Chapter - 3

CAPACITORS

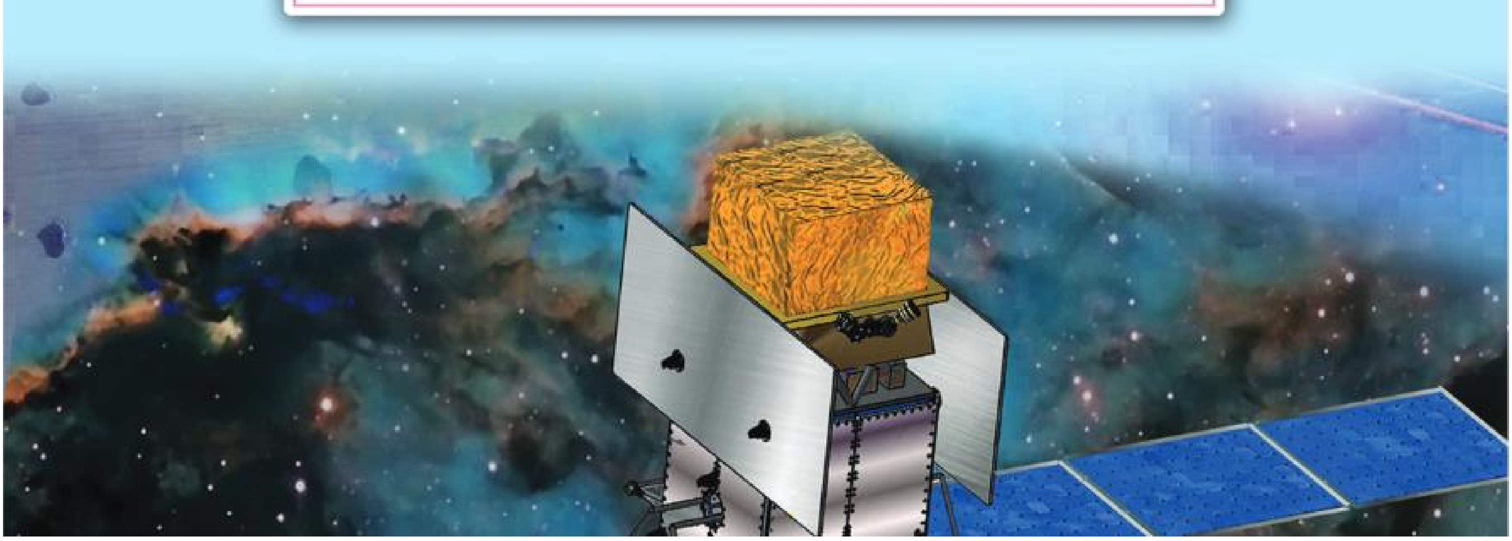
✤ Capacitance - Principle ✤

✤ Parallel plate Capacitor ✤

✤ Spherical and Cylindrical capacitors ✤

✤ Energy stored in capacitor ✤

✤ Types of capacitors and uses, Dielectrics, Polarisation ✤



3.1 INTRODUCTION

Capacitor is a device which is used to store electrical energy in the same way that a tank is a device for storing water or gas. Different containers with different volumes will have different capacities to hold a liquid or gas. Similarly different capacitors will have different capacities to hold electric charge. The capacity of a capacitor to hold charge depends on the size, shape and surrounding medium of the conductor. The capacity of a capacitor or condenser to hold charge is called it's capacitance or capacity. A given container will have fixed capacity of holding a liquid or gas which does not depend on the quantity to be stored. Similarly capacity of a capacitor does not depend on the stored charge.

A combination of two conductors of any shape placed close to each other constitutes a capacitor or condenser. When a capacitor is charged, one of the two conductors will be given a positive charge (+q) and the other is given a negative charge (–q) of same magnitude. The charge on the positive plate or conductor is called the charge on the capacitor. The potential difference between the plates of a capacitor is known as potential of the capacitor ($V=V_+ - V_-$). Charge on the capacitor does not mean the total charge (total charge is infact zero). Fig 3.1(a) shows two conductors forming a capacitor and Fig 3.1(b) shows the symbol used to represent a capacitor.

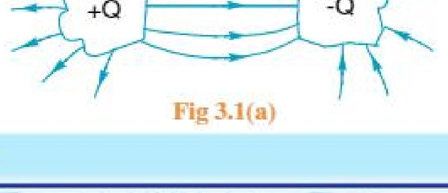


Fig 3.1(a)

Fig 3.1(b)

3.2 CAPACITANCE

If the charge on an isolated conductor is gradually increased, its potential also gets increased. (If the amount of water in a container is increased, level of water also gets increased). At any instant, the charge Q on the conductor or capacitor is directly proportional to the potential V of the capacitor.

$$\Rightarrow Q \alpha V \text{ (or) } Q = CV$$

The proportionality constant C is called the capacitance of the capacitor. It depends on the shape, size of conductors, their nearness to each other and the medium between them. It does not depend on the charge or potential difference.

So capacitance $C = \dfrac{Q}{V}$ .... (3.1)

i.e., ***"The ratio of charge on a capacitor to its potential is known as capacitance or capacity of that capacitor".*** It is numerically equal to the charge to be given to a capacitor to raise its potential by one unit.

The S.I unit of capacitance is coulomb/volt which is called farad (Symbol is F)

$$1F = \frac{1C}{1V}$$

Farad is a large unit for practical purpose. So, its submultiples are used more frequently like

1 micro farad $(1\mu F) = 10^{-6} F$

1 pico farad $(1pF) = 10^{-12} F$

3.3 PRINCIPLE OF A CAPACITOR

The simplest arrangement of a capacitor is two parallel metal plates seperated by air or a dielectric material as shown. The capacitance of a conductor can be increased by increasing its size. But for a given size, the other way of increasing the capacitance is by lowering the potential without altering the charge on the conductor. This is the principle of a capacitor.

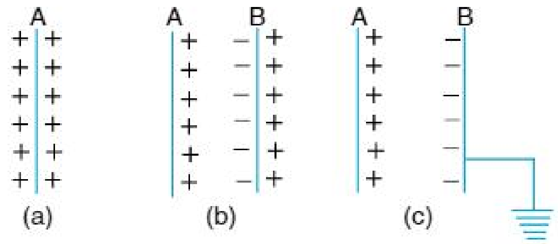


Fig 3.2

Consider a metal plate A to which positive charge Q is given. The charge distributes on A as shown in Fig 3.2(a) its potential raises to V. If C is the capacitance of the conductor, Q = CV. So at this potential V, conductor can not hold charge more than Q. Now as shown in Fig 3.2(b), an uncharged metal plate B is kept parallel to A and closer to it. The positive charge on A induces an equal amount of negative charge on B on its inner side. As the plate B is uncharged, an equal amount of positive charge is induced on the outer side of B. Here the induced negative charge on B lowers the potential of A, where as the induced positive charge on B increases the potential of A. But as the negative charge on B is nearer to the plate A, its effect will be more.

Now the outer side of plate B is earthed as shown in Fig 3.2(c). The induced positive charge on B gets neutralized due to earthing while the induced negative charge on the inner side of B is held by the attraction of positive charge on A. The negative charge on B lowers the potential of A and so some more charge can be added to A to rise its potential to V. It means the capacitance or charge holding ability of plate A is increased. In this case the capacitance can be further increased by filling space between the plates, with a dielectric.

The principle of a capacitor is to increase the capacitance of a conductor by bringing nearer to it, an uncharged conductor and earthing the outer surface of the uncharged conductor. Capacitor can be of any shape, e.g. parallel plate capacitors, spherical capacitors and cylindrical capacitors are in wide use. The capacitance of a capacitor depends on

1) Geometry of the plates
2) Separation between the plates and
3) The dielectric medium between the plates.

- In a capacitor one plate need not necessarily be earthed. The two plates must have equal and opposite charges.
- The method for the calculation of capacitance requires integration of the electric field between two conductors or the plates which are seperated with a potential difference V_{ab}

  i.e., $V_{ab} = -\int_b^a \overline{E}.d\overline{r}$ or $V_+ - V_- = -\int \overline{E}.d\overline{r}$

  from this $C = \dfrac{q}{V_{ab}}$

Example-3.1

Two conductors carrying equal and opposite charges produce a non uniform electric field along X - axis given by $E = \dfrac{Q}{\epsilon_0 A}(1+Bx^2)$ where A and B are constants. Separation between the conductors along X-axis is X. Find the capacitance of the capacitor formed.

Solution :

Potential difference between the conductors is given by

$$V = V_+ - V_- = \int_0^X E dx \Rightarrow V = \int_0^X \frac{Q}{\epsilon_0 A}(1+Bx^2)dx$$

$$\text{or } V = \frac{Q}{\epsilon_0 A}\left(X + \frac{BX^3}{3}\right)_O^X = \frac{Q}{\epsilon_0 A} X\left(1+\frac{BX^2}{3}\right)$$

$$\text{Capacity } C = \frac{Q}{V} = \frac{\epsilon_0 A}{X\left(1+\frac{Bx^2}{3}\right)}$$

Example-3.2

Find the capacitance of a system of two identical metal balls of radius a if the distance between their centres is equal to b, with b>>a. The system is located in a uniform dielectric with permittivity K.

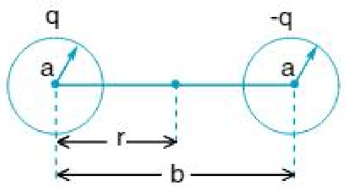
Solution :

Let q and –q be the charges on two balls. Then

$V_1 = V_{ball} - V_\infty = V$

$V_2 = V_{ball} - V_\infty = -V$

The potential difference between the balls



$$V_1 - V_2 = 2V = 2\int_a^{b-a} E\,dr$$

$$= 2\int_a^{b-a}\left(\frac{1}{4\pi\epsilon_0 K}\frac{q}{r^2}\right)dr = \frac{2q}{4\pi\epsilon_0 K}\left[\frac{1}{a} - \frac{1}{b-a}\right]$$

$$C = \frac{q}{V_1 - V_2} = \frac{q}{\left[\frac{2q}{4\pi\epsilon_0 K}\frac{(b-2a)}{a(b-a)}\right]}$$

$$= \frac{2\pi\epsilon_0 K a(b-a)}{(b-2a)}$$

For b >> a, we can write $C = 2\pi\epsilon_0 Ka$.

3.4 PARALLEL PLATE CAPACITOR

Let us consider a parallel plate capacitor which consists of two parallel plates each of area A, seperated by a distance 'd' as shown in figure.

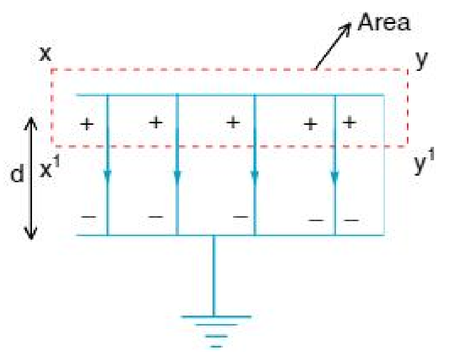


Fig 3.3

Let the upper plate be provided with charge +q and the bottom plate earthed. Then there will be -ve charge induced on the side which faces the positively charged plate. As the plates are uniform and parallel the distribution of charge will be uniform throughout its face (one face only). Hence the electric field will be uniform between the two plates. Let 'E' be the electric field intensity at any point between the plates.

Let a medium of relative permitivity K occupy the entire space between the two plates. Let xyy^1x^1 be a rectangular Gaussian surface enclosing the charged plate. From Gauss law entire flux coming from the Gaussian surface is

$$\phi = \int \overline{E}.\overline{ds} = E\int \overline{ds} = EA \qquad .... (a)$$

$$\text{and } \int E.ds = \frac{q}{K\epsilon_0} \qquad .... (b)$$

equating (a) and (b)

$$EA = \frac{q}{K\epsilon_0};\quad E = \frac{q}{A\varepsilon_0 K}$$

If 'V' is the potential difference between the two plates then $E = \frac{V}{d}$.

$$\left(\frac{V}{d}\right) = \frac{q}{A\epsilon_0 K}$$

Hence the capacitance of parallel plate capacitor.

$$C = \frac{q}{V} \text{ or } C = \frac{\epsilon_0 KA}{d} \qquad ....(3.2)$$

3.5 PARALLEL PLATE CAPACITOR PARTLY FILLED WITH MEDIUM

Let us consider a case of parallel plate capacitor in which a medium of dielectric constant K is partly filled as shown in figure.

Then the field though uniform in air as well as in medium it will have different values. Let 't' be the thickness of the medium whose relative permitivity is K. The remaining space of (d – t) thickness will be occupied by air.

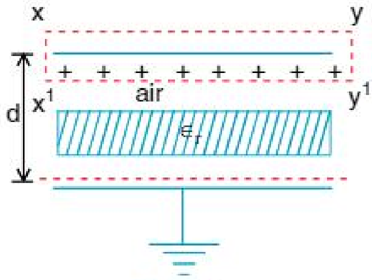


Fig 3.4(a)

Imagine a Gaussian surface enclosing the plate as shown.

