6. SETS & RELATIONS

SYNOPSIS

Set Definition : Any collection of well defined objects is called a set. The objects in a set are called its ***members or elements.***

Empty Set, Non–Empty Set:

A set consisting of no elements is called an empty set or **null set** or **void set** and is denoted by the symbol ϕ or { }. Ex: (1) $\{x \in R / x^2 < 0\}$ Ex: (2) $\{x \mid x \text{ is a real number and } x^2 + 1 = 0\}$

A set which has atleast one element is called a **non-empty** set.

Singleton set : A set consisting of only one element is called a **singleton** set.

Finite Set : A set in which the process of counting of elements surely comes to an end is called a **finite set.**

Infinite Set : A set which is not finite is called an infinite set. In other words, a set in which the process of counting of elements does not come to an end is called an **infinite set.**

Cardinal Number of a finite set :

The number of distinct elements contained in a finite set A is called its **cardinal number** to be denoted by $n(A)$.

Equal Sets :

Two sets A and B are said to be equal if every elements of A is an element of B and every element of B is an element of A. i.e, $x \in A \Rightarrow x \in B$ and $y \in B \Rightarrow y \in A$, then A and B are equal, and we write $A = B$

Equivalent sets :

Two finite sets A and B are said to be equivalent, if $n(A) = n(B)$.

Clearly equal sets are equivalent, but equivalent sets need not be equal.

Equivalence of two sets is denoted by the symbol '$\sim$'. Thus if A and B are equivalent sets, we write $A \sim B$ which is read as 'A is equivalent to B'.

Subset and super set :

The set B is said to be subset of set A if every element of set B is also an element of set A. Symbolically we write it as, $B \subseteq A$ or $A \supseteq B$, where A is super set of B.

i) $B \subseteq A$ is read as B is contained in A or B is subset of A or A is super set of B.

ii) $A \supseteq B$ is read as A contains B or B is a subset of A or A is super set of B.

Proper Subset :

The set B is said to be a proper subset of set A if every element of set B is an element of A whereas some element of A is not an element of B.

We write it as $B \subset A$ and read it as 'B is a proper subset of A'. Thus B is a proper subset of A if every element of B is an element of A and there is atleast one element in A which is not in B.

Note : $\mathrm{N} \subset \mathrm{W} \subset \mathrm{Z} \subset \mathrm{Q} \subset \mathrm{R} \subset \mathrm{C}$

Power Set :

The set formed by all the subsets of a given set A is called the power set of A, it is usually denoted by $P(A)$. **If A contains n element, then $P(A)$ contains 2^n subsets.**

Universal set :

A set containing all the elements under consideration is known as universal set and it is denoted by either 'U' or 'X'.

Union of sets :

The union of two sets A and B, denoted by $A \cup B$ is the set of all those elements, each one of which is either in A or in B or in both A and B.

Intersection of sets :

The intersection of two sets A and B, denoted by $A \cap B$ is the set of all elements, common to both A and B.

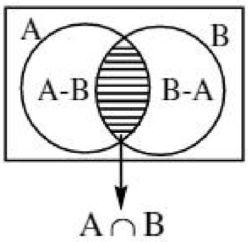
Disjoint Sets and Intersecting Sets:

i) Two sets A and B are said to be disjoint, if $A \cap B = \phi$.

ii) If $A \cap B \neq \phi$, then A and B are said to be intersecting or overlapping sets.

Difference of Sets :

If A and B are two sets, then their difference $A - B$ is the set of all those elements of A which do not belong to B.



A,B are two sets, then

(1) $A - B = \{x \mid x \in A \text{ but } x \notin B\}$

(2) $B - A = \{x \mid x \in B \text{ but } x \notin A\}$ Generally $A - B \neq B - A$

(3) $(A - B) \cap B = \phi$, $(A - B) \cup B = A \cup B$ (4) $A - (A - B) = A \cap B$

Symmetric Difference of two sets :

The symmetric difference of two sets A and B denoted by $A \Delta B$ is the set $(A - B) \cup (B - A)$.

i.e,

$$A \Delta B = (A - B) \cup (B - A) = \{x \mid x \in A \text{ or } B \text{ but } x \notin A \cap B\} = (A \cup B) - (A \cap B)$$

1) $A \Delta B = B \Delta A$ 2) $A \Delta A = \phi$ 3) $A \Delta \phi = \phi \Delta A = A$ 4) $A \Delta B = \phi \Leftrightarrow A = B$

Compliment of a set :

Let U be the universal set and $A \subset U$, then the complement of A, denoted by A' or A^C *or* $\overline{A}$ *or* $U - A$ is defined as $A' = \{x : x \in U \text{ and } x \notin A\}$

(1) $A \cup \overline{A} = U$ (2) $A \cap \overline{A} = \phi$ (3) $(A^C)^C = A$

Idempotent Laws : For any set A, we have

(i) $A \cup A = A$ (ii) $A \cap A = A$

Identity Laws : For any set A, we have (i) $A \cup \phi = A$ (ii) $A \cap U = A$

Commutative Laws : For any two sets A and B we have

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$.

