode of vibration.

b) Damped Vibrations

If a body is vibrating with decreasing amplitude, due to the resistance of the surrounding medium, such oscillations are said to be damped vibrations. The amplitude gradually decreases and oscillations die finally.

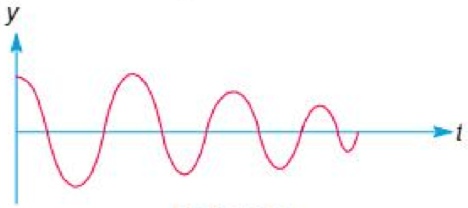


Fig 1.38(a)

e.g : when bob of a pendulum is pulled to a side and left free, it vibrates with its amplitude decreasing gradually due to air resistance (dissipative force).

Here the energy of vibrating body is utilised in overcoming the resistance. If we assume that dissipative force is non-periodic and varies linearly with velocity of the object. The analytical equation is $F_{net} = ky - \gamma v$, where y is displacement from mean position and γ is called damping constant, that depends on characteristic of dissipative medium.

The solution is found to be of the form

$$y = Ae^{\frac{\gamma t}{2m}} \cos(\omega_d t + \phi)$$

where A is the amplitude at time t = 0 and ω_d is the angular frequency of the damped oscillator given by $\omega_d = \sqrt{\frac{K}{m} - \frac{\gamma^2}{4m^2}}$.

From the expression of ω_d, it is clear that frequency of this body is slightly less than natural frequency i.e., $(\omega_d < \omega_0)$. The mechanical energyof the body after time 't'.

$$E = E_0 e^{-\frac{\gamma}{m}t}, \text{ with } E_0 = \frac{1}{2}KA^2$$

If the energy lost due to damping is supplemented by an external periodic force, sustained oscillations can be obtained. Such oscillations are known as maintained oscillations.

c) Forced Vibrations

If a body is made to vibrate under the influence of an external periodic force, such that it vibrates with the frequency of the periodic force impressed on it, such oscillations are known as forced vibrations. The natural frequency of the vibrating body need not be equal to the frequency of the periodic force. The amplitude of vibration is finite and constant. It depends on frequency of applied force, body and damping. Lesser the difference in frequencies and lesser the damping greater will be the amplitude of vibration. The resultant displacement of the body is not in phase (lead or lag) with the applied force.

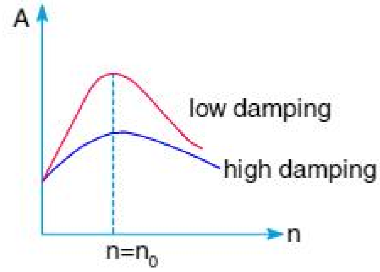


Fig 1.38(b)

Suppose an external force varying periodically with time is applied to a damped oscillator as $F_e = F_0 \cos\omega_e t$ where ω_e = angular frequency of external force. Analytically $F_{net} = -ky - \gamma v + F_e$

The solution is found to be of the form $y = A\cos(\omega_e t \pm \phi)$

The amplitude 'A' is a function of the external periodic force frequency and the natural frequency ω_0

$$\therefore A = \frac{F_0}{\left[m^2(\omega^2{}_0 - \omega_e{}^2)^2 + \omega_e{}^2\left(\frac{\gamma}{m}\right)^2\right]^{1/2}}$$

$$\text{and } \operatorname{Tan}\phi = \frac{\gamma\omega_e}{m(\omega_0{}^2 - \omega_e{}^2)}$$

From this equation it is clear that the resultant displacement lead, when $\omega_0 > \omega_e$ and lag when $\omega_0 < \omega_e$.

eg : When the stem of a vibrating tuning fork is held on the top of a table, a much louder sound is heard due to forced vibration. Here frequency of the table is equal to frequency of the tuning fork.

When the string of veena or violin is plucked and released to vibrate, the air in the sounding box (hollow box) sets into forced vibrations.

The forced vibrations cease when the applied periodic force is removed.

Note : However, if damping is absent $(\gamma = 0)$

$$A = \frac{F_0}{m(\omega_0{}^2 - \omega_e{}^2)} \text{ and } \phi = 0$$

d) Resonance

In the absence of damping if a body is made to vibrate under the influence of an external periodic force and if the natural frequency of that body coincides with the frequency of the periodic force on it, that body vibrates with increasing amplitude. "This phenomenon is known as "resonance".

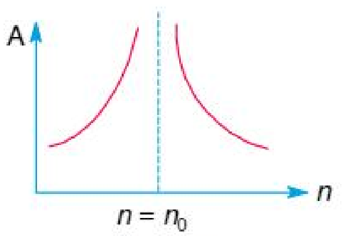


Fig 1.38(c)

Resonance is a special case of forced vibrations with $\gamma = 0$ and $\omega_0 = \omega_e$. Theoretically at resonance amplitude of vibration is infinite. Resonant forced vibrations are also known as sympathetic vibrations.

The following examples explain about resonance.

e.g (i) Consider a cylindrical vessel filled with water. A tuning fork is excited and kept above the open end of the vessel with the prongs parallel to free surface of water.

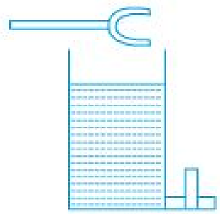


Fig 1.38(d)

If the level of water is gradually decreased, at one stage loud sound can be heard. The reason is, at that instant, air column of that particular length resonates with the tuning fork. Using this, speed of sound in air can be found.

e.g (ii) Resonance is not desired sometimes. When a band of soldiers are marching on a bridge,they are asked to go out of step. If the solides march in step, the natural frequency of the bridge may be equal to the frequency of the footsteps of the soldiers. Due to resonance, the amplitude of vibrations of the bridge may increase enormously. Then, bridge may vibrate violently and collapse.

1.24 DIFFERENT FORMS OF LONGITUDINAL WAVE

A sound wave is a longitudinal mechanical wave. As we know, during a longitudinal wave propagation the particles of the medium oscillate to produce pressure and density variation along the direction of the wave. These variations result in series of high and low pressure (and density) regions called compression and rarefactions respectively. Hence the longitudinal wave can be in terms of displacement of particles called displacement wave y(x, t) or in terms of change in pressure called pressure wave $\Delta P(x,t)$ or change in density called density wave $\Delta d(x,t)$. In a sinusoidal sound wave, the pressure fluctuates above and below the normal pressure of the medium with the same frequency as the motion of the particles. But this pressure fluctuation in the medium is very small compared to normal pressure of the medium. Similarly the density of the medium also vibrates sinusoidally above and below its normal level.

(a) Pressure wave form

Assume a harmonic displacement sound wave $y = A\sin(kx - \omega t)$ travelling in the x-direction in a medium of normal pressure P_0 and density d_0.

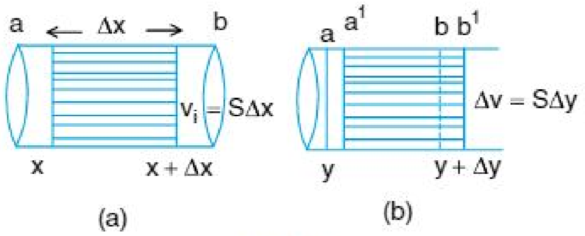


Fig 1.39

Now consider an element of medium which is confined within x and x+ Δx in the undistributed state as shown in Fig 1.39(a). If 'S' is the cross section, the volume of element in undistributed state will be $v_i = S\,\Delta x$.

As the wave passes, the ends at x and $x + \Delta x$ are displaced by amounts y and $y + \Delta y$ as shown in Fig 1.39(b). So that change in volume of the element will be $\Delta v = S\Delta y$. The volume strain for the element is

$$\frac{\Delta v}{v_i} = \frac{S.\Delta y}{S.\Delta x} = \frac{\Delta y}{\Delta x} \qquad .... (1)$$

If 'B' is the Bulk modulus of the medium, from the definition of Bulk modulus, the pressure variation in the medium is

$$\Delta P = -B.\frac{\Delta v}{v_i} \qquad .... (2)$$

$\left(\because B = -\Delta P.\frac{v_i}{\Delta v}\right)$ from (1) and (2)

(or) $\Delta P = -B\left(\frac{\Delta y}{\Delta x}\right) \approx -B\left(\frac{dy}{dx}\right)$

$$\Delta P = -BAk\cos(kx - \omega t)$$

$$\therefore \Delta P = -(\Delta P)_{max}\cos(kx - \omega t),$$

with $(\Delta P)_{max} = BAk$ is the amplitude of pressure variation or pressure amplitude.

From the above we see that pressure wave is 90^o out of phase (lags) with respect to the displacement wave. When the displacement is zero, the pressure variation is either maximum or minimum and vice versa as shown in Fig 1.40.

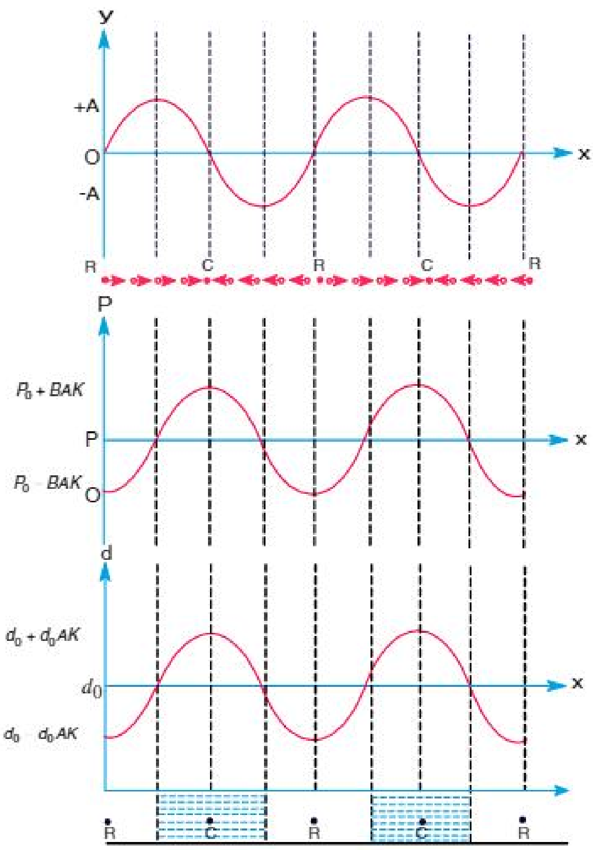


Fig 1.40

From the Fig 1.40 it is clear that

(i) Displacement of the particles is zero either at centre of compression or rarefaction and maximum at boundary of compression or rarefaction.

(ii) In region of compression pressure is above the normal pressure of the medium and is maximum (P_0+BAK) at centre of compression.

In the region of rarefaction, pressure is below the normal pressure of the medium and is minimum $(P_0 - BAK)$ at centre of rarefaction.

(iii) At the boundary of compression and rarefaction pressure is normal pressure of the medium.

(iv) At the centre of compression and rarefaction particle velocity is maximum and at the boundary of compression and rarefaction particles are momentarily of rest. This is explained as in a harmonic progressive wave $V_P = -(\text{slope of } y-x)v$

Since the change in pressure of the medium

$$\Delta P = -B\left(\frac{dy}{dx}\right) \text{ (or) } \Delta P = B\left(\frac{V_P}{v}\right)$$

i.e., change in pressure of the medium is equal to the product of Bulk modulus and ratio of particle speed to wave speed. for a given medium, B and v are constants. Where V_p is maximum, Δp is also maximum, which is true at $y = 0$

> ***Note:*** As sound sensors (e.g ear or mike) detect pressure changes, description of sound as pressure wave is preferred over displacement wave.

(b) Density wave form

Let d_0 be the normal density of the medium and Δd be the change in density of the medium during the wave propagation.

Then fraction of change in volume of the element $\frac{\Delta V}{V} = -\frac{\Delta d}{d_0}$ (since $m = Vd$)

According to definition of Bulk's modulus

$$B = -\Delta P\left(\frac{V}{\Delta V}\right) = -\Delta P\left(\frac{d_0}{\Delta d}\right)$$

$$\text{(or)} \quad \Delta d = -\frac{d_0}{B}.\Delta p$$

$$\text{(or)} \quad \Delta d = -\frac{d_0}{B}(\Delta p)_{max}\text{Cos}(kx - \omega t)$$

$$\text{(or)} \quad \Delta d = -d_0 Ak\ \text{Cos}(kx - \omega t)$$

$$(\therefore \Delta p_{max} = BAk)$$

$$\text{(or)} \quad \Delta d = -(\Delta d)_{max}\ \text{Cos}(kx - \omega t),$$

where $(\Delta d)_{max} = d_0 Ak$

is called density amplitude. Thus the density wave is in phase with the pressure wave and this is 90^0 out of phase (lags) with the displacement wave as shown in the Fig 1.40.

> ***Note :*** The relation between density amplitude and pressure amplitude is $(\Delta d)_{max} = (\Delta p)_{max}\left(\frac{d}{B}\right)$

1.25 SPEED OF LONGITUDINAL WAVE IN A MEDIUM (LIQUID OR GAS)

Consider again a longitudinal wave going in x - direction in a fluid, i.e., $y = A\sin(kx - \omega t)$. Let B and d be the Bulk modulus and normal density of the fluid.

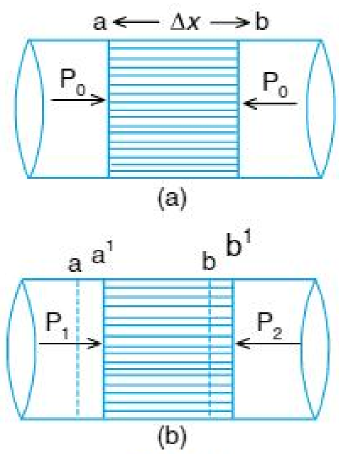


Fig 1.41

Consider the elemental medium of length Δx in the undistributed state as shown in Fig 1.41(a). If S is the area of the cross section, the volume of the element is $S\Delta x$ and mass is $(S\Delta x)d$. The element is in equilibrium with the pressure P_0.

Because of the disturbance, let the pressure difference across the element be $(P_1 - P_2)$ as shown in Fig 1.41(b)

$\therefore$ The net force on the element is

$F = (P_1 - P_2)S = ma$

$\therefore (P_1 - P_2)S = d(S\Delta x)a$

$$\text{(or)} \quad \frac{\partial p}{\partial x} = d\frac{\partial^2 y}{\partial t^2} \qquad ....(1)$$

If Δy is the change in length of the element, then

$$\frac{\Delta V}{V}=\frac{\Delta y}{\Delta x} \quad ....(2)$$

By the definition of Bulk modulus the excess pressure ΔP may be written as

$$\Delta P=-B.\frac{\Delta y}{\Delta x} \quad ....(3)$$

From the equations 1, 2, and 3

$$\frac{\partial^2 y}{\partial x^2}=\frac{d}{B}.\frac{\partial^2 y}{\partial t^2} \text{ (or) } \frac{\partial^2 y}{\partial t^2}=\frac{B}{d}\frac{\partial^2 y}{\partial x^2} \quad ....(4)$$

Comparing the equation (4) with the wave equation.

$$\frac{\partial^2 y}{\partial t^2}=v^2\frac{\partial^2 y}{\partial x^2} \quad ....(5)$$

We have $v=\sqrt{\frac{B}{d}}$ ....(6)

This is the speed of longitudinal wave in a gas or a liquid. In case of gases, the Bulk modulus of a given gas is not constant. It depends on the thermodynamic process.

In adiabatic process $B=\gamma P$, where γ is the ratio of molar heat capacities Cp/Cv and 'P' is the pressure of the gas. Thus, $v=\sqrt{\frac{\gamma p}{d}}$

$$=\sqrt{\frac{\gamma RT}{M}} \quad \left(\because p=\frac{dRT}{M}\right)$$

According to Laplace, sound propagation through the gas is an adiabatic process, which is in good agreement with the experimental values.

In isothermal process B = P, then

$$v=\sqrt{\frac{P}{d}}=\sqrt{\frac{RT}{M}}$$

According to Newton, sound propagates through the gas under isothermal condition, But this is not in agreement with the experimental values. Velocity of sound in air is measured by resonance tube, while in gases by Quinke's tube. Kundt's tube is used to determine velsa'm of sound in solid and gases. The factors affecting the speed of longitudinal wave in gas are

(i) The speed of wave depends on the nature of the gas lighter the gas greater will be the velocity.

i.e., $v\alpha\frac{1}{\sqrt{M}}$, when γ and T are constant

e.g. $v_{Hydrogen}>v_{Oxygen}$

(ii) For a given gas, with rise in temperature, speed of wave in the gas increases

i.e., $v\alpha\sqrt{T}$, when γ and M are constant

(iii) Since moist air has lower density than dry air, the speed of sound in moist air is more than that in dry air. This is because $v\alpha\frac{1}{\sqrt{d}}$

iv) Speed of longitudinal wave is independent of pressure at constant temperature. This is because $\frac{P}{d}$ is a constant at constant temperature.