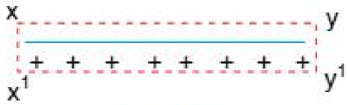


Fig 3.4(b)

If E_0 is the field in air, then

$$\int E_0 ds = E_0 A = \frac{q}{\varepsilon_0} \text{ or } E_0 = \frac{q}{A\varepsilon_0} \quad \text{.... (a)}$$

Similarly by considering a Gaussian surface through the medium, we get from Gauss's law,

$$\int E.ds = \frac{q}{\varepsilon_0 K} = EA$$

where E is a field in the medium

$$\therefore E = \frac{q}{A\varepsilon_0 K} \quad \text{.... (b)}$$

The P.D. between the two plates of the capacitor.

$$V = E_0(d-t) + E.t$$

$$V = \frac{q}{A\varepsilon_0}(d-t) + \frac{q}{A\varepsilon_0 K}t$$

$$= \frac{q}{A\varepsilon_0}\left[(d-t) + \frac{t}{K}\right]$$

$$\text{or } C = \frac{q}{V} = \frac{q}{\frac{q}{A\varepsilon_0}[d - t + t/K]}$$

$$C = \frac{A\varepsilon_0}{[(d-t) + (t/K)]} \quad \text{.... (3.3)}$$

- On comparison we can conclude that capacitance of capacitor with multiple media is given by

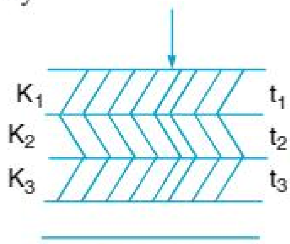


Fig 3.4(c)

$$C = \frac{A\varepsilon_0}{\sum \frac{\text{Thickness of medium}}{\text{Relative permitivity}}}$$

$$C = \frac{\varepsilon_0 A}{\left[\frac{t_1}{K_1} + \frac{t_2}{K_2} + \dots\dots\right]} \quad \text{.... (3.4)}$$

3.6 CHARGING A CAPACITOR

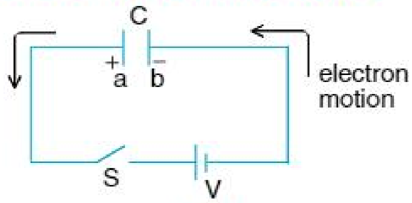
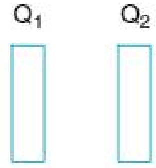


Fig 3.5

Consider a capacitor C connected in a circuit with a battery as shown. Here battery provides the energy and pulls the electrons from plate 'a' and send them to plate b as shown. As a result positive charge on plate 'a' and negative charge on plate 'b' start increasing. The potential difference between the plates increases until it equals the applied voltage V between the terminals of the battery. Finally plate 'a' and the terminal of the battery connected to that plate will be at the same potential. There will be no longer an electric field in the conducting wires. Similarly plate b and the terminal of the battery connected to that plate also will be at the same potential and no electric field in the connecting wire between them. As there is no electric field, no further motion of electrons will be there in the connecting wires. Now we say that capacitor is fully charged with a potential difference V and charge Q = CV.

Example-3.3

Charges Q_1 and Q_2 are given to two identical metal plates. Find the resultant charge distribution on the four surfaces of the plates. Find the capacitance of such an arrangement shown in the figure.

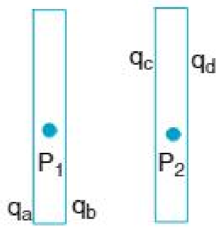


(Area of each plate is A and separation between the plates is d. Ignore fringing effect)

Solution :

Let us assume the charges on the four surfaces of the plates as shown. From the given data,

$Q_1 = q_a + q_b$; $Q_2 = q_c + q_d$



Here $q_b = -q_c$ (due to induction)

$\Rightarrow q_b = Q_1 - q_a$ and $q_d = Q_2 + q_b$

So $q_d = \dfrac{Q_1 + Q_2}{2}; q_b = \dfrac{Q_1 - Q_2}{2}$

$q_c = \dfrac{Q_2 - Q_1}{2}; q_a = \dfrac{Q_1 + Q_2}{2}$

We can get this by putting E = 0 at P_1

$$\frac{q_a}{2A\in_0} - \frac{q_b}{2A\in_0} - \frac{q_c}{2A\in_0} - \frac{q_d}{2A\in_0} = 0$$

or $q_a - q_b - q_c - q_d = 0$

Similarly at P_2 also E = 0

$$\text{or } \frac{q_a}{2A\in_0} + \frac{q_b}{2A\in_0} + \frac{q_c}{2A\in_0} - \frac{q_d}{2A\in_0} = 0$$

$q_a + q_b + q_c - q_d = 0$

$\Rightarrow q_a - q_b = Q_2$ and $q_d - q_c = Q_1$

$q_a + q_d = \dfrac{Q_1 + Q_2}{2}$ and $q_b - q_c = Q_1 - Q_2$

from which we can get q_a, q_b, q_c and q_d.

The charges on the inner surfaces are equal and opposite. The field between the plates is

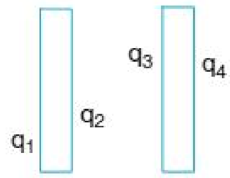
$$E = \frac{(q_b - q_c)}{A\in_0} = \frac{(Q_1 - Q_2)}{A\in_0}$$

The potential difference between the plates is given by $V = Ed = \dfrac{(Q_1 - Q_2)d}{A\in_0}$

Capacitance $C = \dfrac{q}{V} = \dfrac{(Q_1 - Q_2)}{V} \Rightarrow C = \dfrac{\in_0 A}{d}$

Example-3.4

An isolated parallel plate capacitor of capacitance C has four surfaces with charges q_1, q_2, q_3 and q_4 as shown. Find the potential difference between the plates?



Solution :

We know that conducting surfaces facing each other must have equal and opposite charges

$\Rightarrow q_2 = -q_3$

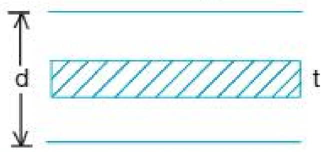
We also know that charge on capacitor means charge on the inner surface of the positive plate, ie q_2 here

Now potential difference between the plates is

$$V = \frac{q_2}{C} = \frac{2q_2}{2C} = \frac{q_2 - (-q_2)}{2C} \text{ or } V = \frac{q_2 - q_3}{2C}$$

Example-3.5

The capacity of an air filled parallel plate capacitor is C_0. A dielectric slab of thickness t with dielectric constant k is filled between the plates as shown.



To have the capacitance of the arrangement C_0 again, the plate separation must be increased by x. Find x.

Solution :

Initially $C_0 = \dfrac{\in_0 A}{d}$

After introducing dielectric

$$C = \frac{\in_0 A}{d - t + \frac{t}{k}} = \frac{\in_0 A}{d - t(1 - \frac{1}{k})}$$

If the plate seperation is increased by x

$$C^1 = \frac{\in_0 A}{d + x - t(1 - \frac{1}{k})}$$

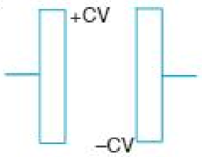
But $C^1 = C_0 \Rightarrow d = d + x - t\left(1 - \dfrac{1}{k}\right)$ or $x = t\left(1 - \dfrac{1}{k}\right)$

Example-3.6

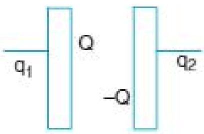
A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. Now a charge +q is given to its positive plate. Then find the potential difference across the capacitor.

Solution :

Initially the charges on the capacitor plates are as shown ie +CV and –CV on the surfaces of the plates facing each other.



When +q is given to +ve plate, the charges will be as shown



From conservation of charge,

$q_1 + Q = CV + q$ and $q_2 - Q = -CV$

Charges on the outer surfaces are

$q_1 = (q + CV) - Q$ and $q_2 = Q - CV$

$\Rightarrow Q = CV + \frac{q}{2} \quad (\because q_1 = q_2)$

Potential difference $= \frac{Q}{C} = V + \frac{q}{2C}$

3.7 CAPACITORS IN SERIES

Let us consider three capacitors connected in series and are kept across a source 'V' as shown in figure.

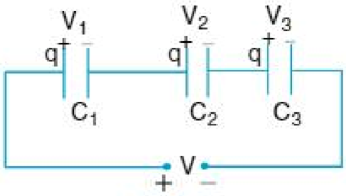


Fig 3.6

The moment the system is connected to the source, left plate of first condenser acquires positive charge. This inturn will produce negative charge of equal magnitude, on the left face of second plate of first condenser. The process continues for the remaining two condensers. Hence the charge acquired by all the three capacitors will be same.

As the capacitors are different, the potentials developed across them will be different.

$$q = C_1V_1 = C_2V_2 = C_3V_3$$

$$V_1 = \frac{q}{C_1},\ V_2 = \frac{q}{C_2},\ V_3 = \frac{q}{C_3}$$

But $V = V_1 + V_2 + V_3$

$$V = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad \text{.... (a)}$$

If a single capacitor when connected across the same supply draws the same charge, that capacitance is said to be the equivalent capacitance of the three capacitors. If C_S is the equivalent capacitance.

$$C_S = \frac{q}{V};\quad V = \frac{q}{C_S} \quad \text{.... (b)}$$

Substituting (b) in (a)

$$\frac{q}{C_S} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3};\quad \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_S} = \sum \frac{1}{C_n} \quad \text{.... (3.5)}$$

3.8 CAPACITORS IN PARALLEL

Capacitors are said to be connected in parallel if the two plates of any capacitor are connected one to positive terminal and the other to negative terminal of the source, as shown, the connection is said to be parallel connection.

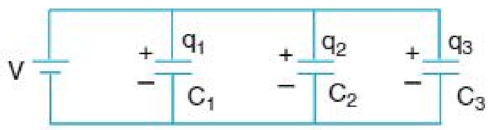


Fig 3.7

The moment capacitors are connected, charge is drawn from the voltage source and this charge is drawn along three branches and thus gets shared. As all capacitors are connected in parallel, the potential across any of the capacitors is same. Here charge gets shared depending upon their capacitances for maintaining same potential.

$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3}$$

$$\therefore q_1 + q_2 + q_3 = C_1V + C_2V + C_3V$$

$$q = V(C_1 + C_2 + C_3)$$

$$\frac{q}{V} = C_1 + C_2 + C_3$$

If a single capacitor when connected to the same potential difference draws a charge q then that capacitor is said to be the effective or equivalent capacitor for the three parallel capacitors.

If the effective capacitance is C_p;

$$C_P = \frac{q}{V} \quad .... (b)$$

from (a) and (b)

$$C_P = C_1 + C_2 + C_3 \,;\; C_P = \sum C_n \quad .... (3.6)$$

3.9 ENERGY STORED IN CAPACITOR

The energy of a capacitor is the amount of work done to charge it. Any charged conductor will store electric potential energy in the form of electrostatic field.

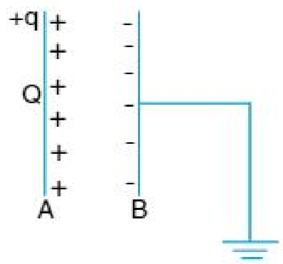


Fig 3.8

During charging, let 'q' be the charge at any instant. If a further charge dq is to be given to this capacitor, work is to be done against the potential it acquired. If 'C' is the capacitance of capacitor q = CV where V is the potential when the charge is q. The work done 'dw' to increase its charge from q to q + dq is

$$dW = Vdq = \frac{q}{C}dq$$

Hence the total work done in increasing the charge to +Q, we have to integrate the function from 0 to Q.

$$\int_0^W dW = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left(\frac{q^2}{2}\right)_0^Q$$

$$W = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \quad .... (3.7)$$

$$(\because Q = CV)$$

Example-3.7

If a battery is used to charge a capacitor, work done by the battery is W and energy stored in the capacitor is U. How these two are related ?

Solution :

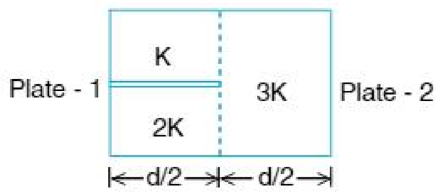
Let q be the charge delivered by the battery to the capacitor. Then work done by the battery is

$W = Vq$ and energy stored is $U = \frac{1}{2}qV \Rightarrow U = W/2$

The remaining energy, W – U, which is equal to $\frac{1}{2}qV$ or U, will be lost as heat and light.

