4. AREAS

SYNOPSIS

1. The area of the region bounded by the curve $y = f(x)$, X-axis and the lines $x = a$, $x = b$ is $\left|\int_a^b f(x)dx\right|$. (If curve does not cross x - axis between $x = a$ and $x = b$)

2. The area of the region bounded by the curve $x = f(y)$, Y-axis and the lines $y=c$, $y=d$ is $\left|\int_c^d f(y)dy\right|$. (If curve does not cross y-axis between $y=c$ and $y = d$.)

3. If $f(x) > 0 x \forall \in [a,c]$ and $f(x) < 0, \forall x \in [c, b]$, then the area bounded by the curve $y = f(x)$, X–axis, the lines $x = a$, $x = b$ is $\int_a^c f(x)\,dx - \int_c^b f(x)\,dx$.

4. Let $y = f(x)$ and $y = g(x)$ are two curves. Then the area between the two curves and the lines $x = a, x = b$ is $\left|\int_a^b \big(f(x)-g(x)\big)\,dx\right|$.

5. Let $y = f(x)$, $y = g(x)$ are two curves intersect at $x = c(a < c < b)$ then the area bounded between the given curves and $x = a$ & $x = b$ is $\left|\int_a^c \big(f(x)-g(x)\big)dx\right| + \left|\int_c^b \big(f(x)-g(x)\big)dx\right|$

6. The area of the region bounded by $y^2 = 4ax$ and $x^2 = 4by$ is $\frac{16ab}{3}$ sq.units.

7. The area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab sq. units.

8. The area of the circle $x^2 + y^2 = a^2$ is πa^2 sq. units.

9. The area of the region bounded by one arch of Sin ax or Cos ax and X-axis is $\frac{2}{a}$ sq.units.

10. The area of the region bounded by the curve $y = \sin ax$ or $\cos ax$ and X-axis in $[0,n\pi]$ is $\frac{2n}{a}$ sq. units.

11. The area of the region bounded by $y = ax^2 + bx + c$ and $y = mx + k$ is $\frac{\Delta^{\frac{3}{2}}}{6a^2}$. Where Δ is the discriminant of $ax^2 + (b - m)x + (c - k)$

12. The area of the region bounded by $x=ay^2+by+c$ and $x = my + k$ is $\frac{\Delta^{\frac{3}{2}}}{6a^2}$, where Δ is discriminant of $ay^2 + (b - m)y + (c - k)$

13. The area of the region bounded by $y = ax^2 + bx + c$ and $y = px^2 + qx + r$ is $\left|\frac{\Delta^{\frac{3}{2}}}{6(a-p)^2}\right|$ where Δ is the discriminent of $(a–p)x^2 + (b - q)x + (c - r)$

LECTURE SHEET

EXERCISE-I

Determination of Area

LEVEL-I (MAIN)

Single answer type questions

1. The area of the region bounded by $x^2 = 8y$, $x = 4$ and X–axis is

   1) $\frac{2}{3}$ 2) $\frac{4}{3}$ 3) $\frac{8}{3}$ 4) $\frac{10}{3}$

2. The area bounded by $y^2 = 4x$ with the lines $x = 2$ and $x = 5$ is

   1) $\frac{8}{3}\left(5\sqrt{5}-2\sqrt{2}\right)$ 2) $\frac{4}{3}\left(5\sqrt{5}-2\sqrt{2}\right)$ 3) $5\sqrt{5}-2\sqrt{2}$ 4) $\frac{16}{3}$

3. The area cut off from a parbola by any double ordinate is k times the corresponding rectangle contained by that double ordinate and its distance from the vertex then $k =$

   1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) $\frac{4}{3}$ 4) $\frac{3}{4}$

4. The area bounded by the X–axis and the curve $y = 4x - x^2 - 3$ is

   1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{4}{3}$ 4) $\frac{8}{3}$

5. The area bounded by $y = x^2 + 2$, X-axis, $x = 1$ and $x = 2$ is

   1) $\frac{16}{3}$ 2) $\frac{17}{3}$ 3) $\frac{13}{3}$ 4) $\frac{20}{3}$

6. The area bounded by $y = 4 - x^2$, $y = 0$ and $y = 3$ is

   1) $\frac{16}{3}$ 2) $\frac{22}{3}$ 3) $\frac{26}{3}$ 4) $\frac{28}{3}$

7. The area (in square units) of the region bounded by the curves $2x = y^2 - 1$ and $x = 0$ is

   1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) 1 4) 2

8. The area bounded by $y = 3x$ and $y = x^2$ is

   1) 10 2) 5 3) 4.5 4) 9

9. If the area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $\frac{a^2}{3}$ then $m =$

   1) 1 2) 2 3) 3 4) $\frac{1}{2}$

10. The area of the region bounded by $y = x^2$, $y = 3x$ and $x = 1$, $x = 3$ is

    1) $\frac{27}{2}$ 2) 9 3) $\frac{10}{3}$ 4) $\frac{20}{3}$

11. The area of the region bounded by $x = -1$, $x = 2$, $y = x^2+1$ and $y = 2x-2$ is

    1) 8 2) 9 3) 10 4) 7

12. The area (in square units) bounded by the curves $y=\sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is

1) 9 2) 36 3) 18 4) $\frac{27}{4}$

13. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is)

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) 1 4) $\frac{1}{6}$

14. The area of the region enclosed by the curves $y = x$, $x = e$, $y=\frac{1}{x}$ and the positive x-axis is

1) $\frac{3}{2}$ square units 2) $\frac{5}{2}$ square units 3) $\frac{1}{2}$ square units 4) 1 square units

15. The area bounded by the curves $y = x^3$, $y = x^2$ and the ordinates $x = 1$, $x = 2$ is

1) 17/12 2) 12/13 3) 2/7 4) 7/2

16. The area of the region enclosed by the curves $y = x^2$ and $y=x^3$ is

1) $\frac{1}{12}$ 2) $\frac{1}{6}$ 3) $\frac{1}{3}$ 4) 1

17. The area bounded by the curve $y = x(x-1)^2$, Y-axis and the line $y = 2$ is

1) $\frac{10}{3}$ 2) $\frac{5}{3}$ 3) $\frac{20}{3}$ 4) $\frac{40}{3}$

18. The area of the region bounded by the curves $y = x^2$ and $y=\frac{2}{1+x^2}$ is

1) $\pi-\frac{2}{3}$ 2) $\pi+\frac{2}{3}$ 3) $\frac{\pi}{3}$ 4) $\frac{2\pi}{3}$

19. If $x = a(> 0)$ divides the area bounded by X–axis, part of the curve $y = 1+\frac{8}{x^2}$ and the ordinates $x = 2$, $x = 4$ into equal parts then $a =$

1) 2 2) $\sqrt{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) $2\sqrt{2}$

20. The area bounded by $y = e^x$, $y = e^{-x}$ and $x = 1$ is

1) $e+\frac{1}{e}+2$ 2) $e+\frac{1}{e}-2$ 3) $e-\frac{1}{e}+2$ 4) $e-\frac{1}{e}-2$

Numerical value type questions

21. Area of the region bounded by the curve $y = (x-1)(x-2)(x-3)$ lying between the ordinates $x = 0$ and $x = 3$ is ..... .

22. Area of region bounded by $y = \sin^{-1}x$, $y = \cos^{-1}x$ and the x-axis is .....

23. Area bounded by parabola $x = -2y^2$, $x = 1-3y^2$ is ..... .

24. Area of region enclosed by the curves $y = x \log x$ and $y = 2x - 2x^2$ is ..... .

25. The area of the region bounded by $\max(|x|,|y|) \le 1$ and $xy \le \frac{1}{2}$ is ..... .

LEVEL-II (ADVANCED)

Single answer type questions

1. The area bounded by the two branches of curve $(y-x)^2 = x^3$ and the straight line $x = 1$ is
   a) 1/5 sq. units b) 3/5 sq. units c) 4/5 sq. units d) 8/4 sq. units

2. $y=\frac{x^2}{1+x^2}$ divides the area enclosed by $y=\frac{1}{1+x^2}$, x-axis and y-axis in the first quadrant in the ratio
   a) $\frac{\pi-2}{2}$ b) $\frac{4-\pi}{4}$ c) $\frac{2\pi-4}{\pi}$ d) $\frac{\pi-1}{\pi+1}$

3. The area bounded by $y=2-|2-x|$, $y=\frac{3}{|x|}$ is
   a) $\left(\frac{5-4\ln 2}{3}\right)$sq.unit b) $\left(\frac{2-\ln 3}{2}\right)$sq.unit c) $\left(\frac{4-3\ln 3}{2}\right)$sq.unit d) $\left(\frac{4+3\ln 3}{2}\right)$sq.unit

4. Area bounded by the curve $y = x\sin x$ and x-axis between $x = 0$ and $x = 2\pi$ is
   a) 2π sq. unit b) 3π sq. unit c) 4π sq. unit d) 5π sq. unit

5. The area bounded by $y = \sec^{-1} x$, $y = \operatorname{cosec}^{-1} x$ and line $x - 1 = 0$ is
   a) $\log(3+2\sqrt{2})-\frac{\pi}{2}$ sq.units b) $\frac{\pi}{2}-\log(3+2\sqrt{2})$ sq.units
   c) $\pi - \log_e 3$ sq.units d) $\pi + \log_e 3$ sq.units

6. If $f(x)=\max\left\{\sin x, \cos x, \frac{1}{2}\right\}$, then the area of the region bounded by the curves $y = f(x)$, x-axis, y-axis and $x=2\pi$ is
   a) $\left(\frac{5\pi}{12}+3\right)$ sq.unit b) $\left(\frac{5\pi}{12}+\sqrt{2}\right)$sq.unit c) $\left(\frac{5\pi}{12}+\sqrt{3}\right)$sq.unit d) $\left(\frac{5\pi}{12}+\sqrt{2}+\sqrt{3}\right)$sq.unit

7. The area between the curve $y = \cos^2 x$, x-axis and ordinates $x = 0$ and $x=\pi$ in the interval $(0, \pi)$ is
   a) π b) $\pi/4$ c) $\pi/2$ d) 2π

8. If $[x]$ is the greatest integer $\le x$, then $\int_{-2}^{2}\min\{x-[x],-x-[-x]\}dx =$
   a) 1 b) 2 c) 3/2 d) 0

9. The maximum area of a rectangle whose two vertices lie on the x-axis and two on the curve $y = 3|x|$ and $|x| \le 3$ is
   a) 7 b) 7/2 c) 9 d) 9/2

10. Area common to the curves $y = x^3$ and $y=\sqrt{x}$ is
    a) 5/3 b) 5/4 c) 5/12 d) 1/2

11. The area between the curve $y = x\,(x-1)(x-2)$ and x-axis is
    a) 1/4 b) 1/2 c) 1 d) 0

12. The area enclosed by $y = x^3$, its normal at (1, 1) and x-axis is equal to
    a) 7/4 b) 9/4 c) 5/4 d) 8/4

13. If two circles each of unit radius intersect orthogonally, the common area of the circles is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{4}+1$ c) $\frac{\pi}{2}+1$ d) $\frac{\pi}{2}-1$

14. If A is the area lying between the curve $y = \sin x$ and x-axis between $x = 0$ and $x = \pi/2$. Area of the region between the curves $y = \sin 2x$ and x-axis in the same interval is given by

a) $A/2$ b) A c) $2A$ d) $3A$

15. Area bounded by the curves $y = \log_e^x$ and $y = (\log_e x)^2$ is

a) $e-2$ sq. units b) 3–e sq. units c) e sq. units d) $e-1$ sq. units

More than one correct answer type questions

16. Consider the curve $C_1 : y = \cos x$ and the curve $C_2 : y = \sin x$, then

a) Area of one curvilinear triangle formed by C_1 and C_2 is $2-\sqrt{2}$

b) Area enclosed by C_1 and C_2 in successive points of intersections is $2\sqrt{2}$

c) Ratio of the area enclosed by C_1 in $\left[0,\frac{\pi}{2}\right]$ divided by the curve C_2 is $1:\sqrt{2}$

d) Area bounded by one arc of C_1 is equal to one arc of C_2

17. Consider the curve $|x+y-1|+|2x+y-1|=1$ then

a) Area bounded by the curve is 2sq. unit

b) Figure formed by the given curve is a rectangle

c) Figure formed by the given curve in I quadrant is a trapezium

d) Area of the region formed by the given curve in the IV quadrant is 1/3

18. If $f(a)$ be the area enclosed by the curves $xy^2 = a^2(a-x)$ and $(a-x)y^2 = a^2x$ then

a) $f(a) = (\pi+2)a^2$ b) $f(3) = 9(\pi-2)$ c) $f(5) = 25(\pi+2)$

d) $y = f(x)$ is a parabola whose latus rectum is approximately equal to $\frac{7}{8}$

19. Area bounded by the curve $xy^2 = a^2(a-x)$ and y-axis is $\Delta(a)$ then

a) $\Delta(3) = 9\pi$ b) $\Delta(1):\Delta(2) = 1:4$

c) minimum value of $\Delta(a)$ in [1, 3] is π d) maximum value of $\Delta(a)$ does not exist

20. Let the curve is $|x-2|+|y+1|=1$

a) Area of the region is 2 sq. units

b) Area of the region is 1 sq. units

c) Area of the region bounded by the given curve in III quadrant when the origin is translated to (2, –1) is $\frac{1}{2}$ sq. units

d) Area of the region bounded by the given curve in III quadrant when the origin is translated to (2, –1) is 0

Linked comprehension type questions

Passage - I :

Consider the areas S_0, S_1, S_2,... bounded by the x-axis and half-waves of the curve $y=e^{-x}\sin x$ where $x \geq 0$.

