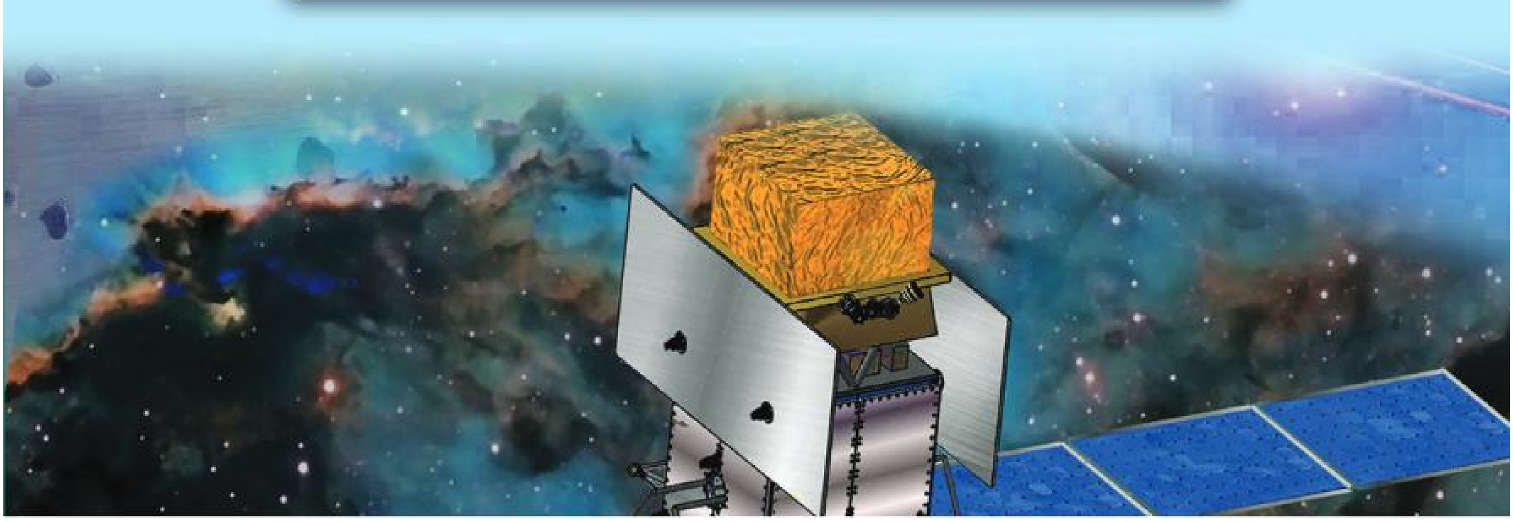
Chapter - 3

ELECTROMAGNETIC INDUCTION

✤ Faraday's experiments ✤ Faraday's laws, Lenz's law ✤
✤ Induced emf and current, motional emf, AC generator ✤
✤ Induced electric field, Eddy currents ✤
✤ Self induction, Mutual induction ✤
✤ Transformer Growth and decay of current in L-R circuit ✤



3.1 INTRODUCTION

Electricity and magnetism were considered as separate and unrelated phenomena for a long time. In the early decades of the nineteenth century, experiments on electric current by Oersted, Ampere and a few others established the fact that electricity and magnetism are inter-related. They found that moving electric charges produce magnetic fields. For example, an electric current deflects a magnetic compass needle placed in its vicinity. This naturally raises the questions like-Is the converse effect possible? Can moving magnets produce electric currents?

Does the nature permit such a relation between electricity and magnetism? The answer is, resounding yes! The experiments of Michael Faraday in England and Joseph Henry in USA, conducted around 1830, demonstrated conclusively that electric currents were induced in closed coils when subjected to changing magnetic fields. In this chapter, we shall study the phenomena associated with changing magnetic fields and understand the underlying principles. The phenomenon in which an emf is generated when there is a change in magnetic flux associated with a closed loop or a moving conductor cuts the magnetic flux lines is called electromagnetic induction. (The phenomenon in which electric current is generated by varying magnetic fields is appropriately called electro magnetic induction)

3.2 FARADAY'S EXPERIMENTS

In 1820, Oersted observed that magnetic field is produced around a current carrying conductor. In the same year, Ampere gave a quantitative analysis for the above observation.

Faraday in 1831 succeeded in verifying the converse of the above phenomenon, i.e. production of electric current by using magnetic field and put forth the laws of electromagnetic induction through a series of very simple experiments.

The simple apparatus used by him is shown in figure. A coil C_1 connected to a galvanometer G.

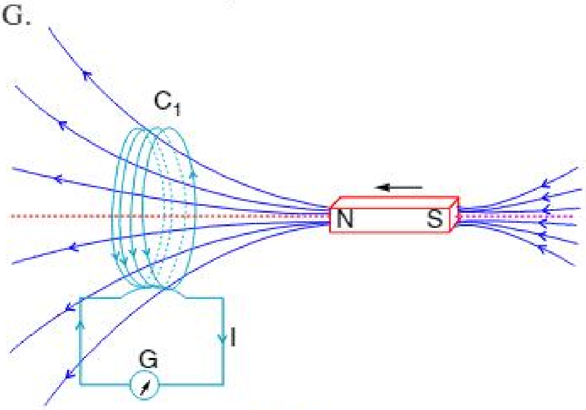


Fig 3.1

When the North - pole of a bar magnet is pushed towards the coil, the pointer in the galvanometer deflects and indicated their presence of electric current in the coil. The deflection lasts as along as the bar magnet is in motion.

When the magnet is pulled away from the coil, the galvanometer shows deflection in the opposite direction. The galvanometer does not show any deflection when the magnet is held stationary.

When the South - pole of the bar magnet is moved towards or away from the coil, the deflections in the galvanometer will be opposite to that observed with the North - pole for similar movements.

The deflection (and hence current) is found to be larger when the magnet is pushed towards or pulled away from the coil faster.

When the bar magnet is held fixed and the coil C_1 is moved towards or away from the magnet, the same effects are observed. **It shows that the relative motion between the magnet and the coil is responsible for generation (induction) of electric current in the coil.**

Faraday further conducted the following simple experiments.

In figure the bar magnet is replaced by a second coil C_2 connected to a battery. The steady current in the coil C_2 produces a steady magnetic field.

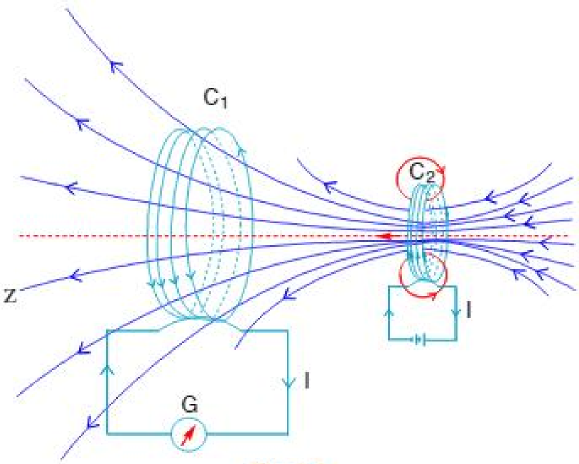


Fig 3.2

As coil C_2 is moved towards the coil C_1, the galvanometer shows a deflection. This indicates that electric current is induced in coil C_1. When C_2 is moved away, the galvanometer shows a deflection again, but in the opposite direction. The deflection lasts as long as coil C_2 is in motion. When the coil C_2 is held fixed and C_1 is moved, the same effects are observed.

The relative motion between the coils is responsible for inducing the electric current.

In another experiment the two coils C_1 and C_2 are held stationary around a common a wooden core. Coil C_1 is connected to galvanometer G while the second coil C_2 is connected to a battery through a tapping key K is just opened or closed, a momentary deflection is observed in the galvanometer. If the key is held pressed for a long time, there is no deflection in the galvanometer.

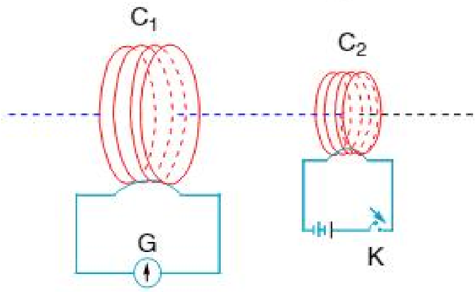


Fig 3.3

He employed an insulated iron core in place of wooden core and found that the magnitude of deflection is increased.

From the above it is concluded that,

i) Higher the relative speed, larger was the deflection.

ii) The deflection is found to be larger when the number of turns in the coil is increased.

iii) The deflection is found to be larger when the core is made of ferromagnetic material.

iv) There must be a change in magnetic flux in order to induce the electric current in a coil.

v) In electromagnetism, a moving electron produces magnetic field. In electromagnetic induction, a changing magnetic field causes to move electrons.

3.3 MAGNETIC FLUX

Consider a small surface of area A. Let $\hat{n}$ be the unit vector which is drawn normal to the surface. If 'θ' is the angle between $\hat{n}$ and the uniform magnetic field $\vec{B}$, then the magnetic flux (ϕ) through the surface is defined as

$$\phi = \vec{B}.\vec{A} = (\vec{B}.\hat{n})A$$

where $\hat{n}$ = unit vector along the normal drawn to the plane of the surface.

A = Area of the magnetic field bounded by the surface.

$\phi = (B\cos\theta)A$

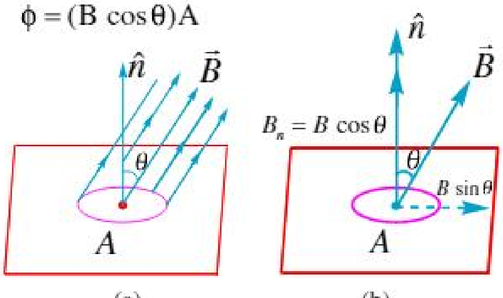


Fig 3.4

Now $B\cos\theta$ is the component of the magnetic field normal to the plane of the surface and can be represented as B_n. Hence $\phi = B_n A$.

Thus, magnetic flux over a given surface is defined as the product of the area of the surface and the component of the magnetic field normal to the surface (B_n).

The SI unit of flux is weber (Wb).

CGS unit of flux is maxwell (Mx)

1 weber = 1 tesla-meter^2

1 weber = 10^8 maxwell

Dimensional formula of the magnetic flux is $ML^2T^{-2}A^{-1}$

Magnetic flux is a scalar.

It can be positive or negative or zero depending upon the angle between area vector and field direction.

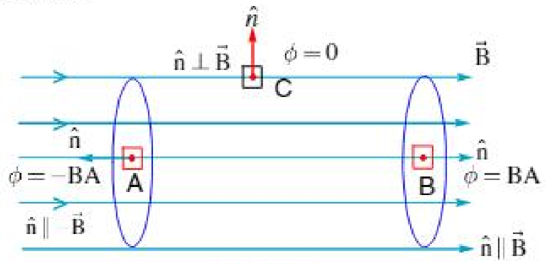


Fig 3.5

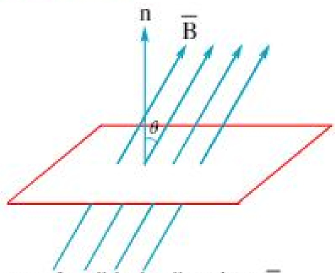
From the above figure,

i) When the plane of the surface is parallel to the direction of the magnetic field (or) normal drawn to the surface is perpendicular to the magnetic field $(\hat{n} \perp \vec{B})$ the magnetic flux linked with the surface is zero i.e., $\phi = 0$ $[\because \theta = 90^0]$.

ii) When the plane of the surface is perpendicular to magnetic field (or) normal drawn to the surface is parallel to the magnetic field $(\hat{n} \parallel \vec{B})$, then magnetic flux linked with the surface is maximum. i.e., $\phi_{max} = BA$ $(\because \theta = 0^\circ)$

iii) When the flux entering the surface is opposite to the area vector ($\hat{n}$), $\phi = -BA$. $(\because \theta = 180^\circ)$

3.3(A) MAGNETIC FLUX ASSOCIATED WITH A COIL

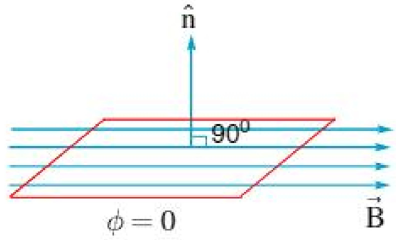


Plane of coil is inclined to $\overline{B}$

Fig 3.6

If a coil having N turns and area A is placed in a magnetic field of induction B with its normal to the plane making an angle θ with $\vec{B}$, then the magnetic flux associated with the coil is, $\phi = NBA\cos\theta$

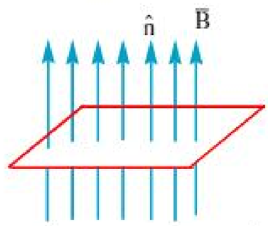
If the plane of the coil is parallel to $\vec{B}$ then the magnetic flux linked with the coil. $\phi = 0$.



Plane of coil parallel to $\vec{B}(\hat{n} \perp \vec{B})$

Fig 3.7

If the plane of the coil is perpendicular to $\vec{B}$ then the magnetic flux linked with the coil $\phi = BAN$



Plane of coil perpendicular to $\overline{B}$ $(\hat{n} \parallel \overline{B})$

Fig 3.8

Note: Magnetic flux linked with a coil $(\phi = NBA\cos\theta)$ can be changed by

a) Changing the no. of turns (N)

b) Varying the magnetic field (B)

c) pulling or pushing the coil into magnetic field (By changing the area of the magnetic field bounded by coil)

d) changing the orientation of the coil (θ) in the magnetic field

3.3(B) CHANGE OF FLUX DUE TO ROTATION OF THE COIL

When the coil is rotated from an angle of θ_1 to an angle of θ_2 (both are measured w.r.t. normal) in a uniform magnetic field then the initial flux through the coil is $\phi_i = NBA\cos\theta_1$

The final flux through the coil after rotation is $\phi_f = NBA\cos\theta_2$

The change in the flux associated with the coil is

$$\Delta\phi = \phi_f - \phi_i$$

$$\Delta\phi = NBA(\cos\theta_2 - \cos\theta_1)$$

if $\theta_1 = 0^\circ$ and $\theta_2 = 90^\circ$ then $\Delta\phi = -NBA$

if $\theta_1 = 90^\circ$ and $\theta_2 = 180^\circ$ then $\Delta\phi = -NBA$

if $\theta_1 = 0^\circ$ and $\theta_2 = 180^\circ$ then $\Delta\phi = -2NBA$

Example-3.1

A rectangular loop of area 0.06 m^2 is placed in a magnetic field of 0.3T with its plane (i) normal to the field (ii) inclined 30^0 to the field (iii) parallel to the field. Find the flux linked with the coil in each case.

Solution :

$\phi = NAB\cos\theta$

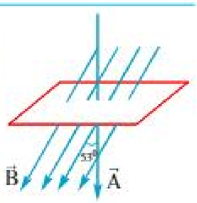
i) $\phi = 1\times 0.06\times 0.3\times\cos 0^\circ = 0.018$ weber

ii) $\phi = 1\times 0.06\times 0.3\times\cos 60^\circ = 0.009$ weber

iii) $\phi = 1\times 0.06\times 0.3\times\cos 90^\circ = 0$

Example-3.2

At a certain location in the northern hemisphere, the earth's magnetic field has a magnitude of 42μT and points downwards at 53^0 to the vertical. Calculate the flux through a horizontal surface of area 2.5 m^2. $[\sin 53^0 = 0.8]$



Solution : $\phi_B = BA\cos\theta$

$= 42\times 10^{-6}\times 2.5\times\cos 53^0 = 63\mu Wb$

3.4 FARADAY'S LAW OF ELECTRO MAGNETIC INDUCTION

Faraday stated his experimental observations in the form of law called Faraday's a law of electromagnetic induction.

a) Whenever the magnetic flux linked with an electric circuit (coil) changes, an emf is induced in the circuit (coil). The induced emf exists as long as the change in magnetic flux continues.

b) The induced emf produced in the coil is equal to the negative rate of change of magnetic flux linked with it.

$e = -\dfrac{d\phi}{dt}$; where ϕ = flux through each turn

If the coil contains N turns, an emf appears in everyturn and all these e.m.f.s are to be added. Then, the induced emf is given by

$e = -N.\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(N\phi)$; Where '$N\phi$' is total flux linked with the coil of N turns.

(or) $e = -\dfrac{d}{dt}(N\phi) = -\dfrac{d}{dt}(NBA\cos\theta)$

Negative sign is in accordance with Lenz's law. The above law is also called **Neumann's law.**

3.5 LENZ'S LAW AND CONSERVATION OF ENERGY

It is a fundamental law involving the law of conservation of energy. It helps us to decide the polarity (direction) of induced emf or induced current in a coil.

Statement : The polarity (direction) of the induced emf is always such that it tends to produce a current which opposes the change in magnetic flux that produced it.

Explanation

The changing magnetic field and magnetic flux induces an electric current in a coil. This induced current itself creates magnetic field and hence magnetic flux is induced around the coil.

If the induced current in the coil is due to increase in flux, then the direction of induced flux is so oriented as to decrease the external flux.

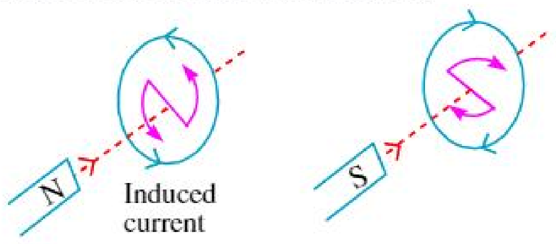
If the induced current in the coil is due to decrease in flux, then the direction of induced flux is so oriented as to increase the external flux.

Therefore, the change in the external magnetic field and flux is always opposed. So some external work is needed to overcome the opposition. The energy needed to do the work is converted into the electrical energy to establish current in the circuit, by obeying law of conservation of energy. If the so called 'opposition' was not there, then there is no need of doing any work to move a charge in the coil, which is against the law of conservation of energy.

Example - 1: Consider a magnet brought towards a circular loop with its north pole facing the loop. As the magnet gets closer to the loop, the magnetic field and the flux of the magnetic field through the area of the loop increases. The induced current should decrease the flux. The original field is away from the magnet, so the induced field should be towards the magnet. Using the right hand thumb rule, the direction of the induced current that produces a field towards the magnet should be anti clock wise.

i.e., when a magnet approaches a coil, with its N - pole towards the coil, the direction of induced current is anti clock wise such that the face of the coil towards the magnet behaves as N - pole.