Example-3.8

A parallel plate capacitor is constructed using three different dielectric materials as shown in the figure. Seperation between the plates of the capacitor is d. Find the equivalent capacitance between the plates



Solution :

Let C_1, C_2 and C_3 be the capacitances of the parts with dielectrics K, 2K and 3K respectively

Then $C_1 = \frac{K \in_0 A/2}{d/2} = \frac{\in_0 KA}{d}$

$$C_2 = \frac{2K \in_0 A/2}{d/2} = \frac{2 \in_0 KA}{d}$$

$$C_3 = \frac{3K \in_0 A}{d/2} \,;\Rightarrow C_3 = \frac{6K \in_0 A}{d}$$

C_1, C_2 in parallel are in series with C_3

$$C_1 + C_2 = \frac{3 \in_0 kA}{d} = C_4$$

$$C_{eff} = \frac{C_3 \times C_4}{C_3 + C_4} = \frac{\in_0 kA}{d}\left(\frac{1}{3} + \frac{1}{6}\right)^{-1} = \frac{2 \in_0 kA}{d}$$

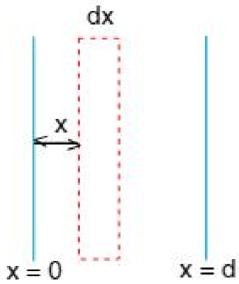
Example-3.9

Calculate the capacitance of a parallel plate capacitor, with plate area A and distance between the plates d, when filled with a dielectric whose permittivity varies as

$$\in(x) = \in_0 + kx\left(0 < x < \frac{d}{2}\right)$$

$$\in(x) = \in_0 + k(d-x)\left(\frac{d}{2} < x \le d\right)$$

Solution :



The given capacitor is equivalent to two capacitors in series. Let C_1 and C_2 be their capacities. Then

$$\frac{1}{C} = \int\left[\frac{1}{dC_1} + \frac{1}{dC_2}\right]$$

Consider an element of width dx at a distance x from the left plate. Then

$$dC_1 = \frac{(\in_0 + kx)A}{dx} \text{ for } 0 < x < \frac{d}{2}$$

$$\text{and } dC_2 = \frac{\{\in_0 + k(d-x)\}A}{dx} \text{ for } \frac{d}{2} < x \le d$$

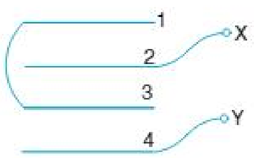
On substituting these two values, we get

$$\frac{1}{C} = \int\frac{1}{dC} = \frac{2}{KA}\ln\left(\frac{2\in_0 + Kd}{2\in_0}\right)$$

$$\Rightarrow C = \frac{KA}{2}\ln\left(\frac{2\in_0 + Kd}{2\in_0}\right)$$

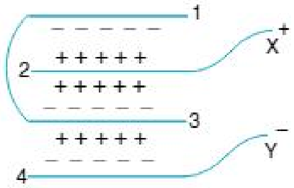
Example-3.10

Four identical metal plates are located in air at equal distance d from one another. The area of each plate is A. Find the equivalent capacitance of the system between X and Y.

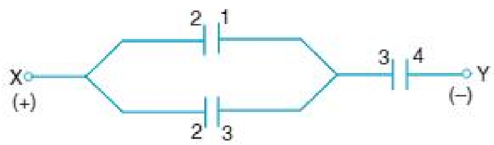


Solution :

Let us give numbers to the four plates. Here X and Y are connected to the positive and negative terminals of the battery (say), then the charge distribution will be as shown



Here the arrangement can be represented as the grouping of three identical capacitors each of capacity $\frac{\in_0 A}{d}$. The arrangement will be as shown



Now the equivalent capacitance between X and Y is

$$C_{XY} = \frac{(C+C)C}{C+C+C} = \frac{2C}{3} = \frac{2\in_0 A}{3d}$$

Example-3.11

Find the charges that appear on the capacitors if a battery of emf V is connected between X and Y as assumed earlier.

Solution :

Here negative charges appear on one surface of 1 as well as 4.

Positive charges appear on both surfaces of 2 and positive and negative charges appear on the surfaces of 3.

Total charge driven by the battery is

$$q = (C_{XY})V = \frac{2\in_0 AV}{3d}$$

Charge that appears on the capacitor between plates 3 and 4 will be q only

$$\Rightarrow \text{charge on 4 is } \frac{-2\in_0 AV}{3d}$$

Charges that appear on capacitors between plates 1 and 2, 2 and 3 will be $\frac{q}{2}$ and $\frac{q}{2}$

So, charge that appears on plate 1 is $\frac{-q}{2} = -\frac{\in_0 AV}{3d}$

Charge that appears on plate 2 will be

$$\frac{q}{2}+\frac{q}{2}=\frac{2\epsilon_0 AV}{3d}$$

Charge that appears on plate 3 will be

$$-\frac{q}{2}+q=\frac{\epsilon_0 AV}{3d}$$

We can check the results from conservation of charges.

3.10 CAPACITY OF AN ISOLATED SPHERE

Let us consider an isolated metal sphere of radius R having a positive charge of +Q. This forms a capacitor with earth as second conductor and air as the dielectric medium between the two conductors.

We know that the potential of an isolated charged sphere is $V=\frac{Q}{4\pi\varepsilon_0 R}$.

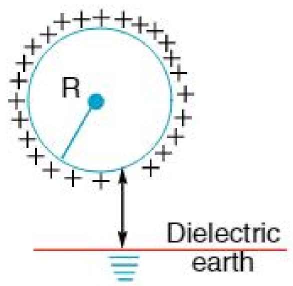


Fig 3.9(a)

By definition of capacitance $C=\frac{Q}{V}$

$\Rightarrow C=4\pi\epsilon_0 R$

3.11 CAPACITANCE OF A SPHERICAL CAPACITOR

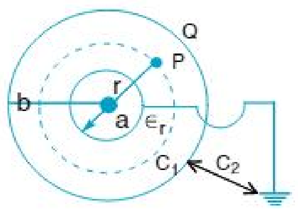


Fig 3.9(b)

Here two cases will arise. In one case charge may be given to the inner sphere and outer sphere is earthed. In the second case charge may be given to outer sphere and inner sphere is earthed.

i) When charge is given to the outer sphere & inner sphere earthed

The charge supplied to the outer sphere will get shared between the outer surface and inner surface of the outer conductor. Then it behaves like a combination of two capacitors C_1 and C_2 connected in parallel.

Hence the total capacitance is C_1+C_2 ....(a)

Considering C_1 as the capacitance between the inner surface of outer sphere and inner sphere then we can calculate C_1 as given below.

Consider a point P which is at a distance 'r' from the centre of system. If E is the field intensity or field strength at P, $E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$.

$$dV=-\overline{E}.\overline{dr}$$

Potential difference between the surfaces of the two spheres facing each other is

$$V=-\int_a^b \overline{E}.\overline{dr}=-\int_a^b \frac{q}{4\pi\varepsilon_0 r^2}dr$$

$$=-\frac{q}{4\pi\varepsilon_0}\int_a^b\frac{dr}{r^2}=-\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_a^b$$

$$=-\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{b}-\frac{1}{a}\right]$$

$$V=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]=\frac{q}{4\pi\varepsilon_0}\left[\frac{b-a}{ab}\right]$$

Hence capacitance of the capacitor between the two spherical surfaces,

$$C_1=q/V=\frac{q}{\cancel{q}/4\pi\varepsilon_0\left(\frac{b-a}{ab}\right)}$$

$$C_1=4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) \qquad .....(3.8)$$

C_2 is the capacitance of an isolated sphere and it will be given by $4\pi\varepsilon_0 b$.

Hence total capacitance when the inner sphere is earthed will be

$$C = C_1 + C_2$$

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) + 4\pi\varepsilon_0 b$$

$$= 4\pi\varepsilon_0\left(\frac{b^2}{b-a}\right) \quad ....(3.9)$$

ii) When charge is given to the inner sphere and outer sphere is earthed

Consider two concentric conducting shells of radii a and b (b > a). Let q be the charge given to the inner shell if its outer surface is earthed. Electric field inside the inner shell is zero. So there will be electric field only between the two shells.

Electric field at a distance r from the centre of the shells (r<b) is $E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2}$

Now potential differences between the two shells is

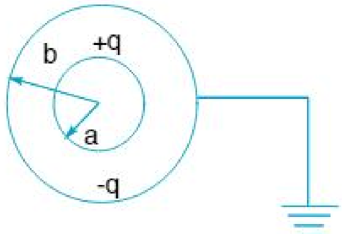


Fig 3.9(c)

$$V = V_+ - V_- = -\int_b^a E dr$$

$$\Rightarrow V = \int_a^b E dr = \frac{q}{4\pi \epsilon_0}\int_a^b \frac{dr}{r^2} = \frac{q}{4\pi \epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Capacitance of the spherical capacitor $C = \frac{q}{V}$

$$\Rightarrow C = 4\pi \epsilon_0\left(\frac{ab}{b-a}\right)$$

- If there is a medium of dielectric constant K between the two spheres, we can write

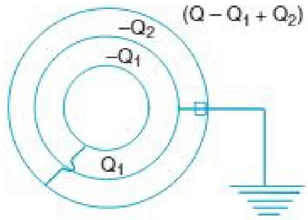
$$C = 4\pi \epsilon_0 K\left(\frac{ab}{b-a}\right) \quad .... (3.10)$$

Example-3.12

Three concentric thin spherical shells are of radii r_1, r_2, r_3 ($r_1 < r_2 < r_3$). The first and third are connected by a conducting wire through a small hole in the second and the second is connected to earth through a small hole in the third. Find the capacitance of the system so formed ?

Solution :

Let Q be the charge given to the outermost shell 3, some of the charge will go to the outer surface of the innermost shell because they are connected together by a wire.



Charge Q_1 on the inner shell will induce a charge $-Q_1$ on the inner surface of the shell 2 and $+Q_1$ on its outer surface.

Shell 3 also induces the charge on shell 2. So that the charges on surfaces of 2 and 3 facing each other are Q_2 and $-Q_2$

Here we can observe three different capacitors between shells 1 –2, 2 – 3 and 3 – ∞ denoted by C_1, C_2, C_3

$$C_1 = 4\pi \epsilon_0\left(\frac{r_1 r_2}{r_2 - r_1}\right)$$

$$C_2 = 4\pi \epsilon_0\left(\frac{r_2 r_3}{r_3 - r_2}\right) \text{ and } C_3 = 4\pi \epsilon_0 r_3$$

Effective capacitance is $C = C_1 + C_2 + C_3$

$$C = 4\pi \epsilon_0\left\{\frac{r_1 r_2}{r_2 - r_1} + \frac{r_2 r_3}{r_3 - r_2} + r_3\right\}$$

3.12 CYLINDRICAL CAPACITOR

A cylindrical capacitor consists of two coaxial cylinders of radii a & b as shown in figure. The cylinders are conducting. Let q be the charge given to the inner cylindrical. Let the outer cylinder be earthed.

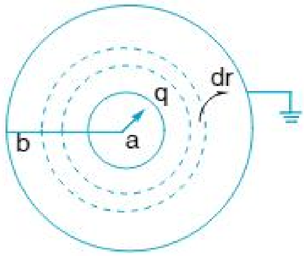


Fig 3.10

In order to calculate the capacitance let us consider a coaxial cylindrical shell of thickness dr. The surface of the imaginary cylinder is Gaussian surface. The electric flux eminating from charged cylinder will pass through the Gaussian surface and hence

$$\int \overline{E}.d\overline{s} = \left(\frac{q}{\varepsilon_0 K}\right)$$

Where K is relative permitivity of the medium between the two cylindrical surfaces

$$\int \overline{E}.d\overline{s} = \left(\frac{q}{\varepsilon_0 K}\right)$$

Where K is relative permitivity of the medium between the two cylindrical surfaces.

$$E\int ds = \left(\frac{q}{\varepsilon_0 K}\right)$$

$$E(2\pi rL) = \frac{q}{\varepsilon_0 K}$$

$$E = \left(\frac{q}{2\pi\varepsilon_0 KL}\right)\frac{1}{r}$$

If 'V' is the potential difference between the inner and outer cylinders, work done in moving unit +ve charge from the outer cylinder to inner cylinder can be calculated by integrating dV where dV = –Edr.

$$\int_V^0 dV = -\int_a^b E.dr \Rightarrow V = \int_a^b E.dr$$

$$V = \int_a^b \left(\frac{q}{2\pi\varepsilon_0 KL}\right)\frac{1}{r}dr$$

$$= \left[\frac{q}{2\pi\varepsilon_0 KL}\right](\log b - \log a)$$

$$V = \frac{q}{2\pi\varepsilon_0 KL}\left(\log_e\left(\frac{b}{a}\right)\right)$$

from $C = \frac{q}{V}$

$$C = \frac{q}{\frac{q}{2\pi\varepsilon_0 KL}\left(\log_e\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0 KL}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow C = \frac{2\pi\varepsilon_0 K L}{2.303\log_{10}\frac{b}{a}} \qquad \text{.... (3.11)}$$

3.13 CAPACITANCE BETWEEN TWO LONG PARALLEL CONDUCTORS

Let 'λ' be the charge per unit length on each conductor. Thus for a conductor of length 'L' charge $Q = \lambda L$. Consider a point P at a distance x from the axis of conductor A. Electric field at P

$$E = E_A + E_B$$

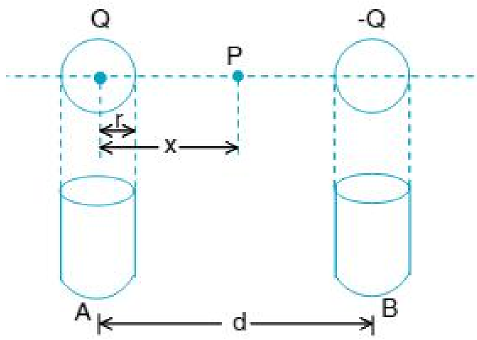


Fig 3.11

$$= \frac{\lambda}{2\pi\epsilon_0 x} + \frac{\lambda}{2\pi\epsilon_0 (d-x)}$$

$$= \frac{\lambda}{2\pi\epsilon_0}\left[\frac{1}{x} + \frac{1}{d-x}\right]$$

Potential difference between the conductors

$$V = V_A - V_B = \int_r^{d-r} E\,dx$$

$$= \frac{\lambda}{2\pi\epsilon_0}\int_r^{d-r}\left[\frac{1}{x}+\frac{1}{d-x}\right]dx$$

$$= \frac{\lambda}{2\pi\epsilon_0}\left[\log(x)-\log(d-x)\right]_r^{d-r}$$

$$= \frac{\lambda}{\pi\epsilon_0}\log\left[\frac{d-r}{r}\right]$$

$$\therefore \text{ Capacitance } C = \frac{Q}{V} = \frac{\lambda L}{\frac{\lambda}{\pi\epsilon_0}\log_e\left[\frac{d-r}{r}\right]}$$

$$C = \frac{\pi\epsilon_0 \lambda L}{\log_e\left(\frac{d-r}{r}\right)} \quad \text{.... (3.12)}$$

3.14 TYPES OF CAPACITORS AND THEIR USES

The simplest form a capacitor is parallel plate capacitor. But for different applications, different types of capacitors are used. Some of the special types of capacitors are described below.

