6. MATHEMATICAL REASONING

SYNOPSIS

I. Statement (or) Propositions :

A sentence is called a statement if it is either **True(T)** or **False(F)** but **not both**.

The letters p,q,r .... etc are used to denote statements.

True Statements : **i)** Two plus Two is equal to Four **ii)** New Delhi is the capital of India

iii) The sum of all interior angles of a triangle is 180^0

False Statements :

i) All prime numbers are odd numbers

ii) Every one in India speak's Hindi

Not a Statements :

i) The square of the integer is an even integer.

ii) Mathematics is difficult

iii) The sum of two integers is greater than zero. [(i), (ii), (iii) statements are always not true]

Negation: (~)Let p be a statement then **'not p'** is called the negation of p. Negation of p is denoted by($\sim p$)

Negation Truth Table $\Longrightarrow$

P	~P
T	F
F	T

Ex (i) p **:** Evey one in india speak's English

$\sim p$ **:** Not evey one in india speak's English

Conjunction $(\wedge)$**:** Let p and q be two statements. The conjunction of p and q is denoted by $p \wedge q$

$p \wedge q$ **is the statement that is true when both p and q are true and is false otherwise**

Conjunction Truth Table $\Longrightarrow$

p	T	T	F	F
q	T	F	T	F
$p \wedge q$	T	F	F	F

Ex (i) p **:** 42 is divisible by 2

q **:** 42 is divisible by 7

$p \wedge q$ **:** 42 is divisible by 2 and 7

Ex (ii) p **:** Roses are Red

q **:** Lilli's are White $p \wedge q$ **:** Roses are red and Lilli's are White

Note: Rama **and** Lakshmana are bothers where **and** – not conjunction

Disjunction($\vee$ **):** Let p and q be two statements the disjunction of p and q is denoted by $p \vee q$.

' $p \vee q$ **' is thestatement that is false when p and q are both false and true otherwise**

Disjunction Truth Table $\Longrightarrow$

p	T	T	F	F
q	T	F	T	F
$p \vee q$	T	T	T	F

Ex (i) p **:** 125 is a multiple of 5

q **:** 125 is a multiple of 7

$p \vee q$ **:** 125 is a multiple of 5 or 7

Ex (ii) **p :** There is a something wrong with the bulb

q : There is a something wrong with the wiring

$p \vee q$ **:** There is a something wrong with the bulb or with the wiring.

Implication($\rightarrow (or) \Rightarrow$): Let p and q be two statements the implication of p and q is denoted by $p \rightarrow q$

'$p \rightarrow q$' is the statement that is the false when p is true and q is fale and true otherwise.

Impliction Truth Table $\Longrightarrow$

p	T	T	F	F
q	T	F	T	F
$p \rightarrow q$	T	F	T	T

Ex (i) p **:** An integer is a multiple of 9

q **:** An integer is a multiple of 3

$p \rightarrow q$ **:** An integer is a multiple of 9 then it is a multiple of 3

Ex (ii) p **:** Triangle ABC is equilateral

q **:** Triangle ABC is Isosceles

$p \rightarrow q$ **:** If a triangle ABC is equilateral than it is Isosceles.

Converse, Inverse: **i)** The statement $q \rightarrow p$ is called the converse of $p \rightarrow q$.

ii) The statement $\sim p \rightarrow \sim q$ is called the inverse of $p \rightarrow q$.

Ex (i) p **:** x is an even integer

q **:** x^2 is divisible by 4

$p \rightarrow q$ **:** If x is even integer then x^2 is divisible by 4.

$q \rightarrow p$ **:** x is an integer and x^2 is divisible by 4 then x is even.

$\sim p \rightarrow \sim q$ **:** If x is not even integer then x^2 is not divisible by 4.

Contrapositive: The statement $\sim q \rightarrow \sim p$ is called the contrapositive of $p \rightarrow q$.

Ex (i) $p \rightarrow q$ **:** If x, y are integers such that x and y are odd then xy is even.

$\sim q \rightarrow \sim p$ **:** If x, y are integers such that xy is even than x or y is even.

Conditional, Converse Inverse, Contropositive Truth Table $\Longrightarrow$

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

BiConditional($\leftrightarrow$ (or) $\Leftrightarrow$): Let p and q be two statements the biconditional of p and q is denoted by $p \leftrightarrow q$.