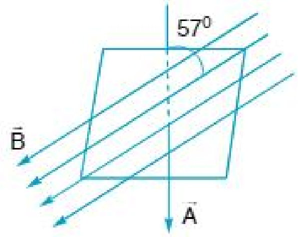
Example-2 : Similarly if the magnet is taken away from the coil as shown then in the face of coil towards the magnet, the induced current flows in clockwise direction. Hence that face of the coil behaves like south pole.



Note : Induced emf can exist whether the circuit is opened or closed. But induced current can exist only in the closed circuits.

Example-3.3

At a certain location in the northern hemisphere, the earth's magnetic field has a magnitude of $42\mu T$ and points downwards at 57^0 to the vertical. Calculate the flux through a horizontal surface of area 2.5 m^2.



Solution :

$\phi_B = BA\cos\theta = 42\times10^{-6}\times2.5\times\cos57^0\,Wb$

$= 57.2\times10^{-6}\,Wb = 57.2\mu Wb$

3.6 EXPRESSION FOR INDUCED EMF AND INDUCED CURRENT

Let, ϕ_1 = magnetic flux linked with a circuit at any instant ϕ_2 = magnetic flux linked with the same circuit after time t.

Applying Faraday's second law and Lenz's law, the induced emf is given by

$$e \propto -\frac{\phi_2-\phi_1}{t} \text{ or } e \propto -\frac{d\phi_B}{dt}$$

where $d\phi_B$ is small change in magnetic flux in a small time interval dt.

Now $e = -k\dfrac{d\phi_B}{dt}$

Let us choose units in such a way that k = 1.

Then $e = -\dfrac{d\phi_B}{dt}$

If the coil has N turns then $e = -N\dfrac{d\phi_B}{dt}$

–ve sign indicates that the induced emf is produced in such a way that the current due to it opposes the change in magnetic flux. This is according to Lenz's law.

If $\dfrac{d\phi_B}{dt}$ is measured in weber per second then e is measured in volt.

Induced current is

$$I = \frac{\text{Induced emf}}{\text{Resistance in the circuit}} = -\frac{1}{R}\left(\frac{d\phi_B}{dt}\right)$$

Expression for induced charge

$$I = \frac{e}{R} = \frac{1}{R}\left(-\frac{d\phi_B}{dt}\right)$$

$$\frac{dQ}{dt} = -\frac{1}{R}\frac{d\phi_B}{dt} \text{ or } dQ = -\frac{1}{R}d\phi_B$$

$\therefore$ Induced charge, $Q = -\frac{1}{R}\int_{\phi_i}^{\phi_f} d\phi_B$

$Q = -\frac{1}{R}\left[\phi_f - \phi_i\right]$ or $Q = \frac{\phi_i - \phi_f}{R}$

We are generally interested only in the value of the charge.

$\therefore$ In general, Induced charge is given by

$$Q = \frac{\text{Change of magnetic flux}}{\text{Resistance}}$$

Example-3.4

The flux of magnetic field through a closed conducting loop changes with time according to the equation $\phi = 0.2t^2 + 0.4t + 0.6$. Find the induced e.m.f at t = 2s.

Solution :

$\phi = 0.2t^2 + 0.4t + 0.6$

$e = -\frac{d\phi}{dt} \Rightarrow e = -0.4t - 0.4$

when t = 2 s ; $e = -0.8 - 0.4 = -1.2$ V

Example-3.5

The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi_B = (5t^3 + 4t^2 + 2t - 5)$ weber. Calculate the induced current through the coil at t = 2 second. The resistance of the coil is 5Ω.

Solution :

$\phi = 5t^3 + 4t^2 + 2t - 5$

$|e| = \frac{d\phi}{dt} = 15t^2 + 8t + 2$

$iR = 15t^2 + 8t + 2$

$i \times 5 = 15 \times 4 + 8 \times 2 + 2 \Rightarrow i = 15.6A$

Example-3.6

A circular coil of 500 turns of wire has an enclosed area of 0.1m² per turn. It is kept perpendicular to a magnetic field of induction 0.2T and rotated by 180^0 about a diameter perpendicular to the field in 0.1 s. How much charge will pass when the coil is connected to a galvanometer with a combined resistance of 50Ω.

Solution :

$\text{Charge} = \frac{\text{change in flux}}{\text{resistance}}$

$q = \frac{\phi_i - \phi_f}{R} = \frac{NBA - (-NBA)}{R} = \frac{2NBA}{R}$

$q = \frac{2 \times 500 \times 0.2 \times 0.1}{50} = 0.4$ C.

3.7 METHODS OF PRODUCING INDUCED EMF

We know that whenever magnetic flux through a coil changes, an induced emf is set up in the coil. Following are the three methods of changing the magnetic flux through a coil and producing an induced emf.

i) Changing the area A of the loop.

ii) Changing the relative orientation of the coil with respect to the magnetic field.

iii) Changing the value of magnetic field B.

Induced emf is two types :

1) Motional emf.

2) Changing field emf.

3.8 MOTIONAL ELECTROMOTIVE FORCE

It is the induced emf developed when a moving conductor cuts the magnetic field lines. We can gain additional insight into the origin of the induced emf by considering the magnetic force (Lorentz force) on mobile charges in the conductor.

The figure shows a uniform magnetic field $\bar{B}$ directed into the plane of the paper. A rod is moved towards the right with a constant velocity $\bar{v}$. A charged particle +q in the rod then experiences a magnetic force $F = q\bar{v} \times \bar{B}$ in the upward direction, i.e., from 'b' to 'a'.

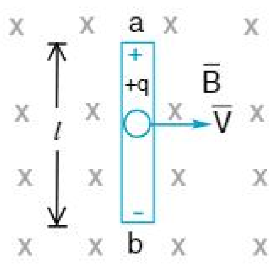


Fig 3.9

The magnetic force causes the charges in the rod to move, creating an excess of positive charge at the upper end 'a' and negative charge at the lower end 'b'. This in turn creates an electric field

$\overline{E}$ within the rod, in the direction from 'a' to 'b' (opposite to the magnetic force). Charge continues to accumulate at the ends of the rod until $\overline{E}$ becomes large enough for the downward electric force to cancel exactly the upward magnetic force (with magnitude qvB). Then qE = qvB and the charges are in equilibrium.

The magnitude of the potential difference $V_{ab} = V_a - V_b$ is equal to the electric field magnitude E multiplied by the length l of the rod. From the above discussion,

$E = vB$ and

$V_{ab} = El = vBl.$

with point 'a' at higher potential than point 'b'.

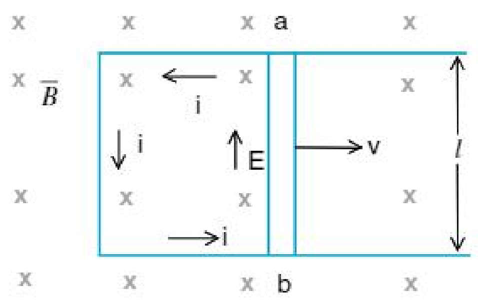


Fig 3.10

Now suppose the moving rod slides along a stationary U-shaped conductor, forming a complete circuit. No magnetic force acts on the charges in the stationary U- shaped conductor, but the charge that was near points 'a' and 'b' redistributes itself along the stationary conductors, creating an electric field within them. This field establishes a current in the direction shown.

The moving rod has become a source of emf; We call this emf a motional electromotive force, denoted by e. The magnitude of this emf is given by $e = Bvl$.

The emf associated with the moving rod is similar to that of the battery with positive terminal at 'a' and negative terminal at 'b', although the origins of the two e.m.f.s are quite different. In each case a non electrostatic force acts on the charges in the device, in the direction from 'b' to 'a' and the emf is the work per unit charge done by this force when a charge moves from 'b' to 'a' in this device. When the device is connected to an external circuit, the direction of current is from 'b' to 'a' in the device and from 'a' to 'b' in the external circuit. The direction of the induced emf can be deduced by using Lenz's law.

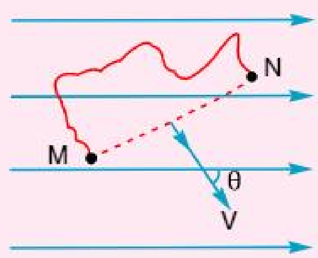
We can generalize the concept of motional emf for a conductor with any shape, moving in any magnetic field, uniform or non uniform.

For an element $\overline{dl}$ of the conductor, the contribution due to the emf is the magnitude $\overline{dl}$ multiplied by the component of $\overline{v} \times \overline{B}$ (the magnetic force per unit charge) parallel to $\overline{dl}$, that is $de = (\overline{v} \times \overline{B}).\vec{dl}$

For any conducting loop, the total emf is $e = (\overline{v} \times \overline{B}).\overline{dl}$

Note :

i) When a conductor is not straight, and if it moves with a velocity $\overline{v}$ in a field $\overline{B}$ then the conductor of irregular shape behaves, as a straight conductor $\overline{MN}$ obtained by joining the end points M and N. If the length MN = l, then induced emf is given by $e = Bvl \sin\theta$.



ii) The direction of the induced emf is given by Lenz's Law. However it is more convenient to apply Fleming's right hand rule.

3.9 FLEMING'S RIGHT HAND RULE

According to this rule, stretch the right hand such that the first finger, the central finger and the thumb are mutually perpendicular to each other. If the first finger points along the direction of magnetic field and the thumb points along the direction of the motion of the conductor, then the direction of induced emf and hence current will be along the direction of the central finger.

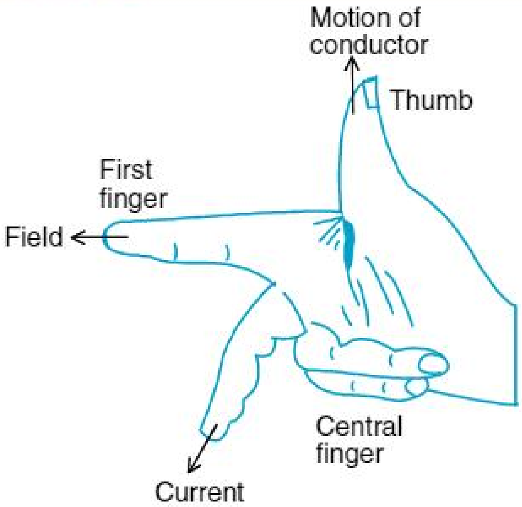
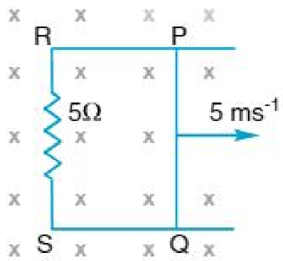


Fig 3.11

Example-3.7

Figure shows a conducting rod PQ in contact with metal rails RP and SQ, which are 0.25m apart in a uniform magnetic field of flux density 0.4T acting perpendicular to the plane of the paper. Ends R and S are connected through a 5Ω resistance. What is the emf when the rod moves to the right with a velocity of 5 ms^{-1} ? What is the magnitude and direction of the current through the 5Ω resistance? If the rod PQ moves to the left with the same speed, what will be the new current and its direction?



Solution :

$$|e| = Blv = 0.4\times0.25\times5 = 0.5V$$

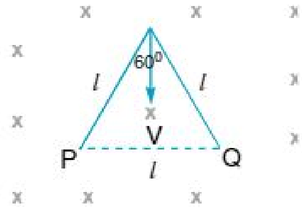
$$\text{Current, } I = \frac{|e|}{R} = \frac{0.5V}{5\Omega} = 0.1A$$

Applying Fleming's right hand rule, the current in the rod shall flow from Q to P.

If the rod moves to the left with the same speed, then the current of 0.1 A will flow from P to Q

Example-3.8

A wire of length $2l$ is bent at mid point so that the angle between the two halves is 60^0. If it moves as shown with a velocity v in a magnetic field B, find the induced emf.



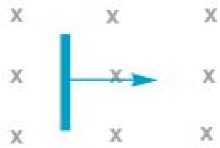
Solution :

$e = Blv.$

Here l = Effective length = PQ.

Example-3.9

A conductor of length 0.1m is moving with a velocity of 4m/s in a uniform magnetic field of 2T as shown in the figure. Find the emf induced?

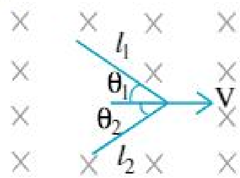


Solution :

$e = Bl\text{v} \sin 90^o = (2)(0.1)(4) = 0.8$ Volt

Example-3.10

Find the emf induced across the ends of the conductor shown in the figure.

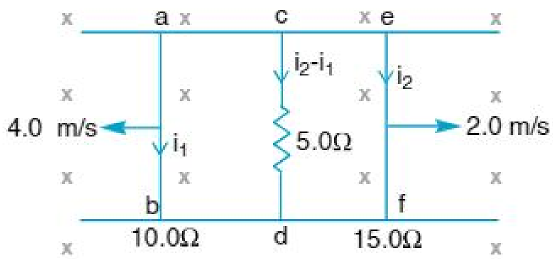


Solution :

$$e = BVl = BV(l_1\sin\theta_1 + l_2\sin\theta_2)$$

Example-3.11

Two parallel rails with negligible resistance are 10.0cm apart. They are connected by a 5.0Ω resistor. The circuit also contains two metal rods having resistances of 10.0Ω and 15.0Ω across the rails. The rods are pulled away from the resistor at constant speeds 4.00 m/s and 2.00 m/s respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5.0Ω resistor.



Solution :

In the figure

$R = 5.0\Omega, r_1 = 10\Omega, r_2 = 15\Omega,$

$e_1 = Blv_1 = 0.01\times 0.1\times 4 = 4\times 10^{-3}\,V$

$e_2 = Blv_2 = 0.01\times 0.1\times 2 = 2\times 10^{-3}\,V$

Applying Kichoff's law to the left loop :

$10i_1 + 5(i_1 - i_2) = 4\times 10^{-3}$

$\Rightarrow 15i_1 - 5i_2 = 4\times 10^{-3}$ .... (i)

Right loop : $15i_2 + 5(i_2 - i_1) = 2\times 10^{-3}$

$\Rightarrow 20i_2 - 5i_1 = 2\times 10^{-3}$ .....(ii)

Solving (i) and (ii) gives :

$i_1 = \frac{18}{55}\times 10^{-3}\,A$

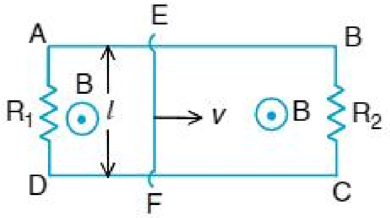
$\Rightarrow i_2 = \frac{10}{55}\times 10^{-3}\,A$

$\Rightarrow$ Current through $5\Omega = i_1 - i_2$

$= \frac{8}{55}\times 10^{-3}\,A = \frac{8}{55}\,mA$

Example-3.12

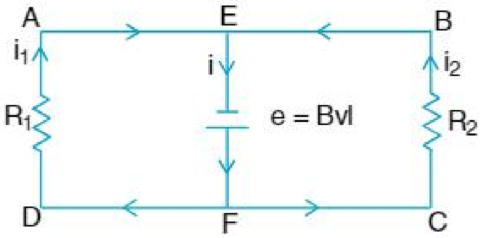
A loop ABCD containing two resistors is placed in a uniform magnetic field B directed out of the plane of the page. A sliding conductor EF of length l and of negligible resistance moves to the right with a uniform velocity v as shown in the figure. Determine the current in each branch.



Solution :

The magnetic field induction B, length l and the velocity v of the conductor EF are mutually perpendicular, hence the emf induced in it is $e = Bvl$ (from E to F)

$\therefore$ The effective electric circuit can be redrawn as shown in Fig.



Let i_1, i_2 and i be the currents through the branches DA, CB and EF respectively.

Applying Kirchhoff's law to the junction E or F, we have

$i_1 + i_2 = i$ ..... (i)

Applying Kirchhoff's loop rule to the loops DAEFD and EBCFE respectively, we have

$i_1R_1 - Bv\ell = 0$ or, $i_1R_1 = Bv\ell$ ..... (ii)

and $-i_2R_2 + Bv\ell = 0$ or, $i_2R_2 = Bv\ell$ ..... (iii)

From Eq. (ii) and (iii)

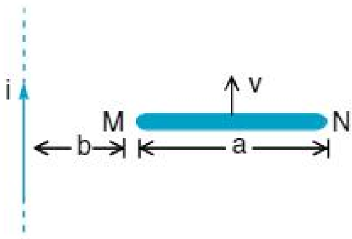
$i_1 = \frac{Bv\ell}{R_1}$ and $i_2 = \frac{Bv\ell}{R_2}$

Finally from Eqn. (i),

$i = i_1 + i_2 = Bv\ell\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

Example-3.13

A conducting rod MN moves with a speed v parallel to a long straight wire which carries a constant current i, as shown in Fig. The length of the rod is normal to the wire. Find the emf induced in the total length of the rod. State which end will be at a lower potential.

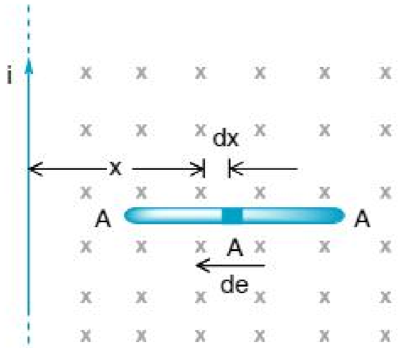


Solution :

The magnetic field induction due to current i is different at different sections of the rod, because they are at different distances from the wire.

Let us, first of all, subdivide the entire length of the conductor MN into elementary sections. Consider a section (shown shaded in the Fig.) of thickness dx at a distance x from the wire. As all the three, v, B and (dX) are mutually perpendicular to each other, the emf induced in it is de = Bvdx.