A) Multiple Capacitor

This is a parallel combination of several parallel plate capacitor and is of fixed capacitance. Mica is used as the dielectric between a number of tin foils arranged in parallel together as shown in figure.

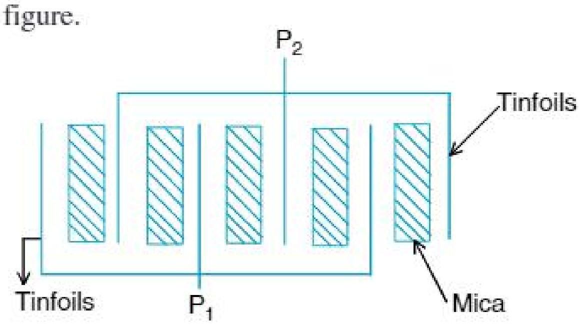


Fig 3.12(a)

If the capacitance between two successive plates (with mica as dielectric) is C, then the capacitance of the multiple capacitor is (n – 1).C Where *n* is the number of parallel plates used.

The whole arrangement is sealed in a plastic case. These capactiors are used in high frequency oscillating circuits. The dielectric constant of mica does not change much with temperature and hence these capacitors are used as standard capacitors in the laboratory.

B) Paper Capacitor

In the paper capacitor a paper soaked in oil or wax is used as dielectric in between tin foils that serve as capacitor plates. This is shown in figure. To increase the capacitance to a large extent several strips of metal foils and waxed paper are arranged alternately like multiple capacitor as shown in figure.

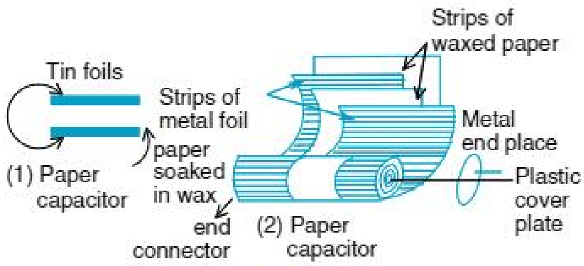


Fig 3.12(b)

C) Variable Capacitor

As the name itself indicates, the capacitance of a variable capacitor can be varied gradually. This is achieved by varying the effective area induced between the plates. The plates are usually made of brass or aluminium and semi circular in shape. Variable capacitors are widely used for tuning circuits in radio and T.V receivers.

As shown in figure a variable capacitor consists of two sets of plates. One set of plate is fixed in position and is called the stator. The other set of plates can be rotated over the stator by rotating the piston. This set is called rotor. During rotation of the rotor the area common to the plates of stator and rotor is varied and consequently the capacitance is varied.

D) Electrolyte Capacitors

An electrolyte capacitor is obtained by passing a direct current between two sheets of aluminium foils (A and C) with a suitable electrolyte like ammonium borate between the foils.

Due to electrolysis a very thin film of thickness of the order of 10^{-6} cm of alluminium oxide is formed on the anode plate and acts as a dielectric between the two plates.

The electrolyte is a good conductor and hence along with C it forms the cathode. As the dielectric thickness is very small the capacitance will be very high. Care must be taken to connect this capacitor with proper polarity in a circuit other wise the oxide film will break down. For this reason in an electrolyte capacitor the polarity of terminals will be indicated.

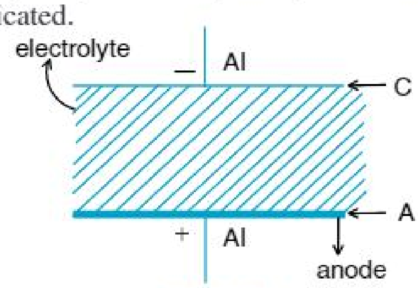


Fig 3.12(c)

These are widely used when high capacitance is required. Capacitance of the order of $10^3\mu F$ can be easily obtained with electrolyte capacitance of small volumes.

3.15 DIELECTRICS

Substances which do not possess free electrons or posses very less number of free electrons to constitute electric current are known as dielectric materials. In such materials the electrons will be tightly bound to the nucleus.

Whenever such a material is placed in an electric field the field gets modified. This effect can be felt by introducing a dielectric slab in between the plates of a parallel plate capacitor where by the charge on the capacitor gets modified. By introducing a dielectric the capacity of the capacitor increases due to this factor alone.

When a conductor is placed in an external electric field, the charge distribution in the conductor adjusts in such a way that the electric field due to the changes induced opposes the external field. This will continue till the net electrostatic field inside the conductor is zero.

When a dielectric is kept in an external electric field, such free movement of charges is not possible. In this case the external field induces dipole moment by re orienting or modifying the molecules of the dielectric. The net effect of all the molecular dipole moments produces an induced field that opposes the external field. But this field induced will not cancel the external field. The induced field, inside the dielectric net electric field will be less than that of external field in magnitude. The extent of this opposing effect depends on the nature of dielectric.

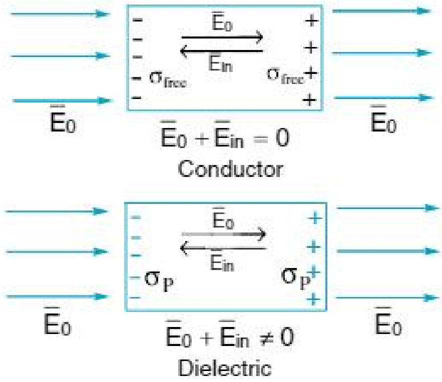


Fig 3.13

We know that the atom consists of central nucleus with +ve charge surrounded by revolving electrons. The centre of the +ve charge and the centre of the –ve charge are supposed to be concentrated at a single point. This will be the case in certain molecules only. Such molecules whose +ve charge centre and –ve charge centre coincide is known as non polar molecule. Non polar molecules will have symmetrical structure of atoms with regards to electrical effect. They have symmetric structure so that the electric dipole moment of the molecule is zero. Examples of such molecules are CO_2, H_2, O_2 etc. If the centres of positive charge and negative charge of a molecule do not coincide such molecule is said to be a polar molecule. These polar molecules will have unsymmetric structure and posses net dipole moment. Example of such molecules is H_2O.

In an external electric field, the positive and negative charges of a non polar molecules and displaced in opposite directions. Due to internal fields in the molecules restoring forces develope. Displacement of the charges stops till he external force is balanced by the restoring force. Finally non polar molecules develope induced dipole moments, which is known as polarisation of dielectric. The induced dipole moments of various molecules add up such that net dipole moment will be produced in the dielectric.

In the absence of external field, different molecular dipoles due to polar molecules of dielectric will be oriented randomly due to thermal agitation. This random distribution results in net dipole moment zero. When external electric field is applied, individual dipoles align such that a net dipole moment will result in parallel to the direction of external field. This is also known as polarisation. Here the extent of polarisation depends on two factors: (a) dipole potential energy in the external field which tends to align the dipoles with the field (b) thermal energy tending to distribute the alignment.

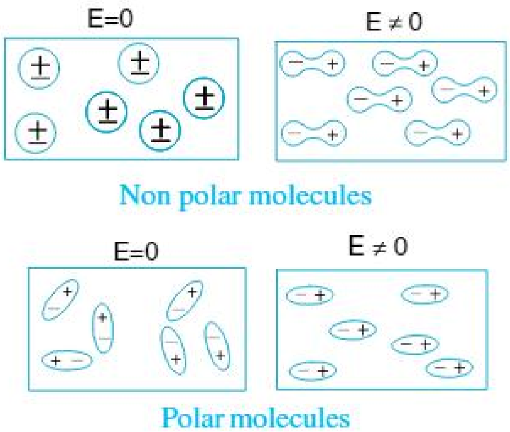
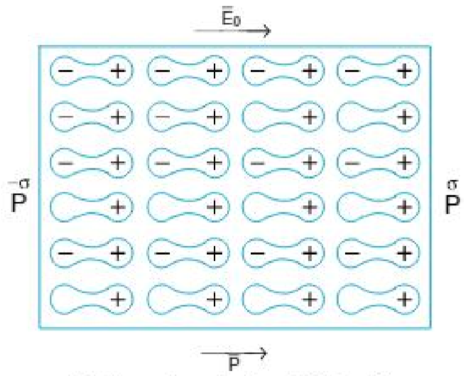


Fig 3.14

Whether it is made of polar or non polar molecules, a dielectric always develops net dipole moment in external electric field. The net dipole moment per unit volume is called polarisation denoted by $\vec{P}$ for a linear isotropic dielectric, $\vec{P} = \chi\overline{E}$

Here χ is a constant known as electric susceptibility of dielectric medium. It is characteristic of the given medium.

It is observed that the polarised dielectric modifies the original external field inside, it. Let $\overline{E}_0$ be uniform external field in which a dielectric slab is kept. The field causes a uniform polarisation $\overline{P}$ of the dielectric. Every volume element ΔV is very small volume element but it contains a very large number of molecular dipoles. Inside the dielectric the positive charge of one dipole sits close to the negative charge of adjacent dipole. As a result anywhere inside the dielectric the small volume element ΔV will have no net charge. But at the surface of the dielectric, normal to the electric field there will be a net charge density. So, the polarised dielectric is equivalent to two charged surfaces with induced surface charge densities $\overset{\sigma}{P}$ and $\overset{-\sigma}{P}$ as shown. The field produced by these surface charges opposes the external electric field. The surface charge densities arise from the bond charges but not from free charges in the dielectric.



Uniformly polarised Dielectric

Fig 3.15

3.16 DIELECTRIC STRENGTH

If the applied electric field E_0 becomes very high in such a strong field the electrons of the atoms inside the dielectric get detached from their atoms. In such a situation the dielectric no longer can behave as an insulator. It behaves just like a conductor. This is known as electrical breakdown or dielectric breakdown. The electric field strength at which the electrical (dielectric) breakdown occurs is called the dielectric strength of the material.

3.17 DIELECTRIC POLARISATION

Dielectric polarisation is defined as the dipole moment per unit volume of the dielectric

We know that a dielectric slab when kept between the plates of a capacitor, the electric field applied across the plate will displace the centres of negative and positive charges and this displacement causes dipole moment effect in every atom. If we consider the entire dielectric substance the total dielectric will develop certain dipole moment with all dipoles put together.

The electric dipole moment per unit volume of the dielectric is known as dielectric polarisation P. Let us consider a slab of cross-sectional area A and length 'L' (for convenience). Then the molecular charges developed on the edge faces of the dielectric convenience are as shown in figure.

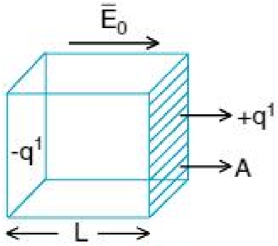


Fig 3.16(a)

The dipole moment of the total slab = Lq^1 and volume of the slab LA. By definition, polarization $P = \frac{Lq^1}{LA} = \frac{q^1}{A}$ ie charge per unit area. Hence dielectric polarization is numerically equal to surface charge density on it.

3.18 INDUCED OR BOUND CHARGE

Consider a parallel plate capacitor with charge q on its positive plate and -q on its negative plate. So, electric field between the plates will be

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0}$$

When dielectric slab is introduced between the plates, polarisation takes place. Let q^1 be the magnitude of induced charge. Due to the induced charges there will be induced electric field E_i. This field opposes the external field E_0. The net electric field will be $\bar{E} = \bar{E}_0 + \bar{E}_i$.

