Chapter - 1

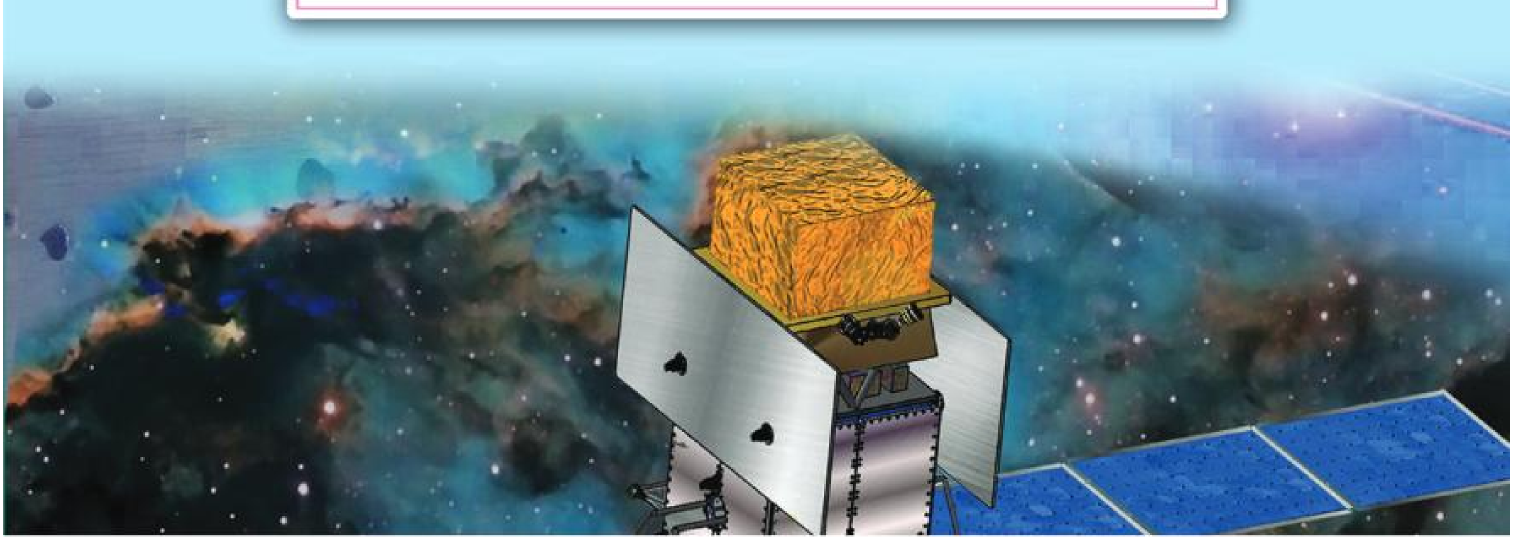
ELECTRIC FIELD AND POTENTIAL

✤ Charge, Coulomb's law, Electricfield, Electric potential ✤

✤ Electric field and electric potential due to different charge distributions ✤

✤ Electric potential energy, Electric dipole ✤

✤ Field and potential due to dipole, Dipole in electric field ✤



1.1 WHAT IS CHARGE ?

Just as masses are responsible for the gravitational force, charges are responsible for the electric force. If we keep two electrons separated by 1cm, the gravitational force between them is about 5.5×10^{-67} N whereas the electric force of repulsion between them is 2.3×10^{-24} N. So, it can be observed that electric force is very large compared to the gravitational force.

Every substance is made of atoms. An atom consists of positively charged nucleus at the centre and around it negatively charged electrons revolve in different orbits. Here the amount of total positive charge is equal to amount of total negative charge in an atom. So, atom is electrically neutral. If a body loses some of the electrons, the deficiency creates excess of positive charge on it and the body is said to be positively charged. Similarly if a body acquires an additional number of electrons, the body becomes negatively charged. When glass rod is rubbed with silk cloth, glass rod loses electrons and silk cloth gains those. As a result glass rod gets negative charge and silk cloth gets positive charge. Electrification carried out by rubbing bodies like this is known as frictional electrification.

Bodies can be charged by contact or induction. If an uncharged body is kept in contact with a positively charged body (say), the latter also gets positively charged as some of the electrons move form it to the first body. So, in this case charge is shared. When a positively charged body (+Q) is brought nearer to an uncharged conductor, an equal and opposite charge (–Q) will appear on the conductor on the near side and on the other side of the conductor a charge (+Q) will appear in order to maintain the original nature (neutral) of the conductor. This is known as induction. A charged body can attract an uncharged body due to induction. So, "induction precedes attraction".

- Charging an object by induction requires no contact with the object inducing the charge.

1.2 CHARGING BY INDUCTION

Consider an insulated neutral spherical conductor. A negatively charged rubber rod is brought near the sphere. The repulsive force between the electrons in the rod and those in the sphere result in redistribution of charge on the latter. As a consequence some electrons move to the side of the sphere farthest away from the rod. The region of the sphere nearest to the rod has an excess of positive charge due to the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere near the accumulation of electrons, some of the electrons leave the sphere and travel to the earth. If that wire is removed, the conducting sphere will be left with an excess of induced positive charge. Now if the rod is removed from the vicinity of the sphere, the induced positive charge remains on that isolated sphere. This excess positive charge will be uniformly distributed over the surface of the sphere. Hence it must be noted that the charged rod loses none of the charge on it.

A process similar to the first step in charging by induction can take place in insulators also. In neutral atoms and molecules of the insulator the average positions of positive and negative charges coincide. But if a charged object is brought closer, these positions may shift slightly due to attractive and repulsive forces from the charged object. Here each molecule gets more positive charge on one side than the other. This effect is known as polarization. This polarization of individual, molecules produces a layer of charge on the surface of the insulator as shown in figure. The attractive force between the dissimilar charges is larger than the repulsive force between similar charges.

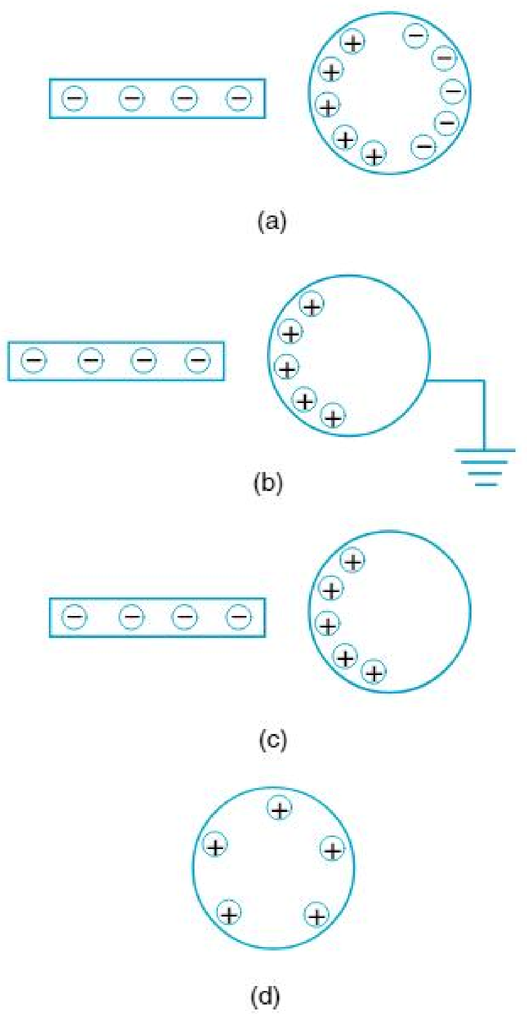


Fig 1.1 (a) Charging a metal sphere by induction

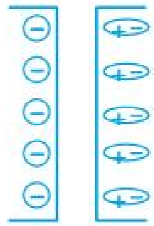


Fig 1.1 (b) Polarization by induction

Knowledge Plus 1.1

☺ A positively charged ball, hanging from a string is brought near a non conducting object. The ball is attracted by the object. From this experiment it is not possible to decide whether the object is negatively charged or neutral. Why?

☛ The attraction between the ball and the object could be an attraction of unlike charges or it could be an attraction between a charged object and a neutral object due to polarization of the molecules of the neutral object. Hence repulsion is the sure test to detect charge on a body. To check the charge on an object one can proceed like this. First a neutral ball could be brought near the object. If the ball is attracted to the object, the object is negatively charged. Another possibility is to bring a known negatively charged ball near the object. If the ball is repelled by the object, the object is negatively charged. If the ball is attracted, it is neutral.

1.3 PROPERTIES OF CHARGES

a) Like charges repel and unlike charges attract each other.

b) Any excess charge given to a conductor, always resides on the outer surface of the conductor

c) Charge is invariant, i.e. charge does not undergo any change due to its motion

d) Stationary charges are not affected by magnetic fields.

e) A charge in motion will be affected by magnetic field provided direction of motion of the charge is neither parallel nor antiparallel to the field direction.

f) The charge density (charge per unit surface area) tends to be very large at sharp points on an isolated conductor. Lightning rods used on tall buildings to prevent lightning from striking the building work on this principle.

g) Charge is additive. The total charge on a body means algebraic sum of positive and negative charges on it. This property is a consequence of the fact that charge is a scalar.

h) The total charge in an isolated system remains constant. Electric charge can neither be created nor destroyed. This is known as law of conservation of charge

Sometimes it seems that charged particles are created. For example a neutron may convert into an electron and proton during β^- emission. These two particles produced have equal and unlike charges before and after this conversion. Similarly proton converts into neutron and position during β^+ emission. In these two cases total charge is conserved.

i) Charge is always quantised i.e., $Q = \pm ne$. Here n is an integer and e is electron charge. The quantization of electric charge was experimentally verified by R.A. Millikan drop experiment $e = 1.6\times10^{-19}$ C. So, 1C (coulomb) = 6.25×10^{18} electrons.

A fraction of charge e is not possible in free state. But in transient state quarks can have $\pm\frac{1}{3}e$ and $\pm\frac{2}{3}e$

If a body is positively charged with one coulomb, 6.25×10^{18} electrons must be removed from it.

