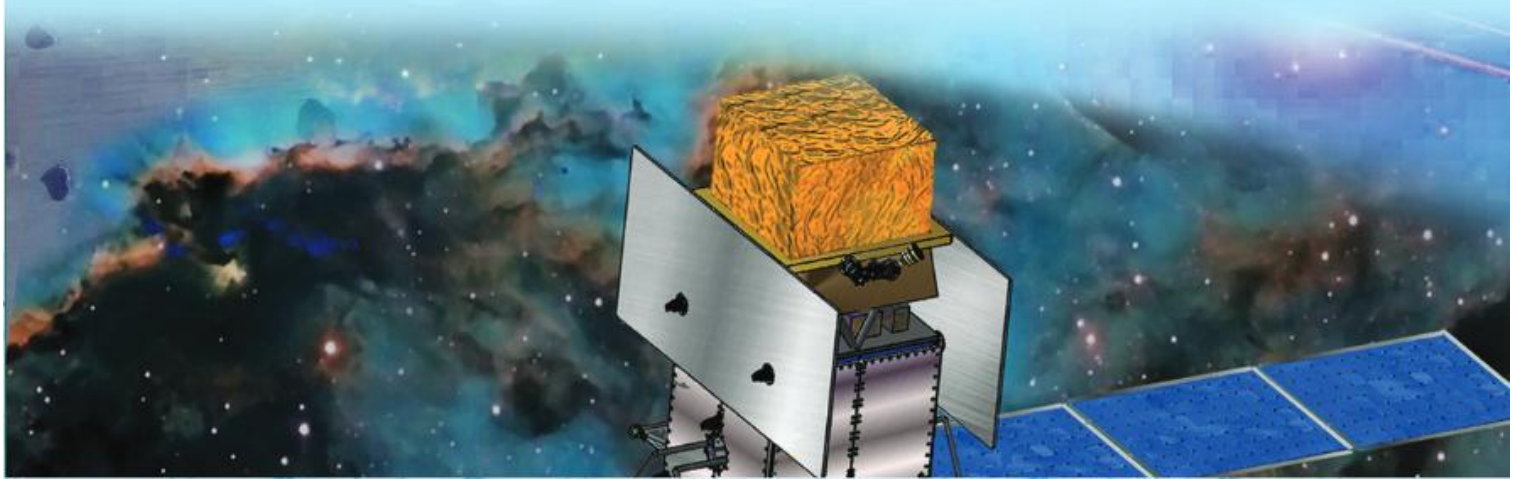
Chapter - 2

RAY OPTICS

- Reflection of light at Plane and Curved Surfaces
- Refraction of light at Plane Surface
- Total Internal Reflection of light
- Refraction of light at Curved Surface
- Lens theory
- Prism theory
- Pure and Impure Spectrum



2.1 INTRODUCTION

Light is a form of energy that produces the sensation of vision. Light itself is invisible. When it falls on an object the scattered light enters the eye and the object becomes visible. In this process the light undergoes modifications exhibiting its properties.

The branch of physics which describes about phenomenon and laws associated with generation and propagation of light and its interaction with matter is called optics and it is divided into two parts namely geometrical optics (ray optics) and physical optics.

The path along which light energy travels is called a ray. In a homogeneous medium a ray may be represented by a straight line with an arrow head in the direction in which the energy travels. The study of light based on the concept of rays with the help of geometrical principles is called Geometrical optics.

Geometrical optics is based on three general laws. (1) The law of rectilinear propagation of light. (2) The laws of refection. (3) The laws of refraction.

Law of rectilinear propagation of light : In a homogeneous medium light travels along a straight line.

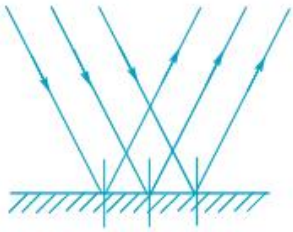
Reflection and Refraction : When light travelling in a medium is incident on a surface separating the medium from another medium it will undergo the following changes :

i) a part of the light comes back in to the same medium from the surface in a definite direction. This part of the light is called reflected part and this phenomenon is called reflection.

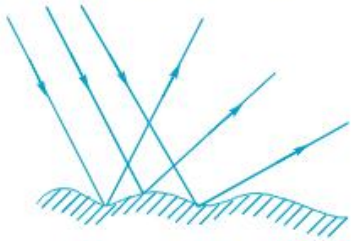
ii) Another part of the light enters the second medium at the surface which is the refracted part and the phenomenon is called refraction,

Types of Reflection

1) When the separating surface is plane, a parallel beam of light incident on it will remain parallel even after reflection as shown in Fig [2.1 (a)]. This type of reflection is said to be regular. Reflection from a plane mirror is an example of this type.



(a) Regular Reflection



(b) Diffuse Reflection

Fig 2.1

2) If the separating surface is uneven, the normal at different points will be in different directions. But the incident rays of light at different points

obey the laws of reflection. So, the rays that were parallel before reflection will be reflected in random directions as shown in Fig 2.1(b). This kind of reflection is called diffuse reflection. We can see an object due to diffuse reflection of light by the object.

Refraction of light is the bending of light when it travels from one medium to another medium. The various phenomena that occur when light travels from a denser medium to a rarer medium have been discussed in this chapter.

Refraction through prisms, defects in images due to lenses, optical instruments, dispersion of light and spectra have also been discussed in this chapter

Light exhibits a dual nature. In some phenomena like interference, diffraction and polarization, it behaves like waves; and in others like photoelectric effect, it behaves as particles. Correspondingly, there are two groups in physical optics-wave optics and quantum optics. Wave optics deals with the wave nature of light; interference, diffraction, and polarization are discussed in subsequent chapters. Quantum optics deals with light as made up of tiny packets of energy called quanta. Photoelectric effect and atomic spectra requiring quantum nature of light for their explanation, are discussed under modern physics.

2.2 THEORIES OF LIGHT

Explanation of known facts about light such as (1) light is a form of energy (2) light travels through vacuum (3) light travels in straight lines (4) reflection of light and (5) refraction of light led to different theories regarding the nature of light. The fact that light, as a form of energy, is propagated from one point to another led to two general theories corresponding to the two methods of transfer of energy namely either by a stream of particles in motion or by means of waves. Two theories of light thus came into existence, one called the corpuscular theory and the other called the wave theory.

2.2.1 NEWTON'S CORPUSCULAR THEORY OF LIGHT

The corpuscular theory of light proposed by Newton assumed that light consists of tiny particles called corpuscles. According to this theory of light.

1. A source of light emits tiny particles called corpuscles in all directions.
2. Different colours of light are due to different sizes of corpuscles.
3. The corpuscles travel in straight lines with great speed, when no force acts. This explains the law of rectilinear propagation of light.
4. The mechanical action of the corpuscles on the retina causes the sensation of light.
5. When the corpuscles approach close to the surface of a material medium, the path of the corpuscles is modified.
6. Reflection of light is due to the force of repulsion applied by the particles of the denser medium on the corpuscles.
7. Refraction of light is due to the force of attraction of the particles of the denser medium such as glass or water on the corpuscles.

Failures of the Theory

1. Partial reflection and refraction is explained by assuming the some of the particles of the denser medium are in a 'fit of repulsion' and others are in a 'fit of attraction' so that reflection and refraction are both produced simultaneously. This explanation is not convincing.
2. According to Newton's corpuscular theory, when light is refracted due to force of attraction, the velocity of light in denser medium must be greater than the velocity of light in rarer medium. But, Foucault's rotating mirror experiment showed that the velocity of light in denser medium is less than the velocity of light in rarer medium.
3. When a source of light emits corpuscles, in due course of time the mass of the source of light should decrease which is not true experimentally.

2.2.2 WAVE THOERY

The wave theory was first stated in 1678, in a scientific manner by Christian Huygens (1629-695), a dutch physicist. According to this theory, a luminous body is a source of disturbance and the disturbance is propagated in the form of waves. As it is difficult to think of wave motion without a medium, Huygens postulated the existence of a hypothetical medium called ether. This 'ether' fills empty space and also material media. Hence light wave considered to consist of elastic or mechanical waves moving through the hypothetical medium ether. This theory explains reflection and refraction. But the explanation of refraction requires that the velocity of light in a rarer medium be greater than that in a denser medium. This was supported by Foucault's experiment in 1850. The phenomenon of interference was explained by Thomas Young (1773 - 1829), a British physicist on the basis of wave theory. The wave theory could not explain the rectilinear propagation of light. This was a great stumbling block for wave theory.

In 1818, Fresnel (1788 - 1827), a French physicist explained successfully not only the approximate rectilinear propagation of light but also the diffraction phenomenon on the basis of Huygen's wave theory of light. Polarization of light cannot be explained on the basis of Huygen's wave theory which assumes light waves to be longitudinal. This difficulty was overcome when Fresnel suggested that light waves are trasverse and not longitudinal.

A wave or disturbance starting from a source (or origin in general) propagates in all possible directions and arrives at a large number of points during the same time. A surface passing through these points in the same phase, is called a wave front. For a point source, at finite distance, the wave front is spherical and is a plane one at a very large (almost infinite) distance (Fig 2.2). In the case of a narrow linear source (such as a narrow illuminated slit), the wavefront is cylindrical (Fig 2.3.)

A ray of light is a line drawn to indicate the direction of light energy; hence at any point, ray will be normal to the wave front.

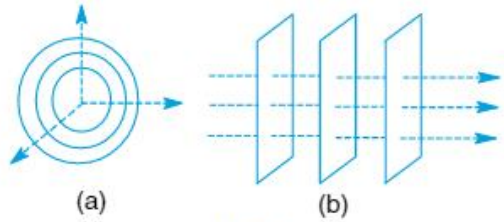


Fig 2.2

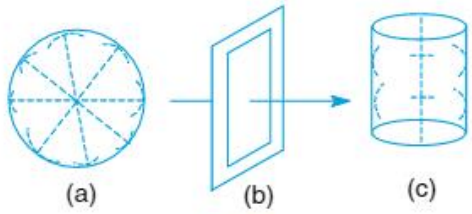


Fig 2.3

In order to explain various phenomena, Huygens assumed that every point on a wave front, acts as a source of secondary wavelets. This principle is referred to as Huygen's principle. To find the wave front at a later instant, each point on the wave front at a given instant, is considered as centre, and spheres of radii equal to distance travelled by light in the given time are constructed. The tangential surface to these spheres, gives the wave front at the desired instant. This construction called Huygen's construction, enables various phenomena like reflection, refraction, double refraction, etc., to be explained. (Fig 2.4.)

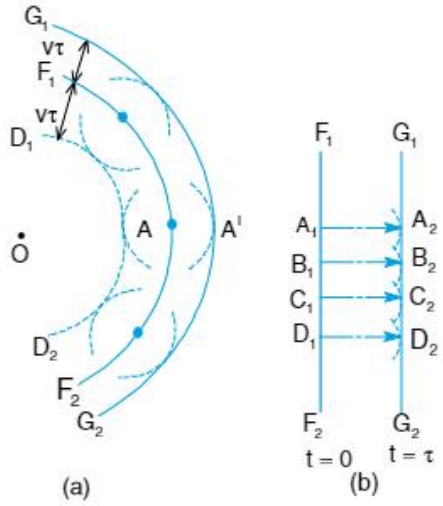


Fig 2.4

The wave theory stated by Huygens was finally established by the efforts of Young, Fresnel and others.

2.2.3 MAXWELL'S ELECTROMAGNETIC THEORY

Maxwell proposed that light is propagated in the form of electromagnetic waves. This is known as Maxwell's electromagnetic wave theory of light.

Electromagnetic waves are generated by accelerated charges. An accelerated charge produces sinusoidal time varying electric field which is associated with sinusoidal time varying magnetic field. The two fields are mutually perpendicular to each other and each varying field producing the other, constituting an electromagnetic wave. If the wave is propagating along X - axis, the electric field is represented by

$$E = E_0 \text{Sin}\omega\left(t - \frac{x}{c}\right)$$

Where c is the speed of light in vacuum and E_0 is the maximum value of the electric field. This is similar to the progressive wave equation of a sound wave. The magnetic field is represented by

$$B = B_0 \text{Sin}\,\omega\left(t - \frac{x}{c}\right)$$

Where B_0 is the amplitude of the magnetic field. Both E and B are functions of position x and time t. If E is along Y - axis then B will be along Z - axis as shown in Fig 2.5.

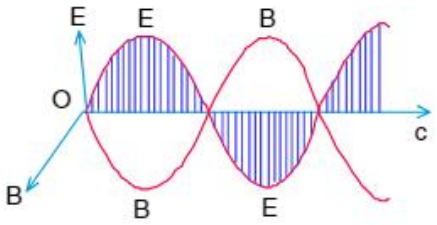


Fig 2.5

The characterisitics of electromagnetic waves are

1. Sinusoidal time varying electric and magnetic fields that are perpendicular to each other and also perpendicular to the direction of propagation of the wave constitute an electro-magnetic wave i.e., electromagnetic waves are transverse by nature.
2. Electromagnetic waves do not require any material medium for their propagation.
3. The relation between the maximum values of the electric and magnetic fields is $E_0 = cB_0$ where c is the velocity of light in vacuum.
4. The average electric energy density is equal to the average magnetic energy density.
5. The intensity (I) of an electromagnetic wave is equal to the product of average electric or magnetic energy density and velocity of light.
6. Electromagnetic waves travel through a medium with the velocity of light in that medium.

The velocity of light in vacuum is related to the absolute permeability μ_0 and absolute permittivity ε_0 of free space as $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$.

The velocity of light in any medium is $v = \frac{1}{\sqrt{\mu\varepsilon}}$ where μ and $\in$ are the absolute permeability and absolute permittivity of the medium respectively. Only the electric field vector is responsible for all the optical properties. So, the electric field vector is called light vector.

Electromagnetic Spectrum: The electromagnetic waves have wavelengths between 10^{-15} m and 30 km. The electromagnetic waves known are separated into different categories : (i) γ-rays, (ii) X-rays, (iii) Ultra - violet rays, (iv) Visible light, (v) Infrared rays, (vi) Micro waves and (vii) Radio waves. The boundaries in terms of wave length separating different types of radiations are not sharply defined. Even though on an average wavelength of γ - rays are shorter than X-rays, some of the wavelengths of X-rays overlap with that of γ-rays.

The arrangement of electromagnetic waves according to different categories is called electromagnetic spectrum. It is shown below.

γ rays	x rays	U.V Region	Light	I.R Region	Micro waves	Radio waves

0.1Å 100Å 4000Å 7000Å 1mm 1m

Fig 2.6

As we move from the left end to the right end of the electromagnetic spectrum the frequency decreases and the wavelength increases. The relation between the frequency v and wavelength λ is given by

$$v\lambda = c$$

2.2.4 QUANTUM THEORY

Although electromagnetic theory was found to explain all phenomena connected with the propagation of light, it failed completely to give satisfactory explanation of the phenomena which result from the interaction of light with matter, such as photoelectric effect, emission and absorption spectra and black body radiation.

Photoelectric effect was discovered as early as 1887 by Hertz. This phenomenon could not be explained on the basis of wave theory. Einstein in 1905, gave a satisfactory explanation of this effect, by extending the ideas of quantum hypothesis proposed by a German physicist Max Planck in 1900, in connection with black body radiation. He postulated that radiation consists of small packets of energy which he called quanta. These quanta travel with the velocity of light. The energy E of a quantum is given by relation $E = h\upsilon$, where υ is the frequency of radiation and $h = 6.625 \times 10^{-34}$ **joule-sec.** is called the Planck's constant. These considerations formed the basis for Quantum theory. This theory also helped in the explanation of the origin of spectra given by a Danish physicist Niels Bohr in 1913. Compton scattering of X-rays, discovered in 1923 and Raman effect discovered in 1928, confirmed the quantum nature of radiation.

2.2.5 REFRACTION OF A PLANE WAVE AT PLANE SURFACE

We use Huygen's principle to derive the laws of refraction. Let PP' represents the surface separating medium 1 and medium 2 as shown in figure. Let v_1 and v_2 represent the speed of light in medium 1 and medium 2 respectively. We assume a plane wavefront A'B' propagating in the direction A', incident on the interface at an angle 'i' as shown in the Fig 2.7. Let 'τ' be the time taken by the wave front to travel the distance BC, then $BC = v_1\tau$.

In order to determine the shape of the refracted wavefront, we draw a sphere of radius $v_2\tau$ from the point A in the second medium. Let CE represent a tangent plane drawn from the point 'C' on to the sphere. Then $AE = v_2\tau$ and CE would represent the refracted plane wavefront. If we now consider the triangles ABC and AEC, we obtain

$$\sin i = \frac{BC}{AC} = \frac{v_1\tau}{AC} \text{ and } \sin r = \frac{AE}{AC} = \frac{v_2\tau}{AC}$$

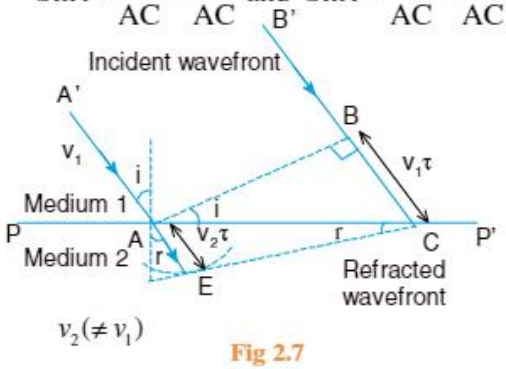


Fig 2.7

where i and r are the angles of incidence and refraction respectively.

Thus we obtain $\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$

If c represents the speed of light in vacuum, then $\mu_1 = \frac{c}{v_1}$ and $\mu_2 = \frac{c}{v_2}$ are known as the refractive indices of medium 1 and medium 2 respectively.

$$\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} \text{ Hence } \mu_1 \sin i = \mu_2 \sin r$$

This is the Snell's law of refraction.

2.2.6 REFLECTION OF A PLANE WAVE BY A PLANE SURFACE

We consider a plane wave A'B' incident at an angle 'i' on a reflecting surface MN. If v represents the speed of the wave in the medium and if 'τ' represents the time taken by the wavefront to advance from the point B to C, then the distance BC = $v\tau$. In order to construct the reflected wavefront we draw a sphere of radius $v\tau$ from the point 'A' as shown in Fig 2.8.

Let CE represent the tangent plane drawn from the point 'C' to this sphere. Obviously $AE = BC = v\tau$ and CE represents the reflected plane wavefront.

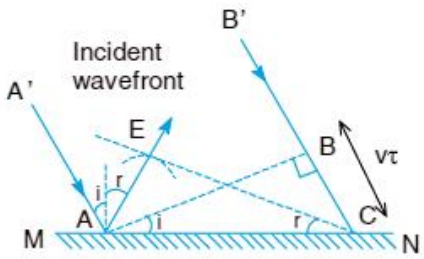


Fig 2.8

From triangle BAC, $\sin i = \frac{BC}{AC}$ and triangle EAC, $\sin r = \frac{AE}{AC} = \frac{BC}{AC}$ (as AE = BC)

Hence Sin i = Sin r ; Therefore $\angle i = \angle r$

This is the law of reflection.

Example-2.1

The magnetic field of an electromagnetic wave in a substance is given by $B = (2 \times 10^{-6}T) \cos [\pi(0.04x + 10^7 t)]$. Find the electromagnetic wave speed.

Solution :

Comparing the given equation with the standard wave equation $B = B_0 \cos(\omega t + kx)$

we have, $\omega = \pi \times 10^7$ rad/s and $k = \pi \times (0.04) m^{-1}$

∴ Speed of electromagnetic wave in this medium is

$$v = \frac{\omega}{k} = 2.5 \times 10^8 m/s$$

2.3 REFLECTION

When light travelling in a medium is incident on a surface separating the medium (boundary) most of incident light comes back in the same medium. This phenomenon is called reflection and the boundary as reflector. Because of reflection only non-luminous objects become visible.

At the point of incidence O, the angle between incident ray and normal is called angle of incidence (i) while the angle between reflected ray and normal angle of reflection (r). The angle (90–i), i.e., the angle which the incident ray makes with the plane reflecting surface is called glancing angle. The plane containing incident ray and normal is called plane of incidence. In case of reflection,

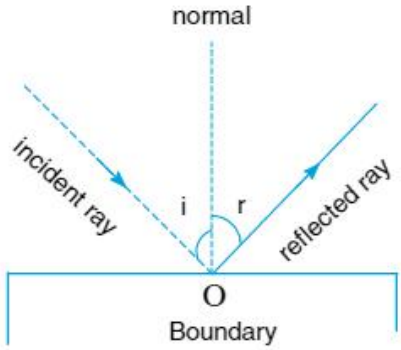


Fig 2.9

(i) The incident ray, reflected ray and normal to the reflection surface at the point of incidence all lie in the same plane.

(ii) The angle of incidence is equal to the angle of reflection. i.e., $\angle i = \angle r$.

These laws are called laws of reflection and hold good for all reflecting smooth surfaces either plane or curved.

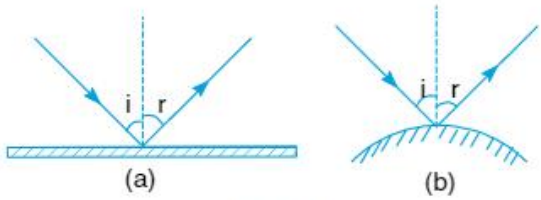


Fig 2.10

Here it is note worthy that in case of reflection

(a) If $\angle i = 0$ then $\angle r = 0$ i.e., if a ray is incident normally on a boundary, after reflection it retraces its path as shown in Fig 2.11.

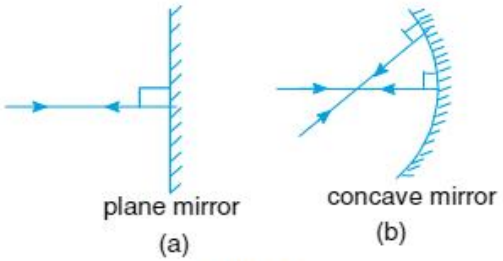


Fig 2.11

(b) There is a phase change 'π' if reflection occurs from a denser medium, no phase change if reflection occurs from a rarer medium.

(c) None of frequency, wavelength and speed changes.

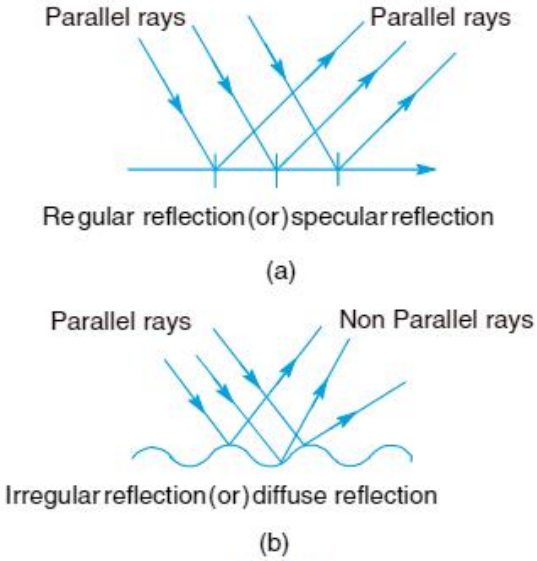


Fig 2.12

(d) On a smooth plane boundary, a parallel beam of light incident will remain parallel even after reflection as shown in Fig 2.12(a). This is known as regular reflection (or) specular reflection and is useful in determine the quality of mirrors. If the boundary is uneven (rough), parallel beam of light is reflected in random directions as shown in Fig 2.12(b). This kind of reflection is called diffuse reflection.

The laws of reflection are obeyed in specular reflection. They do not hold in case of irregular or diffuse reflection. We see non-luminous objects by diffuse reflection only.

2.4 REAL AND VIRTUAL OBJECT

If a surface is incident with a divergent beam, it means a real object is placed in front of the surface at the position wherefrom the rays are diverging as shown in Fig 2.13(a).

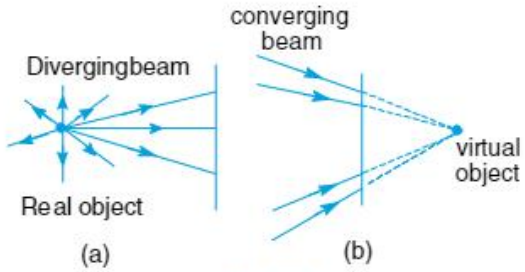


Fig 2.13

If a surface is incident with a converging beam, it means a virtual object is placed behind the surface at the position where the rays appear to converge as shown in the Fig 2.13 (b). In case of image formation, unless stated, object is taken to be real, It may be point object denoted by dot (•) or extended object denoted by an arrow (↑).

2.5 REAL AND VIRTUAL IMAGE

The optical image is a point, where the rays of light either intersect or appear to intersect. If the real rays after reflection or refraction actually converge at a point, the image is said to be real as shown in Fig 2.14 (b). However if the rays do not actually converge but appear to do so, the image is said to be virtual. This is as shown in Fig 2.14 (a).

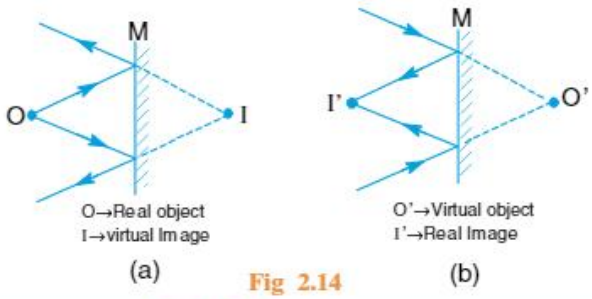


Fig 2.14

The eye sees a virtual image as well as it sees a real image. A real image contains light energy, hence it can be seen on a screen and can be photographed. On the other hand, a virtual image cannot be received on a screen.

2.6 REFLECTION FROM PLANE SURFACES

When you look into a plane mirror, you see an image of yourself that has three properties

(i) The image is up right

(ii) The image is of the same size as you are

(iii) The image is located as far behind the mirror as you are infront of it. This is shown in Fig 2.15 (b).

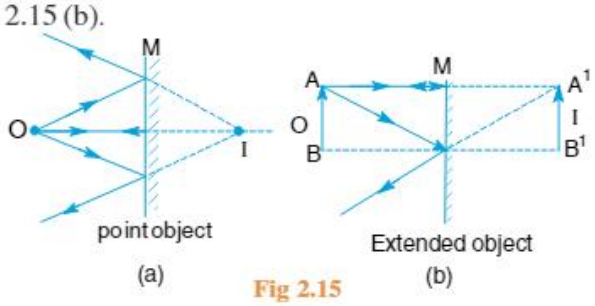


Fig 2.15

It is worthy to note that

1. A plane mirror always forms a virtual image of a real object and vice versa and the line joining object and image is perpendicular to the mirror as shown in Fig 2.16(a).

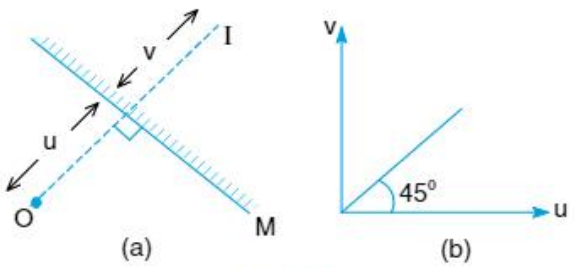


Fig 2.16

The graph between image distance (v) and object distance (u) for a plane mirror is a straight line as shown in Fig 2.16 (b).

The ratio of image height to the object height is called lateral magnification (m). Thus in case of plane mirror 'm' is equal to one.

2. The principle of reversibility states that rays retrace their path when their direction is reversed. In accordance with the principle of reversibility object and image positions are interchangeable. The points corresponding to object and image are called conjugate points. This is illustrated in Fig 2.17.

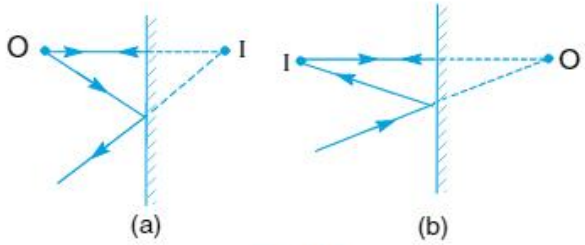


Fig 2.17

3. A mirror, whatever may be the size, forms the complete image of the object lying in front of it. Large mirror gives more bright image than a small one. It is seen that the size of the reflector must be much larger than the wavelength of the incident light, otherwise the light will be scattered in all direction.

4. The angle between directions of incident ray and reflected or refracted ray is called deviation (δ). A plane mirror deviates the incident light through angle $\delta = 180 - 2i$ where 'i' is the angle of incidence. The deviation is maximum for normal incidence, hence $\delta_{max} = 180^0$.

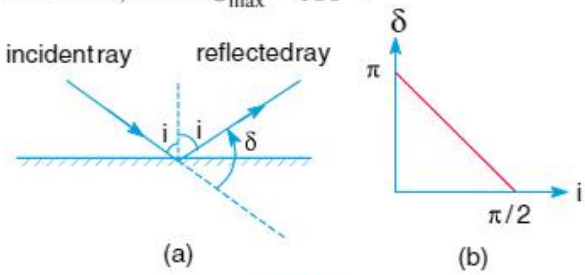


Fig 2.18

Note that, generally, anti-clock wise deviation is taken as positive and clock wise deviation as negative.

5. Every object has its own field of view for the given mirror. The field of view is the region between the extreme reflected rays and depends on the location of the object in front of the mirror.

If our eye lies in the field of view only we can see the image of the object, not otherwise. This is illustrated in Fig 2.19.

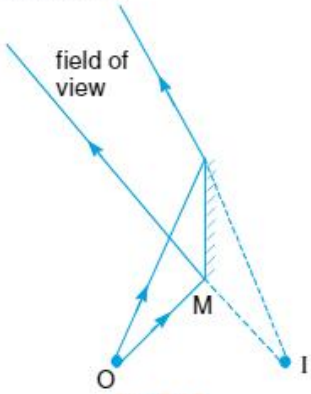


Fig 2.19

6. Keeping incident ray fixed, when the mirror is rotated such that the normal rotates through an angle (θ) in the plane of incidence, then the reflected ray rotates by 2θ along the same sense.

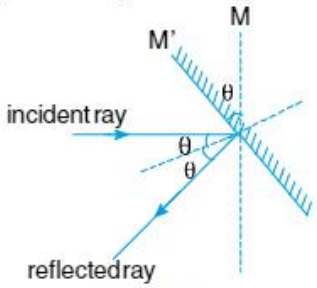


Fig 2.20

7. A plane mirror produces front-back reversal rather than left-right reversal. It must be kept in mind that the mirror produces the reversal effect in the direction perpendicular to plane of the mirror. The Fig 2.21 (a) shows that the right handed co-ordinate system is converted into left handed co-ordinate system.

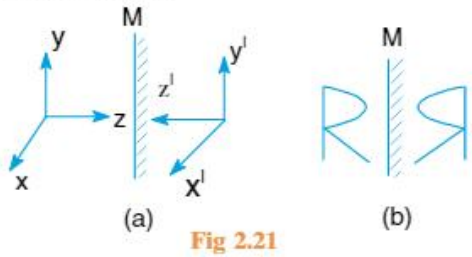


Fig 2.21

i.e., in the image formed by a plane mirror, left is turned into right and vice versa with respect to object as shown in Fig 2.21(b).

