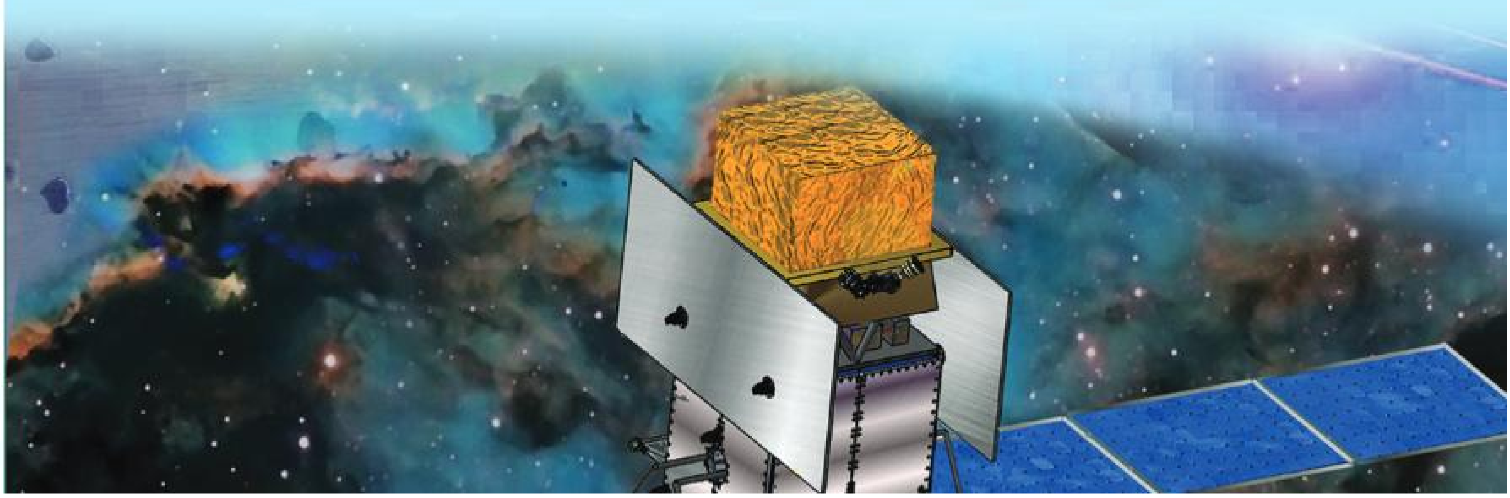
Chapter - 2

ELECTROMAGNETICS

✤ Oersted's experiment, Ampere's circuital law and it's applications ✤ ✤ Biot savart law and it's applications ✤ Force on a moving charge in magnetic field ✤ Force on a current carrying wire in magnetic field ✤ ✤ Flemings left hand rule–Force between two parallel conductors carrying currents, Magnetic moment of current loop-Torque on current loop in a magnetic field, M.C.G–Sensitivity of M.C.G– shunt–ammeter–voltmeter ✤ ✤ Tangent Galvanometer ✤



2.1 INTRODUCTION

In this chapter we begin to look into the connection between electricity and magnetism. For several centuries electricity and magnetism were considered as separate and as unrelated phenomena. We have already discussed about stationary charge, moving charges in the previous chapters

In 1820 Oersted, observed that magnetic field is produced by a current carrying conductor. The relationship between the current in the conductor and the strength of the magnetic field around the conductor was established by Ampere.

Ampere gave the expression for intensity of magnetic field produced due to steady current and also showed how an electric circuit behaves as a magnetic shell.

In 1831 Michael Faraday found that a moving magnetic field causes e.m.f.. The phenomenon is known as 'Electromagnetic Induction'.

2.2 OERSTED'S EXPERIMENT

The first discovery showing connection between electric current and magnetism was made by Oersted in 1820. He noted during a lecture demonstration that a magnetic compass needle brought close to a straight wire carrying an electric current aligned itself perpendicular to the wire. Upon reversing the current in the wire the needle deflected in the opposite direction. From these observations Oersted concluded that a magnetic field is associated with an electric current and that it is the magnetic field which tends to deflect the compass needle.

The direction of magnetic field around a current carrying conductor can be determined by the following rules :

2.3 AMPERE'S SWIMMING RULE

Imagine a person swimming along a current carrying wire in the direction of the current facing a magnetic needle placed under the wire, then the magnetic north pole of the needle deflects towards his left hand.

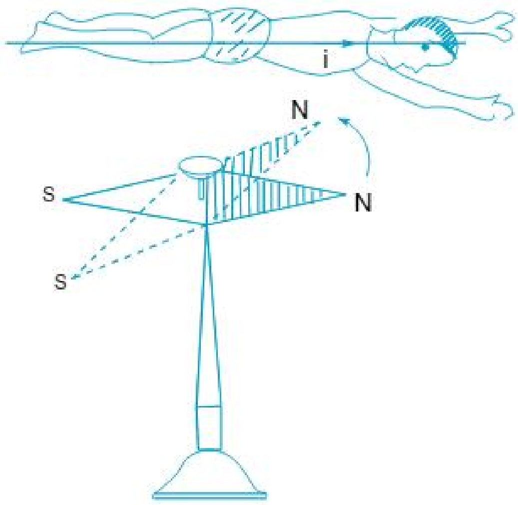


Fig 2.1

2.4 MAGNETIC FIELD DUE TO A STRAIGHT CONDUCTOR

A straight conductor carrying current produces a magnetic field around it which is radially symmetric. The lines of force are circular in a plane perpendicular to the conductor as shown in figure. Tangent drawn to the line of force at any point gives the direction of magnetic field produced at that point by the current carrying conductor.

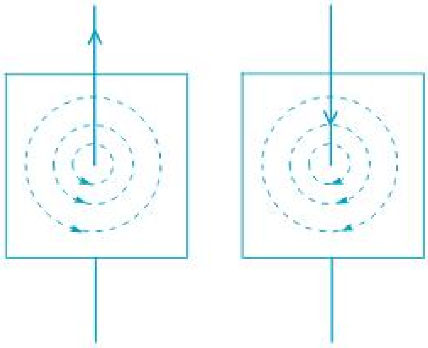


Fig 2.2

The deflection of small compass needle around a straight conductor carrying current will be as shown in figure. The locus of its north pole would give the line of force. The wire is perpendicular to the plane of the paper in figure (a) The current is normally outwards it is represented by figure (b) The current is normally inwards. In each case, the magnetic field direction is given by the arrow shown by the north pole of the compass needle.

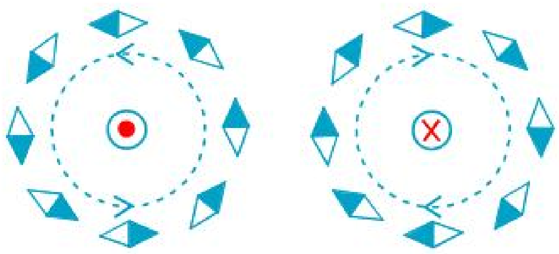


Fig 2.3

2.5 MAXWELL'S CORK SCREW RULE

Imagine a right handed cork screw advancing in the direction of current then the direction of rotation of the head gives the direction of lines of force

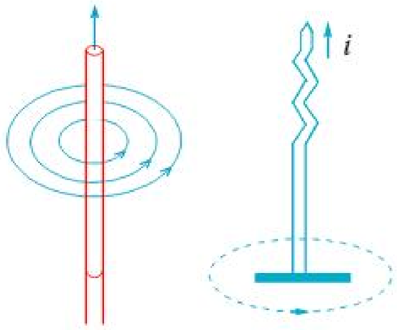


Fig 2.4

2.6 AMPERE'S RIGHT HAND THUMB RULE

When a straight conductor carrying current is held in the right hand such that the thumb is pointing along the direction of current then the direction in which fingers curl round it gives the direction of lines of force.

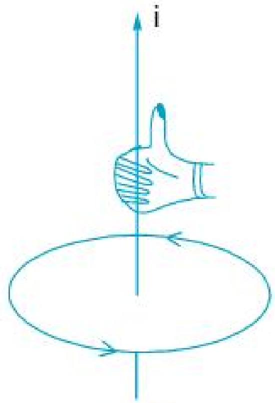


Fig 2.5

2.7 AMPERE'S CIRCUITAL LAW

This law gives the relationship between the current (i) and intensity of magnetic induction B. This law is an alternative to Biot-Savart's Law.

Statement

The line integral of the magnetic field (B) along any closed path in air (or) vaccum is equal to μ_0 times the net current i through the area bounded by the curve

$$\oint \vec{B}.d\vec{l} = \mu_0 i \qquad .... (2.1)$$

Consider a closed plane curve as shown in Fig. Here $\overline{dl}$ is a small length element on the curve. Let $\overline{B}$ be the resultant magnetic field at the position of $\overline{dl}$. If the scalar product $\overline{B}.\overline{dl}$ is integrated by varying $\overline{dl}$ on the closed curve it is called line integral of $\overline{B}$ along the curve and it is represented by $\oint \overline{B}.\overline{dl}$.

The various currents crossing the area bounded by the curve are as shown. Currents i_1, i_3, i_4, i_5, which are normally into the plane of the paper are positive and the current i_2, i_6 and i_7 which are normally outwards are negative so the total current crossing the area bounded by the closed curves $i_1 - i_2 + i_3 + i_4 - i_6$. Here current i_5 and i_7 are outside the enclosed area.

$$\therefore \oint \overline{B}.\overline{dl} = \mu_0 \left(i_1 - i_2 + i_3 + i_4 - i_6\right) \quad .... (2.2)$$

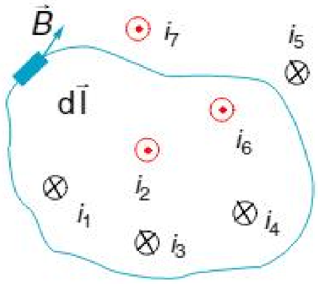


Fig 2.6

The line integral does not depend on the shape of the path or on the position of the wire carrying the current in it. If a conductor carrying current is outside the closed path the line integral of B due to that conductor is zero i.e., we need not consider the current flowing through conductors outside the area of the closed path and we need not consider the currents that do not pierce the area of the closed path.

Ampere's circuital law is always true no matter how distorted the path or how complicated the magnetic field. In most cases even though Ampere's circuital law is true it is inconvenient because it is impossible to perform the path integral in a few special symmetric situations. However wherever applicable, it is easy to perform path integral using Ampere's law.

Points to remember regrading Ampere's Law

1) The line integral does not depend on the shape of the closed path or on the position of the current carrying wire in the loop.

2) If a conductor carrying current is outside the closed path, the line integral of B due to that conductor is zero i.e., we need not consider the currents that do not pierce the area of the closed path.

3) Ampere's circuital law is always true no matter how distorted the path or how complicated the magnetic field is. In most cases even though Ampere's circuital law is true it is inconvenient because it is impossible to perform the path integral. However in few special symmetric cases it is easy to perform path integral using Ampere's law.

4) Ampere's circuital law is applicable for conductors carrying steady current.

5) Ampere's circuital law is analogous to Gauss law.

6) Ampere's circuital law is not independent of Biot-Savart's law. It can be derived from Biot-Savart's law. Its relation with Biot-Savart's Law is similar to the relation between Gauss Law and Coulomb's Law in electrostatics.

- Ampere's circuit law is applicable for the steady current carrying conductors.

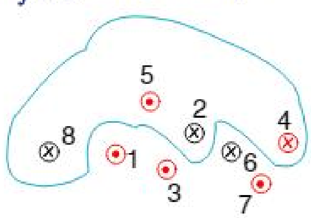
Knowledge Plus 2.1

☺ Magnetic field due to neutral current carrying conductor is independent of frame of reference. Why?

☛ In the ground frame of reference electrons are in motion and protons are at rest. In the moving frame of reference if electrons appears at rest protons are in motion.

Example-2.1

Eight wires cut the page perpendicularly at the points shown. Each wire carries current i_0. Odd currents are out of the page and even currents into the page. Find the line integral $\oint \vec{B}.\, d\vec{l}$ along the loop.



Solution :

According to ampere's, circuital law

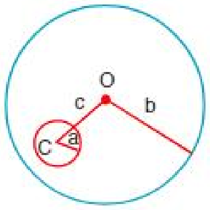
$$\oint \vec{B}.\, d\vec{l} = \mu_0 i \ \text{ enclosed}$$

$$\oint \vec{B}.\, d\vec{l} = \mu_0 \left[i_0 - i_0 + i_0 + i_0\right]$$

$$\oint \vec{B}.d\vec{l} = 2\mu_0 i_0$$

Example-2.2

A long straight metal rod has a very long hole of radius 'a' drilled parallel to its axis as shown in the figure. If the rod carries a current i, find the magnetic field on the axis of the hole given C is the centre of the hole and OC = c.x



Solution :

In the rod current density $j = \frac{i}{\pi(b^2 - a^2)}$.

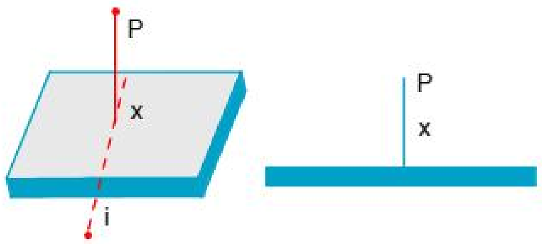
On the hole axis, only the larger rod contributes magnetic field. Imagine an amperean loop of radius c and apply Ampere's law.

$$B_{Total} = \underbrace{\vec{B_1}}_{\text{complete solid}} + \underbrace{\vec{B_2}}_{\text{cavity}} = \frac{\mu_0 j\pi c^2}{2\pi c} + O$$

$$= \frac{\mu_0 ic}{2\pi(b^2 - a^2)} = \frac{\mu_0}{2\pi}\frac{ic}{(b^2 - a^2)}$$

Example-2.3

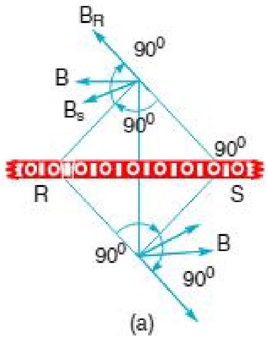
Figure shows a cross-section of a large metal sheet carrying an electric current along its axis. The current in a strip of width is k where k is a constant. Find the magnetic field at a point P at a distance x from the metal sheet.



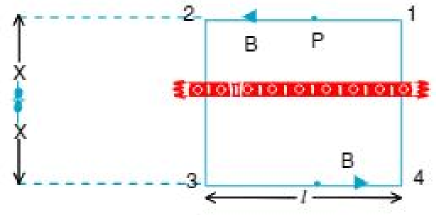
(a) (b)

Solution :

Consider two strips R and S of the sheet situated symmetrically on the two sides of P. The magnetic field at P above sheet and below sheet is parallel to sheet as shown in figure. There is no field perpendicular to the sheet.



(a)



(b)

Now applying Ampere's law to the closed path 1-2-3-4-1 as shown in fig.

$$\oint \vec{B}.d\vec{\ell} = \mu_0 i_{in}$$

$$\int_1^2 \vec{B}.d\vec{\ell} + \int_2^3 \vec{B}.d\vec{\ell} + \int_3^4 \vec{B}.d\vec{\ell} + \int_4^1 \vec{B}.d\vec{\ell} = \mu_0(k\ell)$$

or $B\ell + 0 + B\ell + 0 = \mu_0 k\ell$ or $B = \frac{\mu_0 k}{2}$.

2.8 INTENSITY OF MAGNETIC INDUCTION (B) NEAR A LONG STRAIGHT CONDUCTOR

Consider an infinitely long wire carrying current i as shown in figure. P is a point at a perpendicular distance r from the conductor. The magnetic induction field produced by the conductor is radially symmetric i.e, magnetic lines of force are concentric circles centred at the conductor. The tangent drawn to the line of force at any point gives the direction of magnetic induction field $\overline{B}$ at that point. $\overline{dl}$ is a small element on the circle of radius r angle between $\overline{B}$ and $\overline{dl}$ is 0^0 everywhere on this path. From Ampere's circuital law

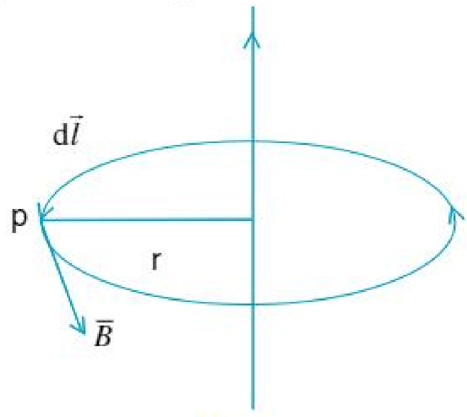


Fig 2.7

$$\oint \overline{B}.\overline{dl} = \mu_0 i \ ; \ \oint B dl \cos 0^0 = \mu_0 i$$

$$B\oint dl = \mu_0 i$$

$$B(2\pi r) = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r} \qquad ..... (2.3)$$

Here r must be much less than the length of conductor. It can also be shown by Ampere's circuital law that the magnetic induction is zero at any point along the axis of the conductor.

2.9 BIOT-SAVART LAW

Biot and Savart conducted several experiments and established the relation between magnetic induction $(\overline{B})$ and current (i).

They obtained a relation using which the magnetic induction field $\overline{B}$ can be calculated at any point near a current carrying conductor.

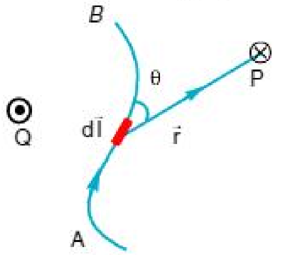


Fig 2.8

AB is a conductor of any arbitrary shape and i is the electric current flowing through it P is a point at which the magnetic field is to be determined. Assume that the current carrying conductor is divided into very small current elements. The magnetic induction field dB at P due to each element is calculated. The total field B would be the resultant of the infinitesimal fields of all such elements of the conductor. Let dl be the length of one such element and 'r' be the distance of point P from this element, (θ) the angle between length of this element and the line joining the element to the point P.

Biot-Savarts law states that the magnetic induction field dB at P due to the current carrying element

a) is directly proportional to the current (i) flowing through the conductor i.e,

$$dB \propto i \qquad .... (i)$$

$\Rightarrow$ is directly proportional to the length (dl) of the element i.e.,

$$dB \propto dl \qquad .... (ii)$$

$\Rightarrow$ is directly proportional to the sine of the angle (θ) between length of the element and the line joining the element to the point P.

$$dB \propto \sin\theta \qquad .... (iii)$$

is inversely proportional to the square of the distance (r) of the point from the element.

$$dB \propto \frac{1}{r^2} \qquad .... (iv) \qquad \therefore dB \propto \frac{i\,dl\sin\theta}{r^2}$$

If the conductor is in vaccum (or) air then

$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}$$

Here $\frac{\mu_0}{4\pi}$ is the proportionality constant.

The above equation gives the magnitude of the magnetic field produced due to small current element

If current flows in the direction as shown in the Fig 2.8, the direction of dB at P is directed perpendicular to the plane of the paper in the inward direction.

In vector form the above equation can be written as $d\overline{B} = \frac{\mu_0}{4\pi}\frac{i\,d\overline{l}\times\overline{r}}{r^3}$ .... (2.4)

The resultant field at P due to the entire conductor can be obtained by integrating the above equation.

$$\vec{B} = \int \frac{\mu_0 i}{4\pi}\frac{d\vec{l}\times\vec{r}}{r^3}$$

The Biot-Savart law for the magnetic field has certain similarities as well as differences with the Coulomb's law for the electrostatic field. Some of these are:

(i) Both are long range, since both depend inversely on the square of distance from the source to the point of interest. The principle of superposition applies to both fields.

(ii) The electrostatic field is produced by a scalar source, namely, the electric charge. The magnetic field is produced by a vector source $i\,dl$.

(iii) The electrostatic field is along the displacement vector joining the source and the field point. The magnetic field is perpendicular to the plane containing the displacement vector r and the current element id$l$.

(iv) There is an angle dependence in the Biot - Savart law which is not present in the electrostatic case. The magnetic field at any point in the direction of dl (the dashed line) is zero. Along this line, $\theta = 0$,

$\sin\theta = 0$ so $|d\vec{B}| = 0$.