'$p \leftrightarrow q$' is the statement that is true when p and q have the same truth values and false otherwise

BiConditional Truth Table $\Longrightarrow$

p	T	T	F	F
q	T	F	T	F
$p \leftrightarrow q$	T	F	F	T

Ex (i) An integer n is odd $\leftrightarrow$ n^2 is odd.

Tautology, Contradiction:

i) A compound statement that is **always true** is called **tautology**.

ii) A compound statement that is **always false** is called a **contradiction**.

Ex : Let p be a statement

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

In this truth table

i) $p \vee \sim p$ is always true $\Rightarrow$ it is tautology

ii) $p \wedge \sim p$ is always false $\Rightarrow$ it is contridiction

Logical Equivalence: The statements p and q are called logically equivalent if they have the same entries in the last column of the truth tables must be same.

Ex (i): $\sim (p \vee q)$ and $(\sim p) \wedge (\sim q)$ are logically equivalent. **Ex (ii):** $p \rightarrow q$ and $\sim p \vee q$ are logically equivalent.

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

p	q	$\sim p$	$(\sim p) \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Algebra of statements :

1. **Idempotent laws :** For any statement p, i) $p \vee p \equiv p$ ii) $p \wedge p \equiv p$
2. **Commutative laws :** For any two statements p and q, i) $p \vee q \equiv q \vee p$ ii) $p \wedge q \equiv q \wedge p$
3. **Associative laws :** For any three statements p, q, r,
   i) $(p \vee q) \vee r \equiv p \vee (q \vee r)$ ii) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
4. **Distributive laws :** For any three statements p, q, r,
   i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
5. **DeMorgan's laws :** If p and q are two statements, then
   i) $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$ ii) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

6. **Identity laws :** If t and c denote a tautology and a contradiction respectively, then for any statement p,
 i) $p \wedge t \equiv p$ ii) $p \vee c \equiv p$ iii) $p \vee t \equiv t$ iv) $p \wedge c \equiv c$

7. **Complement laws :** For any statement p,
 i) $p \vee (\sim p) \equiv t$ ii) $p \wedge (\sim p) \equiv c$
 iii) $\sim t \equiv c$ iv) $!c \equiv t$ iv) $\sim(\sim p) \equiv p$

8. **Law of contrapositive :** For any two statements p and q, i) $p \Rightarrow q \equiv \sim q \Rightarrow \sim p$

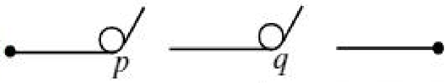
9. **Involution Laws :** For any statements p, i) $\sim(\sim p) \equiv p$

Duality : Two statements S_1 and S_2 are said to be duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

Applications of Logic in switching circuits *(Application of truth tables to switching networks):*

Let $p, q, ...$ denote electrical switches. Two switches say p and q, can be connected by wire in a series or parallel combination as follows.

1. **Series combination ($p \wedge q$):**

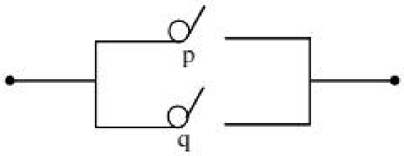


Series Network $\Longrightarrow$

p	T	T	F	F
q	T	F	T	F
$p \wedge q$	T	F	F	F

Note : Sereis network is identical with the table of conjunction.

2. **Parallel Combination($p \vee q$):**



Parallel Network $\Longrightarrow$

p	T	T	F	F
q	T	F	T	F
$p \vee q$	T	F	T	F

Note : Parallel network is identical with the table of disjunction.

A switching network is an arrangement of wires and switches that can be constructed by repeated use of series and parallel combinations.

A switch allows only two possibilities (i) **it is either open**(**F**), in which case there is **no flow of current** (ii) **it is closed**(**T**), in which there is **flow of current.** $\therefore$ every switch has only two truth values T or F.