8. When the object moves in front of stationary mirror, the relative speed between object and its image along the plane of the mirror is zero and in perpendicular direction to plane of mirror, the relative speed is twice that of the object speed.

$(V_{IO})_y = 0$ and $(V_{IO})_x = 2v_x$

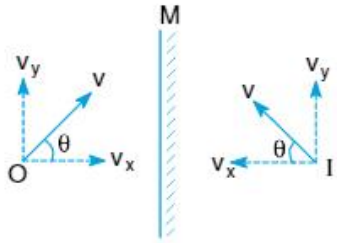


Fig 2.22

9. If an object moves toward (or away from) a plane mirror at speed v, the image will also approach (or recede) at the same speed v, and the relative velocity of image with respect to object will be 2v as shown in Fig 2.23 (a). If the mirror moved toward (or away from) the stationary object with speed v, the image will also move toward (or away from) the object with a speed 2v, as shown Fig 2.23 (b).

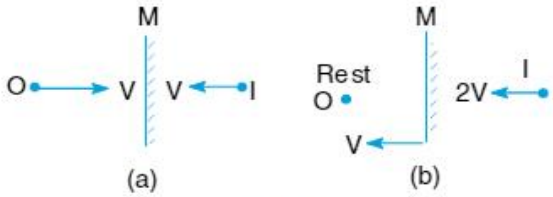


Fig 2.23

10. (a) A person of height 'h' can see his full image in a mirror of minimum length $l = \frac{h}{2}$

(b) A person standing at the centre of room looking towards a plane mirror hung on a wall, can see the whole height of the wall behind him if the length of the mirror is equal to one-third of the height of the wall.

11. If two plane mirrors are inclined to each other at an angle θ, the number of images of a point object formed are determined as follows

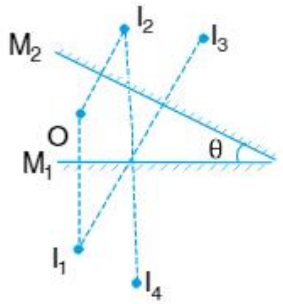


Fig 2.24

(a) If $\frac{360}{\theta}$ is even number (say m), number of images formed n = (m - 1), for all positions of object in between the mirrors.

(b) If $\frac{360}{\theta}$ is odd integer (say m), number of images formed n = m, if the object is not on the ; bisector of mirrors ; n = (m - 1), if the object is on the bisector of mirrors.

(c) If $\frac{360}{\theta}$ is a fraction (say m), the number of images formed will be equal to its integral part i.e. n = [m].

Example : If m = 4.3, the total number of images n = [4.3] = 4

(d) All the images lie on a circle whose radius is equal to the distance between the object 'O' and the point of intersection of mirrors C. If θ is less, more number of images on circle with large radius.

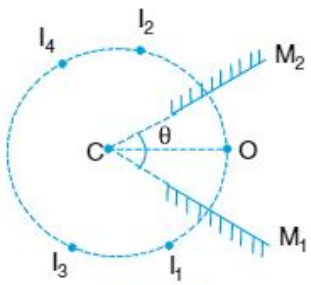


Fig 2.25

(e) If the object is placed in between two parallel mirrors $\theta = 0^0$, the number of images formed is infinite, but of decreasing intensity in according with $I \alpha r^{-2}$.

(f) If 'θ' is given n is unique, but if 'n' is given θ is not unique. Since same number of images can be formed for different θ.

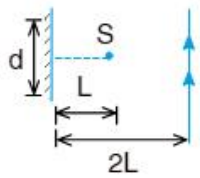
(g) The number of images seen may be different from number of images formed and depends on the position of the observer relative to object and mirror.

12. When a light ray vector incident on a mirror, only the component vector which is parallel to normal of the mirror changes its sign without change of its magnitude on reflection. As mirror can reflect entire energy incident on it, the magnitude of reflected vector is same as that of incident vector. Incident vector corresponds to an object and reflected vector correponds to an image. This vector may be position, velocity or acceleration.

Ex : If a plane mirror lies on x–z plane, a light vector $2\hat{i} + 3\hat{j} - 4\hat{k}$ on reflection becomes $2\hat{i} - 3\hat{j} - 4\hat{k}$.

Example-2.2

A point source of light S, placed at a distance L in front of the centre of a mirror of width d, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2L from it as shown in figure. Find the greatest distance over which he can see the image of the light source in the mirror.



Solution :

The ray diagram will be as shown in Figure.

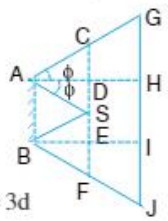
HI = AB = d, DS = CD = d/2

Since, AH = 2AD,

$\therefore$ GH = 2CD = $2\frac{d}{2} = d$

Similarly IJ = d

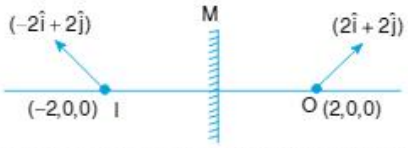
GJ = GH + HI + IJ = d + d + d = 3d



Example-2.3

A plane mirror is placed at origin parallel to y-axis, facing the positive x-axis. An object starts from (2m, 0, 0) with a velocity of $(2\hat{i}+2\hat{j})$ m/s. Find the relative velocity of image with respect to object.

Solution :

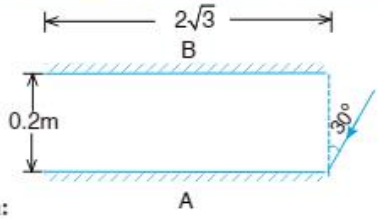


The relative velocity of image with respect to object along normal = $-4\hat{i}$ The relative velocity of image with respect to object along plane of mirror = 0.

Hence the relative velocity of image with respect to object = $-4\hat{i}$.

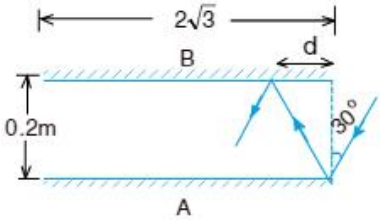
Example-2.4

Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. Find the maximum number of times the ray undergoes reflections (including the first one) before it emerges out .



Solution:

$$d = 0.2\tan 30^\circ = \frac{0.2}{\sqrt{3}}$$



if there are N reflections, then

$$Nd = 2\sqrt{3} \;\therefore\; \frac{0.2}{\sqrt{3}}N = 2\sqrt{3} \Rightarrow N = 30$$

The maximum number of reflections are 30.

2.7 REFLECTION FROM CURVED SURFACE

A curved mirror is a smooth reflecting part (in any shape) of a symmetrical curved surface such as spherical, cylindrical or ellipsoidal. In this chapter we consider a piece of spherical suface only.

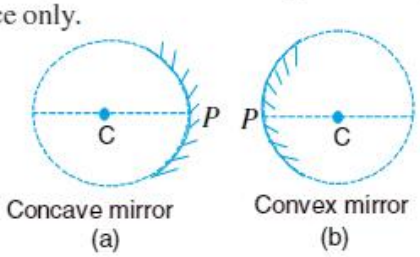


Fig 2.26

If the reflection take place from the inner surface, the mirror is called concave and if its from outer surface, it is convex as shown in the Fig 2.26. In case of thin spherical mirror, the centre 'C' of the sphere of which the mirror is a part is called the centre of curvature of the mirror. P, the centre of the mirror surface, is called the pole. The line CP produced is the principal axis. AB is the aperture which means the effective diameter of the light reflecting area of the mirror. The distance CP is radius of curvature (R). The point F is the focus and the distance PF is called focal length (f) and it is related to R as f = R/2. (Fig 2.27)

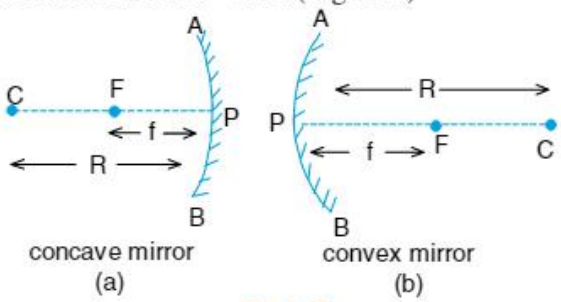


Fig 2.27

2.8 SIGN CONVENTION

To derive the relevant formula for reflection by spherical mirrors and refraction by spherical surfaces, we must adopt a sign convection for measuring distances. In this book, we shall follow the Cartesian sign convention. According to this convection all distances are measured from the pole of the mirror.

(i) The distances measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of light are taken as negative.

(ii) The heights measured on one side with respect to principal axis of the mirror are taken as positive and the heights measured on the other side are taken as negative.

(iii) Acute angles measured from the normal (principal axis) in the anti-clock wise sense are positive, while that in the clockwise sense are negative.

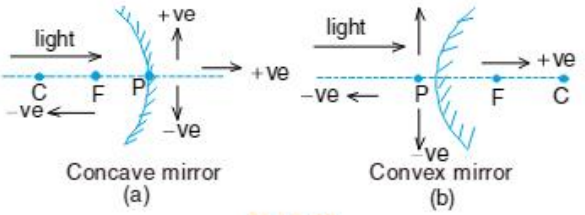


Fig 2.28

2.9 PARAXIAL APPROXIMATION

Rays which are close to the principal axis or make small angle ($\theta < 10°$) with it i.e., they are nearly parallel to the axis, are called paraxial rays. Accordingly we set $\cos\theta \approx 1, \sin\theta \approx \theta$ and $\tan\theta \approx \theta$. This is known as paraxial approximation or first order theory or "Gaussian" optics. In spherical mirrors we restrict to mirror with small aperture and to paraxial rays.

Note - 1 : A beam of parallel paraxial rays is reflected from a concave mirror so that all rays converge to a point F on the principal axis called principal focus of the mirror and it is real focus. Concave mirror is also known as converging mirror. They are used in car head lights, search lights and telescopes.

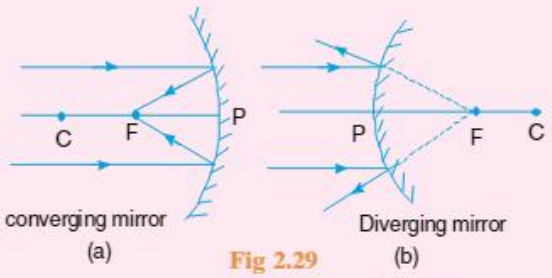


Fig 2.29

Note - 2 : A narrow beam of paraxial rays falling on a convex mirror is reflected to form a divergent beam which appears to come from a point 'F' behind the mirror. Thus a convex mirror has a virtual focus 'F'. Convex mirror is also called a diverging mirror. The distance PF is called the focal length 'f'. Convex mirror gives a wider field of view than a plane mirror and concave mirror. Convex mirrors are used as rear view mirrors in vehicles.

Note - 3 : According to Cartesian sign convention with real object, the focal length of concave mirror is negative, because the distance PF(P to F) is measured in opposite direction of light. As miled reason, focal length of convex mirror is positive.

Note - 4 : The same sign convention is also applicable to virtual objects excess that the light rays from have to be treated as imaginary.

2.10 RELATION BETWEEN *f* AND R

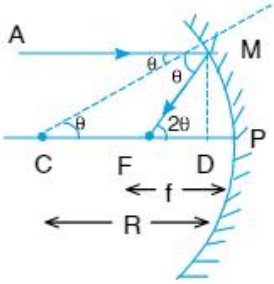


Fig 2.30

A ray AM parallel to the principal axis of a concave mirror is reflected through the principal focus 'F'. If C is the centre of curvature, CM is the normal to the mirror at M. $\angle MCP = \theta$ and $\angle MFP = 2\theta$

Now $\tan\theta = \dfrac{MD}{CD}$ and $\tan 2\theta = \dfrac{MD}{FD}$,

For small 'θ' $\tan\theta \approx \theta$ and $\tan 2\theta \approx 2\theta$,

$$\therefore \tan 2\theta \approx 2\tan\theta \text{ (or) } \frac{MD}{FD} = 2.\frac{MD}{CD}$$

$$\therefore FD = \frac{CD}{2},$$

Now for small θ, the point 'D' is very closed to the point 'P'. Therefore FD = f and CD = R.

$$\therefore f = \frac{R}{2}$$

The focal length of the mirror is independent of the medium in which it is placed and wavelength of incident light. To a plane mirror, focal length 'f' is infinite (as $R = \infty$).

2.11 RULES FOR IMAGE FORMATION

In general, position of image and its nature [i.e. whether it is real or virtual, erect or inverted, magnified or diminished] depend on the distance of the object from the mirror. Nature of the image can be obtained by drawing a ray diagram. In case of image formation, unless other wise stated, object is taken to be real, It may be point object or extended.

(i) A ray parallel to principal axis after reflection from the mirror passes or appear to pass through its focus F.

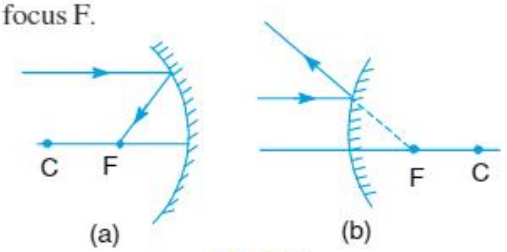


Fig 2.31

(ii) A ray passing through or directed towards focus, after reflection from the mirror becomes parallel to the principal axis (by principle of reversibility).

Fig 2.32

(iii) A ray through or directed towards the centre of curvature C, after reflection from the mirror, retraces its path.

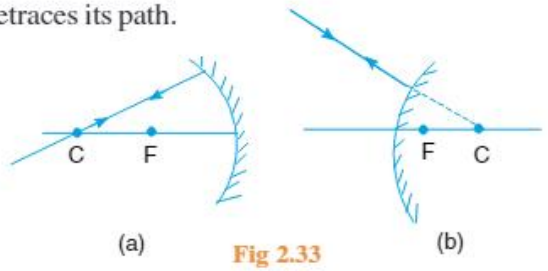


Fig 2.33

(iv) A ray striking at pole P is reflected symmetrically back in the opposite side.

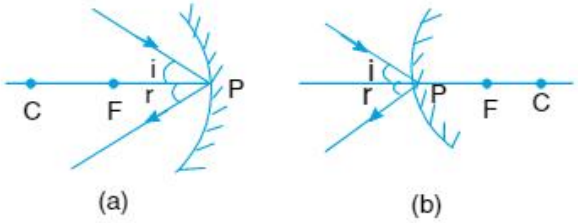


Fig 2.34

2.12 IMAGE FORMATION BY SPHERICAL MIRRORS

a) Convex mirror

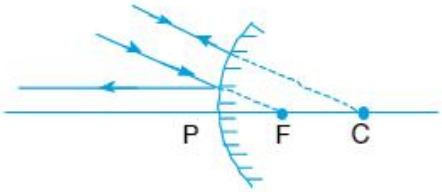
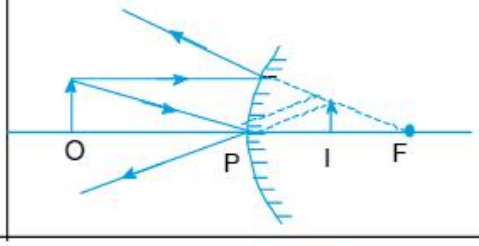
Position of the object	Ray diagram	Image details
At Infinity		Virtual, erect, very small at F
Infront of mirror		Virtual, erect, diminished between P and F

Fig 2.35

b) Concave mirror

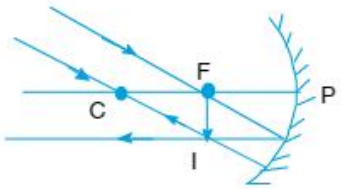
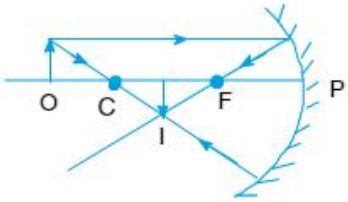
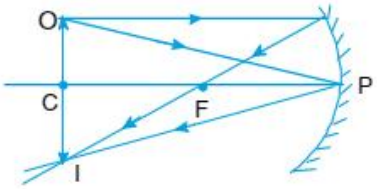
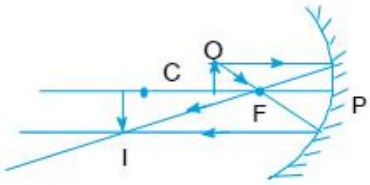
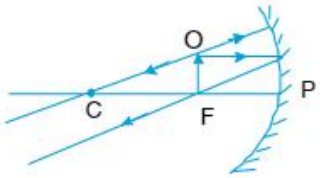
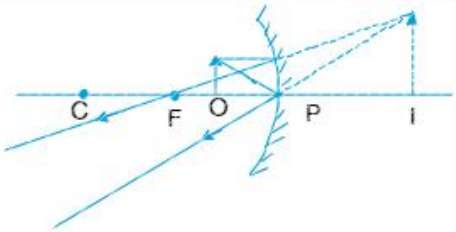
Position of the object	Ray diagram	Image details
At Infinity		Real, inverted, very small, at F
Between ∞ and C		Real, inverted, diminished, between F and C
At C		Real, inverted, equal, at C
Between F and C		Real, inverted, enlarged, beyond C
At F		Real, inverted, very large, at infinity
Between F and P		Virtual, erect, enlarged, behind the mirror

Fig 2.36

2.13 THE MIRROR EQUATION

Fig 2.37 shows the ray diagram considering two rays and the image A'B' (in this case real image) of an object AB formed by a concave mirror.

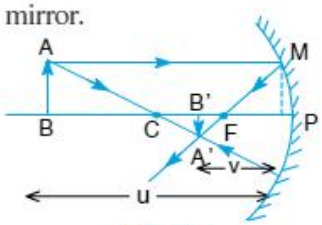


Fig 2.37

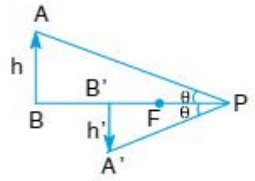


Fig 2.38

Δ^{les} ABC and A'B'C are similar and hence

$$\frac{AB}{A'B'} = \frac{CB}{CB'} \qquad \text{..... (1)}$$

From figure 2.38 Δ^{les} ABP and A'B'P are similar

Hence $\frac{AB}{A'B'} = \frac{PB}{PB'}$

From equations (1) & (2) we get $\frac{CB}{CB'} = \frac{PB}{PB'}$ (or)

$$\frac{PB - PC}{PC - PB'} = \frac{PB}{PB'}$$

$$\therefore \frac{u - R}{R - v} = \frac{u}{v} \text{ (or) } uR + vR = 2uv$$

Dividing through out by uvR, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

Using the sign convention u = –ve, R = –ve and v = –ve into the above equation,

we get $\frac{1}{-u} + \frac{1}{-v} = \frac{2}{-R}$

(or) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \left(\because f = \frac{R}{2}\right)$

This relation is known as Gauss's formula for a spherical mirror. It is valid in all other situations with a spherical mirror and also for a convex mirror. In this formula, to calculate an unknown, known quantities are substituted with proper sign.

2.14 MAGNIFICATION

The size of the image relative to the size of the object is another important quantity to consider. Hence we define magnification. Note that magnification does not mean that the image is enlarged. The image formed by optical system may be larger than, smaller than or of the same size of the object.

a) Lateral magnification

The ratio of the transverse dimension of the final image formed by an optical system to the corresponding dimension of the object is defined as transverse or lateral or linear magnification (m). Hence it is the ratio of the height of image (h') to the height of the object (h). From the Fig 2.39.

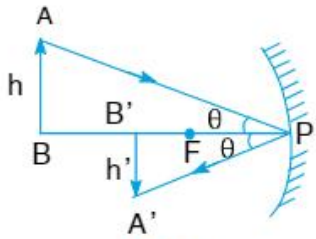


Fig 2.39

Lateral magnification $m = \frac{A'B'}{AB} = \frac{h'}{h}$

Here h and h' will be taken positive or negative in accordance with the accepted sign convention.

In triangles A'B'P and ABP, we have $\frac{B'A'}{BA} = \frac{B'P}{BP}$, with sign convention this becomes

$$\frac{-h'}{h} = \left(\frac{-v}{-u}\right) \text{ so that, } m = \frac{h'}{h} = -\frac{v}{u}$$

Here negative magnification implies that image is inverted with respect to object, while positive magnification means that image is erect with respect to object. i.e., m is negative means for real object, real image is formed and for virtual object virtual image is formed ; m is positive means for real object, virtual image is formed and for virtual object real image is formed.

Ex : m = –2, means, if the object is real, image is real, inverted, magnified and mirror used is concave.

b) Longitudinal magnification

If a one dimensional object is placed with its length along the principal axis, the ratio of length of image to length of object is called longitudinal magnification (m_L). Longitudinal magnification can be expressed as

$$m_L = -\frac{(v_2 - v_1)}{(u_2 - u_1)}$$

where v_1 and v_2 are image positions corresponding to u_1 and u_2 positions.

For small objects $m_L = -\frac{dv}{du}$

We have $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

In case of small linear objects $-\frac{dv}{v^2} - \frac{du}{u^2} = 0$

$$\therefore m_L = -\frac{dv}{du} = \left[\frac{v}{u}\right]^2 = m^2$$

c) Areal magnification

If a two dimensional object is placed with its plane perpendicular to principal axis, its magnification is called areal or superficial magnification. If m is the lateral magnification and m_A is the areal magnification.

$$m_A = \frac{\text{area of image}}{\text{area of object}} = \frac{(ma)(mb)}{ab} = m^2$$

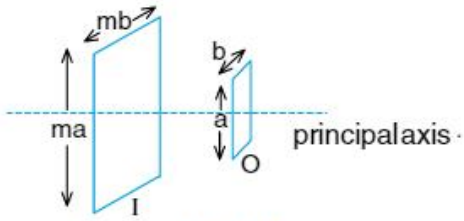


Fig 2.40

d) Over-all magnification

In case of more than one optical component, the image formed by first component will act as an object for the second and image of second acts as an object for third and so on, the product of all individual magnifications is called over all magnification.

$$m_O = \frac{I}{O} = \frac{I_1}{O_1} \times \frac{I_2}{O_2} \times -- \frac{I_n}{O_n} = m_1 \times m_2 \times -- \times m_n$$

Note : In case of spherical mirror, if the object distance (x_1) and image distance (x_2) are measured from focus instead of the pole of the mirror, then mirror formula reduces to a simple form called the Newton's formula.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ reduces to } \frac{1}{f + x_2} + \frac{1}{f + x_1} = \frac{1}{f}$$

which on simplification gives $x_1 x_2 = f^2$ (Newton's formula)

2.15 MOTION OF OBJECT IN FRONT OF MIRROR ALONG THE PRINCIPAL AXIS

When the position of the object changes with time on the principal axis relative to the mirror, the image position also changes with time relative to the mirror. Hence to know the relation between object and image speed we use the mirror equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Differentiate with respect to time, we get

$$-\frac{1}{v^2}.\frac{dv}{dt} - \frac{1}{u^2}.\frac{du}{dt} = 0 \text{ (or) } \frac{dv}{dt} = -\left(\frac{v}{u}\right)^2.\frac{du}{dt}$$

$$\text{(or) } V_I = -\left(\frac{v}{u}\right)^2.V_o$$

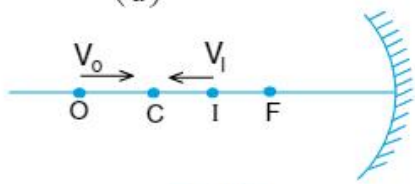


Fig 2.41

where V_I is velocity of image with respect to mirror and V_o is the velocity of object with respect to mirror along the principal axis. Here negative sign indicates the object and image are always moving opposite to each other. In concave mirror, depending on the position of the object, image speed may be greater or lesser or equal to the object speed.

(a) $R < u < \infty$	$\lvert m \rvert < 1$	$V_I < V_o$
(b) $u = R$	$\lvert m \rvert = 1$	$V_I = V_o$
(c) $f < u < R$	$\lvert m \rvert > 1$	$V_I > V_o$
(d) $u < f$	$\lvert m \rvert > 1$	$V_I > V_o$
(e) $u \approx 0$	$\lvert m \rvert \approx 1$	$V_I \approx V_o$

Relation between object and image velocity given above is also valid for convex mirror. In convex mirror, speed of image is slower than the object, whatever the position of the object may be. Above relation is not true in case of acceleration of object and image.

2.16 MOTION OF THE OBJECT TRANSVERSE TO THE PRINCIPAL AXIS

If the object moves transverse to principle axis, image also moves transverse to principle axis.

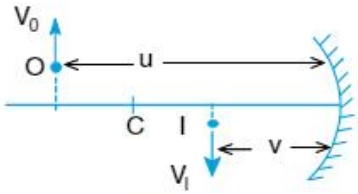


Fig 2.42

Consider the Fig 2.42. In a mirror

$$\frac{h_i}{h_0} = \frac{v}{u} = \text{constant}\,(-m)$$

$$\therefore \frac{\frac{dh_i}{dt}}{\frac{dh_0}{dt}} = -m \quad (\text{or})\ V_I = -m\,V_0$$

2.17 POWER OF CURVED MIRROR

Every optical instrument has power, It is the ability of optical instrument to deviate the path of rays incident on it. If the instrument converges the rays parallel to principal axis, its power is said to be positive and if it diverges, its power is said to be negative.

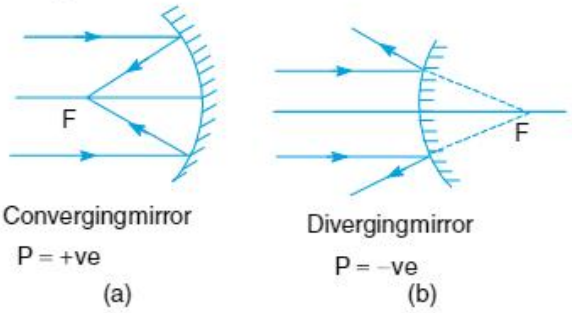


Fig 2.43

For a mirror

$$\text{Power 'P'} = -\frac{1}{f(\text{metre})} \quad (\text{or})\ P = -\frac{100}{f(\text{cm})}$$

S.I. unit of power is dioptre (D) = m^{-1}

For concave mirror (converging mirror), power is positive and for convex mirror (diverging mirror), power is negative.

2.18 $\frac{1}{v}$ vs $\frac{1}{u}$ GRAPH FOR MIRRORS

(a) The graph between $\frac{1}{v}$ and $\frac{1}{u}$ to a concave mirror is shown in Fig 2.44.

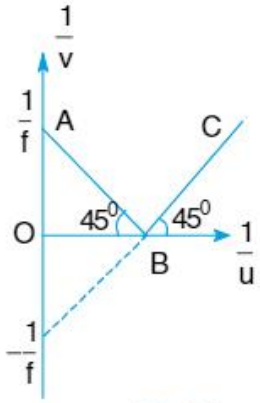


Fig 2.44

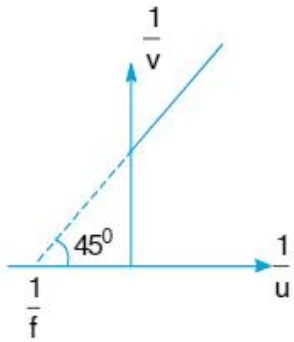


Fig 2.45

Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

For real image $-\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$ $\therefore$ $\frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$

This is a straight line equation with slope -1 .

Hence line AB.

For virtual image , $\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$

$$\frac{1}{v} = \frac{1}{u} - \frac{1}{f}$$

This is a straight line equation with slope +1. Hence line BC

(b) The graph between 1/v and 1/u to a convex mirror as shown in figure 2.45.

Since convex mirror always form virtual image to a real object.

$$\frac{1}{v}+\frac{1}{-u}=\frac{1}{f} \quad \therefore \quad \frac{1}{v}=\frac{1}{u}+\frac{1}{f}$$

This is a straight line equation with slope +1

2.19 u vs v GRAPH FOR CURVED MIRROR

(a) In case of concave mirror, the graph between u and v is hyperbola as shown in Fig 2.46.

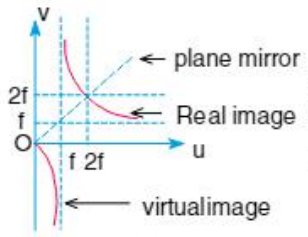


Fig 2.46

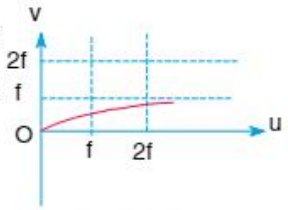


Fig 2.47

For real image $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ (or) $\frac{1}{v}=\frac{u-f}{uf}$

$$v=\frac{f}{1-\frac{f}{u}}$$

For virtual image $\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$

$$\frac{1}{v}=\frac{f-u}{fu} \text{ (or) } v=\frac{f}{\frac{f}{u}-1}$$

(b) In case of convex mirror, the graph between u and v is hyperbola as shown in Fig 2.47.

Since convex mirror form only virtual image.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \text{ (or) } v=\frac{f}{1+\frac{f}{u}}$$

2.20 m-v GRAPH FOR SPHERICAL MIRROR

In a spherical mirror $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\therefore 1+\frac{v}{u}=\frac{v}{f} \text{ (or) } \frac{v}{u}=\frac{v}{f}-1$$

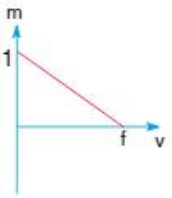


Fig 2.48

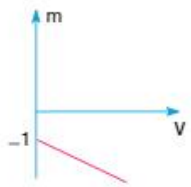


Fig 2.49

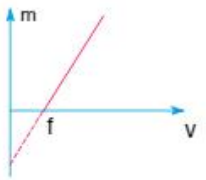


Fig 2.50

a) Concave mirror

If the object is real,

(i) for real image, u = -ve, v = -ve, f = -ve,

$$\therefore -m=\frac{v}{f}-1 \text{ (or) } m=-\frac{v}{f}+1$$

$\Rightarrow$ graph is as shown in Fig 2.48.

(ii) For virtual image, u = -ve, v = +ve, f = -ve

$$\therefore m=-\frac{v}{f}-1$$

$\Rightarrow$graph is as shown in figure 2.49.

b) Convex mirror

Since convex mirror always form virtual image to a real object, u = –ve, v = +ve, f = +ve,

$\therefore m=\frac{v}{f}-1$, graph is as shown in Fig 2.50.

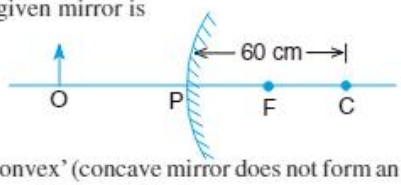
From the above graphs it is observed that for $v \approx 0$, m = 1. i.e., when an object is very near to pole of the mirror $(u \approx 0)$, then the curved mirror behaves like a plane mirror.

Example-2.6

An object of length 10cm is placed at right angles to the principal axis of a mirror of radius of curvature 60 cm such that its image is virtual, erect and has a length 6 cm. What kind of mirror is it and also determine the position of the object?

Solution :

Since the image is virtual, erect and of a smaller size, the given mirror is



'convex' (concave mirror does not form an image with the said description).

Given R = + 60 cm $\qquad f=\frac{R}{2}=30\text{cm}$

Transverse magnification,

$m=\frac{I}{O}=\frac{6}{10}=+\frac{3}{5}$ Further $m=-\frac{v}{u}=\frac{3}{5}$

$\therefore v=-\frac{3u}{5}$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad \frac{-5}{3u}+\frac{1}{u}=\frac{1}{30}$

$\frac{-5+3}{3u}=\frac{1}{30} \qquad \therefore u=-20$ cm

Thus the object is at a distance 20 cm (from the pole) in front of the mirror.

