4. RANDOM VARIABLE & DISTRIBUTIONS

SYNOPSIS

1. Let S be a sample space of a random experiment. A real valued function $X : S \to R$ is called a random variable.

2. Let S be a sample space and $X : S \to R$ be a random variable. The function $F : R \to R$ denoted by $F(x) = P(X \leq x)$, is called probability distribution function of the random variable X.

3. A set E is said to be countable, if there exists a one - one correspondence between E and a sub-set of Natural numbers N

4. If a sample space is countable then it is called a discrete sample space. A real valued function defined on a discrete sample space is called a discrete random variable.

5. If $X:S \to R$ is a discrete random variable with range $\{x_1, x_2, x_3, ...\}$ then $\sum_{r=1}^{\infty} P(X = x_r) = 1$

6. Let $X : S \to R$ be a discrete random variable with range $\{x_1, x_2, x_3, ...\}$.

   If $\sum x_r P(X = x_r)$ exits,then $\sum x_r P(X = x_r)$ is called the mean of the random variable X. It is dentoed by μ or $\bar{x}$ or $E(x)$. If $\sum (x_r - \mu)^2 P(X = x_r)$ exist, then

   $\sum (x_r - \mu)^2 P(X = x_r)$ is called variance of the random variable X. It is denoted by σ^2. The positive square root of the variance is called the standard deviation of the random variable X. It is denoted by σ

7. If the range of discrete random variable X is $\{x_1, x_2,..., x_n,....\}$ and $P(X=x_n)=P_n$ for every positive integer n is given then $\sigma^2 + \mu^2 = \sum x_n^2 P_n = E(x^2)$ known as expectation of x^2.

8. Let n be a positive integer and p be a real number such that $0 \leq p \leq 1$. A random variable X with range $\{0, 1, 2, ..., n\}$ is said to follow (or have) binomial distribution or Bernoulli distribution with parameters n and p if $P(x = r) = {}^nc_r.\, p^r q^{n-r}$ for $r = 0, 1, 2, ... , n.$ where $q = 1-p$.

9. If the random variable X follows a binomial distribution with parameters n and p then mean of X is "np" and the variance is "npq" where $q = 1-p$.

10. Let $\lambda > 0$ be a real number. A random variable X with range $\{0, 1, 2, ...\}$ is said to follow (have) Poisson distribution with parameter λ if

    $P(X = r) = \dfrac{e^{-\lambda}.\lambda^r}{r!}$ for r = 0, 1, 2,....

11. If a random variable X follows Poisson distribution with parameter λ, then mean of X = variance of $X = \lambda$.

LECTURE SHEET

EXERCISE

Random Variable:

1. A random variable has the following distribution

$X(=x_i)$	1	2	3	4
$P(X = x_i)$	k	$2k$	$3k$	$4k$

The value of k and $P(x < 3)$ are equal to

1) $k=\frac{1}{10}$, $P(x<3)=\frac{3}{5}$ 2) $k=\frac{1}{10}$, $P(x<3)=\frac{3}{10}$

3) $k=\frac{3}{10}$, $P(x<3)=\frac{1}{10}$ 4) $k=\frac{1}{10}$, $P(x<3)=\frac{5}{12}$

2. The value of c for which $P(x= k) = ck^2$ can serve as the probability distribution function of a random variable X that takes values 0,1,2,3,4 is

1) $\frac{1}{10}$ 2) $\frac{1}{15}$ 3) $\frac{1}{20}$ 4) $\frac{1}{30}$

3. The range of random variable $x=\{1,2,3 \,.....\}$ and the probabilities are given by $P(x=k)=\frac{c^k}{k!}$ then $c =$

1) $\log_e 2$ 2) $\log_e 3$ 3) $\log_3 2$ 4) $\log_2 3$

4. Two coins whose faces are marked 3 and 4 are tossed. The mean value of the total value of the numbers is

1) 7 2) 6 3) 5 4) 3

5. A pair of dice is thrown at a time. X is the maximum of the two numbers shown on the dice. Then mean of X is

1) $\frac{151}{36}$ 2) $\frac{161}{36}$ 3) $\frac{141}{36}$ 4) $\frac{131}{36}$

6. A sample of 2 items is selected at random from a bag containing 5 items of which 2 are defective. Then mean of number of defective items is

1) $\frac{4}{5}$ 2) $\frac{1}{5}$ 3) $\frac{2}{5}$ 4) $\frac{3}{5}$

7. Let x denote the profit of a business man. The probability of getting profit Rs 3000 is 0.6. The probability of getting loss Rs. 4000 is 0.3. The probability of getting neither profit nor loss is 0.1; the mean and variance of x are