21. The value of S_0 is

a) $\frac{1}{2}(1+e^{\pi})$ sq.units b) $\frac{1}{2}(1+e^{-\pi})$ c) $\frac{1}{2}(1-e^{-\pi})$ d) $\frac{1}{2}(e^{\pi}-1)$ sq.units

22. The sequence S_0, S_1, S_2 .... , forms a G.P. with common ratio

a) $\frac{e^{\pi}}{2}$ b) $e^{-\pi}$ c) e^{π} d) $\frac{e^{-\pi}}{2}$

23. $\sum_{n=0}^{\infty} S_n$ is equal to

a) $\frac{1+e^{\pi}}{1-e^{-\pi}}$ b) $\frac{\frac{1}{2}(1+e^{\pi})}{1-e^{-\pi}}$ c) $\frac{1}{2(1-e^{-\pi})}$ d) $\frac{1}{2(1+e^{-\pi})}$

Passage - II :

Consider the two curves C_1 : y = 1 + cos x and $C_2 : y = 1+\cos(x-\alpha)$ for $\alpha \in \left(0, \frac{\pi}{2}\right)$; where $x \in [0, \pi]$. Also the area of the figure bounded by the curves C_1, C_2 and x = 0 is same as that of the figure bounded by C_2, y = 1 and $x = \pi$.

24. The value of α is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{8}$

25. For the values of α, area bounded by C_1, C_2, $x = 0$ and $x = \pi$ is

a) 1 sq.units b) 2 sq.units c) $2+\sqrt{3}$ sq.units d) $2-\sqrt{3}$ sq.units

Matrix matching type questions

26.

COLUMN - I	COLUMN - II
A) Area enclosed by $y = [x]$ and $y=\{x\}$ where [.] and {.} represent greatest integer and fractional part functions	p) 32/5 sq. units
B) The area bounded by the curves $y^2=x^3$ and $\lvert y\rvert = 2x$	q) 1 sq. units
C) The smaller area included between the curves $\sqrt{x}+\sqrt{y}=1$ and $\lvert x\rvert+\lvert y\rvert=1$	r) 4 sq. units
D) Area bounded by the curves $y=\left[\frac{x^2}{64}+2\right]$ (where [.] denotes the greatest integer function, $y = x - 1$ and $x = 0$ above the x-axis	s) 2/3 sq. units

27. Column - I gives the regions and Column - II gives the areas

COLUMN - I	COLUMN - II
a) $x^2 + y^2 \le 2$, max $\{\lvert x\rvert,\lvert y\rvert\} > 1$	p) 2
b) $x^2 + y^2 \le 2$, $\lvert x\rvert+\lvert y\rvert > 1$	q) 4
c) max $\{\lvert x\rvert,\lvert y\rvert\} < 2, \lVert x\rvert-\lvert y\rVert > 1$	r) $2(\pi-1)$
d) $\lvert x\rvert+\lvert y\rvert < 2, \lVert x\rvert-\lvert y\rVert > 1$	s) $2(\pi-2)$

28. **COLUMN - I** **COLUMN - II**

[.] represents greatest integer function

a) Area enclosed by $[x]^2=[y]^2$ *for* $1\le x\le 4$	p) 8 sq.units
b) Area enclosed by $[\lvert x\rvert]+[\lvert y\rvert]=2$	q) 6 sq.units
c) Area enclosed by $[\lvert x\rvert][\lvert y\rvert]=2$	r) 4 sq.units
D) Area enclosed by $\frac{[\lvert x\rvert]}{[\lvert y\rvert]}=2, -5\le x\le 5$	s) 12 sq.units

Integer answer type questions

29. Area bounded by the curves $y^2(2a-x)=x^3$ and the line $x = 2a$ is $k\pi a^2$ then $\{k\}$ is (where $\{.\}$ denotes fractional part of x)

30. The area bounded by the curves $y = lnx$, $y = \sin^4 \pi x$ and $x = 0$ is k, then $8k - 9$ is equal to

31. Area bounded by the curves $y=\left[\frac{x^2}{64}+2\right]$, $y = x- 1$ and $x = 0$ above x-axis is ([.] denotes the greatest integer function)

32. A point P moves in xy plane in such a way that $[\lvert x\rvert]+[\lvert y\rvert]=1$, where [.] denotes the greatest integer function. Area of the region representing all possible positions of the point P is equal to

33. Area bounded by the lines $y = 2 + x$, $y = 2 - x$ and $x = 2$ is :

EXERCISE-II

Area Bounded By Some Standard Curves

LEVEL-I (MAIN)

Single answer type questions

1. The area bounded by the curve $y^2 = 4ax$, Y-axis between $y = -a$ and $y = a$ is

1) a^3 2) $\frac{a^3}{6}$ 3) a^2 4) $\frac{a^2}{6}$

2. Let A_1 be the area of the parabola $y^2 = 4ax$ lying between vertex and latus rectum and A_2 be the area between latus rectum and double ordinate $x = 2a$. Then $A_1/A_2=$

1) $\frac{2\sqrt{2}-1}{7}$ 2) $\frac{2\sqrt{2}+1}{7}$ 3) $\frac{2\sqrt{2}+1}{3}$ 4) $\frac{2\sqrt{2}-1}{9}$

3. The area bounded by the curve $y^2 = x - 1$ and the line $y = x - 3$ is

1) $\frac{9}{2}$ 2) $\frac{8}{3}$ 3) $\frac{1}{2}$ 4) $\frac{10}{3}$

4. The area bounded by $y = 2x - x^2$ and $y = -x$ is

1) 9/2 2) 43/6 3) 35/6 4) 10/3

5. The area bounded by the curve $y = |4-x^2|$ and X-axis is

1) $\frac{32}{3}$ 2) $\frac{16}{3}$ 3) $\frac{8}{3}$ 4) 4

6. The area of the region bounded by $1-y^2 = |x| + |y| = 1$ is

1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) $\frac{4}{5}$ 4) 9

7. The area bounded by the curves $x = y^2$ and $x = 3-2y^2$ is

1) 3 2) 4 3) 1 4) 2

8. The parabolas $y^2 = 4x$, $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the co-ordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom then $S_1 : S_2 : S_3$ is

1) 2:1:1 2) 1:1:1 3) 1:2:1 4) 1:2:3

9. The area between the curves $y = \sqrt{x}$, $y=x^2$ is

1) $\frac{1}{3}$ 2) $\frac{8}{3}$ 3) $\frac{1}{2}$ 4) $\frac{4}{3}$

10. The area bounded by $y = \sqrt{a^2 - x^2}$, $x + y = 0$ and Y-axis is

1) $\frac{\pi a^2}{2}$ 2) $\frac{\pi a^2}{4}$ 3) $\frac{\pi a^2}{8}$ 4) πa^2

11. The area of the region between the curve $x^2 + y^2 = 4$ and $x = 0$, $x = 1$ is

1) $\sqrt{3}+\frac{2\pi}{3}$ 2) $\sqrt{3}$ 3) $\frac{2\pi}{3}$ 4) 3

12. The area of the ellipse $4(x-1)^2+5(y+2)^2 = 20$ is

1) 10π 2) $5\sqrt{2}\pi$ 3) $2\sqrt{5}\pi$ 4) 6π

13. The area between $y = \sin x$, $y = \cos x$ and Y-axis is (in first quadrant)

1) $\sqrt{2}$ 2) $\sqrt{2}+1$ 3) $\sqrt{2}-1$ 4) $\frac{1}{\sqrt{2}}$

14. The line $x = \frac{\pi}{4}$ divides the area of the region bounded by $y = \sin x$, $y = \cos x$ and x-axis $\left(0 \le x \le \frac{\pi}{2}\right)$ into two regions of areas A_1 and A_2. Then $A_1 : A_2 =$

1) 4 :1 2) 3 : 1 3) 2 : 1 4) 1 : 1

15. The area of one arc of $y = \sin x + \cos x$ is

1) $2\sqrt{2}$ 2) 2 3) $\sqrt{2}$ 4) 4

16. The area of the region between $y = \cos x$ and $y = \sin 2x$ in $0 \le x \le \frac{\pi}{2}$ is

1) $\frac{1}{4}$ 2) $\frac{1}{2}$ 3) $\frac{3}{4}$ 4) 1

17. The area bounded by $y = x \sin x$, x-axis and $x = 0$, $x = 2\pi$ is

1) π 2) 2π 3) 3π 4) 4π

18. The area of the region bounded by the x-axis and the curves $y = \tan x\left(-\frac{\pi}{3} \le x \le \frac{\pi}{3}\right)$ and $y = \cot x\left(\frac{\pi}{6} \le x \le \frac{3\pi}{2}\right)$ is

1) log 2 2) 2log2 3) $\log\sqrt{2}$ 4) $\log\left(\frac{3}{2}\right)$

19. The area bounded by by $y = \cos x$, $y = 0$, $|x| = 1$ is

1) 4 sin1 2) 3 sin1 3) 2 sin1 4) sin1

20. The area of the region enclosed between two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$ is

1) $\frac{\pi}{2} - \frac{\sqrt{3}}{2}$ 2) $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ 3) $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ 4) $\frac{\pi}{2} + \frac{\sqrt{3}}{2}$

Numerical value type questions

21. Area bounded by $f(x) = \frac{(x-1)(x+1)}{(x-2)}$, x-axis and ordinates $x = 0$ and $x = 3/2$ is

22. If the line $y = mx$ bisects the area enclosed by the lines $x = 0$, $y = 0$, $x=3/2$ and the curve $y=1+ 4x -x^2$. Then the value of m = ...

23. Area enclosed by the curves $x^2 = y$, $y = x + 2$ and x–axis is ..... .

24. The area of region for which $0 < y < 3 - 2x - x^2$ and $x > 0$ is ..... .

LEVEL-II (ADVANCED)

Single answer type questions

1. If the line $y = mx$ bisects the area enclosed by the curve $y = 1 + 4x - x^2$ and the lines $x = 0$, $y = 0$, $x = \frac{3}{2}$, then m =

a) $\frac{13}{6}$ b) $\frac{13}{16}$ c) $\frac{11}{8}$ d) $\frac{11}{6}$

2. Let $f(x) = 2\sqrt{x}$ and $g(x) = 2\sqrt{(1-x)}$ be two functions. Then the area bounded by $f(x)$, $g(x)$ and the x–axis is

a) $\frac{1}{3\sqrt{2}}$ sq. unit b) $\frac{2}{3\sqrt{2}}$ sq. unit c) $\frac{1}{\sqrt{2}}$ sq. unit d) $\frac{4}{3\sqrt{2}}$ sq. unit

3. Area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ is

a) $\frac{3\pi - 8}{3}$ sq.units b) $\frac{\pi - 8}{3}$ sq.units c) $\frac{2\pi - 8}{3}$ sq.units d) $\frac{\pi + 2}{3}$ sq.units

4. The area bounded by the curve $x = |y^2 - 1|$ and the line $y = x - 5$ is

a) $\frac{73}{6}$ b) $\frac{85}{6}$ c) $\frac{109}{6}$ d) $\frac{125}{6}$

5. The area enclosed by the curve $y = \sqrt{4 - x^2}, y \ge \sqrt{2}\sin\left(\frac{x\pi}{2\sqrt{2}}\right)$ and x–axis is divided by y-axis in the ratio

a) $\frac{\pi^2 - 8}{\pi^2 + 8}$ b) $\frac{\pi^2 - 4}{\pi^2 + 4}$ c) $\frac{\pi - 4}{\pi - 4}$ d) $\frac{2\pi^2}{2\pi + \pi^2 - 8}$

6. The area of the region of the plane bounded by $\max(|x|,|y|) \le 1$ and $xy \le \frac{1}{2}$ is

a) 1/2 + ln 2 sq.units b) 3 + ln 2 sq. units c) 31/4 sq. units d) 1+2 ln 2 sq. units

7. The area common to the curves $y = \sqrt{9 - x^2}$, $x^2 + y^2 = 6x$ with x-axis is

a) $6\pi - \frac{9\sqrt{3}}{2}$ b) $3\pi - \frac{9\sqrt{3}}{4}$ c) $3\pi - \frac{9\sqrt{3}}{2}$ d) $\frac{3\pi}{2} + \frac{9\sqrt{3}}{2}$

8. The area of the triangle formed by the positive x-axis and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

a) $\sqrt{3}$ b) $1/\sqrt{3}$ c) $2\sqrt{3}$ d) 1

9. The area bounded by the curve $y = 2\cos x$ and the x-axis from $x = 0$ to $x = 2\pi$ is

a) 2 sq.units b) 4 sq.units c) 6 sq.units d) 8 sq.units

10. Area bounded by the curves $y = \sin x$, tangent drawn to it at $x = 0$ and the line $x = \pi/2$, is equal to

a) $\frac{\pi^2 - 4}{2}$ sq.units b) $\frac{\pi^2 - 8}{8}$ sq.units c) $\frac{\pi^2 - 2}{4}$ sq.units d) $\frac{\pi^2 - 2}{2}$ sq.units

11. The area of the region bounded by $x^2 + y^2 - 6x - 4y + 12 < 0$, y < x and $x < \frac{5}{2}$ is

a) $\frac{\pi}{6} - \frac{\sqrt{3} - 1}{8}$ b) $\frac{\pi}{6} - \frac{\sqrt{3} + 1}{8}$ c) $\frac{\pi}{6} - \frac{\sqrt{3} - 1}{8}$ d) $\frac{\pi}{6} + \frac{\sqrt{3} - 1}{8}$