Example-2.7

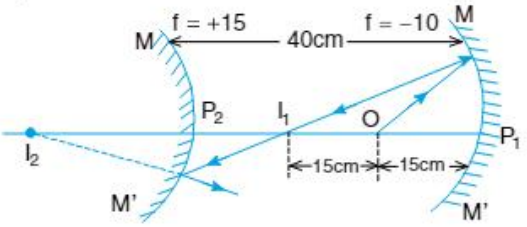
A concave mirror of focal length 10 cm and a convex mirror of focal length 15 cm are placed facing each other 40 cm apart. A point object is placed between the mirrors, on their common axis and 15cm from the concave mirror. Find the position, nature of the image, and over all magnification produced by the successive reflections, first at concave mirror and then at convex mirror.

Solution :

According to given problem, for concave mirror, $u = -15$ cm and $f = -10$ cm

So $\frac{1}{v} + \frac{1}{-15} = \frac{1}{-10}$, i.e., $v = -30$cm

i.e., concave mirror will form real, inverted and enlarged image I_1 of object O at a distance 30 cm from it, i.e., at a distance $40 - 30 = 10$ cm from convex mirror.



For convex mirror the image I_1 will act as an object and so for it $u = -10$ cm and $f = +15$cm.

$\frac{1}{v} + \frac{1}{-10} = \frac{1}{15}$, i.e., $v = +6$ cm

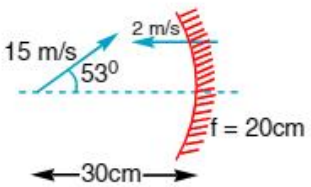
So final image I_2 is formed at a distance 6 cm behind the convex mirror and is virtual as shown in figure

Over all magnification $= m_1 \times m_2 = -2 \times 6/10 = -6/5$

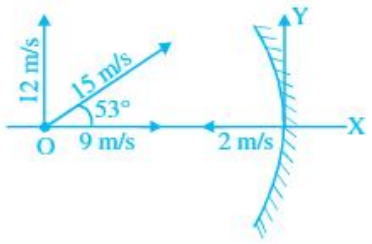
Negative indicates final image is inverted w.r.t given object.

Example-2.8

The velocity of image in a situation as shown in the figure is



Solution:



$\vec{V}_O$ Velocity of object $= (9\hat{i} + 12\hat{j})$ m/s

$\vec{V}_m$ Velocity of mirror $= -2\hat{i}$ m/s

$$m = \frac{f}{f-u} = \frac{-20}{-20-(-30)} = -2$$

For velocity component parallel to optical axis

$$(\vec{V}_{I/m})_{\parallel} = -m^2(\vec{V}_{o/m})_{\parallel}$$

$$(\vec{V}_{I/m})_{\parallel} = -(-2)^2(9\hat{i} - (-2\hat{i})) = -44\hat{i}\text{ m/s}$$

For velocity component perpedicular to optical axis

$$(\vec{V}_{I/m})_{\perp} = m(\vec{V}_{O/m})_{\perp} = (-2)12\hat{j} = -24\hat{j}\text{ m/s}$$

$\because \vec{V}_{I/m}$ = Velocity of image w.r.t. mirror

$= (\vec{V}_{I/m})_{\parallel} + (\vec{V}_{I/m})_{\perp} = (-44i - 24\hat{j})$ m/s

also, $\vec{V}_{I/m} = \vec{V}_I - \vec{V}_m$ or

$\vec{V}_I = (-44\hat{i} - 24\hat{j}) - 2\hat{i}$

$= (-44\hat{i} - 26\hat{j})$ m/s

2.21 GRAPHICAL METHOD OF DETERMINING THE FOCAL LENGTH OF A CONCAVE MIRROR

It forms real and inverted image of an object placed beyond its focus. From mirror equation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Using Cartesian sign convension,

we have $u = -x$, $v = -y$ and $f = -f$

$$\text{or } \frac{1}{-y} + \frac{1}{-x} = \frac{1}{-f}$$

A graph between $\frac{1}{-v}$ and $\frac{1}{-u}$ is a straight line, as shown in figure.

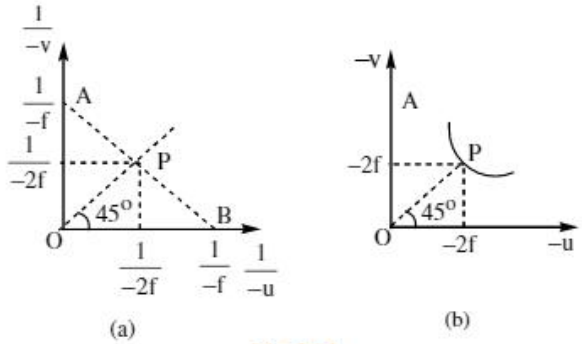


Fig 2.51

Note that the slope of the straight line is –1 and the intercepts on the horizontal and vertical axes are equal. It is equal to $\frac{1}{-f}$. A straight line OP at an angle 45^0 with the horizontal axis is drawn which intersects the line AB at P. The coordinates of the point P are $\left(\frac{1}{-2f}, \frac{1}{-2f}\right)$

The focal length of the mirror can be calculated by measuring the coordinates of either of the points A, B or P.

Alternatively, a graph between –v and –u can also be plotted, which is a curve as shown in figure. A line drawn at an angle of 45^0 from the origin intersects at the point P whose coordinates are (–2f, –2f). By measuring the coordinates of this point, the focal length of the mirror can also be measured.

2.22 REFRACTION OF LIGHT

Refraction is the phenomenon in which direction of propagation of light changes at the boundary when it passes from one medium to an other. When light passes from one medium to the an other, velocity, wavelength and intensity may change, but frequency, phase and the colour of light do not change. The colour of light is determined by frequency and not by wavelength. Velocity of light for all wavelengths in free space is same and is equal to 3×10^8 m/s. However in a medium the speed of light is different for different wavelengths. In a given medium greater the wavelength, greater will be the velocity of light, in accordance with the relation $v \alpha \lambda$.

e.g. as $\lambda_{red} > \lambda_{violet}$, in water $v_{red} > v_{violet}$.

According to Huygen's principle, when a wavefront passes from one medium into another, the speed of secondary wavelets originating from the wavefront changes in second medium. As a result of this, the refracted wavefront in the second medium bends with respect to the incident wavefront in the first medium. The speed of light in a medium is less than the speed of light in vacuum. If the same light passes through two different media, the medium in which speed is less relative to the other medium is called denser, other one is said to be rarer.

Example : Since $V_{air} > V_{water} > V_{glass}$, in case of propagation of light from air to water, water to glass, water is denser relative to air and water is rarer relative to glass. If the speed of the light in a medium (such as water) is same in all directions the medium is said to be isotropic, otherwise it is anisotropic (such as calcite).

2.23 REFRACTIVE INDEX OR OPTICAL DENSITY (μ or n)

The ratio of speed of light in free space to that in a given medium is called absolute refractive index or refractive index of the given medium.

i.e., $\mu = \frac{c}{v} = \frac{\lambda_o}{\lambda}$

Here λ and λ_o are the wave length of the light in medium and vacuum respectively.

Example: The refractive index of free space $\mu = 1$

The refractive index of water $\mu_w = 4/3$

The refractive index of glass $\mu_g = 3/2$

(i) The product of refractive index and wavelength corresponding to that medium is constant.

i.e. $\mu_1\lambda_1 = \mu_2\lambda_2 = \text{constant}$

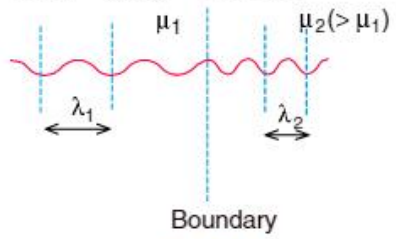


Fig 2.52

In denser medium lesser will be the speed (less wave length) of light and so greater will be the refractive index.

(ii) The refractive index depends not only on the substance but also on the wavelength of the light. This dependence of refractive index on wavelength is called dispersion.

According to Cauchy, the refractive index (μ) of a medium varies with wave length (λ) as

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ----$$

Where A, B and C are constants which are characteristic of the medium. That is why two different wavelengths of light do not have the same refractive index in the same medium.

Example: In glass $\mu_V > \mu_R$ since $\lambda_V < \lambda_R$.

2.24 RELATIVE REFRACTIVE INDEX

When light passes from one medium to the other, the refractive index of medium 2 relative to 1 is written as ${}_1\mu_2$.

$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

According to principle of reversibility of light, if the path of a light ray after going through a number of reflections and refractions is reversed, then it retraces its path in the opposite direction.

Hence ${}_1\mu_2 = \frac{1}{{}_2\mu_1}$ i.e $({}_1\mu_2) \cdot ({}_2\mu_1) = 1$

2.25 OPTICAL PATH

Consider two points A and B in a medium as shown in Fig 2.53. The shortest distance between any two points A and B is called geometrical path. The length of geometric path is independent of the medium that surrounds the path A B. When a light ray travels from the point A to point B it travels with the velocity c if the medium is vacuum and with a lesser velocity *v* if the medium is other than vacuum. Therefore the light ray takes more time to go from A to B located in a medium. The optical path to a given geometrical path in a given medium is defined as the distance travelled by light in vacuum in the same time in which it travels the given path length in that medium.

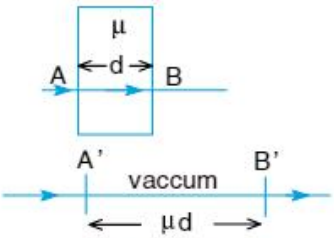


Fig 2.53

AB = real path or geometrical path

A'B'=optical path. If the light travels a path length 'd' in a medium at speed v, the time taken by it will be $\left(\frac{d}{v}\right)$ So optical path length

$$\Delta = c \times t = c \times \left[\frac{d}{v}\right] = \mu d \left(\text{as } \mu = \frac{c}{v}\right)$$

Therefore optical path is μ times the geometrical path. As for all media $\mu > 1$, optical path length is always greater than actual path length.

2.26 REFRACTION AT PLANE SURFACE

When light ray travels from one medium to another, the ray in first medium is called incident ray and in second medium is called refracted ray. The angle made by the refracted ray with normal to the boundary is called angle of refraction (r). The laws of refraction are

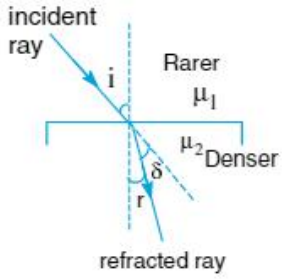
(i) Incident ray, refracted ray and normal always lie in the same plane.

(ii) The product of refractive index and sine of angle of incidence (i) at a point in a medium is constant.

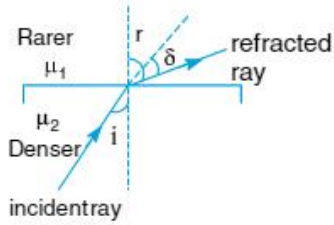
i.e $\mu \times \sin i$ = constant (or) $\mu_1 \sin i_1 = \mu_2 \sin i_2$

If $i_1 = i$ and $i_2 = r$, $\mu_1 \sin i = \mu_2 \sin r$

This law is called Snell's law.



(a)



(b)

Fig 2.54

The Snell's law in vector form $\mu_1(\hat{i}\times\hat{n})=\mu_2(\hat{r}\times\hat{n})$, where $\hat{i}, \hat{r}$ and $\hat{n}$ are the unit vectors along the directions of incident ray, refracted ray and normal.

> ***Note :*** After refraction of light ray, the components sign in the refracted ray is same as the components sign in incident ray. The components of refracted ray changes in such a way that the magnitude of refracted ray vector is same as that of incident any vector.

According to Snell's law if light passes from rarer to denser medium, the ray bends towards the normal as shown in the Fig 2.55 (a). The deviation suffered by incident ray $\delta = i - r$.

For $\mu_1 = \mu_R$ or $\mu_2 = \mu_D$, according to Snell's law $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{\mu_D}{\mu_R} > 1$ i.e., $\angle r < \angle i$

Similarly if light passes from denser to rarer medium, the ray bends away from the normal as shown in the Fig 2.55 (b). The deviation suffered by incident ray $\delta = r - i$.

Refraction does not take place when light is incident normal on a boundary or refractive index of two media are equal as shown in the Fig 2.55. Here the ray is said be transmitted

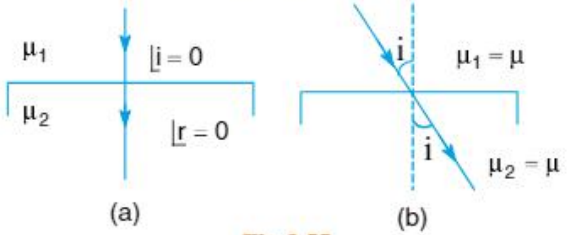


Fig 2.55

When the ray passes undeviated from the boundary, the boundary will not be visible. This is why a transparent solid is invisible in a liquid, if $\mu_{solid} = \mu_{liquid}$.

Note that a virtual ray cannot deviate at the boundary of two media when it passes.

2.27 SINGLE REFRACTION FROM A PLANE SURFACE

Case (i)

Consider an object 'O' placed at a distance 'x' from A in denser medium of refractive index μ as shown in Fig 2.56. Ray OA, which falls normally on the plane surface, passes undeviated as AD. Ray OB, which falls at angle 'r' (with normal) on the plane surface, bends away from the normal and passes as BC in air. Rays AD and BC meet at 'I' on extending these two rays backwards. This 'I' is the virtual image of real object 'O' to an observer in rarer medium close to transmitted ray.

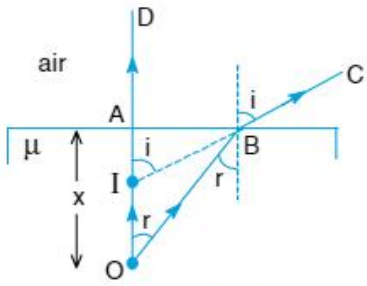


Fig 2.56

$$\sin i \approx \tan i = \frac{AB}{AI} \quad ..... (i)$$

$$\sin r \approx \tan r = \frac{AB}{AO} \quad ..... (ii)$$

Dividing eq. (i) and (ii)

$$\frac{\sin i}{\sin r} = \frac{AO}{AI}$$

According to Snell' law $\mu = \frac{\sin i}{\sin r}$

$$\therefore \mu = \frac{AO}{AI} \qquad \therefore AI = \frac{AO}{\mu} = \frac{x}{\mu}$$

The distance of image AI is called apparent depth or apparent distance. Therefore apparent depth is μ times less than the real depth

i.e., $x_{app} = \frac{x_{real}}{\mu}$

Hence the apparent shift (OI) $= \left(1 - \frac{1}{\mu}\right)x$

If the observer is in a medium, other than air, of refractive index $\mu'(<\mu)$.

Then apparent depth $= \frac{\text{real depth}}{\mu_{relative}} = \frac{\text{real depth}}{\left(\frac{\mu}{\mu'}\right)}$

$\therefore$ apparent depth $= \frac{\mu'}{\mu}$ (real depth)

Fig 2.57 shows variation of apparent depth with real depth of the object.

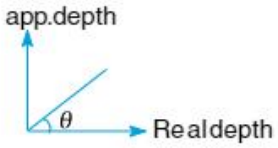
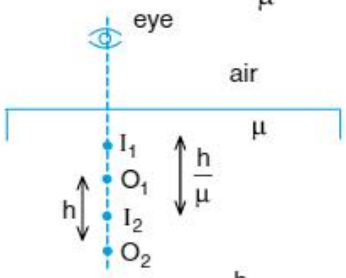


Fig 2.57

$$\text{Slope} = \tan\theta = \frac{\mu'}{\mu}(<1)$$

- Let two objects O_1 and O_2 be separated by 'h' on normal line to the boundary in a medium of refractive index μ. These objects are observed from air close to the normal line of boundary. The distance between the images I_1 and I_2 of O_1 and O_2 is $\frac{h}{\mu}$.



$$O_1O_2 = h \Rightarrow I_1I_2 = \frac{h}{\mu}$$

Fig 2.58

- Apparent depth of object due to composite slab is $x_a = \frac{x_1}{\mu_1} + \frac{x_2}{\mu_2} + \frac{x_3}{\mu_3}$ (see Fig 2.59)

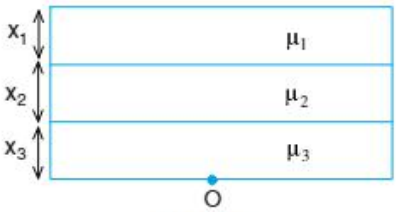


Fig 2.59

- If 'n' number of parallel slabs which may or may not be in contact, with different refractive indices, are placed between the observer and the object, then the total apparent shift

$$s = \left(1 - \frac{1}{\mu_1}\right)x_1 + \left(1 - \frac{1}{\mu_2}\right)x_2 + \text{----} + \left(1 - \frac{1}{\mu_n}\right)x_n$$

where x_1, x_2---- x_n are the thickness of the slabs and μ_1, μ_2 ------ μ_n are the corresponding refractive indices.

- When there is a series of layers with different refractive indices, then the Snell's law in the form $\mu \sin\theta$ = constant, is valid across them only when all the surfaces are parallel to each other.

Ex:

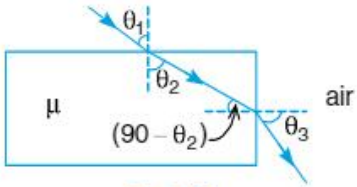


Fig 2.60

$1 \times \sin\theta_1 = \mu.\sin\theta_2 = 1.\sin\theta_3$ (wrong)

$1 \times \sin\theta_1 = \mu.\sin\theta_2$ (correct)

and $\mu \sin(90 - \theta_2) = 1.\sin\theta_3$ (correct)

Case (ii) :

Let the object be in rarer medium (air) at a distance 'y' from boundary and an observer be near the normal in denser medium of refractive index 'μ'. By ray diagram in Fig 2.61, it is observed that the image is virtual, same side to boundary and its distance from the boundary is μ times the object distance. Since $\mu > 1$, image distance is longer than object distance.

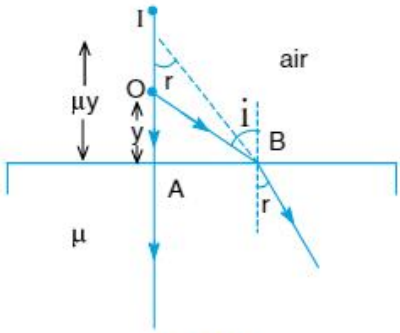


Fig 2.61

$$\sin i \approx \tan i = \frac{AB}{AO}, \sin r \approx \tan r = \frac{AB}{AI}$$

According to Snell's law $1.\sin i = \mu \sin r$

$$\frac{AB}{AO} = \mu\frac{AB}{AI}, AI = \mu.AO$$

Therefore apparent height of object= μ (real height of object)

i.e. $y_{app} = \mu . y_{real}$

Apparent shift = $(\mu - 1)y$.

If the object is in a medium, other than air, of refractive index $\mu'(<\mu)$, then apparent height $= \mu_{rel}$ (real height)

i.e., $y_a = \left(\frac{\mu}{\mu'}\right)y$

Fig 2.62 shows variation of apparent height with real height of the object.

slope = $\tan\phi = \frac{\mu}{\mu'}(>1)$

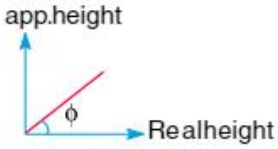


Fig 2.62

Apparent shift = $\left(\frac{\mu}{\mu'} - 1\right)y$

1) When a convergent beam of high passes from denser to rarer medium as shown in Fig 2.63 real image (I) is formed in rarer medium which is nearer to the boundary than the virtual object (O).

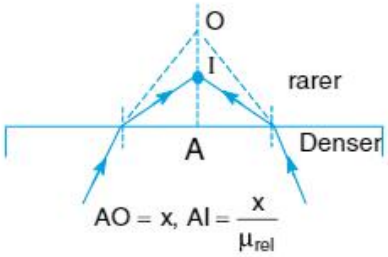


Fig 2.63

2) When a convergent beam of rays passes from rarer to denser medium as shown in Fig 2.64, real image (I) is formed in denser medium which is far their from the to boundary than the virtual object (O).

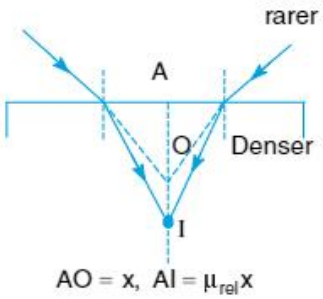


Fig 2.64

2.28 REFRACTION WITH VECTOR NOTATION

The angle of incidence of a light ray on an interface is usually determined geometrically by simply drawing the ray diagram, However, in some situations, the vector representation of the line along which the light ray travels is given. The ray may then reflect or refract at an interface and emerge. How do we find the vector representation of the line along which the emergent ray travels?

Following are the steps for finding this

i) Determine the unit vector representation of the normal to the interface where the incident ray strikes the surface, say $\hat{e}_n$.

ii) Find the component of the incident ray along the normal.

iii) Subtract this component from the original ray to compute the plane in which the incident ray travels. Calculate the unit vector that represents the plane of the incident ray and the normal, say $\hat{e}_p$.

iv) Using the vector dot product, calculate the angle between the incident ray and the normal.

v) From the governing equation calculate the angle of reflection/refraction, say r.

vi) We known that the emergent ray will be in the same plane as that of then incident ray and the normal. Thus the unit vector representing the direction of the emergent beam is simply $\cos(r)\hat{e}_n + \sin(r)\hat{e}_p$

2.29 RELATION BETWEEN VELOCITIES OF OBJECT AND IMAGE

(a) The Fig 2.65 shows an object O moving towards the plane boundary of a denser medium.

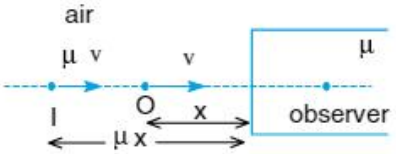


Fig 2.65

To an observer in the denser medium, the object appears to be more distant but moving faster. If the speed of the object is *v*, then the speed of the image will be μv .

(b) Similar to an observer in rarer medium and object in denser medium, the image appears to be closer but moving slowly. If the speed of the object is *v*, then the speed of the image will be $\frac{v}{\mu}$.

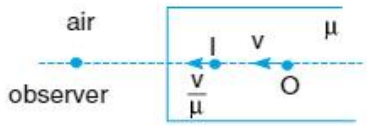


Fig 2.66

2.30 LATERAL SHIFT ON PASSING THROUGH A GLASS SLAB

Fig 2.67 shows a ray AO incident on a slab of thickness 't' at an angle of incidence 'i'. After two refractions at the boundary, the ray emerges parallel to the incident ray. The perpendicular distance between incident ray direction and emergent ray direction is called lateral shift or lateral displacement (x).

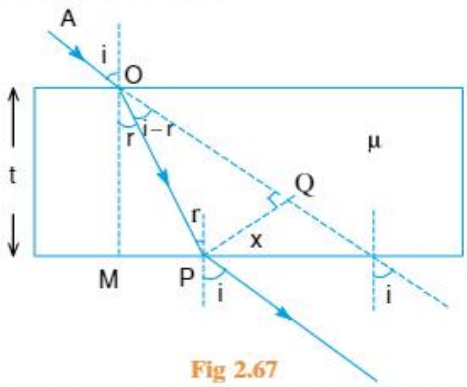


Fig 2.67

From the figure, lateral displacement, x = PQ = OP sin (i - r)

$$x = \frac{OM}{\cos r}\sin(i-r) \quad \left(\because \cos r = \frac{OM}{OP}\right)$$

$$\therefore x = t\left[\frac{\sin(i-r)}{\cos r}\right]$$

For small angle of incidence $\sin i \rightarrow i$, $\sin r \rightarrow r$ and $\cos r \simeq 1$.

$$x = t(i-r) = ti\left(1-\frac{r}{i}\right)$$

$$\text{Lateral shift } x = \left(1-\frac{1}{\mu}\right)ti \quad \left(\because \frac{\sin i}{\sin r} = \mu = \frac{i}{r}\right)$$

2.31 NORMAL SHIFT DUE TO GLASS SLAB

Consider an object placed on the normal line to the boundary of a slab whose thickness is 't' and refractive index 'μ'. On observing this object (real) from other side of the slab, due to double refraction, it appears to have shifted on the normal line. This shift value is called normal shift. This shift is towards the slab, if the slab is denser relative to the surroundings and shift is away from the slab, if the slab is rarer relative to the surrounds. By ray diagram it can be proved that : Normal shift

$$OI = \left(1-\frac{1}{\mu_{rel}}\right)t = \left(1-\frac{\mu'}{\mu}\right)t$$

$$\text{for } \mu' = 1, \text{ normal shift } OI = \left(1-\frac{1}{\mu}\right)t.$$

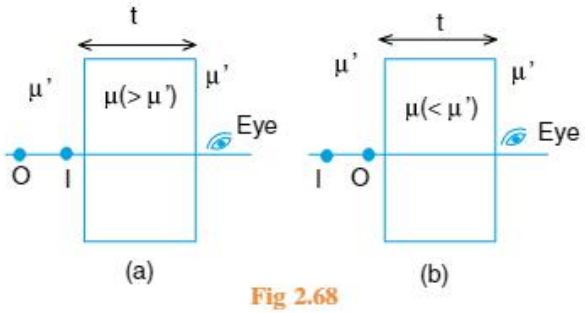


Fig 2.68

By that principle of reversibility, in case of a convergent beam (virtual object) normal shift occurs is opposite way to that of real object.

Some examples of refraction

(i) Oval shape of sun in the morning and evening: In the morning and evening, the sun is at the horizon and refractive index in the atmosphere of earth decreases with height. Therefore unequal refraction occurs for rays diverging from the lower and upper ends of the sun. Rays from the lower edge have to go through a greater thickness of air than rays from the upper edge. So the vertical diameter appears to be diminished in size, whereas horizontal diameter remains unaltered.

(ii) Visibility of two images of an object : When an object in a glass container filled with a liquid is in side view from a level higher than that of the liquid, there will be two images, one due to refraction through liquid and the other due to refraction through glass.

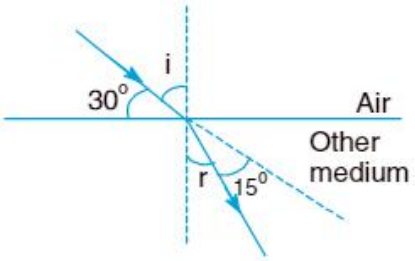
(iii) Twinkling of stars : Due to fluctuations in refractive index of atmosphere, the refraction becomes irregular and the light some times reaches the eye and some times it does not. This gives rise to twinkling of stars.

(iv) Bending of linear object in a liquid : If a linear object is dipped in a liquid inclined to the surface of liquid, its Actual depth will be different for different points and so apparent depth. Due to this, the object appears to be bent.

Example-2.9

Light of wavelength 4500 Å in air is incident on a plane boundary between air and another medium at an angle 30° with the plane of boundary. As it enters from air into the other medium, it deviates by 15° towards the normal. Find refractive index of the medium and also the wavelength of given light in the medium.

Solution :



Angle of incidence i = 90° – 30° = 60°. As the ray bends towards the normal, it deviates by an angle i – r = 15° (given)

$\therefore$ r = 45° Applying Snell's law

$\mu_{air} \sin i = \mu_{med} \sin r$

$\therefore \quad 1 \times \sin 60^0 = \mu \times \sin 45^0$

$$\therefore \mu = \frac{\sin 60^0}{\sin 45^0} = \frac{\sqrt{3}/2}{1/\sqrt{2}} \text{ (or) } \mu = \sqrt{1.5}$$

In terms of wavelengths,

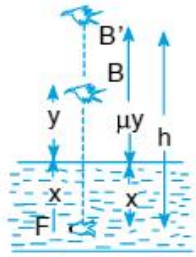
$$\mu = \sqrt{1.5} = \frac{\lambda_{air}}{\lambda_{med}} \text{ (or) } \lambda_{med} = \frac{\lambda_{air}}{\sqrt{1.5}} = \frac{4500}{\sqrt{1.5}}$$

$\lambda_{med} = 3674 \text{ Å}$

Example-2.10

A fish rising vertically to the surface of water in a lake uniformly at the rate of 3 m/s observes a bird diving vertically towards the water at a rate of 9 m/s vertically above it. If the refractive index of water is (4/3), find the actual velocity of the dive of the bird.

Solution :



If at any instant, the fish is at a depth x below the water surface while the bird is at a height y above the surface, then the apparent height of the bird above the surface as seen by fish will be $y_{App} = \mu y$

So the total apparent distance of the bird as seen by fish in water at depth x will be $h = x + \mu y$ (or)

$$\frac{dh}{dt} = \frac{dx}{dt} + \mu \frac{dy}{dt}, \text{ i.e., } 9 = 3 + \mu \frac{dy}{dt}$$

$$\text{i.e., } \frac{dy}{dt} = \frac{6}{(4/3)} = 4.5 \text{m/s}$$

Example-2.11

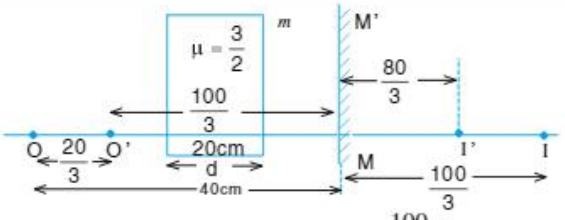
A 20 cm thick glass slab of refractive index 1.5 is kept in front of a plane mirror. Find the position of the image (relative to mirror) as seen by an observer through the glass slab when a point object is kept in air at a distance of 40 cm from the mirror.

Solution :

As thickness of glass slab is 20 cm and its $\mu = (3/2)$, so shift produced by it will be

$$x = d\left[1 - \frac{1}{\mu}\right] = 20\left[1 - \frac{2}{3}\right] = \frac{20}{3} \text{cm}$$

(towards the mirror)



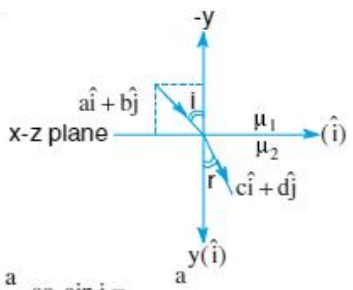
The image of object in the mirror is $\frac{100}{3}$ cm from the mirror. The glass slab shift this image towards the observer by $\frac{20}{3}$ cm.