Associative Laws :

If A, B and C are any three sets, then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $A \cap (B \cap C) = (A \cap B) \cap C$

Distributive Laws :

If A, B and C are any three sets, then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De-morgan's Laws : If A and B are any two sets, then (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Some very important results on Cardinal numbers :

i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A \text{ and } B$ are disjoint non void sets.

iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

iv) $n(A - B) = n(A) - n(A \cap B) = n(A \cap B') = n(A \cup B) - n(B)$

v) $n(B - A) = n(B) - n(A \cap B) = n(A' \cap B) = n(A \cup B) - n(A)$

vi) $n(A \Delta B) = n[(A - B) \cup (B - A)] = n(A) + n(B) - 2n(A \cap B) = n(A \cup B) - n(A \cap B)$

vii) $n(A') = n(U) - n(A)$

viii) $n(A' \cup B') = n(U) - n(A \cap B)$

ix) $n(A' \cap B') = n(U) - n(A \cup B)$

x) Let $n(A) = p$ and $n(B) = q$, where A and B are two sets having different elements. Then i) Max{p,q} $\leq n(A \cup B) \leq p + q$

ii) $0 \leq n(A \cap B) \leq Min\{p, q\}$

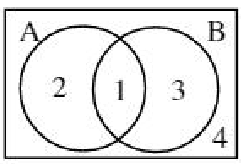
xi) No. of elements in exactly one of the sets A, B,

$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

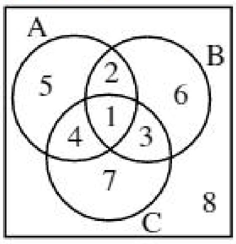
xii) No. of elements in exactly two of the sets $A, B, C = n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

III. VEN Diagrams

1) A,B are two sets then



2) A,B,C are 3 sets then



1) $A \cap B$ 2) $A \cap B'$ 1) $A \cap B \cap C$ 2) $A \cap B \cap C'$ 3) $A' \cap B \cap C$ 4) $A \cap B' \cap C$

3) $A' \cap B$ 4) $A' \cap B'$ 5) $A \cap B' \cap C'$ 6) $A' \cap B \cap C'$ 7) $A' \cap B' \cap C$ 8) $A' \cap B' \cap C'$

SYNOPSIS ON RELATIONS

1. **Ordered Pair:**

   A pair of elements listed in order in brackets is called an ordered pair denoted by (a, b)

   i) $(a,b) \neq (b,a)$ ii) $(a,b) = (c,d) \Leftrightarrow a = c, b = d$

2. If A, B are non empty sets then set of all ordered pairs (a, b) where $a \in A, b \in B,$ is called Cartesian Product of A and B. It is denoted by A×B. $A \times B = \{(a,b) / a \in A, b \in B\}$

   i) In General $A \times B \neq B \times A$

   ii) If $A \times B = B \times A \Rightarrow A = B$

   iii) $n(A) = p$, $n(B) = q$ then $n(A\times B) = pq = n(B\times A)$

   iv) $n(A) = p$, $n(B) = q$, $n(C) = r$ then $n(A\times B\times C) = pqr$

   v) $n(A \cap B) = m$ then $n\{(A \times B) \cap (B \times A)\} = m^2$

3. If A, B are non empty sets then every sub set of $A\times B$ is a relation R from A to B.

4. If R is a relation from A to B then

i) **Domain of R :** $\{a / (a,b) \in R\}$ is called the domain of R. i.e., the set of all 1st elements of all order pairs in the relation R.

ii) **Range of R :** $\{b / (a,b) \in R\}$ is the called the Range of R which is a subset i.e., the set of all 2nd elements of all ordered pairs in the Relation R.

5. i) If R is a relation from A to B and $(x, y) \in R$, then x is related to y under the relation R we write this as ***xRy***

   ii) If $n(A) = m, n(B) = n$ then

   1) the number of possible relations from A to B is 2^{mn}

   2) the number of possible relations from A to A is $2^{(m^2)}$

   3) the number of possible relations from B to B is $2^{(n^2)}$

   Note : If R, S are two relations from A to B, then $R \cup S$, $R \cap S$, $R - S$ are relations from A to B.

6. **Void Relation:** $\phi \subseteq A \times B$. So ϕ is also a relation from A to B. This relation ϕ is called as null relation or void relation.

7. **Universal Relation :** $A \times B \subseteq A \times B$. So $A \times B$ is also a relation from A to B and it is called as Universal Relation.

8. **Inverse of a Relation :** If R is a relation from A to B then R^{-1} is a relation from B to A and is defined by $R^{-1}=\{(b,a)/(a,b)\in R\}$.
   i) $aRb \Leftrightarrow bR^{-1}a$ ii) $(R^{-1})^{-1}=R$
   iii) a) Domain of R^{-1} = Range of R,   b) Range of R^{-1} = Domain of R
   iv) If $R\subseteq A\times B$ and $(a,b)\in R$ then $R^{-1}\subseteq B\times A$ and $(b,a)\in R^{-1}$
   v) If R is a relation from A to B and $R\subseteq R^{-1}$ then $R = R^{-1}$

9. **Identity Relation :** Let A be a non–empty set then the Relation $\{(a,a)/a\in A\}$ is called the identity relation on A. It is denoted by I_A
   Ex: Let $A = \{a,b,c,d\}$ then $I_A = \{(a,a), (b,b), (c,c), (d,d)\}$ is the identity relation on A.