2.10.1 MAGNETIC FIELD AT THE CENTRE OF A CIRCULAR COIL CARRYING CURRENT

Consider a circular coil of radius R carrying a current i. We have to find the magnetic field due to this current at the centre of the loop. Consider any small element dl of the wire (Fig 2.9). The magnetic field at the centre O due to the current element id$\bar{l}$ is

$$dB = \frac{\mu_0}{4\pi}\frac{idl}{R^2}$$

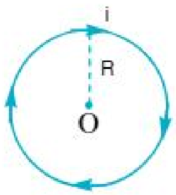


Fig 2.9

where $\bar{r}$ is the vector joining the element to the centre O. The direction of this field is perpendicular to the plane of the diagram and is going into it. The magnitude is

$$dB = \frac{\mu_0}{4\pi}\frac{idl}{R^2} \quad \text{... (i)}$$

As the fields due to all such elements have the same direction, the net field is also in this direction. It can, therefore, be obtained by integrating (i) under proper limits. Thus,

$$B = \int dB = \int \frac{\mu_0 i}{4\pi R^2} dl$$

If the coil has N turns $\int dl = 2\pi RN$

$$= \frac{\mu_0 i}{4\pi R^2}\int dl = \frac{\mu_0 i}{4\pi R^2}\times 2\pi RN = \frac{\mu_0 iN}{2R} \quad \text{...(2.5)}$$

2.10.2 FIELD AT AN AXIAL POINT OF A CIRCULAR COIL

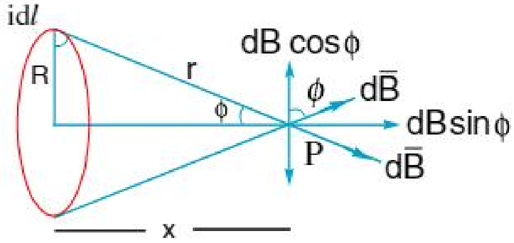


Fig 2.10

In case of a point P on the axis of circular coil for every current there is a symmetrically situated opposite element. The component of the field B perpendicular to the axis cancel each other while along the axis add up

$$\text{i.e., } B = \int dB\sin\phi = \frac{\mu_0}{4\pi}\int \frac{I\,dl\sin\theta}{r^2}\sin\phi$$

And as here angle θ between the element d$\bar{l}$ and $\bar{r}$ is $\pi/2$ everywhere and r is same for all elements while $\sin\phi = (R/r)$ so

$$B = \frac{\mu_0}{4\pi}\frac{iR}{r^3}\int dl$$

Now if the coil has N turns $\int dl = 2\pi RN$ and as $r^3 = (x^2+R^2)^{3/2}$

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{(x^2+R^2)^{3/2}}$$

$$B = \frac{\mu_0}{2}\frac{NiR^2}{(x^2+R^2)^{3/2}} \quad \text{....(2.6)}$$

i) B varies non linearly with x as shown in figure and is maximum when $x^2 = \min = 0$ i.e., the point is at the centre of the coil and then

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NI}{R} = \frac{\mu_0 Ni}{2R}$$

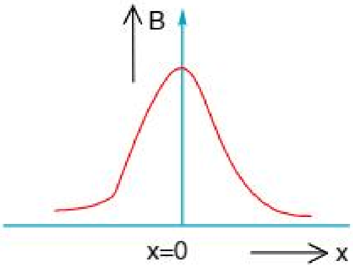


Fig 2.11

ii) if $x >> R$

$$B=\frac{\mu_0}{4\pi}\frac{2\pi NIR^2}{x^3}=\frac{\mu_0}{4\pi}\frac{2NIA}{x^3} \quad \text{with } A=\pi R^2$$

$$\bar{B}=\frac{\mu_0}{4\pi}\frac{2M}{x^3} \qquad \text{.... (2.7)}$$

(where M = magnetic moment)

This result is same as for the field of a magnetic dipole for an axial point. So a current loop for a distant point behaves as a magnetic dipole of moment $=\bar{M}=N\bar{S}$ (where $\bar{S}=i\bar{A}$).

Example-2.4

A current of 2 A is flowing through a circular coil of radius 10 cm containing 100 turns. Find the magnetic flux density at the centre of the coil.

Solution :

Given, $i = 2A$; $r = 10\text{ cm} = 10 \times 10^{-2}\text{m}$; $N = 100$

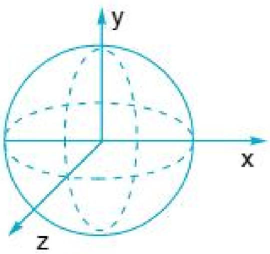
Flux density at the centre of 'N' turn coil is given by,

$$B = N\frac{\mu_0 i}{2r} = 100\times\frac{2\pi\times10^{-7}\times2}{10\times10^{-2}}$$

$$= 1.26 \times 10^{-3}\text{ Wb/m}^2$$

Example-2.5

Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its centre at the origin of co-ordinate system. If current I is flowing through each ring then find the magnitude of the magnetic field at the common centre.



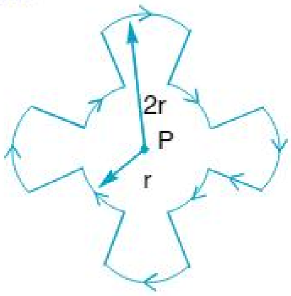
Solution :

B due to the ring lying in XY-plane is : $\frac{\mu_0 I}{2R}$ along Z-axis. B due to the ring lying in YZ - plane is $\frac{\mu_0 I}{2R}$ along X - axis, and B due to the ring lying in XZ - plane is $\frac{\mu_0 I}{2R}$ along Y-axis.

$$\therefore B_{net}=\sqrt{3}\frac{\mu_0 I}{2R}$$

Example-2.6

A current I flows around a closed path in the horizontal plane of the circle as shown in the figure. The path consists of eight arcs with alternating radii r and 2r. Each segment of arc subtends equal angle at the common centre P. Find the magnetic field produced by current path at point P.



Solution :

B due to straight parts of the wire is zero. Each arc subtends an angle

$$\theta=\frac{2\pi}{8}=\pi/4$$

$$\therefore B_{net}=4\left(\frac{\mu_0 I}{4\pi r}\times\pi/4\right)+4\left(\frac{\mu_0 I}{4\pi(2r)}\times\pi/4\right)$$

$=\frac{3\mu_0 I}{8r}$; perpendicular to the plane of the paper and directed inward.

Example-2.7

A thin insulated wire form a spiral of N=100 turns carrying a current of i=8mA. The inner and outer radii are equal to a=5cm and b=10cm. Find the magnetic field at the centre of the coil.

Solution :

Let n = no.of turns per unit length along the radial of spiral. Consider a ring of radii x and x + dx .

No. of turns in the ring = ndx.

$$n=\frac{N}{(b-a)}$$

Magnetic field at the centre due to the ring.

$$dB = \frac{\mu_0 (ndx) i}{2x}$$

So net field

$$B = \int dB = \int_a^b \frac{\mu_0 nidx}{2x} = \frac{\mu_0 ni}{2}\int_a^b \frac{dx}{x}$$

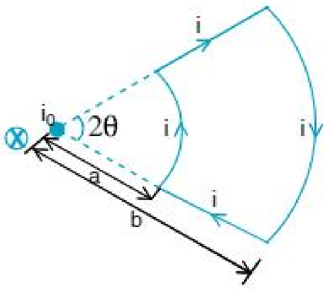
$$\text{or } B = \frac{\mu_0 ni}{2}\ln\frac{b}{a} \text{ or } B = \frac{\mu_0 Ni}{2(b-a)}\ln\frac{b}{a}$$

$$= \frac{4\pi\times10^{-7}\times100\times8\times10^{-3}}{2(10-5)\times10^{-2}}\ln\frac{10}{5}$$

$$B = 6.96\times10^{-6}\,T$$

Example-2.8

A loop, carrying a current i, lying in the plane of the paper, is in the field of a long straight wire with constant current i_0 (inward) as shown in fig. Find the torque acting on the loop.

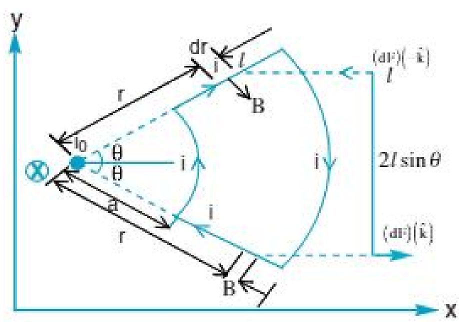


Solution :

The field due to current carrying wire is tangential to every point on the circular portion of the loop and hence the forces acting on these segments are zero.

Now consider two small elements of length dr at a distance r from the axis symmetrically as shown in fig.

The magnitude of the force experienced by each element is $dF = B\,i\,dr = \left(\frac{\mu_0}{2\pi}\frac{i_0}{r}\right) idr$



On element 1 it is into the page and on 2 it is out of the page

$$d\tau = dF\times2r\sin\theta = \left(\frac{\mu_0 i_0 i}{2\pi r}dr\right)\times2r\sin\theta$$

Now total torque,

$$\tau = \frac{\mu_0 i_0 i\sin\theta}{\pi}\int_a^b dr = \frac{\mu_0 i_0 i}{\pi}\sin\theta(b-a)$$

2.11 FIELD AT AN AXIAL POINT OF SOLENOID

If many turns of an insulated wire are wound around a cylinder the resulting coil is called a solenoid

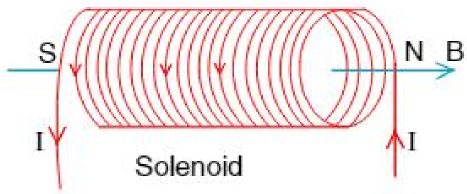


Fig 2.12

The wire is coated with an insulating material so that although the adjacent turns physically touch each other they are electrically insulated.

The field at a point on the axis of a solenoid can be obtained by superposition of fields due to a large number of identical coils all having their centre on the axis of the solenoid. In the solenoid the field at an outside point due to the neighbouring loops oppose each other where as at an inside point the fields are in the same direction. These tendencies to have zero field outside and a uniform field inside become more and more effective as the solenoid is more and more tightly wound

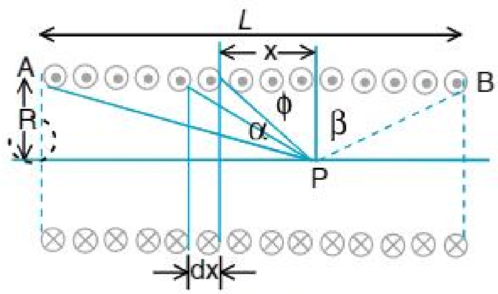


Fig 2.13

Consider a coil of width dx at a distance x from the point P on the axis of the solenoid of length L radius R as shown in figure.

$$dB=\frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{(R^2+x^2)^{3/2}}$$

If n is the number of turns per unit length

$N = ndx$ and as $x = R\tan\phi$

$dx = R\sec^2\phi\, d\phi$

$$\text{so } dB=\frac{\mu_0}{4\pi}\left(\frac{2\pi(ndx)iR^2}{\left(R^2+R^2\tan^2\phi\right)^{3/2}}\right)$$

$$d\overline{B}=\frac{\mu_0}{4\pi}\left[\frac{2\pi niR^2R\sec^2\phi\, d\phi}{R^3\left(1+\tan^2\phi\right)^{3/2}}\right]$$

$$=\frac{\mu_0}{4\pi}\left[\frac{2\pi ni\sec^2\phi\, d\phi}{\sec^3\phi}\right]$$

$$=\frac{\mu_0}{4\pi}(2\pi ni)\cos\phi\, d\phi$$

$$\text{and hence } B=\frac{\mu_0}{4\pi}(2\pi ni)\int_{-\infty}^{\beta}\cos\phi\ d\phi$$

$$B=\frac{\mu_0}{4\pi}(2\pi ni)[\sin\alpha+\sin\beta]$$

$$=\frac{\mu_0}{2}ni(\sin\alpha+\sin\beta) \qquad \text{.... (2.8)}$$

This is the desired result and from this it is clear that

i) if the solenoid is of infinite length and the point is well inside the solenoid $\alpha=\beta=\frac{\pi}{2}$

$$\text{so } B=\frac{\mu_0}{2}ni\left(\sin\frac{\pi}{2}+\sin\frac{\pi}{2}\right)$$

$$B=\mu_0 ni \qquad \text{.... (2.8a)}$$

ii) if the solenoid is of finite length and at mid point

$$\alpha=\beta,\ \ B=\frac{\mu_0}{4\pi}(4\pi ni)\sin\alpha \qquad \text{....(2.8b)}$$

$$\text{and } \sin\alpha=\frac{L}{\sqrt{L^2+4R^2}}$$

iii) if the solenoid is of infinite length and the point is near one end $\alpha=0$ and $\beta=\pi/2$.

$$\text{so } B=\frac{\mu_0}{4\pi}(2\pi ni)(1+0)\Rightarrow B=\frac{1}{2}(\mu_0 ni) \qquad \text{....(2.8c)}$$

iv) the field and its variation with distance along the axis of a solenoid is shown in figure.

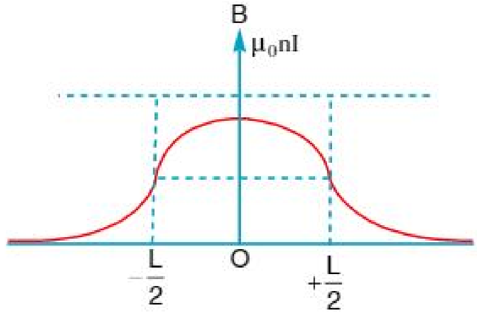


Fig 2.14

2.12 TOROID

A toroid is a long solenoid bent round in the form of a closed ring as shown in figure; i_0 is the current in it's wire. The direction of magnetic field B at any point is tangent to a circle passing through the point and concentric with the toroid. The magnitude of B on any point of a circle will be constant. Consider a point P within space enclosed by the winding of the toroid. A circle of radius r is drawn through point P. Applying Ampere's law to this circle we have

$$\int \overline{B}.d\overline{l}=\mu_0 i \ \ \text{............}$$

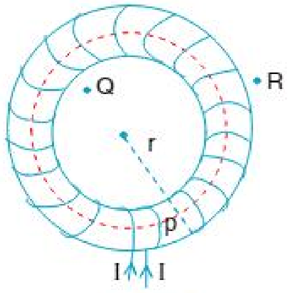


Fig 2.15

where i is the amount of current enclosed by the circle but not the current I_0 through the toroid.

$$\text{now } \int \vec{B}d\vec{l}=B(2\pi r) \ \ \text{...........}$$

and $i = Ni_0$ ....

where N is the total number of turns in the toroid.

$$B(2\pi r) = \mu_0 N i_0$$

$$B = \frac{\mu_0}{2\pi}\frac{Ni_0}{r} \quad .... (2.9)$$

Shows that the field B varies with r. If L be the mean circumference of the toriod then $L = 2\pi r$

$$B = \frac{\mu_0 N i_0}{L}$$

Now consider a point Q as shown in figure. The field at this point is zero. The reason is that no current is enclosed by the circle through Q. The field at an outside point R is also zero because each turn of the winding passes twice through the area enclosed by the circle carrying equal current in opposite directions i.e., the current enclosed in the circle through R is zero.

2.13 MAGNETIC FIELD DUE TO A STRAIGHT CURRENT-CARRYING WIRE

According to Biot-Savart is law

$$\bar{B} = \frac{\mu_0}{4\pi}\int \frac{i d\bar{l} \times \bar{r}}{r^3} \quad .... (1)$$

Proof

As here every element of the wire contributes to $\vec{B}$ in the same direction (say as point P), Eq. (1) for the case becomes

$$B = \frac{\mu_0}{4\pi}\int \frac{i dy\, r \sin\theta}{r^3} = \frac{\mu_0 i}{4\pi}\int \frac{dy \sin\theta}{r^2}$$

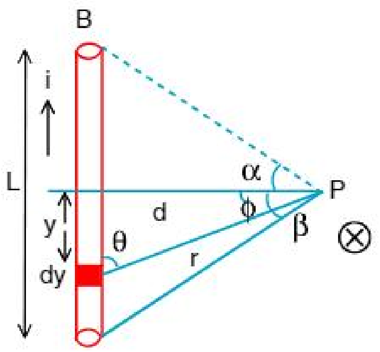


Fig 2.16

$$\tan\phi = y/d$$

$$y = d\tan\phi \Rightarrow dy = d\sec^2\phi d\phi$$

$$= \frac{\mu_0 i}{4\pi}\int \frac{d\sec^2\phi . d\phi \sin\theta}{d^2\sec^2\phi}. \qquad \theta = (90 - \phi)$$

$$B = \frac{\mu_0}{4\pi}\frac{i}{d}\int_{\beta}^{\alpha} \frac{d.\sec^2\phi . d\phi . \sin(90^0 - \phi)}{d^2\sec^2\phi}$$

$$B = \frac{\mu_0 i}{4\pi d}\int_{-\beta}^{\alpha} \cos\phi \, d\phi$$

$$B = \frac{\mu_0 i}{4\pi d}(\sin\alpha + \sin\beta) \quad .... (2.10)$$

- So if the point is along the length of the wire (but not on it then as $d\bar{l}$ and $\bar{r}$ will be either parallel (or) antiparallel i.e. $\theta = 0$ (or) π so $d\bar{l} \times \bar{r} = 0$ and here $\bar{B} = \int_A^B d\bar{B} = 0$

$\Rightarrow$ if a point is at a perpendicular distance d from the wire field B varies inversely with distance d.

$$B \propto \frac{1}{d}$$

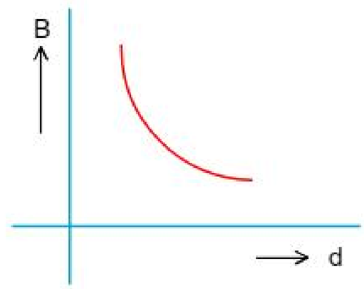


Fig 2.17

$\Rightarrow$ if the wire is of finite length 'L' and the point is on its perpendicular bisector, at a distance 'd' from the wire. $\alpha = \beta$

$$B = \frac{\mu_0}{4\pi}\frac{2i}{d}\sin\alpha \text{ with } \sin\alpha = \frac{L}{\sqrt{L^2 + 4d^2}}$$

$\Rightarrow$ if wire is of infinite length and the point P is on it's perpendicular bisector at a distance 'd' from the wire as shown in figure, $\alpha = \beta = \pi/2$

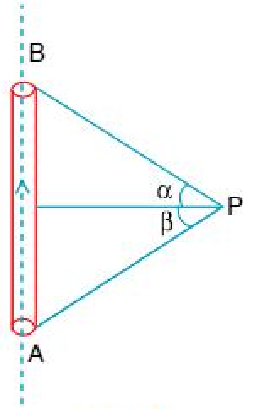


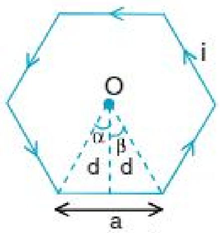
Fig 2.18

$$B=\frac{\mu_0}{4\pi}\frac{i}{d}(2)=\frac{\mu_0}{4\pi}\frac{2i}{d}=\frac{\mu_0 i}{2\pi d} \qquad .... (2.10)(a)$$

Example-2.9

A regular polygon of 'n sides is formed by bending a wire of length $2\pi R$ which carries a current i. Find the magnetic field at the centre of the polygon?

Solution :



One side of polygon $a=\frac{2\pi R}{n}$

$\alpha=\beta=\left(\frac{2\pi}{n}\right)/2=\frac{\pi}{n}$; $\cot\alpha=\frac{d}{a/2}$

$d=\frac{a}{2}\cot\alpha=\frac{\pi R}{n}\cot\left(\frac{\pi}{n}\right)$

All sides of polygon produce magnetic field at O in same direction,

so, $B=n(\text{magnetic field due to one side})$

$$=n\frac{\mu_0 i}{4\pi d}(\sin\alpha+\sin\beta)$$

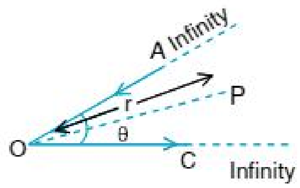
$$=n.\frac{\mu_0 i}{4\pi}\frac{n}{\pi R\cot\frac{\pi}{n}}\left(\sin\frac{\pi}{n}+\sin\frac{\pi}{n}\right)$$

$$=\frac{\mu_0 i}{4R}\frac{n^2}{\pi^2}\frac{2\sin\pi/n}{\cos\pi/n}.\sin\frac{\pi}{n}$$

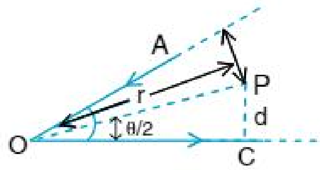
$$B=\frac{\mu_0 i}{2R}\times\frac{\left\{\frac{\sin\pi/n}{\pi/n}\right\}^2}{\cos\pi/n}$$

Example-2.10

Two wires AO and OC carry equal currents i as shown in the figure where $\underline{|AOC}=\theta$. Find the magnitude of the magnetic field at the point P on the bisector of angle θ at a distance 'r' from O. Assume that the other ends of both wires extend to infinity



Solution :



The direction of magnetic field due to both the wires AO and OC are out of page of paper

Due to wire OC

$$B=\frac{\mu_0 i}{4\pi d}\{\sin\phi_1+\sin\phi_2\}$$

$$B=\frac{\mu_0 i}{4\pi r\sin\frac{\theta}{2}}\left\{\sin\left(90-\frac{\theta}{2}\right)+\sin 90\right\}$$

$$B=\frac{\mu_0 i}{4\pi r\sin\frac{\theta}{2}}\left(1+\cos\frac{\theta}{2}\right)$$

Due to both the wires, the net magnetic field is

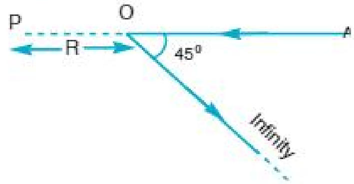
$$B=\frac{2\times\mu_0 i}{4\pi r\sin\frac{\theta}{2}}\left(1+\cos\frac{\theta}{2}\right)$$

$$B=\frac{\mu_0 i}{2\pi r\sin\frac{\theta}{2}}\left(1+\cos\frac{\theta}{2}\right)$$

$$B=\frac{\mu_0}{2\pi}\frac{i}{r}\cot\frac{\theta}{4}$$

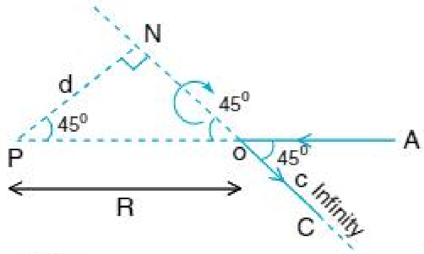
Example-2.11

An infinitely long wire carrying a current i is bent at its mid point O to form an angle 45°. P is a point at a distance R from the point of bending. Find the magnetic field at P.



