1. CIRCLES

SYNOPSIS

General equation - parametric form

1. **Def :** The locus of the point moving in a plane such that it is at a constant distance from a fixed point is called a circle.

   Here the constant distance is called the radius and the fixed point is called the centre.

2. **Some important concepts on circle :**
   i) Angle in a semi circle is a right angle.
   ii) The angle made by a chord of a circle at the centre of the circle is double the angle made by the chord at any point on the circumference of the circle lying on the same side of it.
   iii) Through three non-collinear points, there exists one and only one circle.
   iv) The line joining mid point of a chord and centre of the circle is perpendicular to the chord.
   v) Perpendicular bisector of a chord of a circle passes through the centre of the circle.
   vi) If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.
   vii) The chord of a circle with maximum length is a diameter.
   vii) Circles with same centre are called concentric circles.

3. If (x_1,y_1) is centre and 'r' is radius, then the equation of the circle is $(x-x_1)^2 + (y-y_1)^2 = r^2$.

4. The circle with centre as origin and radius 'r' is $x^2 + y^2 = r^2$.

5. Equation of every circle is
   i) second degree in x and y and in it
   ii) coefficient of x^2 = coefficient of y^2
   iii) coefficient of $xy = 0$

6. Standard form of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$
   i) $g^2 + f^2 - c > 0 \Rightarrow$ real circle.
   ii) $g^2 + f^2 - c = 0 \Rightarrow$ point circle (represented by its centre)
   iii) $g^2 + f^2 - c < 0 \Rightarrow$ imaginary circle (not a real circle)

7. If $x^2+y^2+2gx+2fy+c=0$ represents a circle, then its centre is $(-g, -f)$ and radius = $\sqrt{g^2 + f^2 - c}$.

8. The equation of the circle $x^2+y^2+2gx+2fy+c=0$ contains three arbitrary constants, hence to fix a circle at least three conditions are required.

9. The general equation of a circle is $ax^2 + ay^2 + 2gx + 2fy + c = 0$ its centre is $\left(-\frac{g}{a}, -\frac{f}{a}\right)$,
   radius = $\frac{\sqrt{g^2 + f^2 - ac}}{|a|}$

10. The equation of the circle with (x_1,y_1) and (x_2,y_2) as extremities of a diameter is
    $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ (or) $x^2+y^2-(x_1+x_2)x - (y_1+y_2)y + (x_1x_2+y_1y_2)= 0$.

11. If 'r' is radius of a circle then a line which is at a distance 'd' from centre of the circle, cuts (a chord) an intercept of length $2\sqrt{r^2 - d^2}$

12. let C be the centre and r be the radius of a circle. Let P be a point in the plane of the circle and $\overrightarrow{CP}$ meets the circle at A, B. Let A be nearer to P.
   i) shortest ditance from P to the circle = PA = |CP – r|
   ii) longest distance from P to the circle = PB = CP + r
   iii) The nearest point of on the circle divides $\overline{CP}$ in the ratio r : CP – r
   iv) The farthest point B on the circle divides $\overline{CP}$ in the ratio r : CP + r externally
13. Parametric equations of the circle
   i) $x^2 + y^2 = r^2$ are $x = r\cos\theta$, $y = r\sin\theta$, parameter is θ, point 'θ' is $(r\cos\theta, r\sin\theta)$
   ii) $(x-x_1)^2 + (y-y_1)^2 = r^2$ are $x = x_1 + r\cos\theta$, $y = y_1 + r\sin\theta$ point 'θ' is $(x_1 + r\cos\theta, y_1 + r\sin\theta)$
   iii) Equation of the chord joining the two points θ_1 and θ_2 of the circle $x^2 + y^2 = r^2$ is
   $$x\cos\left(\frac{\theta_1+\theta_2}{2}\right) + y\sin\left(\frac{\theta_1+\theta_2}{2}\right) = r\cos\left(\frac{\theta_1-\theta_2}{2}\right).$$
   iv) The length of chord $\overline{AB}$ joining $A(\theta_1), B(\theta_2)$ of the circle $x^2+y^2=r^2$ (or) $(x-x_1)^2+(y-y_1)^2=r^2$ is $2r\left|\sin(\theta_1-\theta_2)\right|$
14. **Notations :** $S = x^2 + y^2 + 2gx + 2fy + c$, then $S_1 = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$
   $S_{12} = x_1x_2 + y_1y_2 + g(x_1 + x_2) + f(y_1 + y_2) + c$,
   $S_{11} = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$.
15. If $S = 0$ is a circle $P(x_1, y_1)$ is a point, then
   i) $S_{11} > 0 \Leftrightarrow P$ lies outside the circle
   ii) $S_{11} = 0 \Leftrightarrow P$ lies on the circle
   iii) $S_{11} < 0 \Leftrightarrow P$ lies inside of the circle
   iv) For the circle $(x - x_1)^2 + (y - y_1)^2 = r^2$, tangent with slope '$m$' is $(y - y_1) = m(x - x_1) \pm r\sqrt{m^2+1}$.
16. Power of a point (Def) : If 'C' is centre and 'r' is radius of a circle then the power of the point P is defined as $CP^2 - r^2$. (or) If a secant through P cuts a circle in any two points A, B then $PA. PB$ is called as power of P.
17. If $S=x^2+y^2+2gx+2fy+c=0$, is a circle in standard form and $P(x_1, y_1)$ is a point, then the power of P is S_{11}.
   Some important points on power :
   i) power of a point on the circle is zero.
   ii) power is positive if point lies outside the circle.
   iii) power is negative if point lies inside the circle.
18. The point (x, y) is called a lattice point, if x, y both are integers.
19. If the circle $S = 0$ and $L = 0$ is a line intersecting the circle in A and B, then any circle passing through A, B is of the form $S + \lambda L = 0.1$

Equation of normal - equation of tangent - length of tangent - chord of contact - area of triangle formed - circles touching the axes

1. If 'C' is centre 'r' is radius of a circle and the perpendicular distance from centre C to a line 'l' is d, then
   i) $d > r \Leftrightarrow l$ is outside of the circle.
   ii) $d = r \Leftrightarrow l$ is a tangent (touches the circle)
   iii) $d < r \Leftrightarrow l$ cuts the circle in two distinct points

2. Any circle touching
   i) x-axis is of the form $x^2+y^2+2gx+2fy+g^2=0$ (i.e. $c = g^2$), radius $=|f|$
   ii) y-axis is of the form $x^2+y^2+2gx+2fy+f^2=0$, (i.e. $c = f^2$) radius $= |g|$
   iii) both the axes is $x^2+y^2+2gx+2fy+c=0$ with $c = g^2 = f^2$. radius $= |g| = |f|$
3. Equation of the circle with centre (a, b) and touching
   i) x-axis is $x^2+y^2-2ax-2by+a^2=0$
   ii) y-axis is $x^2+y^2-2ax-2by+b^2=0$
   iii) both the axes with radius r are $x^2+y^2 \pm 2rx \pm 2ry + r^2 = 0$.
4. **Conditions for a line to touch a circle :**
   1) $y = mx + c$ to touch $x^2 + y^2 = r^2$ is $c^2 = r^2(m^2+1)$
   2) $lx + my + n = 0$ to touch $x^2 + y^2 = r^2$ is $n^2 = r^2(l^2 + m^2)$ and the point of contact is $\left(-\frac{\ell r^2}{n}, -\frac{mr^2}{n}\right)$
   3) $lx + my + n = 0$ to touch $x^2 + y^2 + 2gx + 2fy + c = 0$ is $(l^2 + m^2)(g^2 + f^2 - c) = (lg + mf - n)^2$.
5. For the circle $x^2 + y^2 = r^2$ tangent with slope 'm' is $y = mx \pm r\sqrt{m^2+1}$.
6. For the circle $(x - x_1)^2 + (y - y_1)^2 = r^2$, tangent with slope '$m$' is $(y - y_1) = m(x - x_1) \pm r\sqrt{m^2+1}$.
7. For the circle $(x-x_1)^2 + (y-y_1)^2 = r^2$
   i) tangents parallel to x-axis : $y = y_1 \pm r$
   ii) tangents parallel to y-axis : $x = x_1 \pm r$
8. For the circle $(x-x_1)^2 + (y-y_1)^2 = r^2$ tangents parallel to $\ell x + my + n = 0$ are
   $\ell x + my = \ell x_1 + my_1 \pm r\sqrt{\ell^2 + m^2}$
9. The intercept made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on
   i) x-axis is $2\sqrt{g^2-c}$ ii) y-axis is $2\sqrt{f^2-c}$
10. The number of circles touching all the three given lines, which are
    i) forming a triangle is Four
    ii) such that two of the lines are parallel is Two
    iii) concurrent is One (The point circle, represented by the point of concurrence)
    iv) all parallel Zero
11. In a circle, normal at any point of the circle passes through its centre.
12. Equation of the tangent at 'θ' to the circle $x^2 + y^2 = r^2$ is $x\cos\theta + y\sin\theta = r$.
13. Equation of the normal at 'θ' to the circle $x^2 + y^2 = r^2$ is $x\sin\theta - y\cos\theta = 0$.
14. The equation of normal at $P(x_1, y_1)$ to $x^2 + y^2 = r^2$ is $y_1x - x_1y = 0$
15. If $P(x_1, y_1)$ is a point lying outside of the circle $S = 0$ then the length of the tangent from P is $\sqrt{S_{11}}$.
16. The equation of the tangent at $P(x_1, y_1)$ of the circle $S = 0$ is $S_1 = 0$.
17. **Chord of contact (Def) :** If P is a point outside the circle and the tangents from P touch the circle in A and B, then the chord joining the points A and B (points of contact) is called as chord of contact of P, with respect to that circle.
    The chord of contact of $P(x_1, y_1)$ with respect to circle $S = 0$ is $S_1 = 0$

18. i) Two tangents can be drawn from an external point to a circle.
ii) Only one tangent can be drawn at a given point of the circle.
iii) No real tangent can be drawn from an internal point to a circle.

19. Locus of point of intersection of perpendicular tangents to the circle
i) $x^2 + y^2 = r^2$ is $x^2 + y^2 = 2r^2$
ii) $(x - x_1)^2 + (y - y_1)^2 = r^2$ is $(x - x_1)^2 + (y - y_1)^2 = 2r^2$
iii) $x^2 + y^2 + 2gx + 2fy + c = 0$ is $x^2 + y^2 + 2gx + 2fy + c = g^2 + f^2 - c$

20. Locus of point of intersection of perpendicular tangents one to each of the circles
i) $x^2+y^2 = a^2$, $x^2+y^2 = b^2$ is $x^2+y^2 = a^2+b^2$
ii) $(x-x_1)^2 + (y-y_1)^2 = a^2$, $(x-x_1)^2+(y-y_1)^2 = b^2$ is $(x-x_1)^2 + (y-y_1)^2 = a^2 + b^2$
iii) $x^2 + y^2 + 2gx + 2fy + c=0$, $x^2 +y^2 + 2gx + 2fy + c^1= 0$ is $x^2+y^2+2gx+2fy+c=g^2+f^2-c^1$.

21. Area of the quadrilateral formed by the to tangents drawn from an external point to a circle and a pair of radii through their points of contact is $r(\sqrt{S_{11}})$.

22. The length of the chord of contact of the point $P(x_1,y_1)$ w.r.t to the circle $S = 0$ is $2r\sqrt{\dfrac{S_{11}}{S_{11}+r^2}}$.

23. $S = 0$ is a circle in standard form, with centre C and radius r. If $P(x_1, y_1)$ is a point then the area of the triangle formed by pair of tangents from P and chord of contact of P is $\dfrac{r(S_{11})^{3/2}}{S_{11}+r^2}$.

24. Equation of the pair of tangents from (x_1, y_1) to the circle $S = 0$ is $S_1^2 = S_{11}S$.

Relative positions of two circles - common tangents - lengths of common tangents.

1. If C_1, C_2 are centres of two circles, whose radii are r_1, r_2 $(r_1 > r_2)$ then the number of common tangents to these two circles is
i) zero if $r_1 - r_2 > C_1C_2$ ii) one if $r_1 - r_2 = C_1C_2$
iii) two if $r_1 - r_2 < C_1C_2 < r_1 + r_2$ iv) three if $C_1C_2 = r_1 + r_2$
v) four if $C_1C_2 > r_1 + r_2$

2. Two circles with centres C_1, C_2 and radii r_1, r_2 touch each other if $C_1C_2 = r_1 \pm r_2$

3. **Centres of similitude :**
i) The point of intersection of direct common tangents of two circles is called as external centre of similitude.
If C_1, C_2 are centres r_1, r_2 are radii, then external centre of similitude divide the line joining C_1, C_2 in the ratio $r_1 : r_2$ externally.
ii) The point of intersection of transverse common tangents of two circles is called as internal centre of similitude.
If C_1, C_2 are centres r_1, r_2 are radii, then internal centre of similitude divide the line joining C_1, C_2 in the ratio $r_1 : r_2$ internally.

4. If d is the distance between centres of two circles whose radii are r_1 and r_2 then length of direct common tangent of two circles is $\sqrt{d^2 - (r_1 - r_2)^2}$ and length of transverse common tangent of two circles is $\sqrt{d^2 - (r_1 + r_2)^2}$.

Chord with midpoint - pole, polar - conjugate points, lines - inverse point - miscellaneous problems

1. **Pole and Polar (Def) :** If 'P' is a point (other than centre) in the plane of a circle and a secant line through P cuts the circle in A and B. If the tangents at A and B intersect in Q, then the locus of 'Q' is a straight line called the polar of P w.r.t the circle.
   To this polar P is called the pole w.r.t the circle.
   Equation Polar of the point $P(x_1, y_1)$ with respect to the circle $S = 0$ is $S_1 = 0$

2. **Some important aspects of pole and polar :**
   i) If P lies outside the circle, then polar is chord of contact of P.
   ii) If P lies on the circle, then polar is tangent at P to the circle.
   iii) If P lies inside the circle, then polar completely lies outside the circle.
   iv) The polar of centre of circle w.r.t the same circle does not exist.
   v) For any diameter pole does not exist.
   vi) Polar is a straight line and it is perpendicular to the line joining centre and the point.
   vii) The polar of a point if exists is unique
   viii) Polars of collinear points are concurrent.
   ix) Poles of concurrent lines are collinear.

3. Pole of the line $lx + my + n = 0$ with respect to the circle $x^2 + y^2 = r^2$ is $\left(-\frac{lr^2}{n}, -\frac{mr^2}{n}\right)$.

4. The pole of the line $lx + my + n = 0$ w.r.t the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is
   $$\left(-g+\frac{-lr^2}{N}, -f+\frac{-mr^2}{N}\right) \text{where } N = l(-g) + m(-f) + n$$

5. **Conjugate points (Def) :** If polar of a point P w.r.t a circle passes through another point Q, then the polar of Q passes through P. Two such points are called a pair of conjugate points.
   $P(x_1, y_1)$ and $Q(x_2, y_2)$ are conjugate points with respect to the curve (circle) $S = 0 \Leftrightarrow S_{12} = 0$
   For a point there exists an infinite number of conjugate points.

6. **Conjugate lines (Def) :** If pole of the line l_1 with respect to a circle lies on the line l_2, then the pole of l_2 with respect to same circle lies on l_1.
   Two such lines are called conjugate lines.
   The condition that the lines $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$ to be conjugate w.r.t the circle
   i) $x^2 + y^2 = r^2$ is $r^2(l_1l_2 + m_1m_2) = n_1n_2$
   ii) $x^2+y^2+2gx+2fy+c=0$ is $(g^2+f^2-c)(l_1l_2 + m_1m_2) = (l_1g + m_1f - n_1)(l_2g + m_2f - n_2)$
   For a line there exists infinite number of conjugate lines.

7. **Inverse point (Def) :** C is centre and 'r' is radius of a circle. If P is a point, then the point of intersection of the lines CP and polar of P is called inverse point of P w.r.t the circle.

8. **Some important aspects of inverse points :**
   i) If Q is inverse point of P, then P is inverse point of Q, then P, Q is a pair of inverse points.
   ii) If P lies inside a circle, then its inverse point lies outside the circle.
   iii) If P lies on the circle then inverse point of P is itself.
   iv) Inverse point of centre does not exist.
   v) Every pair of inverse points is a pair of conjugate points, but the converse is not true.

vi) Inverse point of a point if exists is unique.

vii) If 'C' is centre 'r' is radius of a circle and P, Q is a pair of inverse points then $CP.CQ = r^2$.

viii) P and Q ; lie on same side of the centre C.

9. Equation of the chord of the circle $S = 0$ whose mid point is (x_1, y_1) is $S_1 = S_{11}$ and its length is $2\sqrt{|S_{11}|}$.

10. The locus of mid points of chords of circle $x^2 + y^2 = r^2$ which subtend angle 'θ' at centre is $x^2 + y^2 = r^2 \cos^2 \theta/2$

11. The locus of the midpoints of the chords of the circle $S = 0$ passing through a given point is a circle with the given point and the centre of $S = 0$ as ends of diameter.

LECTURE SHEET

EXERCISE-I

General equation - parametric form

LEVEL-I (MAIN)

Single answer type questions

1. The radius of the circle passing through (6, 2) and the equations of two normals for the circle are $x + y = 6$ and $x + 2y = 4$ is

1) $\sqrt{5}$ 2) $2\sqrt{5}$ 3) $3\sqrt{5}$ 4) $4\sqrt{5}$

2. Origin is the centre of circle passing through the vertices of an equilateral triangle whose median is of length $3a$ then equation of the circle is

1) $x^2 + y^2 = a^2$ 2) $x^2 + y^2 = 2a^2$ 3) $x^2 + y^2 = 3a^2$ 4) $x^2 + y^2 = 4a^2$

3. The equation of the circle passing through (2, 0) and (0, 4) and having the minimum radius is ......

1) $x^2 + y^2 = 4$ 2) $x^2 + y^2 - 2x + 4y = 0$ 3) $x^2+y^2-x-2y = 0$ 4) $x^2 + y^2 - 2x - 4y = 0$

4. The abscissae of two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $y^2 + 2py - q^2 = 0$ then the radius of the circle with AB as diameter is

1) $\sqrt{a^2+b^2+p^2+q^2}$ 2) $\sqrt{a^2+p^2}$ 3) $\sqrt{b^2+q^2}$ 4) $\sqrt{a^2+b^2-p^2-q^2}$

5. A rod AB of length 4 units moves horizontally with its left end A always on the circle $x^2 + y^2 - 4x - 18y - 29 = 0$ then the locus of the other end B is

1) $x^2 + y^2 - 12x - 8y + 3 = 0$ 2) $x^2 + y^2 - 12x - 18y + 3 = 0$

3) $x^2 + y^2 + 4x - 8y - 29 = 0$ 4) $x^2 + y^2 - 4x - 16y + 19 = 0$

6. A circle of constant radius $3k$ passes through (0, 0) and cuts the axes in A and B then the locus of centroid of triangle OAB is

1) $x^2 + y^2 = k^2$ 2) $x^2 + y^2 = 2k^2$ 3) $x^2 + y^2 = 3k^2$ 4) $x^2 + y^2 = 4k^2$

7. A line is at a distance 'c' from origin and meets axes in A and B. The locus of the centre of the circle passing through O, A, B is

1) $x^{-2} + y^{-2} = c^{-2}$ 2) $x^{-2} + y^{-2} = 2c^{-2}$ 3) $x^{-2} + y^{-2} = 3c^{-2}$ 4) $x^{-2} + y^{-2} = 4c^{-2}$

8. If an equilateral triangle is inscribed in the circle $x^2 + y^2 - 6x - 4y + 5 = 0$ then length of its side is

1) $\sqrt{6}$ 2) $2\sqrt{6}$ 3) $3\sqrt{6}$ 4) $4\sqrt{6}$

9. The locus of the foot of the perpendicular drawn from origin to a variable line passing through fixed point (2, 3) is a circle whose diameter is

1) $\sqrt{13}$ 2) $\frac{\sqrt{13}}{2}$ 3) $2\sqrt{13}$ 4) $\sqrt{26}$

10. A square is inscribed in the circle $x^2 + y^2 - 4x + 6y - 5 = 0$ whose sides are parallel to the co-ordinate axes then vertices of square are

1) (5, 0), (5, –6), (–1, 0), (–1, –6) 2) (5, 1), (5, –6), (–1, 1), (-1, –6)

3) (5, –1), (5, 6), (–1, 0), (1, –6) 4) (0, 5), (–6, 5), (0, –1), (6, 1)

11. The line $x + y = 1$ cuts the coordinate axes at P and Q and a line perpendicular to it meet the axes in R and S. The equation to the locus of the point of intersection of the lines PS and QR is

1) $x^2 + y^2 = 1$ 2) $x^2+y^2-2x-2y=0$ 3) $x^2+y^2-x-y=0$ 4) $x^2+y^2+x+y=0$

12. Let $f(x, y) = 0$ be the equation of a circle. If $f(0,\lambda)$ has equal roots $\lambda = 2,2$ and $f(\lambda,0)$ has roots $\lambda = \frac{4}{5}, 5$ then centre of the circle is

1) $\left(2,\frac{29}{10}\right)$ 2) $\left(\frac{29}{10},2\right)$ 3) $\left(-2,\frac{29}{10}\right)$ 4) $\left(\frac{29}{10},-2\right)$

13. If $\left(m_i,\frac{1}{m_i}\right)$, i = 1,2,3,4 are concyclic points then the value of $m_1m_2m_3m_4$ is

1) 1 2) –1 3) 0 4) ∞

14. If $O = (0, 0)$, $A = (1, 0)$ and $B = \left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ then centre of the circle for which the lines OA, OB and AB are tangents is

1) $\left(\frac{1}{2},\frac{1}{2\sqrt{3}}\right)$ 2) $\left(\frac{1}{2},\frac{1}{\sqrt{3}}\right)$ 3) $\left(\frac{1}{\sqrt{3}},\frac{1}{2}\right)$ 4) $\left(\frac{1}{2\sqrt{3}},\frac{1}{2}\right)$

15. If the lines $x-2y+3=0$, $3x+ky+7=0$ cut the coordinate axes in concyclic points, then $k =$

1) 3/2 2) 1/2 3) –3/2 4) –4

16. Two rods of lengths 'a' and 'b' slide along coordinate axes such that their ends are concyclic. Locus of the centre of the circle is

