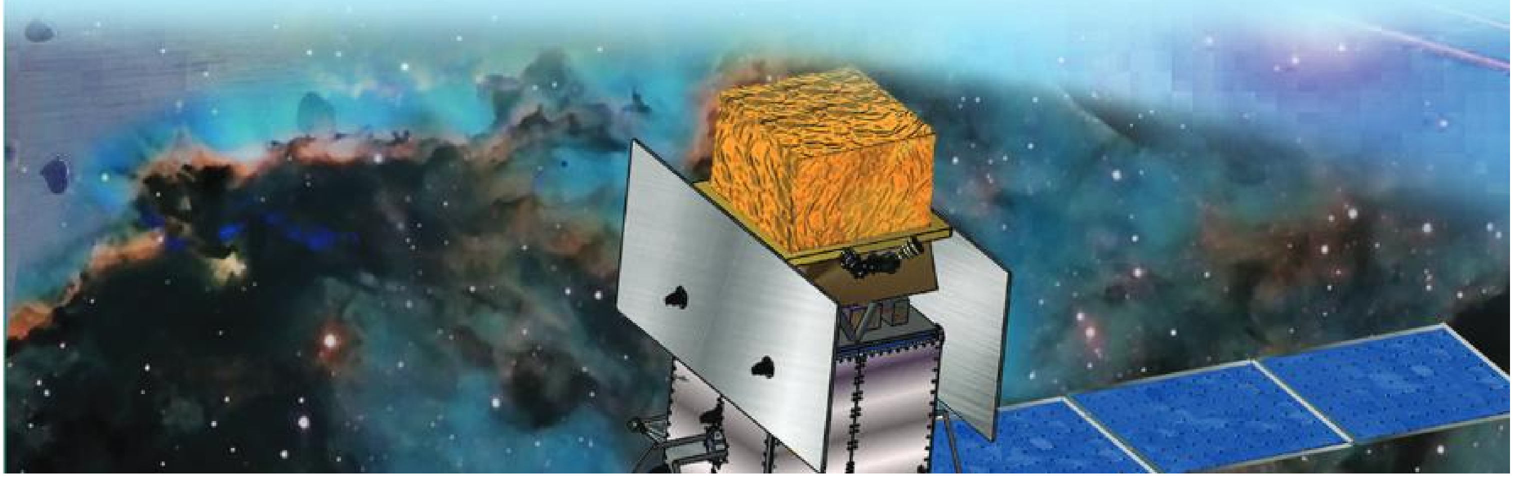
Chapter - 2

NUCLEAR PHYSICS

- Nuclear Forces
- Mass Defect And Binding Energy
- Nuclear Stability
- Modes of Decay
- Radioactivity
- Nuclear Reactions



2.1 INTRODUCTION

We have learnt that all matter, right from a tiny speck of dust to a gigantic cosmic entity consists of atoms. Does an atom have a structure? An atom consists of a central part called nucleus around which electrons will revolve.

The overall dimensions of a nucleus are much smaller than those of an atom. Experiments on scattering of α-particles demonstrated that the radius of a nucleus was smaller than the radius of an atom by a factor of about 10^4. This means the volume of a nucleus is about 10^{-12} times the volume of the atom. In other words, an atom is almost empty. If an atom is enlarged to the size of a classroom, the nucleus would be of the size of a pinhead. But, the nucleus contains most (more than 99.9%) of the mass of an atom.

Another question comes to our mind is that whether a nucleus too have a structure like an atom? If so, what is the nature of its constituents and how are they held together in the nucleus? In unfolding such series of questions, a separate branch in physics called Nuclear physics has evolved.

The year 1896, marks the beginning of nuclear physics. It is in this year, the French physicist Henri Becqurel discovered the phenomenon of radioactivity in one of the Uranium compounds, quite accidentally. The phenomenon of radioactivity refers to a particular type of invisible radiation emerging from certain specific substances. At first sight, this radiation was quite puzzling for scientists. Later on the experiments conducted by Ernst Rutherford and others proved that this radiation consists of alpha, beta and gamma rays. These rays were found to be originating in the nucleus. The experiments conducted by Geiger and Ernst Marsden, involving the scattering of alpha particles have revealed that there exists a positively charged nucleus at the centre of each atom. It was James Chadwick's discovery of neutron in 1932 that clinched the issue of nuclear structure. Soon after the discovery of neutron Werner Heisenberg proposed that the nucleus consists of neutrons and protons. The discovery of artificial radio-activity in 1933 and nuclear fission in 1938, development of first controlled fission reactor in 1942 are some of the important milestones in the field of nuclear physics. More recently, the scientists are focusing on the nature of nuclear forces and are trying to integrate electro weak force (which resulted after integrating electromagnetic force and weak nuclear force), strong nuclear force and gravitational force.

2.2 CONSTITUENTS OF THE NUCLEUS

All atomic nuclei are made up of elementary particles called protons and neutrons. Proton is the nucleus of the hydrogen atom. It has a positive charge of 1.6×10^{-19} C having a mass of 1.6726×10^{-27} kg. This is nearly equal to 1836 times the electron mass. Neutron is electrically neutral (i.e., neutron carries no charge). Mass of neutron is slightly greater than that of the proton (1.6750×10^{-27} kg). Both the proton and neutron together constitute the nucleus. They are called nucleons.

The number of protons inside the nucleus is called the atomic number (Z). The total number of nucleons (sum of protons and neutrons) is called the mass number (A). (A–Z) gives the number of neutrons in a nucleus. The stability of a nucleus depends on the relative number of protons and neutrons.

Nucleus is represented by the symbol $_{z}x^{A}$. This symbol clearly explains the composition of the nucleus.

For example, the symbol $_{92}U^{238}$ indicates that the uranium nucleus contains 92 protons and 238 – 92 = 146 neutrons.

Depending upon the relative number of nucleons contained by the nuclei , they are named as isotopes, isobars, isotones and isomers.

Isotopes : The nuclides having same number of protons but different number of neutrons, are called isotopes. They have same Z but different value of (A – Z). Their chemical properties are similar, so they have same position in periodic table. But their nuclear properties are different.

$_{1}H^{1}$, $_{1}H^{2}$, $_{1}H^{3}$ are the isotopes of Hydrogen atom.

$_{2}He^{3}$, $_{2}He^{4}$, $_{2}He^{6}$ are the isotopes of Helium atom.

$_{8}O^{16}$, $_{8}O^{17}$, $_{8}O^{18}$ are the isotopes of oxygen atom.

Isobars : The nuclides having same number of atomic mass number and different atomic number are called isobars. They have same A but different Z. They possess different neutron number N.

e.g: $_{6}C^{14}$, $_{7}N^{14}$ are isobars.

Isotones : The nuclides having same number of neutrons but different number of protons are called Isotones. They have same N but different Z.

e.g: $_{7}N^{17}$, $_{8}O^{18}$, $_{9}F^{19}$ are isotones.

Isomers: The nuclides having same number of protons and same number of nucleons are called isomers. They have same Z and same A. Their nuclear properties are different. They have different radioactive decay and possess different magnetic momenta.

e.g: $m_{35}Br^{80}$ metastable Bromine and $g_{35}Br^{80}$ ground state Bromine are isomers.

2.3 MASS AND ENERGY

The mass of a proton is about 1836 times the mass of the electron whereas the neutron is slightly more massive than proton and its mass is about 1839 times the mass of electron.

Mass of elementary particles may be expressed in atomic mass units denoted by amu or simply 'u'. One atomic mass unit is defined as $\frac{1}{12}$ mass of ${}_{6}^{12}C$, an isotope of carbon atom or $\frac{1}{16}$th of the mass of a single oxygen atom $_{8}O^{16}$. The exact value for the mass of ${}_{6}^{12}C$, in atomic mass unit is 12 amu.

1 amu = 1u = 1.660539×10^{-27} kg

In nuclear physics the mass is also expressed as million electron volt/(speed of the light)^2 or simply MeV/c^2

1u = 931.5 MeV/c^2

The masses of electron, proton and neutron in terms of various units are :

Mass of the electron = $m_e = 9.1095 \times 10^{-31}$ kg
= 0.000549 u = 0.511 MeV/c^2

Mass of the proton = $m_p = 1.6726 \times 10^{-27}$ kg
=1.007276 u=938.28 MeV/c^2

Mass of the neutron = $m_n = 1.6750 \times 10^{-27}$ kg
=1.008665 u=939.573 MeV/c^2

2.4 SIZE OF THE NUCLEUS

The scattering experiments conducted by Rutherford and his co-workers gave the first evidence of the nuclear size. It is a very tiny fraction of the size of the atom. A nucleus is almost spherical in appearance with no clear boundary. Its charge distribution is not uniform. The scattering experiment of Rutherford has shown that the estimated volume of the nucleus is proportional to mass number (A). If ‘R’ is the radius of the nucleus

$$\frac{4}{3}\pi R^3 \alpha A \text{ (or) } R \alpha A^{1/3} \text{ (or) } R = R_0 A^{1/3}$$

where $R_0 \approx 1.2\times10^{-15}\,m = 1.2$ fm

Hence $R = 1.2\times10^{-15} A^{1/3} m$

It is interesting to notice from above equation that the volume per nucleon $\left(\frac{4\pi}{3}\frac{R^3}{A}=\frac{4\pi}{3}R_0^3\right)$ is the same for all nuclei. In other words, the density of the nuclear matter, which is a measure of the number of nucleons per unit volume, is independent of A. This leads to another result that the nuclear forces are independent of the charge of the nucleons.

2.5 NUCLEAR DENSITY

Density stands for mass per unit volume. Let us calculate the density of a nucleus of mass M and size R, considering that it is almost a spherical distribution.

Taking nucleonic mass ≈ 1 a.m.u, we have mass of the nucleus

$M = \text{mass of A nucleons} = A \times 1.66 \times 10^{-27}\,\text{kg}$

$\therefore$ Volumetric density

$$\rho = \frac{m}{V} = \frac{A \times 1.66 \times 10^{-27}}{\frac{4\pi}{3}(1.2 \times 10^{-15})^3 A}$$

(or) $\rho \approx 2.3 \times 10^{17}\,\text{kg}/\text{m}^3$

We can notice that not only 'ρ' is very high in value, but also independent of mass number A. Thus all nuclei have almost the same density. Nuclear density is about 10^{15} times greater than the density of matter in bulk. Such extra-ordinary high densities are not seen in our everyday observations, but such values are possible in neutron stars (white dwarfs).

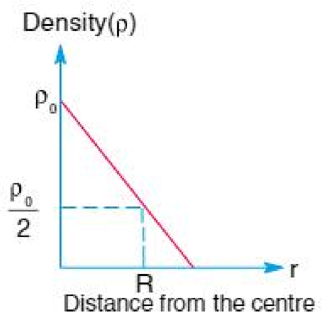


Fig 2.1

Density distribution is not uniform through out a given nucleus, but it varies, It is represented in figure. It is maximum at the center of the nucleus and its value falls off with distance r. As is well known, a nucleus has no well defined boundary. As such, the radius (R) of a nucleus has also been defined as the distance at which the density (ρ) of nuclear matter falls to one half of its maximum value at the centre. In view of Einstein's mass energy equivalence ($E = mc^2$), atomic nuclei represent a region where concentration of energy is extremely large.

2.6 NUCLEAR FORCES

The atomic nucleus contain two types of particles, protons and neutrons. While a proton has a positive charge, a neutron is electrically neutral. Because of repulsive force between the protons, we can expect nucleus to be unstable, but it is stable. This means that a very strong attractive force should exist between the nucleons in a nucleus and is called nuclear force. This force is certainly different from the gravitational force of attraction between nucleonic masses because gravitational forces are too weak compared to electrostatic forces for a given separation.

Regarding nuclear force

(i) Nuclear force exists only inside a nucleus . Nuclear force is attractive and 100 times stronger than Coulomb's force and appears to be independent of electric charge. The nuclear interaction between two protons, two neutrons, or one proton and one neutron are basically same i.e.,

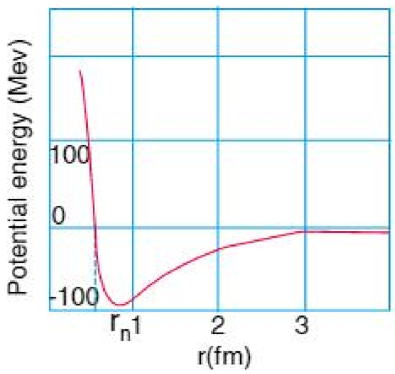
$$F_{nn} \approx F_{pp} \approx F_{np}$$

(ii) The average binding energy per nucleon is constant for most of the nuclei, except for lighter that means the total binding energy of a nucleus is nearly proportional to the total number of nucleons present in it. If every nucleon is assumed to interact with every other nucleon, this interaction and hence binding energy must be proportional to the number of interaction pairs. For 'A' nucleons in nucleus, there are $\frac{A(A-1)}{2}$ distinct pairs. Then binding energy should be proportional to nearly

A^2. But the binding energy is proportional to A only. This suggest that a nucleon must be interacting with only a limited number of neighboring nucleons and not with all of them. This property of nuclear force is known as saturation force.

(iii) Nuclear forces must have a short range. It means that the strength of nuclear force and interaction energy between two nucleons must fall off very rapidly as their separation is increased. Since this force exists only inside the nucleons, the range of such a force must be of the order of nuclear dimensions 10^{-14} m. As such, the de-Broglie wavelength $\left(\lambda = \frac{h}{p}\right)$ of nucleons (or) their Compton wavelength $\left(\lambda = \frac{h}{mc}\right)$ must never exceed the value $R \approx 10^{-14} m$.

When the interacting nucleons come very close to each other, attraction changes into strong repulsion and the natural adjustment follows. In other words the distance between two nucleons adjusts so as to give stability. In the given Fig 2.2 for r < 0.8 fm, the force between two nucleons is repulsive.



(b)

Fig 2.2

(iv) Nuclear forces are spin dependent in the sense that they very much depend on the mutual orientation of the spin of the interacting nucleus. This fact has been confirmed from the nuclear energy levels. It has been found that the interaction energy of a two nucleon system where the two nucleons have their spins parallel is different from the energy of such a system where the spins are anti - parallel. In neutron - proton system (deuterium nucleus) a spin - 1 bound state is observed, no such bound state observed with spin - 0.

(v) They are non central because such forces can be represented along the straight line joining the centres of the two interacting nucleons and that force $F \alpha r^{-7}$. Hence the orbital angular momentum of the two nucleons relative to their centre of mass is not - conserved whereas in central forces angular momentum is conserved.

(vi) They are exchange forces. According to Meson theory of Yukawa, all nucleons consist of identical cores surrounded by a cloud of one or more mesons. Mesons are charged as positive or negative or neutral. Pions (π^+, π^- or π^0) and kaons (k^+, k^-, k^0) belong to this category. The difference between neutron and proton is due to difference in the composition of their respective meson clouds. Every nucleon continuously emits and absorbs pions. The transfer of momentum associated with the shift of pion is equivalent to the action of force of attraction or repulsion. Thus nuclear force is the result of the exchange of pions between the nucleons. Conversion of nucleons is given below.

$$n \rightarrow p + \pi^- \text{ and } p + \pi^- \rightarrow n$$
$$p \rightarrow n + \pi^+ \text{ and } n + \pi^+ \rightarrow p$$

The emission of a pion by a nucleon with no change of its mass is a violation of the law of conservation of mass and energy. But this violation is allowed only very temporarily because this nucleon reabsorbs the same or another pion available so soon. A pion is not stable :The mean life of charged pions and neutral pion are about 10^{-8} s and 10^{-16} s respectively. The rest mass of charged pion is $273m_e$ and that of neutral pion is $264m_e$, where m_e is the rest mass of electron.

2.7 NUCLEAR SPIN AND MAGNETIC MOMENT

Experimental facts such as α spectra, γ-spectra show that the nucleons of the nucleus are in non - stop motion in discrete quantized orbits. Orbital motion of the nucleons will lead to having certain mechanical angular momentum. In addition, charged particles protons and neutral particles neutrons will also posses certain magnetic moment. The magnetic moment due to neutrons is explained by internal structure of neutrons. The nucleus as a whole, therefore will posses (a) an intrinsic angular momentum (which is the resultant of orbital and spin momenta of the different nucleons) plus (b) a magnetic moment (only for protons). While (a) is popularly called the nuclear spin (b) is the nuclear magnetic moment for the nucleus as a whole.

2.8 MASS DEFECT AND NUCLEAR BINDING ENERGY

Though the atomic masses are considered to be integer numbers, the fact remain that they are not integral always. They differ from the integers marginally. It may be due to some deviations of the mass of the nucleus (less) than from the mass of its constituents. The difference between mass of the nucleus and the sum of the individual masses of its nucleons is called mass defect.