Solution :

Since point P lies on the axis of straight part OA, magnetic field at point P is zero due to path OA of wire



For part OC

From $\Delta OPN, d = R\cos 45^0$

Since both the ends O and C are on the same side of normal PN,

$\phi_1 = -45^0$ and $\phi_2 = +90^0$

So $B = \dfrac{\mu_0 i}{4\pi d}(\sin\phi_1 + \sin\phi_2)$

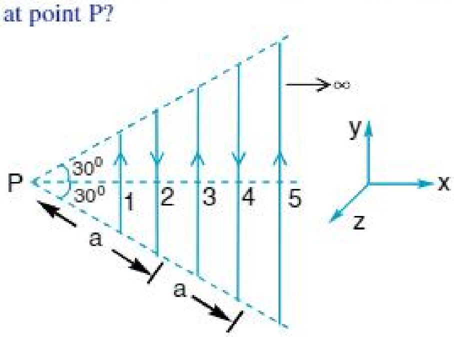
$$= \frac{\mu_0 i}{4\pi R\cos 45^0}\left[\sin(-45^0) + \sin 90^0\right]$$

$$= \frac{\mu_0 i \times \sqrt{2}}{4\pi R}\left(-\sin 45^0 + 1\right) ; = \frac{\sqrt{2}\mu_0 i}{4\pi R}\left(1 - \frac{1}{\sqrt{2}}\right) \otimes$$

$$= \frac{\left(\sqrt{2}-1\right)\mu_0 i}{4\pi R} \otimes \text{ (into the page)}$$

Example-2.12

Infinite number of straight wires each carrying current I are equally placed as shown in the figure. Adjacent wires have current in opposite direction. Find net magnetic field at point P?



Solution :

$$B_{net} = \frac{\mu_0 I}{4\pi}\left(\sin 30^0 + \sin 30^0\right)\hat{k}\left[\frac{1}{d} - \frac{1}{2d} + \frac{1}{3d} - \frac{1}{4d} .... + \infty\right]$$

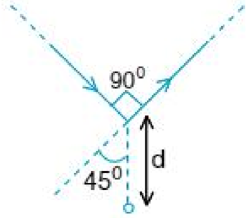
$$\left(\text{where } \quad d = a\cos 30 = \frac{\sqrt{3}a}{2}\right)$$

$$\therefore B_{net} = \frac{\mu_0 I}{2\sqrt{3}\pi a}\hat{k}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...\infty\right]$$

$$= \frac{\mu_0 I}{2\sqrt{3}\pi a}\ln 2\,\hat{k} = \frac{\mu_0 I}{4\pi}\frac{\ln 4}{\sqrt{3}a}\hat{k}$$

Example-2.13

Find the magnetic field at P due to the arrangement shown.



Solution :

$$B_{net} = 2\times\frac{\mu_0 I}{4\pi r}\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right)$$

$$\text{where } r = d/\sqrt{2} = \frac{\mu_0 I}{\sqrt{2}\pi d}\left(1 - \frac{1}{\sqrt{2}}\right)$$

Example-2.14

Equal currents each 'i' are flowing in three infinitely long wires along positive x, y and z axes. Find the magnetic field at the point (0, 0, –a).

Solution :

$\overline{B}$ due to current in x-direction $\vec{B}_x = \dfrac{\mu_0 i}{2\pi a}\hat{j}$

$\overline{B}$ due to current in y-direction

$$\overline{B}_y = \frac{\mu_0 i}{2\pi a}(-\hat{i})$$

$\overline{B}$ due to current in Z- direction is zero since the point is present on the line of the current carrying conductor. $B_Z = 0$.

$$\overline{B} = \overline{B}_x + \overline{B}_y + \overline{B}_z$$

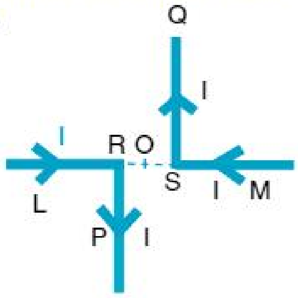
$$\overline{B}_{(0,0,-a)} = \frac{\mu_0 i}{2\pi a}\hat{j} - \frac{\mu_0 i}{2\pi a}\hat{i}$$

$$\left|\overline{B}_{(0,0,-a)}\right| = \frac{\mu_0 i}{2\pi a}\sqrt{2}$$

$$\left|\overline{B}\right| = \frac{\mu_0 i}{2\pi a}\sqrt{2}$$

Example-2.15

A pair of stationary and infinitely long bent wires are placed in the x-y plane as shown in Figure. The wires carry current of 10 ampere each as shown. The segment L and M are along the x-axis. The segment P and Q are parallel to the Y-axis such that OS=OR=0.02m. Find the magnitude and direction of the magnetic induction at the origin O.



Solution :

Since point O is along the length of segments L and M the field at O due to these two segments will be zero

$\therefore$ Magnetic field at O is due to QS and RP.

$$\therefore B_{SQ} = \frac{\mu_0}{4\pi} \times \frac{I}{OS} \odot = 10^{-7} \times \frac{10}{0.02} \odot$$

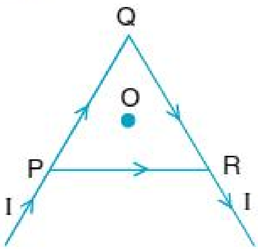
$$B_{RP} = \frac{\mu_0}{4\pi} \times \frac{I}{OR} \odot = 10^{-7} \times \frac{10}{0.02} \odot$$

$$\therefore B_0 = B_{SQ} + B_{RP} = 10^{-7} \times \frac{10}{0.02} \times 2 \odot$$

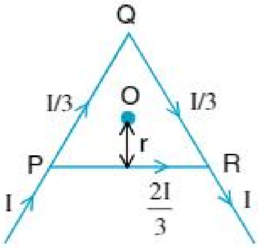
$$= 10^{-4} T \odot$$

Example-2.16

An equilateral triangle of side length l is formed from a piece of wire of uniform resistance. The current I is as shown in figure. Find the magnitude of the magnetic field at its center O.



Solution :



The magnetic field induction at O due to current through PR is

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I/3}{r} \left[\sin 60^0 + \sin 60^0 \right]$$

$$= \frac{\mu_0}{4\pi} \frac{2I}{3r} \odot \qquad \text{(directed outward)}$$

The magnetic field induction at O due to current through PQR is

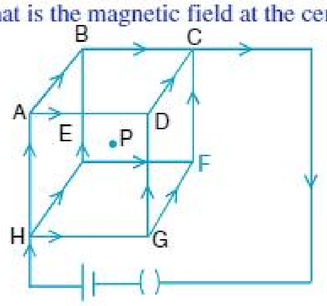
$$B_2 = 2 \times \frac{\mu_0 (I/3)}{4\pi r} \left[\sin 60^0 + \sin 60^0 \right]$$

$$= \frac{\mu_0 2I}{4\pi\, 3r} \otimes \ \text{(directed inward)}$$

$\therefore$ Resultant magnetic induction at O $\Rightarrow B_1 - B_2 = 0$.

Example-2.17

A steady current is set up in a network composed of wires of equal resistance and length 'd' as shown in Figure. What is the magnetic field at the centre P ?

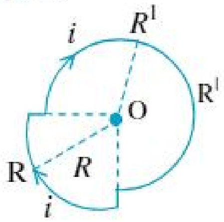


Solution :

By symmetry of the circuit, it is clear that for each current carrying wire there is another wire such that it cancels the field of the first. For e.g. the field at P due to wire AB is cancelled by field of wire GF. Field of wire CD is cancelled by wire HE. Similarly, AH and CF, DG and BE, BC and HG, AD and EF fields are cancelled and therefore net field is zero at 'P'.

Example-2.18

A current of i amperes is flowing through each of the bent wires as shown. Find the magnitude and direction of magnetic field at O.



Solution :

$$B(\theta) = \frac{\mu_0 I}{4\pi R}\theta \ ; \ B_{net} = \frac{\mu_0 I}{4\pi}\left[\frac{\pi/2}{R} + \frac{3\pi/2}{R^1}\right] = \frac{\mu_0 I}{8}\left[\frac{1}{R} + \frac{3}{R^1}\right]$$

(normally into the page)

Example-2.19

Two parallel conductors A and B separated by 5 cm carry electric current of 6A and 2 A in the same direction. Find the point between A and B where the field is zero.

Solution :

When the field is zero

$B_1 = B_2 \therefore \frac{\mu_0 i_1}{2\pi x} = \frac{\mu_0 i_2}{2\pi(d-x)}$

where 'd' is the distance between them

$\therefore \frac{i_1}{x} = \frac{i_2}{(d-x)}$

$\frac{6}{x} = \frac{2}{(5-x)} \Rightarrow 3(5-x) = x$ or $4x = 15$

$x = 3.75$ cm

$\therefore$ Field is zero at a distance of 3.75 cm from A

2.14 FORCE ON A MOVING CHARGE IN MAGNETIC INDUCTION B

When a charged particle having charge q travels with velocity $\bar{V}$ in magnetic field $\bar{B}$ it experiences a force F given by

$$\bar{F} = q(\bar{V} \times \bar{B}) \qquad .... (2.11)$$

i) If the charged particle is at rest in magnetic field (i.e V = 0) then it will experience no force.

And also if the motion of the charged particle is collinear with the field i.e. $\theta = 0^0$ or 180^0

$$|F| = qvB\sin\theta = 0 \quad (\sin\theta = \sin 180^0 = 0)$$

i.e., a moving charged particle does not experience any force in a magnetic field if its motion is parallel (or) anti-parallel to the field.

$\Rightarrow$ As the magnitude of the force experienced by a charged particle in a magnetic field

$F = qVB \sin\theta$

$\sin\theta = 1$ where $\theta = 90^0$

$\therefore$ Force will be maximum, $F_{Max} = qvB$

i.e., the particle is moving perpendicular to the field in this situation, all the three vectors, $\bar{F}, \bar{v}$ and $\bar{B}$ are mutually perpendicular to each other

$\Rightarrow$ In case of motion of charged particle in a magnetic field, as force is always perpendicular to the motion, i.e., displacement so the work done

$$w = \int \bar{F}.d\bar{s} = \int F\,ds\cos 90^0 = 0$$

i.e., work done on a charged particle in a magnetic field is always zero. And from work-energy theorem $w = \Delta(KE)$, the kinetic energy $l = \frac{1}{2}mv^2$ remains constant, but the direction of velocity will be changing

When the charged particle is moving perpendicular to the field

In this situation as the angle between $\bar{B}$ and $\bar{v}$ is $\theta = 90^0$, so the force will be maximum F = qvB and always perpendicular to the motion. So the path will be a circle (with its plane perpendicular to the field) as in a circle tangent and radius are always perpendicular to each other. Here centripetal force is provided by the force qVB.

$$\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB} \qquad .... (2.11a)$$

$$r = \frac{mv}{qB} = \frac{P}{qB} \Rightarrow r = \frac{\sqrt{2mK}}{qB} \qquad ..... (2.11b)$$

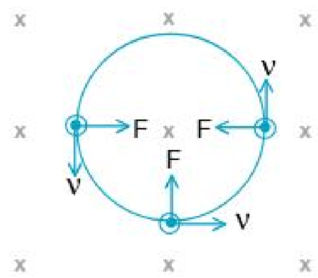


Fig 2.19

i.e., in case of circular motion of a charged particle in a given field $r \propto v \propto p \propto \sqrt{K}$ i.e., with increase in speed or kinetic energy the radius of the orbit increases

$\Rightarrow$ As in uniform circular motion $v = r\omega$. So the angular frequency of circular motion charged particle in is

$$\omega = \frac{v}{r} = \frac{qB}{m} \quad \text{.... (2.11c)}$$

$\therefore$ The time period of the charged particle is $T\frac{2\pi}{\omega} = \frac{2\pi m}{\omega}$.

$\therefore$ Time period (frequency) is independent of speed of particle and radius of the orbit and depends only on the field B and the nature i.e., specific charge (q/m) of the particle.

When the charged particle is moving at an angle to the field (other than 0^0, 90^0 (or) 180^0)

In this situation resolving the velocity of the particle along and perpendicular to the field, we find that the particle moves with constant velocity $v\cos\theta$ along the field (as no force acts on a charged particle when it moves parallel to the field) and at the same time it is also moving with velocity $v\sin\theta$ perpendicular to the field due to which it will describe a circle (in a plane perpendicular to the field) of radius

$$r = \frac{m(v\ \sin\theta)}{qB}$$

and time period $T = \frac{2\pi r}{v\ \sin\theta} = \frac{2\pi m}{qB}$ ....(2.11d)

So the resultant path will be a helix with its axis parallel to the field $\overline{B}$ shown in figure.

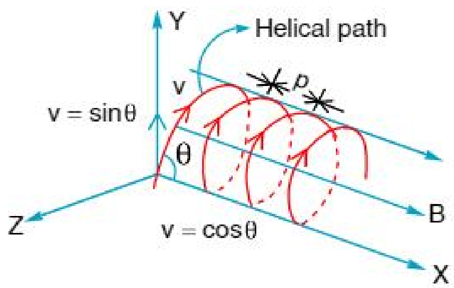


Fig 2.20

1) The projection of the path in to y-z plane will be a circle while along x-y plane (or) x-z plane will be sinusoidal.

2) The pitch of the helix ($\therefore$ i.e., linear distance travelled in one rotation) will be given by

$$P = T(v\ \cos\theta) = \frac{2\pi m(v\ \cos\theta)}{qB} \quad \text{.....(2.11e)}$$

2.15 MOTION OF CHARGED PARTICLE IN COMBINED ELECTRIC AND MAGNETIC FIELDS

When the moving charged particle is subjected simultaneously to both electric field $\overline{E}$ and magnetic field $\overline{B}$, the moving charged particle will experience electric force $\overline{F}_e = q\overline{E}$ and magnetic force $\overline{F}_m = q(\overline{v} \times \overline{B})$. So the net force on it will be $\overline{F} = q\left[\overline{E} + (\overline{v} \times \overline{B})\right]$, which is the famous 'Lorentz-force equation'. Depending on the direction of $\overline{v}, \overline{E}$ and $\overline{B}$, various situations are possible and the motion in general is quite complex. Here we consider the following cases.

Case : I

When $\overline{v}, \overline{E}$ and $\overline{B}$ are collinear :

In this situation as the particle is moving parallel or antiparallel to the fields, the magnetic force on it will be zero and only electric force will act and so

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} \quad \text{.... (2.11f)}$$

Hence the particle will pass through the field following a straight line path (parallel to the field) with change in its speed, velocity, momentum and kinetic energy. Magnitudes of all these quantities will change without change in direction of motion as shown in figure

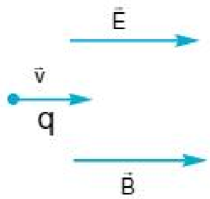


Fig 2.21(a)

Case : II (Velocity selector)

$\vec{v}, \vec{E}, \vec{B}$ are mutually perpendicular :

In this situation if $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0$$

$$\text{i.e} = \vec{a} = \vec{F}/m = \vec{0}$$

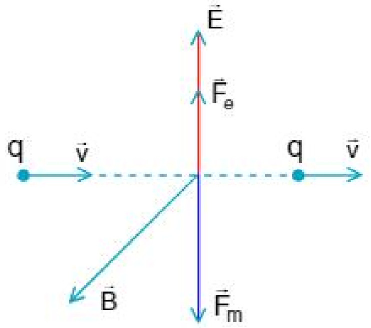


Fig 2.21(b)

as shown in figure the particle will pass through the field with same velocity in the same direction. In this situation as

$$F_e = F_m \quad \text{i.e.} = qE = qvB$$

$$v = E/B \qquad \text{.... (2.11g)}$$

This principle is used in "Velocity selector" to get a charged beam having a specific velocity.

This principle is also used in J.J.Thamson experiment for determination of specific charges.

Case : III

When $\vec{E} \parallel \vec{B}$ and charged particle velocity is perpendicular to both of these fields :

Consider a particle of positive charge "q" and mass "m" is released from the origin with velocity $\vec{v} = v_0\hat{i}$ into a region of uniform electric and magnetic fields parallel to –ve z axis $\vec{E} = -E_0\hat{k}$ and $\vec{B} = -B_0\hat{k}$.

The electric field accelerates the particle in -ve Z - direction. -ve Z component of velocity goes on increasing with acceleration of $\frac{qE_0}{m}$ along -ve z-axis.

The magnetic field rotates the charged particle in a circle in x-y plane with angular frequency $\omega = \frac{Bq}{m}$

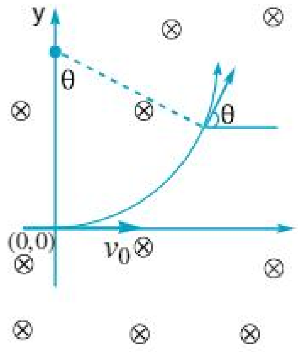


Fig 2.21(c)

Hence the nature of the path is helix with increasing pitch (or Non-uniform pitch) and the axis of the plane is parallel to -ve Z-axis .

After "t" sec. from the projection in mixed field, it's velocity vector is

$$\vec{v}_t = v_0 \cos\left(\frac{B_0 q}{m}t\right)\hat{i} + v_0 \sin\left(\frac{B_0 qt}{m}\right)\hat{j} - \frac{E_0 qt}{m}\hat{k} \text{ ...(2.11h)}$$

- i) While moving in helical path the particle touches the -Z-axis after every "T" sec

$$T = \frac{2\pi m}{B_0 q}$$

- ii) t = 0, velocity is along positive x-axis and magnetic field is along -ve Z-axis. Therefore - magnetic force along positive y-axis and the particle rotates is x-y plane.

Case : IV

When $\vec{E} \perp \vec{B}$ and the charged particle is released at rest from origin :

Consider a particle of positive charge q and mass m released from the origin with zero initial velocity into a region of uniform electric and magnetic fields. The field $\vec{E}$ is acting along x-axis and field $\vec{B}$ along y-axis. $\vec{E} = E_0 i$ and $\vec{B} = B_0\hat{J}$

Electric field will provide the particle an acceleration (and therefore a velocity component) in x-direction and the magnetic field will rotate the particle is x-z plane. (perpendicular to $\vec{B}$). Hence, at any instant of time of its velocity will have only x and z components.

$$\vec{v} = v_x\hat{i} + v_z\hat{k}$$

$$\vec{F}_q + \vec{F}_m = q\vec{E} + q\left(\vec{v}\times\vec{B}\right)$$

$$\vec{a} = \frac{\vec{F}}{m} = a_x\hat{i} + a_z\hat{k} = \frac{qE_0\hat{i} + q\left(v_x\hat{i} + v_z\hat{k}\right)\times B_0\hat{j}}{m}$$

$$a_x\hat{i} + a_z\hat{k} = \frac{q\left(E_0 - v_zB_0\right)}{m}\hat{i} + \frac{qv_xB_0\hat{k}}{m}$$

$$\frac{dv_x}{dt} = a_x = \frac{q}{m}\left(E_0 - V_ZB_0\right) \qquad \text{.... (2.11i)}$$

$$\frac{dv_z}{dt} = a_z = \frac{q}{m}v_xB_0 \qquad \text{....(2.11j)}$$

Differentiating Eq (2.11i) w.r.t time ,

$$\frac{d^2v_x}{dt^2} = \frac{-Bq_0}{m}\left(\frac{dv_z}{dt}\right) = -\left(\frac{B_0q}{m}\right)^2 v_x$$

$$\left[\sin ce\frac{dv_z}{dt} = \frac{B_0qx}{m}\right]$$

$$\frac{d^2v_x}{dt^2} + \left(\frac{B_0q}{m}\right)^2 v_x = 0 \qquad \text{.... (2.11k)}$$

Hence v_x is in S.H.M.

with angular frequency $\omega = \left(\frac{B_0q}{m}\right)$ with initial velocity zero.

$$v_x = A\sin\omega t.$$

$$\frac{dv_x}{dt} = A\omega\cos\omega t$$

$$\text{At } t = 0 \; v_x = 0 \; V_z = 0$$

$$\frac{dv_x}{dt} = \frac{E_0q}{m} = \frac{A.B_0q}{m}\cos\omega t$$

$$A = \frac{E_0}{B_0}$$

$$V_x = \frac{E_0}{B_0}\sin\frac{B_0q.t}{m} \qquad \text{.... (2.11}l\text{)}$$

Substituting value of V_X is Eq (2.11j) we get

$$\frac{dv_z}{dt} = \frac{q\cancel{B_0}}{m}\times\frac{E_0}{\cancel{B_0}}\sin\left(\frac{B_0qt}{m}\right)$$

$$dv_z = \frac{E_0q}{m}\sin\frac{B_0q}{m}t.dt$$

Integrating on both sides.

$$\int_0^{v_z} dv_z = \int_0^t \frac{E_0q}{m}\sin\frac{B_0qt}{m}.dt = \int_0^t \frac{E_0q}{m}\sin\omega t.dt$$

$$v_z = \frac{E_0q}{mw}\left[-\cos\omega t\right]_0^t$$

$$v_z = \frac{E_0q}{m\omega}\left(1 - \cos\omega t\right) \qquad \text{..... (2.11m)}$$

where $\omega = \frac{B_0q}{m}$ and $V_x = \frac{E_0}{B_0}\sin\omega t$

These equations are the equations for a cycloid. which is defined as the path generated by the point on the circumference of a wheel rolling on a ground.

Example-2.20

A magnetic field of $(4.0\times10^{-3}\hat{k})T$ exerts a force $(4.0\hat{i} + 3.0\hat{j})\times10^{-10}N$ on a particle having a charge 10^{-9} C and moving in the x-y plane. Find the velocity of the particle.