Velocity of longitudinal wave is maximum in solids and minimum in gases.

i.e., $v_{solid}>v_{liquid}>v_{gases}$

Example-1.16

Determine the speed of sound waves in water, and find the wavelength of a wave having a frequency of 242 Hz. Take $B_{water}=2\times10^9$ Pa.

Solution :

Speed of sound wave,

$$v=\sqrt{\frac{B}{\rho}}=\sqrt{\frac{(2\times10^9)}{10^3}}=1414 m/s$$

Wavelength $\lambda=\frac{v}{f}=5.84m$

1.26 ENERGY, POWER AND INTENSITY OF SOUND WAVE

When a sound wave travels through a medium, it will set particles of the medium into vibrations as it pass through them. For this to happen the medium must posses both inertia (i.e., mass, density) and elasticity, so that kinetic energy and potential energy can be stored.

Energy density $u=\frac{1}{2}d\omega^2A^2$

Average power $P = \frac{1}{2} d\omega^2 A^2 S v$

Average Intensity $I = \frac{P}{S} = \frac{1}{2} d\omega^2 A^2 v$

In terms of pressure amplitude, sound intensity

$$I = \frac{1}{2} d\omega^2 \left(\frac{\Delta p_{max}}{Bk} \right)^2 v \text{ (or) } I = \frac{1}{2} \frac{(\Delta P)^2_{max}}{dv}$$

$$[\because (\Delta P)_{max} = BAk, k = \frac{\omega}{v} \text{ and } B = dv^2]$$

Above equations explain that intensity of wave is proportional to square of pressure amplitude or displacement amplitude or density amplitude and is independent of frequency. The S.I. unit of intensity is Watt/m^2.

1.27 INTERFERENCE OF WAVES

When two waves of same frequency which are in phase or have a constant phase difference at the time of generation, arrive at a point simultaneously almost along the same line, the resultant intensity is found to be different from the sum of intensities due to each wave. This redistribution of intensity is called interference and this is based on law of conservation of energy.

Two sources of wave which are always in phase or have a constant phase difference at the time of generation are called coherent sources. For the two sources to be coherent their frequencies must be same. But the converse is not always true. Two independent sources of sound producing same frequency can be coherent but not so in light. If the phase difference of the sources changes erratically with the time, even if they have the same frequency, the sources are said to be incoherent.

In interference at some points the waves superpose in such way that the resultant intensity is greater than the sum of the intensities due to separate waves i.e., $I_r > I_1 + I_2$. This is said be constructive interference. At some other points the resultant intensity is less than the sum of the separate intensities i.e., $I_r < I_1 + I_2$. This is said to be destructive interference.

Consider to two coherent harmonic waves of amplitude A_1 and A_2 with a phase difference ϕ at the time of superposition.

$$y_1 = A_1 \sin(kx - \omega t); \quad y_2 = A_2 \sin(kx - \omega t + \phi)$$

According to the principle of superposition, the resultant wave is represented by $y = y_1 + y_2$. The addition of the two waves can be understood in terms of the phase diagram, as shown in the Fig 1.44.

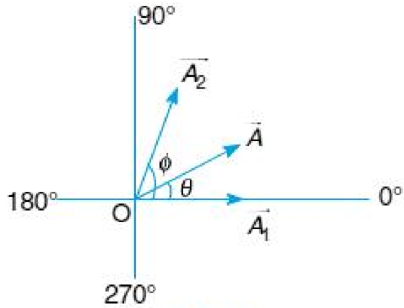


Fig 1.44

The resultant wave is also a travelling wave and is given by $y = A\sin(kx - \omega t + \theta)$, where and

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi} \right)$$

Since, Intensity $\propto$ (amplitude)^2

The resultant intensity is given by

$$I_r = kA^2 = kA_1^2 + kA_2^2 + \sqrt{k}A_1.\sqrt{k}A_2\cos\phi$$

$$\text{(or) } I_r = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

where I_1 and I_2 are the intensities of the individual waves. Here we can see that the resultant amplitude or intensity of the medium particles after super-position depends on the amplitude of the component waves and on the phase difference 'ϕ' between the two component waves. Thus if the phase difference between the two waves changes at the point of superposition, the resultant amplitude of that medium particle also changes.

Note : The phase difference between two waves generated by the same source when they travel along paths of unequal length Δx is $\phi = \frac{2\pi}{\lambda}\Delta x$, λ is the wavelength of each wave.

(a) Constructive interference

The resultant amplitude (or intensity) is maximum when $\cos\phi = 1$ or

$\phi = 2n\pi, n = 0,1,2,3, ...$

i.e., $\phi = 0, 2\pi, 4\pi, 6\pi........$

Using $\phi = \frac{2\pi}{\lambda}\Delta x$, the above condition may be written in terms of the path difference as

$\Delta x = n\lambda$, n = 0, 1, 2, 3, ...

i.e., $\Delta x = 0, \lambda, 2\lambda, 3\lambda, ...$

Thus waves interfere constructively when the path difference between them while reaching a point is an integral multiple of λ.

Then $A_{max} = A_1 + A_2$ and $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

In transverse waves the interference is constructive when the crest of one wave falls over the crest of another wave or trough of one wave falls on through of another wave. This is shown in the Fig 1.45. In longitudinal wave, the interference is constructive when the compression of one wave coincide with the compression of another wave or rarefaction coincides with rarefaction of the other. If the given wave is sound the maximum sound is heard.

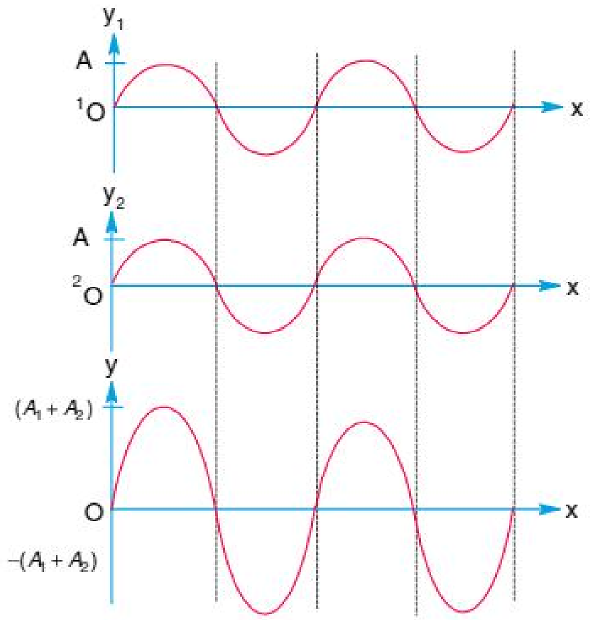


Fig 1.45

(i) For n = 0, $\Delta x = 0$ the maximum obtained is called zero order maxima.

(ii) For n = 1, $\Delta x = \lambda$ the maxima obtained is called first order maxima.

(iii) For n = 2, $\Delta x = 2\lambda$ the maxima is called second order maxima and so on

(b) Destructive interference

The resultant amplitude (or intensity) is minimum when $\cos\phi = -1$

(or) $\phi = (2n-1)\pi$, where n = 1, 2, 3, ...

(or) $\phi = \pi, 3\pi, 5\pi, 7\pi........$

In terms of path difference $\Delta x = (2n-1)\frac{\lambda}{2}$, n = 1,2,3, ...

i.e., $\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}.......$

Thus the waves interfere destructively when the path difference between them while reaching a point is odd integral multiple of $\frac{\lambda}{2}$.

Then $A_{min} = A_1 - A_2$ and $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

In transverse waves, the interference is destructive when the crest of one wave fall over the trough of another wave. This is shown in Fig 1.46.

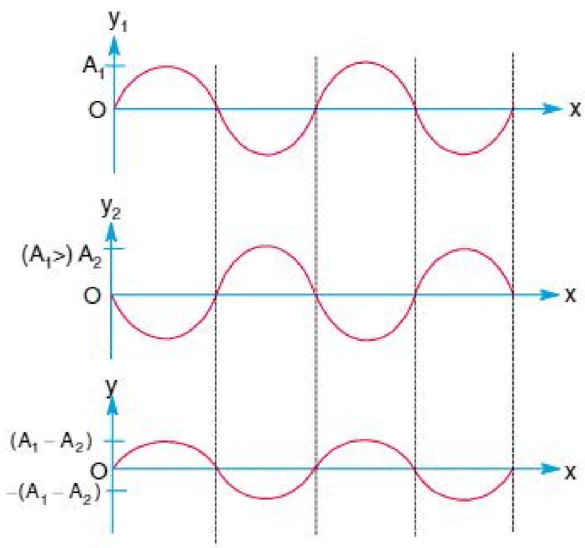


Fig 1.46

In longitudinal wave, the interference is destructive when the compression of one wave coincide with the rarefaction of another wave. If the given waves are sound minimum sound is heard.

(i) For n = 1, $\Delta x = \frac{\lambda}{2}$, the minima obtained is called first order minima.

(ii) For n = 2, $\Delta x = \frac{3\lambda}{2}$, the minima obtained is called second order minima.

(iii) For n = 3, $\Delta x = 5\frac{\lambda}{2}$ the minimum obtained is called third order minima and so on.

1.27.1 WHEN THE DETECTOR MOVES PARALLEL TO LINE JOINING TWO SOUND SOURCES

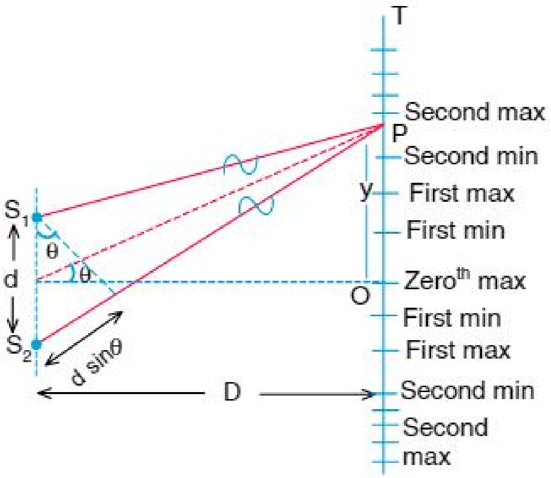


Fig 1.47

Consider a detector D moving along the line OT, which is parallel to line of sources S_1 and S_2. Let P be a point of superposition and θ be the angular position P at center of line joining sources.

The geometrical path difference between the two waves at P is $\Delta x = S_2P - S_1P = d\sin\theta$.

(a) For maxima (maximum sound) at P

$d\,\text{Sin}\,\theta = n\lambda$, n = 0, 1, 2, 3 ....

If 'θ'is small, $\text{Sin}\theta \approx \text{Tan}\theta = \frac{y}{D}$

$$d\left(\frac{y}{D}\right) = n\lambda \quad (\because d\sin\theta = n\lambda)\,(\text{or})\; y = \frac{n\lambda D}{d}$$

$\therefore$ position of n^{th} order maxima $y_n = \frac{n\lambda D}{d}$.

(i) It is observed that (i) Zeroth maxima is at O.

When the detector moves away from O the order of maxima increases as shown in the Fig 1.47. This is because $n = \frac{d}{\lambda}\sin\theta$, as θ increases n increases.

(ii) The spacing between two successive maxima positions is constant.

(iii) Since $\sin\theta = \frac{n\lambda}{d} \le 1$ (or) $n \le \frac{d}{\lambda}$

$\therefore$ The maximum order of maxima on one side to O is n.

Hence the total number of maxima that can be detected by detector is $(2n+1)$

(b) For minima (minimum sound) at P

$$d\sin\theta = (2n-1)\frac{\lambda}{2},\; n = 1, 2, 3....$$

Position of n^{th} order minimum

$$y_n = \frac{(2n-1)}{2}\frac{\lambda D}{d}$$

$$(\therefore \sin\theta \approx \tan\theta \approx \frac{y}{D}).$$

It is observed that in between two successive maxima, there will be a minima in equal spacing and order of minima increases as detector moves away from O.

1.27.2 WHEN THE DETECTOR MOVES PERPENDICULAR TO LINE JOINING TWO SOUND SOURCES

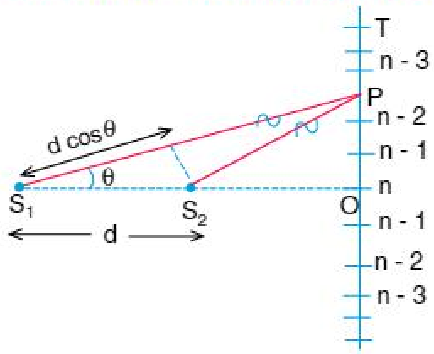


Fig 1.48

Consider a detector D moving along the line OT which is perpendicular to line of S_1 and S_2 sources.

The geometrical path difference between two waves on reaching 'P' is

$$\Delta x = S_1P - S_2P = d\cos\theta$$

(a) For maximum $d\cos\theta = n\lambda$,

$n = 0, 1, 2, 3$.............

(b) For minimum $d\cos\theta = (2n-1)\frac{\lambda}{2}$,

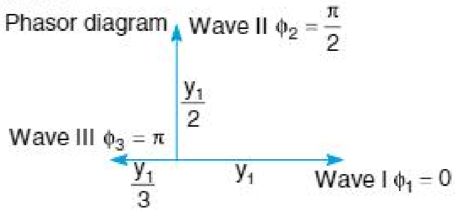
$n = 1, 2, 3$ ....

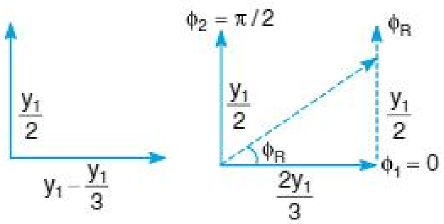
It observed that when the detector moves away from 'O' the order of maximum or minimum decreases. The spacing between two successive maxima or two successive minima is not constant. In this case zeroth maxima is obtained at infinity.

Example-1.17

Three sinusoidal waves of the same frequency travel along a string in the positive direction of x axis. Their amplitudes are $y_1, \frac{y_1}{2}$ and $\frac{y_1}{3}$, and their phase constants are $0, \frac{\pi}{2}$, and π respectively. What are (a) the amplitude and (b) the phase constant of the resultant wave?

Solution :





$$A_R = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = y_1\sqrt{\frac{4}{9}+\frac{1}{4}} = \frac{5}{6}y_1$$

$$\phi_R = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}(3/4) = 37^0$$

The phase constant of the resultant wave with respect to first wave is 37^0.

Example-1.18

Two sources of intensities I and 4I are used in interference experiment. Find the intensity at points where waves from the two sources superimpose with a phase erence of (a) zero (b) $\left(\frac{\pi}{2}\right)$ and (c) π .

Solution :

In case of interference of two waves of intensities I_1 and I_2 with phase difference ϕ

$$I_R = I_1 + I_2 + 2(\sqrt{I_1 I_2})\cos\phi$$

Here, $I_1 = I$ and $I_2 = 4I$,

$$I_R = I + 4I + 2\sqrt{4I \times I}\cos\phi = 5I + 4I\cos\phi$$

So (a) For $\phi = 0$, $I_R = 5I + 4I \times I = 9I$ [as $\cos 0 = 1$]

(b) For $\phi = (\pi/2), I = 5I$ [$\cos\ \phi = (\pi/2) = 0$]

(c) For $\phi = \pi$, $I = 5I + 4I(-1) = I$ [as $\cos(\pi) = -1$]

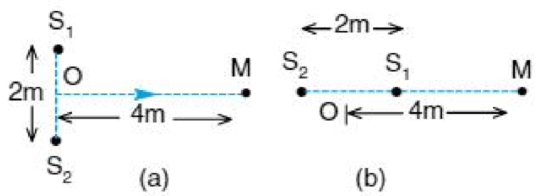
Example-1.19

Two speakers connected to the same source of frequency are placed 2.0 m apart in a box. A sensitive microphone placed at a distance of 4.0 m from their mid-point along the perpendicular bisector shows maximum response. The box is slowly rotated till the speakers are in line with the microphone. The distance between the mid-point of the speakers and the microphone remains unchanged. Exactly 5 maximum responses are observed in the microphone in the process. Calculate the wavelength of the sound wave.