3. **Nature of Switches p and p^1:**

p and p^1 denote switches with the property that if one is **'on'**, the other is **'off'** and vice versa.

p	p^1
T	F
F	T

Note : Nature of switches is identical with the table of nagation

LECTURE SHEET

EXERCISE

1. Which of the following is a proposition
   1) Logic is an interesting subject
   2) He is very talented
   3) I am a lion
   4) A triangle is a circle and 10 is a prime number

2. Which of the following is not a proposition
   1) 3 is a prime
   2) Mathematics is interesting
   3) 5 is an even integer
   4) $\sqrt{2}$ is irrational

3. Let p : Mathematics is interesting and let q:Mathematics is difficult, then the symbol $p \wedge q$ means
   1) Mathematics is interesting implies that Mathematics is difficult
   2) Mathematics is interesting implies and is implied by Mathematics is difficult
   3) Mathematics is interesting and Mathematics is difficult
   4) Mathematics is interesting or Mathematics is difficult

4. Let p and q be two propositions given by p : The sky is blue, q : milk is white. Then, $p \wedge q$ is
   1) The sky is blue or milk is white
   2) The sky is blue and milk is white
   3) The sky is white and milk is blue
   4) If the sky is blue, then milk is white

5. p : It is hot, q : He wants water. Then, the verbal meaning of $p \to q$ is
   1) It is hot or he wants water
   2) It is hot and he wants water
   3) If it is hot, then he wants water
   4) If and only if it is hot, he wants water

6. p : I take medici ne, q : I can sleep. Then, the compound statement $\sim p \to \sim q$ means
   1) If I do not take medicine, then I cannot sleep
   2) If I do not take medicine, then I can sleep
   3) I take medicine iff I can sleep
   4) I take medicine if I can sleep

7. The contrapositive of the statement "if $2^2 = 5$, then I get first class" is
   1) If I do not get a first class, then $2^2 = 5$
   2) If I do not get a first class, then $2^2 \neq 5$
   3) If I get a first class, then $2^2 = 5$
   4) If I get a first class, then $2^3 = 5$

8. The negative of the proposition : "If a number is divisible by 15, then it is divisible by 5 or 3".
   1) If a number is divisibley by 15, then it is not divisible by 5 and 3
   2) A number is divisible by 15 and it is not divisible by 5 and 3
   3) A numberis divisible by 15 and it is not divisible by 5 or 3
   4) A number is not divisible by 15 or it is not divisible by 5 and 3

9. "If the pressure increases, then the volume decreases". The negation of this propositions is
   1) If the pressure does not increase the volume does not decrease
   2) If the volume increases, the pressure decreases
   3) Pressure increases and volume does not decrease.
   4) If the volume decreases, then the pressure increases

10. The negation of the proposition “If 2 is prime, then 3 is odd” is
    1) If 2 is not prime then 3 is not odd
    2) 2 is prime and 3 is not odd
    3) 2 is not prime and 3 is odd
    4) If 2 is not prime then 3 is odd

11. p : I take only bread and butter in breakfast. q : I do not take any thing in breakfast. Then the compound proposition "I take only bread and butter in breakfast or I do not take any thing" is represented by

1) $p \wedge q$ 2) $p \vee q$ 3) $p \rightarrow q$ 4) $p \leftrightarrow q$

12. p : To become an airforce officer one should be graduate. q : To become an airforce officer one should have good health. The compound proposition "To become an airforce officer one should be a graduate and should have good health" is represented by

1) $p \vee q$ 2) $p \rightarrow q$ 3) $p \wedge q$ 4) $p \leftrightarrow q$

13. p : It rains, q : The street gets flooded. The proposition "If it does not rain, then the street does not get flooded" is represented by

1) $p \rightarrow \sim q$ 2) $\sim p \rightarrow q$ 3) $p \leftrightarrow q$ 4) $\sim p \rightarrow \sim q$

14. Given that water freezes below zero degree celsius. p : Water froze this morning, q : This morning temperature was below 0^0C. Which of the following is correct ?

1) p and q are logically equivalent 2) p is the inverse of q

3) p is the converse of p 4) p is the contrapositive of q

15. The contrapositive of $2x+3=9 \Rightarrow x \neq 4$ is

1) $x=4 \Rightarrow 2x+3 \neq 9$ 2) $x=4 \Rightarrow 2x+3=9$ 3) $x \neq 4 \Rightarrow 2x+3 \neq 9$ 4) $x \neq 4 \Rightarrow 2x+3=9$

16. If p and q are two simple propositions, then $p \rightarrow q$ is false when

1) p is true and q is true 2) p is false and q is true

3) p is true and q is false 4) both p and q are false

17. For any three propositions p, q and r, the proposition $(p \wedge q) \wedge (q \wedge r)$ is true when

1) p, q, r are all false 2) p, q, r are all true

3) p, q are true and r is false 4) p is true and q and r are false

18. Which of the following is true for the propositions p and q ?

1) $p \wedge q$ is true when at least one of p and q is true 2) $p \rightarrow q$ is true when p is true and q is false