(from N to M by Fleming's right hand rule)



For the rest of sections, the induced emf is in the same sense, (i.e, from N to M)

$\therefore$ Total emf induced in the conductor is

$$e = \int de = \int_b^{b+a} Bv\,dx$$

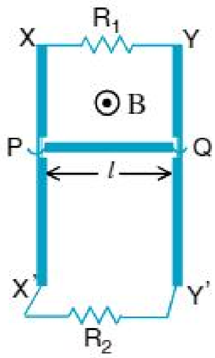
Substituting for $B = \frac{\mu_0 i}{2\pi x}$, the above equation gets changed to

$$e = \int_b^{b+a} \frac{\mu_0\, iv\, dx}{2\pi x}$$

$$e = \frac{\mu_0 iv}{2\pi}[\ln x]_b^{b+a} \quad \text{or,} \quad e = \frac{\mu_0 iv}{2\pi}\ln(1 + a/b)$$

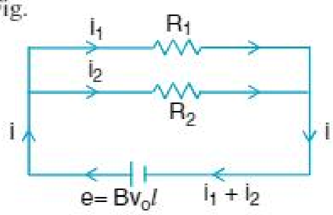
Example-3.14

Two parallel vertical metallic bars XX' and YY', of negligible resistance are separated by a length 'l', as shown in Fig. The ends of the bars are joined by resistances R_1 and R_2. A uniform magnetic field of induction B exists in space normal to the plane of the bars. A horizontal metallic rod PQ of mass m starts falling vertically, making contact with the bars. It is observed that in the steady state the powers dissipated in the resistances R_1 and R_2 are P_1 and P_2 respectively. Find an expression for R_1, R_2 and the terminal velocity attained by the rod PQ.



Solution :

Let v_0 be the terminal velocity attained by the rod PQ (in the steady state). If i_1 and i_2 be the currents flowing through R_1 and R_2 in this state, then current flowing through the rod PQ is $i=i_1+i_2$ (see the circuit diagram) shown in Fig.



$\therefore$ Applying Kirchhoff's loop rule, yields.

$i_1R_1 = Bv_0\ell$ and $i_2R_2 = Bv_0\ell$

$$\therefore i_1 + i_2 = Bv_0\ell\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{.... (i)}$$

Given that, $P_1 = i_1^2R_1 = \frac{B^2v_0^2\ell^2}{R_1}$ .... (ii) and

$$P_2 = i_2^2R_2 = \frac{B^2v_0^2\ell^2}{R_2} \quad \text{...... (iii)}$$

Also in the steady state, the acceleration of PQ = 0.

$mg = B\,(i_1+i_2)l$

$$mg = B^2\ell^2v_0\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{...... (iv)}$$

Multiplying both sides by v_0

$$mgv_0 = B^2\ell^2v_0^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = P_1 + P_2.$$

[From Eq. (ii) and (iii)]

$\therefore$ The terminal velocity is $v_0 = \frac{P_1 + P_2}{mg}$

Substituting for v_0 in Eq. (ii),

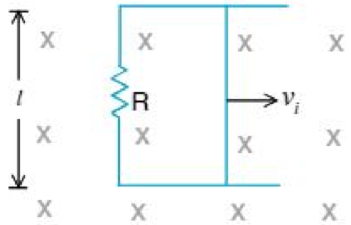
$$P_1 = \frac{B^2\ell^2}{R_1}\left(\frac{P_1 + P_2}{mg}\right)^2 ; \quad R_1 = \left[\frac{B\ell(P_1 + P_2)}{mg}\right]^2 \times \frac{1}{P_1}$$

Similarly from Eq. (iii).

$$R_2 = \left[\frac{B\ell(P_1 + P_2)}{mg}\right]^2 \times \frac{1}{P_2}$$

Example-3.15

A bar of mass m and length l moves on two frictionless parallel rails in the presence of a uniform magnetic field directed into the plane of the paper. The bar is given an initial velocity v_i to the right and released. Find the velocity of bar, induced emf across the bar and the current in the circuit as functions of time.



Solution :

The induced current is in the counter clock-wise direction and the magnetic force on the bar is given by $F_B = -ilB$.

The negative sign indicates that the force is towards the left and retards motion.

$F = ma$

$$-ilB = m.\frac{dv}{dt}$$

Because the force depends on current and the current depends on the speed, the force is not constant and the acceleration of the bar is not constant. The induced current is given by

$$i = \frac{Blv}{R} \text{ and } -ilB = m.\frac{dv}{dt}$$

$$\Rightarrow -\left(\frac{Blv}{R}\right)lB = m.\frac{dv}{dt} \Rightarrow \frac{dv}{v} = -\frac{B^2l^2}{mR}dt$$

$$\int_{v_i}^{v}\frac{dv}{v} = -\frac{B^2l^2}{mR}\int_0^t dt$$

$$\ln\left(\frac{v}{v_i}\right) = -\frac{B^2l^2}{mR}t = \frac{-t}{T} \text{ where } T = \frac{mR}{B^2l^2}$$

$$v = v_i e^{-\frac{t}{T}}.$$

The speed of the bar therefore decreases exponentially with time under the action of magnetic retarding force.

$$\text{emf} = iR = Blv_i e^{-\frac{t}{T}}$$

$$\text{current : } i = \frac{Blv}{R} = \frac{Bl}{R}v_i e^{-\frac{t}{T}}$$

3.10 MOTIONAL EMF INDUCED IN A ROTATING BAR

A conducting bar of length l rotates with a constant angular speed 'ω' about a pivot at one end. A uniform magnetic field B is directed perpendicular to the plane of rotation as shown in the figure.

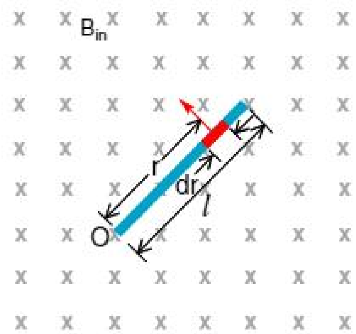


Fig 3.12

Consider a segment of the bar of length dr, whose velocity is v. The emf induced in a conductor of length dr moving perpendicular to the field B is given by. $dE = Bvdr$.

Each segment of the bar is moving perpendicular to B, so an emf is generated across each segment, the value of which is given by the above equation. Summing up the e.m.f.s induced across all the elements, which are in series, gives the total emf between the ends of the bar.

$$e = \int_0^l Bvdr = \int_0^l B(r\omega)dr$$

$$= B\omega\int_0^l rdr \Rightarrow e = \frac{1}{2}B\omega l^2$$

From Fleming's right hand rule we see that 'P' is at lower potential and 'O' is at higher potential.

Method-2

For a second approach consider that at any instant the flux enclosed by the sector aob is given by

$$\phi = BA = B\left(\frac{1}{2}l^2\theta\right) = \frac{1}{2}Bl^2\theta$$

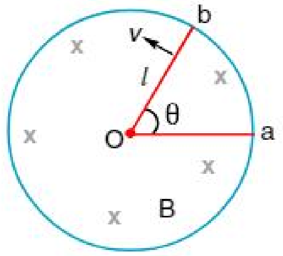


Fig 3.13

$$|e| = \frac{d\phi}{dt} = \frac{1}{2}Bl^2.\frac{d\theta}{dt} = \frac{1}{2}Bl^2\omega\left(\because \omega = \frac{d\theta}{dt}\right).$$

Example-3.16

A copper rod of length 2m is rotated with a speed of 10 rps in a uniform magnetic field of 1 tesla about a pivot at one end. The magnetic field is perpendicular to the plane of rotation. Find the emf induced across its ends.

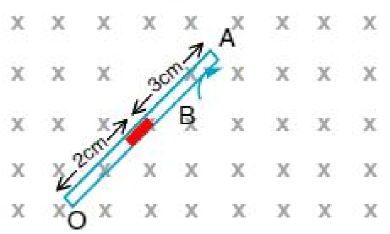
Solution :

$$e = \frac{1}{2}B\omega l^2 = \frac{1}{2}B(2\pi n)l^2 = \pi B n l^2$$

$$e = 3.14 \times 1 \times 10 \times 2 \times 2 = 125.6 \text{ volt}$$

Example-3.17

A rod of length 10 cm is made up of conducting and non-conducting material (shaded part is non-conductiing). The rod is rotated with constant angular velocity 10 rad/s about point O, in a constant magnetic field of 2T as shown in the figure. Find induced emf between the points A and B ?



Solution :

$$e_{AB} = \int_{0.07}^{0.1}(2)(10r)\,dr = 0.051\text{V}$$

3.11 MOTIONAL EMF INDUCED IN A ROTATING DISC

Faraday disc dynamo or homopolar generator

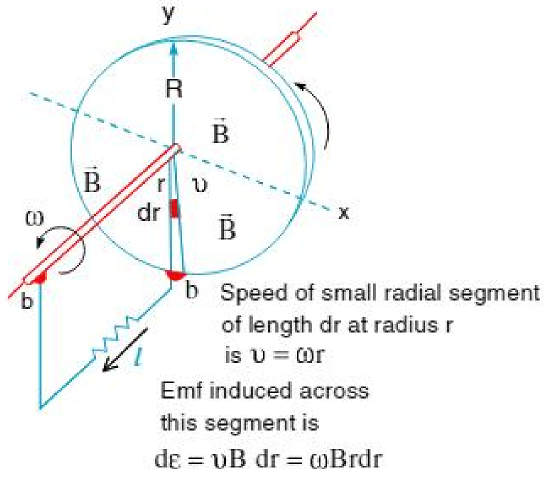


Fig 3.14

A conducting disc with radius R lies in the XY plane and rotates with a constant angular velocity ω about the z-axis. The disc is in a uniform constant $\overline{B}$ field parallel to z-axis. A motional emf is present because the conducting disc moves relative to the $\overline{B}$ field. The complication is that different parts of disc move at different speeds v, depending on their distance from the rotation axis. We determine the emf between the centre and the rim by considering small segments of the disc and add their contribution to determine the emf.

Consider a small segment of the disc labelled by its velocity vector $\overline{v}$. The magnetic force per unit charge on this segment is $\overline{v} \times \overline{B}$,which points radially outward from the centre. Hence, the induced emf tends to make a current flow radially outward, which tells, us that the moving conducting path to be considered here is a straight line from the centre to the rim. We can find the emf from each small disc segment along this line using the expression $de = (\overline{v} \times \overline{B}).\overline{dl}$, then integrate to find the total emf.

$$de = (\overline{v} \times \overline{B}).\overline{dr} \;;\; \int de = \omega B\int_0^R r\,dr \;\; (\because v = r\omega)$$

$$e = \frac{1}{2}B\omega R^2$$

We can use this device as a source of emf in a circuit by completing the circuit through stationary brushes (b in the figure) that contact the disc and its conducting shaft as shown. The emf in such a disc was studied by Faraday.

Hence the device is called a Faraday disc dynamo or a homopolar generator. Faraday disc dynamo is a direct current generator. It produces an emf that is constant in time.

Example-3.18

A copper disc of radius 1m is rotated about its natural axis with an angular velocity 2 rad/sec in a uniform magnetic field of 5 tesla with its plane perpendicular to the field. Find the emf induced between the centre of the disc and its rim.

Solution :

$$e=\frac{1}{2}B\omega r^2 \;;\quad e=\frac{1}{2}\times5\times2\times1\times1=5\,\text{volt}$$

Example-3.19

A wheel with 10 metallic spokes, each 0.50 m long, is rotated with a speed of 120 rev/minute in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 0.40 gauss, what is the induced emf between the axle and the rim of the wheel ?

Solution :

$f = 120$ rev/min $= 2$ rev/second,

$B = 0.40$ gauss $= 0.4\times10^{-4}$ T,

Area swept, by each spoke per second, $A=\pi r^2 f$

Magnetic flux cut by each spoke per second,

$$\frac{d\phi_B}{dt}=BA=B\pi r^2 f$$

Induced emf, $e=B\pi r^2 f$ (numerically)

$$e=0.4\times10^{-4}\times\frac{22}{7}\times0.5\times0.5\times2$$

$$e=6.29\times10^{-5}\,\text{volt}$$

Note : $e=\frac{1}{2}Br^2\omega$ in each spoke

- The spokes are connected with their one end at the rim and the other end at the axle. So the spokes are connected in parallel. Thus, the emf developed across the ten spokes would be the same as the emf developed across a single spoke.

3.12 MOTION OF RECTANGULAR LOOP IN MAGNETIC FIELD

An external agent pulls the loop to the right at constant speed v by exerting a force F. The uniform magnetic field B is perpendicular to the plane of the loop and directed inwards.

Let us calculate the mechanical power expended by the external agent.

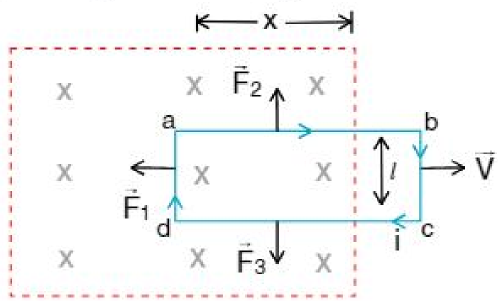


Fig 3.15

Flux enclosed by the loop at any instant=$Bl(x)$

Here lx is the area of that part of the loop in which B is not zero.

$$\text{Induced emf, } e=-\frac{d\phi_B}{dt}$$

$$e=\frac{-d}{dt}(Blx)=Bl\left(-\frac{dx}{dt}\right)=Blv$$

Since x is decreasing, therefore, we have set $\left(-\frac{dx}{dt}\right)$ equal to the speed v at which the loop is pulled out of the magnetic field.

$$\text{Induced current, } i=\frac{e}{R}=\frac{Blv}{R}$$

where R is the resistance of the loop. From Lenz's law, the direction of the induced current must be clockwise. It opposes the decrease in magnetic flux by setting up a field that is parallel to the external field within the loop.

The induced current in the loop gives rise to magnetic forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ that act on the three conductors according to the equation: $\vec{F}_1=i(\vec{l}\times\vec{B})$

Since $\vec{F}_2$ and $\vec{F}_3$ are equal and opposite, therefore, they cancel each other.

The force $\vec{F}_1$ opposes the effort of the external agent to move the loop.

$$F_l = Bil \sin 90^0$$

$$\text{or} \quad F_l = Bil = B\left(\frac{Blv}{R}\right)l \quad \text{or} \quad F_l = \frac{B^2l^2v}{R}$$

The loop shall move with a uniform velocity $\vec{\upsilon}$ only if the external agent that pulls the loop exerts a force F equal in magnitude to F_l.

Power of agent, $P = F_l V$

$$\text{or} \quad P = \frac{B^2l^2V^2}{R}$$

This gives us the rate at which work is done by the external agent. Let us now calculate the rate at which energy is dissipated in the loop as a result of Joule heating by the induced current.

$$P = i^2R = \left(\frac{BlV}{R}\right)^2 R = \frac{B^2l^2V^2}{R}$$

This result agrees precisely with the rate at which mechanical work is done on the loop. The work done by the external agent is eventually dissipated as Joule heating in the loop.

3.13 INDUCED emf BY CHANGING THE RELATIVE ORIENTATION OF THE COIL AND THE FIELD (PRINCIPLE OF AC GENERATOR)

Consider a coil of area A and having N turns placed in a magnetic field. If the normal to the plane of the coil makes an angle θ with the direction of the magnetic field, then the magnetic flux linked with the coil at any instant is given by

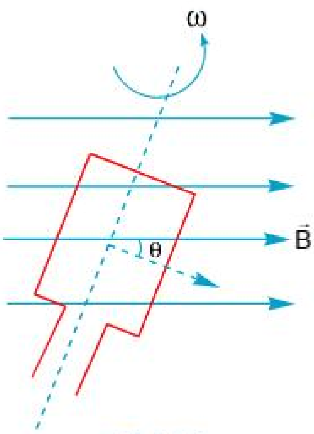


Fig 3.16

$$\phi_B = NBA \cos\theta$$

As the coil rotates about an axis perpendicular to the magnetic field, it keeps on changing its relative orientation with respect to the field. Thus, the flux ϕ_B will keep on changing continuously with time. This will cause an induced emf in the coil. The direction of the emf will reverse after every half rotation of the coil. If the outer terminals of the coil are connected to an outside circuit, an electric current will flow in the circuit which will reverse its direction in a time interval of T/2, where T is the time taken by the coil to complete one rotation.

Let ω be the angular velocity of the coil. Let us measure time from the instant when the coil is perpendicular to the field i.e. the normal to the plane is along the magnetic field so that $\theta = 0^0$.

Then, at time t, $\theta = \omega t$.

$$\therefore \phi_B = NBA \cos \omega t$$

Differentiating w.r.t. 't', we get

$$\frac{d\phi_B}{dt} = -NBA\ \omega \sin\omega t$$

Induced emf,

$$e = -\frac{d\phi_B}{dt} = -(-NBA\ \omega \sin\omega t)$$

$$\text{or} \quad e = NBA\ \omega \sin\omega t \qquad \text{..... (1)}$$

So, the induced emf is alternating in nature.

The value of e is maximum i.e., e_0 if the value of $\sin\omega t$ is maximum i.e., 1. This will happen when the plane of the coil is parallel to the magnetic field i.e., when $\omega t = 90^0$

$$\therefore e_0 = NBA\ \omega$$

e_0 is called the amplitude or peak value of emf. It depends upon (i) strength of the magnetic field, (ii) area of the coil, (iii) speed of rotation, and (iv) the number of turns of the coil.

Now, from equation (1), $e = e_0 \sin \omega t$

If f be the frequency of rotation, then

$$e = e_0 \sin 2\pi ft \qquad \text{..... (2)}$$

If we plot a graph between e and ωt, the graph is a sine curve as shown in Fig.

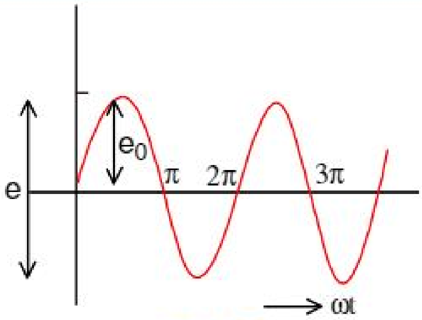


Fig 3.17

The emf represented by equation (2) is called sinusoidal emf or alternating emf. This emf appears at the outer terminals of the rotating coil and can be transferred to the external circuit by suitable means.

If we count time from that instant when the coil is parallel to the magnetic field i.e., at t = 0

$\theta = \dfrac{\pi}{2}$, then at time t, $\phi_B = NBA\cos\left(\omega t + \dfrac{\pi}{2}\right)$

This would lead to the following equation of alternating emf: $e = e_0 \cos \omega t$

Example-3.20

Chaitanya pedals a stationary bicycle at one revolution per second. The pedals are attached to 100 turns coil of area 0.1m^2 and placed in a uniform magnetic field of 0.1 T. What is the maximum voltage generated in the coil ?