Let Z protons and (A - Z) neutrons constitute a nucleus of atomic number Z and mass number A. If 'M' is the mass of the nucleus, m_p and m_n are the masses of proton and neutron respectively, then the mass defect

$$\Delta m = [Zm_p + (A - Z)m_n] - M.$$

This mass defect is released in the form of energy according to Einstein's equation $E = mc^2$. Nuclear binding energy is defined as the energy released during the formation of the nucleus or it is the energy required to break up the nucleus into its components and place them infinitely apart at rest. Hence binding energy

$$BE = \left[[Zm_p + (A - Z)m_n] - M\right]c^2$$

If all the masses are measured in atomic mass unit i.e in a.m.u (u) where

$$1u = 1.67 \times 10^{-27}\,kg = 931.4MeV$$

Then $BE = \Delta m \times 931.4\,MeV$.

Greater this loss of energy (binding energy) more stable is the nucleus formed and vice versa.

The stability of the nucleus is indicated by the average binding energy per nucleon rather than by the net binding energy

$$\text{BE per nucleon} = \frac{BE}{A} = \frac{\Delta m \times 931.4}{A} MeV$$

It may also be defined as the amount of energy required to remove one nucleon out from the nucleus or vice versa.

If for any nucleus, the B.E per nucleon is large it is stable. For unstable nuclei the B.E per nucleon will be small. The curve obtained on plotting the binding energy per nucleon versus the mass number is known as the binding energy curve.

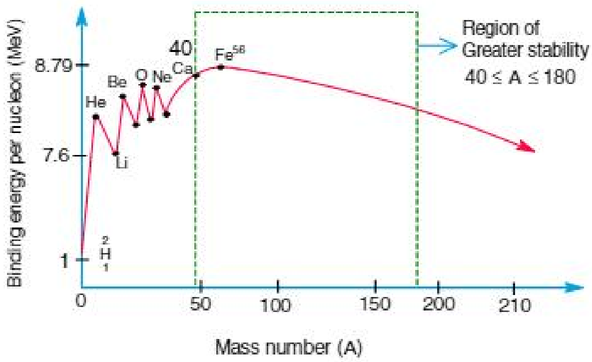


Fig 2.3

The main features of binding energy curve shown in Fig 2.3 are :

1) The minimum value of binding energy per nucleon is in the case of deuteron (1.11MeV).

2) The maximum value of $\frac{BE}{A}$ is 8.75MeV for the nuclide $_{28}Fe^{56}$ (iron) which is the most stable.

For A = 238, the value is about 7.6 Mev

3) The average $\frac{BE}{A}$ for all the elements is about 8 MeV.

4) The most stable nuclides fall in the range $30 < A < 170$ as their $\frac{BE}{A}$ cover the upper most constant level values.

5) Bellow A = 28 there are peaks in the curve. They correspond to $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$ nuclei. All of these represent nuclei having equal number of protons and neutrons (even - even nuclei). They are more stable than their immediate neighbours of odd - odd nuclei and odd - even nuclei.

6) Nuclei with A > 220 are unstable. That means from A > 220, single heavy nucleus breaks into two nearly equal nuclei with mass number A < 150 which are most stable. This process occurs at right of the BE curve as shown in Fig 2.4. This process is called nuclear fission. The is concept is explained is detail under nuclear fission section.

7) Nuclei with A < 30, some are distinctly unstable. That means for A < 30 two light nuclei combine to form a single nucleus with mass number A < 60 which are more stable. This process occurs at left of the BE curve as shown in Fig 2.4. This process is called nuclear fusion.

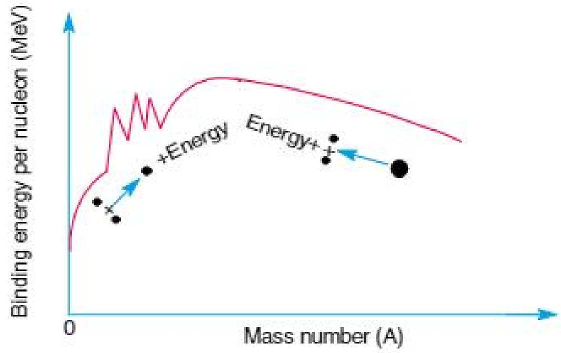


Fig 2.4

Therefore it may be concluded that the stability of a nucleus depends upon binding energy per nucleon rather than on the total binding energy of the nucleus.

Let us consider the nuclei of uranium, helium and iron as examples so as to explain the relationship between binding energy per nucleon and stability of the nuclei. Uranium has a relatively low binding energy per nucleon as 7.6 Mev. Hence to attain greater stability uranium breaks up into nuclei of intermediate mass resulting in a phenomenon called fission. On the other hand lightrt nuclei such as hydrogen combine to form heavy nucleus to form helium for greater stability, resulting in a phenomenon called fusion. Iron whose binding energy per nucleon stands maximum at 8.7 Mev is the most stable and will undergo neither fission nor fusion.

2.9 PACKING FRACTION (f)

Packing fraction has been a useful parameter in the study of nuclear stability and isotopic masses, eventhough it does not have a precise physical meaning.

Packing fraction

$$f = \frac{\text{Atomic mass of nucleus} - \text{mass number}}{\text{mass number}}$$

$$= \frac{_ZM^A - A}{A}$$

where $_ZM^A$ is the mass of the nucleus in a.m.u

$$\text{(or) } f = \frac{\Delta m}{A} = \frac{\text{mass defect}}{\text{mass number}}, \text{ so that}$$

$$\frac{B.E}{A} = \frac{\Delta m(931.5)}{A} = f \times 931.5\text{MeV}$$

The packing fraction may be negative, zero or positive. A nucleus with a negative f has an atomic mass less than its mass number. Here some mass has been converted into energy and that nucleus is stable. Similarly positive value of packing fraction implies a tendency towards instability in the nucleus. In general, a lower value of packing fraction implies greater stability. Transition elements or iron have lowest value of packing fraction, showing maximum stability.

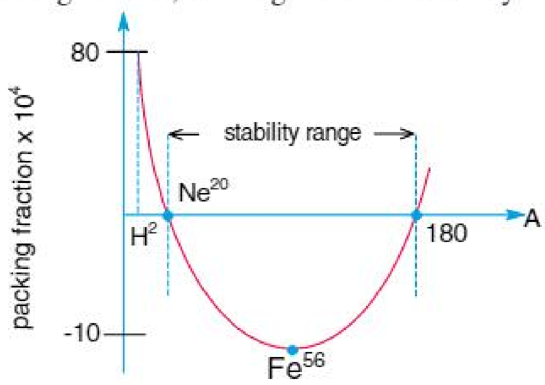


Fig 2.5

A curve is plotted between the packing fraction (f) and the mass number (A) as shown in Fig 2.5 for different nucleide. For A < 20, f is +ve, from A = 20 to A =180, f is –ve, and from A > 180, f is again +ve.

Example-2.1

Calculate the average energy required to extract a nucleon from the nucleus of an α-particle in MeV. It is given that the masses of α-particle, proton and neutron are 4.00150 a.m.u, 1.00728 a.m.u and 1.00867 amu respectively.

Solution :

Mass defect, $(\Delta m) = 2(m_p + m_n) - m_N$

$= 4.0319 - 4.00150$

$= 0.0304$ a.m.u

Binding energy of α-particle $= \Delta m \times 931.5$

$= 0.034 \times 931.5$

$= 28.3$ MeV

Average energy required to extract nucleon $= \dfrac{28.3}{4}$

$= 7.07$MeV .

Example-2.2

The atomic mass of an alpha particles 4.002603 amu and that of oxygen is 15.994915 amu. Find the energy required to split up the oxygen -16 nucleus into 4 alpha particles.

Solution : $\Delta m = m(4He_2^4) - m(O_8^{16})$

$= 4 \times 4.002603 - 15.994915 = 0.015497$ amu

Hence BE = $\Delta m \times 931.5$ MeV = 14.435 MeV

2.10 NUCLEAR STABILITY

A nucleus is stable if its density remains the same with the passage of time, i.e it does not transform itself into another nucleus without release or the addition of some energy. Among all the nuclides (nearly 1500 known) only a total of 274 are found to be stable. It has been found that the stability of a nucleus very much depends upon both the number of protons and the number of neutrons in the nucleus.

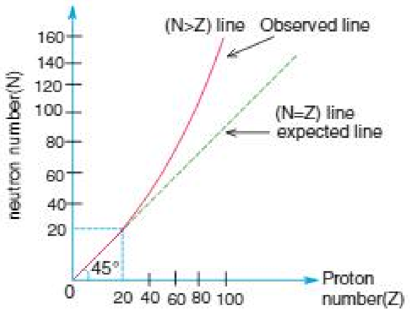


Fig 2.6

Fig 2.6 shows a plot of the neutron number N verses the proton number Z for the stable nuclides and the known unstable nuclei that live long enough to be observed. In the graph the expected shape of the line of stability is at N=Z, but the actually observed stability line is N > Z. The two lines run together upto Z = 20 (i.e., A = 40); thereafter they separate into two. Notice that the slope of N > Z line is more than 45^0 after Z = 20. Upto Z = 20, we have the number of neutrons equal to the number of protons. Beyond that the stable nuclei show an excess of neutrons.

As the number of protons increases, the coulombic repulsive force increases, which increases the energy of the nucleus and so decreases the stability. Hence the excess neutrons keep the nucleus stable by attraction between neutrons and protons.

For large values of Z > 83 the nuclear force (short range) is unable to hold the nucleus together against the electrostatic repulsion (long range) of the protons. Hence to attain stability, it spontaneously emits various particles, i.e., α, β and / or γ–radiations.

Like electron energy levels in atoms, nuclear energy levels also exist in nucleus, which are filled by nucleons in sequence to achieve configuration of minimum energy and maximum stability. Nucleons which have spins of 1/2, obey the exclusion principle. Hence each nuclear energy level can contain two neutrons of opposite sign and two protons of opposite sign.

Nuclides with even number of protons and even number of neutrons are more stable than with odd number of protons and odd number of neutrons. Also the even - odd nuclei and the odd - even nuclei have the intermediate levels of stability between these two. We find that 60% of total nuclei come under the even - even type and only negligible number belongs to the odd - odd category. These are ${}_1H^2, {}_3Li^6, {}_5B^{10}$ and ${}_7N^{14}$ nuclei. This shows that a nucleus in which the nucleons of the same kind occur in pairs is particularly stable and a nucleus that contains an unpaired proton or unpaired neutron is much less stable.

Ex : ${}_6C^{12}$ more stable than ${}_5B^{12}$

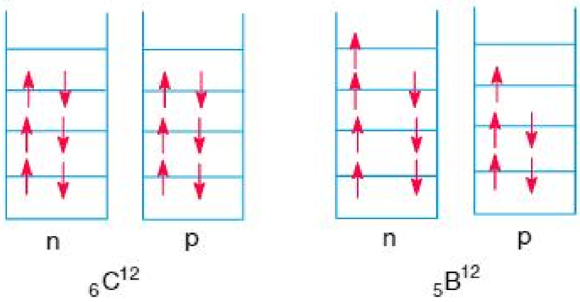


Fig 2.7

Nuclei with partly filled levels have greater tendency to pickup or lose neutrons or convert neutron to proton and vice versa than those with filled levels. Hence they are likely to participate in the nuclear reactions involved in the formation of elements.

The experimental facts reveal that when either number of protons (Z) or the number of neutrons (N) is equal to one of the numbers 2, 8, 20, 28, 50, 82 and 126, the nuclei becomes stable. These numbers are commonly referred to as magic numbers.

2.11 RADIO ACTIVITY

Spontaneous decay of unstable nuclei by emission of certain sub particles like α, β and γ radiation is called natural radio activity.

In general natural radioactivity occurs in heavy nuclei which tend to become stable. The nuclei which can be transformed into radioactive nuclei by various means is called artificial radioactivity. Regarding radioactivity.

(i) It is completely unaffected by the physical and chemical conditions to which the nucleus is subjected, i.e., we cannot change the radio activity by applying high temperature, high pressure and strong electric field etc.

(ii) A nucleus can disintegrate immediately or it may take infinite time. Hence we can define a probability for the number of nuclei that may disintegrate in a given time interval.

(iii) The energy liberated during the radioactive decay comes from within individual nuclei.

2.12 MODES OF DECAY

A radioactive nucleus is called a parent nucleus, the nucleus resulting from its decay by particles emission is called daughter nucleus. This daughter nuclei may be stable or unstable.

$$\underset{\text{Parent}}{X} \longrightarrow \underset{\text{Daughter}}{Y} + \underset{\text{Radiation}}{R} + \underset{\text{Energy}}{Q}$$

Here R may be either α particle or β particle or γ radiation. Q is the energy of the emitted particles (or radiation).

During the radioactivity all conservation laws must be observed : mass - energy, linear momentum, angular momentum, electric charge, spin, nucleon number etc. In certain decay process proton number also conserved.

Let M_x, M_y and M_R be the masses of the parent nucleus, daughter nucleus and the radiated particles respectively. By conservation of energy

$$M_X c^2 = M_Y c^2 + M_R c^2 + Q.$$

where Q is the energy released and is equal to the total energy carried by the reaction products. $Q = [M_x - M_y - M_R]c^2$ or $Q = \Delta mc^2$, and the disintegration energy is negative of binding energy. If $Q > 0$, the nuclei is said to be unstable. In natural radio activity $Q > 0$.

2.12.1 α -DECAY

An α - particle is a helium nucleus. Thus a nucleus emitting an α -particle loses two protons and two neutrons. Therefore atomic number Z decrease by 2, the mass number A decrease by 4 and the neutron number N decrease by 2.

$${}_zX^A \rightarrow {}_{z-2}Y^{A-4} + {}_2He^4 + Q$$

$$eg:\ {}_{88}Ra^{226} \rightarrow {}_{86}Rn^{222} + {}_2He^4 + 4.87\text{MeV}$$

'α'-decay is possible only if the mass of the parent nucleus is greater than the mass of the daughter nuclei.

If M_x, M_y and M_α are the masses of parent nuclei, the daughter nuclei and the α-particle respectively, then disintegratation energy

$$Q=[M_x-M_y-M_\alpha]c^2 \text{, i.e., } Q=K_y+K_\alpha$$

if the parent nuclei is assumed to be at rest initially. By conservation of momentum $P_y = P_\alpha$

Hence the ratio of their kinetic energies is $\frac{K_\alpha}{K_y}=\frac{M_y}{M_\alpha}$. Then the K.E of the α particle is $K_\alpha=\left[\frac{M_y}{M_\alpha+M_y}\right]Q$. Since the mass of the daughter nuclei is (A - 4) a.m.u., while that of the α-particle 4 a.m.u., we may write $K_\alpha=\left[\frac{A-4}{A}\right]Q$. The kinetic energy of α-particle emitted is nearly equal to the disintegration energy (Q), because the daughter nucleus recoils with a negligibly small speed. However decayed α-particles have energies that range from 4MeV to 9MeV, depending on the particular nuclide involved. After traversing certain distance the α-particles lose their power of ionization as well as the power of exciting fluorescence.

2.12.2 β-DECAY

It is of three types. Beta decay can involve the emission of either electrons or positrons. A positron is an antiparticle of electron which has a charge +e and mass equal to that of electron. The electron or positron emitted in β-decay do not exist inside the nucleus. They are created at the time of emission just as photons are created when an electron in atom makes a transition from a higher to lower energy state. The conservation principle of energy, linear momentum and angular momentum all are apparently violated in β-decay.

(i) In negative β- decay (β^-) a neutron is transformed into proton and an electron with emission of antineutrino (ν^*). β^- decay is due to nucleus having too many neutrons relative to number of protons. ${}_0n^1 \to {}_1p^1 + {}_{-1}e^0 + \nu^*$.

To conserve momentum, the spin and angular momentum the anti neutrino is emitted. Before emission of β- particle, the momentum of the parent nucleus is zero. The antineutrino is emitted along with the β- particle, with a momentum which exactly balances the sum of the momentum of the β- particle and the recoiling daughter nucleus. It has zero electric charge and its rest mass is very small, closest to zero.

$${}_ZX^A \xrightarrow{\beta^-} {}_{Z+1}Y^A + {}_{-1}e^0 + \nu^* + Q$$

Thus in β^- decay, atomic number of daughter nuclei increases by one.

eg : ${}_6C^{14} \xrightarrow{\beta^-} {}_7N^{14} + {}_{-1}e^0 + \nu^* + Q$

(ii) In positive β- decay (β^+) a proton is transformed into neutron and a positron with emission of neutrino (ν). Neutrino is the anti particle of anti neutrino. β^+- decay is due to nucleus having too many protons relative to number of neutrons.