These are six types of quarks. Out of these four have charge $+\frac{2}{3}e$ and the remaining two $-\frac{1}{3}e$ Protons and neutrons are now known to be made up of quarks. Free quarks have not been detected. But there are experimental evidence for the existence of all six quarks and corresponding antiquarks within the nucleus. A proton consists of two up quarks and sone down quark and a neutron consists of one up quark and two down quarks. Charge carried by upquark is $+\frac{2}{3}e$ and charge carried by down quark is $-\frac{1}{3}e$

At the microscopic level, practically quantisation of charge has no consequence. So that it may be ignored. But at the macroscopic level charges appear as discrete lumps and so quantisation of charge cannot be ignored.

j) A stationary charge produces electrostatic field and an isolated charge in motion produces magnetic field also in addition to electrostatic field. Charge subjected to acceleration produces electromagnetic field.

k) A body can have mass without charge whereas a body cannot have charge without mass.

l) SI unit of charge is coulomb. Charge of electron is 1.602×10^{-19}C (Negative)

1.4 GOLD LEAF ELECTROSCOPE

Gold leaf electroscope is used to detect charge on a body. It consists of a vertical metal rod with a metal knob at the top and two thin gold leaves attached at the bottom end. This rod is arranged in a box with glass windows as shown. If a charged body touches the metal knob at the top, the charge flows on to the leaves through the rod. As a result the leaves diverge. The amount of charge is indicated by the degree of divergence of the leaves. Here both halves of the foil (i.e., leaves) get similar charge and so repel each other.

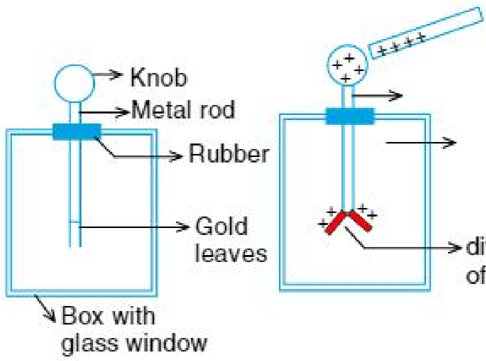


Fig 1.2 Gold Leaf Electrolysis

1.5 CONDUCTORS AND INSULATORS

Substances can be classified into conductors and insulators based on their ability to allow charge to flow. Substances which allow electricity i.e., charges to pass through them easily are called conductors. All metals are conductors. Conductors have charges (electrons) which are relatively free to move inside them. Human body and the earth are also considered as conductors. Some substances oppose or resist the passage of electricity or charges through them which are known as insulators. Non metals like glass, nylon, plastic, wood etc., are insulators. If some charge is given to a conductor, it gets distributed over the entire surface of that conductor. But in the case of insulator, it will not be distributed and stays at the same place.

1.6 EARTHING OR GROUNDING

If a charged body is kept in contact with the earth, by a conductor, the excess charge on that body disappears. Here that charge gets transferred into the earth. This process is known as grounding or earthing. Earthing is done for the safety of electrical appliances and circuits. The household electric wiring will have three wires which are known as live, neutral and earth. The first two carry electric current from the power station. The third wire is earthed by connecting it to a thick metal plate burried deep into the earth. Metallic bodies or surfaces of the electric appliances like TV, refrigerator etc., are connected to the earth wire. If live wire touches the metallic body by fault, the charge flows to the earth without damaging that appliance. This prevents the injury to the person in contact with that appliance also.

1.7 COULOMB'S LAW (INVERSE SQUARE LAW)

Electric force between charged objects were measured quantitatively by Charles Coulomb. The force of attraction or repulsion between charges exists even in vacuum. The electrostatic force between two protons or electrons is about 10^{36} times stronger than the gravitational force. As the gravitational force between charges is very small it may be usually neglected in comparison with the electrostatic force. The statement of this law can be stated as given below.

"The force of attraction or repulsion between two stationary point charges is directly proportional to the product of the magnitude of the two charges and is inversely proportional to the square of the distance between them. This force acts along the line joining those two charges".

Consider two point charges q_1 and q_2 at rest at points A and B. The seperation between those two charges is r. As per the statement, the force F applied by one charge on the other is proportional to $q_1 q_2$ and inversely proportional to r^2.

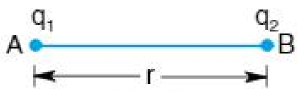


Fig 1.3

$$\Rightarrow F \propto q_1 q_2 \text{ and } F \propto \frac{1}{r^2}$$

$$\Rightarrow F \propto \frac{q_1 q_2}{r^2}$$

$$\text{or } \boxed{F = K_0\left(\frac{q_1 q_2}{r^2}\right)} \qquad \text{.... 1.1(a)}$$

Here K_0 is proportionality constant. The value of K_0 depends on the medium between the charges and also on the system of units in which the charges and distance r are expressed. K_0 is known as coulomb's constant.

In SI, for free space i.e., air or vacuum, $K_0 = \frac{1}{4\pi \epsilon_0}$ where ϵ_0 is called the permittivity of free space. Where $\epsilon_0 = 8.854 \times 10^{-12}\ C^2/Nm^2$. In free space we can write

$$\boxed{F_0 = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2}} \qquad \text{..... 1.1(b)}$$

This is the mathematical form of coulomb's inverse square law in free space.

In SI, the charges are expressed in coulombs and distance in metres. The value of K_0 is $9\times10^9\ Nm^2/C^2$ in that system. Now coulomb's law can be written as

$$\boxed{F_0 = 9\times10^{19}\frac{q_1q_2}{r^2}\ (\text{in newton})} \quad \text{.... 1.1(c)}$$

From this we can define coulomb as given below

"Coulomb is that charge which when placed at a distance of one meter in air or vacuum from an identical charge, repels it with a force of 9×10^9 N".

Each charge exerts a force of this magnitude on the other charge such that the two forces form Newton's third law force pair.

(Coulomb is the amount of charge which will be transferred through the cross section of a wire in 1 second when there is a current of 1 ampere in the wire)

1.8 PERMITTIVITY OF MEDIUM

The medium surrounding charged bodies affect the electric force. This is verified experimentally. It is observed that the force between two point charges is maximum in free space. A medium always reduces this force. The permittivity of a medium is denoted by $\in$. Permittivity of a medium explains about its response when charges are kept in it.

In SI, for medium other than free space, $K_0 = \frac{1}{4\pi\in}$ so that we can write the equation for the force between the charges as

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}} \quad \text{.... 1.2(a)}$$

from equations 1.1(b) and 1.2(a) we can write

$$\boxed{\frac{F_0}{F} = \frac{\in}{\in_0} = \in_r} \quad \text{.... 1.2(b)}$$

$\in_r$ is known as the relative permittivity of the medium. It is a constant for a given medium and it gives the extent to which the force between two charges separated by a medium decreases compared with the force between the same charges in free space separated by the same distance. Relative permittivity is also known as dielectric constant K of the medium.

So, relative permittivity or dielectric constant

$$K = \frac{\text{permittivity of the medium}}{\text{permittivity of free space}}$$

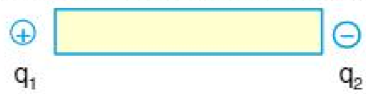
So, the mathematical form of inverse square law is given as

$$\boxed{F = \frac{1}{4\pi\in}\frac{q_1q_2}{r^2} = \frac{1}{K}\frac{1}{4\pi\in_0}\frac{q_1q_2}{r^2}} \quad \text{.... 1.2(c)}$$

For free space or vacuum or air K = 1 and for a good conductor $K \to \infty$

Knowledge Plus 1.2

An insulator in the form of a thin rod is placed between two unlike point charges $+q_1$ and $-q_2$. How will the force acting on the charges change?



Due to polarization of the rod, charge $+q_1$ will experience forces due to induced charges. The attractive force exerted by the negative charge at the near end will be stronger than the repulsion by the positive charge at the far end. Thus total force acting on $+q_1$ will increase.

1.9 LIMITATIONS OF COULOMB'S LAW

a) Coulomb's law is valid for point charges only. Suppose if two large conducting spheres having charges q_1 and q_2 are seperated by certain distance, the actual force between those will be different from the value obtained from the formula. The reason behind this is induced charges.

b) Coulomb's law is valid only for static charges. Suppose if two charges are moving, both will have an associated magnetic field also in addition to electrostatic field. So the net force will be vector sum of electrostatic force and the magnetic force.

$$\boxed{\overline{F} = \overline{F}_e + \overline{F}_m}$$

If one of the charges is at rest here, $\vec{F}_m = 0$ and now coulomb's law can be applied. So, we deduce that coulomb's law can be applied to point charges if at least one of them is at rest.

Coulomb's law is valid upto nuclear distance of 10^{-15}m to 10^{-16}m.

Example-1.1

Two charges $4\mu C$ and $1\mu C$ are placed at a distance of 10 cm. Where should a third charge be placed between them so that it does not experience any force.

Solution :

$Q_1 = 4\mu C = 4\times10^{-6}C$; $Q_2 = 1\mu C = 1\times10^{-6}C$

d = 10 cm

Let the third charge Q be placed at a distance of x from Q_1 then $x = ?$

$$\frac{Q_1}{x^2} = \frac{Q_2}{(10-x)^2};\ \frac{4\times10^{-6}}{x^2} = \frac{1\times10^{-6}}{(10-x)^2},\ \frac{2}{x} = \frac{1}{10-x}$$

$x = \frac{20}{3}$ cm from $4\mu C$.

Example-1.2

Calculate the ratio of electric and gravitational force between two protons. Charge of each proton is 1.6×10^{-19} C, mass is 1.672×10^{-27} kg and $G = 6.67\times10^{-11}\ Nm^2kg^{-2}$.