10. **Compositive Relation :** If R is a relation from A to B and S is a relation from B to C respectively then the set of all ordered pairs (a, c) whenever $(a,b)\in R$ and $(b,c)\in S$ is called composite relation of R and S from A to C is denoted by SoR.
    $\therefore SoR=\{(a,c)/(a,b)\in R \text{ and } (b,c)\in S\}\subseteq A\times C$

Types of Relations :

1. **Reflexive Relation:** A Relation R on a set A is said to be reflexive if every element of A is related to it self. i.e., $\forall a\in A\Rightarrow(a,a)\in R$
   Note: **If a set A contains n elements then the number of reflexive relations from A to A is $2^{n(n-1)}$**
   Ex: Let $A = \{1, 2, 3\}$ be a set then $R = \{(1,1), (2,2), (3,3), (1,3), (2,1)\}$ is a Reflexive Relation on A
   But $R =\{(1,1), (3,3), (2,1), (3,2), (1,2)\}$ is not a reflexive relation on set A because $2\in A$, but $(2,2)\notin R_1$

2. **Symmetric Relation:** A Relation R on a set A is said to be symmetric **if $(a,b)\in R\Leftrightarrow(b,a)\in R$**
   Note: i) **If a set A contains n elements then number of symmetric relations from A to A is $2^{\frac{n(n+1)}{2}}$**
   ii) If R is a symmetric Relation then $R = R^{-1}$
   Ex: Let $A = \{1, 2, 3, 4\}$ be a set then $R = \{(1,3)(1,4)(3,1)(2,2)(4,1)\}$ is a Symmetric Relation on A
   But $R_1 = \{(1,1), (2,2), (3,3), (1,3)\}$ is not a Symmetric relation on set A because $(1,3)\in R_1$, but $(3,1)\notin R_1$

3. **Anti Symmetric Relation :** If R is a Relation on a set A such that **If aRb and bRa then $a = b$,** then R is called Anti Symmetric Relation on A.
   Note : A relation which not Symmetric need not be Anti Symmetric.
   Ex: The relation '$\leq$' on the set R of real numbers is Anti Symmetric because **$a\leq b$ and $b\leq a$ $\Rightarrow a=b\ \forall a,b\in R$**

4. **Transitive Relation :** A Relation R on a set A is said to be transitive if **aRb and $bRc\Rightarrow aRc\ \forall a,b,c\in A$**
   Ex: On the set N,R is defined by $aRb\Rightarrow a<b$ is transitive because $a,b,c\in N$, $a<b$ **and** $b<c\Rightarrow a<c$

5. **Equivalence Relation :** A relation on a set A is said to be an equivalence relation on A, if it is
   i) Reflexive   ii) Symmetric and   iii) Transitive

6. **Partial Order Relation :** A Relation R on a set A is said to be a partial ordered relation on A if it is
   i) Reflexive   ii) Anti Symmetric   and   iii) Transitive
   Note : In Real Numbers $<$, $\leq$; in sets $\subset,\subseteq$ are not equivalence relations.

LECTURE SHEET

EXERCISE

1. The number of elements in the set. $\{(a, b)/\ 2a^2 + 3b^2 = 35,\ a,\ b \in z\}$ when z is the set of all integers is
 1) 2 2) 4 3) 8 4) 12

2. If sets A and B are defined as $A = \{(x,y)/y = e^x, x \in R\}$, $B = \{(x,y)/y = x, x \in R\}$, then
 1) $B \subset A$ 2) $A \subset B$ 3) $A \cap B = \phi$ 4) $A \cup B = A$

3. If $A = \{(x,y)/x^2 + y^2 \le 4;\ \ x, y \in R\}$ and $B = \{(x,y)/x^2 + y^2 \ge 9;\ x, y \in R\}$, then
 1) $A - B = \phi$ 2) $B - A = \phi$ 3) $A \cap B \neq \phi$ 4) $A \cap B = \phi$

4. If sets A and B are defined as $A = \left\{(x,y)/y = \frac{1}{x}, 0 \neq x \in R\right\}, B = \{(x,y)/y = -x,\ \ x \in R\}$, then
 1) $A \cap B = A$ 2) $A \cap B = B$ 3) $A \cap B = \phi$ 4) $A \cap B = A \cup B$

5. Let $A = \{(x,y)/y = e^x, x \in R\}, B = \{(x,y)/y = e^{-x}, x \in R\}$, then
 1) $A \cap B = \phi$ 2) $A \cap B \neq \phi$ 3) $A \cup B = N$ 4) $A \cup B = Z$