12. The area of the closed figure formed by $y = \frac{x^2}{2} - 2x + 2$ and the tangents to it at $(1, \frac{1}{2})$ and (4, 2) is ..... (in sq unit)

a) $\frac{9}{8}$ b) $\frac{3}{8}$ c) $\frac{3}{2}$ d) $\frac{9}{4}$

13. The area of the region in 1st quadrent bounded oy y-axis, $y = \frac{x}{4}$, $y = 1 + \sqrt{x}$ and $y = \frac{2}{\sqrt{x}}$ is ..... (in sq. unit)

a) $\frac{2}{3}$ b) $\frac{8}{3}$ c) $\frac{11}{3}$ d) $\frac{13}{6}$

14. The are enclosed by the curves $3x^2 + 5y = 32$ and $y = |x - 2|$ is

a) $\frac{13}{2}$ b) $\frac{17}{2}$ c) $\frac{23}{2}$ d) $\frac{33}{2}$

15. Let $f(x) = x^2$, $g(x) = \cos x$ and $\alpha, \beta (\alpha < \beta)$ be the roots of the equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area bounded by the curves $y = (fog)(x)$, the ordinates $x = \alpha, x = \beta$ and x-axis is

a) $\frac{\pi - 3}{2}$ b) $\frac{\pi + 3}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{12}$

More than one correct answer type questions

16. Let A_1 be the area bounded by the curves $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x = 0$ lying in the first quadrant and A_2 be the remaining region in the first quadrant then

a) $A_1 = 2 + \frac{25}{2}\tan^{-1}\frac{4}{3}$ b) $A_1 = 2 + \frac{25\pi}{4} - \frac{25}{2}\tan^{-1}\frac{3}{4}$ c) $A_2 = 12 - \frac{25}{3}\tan^{-1}\frac{4}{3}$ d) $A_2 = \frac{25}{2}\tan^{-1}\frac{3}{4} - 2$

17. Let the circle $x^2 + y^2 = 4$ divide the area bounded by the tangent and normal at $(1,\sqrt{3})$ and x-axis is A_1 and A_2 then

a) $A_1 = \frac{2\pi}{3}$ b) $A_1 = \frac{4\pi}{3}$ c) $A_2 = 2 - \frac{2\pi}{3}$ d) $A_2 = 2\sqrt{3} - \frac{2\pi}{3}$

18. The area enclosed between the curves, $x^2 = y$ and $y^2 = x$ is equal to

a) $\frac{1}{3}$ sq.unit b) $2\int_0^1 (x - x^2)dx$

c) area of the region $\{(x,y) : x^2 \le y \le |x|\}$ d) 1

19. Let $A(k)$ be the area bounded by the curves $y = x^2 - 3$ and $y = kx + 2$

a) The range of $A(k)$ is $\left[\frac{10\sqrt{5}}{3}, \infty\right]$ b) The range of $A(k)$ is $\left[\frac{20\sqrt{5}}{3}, \infty\right]$

c) If function $k \to A(k)$ is defined $k \in [-2, \infty]$, then $A(k)$ is many-one function

d) The value of k for which area is minimum is 1

20. If A is the area bounded by the curves $y = \sin x$ and x-axis in the interval $\left[0, \frac{\pi}{4}\right]$, in the same interval, area between the curves $y = \cos x$ and x-axis is A_1 and area between the curves $y = \sin x$, $y = \cos x$ and y-axis is A_2 then

a) $A_1 + A_2 = A$ b) $A_2 + A = A_1$ c) $A_2 = \sqrt{2} - 1$ d) $A_2 = \sqrt{2} + 1$

Linked comprehension type questions

Passage - I :

Let $f(x) = x^2 - 3x + 2$ be a function, $\forall x \in R$.

21. The area bounded by $f(x)$, the x-axis and y-axis is

a) $\frac{1}{3}$sq.unit b) $\frac{2}{3}$sq.unit c) $\frac{3}{5}$sq.unit d) $\frac{5}{6}$sq.unit

22. The area bounded by the curve $f(|x|)$ and x-axis is

a) $\frac{2}{5}$ sq.unit b) $\frac{5}{2}$ sq.unit c) $\frac{3}{5}$ sq.unit d) $\frac{5}{3}$ sq.unit

23. The area bounded by the curve $|f(|x|)|$ between $1 \le |x| \le 2$ and x-axis is

a) $\frac{1}{5}$ sq.unit b) $\frac{1}{4}$ sq.unit c) $\frac{1}{3}$ sq.unit d) $\frac{1}{2}$ sq.unit

24. The number of solutions of $|y| = f(|x|)$ and $x^2 + y^2 = 2$ is

a) 4 b) 6 c) 8 d) 5

Passage - II :

Consider the ellipse E, $\frac{x^2}{4} + y^2 = 1$. Let PM be the ordinate of P, a point on E in the first quadrant. Let Q be a point on PM such that QM is equal to the length of the subtangent at P. Let Q trace the curve C as P varies

25. The equation of C is

a) $x^2 + xy = 4$ b) $x^2 + y^2 = 4$ c) $x^2 = 4y$ d) $y^2 + xy = 4$

26. If the curve C intersects E at the points A and B, the area of triangle OAB is

a) $\sqrt{5}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{5}}{2}$

27. The area enclosed by C and E is

a) $\cos^{-1}\frac{2}{\sqrt{5}} + 2\ ln\frac{5}{4}$ b) $\sin^{-1}\frac{2}{\sqrt{5}} + 2\ ln\frac{5}{4}$ c) $\sin^{-1}\frac{2}{\sqrt{5}} - 2\ ln\frac{5}{4}$ d) $\cos^{-1}\frac{2}{\sqrt{5}} - 2\ ln\frac{5}{4}$

Matrix matching type questions

28.

COLUMN - I	COLUMN - II
A) Let a function is defined in [–2, 2] as $f(x)=\begin{cases}\{x\} & -2\le x<-1\\ \lvert\operatorname{sgn} x\rvert & -1\le x\le 1\\ \{-x\} & 1<x\le 2\end{cases}$ Then the area bounded by the graph of $f(x)$ and x-axis is	p) 4
B) Area bounded by the min $\{\lvert x\rvert,\lvert y\rvert\}=1$ and the max $\{\lvert x\rvert,\lvert y\rvert\}=2$ is	q) 3
C) Area bounded by $y=\sqrt{4-x^2}$ and its diameter $y = 0$ is	r) 6
D) Let 'R' be the region containg the points (x,y) on the xy-plane satisfying $2\le\lvert x+3y\rvert+\lvert x-y\rvert\le 4$ then the area of this region is	s) 2π

29.

COLUMN - I	COLUMN - II
A) Area bounded by $y = 4x - x^2$ with X–axis is	p) 2 sq. units
B) Area bounded by $y = x^3$ between $x = 1$ and $x = 2$ with X–axis is	q) 4 sq. units
C) Area bounded by $y=1+\frac{8}{x^2}$ between $x = 2$ and $x = 4$, X-axis is	r) $\frac{15}{4}$ sq. units
D) Area bounded by $y = \sin x$ between $x = 0$ and $x = \pi$ with X-axis is	s) 32/3 sq. units

Integer answer type questions

30. If the area between the curves $y = x - x^2$ and $y = mx$ is $\frac{9}{2}$, then the sum of all values of m is

31. The area bounded by the curves $y=-\sqrt{4-x^2}$, $x^2=-\sqrt{2}y$ and $x = y$ is ℓ then $[\ell]$ (where [.] denotes G.I.F) =

32. If the area included between the two parabolas $y^2 = 4a(x + a)$ and $y^2 = 4b(b - x)$ is 16/3. Then the product of A.M. and the G.M. of a and b is

33. The area of the figure enclosed by the curve $5x^2 + 6xy + 2y^2 + 7x + 6y + 6 = 0$ is k then $[k]$ = (where [.] denotes G.I.F) is equal to

34. Area of the region bounded by the curve $x^2 + y^2 = 2ax$ is $\frac{1}{3}K\pi a^2$ then K =

EXERCISE-III

Miscellaneous Models

LEVEL-I (MAIN)

Single answer type questions

1. The area of the region bounded by the curve $y = (x^2 + 2)^2 + 2x$ between the ordinates $x = 0$, $x = 2$ is

   1) $\frac{436}{15}$ 2) $\frac{208}{3}$ 3) $\frac{236}{5}$ 4) $\frac{340}{13}$

2. The area between the curve $xy = a^2$, $x = a$, $x = 4a$ and X–axis is

   1) $a\log 2$ 2) $2a^2\log 2$ 3) $a^2\log 2$ 4) $\log a$

3. The area bounded by the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ and the coordinate axes is

   1) $\frac{a^2}{6}$ 2) $\frac{a^2}{8}$ 3) $\frac{a^2}{4}$ 4) $\frac{a}{6}$

4. The area bounded by $y = \sin x$ in $[0, 2\pi]$

   1) 1 2) 2 3) 4 4) 3

5. The positive value of the parameter "a" for which the area of the figure bounded by $y = \sin ax$, $y = 0$, $x = \frac{\pi}{a}$ and $x = \frac{\pi}{3a}$ is 3 is equal to

   1) 1 2) $\frac{1}{3}$ 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

6. The area bounded by $y = |\sin x|$, X-axis and the lines $|x| = \pi$ is

   1) 1 2) 2 3) 3 4) 4

7. The area enclosed between the curves $y = \log_e(x+e)$, $y = e^{-x}$ and the X-axis is

   1) 2 2) 1 3) 3 4) 4

8. The area of the region bounded by $y = 2^x$, $y = 2x - x^2$ and $x = 0$, $x = 2$ is

   1) $\frac{3}{\log 2} + \frac{4}{3}$ 2) $\frac{3}{\log 2} - \frac{4}{3}$ 3) $\frac{3}{\log 2}$ 4) $3\log 2 - \frac{4}{3}$

9. The area bounded by the curves $x^2+y^2 \le 1$ and $|x| + |y| \ge 1$ is

   1) 2 2) $\pi + 2$ 3) $\pi - 2$ 4) π

10. The area of the region $\{(x,y) / x^2+y^2 \le 1 \le x+y\}$ is

1) $\frac{\pi}{4}+\frac{1}{2}$ 2) $\pi+1$ 3) $\frac{\pi}{4}-\frac{1}{2}$ 4) $\frac{\pi}{4}+\frac{3}{4}$

11. The area of the region bounded by $a^2y^2 = x^2(a^2-x^2)$ is

1) $\frac{4a^2}{5}$ 2) $\frac{4a}{3}$ 3) $\frac{4a^2}{3}$ 4) $\frac{5a^2}{8}$

12. The area bounded by $y = f(x) = x^4 - 2x^3 + x^2 + 3$, X-axis and ordinates corresponding to minimum of the function $f(x)$ is

1) 10/3 2) 91/30 3) 2/3 4) 1/2

13. The slope of the tangent to a curve $y = f(x)$ at $(x, f(x))$ is $2x + 1$. If the curve passes through the point (1, 2) then the area of the region bounded by the curve, X–axis and the line $x = 1$ is

1) 5/6 2) 6/5 3) 6 4) 1/6

14. The area bounded by the curve $y = f(x)$ the coordinate axes and the line $x = t$ is given by te^t then $f(x)$=

1) $e^x(x+1)$ 2) $e^x(x-1)$ 3) $x(1+e^x)$ 4) $x(1-e^x)$

15. The area bounded by the curve $f(x) = x+\sin x$ and its inverse between the ordinates $x = 0$, $x = 2\pi$ is

1) 2 2) 4 3) 6 4) 8

Numerical value type questions

16. The area bounded by the curve $y = \sin^{-1}x$ and the line $x = 0$, $|y| = \frac{\pi}{2}$ is ..... .

17. The area bounded by $y = \tan^{-1}x$, $y = \cot^{-1}x$ and y-axis in 1st quadrant is ..... .

18. Let S be the area bounded by the curve $y = \sin x$ $(0 \le x \le \pi)$ and x-axis, and T be the area of region bounded by the curve $y = \sin x$ $(0 \le x \le \pi/2)$, $y = a\cos x$ $(0 \le x \le \pi/2)$ and x-axis. Where $a \in R^+$, then value of a such that $S:T = 1:\frac{1}{3}$ is .

