1. BINOMIAL THEOREM

SYNOPSIS

Binomial theorem for positive integral index and some key factors related to Binomial theorem

1. If n is a positive integer then

$$(x+a)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} a^r = {}^nC_0 x^n + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + \ldots\ldots\ldots\ldots + {}^nC_n a^n$$

i) The number of terms in the expansion $(x+a)^n$ is $n+1$

ii) The sum of the powers of x and a in any term in the expansion of $(x+a)^n$ is n

iii) The general term in the expansion of $(x+a)^n$ is $T_{r+1} = {}^nC_r x^{n-r} a^r$

iv) ${}^nC_0, {}^nC_1, {}^nC_2, \ldots\ldots\ldots\ldots {}^nC_n$ are binomial coefficients in the expansion of $(x+a)^n$

v) The binomial coefficients which are equidistant from the begining and from the ending are equal.

i.e. ${}^nC_0 = {}^nC_n$, ${}^nC_1 = {}^nC_{n-1}$; ${}^nC_2 = {}^nC_{n-2} \ldots\ldots\ldots$ etc.

vi) In the expansion of $(x+a)^n$, r^{th} term from the end is equal to $(n-r+2)^{th}$ term from the begining.

2. $$(x-a)^n = \sum_{r=0}^{n} (-1)^r\, {}^nC_r x^{n-r} a^r = {}^nC_0 x^n - {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 - \ldots\ldots\ldots + (-1)^n\, {}^nC_n a^n$$

The general term in this expansion is $T_{r+1} = (-1)^r\, {}^nC_r\, x^{n-r} a^r$

Note : **The expansions of $(x+a)^n$ and $(a+x)^n$ are equal but their respective terms are not equal.**

3. $$(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots\ldots + {}^nC_n \cdot x^n$$

The general term in the expansion $T_{r+1} = {}^nC_r x^r$

4. $$(1-x)^n = \sum_{r=0}^{n} (-1)^r\, {}^nC_r\, x^r = {}^nC_0 - {}^nC_1 \cdot x + {}^nC_2\, x^2 - \ldots\ldots + (-1)^n\, {}^nC_n x^n$$

The general term in the expansion $T_{r+1} = (-1)^r\, {}^nC_r x^r$

Number of terms

5. a) The number of non-zero terms in the expansion of $\{(x+a)^n + (x-a)^n\}$ is

i) $\frac{n+1}{2}$, if n **is an odd** integer. ii) $\frac{n}{2}+1$, if n **is even** integer.

b) The number of non-zero terms in the expansion of $\{(x+a)^n - (x-a)^n\}$ is

i) $\frac{n+1}{2}$, if n **is an odd** integer. ii) $\frac{n}{2}$, if n **is even** integer.

c) The number of terms in the expansion of $(x+a)^n + (x-a)^n + (x+ai)^n$ is $\left[\frac{n+4}{4}\right]$. When [.] is G.I.F.

6. i) If **n is odd** there will be **two middle** terms in the expansion $(x+a)^n$ which are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms

ii) If **n is even** there will be only **one middle** term in the expansion $(x+a)^n$, which is $\left(\frac{n}{2}+1\right)^{th}$ term

7. The coefficient of the middle term is the greatest binomal coeffcient in the expansion of $(x+a)^n$

8. i) If **n is odd,** there are **two** greatest binomial co-efficients in the expansion which are ${}^nC_{\frac{n-1}{2}}$ and ${}^nC_{\frac{n+1}{2}}$ also ${}^nC_{n-1/2} = {}^nC_{\frac{n+1}{2}}$.

ii) If **n is even**, there is only **one** greatest binomial coefficient in the expansion $(x+a)^n$ which is ${}^nC_{n/2}$

9. The coefficient of x^k in the expansion of $\left(ax^p+\frac{b}{x^q}\right)^n$ is ${}^nC_r\, a^{n-r} b^r$ where $r=\frac{np-k}{p+q}$

10. The term independent of x in the expansion of $\left(ax^p+\frac{b}{x^q}\right)^n$ is ${}^nC_r\, a^{n-r} b^r$ where $r=\frac{np}{p+q}$

11. a) A generalised multinomial theorem

$$(x_1+x_2+x_3+\ldots\ldots x_m)^n = \sum \frac{n!}{n_1!n_2\ldots\ldots n_m!} x_1^{n1} x_2^{n2}\ldots\ldots x_n^{n_m}$$

Where the summation is taken over all non negetive integers n_1, n_2 .... n_m such that $n_1 + n_2 + \ldots + n_m = n$

b) The general term in the expansion of $(x_1+x_2+\ldots\ldots+x_m)^n$ is $\frac{(n_1+n_2+\ldots\ldots+x_m^{n_m})!}{n_1!n_2!\ldots n_m!}\left(x_1^{n_1}x_2^{n_2}\ldots x_p^{n_p}\right)$

c) No. of terms in the expansion of $(x_1+x_2+\ldots\ldots+x_m)^n$ is ${}^{(n+m-1)}C_{(m-1)}$

d) The number of terms in the expansion of $(x+y+z)^n$ is $\frac{(n+1)(n+2)}{2}$.

e) The greatest coefficient in the expansion of $(x_1+x_2+\ldots\ldots x_m)^n$ is equal to $\frac{n!}{(q!)^{m-r}((q+1)!)^r}$

where q is the quotient and r is the remainder when n is divided by m

12. **Numerically greatest term** (N.G.T.) in the expansion of $(1+x)^n$

i) If $\frac{(n+1)|x|}{|x|+1}=P+f$, then there exists only **one** N.G.T. which is $(P+1)^{th}$ term and its value is $|T_{P+1}|$. (Where P is an integer and f is a proper fraction, $0<f<1$)

ii) If $\frac{(n+1)|x|}{|x|+1}=P$ is an integer then there are **two** numerically greatest terms which are P^{th} and $(P+1)^{th}$ terms. Also $|T_P|=|T_{P+1}|$.

Note : To find numerically greatest term of $(a+b)^n$ we write $(a+b)^n = a^n(1+x)^n$ where $x = \frac{b}{a}$ and proceed.

13. i) If $n>2$, $n \in N$, then $(2n-1)^n + (2n)^n < (2n+1)^n$

ii) If the coefficients of x^{r-1}, x^r, x^{r+1} in $(1+x)^n$ are in A.P. then $(n-2r)^2 = n+2$.

14. i) $(1+\alpha)^n - 1$ is divisible by $M(\alpha)$

ii) $(1+\alpha)^n - n\alpha - 1$ is divisible by $M(\alpha^2)$

iii) $(1+\alpha)^n - {}^nC_2\alpha^2 - n\alpha - 1$ is divisible by $M(\alpha^3)$

15. i) Coefficient of x^{n-1} in $(x-\alpha_1)(x-\alpha_2).......(x-\alpha_n)$ is $-(\alpha_1+\alpha_2+.......+\alpha_n)$

ii) Coefficient of x^{n-1} in $(x+\alpha_1)(x+\alpha_2).......(x+\alpha_n)$ is $(\alpha_1+\alpha_2+.......+\alpha_n)$

iii) Coefficient of x^{n-2} in $(x-\alpha_1)(x-\alpha_2).......(x-\alpha_n)$ is $\frac{(\alpha_1+\alpha_2+.......+\alpha_n)^2-(\alpha_1{}^2+\alpha_2{}^2+.........+\alpha_n{}^2)}{2}$

Binomial Coefficients :

$(x+a)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1}a + {}^nC_2\, x^{n-2}a^2 + ... + {}^nC_r\, x^{n-r}a^r + ... + {}^nC_n\, a^n$, Here the coefficients ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_r, ...\ {}^nC_n$ are called binomial coefficients.

Note :

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ...+ {}^nC_r\, x^r + ... + {}^nC_n\, x^n$ the coefficients ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_r, ..., {}^nC_n$ are simply denoted by $C_0, C_1, C_2, ..., C_r, ..., C_n$ respectively

i.e., $(1+x)^n = C_0 + C_1x + C_2x^2 + ...+ C_rx^r + ...+ C_nx^n$

Standard results on Binomial coefficients

1. $C_0 + C_1 + C_2 + ...+ C_n = 2^n \Rightarrow \sum_{r=0}^{n} c_r = 2^n$

2. $C_0 - C_1 + C_2 - C_3 + ... + (-1)^n.C_n = 0$

3. $C_0 + C_2 + C_4 + ...= C_1 + C_3 + C_5 + ...= 2^{n-1}$

4. $a.C_0 + (a+d).C_1 + (a+2d).C_2 + ...+ (a+nd).C_n = (2a+nd)\ 2^{n-1}$

5. $a.C_0 - (a+d).C_1 + (a+2d).C_2 - .... = 0$

6. $C_1 + 2.C_2 + 3.C_3 + ...+ n.C_n = n.2^{n-1} \Rightarrow \sum_{r=1}^{n} r.{}^nC_r = n.2^{n-1}$

7. $C_1 - 2.C_2 + 3.C_3 - ....= 0 \Rightarrow \sum_{r=1}^{n} (-1)^{r-1} r.{}^nC_r = 0$

8. $a.C_0^2 + (a+d).C_1^2 + (a+2d).C_2^2 + ...+ (a+nd).C_n^2 = \frac{1}{2}(2a+nd)\ {}^{2n}C_n$

9. ${}^{m}C_0{}^{n}C_r + {}^{m}C_1{}^{n}C_{r-1} + \ldots\ldots + {}^{m}C_r{}^{n}C_0 = {}^{(m+n)}C_r$

10. $C_0 + (C_0 + C_1) + (C_0 + C_1 + C_2) + \ldots + (C_0 + C_1 + \ldots\ldots + C_{n-1}) = n.2^{n-1}$

11. $C_0C_r + C_1C_{r+1} + \ldots\ldots + C_{n-r}C_n = {}^{2n}C_{n-r}$ or ${}^{2n}C_{n+r}$

12. $C_0^2 + C_1^2 + C_2^2 + C_3^2 + \ldots + C_n^2 = {}^{2n}C_n = \dfrac{(2n)!}{(n!)^2}$

13. $C_0^2 - C_1^2 + C_2^2 - \ldots\ldots + (-1)^n C_n^2 = \begin{cases} 0 & , \text{if } n \text{ is odd} \\ (-1)^{n/2}.{}^{n}C_{\frac{n}{2}}, & \text{if } n \text{ is even} \end{cases}$

14. $C_0 + \dfrac{C_1}{2}x + \dfrac{C_2}{3}x^2 + \ldots\ldots + \dfrac{C_n}{n+1}x^n = \dfrac{(1+x)^{n+1} - 1}{(n+1)x}$

15. $\dfrac{C_0}{1} + \dfrac{C_1}{2} + \dfrac{C_2}{3} + \ldots\ldots + \dfrac{C_n}{n+1} = \dfrac{2^{n+1} - 1}{n+1}$

16. $\dfrac{C_1}{2} + \dfrac{C_3}{4} + \ldots\ldots = \dfrac{2^n - 1}{n+1}$

17. $\dfrac{C_0}{1} - \dfrac{C_1}{2} + \dfrac{C_2}{3} - \ldots\ldots + \dfrac{(-1)^n \cdot C_n}{n+1} = \dfrac{1}{n+1}$

18. $\dfrac{C_0}{1} + \dfrac{C_2}{3} + \dfrac{C_4}{5} + \ldots\ldots = \dfrac{2^n}{n+1}$

19. a) $\sum_{r=0}^{n} r^2.c_r = n(n+1).2^{n-2}$ b) $\sum_{r=0}^{n} (-1)^r r^2.c_r = 0$

20. a) $\sum_{r=0}^{n} r^3.c_r = n^2(n+3).2^{n-3}$ b) $\sum_{r=0}^{n} (-1)^r r^3.c_r = 0$

21. Let $f(x)$ is any polynomial function which is expansion of any multinomial raised to some power, $f(x) = a_0 + a_1x + a_2x^{2l + \cdots a}{}_n x_n$ is identity in x and they true for all real (or) complex

$a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$

i) Sum of the Coefficients $= f(1)$

ii) Sum of the Coefficients of x having even powers is $\dfrac{f(1) + f(-1)}{2}$

iii) Sum of the Coefficients of x having odd powers is $\dfrac{f(1) - f(-1)}{2}$

iv) $a_0 - a_2 + a_4 + \ldots\ldots = \dfrac{f(i) + f(-i)}{2}$

v) $a_0 + a_3 + a_6 + ... = \dfrac{f(1) + f(w) + f(w^2)}{3}$

vi) $a_0 + a_4 + a_8 + ... = \dfrac{f(1) + f(-1) + f(i) + f(-i)}{4}$

vii) $a_0 + a_n + a_{2n} + ... = \dfrac{f(1) + f(\alpha) + f(\alpha^2) + ... + f(\alpha^{n-1})}{n}$

where $1, \alpha, \alpha^2 ......\alpha^{n-1}$ the n^{th} roots of unity

viii) $(a_0 - a_2 + a_4 ....)^2 + (a_1 - a_3 + a_5 ...)^2 = f(i) f(-i)$

Binomial theorem for rational Index :

It n is not a positive integer and $|x| < 1$ then

1. $(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \text{........} + \dfrac{n(n-1)(n-2).....(n-r+1)}{r!}x^r + ...\infty$

2. $(1-x)^{-n} = 1 + nx + \dfrac{n(n+1)}{2!}x^2 + \dfrac{n(n+1)(n+2)}{3!}x^3 + ... + \dfrac{n(n+1)(n+2)....(n+r-1)}{r!}x^r + ..\infty$

3. $(1-x)^{-n} = 1 + {}^{n}C_1 x + {}^{(n+1)}C_2 x^2 + ... + {}^{(n+r-1)}C_r x^r + ...\infty$, if n is a positive integer

4. $(1-x)^{-1} = 1 + x + x^2 + x^3 + ... + x^r + ... + \infty$

5. $(1+x)^{-1} = 1 - x + x^2 - x^3 + ... + (-1)^r x^r + ... + \infty$

6. $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 ..... + (r+1)x^r + ...\infty$

7. $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + ... + (-1)^r (r+1) x^r + ...\infty$

8. $(1-x)^{-3} = \dfrac{1}{1.2}[1.2 + 2.3x + 3.4x^2 .... + (r+1)(r+2)x^r + ...\infty]$

9. $(1+x)^{-3} = \dfrac{1}{1.2}[1.2 - 2.3x + 3.4x^2 ...... + (-1)^r (r+1)\ (r+2)x^r + ...\infty]$

10. $(1-x)^{\frac{-p}{q}} = 1 + \dfrac{p}{1!}\left(\dfrac{x}{q}\right) + \dfrac{p(p+q)}{2!}\left(\dfrac{x}{q}\right)^2 + \dfrac{p(p+q)(p+2q)}{3!}\left(\dfrac{x}{q}\right)^3 + ... + ...$

$$..... + \frac{p(p+q)(p+2q).........(p+(r-1)q)}{r!}\left(\frac{x}{q}\right)^r + ...\infty$$

11. $(1+x)^{\frac{-p}{q}} = 1 - \dfrac{p}{1!}\left(\dfrac{x}{q}\right) + \dfrac{p(p+q)}{2!}\left(\dfrac{x}{q}\right)^2 - \dfrac{p(p+q)(p+2q)}{3!}\left(\dfrac{x}{q}\right)^3 + ... + ...$

$$..... + (-1)^r \frac{p(p+q)(p+2q)........(p+(r-1)q)}{r!}\left(\frac{x}{q}\right)^r + ...\infty$$

LECTURE SHEET

EXERCISE-I

Binomial expansion for positive integral index, Middle term, Numerically greatest term, R-f factor relation & Multinomial theorem

LEVEL-I (MAIN)