${}_1p^1 \to {}_0n^1 + {}_{+1}e^0 + \nu + Q$. Hence, in β^+ decay atomic number of daughter nuclei decreases by 1

$${}_ZX^A \xrightarrow{\beta^+} {}_{Z-1}Y^A + {}_{+1}e^0 + \nu + Q$$

eg : ${}_7N^{13} \xrightarrow{\beta^+} {}_6C^{13} + {}_{+1}e^0 + \nu + Q$

in β^- and β^+ emission, the rest mass of parent nucleus must be greater than that of the daughter nucleus.

Hence in β^- -decay

$$Q=\left[M_x-M_y-M_\beta\right]c^2 \text{ and}$$

in β^+ -decay $Q=\left[M_x-M_y-M_{\beta^+}\right]c^2$

Since $Q>0$, in both decay processes emitted β-particles have kinetic energy. Experimentally it is found that in a given sample, the β- particles are emitted with a continuous range of kinetic energies from 0 to some maximum value K_{max} as shown in Fig 2.8.

The maximum K.E value is characteristic of the nuclide and its value is Q_{max}. Even for same type nuclides, the emitted β- particles have different KE due to unequal sharing of energy by the emitted neutrinos. The average energy released per particle

$$= \frac{\text{Area under the curve}}{\text{Total number of } \beta - \text{particles}}$$

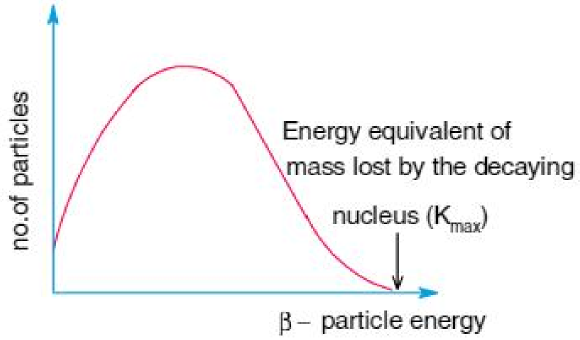


Fig 2.8

(iii) Electron capture : In this decay mode the parent nucleus captures one of its own orbital atomic electron and emits a positron with neutrino. The final product after decay is a nucleus whose charge is (Z – 1). Electron capture is also due to excess protons relative to neutrons. This is similar to β^+ decay. ${}_1p^1 + {}_{-1}e^0 \rightarrow {}_0n^1 + \nu$.

Hence ${}_ZX^A + {}_{-1}e^0 \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + \nu + Q$

where $Q = [M_x - M_y]c^2$. The electron capture involves the electron (s) of the inner-most K-shell normally and this is referred as K-capture.

eg : ${}_4Be^7 + {}_{-1}e^0 \rightarrow {}_3Li^7 + {}_{+1}e^0 + \nu + Q$

In heavy nuclides electron capture is much more likely than the positron emission.

2.12.3 γ-DECAY

A nucleus can exist in one of a number of quantized energy states, the excited states, similar to an atom. A stable nucleus is normally found in its lowest energy state called ground state. An excited nucleus (unstable) can return to its ground state by emitting photon (s) of MeV energy ranges. Such photons are called gamma γ- rays or γ-photons. There is no change of A and Z values of a given nucleus during γ-decay. Hence γ-decay due to nucleus has excess energy.

$${}_Z{}^*X^A \rightarrow {}_ZX^A + \text{Photon(s)}$$

eg : ${}_{38}{}^*Sr^{87} \rightarrow {}_{38}Sr^{87} + \gamma$.

The γ-ray energy is the difference of the higher energy state and lower energy state. In a case where an excited nucleus returns to its ground state in a number of steps, a cascade of photons is released, one after the other. But only those nuclear transitions are allowed for which conservation laws hold.

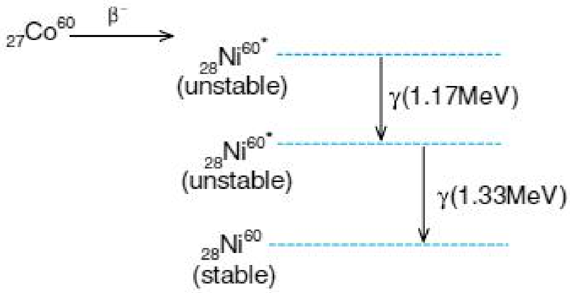


Fig 2.9

Most radio active isotopes after their α-decay and /or β-decay leave the daughter nucleus in an excited state. A -emission is the next step in all such cases. This is shown in Fig 2.9 with example. Most excited nuclei have very short half-life against -decay, but a few remain excited for as long as several hours. A long lived excited nucleus is called an isomer of the same nucleus in its ground state for example ${}_{38}Sr^*$ is an isomer of ${}_{38}Sr$.

Example-2.3

Neon-23 decays in the following way ${}^{23}_{10}Ne \rightarrow {}^{23}_{11}Na + {}_{-1}e^0 + \bar{\nu}$. Find the minimum and maximum kinetic energy that the beta particle (${}_{-1}e^0$) can have. The atomic masses of ${}^{23}Ne$ and ${}^{23}Na$ are 22.9945 u and 22.9898 u, respectively.

Solution :

Here, atomic masses are given (not the nuclear masses), but still we can use them for calculating the mass defect because mass of electrons get cancelled both sides. Thus,

Mass defect $\Delta m = (22.9945 - 22.9898) = 0.0047u$

$\therefore$ $Q = (0.0047u)(931.5MeV/u) = 4.4$ MeV

Hence, the energy of beta particles can range from 0 to 4.4 MeV.

Example-2.4

Find the Q value of the reaction $p + {}^7Li \rightarrow {}^4He + {}^4He$. Determine whether the reaction is exothermic or endothermic. The atomic mass of ^1H, ^4He and ^7Li are 1.007825 u, 4.002603 u and 7.016004 u respectively

Solution :

The total mass of the initial particles

$m_i = 1.007825 + 7.016004 = 8.023829$ u and the total mass of the final particles

$m_f = 2 \times 4.002603 = 8.005206$ u

Difference between initial and final mass of particles

$\Delta m = m_i - m_f = 8.023829 - 8.005206 = 0.018623$ u

This mass is converted into energy and the reaction is exothermic. The Q value is positive and given by

$Q = (\Delta m)c^2 = 0.018623 \times 931.5$

$= 17.35$ MeV

Example-2.5

The kinetic energy of α-particles emitted in the decay of $_{88}Ra^{226}$ into $_{86}Rn^{222}$ is measured to be 4.78 MeV. What is the total disintegration energy or the 'Q-value of this process'?

Solution :

The standard relation between the kinetic energy of the α-particle (T_α) and the Q-value (or total disintegration energy) is

$$T_\alpha = \left(\frac{A-4}{A}\right).Q$$

$$Q = \left(\frac{A}{A-4}\right).T_\alpha = \left(\frac{226}{226-4}\right) \times 4.78\text{MeV}$$

$$= \frac{226}{222} \times 4.78\text{MeV}$$

$$\therefore Q = 4.865\text{MeV} \approx 4.87\text{MeV}$$

Notice that T_α is very close to (but smaller than) Q.

Table-1 Properties of α, β, γ -Rays

α - rays	β - rays	γ - rays
1) They are Helium nuclei or doubly ionized Helium atoms	1) They are fast moving electrons coming from the nucleus	1) They are electromagnetic waves of wavelength less than 1 Å having no charge
2) They are deflected by electric and magnetic fields through small angles.	2) They are deflected in electric and magnetic fields in a direction opposite to that of α-rays but through large angles	2) They are not deflected by electric and magnetic fields
3) They can ionize the gas through which they pass and the ionizing power is 100 times to that of β - rays and 10000 times of γ rays	3) They can ionise the gas through which they pass. Their ionising power is 1/100 that of α - rays and 100 times that of γ rays	3) They can ionise the gas through which they pass, but with much lower ionising power
4) They can penetrate through matter. The penetrating power is 1/100th of β - rays and 1/10000 of γ - rays.	4) They can penetrate through matter. The penetrating power is 100 times that of α-rays but 1/100th of γ- rays.	4) They can penetrate through matter. The penetrating power is more than that of α and β
5) These are particles	5) These are particles	5) These are photons.
6) Their velocities are in the order of 10^6 m/s	6) Their velocity is 1/10th of the speed of light	6) They travel with the velocity of light
7) They effect photographic plate	7) They effect photographic plate.	7) They effect photographic plate.
8) They cause fluorescence.	8) They cause fluorescence.	8) They cause fluorescence.

2.13 ARTIFICIAL TRANSMUTATION OF ELEMENTS

The conversion of one element into another by artificial means is called artificial transmutation of the element. Rutherford performed number of experiments in which the atoms of different stable elements, such as nitrogen, aluminum, phosphorus, etc, were bombarded by high sped $\alpha-$particles from natural radioactive substances. The apparatus used by Rutherford to demonstrate the first artificial transmutation of elements consisted of a chamber 'A' with a window on one side. It was covered with a thin sheet of silver foil F and a zinc sulphide screen S was placed next to it as shown in Fig 2.10.

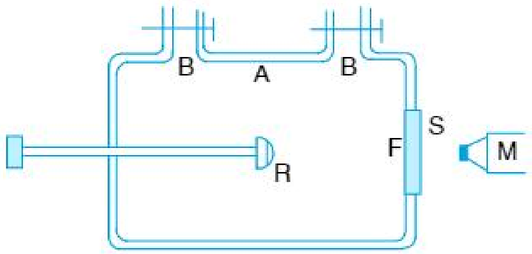


Fig 2.10

The scintillations on the screen were observed by a microscope M. The side tubes B, B were used to fill various gases in the chamber. The source of $\alpha-$particles, Ra was placed on a small disc at R. Its distance from F was adjustable. The $\alpha-$particles from the radioactive Ra were of energy about 8 MeV with a range of about 7cm in air. When the $\alpha-$particles pass through the different gases in the chamber the resultant effect could be observed as scintillations on S. When the chamber was filled with oxygen or carbon dioxide, there were no scintillations on S provided the distance between R and F was greater than the range of the $\alpha-$particles. But when the chamber was filled with nitrogen, scintillations were observed on S even if the distance between R and F was increased to 40 cm much greater than the range of $\alpha-$particles in nitrogen. These particles producing scintillations cannot be $\alpha-$particles as they cannot have such a long range. Rutherford found that the long range particles were nothing but protons. Rutherford concluded that the nitrogen nucleus hit by an $\alpha-$particle $({}_2^4He)$ transmutes into oxygen nucleus along with a proton ${}_1^1H$. The nuclear reaction causing this artificial transmutation can be written as ${}_7^{14}N+{}_2^4He \rightarrow {}_8^{17}O+{}_1^1H$. This reaction shows that a stable isotope of nitrogen is converted into a rare isotope of oxygen. Thus, artificial transmutation was made possible.

2.14 LAW OF RADIOACTIVE DECAY

A macroscopic sample of any radioactive isotope consists of a vast number of radioactive nuclei. These nuclei do not decay all at a time. Rather, they decay one by one over a period of time. This decay is a random and statistical process i.e., each and every nucleus has an equal chance of disintegration at any time. We can't predict exactly which nucleus in a sample will decay, but we can determine statistically how many nuclei in a sample will decay over a period of time.

Based on their experimental observations and analysis of certain radioactive materials, Rutherford and Soddy formulated a theory of radioactive decay. According to them

(i) After decay of a nucleus, the new product (daughter nucleus) has totally different physical as well as chemical properties.

(ii) The decay is spontaneous in the sense that it can't be controlled or suppressed by any, means physical or chemical.

(iii) The number of nuclei breaking up at any instant (also called the intensity of radioactivity or activity) is proportional to the number of atoms present (undecayed) at that very instant.

If N is the number of radioactive nuclei present at some instant t, and if dN atoms decay in an infinitesimally small time dt, then

$$\frac{dN}{dt} \alpha N \text{ (or) } \frac{dN}{dt} = -\lambda N,$$

where λ is called decay constant. The minus sign indicates that N decreases with time. If N_0 is the initial number of nuclei (i.e., at $t=0$).

Then $\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$ (or)

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\therefore N = N_0 e^{-\lambda t}$$

Above equation is known as the decay law or the law of radio-active decay. It is an exponential law.

Let $\underset{\text{(unstable)}}{X} \longrightarrow \underset{\text{(stable)}}{Y} + \text{radiation}$

Consider at $t = 0$, the number of atoms of radio active substance X is N_0 and the number of atoms of Y (stable) is zero.

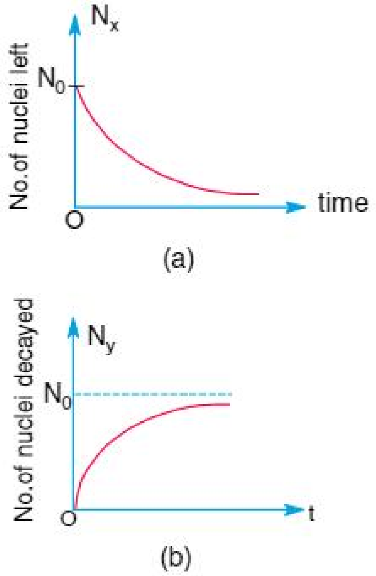


Fig 2.11

If N_x is the number of parent nuclei left after time 't', then $N_x = N_0 e^{-\lambda t}$.

It can be seen that $N_x \to 0$ as $t \to \infty$, This indicates that number of atoms of the parent element goes on decreasing and the number of atoms of daughter element goes on increasing. Hence the number of daughter nuclei (N_y) formed in that time $N_y = N_0 - N_x$ (or) $N_y = N_0(1 - e^{-\lambda t})$

Because the decay constant (λ) is characteristic of a particular radioactive species, we have taken same value of λ in N_x and N_y. The curves are, therefore symmetrical as shown in Fig 2.12.

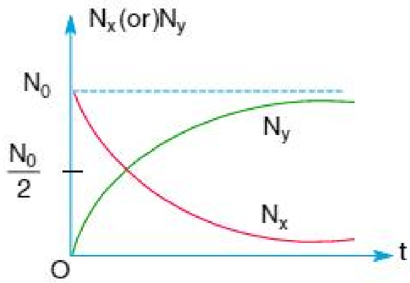


Fig 2.12

2.14.1 ACTIVITY (R)

The number of decays per unit time or decay rate is called activity (R). i.e.

$$|R| = \left|\frac{dN}{dt}\right| = \frac{d}{dt}\left(N_0 e^{-\lambda t}\right) \text{ (or)}$$

$$R = \lambda N_0 e^{-\lambda t} \text{ (or) } R = R_0 e^{-\lambda t}$$

where $R_0 = \lambda N_0$ is the decay rate at $t = 0$, called initial activity.

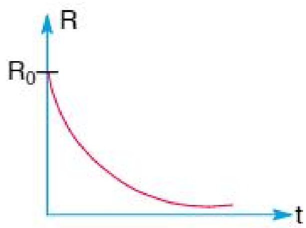


Fig 2.13

If a nuclide can decay simultaneously by n processes, which have activities R_1, R_2, ... and R_n, then the resultant activity $R = R_1 + R_2 + ... + R_n$. Nucleus decaying simultaneously by more than one process is called "parallel decay".

The S.I unit of activity is Becqerel (Bq) or Curie (Ci) or Rutherford (Rd).

1 Ci = 3.7×10^{10} decays per second, 1 Bq = 1 decay per second, 1 Rd = 10^6 decays per second.

2.14.2 DECAY CONSTANT (λ)

It gives the ability of decay of a nucleus. The decay constant λ for a given radioactive species is the reciprocal of the time during which the number of atoms decreases to $\frac{1}{e}$ times their original value.

Larger value of λ corresponds to decay in smaller time and vice versa. λ = 0 for stable nuclei. Decay constant is the characteristic of the sample taken and does not vary with time. If a nucleus can decay simultaneously by more than one process (say n), which have decay constants λ_1, λ_2......and λ_n, then the effective decay constant is $\lambda = \lambda_1 + \lambda_2 + ...+ \lambda_n$.

2.14.3 HALF LIFE (T)

The half life of a radioactive species is defined as the time interval during which the activity of a radioactive sample falls to half of its initial value, (or) half life of radioactive substance is the time taken for half of a given number of nuclei to decay. Half lives vary from isotope to isotope. While T may be as small as 10^{-16} s, its largest value may be as big as 10^9 years.