Therefore the position of the final image from the mirror w.r.t observer is $\frac{100}{3} - \frac{20}{3} = \frac{80}{3}$ cm

Example-2.12

The x-z plane separates two media A and B of refractive indices $\mu_1 = 1.5$ and $\mu_2 = 2$. A ray of light travels from A to B. Its directions in the two media are given by unit vectors $\vec{u_1} = a\hat{i} + b\hat{j}$ and $\vec{u_2} = c\hat{i} + d\hat{j}$. Then

Solution :



$\tan i = \frac{a}{b}$ so $\sin i = \frac{a}{\sqrt{a^2 + b^2}}$

and $\tan r = \frac{c}{d}$, $\sin r = \frac{c}{\sqrt{c^2 + d^2}}$

$\mu_1 \sin i = \mu_2 \sin r$

$$\left(\frac{3}{2}\right)\left(\frac{a}{\sqrt{a^2 + b^2}}\right) = 2\left(\frac{c}{\sqrt{c^2 + d^2}}\right)$$

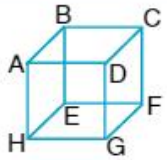
But as $a\hat{i} + b\hat{j}$ and $c\hat{i} + d\hat{j}$ are unit vectors so

$\sqrt{a^2 + b^2} = \sqrt{c^2 + d^2} = 1$

Hence $\frac{3}{2}a = 2c$, so $\frac{a}{c} = \frac{4}{3}$

Example-2.13

An air bubble is trapped inside a glass cube of edge 30 cm. Looking through the face ABEH, the bubble appears to be at normal distance 12 cm from this face and when seen from the opposite face CDGF, it appears to be at normal distance 8 cm from CDGF. Find refractive index of glass and also the actual position of the bubble.



Solution :

Let the actual distance of the bubble from the face ABEH is 'x' then its actual distance from the face CDGF is 30 – x

$$\text{But } \mu = \frac{\text{Actual distance of bubble from refracting surface}}{\text{Apparent distance of bubble from refracting surface}}$$

The apparent distance of bubble from refracting surface ABEH $= \frac{x}{\mu}$; $12 = \frac{x}{\mu}$ ..... (1)

The apparent distance of bubble from refracting surface CDGF $= \frac{30 - x}{\mu}$; $8 = \frac{30 - x}{\mu}$ ..... (2)

Adding Eqs. (1) and (2) $\frac{x}{\mu} + \frac{30 - x}{\mu} = 20$,

we get $\mu = 1.5$

From Eq. (1); $\frac{x}{\mu} = 12$ (or) $\frac{x}{1.5} = 12$.

so x = 18 cm

This actual position of bubble is at normal distance 18 cm from face ABEH or at a distance 12 cm from face CDGF.

Example-2.14

The XY plane is the boundary between two transparent media. Medium 1 with $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z \leq 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium 2.

Solution :

Unit vector representing the normal to the plane $\hat{e}_n = \hat{k}$.

Component of the incident ray along the normal is $-10\hat{k}$.

The unit vector that represents the plane of the incident ray and the normal.

$$\hat{e}_p = \frac{\left(6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j}\right)}{\sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2}} = 0.6\,\hat{i} + 0.8\,\hat{j}$$

Angle between the incident ray and the normal is given by

$$\cos\theta = \left(6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}\right).\hat{k} / \sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2 + 10^2}$$

(or) $\cos\theta = -0.5$

Therefore, the angle $\theta = 120^0$

The angle of incidence is $\theta = 180^o - 120^o = 60^o$

The angle of the refracted beam is given by $\sqrt{2}\sin(\theta) = \sqrt{3}\sin(r)$ or $r = 45^o$

The equation of the emergent ray is

$\cos(45)(-\hat{k}) + \sin(45)[0.6i + 0.8j]$

$= 1/\sqrt{2}\,(0.6\,\hat{i} + 0.8\,\hat{j} - \hat{k})$

2.32 FINDING REFRACTIVE INDEX OF A LIQUID-CONCAVE MIRROR METHOD

A concave mirror of large radius of curvature is placed on a table with its principle axis vertical, as shown in figure 2.69.

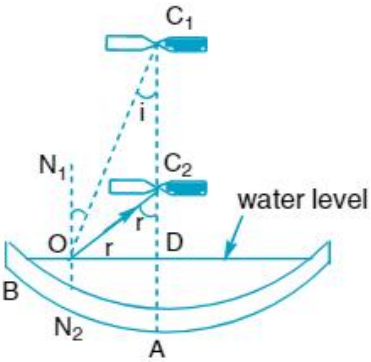


Fig 2.69

A horizontal pin is placed with its tip on the principal axis of the mirror. The pin is moved till there is no parallax between the tip of the pin and its image, when the pin lies at the center of curvature of the mirror.

A small quantity of liquid whose refractive index is to be measure is poured into the mirror. The pin is moved down in order to remove the parallax between the tip of the pin and its image.

In the figure, C_1 is the position of the pin when its image coincides with itself without liquid, and C_2 is the position of the pin when its image coincides with itself after pouring the liquid into the concave mirror.

The ray BO that is normal to the mirror passes through C_1 before pouring the liquid. It is refracted away from the normal when the liquid is poured, now it passes through C_2.

In figure $\sin i = \dfrac{OD}{OC_1}$, $\sin r = \dfrac{OD}{OC_2}$

From Snell's law,

$$\mu = \frac{OD}{OC_2} \times \frac{OC_1}{OD} = \frac{OC_1}{OC_2}$$

Taking paraxial ray assumption,

$OC_1 = DC_1$; $OC_2 = DC_2$;

Thus, $\mu = \dfrac{DC_1}{DC_2}$

If we neglect the depth of the liquid, we have

$$\mu = \frac{AC_1}{AC_2}$$

2.33 TOTAL INTERNAL REFLECTION

If light is passing from denser to rarer medium through a boundary, as angle of incidence (i) increases angle of refraction (r) also increases and for certain value of i (< 90°), r will become 90° as shown in the Fig 2.70. The value of angle of incidence for which r = 90° is called critical angle (θ_c).

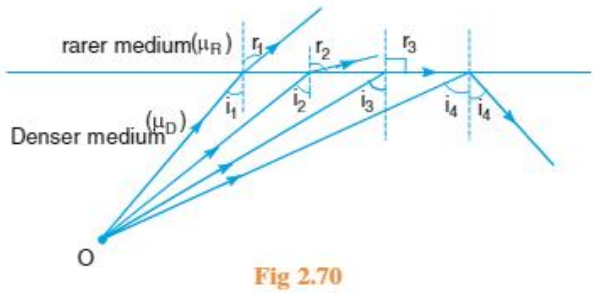


Fig 2.70

$i_1 < \theta_c$	$r_1 < r_2 < 90°$
$i_2 < \theta_c$	$r_3 = 90°$
$i_3 = \theta_c$	$i_4 > \theta_c$

According to Snell's law, at critical angle of incidence

$$\mu_D.\sin\theta_c = \mu_R.\sin 90°$$

$$\sin\theta_c = \frac{\mu_R}{\mu_D}$$

For $\mu_R = 1$ and $\mu_D = \mu$; $\sin\theta_C = \frac{1}{\mu}$

If $i > \theta_C$, sin r > 1. This means that 'r' is imaginary. This situation implies that refracted ray does not exist. So that light incident on the boundary will be reflected back in to the same medium from the boundary. This phenomenon is called total internal reflection.

- For total internal reflection to take place light must be propagating from denser to rarer medium.

  Examples : Ray from water to air, glass to water.
- Total internal reflection will take place only if angle of incidence is greater than critical angle.

  i.e., $i > \theta_C$ with $\theta_C = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right)$
- Critical angle (θ_C) depends on nature of pair of medium. Greater $\frac{\mu_R}{\mu_D}$ ratio greater will be the critical angle.

glass - air	**glass - water**
$\mu_D = \frac{3}{2}, \mu_R = 1$	$\theta_c = \sin^{-1}\left(\frac{2}{3}\right)$
$\theta_c = \sin^{-1}\left(\frac{8}{9}\right)$	$\theta_c = 42°$
$\theta_c \approx 63^0$	

- For a given pair of media, critical angle depends on wavelength of light used i.e greater the wavelength of light, lesser will be the value of μ [as $\mu \propto \frac{1}{\lambda}$] and so greater will be the critical angle.

  This is why critical angle is maximum for red and minimum for violet. Generally, the critical angle of a medium is defined for mean colour i.e., yellow.
- In case of total internal reflection, as all (100%) incident light is reflected back in to the same medium, there is no loss of intensity. This is why images formed by total internal reflection are much brighter than these formed by mirrors and lenses.
- Image due to total internal reflection is and lateral invered with respect to object.

2.34 DEVIATION OF LIGHT UNDER TOTAL INTERNAL REFLECTION

(i) The Fig 2.71 shows a light ray travelling from a denser to rarer medium at an angle i less than the critical angle θ_c.

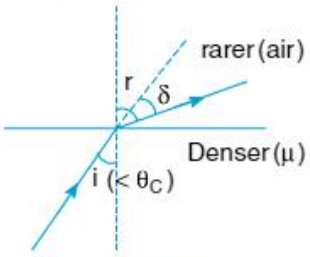


Fig 2.71

The deviation δ of the light ray is given by $\delta = r - i$

Since $\mu \sin i = \sin r$, $\delta = \sin^{-1}(\mu \sin i) - i$

This is a non linear equation. The maximum value of 'δ' occurs when $i = \theta_c$, and is equal to $\frac{\pi}{2} - \theta_C$, i.e. $\delta_{max} = \frac{\pi}{2} - \theta_c$

(ii) If the light is incident at an angle $i > \theta_c$, as shown in the Fig 2.72, then the angle of deviation is given by $\delta = \pi - 2i$. This is linearly decreasing function. The maximum value of δ occurs when $i = \theta_c$ and is equal to $\delta_{max} = \pi - 2\theta_c$.

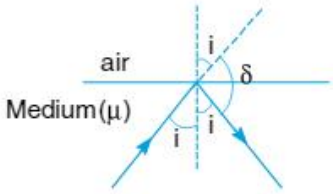


Fig 2.72

The variation of 'δ' with the angle of incidence 'i' is plotted in Fig 2.73.

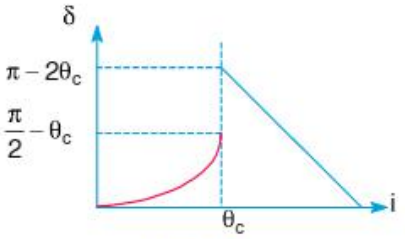


Fig 2.73

Some examples of total internal reflection

(i) An air bubble in water shines due to TIR at its outer surface when light propagating from water to air medium for $i > \theta_C$.

(ii) The faces of a piece of diamond are so cut that light entering into it suffers total internal reflection repeatedly at different faces and comes out though one or two faces only.

So these faces appear very glittering which explains the brilliance of diamonds.

(iii) **Mirage :**

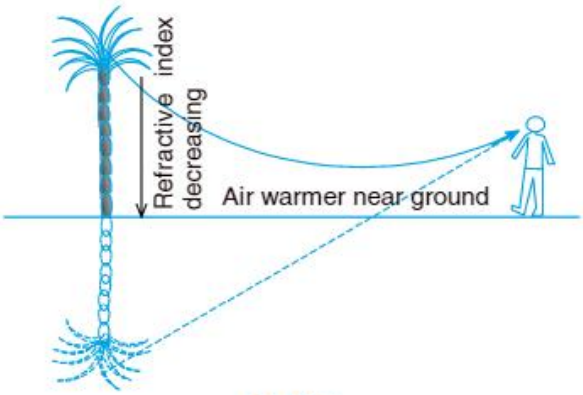


Fig 2.74

On hot summer days, the air near the ground becomes hotter than the air at higher levels. The refractive index of air decreases with its density. Hotter air is less dense, and has smaller refractive index than the cooler air. If the air is still, the optical density at different layers of air increases with height. As a result, light from a tall object such as tree, passes the medium whose refractive index decreases towards the ground. Then a ray of light from the object successively bends away from the normal and under goes total internal reflection if the angle of incidence for the air near the ground exceeds the critical angle. This is as shown in figure. To a distant observer, the light appears to be coming from somewhere below the ground. Such inverted images of distant tall objects causes an optical illusion to the observer. This phenomenon is called mirage.

Note : Similar to mirage, looming occurs in winter.

2.35 OPTICAL FIBRE

Optical fibre is a transmission medium to carry the optical signal without any appreciable loss. When a signal in the form of light is directed at one end of the fibre at a suitable angle, it under goes repeated TIR along the length of the fibre and finally comes out at the other end. This is shown in Fig 2.75. Due to TIR at each stage, there is no appreciable loss in the intensity of the light signal.

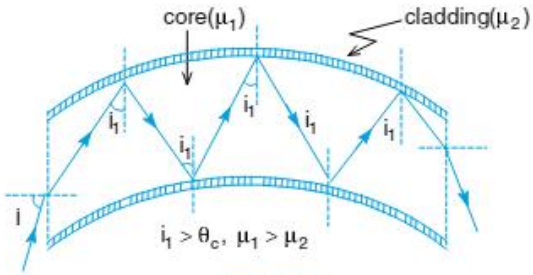


Fig 2.75

The basic structure of an optical fibre consists of a core and it is surrounded by cladding. For extra protection, the cladding is enclosed in an additional layer called coating or buffer. The refractive index of core is greater than that of cladding. The optical signal in the core undergoes TIR, at core and cladding interface. The light is trapped within the rod even if the rod is curved. Such a rod is called a light pipe. The thickness of optical fibre is of the order of $50\mu m$. If μ_1 and μ_2 are the refractive indices of core and cladding, the launching angle of incidence (i) for which light will be confined in the core is $i \le \sin^{-1}\sqrt{\mu_1^2 - \mu_2^2}$.

Formula for maximum launching angle

From the figure, from snell's law

$$\frac{\sin i}{\sin r_1} = \mu_1 \qquad \sin i = \mu_1 \sin r_1 \qquad .....(1)$$

From the figure angle of incidents at core and cladding interface is $i_1 = 90^0 - r_1$

For TIR, $i_1 > C$ $\qquad 90^0 - r_1 > C$

$$\sin(90^0 - r_1) > \sin C \qquad \cos r_1 > \frac{\mu_2}{\mu_1}$$

$$1 - \sin^2 r_1 > \left(\frac{\mu_2}{\mu_1}\right)^2,$$

$$\sin r_1 < \sqrt{1 - \left(\frac{\mu_2}{\mu_1}\right)^2} \qquad ....(2)$$

from eq. (1) and (2) $\frac{\sin i}{\mu_1} < \sqrt{1 - \left(\frac{\mu_2}{\mu_1}\right)^2}$

$$\sin i < \mu_1 \sqrt{1 - \left(\frac{\mu_2}{\mu_1}\right)^2}$$

$$\sin i_{max} = \mu_1 \sqrt{1 - \left(\frac{\mu_2}{\mu_1}\right)^2};$$

$$\sin i_{max} = \sqrt{\mu_1^2 - \mu_2^2}$$

Uses of Optical fibres

1) The optical fibre sensors are used to measure temperature and pressure.

2) Optical fibres are used to transmit large number communication signals through light pipes without any loss and superposition.

3) Optical fibres are used for measuring the blood flow in the heart.

Reflecting prisms

Reflecting prisms are designed to bend light ray by 90° or by 180° make use of total internal reflection as shown in Fig 2.76. Such a prism is also used to invert images without changing their size.

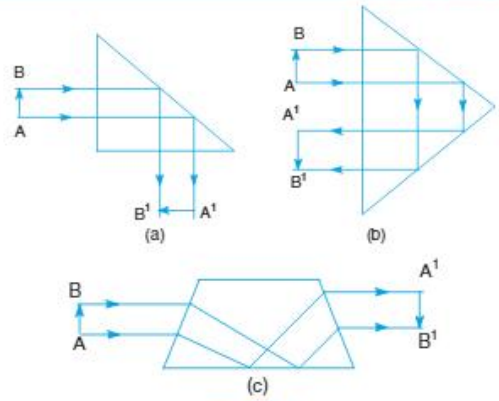


Fig 2.76

Example-2.15

A ray of light travelling in a rarer medium strikes a plane boundary between the rarer medium and a denser medium at an angle of incidence 'i' such that the reflected and the refracted rays are mutually perpendicular. Another ray of light of same frequency is incident on the same boundary from the side of denser medium. Find the minimum angle of incidence at the denser-rarer boundary so that the second ray is totally reflected.

Solution:

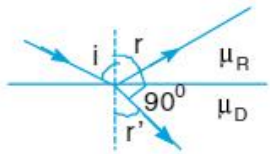


Figure shows incidence of a ray at the rarer-denser boundary such that reflected and refracted rays are mutually perpendicular.

i.e. $r + 90^0 + r' = 180^0$. or

$r' = 90^0 - r = 90^0 - i$[r = i, law of reflection]

Apply Snell's law at the boundary,

$\therefore \mu_R \sin i = \mu_D \sin r'$

$\mu_R \sin i = \mu_D \sin(90^0 - i) = \mu_D \cos i$

or $\frac{\mu_D}{\mu_R} = \tan i$ .....(1)

$$\sin\theta_c = \frac{1}{{}_R\mu_D} = \frac{1}{\mu_D/\mu_R} = \frac{\mu_R}{\mu_D} \qquad .....(2)$$

Using equation (1),

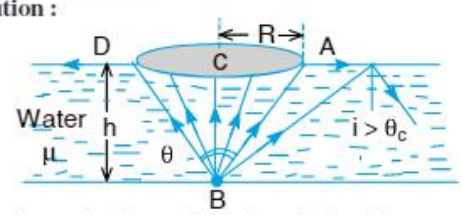
$$\sin\theta_c = \frac{1}{\tan i} = \cot i$$

$$\Rightarrow \theta_c = \sin^{-1}(\cot i)$$

Example-2.16

A bulb is placed at a depth of $2\sqrt{7}$ m in water and a floating opaque disc is placed over the bulb so that the bulb is not visible from the surface. What is the minimum diameter of the disc?

Solution :



As shown in figure, light from bulb will not emerge out of water if at the edge of disc,

$i > \theta_c$ or $\sin i > \sin\theta_c$ ..... (1)

Now if R is the radius of disc and h is the depth of bulb from it, $\sin i = \dfrac{R}{\sqrt{R^2+h^2}}$ and $\sin\theta_c = \dfrac{1}{\mu}$

So equation (1) becomes

$$\frac{R}{\sqrt{R^2+h^2}} > \frac{1}{\mu} \text{ or } R > \frac{h}{\sqrt{\mu^2-1}} \quad .....(2)$$

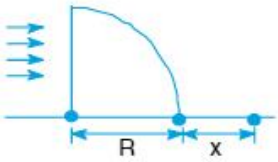
Here $h = 2\sqrt{7}$ m and $\mu = (4/3)$

So $R_{min} = \dfrac{2\sqrt{7}}{\sqrt{(16/9)-1}} = 6\text{m}$

So diameter of disc = 2R = 2× 6 = 12 m

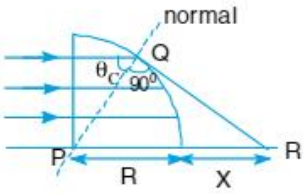
Example-2.17

A uniform, horizontal beam of light is incident upon a quarter cyclinder of radius R = 5cm, and has a refractive index $2/\sqrt{3}$. A patch on the table at a distance 'x' from the cylinder is unilluminated. Find the value of 'x'?

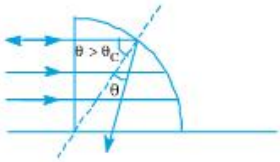


Solution :

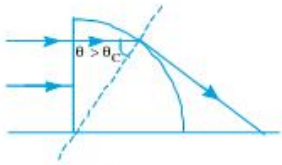
When the angle of incidence is θ_c, 'x' region will remain unillumiated. This is the limiting case.



When the angle of icidence is greater than critical agle T.I.R. will place, In this case also 'x' region is unilluminated.



When angle of incidence $\theta < \theta_c$, then the incident ray will refract and hence 'x' region will be illuminated. So, in limiting case:



In $\Delta PQR; \cos\theta_c = \dfrac{R}{R+x}$

$$\sin\theta_c = \frac{\mu_r}{\mu_d} = \frac{1}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$\therefore \theta_c = 60^0$, R = 5cm (given)

$\Rightarrow \cos 60 = \dfrac{5}{5+x} \Rightarrow 5+x = 10$

$\Rightarrow$ x = 5cm.

2.36 REFRACTION THROUGH A MEDIUM WITH VARIABLE REFRACTIVE INDEX

So far, refraction in a medium with variable refractive index we have assumed that the refractive index of the slab is a constant. This need not be necessarily true. An example of such a situation is the atmosphere. The atmosphere becomes thinner as we go up. Hence, the refractive index of air is highest close to earth and decreases as we move upward. How do we model such a problem?

Divide the medium into different layers. This indicates that each layer has different refractive index having value according to a given expression. If μ is a function of x, then a differential layer will be a thin layer of thickness dx. If μ is a function of y, then a differential layer will be a thin layer of thickness dy.

At a given layer, let the angle of incidence be i. Relate this angle of incidence to the initial condition and the refractive index at that point using the relation.

$\mu \sin(i) = \text{constant}$ ....(i)

From the given ray diagram draw a tangent to the path taken by the ray of light.

Geometrically, relate the slope of this tangent to the angle of incidence.

$$\tan\theta = \frac{dy}{dx} = \tan(90 - i)$$

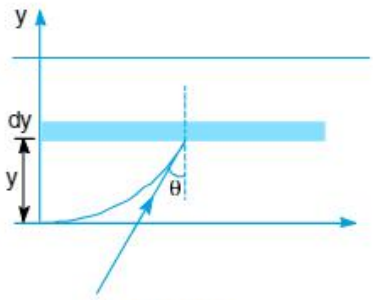


Fig 2.77

Substitute for i from Eq. (i) and determine dy/dx as a function of x and y.

Integrate and obtain an expression for y as a function of x. Let us discuss a case where refractive index is changing in y-direction.

As $1 : \mu = f(y)$

Consider a slab with a medium where the refractive index varies from μ_1 and μ_2 in relation with y. Assume that the outside medium is air. Let light be incident on the glass slab at an angle θ. The light ray strikes the glass slab at the point A. The normal at this point is NN' as shown in figure. Once the ray enters the medium, the refractive index varies continuously as if there are infinite number of successive glass slabs. Therefore, the angle of incidence also varies continuously and the path of the ray is curved. The ray finally reaches the second surface at the point B before exiting the slab, once again refracting at the exit point.

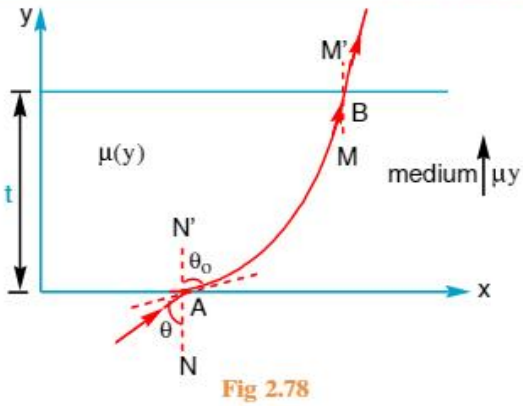


Fig 2.78

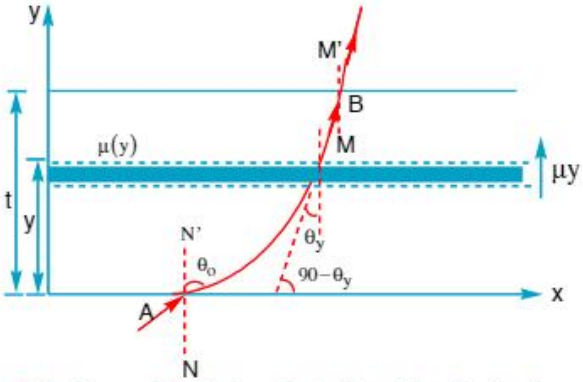
How can we derive a mathematical expression for the equation of the ray in the medium? Consider refraction at some height y. Here, the angle of incidence is θ_y and the refractive index of the medium is μ_y. Across an interface, we have

$\mu_1 \sin(\theta_1) = \mu_2 \sin(\theta_2)$

i.e., product of the refractive index and the since of the angle of incidence is constant.

Therefore, at the height y we can say that

$\mu_y \sin\theta_y = \text{constant}$ .... (1)



Path of a ray of light being refracted in a slab with refractive index as a function of y and y ranges from 0 to 1.

Fig 2.79

Further more, applying the law of refraction at point A which is the interface between air and the medium, we have

$1 \times \sin(\theta) = \mu_o \sin\theta_o = \mu_y \sin\theta_y$ .... (ii)

Therefore, $\sin\theta_y = \dfrac{\sin\theta}{\mu_y}$

Now, the slope of the curve at the point A is given by

$$\tan\phi = \frac{dy}{dx} \quad ..... (iii)$$

But $\phi = 90 - \theta_y$

Therefore, $\frac{dy}{dx} = \cot(\theta_y) = \sqrt{\mu_y^2 - \sin^2\theta}$

$$\frac{dy}{\sqrt{\mu_y^2 - \sin^2\theta}} = dx$$

Integrating both side with proper limits.

If the functional dependence of 'μ_y' is known, we can easily solve for the equation of the curve.

$$\int_{y_2}^{y_1} \frac{dy}{\sqrt{\mu_y^2 - \sin^2\theta}} = \int_{x_1}^{x_2} dx$$

Example-2.18

Due to a vertical temperature gradient in the atmosphere, the index of refraction varies. Suppose index of refraction varies as $n = n_0\sqrt{1+ay}$, where n_0 is the index of refraction at the surface and $a = 2.0\times10^{-6}\,m^{-1}$. A person of height h = 2.0m stands on a level surface. Beyond what distance will he not see the runway?

Solution :

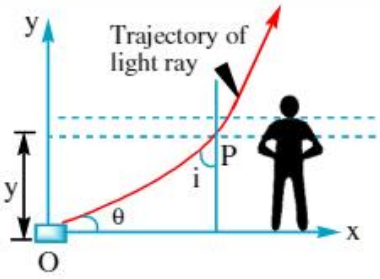
As refractive index is changing along y-direction, we can assume a number of thin layers of air placed parallel to x-axis. Let O be the distant object just visible to the man. Consider a layer of air at a distance y from the ground. Let P be a point on the trajectory of the ray. From figure, $\theta = 90 - i$.

The slope of tangent at point P is

$\theta = dy/dx = \cot i$.

From Snell's law, n sin i = constant

At the surface, $n = n_0$ and $i = 90^0$.



$$n_o \sin 90^o = n\sin i = \left(n_o\sqrt{1+ay}\right)\sin i$$

$$\sin i = \frac{1}{\sqrt{1+ay}} \Rightarrow \cot i = \frac{dy}{dx} = \sqrt{ay}$$

$$\int_0^y \frac{dy}{\sqrt{ay}} = \int_0^x dx \Rightarrow x = 2\sqrt{\frac{y}{a}}$$

On substituting y = 2.0 m and a = 2 × 10–6m^{-1},

we have $x_{max} = 2\sqrt{\frac{2}{2\times10^{-6}}} = 2000\,m$

Example-2.19

A ray of light is incident on a glass slab at grazing incidence. The refractive index of the material of the slab is given by $\mu = \sqrt{(1+y)}$. If the thickness of the slab is d, determine the equation of the trajectory of the ray inside the slab and the coordinates of the point where the ray exits from the slab. Take the origin to be at the point of entry of the ray.

Solution :

From the equation, we have

$$\frac{dy}{dx} = \cot(\theta_y) = \sqrt{\mu_y^2 - \sin^2\alpha}$$

Here, $\mu = \sqrt{(1+y)}$ and $\alpha = 90^0$

Therefore, $\frac{dy}{dt} = y^{1/2}$

Integrating with the boundary condition that y = 0 at x = 0, we get $y = \frac{x^2}{4}$ to be the equation of the path of the ray through the slab. The ray will obviously exit at the point $(2\sqrt{2}, d)$.

2.37 FINDING FOCAL LENGTH OF A CONVEX LENS

Plane Mirror method

A convex lens is placed on a plane mirror figure. An object pin O is moved along the principal axis of the lens. At a certain position, the

image coincides with the object. The distance of object pin O from the lens is the focal length of the lens. When an object is placed at the focus of the lens, the refracted rays become parallel to the principal axis. These parallel rays are incident on the mirror normally, therefore they retrace their path and converge at the focus.

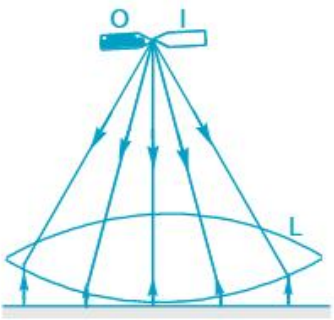


Fig 2.80

2.38 MEASUREMENT OF REFRACTIVE INDEX OF LIQUID BY A CONVEX LENS

Fig shows an equiconvex lens placed on a plane mirror. An object pin is moved up and down. When the pin lies at the focus of the lens, there is no parallax between the object and the image.

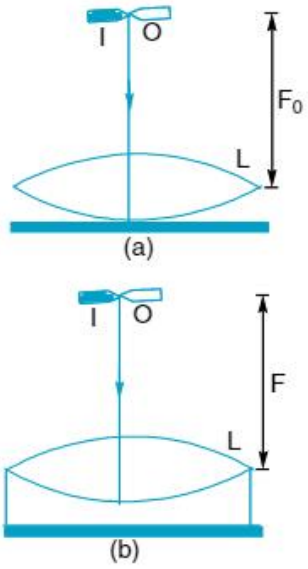


Fig 2.81

When a liquid whose refractive index is to be obtained is placed between a plane mirror and a convex lens, the object pin O has to be shifted downward so that no parallax exists between it and its image. The position of object O from the lens is now equal to the combined focal length of 'lens and liquid' combination. If f is the focal length of the liquid lens, then the combined focal length is given by

$$\frac{1}{F} = \frac{1}{f_0} + \frac{1}{f}$$

The liquid lens is plano-convex type as its lower surface is the plane surface of the mirror and the upper surface is the curved surface of the convex lens. If R is the radius of curvature of the convex lens, then

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) \Rightarrow \frac{1}{f} = -\frac{(\mu - 1)}{R}$$

Thus, $\mu = 1 + \frac{R}{f}$

2.39 REFRACTION AT SPHERICAL SURFACES

We have so far considered refraction at plane surface. We shall now consider refraction at a spherical interface between two transparent media. An infinitesimal part of a spherical surface can be regarded as planar and the same laws of refraction can be applied at every point on the surface. Just as for reflection by a spherical mirror, the normal at the point of incidence is perpendicular to the tangent plane to the spherical surface at that point and therefore, passes through its centre of curvature.

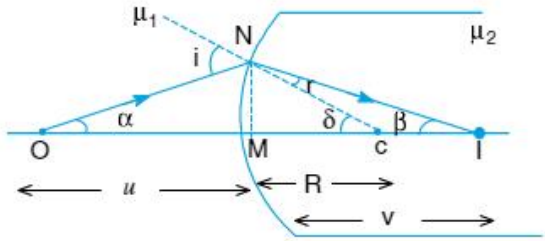


Fig 2.82

Fig 2.82 shows the geometry of formation of image I of an object 'O' on the principal axis of spherical surface with centre of curvature C, and radius of curvature R. The rays are incident from

a medium of refractive index μ_1 to another of refractive index μ_2. As before, we take like in curved mirrors the aperture of the surface to be small compared to other distances involved. Hence NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis.

$$\text{Tan}\,\alpha = \frac{MN}{OM},\ \text{Tan}\,\delta = \frac{MN}{MC},\ \text{Tan}\,\beta = \frac{MN}{MI}$$

Now for ΔNOC, 'i' is the exterior angle.