Single answer type questions

1. The third term in the expansion of $\left(\frac{1}{x}+x\log_{10}x\right)^5$ is 1 then $x =$

1) 1 2) 10 3) 10^2 4) 10^3

2. In the binomial expansion of $(a-b)^n$, $n>5$, the sum of 5th and 6th terms is zero, then $\frac{a}{b}$ equals

1) $\frac{n-4}{5}$ 2) $\frac{5}{n-4}$ 3) $\frac{6}{n-5}$ 4) $\frac{n-5}{6}$

3. If the ratio of the seventh term from beginning in $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ to seventh term from end is 1/6, then $n =$

1) 3 2) 6 3) 12 4) 9

4. If the sum of odd terms and the sum of even terms in $(x+a)^n$ are p and q respectively then $4pq =$

1) $(x+a)^{2n}-(x-a)^{2n}$ 2) $(x^2-a^2)^{2n}+(x+a)^{2n}$

3) $(x^2-a^2)^n-(x-a)^{2n}$ 4) $(x^2+a^2)^n+(x-a)^{2n}$

5. If the sum of odd terms and the sum of even terms in $(x+a)^n$ are p and q respectively then $p^2+q^2=$

1) $\frac{(x+a)^{2n}-(x-a)^{2n}}{2}$ 2) $(x+a)^{2n}-(x-a)^{2n}$

3) $\frac{(x+a)^{2n}+(x-a)^{2n}}{2}$ 4) $(x+a)^{2n}+(x-a)^{2n}$

6. If $T_0, T_1, T_2, \ldots\ldots, T_n$ represent the terms in $(x+a)^n$, then $(T_0-T_2+T_4-T_6+\ldots\ldots)^2+(T_1-T_3+T_5-\ldots\ldots)^2$ is

1) $(x^2-a^2)^2$ 2) $(x^2+a^2)^n$ 3) $(a^2-x^2)^n$ 4) $(x^2+a^2)^{2n}$

7. The expression $\left[x+(x^3-1)^{1/2}\right]^5+\left[x-(x^3-1)^{1/2}\right]^5$ is a polynomial of degree

1) 7 2) 4 3) 5 4) 6

8. If the coefficient of $(2r+4)^{th}$ term and $(r-2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal then $r =$

1) 9 2) 4 3) 6 4) 3

9. If the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ equals the coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$, then a and b satisfy the relation

1) $ab = 1$ 2) $\frac{a}{b}=1$ 3) $a + b = 1$ 4) $a - b = 1$

10. In the expansion of $\left(\frac{1}{x^2}-x^3\right)^n$ $n \in N$, if the sum of the coefficients of x^5 and x^{10} is 0, then n is:

1) 25 2) 20 3) 15 4) None of these

11. If the constant term in the binomial expansion of $\left(x^2-\frac{1}{x}\right)^n, n \in N$ is 15 then the value of n is equal to

1) 6 2) 9 3) 12 4) 15

12. Term independent of x in $\left(x-\frac{1}{x}\right)^4\left(x+\frac{1}{x}\right)^3$ is

1) 1 2) 2 3) 0 4) 4

13. The term independent of x in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is

1) $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2$ 2) ${}^{2n}C_n$

3) $\frac{1.3.5.........(2n-1)}{n!}2^n$ 4) all the above

14. The coefficient of x^{53} in $\sum_{r=0}^{100} {}^{100}C_r(x-3)^{100-r}\cdot 2^r$ is

1) ${}^{100}C_{47}$ 2) ${}^{100}C_{53}$ 3) $-({}^{100}C_{53})$ 4) ${}^{100}C_{100}$

15. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is

1) $(n-1)$ 2) $(-1)^n(1-n)$ 3) $(-1)^{n-1}(n-1)^2$ 4) $(-1)^{n-1}n$

16. Coefficient of x^5 in $(1 + x)^{21}+(1+x)^{22}+\ldots\ldots\ldots\ldots+(1+ x)^{30}$ is

1) ${}^{51}C_5$ 2) ${}^{9}C_5$ 3) ${}^{31}C_6-{}^{21}C_6$ 4) ${}^{30}C_5+{}^{20}C_5$

17. If r^{th} term is the middle term in the expansion of $\left(x^2-\frac{1}{2x}\right)^{20}$, then $(r + 3)^{th}$ term is:

1) ${}^{20}C_{14}\frac{x}{2^{14}}$ 2) ${}^{20}C_{12}x^2 2^{-12}$ 3) $-{}^{20}C_7 x 2^{-13}$ 4) none of these

18. The middle term in the expansion of $(1 - 3x + 3x^2 - x^3)^{2n}$ is

1) ${}^{6n}C_{3n}(-x)^{3n}$ 2) ${}^{6n}C_{2n}(-x)^{2n+1}$ 3) ${}^{4n}C_{3n}(-x)^{3n}$ 4) ${}^{6n}C_{3n}(-x)^{3n-1}$

19. The coefficeint of the middle term in $(1+\alpha x)^4$ and $(1-\alpha x)^6$ is same then α =

1) – 5/3 2) 3/5 3) –3/ 10 4) 10/3

20. The numerically greatest term in the expansion $(5x - 6y)^{14}$ when $x = 2/5$, $y = 1/2$ is

1) $^{14}C_6\, 2^8 .3^6$ 2) $^{14}C_7\, 2^6 .3^8$ 3) $^{14}C_6\, 2^6 .3^8$ 4) $^{14}C_7\, 2^8 .3^6$

21. The greatest coefficient (numerically) in $\left(2x-\frac{1}{3x}\right)^{10}$ is

1) 5120 2) $\frac{1720}{3}$ 3) 1618 4) $\frac{5120}{3}$

22. If the middle term of $(1 + x)^{2n}$ is the greatest term then x lies between

1) $n-1<x<n$ 2) $\frac{n}{n+1}<x<\frac{n+1}{n}$ 3) $n<x<n+1$ 4) $\frac{n+1}{n}<x<\frac{n}{n+1}$

23. The greatest coefficient in $\left(\frac{x^{3/2}y}{2}+\frac{2}{xy^{3/2}}\right)^{12}$ is

1) $12(2^{11})$ 2) $12(2^{10})$ 3) $12(2^{22})$ 4) $33(2^9)$

24. Integral part of $(7 + 4\sqrt{3})^n$ is $(n \in N)$

1) an even number
2) an odd number
3) an even or an odd number depending upon the value of n
4) nothing can be said

25. Integral part of $(7+5\sqrt{2})^{2n+1}$ is $(n \in N)$

1) an even number
2) an odd number
3) an even or an odd number depending upon the value of n
4) nothing can be said

26. If $R=(6\sqrt{6}+14)^{2n+1}$ and $f = R - [R]$, where [.] denotes the G.I.F., then Rf =

1) 20^n 2) 20^{2n} 3) 20^{2n+1} 4) 1

27. The integral part of $(\sqrt{2}+1)^6$ is

1) 198 2) 196 3) 197 4) 199

28. The greatest integer which divides the number $101^{100} - 1$ is

1) 10^2 2) 10^3 3) 10^4 4) 10^5

29. The remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9 is

1) 2 2) 7 3) 8 4) 0

30. The coefficient of x^5 in the expansion of $(x^2 - x - 2)^5$ is

1) –83 2) –82 3) –81 4) 0

31. Coefficient of $x^3y^4z^2$ in $(2x-3y+4z)^9$ is

1) $-\frac{9!}{4!4!}2^3 3^4 4^2$ 2) $-\frac{9!}{3!2!4!}2^3 3^4 4^2$ 3) $\frac{9!}{4!\ 4!}2^3 3^4 4^2$ 4) $\frac{9!}{3!2!4!}2^3 3^4 4^2$

32. No.of terms in $(1+5\sqrt{2}x)^9+(1-5\sqrt{2}x)^9$ if $x>0$, is

1) 3 2) 5 3) 4 4) 6

33. No.of terms in $(1+3x+3x^2+x^3)^6$ is :

1) 17 2) 19 3) 21 4) 16

34. The number of distinct terms in $(a+b+c+d+e)^3$ is

1) 35 2) 38 3) 42 4) 45

35. No.of nonzero terms in $(1+x)^{42}+(1-x)^{42}+(1+ix)^{42}+(1-ix)^{42}$ is

1) 11 2) 12 3) 21 4) 22

36. The number of irrational terms in the expansion of $(\sqrt[5]{3}+\sqrt[3]{7})^{36}$ is

1) 30 2) 34 3) 31 4) 29

37. Sum of rational terms in $(\sqrt{2}+\sqrt[5]{3})^{10}$ is

1) 41 2) 42 3) 32 4) 38

38. $x+y=1 \Rightarrow \sum_{r=0}^{n} r\cdot {}^nC_r x^r y^{n-r} =$

1) $2nx$ 2) nx 3) ny 4) nx^2

39. If the coefficients of $r, (r+1), (r+2)$ terms in $(1+x)^{14}$ are in A.P. then $r=$

1) 3, 2 2) 5, 9 3) 2, 4 4) 5, 3

40. If a_1, a_2, a_3, a_4 are the coefficients of 2^{nd}, 3^{rd}, 4^{th} and 5^{th} terms of $(1+x)^n$ respectively then $\frac{a_1}{a_1+a_2}, \frac{a_2}{a_2+a_3}, \frac{a_3}{a_3+a_4}$ are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

41. For natural numbers m, n if $(1-y)^m(1+y)^n=1+a_1y+a_2y^2+........$, and $a_1=a_2=10$, then $(m,n)$is :

1) (35, 20) 2) (45, 35) 3) (35, 45) 4) (20, 45)

Numerical value type questions

42. If 7 divides $32^{32^{32}}$, then find the remainder

43. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$ (where $\frac{p}{q}=0$, if $p<q$) is maximum when m is the sum of the digits of m is

44. If $f(x)=\sum_{r=1}^{n}\{r^2({}^nC_r-{}^nC_{r-1})+(2r+1){}^nC_r\}$ and $f(30)=30(2)^n$, then value of x is

LEVEL-II (ADVANCED)

Single answer type questions

1. If the 4th term of $\left\{\sqrt{x^{\frac{1}{1+\log_{10}x}}}+\sqrt[12]{x}\right\}^6$ is equal to 200, $x > 1$ and the logarithm is common logarithm, then x is not divisible is

a) 2 b) 5 c) 10 d) 4

2. If $p^4 + q^3 = 2$ $(p > 0, q > 0)$, then the maximum value of term independent of x in the expansion of $\left(px^{\frac{1}{12}}+qx^{-\frac{1}{9}}\right)^{14}$ is

a) ${}^{14}C_4$ b) ${}^{14}C_6$ c) ${}^{14}C_7$ d) ${}^{14}C_{12}$

3. If $f(x)$ is periodic with period 't' such that $f(2x + 3) + f(2x + 7) = 2$, then the coefficient of m^{-3t} in expansion of $\left(m+\frac{b}{m^3}\right)^{4t}$ is

a) ${}^{16}C_7 b^7$ b) ${}^{32}C_{30} b^{30}$ c) ${}^{16}C_5 b^5$ d) ${}^{32}C_4 b^4$

4. The coefficient of x^{301} in the expansion of $(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498} + ... x^{500}$ is

a) ${}^{501}C_{301}$ b) ${}^{500}C_{301}$ c) ${}^{501}C_{300}$ d) none of these

5. The coefficient of x^{70} in the product $(x - 1)(x^2 - 2)(x^3 - 3)(x^4 - 4) ....(x^{12} - 12)$ is

a) 4 b) 6 c) 8 d) 12

6. Coefficient of x^{2016} in $(1+x+x^2+x^3+x^4)^{1001}(1-x)^{1002}(1+7x^{14})$ is

a) 0 b) $-7.{}^{1001}C_{999}$ c) $7.{}^{1001}C_{403}$ d) ${}^{1001}C_{598}$

7. Coefficient of x^6 in $((1+x)(1+x^2)^2(1+x^3)^3....(1+x^n)^n)$ is

a) 26 b) 28 c) 30 d) 35

8. The sum of all the coefficients of those terms in the expansion of $(a+b+c+d)^8$ which contains b but not c is

a) 6305 b) 6561 c) 256 d) 4^8

9. The number of distinct terms in the expansion of $(x + y^2)^{13} + (x^2 + y)^{14}$ is

a) 27 b) 29 c) 28 d) 25

10. If n is an even integer and a, b, c are distinct, the number of distinct terms in the expansion of $(a+b+c)^n + (a+b-c)^n$ is

a) $\left(\frac{n}{2}\right)^2$ b) $\left(\frac{n+1}{2}\right)^2$ c) $\left(\frac{n+2}{2}\right)^2$ d) $\left(\frac{n+3}{2}\right)^2$

11. The coefficient of x^4 in the expansion of $\left(1+2x+\frac{3}{x^2}\right)^6$ is

a) 240 b) 250 c) 260 d) 230

12. The number of terms in the expansion of $\left(x^3+\frac{1}{x^3}+1\right)^{100}$ is

a) 301 b) 201 c) 101 d) None of these

13. The coefficient of $a^{10}b^7c^3$ in the expansion of $(bc + ca + ab)^{10}$ is

a) 30 b) 60 c) 120 d) 240

14. If $x=(2+\sqrt{3})^n, n \in N$ and $f = x - [x]$, then $\frac{f^2}{1-f}$ is

a) an irrational number b) a non-integer rational number

c) an odd number d) an even number

15. If $n > 0$ is an odd integer, and $x=(\sqrt{2}+1)^n$ and $f = x - [x]$, then $\frac{1-f^2}{f}$ is

a) an irrational number b) a non-integer rational number

c) an odd integer d) an even integer

16. If 6th term in the expansion of $\left(\frac{3}{2}+\frac{x}{3}\right)^{11}$ is numerically greatest, when $x = 3$, then the sum of possible integral value of 'n' is

a) 23 b) 24 c) 25 d) 26

17. The algebraically second largest term in the expansion of $(3-2x)^{15}$ at $x=\frac{4}{3}$.

a) 5 b) 7 c) 9 d) 11

18. The remainder when $27^{10} + 7^{51}$ is divided by 10

a) 4 b) 6 c) 9 d) 2

More than one correct answer type questions

19. The 9th term of $\left(\frac{\sqrt{10}}{\left(\sqrt{x}\right)^{5\log_{10}x}}+x.x^{\frac{1}{2\log_{10}x}}\right)^{10}$ is 450, then the rational value of x is

a) 10 b) 100 c) $\frac{1}{10}$ d) $(10)^{-2/5}$

20. If a, b, c, d are any four consecutive coefficients of $(1+x)^n$ then which of the following is (are) correct

a) $\frac{a}{a+b}+\frac{c}{c+d}=\frac{2b}{b+c}$

b) $\left(\frac{b}{b+c}\right)^2>\frac{ac}{(a+b)(c+d)}$

c) $\left(\frac{b}{b+c}\right)^2<\frac{ac}{(a+b)(c+d)}$

d) $\left(\frac{b}{b+c}\right)^2=\frac{ac}{(a+b)(c+d)}$

21. If recursion polynomials $P_k(x)$ are defined as $P_1(x)=(x-2)^2, P_2(x)=((x-2)^2-2)^2$, $P_3(x)=(((x-2)^2-2)^2-2)^2$,... (In general $P_k(x)=(P_{k-1}(x)-2)^2$), then

a) In $P_k(x)$ constant term is 4

b) In $P_k(x)$ coefficient of x is 4^k

c) In $P_k(x)$ coefficient of x is -4^k

d) In $P_k(x)$ coefficient of x^2 is $\frac{4^{2k-1}-4^{k-1}}{3}$

22. Which of the following statements is/are incorrect ?

a) If $(3+a\sqrt{2})^{100}+(3+b\sqrt{2})^{100}=7+5\sqrt{2}$. Number of pairs (a,b) for which the equation is true is one (a,b are rational numbers)

b) The number of distance terms in the expansion of $\left(x^3+\frac{1}{x^3}+1\right)^{200}$ is 401

c) In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$. If l_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$ then $\frac{\ell_2}{\ell_1}$ is 16

d) The sum of the roots (real or complex) of the equation $x^{2001}+\left(\frac{1}{2}-x\right)^{2001}=0$ is 1000.