6. If $A = \left\{(x,y)/y = \frac{4}{x}, x \neq 0\right\}$ and $B = \{(x,y)/x^2 + y^2 = 8,\ \ x, y \in R\}$, then
 I) $A \cap B = \phi$ II) $A \cap B \neq \phi$
 III) $A \cap B$ contains two points only
 IV) $A \cap B$ contains 4 points only
 The true statements are :
 1) I, II 2) II, III 3) III, IV 4) II, IV

7. If $A = \{(x,\ y)/\ x^2 + y^2 = 25\}$ $B = \{(x,y)/x^2+9y^2 = 144\ \}$ then $A \cap B$ contains
 1) one point 2) three points 3) two points 4) four points

8. If $A = \{(x,y)/y^2 = x,\ \ x, y \in R\}$ and $B = \{(x,y)/y = |x|,\ \ x, y \in R\}$, then
 I) $A \cap B = \phi$ II) $A \cap B \neq \phi$
 III) $A \cap B$ contains two points only
 IV) $A \cap B$ contains three points only. The true statements are :
 1) I, II 2) II, III 3) III, IV 4) II, IV

9. A set A has 3 elements and another set B has 6 elements. Then
 1) $3 \le n(A \cup B) \le 6$ 2) $3 \le n(A \cup B) \le 9$ 3) $6 \le n(A \cup B) \le 9$ 4) $0 \le n(A \cup B) \le 9$

10. Let U be the universal set for sets A and B. If $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then $n(A' \cap B')$ is equal to 300, provided that $n(U)$ is equal to
 1) 600 2) 700 3) 800 4) 900

11. If $n(U) = 60$, $n(A) = 35$, $n(B) = 24$ and $n(A \cup B)' = 10$ then $n(A \cap B)$ is

1) 9 2) 8 3) 6 4) 7

12. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total nuber of subsets of the second set. The values of m and n are

1) $m = 7, n = 6$ 2) $m = 6, n = 3$ 3) $m = 5, n = 1$ 4) $m = 8, n = 7$

13. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, then

1) $x = 39$ 2) $x = 63$ 3) $39 \le x \le 63$ 4) $30 \le x \le 80$

14. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of the set of all determinants with value -1, then

1) C is empty 2) B has as many elements as C

3) $A = B \cup C$ 4) $n(B) = 2\ n(C)$

15. Out of 800 boys in a school 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is

1) 160 2) 240 3) 216 4) 128

16. Each student in a class of 40, studies at least one of the subjects English, Mathematics and Economics. 16 study English, 22 Economics and 26 Mathematics, 5 study English and Economic, 14 Mathematics and Economics and 2 study all the three subjects. The number of students who study English and Mathematics but not Economics is

1) 7 2) 5 3) 10 4) 4

17. A group of 123 workers went to a canteen for cool drinks, ice - cream and tea; 42 workers took ice-cream ; 36 tea and 30 cool drinks ; 15 workers purchased ice cream and tea; 10 ice cream and cool drinks; 4 cool drinks and tea but not ice cream; 11 took ice cream and tea but not cool drinks. Number of workers that did not purchase anything is

1) 54 2) 64 3) 56 4) 44

18. In a class of 60 students, 23 play Hockey, 15 play Basket ball and 20 play cricket. 7 play Hockey and BasKet Ball, 5 play cricket and Basket ball, 4 play Hockey and Cricket and 15 students do not play any of these games. Then

1) 4 play Hockey, Basket ball and Cricket

2) 20 play Hockey but not Cricket

3) 1 plays Hockey and Cricket but not Basket ball

4) all above are correct

19. In a certain town 25% families own a phone and 15% own a car, 65% families own neither a phone nor a car, 2000 families own both a car and a phone. Consider the following statements in this regard.

A) 10% families own both a car and a phone B) 35% familes own either a car or a phone

C) 40,000 families live in the town. Which of the following statements are correct

1) A and B 2) A and C 3) B and C 4) A, B and C

Relations

20. If the relation $R : A \to B$ where $A=\{1,2,3,4\}$, $B = \{1, 3, 5\}$ is defined by $R = \{(x,y): x < y,\ \ x \in A,\ \ y \in B\}$ then $RoR^{-1} =$

1) $\{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$ 2) $\{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$

3) $\{(3, 3), (3, 5), (5, 3), (5, 5)\}$ 4) $\{(3, 3), (3, 4), (4, 5)\}$

21. Let A be set of first ten natural numbers and R be a relation on A, defined by $(x, y) \in R \Rightarrow x + 2y = 10$, then domain of R is

1) $\{1, 2, 3, ..., 10\}$ 2) $\{2, 4, 6, 8\}$ 3) $\{1, 2, 3, 4\}$ 4) $\{2, 4, 6, 8, 10\}$

22. Let A be set of first ten natural numbers and R be a relation on A, defined by $(x, y) \in R \Rightarrow x + 2y = 10$. Then range of R is

1) $\{1, 2, 3,....., 10\}$ 2) $\{2, 4, 6, 8\}$ 3) $\{1, 2, 3, 4\}$ 4) $\{2, 4, 6, 8, 10\}$