1) 100, 182000000 2) 4,00, 4560000

3) 400, 12300 4) 600, 984 0000

8. Two cards are drawn successively one by one with out replacement from a pack of cards. The mean of number of kings is

1) $\frac{1}{13}$ 2) $\frac{2}{13}$ 3) $\frac{3}{13}$ 4) $\frac{4}{13}$

Binomial Distribution :

9. If the difference between the mean and variance of a binomial distribution for 5 trails is $\frac{5}{9}$ then the distribution is

1) $\left(\frac{2}{5}+\frac{3}{5}\right)^5$ 2) $\left(\frac{2}{3}+\frac{1}{3}\right)^5$ 3) $\left(\frac{1}{3}+\frac{2}{3}\right)^5$ 4) $\left(\frac{3}{4}+\frac{1}{4}\right)^5$

10. If a binomial distribution has mean 2.4 and variance is 1.44, then $n =$

1) 10 2) 6 3) 16 4) 20

11. If the mean of the binomial distribution is 100. Then standard deviation lies in the interval

1) [0, 7) 2) [1, 7) 3) [0, 10) 4) [1, 11)

12. The least number of times a coin must be tossed so that the probability of getting atleast one head is atleast 0.8 is

1) 6 2) 5 3) 4 4) 3

13. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than one is equal to

1) $\frac{1}{16}$ 2) $\frac{5}{16}$ 3) $\frac{11}{16}$ 4) $\frac{15}{16}$

14. Suppose X follows binomial distribution with parameters n=100 and $p = \frac{1}{3}$ then $P(x = r)$ is maximum when $r =$

1) 50 2) 32 3) 33 4) 67

15. When a coin is tossed n times and the proabability for getting 6 heads is equal to the probability of getting 8 heads, then the value of n is

1) 10 2) 12 3) 14 4) 20

16. One hundred identical coins each with probability p of showing up head, are tossed once. If $0<p<1$ and probability of heads showing on 50 coins is equal to that of showing on 51 coins then the value of p is

1) $\frac{1}{2}$ 2) $\frac{49}{101}$ 3) $\frac{50}{101}$ 4) $\frac{51}{101}$

17. A die is thrown $(2n + 1)$ times. The probability of getting 1 or 3 or 4 atmost n times is

1) $\frac{1}{n}$ 2) $\frac{1}{2n+1}$ 3) $\frac{n}{2n+4}$ 4) $\frac{1}{2}$

18. The probability that a student is not a swimmer is $\frac{1}{5}$. The probability that out of 5 students exactly 4 are swimmers is

1) $\left(\frac{4}{5}\right)^3$ 2) $\left(\frac{4}{5}\right)^4$ 3) $5_{C_4}\left(\frac{4}{5}\right)^4$ 4) $\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^4$

19. The probability that a candidate secure a seat in Engineering through EAMCET is $\frac{1}{10}$. Seven candidate are selected at random from a centre. The probability that exactly two will get seats is

1) $15\ (0.1)^2\ (09)^5$ 2) $20\ (0.1)^2\ (0.9)^5$

3) $21\ (0.1)^2\ (0.9)^5$ 4) $(0.1)^2\ (0.9)^5$

20. In an aveage rain falls on 12 days in every 30 days. The probability that rain will fall on just 3 days of a given week is

1) $35\left(\frac{1}{5}\right)^3\left(\frac{3}{5}\right)^4$ 2) $35\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^4$ 3) $35\left(\frac{1}{5}\right)^3\left(\frac{2}{5}\right)^4$ 4) $35\left(\frac{1}{5}\right)^2\left(\frac{3}{5}\right)^4$

21. The probability of a man hitting a target is $\frac{3}{4}$. He makes 5 trials. The probability that he will hit the target every time he tries is

1) $\frac{243}{1024}$ 2) $\frac{81}{1024}$ 3) $\frac{243}{256}$ 4) $\frac{241}{256}$

22. A box contains tickets numbered from 1 to 20. If 3 tickets are drawn one by one with replacement then the probability of getting prime number exactly 2 times is

1) $\frac{36}{125}$ 2) $\frac{12}{125}$ 3) $\frac{1}{125}$ 4) $\frac{4}{125}$

23. One in 9 ships is likely to be wrecked, when they are set on sail. When 6 ships set on sail, the probability for exactly, 3 will not arrive safely is

1) $\frac{20\times 8^3}{9^6}$ 2) $1-\frac{1}{9^6}$ 3) ${}^6C_3\left(\frac{8^3}{9^6}\right)$ 4) ${}^6C_3\left(\frac{8^6}{9^3}\right)$

24. 12 coins are tossed 4096 times. The expected number of times that one can get atleast 2 heads is