1) $4(x^2+y^2) = a^2+b^2$ 2) $4(x^2+y^2) = a^2-b^2$

3) $4(x^2-y^2) = a^2-b^2$ 4) $xy = ab$

17. The sides of a square are $x = 4$, $x = 7$, $y = 1$, $y = 4$. Then the equation of the circumcircle of the square is

1) $x^2 + y^2 - 11x - 5y + 32 = 0$ 2) $x^2 + y^2 - 11x - 5y + 17 = 0$

3) $x^2 + y^2 - 6x - 5y + 7 = 0$ 4) $x^2 + y^2 - 5x - 2y + 15 = 0$

18. The shortest distance from (–2, 14) to the circle $x^2 + y^2 - 6x - 4y - 12 = 0$ is

1) 4 2) 6 3) 8 4) 10

19. The nearest point on the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ from (–5, 4) is

1) (1, 1) 2) (–1, 1) 3) (–1, 2) 4) (–2, 2)

20. If $d_1 \& d_2$ are the longest and shortest distances of (–7, 2) from any point (α,β) on the curve whose equation is $x^2+y^2-10x-14y = 51$ then G.M of d_1 & d_2 is
1) $\sqrt{11}$ 2) 7 3) 2 4) $2\sqrt{11}$

21. If a line through $P(-2, 3)$ meets the circle $x^2 + y^2 - 4x + 2y + k = 0$ at A and B such that $PA.PB = 31$ then the radius of the circle is
1) 1 2) 2 3) 3 4) 4

22. The locus of centre of a circle which passes through the origin and cuts off a length of 4 units from the line $x = 3$ is
1) $y^2 + 6x = 0$ 2) $y^2 + 6x = 13$ 3) $y^2 + 6x = 10$ 4) $x^2 + 6y = 13$

23. The radius of circle having centre at (2, 1) and whose one of the chord is diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is
1) 1 2) 2 3) 3 4) 4

24. The condition that the chord $x\cos\alpha + y\sin\alpha - p=0$ of $x^2+y^2-a^2=0$ may subtend a right angle at the centre of the circle is
1) $a^2 = 2p^2$ 2) $p^2 = 2a^2$ 3) $a = 2p$ 4) $p = 2a$

25. If OA and OB are two equal chords of the circle $x^2 + y^2-2x + 4y = 0$ perpendicular to each other and passing through the origin, the slopes of OA and OB are the roots of the equation
1) $3m^2 + 8m-3 = 0$ 2) $3m^2-8m-3 = 0$ 3) $8m^2 + 3m-8 = 0$ 4) $8m^2 + 3m-8 = 0$

26. The number of integral values of λ so that $x - 2y + \lambda = 0$ intersects the circle $x^2 + y^2 - 6x + 1 = 0$ in two distinct points is
1) 6 2) 10 3) 11 4) 13

27. The locus of a point which divides the join of $A(-1, 1)$ and a variable point P on the circle $x^2+y^2=4$ in the ratio 3 : 2 is
1) $25(x^2 + y^2) + 20(x + y) + 28 = 0$
2) $25(x^2 + y^2) - 20(x + y) + 28 = 0$
3) $25(x^2 + y^2) + 20(x - y) + 28 = 0$
4) $25(x^2 + y^2) + 20(x - y) - 28 = 0$

28. If $\frac{\pi}{6}$ and $\frac{\pi}{2}$ are the ends of chord of the circle $x^2 + y^2 = 16$ then its length is
1) 2 2) 4 3) 16 4) 8

29. If a straight line through $C(-\sqrt{8},\sqrt{8})$ making an angle 135^0 with the axes and cuts the circle $x = 5\cos\theta$, $y = 5\sin\theta$ in points A and B then $AB=$
1) 5 2) 10 3) 25 4) 16

30. A rectangle $ABCD$ is inscribed in a circle with a diameter lying along the line $3y = x + 10$. If $A = (-6, 7)$, $B = (4, 7)$ then area of the rectangle is
1) 80 sq. units 2) 40 sq.units 3) 160 sq.units 4) 20 sq.units

31. The area bounded by circles $x^2 + y^2 = r^2$, $r = 1, 2$ and rays given by $2x^2-3xy-2y^2=0$, $(y > 0)$ is
1) π 2) $\frac{3\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

32. Let AB be the chord $4x - 3y + 5 = 0$ of the circle $x^2+y^2-2x+4y-20=0$. If $C = (7, 1)$ then the area of triangle ABC is
1) 15 sq.unit 2) 20 sq.unit. 3) 24 sq.unit 4) 45 sq.unit

Numerical value type questions

33. The sum of the squares of the length of the chords intercepted by the $x + y = n$, $n \in N$ on the circle $x^2 + y^2 = 4$ is k then $\frac{k}{4}$ is ......

34. If in a ΔABC (whose circumcentre is at the origin), $a \le \sin A$, then for any point (x, y) inside the circumcircle of ΔABC, we have $|xy| < \frac{1}{\lambda}$ then $\frac{\lambda}{7}$ is

35. The Base of a triangle $AB = 6$ the third vertex C moves such that $\frac{\sin A}{\sin B} = 2$. Then Locus of C is a circle then its radius is

LEVEL-II (ADVANCED)

Single answer type questions

1. The number of lattice points that are interior to the circle $x^2 + y^2 = 25$ is

a) 81 b) 69 c) 12 d) 70

2. Three concentric circles of which biggest is $x^2 + y^2 = 1$, have their radii in A.P. If the line $y = x + 1$ cuts all the circles in real and distinct points, then the interval in which the common difference of A.P will lie, is

a) $\left(0,\left(1-\frac{1}{\sqrt{2}}\right)\right)$ b) $\left(0,\frac{1}{2}\right)$ c) $\left(0,\frac{1}{2}\left(1-\frac{1}{\sqrt{2}}\right)\right)$ d) (1, 1)

3. The set of values of 'c' so that $y = |x|+c$ and $x^2 + y^2 - 8|x| - 9 = 0$ have no solution is

a) $(-\infty,-3)\cup(3,\infty)$ b) $(-3, 3)$

c) $(-\infty,-5\sqrt{2})\cup(5\sqrt{2},\infty)$ d) $(5\sqrt{2}-4,\infty)$

4. In a triangle ABC, the equation of the side $\overline{BC}$ is $2x - y = 3$ and its circumcentre and orthocentre are at (2, 4) and (1, 2) respectively then the circumcircle of the triangle ABC is

a) $(x+2)^2+(y-4)^2=\frac{61}{25}$ b) $(x-2)^2+(y-4)^2=\frac{61}{5}$

c) $(x+2)^2+(y+4)^2=\frac{61}{25}$ d) $(x+2)^2+(y+4)^2=\frac{61}{5}$

5. The range of values of r for which the point $\left(-5+\frac{r}{\sqrt{2}}, -3+\frac{r}{\sqrt{2}}\right)$ is an interior of the major segments of the circle $x^2 + y^2 = 16$, cut off by the line $x + y = 2$ is

a) $(-\infty, 5\sqrt{2})$ b) $(4\sqrt{2}-\sqrt{14}, 5\sqrt{2})$ c) $(4\sqrt{2}-\sqrt{14}, 4\sqrt{2}+\sqrt{14})$ d) $(5\sqrt{2},\infty)$

6. On the line joining the points $A(0, 4)$ and $B(3, 0)$, a square $ABCD$ is constructed on the side of the line away from the origin. Equation of the circle having centre at 'C' and touching the axis of x is

a) $x^2 + y^2 - 14x - 6y + 49 = 0$ b) $x^2 + y^2 + 14x - 6y + 49 = 0$

c) $x^2 + y^2 - 6x - 14y + 49 = 0$ d) $x^2 + y^2 - 6x - 14y + 9 = 0$

7. The equation of a circle is $x^2 + y^2 = 4$. A regular hexagon is inscribed in the circle whose one vertex is (2, 0). Then a consecutive vertex has the coordinates

a) $(\sqrt{3},1)$ b) $(1,-\sqrt{3})$ c) $(\sqrt{3},-1)$ d) $(1,\sqrt{3})$

8. A circle of constant radius r passes through the origin O, and cuts the axes at A and B. Then the locus of the foot of the perpendicular from O to AB is

a) $(x^2 + y^2)(x^{-2} + y^{-2}) = 4r^2$ b) $(x^2 + y^2) = 4r^2$

c) $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = 4r^2$ d) $(x^2 + y^2)(x^{-2} + y^{-2})^2 = 4r^2$

9. Equation of chord AB of circle $x^2 + y^2 = 2$ passing through $P(2, 2)$ such that $\frac{PB}{PA} = 3$ is given by

a) $x = 3y$ b) $x = y$ c) $y - 2 = \sqrt{3}(x-2)$ d) $x + y = 4$

10. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the centre, angles $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then k is

a) 5 b) 12 c) 8 d) 3

11. A rhombus is formed by the radii & chords of a circle whose radius 16cm then the area of rhombus is

a) $128\sqrt{3}$ cm^2 b) 128 cm^2 c) 256 cm^2 d) $256\sqrt{3}$ cm^2

12. The equation of circum circle of a regular hexagon whose two consecutive vertices have the coordinates (–1, 0) and (1, 0) which lies wholly above x-axis is

a) $x^2 + y^2 - 2\sqrt{3}y - 1 = 0$ b) $x^2 + y^2 - \sqrt{3}y - 1 = 0$

c) $x^2 + y^2 - 2\sqrt{3}x - 1 = 0$ d) none of these

13. Let C_1, C_2 and C_3 be three parallel chords of a circle on the same side of the centre. The distance between C_1 and C_2 is the same as the distance between C_2 and C_3. The lengths of the chords are 20, 16 and 8. The radius of the cirle is

a) 12 b) $4\sqrt{7}$ c) $\frac{5\sqrt{65}}{3}$ d) $\frac{5\sqrt{22}}{2}$

14. Six points (x_i, y_i), $i = 1, 2, ..., 6$ are taken on the circle $x^2 + y^2 = 4$ such that $\sum_{i=1}^{6} x_i = 8$ and $\sum_{i=1}^{6} y_i = 4$. The line segment joining orthocentre of a triangle made by any three points and the centroid of the triangle made by other three points passes through a fixed points (h, k), then $h + k$ is

a) 1 b) 2 c) 3 d) 4

15. If a line segment AM = 'a' moves in the plane XOY. remaining parallel to OX. so that left end point A slides along the circle $x^2 + y^2 = a^2$ then the locus of M is

a) $x^2 + y^2 = 4a^2$ b) $x^2 + y^2 = 2ax$ c) $x^2 + y^2 = 2ay$ d) $x^2 + y^2 - 2ax - 2ay = 0$

16. A circle with centre at the origin and radius equal to 'a' meets the axis of X at A and B. $P(\alpha)$ and $Q(\beta)$ are two points on this circle so that $\alpha - \beta = 2\gamma$ where γ is a constant. The locus of the point of intersection of AP and BQ is

a) $x^2 - y^2 - 2ay\tan\gamma = a^2$ b) $x^2 + y^2 - 2ay\tan\gamma = a^2$

c) $x^2 + y^2 + 2ay\tan\gamma = a^2$ d) $x^2 - y^2 + 2ay\tan\gamma = a^2$

17. The circle $x^2 + y^2 = 4$ cuts the line joining the points $A(1, 0)$ and $B(3, 4)$ in two points P and Q. Let $\frac{BP}{PA} = \alpha$ and $\frac{BQ}{QA} = \beta$ then the quadratic equation whose roots are α, β is

a) $x^2 + 2x + 7 = 0$ b) $3x^2 + 2x - 21 = 0$ c) $2x^2 + 3x - 27 = 0$ d) $x^2 + x + 1 = 0$

18. If P is a point on the circle $x^2 + y^2 = 9$ and Q is a point on the line $7x + y + 3 = 0$, and the line $x - y + 1 = 0$ is the perpendicular bisector of PQ, then the coordinates of P are

a) $(3, 0)$ b) $\left(\frac{72}{25}, \frac{-21}{25}\right)$ c) $(0, 3)$ d) $\left(\frac{-72}{25}, \frac{21}{25}\right)$

19. Equation of a diameter of a circle through the origin is $x + y = 1$ and the greatest distance of any point of the circle from the diameter is $\sqrt{5}$. Then an equation of circle is

a) $x^2 + y^2 - 2x + 4y = 0$ b) $x^2 + y^2 - 4x + 2y = 0$

c) $x^2 + y^2 + 4x - 2y = 0$ d) $x^2 + y^2 + 2x + 4y = 0$

20. If a line passes through the point P(1, – 2) and cuts the circle $x^2 + y^2 - x - y = 0$ at A and B Then the maximum value of $PA + PB$ is

a) $\sqrt{26}$ b) 8 c) $\sqrt{8}$ d) $2\sqrt{8}$

21. The number of rational points on the circle $x^2 + (y - \sqrt{3})^2 = 4$ must be [rational points are points whose both co-ordinates are rational]

a) zero b) at most 2 c) 2 d) infinite

22. A and B are two points in xy-plane which are $2\sqrt{2}$ unit distance apart and subtended an angle of 90^0 at the point $C(1, 2)$ on the line $x - y + 1 = 0$ which is larger than any angle subtended by the line segent AB at any other point on the line. The equation of the circle through the points A, B and C is ....

a) $x^2 + y^2 - 6y + 7 = 0$ b) $x^2 + y^2 - 4x - 2y + 3 = 0$

c) $x^2 + y^2 - 6x + 6y + 7 = 0$ d) $x^2 + y^2 - 2x - y + 3 = 0$

23. A circle touches sides AB and AD of rectangle $ABCD$ at P and Q respectively and passes through vertex C. If distance of C from chord PQ is 5 units than area of rectangle is

1) 45 b) 75 c) 50 d) 25

More than one correct answer type questions

24. Circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 8x + 11 = 0$ cut off equal intercepts on a line through the point $\left(-2, \frac{1}{2}\right)$. The slope of the line is

a) $\frac{-1+\sqrt{29}}{14}$ b) $\frac{1+\sqrt{7}}{4}$ c) $\frac{-1-\sqrt{29}}{14}$ d) $\frac{1-\sqrt{7}}{4}$

25. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal then the line L_1 is

a) $x + y = 0$ b) $x - y = 0$ c) $x + 7y = 0$ d) $7x + y = 0$

26. The locus of a point which divides the join of $A(-1,1)$ and a variable point P on the circle $x^2 + y^2 = 4$ in the ratio 3:2 meets the line $y = x$ at the point

a) $\left(\sqrt{\frac{14}{25}},\sqrt{\frac{14}{25}}\right)$ b) $\left(\sqrt{\frac{8}{25}},\sqrt{\frac{8}{25}}\right)$ c) $\left(-\sqrt{\frac{14}{25}},-\sqrt{\frac{14}{25}}\right)$ d) $\left(-\sqrt{\frac{8}{25}},-\sqrt{\frac{8}{25}}\right)$

27. An equation of a circle through the origin making an intercept of $\sqrt{10}$ on the line $y = 2x + 5/\sqrt{2}$ which subtends an angle of 45^o at the origin is

a) $x^2 + y^2 - 4x - 2y = 0$ b) $x^2 + y^2 - 2x - 4y = 0$

c) $x^2 + y^2 + 4x + 2y = 0$ d) $x^2 + y^2 + 2x + 4y = 0$

28. Equation of chord of the circle $x^2 + y^2 - 3x - 4y - 4 = 0$, which passes through the origin such that the origin divides it in the ratio 4:1, is

a) $y = 0$ b) $24x + 7y = 0$ c) $7x + 24y = 0$ d) $7x - 24y = 0$

29. If α, β, γ are the parameters of points A, B, C on circle $x^2 + y^2 = a^2$ and if the triangle ABC be equilateral, then

a) $\sum \cos\alpha = 0$ b) $\sum \sin\alpha = 0$ c) $\sum \tan\alpha = 0$ d) $\sum \cot\alpha = 0$

30. $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$ represents

a) equation of a straight line, if θ is constant and r is variable

b) equation of a circle, if r is constant and θ is a variable

c) a straight line passing through a fixed point and having a known slope

d) a circle with a known centre and a given radius.

31. Equation of a circle with centre $C(h, k)$ such that $h^2 - 3h + 2 = 0$ and $k^2 - 5k + 6 = 0$ having radius 4 can be

a) $x^2 + y^2 - 2x - 4y - 11 = 0$ b) $x^2 + y^2 - 2x - 6y - 6 = 0$

c) $x^2 + y^2 - 4x - 4y - 8 = 0$ d) $x^2 + y^2 - 4x - 6y - 3 = 0$

Linked comprehension type questions

Passage - I :

Consider a circle $x^2 + y^2 = a^2$. Let $A(a, 0)$ and $D(\alpha, \beta)$ be a given interior point of the circle. If BC be an orbitrary chord of the circle through point D. (Where θ is the angle of inclination of chord BC) Then

32. Arithmatic mean of BD & DC is

a) $-(\alpha\cos\theta+\beta\sin\theta)$ b) $2(\alpha\cos\theta+\beta\sin\theta)$

c) $-2(\alpha\sin\theta+\beta\cos\theta)$ d) $2(\alpha\sin\theta+\beta\cos\theta)$

33. Harmonic mean of BD & DC is

a) $\dfrac{\alpha^2+\beta^2-a^2}{2(\alpha\cos\theta+\beta\sin\theta)}$ b) $-\dfrac{\alpha^2+\beta^2-a^2}{(\alpha\cos\theta+\beta\sin\theta)}$ c) $\dfrac{\alpha^2+\beta^2-a^2}{\alpha\sin\theta+\beta\cos\theta}$ d) $\dfrac{2(\alpha^2+\beta^2-a^2)}{\alpha\cos\theta+\beta\sin\theta}$

34. The locus of centroid of ΔABC is a circle $\left(x-\dfrac{a}{3}\right)^2+y^2=\lambda^2$, when $\lambda=$

a) $\dfrac{r_1+r_2}{2}$ b) $\dfrac{r_1+r_2}{3}$ c) $\dfrac{r_1-r_2}{2}$ d) $\dfrac{r_1-r_2}{3}$

Passage - II :

A point 'P' moves in a plane such that $\dfrac{PA}{PB}=\lambda$, where $\lambda\in(0, 1)$ is a constant and $A(0, 0)$, $B(a, 0)$ are fixed points where $AB = a$.

35. The locus of P is a circle whose diameter is

a) $\dfrac{a\lambda}{1-\lambda^2}$ b) $\dfrac{a\lambda}{2(1-\lambda^2)}$ c) $\dfrac{2a\lambda}{1-\lambda^2}$ d) None of these

36. The locus of 'P' will have

a) the point A as it's interior point and the point B as it's exterior point

b) the point B as it's interior point

c) the points A and B both as it's interior points

d) None of these

Passage - III :

Consider the two circles $C_1\equiv x^2+y^2=r_1^2$ & $C_2\equiv x^2+y^2=r_2^2$ $(r_2<r_1)$. Let A be a fixed point on the circle C_1 say $A(r_1, 0)$ & B be a variable point on the circle C_2, the line BA meets the circle C_2 again at C then,

37. The set of values of $OA^2+OB^2+BC^2$ =

a) $\left[5r_2^2-3r_1^2, 5r_2^2+r_1^2\right]$ b) $\left[4r_2^2-4r_1^2, -4r_1^2\right]$

c) $\left[4r_1^2, 4r_2^2\right]$ d) $\left[5r_2^2-3r_1^2, 5r_2^2+3r_1^2\right]$

38. The locus of the midpoint of AB (O being the origin) is

a) $\left(x-\dfrac{r_1}{2}\right)^2+y^2=\dfrac{r_2^2}{2}$ b) $\left(x-\dfrac{r_1}{2}\right)^2+y^2=\dfrac{r_2^2}{4}$

c) $\left(x-\dfrac{r_2}{2}\right)^2+y^2=\dfrac{r_1^2}{2}$ d) $\left(x-\dfrac{r_2}{2}\right)^2+y^2=\dfrac{r_1^2}{4}$

Matrix matching type questions

39. A variable straight line through $A(-1, 1)$ is drawn to cut the circle $x^2 + y^2 = 1$ at the points B and C. A point 'P' is chosen on the line ABC satisfying the condition given the Column - I. Let d be the minimum distance of the origin from the locus of P given in the Column - II

COLUMN - I	COLUMN - II
A) AB, AP, AC are in A.P	p) 0
B) AB, AP, AC are in G.P	q) $1/\sqrt{2}$
C) AB, AP, AC are in H.P	r) $\sqrt{2}$
D) $AB, \frac{AP}{2}, AC$ are in A.P	s) $\sqrt{2}-1$

40. Let $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3, y_3)$ be 3 distinct points lying on circle $S : x^2 + y^2 = 1$, such that $x_1x_2 + y_1y_2 + x_2x_3 + y_2y_3 + x_3x_1 + y_3y_1 = -\frac{3}{2}$.

COLUMN - I	COLUMN - II
A) Let P be any arbitrary point lying on S, then $(PA)^2+(PB)^2+(PC)^2 =$	p) 3
B) Let the perpendicular dropped from point 'A' to BC meets S at Q and $\angle OBQ = \frac{\pi}{k}$, where '$O$' is origin, then $k =$	q) 4
C) Let R be the point lying on line $x + y = 2$ at the minimum distance from S and the square of maximum distance of R from S is $a + b\sqrt{b}$, then $a + b =$	r) 5
D) Let I and G represent incenter and centroid of ΔABC respectively, then $IA + IB + IC + GA + GB + GC =$	s) 6

Integer answer type questions

41. The number of such points $(a + 1, \sqrt{3}a)$ where a is any integer lying inside the region bounded by the curve $x^2 + y^2 - 2x - 3 = 0$ and $x^2 + y^2 - 2x - 15 = 0$ is ....

42. A line is drawn through the point $A(3, 4)$ inclined at an acute angle θ with positive direction of the x-axis. If it cuts the circle $x^2 + y^2 = 4$ in B and C. Then the maximum value of $\left|\frac{AB+AC}{2}\right|$ is ....