If N = N_0 at t = 0, Then

$N = \frac{N_0}{2}$ and t = T

$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T}$ $(\because N = N_0 e^{-\lambda t})$

(or) $\lambda T = \log_e 2$

$\therefore T = \frac{\log_e 2}{\lambda}$

$= \frac{0.693}{\lambda}$ $(\because e \approx 2.718)$

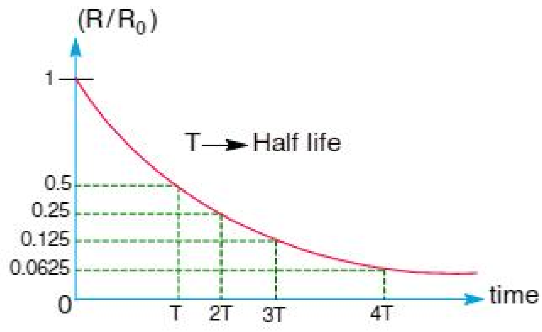


Fig 2.14

From graph 2.14 it is very clear that

After n-half lives (i.e., t = nT), the fraction of nuclides decayed is $\left(\frac{1}{2}\right)^n$, i.e., $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

So the fraction of nuclides undecayed is $1 - \left(\frac{1}{2}\right)^n$.

At any given instant whatever amount of the undecayed sample we have, it is reduced to exactly half its value after a time equal to the half life of the sample.

Half life is the characteristic property of the sample and it cannot be changed by any known method.

In parallel decay $\lambda = \lambda_1 + \lambda_2 + ...\lambda_n$.

Hence $\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2} + ...\frac{1}{T_n}$, where T is the equivalent half-life and T_1, T_2 ....T_n are the half-lives of individual decay.

2.14.4 AVERAGE LIFE OR MEAN LIFE

Radioactivity is a random phenomenon because we just can't predict which of the atoms in a given sample will decay first and when. In decay process some of the atoms of the given sample may have very short life span, and others may not decay even after a very long span of time.

It is very useful to calculate the average life. Hence average life is defined as the average time for which a nucleus survives before it decays

i.e., $\tau = \dfrac{\sum \text{life span of individual nucleus}}{\text{Total number of nucleus}} = \dfrac{\sum t}{N_0}$

Let N_0 be the no.of radioactive nuclei present at t = 0. and the number of atoms decaying in the time interval t and t + dt be dN The life time of each of these nuclei is t. Then the total life time of these nuclei is t dN.

$\therefore$ the total life span of all the nuclei present initially $= \int_{N=N_0}^{0} t dN$

By the definition of mean life

$$\therefore \tau = \frac{1}{N_0}\int_0^\infty t[N_0\lambda(-e^{-\lambda t}).dt]$$

$$(\because N = N_0 e^{-\lambda t} \text{(or) } dN = N_0\lambda(-e^{-\lambda t})dt)$$

On solving this integration we get $\tau = \dfrac{1}{\lambda}$

The mean life or average life of a radio active sample is inversely proportional to decay constant. It is also equal to the time during which the given sample reduce to $\dfrac{1}{e} \simeq 0.37$ of its original amount.

$$\therefore T = \frac{0.693}{\lambda} = 0.693\tau \quad \text{(or)}$$

$$\tau \approx 1.443T$$

Since the nucleus can decay immediately or it may take infinite time, we can predict the probability of the number of nuclei being disintegrated at an instant. The probability of survival of a nucleus after time t is $P_s = \dfrac{N}{N_0} = e^{-\lambda t}$.

The probability of a nucleus to disintegrate in time t is $P_d = 1 - P_s = 1 - e^{-\lambda t}$.

Example-2.6

A radioactive isotope X has a half life of 3 second. Initially a given sample of this isotope contains 8000 atoms. Calculate (a) its decay constant, (b) the time t_1 when 1000 atoms of the isotope X remain in the sample, and (c) the number of decay per second in the sample at $t = t_1$.

Solution :

a) $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{3} = 0.231\ s^{-1}$

b) $N = N_0 e^{-\lambda t_1}$ (or)

$$t_1 = \frac{1}{\lambda}\log_e \frac{N_0}{N}$$

$$= \frac{1}{0.231}\log_e \frac{8000}{1000} = 9 \text{ sec}$$

c) $\left|\dfrac{dN}{dt}\right|_{t=t_1} = \lambda N = 0.231 \times 1000 = 231 s^{-1}$

Example-2.7

At time t = 0, number of nuclei of a radioactive substance is 100. At t = 1s this number becomes 90. Find the number of nuclei at t = 2s.

Solution :

In 1 second 90% of the nuclei have remained undecayed, so in another 1 second 90% of 90.

i.e., 81 nuclei will remain undecayed.

Example-2.8

The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time t_2 ($t_2 > t_1$). Obtain an expression for its mean life.

Solution :

$A_1 = A_o e^{-\lambda t_1}$ and $A_2 = A_o e^{-\lambda t_2}$

$$\Rightarrow \frac{A_1}{A_2} = e^{\lambda(t_2 - t_1)}$$

or $\lambda(t_2 - t_1) = \ln\left|\dfrac{A_1}{A_2}\right|$ (or)

$$\tau = \frac{1}{\lambda} = \frac{t_2 - t_1}{\ln\left|\dfrac{A_1}{A_2}\right|}$$

Example-2.9

The mean lives of a radioactive substance are 1620 year and 405 year for α-emission and β-emission respectively. Find the time during which three-fourth of a sample will decay if it is decaying both by α-emission and β-emission simultaneously

Solution :

The decay constant λ is the reciprocal of the mean life τ

Thus, $\lambda_\alpha = \frac{1}{1620}$ per year and

$\lambda_\beta = \frac{1}{405}$ per year

$\therefore$ Total decay constant, $\lambda = \lambda_\alpha + \lambda_\beta$ (or)

$$\lambda = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$$

per year. We know that $N = N_0 e^{-\lambda t}$

When $\frac{3}{4}$th part of the sample has disintegrated, $N = N_0/4$

$$\therefore \frac{N_0}{4} = N_0 e^{-\lambda t} \text{ (or) } e^{\lambda t} = 4$$

Taking logarithm of both sides, we get

$$\lambda t = \log_e 4 \text{ (or)}$$

$$t = \frac{1}{\lambda}\log_e 2^2 = \frac{2}{\lambda}\log_e 2 = 449 \text{ year}$$

2.15 LAW OF SUCCESSIVE DISINTEGRATION

A parent nucleus may decays into a daughter nucleus, which may decay into another daughter nuclei and so on. Such decay is called successive disintegration or series decay. The chain stops only when the end product is stable.

$$A \to B \to C \to ....X(\text{stable})$$

Any adjacent nuclei may be considered as parent and daughter nuclei.

eg : ${}_{92}u^{238} \to {}_{90}Th^{234} \to {}_{91}Pa^{234} \to ....$

If $A \to B \to C(\text{stable})$,

here A is the parent nucleus and B is the daughter nucleus of A, C is the daughter nucleus of B.

If at $t = 0$, the number of nuclei of $A = N_0$ and those of B of C (yet to be formed) = 0, then after some time t, their respective numbers will have changed.

Let their numbers be now N_1, N_2 and N_3 respectively. If λ_1 and λ_2 are their decay constants, then we can say, rate of decay of $A = \frac{dN_1}{dt} = \lambda_1 N_1$, which is also rate of formation of B.

Rate of decay of $B = \frac{dN_2}{dt} = \lambda_2 N_2$, which is also rate of formation of C

$\therefore$ Rate of increase of number of nuclei of

$$B = \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \text{(or)}$$

$$\frac{dN_2}{dt} = \lambda_1 . N_0 e^{-\lambda_1 t} - \lambda_2 N_2 \quad (\because N_1 = N_0 e^{-\lambda t})$$

$$\text{(or)} \quad \frac{d}{dt} N_2 + \lambda_2 N_2 = \lambda_1 N_0 e^{-\lambda_1 t}$$

Multiplying by $e^{\lambda_2 t}$ on both sides, we get

$$e^{\lambda_2 t}\frac{dN_2}{dt} + \lambda_2 N_2 e^{\lambda_2 t} = \lambda_1 N_0 e^{-\lambda_1 t}.e^{\lambda_2 t} \quad \text{(or)}$$

$$\frac{d}{dt}(N_2 e^{\lambda_2 t}) = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t}$$

On integration we get

$$N_2 e^{\lambda_2 t} = \lambda_1 N_0 \left[\frac{e^{(\lambda_2 - \lambda_1)t}}{\lambda_2 - \lambda_1}\right] t + K,$$

where K is integration constant At $t = 0$ and $N_2 = 0$, Hence

$$\text{hence } N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) \ .$$

This gives the number of daughter nuclei (B) at time t. Thus N_2 depends not only on its decay constant λ_2 but also on parents decay constant λ_1.

Since rate of formation of $C = \frac{dN_2}{dt}$, the number of atoms of C at time t, is gives by

$$N_3 = N_0\left[1+\frac{\lambda_1}{\lambda_2-\lambda_1}e^{-\lambda_2 t}-\frac{\lambda_2}{\lambda_2-\lambda_1}e^{-\lambda_1 t}\right]$$

Obviously, during any intermediate state we shall have only a mixture of various nuclei (A, B, C)

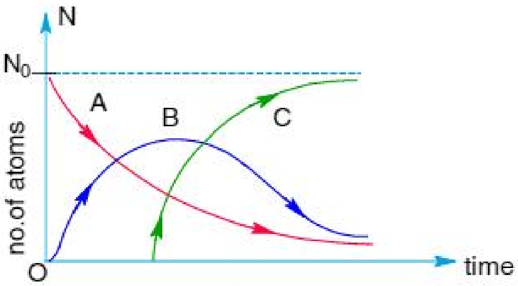


Fig 2.15

If 'C' is the final stable product, then the decay and recovery curves for the substances A, B and C are as shown in Fig 2.15. In nuclide curve of B, before it reaches to its peak value, the rate of formation of B (or rate of disintegration of A) is greater than its rate of disintegration to C.

i.e $\lambda_1 N_1 > \lambda_2 N_2 \left(\because \frac{dN_2}{dt} > 0\right)$.

Beyond the peak value $\frac{dN_2}{dt} < 0$, hence the rate of formation of B is less than its rate of disintegration to C. i.e., $\lambda_1 N_1 < \lambda_2 N_2$.

At peak of B, the rate of formation of B = rate of disintegration of B. Hence $\lambda_1 N_1 = \lambda_2 N_2$.

If 'C' is also an unstable product, then the decay and recovery curves of various pieces are as shown in Fig 2.16.

With the passage of time, the parent atoms decrease that of the daughter increases, hence a situation is reached where by the rates of decay of various nuclei A,B,C... are equal i.e., slopes of N - t curves are equal.

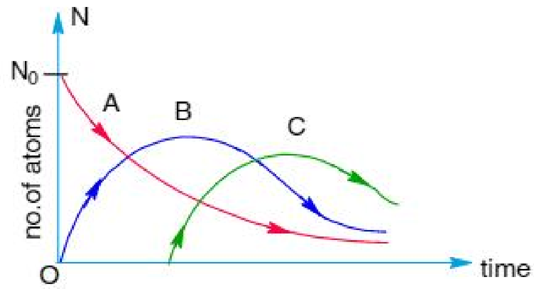


Fig 2.16

$$\therefore \frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dN_3}{dt} = ... = 0$$

(N_1, N_2, N_3.... do not vary with time)

$$\therefore \lambda_1 N_1 = \lambda_2 N_2 = \lambda_3 N_3 = ... \text{ etc}$$

This is said to be radio-active equilibrium law. Now we discuss two cases of radio-active equilibrium.

Note : It is to be noted that if the parent has shorter half-life than that of the daughter $(\lambda_1 > \lambda_2)$, no state of equilibrium is attained. If initially we have only parent atoms, then the parent atoms decay, the daughter atoms increase in number, pass through a maximum and eventually decay with their own half lifes.

Example-2.10

Find the decay constant of ^{55}Co radio nuclide if its activity is known to decrease 4% per hour. The decay product is non-radioactive.

Solution :

Initial activity of nuclei $A_0 = \lambda N_0$

Activity of nuclei at time t, $A = \lambda N = \lambda N_0 e^{-\lambda t}$

$A = A_0 e^{-\lambda t}$ ...... (1)

Since activity decreases at 4.0% per hour, activity of ^{55}Co radio nuclide at $t = 1$ hr.

$A = A_0 - \eta A_0 = A_0(1-\eta)$,

where $\eta = 0.04$ ...... (2)

Taking log of Eq. (1) we get $\lambda = -\frac{1}{t}\ln(1-\eta)$

On substituting values, we get $\lambda = 1.1\times10^{-5}s^{-1}$

Example-2.11

A ^{32}P radio nuclide with half life T = 14.3 days is produced in a reactor at a constant rate q = 2.7 × 10^9 nuclei per second. How soon after the beginning of production of that nuclide will its activity be equal to A = 1.0 × 10^9 dis/s ?

Solution :

According to the problem, the radio nuclide is being produced at a constant rate as well as it decays with rate $-\frac{dN}{dt} = \lambda N$. Hence net rate of accumulation of radio nuclide is given by

$$\frac{dN}{dt} = q - \lambda N \quad ..... (1)$$

$$\text{(or)} \int_0^N \frac{dN}{q-\lambda N} = \int_0^t dt \quad \text{(or)} \quad t = \frac{T_{1/2}}{\ln 2}\ln\left(1-\frac{\lambda N}{q}\right)$$

On substituting numerical values, we get $t = 9.5$ days

Example-2.12

A ^{118}Cd radionuclide goes through the transformation chain.

$$^{118}Cd \xrightarrow[30\,min]{} {}^{118}In \xrightarrow[45\,min]{} {}^{118}Sn \text{ (stable)}$$

The half lives are written below the respective arrows. At time t = 0 only Cd was present. Find the fraction of nuclei transformed into stable 5th over 60 minutes.

Solution:

$$\text{Since } N_1 = N_0 e^{-\lambda_1 t} \text{ and } N_2 = \frac{N_0\lambda_1}{\lambda_2-\lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

$$\therefore N_3 = N_0 - N_1 - N_2$$

$$= N_0\left[1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2-\lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})\right]$$

$$\therefore \frac{N_3}{N_0} = 1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2-\lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

$$\lambda_1 = \frac{0.693}{30} = 0.0231\,min^{-1}$$

$$\lambda_2 = \frac{0.693}{45} = 0.0154\,min^{-1} \text{ and } t = 60 \text{ minutes.}$$

On substituting the given values $\frac{N_3}{N_0} = 0.31$

2.16 DISCOVERY OF NEUTRONS

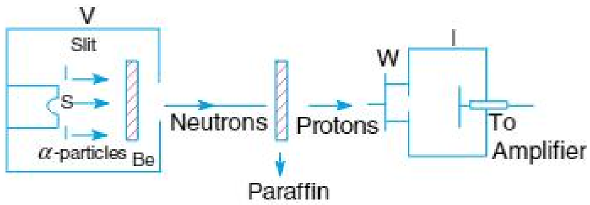


Fig 2.17

Chadwick discovered neutron.

He used a thick plate coated with polonium as the source (S) of α - particles. The α - particles bombarded the beryllium target and the radiation from it was allowed to pass through paraffin wax. From paraffin protons were ejected and the radiation from beryllium was stopped. The high energy protons were allowed to pass through an ionisation chamber I through window W. The ionization current due to protons was amplified and measured with the help of an electrometer.

He applied the elementary principles of elastic collisions. When a body collides with another identical body at rest, the first body comes to rest and the second body moves with the velocity of the first body. Since the radiation from paraffin wax contained protons, the colliding particle must have the same mass as that of proton. He showed that the colliding particle had almost the same energy as proton. He called the penetrating radiation from beryllium as neutrons. Thus neutron was discovered.

$$_4Be^9 + _2He^4 \rightarrow _6C^{12} + _0n^1 + Q$$

Properties of neutrons

(i) Neutron is an uncharged particle and hence it is not deflected by the electric and magnetic fields.

(ii) It has very high penetrating power and has very low ionization power.

(iii) Inside the nucleus neutrons appear to be stable. The average life of an isolated neutron is about 10 min. A free neutron is unstable and spontaneously decays into a proton, electron and an antineutrino $(\bar{\nu})$. $^1_0n \rightarrow {}^1_1H + {}^0_{-1}e + \bar{\nu} + Q$

iv) If fast neutrons pass through substances like heavy water, paraffin wax, graphite etc., they are slowed down.

v) Neutrons are diffracted by crystals.