Solution :

Electrostatic force between two protons is

$$F_1 = 9\times10^9\frac{Q_1Q_2}{r^2}$$

$Q_1 = 1.6\times10^{-19}$ C, $Q_2 = 1.6\times10^{-19}$ C

r is the distance between the two protons

Gravitational force between them : $F_2 = \frac{Gm_1m_2}{r^2}$

$m_1 = m_2 = 1.672\times10^{-27}$ kg

$G = 6.67\times10^{-11}\ Nm^2/kg^2$

$$\therefore\ \frac{F_1}{F_2} = \frac{9\times10^9Q_1Q_2}{G.m_1m_2}$$

$$= \frac{9\times10^9\times1.6\times10^{-19}\times1.6\times10^{-19}}{6.67\times10^{-11}\times1.672\times10^{-27}\times1.672\times10^{-27}}$$

$= 1.23\times10^{36}$

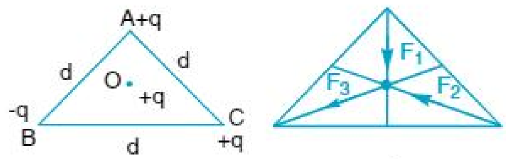
Example-1.3

Three charges +q, –q and +q are kept at the corners of an equilateral triangle of side d. Find the resultant electric force on a charge +q placed at the centroid O of the triangle.

Solution :

Let the force acting on +q charge at O due to +q at A be F_1, +q at B be F_2 and –q atC be F_3

Here $AO = BO = CO = \frac{d}{\sqrt{3}}$



In magnitude $F_1 = F_2 = F_3 = \frac{1}{4\pi\epsilon_0}\frac{3q^2}{d^2}$

Resultant of F_1 and F_2 is $F_4 = \frac{1}{4\pi\epsilon_0}\frac{3q^2}{d^2}$

(as angle between F_1 and F_2 is 120^0)

Direction of F_4 is along the direction of F_3. Hence the resultant force on +q at O is $F = F_3 + F_4 = \frac{3q^2}{2\pi\epsilon_0 d^2}$

Example-1.4

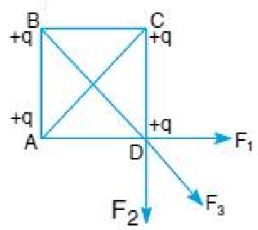
Four charges of +q, +q, +q and +q are placed at the corners A, B, C and D of a square. Find the resultant force on the charge at D.

Solution :

Let side = a, $BD = \sqrt{2}a$

force $F_1 = \frac{1}{4\pi\epsilon_0}.\frac{q^2}{a^2}$ (A to D)

force $F_2 = \frac{1}{4\pi\epsilon_0}.\frac{q^2}{a^2}$ (C to D) $\therefore F_1 = F_2 = F$



Resultant of F_1 and $F_2 = \sqrt{F_1^2 + F_2^2} = \sqrt{2}.F$

force due to charge at B is

$$F_3 = \frac{1}{4\pi\epsilon_0}.\frac{q^2}{(\sqrt{2}a)^2} = \frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\ \text{(B to D)}$$

Resultant force on change at D = $\sqrt{2}F + F_3$

$$=\sqrt{2}\frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{a^2}+\frac{1}{2}\frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{a^2}$$

$$=\frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\left[\sqrt{2}+\frac{1}{2}\right]=\frac{q^2}{8\pi\epsilon_0 a^2}\cdot\left[1+2\sqrt{2}\right]$$

Example-1.5

A charge Q is to be divided on to two small objects. What should be the value of the charges on the objects so that the force between the objects will be maximum.

Solution :

Let q and (Q – q) be the charges on those bodies

Force between the charges

$$F=\frac{1}{4\pi\epsilon_0}\frac{(Q-q)q}{r^2}\Rightarrow q=\frac{Q}{2}$$

For F to be maximum $\frac{dF}{dq}=0$

$$\Rightarrow\frac{Q}{r^2}-\frac{2q}{r^2}=0\quad\Rightarrow q=\frac{Q}{2}$$

Thus we have to divide charges equally on the objects.

Example-1.6

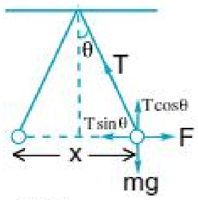
Two small equally charged spheres each of mass m are suspended from the same point by silk threads of length L. The distance between the spheres x << L. Find the rate $\frac{dq}{dt}$ with which the charges leak off each sphere if their approach velocity varies as $\vartheta=\frac{a}{\sqrt{x}}$ where a is a constant.

Solution :

For momentary equilibrium of the spheres,

$T\cos\theta=mg$ and $T\sin\theta=F$

where T is tension in the string



$$\Rightarrow\tan\theta=\frac{F}{mg}$$

For x << L,

we can take $\tan\theta=\frac{x}{2L}$; So $F=\frac{mgx}{2L}$

Here $F=\frac{1}{4\pi\epsilon_0}\frac{q^2}{x^2}$ and $q^2=\frac{2\pi\epsilon_0 mgx^3}{L}$

given $\frac{dx}{dt}=\frac{a}{\sqrt{x}}$ where a is a constant

$$\Rightarrow\frac{dq}{dt}=\frac{3a}{2}\sqrt{\frac{2\pi\epsilon_0 mg}{L}}$$

1.10 COULOMB'S LAW IN VECTOR FORM

Let $\bar{r}_1$ and $\bar{r}_2$ be the position vectors of point charges q_1 and q_2 respectively. Denote force on q_1 due to q_2 by $\bar{F}_{12}$ and force on q_2 due to q_1 by $\bar{F}_{21}$

$$\bar{r}_{21}=\bar{r}_2-\bar{r}_1 \text{ and } \bar{r}_{12}=\bar{r}_1-\bar{r}_2=-\bar{r}_{21}$$

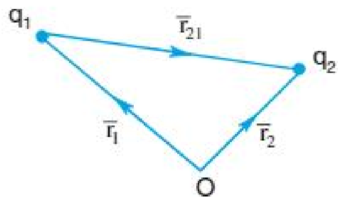


Fig 1.4 (a)

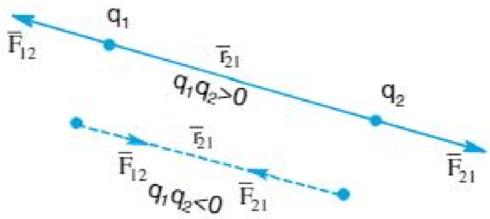


Fig 1.4 (b)

Coulomb's force between q_1 and q_2 located at $\bar{r}_1$ and $\bar{r}_2$ is expressed as

$$\bar{F}_{21}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{21}^2}\hat{r}_{21}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{21}^3}\bar{r}_{21}$$

$$\text{and } \bar{F}_{12}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^3}\bar{r}_{12}$$

The above equation is valid for any sign of q_1 and q_2 (whether positive or negative). If q_1 and q_2 are of the same sign, $\bar{F}_{21}$ is along $\bar{r}_{21}$ which denotes repulsion. If q_1 and q_2 are of opposite signs, $\bar{F}_{21}$ is along $-\bar{r}_{21}$ which denotes attraction.

From the above equations, we can observe that $\bar{F}_{12}=-\bar{F}_{21}$, which explains that coulomb's law agrees with Newton's third law.

1.11 FORCE BETWEEN MULTIPLE CHARGES (PRINCIPLE OF SUPER POSITION)

The resultant force on any point charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges taken one at a time. The individual forces are unaffected due to the presence of other charges. This is known principle of superposition.

Consider a system of stationary point charges $q_1, q_2, ..., q_n$ in vacuum. There is a stationary point charge q also such that its distance from q_1 is r_1, its distance from q_2 is r_2 ..... its distance from q_n is r_n.

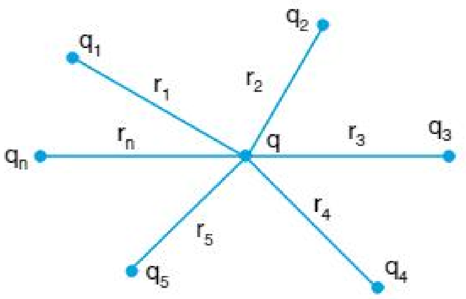


Fig 1.5

The force acting on q due to q_1 is $F_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q}{r_1^2}$

The force acting on q due to q_2 is $F_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2 q}{r_2^2}$

Similarly the force acting on q due to q_n is $F_n = \frac{1}{4\pi\epsilon_0}\frac{q_n q}{r_n^2}$

The net force acting on q is given by

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + ... + \bar{F}_n = \sum \bar{F}_n$$

- If charge q is spread over a region instead of being concentrated at particular point, the force applied by it on point charge Q is

$$\boxed{\bar{F} = \frac{1}{4\pi\epsilon_0}\int\frac{Qdq}{r^2}\hat{r}} \quad .... 1.2(d)$$

- Two point charge q_1 and q_2 are separated by r_0 in vacuum. The same charges are seperated by r in a medium of dielectric constant K. If the force between the charges is same.

$$\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_0^2} = \frac{1}{K}\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

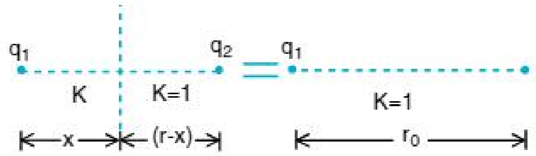
$$r_0 = \sqrt{K}r$$

$\Rightarrow$ a seperation of r in a medium is equivalent to seperation $\sqrt{K}r$ in vacuum.

- The coulomb force between any two point charges is independent of the presence of other charges.
- The force applied by q_1 on q_2 is medium dependent but the net force applied by q_1 on q_2 is medium independent as induced charges are developed.

Example-1.7

Two point charges are seperated by a distance r such that a medium of dielectric constant K is occupied by a length x Now find the coulomb force between those stationary charges.



Solution :

In this case we can remove the dielectric and its effective equivalent distance in free space can be taken as $\sqrt{K}x$

Now the force between the point charges is

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{\left\{(r-x)+\sqrt{K}x\right\}^2}$$

If K = 4 and x = r/2

$$F = \frac{1}{4\pi\epsilon_0}\frac{4q_1q_2}{9r^2} = \frac{4}{9}\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$F = \frac{4}{9}F_0$ where

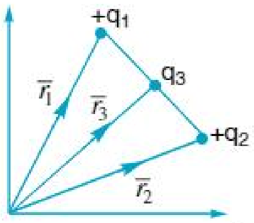
$F_0 = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$ which is the force between the charges if they are separated by r in vacuum.

Example-1.8

Two point charges $+q_1$ and $+q_2$ are located at two points with position vectors $\vec{r}_1$ and $\vec{r}_2$. Find a negative charge q_3 and the position vector $\vec{r}_3$ of the point at which it has to be placed for the force acting on each of the three charges to be equal to zero.