Therefore $i = \alpha + \delta$

similarly $r = \lfloor NCM - \lfloor NIM = \delta - \beta$

Now by snell's law $\mu_1.\sin i = \mu_2.\sin r$ or for small angles $\mu_1.i = \mu_2.r$. Substituting for i & r, we get $\mu_1(\alpha+\delta) = \mu_2(\delta-\beta)$

$$\mu_1\alpha + \mu_2\beta = \delta(\mu_2 - \mu_1)$$

$$\frac{\mu_1}{OM} + \frac{\mu_2}{MI} = \frac{\mu_2 - \mu_1}{MC}.$$

Here OM, MI and MC represent magnitude of distances. Applying Cartesian sign convention

OM = –u, MI = + v, MC = + R

$$\therefore\ \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

This is the Gaussian' relation for a single spherical refracting surface. μ_1 is the refractive index of the medium which comes before the boundary and μ_2 is the refractive index of the medium which comes after the boundary if we move in the direction of light. Though above relation is derived for a convex surface and for a real object and real image, it is equally valid for all other conditions.

If the object or image itself is present at a refracting surface, refraction at that surface is not considered. Note that with respect to real object convex refracting surfaces can form real image (for distant object) as well as virtual image (for nearer object), whereas concave refracting surface forms only virtual image.

2.40 MAGNIFICATION

a) Lateral magnification or transverse magnification

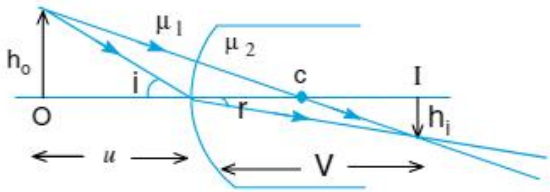


Fig 2.83

The lateral magnification may be obtained with the help of the given Fig 2.83. It is the ratio of height of image to height of object $m = \frac{h_i}{h_0}$

From Snell's law $\mu_1 i = \mu_2.r$

(For small angles)

Therefore $\mu_1 \frac{h_0}{u} = \mu_2 \frac{h_i}{v}$

Thus lateral magnification

$$m = \frac{h_i}{h_0} = \frac{\mu_1}{\mu_2}.\left(\frac{v}{u}\right)$$

b) Longitudinal magnification at refracting curved surface

If a small object of length 'du' placed on the axis, produces an image of length 'dv' along the axis of the refracting surface, then longitudinal magnification

$$m_L = \frac{dv}{du}$$

Since $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

On differentiating, $\frac{dv}{du} = \frac{\mu_1}{\mu_2}.\frac{v^2}{u^2}$

$$\therefore m_L = \frac{\mu_1}{\mu_2}\frac{v^2}{u^2}$$

Longitudinal magnification $m_L = m^2\left(\frac{\mu_2}{\mu_1}\right)$,

where 'm' is transerverse magnification

2.40.1 SPEED OF IMAGE AND OBJECT ALONG THE PRINCIPAL AXIS

Since $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$,

as the object position changes with time, the image position also changes.

Hence $-\frac{\mu_2}{v^2}\frac{dv}{dt} + \frac{\mu_1}{u^2}\frac{du}{dt} = 0$, $\frac{\mu_2}{v^2}v_I = \frac{\mu_1}{u^2}v_0$

$$\therefore \; v_I = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right)^2 v_0$$

v_I = (longitudinal magnification) v_o.

On applying proper sign convention we get direction of motion of image.

2.40.2 MOTION OF OBJECT TRANSVERSE TO THE PRINCIPAL AXIS

If the object moves transverse to the principle axis at distance u with the speed V_0, if m is the magnification, then

$$m = \frac{\mu_1}{\mu_2}\left(\frac{v}{u}\right) = \frac{h_i}{h_0} = \frac{\frac{dh_i}{dt}}{\frac{dh_0}{dt}}$$

$$V_I = \frac{\mu_1}{\mu_2}\left(\frac{v}{u}\right).V_0 \text{ (or) } m = \frac{V_I}{V_0} \text{ (or) } V_I = mV_o$$

2.41 PRINCIPAL FOCI

In the equation $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

(a) If the object is at infinity (i.e., $u = \infty$)

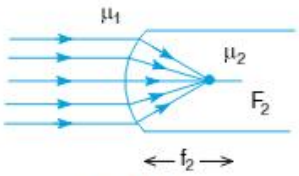


Fig 2.84

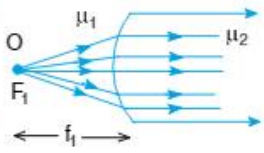


Fig 2.85

$$\frac{\mu_2}{v} - 0 = \frac{\mu_2 - \mu_1}{R} \Rightarrow f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1} \text{ (as } v = f_2)$$

This value of $v = f_2$ is called the second principal focal length. i.e The position of image corresponding to the object at infinity is called the second principal focus of the refracting surface. This is shown in Fig 2.84.

(b) Similarly if $v = \infty$, i.e., the object is so placed that the refracting rays becomes parallel to the principal axis, then $\frac{\mu_2}{\infty} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$u = -\frac{\mu_1 R}{\mu_2 - \mu_1} = f_1$$

This value of $u = f_1$ is called the first principal focal length of the refracting surface. i.e. The position of the object, whose image is formed at infinity is known as the first principal focus of the refracting surface. This is shown in Fig 2.85.

Hence $\frac{f_1}{f_2} = -\frac{\mu_1}{\mu_2}$

It is easy to see that the first focal length f_1 for spherical refracting surface is not equal to the second focal length f_2.

Further $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\therefore \frac{\mu_2 R}{v(\mu_2 - \mu_1)} - \frac{\mu_1 R}{u(\mu_2 - \mu_1)} = 1 \;\therefore\; \frac{f_2}{v} + \frac{f_1}{u} = 1$$

2.42 POWER OF REFRACTING SPHERICAL SURFACE

Any distance divided by the refractive index of the space in which it is measured is called a reduced distance. The refracting power of a spherical surface is defined as the reciprocal of the reduced focal length. Let f_1 and f_2 be the first and second principal focal lengths of refracting surface.

The ratios $\frac{f_1}{\mu_1}$ and $\frac{f_2}{\mu_2}$ are the reduced focal lengths.

Then refracting power

$$P = \frac{\mu_1}{f_1} = -\frac{\mu_2}{f_2} \qquad \left(\text{As } \frac{f_1}{\mu_1} = -\frac{f_2}{\mu_2}\right)$$

$$\text{Power (p)} = \frac{\mu_2 - \mu_1}{R}$$

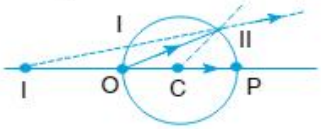
Example-2.20

A small object stuck on the surface of a glass sphere ($\mu = 1.5$) is viewed from the diametrically opposite position. Find transverse magnification

Solution :

Refraction will take place at side II of the surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$



Here $\mu_1 = 1.5, \mu_2 = 1$ and $u = -2R$

$$\frac{1}{v} - \frac{1.5}{(-2R)} = \frac{1-1.5}{-R}$$

$$\frac{1}{v} = \frac{0.5}{R} - \frac{1.5}{2R} = \frac{-0.5}{2R} \text{ or } v = -4R$$

Negative sign indicates that the image is formed to the left of refracting surface as shown.

Magnification, $m = \frac{\mu_1 v}{\mu_2 u} = \frac{1.5(-4R)}{1(-2R)}$

$\therefore\ m = 3$

Example-2.21

One end of a cylindrical glass rod ($\mu = 1.5$) is given the shape of a concave refracting surface of radius 10cm. An air bubble is situated in the glass rod at a point on its axis such that it appears to be at distance 10 cm from the surface and inside glass when seen from the other medium. Find the actual location of air bubble.

Solution :

Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\mu_1 = 1.5,\ \mu_2 = 1;\ \ v = -10$ cm ; $R = +10$ cm

$$\therefore -\frac{1}{10} - \frac{1.5}{u} = \frac{1-1.5}{10} = -\frac{1}{20}$$

$$\frac{1.5}{u} = -\frac{1}{10} + \frac{1}{20} = -\frac{1}{20}$$

$\therefore\ u = -30$ cm

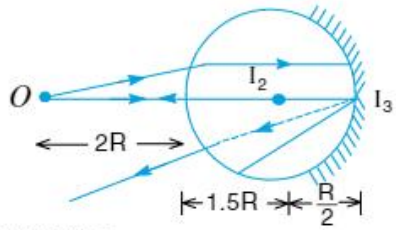
Hence the air bubble is actually located at a distance 30cm from the surface and inside glass.

Example-2.22

A solid glass sphere with radius R and an index of refraction 1.5 is silvered over one hemisphere. A small object is located on the axis of the sphere at a distance 2R to the left of the vertex of the unsilvered hemisphere. Find the position of final image after all refractions and reflections have taken place.

Solution :

The ray of light first gets refracted then reflected and then again refracted. For first refraction and then reflection the ray of light travels from left to right while for the last refraction it travels from right to left. Hence, the sign convention will change accordingly.



First : refraction

Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with proper sign conventions,

we have $\frac{1.5}{v} - \frac{1.0}{-2R} = \frac{1.5-1.0}{+R}$ $\quad \therefore v_1 = \infty$

Second : reflection

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$ with proper sign conventions,

we have $\frac{1}{v_2} + \frac{1}{\infty} = -\frac{2}{R}$

$$\therefore v_2 = -\frac{R}{2}$$

Third : refraction

Again using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with reversed sign convention, we have $\frac{1.0}{v_3} - \frac{1.5}{-1.5R} = \frac{1.0-1.5}{-R}$

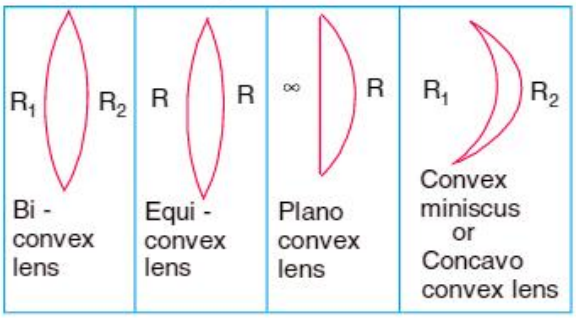
or $v_3 = -2R$ i.e., final image is formed on the vertex of the silvered face.

2.43 LENS THEORY

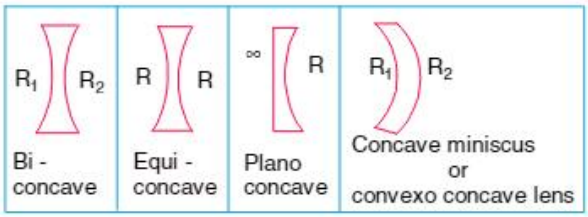
A Lens is a piece of transparent material with two refracting surfaces such that at least one is curved and refractive index of its material is different from that of the surroundings.

If the curved surface (or surfaces) of a lens are spherical, the lens is called spherical lens and if its thickness is small the lens is called thin. Here we shall limit ourselves to thin spherical lenses. Different types of spherical lenses are as shown in Fig 2.86 (a) and (b).

While naming the lens, we first call the shape of the surface which has larger radius of curvature. A thin lens with refractive index greater than that of surroundings behaves as a convergent or convex lens if its central portion is thicker than marginal one. i.e. it converge parallel rays incident on it. However if the central portion of the lens (with $\mu_L > \mu_m$) is thinner than marginal, it diverges parallel rays and behaves as divergent or concave lens. Converging lens is represented as ↕ and divergent lens is represented as ⤲.



(a) Convex Lens



(b) Concave Lens

Fig 2.86

In case of thin spherical lenses

(a) Optical centre or pole O is a point for a given lens through which any ray passes undeviated.

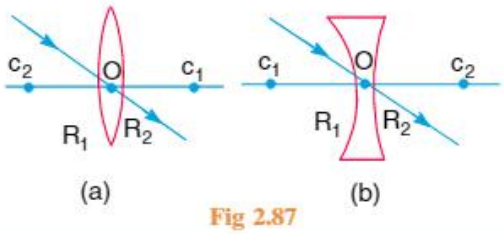


Fig 2.87

(b) Principal axis C_1C_2 is a line passing through optical centre and centres of curvature of the refracting surfaces, which is perpendicular to lens.

(c) A lens has two surfaces and hence two focal points. First focal point (F_1) is an object point (real in case of a convex lens and virtual for concave) on the principal axis for which image is formed at infinity.

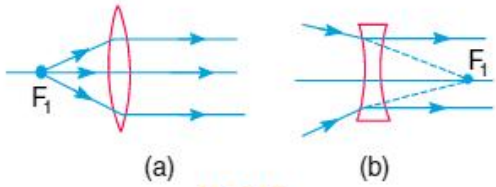


Fig 2.88

Second focal point (F_2) is an image point on the principal axis for which object lie at infinity.

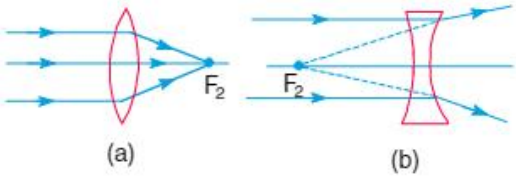


Fig 2.89

(d) The distance between optical centre of a lens and the point where the parallel beam of light converges or appears to converge. i.e., second principal focal point (F_2), is called focal length f.

(e) To a lens, if the media on the two sides is same, then first principal focal distance is equal to second principal focal distance. i.e., $|f_1| = |f_2|$.

We are mainly concerned with the second focus F_2. Thus, wherever we write the focal length 'f', it denotes second principal focal length.

(f) In reference to a lens, aperture is the effective diameter of its light transmitting area. The intensity of image formed by a lens depends on square of aperture. i.e., $I \propto (\text{aperture})^2$

(g) The sign convention to a lens is similar to curved mirror.

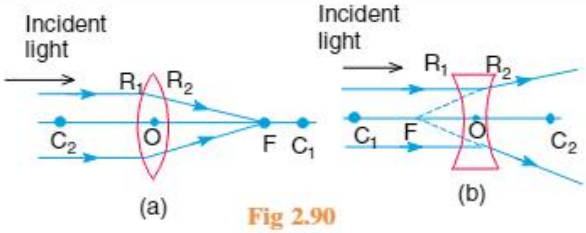


Fig 2.90

R_1 (= OC_1) is +ve	R_1 (= OC_1) is –ve
R_2 (= OC_2) is –ve	R_2 (= OC_2) is +ve
f = (OF) is +ve	f = (OF) is –ve

(h) A thin lens is a lens in which the thickness of the lens is small compared to the object distance, the image distance or either of the two radii of curvature of the lens.

R_1 (= OC_1) is +ve	R_1 (= OC_1) is -ve
R_2 (= OC_2) is -ve	R_2 (= OC_2) is +ve
f = (OF) is +ve	f = (OF) is -ve

(h) A thin lens is a lens in which the thickness of the lens is small compared to the object distance, the image distance or either of the two radii of curvature of the lens.

2.44 RULES FOR IMAGE FORMATION

In order to locate the image and its nature by a lens graphically, the following rules are adopted.

(i) A ray parallel to the principal axis, after refraction, passes through the principal focus or appears to diverge from it.

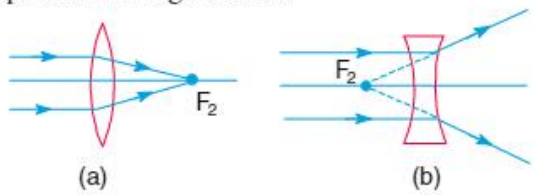


Fig 2.91

(ii) A ray passing through the first focus F_1 become parallel to the principal axis after refraction.

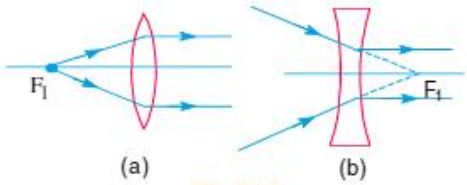


Fig 2.92

(iii) A ray through the optical centre O passes undeviated because the middle of the lens acts like a thin parallel sided slab.

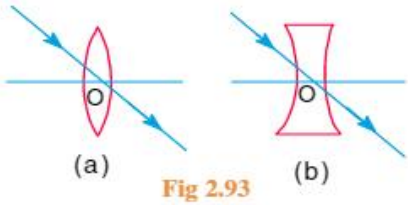
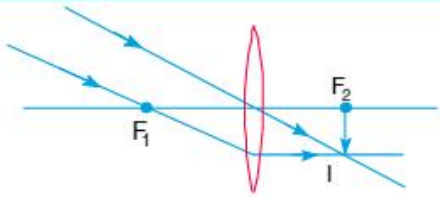
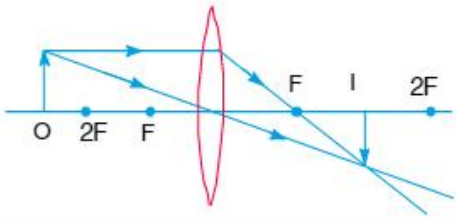


Fig 2.93

2.45 IMAGE FORMATION BY LENS

a) Convex lens

Position of the object	Ray diagram	Image details
At infinity		Real, inverted, diminished at F
Between		Real, inverted, diminished between F and 2F

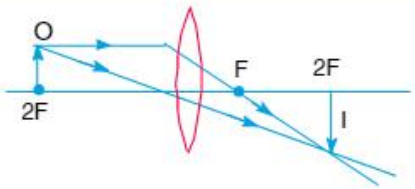
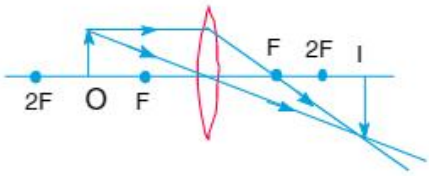
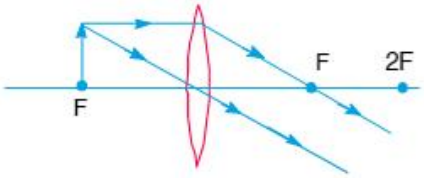
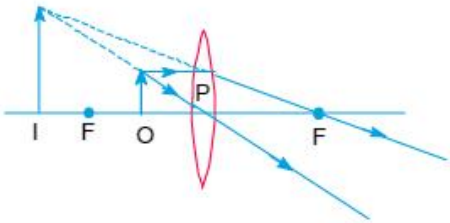
At 2F		Real, inverted, equal, at 2F
Between 2F and F		Real, inverted, enlarged between 2F and infinity
At F		Real, inverted, enlarged, at infinity
Between F and P		Virtual,erect,enlarged between and object on same side

Fig 2.94

b) Concave lens

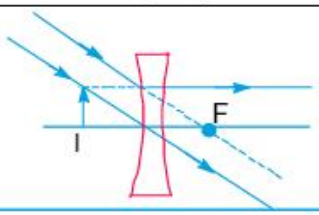
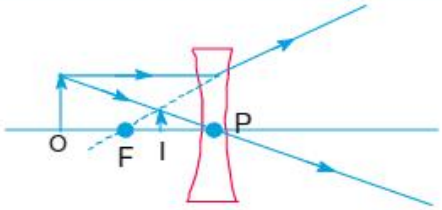
Position of the object	Ray diagram	Image details
At infinity		Virtual, erect, diminished, at F
In front of lens		Virtual, erect, diminished, between F and P

Fig 2.95

From the ray diagrams it is clear that

a) Regarding convex lens

(i) A convex lens will form a real image for a real object when the object is placed beyond focus.

(ii) When the object comes within the focus, then a virtual image is formed for the real object.

(iii) The real image formed is always inverted while virtual image is always erect

b) Regarding concave lens

(i) A concave lens always forms virtual image for a real object.

(ii) The image formed by a concave lens is always erect and diminished in size.

(iii) A concave lens can form a real image as well as virtual image if the object is virtual.

2.46 LENS MAKER'S FORMULA

In case of image formation by a lens, the incident ray is refracted at first and second surfaces respectively. The image formed by the first surface acts as object for the second.

Consider an object O placed at a distance u from a convex lens as shown in Fig 2.96. Let its image be I_1 after refraction through first surface. So from the formula for refraction at curved surface.

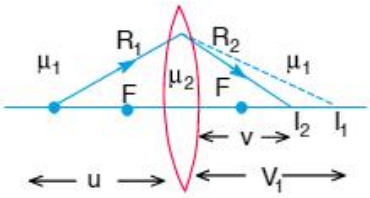


Fig 2.96

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

For first surface $\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$ ..... (1)

The image I_1 acts as object to second surface, and form final image I_2.

For second surface

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \quad ..... (2)$$

So adding (1) and (2) equations, we have

$$\mu_1\left[\frac{1}{v} - \frac{1}{u}\right] = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\text{(or)} \quad \frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

with $\mu_r = \frac{\mu_2}{\mu_1}$ (or) $\frac{\mu_L}{\mu_M}$

Now if object is at infinity, image will be formed at the focus i.e. for $u = -\infty$, $v = f$, so that above equation becomes $\frac{1}{f} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

which is known as Lens-maker's formula and for a lens it becomes $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ which is known as the 'lens-formula' or Gauss's formula for a lens. Though we derived it for a real image formed by a convex lens, the formula is valid for both convex as well as concave lens and for both real and virtual images. The lens maker's formula is applicable for thin lenses only and the value of R_1 and R_2 are to be put in accordance with the Cartesian sign convention.

2.47 MAGNIFICATION

a) Lateral magnification

Magnification produced by a lens is defined as the ratio of the size of image to that of the object. Here the sizes are measured perpendicular to principal axis.

$$m = \frac{II'}{OO'} = \frac{h_i}{h_0} = \frac{v}{u}$$

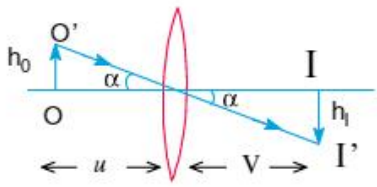


Fig 2.97

When we apply the sign convention, for erect (and virtual) image formed by a convex or concave lens 'm' is positive, and for an inverted (and real) image, m is negative.

b) Longitudinal magnification

Longitudinal magnification is defined as the ratio of infinitesimal axial length (dv) in the region of the image to the corresponding length (du) in the region of the object.

Longitudinal magnification $(m_L) = \frac{dv}{du}$.

On differentiating equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

we get $m_L = -m^2$.

c) Angular magnification of lens

The ratio of the slopes of emergent ray and corresponding incident ray with principal axis is called the angular magnification.

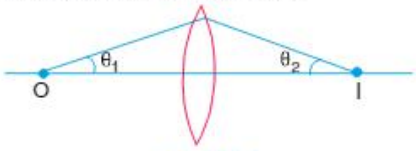


Fig 2.98

Angular magnification$(\gamma) = \frac{\tan\theta_2}{\tan\theta_1}$

It shoud note be that

(i) longitudinal magnification is proportional to the square of the lateral magnification.

(ii) When several lenses or mirrors are used co-axially, the total magnification

$m = m_1 \times m_2 \times ..... \times m_n$.

Example-2.23

What is the refractive index of material of a plano-convex lens, if the radius of curvature of the convex surface is 10 cm and focal length of the lens is 30 cm?

Solution : According to lens-maker's formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here f = 30 cm, $R_1 = 10$ cm and $R_2 = \infty$

so $\frac{1}{30} = (\mu - 1)\left[\frac{1}{10} - \frac{1}{\infty}\right]$

i.e., $3\mu - 3 = 1$ or $\mu = (4/3)$

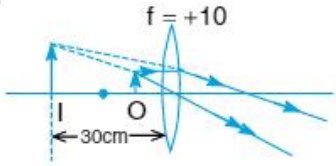
Example-2.24

A convex lens of focal length 10 cm. is used as a magnifying lens.

(a) Where should the object be placed if the image is to be 30 cm from the lens?

(b) What will be the magnification?

Solution :



(a) In case of magnifying lens, the lens is convergent and the image is erect, enlarged, virtual, between infinity and object and on the same side of lens as shown in figure So here f = 10 cm and v= –30 cm and hence from lens-formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we have $\frac{1}{-30} - \frac{1}{u} = \frac{1}{10}$

i.e., u = –7.5 cm

So the object must be placed in front of lens at a distance of 7.5 cm (which is < f) from it.

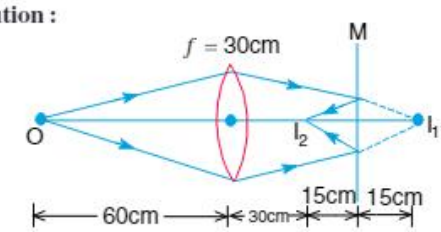
(b) $m = \left[\frac{I}{O}\right] = \frac{v}{u} = \frac{-30}{-7.5} = 4$

i.e., image is erect, virtual and four times the size of object.

Example-2.25

A source of light is located at double focal length from a convergent lens. The focal length of the lens is f = 30 cm. At what distance from the lens should a flat mirror be placed, so that the rays reflected from the mirror are parallel after passing through the lens for the second time?

Solution :



Object is at distance of 2f = 60 cm from the lens. Image formed by lens I_1, should be at a distance 60 cm from the lens.

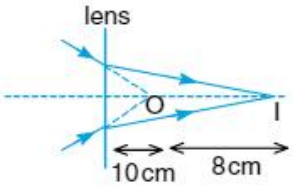
Now I_2, the image formed by plane mirror should lie at focus or a distance of 30 cm from the lens.

Hence, the mirror should be placed at distance 45 cm from the lens as shown in figure.

Example-2.26

A converging beam of light forms a sharp image on a screen. A lens is placed in the path of the beam, at 10cm from the screen. It is found that the screen has to be moved 8 cm further away from the lens to obtain a sharp image. Find the focal length and nature of lens.

Solution :



As shown in figure, in this problem, object O is virtual while image I real,

so that u = 10 cm and v = (10 + 8) = 18 cm and hence

from lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

we have $\frac{1}{18}-\frac{1}{10}=\frac{1}{f}$

which on simplification gives $f=-\frac{90}{4}=-22.5\text{cm}$

i.e., the lens is diverging (concave) of focal length 22.5 cm

Example-2.27

A convex lens of focal length 24 cm ($\mu = 1.5$) is totally immersed in water ($\mu = 1.33$). Find its focal length in water.

Solution :

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

When the lens is in air,

$$\frac{1}{24\text{cm}}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{..... (i)}$$

When the lens is in water

$$\frac{1}{f}=\left(\frac{1.5}{1.33}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{..... (ii)}$$

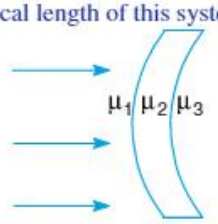
Dividing the expression (i) by (ii), we get

$$\frac{f}{24\text{cm}}=\frac{0.5}{0.125}$$

f = 96 cm

Example-2.28

In the figure, light is incident on the thin lens as shown. The radius of curvature for both the surface is R. Determine the focal length of this system.



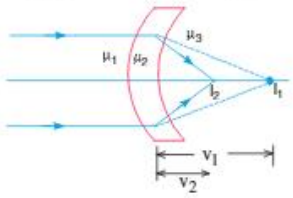
Solution :

For refraction at first surface,

$$\frac{\mu_2}{v_1}-\frac{\mu_1}{-\infty}=\frac{\mu_2-\mu_1}{+R} \quad \text{.... (i)}$$

For refraction at 2nd surface,

$$\frac{\mu_3}{v_2}-\frac{\mu_2}{v_1}=\frac{\mu_3-\mu_2}{+R} \quad \text{.... (ii)}$$



Adding equations (i) and (ii) we get

$$\frac{\mu_3}{v_2}=\frac{\mu_3-\mu_1}{R} \text{ or } v_2=\frac{\mu_3 R}{\mu_3-\mu_1}$$

Therefore, focal length of the given lens system is

$$\frac{\mu_3 R}{\mu_3-\mu_1}$$

2.48 POWER OF A LENS

As we said earlier in mirrors, the power of a lens is the measure of its ability to produce convergence or divergence of a parallel beam of light. The power P of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distant from the optical centre.

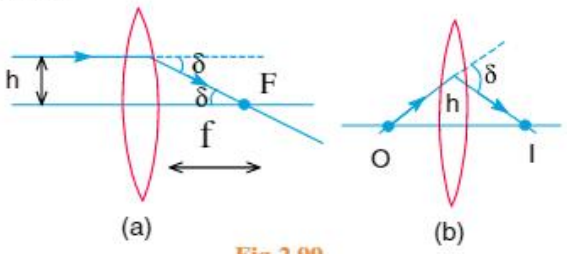


Fig 2.99

$$\tan\delta=\frac{h}{f} \qquad \text{if } h=1,\ \tan\delta=\frac{1}{f}$$

As per definition, power (P) = $\tan\delta = \frac{1}{f}$

If lens is placed in a medium other than air, of refractive index μ, then power $P = \frac{\mu}{f}$

The S.I.unit of power is dioptre (D) and $1D = 1m^{-1}$.

i.e. $P = \frac{1}{f\,(in\,m)} = \frac{100}{f\,(in\,cm)}D$

For of concave meniscus with $R_1 = R_2$, $f = \propto$ and P = 0.

The deviation produced by a lens is independent of the position of the object whatever may be the direction of incidence. All the rays suffer the same deviation in refraction through the lens. Clearly a lens with shorter focal length deviates the incident light more. A convex lens converge the incident rays. Due to this reason, that power of a convex lens is taken as positive. On the other hand, a concave lens diverge the incident rays. Therefore its power is taken as negative.

- Every part of a lens forms complete image even if a portion is obstructed, but intensity of the image decreases. Also if a lens is painted with black strips and a green page is seen through it, the green page will not appear with black strips but will remain green with reduced intensity.
- The minimum distance between a real object and its real image formed by a single lens is 4f. This is proved in section 2.46.
- As focal length of a lens depends on its refractive index i.e. $\frac{1}{f} \propto (\mu - 1)$, the focal length of a given lens is different for different wavelengths and maximum for red and minimum for violet whatever the nature of the lens as shown in the figure.

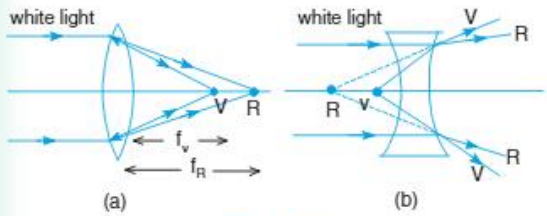


Fig 2.100

Since, on change temperature of the medium refractive index changes, at different temperatures focal length of a lens is different.

- If a lens made of a number of layers of different refractive indices as shown in figure, for a given wavelength of light it will have many focal lengths as $\frac{1}{f} \propto (\mu - 1)$. Hence it will form as many images as there are different μ's.

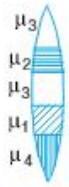


Fig 2.101

According to given Fig 2.101, number of images formed by lens is 4.

- If a lens is shifted from one medium to an other, depending on the refractive index of the lens and the surrounding medium, its focal length as well its nature may change.

As $\mu_r = \frac{\mu_L}{\mu_M}$ and $\frac{1}{f} \propto (\mu_r - 1)$

when μ_L & μ_M change such that

Case (i) If $\mu_r > 1$, focal length changes, but nature remain same.

Case (ii) If $\mu_r = 1$, focal length become infinite, the lens will neither converge nor diverge but will behave as a plane glass plate.

Case (iii) If $\mu_r < 1$, focal length, and nature of the lens changes.