Linked comprehension type questions

Passage - I :

Let $f(n)=3^{2n}+3^n+1$ for every positive integer n, Answer the following questions:

23. Which of the following is true ?

a) $f(n+3)=3^6 f(n)-702.3^n-728$

b) $f(n+3)=3^6 f(n)-701.3^n-729$

c) $f(n+3)=3^6 f(n)+702.3^n-728$

d) None of these

24. Which of the following is false ?

a) $f(100)$ is divisible by 13

b) $f(1001)$ is divisible by 13

c) $f(2007)$ is divisible by 13

d) None of these

25. Which of the following is true?

a) $f(50)$ leaves remainder 1 when divided by 13

b) $f(51)$ leaves remainder 0 when divided by 13

c) $f(51)$ leaves remainder 3 when divided by 13

d) None of these

Passage - II :

To find coefficient of $x^r (0\le r\le n-1)$ in the expansion

$(x+a)^{n-1}+(x+a)^{n-2}(x+b)+....+(x+a)(x+b)^{n-2}+(x+b)^{n-1}$ we first sum up the series

26. The coefficient of $x^r (0\le r\le n-1)$ in the expansion
$E=(x+2)^{n-1}+(x+2)^{n-2}(x+1)+(x+2)^{n-3}(x+1)^2+....+(x+1)^{n-1}$

a) ${}^nC_r$

b) ${}^nC_r(2^{n-r}-1)$

c) ${}^nC_r\, 2^{n-r}$

d) none of these

27. The coefficient of x^{n-1} in the expansion of $E=(2x+1)^{n-1}+(2x+1)^{n-2}(x+1)+....+(x+1)^{n-1}$ is

a) 2^n b) 2^n-1 c) 2^n+1 d) 2^{2n}

28. The coefficient of $x^r (0 \le r \le n)$ in the expansion of $E=2^n+2^{n-1}(x+2)+2^{n-2}(x+2)^2+...+(x+2)^n$

a) ${}^{n+1}C_{r+1}2^{n-r}$ b) ${}^{n}C_{r}2^{n-r}$ c) ${}^{n}C_{r}2^{r}$ d) none of these

Matrix matching type questions

29.

COLUMN - I	COLUMN-II
A) 5^{97} is divided by 52, then the remainder is	p) 5
B) $53^{53}-33^3$ is divided by 10, then the remainder is	q) 6
C) $27^{10}+7^{51}$ is divided by 10, then the remainder is	r) 2
D) $13^{99}-19^{93}$ is divided by 162, then the remainder is	s) 0

30.

COLUMN - I	COLUMN - II
A) Number of terms in $(x+y-z+w)^6$ is	p) 67
B) Last two digits in 7^{2011} is	q) 84
C) Number of rational terms in $\left(\sqrt[3]{7}+\sqrt[5]{11}\right)^{1001}$ is	r) 4
D) Remainder when 2^{2011} is divided by 127 is	s) 43

Integer answer type questions

31. The value of $\left\{\left(\sqrt{3}+\sqrt{2}+1\right)^6+\left(\sqrt{3}+\sqrt{2}-1\right)^6+\left(\sqrt{3}-\sqrt{2}+1\right)^6+\left(-\sqrt{3}+\sqrt{2}+1\right)^6\right\}$ is where $\{x\}$ denotes fractional part of x

32. The number of distinct terms in the expansion of $\left(x+y+z+\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}\right)^2$ is m and that in the expansion of $\left(x+y+z+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)^2$ is n then $|m-n|=$

33. If $x=(3+2\sqrt{2})^n, y=(9+4\sqrt{5})^n, n\in N$ then the value of $(x-y)-(x^2-y^2)+x[x]-y[y]$ is where [.] denotes G.I.F

34. The remainder when $x=5^{5^{5^{.......(24 \text{ times } 5)}}}$ is divided by 24 is

35. If $5^{81}+\lambda$ is divisible by 26, then minimum positive value of $\frac{\lambda}{3}$ must be

36. $s=a+(a+d)+(a+2d)+....+(a+nd)$ and $A=a+(a+d)^nC_1+(a+2d)^nC_2+....+(a+nd)^nC_n$ then $(n+1)A=k^nS$ where $k=$ --------

EXERCISE-II

Properties of Binomial coefficients, Summation of series using multinomial coefficients & Multiple summations

LEVEL-I (MAIN)

Single answer type questions

1. $C_0 + 4\cdot C_1 + 7\cdot C_2 + ...(n+1)$ terms =

1) $(3n+2)\cdot 2^{n-1}$ 2) $(2n+2)\cdot 2^{n-1}$ 3) $(2n+2)\cdot 3^{n-1}$ 4) $(2n-2)\cdot 3^{n+1}$

2. $3\cdot C_0 - 7\cdot C_1 + 11\cdot C_2 - ......(n+1)$ terms =

1) 0 2) –1 3) 1 4) 2

3. $\dfrac{(C_0+C_1)(C_1+C_2)(C_2+C_3).....(C_{n-1}+C_n)}{C_0C_1C_2......C_n} =$

1) $\dfrac{(n+1)^n}{n!}$ 2) $\dfrac{n+1}{n!}$ 3) $\dfrac{(n+1)^{n-1}}{n!}$ 4) $\dfrac{(n-1)^n}{n!}$

4. $\dfrac{{}^{n}C_1 + {}^{(n+1)}C_2 + {}^{(n+2)}C_3 + ...+ {}^{(n+m-1)}C_m}{{}^{m}C_1 + {}^{(m+1)}C_2 + {}^{(m+2)}C_3 + ...+ {}^{(m+n-1)}C_n} =$

1) 1 2) 2 3) 3 4) 4

5. $\displaystyle\sum_{r=0}^{n-1} \frac{C_r}{C_r + C_{r+1}} =$

1) $\dfrac{n}{2}$ 2) $\dfrac{n}{3}$ 3) $\dfrac{n}{4}$ 4) $\dfrac{2n}{3}$

6. Sum of last 8 coefficients in $(1+x)^{16}$ is ____

1) $\left[2^{15} - \frac{1}{2}\cdot {}^{16}C_8\right]$ 2) $\left[2^{15} + \frac{1}{2}\cdot {}^{6}C_2\right]$ 3) $\left[2^{15} - \frac{1}{2}\cdot {}^{6}C_2\right]$ 4) $\left[2^{15} - \frac{1}{4}\cdot {}^{6}C_2\right]$

7. ${}^{(2n+1)}C_0 - {}^{(2n+1)}C_1 + {}^{(2n+1)}C_2 - .... + {}^{(2n+1)}C_{2n} =$

1) 0 2) 1 3) –1 4) 2

8. ${}^{15}C_2 + 2\cdot{}^{15}C_3 + 3\cdot{}^{15}C_4 + ... + 14\cdot{}^{15}C_{15} =$

1) $13.2^{14}-1$ 2) $13.2^{14}+1$ 3) $12^{14}+1$ 4) $12^{14}-1$

9. $C_0 - [C_1 - 2\cdot C_2 + 3\cdot C_3 - .... + (-1)^{n-1}\cdot n\cdot C_n] =$

1) 0 2) 1 3) –1 4) 2

10. $2\cdot C_2 + 6\cdot C_3 + 12\cdot C_4 + ... + n(n-1)\cdot C_n =$

1) $n(n-1)\cdot 2^{n-1}$ 2) $2n(n-1)\cdot 2^{n-2}$ 3) $n(n-1)\cdot 2^{n-2}$ 4) $2n(n+1)\cdot 2^{n-1}$

11. $\frac{C_0}{1}+\frac{C_2}{3}+\frac{C_4}{5}+.......+\frac{C_{16}}{17}=$

1) $\frac{2^{15}}{14}$ 2) $\frac{2^{16}}{17}$ 2) $\frac{2^{15}}{16}$ 4) $\frac{2^{20}}{22}$

12. $\frac{C_1}{2}+\frac{C_3}{4}+.......+\frac{C_{15}}{16}=$

1) $\frac{2^{15}-1}{16}$ 2) $\frac{2^{15}+1}{16}$ 3) $\frac{2^{14}+1}{16}$ 4) $\frac{2^{20}+1}{16}$

13. $\frac{C_0}{2}+\frac{C_1}{3}+\frac{C_2}{4}+.....+\frac{C_n}{n+2}=$

1) $\frac{2n\cdot 2^{n+1}-1}{(n-1)(n-2)}$ 2) $\frac{n\cdot 2^{n+1}-1}{(n-1)(n-2)}$ 3) $\frac{n\cdot 2^{n+1}-1}{(n+1)(n-2)}$ 4) $\frac{n\cdot 2^{n+1}+1}{(n+1)(n+2)}$

14. $\frac{C_0}{2}+\frac{C_1}{6}+\frac{C_2}{12}+.......+\frac{C_n}{(n+1)(n+2)}=$

1) $\frac{2^{n+2}-n-2}{(n+1)(n+2)}$ 2) $\frac{2^{n+2}-n-3}{(n+1)(n+2)}$ 3) $\frac{2^{n+1}-n-3}{(n+1)(n+2)}$ 4) $\frac{2^{n+1}-n-3}{(n-1)(n-2)}$

15. $C_1^2+2\cdot C_2^2+3\cdot C_3^2+......+n\cdot C_n^2=$

1) $n\cdot {}^{2n}C_n$ 2) $\frac{n}{2}\cdot {}^{2n}C_{n-1}$ 3) $\frac{n}{2}\cdot {}^{2n}C_n$ 4) $\frac{n}{2}\cdot {}^{2n}C_{n+1}$

16. If ${}^{2n}C_r=C_r$, then $C_1^2-2.C_2^2+3.C_3^2-4.C_4^2+......+2n.C_{2n}^2=$

1) $\frac{(-1)^{n-1}\cdot(2n)!}{(n-1)!}$ 2) $\frac{(-1)^n\cdot(2n)!}{(n+1)!}$ 3) $\frac{(-1)^{n-1}\cdot(2n)!}{n!(n-1)!}$ 4) $\frac{(-1)^n\cdot(2n)!}{(n+1)!n!}$

17. $C_0C_2+C_1C_3+C_2C_4+........+C_{n-2}C_n=$

1) ${}^{2n}C_{n-2}$ 2) ${}^{2n}C_n$ 3) ${}^{2n}C_{n-1}$ 4) ${}^{2n}C_{2n-2}$

18. $({}^{2n}C_0)^2-({}^{2n}C_1)^2+({}^{2n}C_2)^2-....+({}^{2n}C_{2n})^2=$

1) $(-1)^n\cdot {}^{2n}C_n$ 2) $(-1)^{2n}\cdot {}^{2n}C_n$ 3) $(-1)^n\cdot {}^{3n}C_n$ 4) $(-1)^n\cdot {}^{n}C_n$

19. ${}^{2n+1}C_0^{\,2}-{}^{2n+1}C_1^{\,2}+{}^{(2n+1)}C_2^{\,2}-.....-{}^{(2n+1)}C_{2n+1}^{\,2}=$

1) 0 2) ${}^{(2n+1)}C_n$ 3) $-({}^{2n+1}C_n)$ 4) $-\frac{1}{2}({}^{2n}C_n)$

20. $C_0+(C_0+C_1)+(C_0+C_1+C_2)+.......+(C_0+C_1+C_2+.......+C_n)=$ (n is even)

1) $(n+2)2^{n-1}$ 2) $(n+1)2^{n-1}$ 3) $(n-2)2^{n-1}$ 4) $(n-2)2^{n+1}$

21. If ${}^{10}C_1 \cdot {}^9C_5 + {}^{10}C_2 \cdot {}^9C_4 + {}^{10}C_3 \cdot {}^9C_3 + {}^{10}C_4 \cdot {}^9C_2 + {}^{10}C_5 \cdot {}^9C_1 + {}^{10}C_6 = {}^{19}C_6 + x$ then $x =$

1) –84 2) 84 3) 81 4) –81

22. $\sum_{r=1}^{n} (-1)^{r-1}\ {}^nC_r(a-r) =$

1) a 2) $-a$ 3) $2a$ 4) $3a$

23. $\sum_{r=0}^{n} \frac{1}{{}^nC_r} = S_n$, $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$ then $\frac{t_n}{S_n} =$

1) $\frac{1}{4}n$ 2) $\frac{1}{3}n$ 3) $\frac{1}{2}n$ 4) n

24. Sum of coefficients of terms of even powers of x in $(1 + x + x^2 + x^3)^5$ is

1) 512 2) 516 3) 612 4) 234

25. Sum of coefficients of terms of odd powers of x in $(1 + x - x^2 - x^3)^8$ is

1) 0 2) 1 3) 2 4) –1

26. Sum of coefficients of all the integral powers of x in $(1+2\sqrt{x})^{40}$ is

1) $\frac{3^{40}-1}{2}$ 2) $\frac{3^{40}+1}{2}$ 3) $\frac{3^{38}-1}{2}$ 4) $\frac{3^{38}+1}{2}$

27. If the sum of all the Binomial coefficients in $(x+y)^n$ is 512, then the greatest Binomial coefficient is

1) ${}^{10}C_5$ 2) ${}^9C_4$ or ${}^9C_5$ 3) ${}^{11}C_5$ or ${}^{11}C_6$ 4) ${}^{12}C_6$

28. $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + ... + a_{20}x^{20} \Rightarrow \frac{a_2}{a_1} =$

1) 10.5 2) 21 3) 10 4) 5.5

29. If a_r is the coefficient of x^r in the expansion of $(1 + x + x^2)^n$ then $a_1 - 2a_2 + 3a_3 - .... -2na_{2n} =$

1) 0 2) n 3) $-n$ 4) $2n$

30. If a_k is the coefficient of x^k in the expansion of $(1+x+x^2)^n$ for $k = 0,1,2,.....,2n$ then $a_1 + 2a_2 + 3a_3 + ... + 2n\ a_{2n} =$

1) $-a_0$ 2) 3^n 3) $n.3^n$ 4) $-n.3^n$

31. $(1+x+x^2+......+x^p)^n = a_0 + a_1x + a_2x^2 + ... + a_{np}x^{np} \Rightarrow a_1 + 2a_2 + 3a_3 + ... + np\ \ a_{np} =$

1) $\frac{np(p+1)^n}{2}$ 2) $\frac{np(p+1)^n}{4}$ 3) $\frac{np(p-1)^n}{4}$ 4) $\frac{np(p-1)^{2n}}{4}$

32. If $(1+x-2x^2)^8 = 1 + a_1x + a_2x^2 + ... + a_{16}x^{16}$, then $a_2 + a_4 + a_6 + ... + a_{16} =$

1) 120 2) 123 3) 127 4) 231

33. If $(1+x-2x^2)^8 = 1+a_1x+a_2x^2+.......+a_{16}x^{16}$, then $a_1+a_3+a_5+.......+a_{15}=$

1) 2^7 2) -2^7 3) 3^2 4) 4^6

34. If $(1+x+x^2)^8 = a_0+a_1x+......+a_{16}x^{16}$ then $a_0-a_2+a_4-a_6+.......+a_{16}=$

1) 1 2) 2 3) 3 4) 4

35. If $(1+x+x^2)^8 = a_0+a_1x+......+a_{16}x^{16}$ then $a_1-a_3+a_5-a_7+.......-a_{15}=$

1) 1 2) 2 3) 3 4) 0

36. If $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$ then $a_0+a_3+a_6+........=$

1) 3^{n-1} 2) 0 3) 1 4) 3^n

37. If $(1+x)^{10} = \sum_{r=0}^{10} C_r x^r$ then $(C_0-C_2+C_4-C_6+C_8-C_{10})^2 + (C_1-C_3+C_5-C_7+C_9)^2 =$

1) 2^{10} 2) -2^{10} 3) 2^{12} 4) -2^{12}

38. If $(1+x)^n = C_0+C_1x+C_2x^2+......C_nx^n$ then $C_0+C_4+C_8+........=$

1) $2^{n-2}+2^{\frac{n}{2}-1}\cos\frac{n\pi}{4}$ 2) $2^{n-2}+2^{\frac{n}{2}-1}\sin\frac{n\pi}{4}$ 3) $2^{n-1}+2^n\cos\frac{n\pi}{4}$ 4) $2^{n-1}+2^{\frac{n}{2}}\sin\frac{n\pi}{4}$