Solution :

$E_0 = NBA\omega = NBA(2\pi v)$

$E_0 = 100 \times 0.1 \times 0.1(2 \times 3.14 \times 1)V = 6.28V$

Example-3.21

A coil of 800 turns and 50 cm^2 area makes 10 rps about an axis in its own plane in a magnetic field of 100 gauss perpendicular to this axis. What is the instantaneous induced emf in the coil ?

Solution :

$A = 50\text{cm}^2 = 50 \times 10^{-4}\text{m}^2$

n = 10 rps, N = 800

$B = 100\text{gauss} = 100 \times 10^{-4}T = 10^{-2}T$

Now, $E = E_0 \sin\omega t = NBA\omega\sin\omega t$

$= 800 \times 10^{-2} \times 50 \times 10^{-4} \times 2\pi \times 10\sin(20\pi t)$

or $E = 2.5\sin(20\pi t)$ volt

3.14 INDUCED ELECTRIC FIELDS OR INDUCED EMF DUE TO A CHANGING MAGNETIC FIELD

A changing magnetic flux induces an emf and a current in a conducting loop. The normal flow of charges in a circuit is due to an electric field in the wires set up by a source such as a battery. We can interpret the changing magnetic field as creating an induced electric field.

The electric field applies a force on the charges to cause them to move. With this approach, then, we see that an electric field is created in the conductor as a result of changing magnetic field. In fact the law of electromagnetic induction can be interpreted as follows. An electric field is always generated by a changing magnetic field, even in free space where no charges are present.

This induced electric field, however has quite different properties from those of electrostatic field produced by a stationary charge. Consider a conducting loop of radius R, situated in a uniform magnetic field $\overline{B}$ that is perpendicular to the plane of the loop as shown in the figure.

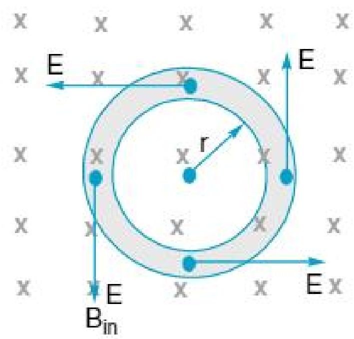


Fig 3.18

If the magnetic field changes with time, then an emf $e = \dfrac{-d\phi}{dt}$ is induced in the loop. The induced current thus produced implies the presence of an induced electric field E that must be tangential to the loop in order to provide an electric force on the charge around the loop.

The work done by the electric field on the loop in moving a test charge q once around the loop = qe. Because the magnitude of electric force on the charge is qE, the work done by the electric field can also be expressed as $qE(2\pi r)$, where $2\pi r$ is the circumference of the loop. These two expressions for the work must be equal; therefore, we see that

$$qe = qE(2\pi r)$$

$$E = \frac{e}{2\pi r}$$

Using this result along with Faraday's law and the fact that $\Phi_B = BA = B\pi r^2$ for a circular loop, the induced electric field can be expressed as

$$E = \frac{1}{2\pi r}\left(-\frac{d\Phi_B}{dt}\right) = -\frac{1}{2\pi r}\frac{d}{dt}(B\pi r^2) = -\frac{r}{2}\frac{dB}{dt}$$

The expression can be used to calculate the induced electric field if the time variation of the magnetic filed is specified. The negative sign indicates that the induced electric field E results in a current that opposes the change in the magnetic field.

In general, the emf for any closed path can be expressed as the line integral of $E.d\vec{l}$ over that path. Hence, the general form of Faraday's law of induction is

$$e = \oint E.d\vec{l} = \frac{d\Phi_B}{dt}$$

It is important to recognize that **the induced electric field E that appears in the equation is a nonconservative field that is generated by a changing magnetic field.**

(a) It is a non-conservative field in the sense that work done in circulating a unit positive charge round the loop is given by W = e, where e = emf induced in the loop, rather than zero, as in case of a conservative field produced by static electric charges.

(b) Since the electric field due to a changing magnetic field is non-conservative, the concept of potential does not apply to it.

- Electric field is necessarily established due to a time varying magnetic field even in the absence of any material object. Such a field is known as vortex field.

Example-3.22

A magnetic field directed into the page changes with time according to the expression $B = (0.03t^2 + 1.4)T$, where t is in seconds. The field has a circular cross-section of radius R = 2.5cm. What is the magnitude and direction of electric field at P, when t = 3.0s and r = 0.02 m.

Solution :

$$e = \oint E.dl = \frac{+d\phi}{dt}$$

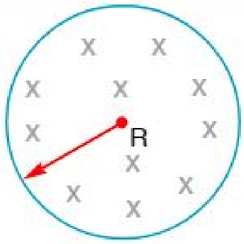
$$E(2\pi r) = A.\frac{dB}{dt} = \pi r^2 \times \frac{d}{dt}(0.03t^2 + 1.4)$$

$$E = \frac{\pi r^2}{2\pi r} \times (0.06t) = \frac{r}{2}(0.06t)$$

$$|E| = \frac{0.02}{2} \times 0.06 \times 3 = 18 \times 10^{-4} N/columns$$

Example-3.23

The magnetic field at all points within the cylindrical region whose cross-section is indicated in the accompanying figure starts increasing at a constant rate 'α'. Find the magnitude of electric field as a function of r, the distance from the geometric centre of the region.



Solution :

Case-1 : For r < R

$$E.2\pi r = -A\frac{dB}{dt}$$

$$E.2\pi r = -\pi r^2 \frac{dB}{dt}$$

$$E = -\frac{r}{2}\frac{dB}{dt} = -\frac{r}{2}\alpha$$

$$E \alpha r$$

Case -2 : $r = R$

$$E.2\pi R = -\pi R^2 \frac{dB}{dt}$$

$$E = -\frac{R}{2}\frac{dB}{dt}$$

$$E = -\frac{R\alpha}{2}$$

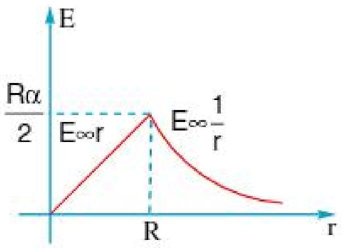
Case - 3 : $r > R$

$$E.2\pi r = -\pi R^2 \frac{dB}{dt}$$

$$E = -\frac{R^2}{2r}\frac{dB}{dt}$$

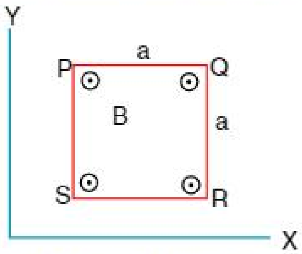
$$E = -\frac{R^2}{2r}\alpha$$

$$E_{out}\alpha \frac{1}{r}$$



Example-3.24

A wire is bent in the form of a square of side a in a varying magnetic field $\overline{B} = \propto B_0 t\hat{k}$. If the resistance per unit length is λ, then find the following.



i) The direction of induced current

ii) The current in the loop

iii) Potential difference between P and Q

Solution :

i) Direction of current is clockwise.

ii) $|E| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = a^2\frac{d}{dt}(\propto B_0.t) = a^2 \propto B_0$

Current : $i = \frac{E}{R} = \frac{a^2 \propto B_0}{4a\lambda} = \frac{a \propto B_0}{4\lambda}$

iii) $V_p + \frac{E}{4} - i.a\lambda = V_Q$

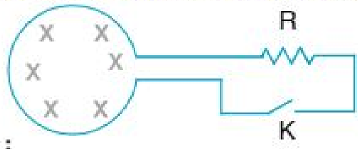
$$V_P - V_Q = ia\lambda - \frac{E}{4}$$

$$V_P - V_Q = \frac{E}{4a\lambda}.a\lambda - \frac{E}{4}$$

$$V_P - V_Q = \frac{E}{4} - \frac{E}{4} = 0$$

Example-3.25

Shown in the figure is a circular loop of radius r connected to a resistance R. A variable magnetic field of induction $B = e^{-1}$ is established inside the coil. If the key (K) is closed, find the electric power developed



Solution :

$$E = \frac{-d\phi}{dt} = -A.\frac{dB}{dt}$$

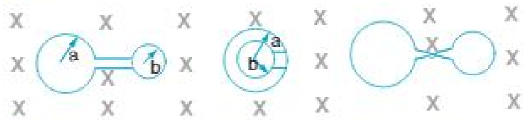
$$E = -\pi r^2\frac{d}{dt}(e^{-1});\ E = \pi r^2\ e^{-t}$$

$$P = \frac{E^2}{R} = \frac{\pi^2 r^4 e^{-2t}}{R}$$

Note : At $t = 0$; $P = \frac{\pi^2 r^4}{R}$

Example-3.26

The figure shows two circular rings of radii a and b (a>b) made of a wire whose resistance is λ per unit length.



All the three arrangements are placed in a uniform time varying magnetic field $\frac{dB}{dt} = k$, perpendicular to the plane of the loops. Determine the current induced in each case.

Solution :

i) $\phi = \pi(a^2 + b^2)B$

$$|E| = \frac{d\phi}{dt} = \pi(a^2 + b^2)\frac{dB}{dt} = \pi(a^2 + b^2)k$$

$$i = \frac{E}{R} = \frac{\pi k(a^2 + b^2)}{2\pi\lambda(a + b)} = \frac{k(a^2 + b^2)}{2\lambda(a + b)}$$

ii) $\phi = \pi(a^2 - b^2)B$

$$|E| = \frac{d\phi}{dt} = \pi(a^2 - b^2)\frac{dB}{dt} = \pi(a^2 - b^2)k$$

$$i = \frac{E}{R} = \frac{\pi(a^2 - b^2)k}{2\pi\lambda(a + b)} = \frac{(a - b)k}{2\lambda}$$

iii) $\phi = \pi(a^2 - b^2)B$

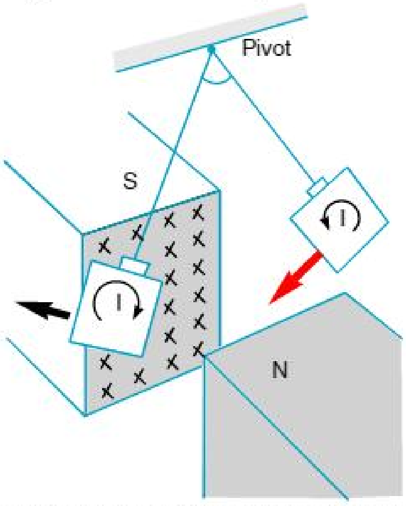
$$|E| = \frac{d\phi}{dt} = \pi(a^2 - b^2)\frac{dB}{dt} = \pi(a^2 - b^2)k$$

$$i = \frac{E}{R} = \frac{\pi(a^2 - b^2)k}{2\pi\lambda(a + b)} = \frac{(a - b)k}{2\lambda}$$

3.15 EDDY CURRENTS

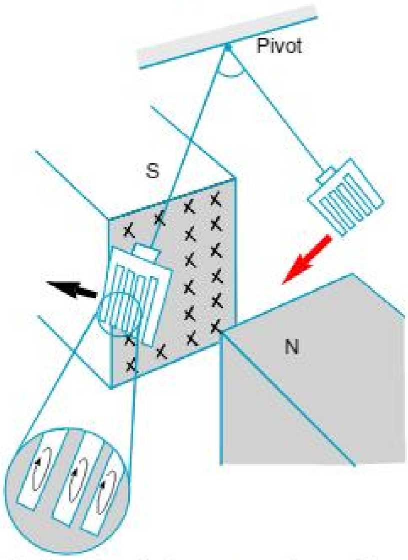
When bulk pieces of conductors are subjected to changing magnetic flux, induced currents are produced in them. The flow patterns of induced currents resemble the swirling eddies in water. This effect was discovered by Foucault and these currents are called *eddy currents (or) Foucault currents*.

A copper plate is allowed to swing like a simple pendulum between the pole pieces of a strong magnet as shown in figure.



Eddy currents are generated in the copper plate while entering and leaving the region of magnetic field

Fig 3.19



Cutting slots in the copper plate reduces the effect of eddy currents

Fig 3.20

It is found that the motion is damped and the plate comes to rest in the magnetic field. This phenomenon can be explained on the basis of electromagnetic induction. As the plate moves in and out of the region between magnetic poles, the magnetic flux associated with the plate keeps on changing and the flux change induces eddy currents in the plate.

The directions of eddy currents are opposite when the plate swings into the region between the poles and when it swings out of the region.

If rectangular slots are made in the copper plate as shown in figure, area available to the flow of eddy currents is less. Thus, the pendulum plate with holes or slots reduces electro magnetic damping and the plate swings more freely.

We know that the magnetic moments of the induced currents (which oppose the motion) depend upon area enclosed by the currents (M=IA) Eddy currents heat up the metallic cores and dissipate electrical energy in the form of heat in the devices like transformers, electric motors and other such devices.

Eddy currents are minimized by using laminations of metal to make metal core. Since the dissipation of electrical energy into heat depends on the square of the strength of electric current, heat loss is substantially reduced.

3.16 MINIMISATION OF LOSSES DUE TO EDDY CURRENTS

Metallic cores are used in electrical devices like transformer, dynamo, choke etc. Due to changing magnetic field, large eddy currents are produced in the core which cause large amount of heat in the core. It results into loss of useful energy.

When bulk pieces of conductors are subjected to changing magnetic flux, induced currents are produced in them. The flow patterns of induced currents resemble the swirling eddies in water. This effect was discovered by Foucault and these currents are called *eddy currents (or) Foucault currents*.

A copper plate is allowed to swing like a simple pendulum between the pole pieces of a strong magnet as shown in figure.

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The directions of eddy currents are opposite when the plate swings into the region between the poles and when it swings out of the region.

If rectangular slots are made in the copper plate area available to the flow of eddy currents is less. So, electromagnetic damping is reduced and the plate swings more freely.

We know that the magnetic moments of the induced currents (which oppose the motion) depend upon area enclosed by the currents (M=IA) Eddy currents heat up the metallic cores and dissipate electrical energy in the form of heat in the devices like transformers, electric motors and other such devices.

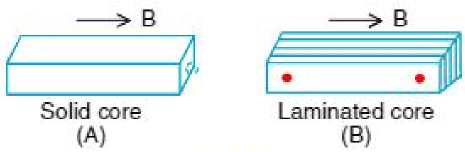


Fig 3.21

To minimise losses due to eddy currents, the solid metallic core (fig.A) is replaced with a large number of thin sheets (fig.B). These sheets are electrically insulated from one another and are called as laminations. Such a core is called laminated core. These sheets are arranged parallel to the magnetic flux. The insulation breaks the paths of the eddy currents and keeps the eddy currents confined to the individual sheets. As a result of this, eddy current produced in one sheet is not added to the current produced in the other sheet. In other words, eddy currents in the core as a whole are reduced to a large extent.

Induction Furnace : Induction furnace is based on the heating effect of eddy currents. A metallic block to be melted is placed in a high frequency changing magnetic field. Strong eddy currents are induced in the block. Due to the high resistance of the metal, a large amount of heat is produced in it. This heat ultimately melts the metallic block. The induction furnace is used to separate metals from their ores and to make some alloys.

1) ***Electric power meters*** **:** The shiny metal disc in the electric power meter (analogue type) rotates due to the eddy currents. Electric currents are induced in the disc by magnetic fields produced by sinusoidally varying currents in a coil. We can observe the rotating shiny disc in the power meter of your house.
2) ***Magnetic braking in trains*** **:** Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train and the braking effect is smooth.
3) ***Electromagnetic damping*** **:** Certain galvanometers have a fixed core made of nonmagnetic metallic material. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.

Disadvantages/Undesirable effects of eddy currents

1. The production of eddy currents in a metal block leads to the loss of electrical energy in the form of heat.
2. The heat produced due to eddy currents break the insulation used in the electric machine (or) appliance.
3. Eddy currents cause the damping effect.

3.17 SELF INDUCTION

Self induction is the property of a coil by virtue of which it opposes the growth or decay of the current flowing through it.

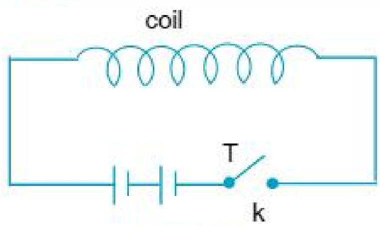


Fig 3.22

Consider a coil connected to a battery through a key (K). When the key is closed, due to increasing current in the coil, the magnetic field and hence flux linkage around the coil also increases. As a result of this, induced emf is set up in the coil. According to Lenz's law, the direction of induced emf is such that it opposes the growth of current in the coil. This delays the current to acquire the maximum value.

When the key (K) is released, the current in the coil starts decreasing. So the magnetic flux linked with the coil decreases. As a result of this change in magnetic flux, induced emf is set up in the coil itself.

According to Lenz's law, the direction of induced emf is such that it opposes the decay of current in the coil. This delays the current to acquire minimum (or) zero value.

This property of the coil which opposes the growth (or) decay of the current is called self induction.

Self induction is also known as Inertia of electricity as it opposes the growth or decay of the current in the circuit.

3.18 COEFFICIENT OF SELF INDUCTION (OR) SELF INDUCTANCE

Let I be the current flowing through a coil. Then a magnetic field will be set up in the coil.

The magnetic flux $(N\phi)$ linked with the coil is found to be proportional to the strength of the current (I)

i.e., $N\phi \propto I \Rightarrow N\phi = LI$

where 'L' is the constant of proportionality and is known as co-efficient of self induction (or) self inductance.

If $I = 1A$, $N\phi = L$

Thus co-efficient of self induction of a coil is defined as the magnetic flux linked with a coil through which a unit current flows.

Also, according to Faraday's law of electromagnetic induction, induced emf in the coil is given by

$$e = \frac{-dN\phi}{dt} \Rightarrow e = \frac{-d(LI)}{dt}$$

$$e = -L\frac{dI}{dt}$$

$$\left(\therefore e = -L\frac{dI}{dt}\right)$$

If $-\frac{dI}{dt} = 1$ i.e., if rate of decreases of current is unity, we get $L = e$.

Thus, coefficient of self induction of a coil is defined as the induced emf produced in the coil through which the rate of decrease of current is unity.