1) 4080 2) 4081 3) 4082 4) 4083

Poission Distribution :

25. A poisson variate x is such that $P(x=2) = 9\ P(x=4) + 90.P(x=6)$ then mean and standard deviation are

1) 1,1 2) 1, 2 3) 2, 2 4) $2, \sqrt{2}$

26. Suppose on an average 1 house in 1000 in a certain district has a fire during a year. If there are 2000 houses in that district, the probability that exactly 5 houses will have a firing during a year

1) $\frac{1}{15e^2}$ 2) $\frac{14}{15e^2}$ 3) $\frac{4}{15e^2}$ 4) e^{-2}

27. If the number of telephone calls an operator receives between 10.00 pm to 10.10 pm follow poisson distrbution with mean 3. The probability that the operator receives one call during that interval the next day is

1) $\frac{e^3}{3}$ 2) e^{-3} 3) $\frac{e^{-3}}{3}$ 4) $3e^{-3}$

28. The probability that an individual suffers a bad reaction from an injection is 0.001. The probability that out of 2000 individuals more than 2 suffer a bad reaction is

1) $\frac{5}{e^2}$ 2) $1-\frac{5}{e^2}$ 3) $1-\frac{4}{e^2}$ 4) $\frac{4}{e^2}$

29. In a book of 500 pages it is found that there are 250 typing errors. Assume that poission law holds for the number of errors per page. Then the probability that a random sample of 2 pages will contain no error is

1) $e^{-0.3}$ 2) $e^{-0.5}$ 3) e^{-1} 4) e^{-2}

30. A firm hires out 3 cars every day. The demand for a car on each day is distributed as a poisson distribution with mean 2.0. The probability that some demand is refused on a day is ($e^{-2} = 0.135$)

1) 0.2177 2) 0.1912 3) 0.1455 4) 0.0192

31. Six coins are tossed 6400 times. The probability of getting 6 heads x times using poisson distribution is

1) $6400e^{-x}$ 2) $\frac{6400e^{-x}}{\angle x}$ 3) $\frac{e^{-100}100^x}{x!}$ 4) e^{-100}

32. Cycle tyres are supplied in lots of 10 and there is a chance of 1 in 500 tyres to be defective. Using Poisson's distribution the approximate number of lots containing no defective tyres in a consignment of 10,000 lots is

1) 9980 2) 9998 3) 9802 4) 9982

33. In a poisson distribution, the variance is m. The sum of terms in odd places in the distribution is

1) e^{-m} 2) $e^{-m}\cosh(m)$ 3) $e^{-m}\sinh(m)$ 4) $e^{-m}\coth(m)$

34. If three letters are placed in three addressed envelopes then the mean and variance of X where X denotes the number of correct dispatches.

1) 1,1 2) 1,2 3) 2, 1 4) 2, 2

Numerical value type questions

35. X follows a binomial distribution with parameters $n = 6$ and p. If $4P(x = 4) = P(x = 2)$ then $p =$

36. A die is rolled 3 times. If getting six is considered as success. The probability of atleast 2 successes is

37. The probability that a bomb dropped from a plane strikes the target is $\frac{1}{5}$. The probability that out of six bombs dropped atleast 2 bombs strike the target is

38. The probability of happening of an event in a trial is 0.5. The probability of happening of the event atleast once if the experiment is repeated 3 times is

PRACTICE SHEET

EXERCISE

Random Variable:

1. A random varibale x has its range {0, 1, 2} and the probabilities are given by $P(x=0) = 3k^3$; $P(x=1) = 4k-10k^2$; $P(x=2)= 5k-1$ where k is constant then $K =$

   1) 1 2) –1 3) $\frac{1}{3}$ 4) $\frac{2}{3}$

2. The range of a random variable $x=$ {1,2,3....} and the probabilities are given by $P(x=k)=\frac{3^{CK}}{K!}$ (k=1,2,3.....) and C is a constant. Then $C =$

   1) $\log_3(\log 2)$ 2) $\frac{1}{2}\log(\log 2)$ 3) $\frac{\log_e(\log 2)}{\log_3^e}$ 4) $\log_2(\log 3)$

3. If the range of a random variable X is {0,1,2, 3, 4......} with $P(x=k) = \frac{(k+1)a}{3^k}$ for $k \geq 0$ then $a =$

   1) $\frac{2}{3}$ 2) $\frac{4}{9}$ 3) $\frac{8}{27}$ 4) $\frac{16}{81}$

4. A random variable x takes the values 0, 1, 2, 3 and its mean is 1.3. If $P(x=3)=2\,P(x=1)$ and $P(x = 2) = 0.3$, then $P(x = 0) =$