EXERCISE-II

Equation of normal - equation of tangent - length of tangent - chord of contact - area of triangle formed - circles touching the axes

LEVEL-I (MAIN)

Single answer type questions

1. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then the radius of the circle is

1) $\frac{3}{4}$ 2) $\frac{1}{2}$ 3) $\frac{7}{8}$ 4) $\frac{11}{10}$

2. The normal of the circle $(x-2)^2+(y-1)^2=16$ which bisects the chord cut off by the line $x-2y+3=0$ is

1) $2x+y+3=0$ 2) $2x+y-4=0$ 3) $2x+y-5=0$ 4) $2x+y-7=0$

3. If $y=3x$ is a tangent to a circle with centre (1, 1) then the other tangent drawn through (0, 0) to the circle is

1) $3y=x$ 2) $y=-3x$ 3) $y=2x$ 4) $3y=-2x$

4. Locus of point of intersection of tangents to the circle $x^2+y^2=a^2$ which makes complimentary angles with x-axis is

1) $x^2-y^2=0$ 2) $x^2+y^2=0$ 3) $xy=0$ 4) $x^2+y^2=2a^2$

5. Tangents AB and AC are drawn to the circle $x^2+y^2-2x+4y+1=0$ from $A(0, 1)$ then equation of circle passing through A, B and C is

1) $x^2+y^2+x+y+2=0$ 2) $x^2+y^2-x+y-2=0$
3) $x^2+y^2-x-y-2=0$ 4) $x^2+y^2-x-y+2=0$

6. Angle between tangents drawn from a point P to circle $x^2+y^2-4x-8y+8=0$ is 60^0 then length of chord of contact of P is

1) 6 2) 4 3) 2 4) 3

7. Locus of the point of intersection of tangents to the circle $x^2+y^2+2x+4y-1=0$ which include an angle of 60^0 is

1) $x^2+y^2+2x+4y-19=0$ 2) $x^2+y^2+2x+4y+19=0$
3) $x^2+y^2-2x-4y-19=0$ 4) $x^2+y^2-2x-4y+19=0$

8. The angle between the pair of tangents drawn from a point P to the circle $x^2+y^2+4x-6y+9\sin^2\alpha+13\cos^2\alpha=0$ is 2α then locus of 'P' is

1) $x^2+y^2+4x-6y+9=0$ 2) $x^2+y^2-4x-6y+9=0$
3) $x^2+y^2+4x+6y-9=0$ 4) Does not exists

9. Locus of point of intersection of perpendicular tangents to the circle $x^2+y^2-4x-6y-1=0$ is

1) $x^2+y^2-4x-6y-15=0$ 2) $x^2+y^2-4x-6y+15=0$
3) $x^2+y^2-4x-3y-15=0$ 4) $x^2+y^2+4x+6y-15=0$

10. Locus of the point of intersection of perpendicular tangents drawn one to each of the circles $x^2+y^2=8$ and $x^2+y^2=12$ is

1) $x^2+y^2=4$ 2) $x^2+y^2=20$ 3) $x^2+y^2=208$ 4) $x^2+y^2=16$

11. If two tangents are drawn from a point on $x^2+y^2=16$ to the circle $x^2+y^2=8$ then the angle between the tangents is

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{2\pi}{3}$ 4) π

12. No. of circles touching all the lines $x+4y+1=0$, $2x+3y+3=0$ and $x-6y+3=0$ is

1) 0 2) 2 3) 4 4) Infinite

13. No. of circles touching all the lines $x-2y+1=0$, $2x+y+3=0$ and $4x-8y+3=0$ is

1) 1 2) 2 3) 4 4) Infinite

14. If the line $y=x$ touches the circle $x^2+y^2+2gx+2fy+c=0$ at P where $OP=6\sqrt{2}$ then $c=$

1) 36 2) 72 3) 18 4) 144

15. The length of tangent from any point on the circle $x^2+y^2+4x-6y-12=0$ to the circle $x^2+y^2+4x-6y+4=0$ is

1) 2 2) 16 3) 8 4) 4

16. The locus of point of Intersection of tangents to the circle $x=a\cos\theta,\ y=a\sin\theta$ at points whose parametric angles differ by $\pi/4$ is

1) $x^2+y^2=2(\sqrt{2}-1)^2a^2$ 2) $x^2+y^2=2(2-\sqrt{2})\ a^2$

3) $x^2+y^2=(\sqrt{2}+1)^2\ a^2$ 4) $9(x^2+y^2)=4a^2$

17. The equation of the circle of radius 3 that lies in 4th quadrant and touching the lines $x=0$, $y=0$ is

1) $x^2+y^2-6x+6y+9=0$ 2) $x^2+y^2+6x+6y+9=0$

3) $x^2+y^2-6x-6y+9=0$ 4) $x^2+y^2+6x-6y+9=0$

18. If two circles touching both the axes intersect at two points P and Q where $P=(3, 1)$ then $PQ=$

1) $\sqrt{2}$ 2) $2\sqrt{2}$ 3) $3\sqrt{2}$ 14) $4\sqrt{2}$

19. The radius of the largest circle lying in the first quadrant and touching $4x+3y=12$ and co-ordinate axes is

1) 5 2) 6 3) 7 4) 8

20. Equation of circle which touch x-axis at (3, 0) and making y-intercept of length 8 units is

1) $x^2+y^2-6x\pm 10y+9=0$ 2) $x^2+y^2-6x\pm 10y-9=0$

3) $x^2+y^2\pm 6x\pm 10y+9=0$ 4) None

21. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is

1) an ellipse 2) a circle 3) a hyperbola 4) a parabola

22. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through 'A' is

1) $(x-p)^2=4qy$ 2) $(x-q)^2=4py$ 3) $(y-p)^2=4qx$ 4) $(y-q)^2=4px$

23. The area of the triangle formed by two tangents from (1, 1) to $x^2+y^2+4x+6y+4=0$ and their chord of contact is

1) $\frac{192}{5}$ 2) $\frac{192}{15}$ 3) $\frac{192}{25}$ 4) $\frac{96}{25}$

24. The area of the triangle formed by the tangent drawn at the point (–12, 5) on the circle $x^2+y^2=169$ with the coordinate axes is

1) $\frac{625}{24}$ 2) $\frac{28561}{120}$ 3) $\frac{225}{23}$ 4) $\frac{8561}{20}$

25. If OA and OB are the tangents from origin to the circle $x^2+y^2+2gx+2fy+c=0$ and C is the centre of the circle then the area of the quadrilateral $OACB$ is

1) $\sqrt{g^2+f^2-c}$ 2) $\sqrt{c\left(g^2+f^2-c\right)}$ 3) $\frac{\sqrt{g^2+f^2-c}}{c}$ 4) $\frac{\sqrt{g^2+f^2-c}}{2}$

26. A line meets the coordinate axes in A and B. A circle is circumscribed about the triangle OAB. If m and n are distances of tangent to circle at origin from the points A and B respectively then diameter of the circle is

1) $m + n$ 2) $m - n$ 3) mn 4) m/n

27. The length of the chord of contact of (–2, 3) w.r.t to the circle $x^2 + y^2 - 2x + 4y + 1 = 0$ is

1) $15\sqrt{\frac{13}{3}}$ 2) $5\sqrt{\frac{3}{13}}$ 3) $4\sqrt{\frac{15}{17}}$ 4) $15\sqrt{\frac{3}{13}}$

28. The locus of the point the chord of contact which w.r.t to the circle $x^2 + y^2 = a^2$ subtendes a right angle at the centre of the circle is

1) $x^2 + y^2 = a^2/2$ 2) $x^2 + y^2 = a^2/3$ 3) $x^2 + y^2 = 2a^2$ 4) $x^2 + y^2 = 3a^2$

Numerical value type questions

29. A circle $x^2 + y^2 + 4x - 2\sqrt{2}y + c = 0$ is the directr circle of circle S_1 and S_2 is the director circle of circle S_1 and so on. If the sum of radii of all these circles is 2, then the value of C is ....

30. Lines are drawn through the point $P(-2, -3)$ to meet the circle $x^2 + y^2 - 2x - 10y + 1 = 0$. The length of the line segment PA, A being the point on the circle when the line meets circle at coincidental points, is

31. If the radius of the circumcircle of the triangle TPQ, where PQ is chord of contact corresponding to point T with respect to circle $x^2 + y^2 - 2x + 4y - 11 = 0$, is 2 units, then minimum distance of T from the director circle of the given circle is

LEVEL-II (ADVANCED)

Single answer type questions

1. A ray of light incident at the point (–2,–1) gets reflected from the tangents at (0, –1) to the circle $x^2 + y^2 = 1$ The reflected ray touches the circle, the equation of the line along which the incident ray moved is

a) $4x - 3y + 11 = 0$ b) $4x + 3y + 11 = 0$ c) $3x + 4y + 11 = 0$ d) None of these

2. A chord of the circle $x^2 + y^2 - 4x - 6y = 0$ passing through the origin subtends an angle $\tan^{-1}(7/4)$ at the point where the circle meets positive y–axis. Equation of the chord is

a) $2x + 3y = 0$ b) $x + 2y = 0$ c) $x - 2y = 0$ d) $2x - 3y = 0$

3. In an equilateral triangle, 3 coins of radii '1' are kept so that they touch each other and also the sides of the triangle, then the area of the triangle is

a) $4 + 2\sqrt{3}$ b) $6 + 4\sqrt{3}$ c) $12+\frac{7\sqrt{3}}{4}$ d) $3+\frac{7\sqrt{3}}{4}$

4. Tangents are drawn to the circle $x^2 + y^2 = 50$ from a point 'P' lying on the x-axis. These tangents meet the y-axis at points 'A' and 'B'. Possible coordinates of 'P' so that area of triangle PAB is minimum is

a) (10, 0) b) $(10\sqrt{2},0)$ c) $(-10\sqrt{2},0)$ d) (0, 0)

5. The value of c for which the sets $\{(x,y)/x^2 + y^2 + 2x \le 1\}$, $\{(x,y)/x - y + c \ge 0\}$ contains only one point in common is

a) $(-\infty,-1]\cup[3,\infty)$ b) $\{-1, 3\}$ c) $\{-3\}$ d) $\{-1\}$

6. If the pair of straight lines $xy\sqrt{3}-x^2=0$ is tangent to the circle at P and Q from origin O such that area of smaller sector formed by CP and CQ is 3π sq. unit, where C is the centre of circle, then OP equals to

a) $(3\sqrt{3})/2$ b) $3\sqrt{3}$ c) 3 d) $\sqrt{3}$

7. If the area of the triangle formed by two tangents are drawn from $P(6, 8)$ to the circle $x^2 + y^2 = r^2$ and chord of contact is maximum then the radius is

a) 2 b) 3 c) 4 d) 5

8. A tangent to the circle $x^2 + y^2 = 1$ through the point (0, 5) cuts the circle $x^2 + y^2 = 4$ at A and B. The tangents to the circle $x^2 + y^2 = 4$ at A and B meet at C. The coordinates of C are

a) $\left(\frac{8\sqrt{6}}{5},\frac{4}{5}\right)$ b) $\left(\frac{8\sqrt{6}}{5},\frac{-4}{5}\right)$ c) $\left(\frac{-8\sqrt{6}}{5},\frac{-4}{5}\right)$ d) (8, 4)

9. If C_1 and C_2 are the two concentric circles with radii r_1 and r_2 $(r_1 < r_2)$. If the tangents drawn from any point of C_2 to C_1 meets again C_2 at the ends of its diameter then

a) $r_2 = 2r_1$ b) $r_2 = \sqrt{2}\, r_1$ c) $r_2^2 < 2r_1^2$ d) $r_2 = 3r_1$

10. The tangents PA and PB are drawn from any point P on the circle $x^2+y^2=2a^2$ to the circle $x^2+y^2= a^2$. The chord of contact AB on extending meets again the first circle at the points A' and B'. The locus of the point of intersection of tangents at A' and B' may be given by

a) $x^2 + y^2 = 3a^2$ b) $x^2 + y^2 = 4a^2$ c) $x^2 + y^2 = 6a^2$ d) $x^2 + y^2 = 8a^2$

11. A point P moves such that chord of contact of P w.r.t the circle $x^2 + y^2 = 4$ passes through the point (1, 1). The point on the locus of P nearest to the origin is

a) (1, 2) b) (2, 2) c) (3, 3) d) none

12. Let BC be the chord of contact of the tangents from a point A to the circle $x^2 + y^2 = 1$. P is any point on the arc BC. Let PX, PY, PZ be the lengths of the perpendiculars from P on AB, BC and CA respectively then PX, PY, PZ are in ....

a) A.P b) G.P c) H.P d) none

13. If C_1, C_2, C_3 is a sequence of circles such that C_{n+1} is the director circle of C_n. If the radius of C_1 is a then the area bounded by the circle C_n and C_{n+1} is ....

a) $\pi\, 2^n a^2$ b) $\pi\, 2^{2n-2} a^2$ c) $\pi\, 2^{2n-1} a^2$ d) none

14. If normal at $P(1, -2)$ to the circle intersects it again at Q and equation of tangent at Q to the circle is $4y = 3x + 14$ than equation of circle is :

a) $(2x - 1)^2 + 4y^2 = 25$ b) $(2x + 1)^2 + 4y^2 = 25$ c) $\left(x-\frac{1}{2}\right)^2+y^2=15$ d) $\left(x+\frac{1}{2}\right)^2+y^2=25$

More than one correct answer type questions

15. A point $P(\sqrt{3},1)$ moves on the circle $x^2 + y^2 = 4$ and after covering a quarter of the circle leaves it tangentially. The equation of a line along which point moves after leaving the circle is

a) $y=\sqrt{3}x+4$ b) $\sqrt{3}y=x+4$ c) $\sqrt{3}y=x-4$ d) $y=\sqrt{3}x-4$

16. No. of points of intersections of director circle of $x^2 + y^2 = 4$ and the curve

a) $|x|+|y|=4$ are 4 b) $|x|+|y|=2$ are 8 c) $|x|+|y|=6$ are 0 d) $|x|+|y|=2\sqrt{2}$ are 4

17. Equation of a circle of radius 1 touching the circles $x^2 + y^2 - 2|x| = 0$ is

a) $x^2 + y^2 + 2\sqrt{2}\,x + 1 = 0$
b) $x^2 + y^2 - 2\sqrt{3}\,y + 2 = 0$
c) $x^2 + y^2 + 2\sqrt{3}\,y + 2 = 0$
d) $x^2 + y^2 - 2\sqrt{2}\,y + 1 = 0$

18. From any point P on the circle $x^2 + y^2 - 6x - 8y + 15 = 0$ tangents PA, PB are drawn to the circle $x^2 + y^2 - 6x - 8y + 20 = 0$ and the locus of the orthocentre of the Δ^{le} PAB is a circle then

a) centre is (3, 4)
b) centre is (–3, 2)
c) radius is $\sqrt{5}$
d) radius is $\sqrt{10}$

Linked comprehension type questions

Passage - I :

Let P is a point on the circle $x^2 + y^2 - 6x - 8y + 16 = 0$, where O is the origin and OX is the positive side of the x-axis then the cooradinates of the point P such that

19. OP is minimum

a) $\left(\frac{4}{5},\frac{32}{5}\right)$
b) $\left(\frac{8}{5},\frac{6}{5}\right)$
c) $\left(\frac{6}{5},\frac{8}{5}\right)$
d) $\left(\frac{32}{5},\frac{24}{5}\right)$

20. $\angle POX$ is maximum

a) (0, 4)
b) $\left(\frac{28}{25},\frac{96}{25}\right)$
c) $\left(\frac{24}{5},\frac{32}{5}\right)$
d) $\left(\frac{8}{5},\frac{6}{5}\right)$

Passage - II :

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}\,x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2},\frac{3}{2}\right)$. Further it is given that the origin and the centre C are on the same side of the line PQ

21. The equation of circle C is

a) $(x-2\sqrt{3})^2 + (y-1)^2 = 1$
b) $(x-2\sqrt{3})^2 + \left(y+\frac{1}{2}\right)^2 = 1$
c) $(x-\sqrt{3})^2 + (y+1)^2 = 1$
d) $(x-\sqrt{3})^2 + (y-1)^2 = 1$

22. Equation of the sides QR, RP are

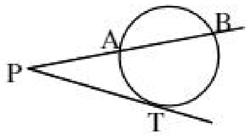
a) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$
b) $y = \frac{1}{\sqrt{3}}x, y = 0$
c) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$
d) $y = \sqrt{3x}, y = 0$

23. Points E and F are given by

a) $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),\left(\sqrt{3},0\right)$
b) $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right),\left(\sqrt{3},0\right)$
c) $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
d) $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

Passage-III :

From a point P outside a circle if a tangent PT and a secant PAB is drawn as shown in figure then $PT^2 = PA.PB$



24. If $PA = 2^{\sqrt{5}}$, $PB = 5^{\sqrt{2}}$ and $\log_{10}^{(PT)^2} = (\sqrt{a} - \sqrt{b})\log_{10}^{2} + \sqrt{b}$ where a, b N then value of $a + b$ equals

    a) 7   b) 9   c) 11   d) 13

25. If $PA = x$; $PT = y$ and $PB = z$ and $a^x = b^y = c^z$ where a, b, c are positive real numbers then

    a) $\log_b^a = \log_c^b$ b) $\log_c^b = \log_a^c$ c) $\log_a^c = \log_b^a$ d) $\log_b^a = 2\log_a^c$

26. If $PT^2 = 8\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$ and $PA = \log_{10}^{\sin x}$ and $PB = \log_{\cos x}^{10}$ then the number of solutions of the equations $PT^2 = PA.PB$ where $x \in [0, 2\pi]$ is/are "

    a) 4   b) 3   c) 2   d) 1

Passage - IV :

A square s inscribed in a circle of radius 2 which touches the line $y = 1$ at $(5, 1)$. One side of the square is parallel to $y = x + 3$.

27. If a circle is inscribed in a square than its area will be

    a) 2π b) 4π c) 16π d) none

28. Equation of the first circle will be

    a) $x^2 + y^2 + 10x + 6y - 30 = 0$ b) $x^2 + y^2 - 10x - 6y + 30 = 0$

    c) $x^2 + y^2 - 10x - 6y - 30 = 0$ d) $x^2 + y^2 - 6x - 10y - 30 = 0$

Matrix matching type questions

29. For the circle $x^2 + y^2 + 4x + 6y - 19 = 0$

COLUMN - I	COLUMN - II
A) Length of the tangent from (6, 4) to the circle	p) $\frac{72\sqrt{226}}{113}$
B) Length of the chord of contact from (6, 4) to the circle	q) $\sqrt{113}$
C) Distance of (6, 4) from the centre of the circle	r) $\sqrt{113} - \sqrt{32}$
D) Shortest distance of (6, 4) from the circle	s) 9

Integer answer type questions

30. The equation of pair of tangents from origin to a circle is $24xy + 7y^2 = 0$. If the radius of the circle is 3 then the length of the tangent drawn from the origin is ....

31. Angle between tangents drawn from a point P to circle $x^2 + y^2 - 4x - 8y + 8 = 0$ is 60^0 then length of chord of contact of P is ....

32. The value of α in $[0, \pi]$ so that $x^2+y^2+2\sqrt{\sin\alpha}\,x+(\cos\alpha-1)=0$ having intercept on x-axis always greater than 2 is $\left(\frac{\pi}{k}, \pi\right]$ then k is

33. No. of circles which touch both the Axes and whose centre lies on $x - 2y = 3$ is

34. A triangle has two of its sides along the axes, its third side touches the circle $x^2+y^2-2ax-2ay+a^2=0$. If the locus of the circumcentre of the triangle passes (38, –37) then $a^2 - 2a$ is equal to K then the digit in ten's place is____

35. A circle of radius 1 unit touches positive x-axis and positive y-axis at A and B, respectively. A variable line passing through origin intersects the circle in two points D and E. If the area of the triangle DEB is maximum when the slope of the line is m, then find the value of m^{-2}.

36. It angle between tangents drawn to $x^2 + y^2 - 12x - 16y = 0$ at points where curve is cut by $5y = 5x + C$ ($C > 0$) is $\frac{\pi}{2}$ and $C = 10k$, then k equals ....

37. From a point A on a circle $x^2 + y^2 = 3$ chords AB and AC, making equal angles with the normal at A, are drawn. The maximum perimeter of the ΔABC ....

EXERCISE-III

Relative positions of two circles - common tangents - lengths of common tangents

LEVEL-I (MAIN)

Single answer type questions

1. The circles $x^2 + y^2 - 2x - 4y - 20 = 0$, $x^2 + y^2 + 4x - 2y + 4 = 0$ are

1) one lies out side the other
2) one lies completely inside the other
3) touch externally
4) touch internally

2. The number of common tangents to $x^2 + y^2 = 256$, $(x-3)^2 + (y-4)^2 = 121$ is

1) one 2) two 3) four 4) zero

3. If the circles $x^2 + y^2 + 2gx + 2fy = 0$, $x^2 + y^2 + 2g^1x + 2f^1y = 0$ touch each other then

1) $fg = f^1g^1$ 2) $fg^1 = f^1g$ 3) $f + g = f^1 + g^1$ 4) $f + f^1 = g + g^1$

4. If the distance between the centres of two circles of radii 3, 4 is 25 then the length of the transverse common tangent is ....

1) 24 2) 12 3) 26 4) 13

5. Lengths of common tangents of the circles $x^2 + y^2 = 6x$, $x^2 + y^2 + 2x = 0$ are

1) $\sqrt{3}$ 2) $\sqrt{3}, 3\sqrt{3}$ 3) $2\sqrt{3}$ 4) $2\sqrt{3}, 3\sqrt{3}$

Numerical value type questions

6. A circle of radius 2 has centre at (2, 0). A circle of radius 1 has centre at (5, 0). A line is tangent to the two circles at points in the first quadrant. which of the following is the y -intercept of the line is ___

7. C_1, C_2, C_3 are circles of radii 5, 3, 2 respectively. C_1 and C_2 touch each other externally and C internally. The radius of circles C_3 which touches C interally and C_2, C_1 externally is ....