3.19 SELF INDUCTANCE OF A SOLENOID

Consider a long solenoid of length l, area of cross section A and number of turns per unit length n and length is very large when compared with radius of cross section.

Let I be the current flowing through the solenoid. The magnetic field inside the long solenoid is uniform and is given by $B = \mu_0 nI$

Total number of turns in the solenoid of length l, $N = nl$.

Now, the magnetic flux linked with each turns of the solenoid $= B \times A = \mu_0 nIA$

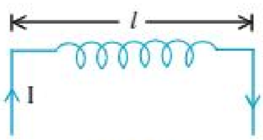


Fig 3.23

$\therefore$ Total magnetic flux linked with the whole solenoid, ϕ = magnetic flux with each turn x number of turns in the solenoid.

$$\phi = \mu_0 nIA \times n\ell = \mu_0 n^2 IA\ell \quad ..... (1)$$

$$\phi = LI \quad ..... (2)$$

$$LI = \mu_0 n^2 IA\ell \text{ from (1) \& (2)}$$

$$\therefore L = \mu_0 n^2 A\ell$$

Since $n = \frac{N}{\ell}, L = \mu_0 \frac{N^2}{\ell} A$

Thus self inductance (L) depends on

i) the number of turns (N) of the solenoid,

ii) the length (ℓ) of the solenoid,

iii) the area of cross-section (A) of the solenoid,

iv) nature of the material of the core of the solenoid.

3.20 SELF INDUCTANCE OF A COIL

Let us consider a circular coil of radius r and containing N-turns. Suppose it carries a current i. The magnetic field due to this current

$$B = \frac{\mu_0 Ni}{2r}$$

And total flux $N\phi_B = NBA$

$$= N\left(\frac{\mu_0 Ni}{2r}\right)\pi r^2 \text{ (or)} = \frac{\mu_0 \pi N^2 ri}{2}$$

Now compare with $N\phi_B = Li$, we get

$$L = \frac{\mu_0 \pi N^2 r}{2}$$

Note : Inductance may be viewed as electrical inertia. It is analogous to inertial in mechanics. It does not oppose the current, but it opposes the change in current.

3.21 ENERGY STORED IN AN INDUCTOR

Consider an inductor of inductance 'L' connected with a battery.

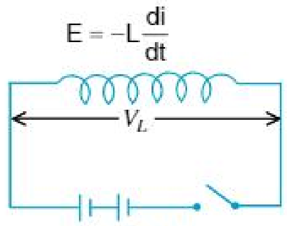


Fig 3.24

When current flows through the inductor, an induced emf is set up in the inductor. This induced emf is given by

$$e = -L\frac{dI}{dt}$$

–ve sign shows that 'E' opposes the change of current I in the inductor.

To drive the current through the inductor against the induced emf 'E', the external voltage is applied. Here external voltage is emf of the battery = E

According to KVL, E + E = 0

E = –e

$$E = L\frac{dI}{dt}$$

Let an infinitesimal charge dq be driven through the inductor. So, the workdone by the external voltage is given by $dW = Edq$

$$dW = L\frac{dI}{dt}dq = LdI\left(\frac{dq}{dt}\right)$$

but $\frac{dq}{dt} = I$

$$\therefore dW = LIdI$$

Total workdone before attaining a current (I) through the inductor is given by

$$W = \int dW = \int_0^I LI\,dI$$

$$W = L\left(\frac{I^2}{2}\right)_0^I = L\left(\frac{I_0^2}{2}\right)$$

$$\Rightarrow W = \frac{1}{2}LI^2$$

The workdone in increasing the current flowing through the inductor is stored as the potential energy (U) in its magnetic field. Hence energy stored in the inductor is given by

$$U = \frac{1}{2}LI^2$$

Example-3.27

The self-inductance of a coil having 200 turns is 10 millihenry. Calculate the magnetic flux through the cross-section of the coil corresponding to current of 4 milliampere. Also determine the total flux linked with the coil.

Solution :

Total magnetic flux linked with the coil,

$\phi = LI = 10^{-2} H \times 4 \times 10^{-3} A$

$\phi = 4 \times 10^{-5} Wb$

Magnetic flux through the cross-section $= \frac{\phi}{200}$

$= 2 \times 10^{-7} Wb$

Example-3.28

A coil of inductance 0.2 henry is connected to 600 volt battery. At what rate will the current in the coil grow when circuit is completed?

Solution :

As the battery and inductor are in parallel, at any instant, emf of the battery and self emf in the inductor are equal

$$|e| = L\frac{dI}{dt} \quad \text{or} \quad \frac{dI}{dt} = \frac{|e|}{L} = \frac{600V}{0.2H} = 3000 A\ s^{-1}$$

Example-3.29

An inductor of 5 H inductance carries a steady current of 2A. How can a 50 V self-induced emf be made to appear in the inductor ?

Solution :

$L = 5H, |e| = 50V$

Let us produce the required emf by reducing current to zero.

$$\text{Now, } |e| = L\frac{dI}{dt} \text{ or } dt = \frac{LdI}{|e|} = \frac{5 \times 2}{50} s$$

$$= \frac{10}{50} s = \frac{1}{5} s = 0.2s$$

So, the desired emf can be produced by reducing the given current to zero in 0.2 second.

Example-3.30

Two different coils have self-inductances L_1 = 16 mH and L_2 = 12mH. At a certain instant, the current in the two coils is increasing at the same rate and power supplied to the two coils is the same. Find the ratio of

i) induced voltage

ii) current

iii) energy stored in the two coils at that instant.

Solution :

i) $V_1 = L_1\frac{dI}{dt}$; $V_2 = L_2\frac{dI}{dt}$

$$\frac{V_1}{V_2} = \frac{L_1}{L_2} = \frac{16}{12} = \frac{4}{3}$$

ii) $V_1 I_1 = V_2 I_2, \frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{3}{4}$

iii) $$\frac{W_1}{W_2} = \frac{\frac{1}{2}L_1 I_1^2}{\frac{1}{2}L_2 I_2^2} = \left(\frac{L_1}{L_2}\right)\left(\frac{I_1}{I_2}\right)^2 = \frac{4}{3}\left(\frac{3}{4}\right)^2 = \frac{3}{4}$$

3.22 MUTUAL INDUCTION

Mutual induction is the phenomenon of production of induced emf in a coil due to the flow of current changing with time in a near by coil.

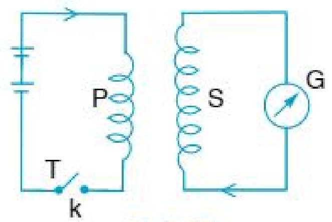


Fig 3.25

Consider a coil connected to a battery through a key (k). This coil is called primary coil (P). Another coil placed near the primary coil and connected to the galvanometer is called secondary coil (S). When key (k) is pressed, galvanometer shows a deflection.

Explanation :

When key (k) is pressed, current through P begins to increase. As a result of this, magnetic field around P increases, so magnetic flux linking with secondary coil also changes. The induced emf is produced in the secondary coil due to the change in magnetic flux. Hence the current through the secondary coil flows which is indicated by the deflection of the galvanometer. This phenomenon of producing e.m.f. is called mutual induction.

Mutual induction is also produced in a set of coils when primary winding is connected to an a.c. source.

3.23 COEFFICIENT OF MUTUAL INDUCTION (OR) MUTUAL INDUCTANCE

It is found that the magnetic flux linked with the secondary coil is directly proportional to the current flowing through the primary coil.

i.e., $\phi_s \propto I_p$ ...... (i)

$$\phi_s = MI_p$$

where M (ie, the ratio ϕ_s / I_p) is the constant of proportionality and is known as co-efficient of mutual induction or Mutual Inductance.

If $I_P = 1$, then $M = \phi_s$

Thus, mutual inductance of two coils can be defined as the magnetic flux linked with the secondary coil due to the flow of unit current in the primary coil.

According to Faraday's law of electromagnetic Induction $e = \frac{-d\phi}{dt}$

$$\therefore \; e_S = \frac{-d\phi_s}{dt}$$

we get $e_S = \frac{-d(MI_p)}{dt} = -M\frac{dI_P}{dt}$

If $\frac{-dI_P}{dt} = 1$, then $M = e_s$

Thus, mutual inductance of two coils can be defined as the induced e.m.f. produced in the secondary coil due to unit rate of decrease of current in the primary coil.

3.24 MUTUAL INDUCTANCE OF TWO LONG CO-AXIAL SOLENOIDS

Consider two solenoids S_1 and S_2 such that the solenoid S_2 completely surrounds the solenoid S_1.

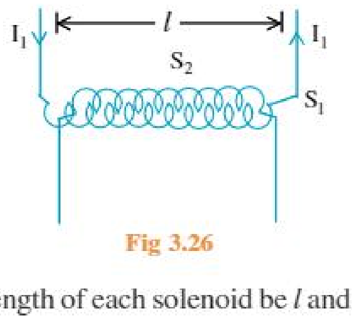


Fig 3.26

Let length of each solenoid be l and of nearly same area of cross-section A. N_1 and N_2 are the total number of turns of solenoid S_1 and S_2 respectively.

$\therefore$ Number of turns per unit length of solenoid S_1, $n_1 = \frac{N_1}{l}$

Number of turns per unit length of solenoid S_2, $n_2 = \frac{N_2}{l}$

Magnetic field inside the solenoid S_1 is given by $B_1 = \mu_0 n_1 I_1 = \mu_0 \frac{N_1}{l} I_1$

$\therefore$ Magnetic flux linked with each turn of solenoid $S_2 = B_1 A = \mu_0 \frac{N_1}{l} I_1 A$

$\therefore$ Total magnetic flux linked with N_2 turns of the solenoid S_2 is

$$\phi_2 = N_2(B_1 A) = \mu_0 \frac{N_1}{l} I_1 A \times N_2$$

$$\phi_2 = \frac{\mu_0 N_1 N_2 I_1 A}{l} \quad \text{.... (i)}$$

But $\phi_2 = M_{12} I_1$ .... (ii)

Where M_{12} is the mutual inductance when current varies in solenoid S_1 and makes magnetic flux linked with solenoid S_2,

from (i) and (ii) we get

$$M_{12} I_1 = \frac{\mu_0 N_1 N_2 I_1 A}{l}$$

$$\therefore M_{12} = \frac{\mu_0 N_1 N_2 A}{l}$$

Similarly, $M_{21} = \frac{\mu_0 N_1 N_2 A}{l}$, where M_{21} is the mutual inductance when current varies in solenoid S_2 and makes magnetic flux linked with solenoid S_1.

It can be proved that $M_{12} = M_{21} = M$

The above equation is treated as a general result If the two solenoids are wound on a magnetic substance of permeability μ_r, then the mutual inductance is given by

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{\ell}$$

3.25 RECIPROCITY THEOREM

Consider two closely wound coils C_1 and C_2. The current I_1 in C_1 creates magnetic field lines some of which pass through C_2. If ϕ_{21} is the flux linked with C_2 in this case, we can write

$$\phi_{21} = M_{21}I_1$$

Where M_{21} is mutual inductance of C_2 with respect to C_1.

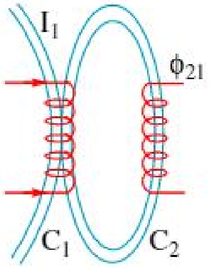


Fig 3.27

If I_1 varies with time t, emf induced in C_2 is

$$e_2 = \frac{-d\phi_{21}}{dt} = -M_{21}\frac{dI_1}{dt}$$

Similarly If ϕ_{12} is flux linked with C_1 due to current I_2 through C_2, we can write $\phi_{12} = M_{12}I_2$

Where M_{12} is mutual inductance of C_1 with respect to C_2

Then emf induced in C_1 is

$$e_1 = \frac{-d\phi_{12}}{dt} = -M_{12}\frac{dI_2}{dt}$$

If $\frac{dI_1}{dt} = \frac{dI_2}{dt}$, we get $e_1 = e_2$

It means $M_{12} = M_{21} = M$

This is known as reciprocity theorem of mutual inductance.

Example-3.31

Calculate the mutual inductance between two coils when a current of 2A changes to 6A in 2 seconds and induces an emf of 20 mV in the secondary coil.

Solution :

$$e = -M\frac{dI}{dt}$$

$$-20\times10^{-3} = -M\frac{6-2}{2} \quad \text{or } M = 10\text{mH}$$

Example-3.32

If the coefficient of mutual induction of the primary and secondary coils of an induction coil is 6 H and a current of 5A is cut off in 1/5000 second, calculate the emf induced in the secondary coil.

Solution :

$$e = -M\frac{dI}{dt};\ e = 6\times\frac{5}{1/5000}V = 15\times10^4 V$$

Example-3.33

A solenoid is of length 50 cm and has a radius of 2 cm. It has 500 turns. Around its central section a coil of 50 turns is wound. Calculate the mutual inductance of the system.

Solution :

$$N_P = 500, N_S = 50$$

$$A = \pi\times0.02\times0.02\ m^2$$

$$\mu_0 = 4\pi\times10^{-7}\,Hm^{-1}, \ell = 50\,cm = 0.5\,m$$

$$\text{Now, } M = \frac{\mu_0 N_P N_S A}{\ell}$$

$$= \frac{4\pi\times10^{-7}\times500\times50\times\pi\times(0.02)^2}{0.5}H$$

$$= 789.8\times10^{-7}H = 78.98\ \mu H$$

Example-3.34

An air-cored solenoid is of length 0.3 m, area of cross section $1.2\times10^{-3}\ m^2$ and has 2500 turns. Around its central section, a coil of 350 turns is wound. The solenoid and the coil are electrically insulated from each other. Calculate the emf induced in the coil if the initial current of 3 A in the solenoid is reversed in 0.25s.

Solution :

$$M = \frac{\mu_0 N_1 N_2 A_2}{\ell}$$

$$= \frac{4\pi\times10^{-7}\times2500\times350\times1.2\times10^{-3}}{0.3}H$$

$$= 4.4\times10^{-3}H$$

$$e = M\frac{dI}{dt}$$

$$\Rightarrow e = 4.4\times10^{-3}\times\frac{3-(-3)}{0.25}V = 0.1056V$$

❖ If two solenoids are of unequal length, then length of bigger solenoid is to be considered.

Example-3.35

A solenoid of length 50cm with 20 turns per centimetre and area of cross-section 40cm^2 completely surrounds another co-axial solenoid of the same length, area of cross-section 25cm^2 with 25 turns per centimetre. Calculate the mutual inductance of the system.

Solution :

$$\ell = 50\text{cm} = \frac{1}{2}\text{m}$$
$$N_1 = 20 \times 50 = 1000, A_1 = 40 \times 10^{-4}\text{m}^2$$
$$N_2 = 25 \times 50 = 1250, A_2 = 25 \times 10^{-4}\text{m}^2$$
$$M = \frac{\mu_0 N_1 N_2 A_2}{l}$$
$$= \frac{4\pi \times 10^{-7} \times 1000 \times 1250 \times 25 \times 10^{-4}}{1/2}\text{H}$$
$$= 7.9 \times 10^{-3}\text{H} = 7.9\,\text{mH}$$

❖ For calculation of M, we have to consider the cross-sectional area of the inner solenoid.

Example-3.36

A solenoidal coil has 50 turns per centimetre along its length and a cross-sectional area of 4×10^{-4} m^2. 200 turns of another wire is wound round the first solenoid co-axially. The two coils are electrically insulated from each other. Calculate the mutual inductance between the two coils.

Solution :

$$n_1 = 50 \text{ turns per cm} = 5000 \text{ turns per metre}$$
$$n_2 l = 200, A = 4 \times 10^{-4}\text{m}^2$$
$$M = \mu_0 n_1 (n_2 l) A$$
$$= 4\pi \times 10^{-7} \times 5000 \times 200 \times 4 \times 10^{-4}\text{H}$$
$$= 5.03 \times 10^{-4}\text{ H}$$

Example-3.37

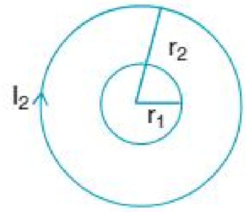
Two circular coils, one of smaller radius r_1 and the other of very large radius r_2 are placed co-axially with centres coinciding. Obtain the mutual inductance of the arrangement.

Solution :

Suppose a current I_2 flows through the outer circular coil. The field at the centre of the coil is

$$B_2 = \frac{\mu_0 I_2}{2r_2}$$

The second co-axially placed coil has very small radius. So B_2 may be considered constant over its cross-sectional area.



Now, $\phi_1 = \pi r_1^2 B_2 = \pi r_1^2 \left(\frac{\mu_0 I_2}{2r_2}\right)$

or $\phi_1 = \frac{\mu_0 \pi r_1^2}{2r_2} I_2$

Comparing with $\phi_1 = M_{12} I_2$, we get

$$M_{12} = \frac{\mu_0 \pi r_1^2}{2r_2}$$

Also, $M_{21} = M_{12} = \frac{\mu_0 \pi r_1^2}{2r_2}$

❖ It would have been difficult to calculate the flux through the bigger coil of the non-uniform field due to the current in the smaller coil and hence the mutual inductance M_{12}. The equality $M_{12} = M_{21}$ is helpful. Note also that mutual inductance depends solely on the geometry.

Example-3.38

(a) A toroidal solenoid with an air core has an average radius of 0.15m, area of cross section 12×10^{-4} m^2 and 1200 turns. Obtain the self inductance of the toroid. Ignore field variation across the cross section of the toroid. (b) A second coil of 300 turns is wound closely on the toroid above. If the current in the primary coil is increased from zero to 2.0 A in 0.05 s, obtain the induced emf in the secondary coil.