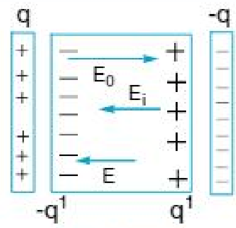


Fig 3.16(b)

$$E = E_0 - E_i$$

$$\text{where } E_i = \frac{\sigma^1}{\epsilon_0} = \frac{q^1}{A\epsilon_0}$$

$$\text{But } E = \frac{E_0}{K} \text{ (K is dielectric constant)}$$

$$E = \frac{q}{A\epsilon_0 K} = \frac{q}{A\epsilon_0} - \frac{q^1}{A\epsilon_0}$$

$$\Rightarrow q^1 = q\left(1 - \frac{1}{K}\right) \qquad \text{.... (3.13)}$$

3.19 ELECTRIC DISPLACEMENT

If σ_P is induced charge density and $\bar{P}$ is polarisation in a dielectric, we can write $\sigma_P = \bar{P}.\hat{n}$

Hence $\hat{n}$ is a unit vector along the outward normal to the surface. This is a general equation which is time for any shape of the dielectric. For the polarised dielectric slab as shown, $\bar{P}$ is along $\hat{n}$ at the right surface and opposite to $\hat{n}$ at the left surface. At the right surface, induced charge density positive and at the left surface it is negative.

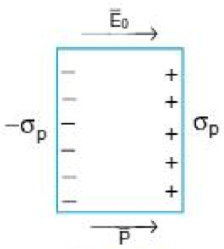


Fig 3.17

$$\bar{E}.\hat{n} = \frac{\sigma - \bar{P}.\hat{n}}{\epsilon_0} \text{ or } \left(\epsilon_0 \bar{E} + \bar{P}\right).\hat{n} = \sigma$$

Here the quantity $\epsilon_0 \bar{E} + \bar{P}$ is called the electric displacement denoted by $\bar{D}$ which is a vector quantity.

$\overline{D} = \epsilon_0 \overline{E} + \overline{P}$ and $\overline{D}.\hat{n} = \sigma$

In vacuum, $\overline{E}$ is related to the free charge density σ. But when a dielectric medium is present, the correspondingly role will be taken by $\overline{D}$. For a dielectric medium, $\overline{E}$ is not directly related to free charge density σ. Instead of that $\overline{D}$ is related. Since $\overline{P}$ is in the same direction as $\overline{E}$, all the time vectors $\overline{P}$, $\overline{E}$ and $\overline{D}$ will be in the same direction.

$$\frac{D}{E} = \frac{\sigma \epsilon_0}{T - T_P} = \epsilon_0 K$$

$$\Rightarrow \overline{D} = \epsilon_0 K\overline{E}$$

and $\overline{P} = \overline{D} - \epsilon_0 \overline{E} = \epsilon_0 (K-1)\overline{E}$

but we know that $\overline{P} = \chi\overline{E}$

so $\chi = \epsilon_0 (K-1)$

χ is electric susceptibility and K is dielectric constant of the medium.

3.20 FORCE BETWEEN THE PLATES OF A CAPACITOR

Consider a parallel plate capacitor with plate area A. Let Q and –Q be the charges on the plates of capacitor. Let F be the force of attraction between the plates. Let E be the field between the capacitor plates. The expression for the force can be derived by energy method. Let the distance between the plates be x.

So electric field energy between the plates is

$$U = \frac{1}{2}\epsilon_0 E^2 (Ax)$$

$$\frac{dU}{dx} = \frac{1}{2}\epsilon_0 E^2 A$$

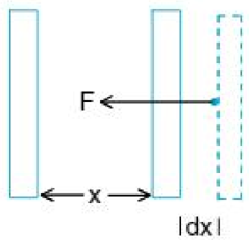


Fig 3.18

By definition $F = -\frac{dU}{dt} = \frac{-1}{2}\epsilon_0 E^2 A$

(Conservative force)

So the force of attraction between the plates is $\frac{1}{2}\epsilon_0 E^2 A$

- For a isolated charged capacitor

$$F = \frac{Q^2}{2\epsilon_0 A} \qquad ...(3.14)$$

This force does not depend on the seperation between the plates, and so the certain amount of force is needed to change the separation.

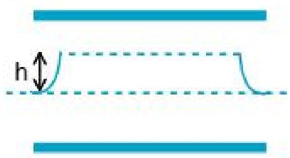
- For a capacitor having constant potential difference across the plates the force

$$F = \frac{1}{2}\epsilon_0 \frac{V^2}{d^2} A \qquad ...(3.15)$$

In this case force depends on the separation between the plates. Thus to change the separation variable force is needed.

Example-3.13

A parallel-plate capacitor is placed in such a way that its plates are horizontal and the lower plate is dipped into a liquid of dielectric constant k and density ρ. Each plate has area A. The plates are now connected to a battery which supplies a positive charge of magnitude Q to the upper plate. Find the rise in the level of the liquid in the space between the plates. (in static equilibrium condition)



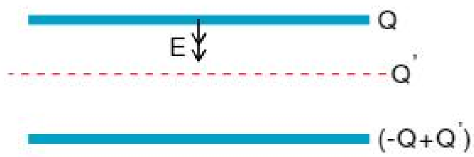
Solution :

Because of the charge of the plates of capacitor, a charge will induce on the upper and lower layers (which is in contact with negative plate of capacitor) of the liquid. The force on upper layer of liquid is F = EQ' in upward direction. Because of this force, liquid rises till force becomes equal to weight of liquid risen. Thus F = EQ' = mg

where $Q' = Q\left(1 - \frac{1}{k}\right)$ and $mg = Ah\rho g$

Charge on lower plate

$$= -Q + Q' = -Q + Q\left(1 - \frac{1}{k}\right) = -\frac{Q}{k}$$



The electric field due to both the plates of capacitor

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma'}{2\epsilon_0}$$

$$= \frac{Q}{2\epsilon_0 A} + \frac{Q}{2k\epsilon_0 A} = \frac{Q}{2\epsilon_0 A}\left(1 + \frac{1}{k}\right)$$

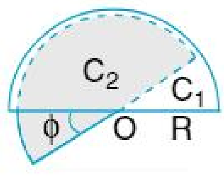
We have $EQ' = mg$

$$\therefore \left[\frac{Q}{2\epsilon_0 A}\left(1 + \frac{1}{k}\right)\right] Q\left(1 - \frac{1}{k}\right) = Ah\rho g$$

or $h = \dfrac{(k^2 - 1)Q^2}{2A^2 k^2 \epsilon_0 \rho g}$

Example-3.14

A capacitor consists of two stationary plates shaped as a semicircle of radius R and a movable plate made of dielectric with relative permitivity $\in$ and capable of rotating about an axis O between the stationary plates. The thickness of the movable plate is equal to d which is practically the separation between the stationary plates. A potential difference V is applied to the capacitor. Find the magnitude of the moment of forces relative to the axis O acting on the movable plate in the position shown in the figure.



Solution :

The area of dielectric which is out of plates is

$$= \frac{R\phi \times R}{2} = \frac{R^2\phi}{2}$$

The area of dielectric inside the plates is

$$= \frac{\pi R^2}{2} - \frac{R^2\phi}{2} = \frac{(\pi - \phi)R^2}{2}$$

The capacitance of the system $C = C_1 + C_2$

$$= \frac{\epsilon_0\left(\dfrac{R^2\phi}{2}\right)}{d} + \frac{\in\epsilon_0(\pi - \phi)R^2}{2d}$$

Because of charges on the plates of capacitor, a charge will induce on the dielectric, due to which it experiences a force towards the plates which constitutes a torque about O. Let this torque rotate the dielectric by $d\phi$. The work done by the field

$dW = -dU = \tau d\phi \qquad \therefore \tau = -\dfrac{dU}{d\phi}$

where $U = \dfrac{1}{2}CV^2$

$$\therefore \tau = -\frac{dU}{d\phi} = -\frac{d}{d\phi}\left(\frac{1}{2}CV^2\right) = -\frac{V^2}{2}\left(\frac{dC}{d\phi}\right)$$

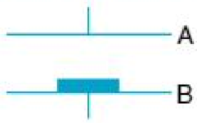
$$= -\frac{V^2}{2}\frac{d}{d\phi}\left[\frac{\epsilon_0 R^2\phi}{2d} + \frac{\in\epsilon_0(\pi - \phi)R^2}{2d}\right]$$

$$= -\frac{V^2}{2}\left[\frac{\epsilon_0 R^2}{2d} + \frac{\in\epsilon_0(-1)R^2}{2d}\right]$$

$$= \frac{(\in - 1)\epsilon_0 R^2 V^2}{4d}$$

Example-3.15

Capacitory is made of two circular plates of radius R each seperated by a distance a << r. A thin conducting disc of radius r << R and thickness t << r is kept at the centre of the bottom plate. Find the minimum voltage required to left the disc if its mass is m.



Solution :

Let V be the pd applied.

Now field on the disc = $E = \dfrac{V}{d}$

Charge on plate B is $CV = \dfrac{\epsilon_0 \pi R^2 V}{d}$

Charge on the disc is $q = \dfrac{CV}{\pi R^2}(\pi r^2) = \dfrac{\pi\epsilon_0 V r^2}{d}$

Force acting on the disc = $F = q\in$

Weight of the disc = mg

If V is minimum voltage to lift the disc, we use F = mg

$$\frac{\pi\epsilon_0 V r^2}{d}\left(\frac{V}{d}\right) = mg$$

$$\Rightarrow V = \sqrt{\frac{mgd^2}{\pi\epsilon_0 r^2}}$$

3.21 EFFECT OF DIELECTRIC ON VARIOUS FACTORS OF PARALLEL PLATE CAPACITOR

Consider an air filled capacitor of capacitance C_0. Such that $C_0 = \frac{\epsilon_0 A}{d}$.

Now a dielectric slab of dielectric constant K is inserted between the plates completely filling the gap. As a result its capacity changes to $C = \frac{K \epsilon_0 A}{d}$

$\Rightarrow C = KC_0$

(a) If the capacitor (air filled) is initially connected between the terminals of a battery of emf V_0, charge on the capacitor is $Q_0=C_0V_0$ without disconnecting the battery if the dielectric is inserted between the plates of the capacitor, its potential $V = V_0$ only.

Now charge on the capacitor is $Q = CV$

i.e., $Q = KC_0V_0 = KQ_0$

$\Rightarrow$ Charge on the capcitor increases to K times

Energy stored in the capacitor before introducing the dielectric is $U_0 = \frac{1}{2}C_0V_0^2$.

After introducing dielectric energy stored in the capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}KC_0V_0^2 = KU_0$$

$\Rightarrow$ Energy stored increases to K times

Electric field between the capacitor plates is $E = \frac{V_0}{d}$.

After introducing dielectric electric field between the plates

$$E = \frac{V}{d} = \frac{V_0}{d}$$

$\Rightarrow$ Electric field between the plates does not change.

(b) If the air filled capacitor is initially charged to potential V_0 and disconnected from the battery charge on it does not change.

$\Rightarrow Q = Q_0 = C_0V_0$ only

Now potential of the capacitor is $V = \frac{Q}{C}$

is $V = \frac{Q_0}{KC_0} = \frac{V_0}{K}$

$\Rightarrow$ Potential of the capacitor reduced to $\frac{1}{K}$ times energy stored in the capacitor before introducing dielectric is

$$U_0 = \frac{Q_0^2}{2C_0}$$

After introducing dielectric $U = \frac{Q_0^2}{2C}$

i.e., $U = \frac{Q_0^2}{2KC_0} = \frac{U_0}{K}$

$\Rightarrow$ Energy stored in the capacitor reduces to $\frac{1}{K}$ times.

Electric field between the plates of the capacitor is $E_0 = \frac{V_0}{d}$ initially.

After introducing dielectric slab, $E = \frac{V}{d} = \frac{V_0}{Kd}$

i.e., $E = \frac{E_0}{K}$

$\Rightarrow$ Electric field decreases to $\frac{1}{K}$ times

3.22 KIRCHHOFF'S LAWS FOR CAPACITOR CIRCUITS

Kirchhoff's laws are useful in circuit analysis which involve capacitors, resistors, inductors and batteries. We will discuss about these in the next topic. For circuits with capacitors we have junction law and loop laws.

(a) From junction law, we can say "the algebraic sum of the charges at any junction is zero".

Similarly total charge of an isolated system remain constant or zero.

At junction O, $\Sigma q = 0$

$\Rightarrow q_1 + q_2 + q_3 = 0$

The dotted line forms isolated system so in that case also $q_1 + q_2 + q_3 = 0$

(Similarly for currents $\Sigma i = 0$ from junction rule)

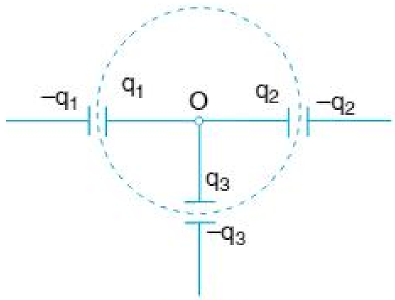


Fig 3.19(a)

(b) In a closed loop, algebraic sum of potential differences across all circuit elements must be equal to zero.

$$\Rightarrow \sum p.d = 0$$

$\sum E + \Sigma iR = 0$ for resistors and batteries

$\sum E + \sum \frac{q}{C} = 0$ for capacitors and batteries

$\sum E + \sum \frac{q}{C} + \Sigma iR = 0$ for all the three

The junction rule is application of principle of conservation of charge, where as loop rule is a consequence of conservation of energy while applying loop rule, potential gain or potential fall will be calculated as mentioned below.