   1) 0.1 2) 0.2 3) 0.3 4) 0.4

5. A die is rolled and let x denote twice the number appearing on its face. Mean of x is

   1) 6 2) 7 3) 4 4) 5

6. A player tosses two coins. He wins Rs.1 if 1 head appears, Rs.2 if 2 heads appear. But he lose Rs 5 if no head appears. The mean of the prized money is

   1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $-\frac{1}{4}$ 4) $\frac{1}{5}$

7. If the standard deviation of the binomial distribution $(q+p)^{16}$ is 2 then mean is

   1) 16 2) 8 3) 4 4) 6

8. The probablity of hitting a target is 1/ 3. The least number of times to fire so that the probability of hitting the target atleast once is more than 90% is

   1) 4 2) 5 3) 6 4) 7

9. Suppose x follows binomial distribution with parameters n and p where $0 < p < 1$. If $\frac{P(x=r)}{P(x=n-r)}$ is independent of n and r then $p =$

   1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) 1

10. Suppose x follows binomial distribution with parameters $n = 100$ and $p = \frac{1}{2}$ then $P(x = r)$ is maximum when $r =$

    1) 50 2) 32 3) 33 4) 67

11. A fair coin is tossed n times. Let x be random variable denoting the number of heads tossed. If $P(x=4)$, $P(x=5)$, $P(x=6)$ are in A.P. then $n=$

1) 7 2) 10 3) 14 4) 7 or 14

12. For a binomial variate X with $n=6$ if $P(x=2)=9(P(x=4))$ then its variance is

1) $\frac{8}{9}$ 2) $\frac{1}{4}$ 3) $\frac{9}{8}$ 4) 4

13. Suppose A and B are two equally strong table tennis players. The probability that A beats B in exactly 3 games out of 4 is

1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{1}{8}$ 4) $\frac{3}{4}$

14. Five coins are tossed 3200 times. The expected number of times we get exactly two heads is

1) 600 2) 1000 3) 2000 4) 1500

Poission Distribution :

15. A manufacturing concern emplyoing a large number of workers over a period of time and the average absentee rate is 2 workers per shift then probability that exactly two workers will be absent is

1) $\frac{1}{e^2}$ 2) $\frac{2}{e^2}$ 3) $\frac{4}{e^2}$ 4) $2e^2$

16. If X is a Poisson variate and $E(X^2)=6$, then $E(X)=$

1) -3 2) 2 3) -3 or 2 4) 3

17. Suppose x follows binomial distribution with parameters $n=8$ and $p=\frac{1}{2}$ then $P(|x-4|\le 2)$ is

1) $\frac{114}{123}$ 2) $\frac{119}{128}$ 3) $\frac{7}{33}$ 4) $\frac{103}{124}$

18. Suppose $X \sim B(n, p)$ and $P(X=3)=P(X=5)$. If $p>\frac{1}{2}$ then

1) $5 \le n \le 7$ 2) $n>8$ 3) $n \ge 9$ 4) $n<10$

19. In a hurdle race a player has to cross 10 hurdles. The probability that he will clear each hurdle is 5/6. The probability that he will knock down fewer than two hurdles is

1) $\frac{2}{5}\times\frac{6^9}{15^{10}}$ 2) $\frac{3\times 6^9}{5^{10}}$ 3) $\frac{3\times 5^{10}}{6^{10}}$ 4) $\frac{1}{2}$

20. Six dice are thrown 729 times. The number of times you expect atleast 3 dice to show either 5 or 6 is

1) 233 2) 249 3) 396 4) 433

Numerical value type questions

21. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X=1)$ is

22. If the mean of a binomial distribution with 9 trials is 6, then its variance is

23. An unbiased die is tossed 6 times. The mean of number of odd numbers is

KEY SHEET

LECTURE SHEET

1) **2**	2) **4**	3) **1**	4) **1**	5) **2**	6) **1**	7) **4**	8) **2**	9) **2**	10) **2**
11) **3**	12) **4**	13) **3**	14) **3**	15) **3**	16) **4**	17) **4**	18) **2**	19) **3**	20) **2**
21) **1**	22) **1**	23) **3**	24) **4**	25) **1**	26) **3**	27) **4**	28) **2**	29) **3**	30) **3**
31) **3**	32) **3**	33) **2**	34) **1**	35) **0.33**	36) **0.07**	37) **0.35**		38) **0.88**	

PRACTICE SHEET

1) **3**	2) **1**	3) **2**	4) **4**	5) **2**	6) **3**	7) **2**	8) **3**	9) **1**	10) **1**
11) **4**	12) **3**	13) **2**	14) **2**	15) **2**	16) **2**	17) **2**	18) **1**	19) **3**	20) **1**
21) **0.03**	22) **2**	23) **1**							