Solution :

For the equilibrium of q_3



$$\frac{1}{4\pi\epsilon_0}\left\{\frac{q_2q_3(\bar{r}_2-\bar{r}_3)}{|\bar{r}_2-\bar{r}_3|^3}+\frac{q_1q_3(\bar{r}_1-\bar{r}_3)}{|\bar{r}_1-\bar{r}_3|^3}\right\}=0$$

but $\frac{\bar{r}_2-\bar{r}_3}{|\bar{r}_2-\bar{r}_3|}=-\frac{\bar{r}_1-\bar{r}_3}{|\bar{r}_1-\bar{r}_3|}$ or $\frac{q_2}{|\bar{r}_2-\bar{r}_3|^2}=\frac{q_1}{|\bar{r}_1-\bar{r}_3|^2}$

or $\sqrt{q_2}(\bar{r}_1-\bar{r}_3)=\sqrt{q_1}(\bar{r}_3-\bar{r}_2)$

$$\Rightarrow \bar{r}_3=\frac{\sqrt{q_2}\,\bar{r}_1+\sqrt{q_1}\,\bar{r}_2}{\sqrt{q_1}+\sqrt{q_2}}$$

For the equilibrium of q_1

$$\frac{1}{4\pi\epsilon_0}\left\{\frac{q_3(\bar{r}_1-\bar{r}_3)}{|\bar{r}_1-\bar{r}_3|^3}+\frac{q_2(\bar{r}_2-\bar{r}_1)}{|\bar{r}_2-\bar{r}_1|^3}\right\}=0 \text{ or}$$

$$q_3=\frac{q_2|\bar{r}_1-\bar{r}_3|^2}{|\bar{r}_2-\bar{r}_1|^2}$$

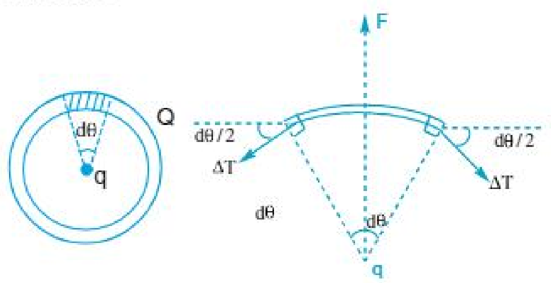
Substituting the value of r_3,

we get $q_3=\frac{-q_1q_2}{\left(\sqrt{q_1}+\sqrt{q_2}\right)^2}$

Example-1.9

A ring of radius R is with a uniformly distributed charge Q on it. A charge q is now placed at the centre of the ring. Find the increment in tension in the ring.

Solution :



Consider an element. Its enlarged view is as shown. For equilibrium of this segment, we can write.

$$F=2\Delta T\sin\left(\frac{d\theta}{2}\right)$$

Here F is the repulsive force between q and elemental charge dQ

$$dQ=\frac{Q}{2\pi R}(Rd\theta)$$

The electric outward force on element is

$$F=\frac{1}{4\pi\epsilon_0}\frac{qdQ}{R^2}$$

From the above three equations, we can write

$$\frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\frac{QRd\theta}{2\pi R}=2\Delta T\left(\frac{d\theta}{2}\right)$$

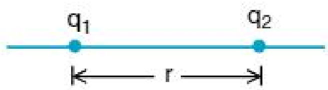
($\because \sin\alpha=\alpha$ for small angle)

$$\Delta T=\frac{Qq}{8\pi^2\epsilon_0 R^2}$$

Example-1.10

Charge q_1 is fixed and another point charge q_2 is placed at a distance r_0 from q_1 on a frictionless horizontal surface. Find the velocity of q_2 as a function of seperation r between them (treat as point charges and mass of q_2 is m)

Solution :



When the seperation between the charges is r, the force between them is $F=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

Acceleration of $q_2=\frac{F}{m}=\frac{q_1q_2}{4\pi\epsilon_0 mr^2}$

$$\frac{dv}{dt}=\frac{dv}{dr}\left(\frac{dr}{dt}\right)=\frac{vdv}{dr}$$

$$\Rightarrow \frac{vdv}{dr}=\frac{q_1q_2}{4\pi\epsilon_0 mr^2}$$

$$\int_0^v vdv=\frac{q_1q_2}{4\pi\epsilon_0 m}\int_{r_0}^{r} r^{-2}dr$$

$$\text{or } \frac{v^2}{2}=\frac{q_1q_2}{4\pi\epsilon_0 m}\left|\frac{-1}{r}\right|_{r_0}^{r}$$

$$\Rightarrow v=\sqrt{\frac{q_1q_2}{2\pi\epsilon_0 m}\left\{\frac{1}{r_0}-\frac{1}{r}\right\}}$$

1.12 SHELL THEOREMS

First Theorem

A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the charge on that conducting shell were concentrated at its centre.

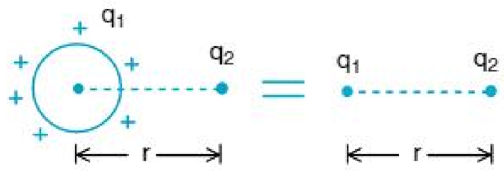


Fig 1.6

Second Theorem

A uniformly charged shell exerts no electrostatic force on a charged particle located inside the shell

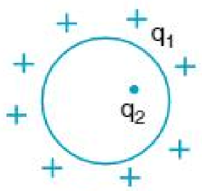


Fig 1.7

Here force between q_1 and $q_2 = 0$

1.13 ELECTRIC FIELD

From coulomb's law we know that a point change q_1 exerts or force on point charge q_2. Here we find this force without any contact between those charges. The action of one charge at a distance can be explained by saying that the charge sets up an electric field.

We know that in a room temperature at every point will have a definite value which can be measured using a thermometer. The resulting distribution of temperature in a given region is known as temperature field. Similarly we can imagine pressure field in atmosphere, which denotes distribution of air pressure. These two are scalar fields. Electric field is a vector field, which is a distribution of vectors, one for each point in the region around a charged particle or distribution of charge.

Consider an electric field in a given region. Bring a charge q_0 to a given point in that field without disturbing any other charge that has produced the field. Let $\vec{F}$ be the electric force experienced by q_0. We define the intensity of electric field $\overline{E}$ at the given point as $\overline{E} = \frac{\overline{F}}{q_0}$. Here q_0 is called test charge and it does not disturb the field already present. So, we can say $q_0 \to 0$ i.e. q_0 is negligibly small. So we can denote electric field intensity as $\overline{E} = \underset{q_0 \to 0}{Lt}\left(\frac{\overline{F}}{q_0}\right)$ .... (1.3)

The intensity of electric field is often called as electric field strength. **"The intensity of electric field or electric field strength E at a point in space is defined as the force experienced by unit positive test charge placed at that point".**

Electric field strength is a vector quantity. Its direction is in the direction along which the force on a positive charge acts. The S.I unit of electric field strength is newton per coulomb (NC^{-1}). It can be expressed in volt per metre (Vm^{-1}). Dimensional formula for E is $MLT^{-3}I^{-1}$.

Electric field can be uniform or non uniform. A uniform electric field is that in which at every point, the intensity of the electric field E is the same both in magnitude and direction. A non-uniform electric field is that in which the intensity of electric field E changes from point to point either in magnitude or in direction or in both.

1.14 ELECTRIC FIELD INTENSITY DUE TO AN ISOLATED POINT CHARGE

Consider a point charge q placed at point A as shown. Let us find the electric field $\overline{E}$ at a point P at a distance r from q. Imagine a positive test charge q_0 at P. The charge q produces a field $\overline{E}$ at P. The force applied by q on q_0 is given by

$$F = \frac{1}{4\pi \epsilon_0}\frac{qq_0}{r^2} \text{ which acts along AP.}$$

By definition E means $\frac{F}{q_0} \Rightarrow E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2}$ ..(1.4)

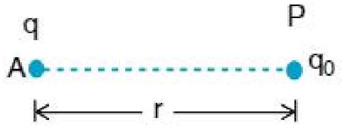


Fig 1.8 (a)

- If q is positive charge, $\overline{E}$ is along A to P and if q is negative charge, $\overline{E}$ is along P to A.

- Electric field due to many charges can be obtained by the principle of superposition i.e., If $\overline{E}_1, \overline{E}_2 .... \overline{E}_n$ be the electric fields produced by $q_1, q_2, ..., q_n$ at a point, then the resultant field at that point is

$$\overline{E} = \overline{E}_1 + \overline{E}_2 + ... + \overline{E}_n$$

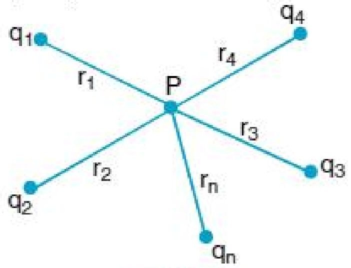


Fig 1.8 (b)

Here $E_1 = \dfrac{1}{4\pi \in_0} \dfrac{q_1}{r_1^2}$, $E_2 = \dfrac{1}{4\pi \in_0} \dfrac{q_2}{r_2^2}$....

- If $\overline{E}$ is electric field at a point, and charge q is kept at that point such that force acting on that charge is $\overline{F}$,

  we can write F = Eq

  Here if q is positive $\overline{F} \parallel \overline{E}$

  if q is negative $\overline{F} \parallel -\overline{E}$

- If m is mass of charged particle which is in a uniform electric field E, its acceleration is

$$a = \frac{F}{m} = \frac{Eq}{m}$$

- A proton and an electron when left in a uniform electric field, both will experience same force in magnitude but opposite in direction. They experience different acceleration in magnitudes along opposite direction.

- E can be expressed in vector form also. Consider charge q at position vector $\overline{r}_1$ with respect to the origin of a coordinate system. Electric field at a point with position vector $\overline{r}_2$ is given by

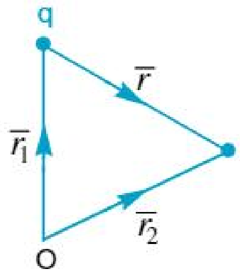


Fig 1.9

$$\overline{E} = \frac{1}{4\pi \in_0} \frac{q}{r^2} \hat{r} = \frac{1}{4\pi \in_0} \frac{q}{r^3} \overline{r}$$

Here $\overline{r} = (\overline{r}_2 - \overline{r}_1)$

if q is positive charge, $\overline{E} \parallel \overline{r}$

if q is negative charge $\overline{E} \parallel -\overline{r}$.