LEVEL-II (ADVANCED)

Single answer type questions

1. Two circles of radii r_1 & r_2 $(r_1 > r_2)$ touch each other externally then the radius of the circle which touches both of them externally & also their direct common tangent is

a) $\frac{r_1 r_2}{(\sqrt{r_1}+\sqrt{r_2})^2}$ b) $\sqrt{r_1 r_2}$ c) $\frac{r_1 r_2}{2}$ d) $r_1 - r_2$

2. If C, C_1, C_2 be the circles of radii 5, 3, 2 respectively. If C_1 & C_2 touch externally & they touch internally with C. The radius of circle C_3 which touches externally with C_1 & C_2 and internally with C is

a) $\frac{30}{19}$ b) 1 c) 3 d) Can not be determined

3. A square $OABC$ is formed by line pairs $xy = 0$ and $xy + 1 = x + y$ where 'O' is the origin . A circle with centre C_1 inside the square is drawn to touch the line pair $xy = 0$ and another circle with centre C_2 and radius twice that of C_1 is drawn to touch the circle C_1 and other line pair. The radius of the circle with centre C_1 is

a) $\frac{\sqrt{2}}{\sqrt{3}(\sqrt{2}+1)}$ b) $\frac{2\sqrt{2}}{3(\sqrt{2}+1)}$ c) $\frac{\sqrt{2}}{3(\sqrt{2}+1)}$ d) $\frac{\sqrt{2}+1}{3\sqrt{2}}$

4. If r_1 and r_2 are the radii of smallest and largest circles which passes through (5, 6) and touches the circle $(x-2)^2 + y^2 = 4$, then $r_1 r_2$ is

a) $\frac{4}{41}$ b) $\frac{41}{4}$ c) $\frac{5}{41}$ d) $\frac{41}{6}$

5. If $C_1 : x^2 + y^2 = (3+2\sqrt{2})^2$ be a circle and PA and PB are pair of tangents to C_1, where P is any point on the director circle of C_1, then the radius of smallest circle which touch C_1 externally and also the two tangents PA and PB is

a) $2\sqrt{3}-3$ b) $2\sqrt{2}-1$ c) $2\sqrt{2}-1$ d) 1

6. If C_1: $x^2 + y^2 - 20x + 64 = 0$ and C_2: $x^2 + y^2 + 30x + 144 = 0$. Then the length of the shortest line segment of a line $\overrightarrow{PQ}$ which touches C_1 at P and to C_2 at Q is

a) 20 b) 15 c) 22 d) 27

7. If two circles touch externally having centres A & B, radii 9, 4 units respectively and one of its direct common tangent touches the circles at P & Q respectively then area of $APQB$ is

a) 78 b) 82 c) 75 d) none

8. Locus of the centre of the circle which touches $x^2 + y^2 - 6x - 6y + 14 = 0$ externally and also y–axis is

a) $y^2 - 6x - 10y - 14 = 0$ b) $y^2 - 6x + 10y + 14 = 0$

c) $y^2 + 6x + 10y + 14 = 0$ d) $y^2 - 10x - 6y + 14 = 0$

9. The locus of the centres of the circles which touch the two circles $x^2+y^2=a^2$ and $x^2+y^2=4ax$ externally is

a) $12x^2 - 4y^2 - 24ax + 9a^2 = 0$ b) $12x^2 + 4y^2 + 24ax + 9a^2 = 0$

c) $12x^2 - 4y^2 + 24ax + 9a^2 = 0$ d) $12x^2 + 4y^2 - 24ax + 9a^2 = 0$

10. The coordinates of the centre of the smallest circle touching the circle $x^2 + y^2 = 4$ and the lines $x + y = 5\sqrt{2}$ are

a) $\left(\frac{7}{2\sqrt{2}}, \frac{7}{2\sqrt{2}}\right)$ b) $\left(\frac{3}{2}, \frac{3}{2}\right)$ 3) $\left(\frac{-7}{2\sqrt{2}}, \frac{-7}{2\sqrt{2}}\right)$ d) $\left(\frac{-3}{2}, \frac{-3}{2}\right)$

11. W.r.t a variable point on the line $x + y = 2a$ chord of contact of the circle $x^2 + y^2 = a^2$ is drawn. If it passes through a fixed point P, the chord of the circle with P as midpoint is

a) parallel to the line $x + y = 2a$ b) perpendicular to the line $x + y = 2a$

c) makes angle 45^0 with line $x + y = 2a$ d) makes angle 135^0 with line $x + y = 2a$

More than one correct answer type questions

12. The centre of the circles passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is (are)

a) (3/2, 1/2) b) (1/2, 3/2) c) $(1/2, \sqrt{2})$ d) $\left(\frac{1}{2}, -\sqrt{2}\right)$

13. If PQR is the triangle formed by the common tangents to the circles $x^2 + y^2 + 6x = 0$ and $x^2 + y^2 - 2x = 0$ then

a) Centroid of ΔPQR is (1, 0) b) In-centre of is (1, 0)

c) Circum-radius of is 2 units d) In radius of is 1 unit

14. The radius of the circle which touch the lines $x + y = 2$ and $x - y = 2$ and the circle $x^2 + y^2 = 1$ is

a) $\sqrt{2} - 1$ b) $\sqrt{2} + 1$ c) $3(\sqrt{2} - 1)$ d) $3(\sqrt{2} + 1)$

15. In the above problem the centre of the circle is

a) $(\sqrt{2}, 0)$ b) $(-\sqrt{2}, 0)$ c) $(-4 + 3\sqrt{2}, 0)$ d) $(-4 - 3\sqrt{2}, 0)$

16. A circle C touchs the x-axis and the circle $x^2 + (y - 1)^2 = 1$ externally, then locus of the centre of the circle is given by

a) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$ b) $\{(x, y) : y = x^2\} \cup \{(0, y) : y \le 0\}$

c) $\{(x, y) : x^2 + (y - 1)^2 - 4\} \cup \{(0, y) : y \le 0\}$ d) $\{(x, y) : x^2 + 4y = 0\} \cup \{(0, y) : y \le 0\}$

Linked comprehension type questions

Passage-I :

Two circles $S_1 = 0$ and $S_2 = 0$ are touching to each other externally at point T with centre C_1, C_2 and radii r_1 and r_2 respectively. If P and Q be the points of contact of a direct common tangent to the two circles and PQ meets the line joining C_1C_2 in S. Tangent at common point T is intersecting to the tangent PQ at point R and to other direct tangent at point V let $S_1 = x^2 + y^2 - 6x = 0$ and $S_2 = x^2 + y^2 + 2x = 0$

17. Angle between the two direct tangents is

a) 90^0 b) 30^0 c) 60^0 d) none

18. Direct tangents are

a) $y=\sqrt{3}x+\sqrt{3}$, $y=-\sqrt{3}x+\sqrt{3}$

b) $y=\frac{x}{\sqrt{3}}-\sqrt{3}$; $y=\frac{-x}{\sqrt{3}}-\sqrt{3}$

c) $y=\frac{x}{\sqrt{3}}+\sqrt{3}$, $y=\frac{-x}{\sqrt{3}}-\sqrt{3}$

d) none

19. A circle $S = 0$ of radius 1 unit rolls on the outside of the circle $S_2 = 0$ touching it externally locus of the centre of this outer circle is

a) circle b) ellipse c) parabola d) none

Matrix matching type questions

20.

COLUMN - I	COLUMN - II
A) The circle $x^2+y^2+2x+c=0$ and $x^2+y^2+2y+c=0$ touch each other	p) if $c = 1$
B) The circle $x^2+y^2+2x+3y+c^2=0$ and $x^2+y^2-x+2y+c^2=0$ intersect orthogonally	q) if $c = -1$
C) The circle $x^2+y^2=9$ contains the circle $x^2+y^2-2x+1-c^2=0$	r) if $c = 1/2$
D) The circle $x^2+y^2=9$ is contained in the circle $x^2+y^2-6x-8y+25-c^2=0$	s) if $c > 8$

21. From the point $P(4, -4)$ tangents PA and PB are drawn to the circle $x^2 + y^2 - 6x + 2y + 5 = 0$ whose centre is C then match the following

COLUMN - I	COLUMN - II
A) Length of AB	p) 5/2
B) Tangent of the angle between PA and PC	q) $\sqrt{10}$
C) Area of triangle PAB	r) 1
D) Absolute difference of slopes of PA and PB	s) 3/4

Integer answer type questions

22. Two circles of radii a and b touching each other externally are inscribed in the area bounded by $y=\sqrt{1-x^2}$ and x - axis. If $b = \frac{1}{2}$, then $4a$ is equal to ....

23. The centres of two circles C_1 and C_2 each of units radius are at a distance of 6 units from each other. Let P be the mid-point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C_1 then the radius of the circle C is

24. From a point P outside of a circle with centre at O, tangent segments PA and PB are drawn. If $\frac{1}{(AB)^2}+\frac{1}{(PA)^2}=\frac{1}{16}$ then the length of the chord AB is ....

25. Inside a semi-circle of radius 1 unit, two circles of radii r_1 and r_2 are drawn, each touching the circumference and the diameter of the semi-circle also touches each other externally. Then $[\max.(r_1 + r_2)]$ where $[x]$ denotes integral value nearest to x ....

EXERCISE-IV

Chord with midpoint - pole, polar - conjugate points, lines - inverse point - miscellaneous problems

LEVEL-I (MAIN)

Single answer type questions

1. The locus of midpoints of the chord of the circle $x^2 + y^2 = 25$ which passes through a fixed point (4, 6) is a circle. The radius of that circle is

1) $\sqrt{52}$ 2) $\sqrt{2}$ 3) $\sqrt{13}$ 4) $\sqrt{10}$

2. Locus of mid points of chords to the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ which are parallel to the line $3x + 4y + 5 = 0$ is

1) $3x + 4y - 25 = 0$ 2) $4x + 3y + 5 = 0$ 3) $4x + 3y - 25 = 0$ 4) $4x - 3y + 25 = 0$

3. The midpoint of chord formed by the polar of (–9,12) w.r.t $x^2 + y^2 = 100$ is

1) $\left(4,-\frac{4}{3}\right)$ 2) $\left(-4,\frac{16}{3}\right)$ 3) $\left(-4,\frac{16}{9}\right)$ 4) $\left(4,\frac{16}{3}\right)$

4. If two distinct chords drawn from the point (p,q) on the circle $x^2+y^2-px-qy=0$ (where $pq \neq 0$) are bisected by x-axis then

1) $p^2 = q^2$ 2) $p^2 = 8q^2$ 3) $p^2 < 8q^2$ 4) $p^2 > 8q^2$

5. Let $x(x - a) + y(y - 1) = 0$ be a circle. If two chords from $(a, 1)$ bisected by X-axis are drawn to the circle then the condition is

1) $a^2 > 8$ 2) $a^2 < 8$ 3) $a^2 > 4$ 4) $a^2 < 4$

6. Let C be the circle with centre (0, 0) and radius 3 units. The equation of locus of mid points of chords of the cricle C which subtend an angle of $\frac{2\pi}{3}$ at its centre is

1) $x^2 + y^2 = \frac{3}{2}$ 2) $x^2 + y^2 = 1$ 3) $x^2 + y^2 = \frac{27}{4}$ 4) $x^2 + y^2 = \frac{9}{4}$

7. The straight line $x - 2y + 5 = 0$ intersects the circle $x^2 + y^2 = 25$ in points P and Q, the coordinates of the point of intersection of tangents drawn at P and Q to the circle is

1) (25, 50) 2) (–5, 10) 3) (25, –50) 4) (–5, –10)

8. If the pole of a line w.r. to the circle $x^2 + y^2 = a^2$ lies on the circle $x^2 + y^2 = a^4$ then the line touches the circle

1) $x^2 + y^2 = 2$ 2) $x^2 + y^2 = 1$ 3) $x^2 + y^2 = 3$ 4) $x^2 + y^2 = 4$

9. The polar of point $(2t, t-4)$ w.r.t the circle $x^2 + y^2 - 4x - 6y + 1 = 0$ passes through the point

1) (1, 3) 2) (1, –3) 3) (–3, 1) 4) (3, 1)

10. In ΔABC the poles of AB, BC and AC w.r.t. to circle lies on AB, BC and AC then for ΔABC, centre of circle is

1) Centroid of ΔABC 2) Ortho centre of ΔABC
3) Incentre of ΔABC 4) Circumcebtre of ΔABC

11. The length of tangents from two points A, B to a circle are 4, 3 respectively. If A, B are conjugate points then $AB =$

1) 5 2) $\sqrt{85}$ 3) $\frac{\sqrt{85}}{2}$ 4) $\frac{\sqrt{85}}{3}$

12. If the inverse of $P(1, 2)$ w.r.t. $x^2 + y^2 - 4x - 6y + 9 = 0$ is Q then $PQ =$

1) 2 2) $\sqrt{2}$ 3) 3 4) 4

LEVEL-II (ADVANCED)

Single answer type questions

1. The number of integral values of y for which the chord of the circle $x^2 + y^2 = 125$ passing through the point $P(8, y)$ gets bisected at the point $P(8, y)$ and has integral slope is

a) 8 b) 6 c) 4 d) 2

2. Tangents are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles. $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0$, λ being the variable. The locus of the point of intersection of these tangents is

a) $2x - y + 10 = 0$ b) $x - 2y - 10 = 0$ c) $x - 2y + 10 = 0$ d) $2x + y - 10 = 0$

3. Locus of the midpoints of the chords of contact of $x^2 + y^2 = 2$ from the point on the line $3x + 4y = 10$ is a circle with centre P. If O be the origin then OP is equal to

a) 2 b) 3 c) $\frac{1}{2}$ d) $\frac{1}{3}$

Matrix matching type questions

4. AB is chord of the circle $x^2 + y^2 = 25$ whose middle point is M. Then match the following loci of M under various conditions

COLUMN - I	COLUMN - II
A) When AB subtends 60^o at centre	p) $x^2 + y^2 + 2y = 0$
B) When AB always passes through (0, –2)	q) $x^2 + y^2 = 9$
C) When AB has constant length 8	r) $x^2 + y^2 = 75/4$
D) When AB subtends 45^0 on the circumference of the circle	s) $2(x^2 + y^2) = 25$

KEY SHEET (LECTURE SHEET)

EXERCISE- I

LEVEL-I

1) 2	2) 4	3) 4	4) 1	5) 2	6) 4	7) 4	8) 2
9) 1	10) 1	11) 3	12) 2	13) 1	14) 1	15) 3	16) 3
17) 1	18) 3	19) 2	20) 4	21) 1	22) 2	23) 2	24) 1
25) 2	26) 4	27) 4	28) 2	29) 2	30) 1	31) 2	32) 3
33) 5.5	34) 1.14	35) 4					

LEVEL-II

1) b	2) c	3) d	4) b	5) b	6) a	7) b	8) c
9) b	10) d	11) a	12) a	13) d	14) c	15) b	16) c
17) b	18) d	19) 2	20) a	21) b	22) a	23) d	24) ac
25) bc	26) ac	27) bd	28) ab	29) ab	30) abcd	31) abcd	32) a
33) b	34) b	35) c	36) a	37) a	38) b		

39) A-p, B-s, C-q, D-r 40) A-s, B-p, C-r, D-s 41) 0 42) 5

EXERCISE-II

LEVEL-I

1) 1	2) 3	3) 3	4) 1	5) 2	6) 1	7) 1	8) 1
9) 1	10) 2	11) 1	12) 1	13) 2	14) 2	15) 4	16) 2
17) 1	18) 2	19) 2	20) 1	21) 4	22) 1	23) 3	24) 2
25) 2	26) 1	27) 3	28) 3	29) 4.24		30) 6.92	31) -18.34

LEVEL-II

1) b	2) c	3) b	4) a	5) d	6) b	7) d	8) a
9) b	10) d	11) b	12) b	13) c	14) b	15) bc	16) abcd
17) bc	18) ad	19) c	20) a	21) d	22) d	23) a	24) b
25) c	26) c	27) a	28) b	29) A-s; B-p; C-q; D-r			30) 4
31) 6	32) 4	33) 2	34) 1	35) 3	36) 6	37) 1	

EXERCISE-III

LEVEL-I

1) 2 2) 1 3) 2 4) 1 5) 3 6) 2.82 7) 1.57

LEVEL-II

1) a	2) a	3) c	4) b	5) d	6) a	7) a	8) 4
9) 1	10) a	11) a	12) cd	13) abcd	14) abcd	15) abcd	16) a
17) c	18) d	19) a	20) A-r; B-p,q; C-p,q;r, D-r				

21) Aq; B-r; C-p; D-p 22) 1 23) 8 24) 8 25) 1

EXERCISE-IV

LEVEL-I

1) **3** 2) **3** 3) **2** 4) **4** 5) **1** 6) **4** 7) **2** 8) **2** 9) **4** 10) **3** 11) **1** 12) **2**

LEVEL-II

1) **b** 2) **a** 3) **c** 4) **A-r, B-p, C-q, D-s**

PRACTICE SHEET

EXERCISE-I

General equation - parametric form - position of a point - power of a point - min/max. distances of a point to the circle - length of chord - concyclic points on axes - Locus problems.

LEVEL-I (MAIN)

Single answer type questions

1. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are two diameters of a circle of area 154 sq. units. Then the equation of circle is

   1) $(x+1)^2+(y+1)^2=49$ 2) $(x-1)^2+(y-1)^2=49$

   3) $(x-1)^2+(y+1)^2=49$ 4) $(x+1)^2+(y-1)^2=49$

2. If the two circles $x^2+y^2+2gx+c=0$ and $x^2+y^2-2fy-c=0$ have equal radius then locus of (g, f) is

   1) $x^2 + y^2 = c^2$ 2) $x^2 - y^2 = 2c$ 3) $x - y^2 = c^2$ 4) $x^2 + y^2 = 2c^2$

3. The equation of the circle concentric with the circle $x^2-y^2-6x+12y+15=0$ and of double its area is

   1) $x^2 + y^2 - 6x + 12y - 15 = 0$ 2) $x^2 + y^2 - 6x + 12y - 30 = 0$

   3) $x^2 + y^2 - 6x + 12y - 25 = 0$ 4) $x^2 + y^2 - 6x + 12y - 20 = 0$

4. If the circles described on the line joining the points (0, 1) and (α,β) as diameter cuts the X-axis in points whose abscissae are roots of equation $x^2-5x+3 = 0$ then $(\alpha,\beta) =$

   1) (5, 3) 2) (3, 5) 3) (–5, 3) 4) (–5, –3)

5. A circle passes through origin and meets the axes at A and B so that (2, 3) lies on $\overline{AB}$ then the locus of centerid of ΔOAB is

   1) $2x-3y = 6xy$ 2) $2x + 3y = 6xy$ 3) $3x-2y = 3xy$ 4) $3x + 2y = 3xy$

6. A rod PQ of length $2a$ slides with its ends on the axes. The locus of the circumcentre of ΔOPQ is

   1) $x^2 + y^2 = 2a^2$ 2) $x^2 + y^2 = 4a^2$ 3) $x^2 + y^2 = 3a^2$ 4) $x^2 + y^2 = a^2$

7. A right angled isosceles triangle is inscribed in the circle $x^2 + y^2 - 4x - 2y - 4 = 0$ then length of the side of the triangle is

   1) $\sqrt{2}$ 2) $2\sqrt{2}$ 3) $3\sqrt{2}$ 4) $4\sqrt{2}$

8. A square is inscribed in the circle $x^2 + y^2 - 2x + 8y - 8 = 0$ whose diagonals are parallel to axes and a vertex in the first quadrant is A then OA is

   1) 1 2) $\sqrt{2}$ 3) $2\sqrt{2}$ 4) 3

9. A rod AB of length 3 units moves vertically with its bottom end B always an the circle $x^2 + y^2 = 25$ then the equation of the locus of A is

1) $x^2 + (y+3)^2 = 25$ 2) $(x-3)^2 + y^2 = 25$ 3) $(x+3)^2 + y^2 = 25$ 4) $x^2 + (y-3)^2 = 25$

10. The abscissae of two points A and B are roots of $x^2 + 4x - 45 = 0$ and ordinates of A, B are roots of $y^2 - 7y + 12 = 0$. Then centre of circle for which A, B as diameter is

1) (2, 7/2) 2) (–2, 7/2) 3) (2, –7/2) 4) (–2, –7/2)

11. If the line $3x-2y + 6 = 0$ meets x-axis and y-axis respectively at A and B, then the equation of the circle with radius AB and centre at A is

1) $x^2 + y^2 + 4x + 9 = 0$ 2) $x^2 + y^2 + 4x - 9 = 0$

3) $x^2 + y^2 + 4x + 4 = 0$ 4) $x^2 + y^2 + 4x - 4 = 0$

12. The circle passing through $(t, 1)$, $(1, t)$ and (t, t) for all values of t also passes through

1) (0, 0) 2) (1, 1) 3) (1, –1) 4) (–1, –1)

13. If the points (0, 0), (2, 0), (0, –2) and (k, –2) are concyclic then k =

1) 2 2) –2 3) 0 4) 1

14. $ABCD$ is a square with side 'a'. If AB and AD are taken as positive coordinate axes then equation of circle circumscribing the square is

1) $x^2 + y^2 - ax - ay = 0$ 2) $x^2 + y^2 + ax + ay = 0$

3) $x^2 + y^2 - ax + ay = 0$ 4) $x^2 + y^2 + ax - ay = 0$

15. Centre of the circle inscribed in a rectangle formed by the lines $x^2-8x+12=0$ and $y^2-14y+40=0$ is

1) (4, 7) 2) (7, 4) 3) (9, 4) 4) (4, 9)

16. $ABCD$ is a rectangle with sides $AB = p$, $BC = q$. If AB and AD are taken as negative directions of coordinate axes, then the equation of the circle circumscribing the rectangle is

1) $x^2 + y^2 + px + qy = 0$ 2) $x^2 + y^2 - px - qy = 0$

3) $x^2 + y^2 + 2px + 2qy = 0$ 4) $x^2 + y^2 - 2px - 2qy = 0$

17. The longest distance from (–3, 2) to the circle $x^2 + y^2 - 2x + 2y + 1 = 0$ is

1) 8 2) 4 3) 18 4) 6

18. Equation of circle with centre (3, –1) and which cuts off a chord of length 6 on the line $x - 5y + 18 = 0$ is

1) $x^2 + y^2 + 6x + 2y + 28 = 0$ 2) $x^2 + y^2 - 6x + 2y - 25 = 0$

3) $x^2 + y^2 - 6x - 2y - 28 = 0$ 4) $x^2 + y^2 - 6x + 2y + 28 = 0$

19. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subetends an angle of mesaure 45^0 at the major segment of the circle then m =

1) 2 2) –1 3) –2 4) 3

20. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If $Q = (3, 4)$ and $R = (-4, 3)$ then $\angle QPR$ =