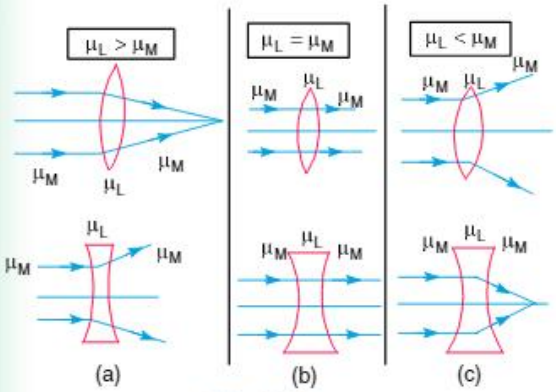


Fig 2.102

- If the two radii of curvatures of a thin lens are not equal, the focal length remain unchanged, whether the light is incident on first face or the other.
- If a equi convex lens of focal length 'f' is cut into two equal parts along its principal axis, as shown in Fig 2.103 (b) then none of μ, R_1, and R_2 will change, the focal length of each part will be equal to that of initial lens, but intensity of image formed by each part will reduce to half.

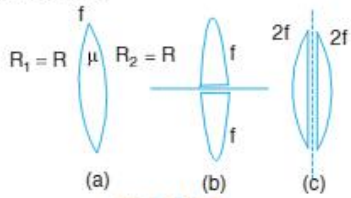


Fig 2.103

However if the same lens is cut in to two equal parts transverse to principal axis, as shown in Fig 2.103(c) the focal length of each part will become double of initial value, but intensity of image remains same.

- (a) On removing a part of lens without disturbing remaining part, the principal axis position of the remaining part is same as earlier.

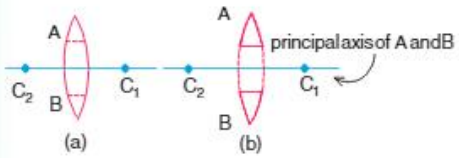


Fig 2.104

(b)If a lens is cut along the principal axis and the separation between them increased in a direction transverse to principal axis, each part has own principal axis.

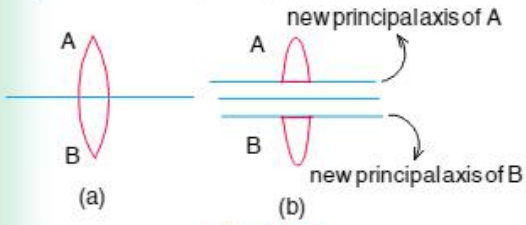


Fig 2.105

To a lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

- Therefore $-\frac{1}{v^2}.\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$ (or)

$V_I = \left(\frac{v}{u}\right)^2 V_0$ where V_I = velocity of image with respect to lens, V_0 = velocity of object with respect to lens.

i.e., $V_I = m^2.V_0 = \left[\frac{f}{u+f}\right]^2.V_0$

- If an object is moved at constant speed towards a convex lens from infinity to focus, the image will move other side of the lens slower in the beginning and faster later on away from the lens. If the object moves from F to optical point, the images moves with greater speed same side of object from infinity towards lens.

2.49 $\frac{1}{v}$ and $\frac{1}{u}$ GRAPHS

a) Convex lens

The graph between 1/v and 1/u in case of convex lens is as shown in Fig 2.106.

For real image

$$\frac{1}{v} - \frac{1}{(-u)} = \frac{1}{f}, \quad \frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$$

It is a straight line with slope –1.

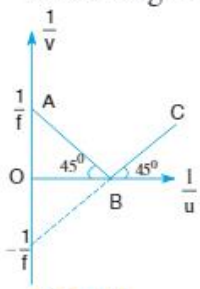


Fig 2.106

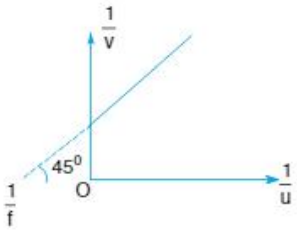


Fig 2.107

For virtual image $\frac{1}{(-v)} - \frac{1}{(-u)} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{u} - \frac{1}{f}$$

It is a straight line with slope +1. Hence AB line when the image is real; BC line when the image is virtual

b) Concave lens

The graph between $\frac{1}{v}$ and $\frac{1}{u}$ in case of concave lens is as shown in Fig 2.107.

Since concave lens only form virtual image.

$$\frac{1}{-v}-\frac{1}{-u}=-\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f}$$

It is a straight line with slope +1

2.50 u and v GRAPHS

a) Convex lens : The graph between v and u is hyperbola to convex lens as shown in the Fig 2.108.

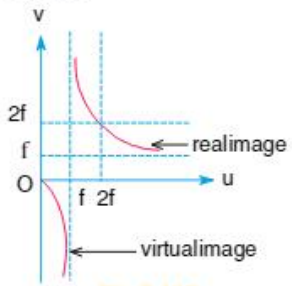


Fig 2.108

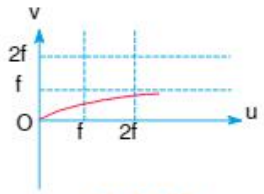


Fig 2.109

b) Concave lens :

The graph between v and u is hyperbola to concave lens as shown in Fig 2.109.

- In case of thin convex lens, if an object is placed at a distance x_1 from first focus and its image is formed at a distance x_2 from the second focus.

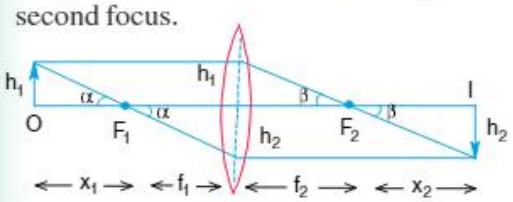


Fig 2.110

From similar triangle to the left of the lens

$$\frac{h_1}{x_1}=\frac{h_2}{f_1}$$

To the right of the lens we have $\frac{h_1}{f_2}=\frac{h_2}{x_2}$

From above two equations $\frac{x_1}{f_1}=\frac{x_2}{f_2}$

$\therefore\ x_1x_2=f_1f_2$

For $f_1=f_2$

$x_1x_2=f^2$ is called Newton's formula or lens user formula. This relation can also be proved by using lens formula.

2.51 DETERMINATION OF THE FOCAL LENGTH OF A CONVEX LENS OR SIZE OF THE OBJECT BY LENS DISPLACEMENT METHOD

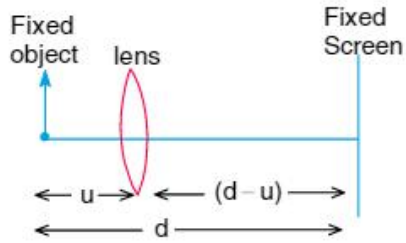


Fig 2.111

If the distance 'd' between an object and screen is greater than four times the focal length of a convex lens, then there are two positions of lens between the object and screen at which a sharp image of the object is formed on the screen. This method is called displacement method and is used in the laboratory to determine the focal length of convex lens.

If the object is at a distance u from the lens, the distance of image from the lens $v=(d-u)$, so from lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{d-u}+\frac{1}{u}=\frac{1}{f}\quad \text{i.e., } u^2-du+df=0$$

So that $u=\frac{d\pm\sqrt{d(d-4f)}}{2}$

Now there are three possibilities

a) If $d<4f$, u will be imaginary, so physically no position of lens is possible

b) If $d=4f$, then $u=\frac{d}{2}=2f$, so only one position is possible and in this case $v=2f$. That is why the minimum separation between the real object and real image is 4f.

c) If $d>4f$, $u_1=\frac{d-\sqrt{d(d-4f)}}{2}$ and

$u_2=\frac{d+\sqrt{d(d-4f)}}{2}$. For these two positions of the object real images are formed for

$$u=u_1, v_1=d-u_1=\frac{d+\sqrt{d(d-4f)}}{2}=u_2$$

For $u = u_2, v_2 = d - u_2 = \dfrac{d - \sqrt{d(d-4f)}}{2} = u_1$

i. e., for two positions of the lens, object and image distances are interchangeable as shown in the Fig 2.112.

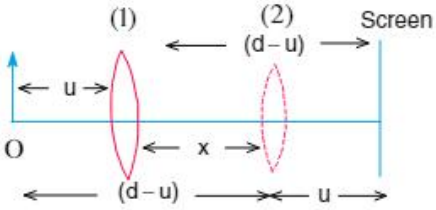


Fig 2.112

So the magnification for the tube positions of the object are related as $m_1 = \dfrac{1}{m_2}$

i.e., $m_1 m_2 = 1$

$$\therefore\ m_1 m_2 = \frac{I_1}{O}.\frac{I_2}{O} = \frac{I_1 I_2}{O^2} = 1$$

Therefore $O = \sqrt{I_1 I_2}$ where I_1 & I_2 are the sizes of images for two positions of the object and O is size of the object. It means that the size of object is equal to the geometric mean of the two images. This method of measuring the size of the object is useful when the object inaccessible.

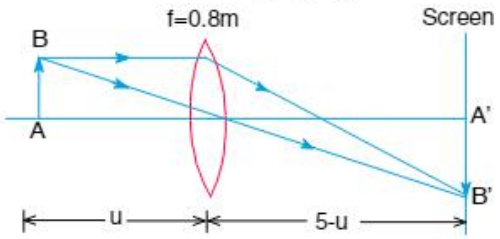
If 'x' is the distance between the two positions of the lens, then $f = \dfrac{x}{m_1 - m_2}$

Example-2.29

An object is 5 m to the left of a flat screen. A converging lens for which the focal length is 0.8m is placed between object and screen. (a) Show that for two positions of lens form images on the screen and determine how far these positions are from the object? (b) How do the two images differ from each other?

Solution :

(a) Using the lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$



we have, $\dfrac{1}{5-u} + \dfrac{1}{u} = \dfrac{1}{0.8}$ or $\dfrac{1}{5-u} + \dfrac{1}{u} = 1.25$

$\therefore\ u + 5 - u = 1.25u\,(5-u)$ or $1.25u^2 - 6.25\,u + 5 = 0$

$u = 4m$ and $1m$

Both the values are real, which means there exist two positions of lens that form images of object on the screen.

(b) $m = \dfrac{v}{u}$

$$\therefore\ m_1 = \frac{(5-4)}{(-4)} = -0.25 \text{ and } m_2 = \frac{(5-1)}{(-1)} = -4.00$$

Hence, both the images are real and inverted, the first has magnification –0.25 and the second –4.00.

2.52 COMBINATION OF LENSES

In case of two thin lenses in contact, if the first lens of focal length f_1 forms the image I_1 at a distance v_1 from it,

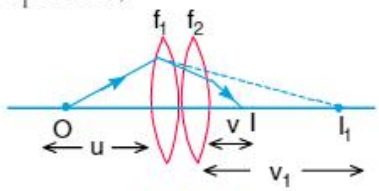


Fig 2.113

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \qquad \text{..... (1)}$$

Now the image I_1 will act as an object for second lens. If the second lens forms image I at a distance 'v' from it, then

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \qquad \text{..... (2)}$$

So adding (1) and (2) equations we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \text{ (or) } \frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

with $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

i.e., the combination behaves as a single lens of equivalent focal length 'F' given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

This derivation is valid for any number of thin lenses in contact co-axially.

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+---+\frac{1}{f_n}$$

In terms of power

$$P_{net}=P_1+P_2+P_3+---+P_n$$

It is worthy to note that :

1. Here focal length values are to be substituted with sign.

2. If the two thin lenses are separated by a distance 'd', then $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}$

So $P_{net}=P_1+P_2-dP_1P_2$

3. If the medium on either side of the lenses is air and the medium between the lenses is one having refractive index 'μ', we can imagine that the rays emerging from the first lens are incident on the second lens as if they have traversed a thickness $\frac{d}{\mu}$ in air.

Hence $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{(d/\mu)}{f_1f_2}$

$$\therefore\ P=p_1+p_2-\left(\frac{d}{\mu}\right)p_1p_2$$

4. If two thin lenses of equal focal length but of opposite nature are put in contact, the resultant focal length of the combinations will be

$$\frac{1}{F}=\frac{1}{f}+\left(-\frac{1}{f}\right)=0$$

i.e., $F=\infty$ and $P=0$

5. If f_1 and f_2 are focal lengths of two lenses (L_1 and L_2) are separated by a distance 'd' on common principal axis and 'F' is the equivalent focal length of the system.

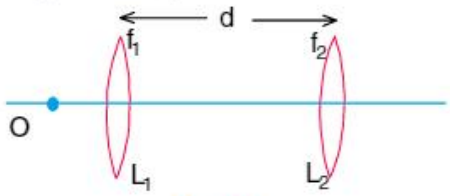


Fig 2.114

Then (i) the distance of equivalent lens from second lens L_2 is $\frac{Fd}{f_1}$ towards the object if the value is positive and away from the object if the value is negative

(ii) The distance of equivalent lens from first lens L_1 is $\frac{Fd}{f_2}$ away from the object, if the value is positive and towards the object if the value is negative. Note that F, f_1 and f_2 are to be substituted according to sign convenction.

6. If a lens of focal length 'f' is divided into two equal parts, and if these parts are put in contact in different combinations as shown in Fig 2.115. Then

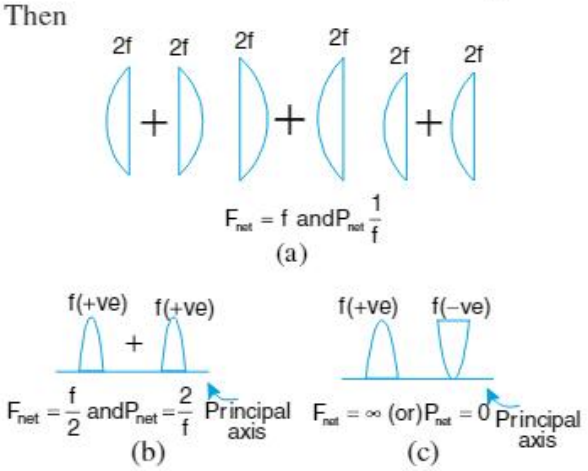


Fig 2.115

Example-2.30

A converging lens of focal length 5.0 cm is placed in contact with a diverging lens of focal length 10.0 cm. Find the combined focal length of the system.

Solution :

Here, $f_1=+5.0$ cm, and $f_2=-10.0$ cm

Therefore, the combined focal length F is given by

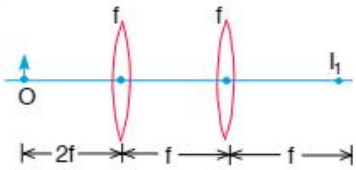
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{5.0}-\frac{1}{10.0}=+\frac{1}{10.0}$$

$\therefore F=+10.0$ cm i.e., the combination behaves as a converging lens of focal length 10.0 cm.

Example-2.31

Two thin converging lenses are placed on a common axis, so that the centre of one of them coincides with the focus of the other. An object is placed at a distance twice the focal length from the left-hand lens. Where will its image be? What is the lateral magnification? The focal length of each lens is f.

Solution :



The image formed by first lens will be at distance 2f with lateral magnification $m_1 = -1$. For second lens this image will behave as a virtual object. Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have, $\frac{1}{v} - \frac{1}{f} = \frac{1}{f}$

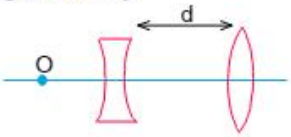
$$\therefore\ v = \frac{f}{2},\ m_2 = \frac{v_2}{u_2} = \frac{(f/2)}{f} = \frac{1}{2}$$

Therefore, final image (real) is formed at a distance f/2 right from the second lens with total lateral magnification,

$$m = m_1 \times m_2 = (-1)\times\left(\frac{1}{2}\right) = -\frac{1}{2}$$

Example-2.32

An object is placed 12 cm to the left of a diverging lens of focal length –6.0 cm. A converging lens of focal length of 12.0 cm is placed at a distance d to the right of the diverging lens. Find the distance d that corresponds to a final image at infinity.



Solution :

Applying lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ twice

we have $\frac{1}{v_1} - \frac{1}{-12} = \frac{1}{-6}$

$\Rightarrow\ v_1 = -4\text{cm}$ and $\frac{1}{\infty} - \frac{1}{v_1 - d} = \frac{1}{12}\ \Rightarrow\ d = 8\text{ cm}$

Example-2.33

A point object is placed at a distance of 12 cm on the axis of a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at a distance of 10 cm from the lens such that the image formed by the combination coincides with the object itself. What is the focal length of convex mirror?

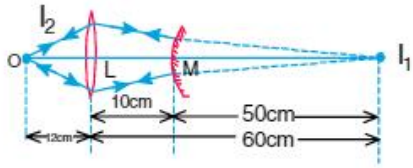
Solution :

For convex lens, $\frac{1}{v} - \frac{1}{-12} = \frac{1}{10}$

i.e., v = 60 cm

i.e., in the absence of convex mirror, convex lens will form the image I_1 at a distance of 60 cm behind the lens.

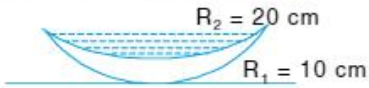
Since, the mirror is at a distance of 10 cm from the lens, I_1 will be at a distance of 60 – 10 = 50 cm from the mirror, i.e., $MI_1 = 50$ cm.



Now as the final image I_2 is formed at the object O itself, the rays after reflection from the mirror retraces its path, i.e., rays on the mirror are incident normally, i.e., I_1 is the centre of the mirror, so that $R = MI_1 = 50$ cm and hence F = (R/2) = (50/2) = 25 cm

Example-2.34

A concavo-convex lens has an index of refraction 1.5 and the radii of curvature of its surface are 10 cm and 20 cm. The concave surface is upwards and it is filled with an oil of refractive index 1.6. Calculate the focal length of the oil-glass combination.



Solution :

For the glass lens $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Here $R_1 = 10\text{cm}$, $R_2 = 20$ cm and $\mu = 1.5$

$$\frac{1}{f_1} = (1.5 - 1)\left[\frac{1}{10\text{cm}} - \frac{1}{20\text{cm}}\right] = \frac{1}{40\text{cm}}$$

For the oil lens $\frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Here $R_1 = 20\text{cm}$ $R_2 = \infty$, and $\mu = 1.6$

$$\frac{1}{f} = (1.6 - 1)\left[\frac{1}{20\text{cm}} - \frac{1}{\infty}\right] = \frac{3}{100\text{cm}}$$

For the oil-glass combination

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{40\text{cm}} + \frac{3}{100\text{cm}} = \frac{11}{200\text{cm}}\ \therefore f = 18.2\text{cm}$$

2.53 LENS WITH ONE SILVERED SURFACE

If the back surface of a lens is silvered and an object is placed infront of it, then the rays are first refracted by lens, then reflected from the silvered surface and finally refracted by lens, so that we get two refractions and one reflection.

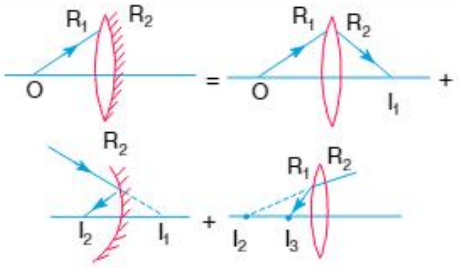


Fig 2.116

In the Fig 2.116 if f_l and f_m are respective focal lengths of lens and mirror, then

$$\frac{1}{F}=\frac{1}{f_l}+\frac{1}{f_m}+\frac{1}{f_l}=\frac{2}{f_l}+\frac{1}{f_m}$$

$$P=P_l+P_m+P_l=2P_l+P_m$$

with $P_l=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ and $P_m=\frac{2}{R_2}$

Here P_l and P_m are substituted with sign. The system will behave as a concave mirror if 'P' is positive and as a convex mirror if 'P'is negative. The replacement with the mirror is due to overall reflection of given rays.

Example A : When the plane surface of a plano convex lens is slivered.

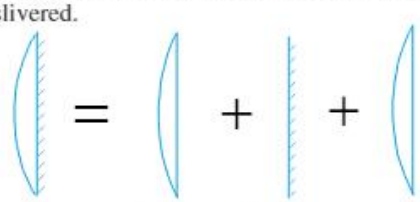


Fig 2.117(a)

$$P=2P_l+P_m.\quad P=2.\left(\frac{\mu-1}{R}\right)+0=\frac{2(\mu-1)}{R}$$

Since $\mu>1$, 'P' is positive, the system behaves as a concave mirror with focal length $\frac{R}{2(\mu-1)}$.

Example B : When curved surface of a plano convex lens is silvered.

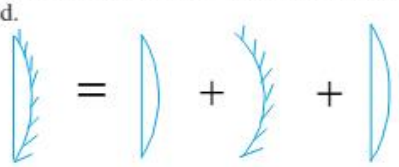


Fig 2.17(b)

$$P=2P_l+P_m=\frac{2(\mu-1)}{R}+\frac{2}{R}=\frac{2\mu}{R}$$

Since 'P' is positive, the system behaves as a concave mirror with focal length $\frac{R}{2\mu}$

Example-2.35

A pin is placed 10 cm in front of a convex lens of focal length 20cm made of material having refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature 22cm. Determine the position of the final image. Is the image real or virtual?

Solution :

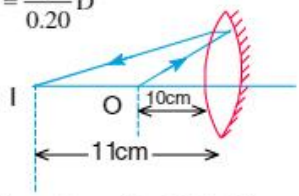
As radius of curvature of silvered surface is 22 cm,

$f_M=\frac{R}{2}=\frac{-22}{2}=-11\text{cm}$ and hence,

$$P_M=-\frac{1}{f_M}=-\frac{1}{-0.11}=\frac{1}{0.11}D$$

Further as the focal length of lens is 20 cm, i.e., 0.20 m, its power will be given by:

$$P_L=-\frac{1}{f_L}=\frac{1}{0.20}D$$



Now as in image formation, light after passing through the lens will be reflected back by the curved mirror through the lens again.

$$P=P_L+P_M+P_L=2P_L+P_M$$

i.e., $P=\frac{2}{0.20}+\frac{1}{0.11}=\frac{210}{11}D$

So the focal length of equivalent mirror

$$F=-\frac{1}{P}=-\frac{11}{210}m=-\frac{110}{21}cm$$

i.e., the silvered lens behaves as a concave mirror of focal length (110/21) cm. So for object at a distance 10cm in front of it, $\frac{1}{v}+\frac{1}{-10}=-\frac{21}{110}$

i.e., $v=-11$ cm i.e., image will be 11 cm in front of the silvered lens and will be real as shown in Fig.

Example-2.36

A biconvex thin lens is prepared from glass of refractive index 3/2. The two bounding surfaces have equal radii of 25 cm each. One of the surfaces is silvered from outside to make it reflecting. Where should an object be placed before this lens so that the image coincides with the object.

Solution :

Here, $R_1=+25$ cm, $R_2=-25$ cm, and $\mu=3/2$

Image coincides with object,

hence, $u = v = -x$ (say)

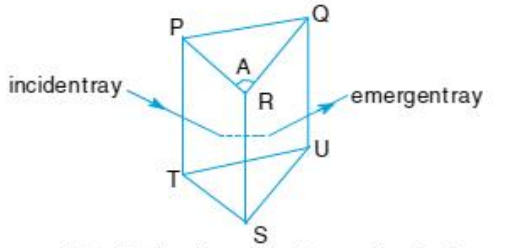
$$\frac{1}{F} = \frac{2}{F_L} + \frac{1}{F_C} = 2\left(\frac{3}{2} - 1\right)\frac{2}{25} + \frac{2}{25} \Rightarrow \frac{1}{F} = \frac{4}{25},$$

By using $\frac{1}{v} - \frac{1}{u} = \frac{1}{F} \Rightarrow -\frac{1}{x} - \frac{1}{x} = \frac{4}{25}$; $x = 12.5$ cm

Hence, the object should be placed at a distance 12.5 cm in front of the silvered lens.

2.54 REFRACTION THROUGH PRISM

Prism is a transparent medium bounded by any number of surfaces in such a way that the surface on which light is incident and the surface from which the light emerges are plane and non-parallel as shown in Fig 2.118.



RS = Refracting edge, $\angle A$ = angle of prism ≠ PQR = principal section

Fig 2.118

Here it is worth noting that

1) Angle of prism or refracting angle of prism means the angle between the faces on which light is incident and from which it emerges. The common edge of these two surfaces of the prism is called refracting edge. A section of the prism by a plane perpendicular to the refracting edge is called principal section.

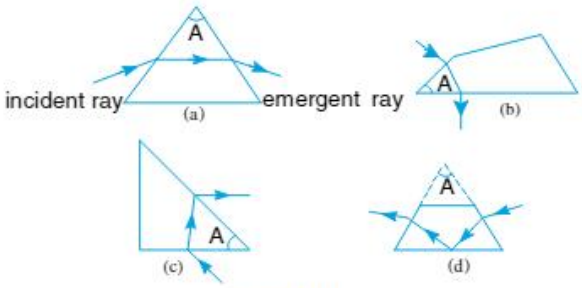


Fig 2.119

2) Angle of deviation means the angle between emergent and incident rays. While measuring the deviation value, anti-clockwise is taken as positive and clockwise as negative.

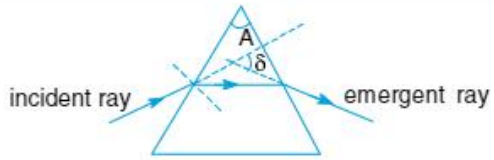


Fig 2.120

δ = deviation angle

3) If refractive index of the material of the prism is equal to that of surroundings, no refraction at its surfaces will take place and light will pass through it undeviated i.e.,[$\delta = 0$]. Generally equilateral, or isosceles right angled prisms are used.

2.55 DETERMINATION OF REFRACTIVE INDEX OF MATERIAL OF THE PRISM FOR MINIMUM DEVIATION

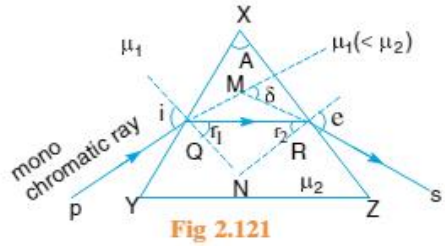


Fig 2.121

Fig 2.121 shows the passage of monochromatic light ray through a triangular prism XYZ. The angles of incidence and refraction at the first face XY are i and r_1, while the angles of incidence and refraction at second face XZ are r_2 and e. The angle between the emergent ray RS and the direction of incident ray PQ is called angle of deviation δ. In the quardrilateral XQNR, if A is the angle of prism.

$$\angle XQN = \angle XRN = 90^0 \Rightarrow \angle A + \angle QNR = 180^0$$

From the triangle QNR,

$r_1 + r_2 + \angle QNR = 180^0$ Comparing these two equations, we get $r_1 + r_2 = A$. The total deviation 'δ' is the sum of deviation at two faces $\delta = (i - r_1) + (e - r_2)$; i.e., $\delta = i + e - A$

Thus the angle of deviation depends on the angle of incidence. A plot between the angle of deviation and angle of incidence is shown in Fig 2.122, which is parabola.

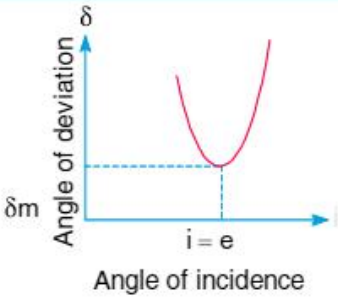


Fig 2.122

We can see that in general, any value of δ except for i = e, correspond to two values of i and hence of e. If δ remains same i and e are interchanged. Physically, this is related to the fact that the path of ray can be traced back resulting in the same angle of deviation 'δ'. At the minimum deviation ($\delta = \delta_m$), the refracted ray travels through the prism symmetrically, that is i = e. Which implies $r_1 = r_2 = r$ (say)

$$2r = A \text{ (or) } r = \frac{A}{2}$$

In the same way $\delta_m = 2i - A$ (or) $i = \frac{A+\delta_m}{2}$

According to Snell's law, at the first interface XY

$\mu_1 \sin i = \mu_2 \sin r$

$$_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

The angles A and δ_m can be measured experimentally. Hence refractive index of prism.

For $\mu_1 = 1$, $\mu_2 = \mu$

$$\therefore \quad \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

This is the prism equation which holds only for light deflected at the angle of minimum deviation. When the angle of prism is very small, the prism is said to be a thin prism. For a thin prism, the values of the angles A and δ are so small that the sines of the angles may be taken to be equal to the angles. i.e.,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} \approx \frac{\frac{A+\delta_m}{2}}{\frac{A}{2}}$$

$\therefore \delta_m = (\mu - 1)A$

It should be noted that

(i) For a given material of prism, light and angle of incidence, when the angle of prism increases angle of deviation also increases as $\delta \alpha A$.

(ii) With increase in wavelength, deviation decreases i.e., deviation for red is least while maximum for violet as deviation $\alpha(\mu - 1)$.

(iii) When a given prism is immersed in a liquid, the angle of deviation changes as $\alpha(\mu_r - 1)$.

2.56 MAXIMUM DEVIATION THROUGH A PRISM

Deviation of the ray will be maximum when the angle of incidence is maximum. i.e., i = 90°. Therefore the maximum deviation $\delta_{max} = 90 + e - A$

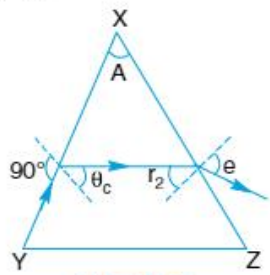


Fig 2.123

To find angle of emergence in this case, apply Snell's law at XZ plane.

$\mu \sin r_2 = 1 x \sin e$

$\mu \sin(A - \theta_c) = 1 x \sin e (\because \theta_c + r_2 = A)$

$\therefore e = \sin^{-1}[\mu \sin(A - \theta_c)]$

Hence $\delta_{max} = 90° + \sin^{-1}[\mu \sin(A - \theta_c)] - A$

Note : (a) Since the law of reversibility is always true, for an angle of incidence $i = \sin^{-1}[\mu \sin(A - \theta_c)]$, the ray grazes at the other surface.

(b) For an angle of incidence $i > \sin^{-1}[\mu \sin(A - \theta_c)]$ only, the ray emerges out of a prism from the other surface.

(c) For an angle of incidence $i < \sin^{-1}[\mu \sin(A - \theta_c)]$, the ray undergoes TIR at the other surface.

2.57 CONDITION FOR NO EMERGENCE FOR ANY ANGLE OF INCIDENCE

It is also possible that the light ray will not emerge out of a prism even though

$i > \sin^{-1}[\mu \sin(A - \theta_c)]$.

For maximum angle of incidence i = 90°, $r_1 = \theta_c$

For no emergence at the other face $r_2 > \theta_c$.

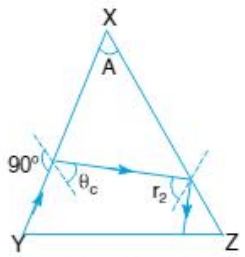


Fig 2.124

$\therefore r_1 + r_2 > 2\theta_c$

But in a prism $r_1 + r_2 = A$

Hence $A > 2\theta_c$ (or) $\sin A/2 > \sin\theta_c$

(or) $\sin A/2 > 1/\mu$ (or) $\mu > \operatorname{cosec}(A/2)$

i.e A ray of light will not emerge out a prism whatever be the angle of incidence.

if $A > 2\theta_c$ (or) $\mu > \operatorname{cosec}\frac{A}{2}$

Example-2.37

A ray of light passing through a prism having $\mu = \sqrt{2}$ suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. What is the angle of prism?