Solution :

Given Magnetic force

$$\vec{F}_m = (4.0\hat{i} + 3.0\hat{j})\times10^{-10}N$$

Let velocity of the particle in x-y plane be,

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

Then from the relation $\vec{F}_m = q\left(\vec{v}\times\vec{B}\right)$

we have $(4.0\hat{i} + 3.0\hat{j})\times10^{-10} =$

$$10^{-9}\left[\left(v_x\hat{i} + v_y\hat{j}\right)\times\left(4\times10^{-3}\hat{k}\right)\right]$$

$$= \left(4v_y\times10^{-12}\hat{i} - 4v_x\times10^{-12}\hat{j}\right)$$

comparing the coefficient of $\hat{i}$ and $\hat{j}$ we have,

$$4\times10^{-10} = 4v_y \times 10^{-12}$$

$\therefore\ v_y = 10^2\,m/s = 100\,m/s$

and $3.0\times10^{-10} = -4v_x \times 10^{-12}$

$\therefore\ v_x = -75\ m/s$

$\therefore\ \vec{v} = -75\hat{i} + 100\hat{j}$

Example-2.21

A charged particle carrying charge $q = 1\mu C$ moves in uniform magnetic field with velocity $v_1 = 10^6$ m/s at angle 45^0 with x-axis in x–y plane and experiences a force $F_1 = 5\sqrt{2}\,mN$ along the negative z-axis. When the same particle moves with velocity $v_2 = 10^6$ m/s along the z-axis it experiences a force F_2 in y-direction. Find

a) the magnitude and direction of the magnetic field

b) the magnitude of the force F_2.

Solution :

F_2 is in y-direction when velocity is along z-axis. Therefore, magnetic field should be along x-axis.

So let, $\vec{B} = B_0\hat{i}$

a) Given $\vec{V}_1 = \dfrac{10^6}{\sqrt{2}}\hat{i} + \dfrac{10^6}{\sqrt{2}}\hat{j}$

and $\vec{F}_1 = -5\sqrt{2}\times10^{-3}\hat{k}$

From the equation, $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$

we have $\left(-5\sqrt{2}\times10^{-3}\right)\hat{k} =$

$$(10^{-6})\left[\left(\frac{10^6}{\sqrt{2}}\hat{i} + \frac{10^6}{\sqrt{2}}\hat{j}\right)\times\left(B_0\hat{i}\right)\right] = -\frac{B_0}{\sqrt{2}}\hat{k}$$

$\therefore\ \dfrac{B_0}{\sqrt{2}} = 5\sqrt{2}\times10^{-3}$ or $B_0 = 10^{-2}\,T$

Therefore, the magnetic field is, $\vec{B} = \left(10^{-2}\hat{i}\right)T$

b) $F_2 = B_0 q v_2 \sin 90^0$

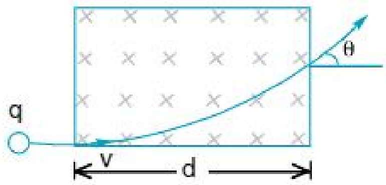
as the angle between $\vec{B}$ and $\vec{v}$ in this case is 90^0.

$\therefore\ F_2 = (10^{-2})(10^{-6})(10^6) = 10^{-2}\,N$

Example-2.22

A particle of mass m and charge q is projected into a region having a perpendicular magnetic field B. Find the angle of deviation of the particle as it comes out of the magnetic field if the width d of the region is slightly less than

a) $\dfrac{mv}{qB}$ b) $\dfrac{mv}{2qB}$ c) $\dfrac{2mv}{qB}$



Solution :

The radius of path $r = \dfrac{mv}{qB}$

For $d \le r$, we have $\sin\theta = \dfrac{d}{r}$

a) $d = \left(\dfrac{mv}{qB}\right) = r$, $\therefore \sin\theta = \dfrac{\left(\dfrac{mv}{qB}\right)}{\left(\dfrac{mv}{qB}\right)} = 1$ or $\theta = \dfrac{\pi}{2}$ rad

b) $d = \left(\dfrac{mv}{2qB}\right) < r$, $\sin\theta = \dfrac{d}{r} = \dfrac{\dfrac{mv}{2qB}}{\dfrac{mv}{qB}} = \dfrac{1}{2}$ or $\theta = \dfrac{\pi}{6}$ rad

c) $d = \left(\dfrac{2mv}{qB}\right) > r$, the deviation of particle is therefore $\theta = \pi$ rad.

2.16 FORCE ON A CURRENT CARRYING WIRE IN A MAGNETIC FIELD

Consider a conducting wire carrying a current i, placed in a magnetic field $\vec{B}$. Consider a small element dl of the wire. The free electron drift with a speed v_d opposite to the direction of the current.

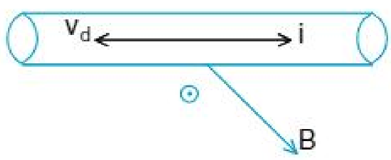


Fig 2.22(a)

The relation between the current i and the drift speed v_d is

$$i = jA = nev_dA \qquad ..... (2.12)$$

Here A is the area of cross-section of the wire and n is the number of free electron is per unit volume. Each electron experiences an average magnetic force

$$\vec{f} = -e\vec{v}_d \times \vec{B}$$

The number of free electrons in the small element considered is nAdl. Thus the magnetic force on the wire of length dl is

$$d\bar{F} = (nAdl)(-e\bar{v}_d \times \bar{B})$$

if we denote the length dl along the direction of the current by $d\bar{l}$, then above equation becomes $d\vec{F} = nAev_d \; d\vec{l} \times \vec{B}$

Using (1), $d\vec{F} = i.d\vec{l} \times \vec{B}$

The quantities $i.d\vec{l}$ is called a current element. If a straight wire of length l carrying a current i is placed in a uniform magnetic field $\bar{B}$ the force on it is

$$\vec{F} = i\vec{l} \times \vec{B} = i(\vec{l} \times \vec{B})$$

$$\bar{F} = ilB\sin\theta \qquad \text{.... (2.12(a))}$$

i) Force on a current element will be minimum when $\sin\theta = \min = 0$ i.e., $\theta = 0^0$ (or) 180^0 i.e., a current element in a magnetic field does not experience any force if the current in it is collinear with the field as in figure.

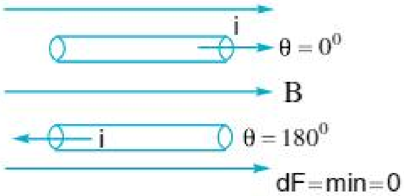


Fig 2.22(b)

ii) The force on the current element will be maximum (Bid$l$) when $\sin\theta = \max = 1$ i.e. $\theta = 90^0$ i.e., force on a current element in a magnetic field is maximum (= Bid$l$) when it is perpendicualr to the field (fig.)

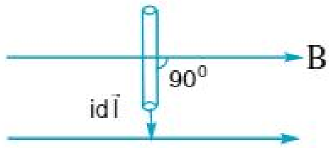


Fig 2.22(c)

$\theta = 90^0$

$dF = \max = Bidl$

The direction of force is always perpendicular to the plane containing $I.d\bar{l}$ and $\bar{B}$ and is same as that of cross product of two vectors $(\bar{A} \times \bar{B})$ with $\bar{A} = id\bar{l}$

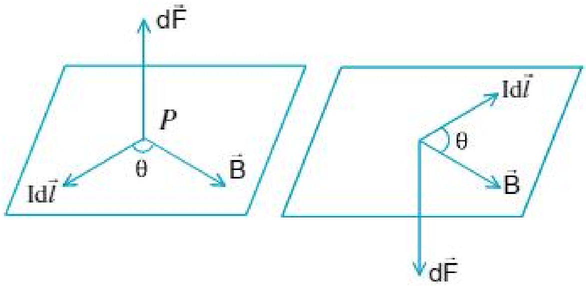
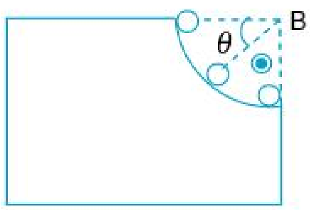


Fig 2.22 (d)

The direction of force when current element $id\vec{l}$ and $\vec{B}$ are perpendicular to each other can also be determined by applying either of the following rules:

Example-2.23

A charged sphere of mass m and charge q starts sliding from rest on a circular track of radius R from the position shown in the figure. There exists a uniform and constant horizontal magnetic field of induction B. Find the maximum force exerted by the track on the sphere.



Solution :

$F_m = qVB$ (directed radially outward)

$$N - mg\sin\theta - qvB = \frac{mV^2}{R}$$

$$N = \frac{mv^2}{R} + mg\sin\theta + qvB$$

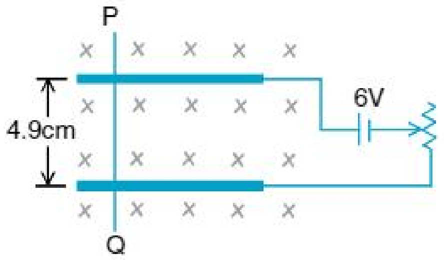
At $\theta = \frac{\pi}{2}$, N is maximum.

$$N = \frac{m}{R}(2gR) + mg + qB\sqrt{2gR}$$

$$N = 3mg + qB\sqrt{2gR}$$

Example-2.24

A wire PQ of mass 10g is at rest on two parallel horizontal metal rails. The separation between the rails is 4.9cm. A magnetic field of 0.80 tesla is applied perpendicular to the plane of the rails, directed inwards. The resistance of the circuit is slowly decreased. When the resistance decreases to below 20 ohm, the wire PQ begins to slide on the rails. Calculate the coefficient of friction between the wire and the rails.



Solution :

Wire PQ begins to slide when magnetic force is just equal to the force of friction, i.e.,

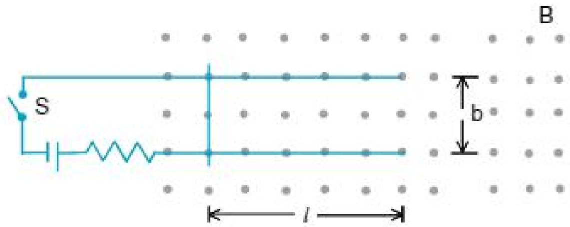
$$\mu\ mg = il\ B\sin\theta \quad (\theta = 90^0)$$

Here, $i = \frac{E}{R} = \frac{6}{20} = 0.3A$;

$$\mu = \frac{ilB}{mg} = \frac{(0.3)(4.9\times10^{-2})(0.8)}{(10\times10^{-3})(9.8)} = 0.12$$

Example-2.25

Two metal strips, each of length *l*, are clamped parallel to each other on a horizontal floor with a separation b between them. A wire of mass m lies on them perpendicular as shown in figure. A vertically upward magnetic field of strength B exists in the space. The metal strips are smooth but the coefficient of friction between wire and the floor is μ. A current i is established when the switch S is closed at the instant t = 0. Discuss the motion of the wire after the switch is closed. How far away from the strips will the wire reach?



Solution :

When current starts flowing in the wire, it experiences a force, F = Bib of constant magnitude due to which it accelerates on the metal strips. Thereafter when wire falls on the floor, it retards due to friction and finally stops. Thus

$$a = \frac{F}{m} = \frac{Bib}{m}$$

The velocity gained by the wire on the strips

$$v^2 = 0 + 2al = 2\frac{Bib\ell}{m}$$

Let x be the distance moved by the wire on the floor. Its final velocity becomes zero, and so

$$0 = v^2 - 2a'x \quad \text{or} \quad x = \frac{v^2}{2a'}$$

Here $a' = \frac{\mu mg}{m} = \mu g$

$$\therefore\ x = \frac{\left[\frac{2Bib\ell}{m}\right]}{2\mu g} = \frac{Bib\ell}{\mu mg}.$$

Note : Try using work-energy theorem

2.17 FLEMING'S LEFT HAND RULE

This is used to find the direction of force on a current carrying conductor when placed in a uniform magnetic induction field.

Statement :

If the fore finger, centre finger and thumb of left hand are stretched mutually perpendicular to each other such that the fore-finger points in the direction of magnetic field and the centre finger in the direction of current, then the thumb points in the direction of force experienced by the conductor.

- Fleming's left hand rule holds only if the magnetic field is uniform over the whole length of the conductor.

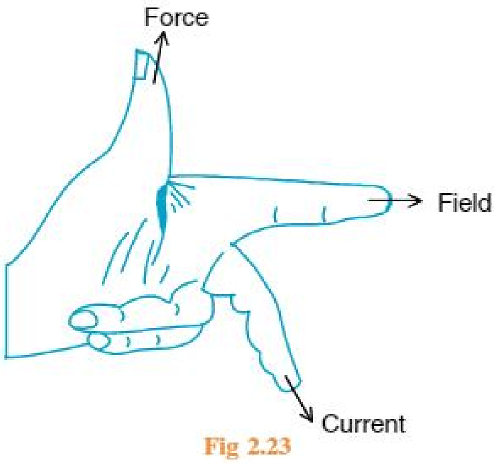


Fig 2.23

Fore figure - Field direction

Centre finger - current direction

Thumb - Thrust (force) direction

- Stretch the first three fingers and thumb of right hand at right angles to each other, then if the fingers point in the direction of field $\overline{B}$ and thumb in the direction of current I, the normal to plane will point in the direction of force.

Regarding the force on a current-carrying conductor in a magnetic field

i) The magnetic force on a current element is non-central because the force $iBdl\sin\theta$ is not function of position r.

ii) $\overline{F}$ is always perpendicular to both $\overline{B}$ and $i\overline{dl}$

iii) If the current-carrying conductor in the form of loop of any arbitrary shape is placed is a uniform magnetic field.

$$\vec{F}=\oint i d\vec{l}\times\vec{B}=I\left[\oint d\vec{l}\right]\times\vec{B}$$

and as for a closed loop the vector sum of $d\vec{l}$ is always zero $\overline{F}=0$, i.e., the net magnetic force on a current loop in a uniform magnetic field is always zero.

2.18 FORCE BETWEEN TWO STRAIGHT PARALLEL CONDUCTORS CARRYING CURRENTS

Consider two infinitely long parallel conductors X and Y carrying currents i_1 and i_2 in same direction as shown in the figure.

The two conductors are separated by a distance r in the plane of the paper.

The magnitude of magnetic Induction field at any point P on the conductor Y due to current i_1 in conductor X is $B_X=\frac{\mu_0}{2\pi}\frac{i_1}{r}$

From right hand grip rule or Maxwell's cork screw rule, the direction of B_X is perpendicular to the plane of the paper and is directed inwards

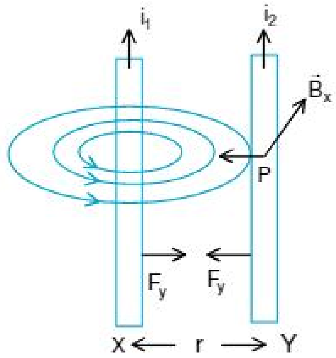


Fig 2.24

Now conductor Y carrying current i_2 is placed in the magnetic induction field B_X produced by conductor X.

Force on length l of conductor Y is given by

$$F_y=B_x i_2 l\sin\theta=B_x i_2 l\ (\theta=90^0)$$

Now $F_y=\frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}l$

so force per unit length is $\frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$

According to Fleming's left hand rule, the force F_y will be in the plane of the paper and it is directed towards the conductor X as shown similarly the force on conductor X for length l is

$$F_x=\frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}l$$

This force will be in the plane of the paper and is directed towards conductor Y. So it is clear that the two conductors attract each other and the force of attraction per unit length would be

$$\frac{F}{l}=\frac{\mu_0}{2\pi}\frac{i_1 i_2}{r} \qquad \text{.... (2.13)}$$

When a current in the parallel conductors are in opposite directions they repel each other and the force would be the same as given in equation. So if current through two parallel conductors are in the same direction they attract each other; if the currents are in opposite directions the conductors repel each other.

Definition of Ampere

$$F=\frac{\mu_0}{2\pi}\frac{i_1 i_2}{r} \qquad i_1=i_2=1\text{A}$$

$$r=1\text{m},\ \mu_0=4\pi\times10^{-7}\text{H/m}$$

$$F=2\times10^{-7}\text{N/m}$$

Ampere is that current which when flowing through each of two parallel conductors of infinite length and placed in free space (or vacuum) at a distance of one metre from each other between them will have a force of 2×10^{-7} newton per meter length of each conductor.

Example-2.26

A long straight conductor carrying a current of 2 A is in parallel to another conductor of length 5 cm and carrying a current 3A. They are separated by a distance of 10 cm. Calculate

a) B due to first conductor at second conductor

b) the force on the short conductor.

Solution :

Given, $i_1 = 2$ A; $i_2 = 3$ A ; $r = 10$ cm $= 10\times10^{-2}$m; $l_2 = 5$ cm

a) $B=\frac{\mu_0 i_1}{2\pi r}=2\times10^{-7}\times\frac{2}{10\times10^{-2}}=4\times10^{-6}$ Tesla

b) $F=\frac{\mu_0 i_1 i_2}{2\pi r}\times l_2=2\times10^{-7}\times\frac{2\times3}{10\times10^{-2}}\times5\times10^{-2}$

$=6\times10^{-7}$ N

Example-2.27

Two parallel horizontal conductors are suspended by light vertical threads 75.0 cm long. Each conductor has a mass of 40.0 gm per metre, and when there is no current they are 0.5 cm apart. Equal magnitude current in the two wires result in a separation of 1.5cm. Find the values and directions of currents.

Solution :

The situation is shown in figure.

Here, we have

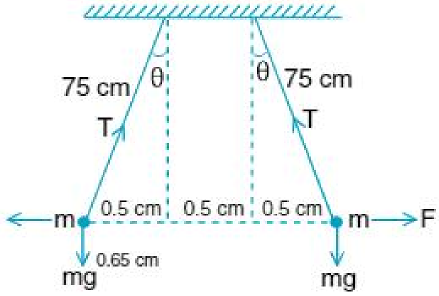
$T\cos\theta = mg$ .... (i)

$T\sin\theta = F = \frac{\mu_0}{4\pi}.\ell.\frac{2i_1i_2}{d}$ or

$T\sin\theta = \frac{\mu_0}{4\pi}.\ell.\frac{2i^2}{d}$ .... (ii)

from eqs. (i) and (ii)

$\tan\theta = \frac{\mu_0}{4\pi}.\ell.\frac{2i^2}{d}.\frac{1}{mg}$ .... (iii)



where θ is small, $\tan\theta \simeq \sin\theta$

From figure $\sin\theta = \frac{0.5\times10^{-2}}{75\times10^{-2}}$

$m = 40.0\times10^{-3}\ \ell$ kg

where l = length of conductor in meter

Substituting in eq. (iii) , we get

$$\frac{0.5\times10^{-2}}{75\times10^{-2}}=10^{-7}.\ell.\frac{2i^2}{(1.5\times10^{-2})}\times\frac{1}{(40\times10^{-3})\ell\times9.8}$$

Solving, we get i = 14 amp.

For repulsion, the currents are in opposite direction.

Example-2.28

A square frame carrying a current I = 0.90A is located in the same plane as a long straight wire carrying a current I_0 = 5.0A. The frame side has a length a = 8.0 cm. The axis of the frame passing through the mid-points of the opposite sides is parallel to the wire and is separated from it by the distance which is $\eta = 15$ times greater than the side of the frame. Find:

a) Ampere force acting on the frame;

b) the mechanical work to be performed in order to turn the frame through 180^0 about its axis, with the currents maintained constant.

Solution :

a) Force of attraction between parallel currents

$$F_1 = \frac{\mu_0}{4\pi}\frac{2I\,I_0}{(2\eta-1)\frac{a}{2}}$$

Similar force of repulsion between antiparallel currents

$$F_2 = \frac{\mu_0}{4\pi}\frac{4I\,I_0}{(2\eta+1)a}$$

Net force of repulsion between the square frame and the long straight wire

$$F = F_1 - F_2 = \frac{2\mu_0}{\pi a}\frac{I\,I_0}{4\eta^2-1}$$

Putting values, $F = 0.05\mu N$.

b) Work performed in turning the frame through 180^0

$$W = \int_0^{\pi}\tau\,d\theta,\ \text{But } d\tau = \overline{dM}\times\overline{B}(r)$$

$$dM = IdA - = 2a\,dr\ n\ B(r) = \frac{\mu_0}{4\pi}\frac{2I_0}{r}n.$$

n = unit normal to the frame into the paper

$\therefore d\tau = dM\ B\sin\theta$

$\theta =$ angle between B(r) and dA during the rotation process.

$$\therefore \tau = \int d\tau\frac{\mu_0}{4\pi}.2I\ I_0\sin\theta\int_{\eta a-a/2}^{\eta a+a/2}(dr/r)$$

$$= \frac{\mu_0}{4\pi}2I\ I_0 a\sin\theta\log_e\frac{2\eta+1}{2\eta-1}\ \therefore \text{Workdone}$$

$$W = \int_0^{\pi}\tau\,d\theta = \frac{\mu_0}{4\pi}2I\,I_0 a\ \log\frac{2\eta+1}{2\eta-1}\int_0^{\pi}\sin\theta\ d\theta$$

$$= \frac{\mu_0}{\pi}I\,I_0 a\ \log\frac{2\eta+1}{2\eta-1} = 0.144\,\mu J$$

Example-2.29

Two horizontal wires PQ and RS of resistance 10Ω and 20Ω are separated by a distance of 10 cm and connected in parallel in a vertical plane across a cell of emf 200V and negligible internal resistance. A wire AB of mass 1g and length 1 cm is balanced exactly, midway between them. What must be the current in it ?

Solution :

AB experience a force of attraction force PQ and RS.

Apply $V = iR$

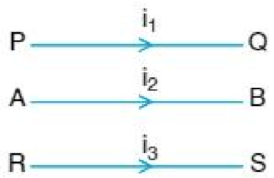
$200 = i_1\times 10 \Rightarrow i_1 = 20A$

$200 = i_2\times 20 \Rightarrow i_1 = 10A$

AB will be in equilibrium if

$$\frac{\mu_0}{2\pi a}i_2 i_3\ell + mg = \frac{\mu_0 i_1 i_3}{2\pi a}$$

$$\frac{\mu_0}{2\pi a}\times(i_1 i_3 - i_2 i_3)\ell = mg$$



$$\frac{4\pi\times10^{-7}}{2\pi\times5\times10^{-2}}(20i_3 - 10i_3)\times\frac{1}{100} = 10^{-3}\times9.8$$

$$\frac{2\times10^{-5}\times10i_3}{5} = 9.8\times10^{-3}$$

$i_3 = 245\,A$.