Solution :

a) $B = \mu_0 n_1 I = \frac{\mu_0 N_1 I}{\ell} = \frac{\mu_0 N_1 I}{2\pi r}$

Total magnetic flux, $\phi_B = N_1 BA = \frac{\mu_0 N_1^2 IA}{2\pi r}$

But $\phi_B = LI$

$$\therefore L = \frac{\mu_0 N_1^2 A}{2\pi r}$$

$$L = \frac{4\pi \times 10^{-7} \times 1200 \times 1200 \times 12 \times 10^{-4}}{2\pi \times 0.15}\text{H}$$

$$= 2.3 \times 10^{-3}\text{H} = 2.3\text{mH}$$

(b) $|e| = \frac{d}{dt}(\phi_2)$, where ϕ_2 is the total magnetic flux linked with the second coil.

$$|e| = \frac{d}{dt}(N_2BA) = \frac{d}{dt}\left[N_2\frac{\mu_0 N_1 I}{2\pi r}A\right]$$

$$\text{or } |e| = \frac{\mu_0 N_1 N_2 A}{2\pi r}\frac{dI}{dt}$$

$$\text{or } |e| = \frac{4\pi\times10^{-7}\times1200\times300\times12\times10^{-4}\times2}{2\pi\times0.15\times0.05}V$$

$$= 0.023\ V$$

Example-3.39

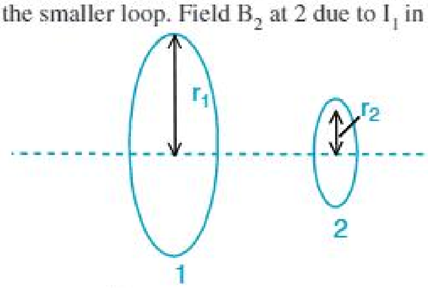
A circular loop of radius 0.3cm lies parallel to a much bigger circular loop of radius 20cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15cm.

a) What is the flux linking the bigger loop if a current of 2.0. A flows through the smaller loop ?

b) Obtain the mutual inductance of the two loops.

Solution :

We know from the considerations of symmetry that $M_{12} = M_{21}$. Direct calculation of flux linking the bigger loop due to the field by the smaller loop will be difficult to handle. Instead, let us calculate the flux through the smaller loop due to a current in the bigger loop. The smaller loop is so small in area that one can take the simple formula for field B on the axis of the bigger loop and multiply B by the small area of the loop to calculate flux without much error. Let 1 refer to the bigger loop and 2 the smaller loop. Field B_2 at 2 due to I_1 in 1 is.



$$B_2 = \frac{\mu_0 I_1 r_1^2}{2(x^2 + r_1^2)^{3/2}}$$

Here x is distance between the centres.

$$\phi_2 = B_2\pi r_2^2 = \frac{\pi\mu_0 r_1^2 r_2^2}{2(x^2+r_1^2)^{3/2}}I_1 \quad \text{But} \quad \phi_2 = M_{21} I_1$$

$$\therefore M_{21} = \frac{\pi\mu_0 r_1^2 r_2^2}{2(x^2+r_1^2)^{3/2}} = M_{12}$$

$$\therefore \phi_1 = M_{12}I_2 = \frac{\pi\mu_0 r_1^2 r_2^2}{2(x^2+r_1^2)^{3/2}}I_2$$

Using the given data

$$M_{12} = M_{21} = 4.55\times10^{-11}H$$

$$\phi_1 = 9.1\times10^{-11}\,Wb$$

Example-3.40

A metallic ring of mass m and radius r is falling under gravity in a region having magnetic field (ring is being horizontal). Assume z axis as vertical direction and z component of magnetic field is $B_z = B_0(1 + \lambda z)$. If R is the resistance of the ring and it falls with velocity V, find the energy lost in the resistance ? If the ring has reached a constant velocity, use conservation of energy to determine V in terms of m, B, λ and acceleration due to gravity.

Solution :

Flux linked with the ring $\phi = B_0(1+\lambda z)\pi r^2$

Magnitude of induced emf in the ring $e = \frac{d\phi}{dt}$

$$\Rightarrow e = \frac{d}{dt}\{B_0(1+\lambda z)\pi r^2\} = B_0\lambda\pi r^2\frac{dz}{dt}$$

$$= \pi r^2\lambda B_0 v \quad \left(\because v = \frac{dz}{dt}\right)$$

$$\text{Energy lost per second} = \frac{e^2}{R} = \frac{\pi r^2\lambda B_0 v}{R}$$

$$\text{Rate of change of potential energy} = \frac{d}{dt}(mgz) = mgv$$

$$\text{From law of conservation of energy, } mgv = \frac{(\pi r^2\lambda B_0 v)^2}{R}$$

$$\text{and } v = \frac{mgR}{(\pi r^2\lambda B_0)^2}$$

3.26(a) SERIES GROUPING OF COILS

n coils of inductances L_1, L_2,.........,L_n are connected in series in such a way that the magnetic flux due to one coil is not linked with the other coil.

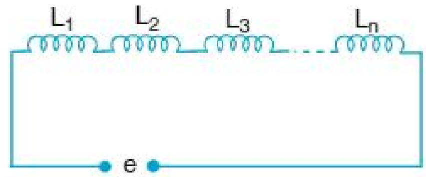


Fig 3.28

$$e = e_1 + e_2 + ...+ e_n$$

$$L.\frac{di}{dt} = L_1.\frac{di}{dt} + L_2.\frac{di}{dt} + ... + L_n.\frac{di}{dt}$$

$$L = L_1 + L_2 + L_3 + ... + L_n$$

3.26(b) PARALLEL GROUPING OF COILS

n coils of inductances L_1, L_2, L_3, ..., L_n are connected in parallel in such a way that the magnetic flux due to one coil is not linked with the other coil.

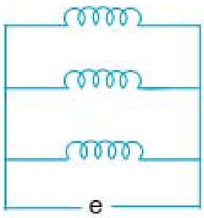


Fig 3.29

$$I = I_1 + I_2 + \ldots\ldots + I_n$$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \ldots\ldots + \frac{dI_n}{dt}$$

$$\frac{e}{L} = \frac{e}{L_1} + \frac{e}{L_2} + \ldots\ldots + \frac{e}{L_n}$$

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots\ldots + \frac{1}{L_n}$$

3.27 AC GENERATOR OR AC DYNAMO

To get sinusoidally varying alternating current, we need a source which can generate sinusoidally varying emf. An **ac** generator, also called an **ac** dynamo, can be used as such a source. It converts mechanical energy into electrical energy, producing an alternating emf.

Construction :

A schematic design of an **ac** dynamo is shown in Fig 3.30(a). A simplified diagram of the same is shown in Fig 3.30(b). It consists of three main parts: a magnet, an armature with slip rings and brushes.

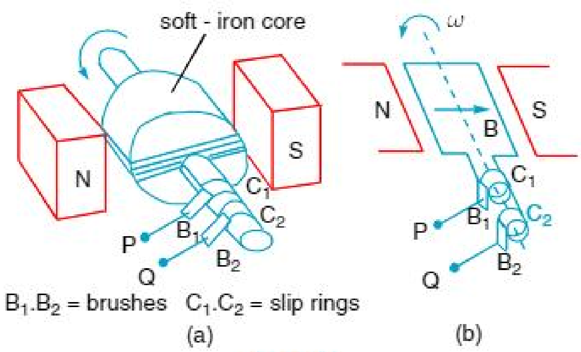


Fig 3.30

Magnet : It may be a permanent magnet or an electromagnet. The poles of the magnet face each other so that a strong uniform magnetic field $\vec{B}$ is produced between the poles.

Armature : It is a coil generally wound over a soft iron core. The core increases the magnetic field due to its magnetization. The two ends of the coil are connected to two slip rings C_1 and C_2. The coil together with the rings can rotate in the magnetic field. The axis of rotation is in the plane of the coil but perpendicular to the magnetic field.

Brushes : Two graphite brushes B_1 and B_2 permanently touch the slip rings. As the armature rotates, the slip rings C_1 and C_2 slip against the brushes so that the contact is maintained all the time. These brushes are connected to two terminals P and Q. The external circuit is connected to these terminals.

emf Induces as the Coil Rotates

Suppose the area of the coil is A, it contains N turns and it is rotated at a constant angular velocity ω. Suppose, the plane of the coil is perpendicular to the magnetic field at t = 0. The total magnetic flux through each turn of the coil is BA in this position. In time t, the coil rotates through an angle $\theta = \omega t$. The flux through each turn of the coil at this time t is $\Phi = BA\cos\omega t$.

Using Faraday's law, the emf induced in each turn of the coil is $-\frac{d\Phi}{dt} = BA\omega\sin\omega t$.

The total emf induced in the coil is,

$$E = NBA\omega\sin\omega t = E_0\sin\omega t$$

We see that the emf varies sinusoidally with time with an angular frequency ω and hence with a time period $T = \frac{2\pi}{\omega}$. The maximum magnitude of the emf, known as peak emf, is E_0.

We know, $E = E_0\sin\omega t = E_0\sin\left(\frac{2\pi}{T}t\right)$

When t = 0 then E = 0

When $t = \frac{T}{4}$ then $E = E_0 \sin\left(\frac{2\pi}{T} \times \frac{T}{4}\right) = +E_0$

When $t = \frac{T}{2}$ then $E = E_0 \sin\left(\frac{2\pi}{T} \times \frac{T}{2}\right) = 0$

When $t = \frac{3T}{4}$ then $E = E_0 \sin\left(\frac{2\pi}{T} \times \frac{3T}{4}\right) = -E_0$

When $t = T$ then $E = E_0 \sin\left(\frac{2\pi}{T} \times T\right) = 0$

If the instantaneous value of alternating emf is $E = E_0 \sin \omega t$, then the corresponding instantaneous value of alternating current is given by $I = I_0 \sin \omega t$. Where I_0 and E_0 denote the peak values of current and emf respectively.

Each cycle of **ac** consists of two half cycles. For one half cycle, the current is entirely positive where as during the next half cycle, the current is entirely negative. One complete cycle of alternating current or alternating voltage is shown in figure (a).

The instantaneous value of alternating current or alternating emf may also be represented by

$I = I_0 \cos \omega t$ and $E = E_0 \cos \omega t$

Figure (b) represents the alternating current (or) alternating emf as cosine function of time.

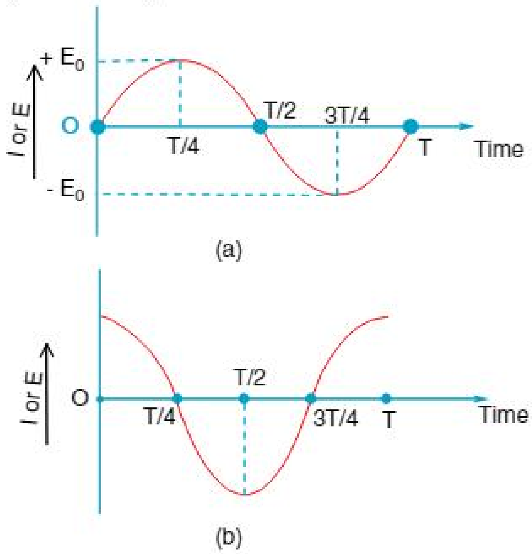


Fig 3.31

3.28 TIME VARYING CURRENTS

These currents are of two types .The first kind of current is that which varies in magnitude with time but not in direction. It is called as transient current. Such type of currents flow in circuits containing the circuit elements like resistor and capacitor or inductor or all the three circuit elements and a source of DC. The other kind of time varying current is that which varies with time both in magnitude and direction periodically .It is called alternating current.

- When a circuit containing only ohmic resistance and source of DC voltage (constant e.m.f) is switched on, the current attains its maximum steady value practically in zero time interval. Similarly, current falls to zero value practically in no time when the circuit is switched off. The maximum steady value attained by the current is given by ohm's law (I=E/R).
- However, if the resistive circuit also contains an inductance or capacitance or both, the current takes some time to attain its maximum or peak value. Similarly, when the battery is removed from a circuit having an inductance or a capacitor or both, the current again takes some time to decay to zero value, thus the current which take some time to grow to a peak value or to fall to zero value, are known as ***transient (or varying) current.***

3.29 GROWTH AND DECAY OF CURRENT IN L-R CIRCUIT

Growth of current

A coil of self inductance L with no resistance and a non-inductive resistance R are connected to a cell of emf E through a two-way switch "s" as shown in the figure. The battery is of negligible internal resistance. When the switch is closed between the points a and b, the battery is included in the circuit and current begins to grow from zero to its maximum value.

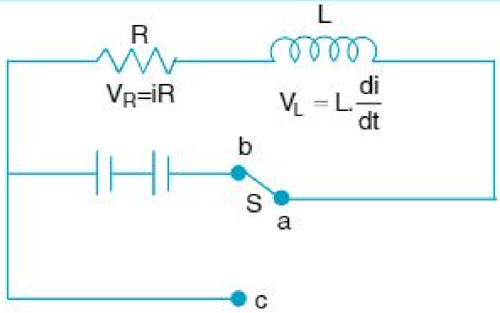


Fig 3.32

On account of the inductor L in the circuit an induced emf is set up in it which acts in opposite direction to that of the applied e.m.f E according to Lenz's law. Thus current in the circuit cannot rise to peak value immediately, but grows at a rate depending upon the resistance and the inductance in the circuits suppose during the growth of current. At any instant, *i* be the current and d*i*/dt, the rate of growth of current in the circuit.

$$E - iR = \frac{L.di}{dt}$$

$$E - iR = \frac{L.di}{dt}$$

$$dt = \frac{L}{E - iR}di$$

$$\int dt = \int \frac{L}{E - iR}di$$

$$t = \frac{-L}{R}\ln(E - iR) + C \quad .... (1)$$

where C is the constant of integration.

At $t = 0,\ i = 0$

$$C = \frac{L}{R}\ln E \quad .... (2)$$

Substitute value of C from (2) in (1)

$$t = \frac{-L}{R}\ln(E - ir) + \frac{L}{R}\ln E$$

$$\ln\left(\frac{E - iR}{E}\right) = \frac{-Rt}{L}$$

$$E - iR = Ee^{\frac{-Rt}{L}}$$

$$iR = E\left[1 - e^{\frac{-Rt}{L}}\right];\ i = \frac{E}{R}\left[1 - e^{\frac{-Rt}{L}}\right]$$

$$t = \alpha, i = i_0 = \frac{E}{R} \quad \therefore i = i_0\left(1 - e^{\frac{-Rt}{L}}\right)$$

The above equation gives the value of current i at any instant of time t during its growth in L.R circuit. This is also known as Helmholtz equation for the growth of current in L-R circuit.

This equation shows that current in L-R circuit rises exponentially with time. The ratio L/R has the dimensions of time and is called inductive time constant (τ_L). Smaller the ratio L/R, the more rapidly does the current attains its maximum value.

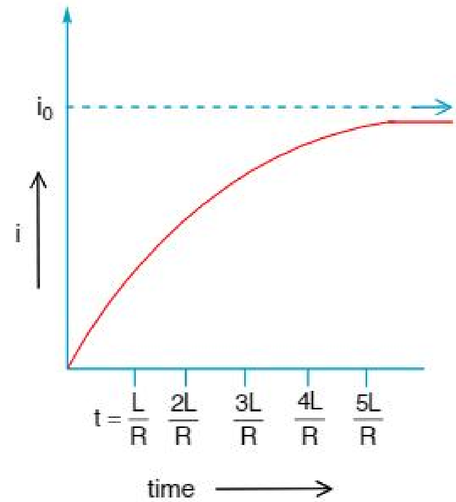


Fig 3.33

$$i = i_0\left(1 - e^{\frac{-Rt}{L}}\right) \text{ when } t = \frac{L}{R}$$

$$i = i_0\left(1 - e^{-1}\right) = i_0\left(1 - \frac{1}{e}\right)$$

$$i = i_0\left(1 - \frac{1}{2.318}\right) = 0.632i_0$$

Thus the inductive time constant of L-R circuit is the time in which the current grows from zero to $0.632i_0$, i.e., 63.2%. of its maximum value.

Thus we conclude that in a L-R circuit, current first rises rapidly, then increases slowly and approaches the final steady state $i_0 = \frac{E}{R}$ after infinite time : However, practically current approaches maximum value after a time equal to few time constants of the L-R circuit.

$$|V_L| = L.\frac{di}{dt} = L.\frac{d}{dt}\left[\left(i_0\left(1-e^{-\frac{Rt}{L}}\right)\right)\right]$$

$$|V_L| = E\,e^{\frac{-Rt}{L}}$$

Thus we notice that the potential difference across the inductor decays exponentially to zero as the current grows to maximum steady value.

Decay of Current

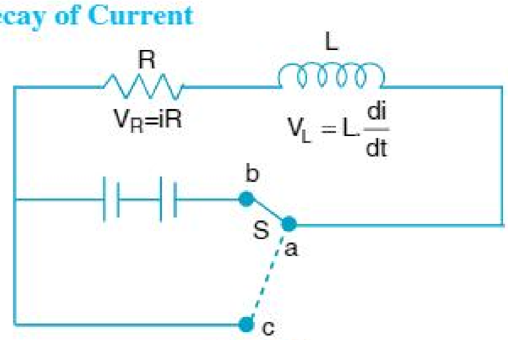


Fig 3.34

Let the switch 's' of the circuit be shifted from 'ab' to 'ac' at the instant (counted at t =0) when the current flowing in the circuit is maximum ($i = i_0 = E/R$). As the switch is used between a and c the cell gets out of the circuit and the current in the circuit starts decaying. In this case though there is no applied emf, the induced emf produced will oppose the decay of current and hence the current will take some time to fall to zero value.

Suppose at any instant of time 't' during the decay of current, i be the value of current in the circuit and $\frac{di}{dt}$ the rate of decay of current.

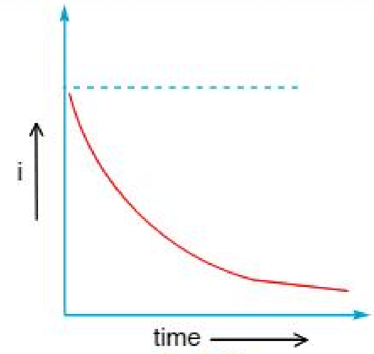


Fig 3.35

$$0 = iR + L.\frac{di}{dt}$$

$$dt = \frac{-L}{R}\frac{di}{i}$$

$$\int dt = \frac{-L}{R}\int\frac{di}{i}$$

$$t = \frac{-L}{R}\log_e i + C \qquad \text{.... (1)}$$

When $t = 0,\ i = i_0$

$$0 = \frac{-L}{R}\log_e i_0 + C$$

$$C = \frac{L}{R}\log_e i_0 \qquad \text{.... (2)}$$

Substitute value of C from (2) in (1)

$$t = \frac{-L}{R}\log_e i + \frac{L}{R}\log_e i_0$$

$$t = \frac{-L}{R}\left[\log_e i + \log_e i_0\right]$$

$$t = \frac{-L}{R}\log_e\left(\frac{i}{i_0}\right)$$

$$\log_e\left(\frac{i}{i_0}\right) = \frac{-Rt}{L}$$

$$\frac{i}{i_0} = e^{\frac{-Rt}{L}}$$

$$i = i_0 e^{\frac{-Rt}{L}}$$

Thus current decreases exponentially with time.

$$\text{When } t=\frac{L}{R},\ i=\frac{i_o}{e}=\frac{i_0}{2.718}=0.368i_0.$$

Therefore, inductive time constant of L-R circuit may also be defined as the time in which the current decays, from maximum to 0.368 times (36.8%) of the maximum value. Theoretically this current will become zero after infinite time.