3) $p \leftrightarrow q$ is true only when both p and q are true 4) $\sim(p \vee q)$ is true only when both p and q are false

19. If $p \rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively

1) T, F, F 2) F, F, F 3) F, T, T 4) T, T, F

20. The compound statement $p \rightarrow (\sim p \vee q)$ is false, then the truth values of p and q are respectively

1) T, T 2) T, F 3) F, T 4) F, F

21. The logically equivalent proposition of $p \leftrightarrow q$ is

1) $(p \rightarrow q) \wedge (q \rightarrow p)$ 2) $(p \rightarrow q) \vee (q \rightarrow p)$ 3) $(p \wedge q) \rightarrow (p \vee q)$ 4) $(p \wedge q) \vee (p \vee q)$

22. Let p and q be two propositions. Then the contrapositive of the implication $p \rightarrow q$ is

1) $\sim q \rightarrow \sim p$ 2) $\sim p \rightarrow \sim q$ 3) $q \rightarrow p$ 4) $p \leftrightarrow q$

23. $p \wedge (q \wedge r)$ is logically equivalent to

1) $p \vee (q \wedge r)$ 2) $(p \wedge q) \wedge r$ 3) $(p \vee q) \vee r$ 4) $p \rightarrow (q \wedge r)$

24. Which of the following is logically equivalent to $p \wedge q$?

1) $p \rightarrow \sim q$ 2) $\sim p \vee \sim q$ 3) $\sim(p \rightarrow \sim q)$ 4) $\sim(\sim p \wedge \sim q)$

25. $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to

1) $\sim p$ 2) p 3) q 4) $\sim q$

26. $\sim p \vee \sim q$ is logically equivalent to

1) $\sim p \to \sim q$ 2) $p \wedge q$ 3) $p \to \sim q$ 4) $p \leftrightarrow q$

27. The negation of the compound proposition $p \leftrightarrow \sim q$ is logically equivalent to

1) $p \leftrightarrow q$ 2) $(p \to q) \wedge (\sim q \to p)$

3) $(\sim q \to p) \vee (\sim p \to q)$ 4) $(p \to q) \wedge (\sim q \to p)$

28. Negation of the statement $p \to (q \wedge r)$ is

1) $\sim p \to \sim(q \vee r)$ 2) $\sim p \to \sim(q \wedge r)$ 3) $(q \wedge r) \to p$ 4) $p \wedge (\sim q \vee \sim r)$

29. Negation of the statement $(p \wedge r) \to (r \vee q)$ is

1) $(p \wedge r) \wedge (\sim r \wedge \sim q)$ 2) $\sim(p \wedge r) \to \sim(r \vee q)$ 3) $\sim(p \vee r) \to \sim(r \vee q)$ 4) $(p \wedge r) \vee (r \vee q)$

30. The contrapositive of $(p \vee q) \to r$ is

1) $p \to (q \vee r)$ 2) $r \to (p \vee q)$ 3) $\sim r \to \sim(p \vee q)$ 4) $\sim r \to (\sim p \vee \sim q)$

31. The contrapositive of $(\sim p \wedge q) \to (q \wedge \sim r)$ is

1) $(p \vee \sim q) \to (\sim q \vee p)$ 2) $(p \vee \sim q) \to (\sim q \vee p)$

3) $(\sim q \vee r) \to (p \vee \sim q)$ 4) $(\sim p \vee r) \to (\sim p \wedge \sim r)$

32. If p and q are two propositions, then $\sim(p \leftrightarrow q)$ is

1) $\sim p \wedge \sim q$ 2) $\sim p \vee \sim q$ 3) $(p \wedge \sim q) \vee (\sim p \wedge q)$ 4) $\sim p \to \sim q$

33. The negation of the propositon $q \vee \sim(p \wedge r)$ is

1) $\sim q \vee (p \wedge r)$ 2) $\sim q \wedge (p \wedge r)$ 3) $\sim p \vee \sim q \vee \sim r$ 4) $q \wedge (\sim p \vee \sim r)$