2.17 NUCLEAR REACTIONS

In this chapter, we have already seen how unstable materials undergo disintegration if Z > 82. We have also seen that such a behavior can be induced by bombarding certain targets (nuclei with any Z) with high energy particles or photons. Such a reaction in which a certain species after suitable bombardment undergoes a change of character is known as a nuclear reaction. In a nuclear reaction, a projectile a strikes the target nucleus X and produces a residual nucleus Y and an outgoing particle b. Each of the particles a and b can be photon, an electron, a proton, a neutron or a nucleus.

$$a + X \rightarrow Y + b \pm Q$$

This is denoted as X (a, b) Y

eg : $$\underset{(KE = 5MeV)}{{}_2He^4} + \underset{(stable)}{{}_7N^{14}} \rightarrow \underset{(unstable)}{\left[{}_9F^{18}\right]} \rightarrow \underset{(stable)}{{}_8^{17}O} + \underset{(stable)}{{}_1^{1}H} + Q$$

In a nuclear reaction, inner nuclear structure of a species changes and very large values of energy (MeV) is released. In nuclear reaction, a projectile can penetrate into the nucleus and interact with a single nucleon in the nucleus so that the nucleon leaves the nucleus. If the nucleon does not leave the nucleus but interacts with several other nucleons, then the nucleus is raised to excited state called a compound nucleus. The compound nucleus can emit a particle identical to the incident particle and with the same KE or emit photons or other particles. Thus decay of the compound nucleus is a statistical process.

A nuclear reaction must ensure that the charge (Z), mass number (A), linear momentum, angular momentum, spin and mass-energy etc are conserved. In a nuclear reaction if Q > 0, it is exoergic reaction. If Q < 0, it is endoergic reaction and is not possible unless the required energy is supplied. The amount of energy so needed in an endoergic reaction is called threshold energy. In endoergic reaction total mass of initial particles is less than that of final particles.

There are so many types of nuclear reactions, depending upon the colliding particles (a, b) involved. Here we restrict only to four types.

2.18 PHOTO NUCLEAR REACTION

The reactions in which a photon causes the target nucleus to change its form are called photo nuclear reaction. In this type of reaction, striking particles are γ -ray photons of high energy and they are completely absorbed by the target nucleus.

$$\gamma + X \rightarrow Y + b + Q$$

The mass energy relation corresponds to

$$h\nu + m_x c^2 = m_y c^2 + m_b c^2 + Q$$

[Q in the form of KE]

$$\therefore h\nu_{min} = (m_y + m_b - m_x)c^2 + Q$$

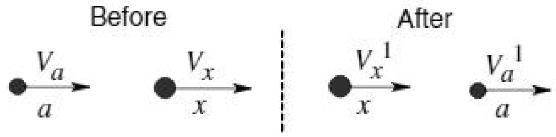
$$\therefore h\nu_{min} = (B.E)c + Q$$

eg : $\gamma + {}_1H^3 \rightarrow {}_1H^2 + {}_0n^1 + Q$

2.19 SCATTERING REACTION

Basically here no reaction is involved, there is only an exchange of energies between target and striking particle. Hence the nature of the target nucleus as well as the product nucleus is the same. When a particle is incident on a nucleus it may be scattered elastically or inelastically.

Consider one dimensional scattering.



By conservation of momentum $P_f = P_i$

$$p_x^1 + p_a^1 = p_x + p_a$$

The K.E before and after collision are

$$K_i = \frac{p_x^2}{2m_x} + \frac{p_a^2}{2m_a} \text{ and } K_f = \frac{p_x^{1^2}}{2m_x} + \frac{p_a^{1^2}}{2m_a}$$

After collision, the internal nuclear energy U of the particles may be different. Hence by conservation of energy.

$K_f + U_f = K_i + U_i$ (or) $K_f - K_i = U_i - U_f$ (or)
$Q = U_i - U_f$ ($\because Q = K_f - K_i$) (or) $Q = -\Delta U$

Case (i) : If $\Delta U = 0$ and hence $Q = 0$, there is no change in KE of system. The collision is said to be elastic.

eg : ${}_2He^4 + {}_{79}Au^{197} \rightarrow {}_{79}Au^{197} + {}_2He^7$

Case (ii) : If $U_f < U_i$, hence $Q > 0$, i.e $K_f > K_i$, the collision is inelastic and exoergic

eg : ${}_1H^1 + {}_3Li^7 \rightarrow {}_3Li^7 + {}_1H^1 + \gamma$

Case (iii) : If $U_f > U_i$, hence $Q < 0$, i.e $K_f < K_i$, the collision is inelastic and endoergic .

Note : In scattering and photo-electric reactions, the minimum energy required to the projectile in endoergic reaction when target is at rest is $E_{min} = \frac{m_a + M_x}{M_x}|Q|$, where |Q| is B.E. (Derivation of this result is similar to atomic collisions)

Example-2.13

Find the minimum kinetic energy of an α-particle to cause the reaction $^{14}N(\alpha, p)^{17}O$. The masses of ^{14}N, ^4He, ^1H and ^{17}O are respectively 14.00307u, 4.00262u, 1.00783u and 16.19913 u.

Solution :

Since, the masses are given in atomic mass unit, it is easiest to proceed by finding the mass difference between reactants and products in the same unit and then multiplying by 931.5 MeV/u. Thus, we have

$Q = (14.00307u + 4.00260u - 1.00783u - 16.99913u)$

$\left(931.5\frac{MeV}{u}\right) = -1.20\ MeV$

Q value is negative, it means reaction is endothermic So, the minimum kinetic energy of α-particle to initiate this reaction would be,

$$K_{min} = |Q|\left(\frac{m_\alpha}{m_N} + 1\right) = (1.20)\left(\frac{4.00260}{14.00307} + 1\right) = 1.54\ MeV$$

2.20 NUCLEAR FISSION

With capture of projectile particle (neutron) by the heavy target nucleus, the target nucleus breaks up into two more or less equal masses, with release of fast neutrons (secondary neutrons), called. It is nuclear fission reaction. In nuclear fission the total mass of the product particles is less than the original mass. Hence energy is always released and it is equal to increase in binding energy of the system.

For instance

${}_0n^1$ + ${}_{92}U^{235}$ $\rightarrow$ $\left[{}_{92}U^{236*}\right]$
(slow neutron) (stable) (unstable)

$\rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + 200MeV$
(stable) (stable) (fast neutrons)

Fission fragments have very large kinetic energies due to the Coulombic force of repulsion. This energy appears as thermal energy, if the fission product cannot escape. This includes the KE of fragments and energy of the γ-rays.

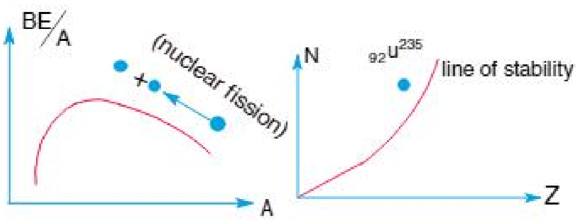


Fig 2.18

From the Fig 2.18 it is observed that $\frac{B.E}{A}$ of products is more than the parent nuclei and hence energy is released.

In target nuclei (fission fragment) neutron - proton ratio is larger than that required for stability. Hence neutrons are spontaneously emitted during the fission and the fragments decay by β^- emission. Fission products do not have unique values of mass number and atomic number and can be formed from the region as shown in Fig 2.19.

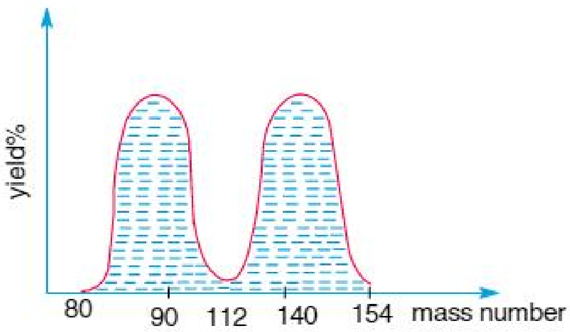


Fig 2.19

eg : ${}_0n^1 + {}_{92}U^{235} \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1 + E$

Depending on the particular reaction, one, two or three neutrons may be emitted. The average number of neutrons emitted in the fission of $_{92}U^{235}$ is about 2.5.

Nuclear fission can also be understood by using liquid drop with positive charge. According to the liquid drop model, the nucleus behaves like a liquid drop. The nucleus tries to be in perfectly spherical shape like a liquid drop. When a neutron bombards the nucleus, an energy called the excitation energy is imparted to it and consequently a compound nucleus is formed. While the nuclear force of the nucleus tries to keep the nucleus in spherical shape, the excitation energy tries to deform it and as a result strong oscillations are set up inside the compound nucleus. These oscillations will distort the shape of the compound nucleus, from sphere to ellipsoid as shown in figure 2.20(b). If the excitation energy is sufficiently large the nucleus in ellipsoid shape attain the dumb bell shape as in figure 2.20(c). In this case the effect of nuclear attractive force is decreased because of increase in the surface area of the nucleus. Further, the electrostatic repulsive force between the two portions of the dump bell drives two portions in opposite directions and finally the nucleus splits into two fragments with the release of few neutrons, as shown in figure 2.20 (d).

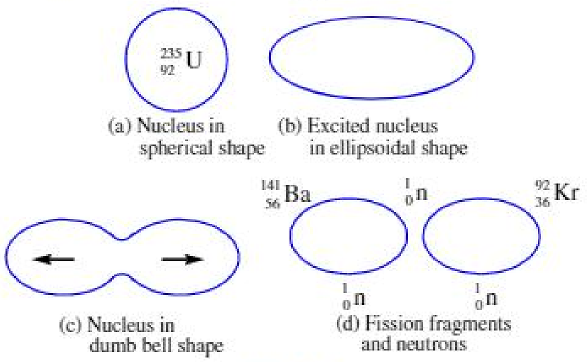


Fig 2.20

Fission of different elements : ^{238}U undergoes fission with fast neutrons whose kinetic energies are greater than 1 Mev, whereas ^{235}U undergoes fission when bombarded with slow neutrons or thermal neutrons whose kinetic energies are nearly 0.025 eV. The probability of producing fission in ^{238}U with fast neutrons is very low and the probability of producing fission in ^{235}U with slow neutrons is very high.

Thorium – 232 undergoes fission with fast neutrons whereas plutonium –239 undergoes fission with fast as well as slow neutrons. U–235 and plutonium –239 are used as fuels in nuclear reactor.

Fission with different projectiles : The projectile that can be used to bombard a heavy nucleus need not be a neutron alone. It can be a proton or a deuteron or an $\alpha-$ particle. Hence fission of uranium and thorium is also possible with the high speed protons of energy 6.9 MeV or by deuterons of energy greater than 8 MeV or by $\alpha-$particles of 32 Mev energy.

Fission products : When uranium –235 is bombarded with thermal neutrons, causing fission, more than 20 elements have been found among the fission products showing that the fission process may take in 30 or more different ways producing a total of about 200 fission products. The products have been found, however, to fall into two groups, one of atomic number in the range 35 to 43 and atomic mass in the range 80 to 110, the other of atomic number from 51 to 57 and atomic mass from 125 to 160.

2.20.1 ENERGY RELEASED IN FISSION PROCESS

In nuclear fission a large amount of energy is released. This energy is released because the original mass of the reactants is greater than the sum of the masses of the products of the reaction. This difference in masses before and after fission is converted into energy according to Einstein's equation, $E = Mc^2$.

Let us consider the fission of $_{92}U^{235}$. The fission reaction is

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236*}$$
$$\rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1 + Q$$

The actual masses before and after the fission reaction are found below :

Mass of $_{92}U^{235}$ = 235 . 045733 amu

Mass of $_{0}n^{1}$ = 1 . 008665 amu

$\therefore$ Total initial mass = 236 . 054398 amu

Mass of $_{56}Ba^{141}$ = 140 . 9177 amu

Mass of $_{36}Kr^{92}$ = 91 . 8854 amu

Mass of 3 neutrons (3×1.008665)

= 3 . 025995 amu

$\therefore$ Total final mass = 235 . 829095 amu

Decrease of mass

= 236.054398 – 235. 829095

= 0.225303 = 0.2253 amu

This decrease in mass is converted into energy.

Since 1 amu = 931 MeV,

Energy released = 0.2253 × 931

= 209.8 MeV.

Thus in the process of fission of one nucleus of a uranium, on the average about 200 MeV energy is released.

The energy released in fission appears as kinetic energy of the fragments and neutrons. Later it is transferred to the surrounding matter appearing as heat.

2.20.2 NUCLEAR CHAIN REACTION

In each fission of $_{92}U^{235}$ reaction, the average number of neutrons emitted is 2.5. These emitted neutrons can initiate other fissions, thereby producing a chain reaction. To sustain the chain reaction in a fission reactor one of the neutrons (on the average) emitted in the fission of U^{235} must hit another U^{235} nucleus and cause it to fission. In the fission reaction,

$$_0n^1 + _{92}U^{235} \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3_0n^1 + Q$$

three more neutrons are produced. These three neutrons may then bombard three more U^{235}, giving rise to 3 × 3 = 9 neutron. Again these 9 neutron with fission gives 9 × 3 = 27 neutrons and so on. Thus the number of neutrons increase in geometric progression. Soon a stage is reached when a very large number of nuclei are split releasing large energy in short time. An atom bomb is an example for this type of fission process.

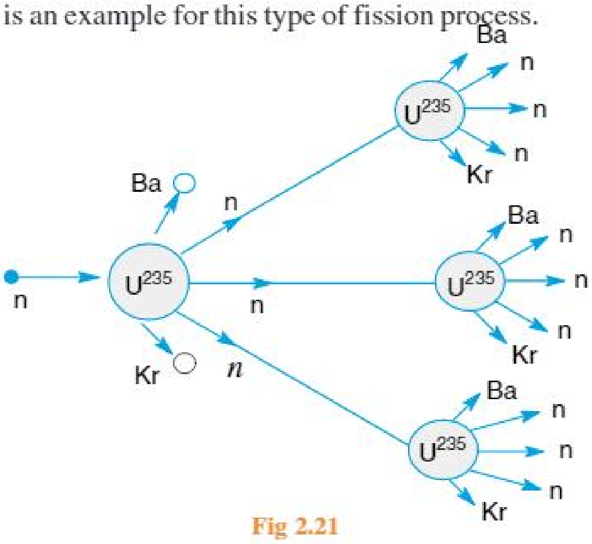


Fig 2.21

The chain reaction can be controlled by slowing down the neutrons and/or by absorbing the neutrons. Neutrons are slowed down using graphite, ordinary water or heavy water. These substances are called moderators. For absorbing neutrons, boron rods are used.

2.20.3 NUCLEAR REACTOR

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and controlled chain reaction. It is also called an atomic pile. By making use of uranium as fuel, the products such as neutrons (useful for producing of uranium), radio isotopes and heat energy (to produce steam and run turbines) are produced.

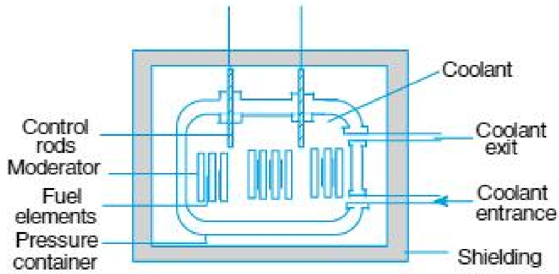


Fig 2.22

Construction : A nuclear reactor consists of thick blocks of carbon surrounded by thick absorbing walls of concrete as shown in Fig 2.22. The other parts of the nuclear reactor are as follows:

1) **Nuclear fuel :** The fissionable material used in the reactor is called nuclear fuel. It generally consists of ${}_{92}U^{235}$ or ${}_{90}Th^{232}$ sealed in aluminium cylinders. These cylinders are inserted in the holes drilled in the carbon blocks.

2) **Moderator :** The material used to slow down the fast moving neutrons produced as a result of nuclear fission is called moderator. The neutrons released in the fission of uranium possess energy of the order of 2 MeV. Moderator reduces their energy to 0.025 eV, which corresponds to the thermal motion of the neutrons and hence called thermal neutrons. This is due to neutrons make elastic collision with the nuclei of moderator. In a nuclear reactor, the moderator is either graphite (carbon) or water or heavy water (deuterium oxide). Moderators have low atomic weight.

3) **Control rods :** The materials that can absorb the neutrons are used to control the nuclear chain reaction. Cadmium or boron rods are used for this purpose. They can be moved in or out of the holes in the carbon blocks. If they are pushed in more neutrons are absorbed and the speed of reaction decreases. When they are pulled out the number of neutrons absorbed decreases and the speed of the reaction increases.

4) **Coolant :** A large amount of heat is developed in the reactor due to chain reaction. Water or liquid sodium is used to remove the heat. These substances which remove heat from the reactor and keep it at a reasonable low temperature are called coolants.