23. Let L denote the set of all straight lines in a plane . Let a relation R defined on L by $XRY \Leftrightarrow X \perp Y;\ \ X, Y \in L$ then R is

1) only reflexive 2) only symmetric 3) only transitive 4) equivalence

24. If R is a relation on Z defined by $xRy \Leftrightarrow x$ divides y then R is

1) reflexive and symmetric 2) reflexive and transitive

3) symmetric, transitive 4) equivalence

25. Consider the non empty set consisting of children in a house, consider a relation R ; xRy iff x is brother of y then R is

1) symmetric but not transitive 2) transitive but not symmetric and reflexive

3) neither symmetric nor transitive 4) both symmetric and transitive

26. Let S be the set of all real numbers. For $a, b \in S$, relation R is defined by aRb iff $|a-b|<1$ then R is

1) only reflexive 2) only symmetric

3) only transitive 4) reflexive &symmetric

27. Let R be a relation defined on the set of real numbers by $aRb \Leftrightarrow 1 + ab > 0$ then R is

1) reflexive & symmetric 2) transitive

3) anti symmetric 4) equivalence

28. Let $A = \{1, 2, 3, 4, 5\}$ and a relation on it is $R = \{(x, y) / x, y \in A$ and $x+y=5\}$ then R is

1) not reflexive, not symmetric but transitive 2) not reflexive, not transitive but symmetric

3) not reflexive, not symmetric, not transitive 4) equivalence

29. For $x, y \in R$, define a relation R by xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then R is

1) an equivalence relation
2) symmetric
3) transitive
4) reflexive but not symmetric & transitive

30. Let n be a positive integer. If R is a relation defined on Z as $xRy \Leftrightarrow x - y$ is divisible by n then R is

1) reflexive
2) symmetric
3) transitive
4) equivalence

31. The minimum number of elements that must be added to the relation $R = \{(1, 2), (2, 3)\}$ on the set $\{1, 2, 3\}$ so that it is an equivalence relation

1) 3
2) 5
3) 6
4) 7

32. S is a relation over the set R of all real numbers and it is given by $(a,b) \in S \Leftrightarrow ab \geq 0$. Then S is

1) symmetric and transitive only
2) reflexive and symmetric only
3) a partial order relation
4) an equivalence relation

33. Two points P and Q in a plane are related if $OP = OQ$, where O is a fixed point. This relation is

1) partial order relation
2) equivalence relation
3) reflexive but not symmetric
4) reflexive but not transitive

34. Let W denote the words in the English dictionary, Define the relation R by : $R = \{(x, y) \in W \times W$ the words x and y have atleast one letter in common$\}$ Then R is

1) reflexive, symmetric and not transitive
2) reflexive, symmetric and transitive
3) reflexive, not symmetric and transitive
4) not reflexive, symmetric and transitive

35. If $R = \{(x, y) / x, y \in Z,\ x^2 + y^2 \leq 4\}$ is a relation in Z, then domain of R is

1) $\{0, 1, 2\}$
2) $\{0, -1, -2\}$
3) $\{-2, -1, 0, 1, 2\}$
4) $\{0, 1, 2, 3\}$

36. R is a relation from $\{11, 12, 13\}$ to $\{8, 10, 12\}$ defined by $y = x-3$. Then R^{-1} is

1) $\{(8, 11), (10, 13)\}$
2) $\{(11, 8), (13, 10)\}$
3) $\{(10, 13), (8, 11)\}$
4) $\{(11, 8), (10, 13), (12, 15)\}$

37. If a relation R is defined on the set Z of integers as follows : $(a, b) \in R \Leftrightarrow a^2 + b^2 = 25$ then, domain(R)=

1) $\{3, 4, 5\}$
2) $\{0, 3, 4, 5\}$
3) $\{0, \pm 3, \pm 4, \pm 5\}$
4) $\{3, 4\}$

38. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is

1) a function
2) reflexive
3) not symmetric
4) transitive

39. Let R = $\{(3, 3), (6, 6), (9, 9)\ (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be relation on the set A = $\{3, 6, 9, 12\}$. The relation is

1) an equialence relation
2) reflexive and symmetric only
3) reflexive and transitive only
4) reflexive only

40. Let X be a non empty set and $P(X)$ be the set of all subsets of X. For $A, B \in P(X)$, power set of X, ARB iff $A \cap B \neq \phi$ then the relation is

1) only reflexive
2) only symmetric
3) only transitive
4) equivalence relation

41. N is the set of natural numbers and R is a relation on $N \times N$ defined by $(a, b)\, R(c, d)$ if and only if $a+d = b+c$ then R is

1) only reflexive
2) only symmetric
3) only transitive
4) equivalence relation

42. Let R be the real line. Consider following subsets of the plane R × R. $S = \{(x, y): y = x + 1$ and $0 < x < 2\}$, $T = \{(x,y): x - y \text{ is an integer}\}$. Which one of the following is true ?