34. Which of the following is logically equivalent to $\sim(\sim p \to q)$?

1) $p \wedge q$ 2) $p \wedge \sim q$ 3) $\sim p \wedge q$ 4) $\sim p \wedge \sim q$

35. Which of the following is logically equivalent to $\sim(\sim q \to p)$?

1) $q \wedge p$ 2) $q \wedge \sim p$ 3) $\sim q \wedge p$ 4) $\sim q \wedge \sim p$

36. Which of the following is logically equivalent to $\sim(p \leftrightarrow q)$?

1) $(p \wedge \sim q) \wedge (q \wedge \sim p)$ 2) $p \vee q$ 3) $(p \wedge \sim q) \vee (q \wedge \sim p)$ 4) $p \wedge q$

37. The negation of the compound proposition $p \vee (\sim p \vee q)$ is

1) $(p \wedge \sim q) \wedge \sim p$ 2) $(p \wedge \sim q) \vee \sim p$ 3) $(p \wedge \sim q) \vee \sim p$ 4) $p \wedge (p \wedge \sim q)$

38. The inverse of the proposition $(p \wedge \sim q) \to r$ is

1) $\sim r \to \sim p \vee q$ 2) $\sim p \vee q \to \sim r$ 3) $r \to p \wedge \sim q$ 4) $\sim p \to (q \wedge r)$

39. The statement $p \to (q \to p)$ is equivalent is

1) $p \to (p \vee q)$ 2) $p \to (p \wedge q)$ 3) $p \to (p \leftrightarrow q)$ 4) $p \to (p \to q)$

40. The statement $\sim p \vee q$ is equivalent is

1) $p \to q$ 2) $\sim p \to q$ 3) $\sim p \to \sim q$ 4) $p \to \sim q$

41. The false statement in the following is

1) $p \wedge (\sim p)$ is a contradiction. 2) $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a contradiction

3) $\sim(\sim p) \leftrightarrow p$ is a tautology 4) $p \vee (\sim p)$ is a tautology

42. The proposition $(p \to \sim p) \wedge (\sim p \to p)$ is

1) a tautology 2) a contradiction

3) neither a tautology nor a contradiction 4) a tautology and a contradiction

43. Which one of the following is not a contradiction

1) $[\sim p \wedge (p \vee \sim q)] \wedge q$ 2) $(\sim p \wedge q) \wedge p$

3) $[(p \to q) \to p] \wedge \sim p$ 4) $(\sim q \to \sim p) \leftrightarrow (p \to q)$

44. Which of the following statements is a tautology

1) $(\sim p \vee q) \sim (p \vee \sim q)$ 2) $(\sim p \vee \sim q) \to p \vee q$ 3) $(p \vee \sim q) \wedge (p \vee q)$ 4) $(\sim p \vee \sim q) \vee (p \vee q)$

45. Which of the following is wrong ?

1) $p \to q$ is logically equivalent to $\sim p \vee q$

2) If the truth values of p, q, r are T, F, T respectively, then the truth value of $(p \vee q) \wedge (q \vee r)$ is T

3) $\sim(p \vee q \vee r) \cong \sim p \wedge \sim q \wedge \sim r$

4) The truth value of $p \wedge \sim(p \vee q)$ is always T

46. Which of the following is false ?

1) $p \vee \sim p$ is a tautology 2) $\sim(\sim p) \leftrightarrow p$ is tautology

3) $(p \wedge (p \to q)) \to p$ is a contradiciton 4) $p \wedge \sim p$ is a contradiciton

47. The proposition $(p \to \sim p) \wedge (\sim p \to p)$ is a

1) tautology 2) contradiction

3) neither a tautology nor a contradiction 4) tautology and contradiction

48. The inverse of the proposition $(p \wedge \sim q) \to r$ is

1) $\sim r \to \sim p \vee q$ 2) $\sim p \vee q \to \sim r$ 3) $r \to p \wedge \sim q$ 4) $\sim q \vee r \to p$

49. If p : 4 is an even prime number q : 6 is a divisor of 12 and r : the HCF of 4 and 6 is 2, then which one of the following is true ?

1) $p \wedge q$ 2) $(p \vee q) \wedge \sim r$ 3) $\sim(q \wedge r) \vee p$ 4) $\sim p \vee (q \wedge r)$

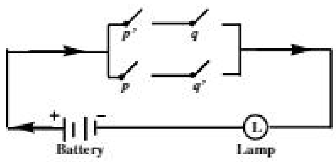
50. Let S be a non–empty subset of R. P : There is a rational number $x \in S$ such that x > 0. Which of the following statement is the negation of the statement P?