Fall in potential

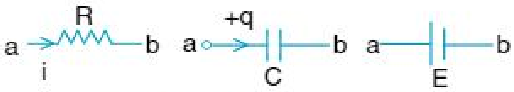


Fig 3.19 (b) Fall in potential

$$V_a - V_b = -iR \text{ or } \frac{-q}{C} \text{ or } -E$$

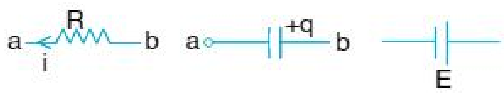


Fig 3.19 (c) Potential rise

$$V_a - V_b = +iR \text{ or } \frac{q}{C} \text{ or } E$$

Example-3.16

In the given branch, if potential of a is 10V higher than b, find the potential of each capacitor and charge on each capacitor ?

Solution :

$V_a - V_b = 10\text{ V}$

Let the charge on each capacitor be 'q' (in μC)

$$V_a - \frac{q}{1} + 10 - \frac{q}{2} = V_b$$

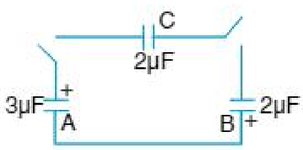
$$q + \frac{q}{2} = 20 \Rightarrow q = \frac{40}{3}\mu C$$

Potential difference across 1 μF is $V_1 = \frac{q}{1} = \frac{40}{3}V$

Potential difference across 2μF is $V_2 = \frac{q}{2} = \frac{20}{3}V$

Example-3.17

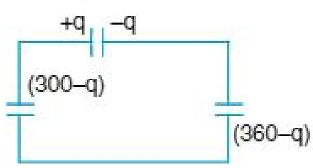
Two capacitors A and B with capacities 3μF and 2μF are charged to a potential difference of 100V and 180V respectively. The plates of the capacitors are connected with the given charge polarities as shown. An uncharged capacitor of capacity 2μF is connected by closing the switches. Calculate the final charge on the three capacitors



Solution :

Let q be the charge flowing during redistribution of charge in the circuit. From Kirchhoff's voltage law, we can write

$$\frac{300-q}{3} - \frac{q}{2} + \frac{360-q}{2} = 0$$

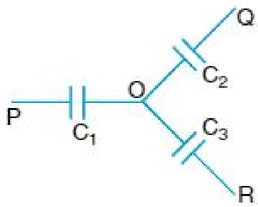


$$\frac{4q}{3} = 280 \Rightarrow q = 210\mu C$$

Final charge on the capacitors are $90\mu C, 210\mu C$ and $150\mu C$

Example-3.18

Three capacitors C_1,C_2 and C_3 are connected as shown. Let V_1, V_2 and V_3 be the potentials of P, Q, R respectively. Then find the potential at the junction O.



Solution :

Let q_1,q_2 and q_3 be the charges on the capacitors C_1, C_2 and C_3 respectively.

$V_P - V_0 = \frac{q_1}{C_1}$; $\quad V_0 - V_Q = \frac{q_2}{C_2}$; $\quad V_0 - V_R = \frac{q_3}{C_3}$

From junction rule applied at O

$q_1 = q_2 + q_3$

$C_1(V_1 - V_0) = C_2(V_0 - V_2) + C_3(V_0 - V_3)$

$$\Rightarrow V_0 = \left(\frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}\right)$$

3.23 WHEATSTONE BRIDGE CIRCUITS

A circuit with five capacitors as shown is known as wheatstone bridge circuit.

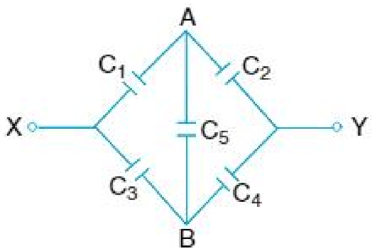


Fig 3.20 (a)

(a) If $\frac{C_1}{C_3} = \frac{C_2}{C_4}$ in this circuit, it is said to be balanced. In such a case, $V_A - V_B = 0$ So, capacitor C_5 will have no charge and while simplifying the circuit, we can ignore C_5.

Now the circuit changes as given below

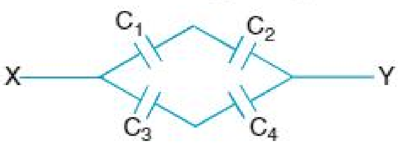


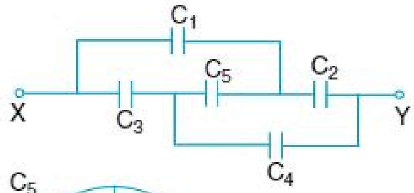
Fig 3.20 (b)

Effective capacity between X and Y will be

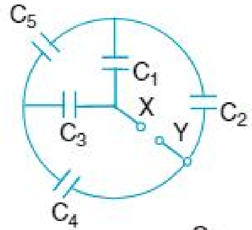
$$C_{XY} = \left(\frac{C_1C_2}{C_1 + C_2}\right) + \left(\frac{C_3C_4}{C_3 + C_4}\right)$$

Based on balanced whetstone bridge principle, the following circuits can be simplified

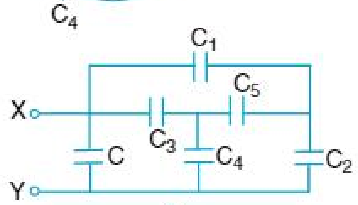
(1)



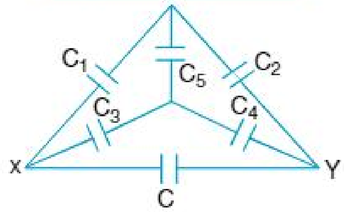
(2)



(3)



(4)



For (1) and (2)

$$C_{XY} = \left(\frac{C_1C_2}{C_1 + C_2}\right) + \left(\frac{C_3C_4}{C_3 + C_4}\right) = C_0$$

For (3) and (4) $C_{XY} = C_0 + C$

(b) If $\frac{C_1}{C_3} \neq \frac{C_2}{C_4}$ in the wheatstone bridge it is said to be unbalanced. Such circuits must be solved by circuit analysis ie we use kirchhoff's laws.

Some times unbalanced bridge circuits can be analysed with reverse symmetry method. Such circuit is given below.

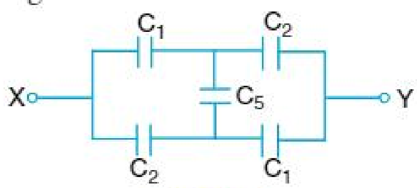


Fig 3.20 (c)

Let q_1 and q_2 be the charges on capacitor C_1 and C_2 when a battery is connected between X and Y the charge distribution will be as shown

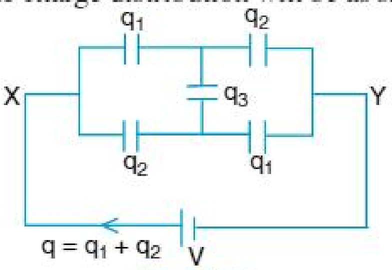


Fig 3.20 (d)

3.24 INFINITE LADDER NETWORKS

A circuit with capacitors connected in a repeated manner upto infinity can be called as infinite ladder network. An example for such network is as given below.

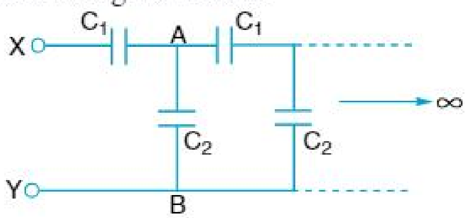


Fig 3.21 (a)

Here $C_{XY} = C_{AB} = C$

Then the given circuit can be modified as

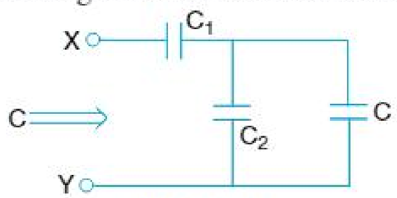


Fig 3.21 (b)

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C + C_2}$$

On simplification, we get

$$C = \frac{-C_2 + \sqrt{C_2^2 + 4C_1C_2}}{2}$$

Similarly you can try the following circuits

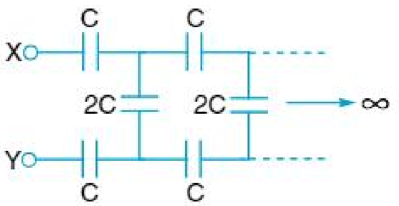


Fig 3.21 (c)

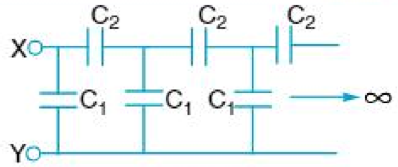


Fig 3.21 (d)

3.25 HIGH VOLTAGE BREAK DOWN (OR) CORONA DISCHARGE

Fig 3.22(a)

Let us consider two conducting spheres A and B connected by a conducting wire. A charge Q is given to the system. This charge will be shared by them in such away that their potentials become equal. Let q_1 and q_2 be the charges on them. Thus

$$V_A = V_B$$

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{R} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r}$$

$$\frac{q_1}{q_2} = \frac{R}{r} \text{ and } \frac{\sigma_1}{\sigma_2} = \frac{\frac{q_1}{4\pi R_1^2}}{\frac{q_2}{4\pi r^2}} = \frac{r^2}{R^2} \times \frac{q_1}{q_2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{r^2}{R^2} \times \frac{R}{r} = \frac{r}{R}$$

we can say $\sigma \propto \frac{1}{r}$

It shows that the field is higher at the surface of small spheres.

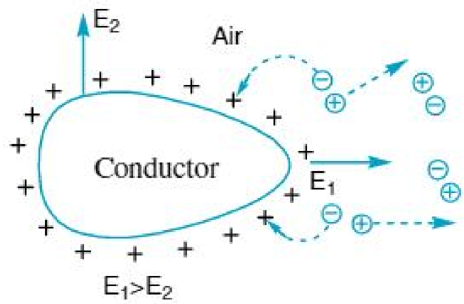


Fig 3.22 (b)

The same can be explained with a conductor having sharp end. It can be shown by simple knowledge of lines of force that field around this sharp end is much higher than the field in the other regions.

Due to this very high field at the sharp ends of the conductor, the air near these points break down and conductor starts discharging. This is known as corona discharge.

3.26 HEAT GENERATED IN CAPACITOR CIRCUITS

For circuits involving capacitors and voltage sources, we can apply conservation of total energy the basic equation for conservation of energy.

The basic equation for conservation of energy will be $U_i + W = U_f + H$

U_i = Initial energy stored in all capacitors of the circuit

U_f = Final energy stored in all capacitors of the circuit

W = Work done by the sources/batteries

H = Heat generated

Here W can be positive or negative

If battery or source supplies charge q, work done will be positive

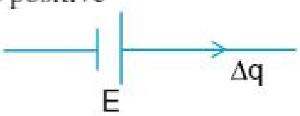


Fig 3.23

$W = E\Delta q \ (>0)$

If a capacitor of capacitance C charged to a potential V ($V \geq 0$) is connected to a source of emf E work done will be positive if E >V.

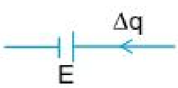
Work done $W = E\Delta q$

Where $\Delta q = (q_{final} - q_{\ initial})$

$W = E\ (CE - CV) = CE\ (E - V)$

If battery/source gets charges, work done by it will be negative

$W = E\Delta q\ (< 0)$



If a capacitor of capacitance C charged to potential V is connected to a source of emf E such that $E < V_1$ work done by the battery will be negative.

Work done $W = E\Delta q$

= where $\Delta q = q_{final} - q_{\ initial}$

or $W = E\ (CE - CV) = -\ CE\ (V - E)$

If $U_i = U_f$, Heat generated = work done by the battery ie H = W

If $U_f > U_i$ then $H = (U_i - U_f) + W$

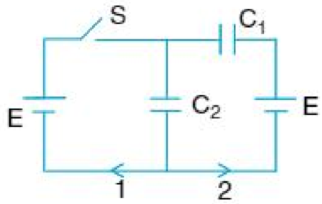
$W = (U_f - U_i) + H$

Here $(U_f - U_i) = H$

i.e., H or $(U_f - U_i) = \dfrac{W}{2}$

Example-3.19

What charge will flow through 1 and 2 if switch S is closed?

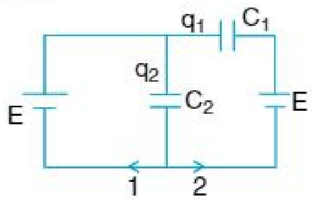


Solution :

Before closing the switch, both capacitors are in series with second source. Now charge on each capacitor is

$q = \left(\dfrac{C_1C_2}{C_1 + C_2}\right)E$

After closing the switch, let q_1 and q_2 be the charges on the capacitors



For the left loop, $\dfrac{-q_2}{C_2} + E = 0$

and for the right loop, $\dfrac{-q_2}{C_2} + E + \dfrac{-q_1}{C_1} = 0$

From these loop equations,

we get $q_1 = 0$ and $q_2 = EC_2$

So flow of charge through section 1 is

$= q_f - q_i = C_2E - 0 = C_2E$

Flow of charge through section 2 is $q_f - q_i$

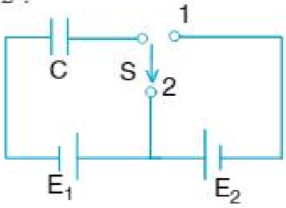
$= -q_1 - q = -(q_1 + q)$

$= \dfrac{C_1C_2E}{C_1 + C_2}$

i.e., charge flows towards the junction at section 2

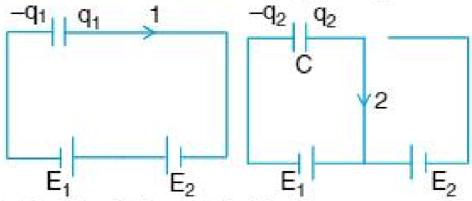
Example-3.20

What amount of heat will be generated in the circuit shown in figure after the switch S is shifted from position 1 to position 2 ?