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

21. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **(AIEEE-2010)**

1) $-35 < m < 15$ 2) $15 < m < 65$ 3) $35 < m < 85$ 4) $-85 < m < -35$

22. The parametric equations $x=\dfrac{2a(1-t^2)}{1+t^2}$ and $y=\dfrac{4at}{1+t^2}$ represent a circle whose radius is

1) a 2) $2a$ 3) $3a$ 4) $4a$

23. $S(x, y) = 0$ represents a circle. The equation $S(x, 2) = 0$ gives two identical solutions $x = 1$ and the equation $S(1, y) = 0$ gives two solutions $y = 0, 2$. Then equation of the circle is

1) $x^2 + y^2 - 2x - 2y + 1 = 0$
2) $x^2 + y^2 + 2x - 2y + 1 = 0$
3) $2x^2 + 2y^2 - 4x - 4y + 5 = 0$
4) $x^2 + y^2 + 3x - 2y + 1 = 0$

LEVEL-II (ADVANCED)

Single answer type questions

1. The number of integral values of λ for which $x^2+y^2+\lambda x+(1-\lambda)y+5=0$ is the equation of a circle whose radius cannot exceed 5, is

a) 14 b) 18 c) 16 d) 10

2. The reflection of a point $A(1, 8)$ in the line $x - 2y + 10 = 0$ is the point B and the reflection of B in the line $x - y + 1 = 0$ is the point C then radius of circumcircle of ΔABC is

a) $\sqrt{2}$ b) $3\sqrt{3}$ c) $5\sqrt{2}$ d) $4\sqrt{3}$

3. The point $([P+1],[P])$ (where [.] denotes the greatest integer function) lying inside the region bounded by the circles $x^2 + y^2 - 2x - 15 = 0$ and $x^2 + y^2 - 2x - 7 = 0$. Then P belongs to

a) $[1,0)\cup[0,1)\cup[1,2)$ b) $[-1, 2)$ c) $(-1, 2)$ d) None of these

4. If $(a, 0)$ is a point on a diameter of the circle $x^2 + y^2 = 4$ then $x^2 - 4x - a^2 = 0$ has

a) exactly one real root in $(-1, 0]$
b) exactly one real root in $[2, 5]$
c) distinct roots greater than 5
d) distinct roots greater than -1 and less than 5

5. If the points $(\sqrt{29},0)$, $(5, 2)$, $(2, -5)$, $(-1, k)$ are concyclic $(k \neq 0)$ then k is ....

a) $\sqrt{25}$ b) $\sqrt{27}$ c) $\sqrt{28}$ d) $\sqrt{24}$

6. Two rods of lengths 'a' and 'b' slide along coordiante axes such that their ends are concyclic. Locus of the centre of the circle is

a) $4(x^2 + y^2)= a^2 + b^2$
b) $4(x^2 + y^2)= a^2 - b^2$
c) $4(x^2 - y^2)= a^2 - b^2$
d) $xy = ab$

7. If the chord of the circle $x^2+y^2=8$ makes equal intercepts of length 'a' on cordinate axes than $|a|<$ .....

a) 12 b) 4 c) $2\sqrt{2}$ d) 8

8. Two vertices of an equilateral triangle are $(-1, 0)$ and $(1, 0)$ and its thrid vertex lies above the X-axis. The equation of the circum circle of the triangle is

a) $x^2 + y^2 = 1$
b) $\sqrt{3}(x^2 + y^2) + 2y - \sqrt{3} = 0$
c) $\sqrt{3}(x^2 + y^2) - 2y - \sqrt{3} = 0$
d) $\sqrt{3}(x^2 - y^2) - 2y - \sqrt{3} = 0$

9. The line $Ax + By + C = 0$ cuts the circle $x^2 + y^2 + gx + fy + c = 0$ at P and Q. The line $A'x + B'y + C' = 0$ cuts the circle $x^2 + y^2 + g'x + f'y + c' = 0$ at R and S. If P, Q, R and S are concyclic, then the value of the determinant $\begin{vmatrix} g-g' & f-f' & c-c' \\ A & B & C \\ A' & B' & C' \end{vmatrix}$ is

a) 0 b) 1 c) 2 d) none

10. A wheel of radius 8 units rolls along the diameter of a semicircle of radius 25 units if bumps into this semicircle what is the length of the portion of the diameter that cannot be touched by the wheel.

a) 12 b) 15 c) 17 d) 20

11. A triangle is inscribed in a circle of radius 1. The distance between the orthocentre and the circum centre of the triangle cannot be

a) 1 b) 2 c) 3 d) 4

12. Two vertices of an equilateral triangle are (1, 0) and (2, 0) and third vertex lies above the x-axis then the equations to the circles described on its sides as diameters is/are

a) $2x^2 + 2y^2 - 6x + 4 = 0$ b) $2x^2 + 2y^2 - 5x - \sqrt{3}y + 3 = 0$

c) $2x^2 + 2y^2 - 7x - \sqrt{3}y + 3 = 0$ d) $2x^2 + 2y^2 - 7x - \sqrt{3}y + 6 = 0$

13. If the conics whose equations are

$S_1 : (\sin^2\theta)x^2 + (2h\tan\theta)xy + (\cos^2\theta)y^2 + 32x + 16y + 19 = 0;$

$S_2 : (\cos^2\theta)x^2 - (2h'\cot\theta)xy + (\sin^2\theta)y^2 + 16x + 32y + 19 = 0$ intersect in four concyclic points where $\theta \in \left[0.\frac{\pi}{2}\right]$ then the correct statement(s) can be ....

a) $h + h' = 0$ b) $h - h' = 0$ c) $\theta = \frac{\pi}{4}$ d) none

14. A family of linear functions is given by $f(x) = 1 + c(x + 3)$ where c R. If a member of this family meets a unit circle centred at origin in two coincident points, then 'c' can be equal to

a) $\frac{-3}{4}$ b) 0 c) $\frac{3}{4}$ d) 1

More than one correct answer type questions

15. If the point (x, y) is called a lattice point if x, y are integers then the total no. of lattice points in the interior of the circle $x^2 + y^2 = a^2$, $a \neq 0$ can not be

a) 2010 b) 2011 c) 2012 d) 2013

16. If $A(-5, 0)$, $B(5,0)$ and P moves such that $\frac{PA}{PB} = \frac{2}{1}$ and maximum area of $\Delta PAB = \frac{100}{3}$ sq. units then

a) Locus of the point P is a circle b) No. of such triangles PAB are 2

c) No. of such triangles PAB are 4 d) Locus of the point P is a Parabola

17. If the curves $ax^2 + 4xy + 2y^2 + x + y + 5 = 0$ and $ax^2 + 6xy + 5y^2 + 2x + 3y + 7 = 0$ intersect at four concyclic points then

a) The value of a is -4

b) centre of the circle is $\left(-\frac{1}{8}, -\frac{3}{8}\right)$

c) centre of the circle is $\left(-\frac{1}{4}, -\frac{3}{4}\right)$

d) radius of the circle is $\sqrt{26}/8$

18. If the line $x + y = n$, $n \in N$ is chord of the circle $x^2 + y^2 = 4$ then

a) Sum of the squares of the length of the chord intercepts by the line on the circle is 22

b) No. of such chords of 4

c) No. of such chords of 2

d) None of these

19. $a,b,c \in \{2,3,4.......30\}$ and g.c.d of (a,c) is 1. Then the circle with (a,c) as centre and b as radius if $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$ is

a) $x^2 + y^2 - 6x - 4y - 23 = 0$

b) $x^2 + y^2 - 8x - 6y - 119 = 0$

c) $x^2 + y^2 - 10x - 8y - 359 = 0$

d) $x^2 + y^2 - 12x - 10y - 839 = 0$

20. $A\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is a point on the cirlce $x^2 + y^2 = 1$ and B is another point of the circle such that are length $AB = \frac{\pi}{2}$ units. Then, co-ordinates of B can be

a) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

c) $\left(-\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

d) none of these

21. An equation of a circle through the origin making an intercept of $\sqrt{10}$ on the line $y = 2x + \frac{5}{\sqrt{2}}$ which subtends an angle of 45^0 at the origin is

a) $x^2 + y^2 - 4x - 2y = 0$

b) $x^2 + y^2 - 2x - 4y = 0$

c) $x^2 + y^2 + 4x + 2y = 0$

d) $x^2 + y^2 + 2x + 4y = 0$

22. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1.

a) $x + y = 0$

b) $x - y = 0$

c) $x + 7y = 0$

d) $x - 7y = 0$

Linked comprehension type questions

Passage - I :

Two variable chords AB and BC of a circles $x^2 + y^2 = a^2$ are such that $AB = BC = a$, and M and N are the midpoints of AB and BC, respectively, such that line joining MN intersect the circles at P and Q, where P is closer to AB and Q is the centre of the circle.

23. $\angle OAB$ is

a) $30°$

b) $60°$

c) $45°$

d) $15°$

24. Angles between tangents at A and C is

a) $90°$ b) $120°$ c) $60°$ d) $150°$

25. Locus of point of intersection of tangents at A and C is

a) $x^2 + y^2 = a^2$ b) $x^2 + y^2 = 2a^2$ c) $x^2 + y^2 = 4a^2$ d) $x^2 + y^2 = 8a^2$

Passage - II :

$A = (2, 2)$, $B = (5, 3)$, $C = (3, -1)$. $P = (x_1 + r\cos\theta, y_1 + r\sin\theta)$ is a point on the circum circle of ΔABC. AB is perpendicular to BC. Then

26. The point $P =$

a) $(4, 0)$ b) $(6, 0)$ c) $(3, 0)$ d) $(8, 0)$

27. $x_1 + y_1 =$

a) 3 b) 4 c) 5 d) 6

28. $\tan\theta =$

a) $\frac{-1}{2}$ b) -1 c) $\frac{1}{2}$ d) -2

Matrix matching type questions

29. Each side of a square has length 4 units and its centre is at (3,4). If one of the diagonals is parallel to the line $y = x$

COLUMN - I	COLUMN - II
A) If P, Q are the vertices of a square and they are farthest and nearest points from origin to the square then PQ is	p) $2\sqrt{2}(\sqrt{2}-1)$
B) The radius of the circle inscribed in the triangle formed by any three vertices is	q) $\sqrt{2}(\sqrt{2}-1)$
C) The radius of the circle inscribed in the triangle formed by any two vertices of square and the centre is	r) $4\sqrt{2}$
D) The radius of circle inscribed in the square is	s) $2(\sqrt{2}-1)$
	t) 2

30. Let C_1 and C_2 be two circles whose equations are $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 + 2x = 0$ and $P(\lambda, \lambda)$ is a variable point.

COLUMN - I	COLUMN - II
A) P lies inside C_1 but outside C_2	p) $\lambda \in (-\infty, -1) \cup (0, \infty)$
B) P lies inside C_2 but outside C_1	q) $\lambda \in (-\infty, -1) \cup (1, \infty)$
C) P lies outside C_1 but outside C_2	r) $\lambda \in (-1, 0)$
D) P does not lie inside C_2	s) $\lambda \in (0, 1)$

Integer answer type questions

31. The difference between the radii of the largest and the smallest circles which have their centres on circumference of $x^2 + y^2 + 4x + 2y = 4$ and passing through (a,b) lying out side the given circle is

32. A circle C_1 is inscribed in a square S_1 of side a_1. Another square S_2 of side a_2 is incribed C_1. A Circle C_2 is inscribed in S_1. The proces is continued.... If $a_1 = 4\sqrt{2}$, and $s = a_1 + a_2 + ...\infty$ then $(\sqrt{2}-1)s =$______

33. The number of points (x, y) having integral coordinates satisying the condition $x^2 + y^2 < 25$ is n then integeral part of $\frac{n}{9} =$

34. Let $A(-2, 2)$ and $B(2, -2)$ be two points and AB subtends and angle of $45°$ at any points P in the plane in such a way that area of ΔAPB is 8 square unit, then number possible position (s) of P is

35. How many ordered pair of integers (a, b) satisfy all the following inequalities $a^2+b^2<16, a^2+b^2<8a, a^2+b^2<8b$?

EXERCISE-II

Equation of normal - equation of tangent - length of tangent - chord of contact - area of triangle formed - circles touching the axes

LEVEL-I (MAIN)

Single answer type questions

1. The tangent to the circle $x^2 + y^2 - 4x + 2y + k = 0$ at (1, 1) is $x - 2y + 1 = 0$ then $k =$
   1) –1 2) 0 3) 1 4) 2

2. The line $y = x$ is a tangent at (0, 0) to a circle of radius is 1, then centre of the circle is
   1) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ 2) $\left(\frac{1}{2\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$ 3) $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ 4) $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

3. The tangents at (5, 12) and (12, –5) to the circle $x^2 + y^2 = 169$ are
   1) coincident 2) perpendicular 3) parallel 4) at an angle of 45^0

4. The angle between the tangents to the circle with centre (4, 5) drawn from $P(-2, -3)$ is 120^0, then length of the tangent to the circle from P is
   1) 4 2) 3 3) 2 4) 5

5. The condition that the pair of tangents drawn from origin to circle $x^2+y^2+2gx+2fy+c=0$ may be at right angles is
   1) $g^2+f^2 = c$ 2) $g^2+f^2 = 2c$ 3) $g^2+f^2+2c = 0$ 4) $g^2-f^2 = 2c$

6. Tangents to $x^2+y^2=a^2$ having inclinations α and β intersect at P. If $\cot\alpha+\cot\beta=0$ then the locus of P is
   1) $x + y = 0$ 2) $x - y = 0$ 3) $xy = 0$ 4) $xy = a^2$

7. From any point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ tangents are drawn to the circle $x^2+y^2+2gx+2fy+c\sin^2\alpha+(g^2+f^2)\cos^2\alpha=0$ then angle between tangents is

1) α 2) 2α 3) $\frac{\pi}{2}$ 4) 0^0

8. No. of circles touching all the lines $x + y-1 = 0$, $x - y -1 = 0$ and $y + 1 = 0$ is

1) 0 2) 2 3) 4 4) Infinite

9. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meet the line $5x - 2y + 6 = 0$ at a point Q on y-axis then $PQ =$

1) 10 2) 15 3) 25 4) 5

10. $P(-9, -1)$ is a point on the circle $x^2 + y^2 + 4x + 8y - 38 = 0$. The equation to the tangent at the other end of the diameter through P is

1) $7x-3y = 60$ 2) $7x + 3y = 56$ 3) $7x-3y = 56$ 4) $7x + 3y = 60$

11. If O is the origin and OP, OQ are the tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, then the circumcentre of the ΔOPQ is

1) $(-g, -f)$ 2) $(-f, -g)$ 3) $(-g/2, -f/2)$ 4) $(-f/2, -g/2)$

12. From any point on the circle $x^2 + y^2 = a^2$ tangents are drawn to the circle $x^2 + y^2 = a^2 \sin^2\theta$. The angle between them is

1) $\theta/2$ 2) θ 3) 2θ 4) 4θ

13. If a circle of radius 2 touches x-axis at (1, 0) then its centre may be

1) (1, 2) or (1, –2) 2) (1, 2) or (2, 1)
3) (–1, 2) or (1, –2) 4) (–1, 2) or (–1, –2)

14. Circle touching both the axes and radius 5 is

1) $x^2+y^2-10x-10y+25=0$ 2) $x^2+y^2-10x+10y+25=0$
3) $x^2+y^2+10x-10y+25=0$ 4) all the above

15. The radius of the circle of least size that passes through (–2, 1) and touches both axes is

1) 1 2) 2 3) 3 4) 5

16. The equations of the circles which touch the y-axis at the origin and also the line $5x+ 12y- 72 = 0$ are

1) $x^2 + y^2 - 6y = 0$, $x^2 + y^2 + 24y = 0$ 2) $x^2 + y^2 + 2y = 0$, $x^2 + y^2 - 18y = 0$
3) $x^2 + y^2 +18x = 0$, $x^2 + y^2 - 8x = 0$ 4) $x^2 + y^2 + 4x = 0$, $x^2 + y^2-16x = 0$

17. The circle passing through origin and making intercepts 6 and –4 on x and y-axis respectively has the centre

1) (3, –2) 2) (–2, 4) 3) (8, –4) 4) Both 1 and 2

18. The centre of the circle touching y-axis at (0, 4) and making an intercept 2 units on the positive x-axis is

1) $(10, \sqrt{3})$ 2) $(\sqrt{17}, 3)$ 3) $(\sqrt{17}, 4)$ 4) $(3, \sqrt{17})$

19. The locus of the point (f, g) such that the length of the tangent from (f, g) to $x^2+y^2=6$ be twice the length of the tangent from the same point to $x^2+y^2+3x+3y=0$ is

1) $x^2 + y^2 + 4x + 4y + 2 = 0$ 2) $x^2 + y^2 + 4x - 4y + 2 = 0$

3) $x^2 + y^2 - 4x + 4y + 2 = 0$ 4) $x^2 + y^2 - 4x - 4y + 2 = 0$

20. If the squares of the lengths of the tangents from a point P to the circles $x^2+ y^2 = a^2$, $x^2 + y^2 = b^2$, $x^2 + y^2 = c^2$, are in A.P. then a^2, b^2, c^2 are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

21. Lines are drawn through point P(–2, –3) to meet the circle $x^2+y^2-2x-10y+1=0$. The length of the line segment PA; A being the point on the circle where the line meets the circle at coincident points; is

1) 16 2) $4\sqrt{3}$ 3) 48 4) none

22. The area of the triangle formed by the tangent drawn at the point (–12, 5) on the circle $x^2 + y^2 = 169$ with the coordinate axes is

1) $\frac{625}{24}$ 2) $\frac{28561}{120}$ 3) $\frac{225}{23}$ 4) $\frac{8561}{20}$

23. Equation of circles which touch both the axes and also the line $x = k$. (k > 0) is

1) $x^2 + y^2 - kx \pm ky + \frac{k^2}{4} = 0$ 2) $x^2 + y^2 + kx \pm ky + \frac{k^2}{4} = 0$

3) $x^2 + y^2 \pm kx + ky + \frac{k^2}{4} = 0$ 4) $x^2 + y^2 \pm kx - ky + \frac{k^2}{4} = 0$

24. If two circles touching both the axes are passing through (2, 3) then length of their common chord is

1) $\sqrt{2}$ 2) $2\sqrt{2}$ 3) $3\sqrt{2}$ 4) $4\sqrt{2}$

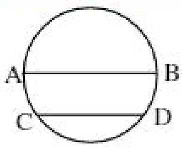
25. The locus of the centre of the circle touching x-axis and the line $y = x$ is

1) $y = (\sqrt{2} - 1)x$ 2) $y = (\sqrt{2} + 1)x$ 3) $y = 2x$ 4) $y = -x$

26. The equation of one of the tangents from (1, 1) to a circle with its centre at (3, 0) is $3x + y - 4 = 0$. The equation of the other tangent is ....

1) $5x - y - 4 = 0$ 2) $3y - x - 2 = 0$ 3) $3y + x - 4 = 0$ 4) $3x - y - 2 = 0$

27. AB is a diameter of a circle. CD is a chord parallel to AB and $2CD = AB$. The tangent at B meets the line AD produced at E then



1) AE = 2AB 2) $\sqrt{3}AB = AE$

3) $\sqrt{2}AB = AE$ 4) $2AB = \sqrt{3}AE$

LEVEL-II (ADVANCED)

Single answer type questions

1. A circle of radius 2 has center at (2, 0) and another circle of radius 1 has center at (5, 0). A line is tangent to the two circles at points in the first quadrant. The equation of the tangent is

a) $\frac{x}{9} + \frac{y}{8} = 1$ b) $\frac{x}{8} + \frac{y}{2\sqrt{2}} = 1$ c) $\frac{x}{8} + \frac{y}{3} = 1$ d) $x + 3y = 9$

2. P is a point outside a circle. If the farthest distance of P from the circle is 4 times to the shortest distance of P. from the circle. Then the Angle between the tangents at P is

a) $2\mathrm{Tan}^{-1}\frac{1}{2}$ b) $2\mathrm{Tan}^{-1}\frac{2}{3}$ c) $2\mathrm{Sin}^{-1}\frac{3}{5}$ d) $2\mathrm{Sin}^{-1}\frac{4}{5}$

3. The range of values of $\lambda(\lambda > 0)$ such that the angle θ between the pair of tangents drawn from $(\lambda, 0)$ to the circle $x^2 + y^2 = 4$ lies in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ is

a) $\left(\frac{4}{\sqrt{3}}, 2\sqrt{2}\right)$ b) $(0, \sqrt{2})$ c) $(1, 2)$ d) $(1, -1)$

4. Equation of a circle touching the line $|x-1| + |y-4| = 6$ is

a) $x^2 + y^2 - 2x - 8y - 1 = 0$ b) $x^2 + y^2 - 2x - 8y - 18 = 0$
c) $x^2 + y^2 - 2x - 8y - 17 = 0$ d) $x^2 + y^2 = 4$

5. The area of the region bounded by the circles which touch the circles of radius one unit and the coordinate axes as tangents is

a) 4π b) $4\sqrt{2}\pi$ c) 8π d) $8\sqrt{2}\pi$

6. If eight distinct points can be found on the curve $|x| + |y| = 1$ such that from each point two mutually perpendicular tangents can be drawn to the circle $x^2 + y^2 = a^2$, then the range of a is.

a) $\left(\frac{1}{\sqrt{2}}, 1\right)$ b) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ c) $\left(\frac{1}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ d) $\left(\frac{1}{2}, 1\right)$

7. A ray of light incident at the point $(-2, -1)$ gets reflected from the tangent at $(0, -1)$ to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle. The equation of the line along which the incident ray moved is

a) $3x + 4y + 11 = 0$ b) $3x + 4y - 11 = 0$
c) $4x + 3y + 11 = 0$ d) $4x - 3y + 11 = 0$

8. P is a point (a, b) in the first quadrant. If the two circles which pass through P and touch both the coordinate axes cut at right angles, than :

a) $a^2 - 6ab + b^2 = 0$ b) $a^2 + 2ab - b^2 = 0$
c) $a^2 - 4ab + b^2 = 0$ d) $a^2 - 8ab + b^2 = 0$