1) There is a rationa number $x \in S$ such that $x \leq 0$

2) There is no rationa number $x \in S$ such that $x \leq 0$

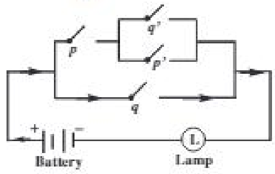
3) Every rational number $x \in S$ satisfies $x \leq 0$ 4) $x \in S$ and $x \leq 0 \Rightarrow$ x is not rational

51. The following circuit when expressed in the symbolic form of logic is



1) $(\sim p \wedge q) \vee (p \wedge \sim q)$ 2) $(\sim p \vee q) \vee (p \vee \sim q)$ 3) $(\sim p \wedge p) \wedge (\sim q \wedge p)$ 4) $(\sim p \wedge \sim q) \wedge (q \wedge p)$

52. The symbolic form of logic of the circuit given below is



1) $[(p\wedge q')\vee p']\wedge q$ 2) $[p\vee(q'\wedge p')]\vee q$ 3) $[(p\wedge p')\vee q']\wedge q$ 4) $[p\wedge(q'\vee p')]\vee q$

PRACTICE SHEET

EXERCISE

1. Which of the following is a proposition
   1) I am an advocate
   2) A half open door is half closed
   3) Delhi is on the Jupiter
   4) $x^2 + y^2 = 100$

2. Let p and q be two propositions given by p : I play cricket during the holidays, q : I just sleep throughout the day, then the compound statement $p\vee q$ is
   1) If I play cricket during the holidays, I just sleep throughtout the day
   2) I play cricket during the holidays and just sleep throughtout the day
   3) I just sleep throughout the day if and only if I play cricket during the holidays
   4) I play cricket during the holidays or I just sleep throughout the day

3. The negation of the proposition "if a quadri-lateral is a square, then it is a rhombus" is
   1) if a quadrilateral is not a square, then it is a rhombus
   2) if a quadrilateral is a square, then it is not a rhombus
   3) a quadrilateral is a square and it is not a rhombus
   4) a quadrilateral is not a square and it is a rhombus

4. If $x = 5$ and $y = -2$, then $x - 2y = 9$. The contrapositive of this proposition is
   1) If $x - 2y \neq 9$, then $x \neq 5$ or $y\neq-2$
   2) If $x - 2y = 9$, $x \neq 5$ and $y \neq -2$
   3) $x - 2y = 9$ if and only if $x = 5$ and $y = -2$
   4) $x - 2y = 9$ if and only if $x = 0$ and $y = 9$

5. "The diagonals of a rhombus are perpendicular". The contrapositive of the above statement is
   1) If the figure is not a rhombus, then its diagonals are not perpendicular
   2) If the diagonals are perpendicular, then the figure is a rhombus
   3) If the diagonals are not perpendicular, then the figure is a rhombus
   4) If the diagonals are not perpendicular, then figure is not a rhombus.

6. "If we control population growth, then we prosper". Negative of this proposition is
   1) If we do not control population growth, we prosper
   2) If we control population, we do not prosper
   3) we control population and we do not prosper
   4) we do not control population but we prosper

7. p : A parallelogram is a rhombus. q : The diagonal are at right angle. The compound proposiiton "A parallelogram is a rhombus iff its diagonals are at right angle" is represented by

1) $p \vee q$ 2) $p \wedge q$ 3) $p \to q$ 4) $p \leftrightarrow q$

8. Consider the propositions : p : I have the raincoat. q : I can walk in the rain. The propositions "If I have the raincoat, then I can walk in the rain" is represented by

1) $p \to q$ 2) $p \vee q$ 3) $p \wedge q$ 4) $p \leftrightarrow q$

9. Cosider the statements : p : I shall pass, q : I study. The symbolic representation of the proposition "I shall pass iff I study" is

1) $p \to q$ 2) $q \to p$ 3) $p \to \sim q$ 4) $p \leftrightarrow q$

10. If p, q, r have truth values T, F, T respectively, which of the following is true?

1) $(p \to q) \wedge r$ 2) $(p \to q) \wedge \sim r$ 3) $(p \wedge q) \wedge (p \vee r)$ 4) $q \to (p \wedge r)$