LEVEL-II (ADVANCED)

Single answer type questions

1. If $f(x+y) = f(x) + f(y) - xy$, $\forall x, y \in R$ and $\lim_{h\to 0}\frac{f(h)}{h} = 3$ then the area bounded by the curves $y = f(x)$ and $y = x^2$ is

a) 1 b) 2 c) 3 d) 4

2. The area bounded by the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ between the lines x = 0, $x = \frac{\pi}{4}$ is

a) $\int_0^{\sqrt{2}-1} \frac{t\,dt}{(1+t^2)\sqrt{1-t^2}}$ b) $\int_0^{\sqrt{2}-1} \frac{4t\,dt}{(1+t^2)\sqrt{1-t^2}}$ c) $\int_0^{\sqrt{2}+1} \frac{4t\,dt}{(1+t^2)\sqrt{1-t^2}}$ d) $\int_0^{\sqrt{2}+1} \frac{t\,dt}{(1+t^2)\sqrt{1-t^2}}$

3. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$. Then $f^1\left(\frac{\pi}{2}\right)$ is

a) $\left(\frac{\pi}{2} - \sqrt{2} - 1\right)$ b) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$ c) $-\frac{\pi}{2}$ d) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

4. Area of the region bounded by the graph $y^2 + 2xy + 40\,|x| = 400$ is

a) 200 b) 400 c) 800 d) 1600

5. P is a variable point in the square formed by the lines $x = \pm 1$ and $y = \pm 1$, P moves such that its distance from the origin is less than its distance from any side of the square. The area traced by the point P is

a) $\frac{4}{3}(4\sqrt{2} + 1)$ b) $\frac{4}{3}(4\sqrt{2} - 1)$ c) $\frac{4}{3}(4\sqrt{2} - 3)$ d) $\frac{4}{3}(4\sqrt{2} - 5)$

6. Let f be a real valued function satisfying $f\left(\frac{x}{y}\right) = f(x) - f(y)$ and $\lim\limits_{x\to 0} \frac{f(1+x)}{x} = 3$ then the area bounded by the curve $y = f(x)$, y-axis and $y = 3$ is

a) $3e$ b) $4e$ c) $5e$ d) $6e$

7. The area bounded by the curves $|y + x| \le 1, |y - x| \le 1$ and $2x^2 + 2y^2 \ge 1$ is

a) $\left(2 + \frac{\pi}{2}\right)$ b) $\left(2 - \frac{\pi}{2}\right)$ c) $\left(4 - \frac{\pi}{2}\right)$ d) $\left(4 + \frac{\pi}{2}\right)$

8. The area of the loop of the curve $x^2 + (y - 1)y^2 = 0$ is equal to

a) 8/15 sq. units b) 2/15 sq. units c) 4/15 sq. units d) 1/15 sq. units

9. The A.M of the areas of three mutually exclusive regions formed by $y = \sin x$, $y = \cos x$, $x = 0$, $x = \frac{\pi}{2}$ and x-axis is

a) $\frac{\sqrt{2}}{3}$ b) $\frac{2}{\sqrt{3}}$ c) $\frac{2}{3}$ d) $\sqrt{\frac{2}{3}}$

More than one correct answer type questions

10. The value of a where $(a > 2)$ for which the reciprocal of the area enclosed between $y = \frac{1}{x^2}; y = \frac{1}{4(x-1)}$; $x = 2$ and $x = a$ is a itself and the value of $b \in (1,2)$, the area of the figure bounded by the lines $x = b$ and $x = 2$ is $1 - \frac{1}{b}$ are

a) $a = 1 + e^2$ b) $a = 1 + e^{-2}$ c) $b = 1 + e^2$ d) $b = 1 + e^{-2}$

11. The area bounded by minimum of $(|x|, |y|) = 2$ and maximum of $(|x|, |y|) = 4$ is

a) 8 b) 16 c) 4 d) 32

12. A polynomial P is positive for $x > 0$ and the area of the region bounded by $P(x)$, the x-axis and the vertical lines $x = 0$ and $x = \lambda$ is $\frac{\lambda^2(\lambda+3)}{3}$ sq. unit. Then

a) $P(x) = x^2 + 2x$ b) $P(x) = x^2 + 2x + 1$ c) $P(3) = 15$

d) area bounded by $y = p(x)$ with the curve $x = y^2 + 2y$ is *1/3*

13. Which of the following have the same bounded area

a) $f(x) = \sin x, g(x) = \sin^2 x$, where $0 \le x \le 10\pi$ b) $f(x) = \sin x, g(x) = |\sin x|$, where $0 \le x \le 20\pi$

c) $f(x) = |\sin x|, g(x) = \sin^3 x$ where $0 \le x \le 10\pi$ d) $f(x) = \sin x, g(x) = \sin^4 x$ where $0 \le x \le 10\pi$

14. Consider the curve $C : y = |x|e^{|x|}$ and lines |x| = 1, $y = 0$ then

a) Area of the region is 2 sq. unit

b) Area of the region is 6 sq. unit

c) Area bounded by C in [–1, 0] is equal to area bounded by C in [0, 1]

d) Area bounded by C in [–1, 0] is not equal to area bounded by C in [0, 1]

15. If the curve $y = ax^{1/2} + bx$ passes through the point (1, 2) and lies above the x-axis for $0 \le x \le 9$ and the area enclosed by the curve, the x-axis and the line $x = 4$ is 8 sq. units. Then

a) $a = 1$ b) $b = 1$ c) $a = 3$ d) $b = -1$

Linked comprehension type questions

Passage - I :

The curve $y = f(x)$ passes through the point (0, 1) and the curve $y = g(x) = \int_{\infty}^{x} f(t)dt$ passes through the point $\left(0, \frac{1}{2}\right)$. The tangents drawn to the curves at the point with equal abscissa intersect on the x-axis. Then

16. The area bounded by the x-axis, the tangent and normal to the curve $y = f(x)$ at the point where it cuts the y-axis is

a) $\frac{3}{4}$ b) 1 c) $\frac{5}{4}$ d) $\frac{3}{2}$

17. The area bounded by the curve $y = f(x)$ and $y = x$ between the ordinates $x = 0$ an $x = 1$ is

a) e b) $\frac{e^2-1}{2}$ c) $\frac{e^2-2}{2}$ d) $e - 1$

18. Area enclosed by the curves $y = f^{-1}(x)$, $y = g^{-1}(x)$ in between the lines $y = 0$, $y = 2$ is

a) 1 b) $e - \frac{1}{2}$ c) $\frac{(e^2+1)}{4}$ d) $\frac{(e^4-1)}{4}$

Passage - II :

Let $f(x)$ be a positive differentiable function defined on [0, 1] such that $f(1) = 0$ and for any $a \in (0, 1)$, $\int_0^a f(x)dx - \int_a^1 f(x)dx = 2f(a) + 3a + b$, where b is a constant

19. $f(x)$ =

a) $\frac{2}{3}(e^{x-1}-1)$ b) $\frac{3}{2}(e^{x-1}-1)$ c) $\frac{2}{3}(1-e^{x-1})$ d) $\frac{3}{2}(1-e^{x-1})$

20. b =

a) $\frac{3}{2e}-3$ b) $\frac{3}{2e}-\frac{3}{2}$ c) $\frac{3}{2e}+3$ d) $\frac{3}{2e}+\frac{3}{2}$

Matrix matching type questions

21.

COLUMN - I	COLUMN - II
Area of the region bounded by	
A) $y = 2x - x^2$ and x–axis	p) $\frac{1}{3}$
B) $\{(x,y)/x^2 < y < \lvert x\rvert\}$	q) $\frac{1}{2}$
C) $y = x$ and $y = x^2$ is	r) $\frac{2}{3}$
D) $y = x\lvert x\rvert$, x – axis and $x = 1$, $y = 1$	s) $\frac{4}{3}$

Integer answer type questions

22. The area of the region enclosed between the curves $x=y^2-1$ and $x=\lvert y\rvert\sqrt{1-y^2}$ is

23. The area of bounded by the curves $y = \lvert\cos 3x\rvert$ and $y = \lvert\sin 3x\rvert$, $0 \le x \le \pi$ is $\frac{a}{b}(\sqrt{2}-1)$. Then a-b is equal to (where g.c.d. of a, b is 1)

24. P(x,y) be a point which moves in xy-plane such that $2[y] = 3[x]$, $-2 < x < 5$, $-3 < y < 7$, where [.] denotes GIF. If 'λ' is the area of the region containing the point P(x, y) then the value of $\sqrt{\lambda\sqrt{\lambda\sqrt{\lambda.....\infty}}}$ must be

25. Area enclosed by the curve $y = (x^2 + 2x)\, e^{-x}$ and the positive direction of x-axis is

26. If $f(x) = x^2 + 6x + 1$ and S denotes the set of points (x,y) in the plane such that $f(x) + f(y) < 0$ and $f(x) - f(y) < 0$, then area of S is $k\pi$, where k is

27. The area bounded by one arc of $y = \sin^2 3x$ and x-axis is k, then $[k]$ ([.] denotes GFI) is

28. If A is the area enclosed by the curve $[x + y] + [x-y] = 5$ for $x > y$, $\forall\, x, y > 0$ then 4A is

29. Let $f(x) = x^3 + 3x + 2$ and $g(x)$ be the inverse of $f(x)$. The area bounded by $y = g(x)$, x-axis and the ordinates $x = -2$ and $x = 6$ is K, then $[K + 1]$ is ([.] denotes GIF

30. If the area of the region enclosed by the curves $y = x \log x$ and $y = 2x - 2x^2$ is K, then $[K]$ ([.] denotes GIF) is

31. If the area enclosed by the curve $\lvert x\rvert + \lvert y\rvert \le k$ is $f(k)$ sq unit, then the value of $\frac{f(10)}{100}$ must be

KEY SHEET (LECTURE SHEET)

EXERCISE-I

LEVEL-I

1) 3 2) 1 3) 2 4) 3 5) 3 6) 4 7) 2 8) 3
9) 2 10) 3 11) 2 12) 1 13) 4 14) 1 15) 1 16) 1
17) 1 18) 1 19) 4 20) 2 21) 2.75 22) 0.4142 23) 1.3333
24) 0.5833 25) 0.6989

LEVEL-II

1) c 2) a 3) c 4) c 5) a 6) d 7) c 8) a
9) d 10) c 11) b 12) a 13) d 14) b 15) b 16) abcd
17) acd 18) bd 19) abcd 20) ac 21) b 22) b 23) d 24) b
25) b 26) A-q;B-p;C-s;D-r 27) A-s;B-r;C-q;D-p
28) A-q;B-s;C-q;D-p 29) 0 30) 2 31) 4 32) 8 33) 4

EXERCISE-II

LEVEL-I

1) 4 2) 2 3) 1 4) 1 5) 1 6) 2 7) 2 8) 2
9) 1 10) 3 11) 1 12) 3 13) 3 14) 4 15) 1 16) 2
17) 4 18) 4 19) 3 20) 3 21) 0.875 22) 2.1666
23) 0.8333 24) 1.6666

LEVEL-II

1) a 2) d 3) a 4) c 5) d 6) b 7) b 8) c
9) d 10) b 11) c 12) a 13) c 14) d 15) d 16) abd
17) ab 18) abc 19) bc 20) bc 21) d 22) d 23) c 24) c
25) a 26) b 27) d 28) A-q;B-p;C-s;D-r 29) A-s;B-r;C-q;D-p
30) 2 31) 3 32) 1 33) 1 34) 3

EXERCISE-III

LEVEL-I

1) 1 2) 2 3) 1 4) 3 5) 3 6) 4 7) 1 8) 2
9) 3 10) 3 11) 3 12) 2 13) 1 14) 1 15) 4 16) 2
17) 0.1505 18) 1.3333

LEVEL-II

1) b 2) b 3) c 4) d 5) d 6) a 7) b 8) a
9) a 10) ab 11) b 12) acd 13) acd 14) ac 15) cd 16) c
17) c 18) d 19) d 20) a 21) A-s;B-p;C-q;D-r 22) 2
23) 7 24) 4 25) 4 26) 8 27) 0 28) 6 29) 2 30) 0
31) 2

PRACTICE SHEET

EXERCISE-I

Determination of Area

LEVEL-I (MAIN)

Single answer type questions

1. The area of the region bounded by the curve $y=|5\sin x|$ from $x = 0$, $x = 4\pi$ and X–axis is
   1) 10 2) 20 3) 40 4) 80
2. The area bounded by the curves $y = xe^x$, $y = xe^{-x}$ and the line $x = 1$ is
   1) $\frac{2}{e}$ 2) $1-\frac{2}{e}$ 3) $\frac{1}{e}$ 4) $1-\frac{1}{e}$
3. The area enclosed between the curve $y = \log_e (x + e)$ and the co-ordinate axes is
   1) 3 2) 4 3) 1 4) 2
4. The area of the region bounded by $y = |x–1|$, $y = 3 – |x|$ is
   1) 1 2) 2 3) 3 4) 4
5. The area of the region bounded by $y = |x – 2|$, $x = 1$, $x = 3$ and X-axis is
   1) 4 2) 2 3) 3 4) 1
6. The area bounded by $y = |x–1|$, $y = 0$ and $|x| = 2$ is
   1) 4 2) 5 3) 3 4) 10
7. The area bounded by the lines $y = |x–2|$, $|x| = 3$ and $y = 0$ is
   1) 7 2) 9 3) 13 4) 15
8. The area of the region bounded by $y = x–[x]$ and $2x–1= 0$ and X–axis is
   1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{1}{8}$ 4) 1
9. The area of the region bounded by $y = x–[x]$, $y = [x]$ in $[0, 2]$ is
   1) 1 2) $\frac{3}{2}$ 3) 2 4) $\frac{5}{2}$
10. The area of the region bounded by $y = [x + 2]$ from $x = 0$ to 3, X-axis is
    1) 8 2) 9 3) 10.5 4) 100

Numerical value type questions

11. The area of region included between the parabola $y=\frac{3x^2}{4}$ and the line $3x – 2y + 12= 0$ is
12. Area is region bounded by curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$ is .....
13. Area is region $\{(x,y)/0\le y\le x^2+1, 0\le y\le x+1, 0\le x\le 2\}$ is ..... .
14. If the area bounded by the parabola $y^2=4\lambda x$ and $y=\lambda x$, $\lambda>0$ is $\frac{1}{9}$. Then $\lambda=$
15. If the area of the region $\{(x,y)/y^2\le 4x, x+y\le 1 x\ge 0, y\ge 0\}$ is $a\sqrt{2}+b$ then $a – b$ = ...