Example-1.11

An infinite number of charges each of value Q are placed on the x-axis at distances of 1, 2, 4, 8,....... meter from the origin. If the charges are alternately positive and negative find the electric field (Intensity of electric field) at origin.

Solution :

The electric field

$$E = \frac{Q}{4\pi \in_0}\left(1 - \frac{1}{2^2} + \frac{1}{4^2} - \frac{1}{8^2} + ...\right)$$

$$= \frac{Q}{4\pi \in_0} \frac{1}{\left[1 - \left(\frac{-1}{4}\right)\right]} = \frac{Q}{4\pi \in_0} \cdot \frac{4}{5}$$

since the expression in the bracket is in GP with a common ratio $= \frac{1}{2^2} = \frac{1}{4}$.

$$E = \frac{4}{5} \cdot \frac{Q}{4\pi \in_0}$$

$$E = \frac{Q}{5\pi \in_0}$$

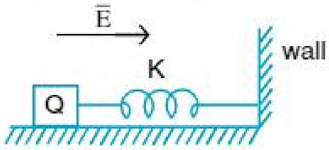
Example-1.12

A point mass m and charge Q is connected with a spring of negligible mass. Initially spring is in its natural length L. Now a horizontal uniform field E is switched on as shown. Find

a) the maximum seperation between the mass and the wall

b) Find the seperation of the point mass and wall at the equilibrium position of mass

c) Find the energy stored in the spring at the equilibrium position of the point mass.



Solution :

At maximum seperation, velocity of point mass is zero. From work energy theorem,

$$W_{spring} + W_{field} = 0$$

$$qEx_0 - \frac{1}{2}Kx_0^2 = 0 \quad (x_0 \text{ is maximum elongation})$$

$$\Rightarrow x_0 = \frac{2qE}{K}$$

$$\therefore \text{seperation } = L + \frac{2qE}{K}$$

b) At equilibrium position, $Eq = Kx \Rightarrow x = \dfrac{qE}{K}$

$$\Rightarrow \text{seperation } = L + \frac{qE}{K}$$

c) $U = \dfrac{1}{2}Kx^2 = \dfrac{1}{2}K\left(\dfrac{qE}{K}\right)^2 = \dfrac{q^2E^2}{2K}$

Example-1.13

A point charge $50\mu C$ is located at a point $2\hat{i}+3\hat{j}$. Find the electric field vector $\bar{E}$ at a point with position vector $8\hat{i}-5\hat{j}$, when the position vectors are expressed in metre.

Solution :

$$\bar{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^3}\bar{r} \text{ ; here } q = 50\times10^{-6}C$$

$$\bar{r} = (\bar{r}_2 - \bar{r}_1) = (8\hat{i}-5\hat{j}) - (2\hat{i}+3\hat{j}) = (6\hat{i}-8\hat{j})$$

then $\bar{E} = 450(6\hat{i}-8\hat{j})NC^{-1}$.

1.15 ELECTRIC FIELD LINES

The electric field in a region can be visualized by drawing imaginary curves known as electric field lines or lines of electric force (*"A line of force is an imaginary line or curve along which a positive point charge free to move would travel such that the tangent to the curve at any point will be parallel to the direction of the electric field at that point".)*

The lines of force due to certain charges are shown in Fig 1.11(a) to 1.11(g).

In a uniform electric field the lines of force will be equidistant, parallel and straight lines directed alike.

The main properties of lines of force are

a) The tangent to the line of force at any point on it gives the direction of the electric field at that point.

b) No two electric lines of force intersect each other. (If they intersect at a point, at that point the electric field should have two different directions which is not possible).

c) Electric line of force will be normal to the surface of a conductor.

d) Electric field lines are open lines. They start from positive charge and end on negative charge. (Magnetic lines of force are closed loops).

e) The number of lines of force per unit cross sectional area at any point is proportional to the magnitude of electric field strength E at that point.

f) The lines of force are crowded where the field is strong and sparse where the field is weak. We can compare the intensities of the field at two points by studying the distribution of field lines.

g) The field lines have no physical existence. They are purely a geometrical construction which help us to visualise the nature of electric field in a region.

h) In a charge free region electric field lines can be taken to be continuous curves without any breaks.

i) Electrostatic field lines do not form any closed loops. This follows from the conservative nature of electric field.

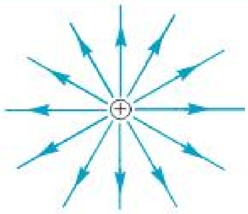


Fig 1.10 (a)

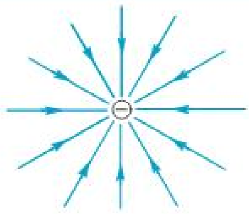


Fig 1.10 (b)

Fig 1.10 (c)

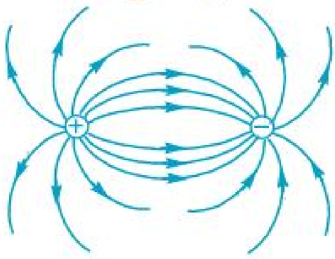


Fig 1.10 (d)

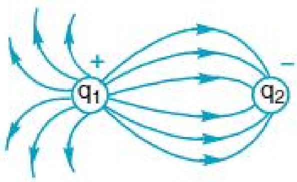


Fig 1.10 (e)

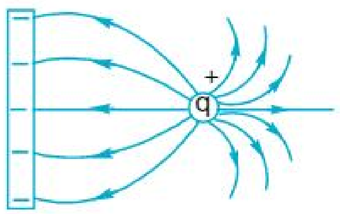


Fig 1.10 (f)

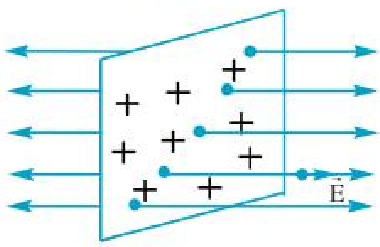


Fig 1.10 (g)

j) The number of field lines passing through the area perpendicular to the field is known as electric flux ϕ.

1.16 SIGNIFICANCE OF ELECTRIC FIELD

In physics 'field' generally refers to a quantity that is defined at every point in space. Field may vary from point to point. Electric field is a vector field just like gravitational field. The greatest speed with which an information of signal can be transmitted from one point to another is speed of light 'C'. Consider force between two distinct charges q_1, q_2 in accelerated motion. Motion of q_1 may be cause and resulting force on q_2 is effect, there will be some time delay between these two. Charge q_1 produces electromagnetic waves due to its acceleration. Those waves propagate with speed of light C, reach q_2 and cause a force on q_2. We must know that electric and magnetic fields can be detected only by their effects on charges. Both fields transport energy.

1.17 EQUILIBRIUM OF SYSTEM OF CHARGES

For any object or the system if net force acting is zero and net torque on it is zero, we can say that it is in equilibrium. For a system of point charges net force $\overline{F} = 0$ for translational equilibrium. It means the net force on each charge of the system must be zero. There are three types of equilibrium stable, unstable and neutral.

A charge is said to be in stable equilibrium if it has a tendency of returning to its position when disturbed. Potential energy of the particle will be minimum if the equilibrium is stable. If the equilibrium is unstable it will not have a tendency of returning to initial position, if disturbed. Potential energy of the particle will be high in this position. Charged particle can be in stable equilibrium for one specific direction and in unstable equilibrium along other directions. This can be explained from the following example.

Let us consider two charges 'Q' each are seperated by distance 'd'. A charge 'q' is placed at the midpoint of line joining them. Along the line, the equilibrium is stable and perpendicular to that line the equilibrium will be unstable. However if the charge at the midpoint is -'q' it will be in unstable equilibrium perpendicular to line joining them and in stable equilibrium along that line.

- In the case of a system of charges if the net electric field is zero at a point, it is known as null point.

Example-1.14

A charge Q is placed at the centre of the line joining two charges q and q. The system of three charges to be in equilibrium, what should be the value of Q ?

Solution :

Here Q is in equilibrium as it is at the centre and net force on it is zero

For equilibrium of q, net force on it must be zero

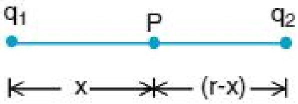
$$\frac{1}{4\pi\epsilon_0}\frac{Qq}{(r/2)^2}+\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}=0$$

where r is seperation between q and q

$$\Rightarrow Q=\frac{-q}{4}$$

Example-1.15

Two point charges q_1 and q_2 (like) are seperated by a distance r and fixed. Locate the point on the line joining those charges where resultant or net field is zero.



Solution :

Let P be the null point where $\bar{E}_{net}=0$

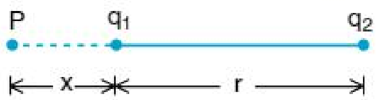
$\Rightarrow \bar{E}_1+\bar{E}_2=0$ (due to those charges)

or $\bar{E}_1=-\bar{E}_2$ and $E_1=E_2$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{q_1}{x^2}=\frac{1}{4\pi\epsilon_0}\frac{q_2}{(r-x)^2} \text{ or } \frac{q_1}{x^2}=\frac{q_2}{(r-x)^2}$$

From this we get x.

If the charges are like, the neutral point will be between the charges; if the charges are unlike, the neutral point will be outside the charge on the line joining them.



In this case $\frac{q_1}{x^2}=\frac{q_2}{(r+x)^2}$.

Example-1.16

Three charges 'q' each are at vertices of an equilateral triangle of side 'r'. How much charge should be placed at the centroid so that the system remains in equilibrium.

Solution :

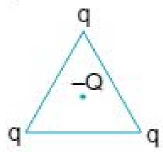
Force on the charge 'q' is $\sqrt{3}\frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{r^2}=F_1$

A charge 'Q' is placed at its centroid. Force applied by this charge on 'q' is $\frac{1}{4\pi\epsilon_0}\cdot\frac{3Qq}{r^2}=F_2$.