Example-2.30

A conductor AB of length 10 cm is at a distance of 10 cm from an infinitely long parallel conductor carrying 10A. What work must be done to move AB to a distance of 20cm if it carries 5A?

Solution :

Force on a conductor at a distance $x = \frac{\mu_0 i_1 i_2\ell}{2\pi x}$

Work done to displace it through a small distance dr = force × direction

$$\Rightarrow dw = \frac{\mu_0 i_1 i_2\ell}{2\pi r}dr$$

$$w = \int_{0.1}^{0.2}\frac{\mu_0 i_1 i_2\ell}{2\pi r}dr\,;$$

$$w = \frac{\mu_0 i_1 i_2 l}{2\pi}\left[\log_e x\right]_{0.1}^{0.2}$$

$$w = \frac{4\pi\times10^{-7}\times10\times5\times10\times10^{-2}}{2\pi}\log_e 2$$

$w = 0.693\times10^{-6}\,J$.

Example-2.31

Two conductors each of length 12 m lie parallel to each other in air. The centre to center distance between the conductors is 15×10^{-2} m and the current in each conductor is 300 amperes. Determine the force in newton tending to pull the conductors together.

Solution :

Force between two conductors is given by

$$F = \frac{\mu_0}{2\pi}\frac{i_1 i_2 \ell}{r}$$

Given ℓ = 12 m, r = 15×10^{-2}m.

$i_1 = i_2 = 300$A

$$\therefore F = 2\times10^{-7}\times\frac{300\times300\times12}{15\times10^{-2}} = 1.44\text{N}.$$

2.19 MAGNETIC MOMENT OF CURRENT LOOP

According to magnetic effects of current in case of a current carrying coil for a distance 'x' on it's axial point

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{\left(x^2+R^2\right)^{3/2}}\hat{n} = \frac{\mu_0}{4\pi}\frac{2\pi NiR^2}{x^3}\hat{n} \quad (x >> R)$$

If we compare this result with the field due to a small bar magnet for a distance 'x' on its axial point

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{x^3}$$

we find that a current-carrying coil for a distant point behave as a magnetic dipole of moment

$$\bar{M} = I\bar{A}N, \text{ with } \bar{A} = \pi R^2 N\hat{n}$$

So the magnetic moment of a current carrying coil is depend as the produced of current in the coil with the area of coil in vector form.

1) Magnetic moment of a current loop is a vector perpendicular to the plane of the loop as shown in figure with dimension (IL^2) and unit A - m^2

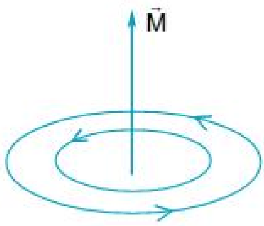


Fig 2.25

2) It depends on the current in the loop and its area but is independent of the shape of the loop (i.e,.) circular (or) rectangular etc.

3) In case of charged particle having charge q and moving in a circle of radius R with velocity $\bar{V}$ is

$$I = qf = q\frac{v}{2\pi R}$$

$$\text{and } \bar{A} = \pi R^2\hat{n} \Rightarrow \bar{M} = I\bar{A} = \frac{1}{2}qvR\hat{n}$$

But as angular momentum of the charged particle in this situation is $\vec{L} = mvR\vec{n}$

$$\vec{M} = \frac{q}{2m}\vec{L} \qquad .... (2.14)$$

i.e., the magnetic moment of a charged particle moving in a circle is (q/2m) times of its angular momentum and is directed opposite to $\vec{L}$ if the charge is negative.

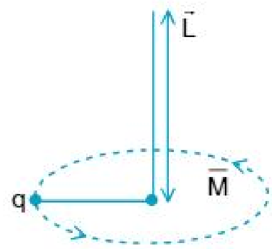


Fig 2.26

2.20 TORQUE ON CURRENT LOOP IN A MAGNETIC FIELD

If electric current flows in a closed wire loop placed in a uniform magnetic field, the forces acting on the loop produces a torque which tends to rotate the loop such that the plane of the loop is perpendicular to the direction of the magnetic field.

Consider a rectangular wire loop PQRS of length l and breadth b. The loop is suspended in a uniform field of magnetic induction field B.

The plane of the loop makes an angle θ with the direction of the magnetic field. Let i be the current flowing through the loop.

There are four forces $\vec{F_1}, \vec{F_2}, \vec{F_3}$ and $\vec{F_4}$ acting on the four arms PS, QR, PQ and RS of the loop respectively as shown in the figure.

Force on PQ is $F_3 = Bib \sin\theta$

Force on RS is $F_4 = Bib \sin\theta$

By fleming's left hand rule, force F_3 acts vertically upwards where as F_4 acts vertically downwards. These two forces are equal in magnitude and act in opposite directions. Hence they cancel each other.

The vertical sides PS and QR of the loop are always perpendicular to the direction of magnetic field irrespective of the position of the loop.

So, force on PS is $F_1 = Bil \sin\theta = Bi\ell$ and

force on QR is $F_2 = Bil \sin 90^0 = Bi\ell$

By fleming's left hand rule, the direction of F_1 is perpendicular to the plane of the loop and is directed outwards. Where as F_2 is also perpendicular to the plane of loop but directed inwards as shown.

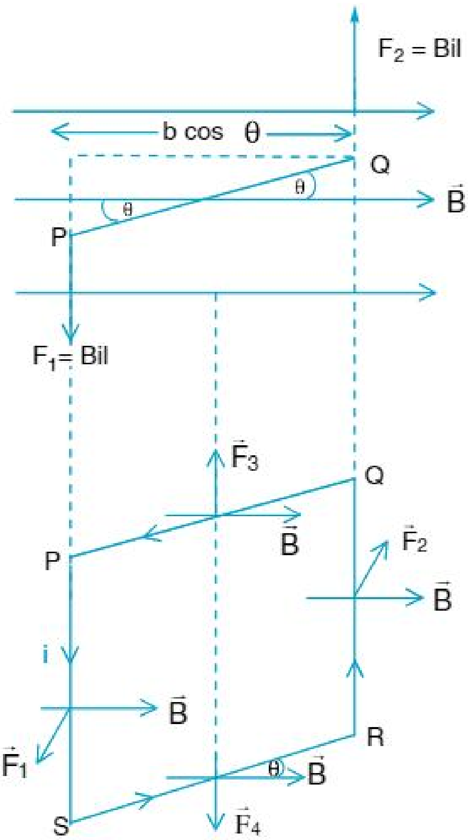


Fig 2.27

Here F_1 and F_2 are equal in magnitude, opposite in direction and have different lines of action.

Therefore they constitute a couple which tends to rotate the loop. (in the above case it is in the anti clockwise direction about the vertical axis)

The moment of the couple or torque is equal to the product of one of the forces and perpendicular distance between the forces.

i.e., torque = Either force × Perpendicular distance between the lines of action of forces

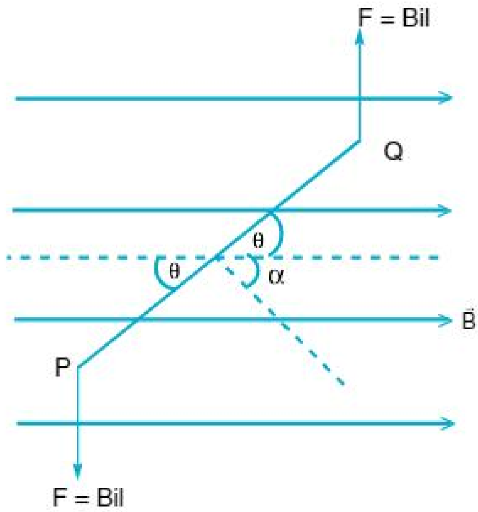


Fig 2.28

So, $\tau = F \times b\cos\theta$ (Here b cos θ is arm length)

Here $F = F_1 = F_2 = Bi\ell$

or $\tau = Bi\ell b\cos\theta$

If the loop has n turns,

then $\tau = n Bi\ell\ b\cos\theta$

But ℓb gives area A of the loop.

$\therefore\ \tau = niAB\cos\theta$ .... (2.15)

This torque is called deflecting torque which rotates the coil.

- If normal to plane of the coil makes an angle α with the direction of magnetic field, then $\tau = niAB\cos(90^0 - \alpha)$ or $\tau = ni\,AB\sin\alpha$ ....(2.15(a))

  We can express this torque in vector notation as $\vec{\tau} = \vec{M} \times \vec{B}$ .... (2.15(b))

  Where niA = M gives the magnitude of magnetic moment of the loop carrying current.

- If the plane of the coil is parallel to the direction of magnetic field, the torque on it is $\tau = BiA\,n\sin 90^0 = Bi\,An$, which is maximum.

  $\therefore \tau_{max} = BiAn$ ....(2.15c)

2.21 MOVING COIL GALVANOMETER

A moving coil galvanometer was first devised by Kelvin and later modified by D'Arsonval. It is used to detect and measure small electric currents of the order 10^{-9}A.

Principle

When a current carrying coil is placed in a uniform magnetic field, it experiences a torque.

Construction

It consists of a rectangular coil of an insulated copper wire wound on a non conducting frame. The coil is suspended between the concave shaped poles of a permanent horse-shoe magnet by a phosphor-bronze wire. The upper end of this wire is connected to torsional head. The lower end of the coil is connected to a phosphor bronze spring. A small concave mirror M is attached to the phosphor bronze wire to measure the deflection of the coil.

The Young's modulus of phosphor bronze is high but rigidity modulus is low. So, it can be twisted very easily but cannot be elongated.

A soft iron cylinder is arranged inside the rectangular frame of the coil. This cylinder increases the field intensity in between the poles. The combination of curved poles and the soft iron produces a radial magnetic field. In radial field, the plane of the coil will be always parallel to the field and experiences a maximum and constant torque. The whole arrangement is kept inside a brass case provided with a glass window. The deflection of the coil can be measured using lamp and scale arrangement.

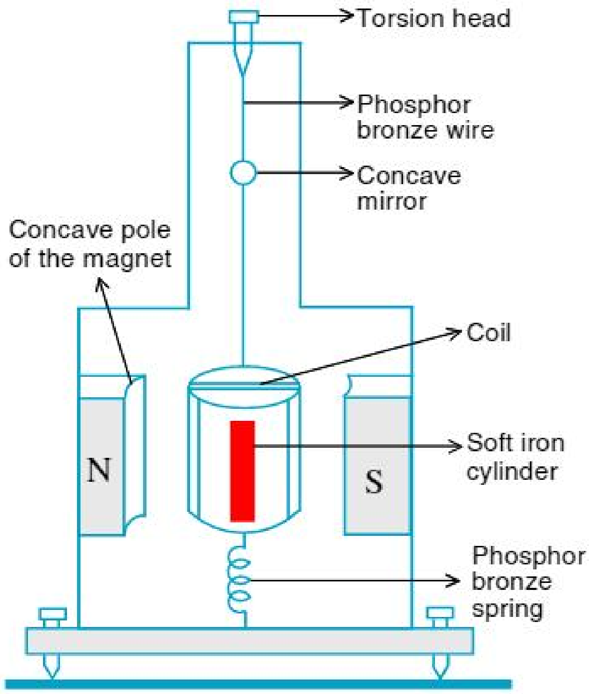


Fig 2.29

Working

When the current to be measured is passed through the coil, the coil experiences a deflecting torque τ_d. Then coil begins to turn. As the coil turns the phosphor bronze wire gets twisted. As a result, an oppositely directed restoring couple develops in the phosphor-bronze wire.

Let n be the number of turns in the coil, A be the area of the coil, B be the intensity of magnetic induction field, and i be the current through the coil.

Fig 2.30

As the field is radial, the plane of the coil is always parallel to the magnetic field. Then the coil experiences a constant and maximum deflecting torque given by

$$\tau_d = niAB \qquad ....(2.16)$$

If C is restoring couple per unit twist and θ is the deflection of the coil, then restoring torque

$$\tau_r = C\theta \qquad .... (2.16a)$$

In equilibrium position, the deflecting torque is equal to the restoring torque.

$$\tau_d = \tau_r \qquad ....(2.16b)$$

i.e $niAB = C\theta$

$$\text{or } i = \frac{C}{nAB}\theta \qquad .... (2.16c)$$

or $i = K\theta$ where $K = \frac{C}{nAB}$ is a constant for the galvanometer.

So, $i \propto \theta \qquad ....(2.16d)$

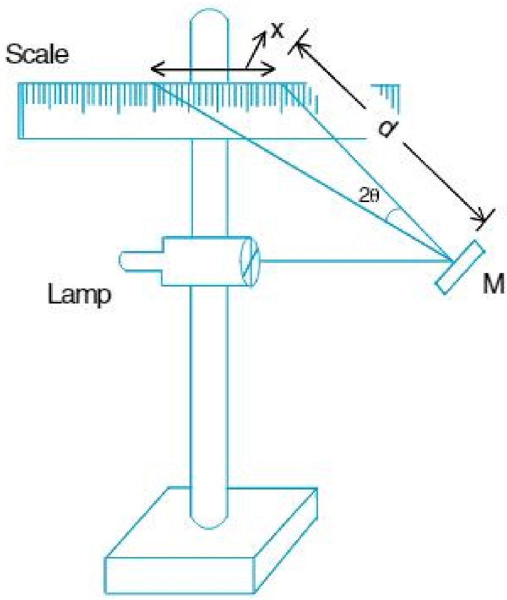


Fig 2.31

Thus the current flowing through the moving coil galvanometer is directly proportional to the deflection of the coil. It means the moving coil galvanometer has a linear scale. It is an important advantage because the instrument can be accurately calibrated.

Measurement of Deflection

The deflection of the coil is usually measured using lamp and scale arrangement. An electric lamp is fixed to a vertical stand. A horizontal scale is arranged to the same stand such that the reflected image of light from the mirror M falls on the scale.

When the coil deflects through an angle θ, the reflected ray will be deflected through 2θ. Corresponding to this, let x be the displacement of image on the scale. If d is the distance between the scale and the mirror,

$$x = d(2\theta) \quad (\because \text{arc length} = \text{radius} \times \text{angle})$$

$$\text{or } \theta = \frac{x}{2d} \text{ in radians}$$

Thus knowing θ and galvanometer constant K, the current passing through the coil can be measured.

2.22 MERITS OF MOVING COIL GALVANOMETER

1) The coil of moving coil galvanometer need not be kept in the magnetic meridian since the magnetic field produced by the permanent magnet is much stronger than the horizontal component of earth's magnetic field.

2) The galvanometer constant $\left(K = \frac{C}{nAB}\right)$ does not depend on the earth's magnetic field. So deflection of the coil is independent of horizontal component of earth's magnetic field.

3) The deflection of the coil is not affected by the presence of magnetic materials in its surroundings.

4) The current is directly proportional to the deflection of the coil. So, the instrument can be accurately calibrated.

5) It can be used to measure very low currents upto 10^{-9} A.

6) Without sacrificing the accuracy, sensitivity of the instrument can be improved.

Demerits

1) It is not a direct reading instrument.

2) The coil may get damaged if large currents are passed through it (Using shunt the galvanometer can be protected).

2.23 ADVANTAGE OF RADIAL FIELD IN M.C.G

If the pole pieces of moving coil galvanometer are flat, the field is not radial. After current passes through the coil, suppose it comes to rest after rotating through an angle θ. In this position, plane of the coil makes an angle θ with the field direction. Then the deflecting torque is given by

$$\tau_d = niAB\cos\theta$$

Now restoring torque is $\tau_r = C\theta$ In equilibrium τ_d and τ_r are equal and opposite to each other.

i.e., $\tau_d = \tau_r$ or $niAB\cos\theta = C\theta$ or

$$i = \frac{C}{nAB}\cdot\frac{\theta}{\cos\theta}$$

As $\frac{C}{nAB} = K$ (galvanometer constant)

$$i = K\frac{\theta}{\cos\theta} \text{ or } i \propto \frac{\theta}{\cos\theta}$$

So the deflection produced is not directly proportional to the current i passing through the coil. On the other hand if the pole pieces are made concave, radial magnetic field will be produced. In radial field, plane of the coil will be always parallel to the field in all positions. Then deflecting torque would be maximum and equal to $niAB$.

Then $niAB = C\theta$ and $i = \frac{C}{nAB}\theta$ or $i = k\theta$ and $i \propto \theta$. So, the deflection produced is directly proportional to the current i passing through the coil due to which current can be measured more accurately.

2.24 SENSITIVITY OF MOVING COIL GALVANOMETER

The sensitivity of an instrument is measured by the minimum input for a standard output. The input is the current through the coil of galvanometer and output is deflection of the coil.

A galvanometer is said to be sensitive, if it gives a large deflection, even when a small current is passed through it or a small voltage is applied across it.

Current sensitivity

It is the deflection produced in the galvanometer when a unit current is passed through it.

If θ is the deflection produced when a current i passes through a galvanometer, then its current sensitivity is $\frac{\theta}{i} = \frac{nAB}{C}$ $\left(i = \frac{C}{nAB}\theta\right)$

In order to have high current sensitivity, n, B and A must be large while C should be small.

Voltage Sensitivity

It is the deflection produced in the galvanometer when a unit voltage is applied across it.

If θ is the deflection produced when a voltage V is applied, then voltage sensitivity of the galvanometer is $\frac{\theta}{V}$.

If G is the resistance of the coil and i is the current passing through the coil, then V = iG. Voltage sensitivity

$$\frac{\theta}{V} = \frac{\theta}{iG} = \frac{nAB}{CG} \quad \left(\because i = \frac{C}{nAB}\theta\right)$$

In order to have high voltage sensitivity same features are required as for high current sensitivity together with low coil resistance.

Example-2.32

The area of the coil in a moving coil galvanometer is 16 cm^2 and has 20 turns. The magnetic induction is 0.2T and the couple per unit twist of the suspended wire is 10^{-6} Nm per degree. If the deflection is 45^0 calculate the current passing through it.

Solution :

Given, $A = 16\ cm^2 = 16 \times 10^{-4} m^2$

$B = 0.2\ T$; $N = 20$, $C = 10^{-6} Nm/degree$; $\theta = 45^0$

From, $C\theta = BiAN$

$$i = \frac{C\theta}{BAN} = \frac{10^{-6} \times 45}{0.2 \times 16 \times 10^{-4} \times 20} = 7 \times 10^{-3} A.$$

Example-2.33

A coil of area $100\ cm^2$ having 500 turns carries a current of 1 mA. It is suspended in a uniform magnetic field of induction $10^{-3}\ Wb/m^2$. Its plane makes an angle of 60^0 with the lines of induction. Find the torque acting on the coil.

Solution :

Given $i = 1\ mA = 10^{-3}\ A$; $N = 500$; $B = 10^{-3}\ Wb/m^2$

$\theta = 60^0$, $\tau = ?$ $A = 100\ cm^2 = 100 \times 10^{-4}\ m^2$

Couple acting on the coil is given by $\tau = BiA\ N\sin\phi$

where ϕ is angle made by normal to the plane of coil with B.

$\phi = 90 - 60 = 30^0$

$\therefore\ C = 10^{-3} \times 10^{-3} \times 100 \times 10^{-4} \times 500 \times \sin 30^0$

$= 250 \times 10^{-8}\ Nm$

2.25 SHUNT

"A low resistance connected in parallel to a moving coil galvanometer to protect it from large currents is known as shunt."

A galvanometer is a very sensitive instrument. It gets damaged if large currents are allowed to pass through it. To prevent the flow of heavy current, a small resistance is connected across the terminals of the galvanometer. It means a low resistance is connected in parallel to the galvanometer. Due to this, a large portion of the current passes through the shunt and only a small portion of the current passes through the galvanometer.

Uses

1) The shunt prevents the flow of large current and protects the moving coil galvanometer. Thus the life of galvanometer can be increased.

2) As it is a small resistance connected in parallel to the galvanometer, the effective resistance is very small. It is an advantage while measuring currents in a circuit.

3) By connecting suitable shunt resistance across the galvanometer, the sensitivity of the galvanometer can be increased.

2.26 CURRENTS IN SHUNT AND GALVANOMETER (PRINCIPLE OF SHUNT)

Let G be the resistance of a galvanometer and S be a small resistance connected parallel to the galvanometer. Let i be the main current which divides into i_g and i_s as shown. i_g is the current through the galvanometer and i_s is the current through the shunt resistance.

So $i = i_g + i_s$ ... (a)

If R is the effective resistance of the parallel combination of galvanometer and shunt resistance, then

$$\frac{1}{R} = \frac{1}{G} + \frac{1}{S} \text{ or } R = \frac{GS}{G+S} \text{ ... (b)}$$

As galvanometer and shunt are connected in parallel, the potential difference of each is the same.

i.e., $V = i_g G = i_s S = iR$ ....(c)

from (c) $i_g G = iR$

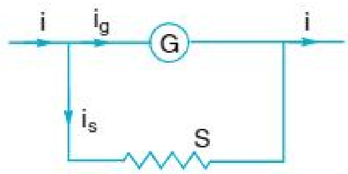


Fig 2.32

$$\text{or } i_g = \frac{iR}{G} = \left(\frac{i}{G}\left(\frac{GS}{G+S}\right)\right) \text{ or } i_g = i\left(\frac{S}{G+S}\right)$$

$\therefore$ current through galvanometer is

$$i_g = \frac{\text{main current} \times \text{Shunt resistance}}{\text{Galvanometer resistance} + \text{shunt resistance}}$$

From (c) $i_s S = iR$

$$\text{or } i_s = \frac{iR}{S} = \frac{i}{S}\left(\frac{GS}{G+S}\right) \text{ or } i_s = i\left(\frac{G}{G+S}\right)$$

∴ Current through shunt resistance is

$$= \frac{\text{main current} \times \text{resistance of the galvanometer}}{\text{Galvanometer resistance+shunt resistance}}$$

Here fraction of the main current which flows through the galvanometer is given by

$$\frac{i_g}{i} = \frac{S}{G+S}$$

from (c) $\frac{i_g}{i_s} = \frac{S}{G}$

So, the ratio of currents through the galvanometer and shunt is equal to the inverse ratio of their resistances.