Solution :

Let q_1 and q_2 be the charges on the capacitor for the positions of switch in 1 and 2 respectively



For the closed loop in the first figure,

$$\frac{q_1}{C} + E_2 - E_1 = 0$$

or $q_1 = C(E_1 - E_2) = CE_1 - CE_2$

For the closed loop in second figure

$\frac{q_2}{C} - E_1 = 0$ or $q_2 = CE_1$

Charge supplied by battery 1 on changing the position of switch is $\Delta q = CE_1 - (CE_1 - CE_2) = CE_2$

So, work is done by battery 1 is given as

$$W = E_1(\Delta q) = CE_1E_2$$

So, heat generated $= W + U_i - U_f = \frac{1}{2}CE_2^{\,2}$

3.27 REDISTRIBUTION OF CHARGE

Two capacitors of capacities C_1 and C_2 are charged to potentials V_1 and V_2 separately and they are connected so that charge flows. Here charge flows from higher potential to lower potential till both capacitors get the same potential.

(a) Of positive plate of one is connected to positive plate of other capacitor

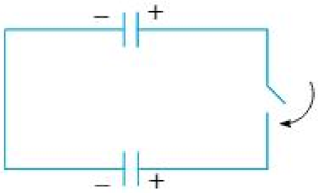


Fig 3.24

Let V be the common potential

Then $Q = Q_1 + Q_2$ (charge conservation)

$(C_1 + C_2)V = C_1V_1 + C_2V_2$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

(b) If positive plate of one capacitor is connected to negative plate of other capacitor, common potential is given by

$$V = \frac{C_1V_1 \sim C_2V_2}{C_1 + C_2}$$

Here charge flow takes place if $V_1 \neq V_2$

In this case there will be loss in energy of the system

$\Delta U = U_f - U_i$

where $U_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$

$U_i = \frac{1}{2}C_1V_1^{\,2} + \frac{1}{2}C_2V_2^{\,2}$

$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$ in the first case

$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 + V_2)^2$ in the second case.

3.28 FORCE ON A DIELECTRIC SLAB

When a dielectric is kept partially inside a capacitor, it experiences a force towards the plates due to the fringing effect. If that dielectric slab is left free, it will move into the capacitor plates. The reason behind this is induced charges with opposite polarities attract the slab.

(a) When the potential difference between the plates is kept constant, the force of attraction on it is $F = \frac{\epsilon_0 bV^2}{2d}(k-1)$.

Here b denotes the breadth of capacitor plate V is potential difference applied across the capacitor, d is the plate seperation and k is dielectric constant of the dielectric slab.

Here the force is independent of x which is length of the dielectric inside the plates.

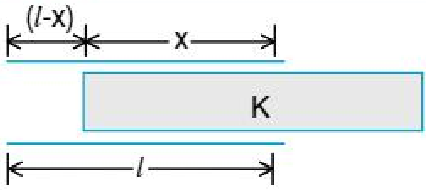


Fig 3.25

(b) When the capacitor is charged and isolated charge on it remains constant. Then the force of attraction on the slab is

$$F = \frac{Q^2 d(K-1)}{2\epsilon_0 b}\left\{\frac{1}{l + x(K-1)}\right\}^{-2}$$

In this case force is function of x.

We can find F in these two cases using energy conservation.

- In case (a) if dielectric slab is released, with a length x inside the capacitor, the slab executes periodic motion. Time period of its oscillation is $T = 8\left(\sqrt{\frac{(l-x)ml d}{\epsilon_0 AV^2(K-1)}}\right)$

3.29 ATMOSPHERIC ELECTRICITY

The electrical behaviour of atmosphere is highly complex. However, following are the electrical properties of atmosphere, on the basis of experimentally observed facts :

a) The conductivity of atmosphere increases with altitude.

b) At ground level, there is a downward vertical electric field of strength 100V/m. The strength of field decreases with altitude; being negligible at a height of 50 km.

c) There is a p.d. of about 4×10^5V between earth and 50 km high layer of atmosphere; the earth being negative. Most of the potential drop occurs at low altitude where the field strength is high.

d) Since the electric field is acting vertically downward, there is a negative surface density of charge all over the earth.

e) The earth and 50km layer of atmosphere form a capacitor of capacitance 0.1 F. We can find the p.d. and electric field.

The experimentally observed values are 4×10^5V and 100 V/m respectively. This discrepancy is explained by the fact that as we go up, conductivity of air increases.

f) There are about 40,000 thunderstorms per day all over the earth. Each storm lasts for about an hour.

g) During each thunderstorm, charged ions separate due to some complex process. The positive charges are carried upward to a height of about 6 km from ground level and negative charges collect at 2-3 km above the ground. Thus the top of thunderstorm has a positive charge and the bottom a negative one.

Knowledge Plus 3.1

☺ A metal sphere of radius 1 cm in air cannot hold a charge of 1 coulomb Why?

☛ If a metal sphere of radius 1cm is charged to hold one coulomb, electric field intensity on its surface will be

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times1}{(10^{-2})^2} = 9\times10^{13}\,\text{Vm}^{-1}\,!!!$$

The dielectric strength of air is 30,000 V cm or $3\times10^6\,\text{Vm}^{-1}$.

The value of E calculated is very large. So electric break down of air takes place and the charge leaks out. O.K, let us find out the size of the sphere which can hold a charge of one coulomb.

$$3\times10^6 = \frac{9\times10^9\times1}{r};\ \ r = 3000\text{m}$$

Imagine the size of that sphere !

Knowledge Plus 3.2

☺ When moulded plastic parts are removed from metal dies, they develop a high voltage why?

☛ When the plastic part is removed, the capacitance of the metal die decreases but the charge developed by friction remains the same. As a result voltage increases (Q = CV).

At a Glance

1. Capacitor is an arrangement of two conductors seperated by a small distance with a medium between them.
2. Capacitance of a conductor is the ratio of its charge to its potential.
3. Capacitance of a capacitor does not depend on the charge on it and the potential to which it is charged.
4. For a parallel plate capacitor $C = \frac{K \epsilon_0 A}{d}$.
5. When a capacitor is charged, energy will be stored in the field between the plates.
6. For capacitors in series $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + ...$
   For capacitors in parallel $C = C_1 + C_2 + ...$
7. When a dielectric is kept in an external field E_0, due to polarization, resultant field reduces to $E = \frac{E_0}{K}$. So, induced field in the dielectric which opposes E_0 is $E_0\left(1 - \frac{1}{K}\right)$.

EXERCISE

LONG ANSWER QUESTIONS

1. Explain principle of a capacitor. What are the uses of capacitors ? What is the effect of filling the space between the plates of a capacitor with a dielectric ?
2. Derive the expression for the energy stored in a capacitor. What is the energy stored when the space between the plates is filled with a dielectric.
   (i) With charging battery disconnected ?
   (ii) With charging battery connected in the circuit ?
3. Describe three different types of capacitors. Their construction and uses.
4. Explain series and parallel combination of capacitors. Derive the formula for equivalent capacitance in each combination.
5. Explain the behaviour of material made of non-polar molecules when it is placed in an external electric field.

SHORT ANSWER QUESTIONS

1. Define capacitance of a conductor and explain the principle of capacitor.
2. Derive an expression for the energy stored in a capacitor. If a dielectric is introduced between the plates how will energy charge?
3. Derive the equation for equivalent capacitor when a number of capacitor are connected in parallel.
4. Derive the equation for the equivalent capacitances when capacitors are connected is series.
5. Write a note on different types capacitors.
6. Explain polar and non-polar molecules and polarization.

VERY SHORT ANSWER QUESTIONS

1. Write down the uses of capacitors.

A. a) To store electrical energy

b) Condensers are used as filters to stop DC and allow AC current

c) Condensers are used in all types of electronic equipment.

2. There capacitors of capacitances $4\mu F, 6\mu F$ and $8\mu F$ are connected in parallel

a) What is the ratio of charges on them

b) What is the ratio of potential differences ?

A. In parallel connection of capacitors

$Q = Q_1 + Q_2 + Q_3$ and the potential difference on all capacitors is same.

Therefore $Q_1 = \frac{C_1}{V}$; $Q_2 = \frac{C_2}{V}$; $Q_3 = \frac{C_3}{V}$

a) $\therefore$ The ratio of charges

$$Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$$
$$Q_1 : Q_2 : Q_3 = 4 : 6 : 8 = 2 : 3 : 4$$

b) The ratio of potentials is

$$V_1 : V_2 : V_3 = V : V : V = 1 : 1 : 1$$

3. If the above capacitors are connected in series, then what is the ratio of

a) Charges on them

b) Potential differences of them

A. If the capacitors are connected in series same charge exists in all the capacitors and potential differences have a across the capacitors be

$$V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$$

Therefore (a) the ratio of charges in the capacitors

$Q_1 : Q_2 : Q_3 = Q : Q : Q = 1 : 1 : 1$

(b) The p.d's across the capacitors

$$V_1 : V_2 : V_3 = \frac{Q}{C_1} : \frac{Q}{C_2} : \frac{Q}{C_3} = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

$$= \frac{1}{4} : \frac{1}{6} : \frac{1}{8} \quad \text{(or)} \quad V_1 : V_2 : V_3 = 6 : 4 : 3$$

4. **What happens to the capacitance of a circular parallel plate capacitor if the radius of plate is doubled?**

A. The capacity of a parallel plate capacitor,

$$C_1 = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \pi r^2}{d}$$

If radius is doubled (2r). The new capacity

$$C = \frac{\varepsilon_0 \pi (2r)^2}{d} = \frac{4\varepsilon_0 \pi r^2}{d} = 4C$$

$\therefore$ The capacitance increases four times.

5. **Two charged conductors are touched mutually and then separated. What will be the charge ratio of on them?**

A. The charge on them will be divided in the ratio of their capacitances. We know that q = CV. When the charged conductors are touched they acquire the same potential. Hence, $q \propto C$

6. **How does a spark discharge occur between two charged objects?**

A. The air between the two charged objects is subjected to an electric field. If the potential gradient in the intervening air column becomes high enough, the air is ionised and conducting path is formed for free electrons, which move across to discharge the surfaces. Stored electric potential energy is dissipated as heat, light and sound.

7. **Given a solid metal sphere and a hollow metal sphere, which will hold more charge? Both spheres are of same radius.**

A. Both spheres will hold the same charge. It is because charge remains on the outer surface of a charged conductor (whether solid or hollow) and the spheres have equal surface areas.

8. **Two capacitors of capacitances $1\mu F$ and $0.01\mu F$ are charged to the same potential. Which will give more intense electric shock if touched?**

A. q = CV. Since V is constant, $q \propto C$. It means that capacitor having large capacitance will store more charge. Hence, when $1\mu F$ capacitor is touched, the discharging current will be high and you will get more intense electric shock than in case of $0.01\mu F$ capacitor.

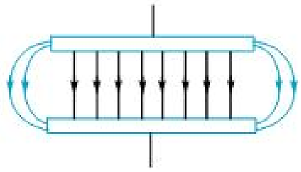
9. **Can there be potential difference between two adjacent conductors which carry the same positive charge?**

A. Yes. We know that V = q/C. The capacitance depends upon the dimensions of the conductor. If the two conductors are of different shapes and sizes, they will be charged to different potentials when given the same charge.

10. **What is 'fringing' in the case of a charged capacitor?**

A. When a capacitor is charged by connecting a battery of emf V between the plates, a uniform electric field E will be formed such that $E = \frac{V}{d}$.

A uniform field means field lines must be straight, equi spaced and parallel. But in real practice, the field will not be uniform at the outer edges of the plates. This is shown by the curved field lines (The reason is, at the edges σ is more). This is known as fringing field. But we ignore this for d << A or by assuming plate size very large.



PROBLEMS

LEVEL - I

1. Two connected bodies having respective capacitances C_1 and C_2 are charged with a total charge Q. Find the potentials of the two bodies. Also find the charges on them individually

[Ans: $Q/(C_1 + C_2)$ each; $C_1Q/(C_1 + C_2)$, $C_2Q/(C_1 + C_2)$]

2. Three connected conductors A, B and C have a total charge of $48\mu C$. The ratio of their capacitances are 1 : 3 : 2. Determine the charges on them individually.