LEVEL-II (ADVANCED)

Single answer type questions

1. The area bounded by the curve $y = x(x-1)^2$, the x-axis and the ordinates of the maximum and minimum points of the curve is

a) $\frac{4}{7}$ b) $\frac{4}{9}$ c) $\frac{4}{81}$ d) $\frac{2}{81}$

2. The area bounded by the curve $y^2 = (1-x^2)^2$ is

a) 3 b) 4 c) $\frac{4}{3}$ d) $\frac{8}{3}$

3. The total area bounded by the curve $y = x^3$, $y = x$, $y = 2x$ is

a) 1 b) $\frac{3}{2}$ c) $\frac{3}{4}$ d) $\frac{1}{2}$

4. The line $x = \lambda$ intersects the curve $y^2 = x$ at A and B. Let $R(\lambda)$ be the ratio of the area of triangle OAB to the area bounded by the curve and AB. Then $\lim_{\lambda \to 0} R(\lambda)$ =

a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{3}{4}$ d) $\frac{4}{5}$

5. The area of the region whose boundaries are defined by the curves $y = 2\cos x$, $y = 3\tan x$ and the y-axis is

a) $1+2ln\left(\frac{2}{\sqrt{3}}\right)$ sq.units b) $1+\frac{3}{2}ln3-3ln2$ sq. units

c) $1+\frac{3}{2}ln3-ln\,2$sq.units d) $ln\,3 - ln\,2$ sq. units

6. The area bounded by $y=\frac{1}{(2x+1)^2}$, $y = 0$, $x = 1$, $x = 2$ is

a) $\frac{1}{3}$ b) $\frac{1}{5}$ c) $\frac{1}{10}$ d) $\frac{1}{15}$

7. The area bounded by the curve $y=\frac{x}{\sqrt{2x^2+1}}$, y=0, x=0 and x=2

a) 1 b) 2 c) $\frac{3}{2}$ d) $\frac{2}{3}$

8. The area bounded by $y=a\cosh\left(\frac{x}{a}\right), x=0, y=0, x=a$ is

a) $\frac{a^2e}{2}$ b) $\frac{a^2}{2e}$ c) $a^2\cosh 1$ d) $a^2\sinh 1$

9. The area between x-axis and the first arc of $y = e^{-ax}\sin ax$ is

a) $\frac{1}{2a}$ b) $\frac{e^{-\pi}}{2a}$ c) $\frac{1-e^{-\pi}}{2a}$ d) $\frac{1+e^{-\pi}}{2a}$

10. The area bounded by y-axis and the curve $x = e^y.\sin \pi y$, $y = 0$, $y = 1$ is

a) $\frac{e+1}{\pi^2+1}$ b) $\frac{e-1}{\pi^2+1}$ c) $\frac{(e+1)\pi}{\pi^2+1}$ d) $\frac{(e-1)\pi}{\pi^2+1}$

More than one correct answer type questions

11. A curve $y = f(x)$ is defined parametrically as $x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$ then

a) Area bounded by $y = f(x)$ is π sq. units

b) Area bounded by $y = f(x)$ with normal at any point on it is $\pi/2$ sq. units

c) Area (smaller area) bounded by $y = f(x)$ and the lines $y=\sqrt{3}x$, $y = 0$ is $\frac{\pi}{3}$ sq.units

d) Area bounded by $y = f(x)$ and the lines $y=\sqrt{3}x$, $y = 0$ is $\frac{\pi}{6}$ sq. units

12. If $f(a)$ be the area enclosed by the curves $xy^2 = a^2(a-x)$ and $(a-x)y^2 = a^2x$ then

a) $f(a) = (\pi+2)a^2$ b) $f(3) = 9(\pi-2)$ c) $f(5) = 25(\pi+2)$

d) $y = f(x)$ is a parabola whose latus rectum is approximately equal to $\frac{8}{7}$

13. Let R be the region represented by $|x - y| + |x + y| \le 8$. Then

a) Area bounded by $xy \ge 2$ and R is $4(7 - \log_e 8)$

b) Area bounded by $xy \ge -2$ and R is $4(7 - \log_e 8)$

c) Area of the region R is 64

d) Area bounded by $xy \le 2$ and R is $12(3 + \log_e 2)$

14. Consider $f(x) = \cos^{-1}(4x^3 - 3x)$. Then

a) $f(x) = \pi - 3\cos^{-1} x$ for $\frac{1}{2} \le x \le 1$

b) $f(x) = 2\pi - 3\cos^{-1} x$ for $-\frac{1}{2} \le x < \frac{1}{2}$

c) $f(x) = 3\cos^{-1} x - 2\pi$ for $-1 \le x < -\frac{1}{2}$

d) Area enclosed by y = f(x), y = 0, $x \ge -\frac{1}{2}$ is $\frac{3\sqrt{3}}{2}$

Linked comprehension type questions

Passage - I :

Let there are two functions defined by $f(x) = \min(|x|, |x-1|, |x+1|)$ and $g(x) = \min\{e^x, e^{-x}\}$. α is the root of $f(x) = g(x)$

15. The area bounded by $f(x)$ in $[-1, 1]$ and x-axis is

a) 1/5 sq.unit b) 1/4 sq.unit c) 1/3 sq.unit d) 1/2 sq.unit

16. The area bounded by $f(x)$, $g(x)$ and $x = 0$ in first quadrant is

a) $e^{-\alpha} - 1$ b) $2 - e^{-\alpha}$ c) $1 + e^{-\alpha}$ d) $e^{-1} - e^{-\alpha} - \frac{\alpha^2}{2}$

17. The sum of roots of equation $f(x) - g(x) = 0$ is

a) 0 b) 2α c) -2α d) 4α

Matrix matching type questions

18.

COLUMN - I	COLUMN - II
a) The area bounded by the curve $y = 2 + \lvert x\rvert - \lvert x+1\rvert - \lvert x-1\rvert$ and the x-axis is	p) $\frac{4}{3}$
b) The area bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is	q) 2
c) The area bounded by the curves $x = 1 - y^2$ and $x - y + 1 = 0$ is	r) $\frac{10}{3}$
d) The area bounded by $y = x(x-1)^2$, $x=0$ and $y = 2$ is	s) $\frac{9}{2}$

19.

COLUMN - I	COLUMN - II
A) If $f(x) = \begin{cases} \sqrt{\{x^2\}}, & x \notin I \\ 1, & x \in I \end{cases}$ and $g(x) = \{x\}^2$ Then area bounded by $f(x)$, $g(x)$ for all $x \in [0,10]$ is .... (where $\{.\}$ represents fractional part of x)	p) $\frac{3}{4}$
B) Area of the region defined by $1 \le \lvert x\rvert + \lvert y\rvert$ and $x^2 - 2x + 1 \le 1 - y^2$ is $k\pi$ where $k =$	q) $\frac{7}{6}$
C) Area bounded by $f(x) = \min\{x+1, \sqrt{1-x}\}$, x-axis and $x = -1$, $x = 1$ is	r) $\frac{128}{5}$
D) Area bounded by the curves $y = 2x \pm \sqrt{x^3}$ and $x = 4$ is	s) $\frac{10}{3}$

Integer answer type questions

20. Area enclosed by the curve $y = f(x)$ defined parametrically as $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ is equal to $\frac{k\pi}{5}$ sq. units then k is equal to

21. Area bounded by $y = \frac{1}{x^2 - 2x + 2}$ and x-axis is $\frac{n\pi}{2}$ then n is

22. The value of the parameter a such that the area bounded by $y = a^2x^2 + ax + 1$, coordinates axes and the line $x = 1$ attains its least value, at $a = k$ then $4k + 3 =$

23. Consider the region formed by the lines $x = 0$. $y = 0$, $x = 2$, $y = 2$. Area enclosed by the curves $y = e^x$ and $y = ln\ x$, within this region, is being removed. Area of the remaining region is $k\log(l/e)$ then $k + l$ is

24. The area bounded by $y = \sin^{-1}x$, $y = \cos^{-1}x$ and x-axis is $\sqrt{a} - \sqrt{b}$ then a–b value is

EXERCISE-II

Area Bounded By Some Standurd Curves

LEVEL-I (MAIN)

Single answer type questions

1. The area of the region between $x - y + 2 = 0$ and the curve $x = \sqrt{y}$, Y-axis is

1) $\frac{5}{3}$ 2) $\frac{10}{3}$ 3) $\frac{8}{3}$ 4) $\frac{20}{3}$

2. The area of the region bounded by $y = x^2 + 2$, $y = -x$, $x = 0$ and $x = 1$ is

1) $\frac{17}{6}$ 2) $\frac{17}{3}$ 3) $\frac{10}{3}$ 4) $\frac{8}{3}$

3. The area bounded by the curve $y = x|x|$, X-axis and the ordinates $x = \pm 1$ is

1) $\frac{2}{3}$ 2) $\frac{1}{3}$ 3) $\frac{4}{3}$ 4) $\frac{5}{3}$

4. The area inside the parabola $5x^2 - y = 0$ but outside the parabola $2x^2 - y + 9 = 0$ is

1) 12 2) $12\sqrt{3}$ 3) 6 4) $6\sqrt{3}$

5. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

1) $\frac{5}{3}$ 2) $\frac{1}{3}$ 3) $\frac{2}{3}$ 4) $\frac{4}{3}$

6. The area bounded by the parabolas $y^2 = 4a(x+a)$ and $y^2 = -4a(x-a)$ is

1) $\frac{16a^2}{3}$ 2) $\frac{8a^2}{3}$ 3) $\frac{4a^2}{3}$ 4) $\frac{a^2}{3}$

7. The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ in the plane is

1) $\frac{8}{3}$ 2) $\frac{16}{3}$ 3) $\frac{32}{3}$ 4) $\frac{34}{3}$

8. The area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 sq.unit. then $a =$

1) $\frac{1}{\sqrt{3}}$ 2) $\frac{2}{\sqrt{3}}$ 3) $\frac{4}{\sqrt{3}}$ 4) $\sqrt{3}$

9. The smaller area of the region cut off by $x + y = 2$ from the circle $x^2 + y^2 = 4$ is

1) $\pi - 2$ 2) $\pi - 4$ 3) $2\pi - 4$ 4) $4\pi - 2$

10. The area bounded by the circle $x^2 + y^2 = a^2$, X-axis and the ordinates $x = a/2$, $x = a$ is

1) $a^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$ 2) $a^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right)$ 3) $\frac{\pi a^2}{4}$ 4) $\frac{\pi a^2}{8}$

11. The area enclosed within the ellipse $4x^2 + 9y^2 = 36$ is

1) 4π 2) 6π 3) 9π 4) 36π

12. AOB is the positive quadrant of the ellipse $x^2/a^2+y^2/b^2=1$, where $OA = a$, $OB = b$. The area between the arc AB and the chord AB of the ellipse is

1) $\frac{\pi ab}{2}$ 2) $\frac{\pi ab}{4}$ 3) $\left(\frac{\pi-1}{4}\right)ab$ 4) $\left(\frac{\pi-2}{4}\right)ab$

13. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

1) $4\sqrt{2}+2$ 2) $4\sqrt{2}-1$ 3) $4\sqrt{2}+1$ 4) $4\sqrt{2}-2$

14. The area bounded by $y = \sin x$, $y = \cos x$ between any two successive intersections is

1) 2 2) $\sqrt{2}$ 3) $2\sqrt{2}$ 4) 4

15. The ratio of the areas bounded by $y = \cos x$, $y = \cos 2x$ between $x = 0$, $x = \frac{\pi}{3}$ and the X-axis is

1) 1 : 2 2) 2 : 1 3) $\sqrt{3}:4$ 4) $2\sqrt{3}:4-\sqrt{3}$

16. The area bounded by $y = \sin x$ and $2x = \pi y$ is

1) $\frac{\pi}{4}-1$ 2) $2-\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) 2

17. The area of the greater region bounded by $y = \cos x$, $y = x + 1$ and $y = 0$ is

1) 1/2 2) 3/2 3) 2 4) 3

18. The area of the smaller region bounded by $y = \cos x$, $x + y = 1$ and $y = 0$ is

1) $\frac{1}{2}$ 2) $\frac{3}{2}$ 3) 2 4) 3

19. The area between the curve $y = \cos x$ and Y-axis from $y = 0$ to $y = 1$ is

1) 1 2) 2 3) $\sqrt{2}-1$ 4) π

20. The area bounded between the parabola $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line $y = 2$ is

1) $20\sqrt{2}$ 2) $\frac{10\sqrt{2}}{3}$ 3) $\frac{20\sqrt{2}}{3}$ 4) $10\sqrt{2}$

Numerical value type questions

21. The area of region $A=\left\{(x,y)/\frac{y^2}{2}\le x\le y+4\right\}$ is .... .

22. The area of region $A=\left\{(x,y)\in R\times R/0\le x\le 3, 0\le y\le 4, y\le x^2+3x\right\}$ is

23. The area in the first quadrant bounde by $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is ..... .

LEVEL-II (ADVANCED)

Single answer type questions

1. The area below the line $y = x + 1$ and bounded by the curve $x^2 + y^2 - 4x - 6y - 19 = 0$ is

a) $\int_{-2}^{6}(x+1+\sqrt{32-(x-2)^2}+3)dx$ b) $\int_{-2}^{6}(\sqrt{32-(x-2)^2}+3(x-1)dx$

c) 16π d) $\frac{32\pi}{3}$

2. The smaller of the areas bounded by the curves $x^2 + y^2 = 4$ and $y^2 = 2(x + 2)$ is

a) $\frac{8}{3}+\pi$ b) $\frac{16}{3}+2\pi$ c) $2\pi-\frac{16}{3}$ d) $\pi-\frac{8}{3}$

3. The area of the smaller part of a disc of radius 10 cut off by a chord which subtends at an angle $\frac{\pi}{4}$ at the circumcentre is

a) $\frac{25}{4}(\pi+2\sqrt{2})$ b) $\frac{25}{2}(\pi+2\sqrt{2})$ c) $\frac{25}{4}(\pi-2\sqrt{2})$ d) $\frac{25}{2}(\pi-2\sqrt{2})$

4. The area bounded by $y = 2 - x^2$ and $y^3 = x^2$ is

a) $\frac{30}{17}$ b) $\frac{16}{15}$ c) $\frac{32}{15}$ d) $\frac{16}{14}$

5. The area bounded by $y = 2x - 2x^2$ and $y = x\ ln\ x$ is

a) $\frac{1}{12}$ b) $\frac{5}{12}$ c) $\frac{17}{12}$ d) $\frac{7}{12}$

6. The area in the first quadrant bounded by the curves $x^2 = 2y$, $y^2 = 2x$ and $x^2 + y^2 = 3$ is $a+b\sin^{-1}\frac{1}{3}$, where ab =