Range of the galvanometer can be increased by connecting shunt in parallel to it. If the range is increased to *n* times then, $\frac{1}{n}$ part of the total current passes through the galvanometer.

$$\frac{i_g}{i} = \frac{1}{n} \text{ or } \frac{1}{n} = \frac{S}{G+S} \text{ or } S = \frac{G}{(n-1)}$$

This is the principle of shunt.

2.27 AMMETER

Ammeter is an instrument used for measuring currents in electrical circuits. A galvanometer can be converted into ammeter by connecting very low resistance in parallel to it. A shunted galvanometer is called an ammeter. When a galvanometer is converted into ammeter, it can measure currents without causing any damage to it.

Value of Shunt

The value of shunt S is chosen according to the maximum current we wish to measure. Suppose that the galvanometer gives full scale deflection when current i_g is passed through it. Assume that it should read current i at full-scale. Let G be the resistance of the galvanometer and S be the small shunt resistance connected in parallel to the galvanometer. The value of S is so adjusted that when current i is passed, only the part i_g passes through the coil and the remaining current $(i - i_g)$ flows through the shunt resistance.

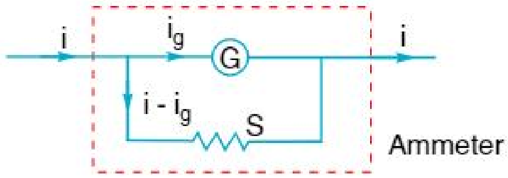


Fig 2.33

Potential difference across shunt = Potential difference across the coil of the galvanometer.

$$(i - i_g)S = i_g G \text{ or } S = \left(\frac{i_g}{i - i_g}\right)G$$

The effective resistance is very small. When it is introduced in the electric circuit, the circuit resistance practically remains unchanged and consequently the current in the circuit to be measured remains unaffected. The resistance of an ideal ammeter is zero. An ammeter is always connected in series in the circuit. The sensitivity of the ammeter $= \frac{i_g}{i} = \frac{S}{G+S}$, Effective resistance of the ammeter is $\left(\frac{GS}{G+S}\right)$

Example-2.34

A galvanometer of resistance $20\,\Omega$ is shunted by a $2\,\Omega$ resistor. What part of the main current flows through the galvanometer?

Solution :

$\frac{i_g}{i} = \frac{S}{G+S}$. Given $G = 20\,\Omega$; $S = 2\,\Omega$

$\therefore \frac{i_g}{i} = \frac{2}{22} = \frac{1}{11}; \frac{1}{11}$th part of current passes through galvanometer.

Example-2.35

A galvanometer has resistance 500 ohm. It is shunted so that its sensitivity decreases by 100 times. Find the shunt resistance.

Solution :

Sensitivity $\propto \frac{1}{\text{range}}$ $\quad\therefore n = 100$

$$S = \frac{G}{(n-1)} = \frac{500}{(100-1)} = \frac{500}{99}\Omega \simeq 5.05\,\Omega$$

Example-2.36

The resistance of a galvanometer is $999\,\Omega$. A shunt of $1\,\Omega$ is connected to it. If the main current is 10^{-2}A, what is the current flowing through the galvanometer.

Solution :

$G = 999\,\Omega, S = 1\,\Omega \; i = 10^{-2}\,A; i_g = ?$

$$i_g = i\left(\frac{S}{G+S}\right) = 10^{-2} \times \left(\frac{1}{999+1}\right) = 10^{-5}\,A.$$

Example-2.37

A galvanometer has a resistance of $98\,\Omega$. If 2% of the main current is to be passed through the meter, what should be the value of the shunt?

Solution :

$G = 98\,\Omega; \frac{i_g}{I} = 2\%$

$$s = \frac{G}{\left(\frac{i}{i_g} - 1\right)}; \therefore \frac{i}{i_g} = \frac{100}{2} = 50 \quad \therefore S = \frac{98}{(50-1)} = 2\Omega$$

2.28 VOLTMETER

A voltmeter is a device used for measuring potential difference between any two points in a circuit.

A galvanometer can be converted into voltmeter by connecting a high resistance in series with it. A galvanometer in series with a high resistance is called a voltmeter.

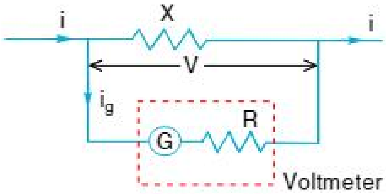


Fig 2.34

Value of Series Resistance :

Let G be the resistance of the galvanometer and R be the resistance connected in series to it. The value of R is so adjusted that when the potential difference across the voltmeter is V, the current through the galvanometer should be i_g, such that $V = i_g(G+R)$.

$$\text{and} \quad G + R = \frac{V}{i_g} \quad \text{or} \quad R = \left(\frac{V}{i_g} - G\right)$$

The effective resistance of the voltmeter is (G+R) which is very high. When voltmeter is in parallel between two points of the circuit, it draws very small current only. So, voltmeter measures the potential difference between any two points in a circuit without effecting the circuit current. The resistance of an ideal voltmeter is infinite. A voltmeter should be always connected in parallel in a circuit.

Note: If the voltage range of a galvanometer is increased to *n* times by connecting a series resistance R, then R=G(n–1)

$[V_G = i_g G, V = i_g(G+R), V = nV_G$ or

$i_g(G+R) = ni_g G, R = G(n-1)]$

Example-2.38

A maximum current of 0.5 mA can be passed through a galvanometer of resistance $20\,\Omega$. Calculate the resistance to be connected in series to convert it into a voltmeter of range 0 – 5V.

Solution :

$R = G(n-1)$, where $n = \frac{V_2}{V_1}$

$V_2 = 5\,V; V_1 = iG = 0.5 \times 10^{-3} \times 20 = 10^{-2}V$

$n = 500$

and $R = 20(500-1) = 9980\,\Omega$

Example-2.39

A galvanometer has a resistance of $100\,\Omega$. A current of 10^{-3}A is permissible through the galvanometer. How can it be converted into

a) an ammeter of range 10 A and

b) a voltmeter of range 10V.

Solution :

$G = 100\,\Omega; i_1 = 10^{-3}\,A$

a) $i_2 = 10A; \; n = \frac{i_2}{i_1} = 10^4$

$$S = \frac{G}{(n-1)} = \frac{100}{(10^4-1)} = \frac{100}{999}\Omega$$

b) $V_1 = i_1 G = 10^{-3} \times 100 = 10^{-1}V$

$$V_2 = 10V \Rightarrow n = \frac{V_2}{V_1} = \frac{10}{10^{-1}} = 100$$

$\therefore R = G(n-1)$

$= 100\,(100-1) = 9900\,\Omega$

Example-2.40

A galvanometer having 30 divisions has current sensitivity of 20 μA/ division. It has a resistance of 25ohm. How will you convert it into an ammeter measuring upto 1 A? How will you now convert this ammeter into a voltmeter reading upto 1 V ?

Solution :

The full scale deflection current

$$i_g = 30 \times (20 \times 10^{-6})$$
$$= 6 \times 10^{-4} A.$$

It S is the required value of the shunt connected in parallel with the galvanometer, then

$$i_g = \frac{S}{S+G} i \text{ or } 6 \times 10^{-4} = \frac{S}{S+25} \times 1$$

After solving, we get $S = \frac{150}{9994} \Omega = 0.0150\Omega$

The resistance of the ammeter

$$R_A = \frac{SG}{S+G} = \frac{0.0150 \times 25}{0.0150 + 25} = 0.0150\Omega$$

To convert this ammeter into the voltmeter, we can use

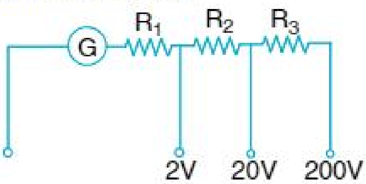
$$V = i_g (R_A + R_0)$$

Here $V = 1V, i_g = 1A$

$\therefore \ 1 = 1\,(0.0150 + R_0)$ or $R_0 = 0.985 \ \Omega$

Example-2.41

A multirange voltmeter can be constructed using a galvanometer circuit as shown. We waNt to construct a voltmeter that can measure 2V, 20V and 200V using a galvanometer of resistance 10Ω and that produces a maximum deflection of current of 1mA. Find R_1, R_2 and R_3 that have to be used.



Solution :

$I_g = 10^{-3}$ A and $G = 10\,\Omega$

$$R_1 + 10 = \frac{2V}{10^{-3} A} = 2000\,\Omega \Rightarrow R_1 = 1990\,\Omega$$

$$R_1 + R_2 + 10 = \frac{20V}{10^{-3} A} \Rightarrow R_2 = 18000\,\Omega = 18K\,\Omega$$

$$R_1 + R_2 + R_3 + 10 = \frac{200\,V}{10^{-3} A} \Rightarrow R_3 = 180\,\Omega$$

2.29 TANGENT GALVANOMETER

Construction

Tangent galvanometer consists of a non-magnetic circular vertical frame on which three separate circular coils of insulated wire having different number of turns are wounded. The ends of these coils are connected to the screws provided over the base of the instrument. The vertical frame (or coil) can be rotated about a vertical axis passing through the centre of the instrument. A circular compass box made of non-magnetic material is fitted at the centre of the vertical frame. Three levelling screws are provided to the base to make the base as horizontal.

Principle and working

The instrument is levelled with the help of levelling screws. The plane of the coil is gently rotated so that it is parallel to magnetic meridian. Now the compass box is rotated till the aluminium pointer reads 0^0–0^0.

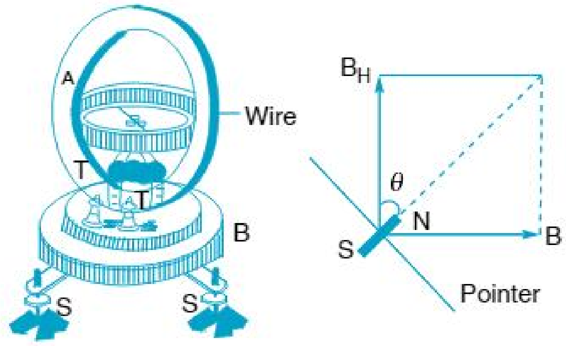


Fig 2.35

When no current flows through the coil, the magnetic needle will be along the magnetic meridian. When a current i is passed through the coil, magnetic induction field B is set up at the centre of the coil. The horizontal component of earth's magnetic field B_H will be at right angles to B since B_H is in the plane of the coil. The magnetic needle is under the action of B and B_H which are mutually perpendicular. Let θ be the deflection of the needle.

From tangent law, $B = B_H \tan\theta$

where $B = \frac{\mu_0}{2}\frac{ni}{r}$

Here n is number of turns of the coil, i is the current through the coil, and r is radius of the coil

so, $\frac{\mu_0}{2}\frac{ni}{r} = B_H \tan\theta$

or $i = \left(\frac{2rB_H}{\mu_0 n}\right)\tan\theta$ or $i = K\tan\theta$

where $K = \left(\frac{2rB_H}{\mu_0 n}\right)$ is a constant called reduction factor of the tangent galvanometer.

So, here current i to be measured is directly proportional to the tangent of angle of deflection. Since the galvanometer works based on tangent law it is called tangent galvanometer.

2.30 HALL EFFECT

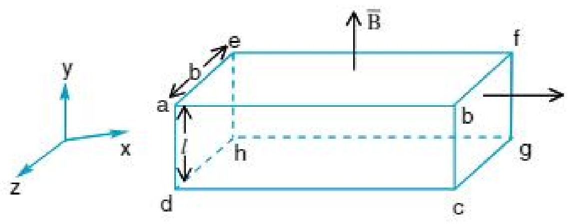


Fig 2.36

If a current carrying conductor is placed in a transverse magnetic field, an emf is set up across the conductor perpendicular to both current and magnetic field. This is known as Hall effect observed by Hall.

Consider a steady current i flowing along n axis in a uniform conducting strip kept as shown. A uniform magnetic field $\overline{B}$ is applied along y-axis as shown. According to Fleming's left hand rule, magnetic field applies a deflecting force on electrons along z-axis. Force on positive ions will be along negative z-axis. Due to this force electrons will be displaced towards front face aberd and positive ions towards back face efgh.

Due to this charge separation, electric field will be developed between those faces. When equilibrium is reached, magnetic deflecting force on charge carries is just balanced by the electric field.

$\Rightarrow e\vartheta_d B = Ee$

$E = B\vartheta_d$

Here ϑ_d is drift speed of electrons.

E is intensity of Hall field.

Hall potential $V = Eb = B\vartheta_d b$

and $\vartheta_d = \frac{i}{neA} (\because A = bl)$

$\Rightarrow V = \frac{i}{neb\ell}Bb$ or $V = \frac{Bi}{ne\ell}$

2.31 DIFFERENCES BETWEEN AMMETER AND GALVANOMETER

Ammeter	Galvanometer
1. It is calibrated in ampere or sub multiples	1. It has only divisions without calibration
2. It is used to measure large currents	2. It is used to measure small currents
3. The zero division in this is at one end of the scale	3. The zero division in this is at the middle of the scale
4. The effective resistance of ammeter is very low	4. The resistance of galvanometer is low

2.31.1 COMPARISON OF AMMETER AND VOLTMETER

Ammeter	Voltmeter
1. It is a device used for measuring currents in electrical circuits	1. It is a device used for measuring potential difference between two points in a circuit
2. A moving coil galvanometer is converted into an ammeter by connecting a suitable low resistance in parallel to it	2. A moving coil galvanometer is converted into a voltmeter by connecting a suitable high resistance in series with it
3. The resistance of an ideal ammeter is zero	3. The resistance of an ideal voltmeter is infinite
4. An ammeter should always be connected in series in the circuit	4. A voltmeter should always be connected in parallel in the circuit

2.31.2 COMPARISON OF M.C.G WITH TANGENT GALVANOMETER

M.C.G.	Tangent Galvanometer
1. It is a moving coil and fixed magnet type galvanometer	1. It is moving magnet and fixed coil type galvanometer
2. It is based on the principle that when a current carrying coil is placed in a uniform magnetic field, the coil expriences a torque	2. It is based on tangent law of magnetism
3. The plane of the coil need not be set in the magnetic meridian	3. The plane of the coil should be in the magnetic meridian
4. The current flowing through the coil is directly proportional to the deflection	4. The current flowing through the coil is directly proportional to the tangent of deflection
5. It can be used to measure the current of order 10^{-9} A	5. It can be used to measure the currents of the order of 10^{-6} A
6. The galvanometer constant does not depend on earth's magnetic field	6. The galvanometer constant depends on earth's magnetic field
7. External magnetic fields have no effect on the deflection	7. External magnetic fields may influence the deflection
8. It is not a portable instrument	8. It is a portable instrument
9. Its cost is high	9. It's cost is low

2.32 BAR MAGNET AS A SOLENOID

We know that a current loop acts as a magnetic dipole. All magnetic phenomena can be explained interms of circulating currents. Magnetic dipole moment associated with a current loop can be defined as $\overline{M} = Ni\overline{A}$. Here N is the number of turns in the loop, i is the current and $\overline{A}$ is the area vector.

The magnetic field lines for a bar magnet resemble those of a solenoid suggesting that a bar magnet may be thought of as a large number of circulating currents like in a solenoid. If we cut a solenoid into two halves we get two smaller solenoids with weaker magnetic properties. The same we can observe if we cut a bar magnet into two equal halves. If we move a small compass needle near a bar magnet and near a current carrying finite solenoid, the deflections are similar in both cases.

Let 2L be the length of a solenoid of radius r. Let n be the number of turns per unit length. Let P be a point on the axis of solenoid at a distance 'd' from the centre of the solenoid.

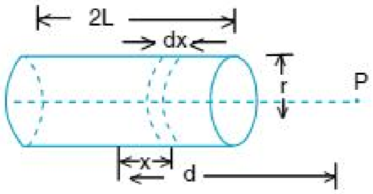


Fig 2.37

Let us consider a circular element of thickness dx of the solenoid at a distance x from the centre. Number of turns in that element are ndx. Let I be the current in the solenoid. The magnetic field at P due to the circular element is

$$dB = \frac{\mu_0 ndxIr^2}{2\{(d-x)^2 + r^2\}^{3/2}}$$

Total magnetic field is obtained by integrating dB from x = –L to x= +L

$$\Rightarrow B = \frac{\mu_0 nIr^2}{2}\int_{-L}^{L}\frac{dx}{\{(d-x)^2 + r^2\}^{3/2}}$$

For d >> r and d >> L, we can take

$$B = \frac{\mu_0 nIr^2}{2d^3}\int_{-L}^{L} dx$$

$$\text{or} \quad B = \frac{\mu_0 nI}{2}\frac{2Ir^2}{d^3}$$

Magnitude of magnetic moment of solenoid is $M = n(2L)I(\pi r^2)$

$$\Rightarrow B = \frac{\mu_0}{4\pi} = \frac{2M}{d^3}$$

This is same as magnetic field on the axial line of a short bar magnet of magnet moment M. So, the magnetic moment of a bar magnet is equal to the magnetic moment of an equivalent solenoid.

2.33 CYCLOTRON

Cyclotron is a device used to accelerate charged particles or ions to high energies. It involves the motion of charge in both electric and magnetic fields. These two fields are mutually perpendicular and so known as crossed fields. Cyclotron consists of two semicircular disc like metal contains D_1 and D_2 which are known as dees. The changed particle moves most of the time inside these dees. Here frequency of revolution of the charge is independent of its energy or speed. Inside these dees the charged particle is subjected to electrostatic shielding. The applied magnetic field makes the charge to move in a circular path inside dees. Whenever the charge moves from one dee to another it will be acted upon by electric field. The polarity of the electric field changes alternately in tune with the circular motion of the charged particle. As a consequence, the particle will be subjected to acceleration by the electric field after completing semicircle in each dee. Due to the acceleration kinetic energy of particle increases. As kinetic energy or speed of the particle increases, radius of its circular path also increases. As a result, path followed by the particle is in the form of a spiral.

To avoid collisions between the ions and the air molecules whole arrangement is evacuated. To change the polarity of electric field, a high frequency alternating voltage will be applied to the dees.

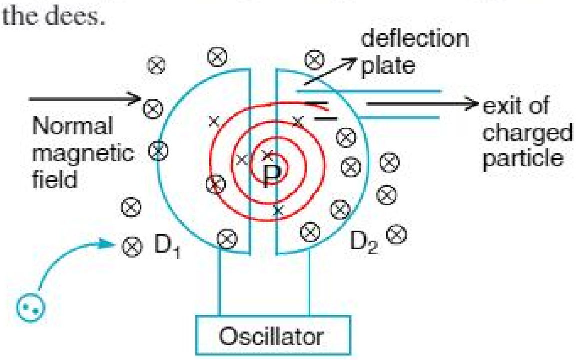


Fig 2.38

The positive ions or positively charged particles like protons will be released at the centre P. Those particles move in one of the dees and reach the gap between the dees in a time interval T/2 where T is period of revolution.

We know that period of revolution of the charged particle $T = \frac{2\pi m}{Bq} = \frac{1}{f_c}$, $f_c = \frac{Bq}{2\pi m}$

This frequency is called cyclotron frequency. Frequency of the applied voltage will be adjusted such that polarity of the dees gets reversed in the same time that it takes the charge or ion to complete one half of the revolution. Frequency of the applied voltage $f_a = f_c$ is called the condition of resonance. Phase of the supply is adjusted in such a way that when the positive charge arrives at the edge of D_1, D_2 will be at a lower potential and the charge will accelerated across the gap. Inside the dees the charge travels in a region free of electric field. Each time kinetic energy of charge increases by Vq as it crosses from one dee to another. As a result radius of the circular path of charge goes on increasing. This continues until charge gets required energy to get a radius approximately that of dees. Then the charges accelerated will be deflected by magnetic field to leave the arrangement via exit slit

$$v = \frac{qBR}{m}$$

Here R is radius of dee or radius of trajectory at the exit.

Kinetic energy of the ion at exits is

$$\frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

The charged particle with more kinetic energy will be used to bombord nuclei in nuclear reactions and to initiate artificial radioactivity. Here time for one revolution of the acceleration charge or ion is independent of its speed or radius of its orbit.

2.34 MAGNETIC CONFINEMENT

If a charged particle enters into a magnetic field at an angle $\theta(0 < \theta < \pi/2)$ it moves in a helical path. If the magnetic field is not uniform and changes slightly during one circular orbit, radius of the helix will decrease as it enters stronger magnetic field. Here radius will increase when it enters weak magnetic field.

Consider two solenoid coils at a distance from each other in an evacuated chamber. Charged particles enter into this chamber and move from one coil to another. At the first coil it starts with a small radius. As the magnetic field decreases, radius of the helical path will increase. Again radius will decrease as field increases. It seems that the two solenoid coils act as mirrors and direction of magnetic force on the charge changes which is responsible for the motion of charge. This arrangement may be called as magnetic bottle or magnetic container. In this case the charge will never touch the sides of the container. This type of magnetic bottles are very useful in confining high energy plasma suitable for fusion reaction. It is very difficult to keep or retain plasma in any container due to the high temperature. Toroid is also a useful container. Tokamak is an equipment for plasma confinement in fusion reactors which contains toroids.

2.35 DIPOLE MOMENT OF A REVOLVING ELECTRON (BOHR MAGNETON)

If a charge q moves along a circular path of radius r it constitutes current I such that $I = \frac{q}{T}$ where T is time period of revolution. Here $T = \frac{2\pi r}{v}$ where *v* is speed of charge in that orbit.