11. If $p \to (q \vee r)$ is false, then the truth values of p, q, r are respectively

1) T, T, T 2) F, T, T 3) F, F, F 4) T, F, F

12. The negation of $q \vee \sim(p \wedge r)$ is

1) $\sim q \vee \sim(p \wedge r)$ 2) $\sim q \vee (p \wedge r)$ 3) $\sim q \wedge (p \wedge r)$ 4) $\sim q \wedge \sim(p \wedge r)$

13. Negation of the statement $\sim p \to (q \vee r)$ is

1) $p \to \sim (q \vee r)$ 2) $p \vee (q \wedge r)$ 3) $\sim p \wedge (\sim q \wedge \sim r)$ 4) $p \wedge (q \vee r)$

14. Logical equivalent proposition to the proposition $\sim (p \wedge q)$ is

1) $\sim p \wedge \sim q$ 2) $\sim p \vee \sim q$ 3) $\sim p \to \sim q$ 4) $\sim p \leftrightarrow \sim q$

15. The logically equivalent proposition of $p \leftrightarrow q$ is

1) $(p \wedge q) \vee (p \vee q)$ 2) $(p \to q) \wedge (q \to p)$ 3) $(p \to q) \vee (q \to p)$ 4) $(p \wedge q) \to (p \vee q)$

16. Logical equivalent proposition to the proposition $\sim(p \vee q)$ is

1) $\sim p \wedge \sim q$ 2) $\sim p \vee \sim q$ 3) $\sim p \to \sim q$ 4) $\sim p \leftrightarrow \sim q$

17. Let p and q be two propositions. Then the inverse of the implication $p \to q$ is

1) $q \to p$ 2) $\sim p \to \sim q$ 3) $q \to p$ 4) $\sim q \to \sim p$

18. $p \to q$ is logically equivalent to

1) $p \wedge \sim q$ 2) $\sim p \to \sim q$ 3) $\sim q \to \sim p$ 4) $\sim p \to q$

19. Which of the following is logically equivalent to $(p \wedge q)$?

1) $p \to \sim q$ 2) $\sim p \vee \sim q$ 3) $\sim(p \to \sim q)$ 4) $\sim(\sim p \wedge \sim q)$

20. The contrapositive of $p \to (\sim q \to \sim r)$ is

1) $(\sim q \wedge r) \to \sim p$ 2) $(q \wedge \sim r) \to \sim p$ 3) $p \to (\sim r \vee q)$ 4) $p \wedge (q \vee r)$

21. The negation of $p \wedge \sim(q \wedge r)$ is :

1) $\sim p \vee (q \wedge r)$ 2) $\sim p \vee (\sim q \vee \sim r)$ 3) $p \vee (q \wedge r)$ 4) $\sim p \wedge (q \vee r)$

22. The contra positive of $(\sim p \wedge q) \to \sim r$ is

1) $(p \wedge q) \to r$ 2) $(p \vee q) \to r$ 3) $r \to (p \vee \sim q)$ 4) $p \to (q \vee \sim r)$

23. Which of the following is logically equivalent to $(p \wedge q)$?

1) $p \to q$ 2) $\sim p \wedge \sim q$ 3) $p \wedge \sim q$ 4) $\sim(p \to \sim q)$

24. $(p \wedge \sim q) \wedge (\sim p \vee q)$ is

1) a tautology
2) a contradiction
3) both a tautology and a contradiction
4) neither a tautoloty nor a contradiction

25. Which of the following is always true ?

1) $(p \to q) \equiv (\sim q \to \sim p)$ 2) $\sim(p \vee q) \equiv (\sim p \vee \sim q)$ 3) $\sim(p \to q) \equiv (p \vee \sim q)$ 4) $\sim(p \wedge q) \equiv (\sim p \wedge \sim q)$

26. Which of the following proposition is a tautology?

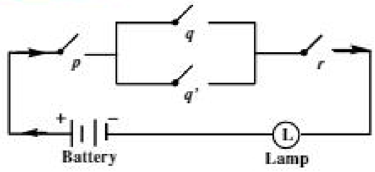
1) $\sim (p \to q) \vee (p \wedge \sim q)$ 2) $(p \to q) \to (p \wedge \sim q)$ 3) $(p \to q) \vee (p \wedge \sim q)$ 4) $(p \to q) \wedge (p \wedge \sim q)$

27. The proposition of $p \to \sim(p \wedge \sim q)$ is

1) a contradiction
2) a tautology
3) either a tautology or a contradiction
4) neither a tautology nor a contradiction

28. When does the current flow through the following circuit ?

1) p, q, r should be closed
2) p, q, r should be open
3) always
4) p, r should be closed



29. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number" and r be the statement "x is a rational number iff y is a transcendental number".