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

7. The area of the region $\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$ is

a) $\frac{11}{6}$ b) $\frac{15}{4}$ c) $\frac{25}{6}$ d) $\frac{23}{6}$

8. The area of the closed figure bounded by $x = -1$, $y = 0$, $y = x^2 + x + 1$ and the tangent to the curve $y = x^2 + x + 1$ at $A(1, 3)$ is

a) 4/3 sq. units b) 7/3 sq. units c) 7/6 sq. units d) 5/2 sq. units

9. The area of the region in the first quadrant included between the curves $x^2 + y^2 = 16$ and $\sqrt{|x|}+\sqrt{|y|}=2$ is

a) $\left(4\pi-\frac{8}{3}\right)$ sq.units b) $\left(4\pi-\frac{4}{3}\right)$ sq. units

c) $\left(8\pi-\frac{4}{3}\right)$ sq.units d) $8\left(\pi-\frac{1}{3}\right)$ sq. units

10. Let $f(x)$ = minimum $(x+1, \sqrt{1-x})$ for all $x \le 1$. Then the area bounded by $y = f(x)$ and the x-axis is

a) $\frac{7}{8}$ sq.units b) $\frac{1}{6}$ sq.units c) $\frac{11}{6}$ sq.units d) $\frac{7}{6}$ sq.units

11. The area bounded by the curve $y^2 = 1 - x$ and the lines $y = \frac{|x|}{x}$, $x = -1$ & $x = \frac{1}{2}$ is

a) $\frac{3}{\sqrt{2}} - \frac{11}{6}$ b) $3\sqrt{2} - \frac{11}{4}$ c) $\frac{6}{\sqrt{2}} - \frac{11}{5}$ d) $3\sqrt{2} + \frac{11}{4}$

12. The area bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ is

a) $\frac{5\pi - 1}{4}$ b) $\frac{5\pi + 1}{4}$ c) $\frac{5\pi - 2}{4}$ d) $\frac{5\pi - 3}{4}$

13. Area bounded by the curve y = max{sinx, cosx} and x-axis between the lines $x = \frac{\pi}{4}$ and $x = 2\pi$ is equal to

a) $\frac{4\sqrt{2} - 1}{\sqrt{2}}$ b) $4\sqrt{2} - 1$ c) $\frac{4\sqrt{2} - 1}{2}$ d) $\frac{4\sqrt{2} + 1}{2}$

More than One correct answer Type Questions

14. Consider two variable parabolas $y^2 = 4ax$, $x^2 = 4ay$, $a \in [1, 2]$ and $A(a)$ is area bounded by them, then

a) $A(1)$ is maximum b) $A(2)$ is maximum
c) $A(1)$ is minimum d) $A(2)$ is minimum

15. Area bounded by the parabola $y = x^2 - 2x + 3$ and tangents drawn to it from the point $P(1, 0)$ is equal to A_1 and the parabola with the chord of contact of P is A_2 then

a) $A_1 = 4\sqrt{2}$ sq. units b) $A_1 = \frac{4\sqrt{2}}{3}$ sq. units
c) $A_2 = \frac{8\sqrt{2}}{3}$ sq. units d) $A_2 = \frac{16}{3}\sqrt{2}$ sq. units

16. The area of the region containing the origin which is bounded by the curves $x^2 + y^2 = 5$ and $||x| - |y|| = 1$ is

a) $10\pi + 4 - 20\tan^{-1} 3$ b) $20 - 5\tan^{-1} 2$
c) $4 + 10\pi - 20\cot^{-1} 1/3$ d) $4 + 20\cot^{-1} 3$

17. Let A be the area bounded by the curve $x = at^2$, $y = 2at$ and the x-axis then A =

a) 296/3 for $3 < t < 4$, $a = 1$ b) 46 for $3 < t < 4$, $a = 1$
c) $\frac{26a^2}{3}$ for $1 < t < 3$ d) $\frac{4}{3}\sqrt{a}((9a)^{3/2} - a^{3/2})$ for $1 < t < 3$

18. If the area bounded by $y = x^2 + 2x - 3$ and the $y = kx + 1$ is the least and is equal to A, then

a) $k = \frac{3}{2}$ b) $k = 2$ c) $A = \frac{32}{3}$ d) $A = \frac{27}{2}$

19. If A is the area of the figure bounded by the curve $y = 8x^2 - x^5$, the straight lines $x = 1$, $x = c$ and x–axis is

a) c = –1 if $A = \frac{16}{3}$

b) c = 0 if $A = \frac{16}{3}$

c) c = 3 if $A = \frac{49}{6}$

d) c = 2 if $A = \frac{49}{6}$

20. Let A_n be the area bounded by the curve $y = \tan^n x$ and the lines $x = 0$, $y = 0$ and $x = \frac{\pi}{4}$. Then

a) $A_n \in \left(\frac{1}{2n+2}, \frac{1}{2n-2}\right)$

b) $A_n = \frac{\pi}{4} + \frac{2}{3}$

c) $A_n = \frac{\pi}{4} - \frac{2}{3}$

d) $A_n \in \left(\frac{1}{8}, \frac{1}{4}\right)$

Linked comprehension type questions

Passage - I :

Let $y = f(x)$ satisfy the relation $xy = x + y + 1, x \in R - \{1\}$ and $g(x) = xf(x)$

21. The range of $g(x)$ has

a) a maximum

b) a minimum

c) both a max. and a min.

d) neither max. nor min.

22. The function y = g(x) is

a) convex in $(1,\infty)$

b) convex in $(-\infty,0)$ only

c) convex in $(0,\infty)$

d) concave in $(1,\infty)$

23. The area bounded by the curve y = g(x) and x-axis is

a) $\frac{3}{2} + \ln 4$

b) $\frac{3}{2} - \ln 4$

c) $\frac{1}{2} + \ln 4$

d) ln4 – 1

Matrix matching type questions

24.

COLUMN - I	COLUMN - II
A) Area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ is	p) $\frac{\Pi a^2}{2}$ Sq. units
B) Area bounded by the curve $x^2+y^2=2ax$ is	q) $(\Pi - 2)a^2$ s.u
C) Area bounded by the curve $xy^2=a^2(a-x)$ and y – axis	r) $2a\Pi$ S.u
D) Area bounded by $x^2+y^2 \le a^2, \lvert x\rvert + \lvert y\rvert \ge a$	s) Πa^2 S.u

25. Match Column I with Column II

COLUMN – I	COLUMN – II
A) Area of the region enclosed by the curve $x^2+4y^2-6x+8y+9=0$ is	p) 4π
B) Area of the region enclosed by the curve $x^2+xy+y^2=1$ is	q) $2\sqrt{3}\pi$
C) Area of the region inside the curves $\frac{x^2}{9}+\frac{y^2}{3}=1$ and $\frac{x^2}{3}+\frac{y^2}{9}=1$ is	r) 2π
D) Area of the region bounded by the curve $y=x\sin x$ and the x-axis between the lines $x=0, x=2\pi$	s) $\frac{2\pi}{\sqrt{3}}$

Integer answer type questions

26. The area of the ellipse $x^2 + 4y^2 - 6x + 8y + 9 = 0$ is "$k\pi$" then k =

27. The area of the closed figure bounded by the curves $y=\sqrt{x}, y=\sqrt{4-3x}$ and $y = 0$ is a/b then $2a-b$ =

28. The area bounded by the parabola $x^2 = 4y$ and the focal chord through the point $(-2,1)$ is a/b then $a - b$ is

29. Consider two curves $C_1 : y^2 = 4[\sqrt{y}]x$ and $C_2 : x^2 = 4[\sqrt{x}]y$, where [.] denotes the greatest integer function. The area of region enclosed by these two curves within the square formed by the lines $x = 1$, $y = 1$, $x = 4$, $y = 4$ is l then $3l - 11$ is

30. If the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x-3$ is $\frac{m}{n}$, where m, n are relatively prime positive integers, then $m + n$ is

31. The area bounded by the curves $y = -x^2 + 6x - 5$, $y = -x^2 + 4x - 3$ and the straight line $y = 3x - 15$ is $\frac{a}{b}$ (G.C.D of a and b is 1). Then the value of $a - 12b$ is

EXERCISE-III

Miscellaneous Models

LEVEL-I (MAIN)

Single answer type questions

1. The area of the region enclosed by the curves $y = x^2$ and $y=x^3$ is

1) $\frac{1}{12}$ 2) $\frac{1}{6}$ 3) $\frac{1}{3}$ 4) 1

2. The area bounded by the curve $x^{2/3} + y^{2/3} = a^{2/3}$ and $\overline{OX}, \overline{OY}$ is

1) $\frac{\pi a^2}{32}$ 2) $\frac{3\pi a^2}{32}$ 3) $\frac{5\pi a^2}{32}$ 4) $\frac{3\pi a^2}{16}$

3. The area of the region bounded by the curves $y = ex \log x$ and $y = \frac{\log x}{ex}$ is

1) $\frac{e}{4} - \frac{5}{4e}$ 2) $\frac{e}{4} + \frac{5}{4e}$ 3) $\frac{e}{3} - \frac{5}{4e}$ 4) 5e

4. The area bounded by $|x| + |y| = 1$ with the coordinate axes is

1) $\frac{1}{2}$ 2) 1 3) $\frac{1}{4}$ 4) 2

5. The area of the region $R = \{(x, y)/x^2 \le y \le x\}$ is

1) $\frac{1}{6}$ 2) $\frac{2}{3}$ 3) $\frac{4}{3}$ 4) 2

6. If the regions A and B are given by $A = \{(x,y)\backslash : y>x\}$ $B = \{(x, y) : y < 2-x^2\}$ then the area of $A \cap B$ is

1) $\frac{1}{3}$ 2) $\frac{9}{2}$ 3) $\frac{2}{5}$ 4) $\frac{3}{4}$

7. The area bounded by the curve $y^2(2a-x) = x^3$ and the line $x = 2a$ is

1) $3\pi a^2$ 2) $\frac{3\pi a^2}{2}$ 3) $\frac{3\pi a^2}{4}$ 4) πa^2

8. The area of the region bounded by the parabola $(y-2)^2 = x - 1$, the tangent to the parabola at the point (2,3) and the x-axis is :

1) 6 2) 3 3) 12 4) 9

9. The area of the region bounded by the curve $y = \tan x$ tangent drawn to the curve at $x = \frac{\pi}{4}$ and the x-axis is

1) $\log 2 - \frac{1}{4}$ 2) $\frac{1}{2}\log 2 - \frac{1}{4}$ 3) 2 4) 4

10. The area bounded by the X-axis, the curve $y = f(x)$ and the lines $x = 1$ and $x = b$ is equal to $\sqrt{b^2+1} - \sqrt{2}$ for all $b > 1$ then $f(x)$ is

1) $\sqrt{x^2+1}$ 2) $\sqrt{x+1}$ 3) $\frac{x}{\sqrt{x^2+1}}$ 4) $\frac{x^2}{\sqrt{x^2+1}}$

Numerical value type questions

11. Area of region $A = \{(x,y)/0 \le x|x|+1, -1 \le x \le 1\}$ is ..... .

12. Area enclosed by the curve $y = f(x)$ defined parametically as $x = \frac{1+t^2}{1+t^2}, y = \frac{2t}{1+t^2}$ is equal to $\frac{K\pi}{5}$. Then K value = ...

13. Area of region defined by $1 \le |x| + |y|$ and $x^2 - 2x + 1 \le 1 - y^2$ is $K\pi$ = ...

LEVEL-II (ADVANCED)

Single answer type questions

1. Let $f(x)$ be a non negative continuous function such that area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = 1$and $x = b > 1$ is $(b - 1)\sin(3b + 4)$ then $f(0) =$

a) 0 b) sin 8 c) sin 4 – 3cos 4 d) cos 4 + 3 sin 4

2. The area bounded by the curve $y = e^{-x^2}$, y = 0 and the maximum ordinate is

a) $\frac{1}{2}$ b) $\frac{1}{2\sqrt{e}}$ c) $\frac{1}{2}(1-\frac{1}{\sqrt{e}})$ d) $\frac{1}{2}(1-\frac{1}{e})$

3. The area bounded by $y=\frac{\sin x}{x}$, x – axis and the ordinates $x = 0$, $x=\frac{\pi}{4}$ is

a) $\frac{\pi}{4}$ b) $<\frac{\pi}{4}$ c) $>\frac{\pi}{4}$ d) $<\int_0^{\pi/4}\frac{\tan x}{x}$

4. For any real number t, x= cosht, y = sinht is a point on the hyperbola $x^2 – y^2 = 1$. The area bounded by the curve and the lines joining the points t and $–t$ to the origin is

a) t b) 2t c) cosht d) sinht

5. Area bounded by $y=\left[\sin^2\frac{x}{2}+\sqrt{\cos\frac{x}{2}}\right]$ and $y=\left|\sin x-\frac{1}{2}\right|$. $x = 0$, $x = \pi$ (where [.] represents GIF is

a) $\frac{1}{6}[7\pi+12(1-\sqrt{3})]$ b) $7\pi+12(1-\sqrt{3})$ c) $\frac{\pi}{2}+1-\sqrt{3}$ d) $\frac{\pi}{2}+1+\sqrt{3}$

More than one correct answer type questions

6. A curve $y = f(x)$ is defined parametrically as $x=\frac{1-t^2}{1+t^2}$, $y=\frac{2t}{1+t^2}$ then

a) Area bounded by $y = f(x)$ is π sq. units

b) Area bounded by $y = f(x)$ with normal at any point on it is $\pi/2$ sq. units

c) Area (smaller area) bounded by $y = f(x)$ and the lines $y=\sqrt{3}x$, $y = 0$ is $\frac{\pi}{3}$ sq.units

d) Area bounded by $y = f(x)$ and the lines $y=\sqrt{3}x$, $y = 0$ is $\frac{\pi}{6}$ sq. units