9. $2x + y = 0$ is the equation of a diameter of the circle which touches the lines $4x - 3y + 10 = 0$ and $4x - 3y - 30 = 0$. The centre and radius of the circle are

a) $(-2, 1)$; 4 b) $(1, -2)$; 8 c) $(1, -2)$; 4 d) $(1, -2)$; 16

10. From a point $P(2, 2\sqrt{2})$ tangents PQ and PR are drawn to the circle $x^2 + y^2 = a^2$. If QR is touching the circle $x^2 + y^2 = 3$ then the value of 'a' is

a) 4 b) $2\sqrt{3}$ c) $3\sqrt{2}$ d) $\sqrt{6}$

11. The line $2x - y + 1 = 0$ is tangent to the circle at the point $(2, 5)$ and the centre of the circle lies on $x - 2y = 4$. The radius of the circle is

a) $3\sqrt{5}$ b) $5\sqrt{3}$ c) $2\sqrt{5}$ d) $5\sqrt{2}$

12. The length of chord of contact of the point (3, 6) w.r.t the circle $x^2 + y^2 = 10$ is

a) $\frac{2\sqrt{70}}{3}$ b) $6\sqrt{5}$ c) $\sqrt{5}$ d) $\frac{12}{\sqrt{5}}$

More than one correct answer type questions

13. Co-ordinates of the centre of a circle, whose radius is 2 unit and which touches the pair of lines $x^2 - y^2 - 2x + 1 = 0$ is (are)

a) (4, 0) b) $(1+2\sqrt{2},0)$ c) (4, 1) d) $(1, 2\sqrt{2})$

14. The points on the line $x = 2$ from which the tangents drawn to the circle $x^2 + y^2 = 16$ are at right angles is

a) $(2,2\sqrt{7})$ b) $(2,2\sqrt{5})$ c) $(2,-2\sqrt{7})$ d) $(2,-2\sqrt{5})$

15. The centre of the circle having radius 5 and touching the line $4x + 3y - 7 = 0$ at (1,1) may be.

a) (5, 4) b) (–5, –4) c) (–3, –2) d) (3, 4)

16. In right angled triangle, the length of the sides are a and b $(0 < a < b)$. A circle passes through the mid point of the smaller side and touches hypotenous at its mid point then the

a) centre of the circle is $\left(\frac{2a^2-b^2}{4a},\frac{b}{4}\right)$

b) centre of the circle $\left(\frac{b^2-a^2}{4a},\frac{b}{4}\right)$

c) radius of the circle is $\frac{b\sqrt{a^2+b^2}}{4a}$

d) radius of the circle is $\frac{a\sqrt{a^2+b^2}}{4a}$

17. The equation of the circle which touches both the axes and the straight line $4x + 3y = 6$ in the first quadrant is

a) $4x^2 + 4y^2 - 4x - 4y + 1 = 0$ b) $x^2 + y^2 - 4x - 4y + 1 = 0$

c) $4x^2 + 4y^2 - x - y - 1 = 0$ d) $4x^2 + 4y^2 + x + y - 1 = 0$

18. The equation of the tangent to the circle $x^2 + y^2 - 2y = 1$ which is perpendicular to the normal which makes equal angles with the coordiante axes is :

a) $x - y = 1$ b) $y - x = 2$ c) $y - x = 3$ d) $x - y = 2$

Linked comprehension type questions

Passage - I :

$f(x, y) = 0$ be the equation of a circle, such that $f(0, y)$ has equal roots and $f(x, 0) = 0$ has two distinct Real roots the angle between the tangents drawn from P to $f(x,y)=0$ is $\pi/3$. Then locus of P is the circle is whose equation is $g(x,y)=x^2+y^2-5x-4y+c=0$ Let Q be a point from which the tangents to $g(x,y)=0$ are perpendicular. AB is the chord of contact of Q w.r.t. $g(x,y)=0$ then

19. Equation of $f(x,y)=0$ is

a) $x^2+y^2-5x-4y+5=0$ b) $x^2+y^2-5x-4y-4=0$

c) $x^2+y^2-5x-4y-5=0$ d) $x^2+y^2-5x-4y+4=0$

20. Area of $\Delta Q\ AB$ is

a) $\frac{25}{2}$ b) $\frac{25}{4}$ c) $\frac{25}{8}$ d) $\frac{25}{12}$

21. The length of X-intercept made by $g(x,y) = 0$ is

a) $2\sqrt{80}$ b) $\sqrt{84}$ c) $\sqrt{74}$ d) $\sqrt{78}$

Passage - II :

Consider the relation $4l^2 - 5m^2 + 6l + 1 = 0$ where $l, m \in R$ then the line $lx + my + 1 = 0$ touches a fixed circle

22. Centre and radius of the circle

a) (2, 0), 3 b) (–3, 0), $\sqrt{3}$ c) (3, 0) $\sqrt{5}$ d) (3,0), $\sqrt{3}$

23. No. of tangents which can be drawn from the point (2, –3) are

a) 0 b) 1 c) 2 d) 1 or 2

24. From a point $P(2, -3)$ two tangents are drawn to the circle and A,B are points of contact then $PA.\ PB$

a) 26 b) 5 c) 15 d) 25

Passage - III :

To the circle $x^2 + y^2 = 4$ two tangents are drawn from $P(-4, 0)$ which touches the circle at T_1 and T_2 and a rhombus $PT_1\ P'T_2$ is completed.

25. Circum centre of the triangle PT_1T_2 is at

a) (–2, 0) b) (2, 0) c) $\left(\frac{\sqrt{3}}{2}, 0\right)$ d) none

26. Ratio of the area of a triangle PT_1, P^1 to that the $P'T_1T_2$ is

a) 2 : 1 b) 1 : 2 c) $\sqrt{3}$: 2 d) none

27. If P is taken to be at (h, 0) such that P' lies on the circle then area of the rhombus is ....

a) $6\sqrt{3}$ b) $2\sqrt{3}$ c) $3\sqrt{3}$ d) none

Matrix matching type questions

28. Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be an equation of circle

COLUMN - I	COLUMN - II
A) If circle lies in first quadrant, then	p) $g < 0$
B) If circle lies above x - axis, then	q) $g > 0$
C) If circle lies on the left of y – axis, then	r) $g^2 - c < 0$
D) If circle touches positive x-axis and does not intersect y-axis, then	s) $c > 0$

Integer answer type questions

29. Radius of the circle that can be drawn to pass through the points (0,7) (0,6) and touching the x–axis is K then $\frac{14K}{13}$ is

30. If a circle of radius r is touching the lines $x^2 - 4xy + y^2 = 0$ in the first quadrant at points A and B then area of ΔOAB is $\frac{K\sqrt{3}r^2}{l}$ then $(k + l)$ is

31. The area of the triangle formed by the pair of tangents drawn from p(6,8) to the circle $x^2 + y^2 = r^2$ and chord of contact of p w.r.t $x^2 + y^2 = r^2$ is maximum then $r =$

32. AB is a diameter of a circle. CD is a chord parallel to AB and $2CD = AB$. The tangent at B meets the line AC produced at E then $AE/AB =$

33. The equation of the circle whose equation of two normals is $x^2 + 3x + 6y + 2xy = 0$ and is large enough to just contain the circle $x(x - 4) + y(y - 3) = 0$ is $x^2 + y^2 + Kx - Ly - 45 = 0$ then $(K + L)$ is

34. A circle touches the hypeotenuse of a right - angled triangle at its middle point and passes through the middle point of the shorter side. If 3 unit and 4 unit 'r' be the radius of the cirlce, then find the value of '$3r$'

35. A circle with centre A and radius 7 is tangent to the sides of an angle of 60^0. A larger circle with centre B is tangent to the sides of the angle and to the first circle. The radius of the larger circle is an integer with unit digit equal to ....

EXERCISE-III

Relative positions of two circles - common tangents - lengths of common tangents

LEVEL-I (MAIN)

Single answer type questions

1. The internal centre of similitude of the circles $x^2 + y^2 - 2x + 4y + 4 = 0$, $x^2 + y^2 + 4x - 2y + 1 = 0$ divides the segment joining their centres in the ratio

   1) 1 : 2 2) 2 : 1 3) –1 : 2 4) –2 : 1

2. For the circles $x^2 + y^2 + 2\lambda x + c = 0$, $x^2 + y^2 + 2my - c = 0$ the number of common tangents when $c \neq 0$ is

   1) one 2) two 3) four 4) zero

3. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

   1) $r > 2$ 2) $2 < r < 8$ 3) $r < 2$ 4) $r = 2$

4. If the circles $x^2+y^2=2$ and $x^2+y^2-4x-4y+\lambda=0$ have exactly three real common tangents the $\lambda =$

   1) –10 2) 6 3) –6 4) 10

5. The common tangents to the circle $x^2+y^2-6x=0$, $x^2 + y^2 + 2x = 0$ form

   1) Right angled triangle 2) Isosceles triangle

   3) equilateral triangle 4) Isosceles right angled triangle

6. If $\left(-\frac{1}{3},-1\right)$ is a centre of similitude for the circles $x^2+y^2=1$ and $x^2+y^2-2x-6y-6=0$, then the length of common tangent of the circle is

1) $\frac{1}{3}$ 2) $\frac{4}{3}$ 3) 1 4) cannot be determined

7. Equation of circle whose radius is 5 and which touches the circle $x^2+y^2-2x-4y-20=0$ at (5, 5) is

1) $(x-9)^2+(y-8)^2=5$ 2) $(x-9)^2+(y+8)^2=25$
3) $x^2+y^2=25$ 4) $(x-9)^2+(y-8)^2=25$

8. Locus of the centre of circle of radius 2 which rolls on out side the rim of the circle $x^2+y^2-4x-6y-12=0$ is

1) $x^2+y^2-4x-6y=0$ 2) $x^2+y^2-4x-6y-36=0$
3) $x^2+y^2-4x-6y+3=0$ 4) $x^2+y^2-4x-6y-25=0$

9. The circles $x^2+y^2+2ax+c=0$ and $x^2+y^2+2bx+c=0$ have no common tangent if

1) $ab=0,\ c<0$ 2) $ab<0,\ c>0$ 3) $ab>0,\ c>0$ 4) $ab>0,\ c<0$

10. The set of all real values of λ for which exactly two common tangents can be drawn to the circles $x^2+y^2-4x-4y+6=0$ and $x^2+y^2-10x-10y+\lambda=0$ is in the interval

1) (18, 42) 2) (12, 24) 3) (12, 32) 4) (18, 48)

LEVEL-II (ADVANCED)

Single answer type questions

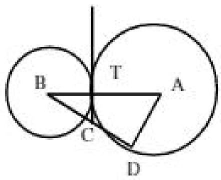
1. The lines given by $x^2-3xy-3x+9y=0$ are normals of a circle which touches the circle $x^2+y^2-6x+6y+17=0$ externally then radius is

a) 3 b) 4 c) 5 d) $\sqrt{2}$

2. Five circle C_1, C_2, C_3, C_4, C_5 with radii r_1, r_2, r_3, r_4, r_5 respectively ($r_1<r_2<r_3<r_4<r_5$) be such that C_{i+1} touch each other externally for all $i=1,2,3,4$ if all the five circles touch each of the two staright lines L_1 and L_2 and $r_1=2$ and $r_5=32$ then r_3 is equal to

a) 8 b) 17
c) $\frac{64}{17}$ d) Depends upon r_2 and r_4

3. Two circles with centres at A and B, touch at T. BD is the tangent at D and TC is a common tangent. AT has length 3 and BT has length 2. The length of CD is



a) 4/3 b) 3/2 c) 5/3 d) 7/4

4. C_1 and C_2 are circles of unit radius with centres at (0, 0) amd (1, 0) respectively C_3 is a circle of unit radius passes through the centres of circles $C_1$and C_2 are have its centre above X-axis. The equation of common tangent to C_1 and C_3 which does nto pass through C_2 is

a) $x-\sqrt{3}y+2=0$ b) $\sqrt{3}x-y+2=0$ c) $\sqrt{3}x-y-2=0$ d) $x+\sqrt{3}y+2=0$

5. If r_1 and r_2 are the radii of smallest and larget circle which passes through (5, 6) and touches the circle $(x-2)^2+y^2=4$ then r_1r_2 is ....

a) $\frac{4}{41}$ b) $\frac{41}{4}$ c) $\frac{3}{41}$ d) $\frac{41}{6}$

6. The equation of the circle of minimum radius which contains the three circles $x^2+y^2-4y-5=0$, $x^2+y^2+12x+4y+31=0$ and $x^2+y^2+6x+12y+36=0$ is ....

a) $\left(x-\frac{31}{18}\right)^2+\left(y-\frac{23}{12}\right)^2=\left(3-\frac{5}{36}\sqrt{949}\right)^2$ b) $\left(x+\frac{23}{12}\right)^2+\left(y+\frac{31}{18}\right)^2=\left(3+\frac{5}{36}\sqrt{949}\right)^2$

c) $\left(x+\frac{31}{18}\right)^2+\left(y+\frac{23}{12}\right)^2=1$ d) none

More than one correct answer type questions

7. Three circles of same radius r are drawn so that each one touches the other two externally. The radius of the circle which touches all the three circles is

a) $(3+2\sqrt{3})\frac{r}{3}$ b) $(2-\sqrt{2})\frac{r}{2}$ c) $(2-\sqrt{3})\frac{r}{\sqrt{3}}$ d) $(2+\sqrt{2})\frac{r}{3}$

8. C_1, C_2 are two circles of radii a, $b(a<b)$ respectively touching both the coordinate axes and have their centres in the first quadrant. Then the true statements among the following are

a) If C_1 and C_2 touch each other then $\frac{b}{a}=3+2\sqrt{2}$

b) If C_1 and C_2 are orthogonal then $\frac{b}{a}=2+\sqrt{3}$

c) If C_1 and C_2 intersect the such a way that their common chord has maximum length then $\frac{b}{a}=3$

d) If C_2 passes through centre of C_1 then $\frac{b}{a}=2+\sqrt{2}$

9. If a circle passes through (0, 0), (4, 0) and touches the circle $x^2+y^2=36$ then

a) The centers of two possible circles are $\left(2,\sqrt{5}\right),\left(2,-\sqrt{5}\right)$

b) The sum of radii of two circles is 8

c) The angle between the two possible circles is $\operatorname{Cos}^{-1}\left(\frac{1}{9}\right)$

d) The length of common chord of the two possible circles is 4

10. The centres(s) of the circles(s) passing through the points (0,0), (1,0) and touching the circle $x^2+y^2=9$ is/are

a) $\left(\frac{3}{2},\frac{1}{2}\right)$ b) $\left(\frac{1}{2},\frac{3}{2}\right)$ c) $\left(\frac{1}{2},2^{1/2}\right)$ d) $\left(\frac{1}{2},-2\frac{1}{2}\right)$

11. The locus of the centre of a circle touching the circle $x^2 + y^2 - 4y - 2x = 2\sqrt{3}-1$ internally and tangents on which from the point (1, 2) are making an angle of 60^o with each other is

a) $(x-1)^2 + (y-2)^2 = 3$ b) $(x-2)^2 + (y-1)^2 = 1 + 2\sqrt{2}$

c) $x^2 + y^2 = 1$ d) $(x-1)^2 + (y-2)^2 = \frac{4}{9}(4+2\sqrt{3})$

12. 'O' is the origin and $A_k(x_k, y_k)$ where $k = 1,2$ are two points. If the circles are described on OA_1 and OA_2 as diameters, then the length of their common chord is equal to

a) $|x_1 y_2 - x_2 y_1|$ b) $\frac{1}{2}|x_1y_2 - x_2y_1|$ c) $\frac{1}{2}A_1 A_2$ d) $\frac{|x_1y_2 - x_2y_1|}{A_1A_2}$

13. Two circles C_1 and C_2 intersect at two distinct points P and Q in a plane. let a line passing through P meets circle C_1 and C_2 in A and B respectively. Let Y is midpoint of AB and QY meets circle C_1 and C_2 in X and Z respectively then :

a) Y is the mid point of XZ b) $\frac{XY}{YZ} = \frac{3}{1}$

c) $XY = YZ$ d) $XY + YZ = 2YZ$

Linked comprehension type questions

Passage - I :

The circles $x^2 + y^2 + 2x = 0$ and $x^2 + y^2 - 6x = 0$ touch each other externally. Then

14. The triangle formed by the direct common tangents and transverse common tangent is

a) Issoless b) Equilateral c) Rt. angled Issoless d) Rt. angled

15. The points of intersection transverse common tangent with direct common tangents are :

a) $(0, \pm\sqrt{3})$ b) $(0, \pm 2)$ c) $(0, \pm 3)$ d) $(0, \pm 2\sqrt{3})$

16. Two circles of radii r_1 and r_2 touch each other externally $r_2 = (3+2\sqrt{2})r_1$. If $2x^2 + 3xy + ky^2 = 0$ is a pair of direct common tangents. Then $K =$ ....

a) 1 b) 2 c) 0 d) – 2

Passage - II :

Let $C = x^2 + y^2 + 4x = 0$ is a given circle and a circle C_1 of radius 2 units rolls on the outer side of the circle 'C' touching it externally. If the line joining the centres of C and C_1 makes an angle 60^0 with the x-axis, then that circle be C_2

17. The locus of centre of the circle C_1 is

a) $x^2 + y^2 + 4x - 12 = 0$ b) $x^2 + y^2 - 4\sqrt{3}y + 8 = 0$

c) $x^2 + y^2 + 4x + 12 = 0$ d) $x^2 + y^2 - 4\sqrt{3}y - 12 = 0$

18. The equation of circle joining the centres C and C_1 as a diameter is

a) $x^2 + y^2 - 2x + 2\sqrt{3}y = 0$ b) $x^2 + y^2 - 2x - 2\sqrt{3}y = 0$

c) $x^2 + y^2 + 2x - 2\sqrt{3}y = 0$ d) None of these

19. The equation of least circle containing both the circles C and C_1 is

a) $x^2 + y^2 - 4x + 12 = 0$ b) $x^2 + y^2 - 4x - 12 = 0$

c) $x^2 + y^2 - 2x + 2\sqrt{3}y + 12 = 0$ d) $x^2 + y^2 + 2x - 2\sqrt{3}y - 12 = 0$

Matrix matching type questions

20. Given a circle S with radIus r and two points P and Q in the plane of S. A right angled triangle is inscribed in the circle such that one of its sides passes through P and other through Q. It is also given that d is the distance between centre C, of S to the midpoint of the segment PQ.

Column-I says about number of such inscribed right angled triangles is/are possible and Column-II is the corresponding condition.

COLUMN - I	COLUMN - II
A) 0	p) not possible
B) 1	q) $2(r-d) > PQ$
C) 2	r) $2(r-d) < PQ$
D) 3	s) $2(r-d) = PQ$

a) A - p; B - q; C - r; D - s b) A - q; B - r; C - s; D - p
c) A - q; B - s; C - r; D - p d) A - s; B - r; C - q; D - p

Integer answer type questions

21. Consider four circles $(x\pm1)^2+(y\pm1)^2=1$. Equation of smaller circle touching these four circles is $x^2+y^2=a-b\sqrt{c}$ then $(a+b+c)$ is

22. The centre of two circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C_1, then the radius of the circle C is

EXERCISE-IV

Chord with midpoint - pole, polar - conjugate points, lines - inverse point - miscellaneous problems

LEVEL-I (MAIN)

Single answer type questions

1. From origin chords are drawn to the circle $x^2+y^2-2px=0$ then locus of midpoints of all such chords is
   1) $x^2+y^2-px=0$ 2) $x^2+y^2+2px=0$ 3) $x^2+y^2+px=0$ 4) Does not exists

2. Locus of midpoints of chords of circle $x^2+y^2=r^2$ having a constant length '$2l$' is
   1) $x^2+y^2=l^2-r^2$ 2) $x^2+y^2=r^2-l^2$ 3) $x^2+y^2=4l^2$ 4) $x^2+y^2=l^2+r^2$

3. If (3,–2) is the midpoint of the chord AB of the circle $x^2+y^2-4x+6y-5=0$ then $AB=$
   1) 4 2) 8 3) 12 4) 16

4. The equation of the circle is $2x(x-a)+y(2y-b)=0$ $(a\neq0, b\neq0)$. The condition on a and b if two chords, each bisected by the x-axis, can be drawn to the circle from $\left(a,\frac{b}{2}\right)$ is
   1) $a^2>2b^2$ 2) $a^2>b^2$ 3) $a^2<b^2$ 4) $a^2<2b^2$

5. If the distance from origin to centres of three circles $x^2+y^2-2\lambda_i x=c^2 (i=1,2,3)$ are in G.P then lengths of tangents drawn to them from any point on the circle $x^2+y^2=c^2$ are in
   1) A.P 2) G.P 3) H.P 4) A.G.P

6. From the point $A(0, 3)$ on the circle $x^2 + 4x + (y - 3)^2 = 0$ a chord AB is drawn and extended to a point P such that $AP = 2AB$ then the locus of P is

1) $x^2 + 4x + (y-3)^2 = 0$ 2) $x^2 + 8x + (y-3)^2 = 0$

3) $x^2 + 4x - (y-3)^2 = 0$ 4) $x^2 + 8x - (y-3)^2 = 0$

7. The polar of the point (1, 2) w.r.t to circle $x^2 + y^2 - 2x - 4y - 4 = 0$

1) Touches the circle 2) Intersects the circle in two points

3) Does not meet the circle 4) None

8. The locus of poles of tangents to the circle $(x-p)^2 + y^2 = b^2$ w.r.t. the circle $x^2 + y^2 = a^2$ is

1) $(a^2-px)^2 = b^2(x^2 + y^2)$ 2) $(a^2-bx)^2 = p^2(x^2 + y^2)$

3) $(a^2+px)^2 = b^2(x^2 + y^2)$ 4) $(a^2+bx)^2 = p^2(x^2 + y^2)$

9. If the pole of a line w.r.t to circle $x^2 + y^2 = c^2$ lies on the circle $x^2 + y^2 = 9c^2$ then the line is tangent to circle with centre origin is

1) $x^2 + y^2 = 9c^2$ 2) $9x^2 + 9y^2 = c^2$ 3) $3x^2 + 3y^2 = c^2$ 4) $81x^2 + 81y^2 = c^2$

10. If P, Q, R are pair wise conjugate points with respect to a circle $(x-3)^2 + (y+1)^2 = 25$ then the ortho centre of ΔPQR is

1) (3, 1) 2) (3, –1) 3) (1, 3) 4) (1, –3)