Example-3.41

In the given circuit, switch S is closed at t = 0 with no charge initially on the capacitor.

a) Just after closing the switch what are the readings of each meter?

b) What does each meter read long time after S is closed?

c) What is the maximum charge on the capacitor?

Solution :

(a) At t = 0, capacitor acts as a short circuit and the inductor acts as open circuit.

Then circuit will have effective resistance $150\,\Omega$

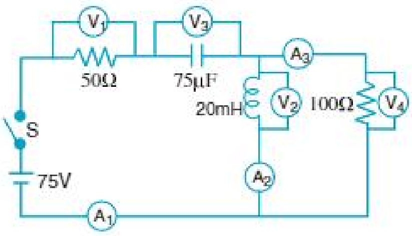
current supplied by battery $I=\frac{75}{150}=0.5A$

Reading $V_1 = IR = (0.5)\,50 = 25V$

Reading $V_2 = V_4 = (0.5)100 = 50$ V (parallel)

Reading $V_3 = 0$ (no charge on capacitor)

$A_1 = A_2 = 0.5$ A and $A_2 = 0$



(b) After long time capacitor acts as open circuit and then $V_3 = 75$ V

All other meters read zero.

(c) After long time or steady state

Charge on capacitor $Q = CV = (75\times10^{-6})\,75$

$= 5625\ \mu C$

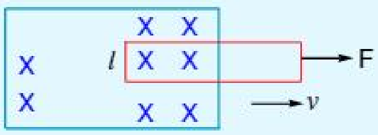
At a Glance

1. The magnetic flux through a surface area A placed in a uniform magnetic field B is defined as $\phi=\bar{B}.\bar{A}=BA\cos\theta$
2. According to Faraday the emf induced in a coil is directly proportional to the rate of change of flux through it.
3. According to Lenz the direction of induced emf is such that it opposes the cause which produces it. Lenz's law is not a new law but a restatement of energy conservation.
4. Faraday's law tells us about the magnitude of induced emf.
5. Lenz's law gives us the magnitude of induced emf.
6. Faraday - Lenz law $E=-N\frac{d\phi}{dt}$
7. There are two basic mechanisms of induced emf generation. (i) The first one involves the motion of a conductor relative to magnetic field lines, called the motional emf. (ii) The second involves the generation of an electric field associated with a time varying - magnetic field.
8. Motional emf induced in a moving conductor can be obtained by integrating $\int(\bar{V}\times\bar{B})dl$ over the length of the conductor.
9. Then a metal rod of length l is placed normal to a uniform magnetic field B and moved with a velocity V perpendicular to the field, the induced emf at its end is $e = BlV$.
10. When a metal rod of length l rotates, with a constant angular speed about one of its ends in a plane perpendicular to the uniform magnetic field B then the emf induced across its end is $e=\frac{1}{2}B\omega l^2$.
11. A wheel of radius R has several spokes in it. It is rotated with a constant angular speed in a plane perpendicular to a uniform magnetic field B, then the emf induced between the axle and rim of the wheel is independent of the number of spokes present in because the spokes are in parallel. The magnitude of emf induced is $e=\frac{1}{2}B\omega R^2$.
12. The induced emf in a circuit does not depend on the resistance of the circuit $e=-\frac{d\phi}{dt}$. whereas the induced current in the circuit depends on resistance $i=-\frac{1}{R}\frac{d\phi}{dt}$.

13. The induced charge that flows in the circuit depends on the change of flux only and not on how fast or slow the flux changes $dq = \frac{-d\phi}{R}$ i.e., $q = -\left(\frac{\phi_2 - \phi_1}{R}\right)$

If the number of turns in the coil is N then $q = -N\left(\frac{\phi_2 - \phi_1}{R}\right)$

14. The rectangular frame of resistance R is being pulled out of a uniform magnetic field B with a uniform velocity V. Then



a) induced emf = BlV

b) induced current $= \frac{BlV}{R}$

c) Force to be applied by external agent to move with uniform speed is $F = \frac{B^2L^2V}{R}$

d) Rate at which work is done by is the external agent is $P = \frac{B^2L^2V^2}{R}$

15. The direction of induced emf (motional emf) is given by Lenz's law. However it is more convenient to apply Fleming's Right hand rule to find direction of induced emf.

16. When a coil of area A having N turns is rotated with a constant angular velocity in a uniform magnetic field B then the emf induced is $E = NBA \sin \omega t$

17. Whenever a magnetic field varies with time an induced electric field is produced in any closed path, whether in matter or in empty space. This induced electric field is non-conservative in nature and is given

$$\oint \bar{E}.d\bar{l} = -A\frac{dB}{dt}$$

18. The currents induced in a thick conductor when the conductor is placed in a changing magnetic field are called Eddy currents or Foucault currents. They dissipate electric energy as heat. Eddy currents can be minimised by using laminated (core).

19. When a current in a coil changes, it induces a back emf in the same coil. The self induced emf is given by $e = -L\frac{di}{dt}$.

Where L is the self inductance of the coil. It is a measure of inertia of the coil against the change of current through it.

20. $\phi = LI$

21. SI unit of L is henry.

22. $1 \text{ henry} = \frac{1 \text{ volt}}{\text{ampere / sec}}$

23. The self inductance of a long solenoid, the core of which consists of a magnetic material of permeability μ_r is given by $L = \mu_r \mu_0 n A l$.

Where A is area of cross-section of solenoid, l its length and n the number of turns, per unit length.

24. Self inductance of a plane coil is $L = \mu_0 n^2 \pi r^2 l$

25. Energy stored in an inductor is $U = \frac{1}{2}LI^2$

26. Energy stored per unit volume is called energy density. Energy density $= \frac{B^2}{2\mu_0}$

27. When n inductances are connected in series, $L = L_1 + L_2 + L_3 + ... + L_n$

28. When n inductances are connected in parallel $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_n}$

29. The phenomenon of production of induced emf in one coil due to the varying current in the neighbouring coil is called mutual induction.

30. The mutual inductance of two coils is equal to the induced emf set up in one coil when rate of change of current in the other is unity.

31. A changing current in coil -2 can induce emf in coil 1. This relation is given by $e_1 = -M_{12}\frac{di_2}{dt}$. The quantity M_{12} is called mutual inductance if coil-1 w.r.t coil-2. Similarly a changing current in coil -1 can induce emf in coil-2. The relationship is $e_2 = -M_{21}\frac{di_1}{dt}$. The quantity M_{21} is called mutual inductance of coil-2 w.r.t coil -1.

32. According to reciprocity theorem of mutual inductance $M_{21} = M_{12}$.

33. The value of mutual inductance depends upon the number of turns of the coils, the extent by which they are separated, the geometrical shape and size of coils and also on angular separation between the coils.

34. The coefficient of coupling of two coils is a measure of the coupling between the two coils. If L_1 and L_2 are the coefficients of self induction of the coils and M is the coefficient of mutual inductance of the two coils, then the coefficient of coupling is given by $k = \dfrac{M}{\sqrt{L_1 L_2}}$

35. Mutual inductance of two long solenoids is given by $M = \mu_0 \mu_r n_1 n_2 A l$

36. $\dfrac{L}{R}$ is called inductive time constant. It has the dimensions of time.

37. CR is called capacitive time constant. It has the dimensions of time.

38. In LR circuit

a) $i = i_0 \left[1 - e^{\frac{-Rt}{L}}\right]$ (growth of current)

b) $i = i_0 e^{\frac{-Rt}{L}}$ (decay of current)

39. In a LCR circuit

a) $q = q_o \left[1 - e^{\frac{-t}{CR}}\right] \rightarrow$ growth of charge

b) $q = q_o e^{\frac{-t}{CR}} \rightarrow$ decay of charge

40. A transformer works on the principle of EMI

41. State Faraday's laws of electromagnetic induction, Lenz's law. Explain self and mutual induction. Derive the equations for L and M

42. Describe the growth and decay of circuit in an inductance, resistance circuit. How the growth and decay are affected with different values of inductance discuss?

43. State the principle on which a transformer works. Describe the working of a transformer with necessary theory.

EXERCISE

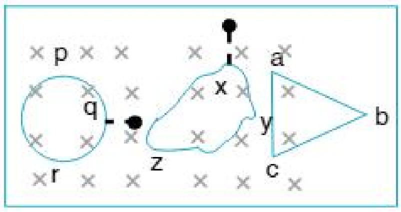
LONG ANSWER QUESTIONS

1. State Faraday's laws of electromagnetic induction, Lenz's law. Explain self and mutual induction. Derive the equations for L and M.

SHORT ANSWER QUESTIONS

1. A current -carrying loop of irregular shape is located in an external magnetic field. If the wire is flexible, why does it become circular?

2. In figure, the loop abc is moving out of the magnetic field, loop pqr is moving inside, while the loop xyz is moving within the field. Give the direction of the induced current. The field is perpendicular to the plane of loops directed inwards away from the reader.



3. Figures1 and 2 show planar loops of different shapes moving out or into a region of magnetic field ,which is directed normal to the plane of the loop and away from reader. Determine the direction of induced current.

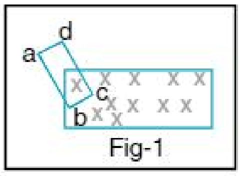


Fig-1

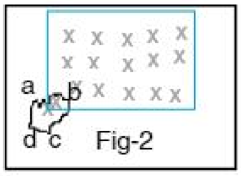


Fig-2

4. Use Lenz's law to determine the direction of induced current in the situations described by the fig 3 and 4 :

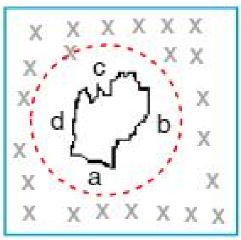


Fig-3

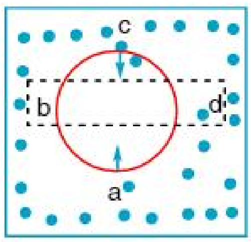


Fig-4

a) In fig. 3, a wire of irregular shape turn into a circular shape and

b) In fig.4 a circular loop being deformed into a narrow straight wire. The cross (x) indicates magnetic field into the paper and the dot (.) indicates magnetic filed out of paper.

5. A conducting loop is held stationary normal to the field between the NS poles of a fixed permanent magnet. By choosing a magnet sufficiently strong, can we hope to generate current in the loop?

6. A closed conducting loop moves normal to the electric field between the plates of a large capacitor. Is a current induced in the loop,when it is (i) wholly inside the capacitor (ii) partially outside the plates of the capacitor? The electric field is normal to the plane of the loop.

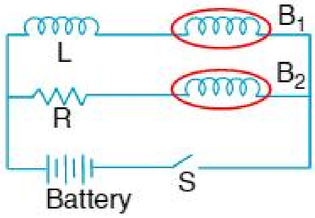
7. A rectangular loop and a circular loop are moving out of a uniform magnetic field region to a field free region with a constant velocity. In which loop do you expect the induced e.m.f to be constant during the passage out of the field region. The field is normal to the plane of loops.

8. Explain, whether an induced current will be developed in a conductor, if it is moved in direction parallel to the magnetic field.

9. An iron bar falling vertically through the hollow region of a thick cylindrical shell made of copper experiences a retarding force and attains a terminal velocity. What can you conclude about the iron bar?

10. A small resistor (say, a lamp) is usually put in parallel to the current carrying coil of an electromagnet. What purpose does it serve?

11. Figure shows an inductor L and a resistor R connected in parallel to a battery through a switch. The resistance R is the same as that of the coil L. Two identical bulbs are put in each arm of the circuit?



(i) Which of the bulb lights up earlier when switch S is closed?

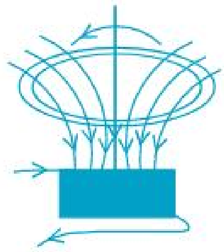
(ii) Will the bulbs be equally bright after some time?

12. As soon as current is switched on in a high - voltage wire, the bird sitting on it flies away, why ?

13. A coil is wound on an iron core and looped back on itself so that the core has two sets of closely wound wires in series carrying current in opposite senses. What do you expect about its self inductance ? Will it be large or small?

14. Why a spark is produced in the switch of a fan, when it is switched off ?

15. The figure shows the magnetic field of a current - carrying coil and a ring falling down the field. Is the direction of induced current in the ring correctly shown?



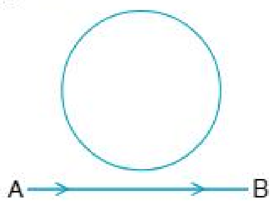
16. A copper ring is suspended by a thread. One end of a magnet is brought horizontally towards the ring .How will it affect the position of the ring?

17. A bar magnet falls down through a coil .Will the acceleration of the magnet the equal to g? Why does the acceleration of a magnet falling through a long solenoid decrease?

18. In the above problem, if the coil is cut some where, what would be the answer?

19. A lamp in a circuit consisting of a coil of large number of turns and a battery does not light up to full brilliance instantly on switching on the circuit. Why?

20. The magnitude of electric current is increasing in a wire at a constant rate in the direction from A to B. Will there be induced current in the conduting loop show in fig? What will be its direction?



21. A metal piece and a stone are dropped from the same height near earth's surface .Which one will reach the earth earlier?

VERY SHORT ANSWER QUESTIONS

1. **Define magnetic flux.**

A. Total number of magnetic lines of force passing through a given area (area being normal to the magnetic line of force) is known as magnetic flux

2. **Is magnetic flux a scalar or vector?**

A. Scalar

3. **Weber is the unit of which physical quantity?**

A. Magnetic Flux

4. **Is an induced e.m.f developed in a conductor, when moved in a direction parallel to the magnetic field?**

A. No, because there is no change of magnetic flux

5. **What do you understand by positive flux?**

A. When normal to the area , points $\vec{B}, \theta = 0^0$ i.e$\phi = BA\cos 0^0 = BA$, magnetic flux is + ve

6. **When the magnetic flux is said to be neutral?**

A. When normal to the area, points perpendicular to $\overline{B}, \theta = 90^0$, i.e $\phi = BA\cos 90^0$; magnetic flux is neutral.

7. **A glass rod of length l moves with velocity v in a uniform magnetic field B. What is the e.m.f induced in the rod?**

Ans Glass is an insulator. So, no induced e.m.f. will be set up.

8. **Two coils are being moved out of magnetic field, one coil is moved rapidly and the other slowly. In which case is more work done and why?**

A. In the case of rapidly moving coil Because induced e.m.f will be more in the rapidly moving coil as compared to slowly moving coil.

9. **Name the scientists who discovered the generation of electric current by means of magnetic field.**

A. Joseph Henry and Michael Faraday

10. **Name the scientist associated with the direction of induced current.**

A. Lenz

11. **State Lenz's law.**

A. According to Lenz's law, induced emf or current is always produced in a direction such that it opposes the cause of its generation.

12. **Why the induced emf is also called as back emf?**

A. It is because the induced e.m.f opposes the applied e.m.f.

13. **What is the basic cause of induced emf?**

A. Change in magnetic flux

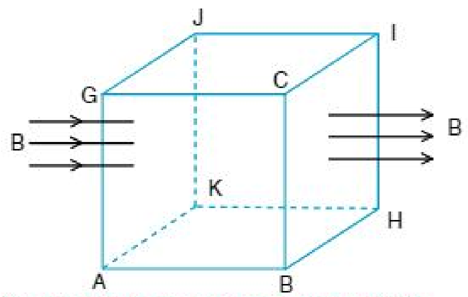
14. **Does Lenz's law violate the law of conservation of energy?**

A. No, Lenz's law is in conformity with the law of conservation of energy

15. **Does Lenz's law hold for an open circuit.**

A. Yes, it can still hold.

16. **Consider a cube ABCGHIJK of side a placed in a uniform magnetic field B acting perpendicular to the face BHIC as shown in figure.**



(i) What is the flux through face ABCG?

(ii) What is the flux through face GCIJ?

(iii) What is the flux throught face AKJG?

(iv) What is the flux through face BHIC?