7. The area enclosed by the curves $x = a\sin^3 t$ and $x = a\sin^3 t$ $y = a\cos^3 t$ is equal to

a) $12a^2\int_0^{\pi/2}\cos^4 t\sin^2 t\,dt$ b) $12a\int_0^{\pi/2}\cos^2 t\sin^4 t\,dt$

c) $2\int_{-a}^{a}(a^{2/3}-x^{2/3})^{3/2}dx$ d) $4\int_0^{a}(a^{2/3}-x^{2/3})^{3/2}dx$

8. If $f(x)=\begin{cases}\sqrt{\{x\}}, & x\notin z\\ 1, & x\in z\end{cases}$ and $g(x)=\{x\}^2$, (where $\{.\}$ dentoes fractional part of x). Let R be the region bounded by $f(x)$ and $g(x)$ then

a) Area of one leaf in R is $\frac{1}{3}$ sq.unit

b) Area of the region R for $x\in[0,10]$ is $\frac{10}{3}$ sq.unit

c) Area of the rectangle formed by the lines $x=0$, $x=5$, $y=0$, $y=1$ excluding R is $\frac{10}{3}$ sq.unit

d) Area of the rectangle formed by the lines $x=0$, $x=5$, $y=0$, $y=1$ excluding R is $\frac{5}{3}$ sq.unit

9. Consider the curves $C_1: y=\sin^{-1}|\sin x|$ and $C_2: y=(\sin^{-1}|\sin x|)^2$, $0\le x\le 2\pi$ then

a) Area bounded by C_1 with x-axis is $\frac{\pi^2}{2}$ sq.unit

b) Area enclosed in between C_1 and C_2 is $\left(\frac{\pi^3}{6}-\frac{\pi^2}{2}+\frac{4}{3}\right)$ sq.unit

c) The graph of C_1 and C_2 intersect on x-axis at $\frac{\pi^2}{2},\frac{3\pi}{2}$

d) The graph of C_1 and C_2 meets the y-axis at only one point

10. If Δ is the area of the triangle formed by the tangent to the curve $f(x)=x^2+bx-b$ at the point (1,1) and the coordinate axes, lies in the first quadrent, then

a) b = –3 if $\Delta=2$ b) b = –2 if $\Delta=1$ c) b = 1 if $\Delta=5$ d) b = –4 or $-\frac{5}{2}$ if $\Delta=\frac{9}{4}$

11. The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is divided into 2 parts by the line $x=2a$. The area of the smaller part is $\Delta(a,b)$. Then

a) $\Delta(a,b)=\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)ab$ b) $\Delta(a,b)=\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)ab$

c) $\Delta(4,3)=4\pi-3\sqrt{3}$ d) $\Delta(3,2)=4\pi-3\sqrt{3}$

12. Let A_n be the area bounded by the curve $y=\tan^n x$ and the lines $x=0$, $y=0$ and $x=\frac{\pi}{4}$. Then

a) $A_n\in\left(\frac{1}{2n+2},\frac{1}{2n-2}\right)$ b) $A_n=\frac{\pi}{4}+\frac{2}{3}$ c) $A_n=\frac{\pi}{4}-\frac{2}{3}$ d) $A_n\in\left(\frac{1}{8},\frac{1}{4}\right)$

Linked comprehension type questions

Passage - I :

Consider the function $f(x)$ such that $f(x)=x^2+\int_0^1(x+t)f(t)dt$

13. The minimum value of $f(x)$ is

a) $\frac{-109}{3}$ b) $\frac{-109}{6}$ c) $\frac{-109}{12}$ d) $\frac{-15}{2}$

14. The area bounded by the x-axis, the tangent and normal at $x = 0$ on the curve $y = f(x)$ is

a) $\frac{3757}{60}$ b) $\frac{3757}{90}$ c) $\frac{3757}{180}$ d) $\frac{3757}{120}$

15. The area bounded by the curve $y = f(x)$ and the x-axis is

a) $\frac{341}{3}\sqrt{109}$ b) $\frac{109}{18}\sqrt{\frac{109}{3}}$ c) $341\sqrt{\frac{109}{3}}$ d) $\frac{109}{9}\sqrt{\frac{109}{3}}$

Passage-II :

Consider the function f(x), a fourth degree polynomial such that $\lim_{x\to 1}\frac{f(x)}{(x-1)^2}=1$, $f^1(0) = -6$, $f^1(2) = 6$

16. The minimum value of $f(x)$ is

a) 0 b) 1 c) –1 d) –2

17. The length of the subtangent of the curve $y=f(x)$ where it cuts y-axis is

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

18. The area bounded by the curve $y = f(x)$ and the coordinate axes is

a) $\frac{3}{5}$ b) $\frac{4}{18}$ c) $\frac{7}{15}$ d) $\frac{8}{15}$

Matrix matching type questions

19.

COLUMN - I	COLUMN - II
A) Area bounded by $\lvert x\rvert-\lvert y\rvert\geq 1$ and $\lvert x\rvert\leq 2$ is	p) $\frac{1}{2}$
B) If $b\in[0,3]$ then the maximum area that can be bounded by the curves $y=1-\lvert x-1\rvert$ and $y=\lvert x-b\rvert$ is	q) $\frac{1}{3}$
C) Let $f(x)=\min\{\sqrt{x(2-x)},2-x\}$. Then area bounded by $y = f(x)$, x-axis is $\frac{\pi}{4}+K$, where K =	r) 2
D) Area bounded by $y=\max\{\lvert x-2\rvert+2,3-\lvert x-2\rvert\}$ and $y=\min\{\lvert x-2\rvert+2,3-\lvert x-2\rvert\}$ is	s) 1

Integer answer type questions

20. Let $O(0,0)$ $A(2,0)$ and $B\left(1,\frac{1}{\sqrt{3}}\right)$ be the vertices of a triangle. Let R be the region consisting of all those points P inside ΔOAB which satisfy $d(P, OA) \le \min\{d((P, OB), d(P, AB)\}$, when '$d$' denotes the distance from the point to the corresponding line. Then the area of the region R is $\sqrt{a}-\sqrt{b}$ then $a + b =$

21. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x=\frac{\pi}{4}, x=\beta>\frac{\pi}{4}$ is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+2\beta\right)$ then $f(0) =$

22. The value of the parameter a such that the area bounded by $y = a^2x^2 + ax + 1$, coordinates axes and the line $x = 1$ attains its least value, at $a = k$ then $4k + 3 =$

23. If, $y=\frac{x^2+3x+2}{x^2-3x+2}$ then the area of the bounded region between the curve and x-axis is $a(\log b-\log c)-1$ then $a - b + c$ is

24. The area defined by $1\le|x-2|+|y+1|\le 2$ is

25. The area of the curve $xy^2=8-4x$ and its asymptote is $k\pi$ then the numerical value of k must be

26. Let $f(x) = x^3 + 3x + 2$ and $g(x)$ be the inverse of $f(x)$. The area bounded by $y = g(x)$, x-axis and the ordinates $x = -2$ and $x = 6$ is K, then $[K + 1]$ is ([.] denotes GIF

27. The area of the region bounded by the curve $y = x^2$ and $y = \sec^{-1}[-\sin^2 x]$, where [.] denotes the greatest integer function, is $\frac{k\pi^{3/2}}{3}$ then k is

KEY SHEET (PRACTICE SHEET)

EXERCISE-I

LEVEL-I

1) **3**	2) **1**	3) **3**	4) **4**	5) **4**	6) **2**	7) **3**	8) **3**
9) **1**	10) **2**	11) **27**	12) **10.5**	13) **3.8333**		14) **24**	15) **6**

LEVEL-II

1) **c**	2) **d**	3) **b**	4) **c**	5) **b**	6) **d**	7) **a**	8) **d**
9) **d**	10) **c**	11) **abc**	12) **bd**	13) **abcd**	14) **bcd**	15) **d**	16) **d**
17) **a**	18) **A-q;B-p;C-s;D-r**			19) **A-r;B-s;C-p;D-p**			20) **5**
21) **2**	22) **0**	23) **6**	24) **1**				

EXERCISE-II

LEVEL-I

1) **2** 2) **1** 3) **1** 4) **2** 5) **4** 6) **1** 7) **2** 8) **1**
9) **1** 10) **1** 11) **2** 12) **4** 13) **4** 14) **3** 15) **4** 16) **2**
17) **2** 18) **1** 19) **1** 20) **3** 21) **18** 22) **7.833** 23) **1.541**

LEVEL-II

1) **c** 2) **c** 3) **d** 4) **c** 5) **d** 6) **c** 7) **d** 8) **c**
9) **a** 10) **d** 11) **a** 12) **c** 13) **a** 14) **bc** 15) **bc** 16) **acd**
17) **ad** 18) **bc** 19) **ad** 20) **acd** 21) **c** 22) **d** 23) **b**
24) **A-r;B-s;C-s;D-q** 25) **A-r;B-s;C-q;D-p** 26) **2** 27) **7**
28) **5** 29) **0** 30) **4** 31) **1**

EXERCISE-III

LEVEL-I

1) **1** 2) **2** 3) **4** 4) **1** 5) **2** 6) **1** 7) **1** 8) **4**
9) **2** 10) **3** 11) **2** 12) **5** 13) **3.33**

LEVEL-II

1) **c** 2) **c** 3) **b** 4) **a** 5) **a** 6) **abc** 7) **acd** 8) **abc**
9) **abd** 10) **ab** 11) **ac** 12) **acd** 13) **c** 14) **c** 15) **b** 16) **a**
17) **c** 18) **d** 19) **A-q;B-p;C-s;D-p** 20) **7** 21) **2** 22) **0**
23) **1** 24) **6** 25) **4** 26) **2** 27) **4**

ADDITIONAL EXERCISE

LEVEL-I (MAIN)

Single answer type questions

1. The area of the parallelogram formed by the lines $4y-3x-a=0$, $3y-4x+a=0$, $4y-3x-3a=0$, $3y-4x+2a=0$ is

   1) $a^2/5$ 2) $a^2/7$ 3) $2a^2/7$ 4) $2a^2/9$

2. Area bounded by $y=\{x\}$, $\{.\}$ is fractional part of function and $x=\pm 1$ is in sq.units.

   1) 1 2) 2 3) 3 4) 4

3. The area bounded by tangent, normal and x-axis at P(2,4) to the curve $y=x^2$ is

   1) 34 2) 32 3) 36 4) 24

4. The area of the region between the curve $y=4x^2$ and the line $y=6x-2$ is in sq. units is

   1) $\frac{1}{9}$ 2) $\frac{1}{12}$ 3) $\frac{3}{2}$ 4) $\frac{1}{5}$

5. Area bounded by $f(x) = \max(\sin x, \cos x)\ \forall\ 0 \le x \le \frac{\pi}{2}$ and the co-ordinate axis is equal to

1) $\frac{1}{\sqrt{2}}$ sq.units 2) $\sqrt{2}$sq.units 3) 2sq.untis 4) 1sq.unit

6. The area bounded by the curve $y = c^2x^2$, y-axis and the lines $y = a$. $y = b$ in sq. units is

1) $\frac{2}{3c}(b^{3/2} - a^{3/2})$ 2) $\frac{3}{3c}(b^{3/2} - a^{3/2})$ 3) $\frac{3}{4c}(b^{3/2} - a^{3/2})$ 4) $\frac{3}{4c}(b^{3/2} + a^{3/2})$

7. The area of the region formed by the curve $\sqrt{x} + \sqrt{y} = \sqrt{4}$ between co-ordinate axes

1) $\frac{4}{3}$ 2) $\frac{8}{3}$ 3) $\frac{16}{3}$ 4) $\frac{5}{3}$

8. Let A_n be the area bounded by the curve $y = (\tan x)^n$ and lines $x = 0$, $y = 0$ and $x = \frac{\pi}{4}$ for $n > 2$, then $A_n + A_{n-2} =$

1) $\frac{1}{n+1}$ 2) $\frac{1}{n-1}$ 3) $\frac{2}{n+1}$ 4) $\frac{2}{n-1}$

9. The plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{1} = 1$ meet the co-ordinate axis at the points A,B,C respectively then the area bounded by the triangle ABC in squnits is

1) 6 2) $\frac{9}{2}$ 3) $\frac{5}{2}$ 4) $\frac{7}{2}$

10. The area bounded by $y = x^4 - 2x^3 + x^2 + 3$, axis of abscissa and two ordinates corresponding to two points of minima of the function $y = f(x)$ is

1)1/30 2) 1/10 3) 91//30 4) 3

11. The area enclosed between the curves $y = \sin^2 x$, $y = \cos^2 x$ in $0 < x < \pi$ and between their consecutive points of intersection is

1) 2 2) 1 3) 4 4) 3

12. Points (0, 4) (2, 2) and (4, 0) with $g(x) < f(x)$ for $0 < x < 2$ and $f(x) < g(x)$ for $2<x< 4$. $\int_0^4 f(x) - g(x)\,dx = 10$ and $\int_2^4 (g(x) - f(x)dx = 5$ then, area between two curves $y=f(x)$ and $y=g(x)$ for $0 < x < 2$ is, ... (in sq. units)

1) 5 2) 10 3) 15 4) 20

13. Area bounded by the curve $y = (x+1)^2$ and $f(x)$ is a differential function such that $f(x+1)+f(z-1) = f(x+z)$ for all real x,z and $f(0) = 0$ and $f'(0) = 4$ is (in sq. units)

1) 32/3 2) 16/3 3) 8/3 4) 28/3

14. The triangle formed by the tangent drawn to the curve $f(x) = ax^2 + (\sin^2 \alpha)x - b, (b \neq 0)$ at point (1,0) and coordinate axes lies in the first quadrant. If its area is 4 then the minimum value of a is