If $F_1=F_2$ the system remains in equilibrium.

$$\sqrt{3}\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}=\frac{1}{4\pi\epsilon_0}\frac{3Qq}{r^2}$$

$$Q=-\frac{q}{\sqrt{3}}$$



Note : In the above problem what is the value of 'Q' so that it remains in equilibrium ?

Ans : Any charge placed at the centroid will be in equilibrium since it doesn't experience any net force.

1.18 MOTION OF CHARGED PARTICLE IN UNIFORM ELECTRIC FIELD

Consider a uniform electric field E in space along Y-axis. A charged particle of mass m and charge q be projected in the XY plane from a point with a velocity u making an angle θ with the X-axis.

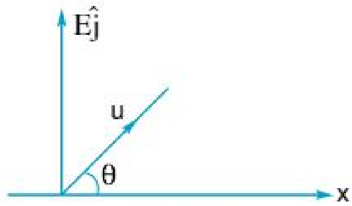


Fig 1.11

Initial velocity of the particle is

$\bar{u}=u\cos\theta\hat{i}+u\sin\theta\hat{j}$

Force acting on the particle is

$\bar{F}=q\bar{E}$ (along + ve Y axis)

$$\bar{a}=\frac{qE}{m}\hat{j}$$

Velocity of the particle after time t is

$\bar{V}=\bar{u}+\bar{a}t=u\cos\theta\hat{i}+(u\sin\theta+at)\hat{j}$

If the point of projection is taken as origin, its position vector after time t is

$\vec{r} = x\hat{i} + y\hat{j}$

where $x = (u\cos\theta)t$

$y = (u\sin\theta)t + \frac{1}{2}at^2$

If the charged particle is projected along the X-axis, $\theta = 0^o$

$\Rightarrow \bar{V} = u\hat{i} + \frac{Eq}{m}t\hat{j}$

Here $x = ut$ and $y = \frac{1}{2}\frac{Eq}{m}t^2$

Direction of motion of particle after time 't' makes an angle α with X axis. Then $\tan\alpha = \frac{Eqt}{mu}$.

- If a charged particle is held stationary or moving with constant speed in uniform electric field, mg = Eq.

1.19 ELECTRIC FIELD INTENSITY DUE TO CHARGE DISTRIBUTION

So far we have discussed about the coulomb force or electric field due to discrete charge distribution. But in many practical situations, we have to deal with charge distributed on bodies like, along a line, over a surface or over a volume. Such charge distributions can be visulaised as very large number of closely spaced point charges. In such case we speak of linear charge density λ, surface charge density σ or volume charge density ρ.

λ is charge per unit length

σ is charge per unit area

ρ is charge per unit volume

We consider the distribution to consist of a very large number of point charges, each dq. The coulomb force due to it or electric field due to it can be calculated. The total force or field due to the entire charge distribution is found by integration.

With linear charge distribution,

$d\bar{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^3}\vec{r}$

Total field is $\bar{E} = \int_L d\bar{E}$

Here $dq = \lambda\ dl$. where dl is elemental length of the line of charge.

With surface charge distribution,

$d\bar{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^3}\bar{r}$

where $dq = \sigma ds$

$\vec{E} = \int_s d\vec{E}$

For volume distribution

$dq = \rho dV$ and $\bar{E} = \int_v d\bar{E}$

where dV is volume of element

Here $\int_L$, $\int_S$, $\int_V$ denote line integral, surface integral and volume integral respectively.

Applications of this integration are given below.

1.20 ELECTRIC FIELD STRENGTH DUE TO A UNIFORMLY CHARGED ROD

At an axial point

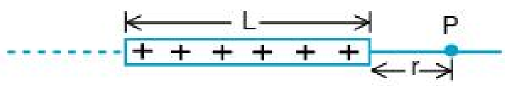


Fig 1.12(a)

Consider a rod of length L, uniformly charged with a charge Q. To calculate the electric field strength at a point P situated at a distance r from one end of the rod, consider an element of length dx on the rod as shown in the figure.

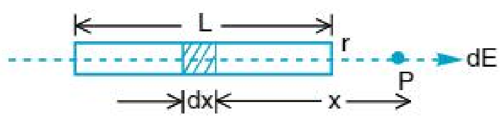


Fig 1.12(b)

Charge on the elemental length dx is

$$dq = \frac{Q}{L}dx$$

$$dE = \frac{dq}{4\pi\varepsilon_0 x^2} = \frac{Qdx}{4\pi\varepsilon_0 Lx^2}$$

The net electric field at point P can be given by integrating this expression over the length of the rod.

$$E_P = \int dE = \int_r^{r+L} \frac{Q}{Lx^2\, 4\pi\varepsilon_0}dx = \frac{Q}{4\pi\varepsilon_0 L}\int_r^{r+L}\frac{1}{x^2}dx$$

$$E_P = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{-1}{x}\right]_r^{r+L}$$

$$E_P = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{r} - \frac{1}{r+L}\right] \qquad .... (1.5)$$

At an equatorial point

Let us find the electric field due to a rod at a point P situated at a distance "r" from centre on its equatorial line.

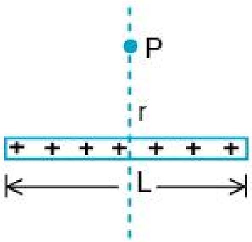


Fig 1.13

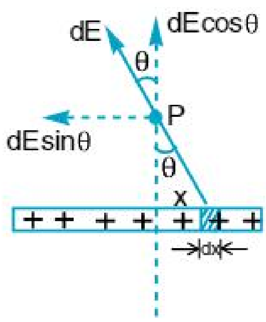


Fig 1.14

Consider an element of length dx at a distance x from centre of rod as in figure (b). Charge on the element is $dq = \frac{Q}{L}dx$.

The strength of electric field at P due to this point charge dq is dE.

$$\Rightarrow dE = \frac{dq}{4\pi\varepsilon_0(r^2 + x^2)}$$

The component $dE\sin\theta$ will get cancelled and net electric field at point P will be due to integration of $dE\cos\theta$ only.

Net electric field strength at point P can be given as

$$E_P = \int dE\cos\theta = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{Qdx}{L(r^2+x^2)} \times \frac{r}{\sqrt{r^2+x^2}} \times \frac{1}{4\pi\varepsilon_0}$$

$$= \frac{Qr}{4\pi\varepsilon_0 L}\int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{dx}{\left(r^2+x^2\right)^{3/2}}$$

From figure (1.14) : $\tan\theta = \frac{x}{r}$

$x = r\tan\theta$

On differentiation $dx = r\sec^2\theta d\theta$

$$E_P = \frac{Qr}{4\pi\varepsilon_0 L}\int \frac{r\sec^2\theta d\theta}{r^3\sec^3\theta}$$

$$= \frac{Q}{4\pi\varepsilon_0 Lr}\int\cos\theta d\theta = \frac{Q}{4\pi\varepsilon_0 Lr}[\sin\theta]$$

Substituting $\theta = \tan^{-1}\frac{x}{r} = \sin^{-1}\frac{x}{\sqrt{x^2+r^2}}$

$$E_P = \frac{Q}{4\pi\varepsilon_0 rL}\left[\frac{x}{\sqrt{x^2+r^2}}\right]_{-\frac{L}{2}}^{\frac{L}{2}}$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\left(\frac{1}{\sqrt{\frac{L^2}{4}+r^2}}\right)$$

$$E_P = \frac{Q}{4\pi\,\epsilon_0\, r}\left\{\frac{2}{\sqrt{L^2+4r^2}}\right\} \qquad .... (1.6)$$

1.21 ELECTRIC FIELD DUE TO INFINITELY LONG CHARGED WIRE

Consider an infinitely long charged wire of negligible thickness as shown in the figure.

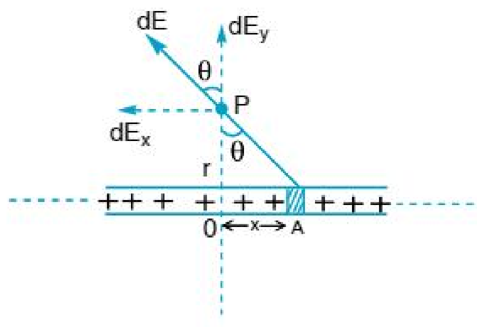


Fig 1.15

Let λ be the charge per unit length.

To find the electric field at point "P" at distance "r" from the wire, consider a small element of length dx at a distance "x" from "O". The charge on this element is $dq = \lambda dx$. The field dE at point P due to this charge dq. is

$$dE = \frac{1}{4\pi\varepsilon_0}\left(\frac{dq}{r^2+x^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda dx}{r^2+x^2}\right)$$

The x and y components of dE are

$dE_x = -dE\sin\theta$ and $dE_y = dE\cos\theta$.

Consider an element of length dx at a distance "x" from "O" towards right.

In this case components of dE along *x* and *y* axes will be $dE\sin\theta$ and $dE\cos\theta$ respectively. Due to symmetry the horizontal components of the field intensity will cancel each other and only vertical, components remain. Hence the total intensity of electric field will be $2\,dE\cos\theta$ in *y*-direction.

$$\therefore E = \int_0^{+\infty} 2dE\cos\theta$$

(as wire extends from 0 to $+\infty$)

$$E = \int_0^{\infty} \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{r^2+x^2} \times 2\cos\theta$$

$$\frac{\lambda}{2\pi\varepsilon_0}\int_0^{\infty}\frac{dx\cos\theta}{r^2+x^2}$$

From figure $\frac{x}{r} = \tan\theta$

$$\therefore \frac{dx}{r} = \sec^2\theta d\theta \text{ or } dx = r\sec^2\theta d\theta$$

When $x = 0, \theta = 0$ and when $x = \infty, \theta = \frac{\pi}{2}$.