5) **Reflector :** The core is surrounded by a substance such as water or graphite. It reduces the loss of neutrons from the reactor as otherwise neutrons would escape from the reactor.

6) **Protective shield :** To prevent the hazardous radioactive radiations spreading out from the reactor, the space around the reactor is covered with thick concrete walls or thick lead sheets.

Reproduction factor : Reproduction factor (K) is defined as the average number of neutrons from each fission that causes a subsequent fission.

$$K = \frac{\text{no.of neutrons in any generation}}{\text{no.of neutrons in immediately preceding generation}}$$

The maximum possible value of K is about 2.5. Due to escape of neutrons from the region containing fissionable nuclei and capture of some of the neutrons, K is reduced below 2.5. If K is exactly 1, the reaction will be self sustaining. If it is less than 1, the reaction will die out. If K is significantly greater than 1, the reaction rate will increase rapidly. In atomic bomb K > 1 and in power reactors K is nearly equal to 1. If K is exactly equal to 1, the reactor is said to be critical, for K < 1, it is said to be subcritical and for K > 1, it is super critical.

2.21.1 BREEDER REACTORS

A breeder reactor converts non - fissionable material, such as U^{238}, into fissionable material and the reactor produces or "breeds" more fuel than it consumes. When ${}^{238}U$ is bombarded with fast neutrons the following reactions take place.

$${}_{92}U^{238} + {}_{0}n^{1} \rightarrow {}_{92}U^{239} + \gamma$$

$${}_{92}U^{239} \rightarrow {}_{93}Np^{239} + {}_{-1}e^{0} + v^*$$

$${}_{93}Np^{239} \rightarrow {}_{94}Pu^{239} + {}_{-1}e^{0} + v^*$$

In this way the non - fissionable U^{238} is converted into fissionable isotope plutonium. The fission properties of plutonium –239 closely resemble that of uranium –235.

2.21.2 POWER REACTORS

In the nuclear reactor, large amount of heat will be generated in the core. These reactors have elaborate cooling systems that use water.

This water absorbs the heat and produces steam. This steam in turn is used to run the steam turbines which ultimately generate electric power. Such reactors are called power reactors. The power generated by a nuclear reactor is,

$$P = \left(\frac{n}{t}\right) x \text{ energy released per fission.}$$

Example-2.14

Calculate the energy released by fission from 2 gm of $_{92}U^{235}$ in kWh. Given that the energy released per fission is 200 MeV.

Solution :

No. of atoms in 1 gm of U_{92}^{235}

$$= \frac{\text{Avagadro number}}{\text{At wt}} = \frac{6.023 \times 10^{23}}{235}$$

Energy released per atom = 200 Mev

$$= 200 \times 10^6 \times 1.6 \times 10^{-19} J$$

Total energy released per gm,

$$E = \frac{6.023 \times 10^{23}}{235} \times 200 \times 10^6 \times 1.6 \times 10^{-19} J$$

$$= \frac{6.023 \times 200 \times 1.6}{235 \times 36 \times 10^5} \times 10^{10} \text{ kWh}$$

$$= 0.2278 \times 10^5 \text{ kWh}$$

Total energy released from 2gm of U_{92}^{235} is

0.4556×10^5KWH

2.22 USES OF ATOMIC ENERGY

1. **Generation of electric power :** The coolant in a nuclear reactor absorbs the heat generated as a result of the chain reaction and it releases the heat to the water which is converted into high pressure steam. This steam is used to drive turbine and operate the electric generator.

2. **Production of radio isotopes :** A small amount of the pure element is placed in an aluminium container and the container is placed in the reactor for a few days. The element absorbs neutrons and the element becomes radioactive isotope. For ex : $_{53}I^{127} +_0 n^1 \rightarrow_{53} I^{128*} + \gamma$

3. **Source of neutrons :** A large number of neutrons are produced in a reactor. They are used in research . The effect of neutrons on biological tissues is studied. A new branch of physics called Neutron Physics has come up.

4. Atomic energy is used to create artificial lakes, to divert the course of a river , to make tunnels for laying new railway tracks etc.

5. Atomic energy is used for driving automobiles, submarines and war - planes.

6. Atomic energy is used in war - fare for creating destructive atom bombs and hydrogen bombs.

2.23 RADIO ISOTOPES AND THEIR USES

Radio isotopes have very short half lives and are used freely for various purposes.

1) Medical applications

a) $_{13}Al^{27} +_0 n^1 \rightarrow_{11} Na^{24*} +_2 He^4$

Radiosodium is used to find out how a given medicine is circulated in the body. It is also used to find out circulatory disorders in blood vessels.

b) $_{15}P^{31} +_1 H^2 \rightarrow_{15} P^{32*} +_1 H^1$

Radiophosphorus is used in the treatment of skin diseases.

c) $_{53}I^{127} +_0 n^1 \rightarrow_{53} I^{128*} + \gamma - \text{rays}$

Radioiodine is used in the treatment of thyroid glands.

d) $_{27}Co^{59} +_0 n^1 \rightarrow_{27} Co^{60*} + \gamma - \text{rays}$

Radiocobalt is used in the treatment of cancer

e) In CT and NMR Imaging techniques

f) Radio - iron is used to detect and treat anaemia.

Thus the radio-isotopes are used both in diagnosis and therapy.

2) Radio - Carbon dating

$$_7N^{14} +_0 n^1 \rightarrow_6 C^{14*} +_1 H^1$$

Radio - carbon is continuously produced in the atmosphere due to neutrons in the atmosphere and also in living plants and animals. This is used to estimate the ages of rocks, the earth etc. Carbon $_6C^{14*}$ is vastly used by archaeology department.

3) Industry

Isotopes are used to find the wear and tear of machine parts ; to detect flaws in metal structures; for treatment of alloys such as quenching, annealing and hardening.

4) Research

Radio - isotopes are used in the study of nuclear disintegrations of elements.

5) In food preservation

By exposing vegetables and other food stuffs to radiations from radio - active isotopes, their shelf life can be increased.

6) In agriculture

Radio - isotopes are used to study the uptake of fertilizers by different plants. They have also been used to develop new species of plants by causing genetic-mutation. In this field, radio - zinc is of immense use.

2.24 NUCLEAR FUSION

Just as the fission of a heavier nucleus releases energy due to conversion of mass into energy, when two lighter nuclei (A < 20) combine to form a heavier nucleus, energy is released. This is because the total binding of the resulting nucleus increases as shown in Fig 2.23. The reaction in which two lighter nuclei combine with the release of energy is called a fusion reaction.

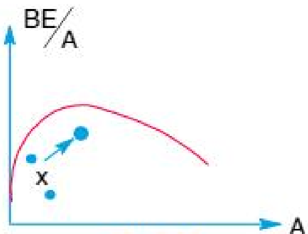


Fig 2.23

A typical fusion reaction is

$$_1H^2 + _1H^3 \rightarrow _2He^4 + _0n^1 + 17.6\text{ MeV}$$

Since the nuclei are positively charged and repel each other, the fusion can be brought about only by making the nuclei collide with one another energetically. The kinetic energy of each nucleus should be high, of the order 1MeV, so as to over come the coulomb repulsion. To obtain this kinetic energy, the primary nuclei should be brought to high temperatures of the order of 10^8K. At these high temperatures the nuclei have finite probability of overcoming the coulomb potential barrier and produce fusion reaction. For this reason this reaction is also known as thermonuclear reaction. At these temperatures atoms are fully ionized and ions and free electrons move about freely. This temperature is called ion temperature. This state of matter is known as plasma. The plasma density n (ions/m^3) must be high to ensure that collision between nuclei are present.

The energy required to heat plasma is proportional to the density of its ions. The fission rate is proportional to square of density n^2. The output energy is proportional to n^2t, where 't' is the confinement time. It is the time for which the nuclei remain together. If the output energy is to exceed the input energy, we must have $C_1n^2t > C_2n$, where C_1 and C_2 are constants and $nt > 10^{20}$ particles / m^3.

This is known as Lawson's criterion and the product nt called confinement quality factor. It is very difficult to hold the plasma particles together for sometime to set the nuclear fusion to start. The large value of gravitational field of sun confines the plasma in the interior of the sun. Hence nuclear fusion takes place in sun and stars.

On comparison with the 200 MeV per fission of U^{235}, this seems small but per unit mass of material it is not. Hydrogen bomb works on fusion principle.

Controlled thermonuclear fusion

The nuclear fusion process in a star is replicated in a thermonuclear fusion device. In controlled fusion reactors, the aim is to generate steady power by heating the nuclear fuel to a temperature in the range of 10^8K. At these temperatures, the fuel is a mixture of positive ions and electrons (plasma). The difficult task is to confine this plasma, since no container can stand such a high temperature. Several countries around the world including India are developing techniques in this connection. If successful, fusion reactors will hopefully supply almost unlimited power to humanity.

2.25 ENERGY OF THE SUN AND STARS

We all know that the sun and stars have been radiating huge amount of energy for several billions of years. This energy of the sun and the brightest stars is produced by nuclear fusion in the core of sun or of the stars, where the temperature is of the order 10^7K or more. Scientists proposed two types of cyclic processes for the sources of energy in the sun and stars. The first is known as carbon-nitrogen cycle and the second is proton - proton cycle.

Carbon - Nitrogen Cycle

According to Bethe carbon - nitrogen cycle is mainly responsible for the production of solar energy. This cycle consists of a chain of nuclear reactions in which hydrogen is converted into Helium, with the help of carbon and nitrogen as catalysts.

Protons interact with C^{12} nuclei to release fusion energy and γ-radiation in accordance with

$${}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13*} + \gamma \quad .... (1)$$

$${}_7N^{13*} \rightarrow {}_6C^{13} + {}_{+1}e^0 + \nu \quad .... (2)$$

$${}_6C^{13} + {}_1H^1 \rightarrow {}_7N^{14} + \gamma \quad .... (3)$$

$${}_7N^{14} + {}_1H^1 \rightarrow {}_8O^{15*} + \gamma \quad .... (4)$$

$${}_8O^{15*} \rightarrow {}_7N^{15} + {}_{+1}e^0 + \nu \quad .... (5)$$

$${}_7N^{15} + {}_1H^1 \rightarrow {}_6C^{12} + {}_2He^4 \quad .... (6)$$

Adding equation (1) to (6)

$$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\nu + Q(= 28\text{MeV})$$

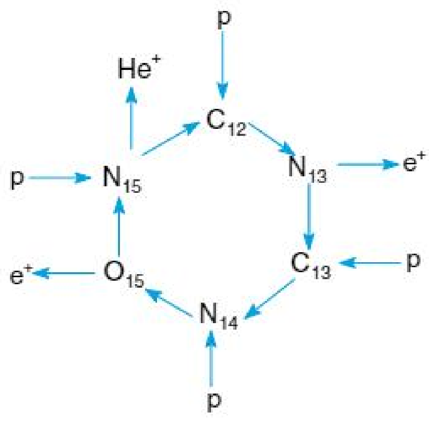


Fig 2.24

This is called **carbon–nitrogen** cycle, Recent evidence suggest that the carbon-nitrogen cycle occurs at very high temperatures of stars, greater than that of sun.

The **proton - proton** cycle occurs at high temperatures as that of sun. It is given as

$${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_{+1}e^0 + \nu + Q_1 \quad .... (1)$$

$${}_1H^1 + {}_1H^2 \rightarrow {}_2He^3 + Q_2 \quad ..... (2)$$

$${}_1H^1 + {}_2He^3 \rightarrow {}_2He^4 + {}_{+1}e^0 + \nu + Q_3 \quad ..... (3)$$

Adding (1) to (3)

$$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\nu + Q(= 24\text{MeV}) \quad .... (4)$$

In addition to this, another cycle is also possible. Which is

$${}_1H^1 + {}_1H^2 \rightarrow {}_2He^3 + Q_2 \quad .... (5)$$

$${}_1H^1 + {}_1H^2 \rightarrow {}_2He^3 + Q_2 \quad .... (6)$$

$${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2\,{}_1H^1 + Q \quad .... (7)$$

This cycle is shown in Fig 2.25

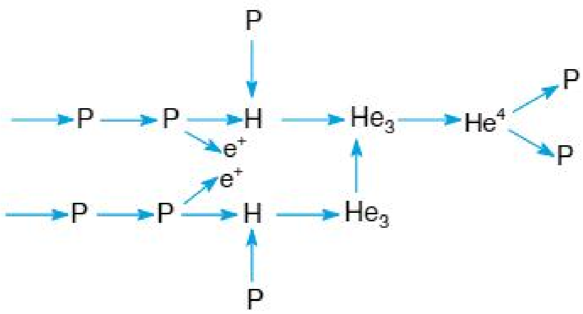


Fig 2.25

Note : It is not the element helium only that can be formed in the interior of a star. As the hydrogen in the core gets depleted and becomes helium, the core starts to cool. The star begins to collapse under its own gravity and temperature of the core increases. If this temperature increases to about 10^8K, fusion takes place again. But this time helium nuclei into carbon. This kind of process can generate through fusion, higher and higher mass number elements. But elements more massive than those near the peak of the binding energy curve in cannot be so produced.

The age of the sun is about 4.5×10^9 years and it is estimated that there is enough hydrogen in the sun to keep it going for another 5 billion years. After that, the hydrogen burning will stop and the sun will begin to cool and will start to collapse under gravity, which will raise the core temperature. The outer envelope of the sun will expand, turning it into a so called red giant.

Example-2.15

Calculate the Q-value for the reaction ${}_1H^2(d,n){}_2He^3$

Solution :

The nuclear reaction given to us is

$${}_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + {}_0n^1 + Q$$

$$Q = [2m({}_1H^2) - m({}_2He^3) - m({}_0n^1)].c^2$$

$$= [2 \times 2.014 - 3.0163 - 1.00867]\text{a.m.u} \times 931\ \text{MeV/a.m.u}$$

$$\cong 3.19\ \text{MeV}.$$

2.26 PAIR PRODUCTION AND PAIR ANNIHILATION

(a) Conversion of Photon energy to mass (Pair production)

It is possible that a photon of certain minimum energy could materialize into an electron (${}_{-1}e^0$) and a positron (${}_{+1}e^0$, also called anti-electron). In this process, electromagnetic energy is converted into mass. It is called the pair production. This is shown in Fig 2.26.

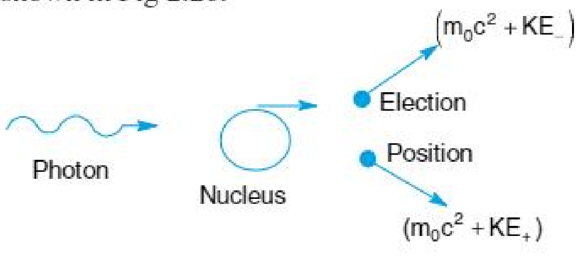


Fig 2.26

Pair production can occur only when the photon passes through matter, because energy and momentum are not conserved when the reaction takes place in isolation. The missing momentum in this process must be supplied by interaction with a massive, object such as a nucleus. As a nucleus is massive it carries only a negligible fraction of energy of photon. No conservation laws are violated when the electron-positron pair is created near an atomic nucleus. By conservation of energy.

$$h\upsilon = E_e + E_p + KE(\text{nucleus})\ \text{But}$$

$$KE\ (\text{nucleus}) \approx 0$$

$$\text{Hence, } h\upsilon = (m_0c^2 + KE_+) + (m_0c^2 + KE_-)$$

where KE_+ and KE_- are the K.E of positron and electron respectively. Since the rest mass of electron is 0.51 MeV, in order to create the electron-positron pair the photon energy must be at least equal to $2m_0c^2 = 1.02\text{MeV}$ i.e., $h\upsilon_{min} = 1.02\text{MeV}$ and the corresponding wave-length.

For $h\upsilon$ > 1.02 MeV, the excess energy is shared equally by electron and positron as kinetic energy. The probability of pair production increases with higher photon energy and with higher atomic number Z of the near by nucleus, because a heavy nucleus can create higher electric field which helps the process.

(b) Conversion of mass into photon energy (Pair Annihilation)

Just as pair production, its inverse process is also possible. An electron and a positron may combine to disappear, producing a pair of photons. The direction of the liberated photons are such as to conserve both energy as well as linear momentum. Here, no nucleus or other particle is required for this process. This is known as pair annihilation.

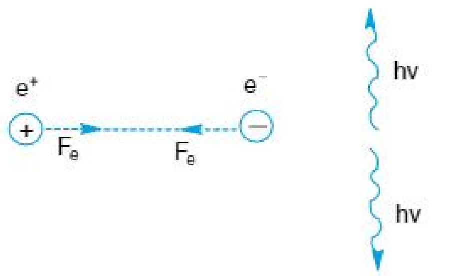


Fig 2.27

i.e., $e^+ + e^- \rightarrow \gamma + \gamma$, here two photons are produced in order to conserve the energy and momentum. The photons move in opposite directions with equal energy.