Solution :

As the prism is in 'the position of minimum deviation' $\delta_m = (2i - A)$ with $r = A/2$

According to given problem, $i = 2r = A$ [as $r = A/2$]

$\delta_m = 2A - A = A$ and hence from

$$\mu = \frac{\sin[(A+\delta_m)/2]}{\sin(A/2)}, \text{i.e.}, \sqrt{2} = \frac{\sin A}{\sin A/2} \text{ (or)}$$

$$\sqrt{2}\sin\frac{A}{2} = 2\sin\frac{A}{2}\cos\frac{A}{2}, \text{ i.e.}, \cos\frac{A}{2} = \frac{1}{\sqrt{2}} \text{ (or)}$$

$$\frac{A}{2} = \cos^{-1}\left[\frac{1}{\sqrt{2}}\right] = 45^0, \text{ i.e.}, A = 90^0$$

Example-2.38

A ray of light is incident at an angle of 60° on one face of prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism.

Solution :

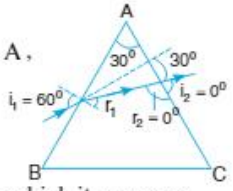
According to given problem,

$A = 30°, i_1 = 60°$ and $\delta = 30°$

and as in a prism, $\delta = (i_1 + i_2) - A$,

$30° = (60° + i_2) - 30°$

i.e., $i_2 = 0°$



So the emergent ray is perpendicular to the face from which it emerges.

Now as $i_2 = 0, r_2 = 0$

But as $r_1 + r_2 = A, r_1 = A = 30°$

So at first face, $1 \times \sin 60° = \mu \sin 30°$ i.e., $\mu = \sqrt{3}$

Example-2.39

One face of a prism with a refracting angle of 30° is coated with silver. A ray incident on another face at an angle of 45° is refracted and reflected from the silver coated face and retraces its path. What is the refractive index of the prism?

Solution :

Given $A = 30°, i_1 = 45°$ and $r_2 = 0$

Since, $r_1 + r_2 = A$

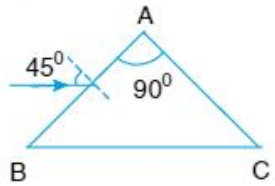
$\therefore r_1 = A = 30°$

Now refractive index of the prism,

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 45°}{\sin 30°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \sqrt{2}$$

Example-2.40

Monochromatic light falls on a right-angled prism at an angle of incidence 45°. The emergent light is found to slide along the face AC. Find the refractive index of material of prism

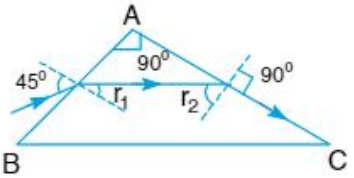


Solution:

Since the emergent light slides along the face AC, angle of emergence is 90°, as shown.

It implies that angle of incidence of the ray that falls on face AC is equal to the critical angle θ_C.

$\therefore r_2 = \theta_C$ ..... (1)



From the prism theory, we know

$r_1 + r_2 = A = 90^o$

$\therefore\ r_2 = 90^o - r_1$ ..... (2)

From equations. (1) and (2), $90^o - r_1 = \theta_C$

$\therefore\ \sin(90^o - r_1) = \sin\theta_C$ (or) $\cos r_1 = \sin\theta_C$

But $\sin\theta_C = \frac{1}{\mu}$ $\therefore \cos r_1 = \frac{1}{\mu}$

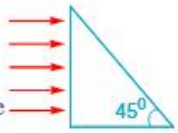
Applying Snell's law at the boundary AB,

$$1 \times \sin 45^0 = \mu \sin r_1 = \mu\sqrt{1 - \frac{1}{\mu^2}}$$

$$\therefore \frac{1}{\sqrt{2}} = \sqrt{\mu^2 - 1} \text{ or } \mu^2 = 3/2 = 1.5\ ;\ \mu = \sqrt{1.5}$$

Example-2.41

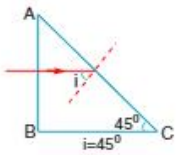
A beam of light consisting of red, green and blue colours is incident on a right-angle prism. The refractive indices of the material of the prism for the red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will



1) separate the red colour from the green and blue colours
2) separate the blue colour from the red and green colours
3) separate all the three colours from one another
4) not separate even partially any colour from the other two colours.

Solution :

The colours for which $i > \theta_c$, will get total int-ernal reflection $i > \theta_c$ or $\sin i > \sin\theta_c$



or $\sin 45^0 > \frac{1}{\mu}$ or $\frac{1}{\sqrt{2}} > \frac{1}{\mu}$

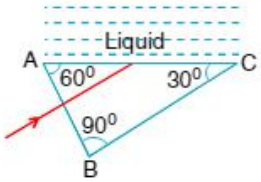
of for which $\mu > \sqrt{2}$ or $\mu > 1.414$

Hence, the rays for which $\mu > 1.414$ will get TIR.

For green and blue $\mu > 1.414$, so they will suffer TIR on face AC only red comes out from this face.

Example-2.42

Light is incident normally on face AB of a prism as shown in figure. A liquid of refractive index μ is placed on face AC of the prism. The prism is made of glass of refractive index 3/2. The limits of μ for which total internal reflection takes place on face AC is

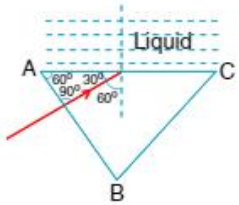


Solution :

Critical angle between glass and liquid face is

$$\sin\theta_c = \frac{\mu}{\frac{3}{2}} = \frac{2\mu}{3}.$$

Angle of incidence at face AC is 60^0.



For the TIR to takes place at the face AC, the angle of incidence at the face AC, $i > \theta_c$,

$$60^0 > \theta_c \text{ or } \sin 60^\circ > \frac{2\mu}{3} \text{ or } \mu < \frac{3\sqrt{3}}{4}$$

2.58 DISPERSION OF LIGHT

When white light passes through a prism it splits up into different component colours. This phenomenon is called dispersion and arises due to the fact that refractive index of prism is different for different wavelengths. So different wavelengths in passing through a prism are deviated through different angles and as $\delta\alpha(\mu - 1)$, violet is deviated most while red least giving rise to display of colours known as spectrum. The spectrum consists of visible and invisible regions.

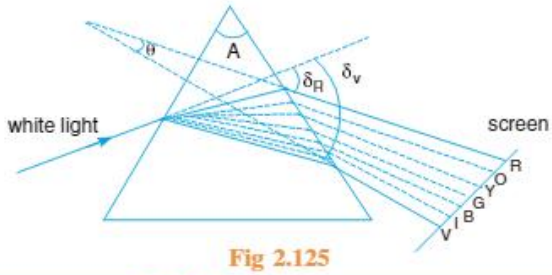


Fig 2.125

In visible spectrum the deviation and the refractive index for the yellow ray are taken as the mean values. If the dispersion in a medium takes place in the order given by "VIBGYOR" it is called normal dispersion. If, however, the dispersion does not follow the rule 'VIBGYOR', it is said to be anomalous dispersion. A medium which brings about dispersion is called dispersive medium. Prism that separate light according to wavelength are known as dispersive prisms. Dispersive prism are mainly used in spectrometers to separate closely adjacent spectral lines. Prisms made of glass are used in the visible region for dispersion. Dispersion can also occur in U.V and I.R regions, but materials used for the dispersion are different.

2.59 ANGULAR DISPERSION

The difference in the angles of deviations of any pair of colours is called angular dispersion (θ) for those two colours. Let the refractive indices of violet, red and yellow be indicated by μ_v, μ_R and μ_y. The deviation δy corresponding to yellow colour is taken as mean deviation. The deviations δ_v, δ_R and δ_y can be written as

$\delta_v = (\mu_v - 1)A$ and $\delta_R = (\mu_R - 1)A$ and

$\delta_y = (\mu_y - 1)A$.

Angular dispersion for violet and red

$\theta = (\delta_v - \delta_R) = (\mu_v - \mu_R)A$.

Thus the angular dispersion depends on the nature of the material of prism and upon the angle of the prism. In general, the angular dispersion refer to angular dispersion of violet and red i.e. the total angle through which the visible spectrum is spread out.

2.60 DISPERSIVE POWER

Dispersive power indicates the ability of the material of the prism to disperse the light rays. It is defined as the ratio of the angular dispersion to the mean deviation.

i.e., Dispersive power: $\omega = \dfrac{\theta}{\delta_y} = \dfrac{\delta_v - \delta_R}{\delta_y}$,

where δ_y = deviation for yellow light.

$$\omega = \frac{\mu_v - \mu_R}{(\mu_y - 1)} = \frac{d\mu}{(\mu - 1)}$$

It is seen that the dispersive power is independent of the angle of prism and angle of incidence, but depends on material of the prism.

The dispersive power is more precisely expressed with reference to C, D and F, Fraunhoffer's lines in the solar spectrum.

The C, D and F lines lie in the red, yellow and blue regions of the spectrum and their wavelengths are 6563Å, 5893Å and 4861Å respectively. Then the dispersive power may be expressed as $\omega = \dfrac{\mu_F - \mu_C}{\mu_D - 1}$.

Note that a single prism produces both deviation and dispersion simultaneously. However, if two prisms (crown and flint) are combined together, we can get deviation without dispersion or dispersion without deviation. The dispersive power of flint glass prism is greater than that of crown glass prism for same refracting angle, i.e., the angular separation of spectral colours in flint glass is more than that of crown glass. If two prisms of prism angles A and A' and refractive indices μ and μ' are placed together,

Total deviation

$\delta = \delta_1 + \delta_2 = (\mu_y - 1)A + (\mu'_y - 1)A'$ and

Total dispersion

$\theta = \theta_1 + \theta_2 = (\mu_v - \mu_R)A + (\mu'_v - \mu'_R)A'$

2.61 DEVIATION WITHOUT DISPERSION OR ACHROMATISM

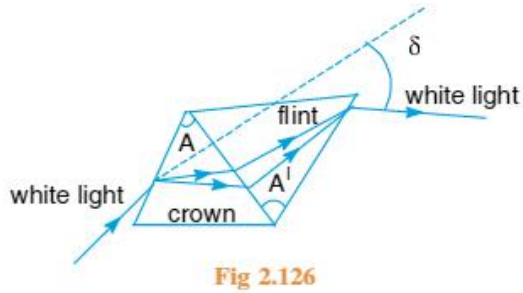


Fig 2.126

An achromatic prism is a combination of two appropriate prisms so constructed that it shows no colours. Flint glasses have higher dispersive power than crown glass. Hence, it is possible to combine two prisms of different materials and specified angles such that ray of white light may pass through the combination without dispersion, though it may suffer deviation. Such a combination is called achromatic combination.

i.e., $\delta \neq 0$ and $\theta = 0$

$$\therefore (\mu_v - \mu_R)A + (\mu'_v - \mu'_R)A' = 0$$

$$\frac{(\mu_v - \mu_R)A}{(\mu_y - 1)}(\mu_y - 1) + \frac{(\mu'_v - \mu'_R)A'}{(\mu'_y - 1)}(\mu'_y - 1) = 0$$

i.e., $\omega_c\delta_c + \omega_f\delta_f = 0$

Since the deviation produced by flint prism here is opposite to crown prism, that of the net deviation $\delta = \delta_c - \delta_f$.

$$\delta = (\mu_y - 1)A - (\mu'_y - 1)A'$$

$$\delta = \frac{(\mu_y - 1)}{(\mu_v - \mu_R)}(\mu_v - \mu_R)A - \frac{(\mu'_y - 1)}{(\mu'_v - \mu'_R)}(\mu'_v - \mu'_R)A'$$

$$\delta = \frac{\theta_c}{\omega_c} - \frac{\theta_f}{\omega_f}$$

2.62 DISPERSION WITHOUT DEVIATION OR DIRECT VISION PRISM

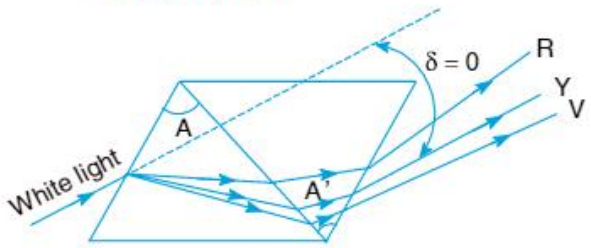


Fig. 2.127

If the angles of the crown and flint glass prism are so adjusted that the deviation produced for the mean rays by the first prism is equal and opposite to that produced by the second prism, then the final beam will be parallel to the incident beam. Such combination of two prisms will produce dispersion of the incident beam without deviation.

i.e., $\delta = 0$ and $\theta \neq 0$

$$\therefore (\mu_y - 1)A + (\mu'_y - 1)A' = 0$$

$$\frac{(\mu_y - 1)}{(\mu_v - \mu_R)}(\mu_v - \mu_R)A + \frac{(\mu'_y - 1)}{(\mu'_v - \mu'_R)}(\mu'_v - \mu'_R)A' = 0$$

i.e., $\dfrac{\theta_c}{\omega_c} + \dfrac{\theta_f}{\omega_f} = 0$

Since the dispersion produced by flint glass prism here is opposite to that of crown glass prism, the net angular dispersion $\theta = \theta_c - \theta_f$.

$$\theta = (\mu_V - \mu_R)A - (\mu'_V - \mu'_R)A' \text{ (or)}$$

$$\theta = \frac{(\mu_v - \mu_R)A}{(\mu_y - 1)}(\mu_y - 1) - \frac{(\mu'_V - \mu'_R)A'}{(\mu'_y - 1)}(\mu'_y - 1)$$

$$\theta = \omega_c\delta_c - \omega_f\delta_f$$

Example-2.43

A crown glass prism of refracting angle 8° is combined with a flint glass prism to obtain deviation without dispersion. If the refractive indices for red and violet rays for crown glass are 1.514 and 1.524 and for the flint glass are 1.645 and 1.665 respectively, find the angles flint glass prism and net deviation.

Solution :

The condition for deviation with out dispersion is $(\mu_v - \mu_R)A = (\mu'_v - \mu'_R)A'$

$$\therefore A' = \frac{(1.524 - 1.514) \times 8^0}{(1.665 - 1.645)} = \frac{0.08^0}{0.02} = 4^0$$

Mean refractive index for crown glass

$$\mu = \frac{1.514 + 1.524}{2} = 1.519$$

Mean refractive index for flint glass

$$\mu = \frac{1.645 + 1.665}{2} = 1.655$$

$\therefore$ The net deviation $(\delta - \delta') = (\mu - 1)A - (\mu' - 1)A'$

$= 0.159 \times 8^0 - 0.655 \times 4^0 = 1.53^o$.

Example-2.44

A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. What is the angle of the prism P_2?

Solution :

In case of thin prism $\delta = (\mu - 1)A$, when two prisms are combined together,

$$\delta = \delta_1 + \delta_2 = (\mu - 1)A + (\mu' - 1)A'$$

For producing dispersion without deviation

$\delta = 0$, i.e., $(\mu' - 1)A' = -(\mu - 1)A$ or

$$A' = -\frac{(1.54-1)}{(1.72-1)} \times 4^0 = -3^0$$

So the angle of the other prism is 3° and opposite to the first.

Example-2.45

White light is passed through a prism of angle 5°. If the refractive indices for red and blue colours are 1.641 and 1.659 respectively, calculate the angle of dispersion between them.

Solution :

As for small angle of prism $\delta = (\mu - 1)A$,

$\delta_b = (1.659 - 1) \times 5^0 = 3.295^0$ And

$\delta_r = (1.641 - 1) \times 5^0 = 3.205^0$ So

$\theta = \delta_b - \delta_r = 3.295^0 - 3.205^0 = 0.090°$.

Example-2.46

Calculate (a) the refracting angle of a flint glass prism which should be combined with a crown glass prism of refracting angle 6^0 so that the combination may have no deviation for D line and (b) the angular separation between the C and F lines, given that the refractive indices of the materials are as follows :

	C	D	F
Flint	1.790	1.795	1.805
Crown	1.527	1.530	1.535

Solution :

Let A_1 and A_2 be the refracting angles of the flint and crown glass prisms respectively. μ_1 and μ_2 be the refractive indices for the D line of flint and crown glasses respectively.

(a) If δ_1 and δ_2 be the angles of deviations due to the flint and crown glass prisms respectively, then for no deviation of D line

$$\delta_1 + \delta_2 = 0;\ A_1(\mu_1 - 1) + A_2(\mu_2 - 1) = 0$$

$$\frac{A_1}{A_2} = -\left(\frac{\mu_2 - 1}{\mu_1 - 1}\right)$$

The negative sign indicates that A_1 and A_2 are oppositely directed.

$$\frac{A_1}{6^0} = \left(\frac{1.530-1}{1.795-1}\right);\ A_1 = 6^0 \times \frac{0.530}{0.795} = -4^0$$

b) Angular dispersion due to the flint glass prism

$= A_1(\mu_F - \mu_C)_1 = -4^0(1.805 - 1.790) = -0.060$

Angular dispersion due to the crown glass prism

$= A_2(\mu_F - \mu_C)_2 = 6^0(1.535 - 1.527) = 0.048$

Net angular dispersion = 0.048 – 0.060 = – 0.012

The negative sign indicates that the resultant dispersion is in the direction of the deviation produced by the flint prism.

2.63 PURE AND IMPURE SPECTRA

If a divergent beam of white light is incident on a prism, each ray in the beam form a spectrum and these spectra overlap. The green of one may coincide with violet of another and red of a third and so on. Such type of spectrum is called impure spectrum.

Ex : Rainbow, solar spectrum.

The spectrum in which each colour is distinctly separated from the other is called pure spectrum.

Ex : Hydrogen spectrum, sodium spectrum.

To get pure spectrum, the experimental arrangement is as shown in figure 2.128. The prism is under minimum deviation position for yellow light.

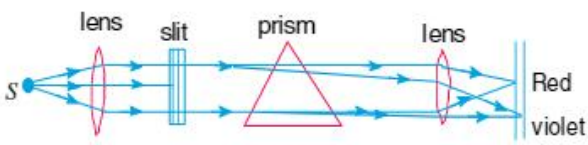
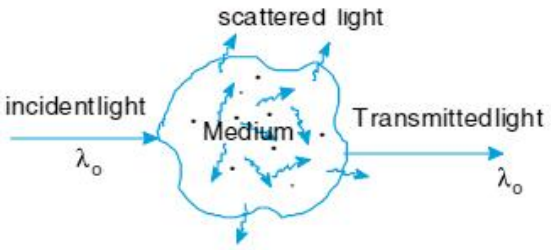


Fig 2.128

2.64 SCATTERING OF LIGHT

If the molecules of a medium after absorbing incoming radiations (light) emit them in all possible directions, this process is called scattering. In scattering if the wavelength of radiation remain unchanged, the scattering is called elastic, otherwise inelastic.



elastic scattering

Fig 2.129

According to Rayleigh, in case of elastic scattering of light by the molecules, the amount of light scattered depends on both nature of molecules and wavelength of light. Light of shorter wavelength is scattered much more than light of longer wavelength.

Intensity of scattered light $\alpha \frac{1}{\lambda^4}$

Therefore blue wavelength are scattered most while red least. Rayleigh scattering takes place when the size of the scatter (say a), is much less than the wavelength of light (λ). i.e., $a << \lambda$.

At large scattering objects (dust or water) all wavelengths are scattered nearly equally.

Regarding scattered light

1. Visibility of an object from all directions is due to scattering of light.
2. Scattered light is plane polarized.
3. Under specific conditions, light can also suffer inelastic scattering from molecules (particles like dust and water droplets) in which its wavelength changes. This effect is called Raman - effect.

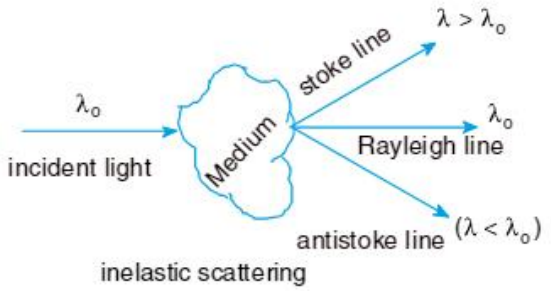


Fig 2.130

4. Scattering explains the blue of sky. When white light from the sun enters the earth's atmosphere, scattering takes place and as scattered intensy is proportional to $\left(\frac{1}{\lambda^4}\right)$, blue is scattered most.

When we look at the sky we receive scattered light which is rich in blue and the sky, appears blue.

5. At sunset or sunrise, the sun rays have to pass through a large distance in the atmosphere. Most of the blue and other shorter wavelengths are removed by scattering. The least scattered light reaching our eyes, therefore, is red. This explains the reddish appearance of the sun and full moon near the horizon.

2.65 RAINBOW

Rainbows are formed by dispersion of sun light falling on rain drops. We can observe the rainbow in a direction facing against the sun. Some times two rainbows are seen. The common rainbow known as the primary rainbow is a coloured band, having red on the outside and violet on the inner side. It is formed due to two refractions and one reflection of light falling on the raindrops. The other rainbow called the secondary rainbow is a coloured band, having violet on the outside and red on the inner side. It is formed due to two refractions and two reflections of the sun light falling on the raindrops.

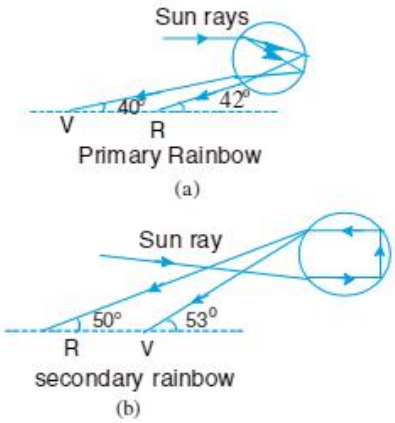


Fig 2.131

It is due to four step process. The intensity of light is reduced at the second reflection and hence the secondary rainbow is fainter than the primary rainbow.

The rainbows are visible only when the altitude of the sun is less than 42°. A complete rainbow can be seen in an aeroplane flying at high altitudes.

EXERCISE

LONG ANSWER QUESTIONS

1. Construct the reflected wavefront when a plane wavefront is incident on a plane reflecting surface. Hence, explain the laws of reflection. Why does an expanding spherical wave continue to expand outward from the source?
2. Deduce the laws of refraction on the basis of Huygens' wave theory. Why is the wavelength of a light wave smaller in a denser medium than in a rarer medium?
3. Define critical angle and explain it with a neat ray diagram? Deduce the relation between critical angle and refractive index.
4. What is total internal reflection? State the conditions to be fulfilled for total internal refection. Describe the construction and working of an optical fibre. State its uses.
5. Discuss refraction at a convex surface when the image formed is real. Derive the necessary formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$
6. Derive Prism formula. On what factors does the angle of minimum deviation depend?
7. Prove the relation $A + \delta = i + e$ for a prism. What does this relation become when the prism is placed in minimum deviation position?
8. Discuss the phenomenon of refraction through a prism. Prove that $\delta = A(\mu - 1)$ for a small angled prism, where the symbols have their usual meanings. How does this formula change if the prism is made of material of refractive index μ_2 kept in a medium of refractive index μ_1.
9. Describe different kinds of spectra.

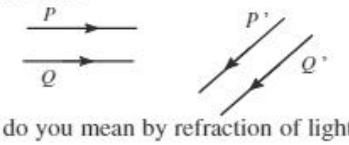
SHORT ANSWER QUESTIONS

1. What is Huygens' wave theory of light? Give its limitations.
2. Define : wavefront and wave normal. State the nature of the wavefronts in the following (i) The source is point source of light. (ii) A narrow slit is illuminated by a source of light.
3. Explain the dual nature of light.
4. State the cause of refraction.
5. State optical path and optical path difference.
6. State the laws of refraction.
7. Find the apparent shift due to refraction at a plane surface when the object in denser and observer in rarer medium.
8. What is lateral shift and normal shift
9. Explain the mirage.
10. Explain how a fish deep inside a pond full of water can see everything above the water near the pond. Show that the field of vision of the fish is concentrated within a cone of apex angle 98°.
11. What is magnification ? Obtain lateral magnification due to refraction at curved surface
12. The equation of refraction at a spherical surface is $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$. Taking $R = \alpha$, show that this equation leads to the equation $\frac{\text{Real depth}}{\text{Apparent depth}} = \frac{\mu_1}{\mu_2}$ for refraction at a plane surface.
13. Define first and second principal focus in lens.
14. Draw the ray diagram for convex lens for the cases (i) $f < u < 2f$ (ii) $u < f$, where u = object distance.
15. Show that the distance between a real object and its real image formed by a thin converging lens is always greater than or equal to four times the focal length of the lens.
16. Derive the equivalent focal length of a plano-convex lens of radius of curvature R and refractive index μ (a) when its curved surface is silvered (b) when its plane surface is silvered?
17. Consider a ray of light passing through a prism (angle A) in the position of minimum deviation δ. Show that the angles of incidence and refraction are given by: $i = (A + \delta)/2$ and $r = (A/2)$.
18. Show that deviation produced by a thin prism of prism angle A and refractive index μ is given by $\delta = (\mu - 1)A$.

19. Obtain the minimum value of refractive index of a prism of angle A so that an incident ray cannot emerge out of it, whatever be the angle of incidence?
20. The perpendicular faces of a right isosceles prism are coated with silver. Prove that the ray's incident at an arbitrary angle on the hypotenuse face will emerge from the prism parallel to the initial direction.
21. A glass prism has a minimum angle of deviation δ for air. State with reason, how the angle of minimum deviation will change, if the prism is immersed in a liquid of refractive index greater than 1. When does the ray of light incident on the prism deviate away from the base?
22. Show that the limiting value of the angle of prism is twice its critical angle.
23. What are pure and impure spectra? State the conditions to produce a pure spectrum.
24. What are Fraunhoffer lines? Explain their origin.
25. State the significance of emission and absorption spectra.

VERY SHORT ANSWER QUESTIONS

1. When a plane wavefront of light is incident on the separating surface between two media, the reflected wavefront is of same width like the incident wavefront whereas the refracted wavefront is of different width. Explain?
2. What is the shape of the wavefront in each of the following cases: (a) Light diverging from a point source. (b) Light emerging out of a convex lens when a point source is placed at its focus. (c) The portion of the wavefront of light from a distant star intercepted by the Earth.
3. What will happen to the image formed by a mirror if half of it is covered with a black paper?
4. For a plane mirror what is the focal length? The magnification?
5. Can two plane mirrors inclined to each other ever form even number of images? Explain?
6. A ray of light is incident on a horizontal plane mirror at an angle of incidence 30°. By what angle should the mirror be rotated so that the reflected ray becomes horizontal?
7. In spherical mirrors describe the shape of curve between
   (i) $\frac{1}{v}$ and $\frac{1}{u}$ (ii) v and u
8. Can a convex mirror ever form a real image? If no, why? If yes, when?
9. A convex mirror of focal length f produces an image $(1/n)^{th}$ of the size of the object. What is the distance of the object from the mirror?
10. What kind of mirror is used in a car to observe the traffic at the back?
11. If an object far away from a convex mirror moves towards the mirror, the image also moves. Does it move faster, slower or at the same speed as compared to the object?
12. Figure shows two rays P and Q being reflected by a mirror and going as 'P' and 'Q'. State which type of mirror is this?



13. What do you mean by refraction of light?
14. What is Snell's law?
15. What is dispersion ?
16. Define refractive index and relative refractive index
17. Give the ratio of velocities of two light waves traveling in vacuum and having wavelengths 5000A° and 6000A°.
18. For the same angle of incidence, the angles of refraction in three different media A, B and C are 30°,60° and 45° respectively. In which medium will the velocity or light be maximum?
19. Can absolute refractive index of a medium be less than unity?
20. Can relative refractive index between two media be less than unity?
21. A beam of light is converging towards a point on a screen. A plane parallel plate of glass is introduced in the path of this converging beam. How will the point of convergence be shifted? Draw a ray diagram?
22. Explain why : (a) A flying bird appears higher in the sky than it really is. (b) Stars appear to twinkle but the sun does not. [Sun is close to the earth so one or the other ray from it always reaches the eye, so it does not appear to twinkle but shines continuously.]
23. What happens to the frequency, wavelengths and speed of light that crosses from a medium with index of refraction μ_1 to one with index of refraction $\mu_2 (< \mu_1)$.

24. Suppose you are inside the water in a swimming pool near an edge. A friend is standing on the edge. Do you find your friend taller or shorter than his usual height?

25. Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

26. On what factors does the critical angle depend?

27. State the consequences of total internal reflection?

28. What is an optical fibre? State its principle.

29. Why do diamonds sparkle?

30. Light travels from denser medium to rarer medium. When the emergent ray grazes the separating surface what is the angle of deviation?

31. If θ_c in the critical angle, write the deviation value when the ray travels from denser to rarer for $i > \theta_c$.

32. A ray of light while traveling from a denser to rarer medium grazes the separating surface. Express the critical angle in terms of the speed of light in the two media.

33. Mention any three uses of optical fibres.

34. It is possible for a lens to act as convergent in one medium and divergent in the other medium?

35. Can a single lens ever form a real and erect image?

36. If the two radii of a thin lens are not equal, will the focal length change with change in surface facing the light?

37. What is the focal length of lenses used in sun-goggles?

38. What will be the equivalent focal length of a system consisting of a thin convex lens and a thin concave lens of same focal length f (a) when they are put in contact (b) when they are separated by a distance d=f (c) when they are separated by a distance, d = 3f ?

39. Will an air bubble inside water behave like a convergent lens or a divergent lens?

40. As an object is moved from the surface of a thin converging lens to a focal point, over what range does the image distance vary?

41. Define refracting angle and principal section of a prism

42. Draw shape of the graph drawn between angle of deviation and angle of incidence in prism.

43. Name the shape of wavefront after refraction of (i) Plane wavefront through convex lens (ii) Divergent spherical wavefront through convex lens (iii) Convergent wavefront through convex lens and concave lens (iv) Plane wavefront through prism.

44. What is angular dispersion? On what factors does it depend?

45. Can we have dispersion without deviation with a pair of prisms of the same material but of different angles?

46. Can a beam of white light when passed through a hollow prism give spectrum? Explain.

47. If three identical prisms are combined, is it possible to pass a beam that emerges undeviated? Undispersed?

48. Distinguish between a pure spectrum and an impure spectrum.

49. What is a line spectrum? What does its study give?

50. What is a band spectrum? What does its study give?

51. What are Fraunhoffer lines and what is their importance?

52. What will be the spectrum of sun during a total solar eclipse?

53. When white light is passed through sodium vapours and the resulting light is dispersed with the help of a prism, what will you observe? Explain.

PROBLEMS

LEVEL - I

1. The magnetic field in a plane electromagnetic wave is given by $B_y = (2\times10^{-7}\,T)\sin[500x + 1.5\times10^{11}t]$
   (a) What is the wavelength and frequency of the wave?
   (b) Write an expression for the electric field vector.

   [Ans: (a) 1.26cm, 2.4×10^{10}Hz
   (b) $E = 60\sin(500x + 1.5\times10^{11}t)$]

2. A sinusoidal electromagnetic plane wave travels through empty space in the x-direction. The electric wave is polarized (plane of vibration in single plane) in the y-direction. Frequency of wave is 1.5 GHz. At a certain point P in space, the oscillating electric field E_y attains a maximum magnitude $E_o = 63$ V/m.
   (a) What is the wavelength of the wave?
   (b) What is the maximum magnitude B_o of the magnetic field at the same point? What is the direction of B ?