1) Neither S nor T is an equivalence relation on R
2) Both S and T are equivalence relations on R
3) S is an equivalence relation on R but T is not
4) T is an equivalence relation on R but S is not

43. Consider the relations $R = \{(x,y) | x,y$ are real numbers and $x = wy$ for some rational number w$\}$;
$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) | m,n,p \text{ and } q \text{ are integers such that } n,q \neq 0 \text{ and } qm = pn\right\}$. Then

1) R is an equivalence relation but S is not an equivalence relation
2) neither R nor S is an equivalence relation
3) S is an equivalence relation but R is not an equivalence relation
4) R and S are both equivalence relation

44. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

1) $A = B$
2) $A = C$
3) $B = C$
4) $A \cap B = \phi$

45. Let R be the set of real numbers

Statement-1 : $A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R

Statement-2 : $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number α $\}$ is an equivalence relation on R.

1) Statement-1 is true, Statement-2 is false
2) Statement-1 is false, Statement-2 is true
3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
4) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

46. Let $X = \{1,2,3,4,5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X, Z \subseteq X$ and $Y \cap Z$ is empty, is

1) 5^2
2) 3^5
3) 2^3
4) 5^3

Numerical value type questions

47. Suppose $A_1, A_2, \ldots\ldots, A_{30}$ are thirty sets each with five elements and $B_1, B_2, \ldots\ldots, B_n$ are n sets each with three elements such that $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$. If each element of S belongs to exactly ten of the $A_i's$ and exactly 9 of the $B_j's$, then the value of n is

48. An investigator interviewed 100 students to determine their preferences for the three drinks ; milk(M), coffee (C) and tea (T). He reported the following : 10 students had all the three drinks M, C, T; 20 had M and C only; 30 had C and T; 25 had M and T; 12 had M only; 5 had C only; 8 had T only. The number of students that did not take any of the three drinks

49. A survey of 500 television viewers produced the following information, 285 watch foot ball, 195 watch hockey, 115 watch basket ball, 45 watch foot ball and basket ball, 70 watch foot ball and hockey, 50 watch hockey and basket ball, 50 do not watch any of the three games. The number of viewers, who watch exactly one of the three games is

50. Let A and B be two sets containing 2 elements respectively. The number of subsets of $A\times B$ having 3 or more elements is

51. If A={1, 2, 3}, the number of reflexive relation in A is

PRACTICE SHEET

EXERCISE

1. If $X=\{8^n-7n-1/n\in N\}$ and $Y=\{49(n-1)/n\in N\}$, then
   1) $X\subset Y$
   2) $Y\subset X$
   3) $X=Y$
   4) information not sufficient

2. Let $A = \{x : x$ is a multiple of $3\}$ and $B = \{x/x$ is a multiple of $5\}$. Then $A\cap B$ is given by
   1) {3, 6, 9, ...} 2) {5, 10, 15, 20, ...} 3) {15, 30, 45, ...} 4) {30, 60, 90, ...}

3. If $aN=\{ax/x\in N\}$. The set $3N\cap 7N=$
   1) {3, 6, 9, 12, ..} 2) {7, 14, 21, 28, ...} 3) {21, 42, 63, 84, ...} 4) {5, 10, 15, ...}

4. If $X=\{4^n-3n-1/n\in N\}$ and $Y=\{9(n-1)/n\in N\}$, then $X\cup Y$ is equal to
   1) X 2) Y 3) N 4) $X–Y$

5. If $aN=\{ax/x\in N\}$ and $bN\cap cN=dN$, where $b,c\in N$ are relatively prime, then
   1) $d = bc$ 2) $c = bd$ 3) $b = cd$ 4) none

6. The set $A = \{x{:}x\in R, x^2=16, \text{ and } 2x = 6\}$ equals
   1) ϕ 2) {14, 3, 4} 3) {3} 4) {4}

7. If A is the set of the divisors of the number 15, B is the set of prime numbers smaller than 10 and C is the set of even numbers smaller than 9, then $(A\cup C)\cap B$ is the set
   1) {1, 3, 5} 2) {1, 2, 3} 3) {2, 3, 5} 4) {2, 5}

8. If $A=\{\phi,\{\phi\}\}$, then the power set of A is
   1) A 2) $\{\phi, \{\phi\}, A\}$ 3) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$ 4) $\{\phi, \{\phi\}\}$

9. Which of the following is a null set ?
   1) {0}
   2) $\{x/x>0 \text{ or } x<0\}$
   3) $\{x/x^2=4 \text{ or } x=3\}$
   4) $\{x/x^2+1=0, x\in R\}$

10. $A = \{n / n$ is a digit in the number 33591$\}$ and $B = \{n / n \in N,\ \ n < 10\}$, then $B - A =$

1) $\{2, 4, 6, 8\}$ 2) $\{7, 2, 4, 8, 6\}$
3) $\{1, 3, 5, 7\}$ 4) $\{(1, 2), (1, 3), (2, 3)\}$

11. Consider the following equations

a) $A - B = A - (A \cap B)$ b) $A = (A \cap B) \cup (A - B)$
c) $A - (B \cup C) = (A - B) \cup (A - C)$

Which of these is / are correct ?