11. If A and B are conjugate points w.r.t to circle $x^2 + y^2 = r^2$ then $OA^2 + OB^2 =$

1) $AB^2 - r^2$ 2) $AB^2 + r^2$ 3) $AB^2 + 2r^2$ 4) $AB^2 - 2r^2$

12. The inverse point of (1, 2) with respect to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$

1) (0, 0) 2) (1, 0) 3) (0, 1) 4) (1, 1)

13. If each side of ΔABC is the polar of the opposite vertex w.r.t a circle with centre P. Then P is .... of ΔABC

1) Circum centre 2) Orthocentre 3) Centroid 4) None

14. If $3x + 2y = 3$ and $2x + 5y = 1$ are conugate lines w.r.t the circle $x^2 + y^2 = r^2$ then $r^2 =$

1) $\frac{3}{16}$ 2) $\frac{16}{3}$ 3) $\frac{4}{\sqrt{3}}$ 4) $\frac{\sqrt{3}}{4}$

15. The inverse point of (2, –3) w.r.t. circle $x^2 + y^2 + 6x - 4y - 12 = 0$ is

1) $\left(\frac{1}{2},\frac{1}{2}\right)$ 2) $\left(-\frac{1}{2},\frac{1}{2}\right)$ 3) $\left(\frac{1}{2},-\frac{1}{2}\right)$ 4) $\left(-\frac{1}{2},-\frac{1}{2}\right)$

16. If the inverse of P(–3, 5) w.r.t a circle is (1, 3) than polar of P w.r.t. the circle is

1) $x + 2y = 7$ 2) $2x - 2y + 11 = 0$ 3) $2x - y + 1 = 0$ 4) $2x - y - 1 = 0$

17. The least length of chord passing through (2, 1) of the circle $x^2 + y^2 - 2x - 4y - 13 = 0$ is

1) 2 2) 6 3) 8 4) 4

KEY SHEET (PRACTICE SHEET)

EXERCISE - I

LEVEL-I

1) 3	2) 2	3) 1	4) 1	5) 4	6) 4	7) 3	8) 2
9) 4	10) 2	11) 2	12) 2	13) 1	14) 1	15) 1	16) 1
17) 4	18) 2	19) 2	20) 3	21) 1	22) 2	23) 1	

LEVEL-II

1) c	2) c	3) d	4) a	5) c	6) c	7) b	8) c
9) a	10) d	11) d	12) abd	13) abc	14) ab	15) d	16) c
17) a	18) ac	19) abcd	20) ab	21) bd	22) bc	23) b	24) b
25) c	26) b	27) c	28) a	29) A–r, B–p, C–s, D–t			
30) A–s, B–r, C–q, D–p			31) 6	32) 8	33) 7	34) 4	35) 6

EXERCISE-II

LEVEL-I

1) 2	2) 1	3) 2	4) 4	5) 2	6) 3	7) 2	8) 3
9) 4	10) 3	11) 3	12) 3	13) 1	14) 4	15) 1	16) 3
17) 1	18) 3	19) 1	20) 1	21) 2	22) 2	23) 1	24) 1
25) 1	26) 2	27) 4					

LEVEL-II

1) b	2) c	3) a	4) a	5) b	6) b	7) c	8) c
9) c	10) d	11) a	12) a	13) bd	14) ac	15) c	16) ac
17) a	18) ac	19) b	20) a	21) c	22) c	23) c	24) b
25) a	26) d	27) a	28) A–prs, B–rs, C–qs, D–ps			29) 7	30) 7
31) 5	32) 2	33) 9	34) 5	35) 1			

EXERCISE-III

LEVEL-I

1) 1	2) 2	3) 2	4) 2	5) 3	6) 3	7) 4	8) 2
9) 3	10) 1						

LEVEL-II

1) a	2) a	3) b	4) b	5) b	6) d	7) ac	8) abcd
9) acd	10) cd	11) d	12) d	13) a	14) d	15) abc	16) abd
17) a	18) c	19) d	20) c	21) 7	22) 2		

EXERCISE-IV

LEVEL-I

1) 1	2) 2	3) 2	4) 1	5) 2	6) 2	7) 3	8) 1
9) 2	10) 2	11) 3	12) 3	13) 2	14) 1	15) 4	16) 1
17) 3							

ADDITIONAL EXERCISE

LEVEL-I (MAIN)

Single answer type questions

1. The equation of the circumcircle of an equilateral triangle is $x^2 + y^2 + 2x + 4y + c = 0$ and one vertex of the triangle is (1,1). The equation of incircle of the triangle is

   1) $4(x^2 + y^2) + 8x + 16y + 17 = 0$ 2) $4(x^2 + y^2) + 8x + 16y - 8 = 0$
   3) $4(x^2 + y^2) + 8x + 16y + 7 = 0$ 4) $x^2 + y^2 + 2x + 4y - 8 = 0$

2. The line $(x - 3)\cos\theta + (y - 3)\sin\theta = 1$ touches a circle $(x - 3)^2 + (y - 3)^2 = 1$ then θ lies in the interval

   1) $\left(0, \frac{\pi}{2}\right)$ 2) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ 3) no value of θ 4) any value of θ

3. The line $9x + y - 28 = 0$ is the chord of contact of the point $P(h, k)$ with respect to the circle $2x^2 + 2y^2 - 3x + 5y - 7 = 0$ then P is

   1) (3, –1) 2) (3, 1) 3) (–3, 1) 4) no position of p

4. The line $4x + 3y - 4 = 0$ divides the circumference of the circle centre at (5, 3) in the ratio 1 : 2 then equation of the circle is

   1) $x^2 + y^2 - 10x - 6y - 66 = 0$ 2) $x^2 + y^2 - 10x - 6y - 50 = 0$
   3) $x^2 + y^2 - 10x - 6y - 6 = 0$ 4) $x^2 + y^2 - 10x - 6y - 55 = 0$

5. Let $f(x, y) = 0$ be the equation of a circle. If $f(0, k) = 0$ has equal roots $k = 2, 2$ and $f(k, 0) = 0$ has roots $k = \frac{4}{5}, 5$, then the centre of the circle is

   1) $\left(-2, \frac{29}{10}\right)$ 2) $\left(-2, \frac{29}{10}\right)$ 3) $\left(2, \frac{29}{10}\right)$ 4) $\left(-2, \frac{-29}{10}\right)$

6. Let $A_0A_1A_2A_3A_4A_5$ be a regular hexagon inscribed in a unit circle with centre at the origin. Then the product of the lengths of the line segments A_0A_1, A_0A_2 and A_0A_4 is :

   1) 3 2) $\frac{3}{4}$ 3) $3\sqrt{3}$ 4) $\frac{3\sqrt{3}}{4}$

7. The value of α in $[0, 2\pi]$ so that $x^2 + y^2 + 2\sqrt{\sin\alpha}\, x + (\cos\alpha - 1) = 0$ having intercept on x–axis always greater than 2 is/are

   1) $(\pi/4, 3\pi/2]$ 2) $(\pi/4, \pi]$ 3) $(\pi/4, 5\pi/4)$ 4) $(0, \pi)$

8. The equation of the circumcircle of the triangle formed by the line $ax + by + c = 0$ $(abc \neq 0)$ and the coordinate axes is ....

   1) $a(x^2 + y^2) + c\,(bx + ay) = 0$ 2) $ab(x^2 + y^2) + c\,(bx + ay) = 0$
   3) $bc(x^2 + y^2) + a\,(bx + ay) = 0$ 4) $c(x^2 + y^2) + b\,(bx + ay) = 0$

9. The least distance of the line $8x - 4y + 73 = 0$ from the circle $16x^2 + 16y^2 + 48x - 8y - 43 = 0$ is

   1) $\frac{\sqrt{5}}{2}$ 2) $2\sqrt{5}$ 3) $3\sqrt{5}$ 4) $4\sqrt{5}$

10. A chord of length 24 units is at a distance of 5 units from the center of a circle then its radius is
    1) 5 2) 12 3) 13 4) 10

11. From a point $R(5, 8)$ two tangents RP and RQ are drawn to a given circle $S = 0$ whose radous is 5. If circumcentre of the triangle PQR is $(2, 3)$, then the equation of circle $S = 0$ is
    1) $x^2 + y^2 + 2x + 4y - 20 = 0$ 2) $x^2 + y^2 + x + 2y - 10 = 0$
    3) $x^2 + y^2 - x - 2y - 20 = 0$ 4) $x^2 + y^2 - 4x - 6y - 21 = 0$

12. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sectors then
    1) $3a^2 - 2ab + 3b^2 = 0$ 2) $3a^2 - 10ab + 3b^2 = 0$
    3) $3a^2 + 2ab + 3b^2 = 0$ 4) $3a^2 + 10ab + 3b^2 = 0$

13. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two sides along the coordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + m\sqrt{x^2 + y^2} = 0$. Then m is equal to
    1) 1 2) -2 3) 2 4) -1

14. If the chord joining the points $(2, -1)$, $(1, -2)$ subtends a right angle at the centre of the circle than its centre and radius are
    1) $(2, -2)$; 2 2) $(2, -2)$; 1 3) $(1, 1)$; 1 4) $(-2, -1)$; 2

15. The radius of the circle wich touches y-axis at $(0, 0)$ and passes through the point (b, c) is
    1) $\dfrac{b^2+c^2}{2|b|}$ 2) $\dfrac{b^2+c^2}{2|c|}$ 3) $\dfrac{b^2+c^2}{2}$ 4) $\dfrac{|b|}{2(b^2+c^2)}$

16. P is a variable point on a circle C and Q is a fixed point outside C. R is a point on PQ dividing it in the ratio $p : q$ where $p > q$ are fixed. Then the locus of R is
    1) circle 2) parabola 3) straight line 4) none

LEVEL-II

LECTURE SHEET (ADVANCED)

Single answer type questions

1. The circum circle of the triangle formed by the lines $x = a$, $y = b$ and $l.x+my = 1$, passes through the origin then $l^2 + m^2 =$
   a) $\dfrac{l+m}{a+b}$ b) $\dfrac{lm}{ab}$ c) $\dfrac{l}{a}+\dfrac{m}{b}$ d) $\dfrac{l}{b}+\dfrac{m}{a}$

2. The line $lx + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$. at points P and Q which lie at finite distances from the origin. The circle on PQ as diameter passes through the origin then $l^2 + m^2 =$
   a) $n(a + b)$ b) $n^2(a + b)$ c) $n(a^2 + b^2)$ d) $n^2(a^2 + b^2)$

3. A pair of tangents are drawn to a unit circle with centre at the origin and these tangents intersect at A enclosing an angle of 60^0. The area enclosed by these tangents and the arc of the circle is
   a) $\dfrac{2}{\sqrt{3}}-\dfrac{\pi}{6}$ b) $\sqrt{3}-\dfrac{\pi}{3}$ c) $\dfrac{\pi}{3}-\dfrac{\sqrt{3}}{6}$ d) $\sqrt{3}\left(1-\dfrac{\pi}{6}\right)$

4. In a triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. If a chord joins A with the point of intersection D of the hypotenuse and the semicircle, then the length of AC equals to

a) $\frac{AB.AD}{\sqrt{AB^2+AD^2}}$ b) $\frac{AB.AD}{AB+AD}$ c) $\sqrt{AB.AD}$ d) $\frac{AB.AD}{\sqrt{AB^2-AD^2}}$

5. A cirlce is inscribed into a rhombus $ABCD$ with one angle 60^0. The distance from the centre of the cirlce to the nearest vertex is equal to 1. If P is any point of the cirlce, then $|PA|^2+|PB|^2+|PC|^2+|PD|^2$ is equal to

a) 12 b) 11 c) 9 d) none

6. Point P and Q are 3 units apart. A cirlce centre at P with a radius $\sqrt{3}$ units at point A and B. The area of the quadrilateral $APBQ$ is :

a) $\sqrt{99}$ b) $\frac{\sqrt{99}}{2}$ c) $\sqrt{\frac{99}{2}}$ d) $\sqrt{\frac{99}{16}}$

7. The equation to the side BC of ΔABC is $x + 5 = 0$. If $(-3,2)$ is the orthocentre of ΔABC. The point where the altitude through A meets the circumcircle of the triangle is

a) (2, 7) b) (2, –7) c) (–7, 2) d) (7, –2)

8. Radii of the smallest and the largest circle passing through a point lying on the sides of a rectangle with vertices $(\pm 2, \pm 1)$ and touching the circle $x^2 + y^2 = 9$, are r_1 and r_2, respectively. Let $d=|r_1-r_2|$ then minimum value of d is

a) $\frac{1}{2}$ b) 1 c) 2 d) 3

9. If $f(x + y) = f(x).f(y)$ for all x and y, $f(1) = 2$ and $a_n = f(n), n \in N$, then the equation of the circle having (a_1, a_2) and (a_3, a_4) as the ends of its one diameter is

a) $(x - 2)(x - 8) + (y - 4)(y - 16) = 0$ b) $(x - 4)(x - 8) + (y - 2)(y - 16) = 0$

c) $(x - 2)(x - 16) + (y - 4)(y - 8) = 0$ d) $(x - 6)(x - 8) + (y - 5)(y - 6) = 0$

10. The value of θ in $[0,2\pi]$ so that circle $x^2+y^2+2(\sin\alpha)x+2(\cos\alpha)y+\sin^2\theta=0$ always lies inside the square of unit side length, is/are

a) $(\pi/3, 2\pi/3)$ b) $(4\pi/3, 5\pi/3)$

c) $(\pi/4, 2\pi/3)$ d) $[\pi/3, 2\pi/3]\cup[4\pi/3, 5\pi/3]$

11. The equation of the circle passing through the point of intersection of the circle $x^2 + y^2 = 4$ and the line $2x + y = 1$ and having minimum possible radius is

a) $5x^2 + 5y^2 + 18x + 6y - 5 = 0$ b) $5x^2 + 5y^2 + 9x + 8y - 15 = 0$

c) $5x^2 + 5y^2 + 4x + 9y - 5 = 0$ d) $5x^2 + 5y^2 - 4x - 2y - 18 = 0$

12. 'O' isthe origin and $A_k(x_k, y_k)$ where $k = 1, 2$ are two points. If the circles are described on OA_1 and OA_2 as diameters, then the length of their common chord is equal to

a) $|x_1y_2 - x_2y_1|$ b) $\frac{1}{2}|x_1y_2 - x_2y_1|$ c) $\frac{1}{2}A_1A_2$ d) $\frac{|x_1y_2 - x_2y_1|}{A_1A_2}$

13. Two circles with radii r_1 and r_2, $r_1 > r_2 \geq 2$, touch each other externally. If 'α' be the angle between direct common tangents, then

a) $\alpha = \sin^{-1}\left(\frac{r_1+r_2}{r_1-r_2}\right)$ b) $\alpha = 2\sin^{-1}\left(\frac{r_1-r_2}{r_1+r_2}\right)$ c) $\alpha = \sin^{-1}\left(\frac{r_1-r_2}{r_1+r_2}\right)$ d) $\alpha = \sin^{-1}\left(\frac{r_1}{r_2}\right)$

14. The locus of the centre of the cirlce such that the point (2, 3) is midpoint of the chord $5x + 2y = 16$ is

a) $2x - 5y + 11 = 0$ b) $2x + 5y - 11 = 0$ c) $2x + 5y + 11 = 0$ d) $2x - 5y - 11 = 0$

15. Triangle ABC is right angled at A. The circle with centre A and radius AB cuts BC and AC internally at D and E respectively. If $BD = 20$ and $DC = 16$ then the length AC equals

a) $6\sqrt{21}$ b) $6\sqrt{26}$ c) 30 d) 32

16. A regular polygon of 9 sides where length of each side is '2' is inscribed in a circle. Then the radius of the circle is

a) $\sec\frac{\pi}{9}$ b) $\operatorname{cosec}\frac{\pi}{9}$ c) $\cot\frac{\pi}{9}$ d) $\tan\frac{\pi}{9}$

17. $ABCD$ is a square of unit area A circle is tangent to two sides of $ABCD$ and passes through exactly one of its vertices. The radius of the circle is ....

a) $2-\sqrt{2}$ b) $\sqrt{2}-1$ c) $\frac{1}{2}$ d) $\frac{1}{\sqrt{2}}$

18. P is a variable point on a circle C and Q is a fixed point on PQ dividing it in the ratio $p : q$ where $p > 0$ and $q > 0$ are fixed. Then the locus of R is

a) a circle b) an ellipse

c) a circle if $p = q$ and an ellipse otherwise d) none

19. Let $ABCD$ be a quadrilateral with area 18 with side AB paralle lto the side CD and $AB = 2CD$. Let AD is perpendicular to AB and CD. If a line is drawn inside the quadrialteral $ABCD$ touching all the sides then its radius is

a) 3 b) 2 c) $\frac{3}{2}$ d) 1

20. CD is the common chord of the two circles of equal radii touching the line L at A and B. C is closer to L then D the ratio of the circum radii of the triangles ACB and ADB is ....

a) less than 1 b) morethan 1 c) equal to 1 d) can't say

More than one correct answer type questions

21. If $(a, 0)$ is a point on a diameter of the circle $x^2 + y^2 = 4$, then $x^2 - 4x - a^2 = 0$ must have

a) exactly one real root in $\left[-\frac{9}{10}, \frac{1}{10}\right]$ b) exactly one real root in $\left[4, \frac{49}{10}\right]$

c) $\left[\frac{3}{2}, \frac{3}{2}\right]$ d) $\left[-\frac{1}{2}, -\frac{1}{2}\right]$

22. Let x, y be real variable satisfying the $x^2 + y^2 + 8x - 10y - 40 = 0$.

Let $a = \max\left(\sqrt{(x+2)^2+(y-3)^2}\right)$ and $b = \min\left(\sqrt{(x+2)^2+(y-3)^2}\right)$, then

a) $a + b = 18$ b) $a+b=4\sqrt{2}$ c) $a-b=4\sqrt{2}$ d) $ab = 73$

23. If the equation of any two diagonals of a regular pentagon belongs to family of lines $(1+2\lambda)y-(2+\lambda)x+1-\lambda=0$ and their lengths are sin 36^0, then locus of centre of circle circumscribing the given pentagon (the triangles formed by these diagonals with sides of pentagon have no side common) is

a) $x^2+y^2-2x-2y+1+\sin^2 72^0=0$
b) $x^2+y^2-2x-2y+\frac{7}{4}\sec^2 36^0=0$
c) $x^2+y^2-2x-2y+1+\cos^2 72^0=0$
d) $x^2+y^2-2x-2y+\sin^2 72^0=0$

24. The equation of circles which passes through the origin and cuts off equal chords of length 'a' from the lines $y = x$ and $y = -x$ are

a) $x^2+y^2 \pm ax \pm ay=0$
b) $x^2+y^2 \pm \sqrt{2ay}=0$
c) $x^2+y^2 \pm \sqrt{2ax}=0$
d) both b and c

25. no of possible common tangents to the circles $x^2+y^2-2kx-2ky+k^2=0$ and $x^2+y^2-4x+6ky+4=0$ can be

a) 1 b) 2 c) 3 d) 4

26. An isosceles triangle ABC is inscribed in a circle $x^2 + y^2 = a^2$ with the vertex A at $(a, 0)$ and the base angles B and C each equal to 75^0 then coordinates of an end point of the base are

a) $\left(\frac{\sqrt{3}a}{2},\frac{a}{2}\right)$ b) $\left(\frac{-\sqrt{3}a}{2},\frac{a}{2}\right)$ c) $\left(\frac{a}{2},\frac{\sqrt{3}a}{2}\right)$ d) $\left(\frac{-\sqrt{3}a}{2},\frac{-a}{2}\right)$

27. A circle of radius r touches the parabola $x^2 + 4ay = 0$ $(a > 0)$ at the vertex of the parabola. The centre of the circle lies below the vertex and the circle lies entirely within the parabola. Then the largest possible value of r is ....

a) a
b) 2a
c) same as semilatusrectm of the parabola
d) none

28. If the locus of the points, the sum of the squares of whose distances from n fixed points $A_i(x_i, y_i)$, $i = 1, 2......n$ is equal to k^2 is the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ then :

a) $g=\frac{-1}{n}\Sigma x_i$ b) $f=\frac{-1}{n}\Sigma y_i$ c) $c = k^2$ d) $c=\frac{1}{n}\left(\Sigma x_i^2+\Sigma y_i^2-k^2\right)$

Linked comprehension type questions

Passage - I :

P is a variable point on the line $L = 0$. Tangents are drawn to the circle $x^2 + y^2 = 4$ from P to touch it at Q and R the parallelogram $PQSR$ is completed.

29. If $L \equiv 2x + y = 6$, then the locus of circum centre of ΔPQR is

a) $2x - y = 4$ b) $2x + y = 3$ c) $x - 2y = 4$ d) $x + 2y = 3$

30. If $P(3, 4)$ then the coordinates of S is

a) $\left(-\frac{46}{25},\frac{-63}{25}\right)$ b) $\left(\frac{-51}{25},\frac{-68}{25}\right)$ c) $\left(\frac{-46}{25},\frac{-68}{25}\right)$ d) $\left(\frac{-68}{25},\frac{-51}{25}\right)$

Passage - II :

$A = (1, 2)$, $B = (7, -6)$ are the ends of a diameter of a circle whose centre is S. C and D are two points on the circle such that $AC = AD = 6$ units. Then

31. Equation of CD is

a) $3x - 4y = 25$ b) $3x - 4y = 13$

c) $3x - 4y = 29$ d) $3x - 4y = 31$

32. Equation of C and D are

a) $(7x + 24y + 95)(x - 7) = 0$ b) $(24x + 7y - 126)(y + 6) = 0$

c) $(24x + 7y - 126)(y - 6) = 0$ d) $(24x + 7y - 126)(x - 7) = 0$

33. Area of quadrilateral $ASCD$ is sq. units is

a) 16 b) 20 c) 24 d) 12

Passage - III

Tangents PA and PB are drawn to the circle $(x - 4)^2 + (y - 5)^2 = 4$ from the point P on the curve $y = \sin x$ where A, B lie on the circle. consider the function $y = f(x)$ representing by the locus of the centre of the circumcentre of the triangle PAB, then answer the following questions.