If both electron-positron are initially at rest, $2m_0c^2 = 2E_r$.

$\therefore E_r = h\upsilon = m_0c^2 = 0.511\text{MeV}$

$\therefore$ The minimum energy of each emitted photon is 0.511 MeV. If electron and positron have some kinetic energy initially, then $E_r > 0.511\text{MeV}$.

Example-2.16

En electron-positron pair is produced when a γ-ray photon of energy 2.36MeV passes close to a heavy nucleus. Find the kinetic energy carried by each particle produced, as well as its total energy.

Solution :

The reaction is represented by

$\gamma \rightarrow ({}_{-1}e^0) + ({}_{+}e^0)$, so that

$E = m_0.c^2 + \text{K.E. (electron)} + m_0c^2 + \text{K.E (positron)}$

$2.36\text{MeV} = 2m_0.c^2 + \text{K.E (electron)} + \text{K.E (positron)}$

$= 1.02\text{ MeV} + \text{K.E}(e^-) + \text{K.E}(e^+)$

$\therefore$ Kinetic energy of electron or position

$\text{KE}(e^-) = \text{K.E}(e^+) = \frac{1}{2}(2.36 - 1.02)\text{MeV},$

K.E. carried by each = 0.67 MeV (motional energy)

Total energy shared by each particle is obviously

$m_0.c^2 + \text{K.E} = 0.51\text{MeV} + 0.67\text{MeV}$

$= 1.18\text{MeV}$

Example-2.17

A gamma ray photon of energy 1896 MeV annihilates to produce a proton-antiproton pair. If the rest mass of each of the particles involved be 1.007276 a.m.u approximately, find how much K.E each particle will carry?

Solution :

Working on the same lines as an electron-positron pair production, we notice that the reaction.

$\gamma \rightarrow$ proton + antiproton, has the energy balance

$E = m_0\text{ (proton)}.\ c^2 + \text{K.E (proton)}$

$+ m_0\text{ (antiproton)}.c^2 + \text{K.E (antiproton)}$

But $m_0.c^2$ = energy equivalent of 1.007276 a.m.u

≈ 938 MeV [$\because$ 1.00726 x 931 $\approx$ 938]

Thus K.E of each particle =

$\frac{1}{2}[1896\text{ MeV} - 2\times938]\text{ MeV} = 10\text{MeV}.$

2.27 MASS - ENERGY INTERCONVERSIONS

A nuclear reaction is similar to chemical reaction in principle. In a chemical reaction energy may be released or absorbed. This energy can be measured as the difference in chemical binding energies of atoms and molecules on the two sides of reaction indicated as a balanced equation.

In the case of nuclear reaction we find it as the difference in nuclear binding energies and chemical binding energy gives negative contribution (mass defect) to the total mass of atom or molecule.

The difference in total mass of atoms or molecules on the two sides of a chemical reaction gets converted into energy or vice versa. The interesting fact is that mass defects involved in a chemical reaction are almost 10^{-6} times than those in a nuclear reaction.

As a result, there is a general impression that mass energy interconversions takes place only in a nuclear reaction and not in a chemical reaction.

In a nuclear reaction, we know that proton number and neutron number are conserved. Here the total rest mass of neutrons and protons is the same on either side of a reaction. In this case total binding energy of nuclei on the left side need not be the same as that on the right hand side. The energy released or absorbed in a nuclear reaction is equal to the difference in these binding energies.

We know that binding energy contributes to mass and so, the difference in total mass of nuclei on the two sides of a reaction represented converts into energy.

So, it can be concluded that nuclear reaction is an example for mass energy interconversion.

2.28 RADIOACTIVE DATING

It is a method to determine the age of a geological or an archaeo-logical specimen by measuring the quantity of a radioisotope in it.

(a) Uranium-235 decays into thorium-234 with a half life of 4.5×10^9 years. Thorium is also radioactive and decays. It continues as a series and lead-206 will be obtained at the end which is stable. Each intermediate step has its own characteristic half life but all these half lives are very short as compared to that of U-238. So we may assume that a given mass of U-238 decays with a half life of 4.5×10^9 years into Pb-206.

Let us consider an example. On analysing an old rock say we find that it contains three U-239 atoms for every Pb-206 atom. Assume that there was no Pb-206 in the rock when it was formed as it will be obvious. It means 1/4th of original U-238 has decayed.

$$\Rightarrow \frac{N_0}{N} = \frac{4}{3}$$

If t is the age of rock, we can write

$$N = N_0 e^{-\lambda t}$$

$$\Rightarrow e^{\lambda t} = \frac{4}{3} \text{ and } \lambda t = \ell n\left(\frac{4}{3}\right)$$

$$\Rightarrow t = \frac{\ell n \frac{4}{3}}{\lambda} = 1.85 \times 10^9 \text{ years}$$

$$(\because \lambda = \frac{0.693}{4.5 \times 10^9 y})$$

So, radioactive U-238 dating helps us to know the age of rock, earth or any planet or stellar object.

(b) To determine the age of organic samples we can use carbon dating technique. Carbon-14 is radioactive with half life of 5730 years. C-14 is produced continuously in the earth's atmosphere due to cosmic radiation. There exists an equilibrium between formation of C-14 and its reduction by decay. When living organisms like plants, animals or human beings die, they stop taking carbon and C-14 in those decreases due to decay. On estimating the present C-14 per unit mass of the substance and based on its half life, we can find age of that sample.

Example-2.18

A timber sample containing 500 g of carbon provides 3070 decays per minute. Activity of atmospheric carbon due to presence of C-14 is 0.255 Bq per gram. What is age of that sample? ($T_{1/2} = 5730$ years).

Solution :

$$\text{Activity of sample} = R = \frac{3070}{60 \times 0.5} = 102 \text{ Bq/Kg}$$

$$\text{Activity of atmospheric carbon} = R_0 = 0.255 = 255 \text{ Bq/Kg}$$

We know that $R = R_0 e^{-\lambda t}$

$$\Rightarrow \frac{R_0}{R} = \frac{5}{2} = e^{\lambda t} \text{ and}$$

$$t = \frac{\log 2.5}{\log 2}(T_{1/2}) = \frac{0.3979}{0.3010} \times 5730 = 7575 \text{ years}$$

EXERCISE

LONG ANSWER QUESTIONS

1. Define binding energy. How does binding energy per nucleon vary with mass number? What is its significance?
2. Define binding energy of nucleus. Draw a curve between mass number and average binding energy. Give the important features of the curve.
3. What are nuclear forces? Discuss their important properties. If nuclear forces are strong attractive forces between nucleons, then why the nucleus does not collapse?
4. What is radioactivity ? State the law of radioactive decay. Show that radioactive decay is exponential in nature. Does the activity of a radioactive element depend on external physical condition?
5. What is artificial transmutation? Describe the experiment that lead to the discovery of artificial transmutation? Why only high energy α - particles were used in the discovery of artificial transmutation and neutron?
6. Explain the discovery of neutron. Mention its properties.
7. Explain the principle and working of a nuclear reactor with the help of a labelled diagram?

8. Explain the source of stellar energy. Explain the carbon - nitrogen cycle and proton - proton cycle occurring in stars.

SHORT ANSWER QUESTIONS

1. Why is the density of the nucleus more than that of the atom? Show that the density of nuclear matter is same for all nuclei .
2. What is meant by mass defect and binding energy. How do you account for the mass defect of nucleus ?
3. What are mass defect and binding energy ? How is mass defect related to binding energy?
4. For greater stability, a nucleus should have greater value of binding energy per nucleon. Why?
5. Draw a diagram to show the variation of binding energy per nucleon with mass number for different nuclei.
6. State the properties of α - rays.
7. State the properties of β - rays.
8. State the properties of γ - rays.
9. Define the term 'decay constant' for a radioactive substance. Deduce the relation between decay constant and half life period.
10. Define average life of a radioactive substance. Obtain the relation between decay constant and average life.
11. What do you mean by half - life and mean life of a radioactive substance? Deduce the relation between them.
12. Write a short note on the discovery of neutron.
13. Mention the uses of radioisotopes.
14. Uranium $^{238}_{92}U$ is more suitable for chain reaction. Why ?
15. What is nuclear fission? Give an example to illustrate it. Find the energy liberated in nuclear fission of $^{238}_{92}U$.
16. Explain the terms : chain reaction and multiplication factor. How is a chain reaction sustained?
17. In a nuclear reactor, what is the function of (i) moderator (ii) the control rods, (iii) the coolant and (iv) protective shielding.
18. Write a note on 'Breeder reactor'.
19. Write a note on 'Power reactor'.
20. Distinguish between nuclear fusion and nuclear fission.
21. Explain the source of energy in the sun.
22. Find the energy released in the fusion of four protons to form a helium nucleus.
23. State the reason why light nuclei usually undergo fusion and heavy nuclei usually undergo fission.

VERY SHORT ANSWER QUESTIONS

1. What are isotopes and isobars?
2. What are isotones and isomers.
3. What will be the ratio of the radii of two nuclei of mass number A_1 and A_2?
4. Does the density of a nucleus depend on the number nucleons it contains?
5. By what factor must the mass number A change for the nuclear radius to double?
6. All the nuclei have about the same density. What property of nuclear force is evident from it?
7. Give two important characteristics of nuclear forces.
8. What do you mean by charge independent mature of nuclear forces?
9. Why a nucleus has mass less than the sum of the masses of the individual nucleons in it.
10. The isotope $^{16}_{8}O$ has 8 protons, 8 neutrons and 11 electrons, while $^{8}_{4}Be$ has 4 protons, 4 neutrons and 4 electrons. Yet the ratio of their atomic masses is not exactly equal to one. Why?
11. Define Binding Energy?
12. What is the difference between binding energy and disintegration energy so that have the same magnitude?
13. Why is the binding energy curve steep for light nuclei and falling off slowly for the heavy nuclei?
14. Why is N approximately equal to Z for stable nuclei? Why is N greater than Z for heavy nuclei?
15. Account for the unstability of heavy nuclei.
16. Write a general equation that represents α -emission?
17. Why do unstable nuclei below the line of stability undergo β^+ decay, whereas unstable nuclei above the line of stability undergo β^- decay?
18. Does the ratio of neutrons to protons in a nucleus increase, decrease or remain the same after the emission of an α -particle?
19. A nucleus contains no electrons but can eject them. How?
20. Which of the following radiations α -rays, β -rays , γ - rays (i) are similar to x- rays (ii) are easily absorbed by matter (iii) travel with greatest speed (iv) are similar in nature to cathode rays.

21. Can a nucleus emit alpha particles with different energies? Explain.
22. A decay by α - emission is often followed by a β decay. Why is it usually β^- decay?
23. Why do all electrons emitted during β - decay not have the same energy?
24. What is the effect of K-capture on N, A and Z? is this the same effect as that of β^+ emission?
25. What are radio isotopes? Give some examples of radio isotopes .
26. What are the characteristics of radioactivity?
27. Natural radioactive nuclei are nuclei of high mass number. Why?
28. State the law of radioactive decay. Sketch a graph to illustrate radioactive decay.
29. What happens to half life of radioactive nuclei at very high temperatures?
30. What are the units and dimensions of disintegration constant?
31. A radioactive substance having N nuclei has activity A. Write down an expression for its half life in terms of A and N?
32. If a radioactive nucleus has a half life of one year, will be completely decayed after two year? Explain.
33. What fraction of a radioactive sample decay after three half lives?
34. Neutrons are the best projectiles to produce nuclear reactions why?
35. Neutrons cannot produce ionisation. Why?
36. State any two properties of neutron.
37. What are delayed neutrons? What is their Importance?
38. What is a thermal neutron? What is its importance?
39. Find the fission fragment X produced in the fission reaction, ${}_0^1n+{}_{92}^{235}U \rightarrow {}_{41}^{99}Nb + X + 4{}_0^1n$
40. What is the role of a moderator in a nuclear reactor?
41. What is the role controlling rods in a nuclear reactor?
42. Why is it useful to slow down neutrons produced by fission in a nuclear reaction?
43. What is the value of neutron multiplication factor in a controlled chain reaction and in an uncontrolled chain reaction?
44. Why is there a critical size of fissile material in a nuclear reactor?
45. Why are nuclear fusion reactions called thermo nuclear reactions?
46. State the conditions necessary for nuclear fusion?
47. What pair production of photon? Give an example?
48. What is annihilation of matter? Give an example.

PROBLEMS

LEVEL - I

1. If radius of ${}_{13}Al^{27}$ nucleus is estimated to be 3.6 fermi, find the radius of ${}_{52}X^{125}$ nucleus (nearly)
[Ans: 6 fermi]

2. Compare the radii of two nuclei with mass numbers 8 and 64. Also compare their densities. **[Ans: 1:2; 1:1]**

3. Calculate the density of nucleus taking the mass of a nucleon as $m = 1.67\times10^{-27}$ kg and $R_0 = 1.4\times10^{-15}$ m.
[Ans : 1.453×10^{17} kg/m^3]

4. Assume that the mass of a nucleus is approximately given by $M = Am_p$ where A is the mass number. Estimate the density of matter in kg/m^3 inside a nucleus. What is the specific gravity of nuclear matter ?
[Ans: 3×10^{17} kg/m^3, 3×10^{14}]

5. Find the BE/A from the following data, mass of ${}_3Li^7$ = 7.01653 amu, mass of proton = 1.00759 amu mass of neutron = 1.00898 amu **[Ans: 5.6 MeV]**

6. Calculate the (i) mass defect, (ii) binding energy and (iii) the binding energy per nucleon of ${}_6^{12}C$ nucleus. Nuclear mass of ${}_6^{12}C$ = 12. 000000 u; mass of proton=1.007825 u and mass of neutron=1.008665 u.
[Ans : 0.09894 u; 92.1 MeV; 7.675 MeV]

7. Calculate the binding energy of an α - particle. Given that mass of proton = 1. 0073 u, mass of neutron = 1.0087 u, and mass of α - particle = 4.0015 u. **[Ans : 28.4 MeV]**

8. Find the energy required to spilt ${}_8^{16}O$ nucleus into four α - particles. The mass of a α - particle is 4.002603 u and that of oxygen is 15. 994915 u.
[Ans:14.43 MeV]

9. Calculate the binding energy per nucleon of ${}_{17}^{35}Cl$ nucleus. Given that mass of ${}_{17}^{35}Cl$ nucleus = 34.98000 u, mass of proton = 1. 007825 u , mass of neutron = 1.008665 u and 1 u is equivalent to 931 MeV. **[Ans: 8.2 V]**

10. Obtain the binding energy of a nitrogen nucleus from the following data : $m_H = 1.00783\,u$, $m_N = 1.00867\,u, m({}_7^{14}N) = 14.00307\,u$ Give your answer in units of MeV. [1u=932.5 MeV / c^2]
[Ans :104.72 Mev]

11. Calculate the mass of an α-particle. Its binding energy is 28.2 MeV. **[Ans : 4.0016 units]**

12. How much energy is released in the following reaction ? $^{7}Li + p \rightarrow \alpha + \alpha$ Atomic mass of $^{7}Li = 7.0160$ u and that of $^{4}He = 4.0026$ u.
[Ans: 17.34 MeV]

13. A radioactive nucleus undergoes a series of decays according to the sequence $A \xrightarrow{\beta} A_1 \xrightarrow{\alpha} A_2 \xrightarrow{\alpha} A_3$. If the mass number and atomic number of A_3 are 172 and 69 respectively, what is the mass number and atomic number of A ? [Ans: 180, 72]

14. The isotope $^{238}_{90}U$ decays successively to form $^{234}_{90}Th, ^{234}_{91}, Pa^{234}_{92}U, ^{230}_{90}Th$ and $^{226}_{88}Ra$. What are the radiations emitted in these steps?
[Ans: $\alpha, \beta, \beta, \alpha, \alpha$]

15. How many α and β - particles are emitted when uranium nucleus $\left(^{238}_{92}U\right)$ decay to $^{214}_{82}Pb$?
[Ans: α -particles and 2 β - particles]

16. If $_{92}U^{238}$ changes to $_{85}At^{210}$ by a series of α and β decays, find the number of α and β decays undergone [Ans: 7, 7]

17. Complete the following reactions.
(a) $^{226}_{88}Ra \rightarrow \alpha +$ (b) $^{19}_{8}O \rightarrow ^{19}_{9}F +$ (c) $^{25}_{13}Al \rightarrow ^{25}_{12}Mg +$
[Ans: (a) $_{84}Rn^{222}$; (b) $e^- + \nu^*$; (c) $e^+ + \nu$]