Solution :

As shown in Figure (a)

initially $S_1M = S_2M$, $\Delta x = 0$., at M, there is zero order maxima.



According to the given problem on rotation of speakers about O when S_1 and S_2 are in line with microphone M as shown in figure (b),

5 maximum responses are observed.

i.e., $S_2M - S_1M = 5\lambda$ or $S_2S_1 = 5\lambda$

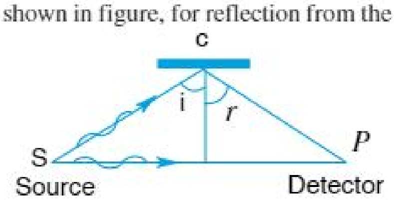
i.e., $\lambda = 2/5 = 0.4\text{m}$ [as $S_1S_2 = 2$ m]

Example-1.20

In a large room a person receives direct sound waves from a source 120 m away from him. He also receives waves from the same source which reach him, being reflected from the 25 m high ceiling at a point halfway between them. For which wavelengths will these two sound waves interfere constructively?

Solution :

As shown in figure, for reflection from the ceiling:



Path $SCP = SC + CP = 2SC$ [as $\angle i = \angle r, SC = CP$] or path $SCP = 2\sqrt{60^2 + 25^2} = 130m$

So path difference between interfering waves along paths SCP and SP,

$\Delta x = 130 - 120 = 10m$

Now for constructive interference at P, $\Delta x = n\lambda$,

i.e., $10 = n\lambda$ or $\lambda = \frac{10}{n}$ with $n = 1, 2, 3, ...$

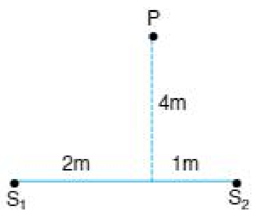
i.e., λ = 10 m, 5 m, (10/3) m and so on.

Example-1.21

Two sound sources S_1 and S_2 emit pure sinusoidal waves in phase. If the speed of sound is 350 m/s,

(a) For what frequencies does constructive interference occur at P?

(b) For what frequencies does destructive interference occur at P?



Solution :

Path difference $\Delta x = S_1P - S_2P$

$$= \sqrt{(2)^2 + (4)^2} - \sqrt{(1)^2 + (4)^2}$$
$$= 4.47 - 4.12 = 0.35 \text{ m}$$

(a) Constructive interference occurs when the path difference is an integer number of wavelength.

or $\Delta x = n\lambda = \frac{nv}{f}$ or $f = \frac{n(v)}{\Delta x}$ where $n = 1, 2, 3, ...$

$$\therefore f = \frac{350}{0.35}, \frac{2 \times 350}{0.35}, \frac{3 \times 350}{0.35}, ...$$

f = 1000Hz, 2000 Hz, 3000 Hz, ..., etc.

(b) Destructive interference occurs when the path difference is a half-integer number of wavelengths

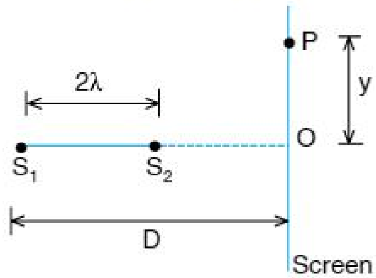
$$\Delta x = (2n+1)\frac{\lambda}{2} \quad n = 0, 1, 2, ...\text{(or)} \quad \Delta x = (2n+1)\frac{v}{2f}$$

$$\therefore f = \frac{(2n+1)v}{2\Delta x} = \frac{350}{2 \times 0.35}, \frac{3 \times 350}{2 \times 0.35}, \frac{5 \times 350}{2 \times 0.35}, ...$$

= 500 Hz, 1500 Hz, 2500 Hz

Example-1.22

Two coherent narrow slits emitting sound of wavelength λ in the same phase are placed parallel to each other at a small separation of 2λ. The sound is detected by moving a detector on the screen at a distance D ($>> \lambda$) from the slit S_1 as shown in figure. Find the distance y such that the intensity at P is equal to intensity at O.



Solution :

Let $\Delta x = \lambda$ at angle θ (see fig 10.58) Path difference between the waves is $\Delta x = 2\lambda \cos\theta$

$\therefore 2\lambda \cos\theta = \lambda$ ($\Delta x = \lambda$) or $\theta = 60^0$

Now, $PO = S_1O \cot 30^0$ or $y = \sqrt{3}$ D

1.28 BEATS

When two waves of slightly different frequencies with equal amplitudes travelling in same direction are superposed, the phenomenon of beats is observed. The resultant intensity at a given point of the wave becomes maximum and minimum periodically. This is called interference with time or beats.

The time interval between two successive maxima or minima is called the beat period.

The reciprocal of the beat period is called the beat frequency. The Fig 1.49 shows the displacement position graph of two waves of equal amplitude with nearly equal frequencies as observed by a stationary observer.

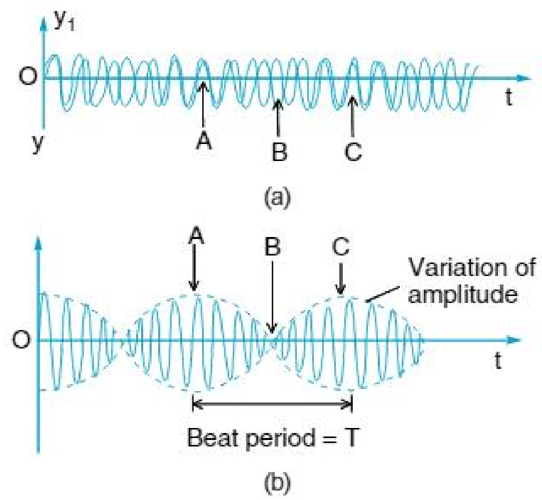


Fig 1.49

At an instant such as 'A' the waves from the source arrive in same phase, or the interference is constructive and the resultant amplitude is double the individual amplitude. But at some later time, because the frequencies are different, the waves will be out of phase or interference will be destructive and the resultant amplitude is zero. After certain time, once again maximum amplitude is obtained.

This type of process will repeat at every position of the wave in regular intervals of time. At given instant of time, some positions are with maximum amplitude and some positions are with minimum amplitude like A,B and C.

Consider two waves with same amplitude A but with slightly different frequencies f_1 and f_2 reaching a stationary observer position at x = 0.

$$y_1 = A\sin\omega_1 t = A\sin 2\pi f_1 t$$

$$y_2 = A\sin\omega_2 t = A\sin 2\pi f_2 t$$

The resultant wave is given by $y = y_1 + y_2$

$$y = 2A\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t\right]\sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t\right]$$

(or) $y = A_b \sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t\right]$,

where $A_b = 2A\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t\right]$ is called beat amplitude.

The resultant wave is a harmonic progressive wave of frequency $f = \frac{\omega_1+\omega_2}{2(2\pi)} = \frac{f_1+f_2}{2}$ and the amplitude at a given position varies periodically with frequency $f_A = \frac{\omega_1-\omega_2}{2(2\pi)} = \frac{f_1-f_2}{2}$

Since, Intensity $\propto$ (amplitude)^2, the resultant intensity $I_r = 4I_0\cos^2\left(\frac{\omega_1-\omega_2}{2}\right)t$, where I_0 is intensity of individual wave.

(a) For maximum intensity $\cos^2\left(\frac{\omega_1-\omega_2}{2}\right)t = 1$

(or) $\cos\left(\frac{\omega_1-\omega_2}{2}\right)t = \pm 1$

$\therefore \left(\frac{\omega_1-\omega_2}{2}\right)t = 0, \pi, 2\pi, 3\pi.......$

i.e., $t = 0, \frac{2\pi}{\omega_1-\omega_2}, \frac{4\pi}{\omega_1-\omega_2}, \frac{6\pi}{\omega_1-\omega_2}......$

$\therefore$ The time interval between two consecutive maxima is $\Delta t = \frac{2\pi}{\omega_1-\omega_2} = \frac{1}{f_1-f_2}$, known as beat period.

(b) For minimum intensity $\cos^2\left(\frac{\omega_1-\omega_2}{2}\right)t = 0$

(or) $\left(\frac{\omega_1-\omega_2}{2}\right)t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}.......$

i.e., $t = \frac{\pi}{\omega_1-\omega_2}, \frac{3\pi}{\omega_1-\omega_2}, \frac{5\pi}{\omega_1-\omega_2}........$

$\therefore$ The time interval between two consecutive minima is $\Delta t = \frac{2\pi}{\omega_1-\omega_2} = \frac{1}{f_1-f_2}$, known as beat period. The beat frequency i.e., number of beats per second or the number of times the resultant intensity becomes maximum or minimum in one second at a given place is $f_b = \frac{1}{\Delta t} = f_1 - f_2$

i.e., beat frequency is equal to the difference of frequencies of two interfering waves. Beats can be obtained in both transverse and longitudinal waves. In case of sound wave beats the maxima is called waxing and minima is called waning.

Note :

(i) As the persistence of human hearing is about 0.1 sec, beats will be detected by the ear only if beat period is $\Delta t \geq 0.1\text{sec}$ or beat frequency $f_b = f_1 - f_2 \leq 10\text{Hz}$.

So if $f_b > 10\text{Hz}$, beats will more than 10 but heard as zero i.e. there will be continuous sound of intensity $I_1 + I_2$ instead of waxing and waning of sound.

(ii) If beat frequency comes out to be fractional do not round it off to nearest integer as over a proper time interval this fractional part will convert into an integer. eg. If beat frequency is 3.8 Hz, then in 5sec, 19 beats will be produced.

Uses of Beats

1. The phenomenon of beats can be used to find the unknown frequency of a sound note.
2. Musicians use the beats phenomenon in tuning their instruments.
3. A gas can be tested to see whether it is poisonous, inflammable or non-dangerous using beats phenomenon.
4. To determine the frequency of a given tuning fork beats phenomenon is used.
5. Beats phenomenon can be used to produce sound waves of small frequency.
6. The phenomenon of beats in electrical circuits is called heterodyne.

   "Heterodyne meter" is used to measure the frequency of electromagnetic wave.

Example-1.23

If two sound waves, $y_1 = 0.3 \sin 596\pi[t - x/330]$ and $y_2 = 0.5 \sin 604\pi[t - x/330]$ are superposed, what will be the (a) frequency of resultant wave (b) frequency at which the amplitude of resultant waves varies (c) frequency at which beats are produced. Find also the ratio of maximum and minimum intensities of beats.

Solution :

Comparing the given wave equation with

$y = A\sin(\omega t - kx) = A\sin\omega[t - (x/v)]$

[as $k/\omega = 1/v$]

we find that here $A_1 = 0.3$ and $\omega_1 = 2\pi f_1 = 596\pi$

i.e., $f_1 = 298$ Hz and $A_2 = 0.5$ and $\omega_2 = 2\pi f_2 = 604\pi$

i.e., $f_2 = 302$ Hz

So (a) The frequency of the resultant wave

$$f_{av} = \frac{f_1 + f_2}{2} = \frac{(298 + 302)}{2} = 300\text{Hz}$$

(b) The frequency at which amplitude of resultant wave varies : $f_A = \frac{f_1 - f_2}{2} = \frac{(298 - 302)}{2} = 2\text{ Hz}$

(c) The frequency at which beats are produced $f_b = 2f_A = f_1 - f_2 = 4\text{ Hz}$

(d) The ratio of maximum to minimum intensities of beat

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(0.3 + 0.5)^2}{(0.3 - 0.5)^2} = \frac{64}{4} = 16$$

Example-1.24

A tuning fork A produces 4 beats with tuning fork B of frequency 256 Hz. When A is filed beats are found to occur at shorter intervals. What was its original frequency?

Solution :

As tuning fork A produces 4 beats with B of frequency ($f_B = 256$ Hz), the frequency of A, f_A will be

$f_A = f_B \pm 4 = 256 \pm 4$, i.e., $f_A = 252$ Hz or 260 Hz

Now on filing due to decrease in inertia frequency of A will increase and occurrence of beats at shorter duration means increase in beat frequency; so if $f_A = 252$ Hz, $256 - f_A = 4$ Hz and so with increase in f_A beat frequency will decrease. If $f_A = 260$ Hz, $f_A - 256 = 4$ Hz and so with increase in f_A beat frequency will increase. and as on filing frequency increases.

$\therefore$ The frequency of A before filing was 260 Hz

1.29 ECHO

Reflection of sound from a distant object such as hill or cliff is called echo. If a sound wave is reflected from an obstacle there will be no change in its velocity, wave length & frequency, but its intensity decreases.

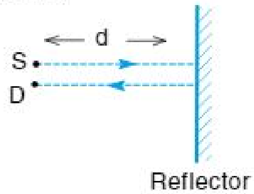


Fig 1.50

It there is a sound reflector at a distance 'd' from the source as shown in the Fig 1.50, the time interval between original sound and its echo at the site of source will be $t = \frac{2d}{v}$. where v is the speed of sound. Since, the effect of original sound remain on our ear for 0.1 second, if the sound returns to the starting point before 0.1 second, then it will not be distinguished from the original sound and no echo will be heard.

Hence $\frac{2d}{v} > 0.1$ i.e., $d > \frac{v}{20}$

If v = 330 m/s, the minimum distance of the reflector to hear echo is 16.5m. Furthermore, multiple or successive echoes will be produced when there are two or more reflectors. If the successive echoes take place at regular intervals of time it is called harmonic echo.

Applications of Echo

1) The phenomenon of echo is used to find depth of the sea and height of an aeroplane.
2) By using an echo, the velocity of sound can be determined.
3) The sound is reflected from a concave surface arranged behind the speaker in an auditorium. This helps all the audience to receive the sound uniformly.
4) Megaphone and stethoscope work on the principle of reflection of sound.

Example-1.25

An engine approaches a hill with a constant speed. When it is at a distance of 0.9 km it blows a whistle, whose echo is heard by the driver after 5 sec. If the speed of sound in air is 330 m/s, calculate the speed of the engine.

Solution :

If the speed of the engine is V, the distance travelled by the engine in 5 sec, will be 5V. And hence, the distance travelled by sound in reaching the hill and coming back to the moving driver = 900 + (900 – 5V) = (1800 – 5V).

So time interval between the original sound and its echo

$$\frac{1800-5V}{330} = t = 5s \text{ (given)}$$

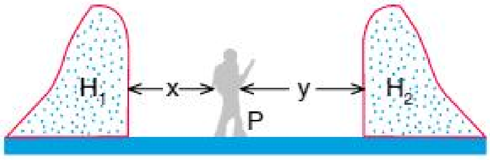
The above equation on solving gives V = 30 m/s.

Example-1.26

A person, standing between two parallel hills, fires a gun. He hears the first echo after 1.5 s and the second after 2.5 s. If the speed of sound is 332 m/s, calculate the distance between the hills. When will he hear the third echo?

Solution :

Let the person P be at a distance x from hill H_1 and y from H_2 as shown in figure. The time interval between the original sound and echoes from H_1 and H_2 will be respectively.



So the distance between the hills

$$x + y = \frac{v}{2}(t_1 + t_2) = \frac{332}{2}[1.5 + 2.5] = 664\text{m}$$

Now as I echo will be from H_1 after time t_1 while II echo from H_2 after time t_2, III echo will be produced due to reflection of sound of I echo from H_2 or of II echo from H_1, i.e., $t_3 = t_1 + t_2 = 1.5 + 2.5 = 4$ sec i.e., III echo will be produced after 4 sec. and in it sound from both I and II echoes will reach simultaneously.

1.30 LONGITUDINAL STANDING WAVE IN AN ORGAN PIPE

An organ pipe is a cylindrical tube of uniform cross section in which a gas or air is trapped as a medium. One end of an organ pipe is always open while the other may be closed or open giving rise to closed end or open end organ pipe respectively. If both ends of a pipe are closed it is known as cavity pipe.

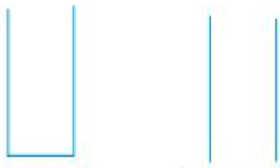


Fig 1.51

Consider a longitudinal wave introduced at the open end of a closed pipe. Upon reaching the closed end of the pipe, it gets reflected from this end. But the reflected pressure wave differ in phase by π with the incident pressure wave i.e., compression is reflected as compression and a rarefaction is reflected as rarefaction. Hence superposition of incident and reflected pressure waves produces longitudinal standing waves. Closed end in always a pressure antinode i.e., pressure fluctuation is maximum, and will be a displacement node as particles at node are permanently at rest as shown in Fig 1.51.