On substituting these values we get

$$E = \frac{\lambda}{2\pi\varepsilon_0}\int_0^{\pi/2}\frac{r\sec^2\theta d\theta\cos\theta}{(r^2+r^2\tan^2\theta)}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\int_0^{\pi/2}\frac{r\sec^2\theta\cos\theta d\theta}{r^2(1+\tan^2\theta)}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\int_0^{\pi/2}\frac{r\sec^2\theta\cos\theta d\theta}{r^2\sec^2\theta}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 r}\int_0^{\pi/2}\cos\theta d\theta$$

$$= \frac{\lambda}{2\pi\varepsilon_0 r}[\sin\theta]_0^{\pi/2} = \frac{\lambda}{2\pi\varepsilon_0 r}[1-0] = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad ....(1.7)$$

1.22 ELECTRIC FIELD DUE TO A SEMI INFINITE UNIFORMLY CHARGED WIRE

Consider a semi infinite uniformly charged wire with linear charge density λ.

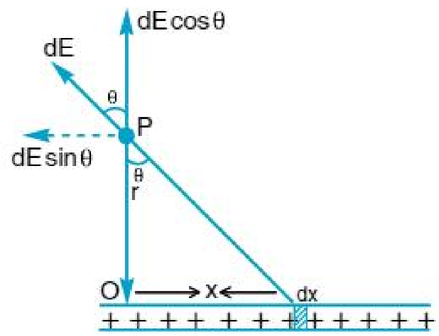


Fig 1.16

Consider a point "P" at a distance "r" from the end "O". To find the electric field strength at P, consider an element of length dx at a distance "x" from "O". Charge on element dx is dq and it can be given as $dq = \lambda dx$. Now due to dq the electric field strength at point P

$$dE = \frac{dq}{4\pi\varepsilon_0(x^2+r^2)}$$

To find net electric field at point P we have to resolve dE in x and y directions and then integrate seperately. In x direction electric field strength due to dq will be

$dE_x = dE\sin\theta$

Net electric field in x direction is

$$E_x = \int dE_x = \int dE\sin\theta$$

$$= \int_0^\infty \frac{dq}{4\pi\varepsilon_0(r^2+x^2)} \times \frac{x}{\sqrt{r^2+x^2}}$$

$$E_x = \int_0^\infty \frac{\lambda x dx}{4\pi\varepsilon_0(r^2+x^2)^{3/2}}$$

Now we substitute $x = r\tan\theta \quad dx = r\sec^2\theta d\theta$ at $x = 0, \theta = 0$ and $x = \infty; \theta = \pi/2$.

On substituting we get

$$E_x = \int_0^{+\pi/2} \frac{\lambda r\tan\theta \times r\sec^2\theta d\theta}{4\pi\varepsilon_0 r^3 \sec^3\theta}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\int_0^{\pi/2} \sin\theta d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\left[-\cos\theta\right]_0^{\pi/2} = \frac{\lambda}{4\pi\varepsilon_0 r}[0+1]$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}$$

$$E_y = \int dE\cos\theta$$

$$= \int_0^\infty \frac{\lambda dx}{4\pi\varepsilon_0(r^2+x^2)} \times \frac{x}{\sqrt{r^2+x^2}}$$

$$= \int_0^\infty \frac{\lambda x dx}{4\pi\varepsilon_0(r^2+x^2)^{3/2}}$$

Now we substitute $x = r\tan\theta$

$dx = r\sec^2\theta d\theta$

$x = 0;\ \theta = 0\ ;\ x = \infty, \theta = \dfrac{\pi}{2}$

$$E_y = \int_0^{\pi/2} \frac{\lambda r\tan\theta\sec^2\theta d\theta}{4\pi\varepsilon_0 r^3\sec^3\theta}$$

$$= \int_0^{\pi/2} \frac{\lambda}{4\pi\varepsilon_0 r}\cos\theta d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\int_0^{\pi/2} \cos\theta d\theta$$

$= \dfrac{\lambda}{4\pi\varepsilon_0 r}$. Thus we finally get

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r} \ \& \ E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$$

Net electric field at point P can be given as

$$E_P = \sqrt{E_x^2+E_y^2} = \sqrt{\left(\frac{\lambda}{4\pi\varepsilon_0 r}\right)^2 + \left(\frac{\lambda}{4\pi\varepsilon_0 r}\right)^2}$$

$$= \sqrt{2}\left(\frac{\lambda}{4\pi\varepsilon_0 r}\right) \quad \text{.... (1.8)}$$

Net field makes angle θ with x-axis which can be given as

$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}(1) = 45^0$$

1.23 ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED RING

The intensity of electric field at a distance *x* from the centre along the axis :

Consider a circular ring of radius "a" having a charge "q" uniformly distributed over it as shown in figure. Let "O" be the centre of the ring.

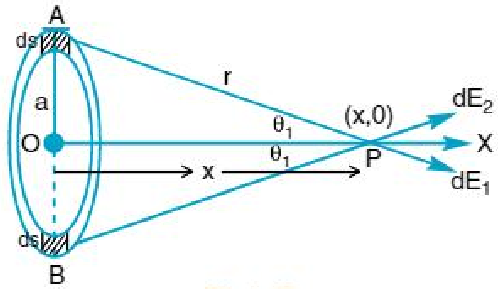


Fig 1.17

Consider an element ds of the ring at point A. The charge on this element is given by

dq=ds×charge density

$$dq = ds\frac{q}{2\pi a} = \frac{qds}{2\pi a}$$

a) The intensity of electric field dE at point P due to the element ds (at A) is given by

$$dE_1 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

The direction of dE_1 is as shown in figure. The component of intensity along *x*-axis will be

$$\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta$$

The component of intensity along *y*-axis will be

$$\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\sin\theta$$

Similarly if we consider an element ds of the ring opposite to A which lies at B, the component of intensity perpendicular to the axis will be equal and opposite to the component of intensity perpendicular to the axis due to element at A. Hence they cancel each other. Due to symmetry of ring the component of intensity due to all elements of the ring perpendicular to the axis will cancel.

So the resultant intensity is only along the axis of the ring. The resultant intensity is given by

$$E = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\int \frac{q\,ds}{2\pi a r^2}\times\frac{x}{r} \text{ (where } \cos\theta = x/r\text{)}$$

$$= \frac{1}{4\pi\varepsilon_0}\times\frac{qx}{(2\pi a)}\times\frac{1}{(a^2+x^2)^{3/2}}\int ds$$

$$[\therefore r^3 = (a^2+x^2)^{3/2}]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qx}{2\pi a}\frac{1}{(a^2+x^2)^{3/2}}\times 2\pi a$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qx}{(a^2+x^2)^{3/2}}$$

At its centre x =0, E = 0

$\therefore$ Electric field at centre is zero.

By symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the element exactly opposite to it. As in the figure the electric field at centre due to segment A is cancelled by that due to segment B. Thus net electric field strength at the centre of a uniformly charged ring is $E_c = 0$.

$$x >> a,\ E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2};$$

$$x << a,\ E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^3}\cdot x$$

$$x = \frac{\pm R}{\sqrt{2}},\ E = \text{Maximum}$$

$$E_{max} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{3\sqrt{3}a^2}$$

1.24 ELECTRIC FIELD STRENGTH DUE TO A CHARGED CIRCULAR ARC AT ITS CENTRE OF CURVATURE

Consider a circular arc of radius R which subtends an angle ϕ at its centre. Let us calculate the electric field strength at C.

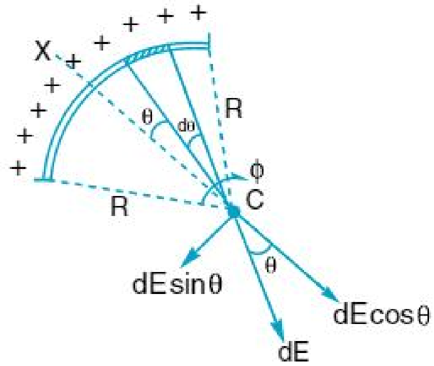


Fig 1.18

Consider a polar segment on arc of angular width $d\theta$ at an angle θ from the angular bisector XY as shown. The length of elemental segment is $Rd\theta$. The charge on this element dq is $dq = \frac{Q}{\phi}d\theta$

Due to this, dq, electric field at centre of curvature of arc C is given as $dE = \frac{dq}{4\pi\varepsilon_0 R^2}$

The electric field component dE due to this segment $dE\sin\theta$ which is perpendicular to the angle bisector gets cancelled out on integration.

The net electric field at C will be along angle bisector which can be calculated by integrating $dE\cos\theta$ within limits from $-\phi/2$ to $\phi/2$. Hence net electric field strength at centre C is

$$E_c = \int dE\cos\theta = \int_{-\phi/2}^{\phi/2}\frac{Q}{4\pi\varepsilon_0\phi R^2}\cos\theta d\theta$$

$$= \frac{Q}{4\pi\varepsilon_0 R^2\phi}\int_{-\phi/2}^{\phi/2}\cos\theta d\theta$$

$$= \frac{Q}{4\pi\varepsilon_0\phi R^2\phi}[\sin\theta]_{-\phi/2}^{\phi/2}$$

$$=\frac{Q}{4\pi\varepsilon_0 R^2\phi}[\sin\phi/2+\sin\phi/2]$$

$$E_c=\frac{2Q\sin\phi/2}{4\pi\varepsilon_0 R^2\phi} \quad \text{.... (1.9)}$$

1.25 ELECTRIC FIELD STRENGTH DUE TO A UNIFORMLY CHARGED DISC

Consider a disc of radius R, charged on its surface with a charge density σ.

Let us find electric field strength due to this disc at a distance *x* from the centre of disc on its axis at point P as shown in figure.