39. $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{a^{i+j+k}}$ is equal to (where $|a|>1$)

1) $(a-1)^{-3}$ 2) $\frac{3}{a-1}$ 3) $\frac{1}{a^3-1}$ 4) None of these

40. The value of $\sum_{1\le i\le j<k\le w\le n}\sum\sum\sum 1$ is

1) ${}^nC_4+{}^nC_3$ 2) ${}^nC_4+{}^nC_3+{}^nC_2$ 3) ${}^nC_4+2\,{}^nC_3+{}^nC_2$ 4) ${}^nC_4+{}^nC_3+2\,{}^nC_2$

LEVEL-II (ADVANCED)

Single answer type questions

1. Consider the sequence $\frac{{}^nC_0}{1.2.3}, \frac{{}^nC_1}{2.3.4}, \frac{{}^nC_2}{3.4.5}$,......, if $n = 50$ then greatest term is

a) 30^{th} b) 24^{th} c) 26^{th} d) 27^{th}

2. If $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ...+ {}^nC_nx^n$ where ${}^nC_0, {}^nC_1, {}^nC_2,....$ are binomial coefficients, then $2(C_0+C_3+C_6+....)+(C_1+C_4+C_7+.....)(1+\omega)+(C_2+C_5+C_8+....)(1+\omega^2)$, where ω is the cube root of unity and n is a multiple of 3, then the above expression is equal to

a) 2^n+1 b) $2^{n-1}+1$ c) $2^{n+1}-1$ d) 2^n-1

3. Value of $\dfrac{\sum_{k=0}^{r} {}^nC_{2k}\,{}^{n-2k}C_{r-k}}{\sum_{k=0}^{n} {}^nC_k\,{}^{2k}C_{2r}\left(\frac{3}{4}\right)^{n-k}\left(\frac{1}{2}\right)^{2k-2r}}(n \geq 2r)$ is

a) 1/2 b) 2 c) 1 d) None of these

4. The sum of the series $\sum_{r=1}^{3n-1} \dfrac{(-1)^{r-1} r}{{}^{3n}C_r}$ is (where n is an even natural number)

a) 0 b) $\dfrac{3n}{3n+1}$ c) $\dfrac{3n+1}{3n+2}$ d) $\dfrac{3n}{3n+2}$

5. If $C_0, C_1, C_2, ...$ are binomial coefficients in the expansion $\sum_{r=0}^{n} C_r x^r$, then value of the expression (series) $\dfrac{2C_0}{1}+\dfrac{3C_1}{2}+\dfrac{4C_2}{3}+\dfrac{5C_3}{4}+.....$ is

a) $\dfrac{2^n+1}{n+1}$ b) $\dfrac{2^n-1}{n+1}$ c) $\dfrac{2^n(n+3)-1}{n+1}$ d) $\dfrac{2^n(n+2)-1}{n+1}$

6. Given ${}^8C_1 x(1-x)^7 + 2^8.C_2 x^2(1-x)^6 + 3^8.C_3 x^3(1-x)^5 + ...8.x^8 = a_0 + a_1 x + a_2 x^2 + ... + a_8 x^8$ then $a_0 + a_1$ is

a) 6 b) 5 c) 8 d) 9

7. If $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$ and $a_k = 1\ \forall\, k \geq n$ then $b_n =$

a) ${}^{2n}C_n$ b) ${}^{2n+1}C_{n+1}$ c) ${}^nC_0$ d) ${}^{2n}C_{n+1}$

8. If $n \in N$, then $\sum_{r=0}^{n} (-1)^r.{}^nC_r.\left[\dfrac{1}{2^r}+\dfrac{3^r}{2^{2r}}+\dfrac{7^r}{2^{3r}}+......\text{to } m \text{ terms}\right]$

a) $\dfrac{2^{mn}}{(2^n-1)2^m}$ b) $\dfrac{2^{mn}+1}{(2^n-1)2^{mn}}$ c) $\dfrac{2^{mn}-1}{(2^n-1)2^{mn}}$ d) None

9. In a ΔABC $\sum_{r=0}^{n} {}^nC_r.a^{n-r}.b^r \cos(rA-(n-r)B) =$

a) $(a+b)^n$ b) $(a-b)^n$ c) c^n d) c^{2n}

10. The largest integer k such that 3^k divides $2^{3^n}+1$, $n \in N$ is

a) 2 b) n c) $n-1$ d) $n+1$

11. If $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r.x^r$ then $6(a_0+a_6+a_{12}+a_{18}+....) =$

a) $3^n-1+2^{n+1}\cos\dfrac{n\pi}{3}$ b) $3^n+1+2^{n+1}.\cos\dfrac{n\pi}{3}$

c) $3^n-1+2^n\sin\dfrac{n\pi}{3}$ d) $3^n+1+2^n\sin\dfrac{n\pi}{3}$

12. The value of $C_3 + C_7 + C_{11} + ...$, is

a) $\frac{1}{2}\left(2^{n-1} - 2^{n/2}\sin\frac{n\pi}{4}\right)$ b) $\frac{1}{2}\left(2^{n-1} + 2^{n/2}\sin\frac{n\pi}{4}\right)$

c) $\frac{1}{4}\left(2^{n+1} - 2^{n/2}\sin\frac{n\pi}{4}\right)$ d) none

13. If k and n are positive integers and $S_k = 1^k + 2^k + 3^k + ... + n^k$ then $\sum_{r=1}^{m} {}^{m+1}C_r.(S_r)$ is equal to

a) $(n+1)^{m+1} - (n+1)$ b) $(n+1)^{m+1} + (n+1)$ c) $(n-1)^{m+1} - (n-1)$ d) none

14. The sum of all the coefficients of those terms in the expansion of $(a + b + c + d)^8$ which contains b but not c is

a) 6305 b) 6561 c) 256 d) 4^8

15. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$ then $a_0^2 - a_1^2 + a_2^2 - a_3^2 + ... + a_{2n}^2$ is equals

a) $\frac{1}{2}a_n$ b) $\frac{1}{2}a_{n+1}$ c) a_{n+1} d) a_n

16. The value of $\sum_{0 \le i < j \le n}\sum i.({}^nC_j)$ is equal to

a) $n(n+1)2^{n-3}$ b) $n^2 2^{n-3}$ c) $n(n-1)2^{n-3}$ d) none

17. The value of the expression $\sum_{0 \le i < j \le n}\sum (-1)^{i+j-1}\, {}^nC_i \cdot {}^nC_j =$

a) ${}^{2n-1}C_n$ b) ${}^{2n}C_n$ c) ${}^{2n+1}C_n$ d) none

18. Let $S_1 = \sum_{0 \le i < j \le 100}\sum C_iC_j, S_2 = \sum_{0 \le j < i \le 100}\sum C_iC_j$ and $S_3 = \sum_{0 \le i = j \le 100}\sum C_iC_j$ where C_r represents coefficient of x^r in the binomial expansion of $(1+x)^{100}$. If $S_1 + S_2 + S_3 = a^b$ where $a, b \in N$, then the least value of $(a+b)$ is

a) 66 b) 72 c) 46 d) 52

More than one correct answer Type Questions

19. ${}^nC_0\,{}^{2n}C_m - {}^nC_1\,{}^{2n-2}C_m + {}^nC_2\,{}^{2n-4}C_m - .... =$

a) $\binom{n}{m-n}2^{2n-m}$ if $m \ge n$ b) 0 if $m < n$ c) $\binom{n}{m-n}2^{2n+m}$ if d) 1 if

20. Which of the following is/are correct?

a) ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - ..... - {}^{20}C_{15} = -{}^{19}C_{15}$

b) ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - ..... - {}^{20}C_{15} = -{}^{20}C_{14}$

c) $16\,{}^{20}C_0 - 15\,{}^{20}C_1 + 14\,{}^{20}C_2 - ..... - 2\,{}^{20}C_{14} - {}^{20}C_{15} = {}^{19}C_{14}$

d) $16\,{}^{20}C_0 - 15\,{}^{20}C_1 + 14\,{}^{20}C_2 - ..... - 2\,{}^{20}C_{14} - {}^{20}C_{15} = {}^{18}C_{15}$

21. Let $(1+\sqrt{2})^n = x_n + y_n\sqrt{2}$ where x_n, y_n are integers, th en

a) $x_n^2 - 2y_n^2 = (-1)^n$ b) $x_n + 2y_n - x_{n+1} = 0$ c) $x_n^2 - 2y_n^2 = 1$ d) $y_{n+1} = x_n + y_n$

22. If $(x^{2006} + x^{2008} + 2)^{2010} = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ then value of $a_0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3 - \frac{1}{2}a_4 - \frac{1}{2}a_5 + a_6 + ...$ is

a) less than 2 b) greater than 0 c) equals 2 d) none of these

23. If $ac>b^2$, then the sum of the coefficients in the expansion of $(a\alpha^2x^2 + 2b\alpha x + c)^n$ is, where $a,b,c,\alpha \in R$ and $n \in N$

a) positive if $a > 0$

b) positive if $c > 0$

c) negative if $a < 0$ and n is odd

d) positive if $c < 0$ and n is even

Linked comprehension type questions

Passage - I :

$(1 + ax + bx^2 + cx^3)^{10} = 1 + p_1x + p_2x^2 + p_3x^3 + ... + p_{30}x^{30}$ And the values of p_1, p_2, p_3 respectively are 20,200,1000 respectively then

24. $b =$

a) -1 b) 2 c) 3 d) 4

25. $c =$

a) 12 b) -15 c) $+30$ d) -32

26. $p_4 =$

a) 540 b) 600 c) 660 d) 480

Passage - II :

Let $(1+x)^{20} = \sum_{r=0}^{20} a_rx^r$ when $a_r = {}^{20}C_r$ Then

27. $\sum\sum_{0\le i<j\le 20} a_i.a_j =$

a) $2^{40} - {}^{40}C_{20}$ b) $2^{39} - {}^{40}C_{20}$ c) $2^{39} - {}^{39}C_{20}$ d) $2^{40} - {}^{39}C_{19}$

28. $\sum\sum_{0\le i<j\le 20} (a_i - a_j)^2 =$

a) $42.{}^{39}C_{19} - 2^{40}$ b) $21.{}^{40}C_{20} - 2^{39}$ c) $21.{}^{39}C_{19} - 2^{40}$ d) $21.{}^{40}C_{20} - 2^{40}$

29. $\sum\sum_{0\le i<j\le 20} (i+j)a_ia_j$

a) $40(2^{39} - {}^{39}C_{20})$ b) $20(2^{39} - {}^{39}C_{20})$ c) $40(2^{40} - {}^{40}C_{20})$ d) $40(2^{40} - {}^{39}C_{19})$

Matrix matching type question

30. **COLUMN - I** **COLUMN - II**

A) $\left({}^{32}C_0\right)^2 - \left({}^{32}C_1\right)^2 + \left({}^{32}C_2\right)^2 = $ ___ p) ${}^{63}C_{32}$

B) $\left({}^{32}C_0\right)^2 + \left({}^{32}C_1\right)^2 + \left({}^{32}C_2\right)^2 + ...\left({}^{32}C_{32}\right)^2 = $ ___ q) 0

C) $\frac{1}{32}\left(1\left({}^{32}C_1\right)^2 + 2\left({}^{32}C_2\right)^2 + 3\left({}^{32}C_3\right)^2 + ...32\left({}^{32}C_{32}\right)^2\right) = $ ___ r) ${}^{32}C_{16}$

D) $\left({}^{31}C_0\right)^2 - \left({}^{32}C_1\right)^2 + \left({}^{32}C_2\right)^2 - \left({}^{31}C_3\right)^2 + ... + \left({}^{31}C_{31}\right)^2 = $ ___ s) ${}^{64}C_{32}$

31. Let $A = \sum_{r=1}^{50} \frac{{}^{50+r}C_r(2r-1)}{{}^{50}C_r(50+r)}, B = \sum_{r=1}^{50}\left({}^{50}C_r\right)^2, C = \sum_{r=1}^{100}(-1)^r\left({}^{100}C_r\right)^2$ then match the following

COLUMN - I **COLUMN - II**

A) A – B = —— p) -1

B) $C - A = $ —— q) 0

C) $B - A = $ —— r) 1

D) $B + C - 2A = $ —— s) 2

Integer answer type questions

32. If ${}^{2015}C_1 - {}^{2015}C_2\left(1+\frac{1}{2}\right) + {}^{2015}C_3\left(1+\frac{1}{2}+\frac{1}{3}\right) - .... + {}^{2015}C_{2015}\left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{2015}\right) = K$, then the sum of the digits of $\left[\frac{1}{k}\right]$, where [.] denotes G.I.F is ________

33. Let $a = 3^{\frac{1}{223}} + 1$ and for all $n \geq 3$, $f(n) = {}^nC_0 a^{n-1} - {}^nC_1 a^{n-2} + {}^nC_2 a^{n-3}$ ....$+(-1)^{n-1}.{}^nC_{n-1}.a^0$. If the value of $f(2007) + f(2008) = 3^k$ where $k \in N$, then $k =$

34. ${}^nC_0.{}^{2n}C_n - {}^nC_1.{}^{(2n-1)}C_n + {}^nC_2.{}^{(2n-2)}C_n + ... + (-1)^n.{}^nC_n.{}^nC_n =$

35. If S be the sum of coefficients in the expansion of $(px + qy - rz)^n$ (where $p, q, r > 0$), then the value of $\lim_{n\to\infty}\frac{S}{(S^{1/n}+1)^n}$ is

36. $C_0, C_1, C_2, ...C_n$ are binomial coefficients in the expansion of $(1 + x)^n$ then

$$\lim_{n\to\infty}\left\{C_n - C_{n-1}\left(\frac{2}{3}\right) + C_{n-2}\left(\frac{2}{3}\right)^2 - .... + (-1)^n C_0\left(\frac{2}{3}\right)^n\right\} =$$

37. In the expansion of $(1+3x+2x^2)^6$, then coefficient of x^{11} is $k \times 2^6$, then k is

38. If for $1 \leq m \leq n$, $f(m, n) = {}^nC_0 - {}^nC_1 + {}^nC_2 - ..... + (-1)^{m-1}.{}^nC_{m-1}$, then $f(7, 8)$ is

EXERCISE-III

Binomial theorem for rational index, Approximations & Summation of series using multinoial

LEVEL-I (MAIN)

Single answer type questions

Rational index :

1. The range of x so that the expansion of $(3-4x)^{1/2}$ is valid is

1) $-3/4 < x < 3/4$ 2) $|x|<3$ 3) $|x|<1/4$ 4) $|x|<1$

2. If the expansion $(4a-8x)^{1/2}$ were to possible then

1) $2<\left|\frac{a}{x}\right|$ 2) $2>\left|\frac{a}{x}\right|$ 3) $2<\left|\frac{x}{a}\right|$ 4) $2>\left|\frac{x}{a}\right|$

3. For $|x|>\frac{3}{2}$, the value of the third term in the expansion of $(3 + 2x)^{3/5}$ is

1) $\frac{27}{50}\cdot 2^{-\frac{3}{5}}\cdot x^{-\frac{9}{5}}$ 2) $\frac{27}{50}\cdot 2^{\frac{3}{5}}\cdot x^{\frac{7}{5}}$ 3) $\frac{27}{50}\cdot 2^{-\frac{2}{5}}\cdot x^{-\frac{7}{5}}$ 4) $-\frac{27}{50}\cdot 2^{-\frac{2}{5}}\cdot x^{-\frac{7}{5}}$