   [Ans: (a) 0.2m ; (b) 2.1×10^{-7} T, Z-axis]

3. If two plane mirrors are inclined at an angle of 120° to each other, how many images of an object are (a) formed? (b) seen?
[**Ans:** (a) 2, if object on the bisector of mirrors, otherwise 3 ; (b) any of 1,2 or 3 depending on the position of observer and object]

4. Two plane mirrors are placed parallel to each other and 40 cm apart. An object is placed 10 cm from one mirror. What is the distance from the object to the image for each of the five images that are closest to the object? [**Ans:** 20cm, 60cm, 80cm, 100cm, 140cm]

5. A ray of light is incident on a horizontal plane mirror at an angle of incidence 30°. By what angle should the mirror be rotated so that the reflected ray becomes horizontal? [**Ans:** 30°]

6. Find the position and nature of the image of an object of height 3 cm when placed 60 cm from a mirror of focal length 15cm, when the mirror is (a) concave, (b) convex.
[**Ans:** (a) Real, inverted, of size 1 cm, at a distance of 20 cm in front of the mirror ; (b) virtual, erect of size 0.6 cm, at a distance of 12 cm behind the mirror]

7. An object is kept at a distance 40 cm from the pole of a mirror and an inverted image 5 times the size of object is formed. Find the nature and the focal length of the mirror. [**Ans:** Concave, $\frac{100}{3}$ cm]

8. An object 2 cm high is placed at right angles to the principal axis of a mirror of focal length 25 cm such that an erect image 6 cm high is formed. What kind of mirror is it and what is the position of the object?
[**Ans:** Concave, at distance $\frac{50}{3}$ cm from the pole]

9. The sun subtends an angle of 0.5° at the vertex of a concave mirror of radius of curvature 18.0m. Find the size of the image of the sun. [**Ans:** 7.85 cm]

10. An object 2 cm high is placed at right angles to the principal axis of a mirror of focal length 25 cm such that an erect image 0.5 cm high is formed. What kind of mirror its is and what is the position of the object?
[**Ans:** Convex, at distance 75 cm from the pole]

11. A person looks at himself in a silvered ball of diameter 64 cm from a distance of 2.7m. Find the position and nature of the image.
[**Ans:** At a distance of about 15 cm behind the reflecting surface, virtual, erect and diminished with m = + (1/18)]

12. What will be the size of image as compared to the object in a rearview mirror of a car if the radius of curvature of the mirror is 1 m and the object is 4.5 m from the mirror?
[**Ans:** (1/10); image will be erect, virtual, diminished and at a distance 45 cm behind the mirror]

13. If the refractive index of diamond is 2.4 find the velocity of light in diamond. ($c = 3 \times 10^8$ m/s)
[**Ans:** 1.25×10^8m/s]

14. The wavelength of blue light in air is 4500 Å . What is its frequency? If the refractive index for blue light is 1.55 in glass what will be the wavelength of blue light in glass? ($c = 3 \times 10^8$ m/s)
[**Ans:** 6.6×10^{14}HZ, 2903Å]

15. A beam of light is incident on a glass slab of refractive index 1.5 at an angle of incidence of 45°. Find the ratio of the width of refracted beam to the incident beam. [**Ans:** 1.24 : 1]

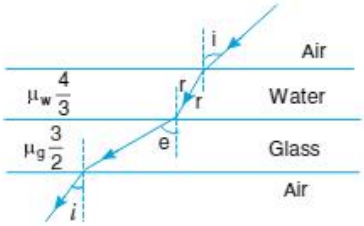
16. Find the time taken by light to travel a distance of 3 cm in glass. ($c = 3 \times 10^8$ m/s and refractive index of water = 4 /3) [**Ans:** 0.13 ns]

17. A glass slab of thickness 4 cm contains the same number of waves as 5 cm of water when both are traversed by the same monochromatic light. If the refractive index of water is (4/3), what is refractive index of glass ? [**Ans:** (5/3)]

18. The refractive index of denser medium with respect to rarer medium is 1.125. The difference between the velocities of light in the two media is 0.25×10^8m/s. Find the velocities of light in the two media. ($c = 3 \times 10^8$ m/s). [**Ans:** 2×10^8 m/s, 2.25×10^8m/s]

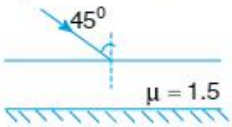
19. A source of yellow light placed in air is observed by a person swimming under water. If the wavelength of yellow light in air is 6000Å, then determine its velocity, wavelength and colour as observed by the person. Take ve locity of light in air ($c=3\times10^8$ m/s^{-1})
[**Ans:** 2.25×10^8 m/s, 4500 Å , Yellow]

20. In the arrangement shown in the figure a light ray is incident at an angle of 37° on the layer of water. Find the angle of emergence (e) through the layer of glass.



[**Ans:** e = $\sin^{-1}$ (0.4)]

21. One side of a glass slab is silvered. A ray of light incident on the other side is as shown. If the refractive index of the slab is 1.5, find the total deviation of the ray when it comes out of the slab.



[Ans: 90°]

22. Calculate the refractive index of material with respect to air if its critical angle is 60°. [Ans: 1.549]

23. A light ray is travelling from a denser medium into rarer medium. The velocities of light in the two media are 2×10^8 m/s and 2.5×10^8 m/s respectively. What is the critical angle at the interface of the two media?

[Ans: $\sin^{-1}\frac{4}{5}$]

24. An optical fibre ($\mu = 1.72$) is surrounded by a glass coating ($\mu = 1.50$). Find the critical angle for total internal reflection at the fibre-glass interface.

[Ans: $\theta_c = \sin^{-1}\frac{75}{86}$]

25. A ray of light traveling in glass ($\mu_g = 3/2$) is incident on a horizontal glass-air surface at the critical angle θ_c. If a thin layer of water ($\mu_w = 4/3$) is now poured on the glass-air surface, at what angle will the ray of light emerges into water at glass-water surface? [Ans: $\sin^{-1}(3/4)$]

26. Glycerine ($\mu = 1.4$) is poured into a large jar of radius 0.2m to a depth of 0.1 m. There is a small light source at the centre of the bottom of the jar. Find the area of the surface of glycerine through which light passes. [Ans: 0.0327 m^2]

27. The radii of curvature of a convex lens are 20 cm and 30 cm. The material of the lens has a refractive index 1.6. Find the focal length of the lens (a) if it is placed in air, and (b) if it is placed in water ($\mu = 1.33$)

[Ans: (a) 20 cm; (b) 60 cm]

28. An object of size 3.0 cm is placed at 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

[Ans: erect, virtual at 8.4 cm]

29. A concave - convex lens has faces of radii 4 cm and 3 cm respectively and is made of glass of refractive index 1.6. Determine (a) its focal length (b) position of image when object is placed at 28 cm in front of the lens (c) magnification in case (b).

[Ans: (a) 20 cm ; (b) 70cm behind the lens ; (c) m=– 2.5]

30. If the angle of prism is 60° and the angle of minimum deviation is also 60°, find the refractive index of the material of the prism. [Ans: $\sqrt{3}$]

31. A ray of light passing through a prism having $\mu = \sqrt{2}$ suffers minimum deviation It is found that the angle of incidence is double the angle of refraction within the prism. What is the angle of the prism? [Ans: 90°]

32. A ray of light passes through an equilateral glass prism, such that the angle of incidence is equal to the angle of emergence. If the angle of emergence is 3/4 times the angle of prism, calculate the refractive index of the glass prism. [Ans: 1.414]

33. A ray of light is incident at an angle of 60° on one face of a prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism. [Ans: $\mu = \sqrt{3}$]

34. A ray of light is incident normally on one of the refracting surfaces of a prism of refracting angle 60°. The emergent ray grazes the other refracting surface. Find the refractive index of the material of the prism.

[Ans: 1.155]

35. For an equilateral prism, it is observed that when a ray strikes grazingly at one face it emerges grazingly at the other. Find the refractive index of the prism.

[Ans: $\mu = 2$]

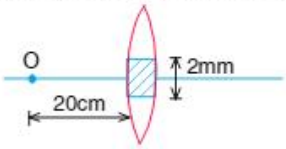
36. A flint glass prism and a crown glass prism are to be combined in such a way that the deviation of the mean ray is zero. The refractive index of flint and crown glasses for the mean ray are 1.62 and 1.518 respectively. If the refracting angle of the flint prism is 6.0°, what would be the refracting angle of the crown prism? [Ans: 7.2°]

37. An air bubble is seen inside a solid sphere of glass (n = 1.5) of 4.0 cm diameter at a distance of 1.0 cm from the surface of the sphere (on seeing along the diameter). Determine the real position of the bubble inside the sphere from that surface.

[Ans: 1.2 cm]

38. A 5.0 diopter lens forms a virtual image which is 4 times the object placed perpendicularly on the principal axis of the lens. Find the distance of the object from the lens. [Ans: 15 cm]

39. A convex lens of focal length 10 cm is painted black at the middle portion as shown in figure. An object is placed at a distance of 20 cm from the lens.



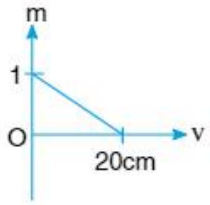
(a) Find the number of images formed.
(b) Find the distance of the image from the lens.
[Ans: (a) 1 ; (b) 20cm]

40. A 35 mm film is to be projected on a 20 m wide screen situated at a distance of 40 m from the film-projector. Calculate the distance of the film from the projection lens and focal length of projection lens.
[Ans: u = 7 cm, f ≈ 7 cm]

41. A small object area 6 cm × 3 cm is placed (plane normal to principal axis) at 10 cm in front of convex mirror of focal length 5 cm. Find the area of the image formed by the mirror. **[Ans: 4 cm × 2 cm]**

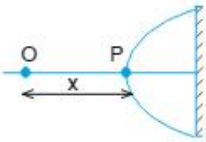
42. What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20cm? Is the system a converging or a diverging lens? **[Ans: diverging, 60 cm]**

43. To a real object, magnification and image distance graph is as shown in the figure. Find (a) the nature of the mirror and image. (b) the power of the mirror.



[Ans: (a) convergent, virtual ; (b) 5D]

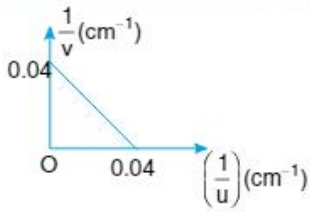
44. A hemispherical surface of radius R and refractive index $\mu = 1.5$ is polished its plane surface. At what distance x from a point P a point object O be placed so that its image coincides with the object itself.



[Ans: 2R]

45. A ray of light is incident on the surface of a spherical glass paper-weight making an angle α with the normal and is refracted in the medium at an angle β. Calculate the deviation. **[Ans: $\delta = 2(\alpha - \beta)$]**

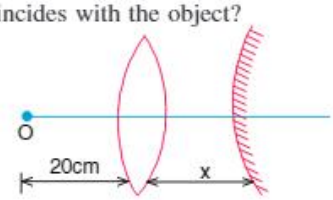
46. The object distance 'u' and the image distance v from a convex lens vary as shown in figure. Find



(a) The nature and focal length of the lens.
(b) The minimum distance between an object and its real image. **[Ans: (a) convex, 25 cm ; (b) 1 m]**

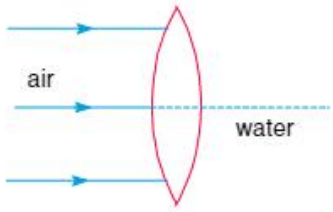
47. A particle executes a simple harmonic motion of amplitude 1.0 cm along the principal axis of a convex lens of focal length 12 cm. The mean position of oscillation is at 20 cm from the lens. Find the amplitude of oscillation of the image of the particle.
[Ans: 2.3 cm]

48. A point object O is placed at a distance of 20 cm from a convex lens of focal length 10 cm as shown in figure. At what distance x from the lens should a mirror of focal length 60 cm, be placed so that final image coincides with the object?



[Ans: 20cm]

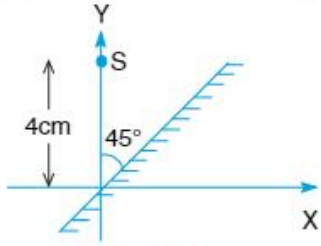
49. A thin equiconvex lens made of glass (n = 3/2) is placed in such a way the that one surface is in contact with water (n = 4/3) and another surface is in contact with air. Find focal length of setup if the rays are incident as shown. Radius of curvature (R = 30 cm)



[Ans: 60 cm]

LEVEL - II

1. What are the co-ordinates of the image of S formed by a plane mirror as shown in figure?

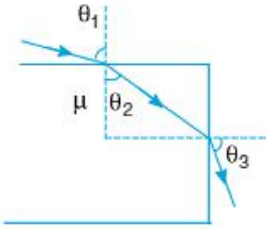


[Ans: 4cm, 0]

2. Two plane mirrors are combined to each other such that one is in y-z plane and the other is in x-z plane. A ray of light travelling along a vector $\hat{i}+\hat{j}+\hat{k}$ is incident on the first mirror. Find the unit vector in the direction of emergent ray after successive reflections through both the mirrors.

[**Ans:** $-\frac{1}{\sqrt{3}}\hat{i}-\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$]

3. Find the relationship between angle of incidence and angle of emergence of light in the adjacent figure. Are angle of incidence and angle of emergence equal?



[**Ans:** $\sin^2\theta_1+\sin^2\theta_3=\mu^2$, No]

4. A bird looking down vertically into a pond from 12cm above the surface sees a fish apparently 18 cm below the surface. What is the actual depth of the fish? What is the apparent height of bird as seen by the fish? [Refractive index of water = 4/3]

[**Ans:** 24 cm and 16 cm from the water surface]

5. There is a dust particle in a glass slab of thickness t and refractive index $\mu = 1.5$. When seen from one side of the slab, the dust particle appears at a distance 6 cm. From opposite side it appears at 4 cm. Find the thickness t of the glass slab.

[**Ans:** 15 cm]

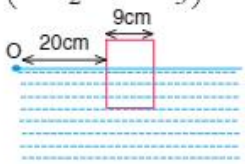
6. A parallel beam of light is incident on a rectangle transparent plate of thickness 8 cm. The angle of incidence and refraction of the beam is 60° and 30° respectively. Find the lateral shift.

[**Ans:** $\frac{8}{\sqrt{3}}$ cm]

7. A pile 4 m high driven into th bottom of a lake is 1 m above the water. Determine the length of the shadow of the pile on the bottom of the lake if the sun rays make an angle 45^0 with the water surface. The refractive index of water is 4/3.

[**Ans:** 2.88 m]

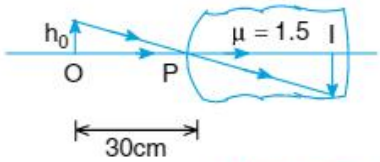
8. A point object is placed at a distance of 20 cm from a glass slab, half immersed in water as shown in figure. Find the distance between two images when seen from the other side of the slab through $\left(\mu_g=\frac{3}{2} \text{ and } \mu_w=\frac{4}{3}\right)$ liquid and air.



[**Ans:** 2 cm]

9. A ray of light travelling in a rarer medium. strikes a plane boundary between the rarer medium and a denser medium at an angle of incident 'i' such that the reflected and the refracted rays are mutually perpendicular. Another ray of light of same frequency is incident on the same boundary from the side of denser medium. Find the minimum angle of incidence at the denser-rarer boundary so that the second ray is totally reflected. [**Ans:** $\theta_c = \sin^{-1}(\cot i)$]

10. A small object of height 0.5 cm is placed in front of a convex surface of glass ($\mu = 1.5$) of radius of curvature 10 cm. Find the height of the image formed in glass.



[**Ans:** 1 cm, real, inverted]

11. A paperweight in the form of a hemisphere of radius 3.0 cm is used to hold down a printed page such that flat surface touch the paper. An observer looks at the page vertically through the paperweight. At what height above the page will the printed letters near the centre appear to the observer?

[**Ans:** no shift is observed]

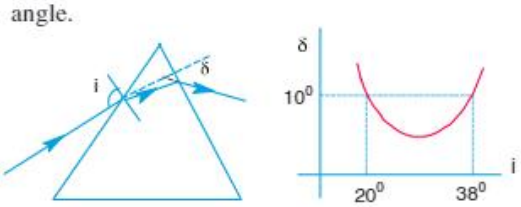
12. A convex glass lens ($\mu = 1.5$) produces a real image of an object. The size of the image equals the size of the object. When the air surrounding the lens is replaced by a liquid, the size of the image is found to be twice that of the object. Find the refractive index of the liquid.

[**Ans:** $\frac{12}{11}$]

13. Two plano concave lenses of glass of refractive index 1.5 have radii of curvature of 20 and 30 cm. They are placed in contact with curved surfaces towards each other and the space between them is filled with a liquid of refractive index (4/3). Find the focal length of the system. [**Ans:** –72 cm]

14. A rectangular block of refractive index μ is placed on a printed page lying on a horizontal surface. Find the minimum value of μ so that the letters on the page is not visible from any of the vertical sides.
[**Ans:** $\mu > \sqrt{2}$]

15. A ray is incident on prism at an angle *i* with normal. When it comes out of prism its angular deviation is δ. Graph between δ and *i* is given. Find the prism angle.



[**Ans:** 48°]

16. The dispersive power of crown and flint glasses are 0.03 and 0.05 respectively. If the difference in the refractive indices of blue and red colours are 0.015 for crown glass and 0.022 for flint glass, calculate the angles of the two prisms for a deviation 2° without dispersion. [**Ans:** 10°, 6.8°]

17. Two prisms of identical geometrical shape are combined with their refracting angles oppositely directed. The materials of the prisms have refractive indices 1.52 and 1.62 for violet. A violet ray is deviated by 1° when passes symmetrically through this combination. What is the angle of the prisms.
[**Ans:** 10°]

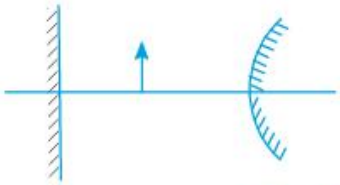
18. A thin prism of crown glass ($\mu_r = 1.515, \mu_v = 1.525$) and a thin prism of flint glass ($\mu_r = 1.612,\ \mu_v = 1.632$) are placed in contact with each other. Their refracting angles are 5.0° each and are similarly directed. Calculate the angular dispersion produced by the combination. [**Ans:** 0.15°]

19. Two small angled prisms A and B deviate the blue rays by 7° and 9° and the red rays by 5° and 7° respectively. Which prism has a greater dispersive power? [**Ans:** A]

20. A beam of light converging towards a point O is intercepted by a convex mirror of focal length 30 cm. The point O is 20 cm behind the mirror from its pole. Find the nature and position of the image. What if the point O is at distance 50 cm from the pole?
[**Ans:** (i) Real, at distance 60 cm from pole in front of reflecting surface and erect and magnified 3 times
(ii) Virtual, at distance 75cm from pole behind the mirror, inverted and magnified 1.5 times]

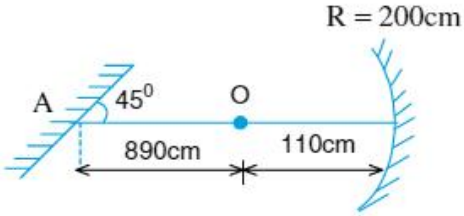
21. When an object is placed at a distance of 25cm from a mirror, the magnification is m_1. The object is moved 15 cm farther away with respect to the earlier position, and the magnification becomes m_2. If $\frac{m_1}{m_2} = 4$, then calculate the focal length of the mirror.
[**Ans:** Concave, 20 cm]

22. An object is placed between a plane mirror and a convex mirror as shown. Let I_1 and I_2 be the images formed by the plane mirror and the convex mirror respectively. It is found that distance between I_1 and I_2 is 100 cm, distance between the object and I_2 is 60 cm and I_2 has magnification 1/2. Find focal length of the convex mirror and also the distance between the two mirrors.



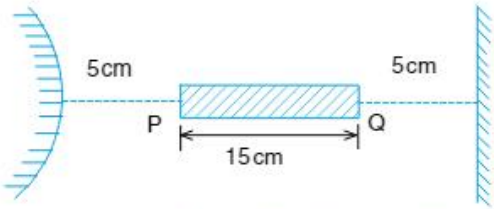
[**Ans:** 40 cm, 60 cm]

23. A plane mirror and a concave mirror are arranged as shown in figure and O is a point object. Find the position of image formed after two reflections, first one taking place at concave mirror.



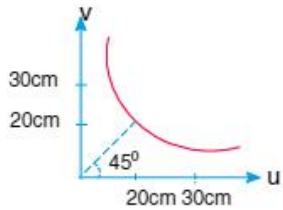
[**Ans:** 100 cm vertically below A]

24. Describe the image in convex mirror, of the image of rod PQ in the plane mirror (radius of curvature of mirror is 60 cm).



[**Ans:** Virtual, erect and 3 cm long]

25. The u - v curve in a concave mirror as shown in figure.



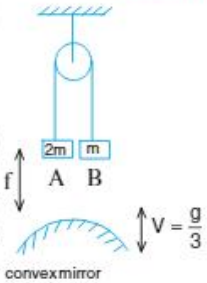
(a) Find the focal length and power of the mirror.
(b) Find the nature and position of the image, when an object is placed at 15 cm from the mirror.

[Ans: (a) 10 cm, 10 D
(b) real, inverted, 30cm from the mirror]

26. A thief is running away in a car with velocity 20m/s. A police jeep is following him which is sighted by thief in his rear view mirror, which is a convex mirror of focal length of 10 m. He observes that the image of the jeep is moving towards him with a velocity of 1 cm/s. If the magnification of mirror for the jeep at that time is 1/10, find actual speed of jeep.

[Ans: 21 m/s]

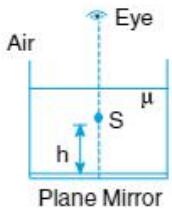
27. A convex mirror of focal length f is approaching an at wood machine with a velocity of $\frac{g}{3}$ as shown, The blocks A and B are moving under gravity. After one second of releasing the blocks they are crossing each other and at the time of crossing they are at a distance f from the mirror. Find the relative speed between the images at that instant.



[Ans: g/6]

28. One face of a rectangular glass plate 6 cm thick is silvered. An object held 8 cm in front of the front face forms an image 12 cm behind the silvered face. Find the refractive index of glass. **[Ans:** $\mu = 1.2$]

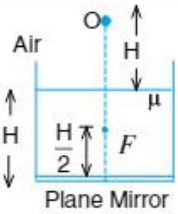
29. In figure a point source S is placed at a height h above the plane mirror in a medium of refractive index μ.



(i) Find the number of images seen for normal view.
(ii) Find the distance between the images.

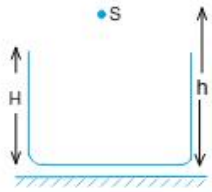
[Ans: (i) 2 (ii) Separation between two images $\frac{2h}{\mu}$]

30. Consider situation in figure. The bottom of the pot is reflecting plane mirror, F is a small fish and O is an object. Refractive index of water is μ. Find the distances from it self the fish can see the image (s) of the object O.



[Ans: H(μ+1/2) above it self,H (μ+3/2) below it self]

31. A point source S is placed at a height h from the bottom of a vessel of height H(<h) and area A. The vessel is polished at the base. Water is gradually filled in the vessel at a constant rate α m^3/s. Find how the distance d of image of the source from the bottom of the vessel varies with time t.



[Ans: $d = \mu h - \frac{(\mu - 1)\alpha t}{A}$]

32. A concave mirror of radius of curvature 40 cm is placed at the bottom of a tank of water ($\mu = 4/3$) with its optic axis vertical and the mirror facing upward. Find the position of the image of a bird flying over the tank at a height 60 cm above the surface of water, which is 20 cm deep.

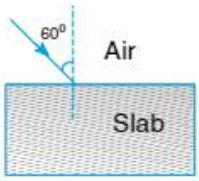
[Ans: 3.75 cm about the water surface]

33. A cylindrical vessel of depth 4m is fully filled with non homogenious liquid whose refractive index (μ) varies with depth y (in meters) a $\mu = 1 + \frac{y}{4}$. Find the depth of the bottom of vessel as seen by an observer just above the top of the vessel. **[Ans:** 4/n 2 m]

34. A point source is sinking down in a liquid of refractive index μ with a constant velocity v_0. Find the rate of change of the area through which light will escape from the liquid when the source is at a depth h_0

[Ans: $\frac{2\pi h_0 v_0}{(\mu^2 - 1)}$]

35. A ray of light enters a glass slab from air as shown in figure. If refractive index of glass slab is given by $\mu = A - Bt$, where A and B are constants and t is the thickness of slab measured from the top surface. Find the maximum depth travelled by ray in the slab. Assume thickness of slab to be sufficiently large.

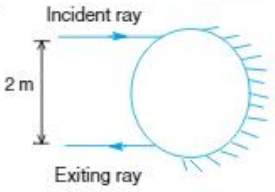


[Ans: $\frac{1}{B}\left(A - \frac{\sqrt{3}}{2}\right)$]

36. A ray incident at a point at an angle of incidence 60° enters a glass sphere of refractive index $\sqrt{3}$ and is reflected and refracted at the farther surface of sphere. Find the angle between the reflected and refracted rays at the second surface. **[Ans: 90°]**

37. A hemisphere (made of material of refractive index $\sqrt{3}$) of radius r is placed on a horizontal surface with its base touching the surface. A vertical beam of cross sectional radius $\frac{\sqrt{3}}{2}r$ is incident symmetrically on its curved surface. Find the radius of the spot of light formed on the horizontal surface **[Ans: $\frac{r}{\sqrt{3}}$]**

38. A transparent cylinder of radius 2m has a mirrored surface on its right half as shown in figure. A light ray travelling in air is incident on the left side of the cylinder.



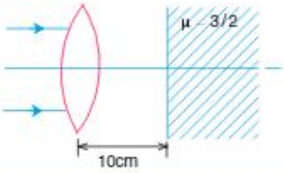
The incident ray and the exiting ray are parallel and at a distance 2m. Determine the refractive index of the material of the cylinder. **[Ans: 1.932]**

39. A point object is placed on the optic axis of a convex lens of focal length f at a distance of $2f$ to the left of it. The diameter of the lens is d. An insect moves at a distance $3f$ to the right of the lens and transverse to the principal axis. Find the maximum distance through which it see the image. **[Ans: d/2]**

40. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam coverage if the lens is (a) a convex lens of focal length 20 cm. and (b) a concave lens of focal length 16 cm ?
[Ans: (a) real, at 7.5cm same side as P, (b) real, at 48 cm same side as P]

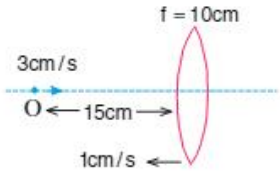
41. The distance between two point sources of light is 24 cm. Find out where you would place a converging lens of focal length 9 cm, so that the images of both the sources are formed at the same point.
[Ans: 6 cm from one of the source]

42. Focal length of a thin convex lens is 30 cm. At a distance of 10 cm from the lens there is a plane refracting surface of refractive index 3/2. Where will the parallel rays incident on lens converge ?



[Ans: 40 cm from lens]

43. Both object and convex lens are approaching each other along the principal axis as shown. Find the speed of image relative to object at the given instant as shown in the figure.



[Ans: 12 cm/s, away]

44. A converging lens of focal length 12 cm and a diverging mirror of focal length 7.5 cm are placed 5.0 cm apart with their principal axes coinciding. Where should an object be placed so that its image falls on itself?
[Ans : 30 cm from the lens further away from the mirror]

45. A converging lens of focal length 15 cm and a converging mirror of focal length 10 cm are placed 50 cm apart with common principal axis. A point source is placed in between the lens and the mirror at a distance of 40 cm from the lens. Find the locations of the two images formed.
[Ans: one at 15 cm and other at 24 cm from the lens away from the mirror]

46. Magnification by a lens of an object at distance 10 cm from it is –2. Now a second lens is placed exactly at the same position where first was kept, without changing the distance between object and lens. The magnification by this second lens is –3. (a) Now both the lenses are kept in contact at the same place.What will be the new magnification. (b) What is the focal length of the combination when both lenses are in contact?
[Ans: (a) $\frac{-6}{11}$; (b) $\frac{60}{17}$cm]

47. The curved surface of a concave spherical mirror with a radius of curvature of 0.2 m is silvered. What is the focal power of this system? Refractive index of water is 4/3.

[Ans: $\frac{40}{3}$D]

48. A thin equiconvex lens of glass ($\mu = 1.5$) with radius of curvature 4m is placed on a horizontal plane mirror. When the space between the lens and mirror is filled with a liquid, an object held at a distance 6m vertically above the lens is found to coincide with its own image. What is the refractive index of the liquid. **[Ans: $\mu = 4/3$]**

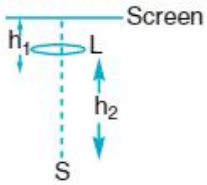
49. Two thin converging lenses are placed on a common axis so that the optic centre of one of them coincides with the focus of the other. An object is placed at a distance twice the focal length from the left-hand lens. Where will its image be? What is the lateral magnification? The focal length of each lens is *f*.

[**Ans:** $\frac{f}{2}$ from right hand lens, m = –1/2]

50. An equiconvex lens of refractive index 1.5 and focal length 10 cm is held with its axis vertical and its lower surface immersed in water, the upper surface being in air. At what distance from the lens will a vertical beam of parallel rays incident on the lens be focussed? $\mu_w = 4/3$.

[**Ans:** 20 cm from the lens in water]

51. In the shown arrangement in a vertical plane, the lens forms a bright image of 'S' on the screen. The lens is suddenly released to fall freely under gravity at time t = 0. The image of source is again obtained on screen at time 't'. Find t.



[**Ans :** $\sqrt{\frac{2(h_2 - h_1)}{g}}$]

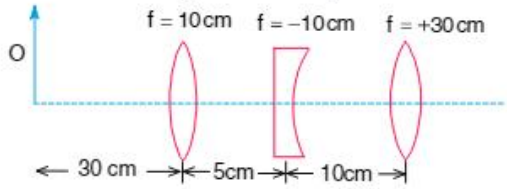
52. A converging lens forms a five fold magnified image of an object. The screen is moved towards the object by a distance 0.5m, and the lens is shifted so that the image has the same size as the object. Find the lens power and the initial distance between the object and the screen. [**Ans:** 6.4D, 1.125m]

53. A plano-convex lens behaves as a concave mirror of focal length 30 cm when its plane surface is silvered and as a concave mirror of focal length 10 cm when its curved surface is silvered. What is the radius of curvature of curved surface and μ of the material of lens? [**Ans:** R = 30 cm and μ = 1.5]

54. A point object is placed at a distance of 20 cm on the principal axis from a thin plano-convex lens of focal length 15 cm. Find the position of final image, if the plane surface is silvered.

[**Ans:** 12 cm, towards curved surface]

55. Find the position of the final image formed by the lens combination given in the figure .

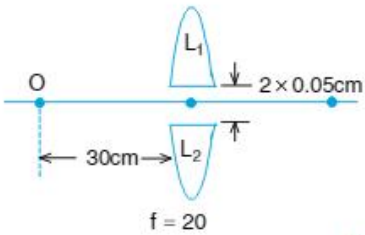


[**Ans:** 30 cm to the right of the third lens]

56. An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, it is found that there is no parallax between the images formed by the two mirrors. What is the radius of curvature of the convex mirror?

[**Ans:** 25cm]

57. A point object O is placed at a distance of 30 cm from a convex lens of focal length 20 cm cut into two halves each of which is displaced by 0.05 cm as shown in figure. Find the position of the image. If more than one image is formed, find their number and distance between them.



[**Ans:** 0.3cm]