1) a and c 2) b only 3) b and c 4) a and b

12. Let A and B be two sets such that $A \times B$ has 6 elements. If three elements of $A \times B$ are $\{(1, 4), (2, 6), (3, 6)\}$, then

1) $A = \{1, 2\}$ and $B = \{3, 4, 6\}$ 2) $A = \{4, 6\}$ and $B = \{1, 2, 3\}$
3) $A = \{1, 2, 3\}$ and $B = \{4, 6\}$ 4) $A = \{1, 2, 4\}$ and $B = \{3, 6\}$

13. Let A be a non-empty set such that $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Then

1) $A = \{-1, 0\}$ 2) $A = \{0, 1\}$ 3) $A = \{-1, 0, 1\}$ 4) $A = \{-1, 1\}$

14. Let A and B be two sets such that $A \times B = \{(a,1),(b,3),(a,3),(b,1),(a,2),(b,2)\}$ then

1) $A = \{1, 2, 3\}$ and $B = \{a, b\}$ 2) $A = \{a, b\}$ and $B = \{1, 2, 3\}$
3) $A = \{1, 2, 3\}$ and $B \subset \{a, b\}$ 4) $A \subset \{a, b\}$ and $B \subset \{1, 2, 3\}$

15. Let A and B be two non-empty sets having n elements in common. Then, the number of elements common to $A \times B$ and $B \times A$ is

1) $2n$ 2) n 3) n^2 4) n^3

16. For any three sets A, B and C, $A \times (B' \cup C')'$ equals

1) $(A \times B) \cap (A \times C)$ 2) $(A \times B) \cup (B \times C)$ 3) $(A \times C) \cap (B \times C)$ 4) $(A \times C) \cup (B \times C)$

17. If $A = \{1,3,5,7,9,11,13,15,17\}$, $B = \{2,4,.......,18\}$ and N is the universal set, then $A' \cup ((A \cup B) \cap B')$ is

1) A 2) N 3) B 4) R

18. Let $S_1 = \{1,2,3,....,20\}$ $S_2 = \{a,b,c,d,e\}$ $S_3 = \{a,c,e,f\}$ then the number of elements of $(S_1 \times S_2) \cap (S_1 \times S_3)$ is

1) 60 2) 80 3) 100 4) 40

19. The number of non – empty subsets of the set $\{1,2,3,4\}$ on

1) 14 2) 15 3) 16 4) 17

20. If $n(A) = 4$, $n(B) = 3$, $n(A \times B \times C) = 24$, then $n(C)$ is equal to

1) 288 2) 1 3) 2 4) 12

21. If $A = \{a, b\}$, $B = \{c, d\}$, $C = \{d, e\}$ then $\{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)\}$ is equal to

1) $A \cap (B \cup C)$ 2) $A \cup (B \cap C)$ 3) $A \times (B \cup C)$ 4) $A \times (B \cap C)$

22. If $A = \{1, 2, 4\}$, $B = \{2, 4, 5\}$, $C = \{2, 5\}$ then $(A - B) \times (B - C) =$

1) $\{(1, 2), (1, 5), (2, 5)\}$ 2) $\{(1, 4)\}$ 3) $(1, 4)$ 4) $\{(1, 2)\}$

23. If $A = \{x / x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$, then $A \times (B \cap C) =$

1) $\{(2, 4), (3, 4)\}$ 2) $\{(4, 2), (4, 3)\}$

3) $\{(2, 4), (3, 4), (4, 4)\}$ 4) $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

24. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 3, 6, 7\}$ then the number of elements in $(A \times B) \cap (B \times A) =$

1) 4 2) 6 3) 2 4) 18

25. In a committee 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. The number of persons speaking at least one of these two languages is

1) 60 2) 40 3) 38 4) 22

26. In a city, three daily newspapers *A, B, C* are published. 42% of the people in that city read *A*, 51% read *B* and 68% read *C*, 30% read *A* and *B*, 28% read *B* and *C* ; 36% read *A* and *C* ; 8% do not read any of the three newspapers. The percentage of persons who read all the three papers is

1) 25% 2) 18% 3) 20% 4) 30%

Relations :

27. If a set A has n elements then number of relations defined on A is

1) $2^{(n^2)}$ 2) $2^{n^2} - 1$ 3) 2^n 4) 2^{2n}

28. $A = \{1, 2, 3, 4, 5\}$, Relation R on A is defined by $R = \{(x, y) / x < y \text{ and } |x^2 - y^2| < 9\}$ then $R =$

1) $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$ 2) $\{(2, 1), (3, 2), (3, 2), (4, 3), (5, 4)\}$

3) $\{(1, 2), (1, 3), (2, 3), (3, 4), (4, 5)\}$ 4) $\{(1, 2), (1, 3), (2, 3), (3, 4)\}$

29. A relation R is defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by $xRy \Leftrightarrow x$ is relatively prime to y. Then domain of R is

1) $\{2, 3, 5\}$ 2) $\{3, 5\}$ 3) $\{2, 3, 4\}$ 4) $\{2, 3, 4, 5\}$

30. If $A = \{1, 2, 3\}$, $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by xRy iff 'x is greater than y'. The range of R is

1) $\{1, 4, 6, 9\}$ 2) $\{4, 6, 9\}$ 3) $\{1\}$ 4) $\{1, 2\}$

31. Let $P = \{(x, y) / x \in R, y \in R, x^2 + y^2 = 1\}$, then P is

1) reflexive 2) symmetric

3) anti-symmetric 4) equivalence

32. Let L denote the set of all straight lines in a plane. Let a Relation R be defined on L by $xRy \Leftrightarrow x$ is parllel to y. Then R is

1) only symmetric 2) only transitive

3) anti symmetric 4) equivalence relation

33. On the set of integers Z, relation R is defined as "$mRn \Leftrightarrow m$ is an integral multiple of n" then R is

1) reflexive, symmetric
2) reflexive, transitive
3) symmetric, transitive
4) equivalence

34. If $A = \{a, b, c, d\}$, then a relation $R = \{(a,b), (b,a), (a,a)\}$ on A is

1) symmetric and transitive only
2) reflexive and transitive only
3) symmetric only
4) transitive only

35. The relation $R = \{(1, 1), (2, 2), (3, 3)\}$ on the set $\{1, 2, 3\}$ is

1) symmetric only
2) reflexive only
3) transitive only
4) an equivalence relation

36. For $x, y \in I$, the relation R is defined by xRy if and only if $x \le y$ then R is

1) partial order relation
2) equivalence relation
3) reflexive and symmetric
4) symmetric and transitive

37. Which of the following is not an equivalence relation on set of integers ?

1) aRb if $a + b$ is an even integer
2) aRb if $a - b$ is an even integer
3) aRb if $a < b$
4) aRb if $a = b$

38. If A is a non empty set then the relation $\subseteq$ (subset) on the power set of A is

1) only reflexive
2) only symmetric
3) only equivalence
4) partial order relation

39. In the set Z of all integers, which of the following relation R is not an equivalence relation ?

1) $xRy : if\ x \le y$
2) $xRy : if\ x = y$
3) $xRy : if\ x - y$ is an even integer
4) $xRy : if\ x = y$ (mod 3)

40. Two sets A and B are as $A = \{(a, b) \in R \times R\ :\ |a - 5| < 1 | b - 5| < 1\}$

$B = \{(a, b) \in R \times R\ :\ 4(a-6)^2 + 9(b-5)^2 \le 36\}$. Then

1) $A \subset B$
2) $A \cap B = \phi$
3) Neither $A \subset B$ nor $B \subset A$
4) $B \subset A$

Numerical value type questions

41. In a group of 1000 people, each can speak eiher Hindi or English. There are 750 people who can speak Hindi and 400 who can speak English. Then number of persons who can speak Hindi only is

42. In a class of 35 students, 17 have taken Mathematics, 10 have taken Mathematics but not Economics. If each student has taken either Mathematics or Economics or both, then the number of students who have taken Economics but not Mathematics is

43. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% buy all the three newspapers, the number of families which buy none of A, B, C is

44. A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. The least number they must have liked both products is

45. Let A be the set of first 10 natural numbers and let $R=\{(x,y)/x\in A, y\in N \text{ and } x+2y=10\}$ then $n\{\text{dom}(R^{-1})\}=$

KEY SHEET

LECTURE SHEET

1) **3**	2) **3**	3) **4**	4) **3**	5) **2**	6) **2**	7) **4**	8) **2**	9) **3**	10) **2**
11) **1**	12) **2**	13) **3**	14) **2**	15) **1**	16) **2**	17) **4**	18) **3**	19) **3**	20) **3**
21) **2**	22) **3**	23) **2**	24) **2**	25) **2**	26) **4**	27) **1**	28) **2**	29) **4**	30) **4**
31) **4**	32) **2**	33) **2**	34) **1**	35) **3**	36) **1**	37) **3**	38) **3**	39) **3**	40) **2**
41) **4**	42) **4**	43) **3**	44) **3**	45) **1**	46) **2**	47) **45**	48) **10**	49) **325**	50) **219**
51) **64**									

PRACTICE SHEET

1) **1**	2) **3**	3) **3**	4) **2**	5) **1**	6) **1**	7) **3**	8) **3**	9) **4**	10) **2**
11) **4**	12) **3**	13) **3**	14) **2**	15) **3**	16) **1**	17) **2**	18) **1**	19) **2**	20) **3**
21) **3**	22) **2**	23) **1**	24) **1**	25) **1**	26) **1**	27) **1**	28) **4**	29) **4**	30) **3**
31) **2**	32) **4**	33) **2**	34) **3**	35) **4**	36) **1**	37) **3**	38) **4**	39) **1**	40) **1**
41) **600**	42) **18**	43) **4000**	44) **170**	45) **4**					