4. If $\frac{1}{(1-2x)(1+3x)}$ is to be expanded as a power series of x, then

1) $|x| < 1/2$ 2) $|x| < 1/6$ 3) $-1/3 < x < 1/2$ 4) $|x| < 1/3$

5. $1+{}^2C_1x+{}^3C_2x^2+{}^4C_3x^3+.......$ to ∞ terms can be summed up if

1) $x<1$ 2) $x>-1$ 3) $-1<x<1$ 4) $-\infty<x<\infty$

6. For $|x|<1$, the $(r+1)^{th}$ term in the expansion of $\sqrt{1-x}$ is

1) $\frac{1\cdot 3\cdot 5..........(2r-3)}{r!}\left(\frac{x}{2}\right)^r$ 2) $-\frac{1\cdot 3\cdot 5..........(2r-3)}{r!}\left(\frac{x}{2}\right)^r$

3) $-\frac{1\cdot 3\cdot 5..........(2r-3)}{r!}(x)^r$ 4) $\frac{1\cdot 3\cdot 5..........(2r-3)}{r!}(x)^r$

7. The general term of $(2a-3b)^{-1/2}$ is

1) $\frac{1.3.5.......(2r-5)}{r!}\frac{1}{\sqrt{2a}}\left(\frac{3b}{4a}\right)^r$ 2) $\frac{1.3.5.......(2r-3)}{r!}\frac{1}{\sqrt{2a}}\left(\frac{3b}{4a}\right)^r$

3) $\frac{1.3.5.......(2r-1)}{r!}\frac{1}{\sqrt{2a}}\left(\frac{3b}{4a}\right)^r$ 4) $\frac{1.3.5.......(2r-3)}{r!}\frac{1}{\sqrt{a}}\left(\frac{3b}{4a}\right)^r$

8. If $|x| < 1$, then the coefficient of x^n in expansion of $(1+x+x^2+x^3+....)^2$ is

1) n 2) $n-1$ 3) $n+2$ 4) $n+1$

9. The coefficient of x^{24} in $(1 + 3x + 6x^2 + 10x^3 + \ldots\ldots \infty)^{2/3}$ is

1) 25 2) 125 3) 50 4) 300

10. If S_n denotes the sum of first n natural numbers then $S_1 + S_2 x + S_3 x^2 + \ldots + S_n x^{n-1} + \ldots \infty$ terms =

1) $(1-x)^{-1}$ 2) $(1-x)^{-2}$ 3) $(1-x)^{-3}$ 4) $(1-x)^{-4}$

11. ${}^4C_1 + {}^5C_2 \cdot \left(\frac{1}{2}\right) + {}^6C_3 \cdot \left(\frac{1}{2}\right)^2 + \ldots\ldots$ to ∞ terms

1) 30 2) 40 3) 900 4) 15

12. The coefficient of x^2 in $(1 + x)^2(8 - x)^{-1/3}$ is

1) $\frac{2167}{4032}$ 2) $\frac{2265}{4132}$ 3) $\frac{313}{576}$ 4) $\frac{3691}{6792}$

13. The coefficient of x^{-n} in $(1 + x)^n \left(1+\frac{1}{x}\right)^n$ is

1) 0 2) 1 3) 2^n 4) ${}^{2n}C_n$

14. If $|x| < \frac{1}{2}$, then the coefficient of x^r in the expansion of $\frac{1+2x}{(1-2x)^2}$ is

1) $r.2^r$ 2) $(2r - 1)\, 2^r$ 3) $r.2^{2r+1}$ 4) $(2r + 1)\, 2^r$

15. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots\ldots$ then a_n is :

1) $\frac{a^n - b^n}{b-a}$ 2) $\frac{a^{n+1} - b^{n+1}}{b-a}$ 3) $\frac{b^{n+1} - a^{n+1}}{b-a}$ 4) $\frac{b^n - a^n}{b-a}$

16. The coefficient of x^{24} in the expansion of $(1+x^2)^{12}(1+x^{12})(1+x^{24})$ is

1) $12C_6$ 2) $12C_6+2$ 3) $12C_6+4$ 4) $12C_6+6$

17. If $0 < x < 1$; then first negative term in the expansion of $(1 + x)^{27/5}$ is

1) 7^{th} term 2) 5^{th} term 3) 8^{th} term 4) 6^{th} term

Approximations :

18. If x is so small that x^2 and higher powers of x are neglected then $\frac{\sqrt{1+x}+\sqrt[3]{1+4x}}{(1+x^2)\ \sqrt[3]{(1-3x)^2}} =$

1) $1+\frac{11x}{12}$ 2) $2+\frac{35x}{6}$ 3) $1-\frac{5x}{12}$ 4) $1+\frac{5x}{12}$

19. If 'c' is small in comparison with l then $\left(\frac{l}{l+c}\right)^{1/2} + \left(\frac{l}{l-c}\right)^{1/2} =$

1) $2+\frac{3c}{4l}$ 2) $2+\frac{3c^2}{4l^2}$ 3) $l+\frac{3c^2}{4l^2}$ 4) $l+\frac{3c}{4l}$

20. If p is nearly equal to q and $n > 1$ such that $\frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q} = \left(\frac{p}{q}\right)^k$ then the value of k is

1) n 2) $1/n$ 3) $n + 1$ 4) $1/\ n+1$

21. If x is numerically so small so that x^2 and higher powers of x can be neglected, then $\left(1+\frac{2x}{3}\right)^{\frac{3}{2}}(32+5x)^{\frac{-1}{5}}$ is approximately equal to:

1) $\frac{32+31x}{64}$ 2) $\frac{32+32x}{64}$ 3) $\frac{31+32x}{64}$ 4) $\frac{1-2x}{64}$

22. If x is nearly equal to 1 then value of $\frac{mx^m-nx^n}{m-n}$ is nearly equal to

1) x^{m+n} 2) x^{m-n} 3) $\frac{1}{1-x}$ 4) $\frac{1}{1+x}$

23. If x is nearly equal to 1 then value of $\frac{ax^b-bx^a}{x^b-x^a}$ is nearly equal to

1) $\frac{1}{1+x}$ 2) $\frac{1}{1-x}$ 3) $\frac{2}{1+x}$ 4) $\frac{2}{1-x}$

Summation of Infinite Series :

24. The sum of the series $\frac{3}{4\cdot 8}-\frac{3\cdot 5}{4\cdot 8\cdot 12}+\frac{3\cdot 5\cdot 7}{4\cdot 8\cdot 12\cdot 16}-......$

1) $\sqrt{\frac{3}{2}}-\frac{3}{4}$ 2) $\sqrt{\frac{2}{3}}-\frac{3}{4}$ 3) $\sqrt{\frac{3}{2}}-\frac{1}{4}$ 4) $\sqrt{\frac{2}{3}}-\frac{1}{4}$

25. $\frac{3}{6}+\frac{3}{6}\cdot\frac{5}{9}+\frac{3\cdot 5\cdot 7}{6\cdot 9\cdot 12}+.......\infty =$

1) $2\sqrt{3}-4$ 2) $3\sqrt{3}-2$ 3) $3\sqrt{3}-4$ 4) $2\sqrt{3}+4$

26. $\frac{5}{9\cdot 18}+\frac{5\cdot 8}{9\cdot 18\cdot 27}+\frac{5\cdot 8\cdot 11}{9\cdot 18\cdot 27\cdot 36}+.......\infty =$

1) $\frac{1}{2}\sqrt[3]{\frac{9}{4}}-\frac{11}{18}$ 2) $\frac{3\sqrt[3]{18}-22}{12}$ 3) $\frac{3\sqrt[3]{9}-11}{12}$ 4) $\frac{\sqrt[3]{10}-5}{6}$

27. $1+\frac{n}{2}+\frac{n(n-1)}{2\cdot 4}+\frac{n(n-1)(n-2)}{2\cdot 4\cdot 6}+.........+\infty =$

1) $1+\frac{n}{3}+\frac{n(n-1)}{3.6}+\frac{n(n+1)(n+1)}{3.6.9}+...$ 2) $1+\frac{n}{3}+\frac{n(n+2)}{3.6}+\frac{n(n+1)(n+1)}{3.6.9}+...$

3) $1+\frac{n}{3}+\frac{n(n+1)}{3.6}+\frac{n(n+1)(n+2)}{3.6.9}+...$ 4) $1+\frac{n}{3}+\frac{n(n+2)}{3.6}+\frac{n(n+1)(n+2)}{3.6.9}+...$

Numerical value type questions

28. If a_1, a_2, a_3, are the last three digits of 17^{256} respectively then the value of $4a_1-2a_2-a_3$ is equal to

29. Coefficient of x^{2009} in $(1+x+x^2+x^3+x^4)^{1001}(1-x)^{1002}$ is

30. Given $(1-2x+5x^2+10x^3)(1+x)^n = 1+a_1x+a_2x^2$ .... and that $a_1^2=2a_2$ then the value of n is

31. If $n\in N$ and $C_k={}^nC_k$, and $\sum_{k=1}^{n}k^3\left(\frac{{}^nC_k}{{}^nC_{k-1}}\right)^2=\frac{n(n+1)^2(n+2)}{3p}$ then p is

KEY SHEET (LECTURE SHEET)

EXERCISE- I

LEVEL-I

1) 2 2) 1 3) 4 4) 1 5) 3 6) 2 7) 1 8) 3
9) 1 10) 3 11) 1 12) 3 13) 4 14) 3 15) 2 16) 3
17) 3 18) 1 19) 3 20) 3 21) 4 22) 2 23) 4 24) 2
25) 1 26) 3 27) 3 28) 3 29)1 30) 3 31) 4 32) 2
33) 2 34) 1 35) 1 36) 2 37) 1 38) 2 39) 2 40) 1
41) 3 42) 4 43) 6 44) 5

LEVEL-II

1) d 2) b 3) a 4) a 5) a 6) d 7) b 8) a
9) c 10) c 11) 3 12) b 13) b 14) c 15) d 16) c
17) b 18) d 19) bc 20) ab 21) acd 22) ad 23) a 24) c
25) c 26) b 27) b 28) a 29) A-p; B-q; C-r; D-s
30) A-q; B-s; C-p; D-r 31)0 32) 1 33) 0 34) 5 35) 7
36) 2

EXERCISE-II

LEVEL-I

1) 1 2) 1 3) 1 4) 1 5) 1 6) 2 7) 2 8) 2
9) 2 10) 3 11) 2 12) 1 13) 4 14) 2 15) 3 16) 3
17) 1 18) 1 19) 1 20) 1 21) 1 22) 1 23) 3 24) 1
25) 1 26) 2 27) 2 28) 1 29) 3 30) 4 31) 3 32) 3
33) 2 34) 1 35) 4 36) 1 37) 1 38) 1 39) 1 40) 3

LEVEL-II

1) b 2) d 3) c 4) d 5) c 6) c 7) b 8) c
9) c 10) a 11) b 12) a 13) a 14) a 15) d 16) c
17) a 18) a 19) ab 20) ad 21) abd 22) ab 23) abc 24) b
25) d 26) c 27) c 28) a 29) b 30) A-r; B-s; C-p; D-q
31) A-p; B-r; C-r; D-s 32) 8 33) 9 34) 1 35) 0 36) 0
37) 9 38) 7

EXERCISE-III

LEVEL-I

1) 1 2) 1 3) 4 4) 4 5) 3 6) 2 7) 3 8) 4
9) 1 10) 3 11) 1 12) 3 13) 2 14) 4 15) 3 16) 2
17) 3 18) 2 19) 2 20) 2 21) 1 22) 1 23) 2 24) 2
25) 3 26) 1 27) 3 28) 7 29) 0 30) 6 31) 4

PRACTICE SHEET

EXERCISE-I

Binomial expansion for positive integral index, Middle term, Numerically greatest term, R-f factor relation & Multinomial theorem

LEVEL-I (MAIN)

Single answer type questions

1. Coefficient of x^{2009} in the expansion of $(1+x+x^2+x^3+x^4)^{1001}(1-x)^{1002}$ is

   1) 0 2) $4.^{1001}C_{501}$ 3) -2009 4) none of these

2. If $\frac{t_2}{t_3}$ in the expansion of $(a+b)^n$ and $\frac{t_3}{t_4}$ in the expansion of $(a+b)^{n+3}$ are equal, then find n.

   1) 3 2) 4 3) 5 4) 6

3. The value of $\dfrac{18^3+7^3+3\cdot18\cdot7\cdot25}{3^6+6\cdot243\cdot2+15\cdot81\cdot4+20\cdot27\cdot8+15\cdot9\cdot16+6\cdot3\cdot32+64}$

   1) 4 2) 3 3) 2 4) 1

4. The value of $(1.02)^4+(0.98)^4$ upto three places of decimal is

   1) 2.048 2) 2.003 3) 2.04 4) 2.004

5. If the coefficient of x in $\left(x^2+\frac{A}{x}\right)^5$ is 270, then A = ?

   1) 3 2) 4 3) 5 4) 6

6. The coefficient of x^4 in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is

   1) $\frac{405}{256}$ 2) $\frac{504}{259}$ 3) $\frac{450}{263}$ 4) none of these

7. In the expansion of $\left(\frac{x^{\frac{1}{3}}}{2}+x^{\frac{-1}{5}}\right)^8$, the term independent of x is :

   1) t_5 2) t_6 3) t_7 4) none of these

8. The coefficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+....+(1+x)^{30}$ is:

   1) ${}^{51}C_5$ 2) ${}^{9}C_5$ 3) ${}^{31}C_6-{}^{21}C_6$ 4) ${}^{30}C_5+{}^{20}C_5$

9. If the coefficient of x^2 and x^3 in the expansion of $(3+\alpha x)^9$ are the same, then the value of a is:

   1) $-\frac{7}{9}$ 2) $-\frac{9}{7}$ 3) $\frac{7}{9}$ 4) $\frac{9}{7}$

10. If the coefficient of the $(n+1)^{th}$ term and $(n+3)^{th}$ term in the expansion of $(1+x)^{20}$ are equal, then the value of n is:

1) 10 2) 8 3) 9 4) 7

11. Given positive integers $r>1$, $n>2$ and that the coefficient of $(3r)^{th}$ and $(r+2)^{th}$ terms in the binomial expansion of $(1+x)^{2n}$ are equal. Then

1) $n=2r$ 2) $n=2r+1$ 3) $n=3r$ 4) none of these

12. The middle term in the expansion of $\left(\frac{2x^2}{3}+\frac{3}{2x^2}\right)^{10}$ is:

1) 251 2) 252 3) 250 4) none of these

13. The middle term in the expansion of $\left(\frac{2x}{3}-\frac{3}{2x^2}\right)^{2n}$ is:

1) ${}^{2n}C_n$ 2) $(-1)^n\, {}^{2n}C_n x^{-n}$ 3) ${}^{2n}C_n x^{-n}$ 4) none of these

14. The middle term in $\left(x^2+\frac{1}{x^2}+2\right)^n$ is

1) $\frac{n!}{((n/2)!)^2}$ 2) $\frac{(2n)!}{((n/2)!)^2}$ 3) $\frac{1.3.5.........(2n+1)}{n!}2^n$ 4) $\frac{(2n)!}{(n!)^2}$

15. For $n\in N$ if two consecutive terms in the expansion of $(p+q)^n$ are equal then $\frac{(n+1)q}{p+q}$ is

1) Negative integer 2) rational number 3) a real number 4) a positive integer

16. The term in $(x+y)^{50}$ which is greatest in absolute value if $|x|=\sqrt{3}|y|$ is

1) T_{17} 2) T_{19} 3) T_{20} 4) T_{21}

17. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are 45, 120 and 210 then the value of n is