We know that magnetic moment of that current is $\mu = IA$ (where $A = \pi r^2$)

$$\mu = I\pi r^2 = \frac{qvr}{2}; \ \vec{\mu} = \frac{-e}{2m}\vec{L}$$

Here negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment

$$\frac{\mu}{L} = \frac{e}{2m}$$

Here $\frac{\mu}{L}$ is equal to half of the specific charge of electron. This ratio is known as gyromagnetic ratio. Its value is equal to $8.8 \times 10^{10} \text{ckg}^{-1}$, which is verified experimentally. From Bohr's second postulate we know that

$$L = \frac{nh}{2\pi}, \text{ where } n = 1,2,3.....$$

for n = 1, we get μ minimum, $\mu_{min} = \frac{eh}{4\pi m}$

μ_{min} is known as Bohr magneton. Its value is equal to $9.27 \times 10^{-24}\,Am^2$

Direction of this magnetic moment is normal to plane of motion of that circle. If the current is clockwise, magnetic moment will be normally into the page and if it is in anticlockwise sense, magnetic moment will be normally outwards

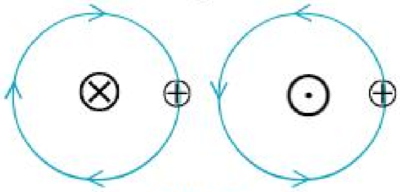


Fig 2.39

In the Bohr's model, electron which is negatively charged revolves around a positively charged nucleus. It also constitutes current. In this case, magnetic moment of orbiting electron will be

$$\mu = \frac{evr}{2} = \frac{e}{2m}(mvr) \text{ (m is mass of electron)}$$

$$= \frac{e}{2m}L$$

Here L is the magnitude of angular momentum of the electron.

At a Glance

1. **Ampere's Circuital law :** Ampere's law states that

   $\oint \vec{B}.d\vec{l} = \mu_0 i_{enc}$ (Ampere's law)

   The line Integral is this equation is evaluated around a closed loop called an amperian loop. The current i is the net current enclosed by the loop.

2. **The Biot - Savart law :** The magnetic field set up by a. Current - Carrying conductor can be found from the Biot - Savart law. This law asserts that the contribution $d\vec{B}$ to the field produced by a current - length element $id\vec{l}$ at a point P, a distance r from the current element is $d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{l} \times \vec{r}}{r^3}$ (Biot - Savart law)

   Here $\vec{r}$ is a vector that points from the element to P. The quantity μ_0, called the permeability constant, has the value

   $4\pi \times 10^{-7}\,T - m/A \cong 1.26 \times 10^{-6}\,T.m/A$

3. **Magnetic field of a circular arc :**

   The magnitude of the magnetic field at the centre of a circular arc, of radius R and central angle θ (in radians) carrying current, i is $B = \frac{\mu_0 I\phi}{4\pi R}$ (at centre of circular arc)

4. **Fields of a solenoid and a toriod :**

   Inside a long solenoid carrying current i, at point not near its ends, the magnitude B of the magnetic field is $B = \mu_0 in$ (ideal solenoid). Where n is the number of turns per unit length. At a point inside a toroid, the magnitude B of the magnetic field is $B = \frac{\mu_0}{2\pi}\frac{iN}{r}$ where is the distance from the centre of the toroid to the point.

5. **Magnetic field of a long straight wire :**

   For a long straight wire carrying a current i, the Biot - Savart law gives, for the magnitude of the magnetic field at a perpendicular distance R from the wire, $B = \frac{\mu_0}{2\pi}\frac{i}{R}$ (long straight wire)

6. **Magnetic field $\vec{B}$:**

   A magnetic field $\vec{B}$ is defined in interms of the force $\vec{F_B}$ acting on a test particle with charge q moving through the field with velocity $\vec{V}$ is $\vec{F_B} = q(\vec{V} \times \vec{B})$

   The SI unit for $\vec{B}$ is the Tesla (T) : $1T = IN/A = 10^4$ gauss.

7. **A charge particle circulating in a magnetic field :**

   A charged particle with mass m and charge magnitude q moving with velocity $\vec{v}$ perpendicular to a uniform Magnetic field $\vec{B}$ will travel is a circle. Applying Newton's second law to the circular motion yields $qvB = \frac{mv^2}{r}$ from which we find the radius r of the circle to be $r = \frac{mv}{Bq}$

   The frequency of revolution f, the angular frequency ω, and the period of the motion T are given by

   $f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{Bq}{2\pi m}$

8. **Mangnetic force on a current - carrying wire :**

   A straight wire carrying a current i in a uniform magnetic field experiences a sideways force.

$\vec{F}_B = i\vec{L}\times\vec{B}$. The force acting on a current element $i\vec{dl}$ in a magnetic field is $d\vec{F}_B = i\vec{dl}\times\vec{B}$. The direction of the length vector $\vec{L}$ (or) $\vec{dL}$ is that of the current i.

9. **The force between parallel wires carrying currents:** Parallel wires carrying currents in the same direction attract each other, where as parallel wires carrying currents is opposite directions repel each other. The magnitude of the force on a length L of either wire is $F = \frac{\mu_0}{2\pi}\frac{Li_1i_2}{d}$

10. **Field of a magnetic dipole :** The magnetic field produced by a current - carrying coil, which is a magnetic dipole, at a point P located a distance Z along the coil's central axis is parallel to the axis and is given by $\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$ where $\vec{\mu}$ is the dipole moment of the coil. This equations applies only when Z is much greater than the dimensions of the coil.

11. **Torque on a current - carrying coil :** A coil (or area A and carrying current i, with N turns) in a uniform magnetic $\vec{B}$ will experience a torque $\vec{\tau}$ given by $\vec{\tau} = \vec{\mu}\times\vec{B}$. Here $\vec{\mu}$ is the magnetic dipole moment of the coil, with magnitude μ = NiA and direction given by the right - hand rule .

EXERCISE

LONG ANSWER QUESTIONS

1. Explain Fleming's left hand rule? Deduce an expression for the force on a current carrying conductor in a magnetic field. Derive expression for the between two parallel conductors carrying current.
2. Obtain an expression for the torque on a loop placed in a uniform magnetic field. Describe the construction and working of moving coil galvanometer?
3. Describe tangent galvanometer with necessary theory? Compare it with moving coil galvanometer.
4. How is a galvanometer converted into an ammeter? Why parallel resistance is smaller than galvanometer resistance? Explain.
5. How is a galvanometer converted into a ammeter? Why series resistance is than galvanometer resistance? Explain.

SHORT ANSWER QUESTIONS

1. State and explain Biot - Savart law.
2. State and prove Ampere's law.
3. Find intensity of magnetic induction of B due to a along current carrying conductor.
4. Deduce an expression for the intensity of magnetic induction field at the centre of a circular current carrying coil using Biot- Savart law.
5. Obtain an expression for the magnetic dipole moment of current loop.
6. Derive an expression for the magnetic dipole moment of revolving electron.

VERY SHORT ANSWER QUESTIONS

1. **What is the importance of Oersted's experiment?**

A. Conclusion of Oersted's experiment is that a current carrying conductor produces a magnetic field around it. This led to various important application.

2. **An electric current flows in a wire from east to west - What will be the direction of the magnetic field due to this wire at a point north of the wire? South of the wire?**

A. Vertically upwards, vertically downwards.

3. **What is the work done in taking a north pole of strength m around a long and straight conductor in a circular path at a perpendicular distance of r from the straight conductor?**

A. μ_0 im, since the force acting on the north pole is $F = mB = \frac{m\mu_0 i}{2\pi r}$ along the tangent and the work done $= \vec{F}.\vec{S} = FS\cos 0$ as both force and displacement are along the same direction.

Hence, work done $= \frac{m\mu_0 i}{2\pi r}(2\pi r) = \mu_0 im.$

4. **Does a current carrying circular coil produce uniform magnetic field?**

A. No, magnetic field produced is not uniform. However, it may be considered as uniform at the centre of the circular coil.

5. **Two parallel long wires separated by a distance 'd' carry equal current 'i'. What is the magnetic induction at a point mid way between the wires if**
(a) currents are in the same direction?
(b) currents are in opposite direction?

A. (a) Zero (b) $\frac{\mu_0 i}{2\pi \frac{d}{2}} + \frac{\mu_0 i}{2\pi \frac{d}{2}} = \frac{2\mu_0 i}{\pi d}$.

6. **What is the force on a conductor of length 'l' carrying current 'i' when it is situated in a magnetic field of induction B? When is it maximum?**

A. Force due a magnetic field on a current carrying conductor is given by $F = Bil\sin\theta$.
Its value is maximum when direction of current and magnetic field are perpendicular to each other.
$\therefore F_{max} = Bil$

7. **When a charged particle moves in a uniform magnetic field at right angles to the direction of the field, which of the following changes? Speed of the particle, Energy of the particle or Path of the particle.**

A. Path of the particle.

8. **An electron is not deflected, while moving through a certain region of space. Can we say that there is no magnetic field in the region?**

A. No. The electron may be moving parallel to the direction of magnetic field.

9. **What will be the path of a charged particle moving along the direction of a uniform magnetic field?**

A. It will be moving along a straight line path because the magnetic force on the charged particle is zero.

10. **An electron beam is moving horizontally towards east. If this beam passes through a uniform magnetic field directed upwards, then in which direction will the beam be deflected?**

A. Towards North.

11. **What is the force on a charged particle of charge strength q moving with a velocity v in a uniform magnetic field of induction B? When is it maximum.**

A. Force on a charged particle $F = Bqv\sin\theta$
F is maximum when $\theta = 90^0$; $F_{max} = Bqv$

12. **Does the torque on a current loop in magnetic field change, when its shape is changed without changing its face area?**

A. No.

13. **A current carrying loop free to turn is placed in a uniform magnetic field. What will be its orientation relative to the direction of magnetic field in the equilibrium state?**

A. The plane of the loop is perpendicular to the direction of magnetic field because the torque on the loop in this orientation is zero.

14. **Is the resistance of an ammeter greater than or less than that of the galvanometer of which it is formed?**

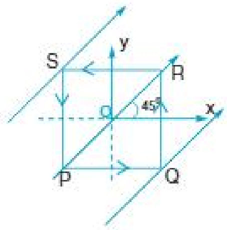
A. It is always less than the resistance of the galvanometer.

15. **Is the resistance of a voltmeter greater than or less that of the galvanometer of which it is formed?**

A. It is always greater than the resistance of the galvanometer.

16. **A moving coil galvanometer can measure a current of 10^{-6}A. What is the resistance of the shunt to measure 1A?**

A. $\frac{i_g}{i} = \frac{S}{G+S} = n$



17. **What is the smallest value of current that can be measured with a moving coil galvanometer, tangent galvanometer?**

A. The smallest value of current that can be measured by a moving coil galvanometer is 10^{-9}.

18. **How do you convert a moving coil galvanometer into an ammeter?**

A. A moving coil galvanometer can be converted into ammeter by connecting a resistance in parallel with the galvanometer.

19. **How do you convert a moving coil galvanometer into a voltmeter?**

A. A moving coil galvanometer can be converted into a voltmeter by connecting a resistance in series with the galvanometer.

PROBLEMS

LEVEL - I

1. A long straight wire carries a current of 50 A. What is the magnitude of intensity of magnetic induction at a point 15 cm from the wire. **[Ans:** 6.7×10^{-5}T]

2. A wire carrying a current of 125 A is bent into the form of a circle of radius 5 cm. Calculate the flux density at the centre of the coil.
[Ans: 157×10^{-5}Wb/m^2]

3. A current of 4A is flowing through a circular coil of radius 20 cm containing 200 turns. Find the magnetic flux density at the centre of the coil.
[Ans: 251.3×10^{-5}Wb/m^2]

4. A wire carrying a current of 12A is in the form of a circle. It is necessary to have a magnetic field of induction 10^{-6} T at the centre. What should be the radius ? **[Ans:** 7.54 m]

5. Two conductors each of length 18 m lie parallel to each other in air. The centre to centre distance between the two conductors is 30×10^{-2} m and the current in each conductor is 450 A. Determine the force in newton tending to pull the conductors together.
[Ans: 2.43 N]

6. A long straight conductor carrying a current of 4 A is in parallel to another conductor of length 5 cm and carrying a current 9 A. They are separated by a distance of 10 cm. Calculate (a) B due to first conductor at second conductor (b) The force on the short conductor.
[Ans: (a) 8×10^{-6} T (b) 36×10^{-7} N]

7. Two parallel conductors A and B separated by 15 cm carry electric current of 9 A and 4 A in the same direction. Find the point between A and B where the field is zero. [Ans: 10.38 cm from A]

8. The area of the coil in a moving coil galvanometer is 8 cm^2 and has 10 turns. The magnetic induction is 0.1 T and the couple per unit twist of the suspended fibre is 10^{-6} Nm per degree. If the deflection is 90^0, calculate the current passing through it.
[Ans: 1.13×10^{-1} A]

9. A coil of area 150 cm^2 having 250 turns carriers a current of 2 mA. It is suspended in a uniform magnetic field of induction 9×10^{-3} Wbm^{-2}. Its plane makes an angle of 60^0 with the line of induction. Find the torque acting on the coil.
[Ans: 3.375×10^{-5} Nm]

10. A galvanometer of resistance 30Ω is shunted by a 3Ω resistor. What part of the main current flows through the meter ? [Ans: $1/11^{th}$ part of current]

11. A maximum current of 1.5 mA can be passed through a galvanometer of resistance 30Ω. Calculate the resistance to be connected in series to convert it into a voltmeter of range 0–9 V. [Ans: $5970\,\Omega$]

12. A galvanometer has a resistance of 500 ohm. It is shunted so that its sensitivity decreases by 100 times. Find the shunt resistance. [Ans: $5.05\,\Omega$]

13. A galvanometer has a resistance of 100Ω. A current of 2×10^{-3} A can pass through the galvanometer. How can it be converted into (a) Ammeter of range 20 A and (b) Voltmeter of range 20 V?
[Ans: (a) $\frac{100}{999}\Omega$ (b) 9900Ω]

14. The resistance of a galvanometer is 999Ω. A shunt of 1Ω is connected to it. If the main current is 10^{-2} A, what is the current flowing through the galvanometer?
[Ans: 10^{-5} A]

15. A galvanometer has a resistance of 96Ω. If 4% of the main current is to be passed through the meter, what should be the value of the shunt? [Ans: 4Ω]

16. A circular coil of wire of radius 'r' has 'n' turns and carries a current 'I'. Find the magnetic induction (B) at a point on the axis of the coil at a distance $\sqrt{3}r$ from its centre ? [Ans : $\frac{\mu_o nI}{16r}$]

17. Two wires A and B are of lengths 40 cm and 30 cm. A is bent into a circle of radius r and B into an arc of radius r. A current i_1 is passed through A and i_2 through B. To have the same magnetic inductions at the centre. What is the ratio of $i_1 : i_2$. [Ans : 3 : 4]

18. A long horizontal rigidly supported wire carriers a current $i_a = 96$ A. Directly above it and parallel to it at a distance, another wire of 0.144 N weight per metre carries a current $i_b = 24$ A, in a direction opposite to that of i_a. If the upper wire is to float in air due to magnetic repulsion, what is its distance (in mm) from the lower wire ? [Ans : 3.2]

19. A circular coil of radius 3cm has 50 turns. It is placed on the horizontal plane and a current of 3A flows through it in clockwise direction as seen from above. Calculate the magnetic field at a point on the axis of the coil, at a distance of 4cm from its centre. Also indicate the direction of the magnetic field.
[Ans : 6.78×10^{-4} T, vertically downward]

20. A cable 5 m above the ground carries a current of 50 A from south to north. Find the direction and magnitude of the magnetic field on the ground directly below the cable.
[Ans : 2.0×10^{-6} T from east to west]

21. The electron in the hydrogen atom circles around the nucleus with a speed of 2.0×10^6 ms^{-1} in an orbit of 5.3×10^{-11} m. What is the magnetic field at the centre of the nucleus? [Ans : 11.39T]

22. A solenoid of infinite length consists of a single layer 1000 turns per unit length of a wire carrying a current of 2mA. Calculate the magnetic field on the axis at the middle of the solenoid. [Ans : 2.5×10^{-6}T]

23. A long straight wire carries a current of 4A. A proton travels with a velocity of 4×10^4 ms–1 parallel to the wire 0.2m from it and in a direction opposite to the current.What is the force which the magnetic field due to current exerts on the moving proton?
[Ans : 2.56×10^{-20}N]

24. A helium ion $(He)^{2+}$ travels at right angles to a magnetic field of intensity 1.2T with a velocity of $2 \times 10^7 cms^{-1}$. Find the magnitude of the force acting on the ion. [Ans : 7.68×10^{-14}N]

25. An electron experiences a force of 2.4×10^{-13} N when it enters a magnetic field with a velocity of 10^6 ms^{-1} at an angle of 30^o. What is the flux density ?

[Ans : 3.0 T]

26. 10 A and 2A currents are passed through two parallel wires A and B respectively in the opposite directions. If the wire A is infinitely long and the length of wire B is 4 m, which is 10cm away from the wire A, calculate the force on the wire 'B'

[Ans : 1.6×10^{-4} N]

27. Two infinitely long parallel wires 5 cm apart in air carry currents of 2A and 4A respectively. Find the magnitude of the force on each metre of wire if currents are (i) in the same direction. (ii) in opposite direction .

[Ans : (i) 3.2×10^{-5} N attractive (ii) 3.2×10^{-5} N repulsive]

28. A horizontal wire of length 0.2m long carries a current of 3A. Find the magnitude of the magnetic field, which can support the weight of the wire. Mass per unit length of wire is 2×10^{-3} kg m^{-1}.

[Ans : 6.53×10^{-3} T]

29. A rectangular coil of sides 12cm and 8cm having 1000 turns and carrying current of 100mA is held in a uniform magnetic field of 0.1 Tesla. What is the maximum torque the coil can experience ?

[Ans : 0.096 Nm]

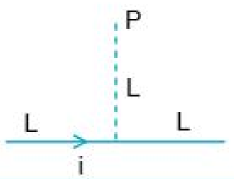
30. A coil of 20 turns of area 8×10^{-2} cm^2 with its plane parallel to the magnetic field of intensity 3000G and carrying a certain current experiences a torque 2.4×10^{-3}Nm .Calculate the value of current.

[Ans : 50A] ($1G = 10^{-4}T$)]

31. The coil of a pivoted coil galvanometer has 25 turns and has area 10 cm2. The coil is placed in a radial magnetic field of flux density 8×10^{-2} T. The torsional constant of the spring is 1.0×10^{-6} Nm per degree. Find the deflection of the coil for a current of 4m A.

[Ans : 8^o]

32. Figure shows a straight wire of length L carrying a current i. Find the magnitude of magnetic field produced by the current at point P.



[Ans: $\frac{\sqrt{2}\,\mu_0 i}{4\pi L}$]

33. A particle of mass 'm' and charge 'q' moves with a constant velocity 'v' along the positive x-direction. It enters a region containing a uniform magnetic field $\vec{B}$ directed along the negative Z- direction, extending from x = a to x = b. Find the minimum value of 'v' required, so that the particle can just enter the region x > b?

[Ans: $\frac{q(b-a)B}{m}$]

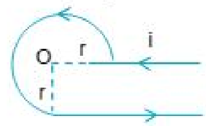
34. A charged particle of chage 4mC enters a uniform magnetic field of induciton $\bar{B} = 3\bar{i} + 6\bar{j} + 6\bar{k}$ tesla with a velocity $\bar{v} = 4\bar{i} - x\,\bar{j} + y\,\bar{k}$. If the particle continues to move undeviated, then find the magnitude of velocity of the particle ? [Ans: 12 m/s]

35. A proton, a deuteron and an α particle having same momentum enter a uniform magnetic field at right angles to the field. Find the ratio of their angular momenta during their motion in the magnetic field.

[Ans: 2:2:1]

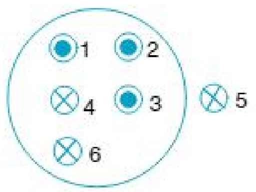
36. The shunt resistance is $\left(\frac{3}{8}\right)$th of that of the galvanometer, what is the fraction of the main current that passes through the galvanometer ? [Ans: 3/11]

37. Find the magnetic induction at point O, if the current carrying wire is in the shape shown in the figure.



[Ans: $\frac{\mu_0 i}{4\pi r}\left[\frac{3}{2}\pi + 1\right]$]

38. Six wires of current I_1 = 1A, I_2 = 2A, I_3 = 3A, I_4 = 1A, I_5 = 5A and I_6 = 4A cut the page perpendicularly at the points 1, 2, 3, 4, 5 and 6. respectively as shown in the figure. Find the value of the integral $\oint \vec{B}.d\vec{l}$ around the closed path.



[Ans: μ_0 weber.m^{-1}]

39. Electric charge q is uniformly distributed over a rod of length l. The rod is placed parallel to a long wire carrying a current i. The separation between the rod and the wire is a. Find the force needed to move the rod along its length with a uniform velocity v.