A longitudinal wave can also reflect at open end. If the longitudinal pressure wave encounters the open end of the pipe, a compression is reflected as a rarefaction and a rarefaction as a compression. Let us see how this reflection take place.

When a rarefaction reaches an open end, the surrounding air or gas rushes towards this region because of its low pressure and creates a compression that travels back along the pipe. Similarly, when a compression reaches an open end, the gas or air in this region expands because of high pressure and creates a rarefaction and travel back along the pipe. Hence superposition of incident and reflected pressure waves results in longitudinal standing waves. Open end is always a pressure node and the particles are free to move, there will be a displacement antinode as shown in Fig 1.52.

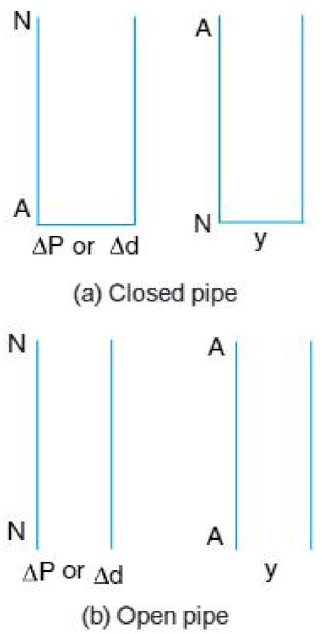


Fig 1.52

If two longitudinal pressure waves of same amplitude and frequency travel in opposite directions, $\Delta P_1 = (\Delta P)_m \operatorname{Sin}(kx - \omega t)$ .....(1)

$\Delta P_2 = (\Delta P)_m \operatorname{Sin}(kx + \omega t)$ .....(2)

By the principle of Superposition

$\Delta P = \Delta P_1 + \Delta P_2$ (or) .....(3)

$\Delta P = 2(\Delta P)_m \operatorname{Sin}kx \operatorname{Cos}\omega t$

(or) $\Delta P = (\Delta P)_s \cos\omega t$, where

$(\Delta P)_s = 2(\Delta P)_m \sin kx$ called amplitude of standing wave, which vary periodically with position with maximum value $2(\Delta P)_m$.

In case of longitudinal wave as pressure and displacement waves have a phase difference of $\pi/2$, at nodes where the displacement is minimum, pressure and density fluctuation will be maximum, while at antinodes where displacement is maximum, pressure and density fluctuation is zero.

1) For pressure nodes or displacement antinodes $\operatorname{Sin} kx = 0$ (or) $kx = n\pi$, $n = 0,1,2,3$........... (or)

$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda$...... (from the open end)

2) For pressure antinodes or displacement nodes

$\operatorname{Sin} kx = 1$ (or) $kx = (2n-1)\frac{\pi}{2}$, $n = 1,2,3$........

(or) $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$...... (from the open end)

The distance between two adjacent nodes or antinodes is $\lambda/2$.

a) Standing wave in closed pipe

In case of closed end organ pipe, as closed end will always be displacement node, while free end antinode, the distance between the successive node and antinode is $\lambda/4$. Let v be the speed of wave in the pipe.

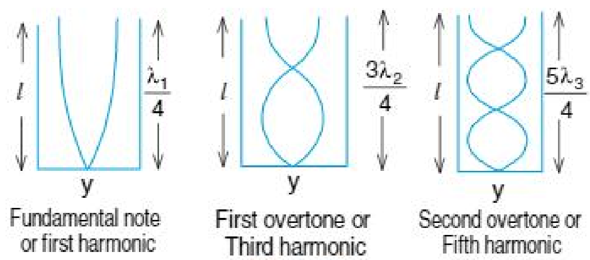


Fig 1.53

If λ_1 is the wavelength of wave produced in first harmonic, then $l = \frac{\lambda_1}{4}$.

The fundamental frequency (or) first harmonic frequency $f_1 = \frac{v}{\lambda_1} = \frac{v}{4l}$

If λ_2 is the wavelength of wave produced in the first overtone, then $l = \frac{3\lambda_2}{4}$

The third harmonic or first overtone frequency $f_2 = \frac{v}{\lambda_2} = 3\left(\frac{v}{4l}\right) = 3f_1$

If λ_3 is the wavelength of wave produced in the second overtone, then $l = \frac{5}{4}\lambda_3$

The fifth harmonic or second overtone frequency $f_3 = \frac{v}{\lambda_3} = 5\left(\frac{v}{4l}\right) = 5f_1$ and so on

Hence the frequency of vibration of closed pipe.

$$f_c = n\left(\frac{v}{4l}\right), n = 1, 3, 5...$$

This mode of vibration frequency is similar to rod clamped at one end.

b) Standing wave in open organ pipe

In case of open end organ pipe, at both ends there will be displacement antinodes.

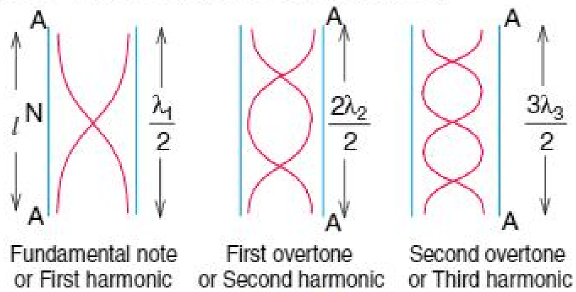


Fig 1.54

If λ_1 is the wavelength of the wave produced in fundamental note of vibration, then $l = \frac{\lambda_1}{2}$

The fundamental or first harmonic frequency

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2l}$$

If λ_2 is the wavelength produced in first over tone, then $l = 2\frac{\lambda_2}{2}$

First overtone or second harmonic frequency

$$f_2 = \frac{v}{\lambda_2} = 2\left(\frac{v}{2l}\right)$$

Similarly second overtone or third harmonic frequency $f_3 = \frac{v}{\lambda_3} = 3\left(\frac{v}{2l}\right)$ and so on.

Hence the frequency of vibration of an open pipe $f_o = n\left(\frac{v}{2l}\right)$, $n = 1, 2, 3, 4.....$

This mode of vibration frequency is similar to that of a rod clamped in the middle.

c) End correction of pipes

Due to inertia of motion of particles in organ pipes, reflection takes place not exactly at open end, but somewhat above it, i.e., at a distance e=0.6r =0.3D called end correction where r is radius of pipe, D is the diameter of pipe.

The effective length of the pipe is therefore, greater from the length of the pipe. So for closed pipe $L_c = l + 0.6$ r, while for open pipe $L_o = l + 1.2$r.

So that $f_c = n\frac{v}{4(l+0.6r)}, n = 1, 3, 5....$ and

$$f_o = n\frac{v}{2(l+1.2r)}, n = 1, 2, 3, 4....$$

A wider tube has greater correction with the fundamental frequency.

1.31 EXPERIMENTAL DETERMINATION OF SPEED OF SOUND IN AIR USING RESONANCE TUBE

Fig 1.55 shows a schematic arrangement of a resonance tube apparatus. It consists of a vertical glass tube T, which is 1m long and about 3cm in diameter. It is fixed on a vertical board along with a meter scale S which is graduated in millimeter. The zero of the scale coincides with the upper end of the tube. The lower end of the glass tube is connected to a reservoir R of water through a rubber pipe P. The water level in the tube can be adjusted by the adjustable screws attached with the reservoir. The vertical adjustment of the tube can be made with the help of leveling screws.

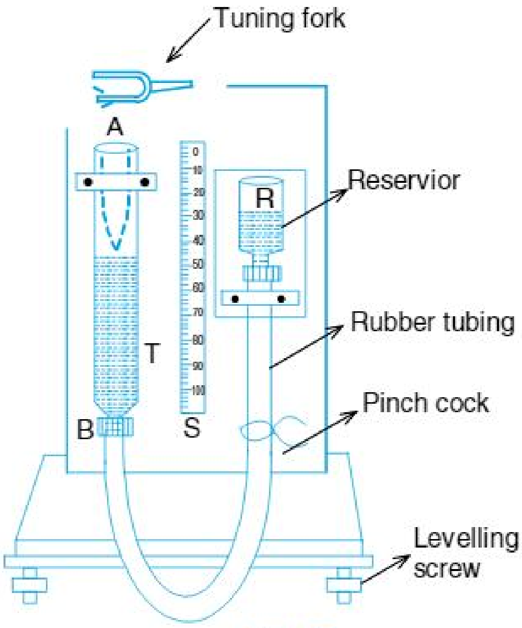


Fig 1.55

A vibrating tuning fork of known frequency is held over the open end of the resonance tube filled with water as shown in Fig 1.55.

The resonance is obtained at some positions as the level of water is lowered as shown in Fig 1.56(a).

In these situations, a loud sound is heard from the pipe, and a stationary wave is set up with the top of the pipe acting as an antinode and the water column side as a node. If v is the speed of sound in air, e is the end correction of the tube and l is the length from the water level to the top of the tube, then

$$l_1 + e = \frac{\lambda}{4} = \frac{1}{4}\left(\frac{v}{f}\right) \qquad .....(1)$$

where v is the velocity of sound in air and f is the frequency of tuning fork and air columns.

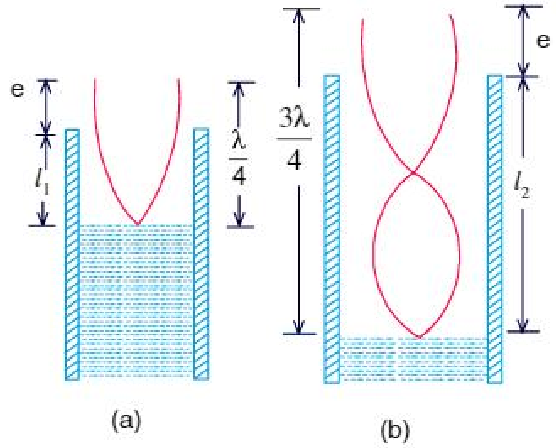


Fig 1.56

Now the water level is further reduced to a lower value as shown in Fig 1.56(b), until once again resonance is obtained with same tuning fork.

If l_2 is new length of air column, then

$$l_2 + e = \frac{3\lambda}{4} = \frac{3}{4}\left(\frac{v}{f}\right) \qquad ....(2)$$

From (1) and (2)

$$l_2 - l_1 = \frac{v}{2f} \text{ (or) } v = 2f(l_2 - l_1) \qquad ....(3)$$

Eliminating $\frac{v}{f}$ from (2) and (3), we get end correction $e = \frac{l_2 - 3l_1}{2}$

Example-1.27

For a certain organ pipe, three successive resonance frequencies are observed at 425, 595 and 765 Hz respectively. Taking the speed of sound in air to be 340 m/s (a) explain whether the pipe is closed at one end or open at both ends (b) determine the fundamental frequency and length of the pipe.

Solution :

a) The given frequencies are in the ratio 425 : 595 : 765, i.e., 5:7:9 and clearly these are odd integers so the given pipe is closed at one end.

b) From part (a) it is clear that the frequency of 5th harmonic (which is 2nd overtone) is 425 Hz

So $425 = 5f_c$ (or) $f_c = 85\text{Hz}$.

Further as $f_C = \frac{v}{4L}, L = \frac{v}{4f_C} = \frac{340}{4 \times 85} = 1\text{m}$

Example-1.28

A tube 1.0m long is closed at one end. A stretched wire is placed near the open end. The wire is 0.3 m long and has a mass of 0.01 kg. It is held fixed at both ends and vibrates in its fundamental mode. It sets the air column in the tube into vibration at its fundamental frequency by resonance. Find

(a) the frequency of oscillation of the air column and

(b) the tension in the wire. Speed of sound in air = 330 m/s

Solution :

a) Fundamental frequency of closed pipe $= \dfrac{v}{4l}$

$$= \frac{330}{4\times 1} = 82.5\text{Hz}$$

b) At resonance, given: fundamental frequency of stretched wire (fixed at both ends) = fundamental frequency of air column

$$\therefore \frac{v}{2l} = 82.5\,\text{Hz}$$

$$\therefore \frac{\sqrt{T/\mu}}{2l} = 82.5 \text{ (or) } T = \mu(2\times 0.3\times 82.5)^2 = 81.675\,\text{N}$$

Example-1.29

Two organ pipes 80 and 81 cm long are found to give 26 beats in 10 sec, when each is sounding its fundamental note find the velocity of sound in air is

Solution :

Fundamental frequency of the first organpipe, $n_1 = \dfrac{V}{2\ell_1}$

Fundamental frequency of the second organ pipe $n_2 = \dfrac{V}{2\ell_2}$

Number of beats per second $= n_1 - n_2 = \dfrac{V}{2\ell_1} - \dfrac{V}{2\ell_2}$

$$\Rightarrow \frac{26}{10} = \frac{V}{160} - \frac{V}{162} \Rightarrow 2.6 = \frac{2V}{160\times 162}$$

$$\Rightarrow V = \frac{2.6\times 160\times 162}{2} = 33696 \text{ cm s}^{-1} \cong 337\,\text{ms}^{-1}$$

Example-1.30

AB is a cylinder of length 1 m fitted with an inflexible diaphragm C at middle and two other thin flexible diapharagms A and B at the ends. The portions AC and BC maintain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of the same frequency. What is the minimum frequency of these vibrations for which diaphragm C is a node? Under the conditions of the experiment velocity of sound in hydrogen is 1100 m/s and oxygen 300 m/s.

Solution :

As diaphragm C is a node, A and B will be antinodes (as is an organ pipe, either both ends are antinode or one end node and the other antinode), i.e., each part will behave as a closed end organ pipe that

$$f_H = \frac{v_H}{4L_H} = \frac{1100}{4\times 0.5} = 550\,\text{Hz} \quad \text{and}$$

$$f_0 = \frac{v_0}{4L_0} = \frac{300}{4\times 0.5} = 150\,\text{Hz}$$

As the two fundamental frequencies are different, the system will vibrate with a common frequency f_C such that $f_C = n_H f_H$

$$= n_0 f_0 \Rightarrow \frac{n_H}{n_0} = \frac{f_0}{f_H} = \frac{150}{550} = \frac{3}{11}$$

Then the third harmonic of hydrogen and 11^{th} harmonic of oxygen or 9^{th} harmonic or hydrogen and 33rd harmonic of oxygen will have same frequency. So the minimum common frequency.

$$f = 3\times 550 \text{ or } 11\times 150\,\text{Hz} = 1650\text{Hz}$$

(as 6^{th} harmonic of H and 22^{nd} of O will not exist.)

1.32 DOPPLER EFFECT

It is of common experience when we stand near a railway track and when a train approaches whistling, the pitch of the whistle appears to be increasing. Similarly as the train moves away from us the pitch appears to be decreasing. A similar effect can be found when we are in a moving train on a track listening to the whistle of a stationary engine on a parallel track. This effect of listening apparently a different frequency than the actual one is known as the Doppler effect.

The phenomenon of apparent change in the frequency of a sound wave due to the relative motion between the source and the listener is called Doppler effect. If the relative motion between the observer and the source brings them closer with time, the apparent frequency heard is greater than the actual frequency. If the relative motion between them is to separate them more with time, the apparent frequency is lower than the actual frequency. The shift in the frequency is known as the Doppler's shift.

Doppler effect is a wave phenomenon, it holds not only for sound waves but also for electromagnetic waves. Here we shall consider only sound waves.

We shall analyse changes in frequency under three different situations: (1) observer is stationary but the source is moving. (2) observer is moving but the source is stationary, and (3) both the observer and the source are moving. The situations (1) and (2) differ from each other because of the absence or presence of relative motion between the observer and the medium. Most waves require a medium for their propagation; however, electromagnetic waves do not require any medium for propagation. If there is no medium present, the Doppler shifts are same irrespective of whether the source moves or the observer moves, since there is no way of distinguishing between the two situations.

Derivation of Relation for Apparent Frequency

a) Source Moving : Observer stationary

Let us choose the convention to take the direction from the observer to the source as the positive direction of velocity. Consider a source S moving with velocity v_s and an observer who is stationary in a frame in which the medium is also at rest. Let the speed of a wave of angular frequency ω and period T_0 both measured by an observer at rest with respect to the medium, be v. We assume that the observer has a detector that counts every time a wave crest reaches it. As shown in Fig. 1.57, at time t = 0 the source is at point S_1 located at a distance L from the observer, and emits a crest. This reaches the observer at time $t_1 = L/v$. At time $t = T_0$ the source has moved a distance v_sT_0 and is at point S_2, located at a distance $(L + v_sT_0)$ from the observer. At S_2 the source emits a second crest. This reaches the observer at $t_2 = T_0 + \dfrac{(L+v_sT_o)}{v}$

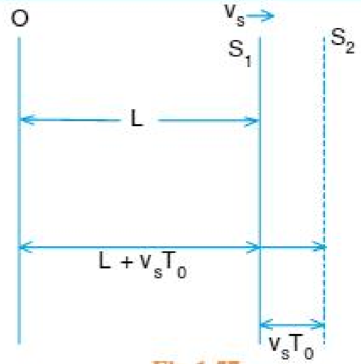


Fig 1.57

At time nT_0 the source emits its $(n+1)^{th}$ crest and this reaches the observer at time

$$t_{n+1} = nT_o + \frac{(L+nv_sT_o)}{v}$$

Hence, in a time interval

$$\left[nT_0 + \frac{(L+nv_sT_0)}{v} - \frac{L}{v}\right],$$

the observer's detector counts n crests and the observer records the period of the wave as T given by

$$T = \left[nT_0 + \frac{(L+nv_sT_0)}{v} - \frac{L}{v}\right]/n$$

$$T = T_0 + \frac{v_sT_0}{v} \text{ (or) } T = T_0\left(1+\frac{v_s}{v}\right)$$

This equation may be rewritten in terms of the frequency f_0 that would be measured if the source and observer were stationary, and the frequency f^1 observed when the source is moving, as

$$f^1 = f_0\left(1+\frac{v_s}{v}\right)^{-1}$$

If v_s is small compared with the wave speed v, taking binomial expansion to terms in first order in v_s/v and neglecting higher power, it may be approximated, giving

$$f^1 = f_0\left(1-\frac{v_s}{v}\right) \qquad ..... (1)$$

For a source approaching the observer, we replace v_s by $-v_s$ to get

$$f^{11} = f_0\left(1+\frac{v_s}{v}\right) \qquad ..... (2)$$

The observer thus measures a lower frequency when the source recedes from him than he does when it is at rest. He measures a higher frequency when the source approaches him.

The corresponding graph is as shown in Fig 1.58.

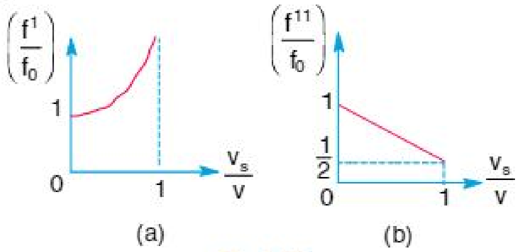


Fig 1.58

The reverse is true when the source moves away from the observer. The apparent frequency is

$$f^{11} = f_0\left(\frac{v}{v+v_s}\right)$$

Since $\frac{f^{11}}{f_0} = \frac{1}{1+\frac{v_s}{v}}$, the corresponding graph is as shown in the figure 1.58 (b).

The apparent wavelength when source move away from the observer is

$$\lambda^{11} = \left(\frac{v+v_s}{v}\right)\lambda$$

b) Observer Moving: Source Stationary

Now to derive the Doppler shift when the observer is moving with velocity v_0 towards the source and the source is at rest, we have to proceed in a different manner. We work in the reference frame of the moving observer. In this reference frame the source and medium are approaching at speed v_0 and the speed with which the wave approaches is $v_0 + v$. Following a similar procedure as in the previous case, we find that the time interval between the arrival of the first and the $(n+1)^{th}$ crests is $t_{n+1} - t_1 = nT_0 - \frac{nv_0T_0}{v_0+v}$

The observer thus, measures the period of the wave to be $T_0\left(1 - \frac{v_0}{v_0+v}\right) = T_0\left(1+\frac{v_0}{v}\right)^{-1}$

giving $f^1 = f_0\left(1+\frac{v_0}{v}\right)$ .... (3)

If $\frac{v_0}{v}$ is small,

the Doppler shift is almost same whether it is the observer or the source moving since Eq.(3) and the approximate relation Eq.(1) are the same.

Its corresponding graph is shown in the Fig 1.59(a). The reverse is true when the observer moves away from the source, is apparent decrease in frequency occurs.

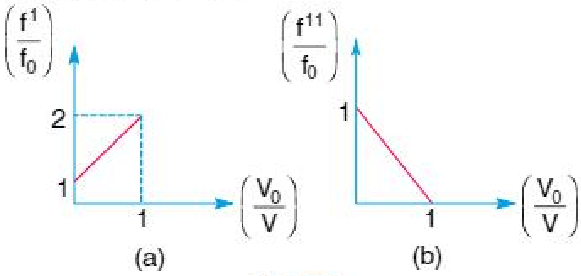


Fig 1.59

i.e., $f^{11} = f_0\left(\frac{v-v_0}{v}\right)$ (or) $\frac{f^{11}}{f_0} = 1 - \frac{v_0}{v}$

Its corresponding graph is shown in the Fig 1.59(b).

c) Both Source and Observer Moving

We will now derive a general expression for Doppler shift when both the source and the observer are moving. As before, let us take the direction from the observer to the source as the positive direction. Let the source and the observer be moving with velocities v_s and v_0 respectively as shown in Fig 1.60.

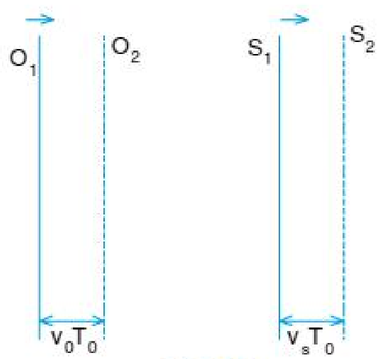


Fig 1.60

Suppose at time t = 0, the observer is at O_1 and the source is at S_1, O_1 being to the left of S_1. The source emits a wave of velocity v of frequency f_0 and period T_0all measured by an observer at rest with respect to the medium. Let L be the distance between O_1 and S_1 at t = 0, when the source emits the first crest. Now, since the observer is moving, the velocity of the wave relative to the observer is $v + v_0$. Therefore, the first crest reaches the observer at time $t_1 = L/(v + v_0)$. At time $t = T_0$, both the observer and the source have moved to their new positions O_2 and S_2 respectively. The new distance between the observer and the source, O_2S_2 would be $L+[(v_s - v_0)T_0]$. At S_2, the source emits a second crest.

This reaches the observer at time. $t_2 = T_0 + [L + (v_s - v_0)T_0]/(v + v_0)$

At time nT_0 the source emits its $(n + 1)^{th}$ crest and this reaches the observer at time

$$t_{n+1} = nT_0 + [L + n(v_s - v_0)T_0]/(v + v_0)$$

Hence, in a time interval $t_{n+1} - t_1$, i.e., $nT_o + [L + n(v_s - v_o)T_o]/(v + v_o) - L/(v + v_o)$,

the observer counts n crests and the observer records the period of the wave as equal to T given by

$$T = T_o\left(1 + \frac{v_s - v_o}{v + v_o}\right) = T_o\left(\frac{v + v_s}{v + v_o}\right)$$

The frequency f observed by the observer is given by

$$f = f_0\left(\frac{v + v_o}{v + v_s}\right).$$

1) The Doppler effect is asymmetric for larger speeds of V_0 and V_s i.e., the change in frequency depends on whether the source is moved towards the observer or the observer is moved towards the source. But when the speed of source and observer are much less than that of sound the Doppler effect is symmetric i.e., the change in frequency becomes independent of whether the source is moving or the observer. Suppose a source is moving towards a stationary observer, with speed v_s.

$$\text{Then } f^1 = f_0\left(\frac{v}{v - v_s}\right) = f_0\left(\frac{1}{1 - \frac{v_s}{v}}\right)$$

$$\text{or } f^1 = f_0\left(1 - \frac{v_s}{v}\right)^{-1} \approx f_0\left(1 + \frac{v_s}{v}\right)$$

$$(\because v_s << v)$$

Suppose an observer is moving towards the stationary source with same speed $v_s (=v_0)$.

$$\text{Then } f^{11} = f_0\left(\frac{v + v_s}{v}\right) = f_0\left(1 + \frac{v_s}{v}\right)$$

which is same as above i.e., $f^{11} = f^1$. Hence when $v_0 << v$ and $v_S << v$, the Doppler's formula reduces the $f^1 = f_0\left(1 \pm \frac{v_{rel}}{v}\right)$

where v_{rel} is the relative speed of the source with respect to observer.

2) When the source and observer do not move along the line joining them, then the components of their velocities along the line joining them must be taken as velocity of observer and velocity of source in Doppler's formula.

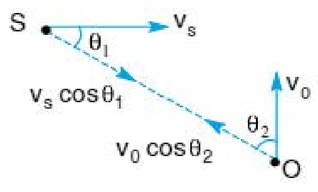


Fig 1.61

For the situation in Fig 1.61

$$f^1 = f_0\left(\frac{v + v_0\cos\theta_2}{v - v_s\cos\theta_1}\right)$$

Here the apparent frequency depends on angle subtended by line of motion of the source and / or observer with line joining source and observer. That angle varies when the source and /or observer moves, hence apparent frequency varies.

3) The velocity of sound in the Doppler formula is affected by the medium velocity or wind velocity. If the wind blows at a speed ω from the source to observer then $f^1 = f_0\left(\frac{v+\omega \pm v_0}{v+\omega \mp v_s}\right)$ a nd in opposite direction from observer to source $f^1 = f_0\left(\frac{v-\omega \pm v_0}{v-\omega \mp v_s}\right)$

4) There is no Doppler effect i.e., no change in frequency.

a) if source and detector both move in same direction with same speed.

i.e., $f^1 = f_0\left(\frac{v-u}{v-u}\right) = f_0$

b) if both are at rest and wind blows at speed ω in any direction. In this situation

$$f^1 = f\left(\frac{v+\omega \pm 0}{v+\omega \mp 0}\right) = f_0$$

c) if one is at the centre of a circle while the other is moving on it with uniform or non uniform speed. In this situation component of their velocity along the line joining them is zero.

Hence $f^1 = f_0\left(\frac{v \pm 0}{v \mp 0}\right) = f_0$

5) If the source is rotating with constant angular velocity 'ω' in circular path of radius R and an observer is stationary out side the path and in the plane of the circular path, then there will be Doppler effect except two positions of the source.

In Fig 1.63(a) at positions 3 and 4 the source moves normal to line of sight, hence there is no Doppler effect. At positions 1 and 2 source velocity is along the line of sight of source and observer, hence the Doppler effect is maximum or minimum. For the given direction of source as in the Fig 1.63(b) the frequency received by the observer is maximum when the source is at position 2 and minimum when the source is at position 1.

$\therefore f_{max} = f_0\left(\frac{v}{v-R\omega}\right)$ at position 2, and

$f_{min} = f_0\left(\frac{v}{v+R\omega}\right)$ at position 1

If we observe the figure 1.63, the positions 1 and 2 are the point of tangents from observer. The reverse is also true when the observer is under rotation and source is at rest out side the loop.

6) In case of reflection of sound from a reflector, the detector will receive two notes, one directly from the source and the other from the reflector, which also acts as a source. If the two frequencies are slightly different, the superposition of these waves will produce beats and the beat frequency can be used to determine the speed of the reflector (or source).

This is the principle used in SONAR and RADAR to determine the speed of moving objects.

Consider the following simple case.

Fig 1.62

If V_s is the velocity of source, V_0 is the velocity of observer and V is the velocity of sound wave, first we calculate the frequency received by the observer due to reflection. The frequency received by the wall $f^1 = f_0\left(\frac{V}{V-V_s}\right)$.

Since the waves are reflected by the wall, and these reflected waves can reach the observer, the wall acts as source for reflected waves. The frequency received by the observer from the wall is

$$f^{11} = f^1\left(\frac{V+V_0}{V}\right) = f_0\left(\frac{V+V_0}{V-V_s}\right)$$

This is the case similar to both source and observer are approaching.

The frequency of wave received by the observer directly from the source is

$$f^{111} = f_0\left(\frac{V-V_0}{V-V_s}\right)$$

Hence the beat frequency received by the observer is $\Delta f = f^{11} - f^{111} = \frac{2f_0 V_0}{V - V_s}$

On knowing number of beats, we can estimate the velocity of source.

7) Doppler effect is applicable even when source and/or observer are under acceleration. Let us consider two cases

Case (i)

Suppose the source starts from the stationary observer at O with a uniform acceleration 'a'. Our interest is to know the apparent frequency heard by the observer after time 't'.

Fig 1.63

The wavefronts received by the observer at 't' are the wavefronts generated by the source earlier at time t^1 $(<t)$.

Let position of the source be S^1 after time t^1 $(<t)$. While the sound waves reach O form S^1, let the source move from S^1 to S in $(t - t^1)$. If 'v' is the speed of sound in the medium, then

$$OS^1 = v(t - t^1) = \frac{1}{2}at^{1^2}$$

$$\text{(or)} \quad \frac{1}{2}at^{1^2} + vt^1 - vt = 0$$

On solving the above equation we get 't^1'. The velocity of source at that instant. $V_s = at^1$. Then according to Doppler effect.

$$f^1 = f_0\left(\frac{v}{v + at^1}\right)$$

Case (ii)

Suppose the observer starts with an acceleration a towards the source at rest as shown in Fig 1.66.

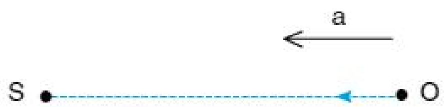


Fig 1.64

At source the distance between two consecutive wavefronts would be $\lambda = vT$ and

$$T = \frac{1}{f_0} \quad ....(1)$$

To know the apparent frequency at time 't', the speed of sound wave relative to the observer is v + at, at that time the wavelength has its normal value. The observer receive the wave-fronts at shorter intervals of time.

$$T^1 = \frac{\lambda}{v + at} = \frac{vT}{v + at} \text{ (or) } \frac{1}{f^1} = \left(\frac{v}{v + at}\right) \cdot \frac{1}{f_0}$$

$$\text{(or) } f^1 = \left(\frac{v + at}{v}\right) f_0 \text{ (or) } \frac{f^1}{f_0} = 1 + \frac{a}{v}t$$

The corresponding graph is as shown in Fig 1.67.

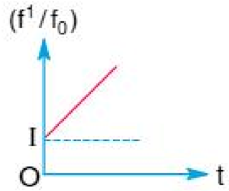


Fig 1.65

Doppler effect holds not only for sound but also for electromagnetic waves.

1.33 DOPPLER EFFECT IN LIGHT

In case of electromagnetic waves there is no medium relative to which the velocity can be defined and hence the Doppler displacement of the frequency of light waves is determined only by the relative velocity of the source and the receiver. This means that source receding from observer and observer receding from source are physically identical situations and must exhibit exactly the same Doppler frequency i.e., Doppler effect for electromagnetic waves is symmetric. Wave speed 'c' is the speed of light and is the same for both source and receiver. In the frame of reference of the observer, if the source is moving away with velocity v, the frequency measured by the observer is given by the theory of relativity,

$$n' = n\sqrt{\frac{1 - (v/c)}{1 + (v/c)}}$$

where n' is the apparent frequency measured by observer. When v is positive the source moves away w.r.t the observer and n' < n. When v is negative the source moves towards the observer and n' > n. This is known as longitudinal Doppler effect. If the relative velocity v is small compared to speed of light c, we can use the same formula which we use for sound waves. When the source is receding $n' = n\left(1-\frac{v}{c}\right)$ and when the source is approaching $n' = n\left(1+\frac{v}{c}\right)$. Therefore the relative change in frequency $\frac{\Delta n}{n} = -\frac{v}{c}$ and

$$\frac{\Delta n}{n} = -\frac{\Delta\lambda}{\lambda} \quad (\because n\lambda = c)$$

$$\therefore \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

When the source is receding v is positive and when the source is approaching v is negative.

If source moves towards the observer the light appears more violet and if it moves away it appears more red.

In case of sound we learned that there is no frequency shift if the relative velocity of source and receiver is perpendicular to the line joining them (For example, when source moves in circular path with its centre at receiver).

This is no longer true in the relativistic case. In addition to the longitudinal effect, a transverse Doppler effect exist for light waves. A decrease in frequency of light is observed, when the source is directed of right angles to the straight line passing through the receiver and source. Let n be the frequency of source and n' be the frequency received by the receiver.

Then $n' = n\sqrt{1-\frac{v^2}{c^2}} \approx n\left[1-\frac{1}{2}\frac{v^2}{c^2}\right]$

The relative change in frequency in transverse Doppler effect is given by $\frac{\Delta n}{n} = -\frac{1}{2}\frac{v^2}{c^2}$

1.34 INTENSITY LEVEL

If I_0 is intensity of sound at threhold of audibility and I is intensity of sound then intensity level of that sound is given by $10\log_{10}\left(\frac{I}{I_0}\right)$ which is known as 'decibel'.

Here decibel (dB) is unity for intensity level. $I_0 = 10^{-12}\ wm^{-2}$.

We can write $L = 10\log_{10}\left(\frac{I}{I_0}\right)$.

If I_1 and I_2 are intensity of sound which are different then their intensity levels are given by

$$L_1 = 10\log_{10}\left(\frac{I_1}{I_0}\right) \text{ and } L_2 = 10\log_{10}\left(\frac{I_2}{I_0}\right)$$

Example-1.31

Two tuning forks with natural frequencies 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3Hz. Find the speed of the tuning fork. (Velocity of sound in air = 340 m/s)

Solution :

As observer is at rest, $f_{Ap} = f\left[\frac{v}{v \mp v_S}\right]$

If speed of tuning fork is u, $f_A = f\left[\frac{v}{v-u}\right]$ while $f_R = f\left[\frac{v}{v+u}\right]$

Now as beat frequency is 3 Hz, so f_A and f_R are very close which is possible only if u<<v. So using binomial theorem, $f_A = f\left[1-\frac{u}{v}\right]^{-1} = f\left[1+\frac{u}{v}\right]$

and $f_R = f\left[1+\frac{u}{v}\right]^{-1} = f\left[1-\frac{u}{v}\right]$ So beat frequency

$\Delta f = f_A - f_R = f\left[\frac{2u}{v}\right]$

i.e., $u = \frac{v}{2}\left[\frac{\Delta f}{f}\right] = \frac{340}{2}\left[\frac{3}{340}\right] = 1.5\,m/s$

Example-1.32

When a train is approaching the observer, the frequency of the whistle is 100 cps while when it has passed the observer, it is 50 cps. Calculate the frequency when the observer moves with the train.

Solution :

According to Doppler effect in case of approaching of source, $f' = f\dfrac{v}{v-v_s}$, i.e., $100 = \dfrac{fv}{v-v_s}$,

i.e., $v - v_s = \dfrac{fv}{100}$, while in case of recession of source.

$f' = f\dfrac{v}{v+v_s}$ i.e., $50 = \dfrac{fv}{v+v_s}$, $v + v_s = \dfrac{fv}{50}$

So adding the two eqns., $2v = \dfrac{3fv}{100}$

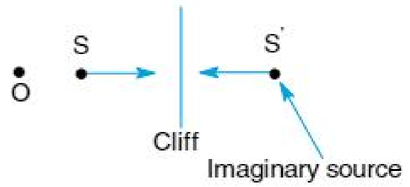
i.e., $f = \dfrac{200}{3} = 66.67\,Hz$

Example-1.33

A siren emitting a sound of frequency 1000 Hz moves away from you towards a cliff at a speed of 10 m/s. (a) What is the frequency of the sound you hear coming directly from the siren? (b) What is the frequency of the sound you hear reflected off the cliff? (c) What beat frequency would you receive? Take the speed of sound in air as, 330 m/s.

Solution :

The situation is as shown in figure.



(a) Frequency of sound reaching directly to us (by S)

$$f_1\left(\frac{v}{v+v_s}\right)f = \left(\frac{330}{330+10}\right)(1000) = 970.6\text{ Hz}$$

(b) Frequency of sound which is reflected of from the cliff is same as from S'

$$f_2 = \left(\frac{v}{v-v_s}\right)f = \left(\frac{330}{330-10}\right)(1000)$$

$$= 1031.3\text{ Hz}$$

(c) Beat frequency $= f_2 - f_1 = 60.7$ Hz

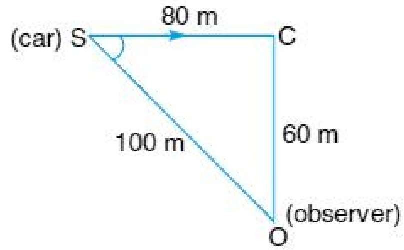
(Too high to be heared as beats)

Example-1.34

A car approaching a crossing at a speed of 20 m/s sounds a horn of frequency 500Hz when 80m from the crossing. Speed of sound in air is 330 m/s. What frequency is heard by an observer 60 m from the crossing on the straight road which crosses car road at right angles?

Solution :

The situation is as shown in figure.



$\cos\theta = \dfrac{80}{100} = \dfrac{4}{5}$

$\therefore$ Apparent frequency is

$$f_{app} = \left(\frac{v}{v - v_s\cos\theta}\right)f = \left(\frac{330}{330 - 20\times\frac{4}{5}}\right)(500)$$

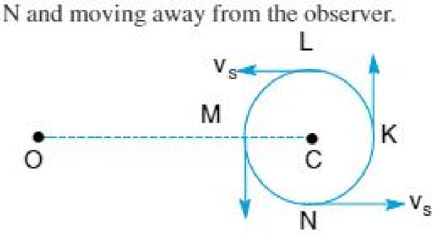
$$= 525.5\text{ Hz}$$

Example-1.35

A whistle of frequency 540 Hz rotates in a circle of radius 2 m at a linear speed of 30 m/s. What is the lowest and highest frequency heard by an observer a long distance away at rest with respect to the centre of circle. Take speed of sound in air as 330 m/s. Can the apparent frequency be ever equal to actual?

Solution :

Apparent frequency will be minimum when the source is at N and moving away from the observer.



$$f_{min} + \left(\frac{v}{v+v_s}\right)f = \left(\frac{330}{330+30}\right)\times 540 = 495\text{Hz}$$

Frequency will be maximum when source is at L and approaching the observer.

$$f_{max} = \left(\frac{v}{v-v_s}\right)f = \left(\frac{330}{330-30}\right)(540) = 594\,\text{Hz}$$

Further when source is at M and K, angle between velocity of source and line joining source and observer is 90^o or $v_s \cos\theta = v_s \cos 90^0 = 0$. So, there will be no change in the apparent frequency. i.e., no Doppler effect.

Example-1.36

With what speed should a galaxy move with respect to us so that the sodium line at 589.0 nm is observed at 589.6 nm?

Solution :

Since $\upsilon\lambda = c, \frac{\Delta\upsilon}{\upsilon} = -\frac{\Delta\lambda}{\lambda}$

(for small changes in υ and λ)

For $\Delta\lambda = 589.6 - 589.0 = +0.6\text{nm}$

$\frac{\Delta\upsilon}{\upsilon} = -\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$ or

$v = +c\left(\frac{0.6}{589.0}\right) = +3.06\text{x}10^5\,\text{ms}^{-1}$

Therefore. the galaxy should move away from us with speed 306 km/s.

1.35 APPLICATIONS OF DOPPLER EFFECT

1) Sound Navigation And Ranging (SONAR)

The phenomenon of Doppler effect is used in combination with sound echo in this device. It is used to find the position, speed and direction of motion of submarines and other bodies under the water (in the sea etc).

2) Radio Detection And Ranging (RADAR)

The phenomenon of Doppler effect in combination with reflection of waves is utilized to find the position, speed and direction of motion of aeroplanes, missiles, satellite etc in the sky. Radio waves are used for the purpose.

3) The phenomenon of Doppler effect is utilized in measuring the speeds of automobiles by traffic police. Microwaves are used for the purpose.

4) The longitudinal Doppler effect is used to determine the speeds at which luminous heavenly bodies are moving towards or receding from us by knowing apparent change in frequency or wavelength. Doppler's red shift shows that the galaxies are moving away from us (expanding universe). Doppler effect is also useful to asses the velocity of thermal motion of the molecules of a luminous gas and consequently, the temperature of the gas.

5) This principle is used for tracking an earth satellite.

EXERCISE

LONG ANSWER QUESTIONS

1. Explain the formation of stationary waves in stretched strings and hance deduce the laws of transverse waves in stretched strings.
2. What are Harmonics and Overtones? How are they formed in an open pipe? Derive the eqautions for the frequencies of the harmonic produced in an open pipe.
3. How are stationary waves formed in a closed pipe? Explain the various modes of vibrations in a closed pipe and establish the relation between their frequencies.
4. What is Doppler effect? Find an expression for the apparent frequency heard when the source is in motion and the listener is at rest. What is the limitation of Doppler effect?
5. What is Doppler's shift? Derive an expression for the apparent frequency heard by a moving listener when the source of sound is at rest. What is the effect of the moving medium on the Doppler effect?

SHORT ANSWER QUESTIONS

1. What are logitudinal waves? Explain with an example.
2. What are transverse waves? Explain with an example.
3. What is a simple harmonic wave? Deduce progressive wave equation.
4. Explain the reflection of waves in string at fixed end and free end.
5. State and explain the laws of transverse waves on a stretched string.
6. Explain the harmonic on a stretched string with sutiable illustrations.
7. What are Free and Forced vibrations? Explain with examples.
8. What are the beats? Explain the importance of beats.
9. What is resonance? Expalin with an example.
10. Explain the reflection of waves at closed and open ends.
11. Explain the modes of vibration in an open pipe with suitable example.

12. Explain the formation of standing wave pattern in closed pipe with suitable figures.
13. What are the applications of Doppler effect?
14. Explain the characteristics of a sound note.

VERY SHORT ANSWER QUESTIONS

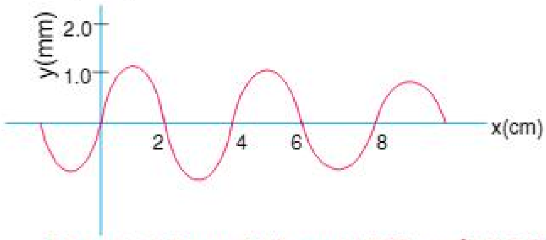
1. Name the parameters of a progressive wave?
2. Establish the relation between the wave velocity (v), frequency (v) and the wave length (λ) of a progressive wave.
3. How are sound different from light waves?
4. What is the distance between two crests with a phase difference of 4π ?
5. What is the phase difference between a compression and next rarefaction?
6. What type of mechanical waves do you expect to exist in (a) vacuum (b) air (c) inside water (d) rock (e) on the surface of a liquid.
7. Is it possible to have longitudinal waves on a string? A transverse wave on a steel rod?
8. How many times a particle will reach maximum displacement during the time taken by the wave to advance by one wavelength?
9. How can you say that the equation $y = A\sin(\omega t + kx)$ represents a progressive wave?
10. What is the significance of $\frac{\omega}{k}$ in the case of a progressive wave given by $y = A\sin(\omega t - kx)$?
11. What is the principle of superposition of waves?
12. What are the conditions required for a wave to get reflected?
13. What is the phase difference between incident and reflected waves at (a) an fixed end (b) free end of string.
14. When sound travels from air into water, does the frequency of the wave change? The wavelength? The speed?
15. What is a stationary wave? What is the distance between a node and the succeeding antinode of a stationary wave?
16. Which type of waves are formed due to vibrations of stretched strings?
17. When stretched string vibrates in two segments, how many nodes and antinodes will be there?
18. What is the frequency of 9^{th} overtone on a stretched of length l and linear density 'm', when the tension is 'T'?
19. A stretched string is plucked and it vibrates transversely. Are the vibrations free or forced?
20. A wire of length 'l' is vibrating in three segments. What is the wavelength of the note emitted?
21. What is the ratio of the frequency of fouth overtone to the fundamental frequency of a stretched string?
22. What happens to the fundamental frequency of a stretched string when the tension is quadrupled?
23. What happens to the fundamental frequency of a stretched string when its linear density becomes 1/4 of its intial value?
24. Two identical wires on a sonometer, are stretched with the same tension 'T'. if their lengths are in the ratio 1 : 2 what is the ratio of their frequencies?
25. When do the paper riders on a sonometer wire fly off?
26. What is the effect of temperature on frequency tuning fork?
27. What happens to the frequency of a tuning fork.
    (a) When it is loaded with little wax and
    (b) When it is filled?
28. What is the influence of resonance on the forced vibrations?
29. What is the condition for resonance to occur?
30. Soldiers marching on a bridge are often ordered to go out of step why?
31. Why sound can not travel in vacuum, while light can?
32. What is Newton's formula for velocity of sound in a gas? How is it corrected by Laplace?
33. Explain why
    (a) Velocity of sound is generally greater in solids than in gases.
    (b) The velocity of sound in oxygen in lesser than in hydrogen.
34. Use the formula $\upsilon = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air
    (a) is independent of pressure and
    (b) Increases with temperature.
35. Three identical sound waves pass through an air column, a brass rod and an oil pipe of same length. In which of the three will it take the least time to reach the other end?
36. Does the law of conservation of energy hold good in case of interference of waves? Explain.

37. Two identical travelling waves moving in the same direction are out of phase by ϕ radians. What is the amplitude of the combined wave in terms of the common amplitude A of the two combining waves?
38. What is the resultant displacement of the particles when a compression falls on a rarefaction?
39. What happens when a crest falls on the crest during superposition of waves?
40. What are beats? Write the expression for beat frequency.
41. Mention any two applications of beats?
42. Distinguish between interference and beats.
43. Two waves of frequencies 256 & 250 are forming beats. What will be the beat frequency?
44. A sound wave travelling along an air column of a pipe gets reflected at the open end of the pipe. What is the phase difference between the incident and reflected pressure waves at the open end?
45. What is the ratio of frequencies of harmonics in an air column of same length in
    (a) a closed pipe and (b) an open pipe.
46. What is the distance between the closed end and open end of a pipe vibrating in the 7^{th} harmonic?
47. What is the distance between two ends of open pipe vibrating in the 3^{rd} overtone?
48. What is "End correction" in resonating air column?
49. If oil of density higher than water is used in a resonance tube, how will the frequency change?
50. What is Doppler effect? What is its limitation?
51. Why do we hear a higher frequency apparently when we approach a stationary sounding railway engine?
52. What is the reason for listening a higher frequency when a source of sound moves towards a stationary listener?
53. Write any two applications of Doppler effect.
54. Does the change in frequency due to Doppler effect depend
    (a) on distance between source and observer
    (b) on the fact that source is moving towards the listener or listener is moving towards the source?
55. What is the characteristic of sound which distinguishes a male voice and a female voice?

PROBLEMS

LEVEL - I

1. For the wave $y = 5\sin 30\pi\{t-(x/240)]$, where x and y are in cm and t is in seconds, find the
   (a) Displacement when t = 0 and x = 2cm,
   (b) Wavelength,
   (c) Velocity of the wave and
   (d) Frequency of the wave
   [Ans: (a) –3.535 cm ; (b) 16 cm ; (c) 240 cm/s ; (d) 15 Hz]
2. Figure shows a plot of the transverse displacement of the particle of a string at t = 0 through which a travelling wave is passing in the positive x-direction. The wave speed is 20 cm/s. Find (a) the amplitude, (b) the wavelength, (c) the wave number and (d) the frequency of the wave.



   [Ans: (a) 1.0 mm (b) 4 cm (c) 0.71 cm^{-1} (d) 5 Hz]
3. A wave of frequency 500 Hz has a wave velocity of 350 m/s.
   (a) Find the distance between two points which are 60^o out of phase.
   (b) Find the phase difference between two displacements at a certain point at times 10^{-3}s apart. **[Ans:** (a) 0.116 m ; (b) 180^o]
4. A sound wave of frequency 100 Hz is travelling in air. The speed of sound in air is 350 m/s.
   (a) By how much is the phase changed at a given point in 2.5 ms?
   (b) What is the phase difference at a given instant between two points separated by a distance of 10.0 cm along the direction of propagation?
   [Ans: (a) $\frac{\pi}{2}$; (b) $\frac{2\pi}{35}$]
5. A progressive wave of frequency 500 Hz is travelling with a speed of 350 m/s. A compressional maximum appears at a place at a given instant. Find the minimum time interval after which of a rarefaction maximum occurs at the same place. **[Ans:** $\frac{1}{1000}$s]

6. Calculate the speed of a transverse wave in a wire of 1.0 mm^2 cross-section under a tension of 0.98 N. Density of the material of wire is 9.8×10^3 kg/m^3. [Ans:10 m/s]

7. Two wires of different densities but same area of cross- section are soldered together at one end and are stretched to a tension T. The velocity of a transverse wave in the first wire is double of that in the second wire. Find the ratio of the density of the first wire to that of the second wire. [Ans : 0.25]

8. A 4.0 kg block is suspended from the ceiling of an elevator through a string having a linear mass density of 19.2×10^{-3} kg/m. Find the speed (with respect to the string) with which a wave pulse can proceed on the string if the elevator accelerates up at the rate of 2.0 m/s^2. Take g = 10 m/s^2 [Ans: 50 m/s]

9. Transverse waves are generated in two uniform wires A and B of the same material by attaching their free ends to a vibrating source of frequency 200Hz. The area of cross section of A is half that of B while tension on A is twice that on B. Find the ratio of wavelengths of the tranvsverse waves in A and B [Ans: 2 : 1]

10. Two progressive transverse waves given by $y_1 = 0.07 \sin\pi(12x - 500t)$ and $y_2 = 0.07 \sin\pi(12x + 500t)$ travelling along a stretched string form nodes and antinodes. What is the displacement at the (i) nodes (ii) antinodes. (iii) What is the wavelength? [Ans: (i) Zero; (ii) 0.14m ; (iii) 1/6 m]

11. A wave of wavelength 2m is superposed on its reflected wave to form a stationary wave. A node is located at x = 3m from fixed end. Find the next node location. [Ans: 4m]

12. The equation for the vibration of a string fixed at both ends vibrating in its third harmonic is given by $y = 2\text{cm} \sin[(0.6\,\text{cm}^{-1})x]\cos[(500\,\pi s^{-1})t]$. Find the length of the string [Ans: 15.7 cm]

13. Two wires are kept tight between the same pair of supports. The tensions in the wires are in the ratio 2:1, the radii are in the ratio 3:1 and the densities are in the ratio 1:2. Find the ratio of their fundamental frequencies. [Ans: 2:3]

14. A one metre long streched string having a mass of 40 g is attached to a tuning fork. The fork vibrates at 128 Hz in a direction perpendicular to the string. What should be the tension in the string if it is to vibrate in four loops? [Ans: 164 N]

15. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. Find the value of M. [Ans: 25kg]

16. A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 186 Hz ? [Ans:The string should be pressed at 60 cm from one end]

17. The frequency of a sonometer wire is 100Hz. When the weights producing the tensions are completely immersed in water the frequency becomes 80Hz and on immersing the weights in a certain liquid the frequency becomes 60Hz. Find the specific gravity of the liquid [Ans: 1.77]

18. When the tension in a string is increased by 44% the frequency increased by 10Hz. Find the frequency of the string . [Ans: 50 Hz]

19. A steel wire of length 1m, mass 0.1kg and uniform cross sectional area 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If the transverse waves are set up plucking the string in the middle, calculate the frequency of the fundamental mode of vibration. ($Y = 2 \times 10^{11}$ N/m^2, $\alpha = 1.21 \times 10^{-5}$/°C) [Ans: 11 Hz]

20. A metallic wire with tension T and at temperature 30°C vibrates with its fundamental frequency of 1 KHz. The same wire the same tension but at 10°C temperature vibrates with a fundamental frequency of 1.001KHz. Find the coefficient of linear expansion of the wire. [Ans: 1×10^{-4}/°C]

21. Three resonant frequencies of a string are 90, 150 and 210 Hz.
    (a) Find the highest possible fundamental frequency of vibration of this string.
    (b) Which harmonics of the fundamental are the given frequencies?
    (c) Which overtones are these frequencies?
    (d) If the length of the string is 80 cm, what would be the speed of a transverse wave on this string?

    [Ans: (a) 30 Hz; (b) 3rd, 5th and 7th ; (c) 2nd, 4th and 6th; (d) 48 m/s]

22. The equation of a travelling sound wave is $y=6.0 \sin (600t-1.8x)$ where y is measured in 10^{-5}m, t in second and x in metre.
    (a) Find the ratio of the displacement amplitude of the particles to the wavelength of the wave.
    (b) Find the ratio of the velocity amplitude of the particles to the wave speed.
    [Ans: (a) 1.7×10^{-5} ; (b) 1.1×10^{-4}]

23. The height of a cloud above the earth is 100 m. If an observer hears the sound of thunder 0.3 s after the lightening is seen what is the velocity of sound on that day? **[Ans:** 333.3 ms^{-1}]

24. In a liquid with density 900 kg/m^3, longitudinal waves with frequency 250 Hz are found to have wavelength 8.0 m. Calculate the bulk modulus of the liquid. **[Ans:**3.6×10^9 Pa]

25. The speed of sound as measured by a student in the laboratory on a winter day is 340 m/s when the room temperature is 17^0 C. What speed will be measured by another student repeating the experiment on a day when the room temperature is 32^0 C? **[Ans:** 349 m/s]

26. At what temperature will the speed of sound be double of its value at 0^0 C? **[Ans:** 819^0C]

27. The ratio of densities of nitrogen and oxygen is 14 : 16. Find the temperature at which the speed of sound in nitrogen will be same as that in oxygen at 55°C? **[Ans:** 14°C]

28. An organ pipe has two successive harmonics with frequencies 400 and 560 Hz. The speed of sound in air is 344 m/s.
    (a) Is this an open or a closed pipe?
    (b) What two harmonics are these?
    (c) What is the length of the pipe?
    [Ans: (a) Closed; (b) 5, 7; (c) 1.08m]

29. The fundamental frequency of a closed organ pipe is 220 Hz.
    (a) Find the length of this pipe
    (b) The second overtone of this pipe has the same wavelength as the third harmonic of an open pipe. Find the length of this open pipe. Take speed of sound in air 345 m/s.
    [Ans: (a) 0.392 m ; (b) 0.470 m]

30. A source of frequency 10 kHz when vibrated over the mouth of a closed organ pipe is in unison at 300K. Find the beats produced when temperature rises by 1K **[Ans:** 16.67 Hz]

31. A closed organ pipe and an open organ pipe of same length produce 2 beats when they are set into vibrations simultaneously in their fundamental mode. The length of open organ pipe is now halved and of closed organ pipe is doubled, find the number of beats produced. **[Ans:** 7]

32. A tuning fork produces 4 beats per second with the other tuning fork of frequency 256 Hz. The first one is now loaded with a little wax and the beat frequency is found to increase to 6 per second. What was the original frequency of the tuning fork? **[Ans:** 252 Hz]

33. Two tuning forks when sounded together produce 5 beats in 2 seconds. Find the time interval between two successive maximum intensities of sound **[Ans:** 0.4s]

34. Two progressive waves $y_1 = 4\sin 400\pi t$ and $y_2 = 3 \sin 404\pi t$ moving in the same direction superpose on each other producing beats. Find the number of beats per second and the ratio of maximum to minimum intensity of the resultant waves. **[Ans:** 2 and $\frac{49}{1}$]

35. Calculate the frequency of beats produced in air when two sources of sound are activated, one emitting a wavelength of 32 cm and the other of 32.2cm. The speed of sound in air is 350 m/s. **[Ans:** 7 per second]

36. Two strings 'x' and 'y' on a Veena playing the same note are slightly out of tune and produce 6 beats per second. The tension in 'x' is slightly decreased and it is found that the beats fall to 3 per second. If the original frequency of 'x' is 324 Hz what is the frequency of 'y'? **[Ans:** 318 Hz]

37. The frequency of a tuning fork 'A' is 2% greater than that of a standard fork 'K'. The frequency of another tuning fork 'B' is 3% less than 'K'. When 'A' and 'B' are vibrated together 6 beats per second are heard per second. Find the frequencies 'A' and 'B'. **[Ans:** 122.4 Hz, 116.4 Hz]

38. A man standing at some distance from a cliff hears the echo of sound after 2s. He walks 495 m away from the cliff. He produces a sound there and recieves the echo after 5s. What is the speed of sound? **[Ans:** 330 m/s]

39. A motor car approaching a cliff with a velocity of 90 kmph sounds the horn and the echo is heard after 20 seconds. Assuming the velocity of sound in air to be 332 ms^{-1}, calculate the distance between the car and the cliff when the horn is sounded. **[Ans:** 3570 m]

40. A person standing between two parallel hills fires a gun. He hears the first echo after 3/2 sec, and a second echo after 5/2 sec. If speed of sound is 332 m/s, calculate the distance between the hills. When will he hear the third, fourth, fifth and sixth echo?

[Ans: 10.664 m, 4 s, 5.5s, 6.5s, 8s]

41. A tuning fork of unknown frequency makes 5 beats per second with another tuning fork which can cause a closed organ pipe of length 40 cm to vibrate in its fundamental mode. The beat frequency decreases when the first tuning fork is slightly loaded with wax. Find its original frequency. The speed of sound in air is 320 m/s. **[Ans:** 205 Hz]

42. A traffic policeman standing on a road sounds a whistle emitting the main frequency of 2.00 kHz. What could be the appparent frequency heard by a scooter-driver approaching the policeman at a speed of 36.0 km/h? Speed of sound in air = 340 m/s.

[Ans: 2.06 KHz]

43. A person riding a car moving at 72 km/h sounds a whistle emitting a wave frequency 1250 Hz. What frequency will be heard by another person standing on the road (a) in front of the car(b) behind the car? Speed of sound in air =340 m/s.

[Ans: (a) 1328 Hz ; (b) 1181Hz]

44. A car moving at 108 km/h finds another car in front of it going in the same direction at 72 km/h. The first car sounds a horn that has a dominant frequency of 800 Hz. What will be the apparent frequency heard by the driver in the front car? Speed of sound in air = 330 m/s. **[Ans:** 827 Hz]

45. A tuning fork of frequency 328 Hz is moved towards a wall at a speed of 2 ms^{-1}. An observer standing on the same side as the fork hears two sounds, one directly from the fork and the other reflected from the wall. How many beats per second can be heard ? (Velocity of sound in air 330 ms^{-1}) **[Ans :** 4]

LEVEL - II

1. A wave is described by the equation

$$y = (1.0\text{ mm})\sin\pi\left(\frac{x}{2.0\text{cm}} - \frac{t}{0.01\text{s}}\right)$$

(a) Find the time period and the wavelength.

(b) Find the speed of the particle at x =1.0 cm at time t = 0.01 s.

(c) What are the speeds of the particles at x = 3.0cm, 5.0cm and 7.0 cm at t = 0.01 s?

(d) What are the speeds of the particles at x=1.0 cm at t = 0.012 and 0.013 s?

[Ans: (a) 20 m/s, 4.0 cm ; (b) Zero ; (c) zero ; (d) 18 cm/s, 25 cm/s]

2. At t = 0, a transverse wave pulse in wire is described by the function; $y = \frac{6}{x^2+3}$ Where x and y in metres. Write the function y(x,t) that describes this wave if it is travelling in the positive x-direction with a speed of 4.50 m/s.

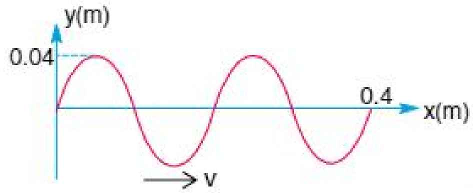
[Ans: $y = \frac{6}{[(x-4.5t)^2+3]}$]

3. A pulse travelling on a string is represented by the function $y = \frac{a^3}{(x-vt)^2+a^2}$ where a = 5 mm and v =20 cm/s. Sketch the shape of the string at t = 0, 1 s and 2 s. Take x = 0 in the middle of the string.

4. The position of a transverse wave travelling in medium along x-axis is shown in figure at time t = 0. Speed of wave is v = 200 m/s. Find

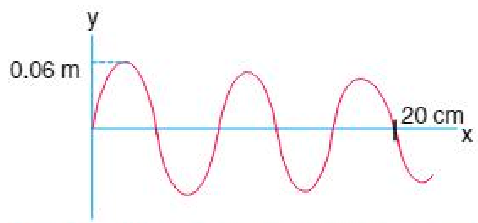
(a) Frequency of the wave

(b) Equation of the wave (in SI unit)



[Ans: (a) 10^3Hz ; (b) $y = 0.04\sin 2\pi(5x - 10^3 t)$]

5. For the wave shown in figure, find its amplitude, frequency and wavelength if its speed is 300 m/s. Write the equation for this wave as it travels out along the negative x-axis if its position at t = 0 is as shown



[Ans: Amplitude = 0.06 m; Frequency = 3750 Hz, Wavelength = 8cm; $y = (0.06\text{m})\sin[(78.5\text{m}^{-1})x + (23562\text{s}^{1})t]$]

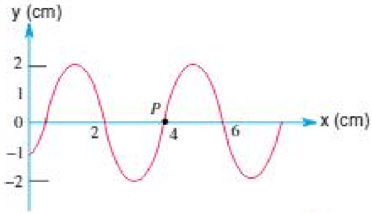
6. Consider a sinusoidal wave travelling in positive x direction as shown in figure. The wave velocity is 40 cm/s. Find:

(a) The frequency

(b) The phase difference between points 2.5 cm apart.

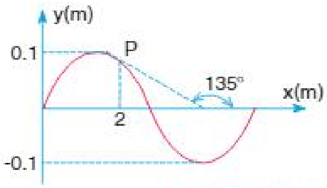
(c) How long it takes for the phase at a given position to change by 60^{o}

(d) The velocity of a particle at point P at the instant shown.



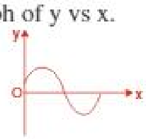
[Ans: (a) 10 Hz; (b) $\frac{5\pi}{4}$ rad; (c) $\frac{1}{60}$ sec ; (d) 1.26 m/s, downward]

7. A wave is travelling along a string. Its equation is given as $y = 0.1\sin 2\pi(100t + 10x)$ (All SI units) Position of different particles at some instant is shown in figure. What is velocity of particle P at this instant?



[Ans : 10m/s downward]

8. A wave propagates on a string in positive x-direction with speed of 40 cm/s. The shape of string at t = 2s is $y = 10\cos\frac{x}{5}$ where x and y are in centimeter. Find the wave equation and draw the graph of y vs x.

[Ans: $y = 10\cos\left(\frac{x}{5} - 8t + 16^{o}\right)$, 

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9. A wave progagates on a string in the positive x-direction at a velocity v. The shape of the string at $t=t_0$ is give by $f(x, t_0) = A\sin(x/a)$. Write the wave equation for a general time t.

[Ans: $f(x, t) = A\sin\frac{x - v(t - t_0)}{a}$]

10. A heavy but uniform rope of length L is suspended from a ceiling.

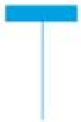
(a) Write the velocity of a transverse wave travelling on the string as a funtion of the distance from the lower end.

(b) If the rope is given a sudden sideways jerk at the bottom, how long will it take for the pulse to reach the ceiling?

(c) A particle is dropped from the ceiling at the instant the bottom end is given the jerk. Where will the particle meet the pulse?

[Ans: (a) $\sqrt{gx}$; (b) $\sqrt{4L/g}$; (c) At a distance L/3 from the bottom]

11. A wire of variable mass per unit length $\mu = \mu_0 x$, is hanging from the ceiling as shown in figure. The length of wire is l_0. A small transverse disturbance is produced at its lower end. Find the time after which the disturbance will reach to the other end.



[Ans: $\sqrt{\frac{8l_0}{g}}$]

12. Three pieces of string, each of length L, are joined together end-to-end, to make a combined string of length 3L. The first piece of string has mass per unit length μ_1, the second piece has mass per unit length $\mu_2 = 4\mu_1$ and the third piece has mass per unit length $\mu_3 = \mu_1/4$.

(a) If the combined string is under tension F, how much time does it take a transverse wave to travel the entire length 3L ? Give you answer in terms of L, F and μ_1.

(b) Does your answer to part (a) depend on the order in which the three piece are joined together? Explain.

[Ans: (a) $\frac{72}{2}\sqrt{\frac{\mu_1}{F}}$; (b) No]

13. A certain 120 Hz wave on a string has an amplitude of 0.160 mm. How much energy exists in an 80 g length of the string? Assume that one wavelength of the string has a mass far smaller than 80 g.

[Ans: 0.58 mJ]

14. A transverse wave of amplitude 0.50 mm and frequency 100 Hz is produced on a wire stretched to a tension of 100 N. If the wave speed is 100 m/s, what average power is the source transmitting to the wire ?

[Ans: 49mW]

15. $y_1 = 8\sin(\omega t - kx)$ and $y_2 = 6\sin(\omega t + kx)$ are two waves travelling in a string of area of cross-section s and density ρ. These two waves are superimposed to produce a standing wave.

(a) Find the energy of the standing wave between two consecutive nodes.

(b) Find the total amount of energy crossing through a node per second.

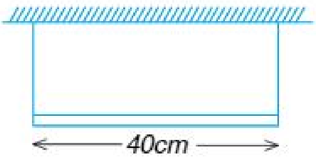
[**Ans:** (a) $\frac{50\pi\rho\omega^2 s}{k}$; (b) $\frac{2\rho\omega^2 s}{k}$]

16. In a stationary wave that forms as a result of reflection of waves from an obstacle, the ratio of the amplitude at an antinode to the amplitude at node is 6. What percentage of energy is transmitted ? [**Ans:** 49%]

17. In a stationary wave that forms as a result of reflection of waves from an ostacle the ratio of the amplitude at an antinode to the amplitude at node is 2. Find the fraction of energy reflected : [**Ans:** $\frac{1}{9}$]

18. A string, fixed at both ends, vibrates in a resonant mode with a separation of 2.0 cm between the consecutive nodes. For the next higher rersonant frequency, this separation is reduced to 1.6cm. Find the length of the string. [**Ans:** 8.0 cm]

19. A uniform horizontal rod of length 40 cm and mass 1.2 kgf is supported by two identical wires as shown.



Where should a mass of 4.8 kg be placed on the rod so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on right into its first overtone? Take g=10 m/s^2.

[**Ans:** 5 cm from the left end]

20. Two wires of radii r and 2r are welded together end to end. The combination is used as a sonometer wire and is kept under a tension T. The welded point is midway between the bridges. Find the ratio of the number of loops formed in the wires, such that the joint is a node when stationary vibrations are set up in the wires. [**Ans:** 1/2]

21. A light string is tied at one end to fixed support and to a heavy string of equal length L at the other end as shown in figure. Mass per unit length of the strings are μ and 9μ and the tension is T. Find the possible values of frequencies such that point A is a node/antinode.

[**Ans:** $\frac{f}{2}, f, \frac{3f}{2}$.... etc, when A is a node and $\frac{3}{2}f, \frac{5}{4}f, \frac{7}{4}f$ ... etc, when A is an antinode, Here, $f = \frac{1}{L}\sqrt{\frac{T}{\mu}}$]

22. The maximum pressure variation that the human ear can tolerate in loud sound is about 30 N/m^2. Find the corresponding maximum displacement for a sound wave in air having a frequency of 10^3 Hz : (take velocity of sound in air as 300 m/s and density of air 1.5 kg/m^3) [**Ans:** nearly 10^{-5} m]

23. Calculate the bulk modulus of air from the following data about a sound wave of wavelength 35 cm travelling in air. The pressure at a point varies between $(1.0 \times 10^5 \pm 14)$ Pa and the particles of the air vibrate in simple harmonic motion of amplitude 5.5×10^{-6} m. [**Ans:** 1.4×10^5 N/m^2]

24. The pressure variation in a sound wave is given by $\Delta P = 12\sin(8.18x - 2700t + \pi/4) N/m^2$. Find the displacement amplitude. [**Ans :** 1.04×10^{-5} m]

25. A point sound source is situated in a medium of bulk modulus 1.6×10^5 N/m^2. An observer standing at a distance 10 m from the source writes down the equation for the wave as $y = A\sin(15\pi x - 6000\pi t)$. Here y and x are in metres and t is in second. The maximum pressure amplitude tolerable by the observer's ear is 24π Pa. Find:

(a) the density of the medium.

(b) the displacement amplitude A of the waves received by the observer and

(c) the power of the sound source.

[**Ans:** (a) 1 kg/m^3; (b) 10 μm ; (c) 288 π^2 watt]

26. A tuning fork of frequency 440 Hz is attached to a long string of linear mass density 0.01 kg/m kept under a tension of 49 N. The fork produces transverse waves of amplitude 0.50 mm on the string.

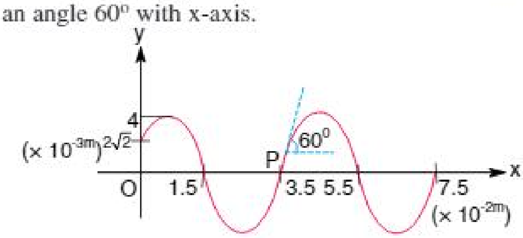
(a) Find the wave speed and the wavelength of the waves.

(b) Find the maximum speed of the particle.

(c) At what average rate is the tunig fork transmitting energy to the string?

[**Ans:** (a) 70 m/s, 16 cm ; (b) 13.8 km/s ; (c) 0.67W]

27. The figure shows a snap photograph of a vibrating string at t = 0. The particle P is observed moving up with velocity $20\sqrt{3}$ cm/s. The tangent at P makes an angle 60^o with x-axis.



(a) Find the direction in which the wave is moving.

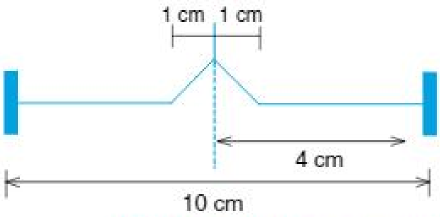
(b) Write the equation of the wave.

(c) Find the total energy carried by the wave per cycle of the string. Assume that μ the mass per unit length of the string = 50 g/m.

[Ans: (a) Negative x ;

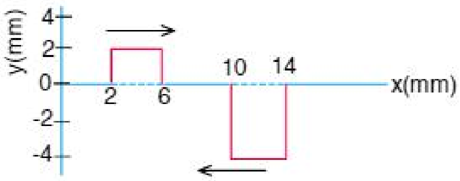
(b) $y = 0.4\sin\left[10\pi t - \frac{\pi}{2}x + \frac{3\pi}{4}\right]$; (c) 1.6×10^{-5} J]

28. A string that is 10 cm long is fixed at both ends. At t = 0, a pulse travelling from left to right at 1 cm/s is 4.0 cm from the right end as shown in figure. Determine the next two times when the pulse will be at that point again. State in each case whether the pulse is upright or inverted.

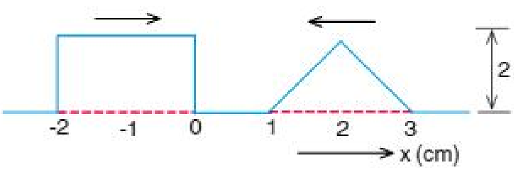


[Ans: 8 sec, Inverted, 20 sec upright]

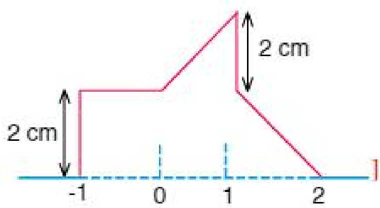
29. Figure shows two wave pulses at t = 0 travelling on a string in opposite directions with the same wave speed 50 cm/s. Sketch the shape of the string at t = 4ms, 6ms, 8ms, and 12ms.



30. Figure shows a rectangular pulse and triangular pulse approaching each other. The pulse speed is 0.5 cm/s. Sketch the resultant pulse at t = 2s.



[Ans :



Resultant Pulse]

31. Two wires of different densities are soldered together end to end then stretched under tension T. The wave speed in the first wire is twice that in the second wire.

(a) If the amplitude of incident wave is A, what are amplitudes of reflected and transmitted waves?

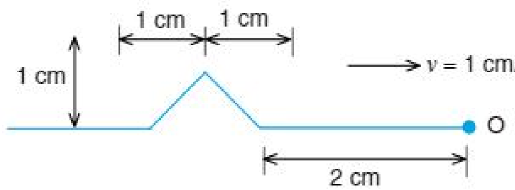
(b) Asssuming no energy loss in the wire, find the fraction of the incident power that is reflected at the junction and fraction of the same that is transmitted.

[Ans: (a) $-\frac{A}{3}, \frac{2}{3}A$; (b) $\frac{1}{9}, \frac{8}{9}$]

32. A wave pulse on a string has the dimensions shown in figure. The wave speed is v = 1 cm/s.

(a) If point O is a fixed end, draw the total wave on the string at t = 3s and t = 4s.

(b) Repeat part (a) for the case in which O is a free end.



33. A wave $y_i = 0.3\cos(0.2x - 40t)$ is travelling along a string toward a boundary at x = 0. Write expressions for the reflected waves if

(a) The string has a fixed end at x = 0 and

(b) The string has a free end at x = 0. Assume SI units.

[Ans: (a) $y(x,t) = 0.3\cos(2.0x + 40t + \pi)$;

(b) $y(x,t) = 0.3\cos(2.0x + 40t)$ SI units]

34. The harmonic wave $y_i = (20\times10^{-3})\cos\pi(2.0x - 50t)$ travels along a string toward a boundary at x = 0 with a second string. The wave speed on the second string is 50 m/s. Write expressions for reflected and transmitted waves. Assume SI units.

[Ans: (a) $(6.67\times10^{-4})\cos\pi\ (2.0x + 50t)$;

(b) $(2.67\times10^{-3})\cos\pi\ (1.0x - 50t)$ SI units]

35. Three source of sound S_1, S_2 and S_3 of equal intensity are placed in a straight line with $S_1S_2 = S_2S_3$ At a point P, far away from the sources, the wave coming from S_2 is 120^0 ahead in phase of that from S_1. Also, the wave coming from S_3 is 120^0 a head of that from S_2. What would be the resultant intensity of sound at P?

[Ans: zero]

36. Two audio speakers are kept some distance apart and are driven by the same amplifier system. A person is sitting at a place 6.0m from one of the speakers and 6.4m from the other. If the sound signal is continuously varied from 500 Hz to 5000Hz, what are the frequencies for which there is a destructive interference at the place of the listener ? Speed of sound in air = 320 m/s.

[Ans: 1200Hz, 2000Hz, 2800Hz, 3600Hz and 4400Hz.]

37. Two loudspeakers radiate in phase at 170 Hz. An observer sits at 8 m from one speaker and 11 m from the other. The intensity level from either speaker acting alone is 60 dB. The speed of sound is 340 m/s.

(a) Find the observed intensity level when both speakers are on together.

(b) Find the observed intensity level when both speakers are on together but one has its leads reversed so that the speakers are 180^o out of phase.

(c) Find the observed intensity level when both speakers are on and in phase but the frequency is 85 Hz. **[Ans:** (a) 0 ; (b) 66dB ; (c) 63 dB]

38. In a resonance tube experiment to determine the speed of sound in air, a pipe of diameter 5 cm is used. The column in pipe resonates with a tuning fork of frequency 480 Hz when the minimum length of the air column is 16 cm. Find the speed of sound in air column in air at room temperature. **[Ans:** 336 m/s]

39. In a resonance column experiment, a tuning fork of frequency 400 Hz. is used. The first resonance is observed when the air column has a length of 20.0 cm and the second resonance is observed when the air column has a length of 62.0 cm.

(a) Find the speed of sound in air?

(b) How much distance above the open end does the pressure node form? **[Ans:** (a) 336 m/s (b) 1 cm]

40. A small source of sound oscillates in simple harmonic motion with an amplitude of 17 cm. A detector is placed along the line of motion of the source. The source emits a sound of frequency 800 Hz which travels at a speed of 340 m/s. If the width of the frequency band detected by the detector is 8 Hz, find the time period of the source. **[Ans:** 0.635]

41. A boy riding on his bike is going towards east at a speed of $4\sqrt{2}$ m/s. At a certain point he produces a sound pulse of frequency 1650 Hz that travels in air at a speed of 334 m/s. A second boy stands on the ground 45^0 south of east from him. Find the frequency of the pulse as received by the second boy.

[Ans: 1670 Hz]

42. A source emitting sound at frequency 4000 Hz, is moving along the Y-axis with a speed of 22 m/s. A listener is situated on the ground at the position (660 m, 0). Find the frequency of the sound received by the listener at the instant the source crosses the origin. Speed of sound in air = 330 m/s.

[Ans: 4018 Hz]

43. A whistle of frequency 1300Hz is at a height 100 m above the ground. A detector is projected upwards with velocity 50 ms^{-1} from the ground along the same line. If the velocity of sound is 300 ms^{-1}, find the frequency detected by the detector after 2s.

[Ans: 1430Hz]

44. A source of sonic oscillations with frequency 1700Hz and a receiver are located at same point. At the moment t = 0, the source starts receding from the receiver with constant acceleration $10 m/s^2$. Assuming the velocity of sound to be 340 m/s, find the frequency registered by the stationary receiver after 10s of the starts of motion. **[Ans:** 1350Hz]

45. A sound source moves with a speed of 80 m/s relative to still air toward a stationary listener. The frequency of sound is 200Hz and speed of sound in air is 340 ms.

(a) Find the wavelength of the sound between the source and the listener.

(b) Find the frequency heard by the listener.

[Ans: (a) 1.3 m; (b) 262 Hz]

46. A railroad train is travelling at 30 m/s in still air. The frequency of the note emitted by the locomotive whistle is 500 Hz. What is the wavelength of the sound waves :

(a) in front of the locomotive?

(b) behind the locomotive? What is the frequency of the sound heard by a stationary listener

(c) In front of the locomotive?

(d) behind the locomotive? Speed o f sound in air is 344 m/s.

[Ans: (a) 0.628m; (b) 0.79 m ; (c) 548 Hz; (d) 460 Hz]

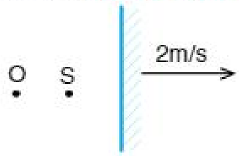
47. Two sound sources are moving in opposite directions with velocities v_1 and $v_2 (v_1 > v_2)$. Both are moving away from a stationary observer. The frequency of both the sources is 900 Hz. What is the value of $v_1 - v_2$ so that the beat frequency observed by the observer is 6 Hz. Speed of sound v = 300 m/s. Given, that v_1 and $v_2 << v$. **[Ans:** 2m/s]

48. A 300 Hz source, an observer and wind are moving as shown in the figure with respect to the ground. What frequency is heard by the observer?

[Ans: 274 Hz]

49. A person standing on a road sends a sound signal to the driver of a car going away from him at a speed of 72 km/h. The signal travelling at 330 m/s in air and having a frequency of 1600 Hz gets reflected from the body of the car and returns. Find the frequency of the reflected signal as heard by the person. [Ans:1417Hz]

50. A stationary sound sources of frequency 334 Hz and a stationary observer 'O' are placed near a reflecting surface moving away from the source with velocity 2m/sec as shown in the figure. If the velocity of sound waves in air is 330 m/sec, find the apparent frequency of the echo



[Ans: 334 Hz]

51. Spherical waves are emitted from a 1.0 watt source in an isotropic non-absorbing medium. What is the wave intensity 1.0 m from the source? [Ans: $\frac{1}{4\pi}$ watt / m^2]

52. The intensity of sound from a point source is $1.0\times10^{-8}\ W/m^2$ at a distance of 5.0m from the source. What will be the intensity at a distance of 25m from the source? [Ans: $4.0\times10^{-10}\ W/m^2$]

53. Most people interpret a 9.0 dB increase in sound intensity level as a doubling in loudness. By what factor must the sound intensity be increased to double the loudness? [Ans: 7.9]

54. About how many times more intense will the normal ear perceive a sound of $10^{-6}\ W/m^2$ than one of $10^{-9}\ W/m^2$? [Ans: Two times]

55. The explosion of a fire cracker in the air at a height of 40 m produces a 100 dB sound level at ground below. What is the instantaneous total radiated power? Assume that it radiates as a point source.

[Ans: 201 watt]

56. The sound level at a point 5.0 m away from a point source is 40 dB. What will be the level at a point 50 m away from the source? [Ans: 20 dB]

57. If the intensity of sound is doubled, by how many decibels does the sound level increase? [Ans: 3dB]

58. Sound with intensity larger than 120 dB. appears painful to a person. A small speaker delivers 4W of audio output. How close can the person get to the speaker without hurting his ears? [Ans: $\frac{1}{\sqrt{\pi}}$ m]

59. Three component sinusoidal waves progressing in the same direction along the same path have the same period but their amplitudes are A, A/2 and A/3. The phases of the variation at any position x on their path at time t = 0 are 0, $-\frac{\pi}{2}$ and $-\pi$ respectively. Find the amplitude and phase of the resultant wave.

[Ans: $\frac{5}{6}A, -\tan^{-1}\left(\frac{3}{4}\right)$]

60. A soldier walks towards a high wall taking 120 steps per minute. When he is at a distance of 90 m from the wall he observes that echo of step coincides with the next step. Find the speed of sound [Ans: 360 m/s]

61. A man stands before a large wall at a distance of 50.0m and claps his hands at regular intervals. Initially, the interval is large. He gradually reduces the interval and fixes it at a value when the echo of a clap merges with the next clap. If he has to clap 10 times during every 3 seconds, find the velocity of sound in air. [Ans:333 m/s]

62. A road runs midway between two parallel rows of buildings. A motorist moving with a speed of 36 Km/h sounds the horn. He hears the echo one second after he has sounded the horn. Find the distance between the two rows of buildings. (velocity of sound in air is 330 m/s)

[Ans: $80\sqrt{17}$m]

[Ans: 485 Hz and 515 Hz]