1) 8 2) 12 3) 10 4) 14

18. The sum of rational terms in the expansion of $\left(\sqrt{2}+3^{1/5}\right)^{10}$ is:

1) 41 2) 40 3) 39 4) 42

19. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is

1) 35 2) 32 3) 33 4) 34

20. In the expansion of $(\sqrt[5]{3}+\sqrt[7]{2})^{24}$, the rational term is

1) T_{14} 2) T_{16} 3) T_{15} 4) T_7

21. No.of terms whose value depend on 'x' in $\left(x^2-2+\frac{1}{x^2}\right)^n$ is

1) $2n$ 2) $2n+1$ 3) $2n-1$ 4) $n+1$

22. Coefficient of $a^8b^6c^4$ in $(a+b+c)^{18}$ is

1) $\frac{18!}{4!\,10!\,5!}$ 2) $\frac{18!}{3!\,8!\,8!}$ 3) $\frac{18!}{2!\,7!\,9!}$ 4) $\frac{18!}{8!\,6!\,4!}$

23. The coefficient of x^9 in $(x-1)(x-4)(x-9)......(x-100)$ is

1) -235 2) 235 3) 385 4) -385

24. Coeff of x^{18} in $(x^2+1)(x^2+4)(x^2+9)......(x^2+100)$ is

1) -385 2) 385 3) 285 4) -285

25. If n is a positive integer then $2^{4n}-15n-1$ is divisible by

1) 64 2) 196 3) 225 4) 256

26. Larger of $99^{50}+100^{50}$ and 101^{50} is

1) 101^{50} 2) $99^{50}+100^{50}$ 3) Both are equal 4) can not be decided

27. If $\{x\}$ denotes the fractional part of x then $\left\{\frac{3^{1001}}{82}\right\}=$

1) $\frac{9}{82}$ 2) $\frac{81}{82}$ 3) $\frac{3}{82}$ 4) $\frac{1}{82}$

LEVEL-II (ADVANCED)

Single answer type questions

1. In the expansion of $(x+y)^{15}$ the eleventh term is geometric mean of ninth and twelfth terms then k^{th} term of the expansion must be greatest then the value of k is

a) 8 b) 6 c) 9 d) 10

2. The term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is

a) 210 b) 105 c) 70 d) 112

3. Middle term in the expansion of $(1-3x+3x^2-x^3)^{3n}$ is

a) $\frac{(6n)!x^n}{(3n)!(3n)!}$ b) $\frac{(6n)!x^{3n}}{(3n)!}$

c) $\frac{(6n)!}{(3n)!(3n)!}(-x)^{3n}$ d) $\frac{(6n)!}{(3n+1)!(3n-1)!}(-x)^{3n+1}$

4. If n is even positive integer, then the condition that the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also is

a) $\frac{n}{n+2}<x<\frac{n+2}{n}$ b) $\frac{n+1}{n}<x<\frac{n}{n+1}$ c) $\frac{n}{n+4}<x<\frac{n+4}{n}$ d) none of these

5. The term independent of x in the product $(4+x+7x^2)\left(x-\frac{3}{x}\right)^{11}$ is

a) $7.{}^{11}C_5$ b) $3^6.{}^{11}C_6$ c) $3^5.{}^{11}C_5$ d) -12.2^{11}

6. The term independent of 'x' in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}, x>0$, is α times the corresponding binomial coefficient. Then 'α' is

a) 3 b) $\frac{1}{3}$ c) $-\frac{1}{3}$ d) 1

7. If $p^2+q = 2$ then maximum value of the term independent of x in the expansion of $(px^{1/6} + qx^{-1/3})^9$ is ($p > 0$, $q > 0$)

a) 42 b) 82 c) 168 d) 84

8. The number of terms in the expansion of $(1+x)^{101}.(1+x^2-x)^{100}$ is

a) 10100 b) 50×101 c) 202 d) 102

9. The coefficient of $a^8b^4c^9d^9$ in $\{ab(c+d)+cd(a+b)\}^{10}$, is

a) $\frac{(10)!}{8!4!9!}$ b) 10! c) 2520 d) none of these

10. The coefficient of x^{50} in the expansion of $(1+x)^{1000}+2x(1+x)^{999}+3x^2(1+x)^{998}+.....+1001x^{1000}$ =

a) ${}^{1002}C_{50}$ b) ${}^{1002}C_{51}$ c) ${}^{1005}C_{50}$ d) ${}^{1005}C_{48}$

11. If the last term in the binomial expansion of $\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3.9^{1/3}}\right)^{\log_3^8}$. Then middle term is

a) ${}^{10}C_5$ b) $-{}^{10}C_5$ c) $\frac{1}{2}({}^{10}C_4)$ d) $-{}^{10}C_6$

12. The sum of rational terms in $\left(\sqrt{2}+\sqrt[3]{3}+\sqrt[6]{5}\right)^{10}$ is equal to

a) 12632 b) 7560 c) 4232 d) 11792

13. Last digit in $2^{2^n}+1\ \forall n \in N, n \neq 1$ is

a) 7 b) 3 c) 5 d) 1

14. If $\{x\}$ represents fractional part of x, then $\left\{\frac{5^{200}}{8}\right\}=$

a) $\frac{1}{4}$ b) $\frac{1}{8}$ c) $\frac{3}{8}$ d) $\frac{5}{8}$

15. The remainder when $(1!)^2+(2!)^2+(3!)^2+....+(100!)^2$ is divided by 144 is

a) 17 b) 31 c) 33 d) 41

More than one correct answer type questions

16. If in the expansion of $\left(\frac{1}{x}+x\tan x\right)^5$ the ratio of 4^{th} term to the 2^{nd} term is $\frac{2}{27}\pi^4$ then the value of x can be –

a) $\frac{-\pi}{6}$ b) $\frac{-\pi}{3}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{12}$

17. If the middle term of $\left(x+\frac{\sin^{-1}x}{x}\right)^8$ is equal to $\frac{630}{16}$ then value of x is (are)

a) $\frac{\pi}{3}$ b) $-\frac{\pi}{3}$ c) $-\frac{\pi}{6}$ d) $\frac{\pi}{6}$

18. The greatest coefficient in the expansion of $(a+b+c)^7$ must be

a) 105 b) odd c) even d) 210

19. In the expansion of $(x^2+2x+2)^n$ when n is a positive integer then which is/are correct

a) coefficient of x is $n.2^n$

b) coefficient of x^3 is $2^n({}^{n+1}C_3)$

c) coefficient of x^2 is $n^2(2^{n-1})$

d) sum of all coefficients of different powers of x is 4^n

Linked comprehension type questions

Passage - I :

The expressions $1+x, 1+x+x^2, 1+x+x^2+x^3, \ldots\ldots, 1+x+x^2+\ldots..+x^{20}$ are multiplied together and the terms of the product thus obtained are arranged in increasing powers of x in the form of $a_0+a_1x+a_2x^2+\ldots.$ then

20. Number of terms in the product is

a) 200 b) 211 c) 231 d) 215

21. If sum of the coefficients of even powers of x is k, sum of the coefficients of odd powers x is l and $m=\frac{a_r}{a_{n-r}}$ where n is degree of the product then the value of $(k+l+m)$ is

a) $\frac{20!}{2}$ b) $21!+1$ c) $\frac{21!}{2}$ d) $19!$

Passage - II :

Let $(x+1)(x+2)(x+3)\ldots.(x+n) = x^n + A_1x^{n-1} + A_2x^{n-2} + A_3x^{n-3} + \ldots.. + A_n$

22. $A_1 + A_n =$

a) $\frac{n}{2}+n!$ b) $\frac{n+1}{2}+n!$ c) $\frac{n(n+1)}{2}+n!$ d) $(n+1)!$

23. $A_2 =$

a) $\frac{(n-1)n(n+1)}{12}$ b) $\frac{n(n+1)(3n+1)}{12}$ c) $\frac{(n+1)(3n+1)}{24}$ d) $\frac{(n-1)n(n+1)(3n+2)}{24}$

24. $A_3=$

a) $\frac{n^2(n-1)(n-2)(n+1)}{24}$ b) $\frac{(n-1)(n-2)(n+1)^2}{24}$

c) $\frac{(n-1)(n-2)n^2(n+1)^2}{24}$ d) $\frac{(n-1)(n-2)(n+1)^2n^2}{48}$

Matrix matching type questions

25. COLUMN - I | COLUMN - II

COLUMN - I	COLUMN - II
A) The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is	p) 0
B) If $(11)^{27} + (21)^{27}$ when divided by 16 leaves the remainder	q) 01
C) Last Two digits of the number $N = 7^{100} - 3^{100}$ are	r) 15
D) The last two digits of the number 3^{400} are	s) 10

26. Remainder when N is divided by A

COLUMN - I	COLUMN - II
A) $N = 99^{100}$ and $A = 10$	p) 7
B) $N = 2^{2007} + 2008$ and $A = 9$	q) 0
C) $N = 9^{2009}-8(2008)-9$ and $A = 64$	r) 1
D) $N = 7^{100}$ and $A = 1000$	s) 19

Integer answer type questions

27. When the terms in the binomial expansion of $\left(\sqrt{x}+\dfrac{1}{2\sqrt[4]{x}}\right)^n$ $(n\in N, n\neq 1, x>0)$ are arranged in decreasing powers of x, the coefficients of the first three terms are in arithmetic progression. The number of terms in the expansion with integer powers of x is
28. If a,b,c and d are the 3rd, 4th, 5th and 6th terms in the expansion of $(1+x)^{100}$ and $\dfrac{b^2-ac}{c^2-bd}=\dfrac{la}{kc}$ then $l+k$ is
29. If in the expansion of $(x^3-1/x^2)^n, n\in N$, sum of the coefficients of x^5 and x^{10} is 0, value of n is $5m$. then the value of m, is
30. The digit in the hundreds place of 3^{100} is
31. If $[x]$ denotes the greatest integer less than or equal to x, then $[(1+0.0001)^{10000}]$ equals
32. The remainder when $(32^{32})^{32}$ is divided by 7 is
33. The last digit of the number $(32)^{32}$ is
34. The unit digit of $17^{1983}+11^{1983}-7^{1983}$ is

EXERCISE-II

Properties of Binomial coefficients, Summation of series using multinomial coefficients & Multiple summations

LEVEL-I (MAIN)

Single answer type questions

1. ${}^{(2n+1)}C_0+{}^{(2n+1)}C_1+{}^{(2n+1)}C_2+.....+{}^{(2n+1)}C_n=$

1) 2^n 2) 2^{-n} 3) 2^{2n} 4) 3^{2n}

2. The sum of the series ${}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.....+{}^{20}C_{10}$ is

1) ${}^{20}C_{10}$ 2) $-({}^{20}C_{10})$ 3) $\frac{1}{2}\cdot({}^{20}C_{10})$ 4) 0

3. $C_0+C_1+2\cdot C_2(3)+3\cdot C_3(3^2)+4\cdot C_4(3^3)+........+n\cdot C_n 3^{n-1}=$

1) $n.4^{n-1}+1$ 2) $2n.4^{n-1}+1$ 3) $n.4^{n-1}-1$ 4) $n.4^{n+1}-1$

4. $k-{}^nC_1(k-1)+{}^nC_2(k-2)-{}^nC_3(k-3)+..........+(-1)^n\,{}^nC_n(k-n)=$

1) 1 2) 2 3) 3 4) 0

5. $\dfrac{1}{1!(n-1)!}+\dfrac{1}{3!(n-3)!}+\dfrac{1}{5!(n-5)!}+......=$

1) $\dfrac{2^{n-1}}{n!}\ \forall n\in N$ 2) $\dfrac{2^{n-1}}{2n!}\ \forall n\in N$ 3) $\dfrac{2^{2n-1}}{n!}\ \forall n\in N$ 4) $\dfrac{2^{2n-1}}{n!}\ \forall n\in N$

6. $C_0+\dfrac{C_1}{2}(4)+\dfrac{C_2}{3}(16)+.............+\dfrac{C_n}{n+1}(2^{2n})=$

1) $\dfrac{5^{n+1}+1}{n-1}$ 2) $\dfrac{5^{n+1}-1}{4(n+1)}$ 3) $\dfrac{5^{n+1}+1}{4(n+1)}$ 4) $\dfrac{5^{n+1}+1}{4(n-1)}$

7. Let P_n denote product of binomial coefficients in $(1+x)^n$ then $\dfrac{P_{n+1}}{P_n}=$

1) $\dfrac{(n+1)^n}{n!}$ 2) $\dfrac{(n+1)^{2n}}{n!}$ 3) $\dfrac{(n+1)^{2n}}{(n!)^2}$ 4) $\dfrac{(n+1)^n}{(n!)^2}$

8. $C_0^2-C_1^2+C_2^2-........-C_{15}^2=$

1) 1 2) 2 3) 3 4) 0

9. If C_K is the coefficient of x^K in $(1+x)^{2005}$ and if $a,d\in R$ then $\sum_{K=0}^{2005}(a+Kd)\cdot C_K=$

1) $(2a+2005d)\ 2^{2004}$ 2) $(2a+2005d)\ 2^{2005}$ 3) $(2a+2004\ d)2^{2005}$ 4) $(2a+2004\ d)\ 2^{2005}$

10. $1\cdot{}^{20}C_1-2\cdot{}^{20}C_2+3\cdot{}^{20}C_3-......-20\cdot{}^{20}C_{20}=$

1) 1 2) 2 3) –1 4) 0

11. $({}^3C_3+{}^4C_3+{}^5C_3+.....+{}^nC_3)\times({}^nC_2+{}^nC_3+.....+{}^nC_n)=$

1) ${}^{(n+1)}C_4\cdot(2^n-n-1)$ 2) ${}^{(n-1)}C_4\cdot(2^n-n-1)$ 3) ${}^{(n-1)}C_4\cdot(2^n-n+1)$ 4) ${}^{(n-1)}C_4\cdot(2^n+n+1)$

12. The sum $S_{10}=\sum_{k=0}^{10}(-1)^k\,{}^{30}C_k$ is

1) ${}^{29}C_9$ 2) ${}^{29}C_{10}$ 3) ${}^{31}C_{11}$ 4) ${}^{30}C_9$

13. The value of ${}^{20}C_0+{}^{20}C_1+{}^{20}C_2+{}^{20}C_3+{}^{20}C_4+{}^{20}C_{12}+{}^{20}C_{13}+{}^{20}C_{14}+{}^{20}C_{15}$ equals to

1) $2^{19}-\dfrac{\left({}^{20}C_{10}+{}^{20}C_9\right)}{2}$ 2) $2^{19}-\dfrac{\left({}^{20}C_{10}+2\times{}^{20}C_9\right)}{2}$

3) $2^{19}-\dfrac{{}^{20}C_{10}}{2}$ 4) none

14. If n is a positive integer and $C_k = {}^nC_k$ then the value of $\sum_{k=1}^{n} k^3\left(\frac{C_k}{C_{k-1}}\right)^2 =$

1) $\frac{n(n+1)(n+2)}{12}$ 2) $\frac{n(n+1)^2(n+2)}{12}$ 3) $\frac{n(n+1)(n+2)^2}{12}$ 4) $\frac{n(n+1)}{2}$

15. Coefficient of x^{10} in $(1+2x)^{21}+(1+2x)^{22}+ ...... +(1+2x)^{30}$ is

1) $2^{10}\left({}^{31}C_{11}-{}^{21}C_{11}\right)$ 2) $2^{10}\left({}^{30}C_{11}-{}^{21}C_{11}\right)$ 3) $2^{9}\left({}^{31}C_{11}-{}^{21}C_{11}\right)$ 4) ${}^{31}C_{11}$

16. If a, b, c are in A.P then the sum of the coefficients of $[1+(ax^2-2bx+c)^2]^{2009}$ is

1) -2 2) -1 3) 0 4) 1

17. If $a_1, a_2, a_3......a_n$ are in A.P. with S_n as the sum of first 'n' terms (S_0=0), then $\sum_{k=0}^{n} {}^nC_kS_k =$

1) $2^{n-2}[na_1+s_n]$ 2) $2^n[a_1+s_n]$ 3) $2[na_1+s_n]$ 4) $2^{n-1}[a_1+s_n]$

18. If $\sum_{r=0}^{n}\left\{\frac{{}^nC_{r-1}}{{}^nC_r+{}^nC_{r-1}}\right\}=\frac{25}{24}$, then n is equal to

1) 3 2) 4 3) 5 4) 6

19. For $n>3$, $1.2\ {}^nC_r-2.3\ {}^nC_{r-1}+.......+(-1)^r(r+1)(r+2)=$

1) ${}^{n-3}C_r$ 2) $2.\ {}^{n-3}C_r$ 3) ${}^{n+3}C_{r+1}$ 4) ${}^{n-2}C_r$

20. Sum of the coefficients of $(x+2y+z)^{10}$=

1) 4^5 2) 5^4 3) 4^{10} 4) 10^4

21. If $(1+x)(1+x+x^2)(1+x+x^2+x^3).......(1+x+x^2+.......+x^{n-1})= a_0+a_1x+a_2x^2+......+a_mx^m$ then $a_0+a_1+......+a_m=$

1) $n!$ 2) $2n!$ 3) $3n!$ 4) $4n!$

22. $(1+x)^{15}=a_0+a_1x+.....+a_{15}x^{15} \Rightarrow \sum_{r=1}^{15} r\frac{a_r}{a_{r-1}}=$

1) 110 2) 115 3) 120 4) 135

LEVEL-II (ADVANCED)

Single answer type questions

1. If the sum of the coefficients in the expansion of $(b+c)^{20}[1+(a-2)x]^{20}$ is equal to square of the sum of the coefficients in the expansion of $[2bcx-(b+c)y]^{10}$ where a, b, c are positive constants, then

a) $a\geq\sqrt{bc}$ b) $\frac{b+c}{2}\geq a$ c) c, a and b are in G.P. d) $\frac{1}{c},\frac{1}{a},\frac{1}{b}$ are in H.P.