[Ans: $\frac{\mu_0 iqv}{2\pi a}$]

40. A charged particle (charge q, mass m) has velocity v_0 at origin in +x direction. In space there is a uniform magnetic field B in -z direction. Find the y coordinate of particle when it crosses y axis. **[Ans:** $\frac{2mv_0}{qB}$ **]**

LEVEL - II

1. Two long straight parallel wires A and B are placed 50 cm apart and carry currents 20 amp and 15 amp respectively. A point 'P' is 40 cm from wire A and 30cm from wire B. Find the magnitude of the resultant magnetic field at 'P'. **[Ans:** $\sqrt{2}\times10^{-5}\text{T}$ **]**

2. Two long straight conductors AOB and COD are perpendicular to each other and carry currents I_1 and I_2. Find the magnitude of the magnetic inductions at a point 'P' at a distance 'a' from the point O in a direction perpendicular to the plane ACBD **[Ans:** $\frac{\mu_0}{2\pi a}\left(I_1^2+I_2^2\right)^{1/2}$ **]**

3. A $2\mu C$ charge moves in a circular orbit of radius 2cm around the nucleus at a frequency 10 rev/sec. Find the magnetic moment associated with the orbital motion of the particle **[Ans:** 4×10^{-9} Amp-m^2**]**

4. The magnetic induction at the centre of a circular coil of radius 10 cm is $5\sqrt{5}$ times the magnetic induction at a point on its axis. Find the distance of the point from the centre of the coil **[Ans: 20cm]**

5. A straight wire is first bent into a circle of radius 'r' and then into a square of side 'a' each of 1 turn. If the currents flowing through them are in the ratio 2 : 3, find the ratio between their effective magnetic moments? **[Ans:** $8/3\pi$ **]**

6. A galvanometer of resistance 40Ω can measure a current of 2mA for full scale deflection. It is converted into an ammeter having range 6 times the previous value by using proper shunt. Find the resistance of ammeter so formed ? **[Ans:** $\frac{20}{3}\Omega$ **]**

7. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milli ampere and voltage sensitivity is 2 divisions per millivolt : In order that each division reads 1 volt, find the resistance in ohms needed to be connected in series with the coil? **[Ans: 9995]**

8. What current would be maintained in a circular coil of wire of 1000 turns and 10 cm in radius in order to just cancel the earth's magnetic field at a place where the horizontal component of Earth's magnetic field is 2×10^{-4}T. **[Ans: 0.4A]**

9. A solenoid 1.0 m long and 4.0 cm in diameter possesses 10 turns/cm. A current of 5.0 A is flowing through it. Calculate the magnetic induction (a) inside and (b) at one end on the axis of solenoid. **[Ans:** $2\pi\times10^{-3}\text{T};\pi\times10^{-3}\text{T}$**]**

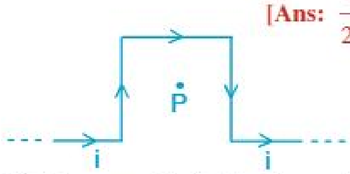
10. A horizontal wire carries 100A current below which another wire of linear density $2\times10^{-3}\text{kg m}^{-1}$ carrying a current is kept at 2cm distance. If the wire kept below hangs in air, what is the current in this wire ? **[Ans: 19.6A]**

11. Two long parallel wires are placed vertically, 10cm apart. One of them carries a current of 20A and other carries a current of 25A. Both the currents flow in the upward direction. A third wire carrying a current of 5A flowing downward is placed between the two parallel wires in such a way that its distance from the wire carrying current 20A is 6 cm. Calculate the force per unit length experienced by the third wire **[Ans:** 2.9×10^{-4} Nm^{-1}**]**

12. A wire of length 5 cm is placed inside the solenoid near its centre such that it makes an angle of 30° with the axis of the solenoid The wire carries a current of 5A and the magnetic field due to solenoid is 2.5×10^{-2} T Calculate the force on the wire. **[Ans:** 3.125×10^{-3}N**]**

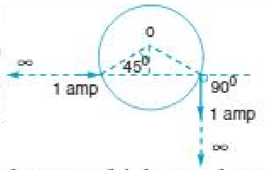
13. A rectangular coil of area 2.0×10^{-4} m^2 and 80 turns is pivoted about one of its vertical sides. The coil is in the radial horizontal magnetic field of 9.0×10^{-3} T. What is the torsional constant of the spring connected to the coil if a current of 0.1 mA produces an angular deflection of 10° ? **[Ans:** 1.44×10^{-9} Nm per degree**]**

14. Two circular coils A and B of radius $\frac{5}{\sqrt{2}}$ cm and 5 cm respectively carry current 5 Amp and $\frac{5}{\sqrt{2}}$ Amp respectively. The plane of B is perpendicular to plane of A and their centres coincide. Find the magnetic field at the centre. **[Ans:** $\frac{\sqrt{5}}{2\sqrt{2}}4\pi\times10^{-5}$T**]**

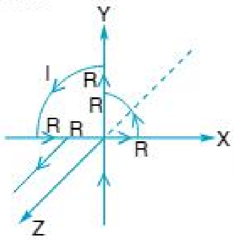
15. 

Find the magnetic field at the centre P of square of side a shown in figure. **[Ans:** $\frac{(2\sqrt{2}-1)\ \mu_0 i}{\pi\ a}$ **]**

16. What is the magnitude of magnetic field at the centre 'O' of the loop of radius $\sqrt{2}$ m made of uniform wire when a current of 1 amp enters the loop and taken out of it by two long wires as shown in the figure. [Ans: zero]

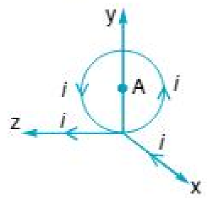


17. Find the magnetic induction at the origin in the figure shown.



[Ans: $\frac{\mu_0 I}{4R}\left(\frac{3}{4}\hat{k}+\frac{1}{\pi}\hat{j}\right)$]

18. Find the magnitude of the magnetic induction B of a magnetic field generated by a system of thin conductors, along which a current i is flowing, at the point A (0, R, 0), which is the centre of the circular conductor of radius R. The ring is in yz plane.



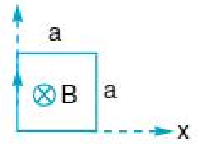
[Ans: $B=\frac{\mu_0 i}{4\pi R}\sqrt{2\left(2\pi^2-2\pi+1\right)}$]

19. Two circular coils of wire each having a radius of 4 cm and 10 turns have a common axis and are 6 cm apart. If a current of 1 A passes through each coil in the opposite direction find the magnetic induction.
(i) At the centre of either coil ;
(ii) At a point on the axis, midway between them.
[Ans: (i) 1.3×10^{-4}T, (ii) zero]

20. An electron is moving with a velocity $5\times10^6\ ms^{-1}\ \hat{i}$ in a uniform electric field of $5\times10^7\ Vm^{-1}\ \hat{j}$. Find the magnitude and direction of minimum uniform magnetic field in tesla that will cause the electron to move undeviated along its original path.
[Ans: $10\hat{k}$]

21. Two coils each of 100 turns are held at right angles such that one lies in the vertical plane with their centres coinciding. The radius of the vertical coil is 20 cm and that of the horizontal coil is 30 cm. How would you neutralize the magnetic field of the earth at their common centre ? What is the current to be passed through each coil? Horizontal component of earth's magnetic induction = 3.49×10^{-5} T and angle of dip = 30°. [Ans: $i_1=0.1110\,A,\ i_2=0.096\,A$]

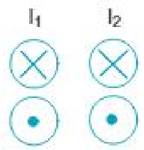
22. A rectangular loop of wire is oriented with the left corner at the origin, one edge along X-axis and the other edge along Y-axis as shown in the figure. A magnetic field is directed into the page and has a magnitude that is given by $B=\alpha y$ where α is a constant. Find the total magnetic force on the loop if it carries current i.



[Ans : $F=\alpha a^2 i\ \hat{j}$]

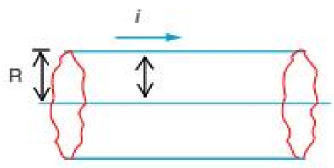
23. Find the ratio of magnetic field magnitudes at a distance 10 m along the axis and at 60° from the axis, from the centre of a coil of radius 1 cm, carrying a current 1 amp.
[Ans: $4/\sqrt{7}$]

24. A system of long four parallel conductors whose sections with the plane of the drawing lie at the vertices of a square carry four equal currents. The directions of these currents are as follows : those marked ⊗ point away from the reader, while those marked with a dot point towards the reader. How is the vector of magnetic induction directed at the centre of the square?



[Ans: In the plane of the drawing from right to left]

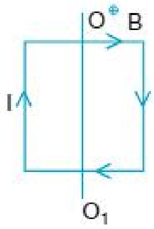
25. A cylindrical conductor of radius R carries a current along its length . The current density J, however, is not uniform over the cross section of the conductor but is a function of the radius according to J = br, where b is a constant. Find an expression for the magnetic field B.



(a) at $r_1 < R$
(b) at distance $r_2 > R$, measured from the axis.
Hint : [use Ampere's Law]

[Ans: $B_1=\frac{\mu_0 b r_1^2}{3},\ B_2=\frac{\mu_0 b R^3}{3r_2}$]

26. A square current carrying loop made of thin wire and having a mass m = 10g can rotate without friction with respect to the vertical axis O O_1, passing through the centre of the loop at right angles to two opposite sides of the loop. The loop is placed in a homogeneous magnetic field with an induction $B = 10^{-1}$ T directed at right angles to the plane of the drawing. A current I = 2A is flowing in the loop. Find the period of small oscillations that the loop performs about its position of stable equilibrium.



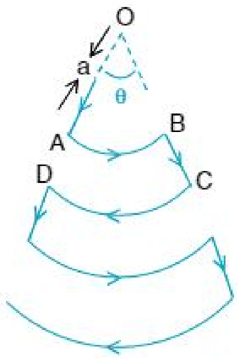
[Ans: $T_0 = 2\pi\sqrt{\frac{m}{6IB}} = 0.57$ s]

[Hint : Restoring torque equation :
$IABSin\theta = -I_0\ddot{\theta}$, where $A = a^2$

and $I_0 = \frac{m}{4}\left[\frac{a^2}{12}\times 2 + \left(\frac{a}{2}\right)^2 \times 2\right] = \frac{m}{4}\times\frac{2}{3}a^2$

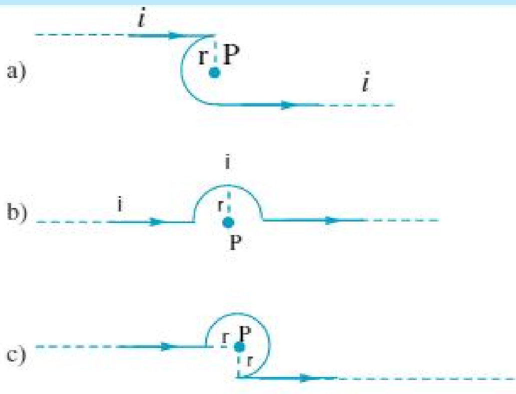
$\Rightarrow \left(\frac{m}{6}a^2\right)\ddot{\theta} = -a^2B\theta \Rightarrow \ddot{\theta} = -\left(\frac{6IB}{M}\right)\theta$]

27. A current i flows through an infinitely long wire having infinite bands as shown. The rad ius of the first curved section is a and the radii of the successive curved portions each increases by a factor η. Find magnetic field at O.



[Ans : $B = \dfrac{\frac{\mu_0 i\theta}{4\pi a}}{1+\frac{1}{\eta}} = \dfrac{\mu_0 i\theta\eta}{4\pi a(\eta+1)}$]

28. The diagram shows three cases. In all the three cases the circular part has radius r and straight ones are infinitely long. For the same current, i find $\overline{B}$ at the centre P.

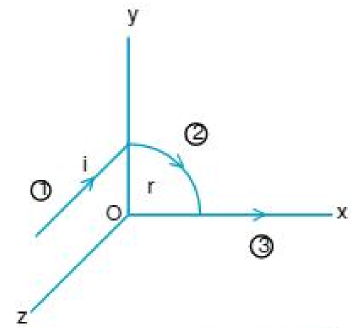


[Ans : a) $\overline{B}_P = \frac{\mu_0 i}{4r}\odot$

b) $\overline{B}_2 = B_p = \frac{1}{2}\frac{\mu_0 i}{2r}\otimes = \frac{\mu_0 i}{4r}\otimes$

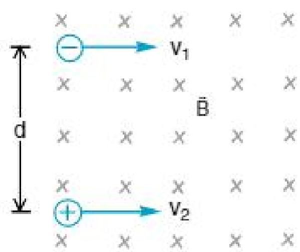
c) $\overline{B}_P = \frac{3}{4}\frac{\mu_0 i}{2r}\otimes + \frac{\mu_0 i}{4\pi r}\odot = \frac{\mu_0}{4}\frac{i}{r}\left(\frac{3}{2}-\frac{1}{\pi}\right)\otimes$]

29. Find magnetic field at O due to the current carrying conductor as shown in the figure.



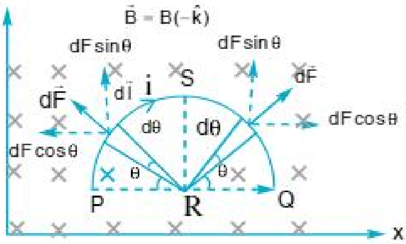
[Ans : $|B_0| = \frac{\mu_0}{4}\frac{i}{r}\sqrt{\frac{1}{\pi^2}+\frac{1}{4}}$]

30. Two identical particles having the same mass m and charges +q and -q separated by a distance d enter a uniform magnetic field $\overline{B}$ as shown in the figure. For what value of d the particles will not collide?



[Ans : $d > \frac{2m}{Bq}(V_1 + V_2)$]

31. In the figure shown, a semi-circular wire loop of radius R is placed in a uniform magnetic field B. The plane of the loop is perpendicular to the magnetic field. Find magnetic force on the loop.



[**Ans:** $Bi \times (2R)$ or $F = Bi(PQ)$ force will be along x-axis]

32. The flat insulating disc of radius 'a' carries an excess charge on its surface with surface charge density $\sigma\ C/m^2$. Let the disc rotate around the axis passing through its centre and perpendicular to its plane with angular speed ω rad/s. If a magnetic field $\vec{B}$ is directed perpendicular to the rotation axis, then find the torque acting on the disc.

[**Ans:** $= \pi\sigma\omega r^3\, B dr$ and $\tau = \pi\sigma\omega B\int_0^a r^3\,dr = \frac{\pi\sigma\omega B a^4}{4}$]

32. Consider a non conducting plate of radius a and mass m which has a charge q distributed uniformly over it, The plate is rotated about its own axis with an angular speed ω. Show that the magnetic moment M and the angular momentum L of the plate are related as $\frac{M}{L} = \frac{q}{2m}$.

[**Ans:** $\frac{M}{L} = \frac{\frac{q\omega a^2}{4}}{\frac{\omega m a^2}{2}} = \frac{q}{2m}$]

34. A thin uniform ring of radius R carrying charge q and mass m rotates about its axis with angular velocity ω. Find the ratio of its magnetic moment and angular momentum.

[**Ans:** $\frac{M}{L} = \frac{q}{2m}$]

35. A wire carrying current i has the configuration as shown in figure. The two semi-infinite straight sections are both tangents to the same circle, with all sections lying in the same plane. What must θ be in order for B to be zero at the centre of the circle?

[**Ans:** $2\left[\frac{\mu_0}{4\pi}\frac{i}{R}\right] = \frac{\mu_0 \frac{\theta}{2\pi}}{2R} \Rightarrow \theta = 2\,\text{rad}$]

36. A uniform constant magnetic field B is directed at an angle of 45^0 to the x - axis in xy-plane. PQRS is a rigid square wire frame carrying a steady current i_0, with its centre at the origin O. At time t=0, the frame is at rest in the position shown in the figure, with its side parallel to x and y axis. Each side of the frame is of mass M and length L.

(a) What is the torque $\vec{\tau}$ about O acting on the frame due to magnetic field?

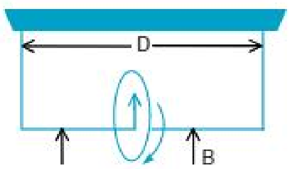
(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about which this rotation occurs (Δt is so short that any variation in the torque during this interval may be neglected).

[**Ans:** (a) $\vec{\tau} = \frac{i_0 L^2 B}{\sqrt{2}}(-\hat{i} + \hat{j})$; (b) $\frac{3}{4}\frac{i_0 B}{M}(\Delta t)^2$]

37. A non-conducting sphere of radius R charged uniformly with surface density σ rotates with an angular velocity ω about the axis passing through its centre. Find the magnetic induction at the centre of the sphere.

[**Ans:** $\mu_0\omega\sigma R\left|\cos\theta - \frac{\cos^3\theta}{3}\right|_0^{\pi/2} = \frac{2}{3}\mu_0\omega\sigma R$]

38. Ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings as shown in fig. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on and the ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension $\frac{3T_0}{2}$.

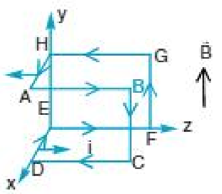


[**Ans :** $T_0 = \frac{\omega Q B R^2}{D}$ or $\omega = \frac{D T_0}{Q B R^2}$]

39. A long straight wire carries a current i. A particle having a positive charge q and mass m, kept at a distance x_0 from the wire is projected towards it with a speed v, as shown in Fig. Find the minimum separation between the wire and the particle.

[**Ans:** $\ell n\frac{x}{x_0} = \frac{-v}{k}$ or $x = x_{0e}^{\frac{-v}{k}} = x_0 e^{\frac{-2\pi m v}{\mu_0 q i}}$]

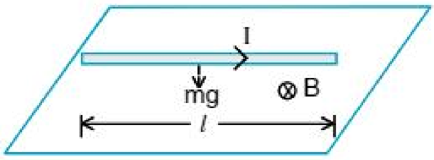
40. Given fig. shows a coil bent with all edges of length 1m and carrying a current of 1A. There exists in space a uniform magnetic field of 2T in the positive y-direction. Find the torque on the loop.



[**Ans :** $\vec{\tau} = F_{AH}\ell(\hat{i}) = Bi\ell \times \ell(\hat{i}) = 2\times1\times1^2 = 2\hat{i}N - m.$]

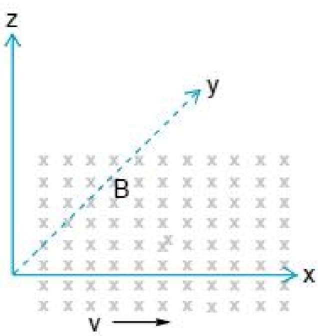
ADDITIONAL EXERCISE

1. A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B (Fig.) What is the magnitude of the magnetic field?



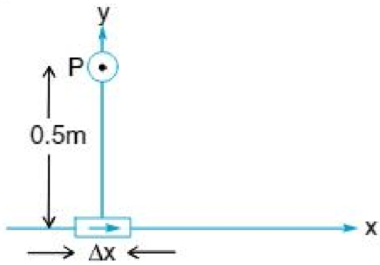
2. What is the radius of the path of an electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} C) moving at a speed of $3{\times}10^7$m/s in a magnetic field of $6{\times}10^{-4}$ T perpendicular to it? What is its frequency? Calculate its energy in keV. ($1eV=1.6{\times}10^{-19}$ J).

3. If the magnetic field is parallel to the positive y-axis and the charged particle is moving along the positive x-axis (Fig.) Which way would the Lorentz force be for (a) an electron (negative charge), (b) a proton (positive charge)



4. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its 'dees' is 60 cm, what is the kinetic energy (in MeV) of the proton beam produced by the accelerator. (e $=1.60 \times 10^{-19}$ C, mp $= 1.67 \times 10^{-27}$ kg, 1 MeV $= 1.6 \times 10^{-13}$ J).

5. An element $\Delta l = \Delta x\hat{i}$ is placed at the origin and carries a large current I = 10 A (Fig.). What is the magnetic field on the y-axis at a distance of 0.5 m. Δx = 1 cm.

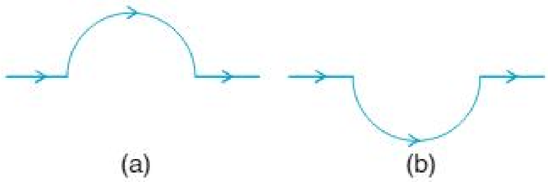


6. A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in Fig. (a). Consider the magnetic field **B** at the centre of the arc.

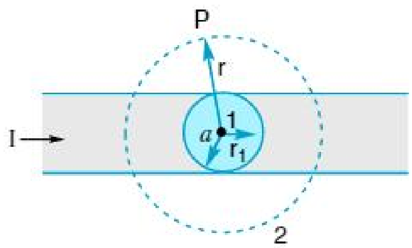
(a) What is the magnetic field due to the straight segments?

(b) In what way the contribution to **B** from the semicircle differs from that of a circular loop and in what way does it resemble?

(c) Would your answer be different if the wire were bent into a semi-circular arc of the same radius but in the opposite way as shown in Fig. (b)?



7. Figure shows a long straight wire of a circular cross-section (radius *a*) carrying steady current *I*. The current *I* is uniformly distributed across this cross-section. Calculate the magnetic field in the region $r < a$ and $r > a$.



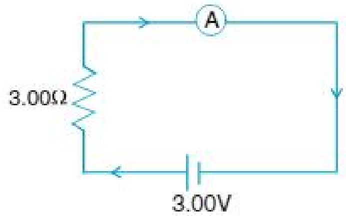
8. Consider a tightly wound 100 turn coil of radius 10 cm, carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil?

9. A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid?

10. The horizontal component of the earth's magnetic field at a certain place is 3.0×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is (a) east to west; (b) south to north?

11. A 100 turn closely wound circular coil of radius 10 cm carries a current of 3.2 A.
(a) What is the field at the centre of the coil?
(b) What is the magnetic moment of this coil? The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of 2T in the horizontal direction exists such that initially the axis of the coil is in the direction of the field. The coil rotates through an angle of 90º under the influence of the magnetic field.
(c) What are the magnitudes of the torques on the coil in the initial and final position?
(d) What is the angular speed acquired by the coil when it has rotated by 90º? The moment of inertia of the coil is 0.1 kg m^2.

12. In the circuit (Fig.) the current is to be measured. What is the value of the current if the ammeter shown
(a) is a galvanometer with a resistance $R_G = 60.00\ \Omega$;
(b) is a galvanometer described in (a) but converted to an ammeter by a shunt resistance $r_s = 0.02\ \Omega$;
(c) is an ideal ammeter with zero resistance?



13. (a) A current-carrying circular loop lies on a smooth horizontal plane. Can a uniform magnetic field be set up in such a manner that the loop turns around itself (i.e., turns about the vertical axis).
(b) A current-carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium? Show that in this orientation, the flux of the total field (external field + field produced by the loop) is maximum.
(c) A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape?