2. DEFINITE INTEGRALS

SYNOPSIS

1. If $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$
2. $\int_a^a f(x)\,dx = 0$
3. $\int_a^b f(x)\,dx = \int_a^b f(y)\,dy = \int_a^b f(t)\,dt$
4. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
5. If $a < c < b$ then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ (where $c \in R$)
6. $\int_a^b f(x)\,dx = \int_a^{c_1} f(x)\,dx + \int_{c_1}^{c_2} f(x)\,dx + ... + \int_{c_n}^{b} f(x)\,dx$ where $a < c_1 < c_2 < .... < c_{n-1} < c_n < b$
7. $\int_0^a f(x)\,dx = \int_0^{a/2} f(x)\,dx + \int_0^{a/2} f(a-x)\,dx$
8. $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$
9. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$
10. $\int_a^b f(x)dx = 0$ if $f(a+x) = -f(b-x)$
11. $\int_a^b f(x)\,dx = 2\int_a^{\frac{a+b}{2}} f(x)\,dx$ if $f(a+x) = f(b-x)$
12. $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$; if $f(x)$ is an even function

    $= 0$; if $f(x)$ is an odd function
13. $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$; if $f(2a-x) = f(x) = 0$; if $f(2a-x) = -f(x)$.
14. $\int_0^a x f(x) dx = \frac{a}{2}\int_0^a f(x) dx$ if $f(a-x) = f(x) = a \cdot \int_0^{a/2} f(x)\,dx$
15. $\int_a^b x\, f(x)\,dx = \frac{a+b}{2}\int_a^b f(x)\,dx$ if $f(a+b-x) = f(x)$
16. If $f(x)$ is a periodic function with period T then $\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx$, $n \in N$.

17. If $f: R \to R$ is a continuous periodic function with period T and $a \in R$ and n is a positive integer then $\int_a^{a+nT} f(x)\,dx = n\int_a^{a+T} f(x)\,dx = n\int_0^T f(x)\,dx \,, n \in N$

18. If $f(x)$ is a periodic function with period T and $a \in R^+$, then $\int_{nT}^{a+nT} f(x)\,dx = \int_0^a f(x)\,dx \,, n \in N$

19. If $f(x)$ is a periodic function with period T then

i) $\int_{a+T}^{b+T} f(x)\,dx = \int_a^b f(x)\,dx$ or $\int_{a+nT}^{b+nT} f(x)\,dx = \int_a^b f(x)\,dx$ where $n \in Z$

ii) $\int_{mT}^{nT} f(x)dx = (n-m)\int_0^T f(x)dx$ where $m, n \in Z$.

20. $\frac{d}{dx}\left(\int_{\varphi(x)}^{\phi(x)} f(t)\,dt\right) = f(\phi(x))\,\phi'(x) - f(\varphi(x))\,\varphi'(x)$

21. If $f(x) \ge 0$ in $[a, b]$ then $\int_a^b f(x)\,dx \ge 0$

22. If $f(x) \le g(x)$ on $[a, b]$ then $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$.

23. If m, M are smallest and greatest values of a function $f(x)$ defined on $[a, b]$ then $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$

24. If $I_n = \int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx$, then $I_n = \frac{n-1}{n} \cdot I_{n-2}$ where $n \in N$.

$I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)....\frac{1}{2}\cdot\frac{\pi}{2}$; n is even$=\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)....\frac{2}{3}$; n is odd.

25. If $I_n = \int_0^{\pi/4} \tan^n x\;dx$ then $I_n = \frac{1}{n-1} - I_{n-2}$ and hence $I_n = \frac{1}{n-1} - \frac{1}{n-3} + \frac{1}{n-5} - \frac{1}{n-7} + ...I_0$ or I_1 according as n is even or odd where $I_0 = \frac{\pi}{4}$ and $I_1 = \frac{1}{2} ln 2$

26. $\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x)dx = \frac{1}{n-1}$

27. $\int_{\pi/4}^{\pi/2} (\cot^n x + \cot^{n-2} x)dx = \frac{1}{n-1}$

28. If $I_n = \int_0^{\pi/4} \sec^n x\;dx$ then $I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$.

29. $\int_0^{\pi/2} Sin^m x \cdot Cos^n x\,dx = \frac{[(m-1)(m-3)(m-5)....][(n-1)(n-3)(n-5)...]}{(m+n)(m+n-2)(m+n-4)(m+n-6).....} \cdot K$ If m and n are both even then $K = \frac{\pi}{2}$, otherwise $K = 1$.

30. $\int_0^{\pi} \sin^n x\,dx = 2\int_0^{\pi} \sin^n x\,dx$ for all positive integral values of n.

Standard Results :

1. $\int_a^b \sqrt{\frac{x-a}{b-x}}\, dx = \frac{\pi}{2}(b-a)$

2. $\int_a^b \sqrt{(x-a)(b-x)}\, dx = \frac{\pi}{8}(b-a)^2$

3. $\int_a^b \frac{1}{\sqrt{(x-a)(b-x)}}\, dx = \pi$

4. $\int_0^{\pi/2} \frac{f(\text{Sin}x)}{f(\text{Sin}x)+f(\text{Cos}x)}\, dx = \frac{\pi}{4}$

5. $\int_0^{\pi/2} \frac{f(\tan x)}{f(\tan x)+f(\cot x)}\, dx = \frac{\pi}{4}$

6. $\int_0^{\pi/2} \frac{f(\sec x)}{f(\sec x)+f(\text{cosec}\, x)}\, dx = \frac{\pi}{4}$

7. $\int_0^{\pi/2} \frac{a\sin x+b\cos x}{\sin x+\cos x}\, dx = (a+b)\frac{\pi}{4}$

8. $\int_0^{\pi/2} \frac{a\tan x+b\cot x}{\tan x+\cot x}\, dx = (a+b)\frac{\pi}{4}$

9. $\int_0^{\pi/2} \frac{a\sec x+b\,\text{cosec}\, x}{\sec x+\text{cosec}\, x}\, dx = (a+b)\frac{\pi}{4}$

10. $\int_0^{\pi/2} ln(\sin x)dx = -\frac{\pi}{2} ln 2$

11. $\int_0^{\pi/2} ln\, Cosx\, dx = -\frac{\pi}{2} ln 2$

12. $\int_0^{\pi/2} ln \tan x\, dx = 0$

13. $\int_0^{\pi/2} ln\, Cotx\, dx = 0$

14. $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x} = \frac{\pi}{2ab}$

15. $\int_0^{\infty} \frac{1}{(x+\sqrt{x^2+1})^n}\, dx = \frac{n}{n^2-1}$

16. $\int_0^{n} [x]dx = \frac{n(n-1)}{2}$

LECTURE SHEET

EXERCISE-I

Properties of Definite integrals

LEVEL-I (MAIN)

Single answer type questions

1. $\int_0^{\pi/2} \frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx =$

   1) 0   2) 3   3) 2   4) 1

2. $\int_0^{\pi} \frac{1}{1+\sin x}dx =$

   1) 1   2) 2   3) –1   4) –2

3. $\int_0^{\pi/4} \frac{\text{Sin}^9 x}{\text{Cos}^{11} x}dx =$

   1) $\frac{1}{9}$ 2) $\frac{1}{10}$ 3) $\frac{1}{99}$ 4) $\frac{1}{90}$

4. $\int_0^1 \sqrt{x(1-x)}\ dx =$

   1) $\frac{\pi}{8}$ 2) $\frac{3\pi}{8}$ 3) $\frac{5\pi}{4}$ 4) $\frac{\pi}{2}$

5. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$ is

   1) $2\sqrt{2}$ 2) 2   3) π 4) $-\sqrt{2}$

6. $\int_0^1 \frac{dx}{x+\sqrt{x}} =$

   1) $ln\ 2$ 2) $ln\ 2+1$ 3) $2ln\ 2$ 4) $2ln\ 2-1$

7. For $a>1$, $b>1$; the value of $\int_0^{\infty}(a^{-x}-b^{-x})dx =$

   1) $ln(ab)$ 2) $\frac{1}{ln(ab)}$ 3) $\frac{1}{ln\,a}+\frac{1}{ln\,b}$ 4) $\frac{1}{ln\,a}-\frac{1}{ln\,b}$

8. If $\int_{ln\,2}^{x}(e^x-1)^{-1}dx = ln\left(\frac{3}{2}\right)$ then $x =$

   1) $ln\ 4$ 2) 1   3) e^2 4) $\frac{1}{e}$

9. $\int_0^{ln5} \frac{e^x\sqrt{e^x-1}}{e^x+3}dx =$

1) $\pi+2$ 2) $\pi-2$ 3) $4-\pi$ 4) $4+\pi$

10. $\int_2^3 \frac{1}{x^2-x}dx =$

1) $ln\frac{2}{3}$ 2) $ln\frac{4}{3}$ 3) $ln\frac{8}{3}$ 4) $ln\frac{1}{4}$

11. $\int_0^3 \frac{3x+1}{x^2+9}dx =$

1) $ln(2\sqrt{2})+\frac{\pi}{12}$ 2) $ln(2\sqrt{2})+\frac{\pi}{2}$ 3) $ln(2\sqrt{2})+\frac{\pi}{6}$ 4) $ln(2\sqrt{2})+\frac{\pi}{3}$

12. $\int_0^{\pi/4} \frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx =$

1) $\frac{1}{3}$ 2) $\frac{1}{6}$ 3) $\frac{1}{2}$ 4) 1

13. $\int_1^{32} \frac{dx}{x^{1/5}\sqrt{1+x^{4/5}}} =$

1) $\frac{2}{5}\left(\sqrt{17}+\sqrt{2}\right)$ 2) $\frac{2}{5}\left(\sqrt{17}-\sqrt{2}\right)$ 3) $\frac{5}{2}\left(\sqrt{17}-\sqrt{2}\right)$ 4) $\frac{5}{2}\left(\sqrt{17}+\sqrt{2}\right)$

14. $\int_2^4 \frac{\sqrt{x^2-4}}{x^4}dx =$

1) $\frac{3}{32}$ 2) $\frac{\sqrt{3}}{32}$ 3) $\frac{3}{8}$ 4) $\frac{\sqrt{3}}{8}$

15. $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx =$

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) π

16. $\int_{\pi/4}^{\pi/2} (\sqrt{\tan x}+\sqrt{\cot x})dx =$

1) $\frac{\pi}{2\sqrt{2}}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{\sqrt{2}}$ 4) $\frac{\pi}{3}$

17. If $\int_0^{\infty} \frac{x^2dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$ then $\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)} =$

1) $\frac{\pi}{60}$ 2) $\frac{\pi}{20}$ 3) $\frac{\pi}{40}$ 4) $\frac{\pi}{80}$

18. $\int_0^{\pi/2} \sin 2x \cdot \tan^{-1}(\sin x)\,dx =$

1) $\frac{\pi}{2}-1$ 2) $\frac{\pi}{2}+1$ 3) $\frac{\pi}{2}$ 4) $1-\frac{\pi}{2}$

19. $\int_0^{\pi/2} e^x(\cos x - \sin x)\,dx =$

1) 1 2) -1 3) 0 4) $\frac{1}{2}$

20. $\int_0^1 \frac{e^x.x}{(x+1)^2}\,dx =$

1) $\frac{e}{2}$ 2) $1+\frac{e}{2}$ 3) $\frac{e}{2}-1$ 4) $1-\frac{e}{2}$

21. $\int_2^e \left(\frac{1}{ln\,x}-\frac{1}{(ln x)^2}\right)dx =$

1) 0 2) e 3) $e-2\log_2 e$ 4) $2\log_2 e$

22. If $A=\int_0^1 \frac{e^t}{1+t}dt$ then $\int_0^1 e^t ln(1+t)dt =$

1) $e\ ln\ 2 - A$ 2) $e\ ln\ 2 + A$ 3) $Aeln\ 2$ 4) $A\ ln\ 2$

23. If $\int_0^1 x f(3x)\,dx = \frac{1}{k}\int_0^3 t f(t)\,dt$ then $k =$

1) 9 2) 3 3) $\frac{1}{9}$ 4) $\frac{1}{3}$

24. If $\frac{d}{dx}f(x)=g(x)$ then $\int_a^b f(x)g(x)dx =$

1) $\frac{f(b)-f(a)}{2}$ 2) $\frac{f(a)-f(b)}{2}$ 3) $\frac{f^2(b)-f^2(a)}{2}$ 4) $\frac{f^2(a)-f^2(b)}{2}$

25. Let $\frac{d}{dx}f(x) = \frac{e^{\sin x}}{x}$, $x>0$. If $\int_1^4 \frac{3}{x}e^{\sin x^3}dx = f(k)-f(1)$ then one of the possible value of k is

1) 63 2) 64 3) 16 4) 32

26. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then $\int_0^1 f(x)g(x)\,dx =$

1) $e+\frac{e^2}{2}+\frac{5}{2}$ 2) $e-\frac{e^2}{2}-\frac{3}{2}$ 3) $e-\frac{e^2}{2}-\frac{5}{2}$ 4) $e+\frac{e^2}{2}-\frac{3}{2}$

27. Let $F(x)=f(x)+f\left(\frac{1}{x}\right)$, where $f(x)=\int_1^x \frac{lnt}{1+t}dt$, then $F(e) =$

1) 2 2) 1/2 3) 0 3) 1

28. If $f(x)=(1+\tan x)\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)$ and $g(x)$ is a function with domain R, then $\int_0^1 x^3 gof(x)dx=$

1) $\frac{1}{2}g\left(\frac{\pi}{4}\right)$ 2) $\frac{1}{4}g(2)$ 3) $\frac{1}{4}g(1)$ 4) 1

29. If $y=\int_0^x \frac{t^2}{\sqrt{t^2+1}}dt$ then $\frac{dy}{dx}$ at $x=1$ is :

1) $\sqrt{2}$ 2) $\frac{1}{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) 2

30. If $\int_0^x f(t)\,dt = x+\int_x^1 t\,f(t)\,dt$ then $f(1)=$

1) 0 2) 1 3) -1 4) $\frac{1}{2}$

31. The greatest value of $f(x)=\int_1^x |t|dt$ on the interval $\left[-\frac{1}{2},\frac{1}{2}\right]$ is :

1) $\frac{3}{8}$ 2) $-\frac{3}{8}$ 3) $-\frac{1}{2}$ 4) $\frac{1}{2}$

Problems on (a - x) Property :

32. $\int_0^{\pi/2}\frac{1}{1+\tan^3 x}dx=$

1) π 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) 2π

33. $\int_0^{\pi/2}\frac{200\sin x+100\cos x}{\sin x+\cos x}dx=$

1) 50π 2) 25π 3) 75π 4) 150π

34. If $\int_0^{b-c} f(x+c)dx = k\int_b^c f(x)dx$ then $k=$

1) 0 2) 1 3) 2 4) -1

35. If $\int_0^{\pi} f(\tan x)dx=\lambda$ then $\int_0^{2\pi} f(\tan x)\;dx=$

1) $\frac{\lambda}{2}$ 2) -2λ 3) 2λ 4) $-\frac{\lambda}{2}$

36. $\int_0^2 \frac{2x-2}{2x-x^2}dx=$

1) 0 2) 2 3) 3 4) 4

37. $\int_0^{\pi/2}\frac{\text{Sin}^2 x}{\text{Sin}x+\text{Cos}x}dx=$

1) $\sqrt{2}\,ln(\sqrt{2}+1)$ 2) $ln(\sqrt{2}+1)$ 3) $\frac{1}{\sqrt{2}}ln(\sqrt{2}+1)$ 4) 1

38. $\int_0^1 \mathrm{Tan}^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx =$

1) 0 2) 1 3) $\pi/4$ 4) $\pi/8$

39. $\int_0^{\infty} \frac{\ln x}{x^2+a^2} dx =$

1) $\left(\frac{\ln a}{a}\right)\frac{\pi}{4}$ 2) 0 3) $\left(\frac{\ln a}{a}\right)\frac{\pi}{3}$ 4) $\left(\frac{\ln a}{a}\right)\frac{\pi}{2}$

40. $\int_0^{\pi/2} \ln(\tan x + \cot x)\, dx =$

1) $\frac{\pi}{2}\ln 2$ 2) 1 3) $\pi \ln 2$ 4) $2\pi \ln 2$

41. $\int_0^1 \frac{\ln(1+x)}{1+x^2} dx =$

1) $\frac{\pi}{4}\ln 2$ 2) $\frac{\pi}{2}\ln 2$ 3) $\frac{\pi}{8}\ln 2$ 4) $\pi \ln 2$

42. $\int_0^{2\pi} \ln(1+\mathrm{Cos}x)dx =$

1) $\pi \ln 2$ 2) $-\pi \ln 2$ 3) $-2\pi \ln 2$ 4) $2\pi \ln 2$

43. $\int_0^{\pi/2} \frac{\sin 8x \cdot \log(\cot x)}{\cos 2x} dx =$

1) 0 2) 1 3) 1/2 4) $\pi/2$

44. $\int_0^{\pi/2} \frac{1}{1+e^{\sqrt{2}\sin(x-\pi/4)}} dx =$

1) 0 2) $\pi/2$ 3) $\pi/8$ 4) $\pi/4$

45. $\int_0^{\infty} \frac{\ln x}{1+x^2} dx =$

1) π 2) 2π 3) 1 4) 0

46. $\int_0^1 x(1-x)^n dx =$

1) $\frac{1}{n+1}-\frac{1}{n+2}$ 2) $\frac{1}{n+1}+\frac{1}{n+2}$ 3) $\frac{1}{n+1}$ 4) $\frac{1}{n+2}$

47. $\int_0^1 \frac{x}{(1-x)^{5/4}} dx =$

1) $\frac{16}{3}$ 2) $-\frac{16}{3}$ 3) $\frac{3}{16}$ 4) $-\frac{3}{16}$

48. If $f(x)$ and $g(x)$ are continuous functions on the interval [0, 4] satisfying $f(x) = f(4 - x)$ and $g(x) + g(4 - x) = 3$ and $\int_0^4 f(x)\,dx = 2$ then $\int_0^4 f(x)g(x)dx =$

1) 0 2) 1 3) 2 4) 3

49. Let $f(x)$ be a continuous function such that $f(a - x) + f(x) = 0$ for all $x \in [0, a]$. Then $\int_0^a \frac{dx}{1+e^{f(x)}}$ is equal to

1) a 2) $\frac{a}{2}$ 3) $f(a)$ 4) $\frac{1}{2}f(a)$

50. If $f(y) = e^y$, $g(y) = y$ and $y > 0$, $F(t) = \int_0^t f(t-y)g(y)dy$ then

1) $F(t) = te^t$ 2) $F(t) = te^{-t}$ 3) $F(t) = 1 - e^{-t}(1+t)$ 4) $F(t) = e^t - (1+t)$

Problems on (a + b – x) Property :

51. $\int_{-1}^{1} (ax^3 + bx)\,dx = 0$ for

1) any values of a and b 2) a > 0, $b > 0$ only

3) $a > 0$, $b < 0$ only 4) $a < 0$, $b < 0$ only

52. $\int_{-\pi/2}^{\pi/2} ln\left(\frac{2-\sin\theta}{2+\sin\theta}\right)d\theta =$

1) 0 2) 1 3) 2 4) –1

53. For $a > 0$, the value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx$ is

1) 2π 2) $\frac{\pi}{a}$ 3) $\frac{\pi}{2}$ 4) $a\pi$

54. $\int_{-1}^{1} \frac{\cosh x}{1+e^{2x}}dx =$

1) 0 2) 1 3) $\frac{e^2-1}{2e}$ 4) $\frac{e^2+2}{2e}$

55. If $f(a + b - x) = f(x)$ then $\int_a^b x\,f(x)\,dx =$

1) $\left(\frac{a+b}{2}\right)\int_a^b f(x)dx$ 2) $\int_a^b f(x)dx$ 3) $(a+b)\int_a^b f(x)dx$ 4) 0

Problems on Even, Odd functions :

56. $\int_{-1}^{1} \frac{2\sin x - 3x^2}{4-|x|}dx =$

1) $6\int_0^1 \frac{x^2}{4-|x|}dx$ 2) $-6\int_0^1 \frac{x^2}{4-|x|}dx$ 3) $-6\int_0^1 \frac{x^2}{4+|x|}dx$ 4) 0

57. $\int_{-\pi}^{\pi} \frac{2x(1+\operatorname{Sin}x)}{1+\operatorname{Cos}^2 x} dx =$

1) π^2 2) $\frac{\pi^2}{2}$ 3) $\frac{\pi^2}{4}$ 4) $2\pi^2$

58. If $f(x) = \begin{vmatrix} x & \cos x & e^{x^2} \\ \sin x & x^2 & \sec x \\ \tan x & x^4 & 2x^2 \end{vmatrix}$ then $\int_{-\pi/2}^{\pi/2} f(x)\,dx =$

1) 0 2) 1 3) –1 4) 2

59. $\sum_{n=1}^{10} \int_{-2n-1}^{2n} \sin^{27} x\,dx + \sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x\,dx =$

1) 0 2) 1 3) 26 4) 27

60. $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} \left((x+\pi)^3 + \cos^2(x+3\pi)\right)dx =$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}-1$ 3) $\frac{\pi^4}{32}$ 4) $\frac{\pi^4}{32}+\frac{\pi}{2}$

61. $\int_{-1}^{1} \frac{1}{(1+x^2)^2} dx =$

1) 0 2) 1 3) $\frac{\pi}{4}+\frac{1}{2}$ 4) $\frac{\pi}{4}-\frac{1}{2}$

62. $\int_{-1}^{1} \sqrt{\left(\frac{x+2}{x-2}\right)^2 + \left(\frac{x-2}{x+2}\right)^2 - 2}\ dx =$

1) $8ln\frac{4}{3}$ 2) $8ln\frac{3}{4}$ 3) $4ln\frac{4}{3}$ 4) 0

63. $f(x) = \int_0^x ln\left(\frac{1-t}{1+t}\right) dt \Rightarrow$

1) $f(x)$ is an even function
2) $f(x)$ is an odd function
3) $f(x)$ is neither even nor odd
4) $f(x)$ cannot be a function

Splitting into intervals :

64. $\int_0^a (f(x)+f(-x))dx =$

1) $2\int_0^a f(x)dx$ 2) $\int_{-a}^{a} f(x)dx$ 3) 0 4) $-\int_{-a}^{a} f(-x)dx$

65. If for every ineger n; $\int_n^{n+1} f(x)\,dx = n^2$ then the value of $\int_{-2}^{4} f(x)\,dx =$

1) 13 2) 15 3) 17 4) 19

66. If $\int_{-3}^{2} f(x)dx = \frac{7}{3}$ and $\int_{-3}^{9} f(x)dx = -\frac{5}{6}$ then $\int_{2}^{9} f(x)dx =$

1) $\frac{3}{2}$ 2) $-\frac{3}{2}$ 3) $-\frac{19}{6}$ 4) $\frac{19}{6}$

67. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4}(3-f(x))dx = 7$ then $\int_{-1}^{2} f(x)dx =$

1) -2 2) 3 3) 5 4) 8

68. $\int_{-1}^{3}\left(\mathrm{Tan}^{-1}\frac{x}{x^2+1} + \mathrm{Tan}^{-1}\frac{x^2+1}{x}\right)dx =$

1) 2π 2) π 3) 4π 4) $\frac{\pi}{2}$

Problems on (2a − x) property :

69. $\int_{0}^{\infty}\frac{x.\ln x}{(1+x^2)^2}dx =$

1) 1 2) −1 3) 0 4) $\frac{\pi}{2}$

70. $\int_{0}^{\pi/2}(\sin^{100}x - \cos^{100}x)dx =$

1) $\frac{1}{100}$ 2) $\frac{100!}{100^{100}}$ 3) $\frac{\pi}{100}$ 4) 0

Problems on x f(x) Models :

71. For $n \in N$, $\int_{0}^{2\pi}\frac{x \ \sin^{2n}x}{\sin^{2n}x + \cos^{2n}x}dx =$

1) π 2) π^2 3) $\pi/2$ 4) $\pi^2/2$

72. $\int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x} =$

1) $\frac{\pi^2}{ab}$ 2) $\frac{\pi^2}{2ab}$ 3) $\frac{\pi^2}{4ab}$ 4) $\frac{2\pi^2}{3ab}$

73. $\int_0^{\pi} x \ln(\mathrm{Sin}x)\,dx =$

1) $\frac{\pi}{2}\ln 2$ 2) $\frac{-\pi^2}{2}\ln 2$ 3) $-\frac{\pi}{2}\ln 2$ 4) −2p $\ln 2$

74. If $\int_{0}^{\pi} x f(\sin x)dx = A\int_{0}^{\pi/2} f(\sin x)dx$ then $A =$

1) 0 2) 2π 3) $\pi/4$ 4) π

75. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} x g(x(1-x))dx$ and $I_2 = \int_{f(-a)}^{f(a)} g(x(1-x))dx$ then $\frac{I_2}{I_1}$ is :

1) 2 2) 1 3) −1 4) −3

Numerical value type Questions

76. $\int_0^{\infty} \frac{dx}{(x+\sqrt{x^2+1})^3} =$ ----------------

77. If $\int_e^x tf(t)dt = \sin x - x\cos x - \frac{x^2}{2}$ for all $x \in R-\{0\}$ then the value of $f\left(\frac{\pi}{6}\right)$ is ________

78. $\int_2^5 \frac{[x^2+49-14x]}{[x^2-14x+49]+[x^2]} dx =$ _______

79. $\int_{-\pi/2}^{\pi/2} \cos^3\theta(1+\sin\theta)^2\, d\theta =$ ________

80. If $f(x) = x^2$ for $0 \le x \le 1$ $= \sqrt{x}$ for $1 \le x \le 2$ then $\int_0^2 f(x)\, dx =$ _________

LEVEL-II (ADVANCED)

Single answer type questions

1. $\int_2^3 \frac{2x^5+x^4-2x^3+2x^2+1}{(x^2+1)(x^4-1)} dx =$

a) $\frac{1}{2}\log 6+\frac{1}{10}$ b) $\frac{1}{2}\log 6-\frac{1}{10}$ c) $\frac{1}{2}\log 3-\frac{1}{10}$ d) $\frac{1}{2}\log 2+\frac{1}{10}$

2. $\int_{-\pi/3}^{0}\left[\cot^{-1}\left(\frac{2}{2\cos x-1}\right)+\cot^{-1}\left(\cos x-\frac{1}{2}\right)\right]dx =$

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{3}$ c) $\frac{\pi^2}{8}$ d) $\frac{3\pi^2}{8}$

3. Number of ordered pair(s) of (a, b) satisfying simultaneously the system of equations $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$ is

a) 0 b) 1 c) 2 d) 4

4. If $I_1 = \int_0^{\pi/2} \frac{\cos^2 x}{1+\cos^2 x}dx$, $I_2 = \int_0^{\pi/2} \frac{\sin^2 x}{1+\sin^2 x}dx$ $I_3 = \int_0^{\pi/2} \frac{1+2\cos^2 x\sin^2 x}{4+2\cos^2 x\sin^2 x}dx$ then

a) $I_1 = I_2 > I_3$ b) $I_3 > I_1 = I_2$ c) $I_1 = I_2 = I_3$ d) $I_1 \ne I_2 \ne I_3$

5. Number of solutions of the equation $\frac{d}{dx}\int_{\cos x}^{\sin x} \frac{dt}{1-t^2} = 2\sqrt{2}$ in $[0, \pi]$ is

a) 4 b) 3 c) 2 d) 0

More than one correct answer type questions

6. $\int_0^1 \frac{2x^2+3x+3}{(x+1)(x^2+2x+2)} dx =$

a) $\frac{\pi}{4} + 2\ln 2 - \tan^{-1} 2$ b) $\frac{\pi}{4} + 2\ln 2 - \tan^{-1}\frac{1}{3}$ c) $2\ln 2 - \cot^{-1} 3$ d) $-\frac{\pi}{4} + \ln 4 + \cot^{-1} 2$

7. Let e be the eccentricity of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, and $f(e)$ be the eccentricity of conjugate hyperbola $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\int_1^3 \underbrace{fff.....f(e)}_{n\ times}\, de$ is equal to

a) 2, if n is even b) 4, if n is even c) $2\sqrt{2}$, if n is odd d) $4\sqrt{2}$, if n is odd

8. Let $f(x)=\int_1^x \frac{3^t}{1+t^2}dt$, where $x > 0$, then

a) for $0<\alpha<\beta, f(\alpha)<f(\beta)$ b) for $0<\alpha<\beta, f(\alpha)>f(\beta)$

c) $f(x)+\pi/4<\tan^{-1}x, \forall x\ge 1$ d) $f(x)+\pi/4>\tan^{-1}x, \forall x\ge 1$

9. If $A_n=\int_0^{\pi/2}\frac{\sin(2n-1)x}{\sin x}dx; B_n=\int_0^{\pi/2}\left(\frac{\sin nx}{\sin x}\right)^2 dx$, for $n\in N$, then

a) $A_{n+1}=A_n$ b) $B_{n+1}=B_n$ c) $A_{n+1}-A_n=B_{n+1}$ d) $B_{n+1}-B_n=A_{n+1}$

10. If $f(x)=\int_a^x [f(x)]^{-1}dx$ and $\int_a^1 [f(x)]^{-1}dx=\sqrt{2}$, then

a) $f(2) = 2$ b) $f'(2) = 1/2$ c) $f^{-1}(2) = 2$ d) $\int_0^1 f(x)dx=\sqrt{2}$

11. The value of $\int_0^\infty \frac{dx}{1+x^4}$ is

a) same as that of $\int_0^\infty \frac{x^2+1dx}{1+x^4}$ b) $\frac{\pi}{2\sqrt{2}}$

c) same as that of $\int_0^\infty \frac{x^2dx}{1+x^4}$ d) $\frac{\pi}{\sqrt{2}}$

12. $\int_0^\alpha \frac{dx}{1-\cos x\cos\alpha}=\frac{A}{\sin\alpha}+B(\alpha\ne 0)$ possible value of A and B are

a) $A=\pi/2\ B=0$ b) $A=\pi/4\ B=\frac{\pi}{4\sin\alpha}$ c) $A=\pi/6\ B=\frac{\pi}{\sin\alpha}$ d) $A=\pi, B=\frac{\pi}{\sin\alpha}$

Linked comprehension type questions

Passage - I :

A function f: $R\to R$ satisfying the following conditions

i) $f(-x)=f(x)$ ii) $f(x+2)=f(x)$ iii) $g(x)=\int_0^x f(t)dt$ and $g(1)$ = a then

13. The function g(x) is

a) even function b) odd function c) neither even nor odd d) none of these

14. The value of $g(x+2)-g(x)$ is

a) $3g(x)$ b) $2g(x)$ c) $g(x)$ d) None of these

Passage - II :

Let the function f satisfies $f(x).f^1(-x)=f(-x)f^1(x)$ for all x and $f(0)=3$

15. $\int_{-1}^{1} f(x)f(-x)dx =$

a) 6 b) 12 c) 18 d) 24

16. $\int_{-21}^{21} \frac{dx}{3+f(x)} =$

a) 0 b) 7 c) 14 d) 42

17. No of roots of $f(x)=0$ in $[-2,2]$ are

a) 0 b) 1 c) 2 d) 4

Matrix matching type questions

18. **COLUMN - I** **COLUMN - II**

Integrals	**Values**
A) $\int_0^{\pi} \frac{1}{4-\sin^2 x}.dx =$	p) $\frac{\pi^2}{6\sqrt{3}}$
B) $\int_0^{\pi} \frac{x}{4-\sin^2 x}.dx =$	q) $\frac{\pi}{3\sqrt{3}}$
C) $\int_0^{\pi} \frac{\sin x}{4-\sin^2 x}.dx =$	r) $\frac{\pi^2}{4\sqrt{3}}$
D) $\int_0^{\pi} \frac{x\sin x}{4-\sin^2 x}.dx =$	s) $\frac{\pi}{2\sqrt{3}}$

19. If $\int_0^{\infty} e^{-x^2 dx} = \frac{\sqrt{\pi}}{2}$ then match the following

Integrals	**Values**
A) $\int_0^{\infty} e^{-3x^2} dx =$	p) $\frac{\sqrt{\pi}}{6}$
B) $\int_{-\infty}^{\infty} e^{-3x^2} dx =$	q) $\frac{1}{2}\sqrt{\frac{\pi}{3}}$
C) $\int_0^{\infty} \frac{1}{x^2} e^{-1/x^2} dx =$	r) $\frac{\sqrt{3\pi}}{3}$
D) $\int_0^{\infty} e^{-tx^2} dx$ at $t=9$ is	s) $\frac{\sqrt{\pi}}{2}$

Integer answer type questions

20. $\int_0^4 \frac{(y^2-4y+5)\sin(y-2)dy}{[2y^2-8y+11]} =$

21. Let $I_1 = \int_0^1 \frac{e^x dx}{1+x}$ and $I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}(2-x^3)}$, then $\frac{I_1}{eI_2} =$

22. If $\int_0^{f(x)} t^2 dt = x\cos\pi x$, then $-9f'(9) =$

23. If $A = \int_1^{\sin\theta} \frac{tdt}{1+t^2}$ and $B = \int_1^{\operatorname{cosec}\theta} \frac{dt}{t(1+t^2)}$, then $\begin{vmatrix} A & A^2 & B \\ e^A e^B & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix} =$

24. Let $f'(x) = \frac{e^{\sin x}}{x}, x > 0$ and if $\int_1^3 \frac{2e^{\sin x^2}}{x} dx = f(a) - f(1)$, then one of the possible value of a is ....

25. $\frac{6\sqrt{3}}{\pi} \int_0^{\pi/4} \frac{dx}{\cos^2 x + 3\sin^2 x} =$

EXERCISE-II

Modulus, Step function & Periodic Property Models

LEVEL-I (MAIN)

Single answer type questions

1. If $a < 0 < b$ then $\int_a^b \frac{|x|}{x} dx =$

1) $a-b$ 2) $b-a$ 3) $a+b$ 4) 0

2. If $f(t) = \int_{-t}^{t} \frac{e^{-|x|}}{2} dx$ then $\underset{t\to\infty}{Lt} f(t) =$

1) 1 2) 1/2 3) 0 4) –1

3. $\int_{-2}^{3} |1-x^2| \, dx =$

1) 28/3 2) 1/3 3) 7/3 4) 14/3

4. $\int_0^{\pi} |\cos\theta - \sin\theta| d\theta =$

1) 2 2) $\sqrt{2}$ 3) $2\sqrt{2}$ 4) 4

5. $\int_0^1 |\sin 2\pi x| \, dx =$

1) 2π 2) $\frac{2}{\pi}$ 3) $\frac{\pi}{2}$ 4) π

6. $\int_{\pi}^{10\pi} |\sin x| dx =$

1) 18 2) 16 3) 14 4) 0

Step function Models :-

7. $\int_0^5 [x]\, dx =$

1) 0 2) 5 3) 10 4) 15

8. $\int_{-2}^{2} |[x]|\, dx =$

1) 1 2) 2 3) 3 4) 4

9. $\int_0^{\pi} [\cot x]\, dx =$ where [.] denotes the greatest integer function, is equal to

1) 1 2) –1 3) $-\frac{\pi}{2}$ 4) $\frac{\pi}{2}$

10. $\int_1^4 ln[x] dx =$

1) $ln\ 4$ 2) $ln\ 6$ 3) $ln\ 8$ 4) $ln\ 3$

11. If [x] represents greatest integer $\le x$, then $\int_0^{\sqrt{2}} [x^2] dx =$

1) $2-\sqrt{2}$ 2) $2+\sqrt{2}$ 3) $\sqrt{2}+1$ 4) $\sqrt{2}-1$

12. If [x] denotes the greatest integer less than or equal to x then $\int_1^{\infty} \left[\frac{1}{1+x^2}\right] dx =$

1) 1 2) $\frac{1}{2}$ 3) 0 4) 2

13. If [x] represents greatest integer function, (x) represents least integer function then $\frac{\int_0^n [x] dx}{\int_0^n (x) dx} =$

1) $\frac{1}{n-1}$ 2) $\frac{1}{n}$ 3) n 4) $\frac{n-1}{n+1}$

14. $\int_1^a [x] f'(x) dx = (a > 1)$, where [x] denotes the greatest integer not exceeding x is

1) $af(a) - \{f(1) + f(2) + ... + f([a])\}$ 2) $[a] f(a) - \{f(1) + f(2) + ... + f([a])\}$

3) $[a] f([a]) - \{f(1) + f(2) + ... + f(a)\}$ 4) $a f([a]) - \{f(1) + f(2) + ... + f(a)\}$

Periodic property :

15. $\int_0^{100\pi} \sqrt{\frac{1-\cos 2x}{2}}\, dx =$

1) 100 2) 200 3) 50 4) 400

16. $\int_0^{50} (x - [x])\, dx =$

1) 50 2) 10 3) 25 4) 100

17. $\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}dx =$

1) $\frac{e^{100}-1}{e-1}$ 2) $\frac{e-1}{100}$ 3) $100(e-1)$ 4) $\frac{e^{100}-1}{100}$

18. $\int_0^{100} \text{Sin}((x-[x])\pi)\,dx =$

1) $\frac{100}{\pi}$ 2) $\frac{200}{\pi}$ 3) 100π 4) 200π

19. $\int_0^{[x]} \frac{2^x}{2^{[x]}}dx =$

1) $-\frac{[2x]}{ln2}$ 2) $\frac{[2x]}{ln2}$ 3) $-\frac{[x]}{ln2}$ 4) $\frac{[x]}{ln2}$

20. $\int_0^{[x]} (x-[x])\,dx =$

1) $\frac{[x]}{2}$ 2) $1+\frac{[x]}{2}$ 3) $[x]$ 4) $2[x]$

Numerical value type Questions

21. $\int_0^{25} [\sqrt{x}]\,dx =$ ____________

22. $\int_{e^{-1}}^{e^2} \left|\frac{ln\,x}{x}\right| dx =$ ____________

23. If $I=\int_{-1}^{1}\frac{e^{|x|}}{1+a^x}dx$ Then I =________

LEVEL-II (ADVANCED)

Single answer type questions

1. $\int_{-1}^{1}\frac{x^3+|x|+1}{x^2+2|x|+1}dx = a\ln 2+b$ then :

a) $a=2\,;\,b=1$ b) $a=2\,;\,b=0$ c) $a=3\,;\,b=-2$ d) $a=4\,;\,b=-1$

2. If $z=x+3i$, then the value of $\int_2^4\left[\arg\left|\frac{z-i}{z+i}\right|\right]dx$, (where [.] denotes the greatest integer function and $i=\sqrt{-1}$, is

a) $3\sqrt{2}$ b) $6\sqrt{3}$ c) $\sqrt{6}$ d) 0

3. $\int_0^{10\pi}[\tan^{-1}x]dx=$ [.] denotes G.I.F

a) $\pi+\tan 1$ b) $10\pi+1$ c) $10\pi+\tan 1$ d) $10\pi-\tan 1$

4. $\int_{-\pi/2}^{2\pi}\left[\cot^{-1}x\right]dx =$

a) $\cot 1+\cot 2$ b) $\pi+\cot 1+\cot 2$ c) $\pi-\cot 1-\cot 2$ d) $\pi+\cot 1$

5. If $\int_{\cos x}^{1} t^2 f(t)\,dt = 1-\cos x \ \forall\ x\in\left(0,\frac{\pi}{2}\right)$, then the value of $\left[f\left(\frac{\sqrt{3}}{4}\right)\right]$ is ([.] denotes the greatest integer function)

a) 4 b) 5 c) 6 d) –7

6. $\int_{0}^{\frac{3\pi}{2}}\frac{\left|\tan^{-1}\tan x\right|-\left|\sin^{-1}\sin x\right|}{\left|\tan^{-1}\tan x\right|+\left|\sin^{-1}\sin x\right|}dx =$

a) $\frac{\pi}{2}$ b) π c) $\frac{3\pi}{2}$ d) 0

7. $I_1=\int_0^{\pi/2}\cos(\sin x)dx$, $I_2=\int_0^{\pi/2}\sin(\cos x)dx$, $I_3=\int_0^{\pi/2}\cos x dx$ then

a) $I_1>I_2>I_3$ b) $I_2>I_3>I_1$ c) $I_3>I_1>I_2$ d) $I_1>I_3>I_2$

8. The value of $\int_0^{3\pi/2}\sin\left[\frac{2x}{\pi}\right]dx$ where [.] denote greatest integer function

a) $\frac{\pi}{2}(\sin 1+\cos 1)$ b) $\frac{\pi}{2}(\sin 1-\sin 2)$ c) $\frac{\pi}{2}(\sin 1-\cos 1)$ d) $\frac{\pi}{2}(\sin 1+\sin 2)$

9. $\int_{-3}^{3}x^8\left\{x^{11}\right\}dx$ where {.} denotes the fractional part function

a) 3^6 b) 3^7 c) 3^8 d) 3^5

10. $\int_{-1}^{1}[x^2-x+1]dx$ where [.] denotes greatest integer function is equal to

a) $5-\sqrt{5}$ b) $5+\sqrt{5}$ c) $\frac{5-\sqrt{5}}{2}$ d) $\frac{5+\sqrt{5}}{2}$

More than one correct answer type questions

11. If $\int_a^b|\sin x|dx=8$ and $\int_0^{a+b}|\cos x|dx=9$, then

a) $a+b=\frac{9\pi}{2}$ b) $|a-b|=4\pi$ c) $\frac{a}{b}=15$ d) $\int_a^b\sec^2 x dx=0$

12. $I=\int_0^1|k-x|\cos\pi x\,dx$ when K is any real number then the value of I is

a) $\frac{-2}{\pi^2}$ if $k\le 0$ b) $\frac{2}{\pi^2}$ if $k\ge 1$ c) 0, if 0<k<1 d) $\frac{2}{\pi^2}$ if k = 1

13. Suppose $I_1 = \int_0^{1/2} \cos\left(\pi \sin^2 x\right)dx \quad I_2 = \int_0^{1/2} \cos\left(2\pi \sin^2 x\right)dx \quad I_3 = \int_0^{1/2} \cos\left(\pi \sin x\right)dx$ then

a) $I_1 = 0$ b) $I_2 + I_3 = 0$ c) $I_1 + I_2 + I_3 = 0$ d) $I_2 = I_3$

Linked comprehension type questions

Passage - I :

If function $f(x)$ is continuous in the interval (a, b) and having same definition between a and b, then we can find $\int_a^b f(x)dx$ and if f(x) is discontinuous and not having same definition between a and b, then we must break the interval such that $f(x)$ becomes continuous and having same defintion in the breaking intervals.

Now, if f(x) is discontinuous at x = c(a < c < b), then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ and also if f(x) is discontinuous at x = a in (0, 2a), then we can write $\int_0^{2a} f(x)dx = \int_0^a \{f(a-x) + f(a+x)\}dx$.

On the basis of above information, answer the following questions

14. $\int_{-1}^{1} [3x-1]dx$ (where [.] denotes greatest integer function) is equal to

a) –5 b) –3 c) –1 d) 0

15. $\int_0^{10} \left[\frac{x^2+2}{x^2+1}\right]dx$ (where [.] denotes the greatest integer function) is equal to

a) 0 b) 2 c) 5 d) 10

Passage - II :

If functions f(x) and g(x) are continuous on the interval [a, b] and g(x) retain the same sign on [a, b] then there is $c \in (a, b)$ such that $\int_a^b g(x)f(x)\,dx = f(c)\int_a^b g(x)\,dx$. This is known as Mean-Value theorem. This result can be used to estimate some definite integrals. Other results which can be used for estimation are :

i) If f increases and has a concave graph in the interval [a, b] then

$$(b-a)\,f(a) < \int_a^b f(x)\,dx < (b-a)\frac{f(a)+f(b)}{2}.$$

ii) If f increases and has a convex graph in the interval [a, b] then

$$(b-a)\frac{f(a)+f(b)}{2} < \int_a^b f(x)\,dx < (b-a)\,f(b)$$

iii) $$\left|\int_a^b f(x)g(x)\,dx\right|^2 \le \left(\int_a^b f^2(x)\,dx\right)\left(\int_a^b g^2(x)\,dx\right)$$

16. Using Mean-value theorem, the best upper bound of $\int_0^1 \frac{\sin x}{1+x^2}dx$ is

a) $(\pi/4)\sin 1$ b) $\pi \sin 1$ c) $(\pi/2)\sin 1$ d) $(\pi/4)\sin 1/2$

17. Using (iii) above the best upper bound of $\int_0^1 \sqrt{1+x^4}\,dx$ is

a) 1.2 b) $\sqrt{1.22}$ c) $\sqrt{1.2}$ d) $\sqrt{1.4}$

Matrix matching type questions

18. **COLUMN - I** **COLUMN - II**

Integrals **Values**

Column - I	Column - II
A) If $f(x)=\lvert x+1\rvert+\lvert x-1\rvert-\lvert x\rvert-1$, then $\int_{-2}^{2} f(x)\,dx =$	p) 1
B) If $\int_0^{x^2} t\,f(t)\,dt = x^5 - x^3$, then $f(1) =$	q) 2
C) If $(x+2y^3)\dfrac{dy}{dx} = y,\ y(1)=1$, then $y(8) =$	r) 3
D) If $I_1=\int_0^{\frac{\pi}{2}} \dfrac{xdx}{\sin x}$ and $I_2=\int_0^1 \dfrac{\tan^{-1} x}{x}dx$ then $\dfrac{I_1}{I_2}=$	s) 4

19. $I=\int_0^1 \dfrac{dx}{\sqrt{4-x^2-x^3}}$, $I_1=\int_0^{1/2} \dfrac{dx}{\sqrt{1-x^4}}$

COLUMN - I	COLUMN - II
A) I less than	p) 1
B) I is more than	q) 1/2
C) I_1 less than	r) $\dfrac{\pi}{4}$
D) I_1 is more than	s) 1/4

Integer answer type questions

20. If $\int_0^{a+2008\pi} \lvert\cos x\rvert\,dx = \lambda + \sin a$, where $a \in \left(0, \dfrac{\pi}{2}\right)$ then the value of K where $K = \dfrac{[\lambda]}{4016}$, where [.] denotes the greatest integer function, is

21. $-\int_{-1}^{2}\left[\dfrac{[x]}{1+x^2}\right]dx$, where [ ] denotes the greatest integer function, is equal to

22. The value of $\int_{-1}^{7} \operatorname{sgn}(\{x\})\,dx$, where {.} denotes the fractional part function, is

23. The value of the definite integral $\int_{a+2\pi}^{a+5\pi/2} (\sin^{1}(\cos x)+\cos^{-1}(\sin x))dx = \dfrac{\pi^2}{k}$ then $k =$ ___

EXERCISE-III

Integration by parts, Reduction formulae and miscellaneous models

LEVEL-I (MAIN)

Single answer type questions

1. $\int_0^{\pi/4} (\tan^4 x + \tan^2 x)\, dx =$

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{1}{4}$

2. $\int_0^{\pi/2} \sin^8 x.\cos^2 x\, dx =$

1) $\frac{\pi}{512}$ 2) $\frac{3\pi}{512}$ 3) $\frac{5\pi}{512}$ 4) $\frac{7\pi}{512}$

3. If $\int_0^{\pi/2} \sin^6 x\, dx = \frac{5\pi}{32}$ then $\int_{-\pi}^{\pi} (\sin^6 x + \cos^6 x)\, dx =$

1) $\frac{5\pi}{8}$ 2) $\frac{5\pi}{16}$ 3) $\frac{5\pi}{32}$ 4) $\frac{5\pi}{4}$

4. $\int_0^3 \sqrt{\frac{x^3}{3-x}}\, dx =$

1) $\frac{17\pi}{8}$ 2) $\frac{27\pi}{8}$ 3) $\frac{34\pi}{17}$ 4) 1

5. $\int_0^a x^3 (ax - x^2)^{3/2}\, dx =$

1) $\frac{9\pi a^7}{2048}$ 2) $\frac{7\pi a^7}{2048}$ 3) $\frac{\pi a^7}{1024}$ 4) $\frac{5\pi a^7}{2048}$

6. $\int_0^{2a} \sqrt{2ax - x^2}\, dx =$

1) $\frac{\pi a^2}{2}$ 2) $\frac{\pi a^2}{4}$ 3) $\frac{2\pi a^2}{3}$ 4) $\frac{\pi a}{4}$

7. $\int_0^{\pi/2} x \sin^8 2x\, dx =$

1) $\frac{35\pi^2}{1024}$ 2) $\frac{3\pi^2}{128}$ 3) $\frac{\pi^2}{32}$ 4) $\frac{5\pi^2}{32}$

8. $\int_0^{2\pi} x \sin^6 x \cos^5 x\, dx =$

1) 0 2) $\frac{5\pi}{32}$ 3) $\frac{8\pi}{693}$ 4) $\frac{35\pi}{64}$

9. $I_n = \int_0^1 x^n(\tan^{-1} x)dx$ then the value of 11 I_{10}+9 I_8 is

1) $\frac{1}{10}-\frac{\pi}{2}$ 2) $\frac{1}{10}+\frac{\pi}{2}$ 3) $\frac{1}{8}+\frac{\pi}{2}$ 4) $\frac{\pi}{2}-\frac{1}{10}$

10. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$, then I_2+I_4, I_3+I_5, I_4+I_6, ... are in :

1) airthmetic progression 2) geometric progression
3) harmonic proression 4) arithmetic-gemetricprogression

11. $\int_0^{\pi/4} (\text{Tan}^n x+\text{Tan}^{n-2}x)d(x-[x]) =$

1) $\frac{2}{n+1}$ 2) $\frac{1}{n+1}$ 3) $\frac{2}{n-1}$ 4) $\frac{1}{n-1}$

12. If $I_n = \int_0^{\pi/4} \tan^n x dx$ then $\underset{n\to\infty}{Lt}\, n(I_n + I_{n+2}) =$

1) 1/2 2) 1 3) ∞ 7 4) 0

13. $I_{m,n} = \int_0^1 x^m (\ln x)^n dx =$

1) 0 2) $\frac{n}{m+1}$ 3) $-\frac{n}{m+1} I_{m-1,n}$ 4) $-\frac{n}{m+1} I_{m,n-1}$

Miscellaneous Models :

14. $2\int_0^1 \frac{\tan^{-1} x}{x} dx =$

1) $\int_0^{\pi/2} \frac{\sin x}{x} dx$ 2) $\int_0^{\pi/2} \frac{x}{\sin x} dx$ 3) $\int_0^{\pi/2} \frac{1}{\sin x} dx$ 4) $\int_0^{\pi} \frac{x}{\sin x} dx$

15. $\underset{x\to 0}{Lt} \frac{x^2}{\int_0^x \tan^{-1} t\, dt} =$

1) 2 2) $\frac{1}{2}$ 3) –2 4) 4

16. $\underset{x\to 0}{Lt} \frac{\int_0^{x^2} \sec^2 t\ dt}{x\sin x} =$

1) 1 2) 0 3) 3 4) 2

17. Let $f = R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \frac{1}{48}$. Then $\underset{x\to 2}{Lt} \int_6^{f(x)} \frac{4t^3}{x-2} dt =$

1) 24 2) 36 3) 12 4) 18

18. $\int_0^{\pi/2} \frac{\sin x}{x} dx$ lies between

1) 0 and 1 2) –1 and 1 3) 1 and $\frac{\pi}{2}$ 4) can not be determined

19. If $I_1 = \int_0^{3\pi} f(\sin^2 x)dx$ and $I_2 = \int_0^{\pi} f(\sin^2 x)dx$ then

1) $I_1 = I_2$ 2) $I_1 = 2I_2$ 3) $I_1 = 4I_2$ 4) $I_1 = 3I_2$

20. If $I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2^{x^3} dx,\ I_3 = \int_1^2 2^{x^2} dx, I_4 = \int_1^2 2^{x^3} dx$ then

1) $I_1 > I_2$ 2) $I_2 > I_1$ 3) $I_3 > I_4$ 4) $I_3 = I_4$

21. $\int_3^4 \frac{dx}{\sqrt[3]{ln\, x}} =$

1) $< \frac{1}{2}$ 2) $> \frac{1}{2}$ 3) $= \frac{1}{2}$ 4) < 0

22. If $a_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin x} dx$ then $a_2 – a_1,\ a_3 – a_2,\ a_4 – a_3$.....are in

1) AP 2) GP 3) HP 4) AGP

23. If for $n \geq 1$, $P_n = \int_1^e (\log x)^n dx$ then $P_{10} – 90P_8$ is equal to

1) –9 2) $10e$ 3) $-9e$ 4) 10

24. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then which one of the following is true ?

1) $I > \frac{2}{3}$ and $J > 2$ 2) $I < \frac{2}{3}$ and $J < 2$ 3) $I < \frac{2}{3}$ and $J > 2$ 4) $I > \frac{2}{3}$ and $J < 2$

25. If $g(x) = \int_0^x \cos 4t\, dt$, then $g(x + \pi) =$

1) $\frac{g(x)}{g(\pi)}$ 2) $g(x) + g(\pi)$ 3) $g(x) - g(\pi)$ 4) $g(x).g(\pi)$

26. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t} \sin t\, dt$ then f has

1) local minimum at π and local maximum at 2π

2) local maximum at π and local minimum at 2π

3) local maximum at π and 2π

4) local minimum at π and 2π

27. $a>0, \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x} dx =$

1) $\frac{\pi}{2}$ 2) π 3) $\frac{a\pi}{2}$ 4) $a\pi$

28. If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$ for $n = 1, 2, 3,.....$ then $I_{n-1} + I_{n+1} = ...$

1) 0 2) 1 3) $\frac{1}{n}$ 4) $\frac{1}{n+1}$

Numerical value type questions

29. $\int_0^{\pi/2} \sin^6 x . \cos^5 x \, dx =$ ____________

30. $\underset{x\to 0}{Lt} \frac{\int_0^x \sin^2 t \cos t \, dt}{x^3} =$ ___________

31. If $\int_0^b \frac{dx}{1+x^2} = \int_b^{\infty} \frac{dx}{1+x^2}$, then $b =$___________

32. If $I_1 = \int_0^{\pi/2} f(\sin 2x)\sin x \, dx$ and $I_2 = \int_0^{\pi/4} f(\cos 2x)\cos x \, dx$, then $\frac{I_1}{I_2}$ =_________

LEVEL-II (ADVANCED)

Single answer type questions

1. If $\beta + 2\int_0^1 x^2 e^{-x^2} dx = \int_0^1 e^{-x^2} dx$ then the value of β is

a) e^{-1} b) e

c) $1/2e$ d) can not be determined

2. $\int_0^{\frac{\pi}{4}} \left(\frac{x}{x\sin x + \cos x}\right)^2 dx =$

a) $\frac{5-\pi}{5+\pi}$ b) $\frac{2}{4+\pi}$ c) $\frac{4-\pi}{4+\pi}$ d) $\frac{4+\pi}{4-\pi}$

3. A function f is continuous for all x (and not every where zero) such that $f^2(x) = \int_0^x f(t)\frac{\cos t}{2+\sin t} dt$, then $f(x)$ is

a) $\frac{1}{2} In\left(\frac{x+\cos x}{2}\right); x \neq 0$ b) $\frac{1}{2} In\left(\frac{3}{2+\cos x}\right); x \neq 0$

c) $\frac{1}{2} In\left(\frac{2+\sin x}{2}\right); x \neq n\pi, n \in I$ d) $\frac{\cos x + \sin x}{2+\sin x}; x \neq n\pi + \frac{3\pi}{4}, n \in I$

4. If for $x \neq 0$, $af(x) + b f\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$, then $\int_1^2 xf(x)dx$ is equal to

a) $\frac{b-9a}{9(a^2-b^2)}$ b) $\frac{b-9a}{b(a^2-b^2)}$ c) $\frac{b-9a}{6(a^2-b^2)}$ d) $\frac{-39a+31b}{6(a^2-b^2)}$

5. For any real number x let $[x]$ denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by $f(x) = \begin{cases} x-[x] & if \quad [x] is\, odd \\ 1+[x]-x \; if & [x] is\, even \end{cases}$ Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is

a) 4 b) 8 c) 10 d) 15

6. Suppose that $F(x)$ is an antiderivative of $f(x) = \frac{\sin x}{x}$ where $x > 0$, then $\int_1^3 \frac{\sin 2x}{x} dx$ can be expressed as

a) $F(6) - F(2)$ b) $\frac{1}{2}(F(6)-F(2))$ c) $\frac{1}{2}(F(3)-F(1))$ d) $2(F(6)-F(2))$

7. If $f(x)$ is monotonic differentiable function on $[a, b]$, then $\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx =$

a) $bf(A) - af(B)$ b) $bf(B) - af(A)$ c) $f(A) + f(B)$ d) cannot be found

8. If $f(\pi) = 2$ and $\int_0^\pi (f(x) + f''(x))\sin x\, dx = 5$, then $f(0)$ is equal to (it given that $f(x)$ is continuous in $[0, \pi]$)

a) 7 b) 3 c) 5 d) 1

9. The value of $\int_1^e \left(\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right)dx$ is

a) $\tan e$ b) $\tan^{-1} e$ c) $\tan^{-1} (1/e)$ d) $\frac{\pi}{4}$

10. $\int_0^1 \frac{x^{Cos\alpha} - 1}{\log x}dx$, where $\alpha \neq (2n+1)\pi$ is

a) $\log (1-\sin\alpha)$ b) $\log (1+\sin\alpha)$ c) $\log (1-\cos\alpha)$ d) $\log (1+\cos\alpha)$

More than one correct answer type questions

11. The maximum and minimum values of the integral $\int_0^{\pi/2} \frac{dx}{(1+\sin^2 x)}$ are

a) $\pi/4$ b) π c) $\pi/2$ d) $3\pi/4$

12. Let $I = \int_1^3 \sqrt{3+x^3}\,dx$, then the values of I will lie in the interval

a) $[4, 6]$ b) $[1, 3]$ c) $[4, 2\sqrt{30}]$ d) $[\sqrt{15}, \sqrt{30}]$

13. If $I_n = \int_0^{\pi/4} \tan^n x dx (n > 1)$ and is an integer), then

a) $I_n + I_{n-2} = \frac{1}{n+1}$

b) $I_n + I_{n-2} = \frac{1}{n-1}$

c) I_2+I_4, I_4+I_6, ... are in H.P

d) $\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}$

14. If $f(x) = \int_1^x \frac{e^t}{1+t^2} dt, \forall x > 0$, then

a) $g(x) = \tan^{-1} x - f(x)$ is a decreasing function for $x > 0$

b) $f(x) \geq \tan^{-1} x - \frac{\pi}{4}, \forall x \geq 1$

c) $f(x) \leq \tan^{-1} x - \frac{\pi}{4}, \forall x \leq 1$

d) $g(x) = \tan^{-1} x - f(x)$ is an increasing function for $x < 1$

15. The integral $\int_{Tan^{-1}\lambda}^{\cot^{-1}\lambda} \frac{\tan x}{\tan x + \cot x} dx, \lambda \in R$ cannot take the value

a) $-\frac{\pi}{4}$ b) $\frac{\pi}{4}$ c) $-\frac{\pi}{2}$ d) $\frac{3\pi}{4}$

Linked comprehension type questions

Passage - I :

$y = f(x)$ satisfies the relation $\int_2^x f(t) dt = \frac{x^2}{2} + \int_x^2 t^2 f(t) dt - 2$

16. The range of $y = f(x)$ is

a) $[0, \infty)$ b) R c) $(-\infty, 0]$ d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

17. The value of $\int_{-2}^{2} f(x) dx$ is

a) 0 b) -2 c) $2\log_e 2$ d) 1

18. The value of x for which $f(x)$ is increasing is

a) $(-\infty, 1]$ b) $[-1, \infty)$ c) $[-1, 1]$ d) R

Passage - II :

Let $f(x)$ and $\phi(x)$ are two continuous functions on R satisfying $\phi(x) = \int_a^x f(t) dt, a \neq 0$ and another continuous function $g(x)$ satisfying $g(x+\alpha) + g(x) = 0 \forall x \in R, \alpha > 0$ and $\int_b^{2k} g(t)\, dt$ is independent of b.

19. If $f(x)$ is an even function, then

a) $\phi(x)$ is also an even function

b) $\phi(x)$ is an odd function

c) If $f(a-x) = -f(x)$, then $\phi(x)$ is an even function

d) If $f(a-x) = -f(x)$, then $\phi(x)$ is an odd function

20. Least positive value of c if c, k, b are in A.P. is

a) 0 b) 1 c) α d) 2α

21. If m, n are even integers and $p, q \in R$, then $\int_{p+m\alpha}^{q+n\alpha} g(t)\,dt$ is equal to

a) $\int_p^q g(x)\,dx$

b) $(n-m)\int_0^{\alpha} g(x)\,dx$

c) $\int_p^q g(x)\,dx + (n-m)\int_0^{\alpha} g(2x)\,dx$

d) $\int_p^q g(x)\,dx + (n-m)\int_0^{\alpha} g(x)\,dx$

Matrix matching type questions

22. Observe the following columns :

COLUMN - I	COLUMN - II
A) If $I = \int_0^1 \frac{x\,dx}{x^3+16}$ then $[I]$ is equal to ([.] represents the greatest integer function)	p) 0
B) If $f(\pi) = 5$ and $\int_0^{\pi} \{f(x) + f''(x)\}\sin x\,dx = 2$, then $f(0)$ is equal to	q) 3
C) The value of the intergal $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ is equal to	r) –3
D) If $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2} = a^2 y$ then a is equal to	s) 1
	t) –1

Integer answer type questions

23. $\dfrac{\int_0^a 4x^4\sqrt{a^2-x^2}\,dx}{\int_0^a (ax)^2\sqrt{a^2-x^2}\,dx} =$

24. If $\frac{\pi}{2} < \alpha < \frac{2\pi}{3}$ and $I = \int_0^{\sin 2\alpha} \frac{dx}{\sqrt{4\cos^2\alpha - x^2}}$ then $5\left(\frac{I+\alpha}{\pi}\right) =$

25. Let g be a differentiable function satisfying $\int_0^x (x-t+1)g(t)\,dt = x^4 + x^2\ \forall\ x \geq 0$ then $\frac{8}{\pi}\int_0^1 \frac{12}{g^1(x)+g(x)+10}dx$

26. $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}\left(1-x^2\right)^5\right\}dx =$

KEY SHEET (LECTURE SHEET)

EXERCISE-I

LEVEL-I

1) 3	2) 2	3) 2	4) 1	5) 4	6) 3	7) 4	8) 1
9) 3	10) 2	11) 1	12) 2	13) 3	14) 2	15) 2	16) 3
17) 1	18) 1	19) 2	20) 3	21) 3	22) 1	23) 1	24) 3
25) 2	26) 2	27) 2	28) 2	29) 3	30) 4	31) 2	32) 2
33) 3	34) 4	35) 3	36) 1	37) 3	38) 1	39) 4	40) 3
41) 3	42) 3	43) 1	44) 4	45) 4	46) 1	47) 2	48) 4
49) 2	50) 4	51) 1	52) 1	53) 3	54) 3	55) 1	56) 2
57) 1	58) 1	59) 1	60) 1	61) 3	62) 1	63) 1	64) 2
65) 4	66) 3	67) 3	68) 2	69) 3	70) 4	71) 2	72) 2
73) 2	74) 4	75) 1	76) 0.38	77) -0.5	78) 1.5	79) 1.6	80) 1.55

LEVEL-II

1) b	2) a	3) b	4) c	5) c	6) acd	7) bc	8) ad
9) ad	10) abc	11) bc	12) ab	13) b	14) c	15) c	16) b
17) a	18) A-s;B-r;C-q;D-p			19) A-q;B-r;C-s;D-p			20) 0
21) 3	22) 1	23) 0	24) 9	25) 2			

EXERCISE-II

LEVEL-I

1) 3	2) 1	3) 1	4) 3	5) 2	6) 1	7) 3	8) 4
9) 3	10) 2	11) 4	12) 3	13) 4	14) 2	15) 2	16) 3
17) 3	18) 2	19) 4	20) 1	21) 70	22) 2.5	23) 1.71	

LEVEL-II

1) b	2) d	3) d	4) b	5) b	6) d	7) d	8) d
9) b	10) c	11) abd	12) abcd	13) abc	14) b	15) d	16) a
17) c	18) A-q;B-p;C-q;D-q			19) A-pr;B-qs;C-pr;D-qs			20) 1
21) 1	22) 8	23) 4					

EXERCISE-III

LEVEL-I

1) 3	2) 4	3) 4	4) 2	5) 1	6) 1	7) 1	8) 1
9) 4	10) 3	11) 4	12) 2	13) 4	14) 2	15) 1	16) 1
17) 4	18) 3	19) 4	20) 1	21) 2	22) 2	23) 3	24) 2
25) 4	26) 2	27) 1	28) 3	29) 0.01	30) 0.33	31) 1	32) 1.41

LEVEL-II

1) a	2) c	3) c	4) d	5) a	6) a	7) b	8) b
9) b	10) d	11) ac	12) ac	13) bcd	14) ab	15) acd	16) d
17) a	18) c	19) d	20) d	21) d	22) A-p; B-r; C-p; D-qr		
23) 2	24) 5	25) 2	26) 2				

PRACTICE SHEET

EXERCISE-I

Properties of Definite integrals

LEVEL-I (MAIN)

Single answer type questions

1. $\int_0^1 \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx =$

   1) $\frac{\pi}{2}$ 2) $-\frac{\pi}{2}$ 3) $\frac{1}{2}$ 4) $-\frac{1}{2}$

2. If $\int_{ln2}^{t} \frac{dx}{\sqrt{e^x-1}} = \frac{\pi}{6}$ then t =

   1) ln 8 2) ln 6 3) ln 4 4) 1

3. $\int_0^1 \frac{dx}{x^2+2x\sin\alpha+1} =$

   1) $\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\operatorname{cosec}\alpha$ 2) $\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\operatorname{cosec}\alpha$ 3) $\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\sec\alpha$ 4) $\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\sec\alpha$

4. $\int_2^3 \frac{2-x}{\sqrt{5x-6-x^2}}dx =$

   1) $\frac{\pi}{2}$ 2) $-\frac{\pi}{2}$ 3) $-\pi$ 4) π

5. $\int_0^{\pi/4} \frac{\operatorname{Sin}x+\operatorname{Cos}x}{9+16\operatorname{Sin}2x}dx =$

   1) $\frac{1}{10}ln3$ 2) $5\ ln\ 3$ 3) $\frac{1}{20}ln\ 3$ 4) $ln\ 3$

6. $\int_0^{\pi/2} \frac{dx}{1+\sin^2 x} =$

   1) $\frac{\pi}{2\sqrt{2}}$ 2) $\frac{\pi}{\sqrt{2}}$ 3) $\frac{\pi}{2}$ 4) 0

7. $\int_0^{\pi/2} \frac{1}{9\cos x+12\sin x}dx =$

   1) $ln\ 6$ 2) $\frac{1}{5}ln\ 6$ 3) $\frac{1}{15}ln\ 6$ 4) $\frac{1}{10}ln\ 6$

8. $\int_0^{\pi/2} x^2 \sin x \, dx =$

1) $\pi - 2$ 2) $\pi + 2$ 3) $\pi + 4$ 4) $\pi - 4$

9. $\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx =$

1) $\frac{\pi}{4} + \ln 2$ 2) $\frac{\pi}{4} - \ln 2$ 3) $\frac{\pi}{4} - \frac{1}{2}\ln 2$ 4) $\ln 2$

10. $\int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx =$

1) 1 2) π 3) 0 4) 2

11. $\int_0^{\pi/2} \left(x \sec^2 \frac{x}{2} + 2\tan\frac{x}{2} \right) dx =$

1) 2π 2) π 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

12. If f is every where continuous function then $\frac{1}{c}\int_{ac}^{bc} f\left(\frac{x}{c}\right) dx =$

1) $\frac{1}{c}\int_a^b f(x)dx$ 2) $c\int_b^a f(x)dx$ 3) $\int_{ac^2}^{bc^2} f(x)dx$ 4) $\int_a^b f(x)dx$

13. Equation of the tangent line to the curve $y = \int_0^x \cos\theta d\theta$ at $x = \frac{\pi}{2}$ is

1) $x = 0$ 2) $y = 0$ 3) $x = 1$ 4) $y = 1$

14. $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = F(K) - F(1)$, then one of the possible values of K is :

1) 4 2) –4 3) 16 4) 8

Problems on (a – x) Property :

15. $\int_0^{\frac{\pi}{2n}} \frac{dx}{1+\text{Cot}^n nx} =$

1) $\frac{\pi}{2n}$ 2) $\frac{\pi}{4n}$ 3) $\frac{\pi}{8n}$ 4) $\frac{\pi}{n}$

16. $\int_0^{\pi/4} \ln(1+\tan x) \, dx =$

1) $\frac{\pi}{8}\ln 2$ 2) $\frac{\pi}{4}\ln 2$ 3) $\pi \ln 2$ 4) $\frac{\pi}{2}\ln 2$

17. If $\int_0^{\pi/2} \ln(\sin x)\,dx = -\frac{\pi}{2}\ln 2$ then $\int_0^{\pi} \ln(1+\cos x)\,dx =$

1) $\pi \ln 2$ 2) $-\pi \ln 2$ 3) $\frac{\pi}{2}\ln 2$ 4) $\ln 2$

18. $\int_0^{\infty} \frac{ln(1+x^2)}{(1+x^2)}dx =$

1) $\ln 2$ 2) $\frac{\pi}{2}\ln 2$ 3) $\pi \ln 2$ 4) $\frac{\pi}{4}\ln 2$

19. $\int_0^1 ln\,\text{Sin}\left(\frac{\pi x}{2}\right)dx =$

1) $\ln 2$ 2) $-\ln 2$ 3) $\ln 4$ 4) 1

20. $\int_0^{\pi/2} \sin 2x.ln(\tan x)\,dx =$

1) 1 2) -1 3) 0 4) $\frac{\pi}{4}$

21. $\int_0^{\pi/2} ln\left(\frac{4+3\,\text{Sin}x}{4+3\,\text{Cos}x}\right)dx =$

1) 1 2) 0 3) $\frac{\pi}{4}$ 4) -1

22. If $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$ then $\int_0^a f(x)\,g(x)\,dx =$

1) $2\int_0^a f(x)\,dx$ 2) $\int_0^a f(x)\,dx$ 3) $2\int_0^a g(x)\,dx$ 4) $\int_0^a g(x)\,dx$

Problems on (a + b − x) Property :

23. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}dx =$

1) 0 2) -1 3) 1 4) 2π

24. $\int_0^{\pi} \frac{dx}{1+3^{\cos x}}$ is equal to

1) π 2) $-\pi$ 3) $\frac{\pi}{2}$ 4) 2π

25. $\int_2^8 \frac{[x^2]}{[x^2-20x+100]+[x^2]}dx =$

1) 0 2) 10 3) 3 4) 12

Problems on Even, Odd functions :

26. $\int_{-\pi}^{\pi} \frac{x\text{Cos}x}{1+\text{Sin}^2 x} dx =$

1) 1 2) 0 3) –1 4) $\frac{1}{2}$

27. $\int_{-1}^{1} \text{Sin}^{-1}\left(\frac{x}{1+x^2}\right) dx =$

1) $\frac{\pi}{4}$ 2) 0 3) 4 4) $\frac{\pi}{2}$

28. $\int_{-\pi}^{\pi} (\text{Cos}ax - \text{Sin}ax)^2\, dx$

1) 0 2) π 3) 2π 4) 4π

29. $\int_{-\pi/2}^{\pi/2} \cos\theta\ (1+\sin\theta)^2\, d\theta =$

1) $\frac{14}{3}$ 2) $\frac{8}{3}$ 3) $\frac{5}{14}$ 4) 0

Splitting into intervals :

30. If $f(x) = x$ for $x < 1 = x - 1$ for $x \geq 1$ then $\int_0^2 x^2 f(x)\, dx =$

1) $\frac{5}{3}$ 2) $\frac{3}{5}$ 3) $-\frac{5}{3}$ 4) $-\frac{3}{5}$

31. If $\int_n^{n+1} f(x)dx = n^2 + n, \forall\, n \in I$ then the value of $\int_{-3}^{3} f(x)\ dx$ is equal to

1) 6 2) 10 3) 16 4) 12

Problems on (2a – x) property :

32. $\int_0^{\pi} \frac{\tan x}{\sec x + \cos x} dx =$

1) π 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) 2π

33. For $m = n$ and $m, n \in N$ then the value of $\int_0^{\pi} \cos mx \cos nx\, dx =$

1) 0 2) $\frac{\pi}{2}$ 3) π 4) 1

Problems on x f(x) Models :

34. $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx =$

1) $\frac{\pi}{2} + 1$ 2) $\frac{\pi}{2} - 1$ 3) $\pi\left(\frac{\pi}{2} - 1\right)$ 4) $\pi\left(\frac{\pi}{2} + 1\right)$

35. $\int_0^{\pi} x f(\sin x) dx$ is equal to

1) $\pi\int_0^{\pi} f(\cos x)\,dx$ 2) $\pi\int_0^{\pi} f(\sin x)\,dx$ 3) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)\,dx$ 4) $\pi\int_0^{\pi/2} f(\cos x)\,dx$

LEVEL-II (ADVANCED)

Single answer type questions

1. If $f(x) = e^{-x} + 2e^{-2x} + 3e^{-3x} + ... + \infty$, then $\int_{\ln 2}^{\ln 3} f(x)\,dx =$

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $ln\ 2$

2. If $\int_0^2 375x^5(1+x^2)^{-4}\,dx = 2^n$ then the value of n is :

a) 4 b) 5 c) 6 d) 7

3. Let $A = \int_0^1 \frac{e^t\,dt}{1+t}$ then $\int_{a-1}^{a} \frac{e^{-t}\,dt}{t-a-1} =$

a) Ae^{-a} b) $-Ae^{-a}$ c) $-ae^{-a}$ d) Ae^{a}

4. If $I_1 = \int_{-100}^{101} \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ and $I_2 = \int_{-100}^{101} \frac{dx}{5+2x-2x^2}$ then $\frac{I_1}{I_2}$ is

a) 2 b) $\frac{1}{2}$ c) 1 d) $-\frac{1}{2}$

5. $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} ((x+\pi)^3 + \cos^2(x+3\pi))\,dx =$

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}-1$ c) $\frac{\pi^4}{32}$ d) $\frac{\pi^4}{32}+\frac{\pi}{2}$

6. $\lim\limits_{x\to\pi/4} \frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2 - \pi^2/16} =$

a) $\frac{8}{\pi}f(2)$ b) $\frac{2}{\pi}f(2)$ c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ d) $4\,f(2)$

7. $\int_0^{\infty} e^{-2x}(\sin 2x + \cos 2x)\,dx =$

a) 1 b) -2 c) 1/2 d) zero

8. $\int_0^{\pi} \frac{(a^2 \sin^2 x + b^2 \cos^2 x)dx}{a^4 \sin^2 x + b^4 \cos^2 x} =$

a) $\frac{\pi}{ab}$ b) $\frac{\pi}{2ab}$ c) $\frac{\pi}{a^2+b^2}$ d) $\frac{2\pi}{a^2+b^2}$

9. $\int_0^{\pi/2} \frac{dx}{\cos^6 x + \sin^6 x} =$

a) zero b) π c) $\pi/2$ d) 2π

10. $\int_1^{\infty} (e^{x+1} + e^{3-x})^{-1} dx =$

a) $\frac{\pi}{4e^2}$ b) $\frac{\pi}{4e}$ c) $\frac{1}{e^2}\left(\frac{\pi}{2} - \tan^{-1}\frac{1}{e}\right)$ d) $\frac{\pi}{2e^2}$

More than one correct answer type questions

11. Let $u = \int_0^{\infty} \frac{dx}{x^4 + 7x^2 + 1}$ & $v = \int_0^{\infty} \frac{x^2 \, dx}{x^4 + 7x^2 + 1}$ then :

a) $v > u$ b) $6v = \pi$ c) $3u + 2v = 5\pi/6$ d) $u + v = \pi/3$

12. If $f(x) = \int_0^x (\sin^4 t + \cos^4 t)dt$, then $f(x+\pi)$ will be equal to

a) $f(x) + f\left(\frac{\pi}{2}\right)$ b) $f(x) + f(\pi)$ or $f(x) + 2f\left(\frac{\pi}{2}\right)$

c) $f(x) - f(\pi)$ d) $f(x) - 2f\left(\frac{\pi}{2}\right)$

13. Number of values of x satisfying the equation $\int_{-1}^{x}\left(8t^2 + \frac{28}{3}t + 4\right)dt = \frac{\left(\frac{3}{2}\right)^{x+1}}{\log_{(x+1)}\sqrt{x+1}}$, is

a) 0 b) 1 c) 2 d) 3

14. If $\int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx = 10$, then

a) $b = 22, a = 2$ b) $b = 15, a = -5$ c) $b = 10, a = -10$ d) $b = 10, a = -2$

15. The value of the integral $\int_0^{\pi/4} \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)}$ is

a) $\frac{1}{ab}\tan^{-1}\left(\frac{b}{a}\right)(a > 0, b > 0)$ b) $\frac{1}{ab}\tan^{-1}\left(\frac{b}{a}\right)(a < 0, b < 0)$

c) $\frac{\pi}{4}(a = 1, b = 1)$ d) $\frac{1}{ab}$

16. $f(x)$ be a non constant twice differentiable function defined on $(-\infty,\infty)$ such that $f(x) = f(1-x)$ and $f^1(1/4) = 0$ then

a) $f^{11}(x)$ vanishes at least twice on $[0,1]$

b) $f^1\left(\frac{1}{2}\right)=0$

c) $\int_{\frac{-1}{2}}^{\frac{1}{2}} f(x+\frac{1}{2})\sin x = 0$

d) $\int_0^{1/2} f(t)e^{\sin \pi t}dt = \int_{\frac{1}{2}}^{1} f(1-t)e^{\sin \pi t}dt$

Linked comprehension type questions

Passage - I :

Let f be a continuous function satisfying $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$ $\forall$ x,y. If $f(0) = 1, f^1(0) = -1$ then

17. $\int_{f(2)}^{f(0)} \frac{f(x)}{1+x^2}dx =$

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) 0

18. $\int_0^1 \sqrt{\frac{f(x)}{1+x}}\, dx =$

a) $\frac{\pi}{2}$ b) $\frac{\pi}{2}-1$ c) $\frac{\pi}{2}+1$ d) π

19. $\int_0^1 \sqrt{\frac{f(x)}{1+x}}\, dx$, $\int_{f(2)}^{f(0)} \frac{f(x)}{1+x^2}dx$, $\int_{f(1)}^{f(0)} \sqrt{\frac{1+x}{f(x)}}dx$ are in

a) A.P b) G.P c) H.P d) None

Integer answer type questions

20. If $I = \int_{-\frac{\pi}{2}}^{2\pi} \sin^{-1}(\sin x)\,dx$ then the value of $-\frac{16I}{\pi^2}$ must be

21. The value of $7+\int_{-4}^{-5} e^{(x+5)^2}dx+3\int_{1/3}^{2/3} e^{9\left(x-\frac{2}{3}\right)^2} dx$ is

22. If $f(x) = \int_a^x \frac{1}{f(x)}dx$ and $\int_a^1 \frac{1}{f(x)}dx = \sqrt{2}$, then $f(2) =$

23. If $f(x)=\int_0^x \frac{dt}{\{f(t)\}^2}$ and $\int_0^2 \frac{dt}{\{f(t)\}^2} = \sqrt[3]{6}$, then $f(9)=$

24. The no.of values α in the interval $[-\pi, 0]$ satisfying $\sin\alpha+\int_{\alpha}^{2\alpha}\cos 2x\, dx = 0$ is

25. The value of $\int_{-4}^{-5} \sin(x^2-3)dx+\int_{-2}^{-1} \sin(x^2+12x+33)dx$ is

EXERCISE-II

Modulus, Step function & Periodic Property Models

LEVEL-I (MAIN)

Single answer type questions

1. $\int_{0}^{\pi/4} \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}\, dx =$

1) $\frac{1}{2}\ln 2$ 2) $-\frac{1}{2}\ln 2$ 3) $\ln 2$ 4) $-\ln 2$

2. $\int_a^b \left(|x-a|+|x-b|\right) dx \ (0<a<b) =$

1) $(b-a)^2$ 2) $(b-a)$ 3) $b+a$ 4) $(b+a)^2$

3. $\int_{-\pi/2}^{\pi/2} \sin|x|\, dx =$

1) 0 2) 1 3) 2 4) 4

4. $\int_0^{\pi/3} |\tan x - 1|\, dx =$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

5. If $f(x)=\int_{-1}^{x}|t|\,dt$ then for any $x\geq 0, f(x)=$

1) $\frac{1}{2}(1-x^2)$ 2) $(1-x^2)$ 3) $\frac{1}{2}(1+x^2)$ 4) $1+x^2$

6. Greatest value of $F(x)=\int_1^x |t|\,dt$ on the interval [–1, 2] is

1) 1 2) 2 3) $\frac{3}{2}$ 4) $\frac{5}{2}$

7. $\int_{-1}^{\frac{3}{2}} |x\sin \pi x|\,dx =$

1) $\frac{3}{\pi}-\frac{1}{\pi^2}$ 2) $\frac{3}{\pi}+\frac{1}{\pi^2}$ 3) $\frac{2}{\pi}+\frac{1}{\pi^2}$ 4) $\frac{3}{\pi}$

8. For $n\in N$, $0<t<\pi/2$; the value of $\int_0^{n\pi+t} (|\cos x|+|\sin x|)\ dx =$

1) $4n+\sin t-\cos t+1$ 2) $4n-\sin t-\cos t+1$
3) $2n-\sin t-\cos t+1$ 4) $2n-\sin t-\cos t-1$

9. $\int_{0.5}^{4.5} [x]dx + \int_{-1}^{1} |x|dx =$

1) 9 2) 8 3) 7 4) 6

10. $\int_0^2 [x^2]\,dx =$

1) 0 2) $5-\sqrt{2}-\sqrt{3}$ 3) $5+\sqrt{2}+\sqrt{3}$ 4) $\sqrt{2}+\sqrt{3}+\sqrt{5}$

11. If $[x]$ denotes the greatest integer less than or equal to x then $\int_0^{\infty}\left[\frac{2}{e^x}\right]dx$ is equal to

1) $\log_e 2$ 2) e^2 3) 0 4) $\frac{2}{e}$

12. The value of $\int_0^{\pi/3}[\sqrt{3}\tan x]\ dx$ (where [.] denotes the greatest integer function)

1) $\frac{5\pi}{6}$ 2) $\frac{5\pi}{6}-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ 3) $\frac{\pi}{2}-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ 4) $\frac{5\pi}{3}$

13. The value of $\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}\,dx$, where $[x]$ is the greatest integer less than or equal to x, is

1) 1 2) 0 3) 4 – sin 4 4) 4 + sin 4

14. The value of $\int_0^1 e^{2x-[2x]}d(x-[x])$ (where [.] denotes the greatest integer function) is

1) $e+1$ 2) e 3) $e-1$ 4) does not exist

15. $\int_0^{100}[\tan^{-1}x]\ dx$

1) 100 + tan 1 2) 100 – tan 1 3) tan1 4) 99 + tan 1

16. $\int_0^{100} e^{x-[x]}\ dx =$

1) 100 e 2) 100 (e – 1) 3) 100 (e + 1) 4) 100 (1 – e)

17. The value of $\int_{-10}^{10}\frac{3^x}{3^{[x]}}dx$ is equal to

1) 20 2) $\frac{40}{ln 3}$ 3) $\frac{20}{ln 3}$ 4) $\frac{60}{ln 3}$

LEVEL-II (ADVANCED)

Single answer type questions

1. $\int_0^{\frac{16\pi}{3}}|\sin x|\,dx =$

a) $\frac{11}{2}$ b) $\frac{15}{2}$ c) $\frac{21}{2}$ d) $\frac{31}{2}$

2. $\int_{-1}^{1}\left(\left[x^2\right]+\log\left(\frac{2+x}{2-x}\right)\right)dx =$ (where $[x]$ denotes the greatest integer $\le x$)

a) –2 b) –1 c) 0 d) 1

3. $\int_0^4 \{\sqrt{x}\} dx$ = (where {.} denotes the fractional part function is)

a) 1/3 b) 1 c) 5/3 d) 7/3

4. Number of positive solutions of the equation, $\int_0^x (t - \{t\})^2 dt = 2(x - 1)$ where { } denotes the fractional part function is :

a) one b) two c) three d) more than three

5. The equation $\int_{-\pi/4}^{\pi/4}\left(a|\sin x| + \frac{b\sin x}{1+\cos x} + c\right)dx = 0$, where a, b, c are constants, gives a relation between

a) a, b and c b) a and c c) a and b d) b and c

6. If $I = \int_{-20\pi}^{20\pi} |\sin x| [\sin x]\, dx$ (where [.] denotes the greatest integer function), then the value of I is

a) –40 b) 40 c) 20 d) –20

7. $f(x)$ be a real valued function $f(x)+f(x+4) = f(x+2)+f(x+6)$ & $g(x) = \int_x^{x+8} f(t)dt$ then $g^1(x) =$

a) f (x) b) f (x+8) c) 8 d) 0

8. If $\int_1^a x\, a^{-\left[\log_a^x\right]} dx = \frac{e-1}{2}$ where $a > 1$, and [.] greatest integer function, then the value of a^2 is

a) $e - 1$ b) e c) $e + 1$ d) $e^2 - 1$

9. If $f(x) = \sin x + \cos x$ and $g(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ then the value of $\int_{-\pi/4}^{2\pi} gof(x)dx =$

a) $\frac{3\pi}{4}$ b) $\frac{\pi}{4}$ c) π d) 3π

More than one correct answer type questions

10. If $\frac{2x}{\pi} < \sin x < x$ for $0 < x < \frac{\pi}{2}$, then $\int_0^{\pi/2} \frac{\sin x}{x} dx$ =

a) > 1 b) < 1 c) $> \frac{\pi}{2}$ d) $< \frac{\pi}{2}$

11. If $f(x) = \int_0^x (\cos(\sin t) + \cos(\cos t))\,dt$, then $f(x+\pi)$ is

a) $f(x) + f(\pi)$ b) $f(x) + 2f(\pi)$ c) $f(x) + f\left(\frac{\pi}{2}\right)$ d) $f(x) + 2f\left(\frac{\pi}{2}\right)$

12. $In = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$, $n = 0,1,2........$ then

a) $I_n = I_{n+2}$

b) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

c) $\sum_{m=1}^{10} I_{2m} = 0$

d) $I_n = I_{n+1}$

Linked comprehension type questions

Passage - I :

$\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = \alpha(b)f(b) - \alpha(a)f(a)$. On the basis of above information, answer the following questions

13. $\int_0^3 (x^2+1)d[x]$ (where [.] denotes the greatest integer function) is equal to

a) 3 b) $\frac{9}{2}$ c) 17 d) $\frac{27}{2}$

14. $\int_{-2}^{3} [|x|]\, d|x|$ (where [.] denotes the greatest integer function) is equal to

a) 0 b) 1 c) 2 d) –1

Passage - II :

If $f(x)$ be an increasing function defined on $[a, b]$. Then max $\{f(t) \mid a \le t \le x, a \le x \le b\} = f(x)$ and min $\{f(t) \mid a \le t \le x, a \le x \le b\} = f(a)$ and If $f(x)$ be a decreasing function defined on $[a, b]$. Then max $\{f(t) \mid a \le t \le x, a \le x \le b\} = f(a)$ and min $\{f(t) \mid a \le t \le x, a \le x \le b\} = f(x)$

On the basis of above information, answer the following questions

15. $\int_0^3 \min\{1, |x|, |x-2|\}dx$ is equal to

a) 1 b) 3/2 c) 2 d) 5/2

16. $\int_{-1}^{1} \max\{x, x^3\}dx$ is equal to

a) 1/2 b) 3/2 c) 1/4 d) 3/4

17. $\int_{-2}^{2} \min\{|x|, [x]\}dx$ (where [.] denotes the greatest integer function) is equal to

a) –2 b) –1 c) 0 d) 1

Matrix matching type questions

18. Match the following where [.] greatest integer function

COLUMN - I Integrals	COLUMN - II Values
A) $\int_{-1}^{1}[x+[x+[x]]]dx =$	p) 3
B) $\int_{2}^{5}([x]+[-x])dx =$	q) 1/5
C) $\int_{-1}^{3} sgn(x-[x])dx =$	r) 4
D) $\int_{0}^{\frac{\pi}{4}}(Tan^6(x-[x])+Tan^4(x-[x])dx =$	s) –3

Integer answer type questions

19. If $\int_{-\pi/2}^{\pi/2}\frac{e^{|\sin x|}\cos x}{(1+e^{\tan x})}dx = Ae-1$ then $A =$ __

20. The value of $\int_{-1}^{1}[x[1+\sin\pi x]+1]\,dx$ is ([.] denotes the greatest integer function)

EXERCISE-III

Integration by parts, Reduction formulae and miscellaneous models

LEVEL-I (MAIN)

Single answer type questions

1. $\int_{0}^{1} x^4(1-x)^{5/2}dx =$

1) $\frac{284}{45045}$ 2) $\frac{384}{45045}$ 3) $\frac{84}{4545}$ 4) $\frac{1384}{4504}$

2. $\int_{0}^{2\pi} x\sin^4 x\cos^6 x\,dx =$

1) 0 2) $\frac{3\pi^2}{128}$ 3) $\frac{5\pi^2}{128}$ 4) $\frac{3\pi^2}{64}$

3. If $I_n = \int_{1}^{e}(\log x)^n dx$ then $I_8 + 8I_7 =$

1) 1 2) e 3) 2 4) 2e

Miscellaneous Models :-

4. If $I_1 = \int_x^1 \frac{1}{1+t^2}dt$ and $I_2 = \int_1^{\frac{1}{x}} \frac{1}{1+t^2}dt$ for $x > 0$, then

1) $I_1 = I_2$ 2) $I_1 > I_2$ 3) $I_1 < I_2$ 4) Cannot be determined

5. If $I_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $I_2 = \int_1^2 \frac{e^x}{x}dx$ then

1) $I_1 = I_2$ 2) $2I_1 = I_2$ 3) $I_1 = 2I_2$ 4) $I_1 I_2 = 1$

6. $\underset{x\to 0}{Lt} \frac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^3} =$

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{4}{3}$ 4) 0

7. $\underset{x\to\infty}{Lt} \frac{\left(\int_0^x e^t\, dt\right)^2}{\int_0^x e^{2t^2}\, dt}$

1) 1 2) -1 3) 0 4) $\frac{1}{2}$

8. $\int_1^\infty \frac{x\,dx}{(a^2+x^2)^3} =$

1) $\frac{1}{(1+a^2)^2}$ 2) $\frac{1}{2(1+a^2)^2}$ 3) $\frac{1}{3(1+a^2)^2}$ 4) $\frac{1}{4(1+a^2)^2}$

9. $\int_0^{\pi/2} \frac{8+7\cos x}{(7+8\cos x)^2}dx =$

1) 1/7 2) 2/7 3) 3/7 4) 4/7

10. $\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\, dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\, dt =$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) π 4) 2π

11. The function $f(x) = \int_{-1}^x t(e^t-1)(t-2)^3(t-3)^5$ dt has a local minimum at $x =$

1) 0 2) 1 3) 2 4) 3

12. The point of extremum of $f(x) = \int_0^x (t-2)^2(t-1)dt$ is a

1) max at $x = 1$ 2) max at $x = 2$ 3) min at $x = 1$ 4) min at $x = 2$

13. The points of extremum of the function $F(x)=\int_1^x e^{\frac{t^2}{2}}(1-t^2)dt$ are

1) 0,1 2) $-1,1$ 3) $-\frac{1}{2},1$ 4) $-1, \frac{1}{2}$

14. The difference between the greatest and least values of $f(x) = \int_0^x (t+1)\,dt$ on [2, 3] is

1) $\frac{7}{2}$ 2) $-\frac{7}{2}$ 3) $\frac{2}{7}$ 4) $-\frac{2}{7}$

15. If $\int_0^a f(x)\,dx = \lambda$ and $\int_0^a f(2a-x)\,dx = \mu$ then $\int_0^{2a} f(x)\,dx =$

1) $\lambda+\mu$ 2) $\lambda-\mu$ 3) $2\lambda-\mu$ 4) $\lambda-2\mu$

16. If f, g, h are continuous functions on $[0, a]$ such that $f(a-x) = f(x)$, $g(a-x) = -g(x)$, $3h(x) - 4h(a-x) = 5$ then $\int_0^a f(x)g(x)h(x)dx =$

1) 0 2) a 3) $a/2$ 4) $2a$

17. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$, for all $x \in [0,1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)dx$ equals

1) 21 2) 41 3) 42 4) $\sqrt{41}$

18. $\int_0^1 \cot^{-1}(1-x+x^2)dx =$

1) $\pi - \ln 2$ 2) $\frac{\pi}{2} - \ln 2$ 3) $\pi + \ln 2$ 4) $\frac{\pi}{2} + \ln 2$

19. $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ can hold if : Observe the following statements :

$S_1 \Rightarrow f(x)$ is periodic with period a

$S_2 \Rightarrow f(2a-x) = f(x)$

$S_3 \Rightarrow f(2a-x) = -f(x)$

the true statements are :

1) S_1, S_2 2) S_2, S_3 3) S_3, S_1 4) S_1, S_2, S_3

20. If $g(x) = \int_0^x \cos^4 t\,dt$ then $g(x+\pi) =$

1) $g(x)+g(\pi)$ 2) $g(x)-g(\pi)$ 3) $g(x)g(\pi)$ 4) $\frac{g(x)}{g(\pi)}$

LEVEL-II (ADVANCED)

Single answer type questions

1. If $I_1 = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$, $I_2 = \int_0^{\pi} x\sin^4 x\, dx$ then $I_1 : I_2 =$

a) 3 : 4 b) 1 : 2 c) 4 : 3 d) 2 : 3

2. If $\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$ then $\int_0^{\infty} e^{-ax^2}dx =$

a) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$ b) $\frac{\sqrt{\pi}}{2a}$ c) $\sqrt{\frac{\pi}{2a}}$ d) $\frac{\sqrt{\pi}}{2}$

3. The value of the integral $\int_0^1 e^{x^2}dx$ is

a) less than e b) greater than e c) less than 1 d) greater than 1

4. $\int_0^{\infty} x^{2n+1}\cdot e^{-x^2}dx = (n \in N)$

a) $n!$ b) $2\,(n!)$ c) $\frac{n\,!}{2}$ d) $\frac{(n+1)!}{2}$

5. $\int_{1/2}^{2} \frac{1}{x}\operatorname{cosec}^{101}\left(x-\frac{1}{x}\right)dx =$

a) 1 b) 1/2 c) 0 d) 1/101

6. $\int_{-1/2}^{1/2} (\sin^{-1}(3x-4x^3) - \cos^{-1}(4x^3-3x))dx =$

a) 0 b) $-\frac{\pi}{2}$ c) $\frac{7\pi}{2}$ d) $\frac{\pi}{2}$

7. $\int_0^{\infty}\left(\frac{\pi}{1+\pi^2x^2} - \frac{1}{1+x^2}\right)\log x\, dx$ is equal to

a) $-\frac{\pi}{2}\ln\pi$ b) 0 c) $\frac{\pi}{2}\ln 2$ d) 1

8. If $A = \int_0^{\pi} \frac{\cos x}{(x+2)^2}dx$ then $\int_0^{\pi/2} \frac{\sin 2x}{(x+1)}dx$ is equal to

a) $A - \frac{1}{2} - \frac{1}{\pi+2}$ b) $\frac{1}{2}+\frac{1}{\pi+2} - A$ c) $\frac{1}{\pi+2} - A$ d) $1+\frac{1}{\pi+2} - A$

9. If $f(x) = x + \int_0^1 (x+t)tf(t)dt$ then $\int_0^1 f(x)dx =$

a) $\frac{18}{23}$ b) $\frac{25}{23}$ c) $\frac{42}{23}$ d) $\frac{21}{23}$

10. If for $K \in N$ $\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + ... + \cos(2k-1)x]$ then $I = \int_0^{\frac{\pi}{2}} \sin 2kx \cot x \, dx$ is

a) $-\pi/2$ b) 0 c) $\pi/2$ d) π

More than one correct answer type questions

11. If $g(x) = \int_0^x 2|t| dt$, then

a) $g(x) = x|x|$
b) $g(x)$ is monotonic
c) $g(x)$ is differentiable at $x = 0$
d) $g'(x)$ is differentiable at $x = 0$

12. Let $f:[1,\infty) \to R$ and $f(x) = x\int_1^x \frac{e^t}{t} dt - e^x$, then

a) $f(x)$ is an increasing function
b) $\lim_{x\to\infty} f(x) \to \infty$
c) $f'(x)$ has a maxima at $x = e$
d) $f(x)$ is a decreasing function

13. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}$, where $n \in N$, which of the followign statements holds good ?

a) $2nI_{n+1} = 2^{-n} + (2n-1)I_n$
b) $I_2 = \frac{\pi}{8} + \frac{1}{4}$
c) $I_2 = \frac{\pi}{8} - \frac{1}{4}$
d) $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

14. The value of $\int_0^1 e^{x^2-x} dx$ is

a) <1 b) >1 c) $> e^{-\frac{1}{4}}$ d) $< e^{-\frac{1}{4}}$

15. f(x) be a continuous function and 'a' is a constant satisfying $\int_0^x f(t)dt = e^x - ae^{2x}\int_0^1 f(t)e^{-t}dt$ then

a) $f(x) = e^x + 2e^{2x}$ b) $f(x) = e^x - 2e^{2x}$ c) $a = \frac{1}{1-2e}$ d) $a = \frac{1}{3-2e}$

Linked comprehension type questions

Passage - I :

Let $f(x) = \sin 2x \sin\left(\frac{\pi}{2}\cos x\right)$ and $g(x) = \frac{f(x)}{2x-\pi}$.

16. $\int_0^\pi f(x)dx =$

a) 0 b) $\frac{8}{\pi}$ c) $\frac{8}{\pi^2}$ d) $\frac{16}{\pi^2}$

17. $\int_0^\pi x^2 g(x)dx =$

a) 0 b) $\frac{8}{\pi}$ c) $\frac{8}{\pi^2}$ d) $\frac{16}{\pi^2}$

Passage - II :

Let $f(x)$ be a derivable function satisfying $x f(x) - \int_0^x f(t)dt = x + \log\sqrt{1+x^2} - x)$ and $f(0) = \log 2$

18. If $g(x) = x f'(x)$ then the range of $g(x)$ is

a) $[0, \infty)$ b) $[0, 1)$ c) $[1, \infty)$ d) $(-\infty, \infty)$

19. The function $f(x)$ is

a) injective b) Transdental

c) neither even nor odd d) symmetric w.r.t. origin

20. $\int_0^1 f(x)dx$

a) $\log(1+\sqrt{2})-1$ b) $2\log(1+\sqrt{2})$ c) $\log(3+2\sqrt{2})-1$ d) 1

Matrix matching type questions

21. Observe the following columns

COLUMN - I	COLUMN - II
A) The values of $\int_{\alpha}^{\pi/2-\alpha} \frac{d\theta}{1+\cot^n\theta}$, where $0 < \alpha < \frac{\pi}{2}$, $n > 0$ is	p) $\frac{\pi}{2}$
B) The value of $\int_{-\pi}^{\pi} \frac{\sin^2 x}{1+\alpha^x}dx$, $\alpha > 0$ is	q) $\frac{\pi}{4}-\alpha$
C) The value of $\int_{\alpha}^{2\pi-\alpha} \frac{x\sin^{2n} x}{\sin^{2n} x+\cos^{2n} x}dx$ is	r) dependent of α
D) The value of $\int_0^{\frac{\pi}{2}} \log_2 \operatorname{cosec} x \, dx$	s) independent of n

Integer answer type questions

22. If $f(x) = 1+\frac{1}{x}\int_1^x f(t)dt$, then the value of $f(e^{-1})$ is

23. If a is a positive integer, then the number of values of a satisfying $\int_0^{\pi/2}\left\{a^2\left(\frac{\cos 3x}{4}+\frac{3}{4}\cos x\right)+a\sin x-20\cos x\right\}dx \le -\frac{a^2}{3}$ are

24. $\dfrac{\int_0^1 (1-x^{50})^{100}dx}{\int_0^1 (1-x^{50})^{101}dx} = \dfrac{5051}{K(5050)}$ then K is

25. Let f : R $\rightarrow$ R be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of f (log5) is

KEY SHEET (PRACTICE SHEET)

EXERCISE-I

LEVEL-I

1) 4	2) 3	3) 4	4) 2	5) 3	6) 1	7) 3	8) 1
9) 3	10) 3	11) 2	12) 4	13) 4	14) 3	15) 2	16) 1
17) 2	18) 3	19) 2	20) 3	21) 2	22) 2	23) 3	24) 3
25) 3	26) 2	27) 2	28) 3	29) 2	30) 1	31) 3	32) 3
33) 2	34) 3	35) 4					

LEVEL-II

1) b	2) b	3) b	4) b	5) a	6) a	7) c	8) d
9) b	10) a	11) bcd	12) b	13) b	14) abc	15) abc	16) abcd
17) b	18) b	19) a	20) 2	21) 7	22) 2	23) 3	24) 3
25) 0							

EXERCISE-II

LEVEL-I

1) 1	2) 1	3) 3	4) 3	5) 3	6) 3	7) 2	8) 1
9) 1	10) 2	11) 1	12) 3	13) 2	14) 3	15) 2	16) 2
17) 2							

LEVEL-II

1) c	2) c	3) d	4) b	5) b	6) a	7) d	8) b
9) b	10) ad	11) ad	12) abc	13) c	14) c	15) b	16) c
17) a	18) A-s;B-s;C-r;D-q			19) 1	20) 2		

EXERCISE-III

LEVEL-I

1) 2	2) 2	3) 2	4) 1	5) 1	6) 2	7) 3	8) 4
9) 1	10) 1	11) 4	12) 3	13) 2	14) 1	15) 1	16) 1
17) 1	18) 2	19) 1	20) 1				

LEVEL-II

1) c	2) a	3) a	4) c	5) c	6) b	7) a	8) b
9) c	10) c	11) abc	12) ab	13) abd	14) ac	15) abcd	16) d
17) b	18) b	19) b	20) c	21) A-qrs, B-ps, C-rs, D-p			22) 0
23) 0	24) 1	25) 0					

ADDITIONAL EXERCISE

LEVEL-I (MAIN)

Single answer type questions

1. $\int_0^{\infty} \frac{45a}{(3+a+at)^4} dt =$

1) $\frac{15}{(3+a)^2}$ 2) $\frac{15}{(3+a)^3}$ 3) $\frac{15}{3(3+a)^3}$ 4) $\frac{15}{a(3+a)^2}$

2. $\int_0^1 \frac{(1-x^2)dx}{x^4+x^2+1} =$

1) $-\frac{1}{2}ln3$ 2) $\frac{1}{2}ln3$ 3) $ln\ 3$ 4) $2\ ln\ 3$

3. If $y = \int_x^{x^2} \sqrt{5-t^2}\, dt$ then the value of $\frac{dy}{dx}$ at $x=\sqrt{2}$ is

1) $1-\sqrt{3}$ 2) $\sqrt{3}(2\sqrt{6}-1)$ 3) $2\sqrt{2}-\sqrt{3}$ 4) $2\sqrt{2}+\sqrt{3}$

4. Let $f(x) = \int_0^x \frac{\cos t}{t} dt\ (x>0)$ then for $x=(2n+1)\frac{\pi}{2}; f(x)$ has

1) maxima when $n = 0, 2, 4, 6, ...$ 2) minima when $n = 0, 2, 4, 6, ...$

3) neither maxima nor minima when $n = -1,-3, -5,...$ 4) Information not sufficient

5. $\int_0^{4014} \frac{2^x}{2^x+2^{4014-x}} dx =$

1) 2^{2007} 2) 2^{4014} 3) 4014 4) 2007

6. $\int_{ln\lambda}^{ln\left(\frac{1}{\lambda}\right)} \frac{f\left(\frac{x^2}{3}\right)(f(x)+f(-x))}{g(3x^2)(g(x)-g(-x))} dx =$

1) 0 2) 1 3) λ 4) $\frac{1}{\lambda}$

7. If f is continuous function then which of the following is correct

1) $\int_{-2}^{2} f(x)dx = \int_0^2 (f(x)-f(-x))dx$ 2) $\int_{-3}^{5} 2f(x)dx = \int_{-6}^{10} f(x-1)dx$

3) $\int_{-3}^{5} f(x)dx = \int_{-6}^{10} f\left(\frac{x}{2}\right)dx$ 4) $\int_{-3}^{5} f(x)dx = \int_{-2}^{6} f(x-1)dx$

8. For $n \in N$, the value of $\int_0^{\pi} \sin^n x \cdot \cos^{2n-1} x \, dx$ is :

1) 0 2) n 3) $2n-1$ 4) $\frac{n\pi}{8}$

9. If $[x]$ represents greatest integer $\leq x$ then $\int_1^{3/2} [2x+1] dx =$

1) 1 2) 3 3) $\frac{1}{2}$ 4) $\frac{3}{2}$

10. $\int_0^2 [x^2 - 1] dx =$

1) $3-\sqrt{3}-\sqrt{2}$ 2) $3+\sqrt{3}+\sqrt{2}$ 3) $\sqrt{3}-1$ 4) $\sqrt{2}+1$

11. If $I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}$ and $I_2 = \int_1^2 \frac{dx}{x}$ then

1) $I_1 = I_2$ 2) $I_1 < I_2$ 3) $I_1 > I_2$ 4) $I_1 I_2 = 1$

12. $I = \int_0^1 e^{x^2} dx \Rightarrow$

1) $I \geq 0$ 2) $1 \leq I \leq e$ 3) $I \geq e$ 4) $I > 4$

13. The value of x in the interval $(-\pi, 0)$ satisfying $\sin x + \int_x^{2x} \cos 2t \, dt = 0$ is

1) $-\frac{\pi}{2}$ 2) $-\frac{\pi}{3}$ 3) $-\frac{\pi}{4}$ 4) $-\frac{\pi}{6}$

14. Let $f(x)$ = maximum $(x + |x|, x - [x])$, where $[x]$ is the greatest integer $\leq x$. Then $\int_{-2}^{2} f(x) \; dx$ is equal to

1) 3 2) 2 3) 1 4) 5

15. If $f(x) = Ax^2 + Bx$ satisfies the conditions $f^1(1) = 8$ and $\int_0^1 f(x) dx = \frac{8}{3}$, then

1) $A = 1; B = -4$ 2) $A = 2; B = 4$ 3) $A = -2; B = 4$ 4) $A = -2; B = -4$

16. If $I = \int_0^1 \sqrt{1+x^3} dx$ then

1) $I > 1$ 2) $I \neq \frac{\sqrt{5}}{2}$ 3) $I > \frac{\sqrt{7}}{2}$ 4) $I = 0$

17. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos^{2n-1} x - \cos^{2n+1} x} \; dx =$

1) $\frac{4}{2n+1}$ 2) $\frac{2n+1}{4}$ 3) $\frac{4}{2n-1}$ 4) $\frac{2n-1}{4}$

18. If $f''(x)$ and $f'(x)$ are continuous on $[a, b]$ then $\int_a^b xf^{11}(x)dx =$

1) $(bf^1(b) - af(a)) - (f(b) - f(a))$
2) $(af^1(a) - bf(b)) - (f(b) - f(a))$
3) $(bf^1(b) - af^1(a)) - (f(b) - f(a))$
4) $(bf^1(b) - f(b)) - (af^1(a) + f(a))$

19. If $\phi(x) = \int_x^{x^2}(t-1)dt, 1 \le x \le 2$ then the greatest value of $\phi(x)$ is

1) 2
2) 4
3) 8
4) 3

20. If $\int_0^{10} f(x)dx = 5,$ then $\sum_{K=1}^{10}\int_0^1 f(K-1+x)dx$ is equal to

1) 50
2) 10
3) 5
4) 15

LEVEL-II

LECTURE SHEET (ADVANCED)

Single answer type questions

1. The value of constant $a > 0$ such that $\int_0^a [\tan^{-1}\sqrt{x}]dx = \int_0^a [\cot^{-1}\sqrt{x}]dx$ is [.] denotes G.I.F

a) $\frac{2(3+\cos 4)}{1-\cos 4}$
b) $\frac{(3-\cos 4)}{1+\cos 4}$
c) $\frac{2(3+\cos 4)}{1+\cos 4}$
d) $\frac{(3+\cos 4)}{1-\cos 4}$

2. If $f(x) = \int_2^{x^2}\frac{(\sin^{-1}\sqrt{t})^2}{\sqrt{t}}dt$, then the value of $((1-x^2)f''(x))^2 - 2f'(x)$ at $x = \frac{1}{\sqrt{2}}$ is

a) $2 - \pi$
b) $3 + \pi$
c) $4 - \pi$
d) $\frac{3}{4}\pi^2$

3. Let a, b, c be non zero real numbers such that $\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx =$ $\int_0^2 (1+\cos^8 x)(ax^2+bx+c)dx$. Then the quadratic equation $ax^2 + bx + c = 0$ has

a) no root in (0, 2)
b) atleast one root in (1, 2)
c) double root in (0, 2)
d) two imaginary roots

4. Let $I_1 = \int_0^{\pi/2} e^{-x^2}\sin(x)dx$; $I_2 = \int_0^{\pi/2} e^{-x^2}dx$; $I_3 = \int_0^{\pi/2} e^{-x^2}(1+x)dx$ and consider the statements

I) $I_1 < I_2$
II) $I_2 < I_3$
III) $I_1 = I_3$

Which of the following is(are) true?

a) I only
b) II only
c) Neither I nor II nor III
d) Both I and II

5. Let $y = \{x\}^{[x]}$ where $\{x\}$ denotes the fractional part of x & $[x]$ denotes greatest integer $\leq x$, then $\int_0^3 y\,dx =$

a) 5/6 b) 2/3 c) 1 d) 11/6

6. The value of $\int_{-2}^{1}\left[x\left[1+\cos\left(\frac{\pi x}{2}\right)\right]+1\right]dx$, where [.] denotes the greatest integer function, is

a) 1 b) 1/2 c) 2 d) –1

7. The equation of the curve is $y = f(x)$. The tangents at $[1, f(1)]$, $[2, f(2)]$ and $[3, f(3)]$ make angle $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}$, respectively, with the positive direction of x-axis, then $\int_2^3 f'(x)f''(x)dx + \int_1^3 f''(x)dx =$

a) $-1/\sqrt{3}$ b) $1/\sqrt{3}$ c) 0 d) 1

More than one correct answer type questions

8. $I = \int_0^{\pi/2} \frac{dx}{1+(Tanx)^{\sqrt{2}}}$ then

a) I < 1

b) I is a rational number

c) I is irrational number

d) None of these

9. $\int_{-\infty}^{a} \frac{(\sin^{-1}e^x + \sec^{-1}e^{-x})dx}{(\mathrm{Tan}^{-1}e^a + \mathrm{Tan}^{-1}e^x)(e^x+e^{-x})}\ (a \in R) =$

a) independent if a b) dependent of a c) $\frac{\pi}{2}\log 2$ d) $\frac{\pi}{2}\log(2\mathrm{Tan}^{-1}e^a)$

10. If $f(x) = \frac{1}{2}a_0 + \sum_{i=1}^{n} a_i\cos(ix) + \sum_{j=1}^{n} b_j\sin(jx)$ then $\int_{-\pi}^{\pi} f(x)\cos kx\,dx =$

a) a_k b) b_k c) πa_k d) πb_k

PRACTICE SHEET (ADVANCED)

Single answer type questions

1. If $f(x)$ is monotonic and differentiable function, then $\int_{f(a)}^{f(b)} 2x(b - f^{-1}(x))dx =$

a) $\int_a^b f^2(x)dx$

b) $\int_a^b \left(f^2(x) - f^2(a)\right)dx$

c) $\int_a^b \left(f^2(x) - f^2(b)\right)dx$

d) $\int_a^b \left(f^2(x) + f^2(b)\right)dx$

2. The range of the function $f(x) = \int_{-1}^{1} \frac{\sin x\,dt}{(1-2t\cos x + t^2)}$ is

a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ b) $[0, \pi]$ c) $\{0, \pi\}$ d) $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

3. Let a,b and c be positive constants. The value of 'a' in terms of 'c' if the value of integral $\int_0^1 acx^{b+1} + a^3bx^{3b+5}dx$ is independent of b equals

a) $\sqrt{\frac{3c}{2}}$ b) $\sqrt{\frac{2c}{3}}$ c) $\sqrt{\frac{c}{3}}$ d) $\sqrt{\frac{3}{2c}}$

4. If x satisfies the equation $\left[\int_0^1 \frac{dt}{t^2+2t\cos\alpha+1}\right]x^2 - \left(\int_{-3}^{3}\frac{t^2\sin 2t\ dt}{t^2+1}\right)x - 2 = 0$ ($0 < \alpha < \pi$) then x =

a) $\pm\sqrt{\frac{\alpha}{2\sin\alpha}}$ b) $\pm\sqrt{\frac{2\sin\alpha}{\alpha}}$ c) $\pm\sqrt{\frac{\alpha}{\sin\alpha}}$ d) $\pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$

5. Let f be integrable over [o,a] for any real value of a if $I_1 = \int_0^{\pi/2} \cos\theta\ f(\sin\theta + \cos^2\theta)d\theta$ and $I_2 = \int_0^{\pi/2} \sin 2\theta\ f(\sin\theta + \cos^2\theta)d\theta$ then

a) $I_1 = -2I_2$ b) $I_1 = I_2$ c) $2I_1 = I_2$ d) $I_1 = I_2$

6. If $c > 0$, then $\int_0^{\infty}\frac{Tan^{-1}(cx)}{x(1+x^2)}dx =$

a) $\frac{\pi}{2}\log(1+c)$ b) $\pi\log(1+c)$ c) $\frac{\pi}{2}\log c$ d) $\pi\log c$

More than one correct answer type questions

7. If $\underset{n\to\infty}{Lt}\sum_{k=1}^{n}\left[\left(\frac{3k}{n}\right)^2 + 2\right]\frac{3}{n} = \int_0^a f(x)dx$ then

a) $a = 1$ b) $f(x) = 9x^2 + 2$

c) $\underset{n\to\infty}{Lt}\sum_{k=1}^{n}\left[\left(\frac{3k}{n}\right)^2 + 2\right]\frac{3}{n} = 15$ d) $\underset{n\to\infty}{Lt}\sum_{k=1}^{n}\left[\left(\frac{3k}{n}\right)^2 + 2\right]\frac{3}{n} = 5$

Integer answer type questions

8. Let $f(x) = \begin{cases} 1-|x|, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ and $g(x) = f(x-1) + f(x+1)\ \forall x \in R$, then $\int_{-3}^{3} g(x)dx =$

9. If $2f(x) + f(-x) = \frac{1}{x}\sin(x - \frac{1}{x})$ then $\int_{1/e}^{e} f(x)dx =$

10. $\frac{8\sqrt{2}}{\pi}\int_0^1\left(\frac{1-x^2}{1+x^2}\right)\frac{dx}{\sqrt{1+x^4}} =$

KEY SHEET (ADDITIONAL EXERCISE)

LEVEL-I (MAIN)

1) **2**	2) **2**	3) **3**	4) **1**	5) **4**	6) **1**	7) **4**	8) **1**	9) **4**	10) **1**
11) **2**	12) **2**	13) **2**	14) **4**	15) **2**	16) **1**	17) **1**	18) **3**	19) **2**	20) **3**

LEVEL-II

LECTURE SHEET (ADVANCED)

1) **a**	2) **d**	3) **b**	4) **d**	5) **4**	6) **c**	7) **a**	8) **ac**	9) **ac**	10) **c**

PRACTICE SHEET (ADVANCED)

1) **b**	2) **d**	3) **a**	4) **d**	5) **b**	6) **a**	7) **ac**	8) **7**	9) **0**	10) **2**