2. Coefficient of x^{10} in $(1+2x)^{21}+(1+2x)^{22}+ ...... +(1+2x)^{30}$ is

a) $2^{10}\left({}^{31}C_{11}-{}^{21}C_{11}\right)$ b) $2^{10}\left({}^{30}C_{11}-{}^{21}C_{11}\right)$ c) $2^{9}\left({}^{31}C_{11}-{}^{21}C_{11}\right)$ d) ${}^{31}C_{11}$

3. If $(1+x+x^2)^{100}=\sum_{r=0}^{200} a_r x^r$ which of the following is true

a) $a_{28}=a_{72}$ b) $a_{56}=a_{144}$ c) $a_{200}=a_{300}$ d) $a_{14}=a_{128}$

4. $\sum\left(\frac{11-3r}{11-r}\right)\frac{{}^{10}C_r}{2^r}$ is $\frac{1}{k}$ then the sum of the digits of k is

a) 7 b) 11 c) 13 d) 17

5. The value of ${}^{404}C_4 - {}^4C_1.{}^{303}C_4 + {}^4C_2.{}^{202}C_4 - {}^4C_3.{}^{101}C_4 =$

a) $(404)^4$ b) $(101)^4$ c) 0 d) 1

6. If $a_n=\sum_{k=0}^{n}\frac{(\log_e 10)^n}{k!(n-k)!}$ for $n\geq 0$ then $a_0+a_1+a_2+a_3+....$ upto ∞ equal to

a) 10 b) 10^2 c) 10^3 d) 10^4

7. $\sum_{r=0}^{10} C_r.\frac{2^{r+1}}{r+1}=$ (where $C_r={}^{10}C_r$)

a) $\frac{3^{11}}{11}$ b) $\frac{2^{11}}{11}$ c) $\frac{3^{11}-1}{11}$ d) $\frac{2^{11}-1}{11}$

8. The value of $\sum_{r=0}^{n} r(n-r)({}^nC_r)^2$ is equal to

a) $n^2.{}^{2n-1}C_{n-1}$ b) $n^2.{}^{2n-2}C_n$ c) $n^2.{}^{2n}C_{n-1}$ d) $n^2.{}^{2n-1}C_n$

9. If $n>3$ then $xy.C_0-(x-1)(y-1).C_1+(x-2)(y-2).C_2-(x-3)\ (y-3).C_3+...+(-1)^n(x-n)(y-n).C_n=$

a) $xy\times 2^n$ b) nxy c) xy d) 0

10. The coefficient of x^{50} in the expansion of $(1+x)^{1000}+2x(1+x)^{999}+3x^2(1+x)^{998}+.....+1001x^{1000}=$

a) ${}^{1002}C_{50}$ b) ${}^{1002}C_{51}$ c) ${}^{1005}C_{50}$ d) ${}^{1005}C_{48}$

11. The coefficient of x^{n-1} in the expansion of $(1+2x+3x^2+4x^3+...+nx^{n-1})^2$ is

a) ${}^nC_3$ b) ${}^{n+1}C_3$ c) ${}^{n+2}C_3$ d) ${}^{n+3}C_3$

12. $\sum_{K=1}^{10}\frac{(-1)^{K-1}}{K}\cdot\left(10_{C_k}\right)=$

a) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.......+\frac{1}{11}$ b) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.......+\frac{1}{10}$

c) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.......+\frac{1}{9}$ d) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.......+\frac{1}{12}$

13. $\sum_{r=0}^{n}\frac{n-3r+1}{n-r+1}\cdot\frac{{}^nC_r}{2^r}$ is equal to

a) $\frac{1}{2^n}$ b) $\frac{1}{3^n}$ c) $\frac{1}{4^n}$ d) $\frac{1}{2^n}+1$

14. The value of $\sum_{r=0}^{n}\sum_{s=1}^{n} {}^{n}C_s\, {}^{s}C_r$ is

a) $3^n - 1$ b) $3^n + 1$ c) 3^n d) none of these

15. If $p > 0, x > 0, p \neq 1$ and $(1-p)(1+3x+9x^2+27x^3+81x^4+243x^5) = (1-p^6)$ then $\frac{p}{x} =$

a) $\frac{1}{3}$ b) 3 c) $\frac{1}{2}$ d) can not be determined

16. If a, b, c are in A.P then the sum of the coefficients of $[1+(ax^2-2bx+c)^2]^{2009}$ is

a) -2 b) -1 c) 0 d) 1

17. If $a_1, a_2, a_3 \ldots\ldots a_n$ are in A.P. with S_n as the sum of first 'n' terms $(S_0 = 0)$, then $\sum_{k=0}^{n} {}^{n}C_k S_k =$

a) $2^{n-2}[na_1 + s_n]$ b) $2^n[a_1 + s_n]$ c) $2[na_1 + s_n]$ d) $2^{n-1}[a_1 + s_n]$

More than one correct answer type questions

18. The value of $\frac{{}^{n}C_0}{n} + \frac{{}^{n}C_1}{n+1} + \frac{{}^{n}C_2}{n+2} + \ldots\ldots + \frac{{}^{n}C_n}{2n}$ is equal to

a) $\int_0^1 x^{n-1}(1-x)^n dx$ b) $\int_0^1 x^{n-1}(1+x)^n dx$ c) $\int_1^2 x^{n-1}(1+x)^n dx$ d) $\int_1^2 x^n (x-1)^{n-1} dx$

19. If $(1+x)^n(1+x^2)^2 = \sum_{k=0}^{n+4} a_k x^k$ and a_1, a_2, a_3 are in A.P then $n =$

a) 2 b) 3 c) 4 d) 5

20. If $C_r = {}^{n}C_r$ then the sum of the series $S = C_0^2 + \frac{(C_1)^2}{2} + \frac{(C_2)^2}{3} + \ldots$ upto $(n+1)$ terms is

a) $\frac{{}^{2n+1}C_{n+1}}{n+1}$ b) $\frac{{}^{2n+2}C_{n+1}}{2(n+1)}$ c) $\frac{{}^{2n+1}C_n}{n+1}$ d) $\frac{(2n+1)!}{(n+1)!^2}$

Linked comprehension type questions

Passage - I :

When $n \in N, (1+x)^n = C_0 + C_1x + C_2x^2 \ldots\ldots + C_rx^r + \ldots\ldots + C_nx^n$ where $C_r = {}^{n}C_r$.

21. If $\frac{C_0}{2^n} + 2.\frac{C_1}{2^n} + 3.\frac{C_2}{2^n} + \ldots\ldots + \frac{(n+1)C_n}{2^n} = 16$ then $n =$

a) 22 b) 32 c) 30 d) 16

22. $\sum_{r=0}^{2n} (-1)^r \frac{8^r}{49^n} {}^{2n}C_r$ is equal to

a) -1 b) 1 c) $\left(\frac{64}{49}\right)^n$ d) 0

23. If ${}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + \ldots\ldots + {}^{2n}C_{2n} = ({}^{n}C_0 + {}^{n}C_0 + {}^{n}C_0 + \ldots\ldots + {}^{n}C_0)^k$ then ${}^{K}C_0 + {}^{K}C_1 + \ldots\ldots {}^{K}C_k =$

a) 1 b) 4 c) 8 d) 16

Passage - II :

Let C_r denotes coefficient of 'x' in the expansion of $(1+x)^{100}$ and

Let $S_1 = \sum\sum_{0\le i<j\le 100} C_iC_j$, $S_2 = \sum\sum_{0\le j<i\le 100} C_iC_j$, $S_3 = \sum\sum_{0\le i=j\le 100} C_iC_j$, then

24. The value of S_1 is

a) $2^{100} - {}^{200}C_{100}$ b) $2^{200} - {}^{200}C_{100}$ c) $\frac{2^{200} - {}^{200}C_{100}}{2}$ d) None

25. The value of S_2 is

a) $2^{100} - {}^{200}C_{100}$ b) $\frac{2^{200} - {}^{200}C_{100}}{2}$ c) $2^{200} - {}^{200}C_{100}$ d) None

26. If $S_1 + S_2 + S_3 = a^b, a, b \in N$ then least value of $(a+b)$ is

a) 66 b) 72 c) 46 d) 52

Matrix matching type question

27. **COLUMN - I** **COLUMN - II**

A) The sum $\sum_{k=0}^{n} \sum_{r=k}^{n} (-1)^k \, {}^nC_r . {}^rC_k \, a^r$ is p) 1

B) The sum ${}^{404}C_4 - {}^4C_1 . {}^{303}C_4 + {}^4C_2 . {}^{202}C_4 - {}^4C_3 . {}^{101}C_4 + {}^4C_4$ q) 2
equals to $(101)^k$, where k is

C) $\frac{T_2}{T_3}$ in the expansion of $(a + b)^n$ and $\frac{T_3}{T_4}$ in r) 5
the expansion of $(a + b)^{n+3}$ are equal, if n is s) 4

D) The remainder when 2^{2009} is divided by 17 is

28. Consider $(1+x+x^2)^{2n} = \sum_{r=0}^{4n} a_r . x^r$, where $a_0, a_1, a_2, ..., a_{4n}$ are real numbers and n is a +ve integer

COLUMN - I **COLUMN - II**

A) The value of $\sum_{r=0}^{n-1} a_{2r}$ is p) ${}^{(2n+1)}C_2$

B) The value of $\sum_{r=1}^{n} a_{2r-1}$ q) $\frac{3^{2n} - 1}{4}$

C) The value of a_2 is r) $\frac{9^n - 2a_{2n} + 1}{4}$

D) The value of a_{4n-1} s) $2n$

Integer answer type questions

29. The sum of the series $3\cdot{}^{2007}C_0 - 8\cdot{}^{2007}C_1 + 13\cdot{}^{2007}C_2 - 18\cdot{}^{2007}C_3 + ...$ up to 2008 terms is K, then K is

30. Given ${}^{8}C_1 x(1-x)^7 + 2\cdot{}^{8}C_2 x^2(1-x)^6 + 3\cdot{}^{8}C_3 x^3(1-x)^5 + ... + 8\cdot x^8 = ax + b$, then $a + b =$

31. Let $1+\sum_{r=1}^{10}(3^r.{}^{10}C_r + r.{}^{10}C_r) = 2^{10}(\alpha.4^5+\beta)$ where $\alpha,\beta \in N$ and $f(x) = x^2-2x-k^2+1$ if α,β lies between the roots of $f(x) = 0$, then the smallest positive integral value of k is

KEY SHEET (PRACTICE SHEET)

EXERCISE-I

LEVEL-I

01) 1	02) 3	03) 4	04) 4	05) 1	06) 1	07) 2	08) 3
09) 4	10) 3	11) 1	12) 2	13) 2	14) 4	15) 4	16) 2
17) 3	18) 1	19) 3	20) 3	21) 1	22) 4	23) 4	24) 2
25) 3	26) 1	27) 3					

LEVEL-II

01) a	02) a	03) c	04) a	05) b	06) b	07) d	08) c
09) c	10) a	11) b	12) d	13) b	14) b	15) d	16) bc
17) ab	18) cd	19) abc	20) b	21) b	22) c	23) d	24) d
25) A-p;B-p;C-p;D-q			26) A-r;B-q;C-q;D-r			27) 3	28) 8
29) 3	30) 0	31) 2	32) 4	33) 6	34) 1		

EXERCISE-II

LEVEL-I

01) 3	02) 3	03) 1	04) 4	05) 1	06) 2	07) 1	08) 4
09) 1	10) 4	11) 1	12) 2	13) 2	14) 2	15) 1	16) 1
17) 1	18) 3	19) 2	20) 3	21) 1	22) 3		

LEVEL-II

01) b	02) a	03) b	04) a	05) b	06) b	07) c	08) b
09) d	10) a	11) c	12) b	13) a	14) a	15) b	16) a
17) a	18) bd	19) ab	20) abcd	21) c	22) b	23) b	24) c
25) b	26) a	27) A-p;B-s;C-r;D-q			28) A-r;B-q;C-p;D-s		
29) 0	30) 8	31) 5					

ADDITIONAL EXERCISE

LECTURE SHEET (ADVANCED)

Single answer type questions

1. Given $(1-x^3)^n = \sum_{k=0}^{n} a_k x^k (1-x)^{3n-2k}$ then the value of $3.a_{k-1} + a_k$ is

a) ${}^nC_k.3^k$ b) ${}^{(n+1)}C_k.3^k$ c) ${}^{(n+1)}C_k.3^{k-1}$ d) ${}^nC_{k-1}.3^k$

2. The constant term in the expansion of $\left(1+x+\frac{2}{x}\right)^6$ is

a) 479 b) 517 c) 569 d) 581

3. If $(1+x)^n = \sum_{r=0}^{n} C_r x^r$ then the value of $\frac{2^2 C_0}{1.2} + \frac{2^3 C_1}{1.2} + \frac{2^4 C_2}{3.4} + ... + ... + \frac{2^{n+2} C_n}{(n+1)(n+2)}$ equals

a) $\frac{3^{n+2}-2n-5}{(n+1)(n+2)}$ b) $\frac{3^n - 2n - 5}{n(n+2)}$ c) $\frac{3^{n+1}+2n-5}{(n+1)(n+2)}$ d) None of these

4. The value of the series, if $C_0, C_1 \ldots C_n$ are binomial coefficients in $(1+x)^n$, then

$C_0 - \frac{C_1 2^3}{2} + \frac{C_2 2^6}{3} - \frac{C_3 2^9}{4} + ...$ up to (n+1) terms equal

a) $\frac{2^{n+1}-1}{n+1}$ b) $\frac{1-(-7)^{n+1}}{8(n+1)}$ c) $\frac{1-(-7)^{n+1}}{3(n+1)}$ d) None of these

5. The sum of the coefficients of all odd exponents of x in the product of

$(1-x+x^2-x^3+x^4+....-x^{49}+x^{50}) \times (1+x+x^2+x^3+....+x^{50})$ equals

a) 1 b) 0 c) -1 d) None of these

6. If $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r = a_0 + a_1 x + a_2 x^2 + ... + a^{2n} x^{2n}$ and

$P = a_0 + a_3 + a_6 + ...;\ Q = a_1 + a_4 + a_7 + ...\ ;\ R = a_2 + a_5 + a_8 + ...$

then the set of values of P,Q are respectively equals

a) (1,1,1) b) $(3^n, 3^n, 3^n)$ c) $(3^{n+1}, 3^{n+1}, 3^{n+1})$ d) $(3^{n-1}, 3^{n-1}, 3^{n-1})$