S - I : r is equivalent to either q or p. S - II : r is equivalent to $\sim(p \leftrightarrow \sim q)$

1) S - I is true, S - II is true, S - II is a correct explanation of S - I
2) S - I is true, S - II is true, S - II is not a correct explanation of S - I
3) S - I is true, S - II is false
4) S - I is false, S - II is false

30. S - I : $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$, S - II : $\sim(p \leftrightarrow \sim q)$ is a tautology.

1) S - I is true, S - II is true, S - II is a correct explanation of S - I
2) S - I is true, S - II is true, S - II is not a correct explanation of S - I
3) S - I is true, S - II is false
4) S - I is false, S - II is true

31. Let S be a non-empty subset of R. Consider the following statement :

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P ?

1) There is no rational number $x \in S$ such that $x \leq 0$
2) Every rational number $x \in S$ satisfies $x \geq 0$
3) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational
4) There is a rational number $x \in S$ such that $x \leq 0$

32. Consider the following statements

P : Suman is brilliant; Q : Suman is rich

R : Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as

1) $\sim Q \leftrightarrow \sim P \wedge R$ 2) $\sim (P \wedge \sim R) \leftrightarrow Q$ 3) $\sim P \wedge (Q \leftrightarrow \sim R)$ 4) $\sim (Q \leftrightarrow (P \wedge \sim R))$

33. The negation of the statement

"If I become a teacher, then I will open a school", is :

1) I will become a teacher and I will not open a school

2) Either I will not become a teacher or I will not open a school

3) Neither I will become a teacher nor I will open a school

4) I will not become a teacher or I will open a school

34. Consider

Statement - 1 : $(p\wedge \sim q)\wedge(\sim p\wedge q)$ is a fallacy

Statement - II : $(p\to q)\leftrightarrow(\sim q\to\sim p)$ is a tautology.

1) Statement-1 is true; Statement-II is true; Statement-II is a correct explanation for Statement-I.

2) Statement-1is true; Statement-II is true; Statement-II is a not correct explanation for Statement-I.

3) Statement-1 is true; Statement-II is false

4) Statement-1 is false; Statement-II is true

35. The statement $\sim(p\leftrightarrow\sim q)$ is :

1) equivalent to $p\leftrightarrow q$ 2) equivalent to $\sim p\leftrightarrow q$

3) a tautology 4) a fallacy

36. The negation of $\sim s\vee(\sim r\wedge s)$ is equivalent to:

1) $s\wedge\sim r$ 2) $s\wedge(r\wedge\sim s)$ 3) $s\vee(r\vee\sim s)$ 4) $s\wedge r$

KEY SHEET

LECTURE SHEET

1) 4	2) 2	3) 3	4) 2	5) 3	6) 1	7) 2	8) 2	9) 3	10) 2
11) 2	12) 3	13) 4	14) 1	15) 1	16) 3	17) 2	18) 4	19) 1	20) 2
21) 1	22) 1	23) 2	24) 3	25) 1	26) 3	27) 1	28) 4	29) 1	30) 3
31) 3	32) 3	33) 2	34) 4	35) 4	36) 3	37) 1	38) 2	39) 1	40) 1
41) 2	42) 2	43) 4	44) 4	45) 4	46) 3	47) 2	48) 2	49) 4	50) 3
51) 1	52) 4								

PRACTICE SHEET

1) 3	2) 4	3) 3	4) 1	5) 4	6) 3	7) 4	8) 1	9) 4	10) 4
11) 4	12) 3	13) 3	14) 2	15) 2	16) 1	17) 2	18) 3	19) 3	20) 1
21) 1	22) 3	23) 4	24) 2	25) 1	26) 3	27) 4	28) 4	29) 4	30) 3
31) 4	32) 4	33) 1	34) 2	35) 1	36) 4				