A. $\phi = BA\cos\theta$ (θ is the angle between the normal to area and B)

(i) $\theta = 90^0, \phi = 0$ (ii) $\theta = 90^0, \phi = 0$

(iii) $\theta = 180^0, \phi = -Ba^2$ (iv) $\theta = 0^0, \phi = +Ba^2$

17. **Who discovered eddy current?**

A. Foucault

18. **What is an ideal inductor?**

A. An ideal inductor is one that has zero resistance

19. **Name the quantity that plays an identical role in an electrical circuit as is played by inertia in mechanics?**

A. Inductance

20. **On which of the following factors the emf induced in the coil does not depend? No.of turns in the coil, resistance of the coil, rate of change of magnetic flux.**

A. Resistance of the coil

21. **Name two factors on which the self -inductance of an air-core coil depends.**

A. Number of turns in the coil and its radius.

22. **What is the effect of metallic core on self inductance?**

A. It increases the self inductance

23. **Why inductors are made of copper?**

A. This is because after silver, copper has lowest resistance and is less costly.

24. **Name two factors on which mutual inductance between a pair of coils depends?**

A. Number of turns in the coils, area of secondary coil.

25. **If two coils are very tightly wound one over the other, will M increase or decrease as compared to the case when the coils are placed some distance apart?**

A. M will increase.

26. **What are the dimensions of L/R?**

A. Time ie [T]

27. **Why the inductance per unit length for a solenoid near its centre is different from inductance per unit length near its ends?**

A. This is because the magnetic field near the centre of the solenoid is two times of its value at the ends.

28. **An artificial satellite with a metal surface is orbiting the earth in equatorial plane. Why no current is induced due to earth's magnetism?**

A. This is because there is no change of magnetic flux linked with the satellite.

29. **Why the coil of a dead beat galvanometer is wound on a metal frame?**

A. The eddy currents will quickly bring the coil to rest.

30. **Two identical coils, one of copper and the other of iron, are rotated with the same angular velocity ω in a uniform magnetic field. In which case the induced e.m.f. is more and why?**

A. The induced emf does not depend upon the nature of material of the coil.

31. **A wire kept along north-south direction is allowed to fall freely. Will an induced e.m.f. be set up?**

A. No, because there will be no change of magnetic flux in this case.

32. **A wire kept in east west direction is allowed to fall freely, will an emf be induced in the wire?**

A. Yes, because there will be a change of magnetic flux.

33. **Why a thick metal plate oscillating about a horizontal axis stops when a strong magnetic field is applied on the plate?**

A. This is because eddy currents are produced and eddy currents oppose mechanical motion (according to Lenz's Law).

PROBLEMS

LEVEL - I

1. A coil of wire enclosing an area of 100 cm^2 is placed at an angle of 70^o with a magnetic field B of 10^{-1} Wbm^{-2}. What is the flux through the coil? B is reduced to zero in 10^{-3} sec. What e.m.f is induced in the coil ? **[Ans: 0.94V]**

2. A given wire is bent into a rectangular loop of size 15 cm × 5 cm and placed perpendicular to a magnetic field of 1.0 Tesla. Within 0.5 sec, the loop is changed into a 10 cm square and the field increases to 1.4 Tesla. Calculate the value of e.m.f. induced in the loop ? **[Ans: 0.013V]**

3. A jet plane is travelling west at the speed of 1800 km/h. What is the voltage difference developed between the ends of the wing 25 m long if the earth's magnetic field at the location has a magnitude of 5.0×10^{-4} Tesla and the dip angle is 30^o ? **[Ans: 3.125V]**

4. A straight rod 2m long is placed in an aeroplane in the east-west direction. The aeroplane lifts itself in the upward direction at a speed of 36km/hour. Find the potential difference between the two ends of the rod if the vertical component of earth's magnetic field is $\left(1/4\sqrt{3}\right)$ gauss and angle of dip=30^o. **[Ans: 5×10^{-4}V]**

5. The magnetic flux threading a coil changes from 12×10^{-3} Wb to 6×10^{-3} Wb in 0.01 sec. Calculate the induced emf. **[Ans: 0.6V]**

6. A wire of length 0.1 m moves with a speed of $10ms^{-1}$ perpendicular to a magnetic field of induction 1 Wbm^{-2}. Calculate induced emf. **[Ans: 1V]**

7. Find the magnitude of emf induced in a 200 turn coil with cross-sectional area of 0.16 m^2 if the magnetic field through the coil changes from 0.10 $\mathrm{Weber/m^2}$ to 0.50 $\mathrm{Weber/m^2}$ at a uniform rate over a period of 0.02 second. **[Ans: 640 V]**

8. A uniform magnetic flux density of $0.1\,Wbm^{-2}$ extends over a plane circuit of area $1m^2$ and is normal to it. How quickly must the field be reduced to zero if an emf of 100 volt is to be induced in the circuit? **[Ans:** $10^{-3}s$]

9. Suppose a coil of 1000 turns of wire is wound around a book, and this book is lying on a table. The vertical component of earth's magnetic field is about $0.6\times10^{-4}\,Wb/m^2$. The area of the coil is $0.15\times0.20m^2$. If this book is turned through 90^0 about a horizontal axis in 0.1 sec, what average emf will be induced in the coil? **[Ans:** 0.018 V]

10. A metal disc of radius 200 cm is rotated at a constant angular speed of 60 rad s^{-1} in a plane at right angle to an external field of magnetic induction 0.05 Wbm^{-2}. Find the e.m.f. induced between the centre and a point on the rim. **[Ans:** 6V]

11. A circular copper disc 10cm in radius rotates at 20πrad/s about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0.2 T acts perpendicular to the disc. Calculate the potential difference developed between the axis of the disc and the rim. **[Ans:** 6.28×10^{-2} V]

12. A 10Ω resistance coil has 1000 turns and at a certain time 5.5×10^{-4} Wb of flux passes through it. If the flux falls to 0.5×10^{-4} Wb in 0.1 second find the emf generated in volts and the charge flown through the coil in coulombs. **[Ans:** 5V, 0.05C]

13. A horizontal straight wire 10 m long is extending east and west and is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of earth's magnetic field 0.30×10^{-4} Wbm^{-2}. What is the instantaneous value of the emf induced in the wire ? **[Ans :** 1.5mV]

14. A train is moving in the north-south direction with a speed of 108kmh^{-1}. Find the e.m.f. generated between two wheels, if the length of the axle is 2m. Assume that the vertical component of earth's field is 8.0×10^{-5} Wbm^{-2}. **[Ans :** 4.8mV]

15. When a wheel with metal spokes 1.2 m long is rotated in a magnetic field of flux density 5×10^{-5}T normal to the plane of the wheel, an e.m.f. of 10^{-2} volt is induced between the rim and the axle. Find the rate of rotation of the wheel. **[Ans :** 44.23 rps]

16. Magnetic flux of 5 microweber is linked with a coil, when a current of 1 mA flows through it. What is the self-inductance of the coil? **[Ans:** 5×10^{-3}H]

17. A closed coil consists of 500 turns wound on a rectangular frame of area 4.0 cm^2 and has a resistance of 50Ω. It is kept with its plane perpendicular to a uniform magnetic field of 0.2 weber/meter^2. Calculate the amount of charge flowing through the coil if it is turned over (rotated through 180^0). Will this answer depend on the speed with which the coil is rotated? **[Ans:** 1.6×10^{-3}C, No]

18. If a rate change of current of 4 As^{-1} induces an emf of 20mV in a solenoid, what is the self inductance of the solenoid? **[Ans:** 5×10^{-3}H]

19. What emf will be induced in a 10 H inductor in which the current changes from 10 A to 7 A in 9.0×10^{-2} seconds? **[Ans:** 333.3V]

20. A coil of inductance 0.5 henry is connected to a 18 volt battery. Calculate the initial rate of growth of current. **[Ans:** 36As^{-1}]

21. A current of 10 A in the primary of a circuit is reduced to zero at a uniform rate in 10^{-3}sec. If the coefficient of mutual inductance is 3 henry, what is the induced emf ? **[Ans:** 3×10^4V]

22. If the current in the primary circuit of a pair of coils changes from 5amp to 1 amp in 0.02 sec, calculate

(i) induced emf in the secondary coil if the mutual inductance between the two coils is 0.5H and

(ii) the change of flux per turn in the secondary, if it has 200 turns. **[Ans:** (i) 100V; (ii) –0.01 Wb]

23. A coil of wire of certain radius has 600 turns and a self inductance of 108mH. What will be the self-inductance of a second similar coil of 500 turns ? **[Ans:** 75mH]

24. What is the self inductance of an air core solenoid 50 cm long and 2 cm radius if it has 500 turns ? Find the magnetic flux when a current of 2 amp passes through it. **[Ans:** 7.89×10^{-4}H, 15.78×10^{-4}Wb]

25. A 2m long solenoid with diameter 4 cm and 2000 turns has a secondary of 1000 turns wound closely near its mid point. Calculate the mutual inductance between the two coils. **[Ans:** 1.58×10^{-3}H]

26. Over a solenoid of 50 cm length and 2 cm radius and having 500 turns is wound another wire of 50 turns near the centre. Calculate

(i) mutual inductance of the two coils

(ii) induced emf in the second coil when the current in the primary changes from 0 to 5A in 0.02 sec.

[Ans: (i) 78.88μH ; (ii) 19.72mV]

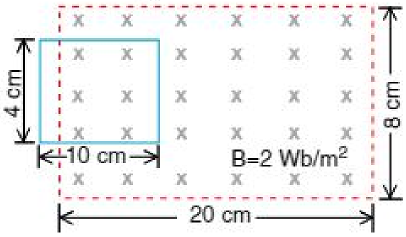
27. A conducting wire of 100 turns is wound over and near the centre of a solenoid of 100 cm length and 2 cm radius having 1000 turns. Calculate the mutual induction of the two coils. **[Ans:157.75 μH]**

28. A coil of area 0.04 m^2 having 1000 turns is suspended perpendicular to a magnetic field of 5.0×10^{-5} Wb/m^2. It is rotated through 90^0 in 0.2 second. Calculate the average emf induced in it. **[Ans: 0.01V]**

LEVEL - II

1. A rectangular loop of sides 8 cm and 2 cm with a small cut is kept stationary in a uniform magnetic field directed normal to the loop. The current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 T^{-1}. If the cut is joined and the loop has a resistance of 1.6Ω, how much power is dissipated by loop as heat? What is the source of this power ? **[Ans: 6.4×10^{-10} W]**

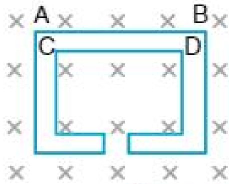
2. A rectangular loop of length 10 cm and width 4 cm is pulled at a constant speed of 1.0 ms^{-1} through a uniform magnetic field of strength 2.0 Weber/m^2 spread over an area 20 cm × 8 cm in such a way that the field remains perpendicular to the plane of loop throughout and directed away from the reader as shown in fig. Discuss the variation of emf in the loop as it moves through the field.



3. An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500 carries a current of 2.5 amp. The current is suddenly switched off in a brief time of 10^{-3}sec. How much is the average back e.m.f. induced across the ends of the open switch in the circuit? Ignore the variation of magnetic field near the ends of the solenoid. **[Ans: 6.54 V]**

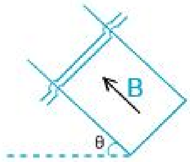
4. A small flat search coil of area 2.0 cm^2 with 25 close turns placed between the poles of a strong magnet normally to the magnetic field is suddenly snatched out of the field. The total charge flown in the coil as measured by a ballistic galvanometer connected to the coil, is 7.5 mC. The resistance of the coil and the galvanometer is 0.50Ω. Find the magnitude of the field of the magnet. **[Ans: 0.75 Wb/m^2]**

5. A wire is bent to form the double loop shown in Fig. There is a uniform magnetic field directed into the plane of the loop . If the magnitude of this field is decreasing, describe the direction of current flow in the loop.



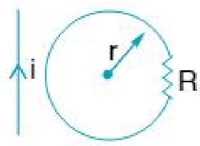
[Ans: A to B and D to C]

6. A wire of length ℓ, mass m and resistance R slides without any friction down the parallel conducting rails of negligible resistance (Fig). The rails are connected to the wire so that the wire and the rails form a closed rectangular conducting loop. The plane of the rails makes an angle θ with the horizontal and a uniform vertical magnetic field of induction B exists throughout the region. Find the steady-state velocity of the wire.



[Ans: $\dfrac{mgR\sin\theta}{B^2\ell^2\cos^2\theta}$]

7. In fig. the mutual inductance of a coil and a very long straight wire is M, the coil has resistance R and self-inductance L. The infinite wire lies in the same plane as that of the coil. The current in the wire varies according to the law i = at, where a is a constant and t is the time. Find the time dependence of current in the coil

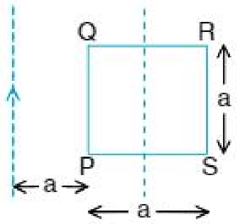


[Ans: $\dfrac{Ma}{R}(1-e^{-tR/L})$]

8. An inductor of 3H is connected to a battery of emf 6V through a resistance of $100\,\Omega$ .Calculate the time constant. What will be the maximum value of current in the circuit? **[Ans: 0.06amp]**

9. A cell of emf 1.5V is connected across an inductor of 2mH in series with a $2\,\Omega$ resistor What is the rate of growth of current immediately after the cell is switched on. **[Ans: 750As^{-1}]**

10. In Fig., a square loop PQRS of side a and resistance r is placed near an infinitely long wire carrying a constant current I. The sides PQ and RS are parallel to the wire The wire and the loop are in the same plane. The loop is rotated by 180^0 about an axis parallel to the long wire and passing through the midpoints of the sides QR and PS. Find the total amount of charge which passes through any point of the loop during the rotation.



[Ans: $\frac{\mu_0}{\pi r} Ia(\ln 2)$ **]**

11. A coil having resistance 15 Ω and inductance 10H is connected across a 90 Volt dc supply. Determine the value of current after 2sec. What is the energy stored in the magnetic field at that instant?

[Ans: 162.45 J]

12. An inductor of 10mH is connected to a 18V battery through a resistor of 10k Ω and a switch. After a long time, when the maximum current is set up in the circuit, the battery is disconnected from the circuit. Calulate the current in the circuit after 15μs.

[Ans: 5.5×10^{-10} **A]**

13. Calculate the back e.m.f of a 10H, 200 Ω coil 40 ms after a 100Vd.c supply is connected to it.

[Ans: $L\frac{dI}{dt} = 100 - 0.275 \times 200 = 45V$ **]**

14. The time constant of a certain inductive coil was found to be 2.5 ms. With a resistance of 80Ω added in series, a new time constant of 0.5 ms was obtained. Find the inductance and resistance of the coil.

[Ans: 50mH]

ADDITIONAL EXERCISE

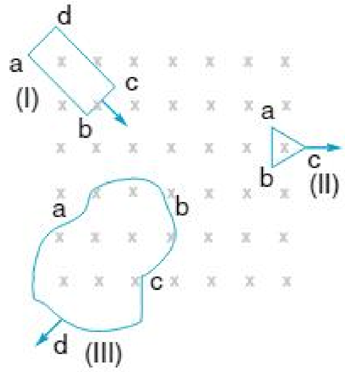
1. Consider Experiment 3.2. (a) What would you do to obtain a large deflection of the galvanometer?

(b) How would you demonstrate the presence of an induced current in the absence of a galvanometer?

2. A square loop of side 10 cm and resistance 0.5 Ω is placed vertically in the east-west plane. A uniform magnetic field of 0.10 T is set up across the plane in the north-east direction. The magnetic field is decreased to zero in 0.70 s at a steady rate. Determine the magnitudes of induced emf and current during this time-interval.

3. A circular coil of radius 10 cm, 500 turns and resistance 2 Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. Estimate the magnitudes of the emf and current induced in the coil. Horizontal component of the earth's magnetic field at the place is 3.0×10^{-5} T.

4. Figure shows planar loops of different shapes moving out of or into a region of a magnetic field which is directed normal to the plane of the loop away from the reader. Determine the direction of induced current in each loop using Lenz's law.



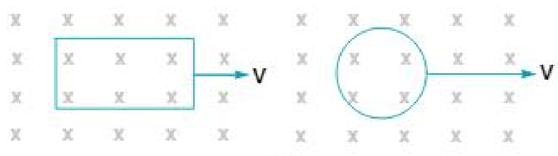
5. (a) A closed loop is held stationary in the magnetic field between the north and south poles of two permanent magnets held fixed. Can we hope to generate current in the loop by using very strong magnets?

(b) A closed loop moves normal to the constant electric field between the plates of a large capacitor. Is a current induced in the loop

(i) when it is wholly inside the region between the capacitor plates?

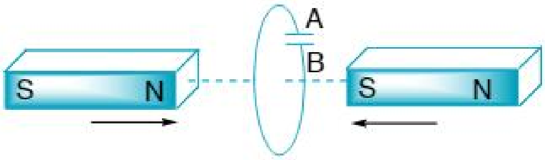
(ii) when it is partially outside the plates of the capacitor? The electric field is normal to the plane of the loop.

(c) A rectangular loop and a circular loop are moving out of a uniform magnetic field region (Fig. (a)) to a field-free region with a *constant velocity* **v**. In which loop do you expect the induced emf to be constant *during* the passage out of the field region? The field is normal to the loops.



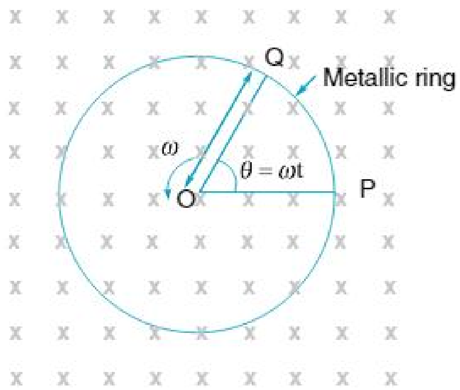
(a)

(d) Predict the polarity of the capacitor in the situation described by Fig. (b)



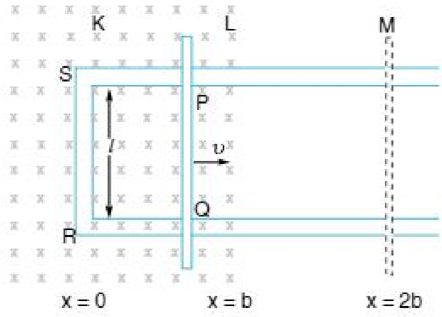
(b)

6. A metallic rod of 1 m length is rotated with a frequency of 50 rev/s, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius 1 m, about an axis passing through the centre and perpendicular to the plane of the ring (Fig.). A constant and uniform magnetic field of 1 T parallel to the axis is present everywhere. What is the emf between the centre and the metallic ring?



7. A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of earth's magnetic field B_H at a place. If $B_H = 0.4$ G at the place, what is the induced emf between the axle and the rim of the wheel? Note that 1 G = 10^{-4} T.

8. Fig. The arm PQ of the rectangular conductor is moved from x = 0, outwards. The uniform magnetic field is perpendicular to the plane and extends from x = 0 to x = b and is zero for x > b.



(a)

Only the arm PQ possesses substantial resistance r. Consider the situation when the arm PQ is pulled outwards from $x = 0$ to $x = 2b$, and is then moved back to $x = 0$ with constant speed v. Obtain expressions for the flux, the induced emf, the force necessary to pull the arm and the power dissipated as Joule heat. Sketch the variation of these quantities with distance.

9. Two concentric circular coils, one of small radius r_1 and the other of large radius r_2, such that $r_1 << r_2$, are placed co-axially with centres coinciding. Obtain the mutual inductance of the arrangement.

10. (a) Obtain the expression for the magnetic energy stored in a solenoid in terms of magnetic field B, area A and length l of the solenoid.

(b) How does this magnetic energy compare with the electrostatic energy stored in a capacitor?

11. Ravi peddles a stationary bicycle and the pedals of the bicycle are attached to a 100 turn coil of area 0.10 m^2. The coil rotates at half a revolution per second and it is placed in a uniform magnetic field of 0.01 T perpendicular to the axis of rotation of the coil. What is the maximum voltage generated in the coil?