34. Range of $y = f(x)$ is

a) $[-2, 1]$ b) $[-1, 4]$ c) $[0, 2]$ d) $[2, 3]$

35. Period of $y = f(x)$ is

a) 2π b) 3π c) π d) None

36. Which of the following is true

a) $f(x) = 4$ has real roots b) $f(x) = 1$ has real roots

c) range of $y = f^{-1}(x)$ is $\left[-\frac{\pi}{4}+2, \frac{\pi}{4}+2\right]$ d) none

Passage-IV :

Let A, B, C be three sets of real numbers (x, y) defined as $A : \{(x,y) : y \geq 1\}$

$B : \{(x, y) : x^2 + y^2 - 4x - 2y - 4 = 0\}$; $C : \{(x, y) : x + y = \sqrt{2}\}$

37. Number of elements in the set $A \cap B \cap C$ is

a) 0 b) 1 c) 2 d) infinite

38. $(x + 1)^2 + (y - 1)^2 + (x - 5)^2 + (y - 1)^2$ has the value equal to

a) 16 b) 25 c) 36 d) 49

39. If the locus of the point of intersection of the pair of perpendicular tangents to the circle B is the curve S, then the area enclosed between B and S is

a) 6π b) 8π c) 9π d) 18π

Matrix matching type questions

40. **COLUMN - I** **COLUMN - II**

A) If two circles $x^2 + y^2 + 2a_1x + b = 0$ and $x^2 + y^2 + 2a_2x + b = 0$ touch each other then triplet (a_1, a_2, b) can be — p) $(2, 2, 2)$

B) If two cirlces $x^2 + y^2 + 2a_1x + b =$ and $x^2 + y^2 + 2a_2x + b = 0$ touches each otehr then triplet (a_1, a_2, b) can be — q) $\left(1,1,\frac{1}{2}\right)$

C) If the straight line $a_1x - by + b^2 = 0$ touches the circle $x^2 + y^2 = a_2x + by$, then triplet (a_1, a_2, b) can be — r) $(2, 1, 0)$

D) If the line $3x+4y-4=0$ touches the circle $(x-a_1)^2+(y-a_2)^2=b^2$ then triplet (a_1, a_2, b) can be — s) $\left(1,1\frac{3}{5}\right)$

41. In the parallelogram $ABCD$ with angle $A = 60^0$, the bisector of angle B is drawn which cuts the side CD at a point E. A circle S_1 of radius 'r' is inscribed in the triangle ECB. Another circle 'S_2' is inscribed in the trapezoid $ABED$.

COLUMN - I **COLUMN - II**

A) The value of radius of S_1 is — p) $2\sqrt{3}r$

B) The value of distance between the centres of S_1 and S_2 is — q) $\frac{\sqrt{3}}{2}r$

C) The value of the length of common tangent of S_1 and S_2 is — r) $\sqrt{7}r$

D) The value of the length CE is — s) $\frac{3}{2}r$

Integer answer type questions

42. If the 4 circles touches the 3 lines $x + 2y - 1 = 0$, $2x - 3y + 5 = 0$, $3x + y - 1 = 0$ of radii a, b, c, d and $a > b > c > d$ then, $\frac{d}{a}+\frac{d}{b}+\frac{d}{c}$ is equal to

43. The circle $x^2 + y^2 + kx + (1 + k)y - (k + 1) = 0$ passes through the same two points for every real value of k and radius of the circle is $\frac{5}{2}$ then k is

44. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If $S = \left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},\frac{-1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$, then the number of points in S lying inside the smaller part is

45. The three vertices of ΔABC are in the circle $x^2 + y^2 = 5$. The point (0,0) in out side the ΔABC and 1 unit away from the nearest side of ΔABC. If the maximum area of the triangle is $K\cos 72^0$. Then the value of k is

46. The sum of all possible integral values of α such that the angle between the pair of tangents drawn from $M(\alpha,\alpha)$ to the circle $x^2 + y^2 - 2x - 2y = 6$ lies in the range $(\pi/3, \pi)$

47. The distance betwen the chords of contact fo the tangents to the circle $x^2 + y^2 + 32x + 24y - 1 = 0$ from the origin and the point (16, 12) is k. The digit of unit place of K is ____

PRACTICE SHEET (ADVANCED)

Single answer type questions

1. The number of integral values of y for which the chord of the circle $x^2 + y^2 = 125$ passing through the point $P(8, y)$ gets bisected at the point $P(8, y)$ and has integral slope is

a) 8 b) 6 c) 4 d) 2

2. The range of values of $\lambda(\lambda > 0)$ such that the angle θ between the pair of tangents drawn from $(\lambda, 0)$ to the circle $x^2 + y^2 = 4$ lies in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ is

a) $\left(\frac{4}{\sqrt{3}}, 2\sqrt{2}\right)$ b) $(0, \sqrt{2})$ c) $(1, 2)$ d) $(1, -1)$

3. The point of concurency of the common chords of the circle $x^2 + y^2 - 4x - 6y - 1 = 0$ and the systemof circles, Passing through the ponts (3,7) and (6,5) is

a) (3, 7) b) (6, 5) c) (0, 9) d) (6, 7)

4. The Ex radii r_1, r_2, r_3 of ABC $(r_1 < r_2 < r_3)$ are the roots of $x^3 - 11x^2 + 36x - 36 = 0$. If a, b, c are the sides of the triangle then length of the common chord of the circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ is

a) $\frac{12}{5}$ b) $\frac{24}{5}$ c) 6 d) $\frac{36}{5}$

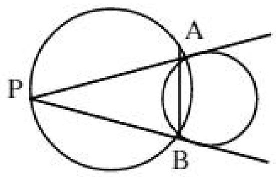
5. The line $3x + 3y = 7$ meets the circle $x^2 + y^2 + 2x + 2y - 14 = 0$ in the points A and B. The equation of the diameters of the circle each making angle 45° with AB are

a) $x + 1 = 0, y - 1 = 0$ b) $x - 1 = 0, y + 1 = 0$ c) $x + 1 = 0, y + 1 = 0$ d) $x - 1 = 0, y - 1 = 0$

6. From point $P(-1,-2)$, PQ and PR are the tangents drawn to the circle $x^2 + y^2 - 6x - 8y = 0$. Then angle subtended by QR on the centre of circle is

a) $\pi - 2\sin^{-1}\left(\frac{5}{2\sqrt{13}}\right)$ b) $\pi - \sin^{-1}\left(\frac{5}{2\sqrt{13}}\right)$ c) $\cos^{-1}\left(\frac{5}{2\sqrt{13}}\right)$ d) $\cos^{-1}\left(\frac{5}{\sqrt{13}}\right)$

7. If the line $x\cos\alpha + y\sin\alpha = p$ represents the common chord $APQB$ of the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2 (a > b)$ as shown figure, then AP is equal to



a) $\sqrt{a^2 + p^2} + \sqrt{b^2 + p^2}$ b) $\sqrt{a^2 - p^2} + \sqrt{b^2 - p^2}$ c) $\sqrt{a^2 - p^2} - \sqrt{b^2 - p^2}$ d) $\sqrt{a^2 + p^2} - \sqrt{b^2 + p^2}$

8. If in a ΔABC (whose circumcentre is at the origin), $a \le \sin A$, then for any point (x, y) inside the circumcircle of ΔABC, we have

a) $|xy| < \frac{1}{8}$ b) $|xy| > \frac{1}{8}$ c) $\frac{1}{8} < xy < \frac{1}{2}$ d) $\frac{1}{8} > xy > \frac{1}{2}$

9. $\underset{x\to0}{Lt}\left(\frac{1+cx}{1-cx}\right)^{\frac{1}{x}}=4,\ x_1=\underset{x\to0}{Lt}\left(\frac{1+2cx}{1-2cx}\right)^{\frac{1}{x}}$ and $r=\underset{x\to0}{Lt}\left(\frac{2+cx}{2-cx}\right)^{\frac{1}{x}}$. Then equation of the circle which touches -axis at $(x_1, 0)$ and radius r is

a) $(x-4)^2+(y-2)^2=4$ b) $(x-8)^2+(y-2)^2=4$

c) $(x-6)^2+(y-2)^2=4$ d) $(x-12)^2+(y-2)^2=4$

10. The minimum distance between the circle $x^2+y^2=9$ and the curve $2x^2+10y^2+6xy=1$ is

a) $2\sqrt{2}$ b) 2 c) $3-\sqrt{2}$ d) $3-\frac{1}{\sqrt{11}}$

11. If $x^2+y^2=16$, $x^2+y^2=36$ are two circles and P and Q move respectively on these circles such that $PQ=4$ then the locus of midpoint of PQ is a circle of radius ....

a) $\sqrt{20}$ b) $\sqrt{22}$ c) $\sqrt{30}$ d) $\sqrt{32}$

12. Let PQ and RS be tangents at the extermities of the diameter PR of a circle of radius 'r'. If PS and RQ intersect at a point 'X' on the circumference of the circle then $2r$ equals............

a) $\sqrt{PQ.RS}$ b) $\frac{PQ+RS}{2}$ c) $\frac{2PQ+RS}{PQ+RS}$ d) $\frac{PQ^2+RS^2}{2}$

13. If the point $(k+1, k)$ lies inside the region bounded by the curve $x=\sqrt{25-y^2}$ and y-axis, then k belongs to the interval

a) $(-1, 3)$ b) $(-4, 3)$ c) $(-\alpha,-4)\cup(3,\alpha)$ d) $(-4, -3)$

14. $ABCD$ is a rectangle A circle passing through vertex C touches the sides AB and CD at M and N respectively. If the distance of the line MN from the vertex C is P units then the area of rectangle $ABCD$ is

a) P b) P^2 c) $2P$ d) $2P^2$

15. Consider a family of circles passing through the intersection point of the lines $\sqrt{3}(y-1)=x-1$ and $y-1=\sqrt{3}(x-1)$ and having its centre on the acute angle bisector of the given lines. Then the common chords of each member of the family and the circle $x^2+y^2+4x-6y+5=0$ are concurrent at

a) $\left(\frac{1}{2},\frac{1}{2}\right)$ b) $\left(\frac{1}{2},\frac{3}{2}\right)$ c) $\left(\frac{3}{2},\frac{3}{2}\right)$ d) $\left(-\frac{1}{2},-\frac{1}{2}\right)$

16. The number of rational points (a point (a, b) is called rational, if a and b both are rational numbers) on the circumference of a circle having centre (π, c) is ....

a) at most one b) at least two c) exactly two d) infinite

17. The number of points inside or on the circle $x^2+y^2=4$ satisfying $\tan^4x+\cot^4x+1=3\sin^2y$ is :

a) 1 b) 2 c) 4 d) 0

More than one correct answer type questions

18. A tangent drawn from the point (4,0) to the circle $x^2+y^2=8$ touches it at a point A in the first quadrant the coordinates of another point B on the circles such that $AB=4$ are

a) $(2, -2)$ b) $(-2, 2)$ c) $(-2\sqrt{2},0)$ d) $(0,-2\sqrt{2})$

19. If $4a^2 - 5b^2 + 6a + 1 = 0$ and the line $ax + by + 1 = 0$ touches fisec circle, then :

a) centre of circle is at (3, 0)
b) the radius of circle is $\sqrt{5}$
c) the radius of circle is $\sqrt{3}$
d) the circle passes through (1, 1)

20. AC is diameter of circle. AB is a tangent. BC meets the circle again at D. $AC = 1$, $AB = a$, $CD = b$, then:

a) $ab > 1$
b) $ab < 1$
c) $\frac{b}{a} > \frac{1}{a^2 + \frac{1}{2}}$
d) $\frac{b}{a} < \frac{1}{a^2 + \frac{1}{2}}$

21. Tangent are drawn to the circle $x^2 + y^2 = 50$ from a point 'P' lying on the x-axis. These tangents meet the y-axis at points 'P_1' and 'P_2'. Possible coordinates of 'P' so that area of triangle PP_1P_2 is minimum, is

a) (9, 0)
b) $(10\sqrt{2}, 0)$
c) (–10, 0)
d) $(-10\sqrt{2}, 0)$

22. If cirlce having centre at (α, β) radius r, completely lies with in two lines $x + y = 2$ and $x + y + 2 = 0$ then min $(|\alpha + \beta + 2|, |\alpha + \beta - 2|)$ is

a) greater than $\sqrt{2}r$
b) less than $\sqrt{2}r$
c) greater than $2r$
d) less than $2r$

23. A circle of radius 2 has center at (2, 0) and another circle of radius 1 has center at (5, 0). A line is tangent to the two circles at points in the first quadrant. The equation of the tangent is

a) $\frac{x}{9} + \frac{y}{8} = 1$
b) $\frac{x}{8} + \frac{y}{2\sqrt{2}} = 1$
c) $\frac{x}{8} + \frac{y}{3} = 1$
d) $x + 3y = 9$

24. If $\alpha, \beta, \gamma, \delta$ be four angles of a cyclic quadrilateral taken in clock wise direction, then the value of $(2 + \sum \cos\alpha \cos\beta)$ will be:

a) $\sin^2\alpha + \sin^2\beta$
b) $\cos^2\gamma + \cos^2\delta$
c) $\sin^2\alpha + \sin^2\delta$
d) $\cos^2\beta + \cos^2\gamma$

Linked comprehension type questions

Passage - I :

If S_1 and S_2 be two concentric circles, the radius of S_2 being double that of S_1. From a point P on S_2, tangents PA and PB are drawn to S_1.

25. The centroid of the ΔPAB lies

a) in the smaller circle S_1
b) in between smaller and bigger circles S_1 and S_2
c) at the centre of both the circles S_1 and S_2
d) any where at the diameter of both the circles other than centre

26. If ΔPQR is inscribed in the circle $S_1 = 0$, where $r = 5$. If Q and R have co-ordinates (3, 4) and (–4, 3) respectively, then the angle $\angle QPR$ is equal to

a) $\frac{\pi}{6}$
b) $\frac{\pi}{4}$
c) $\frac{\pi}{3}$
d) $\frac{\pi}{2}$

27. Centre of the circle obtained from the information of above question is

a) (0, 0)
b) (4, 0)
c) $\left(-\frac{4}{3}, 1\right)$
d) (1, 2)

Passage - II :

Let $ABCD$ be a square of side length 2 units. C_2 is the cirlce through vertices A,B,C,D and C_1 is the circles touching all the sides fo the square $ABCE$. l is a line through A.

28. If P is a point on C_1 and Q is another point on C_2 then $\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$ is equal to

a) 0.75 b) 1.25 c) 1 d) 0.5

29. Touches circles the line L and the circle C_1 externally such that both the circles are on the same side fo the line, then the locus of centre of the circle is

a) ellispe b) hyperbola c) parabola d) pair of straight line

30. A line M through A is drawn parallel to BD. Points S moves such that its distances from the line BD and the vertex A are equal. If locus fo S cuts M at T_2 and T_3 at T_1, then area of triangle $T_1T_2T_3$ is

a) $\frac{1}{2}$ *sq.units* b) $\frac{2}{3}$ *sq.units* c) 1 *sq. units* d) 2 *sq. units*

Passage-III :

Consider a family of circles passing through the points (3, 7) and (6, 5) then

31. Number of circles which belong to the family and also touching X-axis are

a) 0 b) 2 c) 1 d) α

32. If each circle in the family cuts the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ then all the common chords pass through the fixed point

a) (1, 23) b) $\left(2,\frac{23}{2}\right)$ c) $\left(-3,\frac{3}{2}\right)$ d) none

33. If the circle which belongs to the given family cuts the circle $x^2 + y^2 = 26$ orthogonally then the centre of that circle is

a) $\left(\frac{1}{2},\frac{3}{2}\right)$ b) $\left(\frac{9}{2},\frac{7}{2}\right)$ c) $\left(\frac{7}{2},\frac{9}{2}\right)$ d) $\left(3,-\frac{7}{9}\right)$

Passage -IV :

Consider 3 circles $S_1 : x^2 + y^2 + 2x - 3 = 0$; $S_2 : x^2 + y^2 - 1 = 0$; $S_3 : x^2 + y^2 + 2y - 3 = 0$

34. The radius of the circle which bisects the circumferences of the circles $S_1 = 0$; $S_2 = 0$; $S_3 = 0$ is

a) 2 b) $2\sqrt{2}$ c) 3 d) $\sqrt{10}$

35. If the circele $S = 0$ is orthogonal to $S_1 = 0$; $S_2 = 0$ and $S_3 = 0$ and has centre at (a, b) and radius equals to 'r' then the value of $a + b + r$ equals

a) 0 b) 1 c) 2 d) 3

36. The radius of the circle touching $S_1 = 0$ and $S_2 = 0$ at (1, 0) and passing through (3, 2) is

a) 1 b) $\sqrt{12}$ c) 2 d) $2\sqrt{2}$

Passage -V :

The circle $x^2 + y^2 = 8$ is cut by a series of circles all of which pass through the points (4, 0) and (0, 6)

37. All the chords common to the given circle and a member of the famialy of the circles pass through a fixed point sum of whose coordinates is

a) -8 b) $\frac{-16}{5}$ c) 8 d) $\frac{16}{5}$

38. Equation of common chord of the given circle and a member of the family of the circles which passes through the point (12, 4) is $ax + by = 8$ then $a + b =$

a) 0 b) 5 c) -5 d) 8

39. Radius of the member of the family whose common chord with the circle $x^2 + y^2 = 8$ passes through (17, 4) is

a) $\sqrt{27}$ b) 5 c) $\sqrt{26}$ d) $\sqrt{24}$

Matrix matching type questions

40. Consider the circles C_1, of radius a and C_2 of radius b, $b > a$, both lying in the first quadrant and touching the coordinate axes

COLUMN - I	COLUMN - II
A) C_1 and C_2 touch each other	p) $\frac{b}{a}=2+\sqrt{2}$
B) C_1 and C_2 are orthogonal	q) $\frac{b}{a}=3$
C) C_1 and C_2 intersect so that the common chord is longest	r) $\frac{b}{a}=2+\sqrt{3}$
D) C_2 passes through the centre of C_1	s) $\frac{b}{a}=3+2\sqrt{2}$

41. $A(-2, 0)$ and $B(2, 0)$ are two fixed points and P is a point such that $PA - PB = 2$. Let S be the circle $x^2 + y^2 = r^2$ then match the following

COLUMN - I	COLUMN - II
a) If $r = 2$ then the number of points P satisfying $PA - PB = 2$ and lying on $x^2 + y^2 = r^2$ is	p) 2
b) If $r = 1$ then the number of points P satisfying $PA - PB = 2$ and lying on $x^2 + y^2 = r^2$ is	q) 4
c) For $r = 2$ the number of common tangents is	r) 0
d) For $r^2 = \frac{1}{2}$ the number of common tangents	s) 1

a) A - q; B - r; c - s; d - p b) A - r; B - s; c - p; d - q

c) A - s; B - p; c - q; d - p d) A - p; B - s; c - r; d - p

Integer answer type questions

42. If the point $(\sec\alpha, \operatorname{cosec}\alpha)$ moves in the plane of circle $x^2+ y^2 = 3$ and the minimum distance of this point from the circle can be expressed as $a-\sqrt{b}$ where $a,b \in N$. Then value of $a+b$

43. The chord of contact $(a\cos\theta, a\sin\theta)$ of w.r.t $x^2 + y^2 = b^2$ touches $x^2 + y^2 = c^2$. a,b,c are the roots of $x^3 - 14x^2 + 56x + d = 0$ then a is

44. The line $(x + 3)\cos\theta + (y - 4)\sin\theta = 1$ touches a fixed circle $\forall\theta$ whose radius is

45. The tangents to a circle at P and Q intersect at T. AB is a diameter parallel to PQ. The linesjoining P and Q with one of A and B intersectthe diameter $\perp^{lar}$ to AB at R and S. Then RT/ST is

46. If the circle C_1 touches x-axis and the line $y = x\tan\theta, \theta \in \left(0, \frac{\pi}{2}\right)$ in first quadrant and cirlce C_2 touches the ine $y = x\tan\theta$, y-axis and circle C_1 in such a way that ratio of radius of C_1 to radius of C_2 is 2 : 1, then value of $\tan\frac{\theta}{2} = \frac{\sqrt{a} - b}{c}$ where a, b, c are relatively prime natural numbers thus $\frac{(a+b+c)}{11}$ is

47. If (α, β) is a point on the chord PQ of the circle $x^2 + y^2 = 25$, where the coordinates of P and Q are $(3,-4)$ and $(4, 3)$ respectively and $k_1 \le \alpha \le k_2$ and $k_3 \le \beta \le k_4$ then $\sum k_i$ is

KEY SHEET (ADDITIONAL EXERCISE)

LEVEL-I (MAIN)

1) **3** 2) **4** 3) **4** 4) **1** 5) **2** 6) **1** 7) **2** 8) **2** 9) **2** 10) **3**
11) **1** 12) **3** 13) **1** 14) **2** 15) **1** 16) **1**

LEVEL-II

LECTURE SHET (ADVANCED)

1) **c** 2) **b** 3) **b** 4) **d** 5) **d** 6) **d** 7) **c** 8) **b** 9) **a** 10) **d**
11) **d** 12) **d** 13) **b** 14) **a** 15) **b** 16) **b** 17) **a** 18) **a** 19) **b** 20) **c**
21) **abd** 22) **acd** 23) **b** 24) **bd** 25) **cd** 26) **bd** 27) **bc** 28) **abd** 29) **b** 30) **b**
31) **b** 32) **a** 33) **c** 34) **d** 35) **c** 36) **c** 37) **c** 38) **d** 39) **c**
40) **A–r, B–pq, C–qr, D–ps** 41) **A-s; B-r; C-q; D-p** 42) **1** 43) **2** 44) **2** 45) **8**
46) **2** 47) **1**

PRACTICE SHET (ADVANCED)

1) **b** 2) **a** 3) **c** 4) **b** 5) **c** 6) **a** 7) **c** 8) **a** 9) **c** 10) **b**
11) **b** 12) **a** 13) **a** 14) **b** 15) **b** 16) **a** 17) **d** 18) **ab** 19) **abd** 20) **bc**
21) **ac** 22) **a** 23) **b** 24) **ac** 25) **a** 26) **b** 27) **a** 28) **a** 29) **c** 30) **c**
31) **b** 32) **d** 33) **c** 34) **c** 35) **d** 36) **c** 37) **d** 38) **a** 39) **c**
40) **A-s; B-r; C-q; D-p** 41) **d** 42) **5** 43) **8** 44) **1** 45) **1** 46) **2** 47) **6**