18. Plutonium decays with a half - life of 24, 000 years. If plutonium is stored for 72, 000 years, what fraction of it remains ? [Ans: 1/8]

19. A certain substance decays to $\frac{1}{32}$ of its initial activity in 25 days. Calculate its half - life. [Ans: 5 days]

20. The half - life period of a radioactive substance is 20 days. What is the time taken for $\frac{7}{8}$ th of its original mass to disintegrate? [Ans: 60 days]

21. How many disintegrations per second will occur in one gram of $^{238}_{92}U$, if its half - life against α -decay is 1.42×10^{17} s ? [Ans: $1.235 \times 10^4\ s^{-1}$]

22. The half-life of ^{215}At is $100\mu s$. Find the time taken for the radioactivity of a sample of ^{215}At to decay to $\frac{1}{16}$ th of its initial value : [Ans: 400 μs]

23. The half - life of radium is 1600 years. How much time does 1 g of radium take to reduce to 0.125 g ?
[Ans: 4800 MeV]

24. A radioactive sample has 2.0×10^{20} active nuclei at a certain instant of time. How many of them will still be in the same active state after three half - lives ?
[Ans: 2.5×10^{19}]

25. In a sample of a radioactive substance what fraction of the initial number of nuclei will remain undecayed after a time $t = \frac{T}{2}$, where T = half-life of radioactive substance.
[Ans : $\frac{1}{\sqrt{2}}$]

26. A radioactive substance is being produced at a constant rate of 200 nuclei/s. The decay constant of the substance is $1 s^{-1}$. After what time the number of radioactive nuclei will become 100. Initially there are no nuclei present? [Ans: t = ln2 sec]

27. A radioactive element X converts into another stable element Y. Half-life of X is 2h. Initially only X is present. After time t, the ratio of atoms of X and Y is found to be 1 : 4, then find t in hours.
[Ans: $\frac{2}{\ln 2}\ln 5$]

28. A radioactive sample can decay by two different processes. The half - life for the first process is T_1 and that for the second process is T_2. Find the effective half - life T of the radioactive sample.
[Ans: $\frac{T_1 T_2}{T_1 + T_2}$]

29. A radioactive substance contains 10^{15} atoms and has an activity of 6.5×10^{11} Bq. What is its half life.
[Ans: 1.16×10^3 second]

30. N_1 atoms of a radioactive element emit N_2 beta particles per second. Find the decay constant of the element (in s^{-1}) [Ans : $\frac{N_2}{N_1}$]

31. Radioactive ^{131}I has a half-life of 8.0 days. A sample containing ^{131}I has activity 20μ Ci at $t = 0$.
(a) What is its activity at t = 4.0 days? (b) What is its decay constant at t = 4.0 days ?
[Ans: (a) $14\mu Ci$; (b) $1.4 \times 10^{-6} S^{-1}$]

32. After 280 days, the activity of a radioactive sample is 6000 dps. The activity reduces to 3000 dps after another 140 days. Find the initial activity of the sample in dps. [Ans: 24000]

33. If one microgram of $^{235}_{92}U$ is completely destroyed in an atom bomb, how much energy will be released ?
[Ans: 9×10^7 J]

34. Calculate the energy released by fission from 2 g of $^{235}_{92}U$ in kWh. Given that the energy released per fission is 200 MeV. [Ans: 4.54×10^4 kwh]

35. An explosion of atomic bomb releases an energy of 7.6 × 10 [13] J. If 200 MeV energy is released on fission of one ^{235}U atom calculate (i) the number of uranium atoms undergoing fission and (ii) the mass of uranium used in the bomb.

[**Ans:** 2.375×10^{24}; 926.66 g]

36. 200 MeV energy is released when one nucleus of ^{235}U undergoes fission. Find the number of fissions per second required for producing power of 1 megawatt. [**Ans:** 3.125×10^{16}]

37. Find the energy released during the most probable fission of ^{235}U. $^{235}_{9}U + ^{1}_{0}n \to ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n$. Average binding energy per nucleon of ^{235}U, ^{141}Ba and ^{92}Kr are 7.61 MeV, 8.32 MeV and 8.80 MeV respectively. [**Ans:** 194.34 MeV]

38. The binding energies of nuclei X and Y are E_1 and E_2 respectively. Two atoms of X fuse to give one atom of Y and an energy Q is released. Find Q

[**Ans:** $Q = E_2 - 2E_1$]

39. The binding energies per nucleon for deuteron $\left(^{2}_{1}H\right)$ and helium $\left(^{4}_{2}He\right)$ are 1.1 MeV and 7.0 MeV respectively. Calculate the energy released, when two deuterons fuse to form a helium nucleus.

[**Ans:** 23.6MeV]

40. If a star can convert all the He nuclei completely into oxygen nuclei. Find the energy released per oxygen nuclei : [Mass of the nucleus He is 4.0026 amu and mass of oxygen nucleus 15.9994]

[**Ans:** 10.24MeV]

41. How much energy must a gamma ray photon have, if it is to materialize into a pair of electron and positron with each particle having a K.E. of 1 MeV

[**Ans:** 3.02 MeV]

42. A 1MeV positron and a 1 MeV electron meet each moving in opposite directions. They annihilate each other by emitting two photons. If the rest mass energy of an electron is 0.51 MeV find the wavelength of each photon [**Ans:** 8.2×10^{-3} Å]

43. An electron and a positron pair is produced by a gamma ray of 2.1MeV. Find the K.E imparted to each of the charged particle. [**Ans:** 0.54 MeV]

44. Consider a hypothetical annihilation of a stationary electron with a stationary position, what is wave-length of resulting radiation?

[**Ans:** $\frac{h}{m_0 c}$]

LEVEL - II

1. (a) Calculate the energy released if ^{238}U emits an α-particle. (b) Calculate the energy to be supplied to ^{238}U if two protons and two neutrons are to be emitted 0000one by one. The atomic masses of ^{238}U, ^{234}Th and ^{4}He are 238.0508 u, 234.04363 u and 4.00260 u respectively.

[**Ans:** (a) 4.255 MeV ; (b) 24.03 MeV]

2. $^{212}_{83}Bi$ decays as per following equation.

$^{212}_{83}Bi \to ^{208}_{82}Tl + ^{4}_{2}He$. The kinetic energy of α-particle emitted is 6.082 MeV. Calculate the kinetic energy of Tl recoil atoms.

[**Ans:** 0.1308 MeV]

3. A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5MeV, calculate the kinetic energy of the α-particle. [**Ans:** 5.4MeV]

4. In the decay $^{64}Cu \to ^{64}Ni + e^{+} + \nu$, the maximum kinetic energy carried by the positron is found to be 0.650 MeV. (a) What is the energy of the neutrino which was emitted together with a positron of kinetic energy 0.150 MeV ? (b) What is the momentum of this neutrino in kg-m/s? Use the formula applicable to a photon.

[**Ans:** (a) 500 MeV ; (b) 2.67×10^{-22} kg-m/s]

5. A freshly prepared sample of a certain radioactive isotope has an activity of 10 mCi. After 4.0 hr its activity is 8.00 mCi. (a) Find the decay constant and half life (b) How many atoms of the isotope were contained in the freshly prepared sample. (c) What is the sample's activity 30.0 hr after it is prepared.

[**Ans:** (a) 1.55×10^{-5} /sec, 12.4 h
(b) 2.39×10^{13} atoms (c) 1.87 mCi]

6. A sample of radioactive material has a mass m, decay constant λ and molecular weight M. Avogadro constant N_A. (a) Find the initial activity of the samples (b) the activity of the substance after t sec.

[**Ans:** (a) $\frac{\lambda m N_A}{M}$; (b) $\left(\frac{m N_A \lambda}{M}\right)e^{-\lambda t}$]

7. The ratio of molecular mass of two radioactive substances is $\frac{3}{2}$ and the ratio of their decay constant is $\frac{4}{3}$. Then find the ratio of their initial activity per mole [**Ans:** $\frac{4}{3}$]

8. Two radioactive materials X_1 and X_2 contain same number of nuclei. If $6\lambda s^{-1}$ and $4\lambda s^{-1}$ are the decay constants of X_1 and X_2 respectively, find the time after which ratio of number of nuclei undecayed of X_1 to that of X_2 will be 1/e. **[Ans:** $\frac{1}{2\lambda}$ s]

9. Two radioactive elements X and Y have half life periods of 50 minutes and 100 minutes respectively. Initially both of them contain equal number of atoms. Find the ratio of atoms left N_x / N_y after 200 minutes. **[Ans:** 1/4]

10. A certain sample of a radioactive material decays at the rate of 500 per second at a certain time. The count rate falls to 200 per second after 50 minutes.
(a) What is the decay constant of the sample ?
(b) What is its half-life ?
[Ans: (a) 3.05×10^{-4}s; (b) 38 min]

11. The count rate from a radioactive sample falls from 4.0×10^6 per second to 1.0×10^6 per second in 20 hours. What will be the count rate 100 hours after the beginning? **[Ans:** 3.9×10^3 per second]

12. The half-life of a radioisotope is 10 h. Find the total number of disintegrations in the tenth hour measured from a time when the activity was 1 Ci. **[Ans:** 6.91×10^{13}]

13. The half-life $^{238}_{92}U$ against alpha decay is 4.5×10^9 year. How much disintegration per second occurs in 1 g of $^{238}_{92}U$? **[Ans:** 1.23×10^4 dps]

14. Half-life of a radioactive substance A is 4 days. Find the probability that a nucleus will decay in two half-life **[Ans:** 3/4]

15. What is the probability that is radioactive atom having a mean life of 10 days decays during the fifth day ? **[Ans:** 0.39]

16. A small quantity of solution containing Na^{24} radio nuclide (half-life = 15 hour) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hours shows an activity of 296 disintegrations per minute. Determine the total volume of the blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person. (1 curie = 3.7×10^{10} disintegration per second) **[Ans:** 5.95 litre]

17. A free neutron beta-decays to a proton with a half-life of 14 minutes. (a) What is the decay constant? (b) Find the energy liberated in the process. **[Ans:** (a) $8.25 \times 10^{-4}S^{-1}$ (b) 782 keV]

18. Find the probability of survival of a radioactive nucleus for one mean life. **[Ans:** 1/e]

19. The selling rate of a radioactive isotope is decided by its activity. What will be the second-hand rate of a one month $^{32}P(t_{1/2}$ = 14.3 days) source if it was originally purchased for 800 rupees? **[Ans:** 187 rupees]

20. A radioactive isotope X has a half life of 10 s. There are 1000 isotopes which are falling from rest from a height of 500 m. When it is at a height of 100 m for the reference plane, find the activity of the sample **[Ans :** 39]

21. A radioactive isotope is being produced at a constant rate X. Half-life of the radioactive substance is Y. After some time the number of radioactive nuclei become constant (K). Find the value of K **[Ans:** $\frac{XY}{\ln 2}$]

22. A radioactive substance x decays into another radioactive substance y. Initially only x was present λ_x and λ_y are the disintegration constants of X and Y. N_x and N_y are the number of nuclei of X and Y at any time t. At the time of number of nuclei N_y will be maximum write the relation between known quantities. **[Ans:** $\lambda_x N_x = \lambda_y N_y$]

23. A radioactive nucleus is being produced at a constant rate α per second. Its decay constant is λ. If N_0 are the number of nuclei at time t = 0, find the maximum number of nuclei possible **[Ans:** $\frac{\alpha}{\lambda}$]

24. Consider the situation of the previous problem. Suppose the production of the radioactive isotope starts at t = 0. Find the number of active nuclei at time t. **[Ans:** $\frac{R}{\lambda}(1-e^{-\lambda t})$]

25 (a) How much mass is lost per day by a nuclear reactor operated at 10^9 watt power level. (b) If each fission releases 200 MeV, how many fissions occur per second to yield this power level. **[Ans:** (a) 9.6×10^{-4} kg ; (b) 3.125×10^{19}]

26. Assuming the splitting of U^{235} nucleus liberates 200 MeV energy, find (a) the energy liberated in the fission of 1 kg of U^{235} and (b) the mass of the coal with calorific value of 30 kJ/g which is equivalent to 1 kg of U^{235}. **[Ans:** (a) 8.09×10^{13}J; ; (b) 2.7×10^6 kg]

27. Calculate the Q-value of the fusion reaction $^{4}He + {}^{4}He = {}^{8}Be$ In such a fusion energetically favorable? Atomic mass of ^{8}Be is 8.0053 u and that of ^{4}He is 4.0026 u. **[Ans: –93.1 kev, no]**

28. The half-lives of radioisotopes P^{32} and P^{33} are 14 days and 25 days respectively. These radioisotopes are mixed in the ratio of 4 : 1 of their atoms. If the initial activity of the mixed sample is 3.0m Ci, find the activity of the mixed isotopes after 60 year. **[Ans: 0.205 mCi]**

29. A radioactive material of half-life T was produced in a nuclear reactor at different instants, the quantity produced second time was twice of that produced first time. If now their present activities are A_1 and A_2 respectively then find their age difference. **[Ans:** $\frac{T}{\ln 2}\ln\left(\frac{A_2}{2A_1}\right)$**]**

30. There are two radio nuclei A and B. A is an alpha emitter and B a beta emitter. Their disintegration constants are in the ratio of 1 : 2. What should be the ratio of number of atoms of A and B at any time t so that probabilities of getting alpha and beta particles are same at that instant? **[Ans: 2 : 1]**

31. Half-life of a radioactive substance A is two times the half-life of another radioactive substance B. Initially the number of nuclei of A and B are N_A and N_B respectively. After three half lives of A number of nuclei of both are equal. Find the ratio N_A/N_B. **[Ans: 1/8]**

32. ^{57}Co decays to ^{57}Fe by β^+ - emission. The resulting ^{57}Fe is in its excited state and comes to the ground state by emitting γ -rays. The half-life β^+ -decay is 270 days and that of the γ -emission is 10^{-8}s. A sample of ^{57}Co gives 5.0×10^9 gamma rays per second. How much time will elapse before the emission rate of gamma rays drops to 2.5×10^9 per second ? **[Ans: 270 days]**

33. A point source emitting alpha particles is placed at a distance of 1 m from a counter which records any alpha particle falling on its 1 cm^2 window. If the source contains 6.0×10^{16} active nuclei and the counter records a rate of 50000 counts/second, find the decay constant. Assume that the source emits alpha particles uniformly in all directions and the alpha particles fall nearly normally on the window. **[Ans:** $1.05 \times 10^{-7}\ s^{-1}$**]**

34. A radioactive sample decays with an average life of 20 ms. A capacitance 100μ F is charged to some potential and then the plates are connected through a resistance R. What should be the value of R so that the ratio of the charge on the capacitor to the activity of the radioactive sample remains constant in time ? **[Ans : 200Ω]**

35. A charged capacitor of capacitance C is discharged through a resistance R. A radioactive sample decays with an average-life τ . Find the value of R for which the ratio of the electrostatic field energy stored in the capacitor to the activity of the radioactive sample remains constant in time. **[Ans:** $2\tau / C$ **]**

36. Nuclei of a radioactive element A are being produced at constant rate α . The element has a decay constant λ . At time t = 0, there are N_0 nuclei of the element.
(a) Calculate the number N of nuclei of A at time t.
(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A, and also the limiting value of N as $t \to \infty$.
[Ans: (a) $\frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}]$; (b) $\frac{3}{2}N_0, 2N_0$ **]**

37. Consider a radioactive disintegration according to the equation $A \to B \to C$. Deacy constant of A and B is same and equal to λ . Number of nuclei of A, B and C are N_0, 0, 0 respectively at $t = 0$.
Find (a) Number of nuclei of B as function of time t. (b) Time t at which the activity of B is maximum and the value of maximum activity of B.
[Ans: (a) $N_B = \lambda N_0(te^{-\lambda t})$; (b) $t = \frac{1}{\lambda}, R_{max} = \frac{\lambda N_0}{e}$ **]**

38. A radioactive isotope is being produced at a constant rate dN/dt = R in an experiment. The isotope has half-life $t_{1/2}$. Show that after a time $t >> t_{1/2}$, the number of active nuclei will become constant. Find the value of this constant. **[Ans:** $\frac{Rt_{1/2}}{0.693}$ **]**

39. A radioactive nuclide A_1 with decay constant λ_1 transform into a radioactive nuclide A_2 with decay constant λ_2 . Assuming that at the initial moment the preparation contained only the nuclide A_1. Find the time interval after which the activity of the nuclide A_2 reach its maximum value. **[Ans:** $\frac{\ln(\lambda_2 / \lambda_1)}{\lambda_2 - \lambda_1}$ **]**