7. The coefficient of x^2y^2, yzt^2 and $xyzt$ in the expansion of $(x+y+z+t)^4$ are in the ratio

a) 4:2:1 b) 2:4:1 c) 1:2:4 d) 2:3:4

8. If $(1+x+x^2+x^3)^{100} = \sum_{r=0}^{300} b_r x^r$ and $k = \sum_{r=0}^{300} b_r$ then $\sum_{r=0}^{300} rb_r$ is

a) 50.4^{100} b) 150.4^{100} c) 300.4^{100} d) none of these

9. The coefficient of t^8 in the expansion of $(1+2t^2-t^3)^9$ is

a) 1680 b) 2140 c) 2520 d) 2730

10. The largest integer k such that 3^k divides $2^{3^n}+1, n \in N$ is

a) 2 b) n c) $n-1$ d) $n+1$

11. If coefficients of x^{20} in $(1+x-x^2)^{20}$ and $(1+x+x^2)^{20}$ are respectively a and b, then

a) $a=b$ b) $a>b$ c) $a<b$ d) $a+b=0$

12. $(1+x)(1+x+x^2)(1+x+x^2+x^3).....(1+x+x^2+......+x^{100})$ when written in the ascending power of x then the highest exponent of x is

a) 505 b) 5050 c) 100 d) 50

13. The coefficient of x^{20} in the product $(1-x)(1-2x)(1-2^2x)(1-2^3x)---(1-2^{21}x)$ is equal to given that $1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.....+\frac{1}{2^{21}}=p$ and $1+\frac{1}{2}+\frac{1}{2^4}+.....+\frac{1}{2^{42}}=q$

a) $2^{231}(p^2-q)$ b) $2^{230}(p^2-q)$ c) $2^{230}(q-p^2)$ d) $2^{232}(p^2-q)$

More than one correct answer type questions

14. If $(1-x+x^2)^n=a_0+a_1x+a_2x^2+...+a_{2n}x^{2n}$ where $a_0, a_1, a_2,.......a_{2n}$ are in A.P then

a) $a_n=\frac{1}{2n+1}$ b) $a_n=\frac{1}{2n-1}$ c) $a_{2n}=\frac{1-2n}{2n+1}$ d) $a_{2n}=\frac{1+2n}{2n+1}$

15. If $f(\text{m}) = \sum_{i=0}^{m}\binom{30}{30-i}\binom{20}{m-i}$ where $\binom{p}{q}={}^pC_q$ then

a) Maximum value of f(m) is ${}^{50}C_{25}$

b) $f(0)+f(1)+....+f(50)=2^{50}$

c) f(m) is always divisible by 50

d) The value of $\sum_{m=0}^{50}[f(m)]^2={}^{100}C_{50}$

16. The value of $\sum_{k=0}^{7}\left[\frac{\binom{7}{k}}{\binom{14}{k}}\sum_{r=k}^{14}\binom{r}{k}\binom{14}{r}\right]$, where $\binom{n}{r}$ denotes ${}^nC_r$, is

a) 6^7 b) greater than 7^6 c) 8^7 d) greater than 7^8

Linked comprehension type questions

Passage - I :

If $(1+x)^n=\sum_{r=0}^{n}{}^nC_rx^r$ and $\sum_{r=0}^{n}\frac{1}{{}^nC_r}=S_n$, $n\in N, r=0,1,2,...n$ Based on the above information answer the following

17. The value of $\sum\sum_{0\le i<j\le n}\left(\frac{1}{{}^nC_i}+\frac{1}{{}^nC_j}\right)$ is equal to

a) nS_n b) $\frac{(n-1)S_n}{2}$ c) $\frac{nS_n}{2}$ d) $(n-1)S_n$

18. Then the value of $\sum_{0\le i<j\le n}\sum\left(\frac{i}{{}^nC_i}+\frac{j}{{}^nC_j}\right)$ is equal to

a) $\frac{n^2S_n}{2}$ b) $\frac{n^2}{2S_n}$ c) $\frac{n}{2}S_n$ d) None

19. The value of $\sum_{r=1}^{n}\frac{1}{r({}^nC_r)}$ equals

a) $\frac{1}{n}S_n$ b) $\frac{1}{n}S_{n-1}$ c) $\frac{1}{n-1}S_n$ d) $\frac{1}{n-1}S_{n-1}$

Passage - II :

If $\left(1+px+x^2\right)^n=1+a_1x+a_2x^2+......+a_{2n}x^{2n}$, where $n\in N, p\in R$, a_r = Co -efficient of x^r.

20. If $n = 40$, $p = 3$, $r = 5$ then which of the following is true.

a) $135a_5=6a_6+4a_4$ b) $105a_5=6a_6-76a_4$ c) $105a_5=6a_6-36a_4$ d) $135a_5=6a_6-36a_4$

21. The remainder obtained when $a_1+5a_2+9a_3+13a_4+.......+(8n-3)a_{2n}$ is divided by $(p + 2)$ is

a) 1 b) 2 c) 3 d) 0

22. If $p = -3$ and n is even number, the value of $a_1+3a_2+5a_3+7a_4+......+(4n-1)\,a_{2n}$ is

a) n b) $2n - 1$ c) $2n - 2$ d) $2n$

Matrix matching type questions

23. **COLUMN - I** / **COLUMN - II**

COLUMN - I	COLUMN - II
A) $\sum_{0\le i<j\le n}(i+j)(C_i\cdot C_j)$ is	p) $n.2^n$
B) $\sum_{0\le i<j\le n}\sum\left({}^nC_i+{}^nC_j\right)$ is equal to	q) $n.2^{n-1}$
C) $\sum_{0\le i<j\le n}\sum i({}^nC_j)$ is equal to	r) $n(n-1)2^{n-3}$
D) $\sum_{r=0}^{n}r({}^nC_r)$ is equal to	s) $\frac{n}{2}\cdot\left[2^{2n}-{}^{2n}C_n\right]$

24. **COLUMN - I** / **COLUMN - II**

COLUMN - I	COLUMN - II
A) The number of zeros at the end of the sum $101^{11}-1$	p) 11
B) The number of terms in the expansion of $\left(2x^{\frac{1}{3}}+3y^{-\frac{1}{3}}-2z\right)^n$ is 78 then $n =$	q) 2
C) If $\sum_{r=0}^{n}\frac{r}{{}^nC_r}=\sum_{r=0}^{n}\frac{7}{{}^nC_r}$, Then $n =$	r) 9
D) If $\frac{1}{1!9!}+\frac{1}{3!7!}+\frac{1}{5!5!}+\frac{1}{7!3!}+\frac{1}{9!1!}=\frac{2^n}{10!}$. Then $n =$	s) 14

Integer answer type questions

25. $S = {}^3C_0 - {}^4C_1 \cdot \frac{1}{2} + {}^5C_2\left(\frac{1}{2}\right)^2 - {}^6C_3\left(\frac{1}{2}\right)^3 + \ldots\ldots$. If $S = \left(\frac{2}{3}\right)^k$ then value of k is

26. $\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3^k}({}^kC_r) =$

27. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots.. + a_{2n}x^{2n}$, then $\begin{vmatrix} a_{n-3} & a_{n-1} & a_{n+1} \\ a_{n-6} & a_{n-3} & a_{n+3} \\ a_{n-14} & a_{n-7} & a_{n+7} \end{vmatrix}$ is

28. The value of $\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\left(\sum_{l=0}^{r-1}\frac{1}{5^n}{}^nC_r.{}^rC_l.3^l\right)$

29. If $x + \frac{1}{x} = 1$ and $p = x^{4000} + \frac{1}{x^{4000}}$ and q be the digit at unit place in the number $2^{2^n} + 1, n \in N$ and $n > 1$, then $p + q =$

PRACTICE SHEET (ADVANCED)

Single answer type questions

1. The greatest integer less than the number $\left(\frac{2011}{2010}\right)^{2010}$ is

a) 1 b) 3 c) 5 d) 2

2. For all $n \in N, [(\sqrt{3}+1)^{2n}]+1$ is divisible by ...., [.] = G.I.F

a) 2^{n+1} b) 3^{n+1} c) 5^{n+1} d) None

3. If $n \in N$, then $121^n - 25^n + 1900^n - (-4)^n$ is divisible by

a) 1904 b) 2000 c) 2002 d) 2006

4. The coefficient of x^8 in the expansion of $\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!}\right)^2$ is

a) $\frac{1}{315}$ b) 315 c) 8! d) 2^7

5. The coefficient of x^r is $(x+2)^n + (x+2)^{n-1}.(x+1) + (x+2)^{n-2}.(x+1)^2 + \ldots\ldots\ldots + (x+1)^n$ is

a) ${}^nC_r$ b) ${}^{n+1}C_r(2^{n+1-r} - 1)$ c) n^r d) 2^n

6. Sum of the coefficients of the terms of degree m in the expansion of $(1 + x)^n (1 + y)^n (1 + z)^n$ is

a) $({}^nC_m)^3$ b) $3({}^nC_m)$ c) ${}^nC_{3m}$ d) ${}^{3n}C_m$

7. If $\omega \neq 1$ is a cube root of unity and $(\omega + x)^n = 1 + 12\overline{\omega} + 69\omega + \ldots.$ then the values of n and x are respectively

a) 36, 1 b) 12, 2 c) 24, $\frac{1}{2}$ d) 18, $\frac{1}{3}$

8. Then sum $S_n = \sum_{k=0}^{n}(-1)^k \cdot {}^{3n}C_k$, where n = 1, 2, ... is

a) $(-1)^n \cdot {}^{3n-1}C_{n-1}$ b) $(-1)^n \cdot {}^{3n-1}C_n$ c) $(-1)^n \cdot {}^{3n-1}C_{n+1}$ d) None of these

More than one correct answer type questions

9. If the 4^{th} term in the expansion of $\left(2+\frac{3x}{8}\right)^{10}$ has the maximum numerical value, then x can lie in the interval(s)

a) $\left(2,\frac{64}{21}\right)$ b) $\left(-\frac{60}{23},-2\right)$ c) $\left(-\frac{64}{21},-2\right)$ d) $\left(2,-\frac{60}{23}\right)$

10. If $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, then

a) $a_0 - a_2 + a_4 - a_6 + ... = 0$, if n is odd
b) $a_1 - a_3 + a_5 - a_7 + ... = 0$, if n is even
c) $a_0 - a_2 + a_4 - a_6 + ... = 0$, if $n = 4p$, $p \in I^+$
d) $a_1 - a_3 + a_5 - a_7 + ... = 0$, if $n = 4p+1$, $p \in I^+$

11. If $x \in R$, and $S = 1 - C_1\frac{1+x}{1+nx} + C_2\frac{1+2x}{(1+nx)^2} - C_3\frac{1+3x}{(1+nx)^3} + ...$ upto $(n+1)$ terms then S

a) equals x^2 b) equals 1 c) equals 0 d) is independent of x

Linked comprehension type questions

Passage - I :

If $(1+x)^n = C_0 + C_1x + C_2x^2 + C_nx^n$ then

12. The sum of the products of the binomial coefficients C_0, C_1, ... C_n taken two at a time is

a) $2^{2n} - {}^{2n}C_n$ b) $\frac{1}{2}(2^{2n} - {}^{2n}C_n)$ c) $\frac{1}{2}(2^{2n} - 2n)$ d) $2^{2n-1} - {}^{2n}C_n$

13. $\sum\sum_{0 \le i < j \le n} (C_i + C_j)^2 =$

a) $(n-1)\,{}^{2n}C_n + 2^{2n}$ b) $(n-1)\,{}^{2n}C_n - 2^{2n}$ c) $n.{}^{2n}C_n - 2^{2n}$ d) $n.{}^{2n}C_n + 2^{2n}$

14. $\sum\sum_{0 \le i < j \le n} (C_i - C_j)^2 =$

a) $(n-1).{}^{2n}C_n + 2^{2n}$ b) $(n+1)\,{}^{2n}C_n - 2^{2n}$ c) $n.{}^{2n}C_n - 2^n$ d) $n.{}^{2n}C_n - 2^n$

Passage - II :

Consider the binomial expression $(1+x)^n = \sum_{r=0}^{n} a_rx^r$ where a_1, a_2, a_3 are in arithmatic progression.

Consider the binomial expression $A = (\sqrt[3]{2} + \sqrt[4]{3})^{14n}$ the expansion of A contains some rational terms $T_{\alpha_1}, T_{\alpha_2}, T_{\alpha_3}, ...T_{\alpha_m}$ $(\alpha_1 < \alpha_2 < .....\alpha_m)$ Based on the above information answer the following

15. The value of $a_1 + a_2 + a_3$

a) 60 b) 63 c) 70 d) none

16. $\alpha_1, \alpha_2, \alpha_3, ...\alpha_m$ are in arithmetic progression then the common difference of the A.P is

a) 10 b) 12 c) 8 d) 14

17. The value of α_m is

a) 91 b) 92 c) 93 d) none

Matrix matching type questions

18. **COLUMN - I** **COLUMN - II**

COLUMN - I	COLUMN - II
A) If $\sum_{r=0}^{n}\left(\frac{{}^nC_{r-1}}{{}^nC_r+{}^nC_{r-1}}\right)^3=\frac{25}{24}$ then $n=$	p) 2
B) The digit in the units place of the number 3^{400}	q) 0
C) For integer $n>1$, the digit in units place in the number $\sum_{r=0}^{100} r!+2^{2^n}$ is	r) 1
D) The sum of the coefficients in the expansion $(2-3cx+c^2x^2)^{12}$ vanishes then c is	s) 5

Integer answer type questions

19. Let λ denote the term independant of x in the expansion of $\left(x+\frac{\sin\left(\frac{1}{n}\right)}{x^2}\right)^{3n}$ then $\underset{n\to\infty}{lt}\left(\frac{\lambda.n!}{{}^{(3n)}P_n}\right)=$

20. The value $\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\left(\sum_{t=0}^{r-1}\frac{1}{7^n}.n_{c_r}.r_{c_t}.5^t\right)$ is equal to

21. If $\sum_{r=0}^{n}\frac{r+2}{r+1}{}^nC_r=\frac{2^8-1}{6}$ then n is equal to

22. The value of $99^{50}-99.98^{50}+\frac{99.98}{1.2}(97)^{50}+...+99$ is

23. The sum of the series $3\cdot{}^{2007}C_0-8\cdot{}^{2007}C_1+13\cdot{}^{2007}C_2-18\cdot{}^{2007}C_3+....$ up to 2008 terms is K, then K is

KEY SHEET (ADDITIONAL EXERCISE)

LECTURE SHEET (ADVANCED)

1) **b** 2) **d** 3) **a** 4) **b** 5) **b** 6) **d** 7) **c** 8) **c** 9) **c** 10) **a**
11) **c** 12) **b** 13) **b** 14) **ac** 15) **abd** 16) **ab** 17) **a** 18) **a** 19) **a** 20) **b**
21) **c** 22) **d** 23) **A-s;B-p;C-r;D-q** 24) **A-q;B-p;C-s;D-r** 25) **4** 26) **2**
27) **0** 28) **1** 29) **6**

PRACTICE SHEET (ADVANCED)

1) **d** 2) **a** 3) **b** 4) **a** 5) **b** 6) **d** 7) **c** 8) **b** 9) **ac** 10) **ab**
11) **cd** 12) **b** 13) **a** 14) **b** 15) **b** 16) **b** 17) **c** 18) **A-s;B-r;C-q;D-pr**
19) **0** 20) **1** 21) **5** 22) **0** 23) **0**