[Ans: $8\mu C, 24\mu C, 16\mu C$]

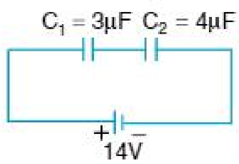
3. A sphere A of radius 5 cm contains a charge of 120 nC. It is now connected in parallel to two other spheres B and C having respective radii of 10 cm and 15 cm. Determine the charges on each of the spheres.

[Ans: 20nC, 40n, C, 60nC]

4. A sphere P of radius 6 cm is charged to a potential of 12V and is connected to an uncharged sphere Q of radius 3 cm. Determine the loss in energy during the process. **[Ans:** 160 p Joules]

5. A body is charged to a certain potential. When, an additional charge of 60 nC is imparted to it, the rise in potential is found to be 15 mV. Find the capacitance of the body. **[Ans:** $4\mu C$]

6. Determine the charges on each of the capacitors shown in the figure.



[Ans: $24\mu C$ each]

7. A $5\mu F$ capacitor is fully charged across a 12V battery. It is then disconnected from the battery and connected to an uncharged capacitor. The voltage across it is found to be 3 volts. What is the capacitance of the uncharged capacitor. **[Ans:** $15\mu F$]

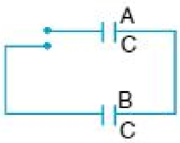
8. The potential difference between the plates of a condenser is increased by 20%. Find the percentage increase in the energy stored in the condenser. **[Ans:** 44%]

9. Capacitance of a capacitor becomes 7/6 times its original value if a dielectric slab of thickness t= 2d/3 is introduced between the plates, where 'd' is the separation between the plates. Find the dielectric constant of the dielectric slab.

[Ans: $\frac{14}{11}$]

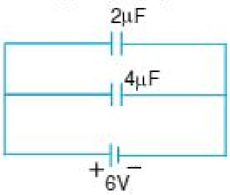
10. A $0.2\ \mu F$ capacitor is charged to 600 V. After disconnecting the battery, it is put in parallel with an uncharged $1\ \mu F$ capacitor. Find the potential. **[Ans:** 100 V]

11. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. Find the charges appearing on the capacitor B a long time after the switch S is closed.



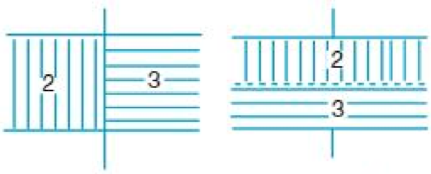
[Ans: q/2]

12. Determine the charges of each of the capacitors shown in figure. Also find the electric energy of the two capacitors system.



[Ans: $12\mu C, 24\mu C, 108\mu J$]

13. A capacitor is filled with two dielectrics of the same dimensions but of dielectric constants 2 and 3 as shown in figure (a) and in figure (b). Find the ratio of the capacitances in the two arrangements.



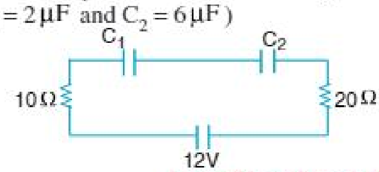
[Ans: 25 : 24]

14. A battery of emf 20 V is connected to two capacitors $1\mu F$ and $3\mu F$ in series. If $1\mu F$ capacitor withstands a maximum of 9V and $3\mu F$ withstands a maximum of 6V, find the effect of connecting the battery.

[Ans: $1\mu F$ capacitor break]

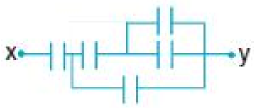
15. A fully charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, find the initial potential difference V across the capacitor. **[Ans:** $\sqrt{\frac{2ms\Delta T}{C}}$]

16. Find the charge and potential difference across capacitor C_1 on steady state in the given circuit ($C_1 = 2\mu F$ and $C_2 = 6\mu F$)



[Ans: $V_1 = 9V$ and $Q_1 = 18\mu C$]

17. If the capacitance of each condenser is $10\mu F$, find the equivalent capacitance between x and y.



[Ans: $\frac{25}{4}\mu F$]

18. A capacitor of capacitance $8\mu F$ is connected across the terminals of a battery of emf 24V. Find the energy stored in the capacitor and the work done by the battery during the process. How do you account for the difference, in terms of work-energy principle?

[Ans: 2.3 mJ, 4.6 mJ]

19. n identical condensers are joined in parallel and are charged to potential V so that energy stored in each condenser is E. If they are seperated and joined in series, find the total energy and total potential difference of the combination. **[Ans:** nE and nV]

20. A parallel plate capacitor of capacity $5\mu F$ and plate separation 6cm is connected to a 1V battery and is charged. A dielectric of dielectric constant 4 and thickness 4 cm is introduced into the capacitor. What is the additional charge that flows into the capacitor from the battery. **[Ans:** $5\mu C$]

LEVEL - II

1. Two identical capacitors are connected in series. Charge on each capacitor is q_o. A dielectric slab is now introduced between the plates of one of the capacitors so as to fill the gap, the battery remaining connected. Find the charge in each capacitor.

[Ans: $\frac{2q_o}{1+\frac{1}{k}}$]

2. A parallel plate capacitor has a capacitance $2\mu F$. A dielectric slab (K = 5) is inserted between the plates and capacitor is charged to 100V and then isolated. What is the potential when dielectric is removed
[Ans: 500 V]

3. Three condensers are connected as shown in series. If the insulated plate of C_1 is at 45V and one plate of C_3 is earthed, find the p.d between the plates of C_2

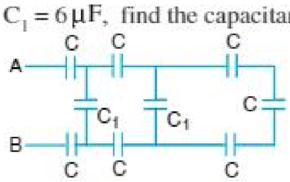
[Ans: 10V]

4. A condenser of capacity 2mF charged to a potential 200V is connected in parallel with a condenser of same capacity but charged to a potential 100V. Find the percentage loss of energy. **[Ans: 10%]**

5. Five equal capacitors connected in series have a resultant capacitance $4\mu F$. What is the ratio of energy stored when the capacitors are connected in series and then parallel and connected to the same source of emf in both the cases. **[Ans: 1:25]**

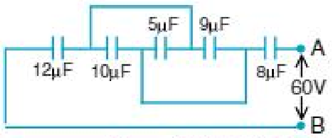
6. A spherical drop of capacitance $1\mu F$ is at 4V. It is broken into eight drops of equal radii. Find the capacitance, potential and energy of each small drop.
[Ans: $\frac{1}{2}\mu F$, 1V & $\frac{1}{4}$ μJ]

7. If in the infinite series circuit, $C = 9\,\mu F$ and $C_1 = 6\,\mu F$, find the capacitance across AB.



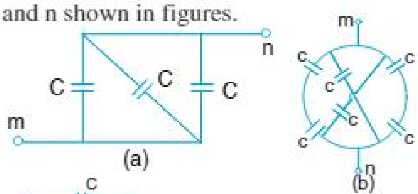
[Ans: 3 μF]

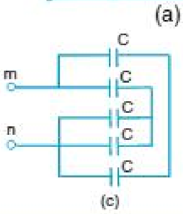
8. Find the energy stored in $5\mu F$ and $8\mu F$ capacitors.



[Ans: 250×10^{-6} J, 36×10^{-4} J]

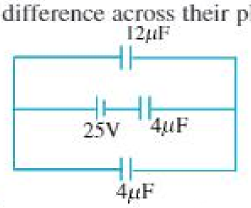
9. Find the effective capacitance between the terminals m and n shown in figures.





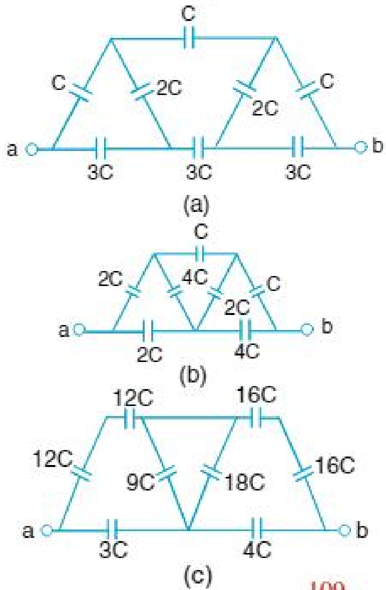
[Ans: (a) 3C; (b) 3C ;(c) 7C/6]

10. In the arrangement of capacitors shown in figure, find the charges appearing on each capacitor, and the potential difference across their plates.



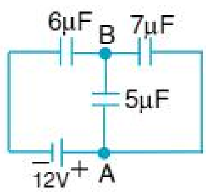
[Ans: $12\mu F$ capacitor $60\mu C, 5V$, $4\mu F$ capacitor $20\mu C, 5V$ and $4\mu F$ capacitor connected with a battery in series $80\mu C, 20V$]

11. Find the effective capacitance between the terminals a and b shown in figures.



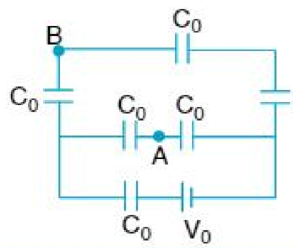
[Ans: (a) 4C/3 ; (b) $\frac{109}{54}$C ; (c) $5\frac{1}{7}$C]

12. In the arrangement shown in figure, find the potential difference between the points A and B.



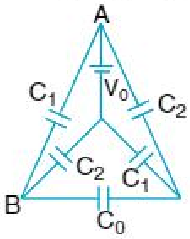
[Ans: 4V]

13. In the arrangement shown in figure, find the potential difference $V_A - V_B$.



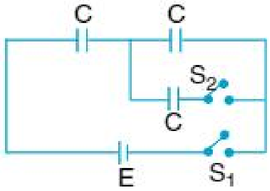
[Ans: $V_0/11$]

14. In the arrangement shown in figure, find the potential difference $V_B - V_C$. Hence, verify that if $C_1 = C_2$, then $V_B - V_C = 0$, irrespective of the value of C_0.



[**Ans:** $\frac{(C_1 - C_2 0V_0}{(C_1 + C_2 + 2C_0)}$]

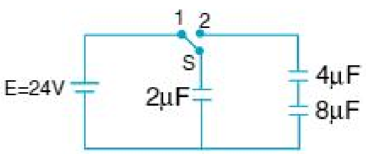
15. Figure shows an arrangement of three identical capacitors and a cell of emf E, along with two switches. Initially, both the switches are open. First switch S_1 is closed. Find (a) the charge flown through the cell, (b) the total energy stored in the capacitors, (c) the work done by the cell, and (d) the heat generated in the system, during the process. (Consider the situation after a long time of closing the switch). Next, the switch S_2 is also closed. Now answer the above four parts, by considering the situation from the beginning.



[**Ans:** (a) $\frac{CE}{2}$; (b) $\frac{CE^2}{4}$; (c) $\frac{CE^2}{2}$; (d) $\frac{CE^2}{4}$

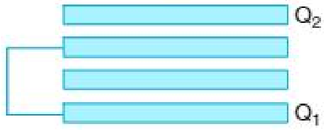
(a) $\frac{2CE}{3}$; (b) $\frac{CE^2}{3}$; (c) $\frac{2CE^2}{3}$; (d) $\frac{CE^2}{3}$]

16. In the situation shown in figure, the switch S is kept in position 1 for a long time, and then shifted to position 2. Find the heat generated after a long time of the shifting.



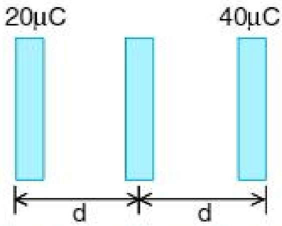
[**Ans:** $329\mu J$]

17. Figure shows an arrangement of four identical rectangular plates of dimensions (a x b) with consecutive separation as d. Charges Q_1 and Q_2 are given to two of the plates and the remaining two are joined by a wire. Find the potential difference between the charged plates.



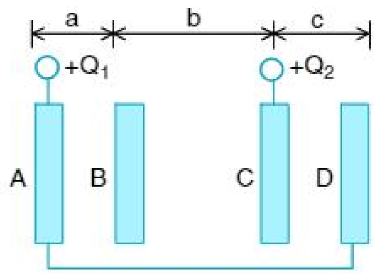
[**Ans:** $\frac{|Q_1 - Q_2| d}{\epsilon_0 \, ab}$]

18. Find the charges appearing in each of the faces (from left to right) of the three plates shown in figure. The outer plates are respectively given charges of $20\mu C$ and $40\mu C$. If the plate dimensions be (a x b) and d << a or b, find the potential difference between the outer plates.



[**Ans:** $+30\mu C, -10\mu C, +10\mu C, -10\mu C, +10\mu C, +30\mu C, (20\mu C)(d/ab\,\epsilon_0)$]

19. Figure shows an arrangement of four identical rectangular plates A, B, C and D each of area S. Find the potential difference $V_A - V_B$ and $V_C - V_A$. Ignore the separation between the plates in comparision to the plate dimensions.



Hint : Find surface-wise change distriloution p.d is due to charges on facing surfaces of respective plates. Note that $q_A + q_d = Q_1$ and $V_A = V_D$

[**Ans:** $\left[\frac{(Q_1 - Q_2)c + Q_1 b}{2\varepsilon_0 S(a+b+c)}\right]a$, $\left[\frac{(Q_2 - Q_1)a + Q_2 b}{2\varepsilon_0 S(a+b+c)}\right]c$]