1) –9/2 2) –2 3) 2 4) –1

15. Area bounded by $y = e^{-x}$ and $y = e^{x}$ and the directrix of the conic $y^2 - 4x - 4y + 12 = 0$ is (in sq. units)

1) $\frac{1}{2}\sum_{r=1}^{\infty}\frac{1}{(2r)!}$ 2) $\sum_{r=0}^{\infty}\frac{1}{(2r)!}$ 3) $\sum_{r=1}^{\infty}\frac{1}{(2r)!}$ 4) $2\sum_{r=1}^{\infty}\frac{1}{(2r)!}$

16. The area enclosed by $\left[\frac{|3x+4y|}{5}\right]+\left[\frac{|4x-3y|}{5}\right]=3$ is (where [.] denotes the G.I.F)

1) 10 2) 12 3) 8 4) 16

LEVEL-II

LECTURE SHEET (ADVANCED)

Single answer type questions

1. The function $f(x) = \max\left\{x^2, (1-x)^2, 2x(1-x)\ \forall\ 0 \le x \le 1\right\}$ then area of the region bounded by the curve $y = f(x)$, x-axis and $x = 0$, $x = 1$ is equals.

a) 27/17 b) 17/27 c) 18/17 d) 19/17

2. The area bounded by the curves $y=\left[\frac{x^2}{64}+2\right]\forall x \in (-8,8), y = x-1$ and co-ordinae axes where [.] is g.i.f

a) 2 b) 4 c) 3 d) 1

3. The aea bounded by parabola $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line is $y = 1/4$ is

a) 4 sq.units b) 1/6 sq.units c) 4/3 sq.units d) 1/3sq.units

4. The area bounded by $y = x^2$.$y=[x + 1]$, $x \le 1$ and the y-axis is where [.] is g.i.f

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) 1 d) $\frac{7}{3}$

5. If A_1 and A_2 respectively represents the area bounded by the curves $f(x,y): 4x^2 \le y \le 3x$ *and* $g(x,y): 4x^2 \le y \le |3x|$ the $A_1 : A_2$ equals.

a) 2 : 1 b) 3 : 1 c) 1 : 2 d) 1 : 3

6. Maximum area of rectangle whose two vertices lies on the x-axis and two on the curve $y = 3 - |x|, \forall |x| < 3$.

a) 9 b) 9/4 c) 3 d) 9/2

7. The area bounded by the curve $|x+y|+|x-y|=1$ is

a) $\frac{1}{2}$ b) 1 c) 2 d) 4

8. The area bounded by the curve $y^2(2 - x) = x^3$ and its asymptote is

a) $\frac{\pi}{2}$ b) π c) 2π d) 3π

9. The area bounded by the curves is $\sqrt{|x|}+\sqrt{|y|}=\sqrt{a}$ and $x^2 + y^2 = a^2$ (where $a > 0$) is

a) $\left(\pi-\frac{2}{3}\right)a^2$ sq.units b) $\left(\pi+\frac{2}{3}\right)a^2$ c) $\left(\pi+\frac{2}{3}\right)a^3$ sq.units d) $\left(\pi-\frac{2}{3}\right)a^3$ sq.units

More than one correct answer type questions

10. If the area enclosed by $y^2 = 4ax$ and line $y = ax$ is $\frac{1}{3}$ sq. unit, then the roots of the equation $x^2 + 2x = a$ are

a) –4 b) 2 c) –2 d) 8

11. The value (s) of 'a' for which the area of the triangle included between the axes and any tangent to the curve is constant, is/are

a) –1/2 b) –1 c) 1/2 d) 1

12. Area of the region bounded by the curve $y = 25^x + 16$ and curve $y = b.\ 5^x + 4$ whose rangent at the point $x = 1$, makes an angle $\tan^{-1}(40 \log 5)$ with the x-axis is .... (in sq. units)

a) $2\log_5\left(\frac{e^4}{27}\right)$ b) $4\log_5\left(\frac{e^4}{27}\right)$ c) $3\log_5\left(\frac{e^4}{27}\right)$ d) None of these

Linked comprehension type questions

Passage - I :

A continuous function $f(x)$ satisfying $x^4 - 4x^2 < f(x) < 2x^2 - x^3$ forall $x \in [0,2]$, such that the each bounded by $y = f(x)$, $y = x^4 - 4x^2$, they y–axis and the line $x = t (0 < t < 2)$ is k times the area bounded by $y = f(x)$, $y = 2x^2 - x^3$, y-axis and the line $x = t$ $(0 < t < 2)$. Answer the following questions.

13. If k = 2, then if $f(x)$ attains point of inflection at

a) 0 b) $\sqrt{2}$ c) $-\sqrt{2}$ d) None of these

14. If k = 0, then $f(x)$ attains local maximum at

a) 0 b) $\sqrt{2}$ c) $-\sqrt{2}$ d) None of these

15. The value of $\int_{-1}^{1} f(x)\,dx$ is

a) $\frac{2}{15(k+1)}(23-10k)$ b) $\frac{2}{15(k+1)}(23+10k)$ c) $\frac{2}{15(k+1)}(10k-17)$ d) $\frac{2}{15(k+1)}(10k+17)$

Passage - II :

Let $f(x)$ be a differentiable function satisfying the condition $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}, y \neq 0, f(y) = 0$ for all x, $y \in R$ and $f'(1)) = 2$. Answer the following questions.

16. The area enclosed by the curves $y = f(x)$, $x^2 + y^2 = 2$ and x-axis, is (in sq. units)

a) $\pi/4$ b) 1/6 c) $\left(\frac{\pi}{4}-\frac{1}{6}\right)$ d) $\left(\frac{\pi}{2}-\frac{1}{3}\right)$

17. The area enclosed by the curves $y = f(x)$ and $y^2 = -x$, is (in sq. units)

a) 1/3 b) –4/3 c) 3/4 d) None of these

18. The area enclosed by the curves $y = f(x)$, $y = 2^x$ and y – axis in first quadrant, is

a) $\frac{69}{6}$ b) $\frac{9 - l\,n\,16}{l\,n\,8}$ c) $\frac{1}{6}$ d) $\frac{12 - l\,n\,256}{l\,n\,8}$

Matrix matching type questions

19.

COLUMN - I	COLUMN - II
A) The area between the curves $y = 2x^4 - x^2$, the x-axis and the ordinates of two minimum of the curve is	p) 2sin 1
B) The area bounded by the curve $x = at^2$, $y=2at$ and the x-axis $(1 < t < 3)$ is	q) $\frac{7}{120}$
C) The area of a circle centred at (1, 2) and passing through (4, 6) is	r) $\frac{104a^2}{3}$
D) The area of the curve bounded by $y = \cos x$, $y = 0$ $\lvert x\rvert = 1$ is given by	s) 25π

Integer answer type questions

20. The Area bounded by loop of $\lvert y\rvert = \sin x$ for $0 < x < \pi$, is _____ .

21. If area bounded by $y = f(x)$, y-axis and the line $2y = \pi(x+1)$ where

$f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ is $\frac{\pi}{k}$ where k = _____

22. If $A(n)$ represents the area bounded by the curve $y = n \log x$, $n \in N$ and $n > 1$, the x-axis and the lines $x = 1$, $x = e$ then the value of $A(n) + n(A)(n-1)$ is kn^2 where k = _____

23. Area bounded by $y = [\cos A + \cos B + \cos C]$ and $y = \left[7\sin\frac{4}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right]$ and $\lvert x-4\rvert + \lvert y\rvert = 2$ (where [.] greatest integer function), and A, B, C are angles of a tgriangle is _____ .

PRACTICE SHEET (ADVANCED)

Single answer type questions

1. The area of the region bounded by the curve $y = x^3$ and $y = x + 6$ and $x = 0$

a) 20 b) 30 c) 10 d) 40

2. The area under the curve $y = 2\sqrt{x}$ included between the lines $x = 0$ and $x = 1$.

a) $\frac{4}{3}$ b) $\frac{3}{4}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

3. Area of the region $\{(x,y): y^2 \le 6ax \text{ and } x^2+y^2 \le 16a^2\}$ is

a) a^2 b) $\sqrt{3}+4\pi$ c) $\frac{2}{3}a^2$ d) $\frac{4}{3}a^2\left(\sqrt{3}+4\pi\right)$

4. Area of the region $y=1+|x+1|, x=-3, x=3$ $y=0$ is

a) 10 b) 16 c) 6 d) 2

5. The area of the region bounded by the curve $y=\sqrt{16-x^2}$ and x–axis is

a) 8π *sq.units* b) 20π *sq.units* c) 16π *sq.units* d) 256π *sq.units*

6. Area of the region bounded by the curves $y=\frac{16-x^2}{4}$ and $y=\sec^{-1}\left[-\sin^2\pi\right]$, (where [.] denotes greatest integer function) is (in sq. units)

a) $\frac{8}{3}(4-\pi)^{\frac{3}{2}}$ b) $2(4-\pi)^{\frac{3}{2}}$ c) $3(4-\pi)^{\frac{3}{2}}$ d) $\frac{7}{3}(4-\pi)^{\frac{3}{4}}$

7. Area of the region, bounded by $y=2+|x|-|x+1|-|x-1|$ and x-axis is (in sq. units)

a) $\frac{3}{2}$ b) 2 c) 1 d) 3

8. The area of the region given by $\max(|x|,|y|) \le 2$ and $e^{|x|}\left(|y|+\frac{1}{2}\right)$ is

a) 14 + *l*n 2 b) 14 – 2 *l*n 2 c) 14 + 2 *l*n 2 d) 14 – *l*n 2

9. The area bounded by $y = \cos^{-1}(\sin x)$ and $y = \sin^{-1}(\sin x)$ on the interval is ... (in sq. units)

a) $\frac{\pi^2}{9}$ b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{4}$ d) $\frac{\pi^2}{8}$

10. Area bounded by $y=\left[\sin^2\frac{x}{2}+\sqrt{\cos\frac{x}{2}}\right]$ and $y=\left[\sin x-\frac{1}{2}\right], x=0, x=\pi$ (where [.] represents the greatest integer function) is (in. sq. units)

a) $\frac{1}{6}\left(7\pi+12(1-\sqrt{3})\right)$ b) $7\pi+12(1+\sqrt{3})$ c) $\frac{\pi}{2}+1-\sqrt{3}$ d) $\frac{\pi}{2}+1+\sqrt{3}$

More than one correct answer type questions

11. If A_1 is the area bounded by $|x-a_1|+|y|=b_1, i \in N$ and $a_{i+1}=a_1+\frac{3}{2}b_1$ and $b_{i+1}=\frac{bi}{2}, a_1=0, b_1=32$ then

a) $A_3=128$ b) $\lim_{n\to\infty}\sum_{i=1}^{n}A_1=\frac{4}{3}(16)^2$ c) $\lim_{n\to\infty}\sum_{i=1}^{n}A_1=\frac{8}{3}(32)^2$ d) $A_3=256$

12. The area enclosed by the curves $x = a\sin^3 t$ and $y = \cos^3 t$ is equal to (in sq. units)

a) $12a\int_0^{\frac{\pi}{2}}\cos^4 t\sin^2 t\,dt$ b) $12a\int_0^{\frac{\pi}{2}}\cos^2 t\sin^4 t\,dt$ c) $2\int_{-a}^{a}\left(a^{\frac{2}{3}}-x^{\frac{2}{3}}\right)^{\frac{3}{2}}dx$ d) $4\int_0^{a}\left(a^{\frac{2}{3}}-x^{\frac{2}{3}}\right)^{\frac{3}{2}}dx$

Integer answer type questions

13. A point O moves in xy plane is such a way that $[1+x+y]=[x], x\in(0,2)$ and where [.] denotes greatest integer function then area of the region represented by all the points P, is equal to.

14. If $f(x)=\sin x, \forall \in \left[0,\frac{\pi}{2}\right]$; $f(x)+f(\pi-x)=2, \forall \in \left[\frac{\pi}{2},\pi\right]$ and $f(x)=f(2\pi-x), \forall \in (\pi,2\pi]$ then area enclosed by the curve $y = f(x)$ and x axis is $a\pi+b$ then find the value of a^2+b^2 = ...

15. If A_n is the area bounded by the curves $y = x$ and $y=x^n, n\in N-\{1\}$ in the first quadrant if (A_2) (A_3) (A_4) ..... $(A_n) = \frac{1}{(an^2+bn).2^{n+c}}$ then $a + b + c$ = ...

16. Let $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ ..... be points of inflection of the curve $x^2 - x + y - 1 = 0$. Area of polygon ABC .... is A_r, then $[A_r]$ is equal to _____ (where [.] denote the G.I.F)

KEY SHEET (ADDITIONAL EXERCISE)

LEVEL-I (MAIN)

1) **3**	2) **1**	3) **1**	4) **2**	5) **2**	6) **1**	7) **2**	8) **2**	9) **4**	10) **3**
11) **2**	12) **3**	13) **1**	14) **1**	15) **4**	16) **4**				

LEVEL-II

LECTURE SHEET (ADVANCED)

1) **b**	2) **b**	3) **d**	4) **b**	5) **c**	6) **b**	7) **c**	8) **2**	9) **a**	10) **ab**
11) **bd**	12) **b**	13) **a**	14) **a**	15) **c**	16) **d**	17) **a**	18) **b**		
19) **A-q;B-r;C-s;D-q**			20) **4**	21) **2**	22) **1**	23) **3**			

PRACTICE SHEET (ADVANCED)

1) **c**	2) **a**	3) **d**	4) **b**	5) **a**	6) **a**	7) **b**	8) **c**	9) **d**	10) **a**
11) **ac**	12) **acd**	13) **3**	14) **4**	15) **0**	16) **0**				