Consider an elemental ring of radius *y* and width dy in the disc as shown in figure. The charge on this elemental ring dq can be given as $dq=\sigma 2\pi y dy$

[Area of elemental ring $ds=2\pi y dy$]

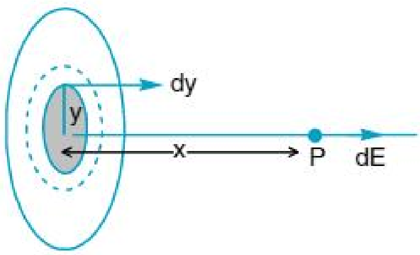


Fig 1.19

Electric field strength due to a charged ring of radius y, charge Q at a distance x from its centre on its axis can be given as

$$E=\frac{Qx}{4\pi\varepsilon_0(x^2+y^2)^{3/2}}$$

Due to the elemental ring electric filed strength dE at point P can be given as

$$dE=\frac{x\,dq}{4\pi\varepsilon_0(x^2+y^2)^{3/2}}$$

$$=\frac{\sigma 2\pi y\pi dyx}{4\pi\varepsilon_0(x^2+y^2)^{3/2}}$$

Net electric field at point P due to this disc is given by integrating above expression within the limits from 0 to R.

$$E=\int dE=\int_O^R\frac{\sigma 2\pi xydy}{4\pi\varepsilon_0\left(x^2+y^2\right)^{3/2}}$$

$$=\frac{\sigma\pi x}{4\pi\varepsilon_0}\int_O^R\frac{2ydy}{\left(x^2+y^2\right)^{3/2}}$$

$$=\frac{2\sigma\pi x}{4\pi\varepsilon_0}\left[\frac{-1}{\sqrt{x^2+y^2}}\right]_O^R$$

$$E=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right] \quad \text{....(1.10)}$$

1.26 ELECTRIC FIELD STRENGTH DUE TO A LARGE CHARGED SHEET

Consider a large charged sheet. Let us find the electric field strength at a point P, at a distance *x* from the sheet.

Consider an elemental ring of radius y and width dy with its centre O as shown in figure.

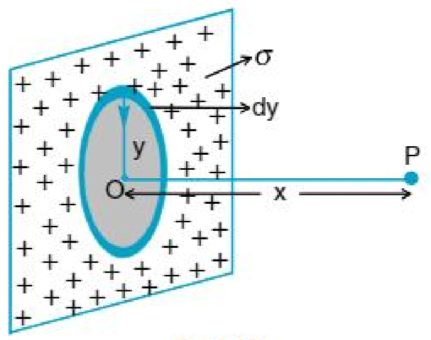


Fig 1.20

The electric field strength due to this elemental ring at point P is

$$dE=\frac{xdq}{4\pi\varepsilon_0(x^2+y^2)^{3/2}}$$

For very large sheet, the above equation can be integrated within limits from 0 to ∞ to get the final result as

$$E=\int\frac{\sigma 2\pi ydy}{4\pi\varepsilon_o(x^2+y^2)^{3/2}}$$

On solving we get

$$E=\frac{\sigma}{2\varepsilon_0} \quad ....(1.11)$$

✤ Electric field strength due to a uniformly charged disc at a distance x from its surface is given as

$$E=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

If we put x = 0 we get $E=\frac{\sigma}{2\varepsilon_0}$

1.27 ELECTRIC FIELD STRENGTH DUE TO A UNIFORMLY CHARGED HOLLOW HEMISPHERICAL CUP

Consider a hollow hemisphere uniformly charged with surface charge density σ. To find electric field strength at its centre C, consider an elemental ring on its surface of angular width $d\theta$ at angle θ from its axis as shown in figure. Surface area of this ring will be $ds=2\pi R\sin\theta R d\theta$. Charge on this elemental ring is $dq=\sigma ds=\sigma 2\pi R^2\sin\theta d\theta$. Due to this ring electric field strength at centre C can be written as

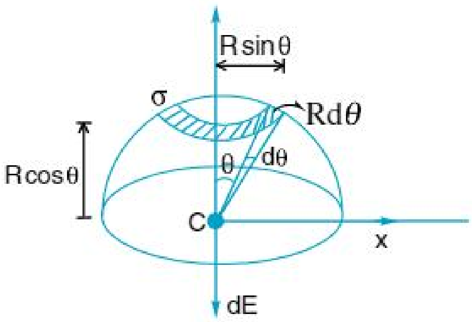


Fig 1.21

$$dE=\frac{dq(R\cos\theta)}{4\pi\varepsilon_0(R^2)^{3/2}[\sin^2\theta+\cos^2\theta]^{3/2}}$$

$$=\frac{\sigma 2\pi R^2\sin\theta d\theta R\cos\theta}{4\pi\varepsilon_0 R^3}$$

$$=\frac{\sigma 2\pi R^3\sin\theta\cos\theta d\theta}{4\pi\varepsilon_0 R^3}$$

$$=\frac{\sigma\pi}{4\pi\varepsilon_0}[\sin 2\theta]d\theta$$

Electric field at centre can be obtained by integrating this expression between limits 0 to $\pi/2$

$$E_C=\int dE=\frac{\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi/2}\sin 2\theta d\theta$$

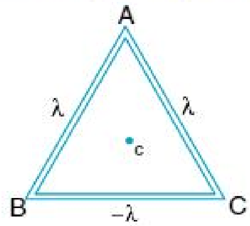
$$=\frac{\pi\sigma}{4\pi\varepsilon_0}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$=\frac{\pi\sigma}{4\pi\varepsilon_0}\left[\frac{1}{2}+\frac{1}{2}\right]=\frac{\pi\sigma}{4\pi\varepsilon_0}=\frac{\sigma}{4\varepsilon_0}.$$

$$E=\frac{\sigma}{4\varepsilon_0} \quad .... (1.12)$$

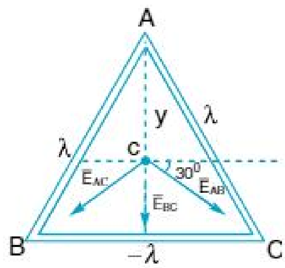
Example-1.17

Given an equilateral triangle with side L, find E at its centroid. The linear charge density is as shown in figure.



Solution :

The electric field strength due to the three rods AB,BC and CA are shown in figure.



$\bar{E}_{AC}=\frac{3\lambda}{2\pi\varepsilon_0 L}$ (towards B)

$\bar{E}_{AB}=\frac{3\lambda}{2\pi\varepsilon_0 L}$ (towards C)

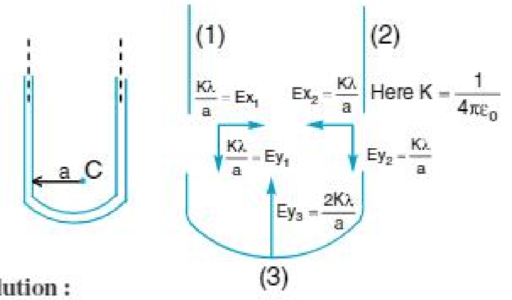
$\bar{E}_{BC}=\frac{3\lambda}{2\pi\varepsilon_0 L}$ (towards midpoint of BC)

$\bar{E}_{Net}=\bar{E}_{AC}+\bar{E}_{AB}+\bar{E}_{BC}$

$=\frac{3\lambda}{\pi\varepsilon_0 L}$ (towards midpoint of BC)

Example-1.18

Find the electric field at point C of the given U shaped wire which is uniformly charged with linear charge density λ . [C is the centre of the semi circular section]



Solution :

The electric fields due to the three parts of u shaped wire are shown in the above figure.

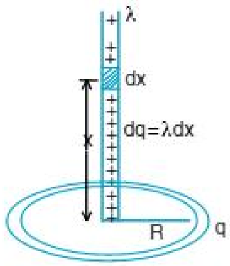
$$\overline{E}_{net} = (Ex_i + Ex_2)i + (Ey_1 + Ey_2 + Ey_3)j$$

$$\overline{E}_{net} = \left(\frac{K\lambda}{a} - \frac{K\lambda}{a}\right)i + \left(\frac{2K\lambda}{a} - \frac{K\lambda}{a} - \frac{K\lambda}{a}\right)j = 0$$

Hence electric field due to given arrangement at C=0.

Example-1.19

A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to q. The charge of the thread (per unit length) is equal to λ . Find the interaction force between the ring and the thread.



Solution :

Force "dF" on the wire = $dq\overline{E}$

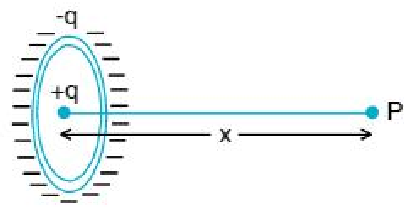
$$= \frac{qx\lambda dx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$

$$= \frac{q\lambda}{4\pi\varepsilon_0}\int_0^\infty \frac{xdx}{(R^2+x^2)^{3/2}}$$

$$F = \frac{\lambda q}{4\pi\varepsilon_0 R}.$$

Example-1.20

A point charge q is located at the centre of a thin ring of radius R with uniformly distributed charge "–q". Find the magnitude of the electric field strength vector at the point lying on the axis of the ring at a distance x from its centre if $x >> R$.



Solution :

Electric field at P due to ring

$$E_1 = \frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}} \text{ (towards centre)}$$

Electric field at P due to +q

$$E_2 = \frac{q}{4\pi\varepsilon_0 x^2} \text{ (away from centre)}$$

Thus net field at P is $E_{Net} = E_1 + E_2$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x^2} - \frac{x}{(x^2+R^2)^{3/2}}\right]$$

For $x >> R$

$$E_{net} = \frac{q\left[(x^2+R^2)^{3/2} - x^3\right]}{4\pi\varepsilon_0 x^2(x^2+R^2)^{3/2}}$$

$$= \frac{3qR^2}{4\pi\varepsilon_0 2x^4} = \frac{3qR^2}{8\pi\varepsilon_0 x^4}$$

(Using Binomial approximation).

1.28 ELECTRIC FIELD STRENGTH DUE TO A NON UNIFORMLY CHARGED ROD

Consider a rod of length "L" charged with a linear charge density which is proportional to distance x from end A of rod. Let us calculate electric field at point "P" shown in figure. For this we have to consider an element of width dx at a distance x from the end A as shown.

Charge on this element is $dq = \lambda dx = cxdx$. Electric field strength at point P due to elemental charge dq is dE, which is given as

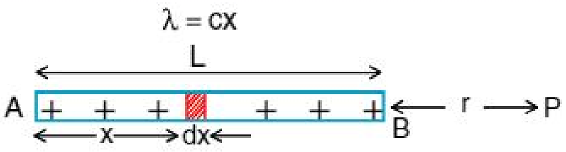


Fig 1.22

$$dE = \frac{dq}{4\pi \in_0 (L+r-x)^2}$$

Net electric field at "P" due to complete rod can be calculated by integrating the above expression within limits from 0 to L.

$$\therefore E = \int dE = \int_0^L \frac{cxdx}{4\pi\varepsilon_0 (L+r-x)^2}$$

Put $t = L + r - x$

$dt = -dx$

at $x = 0, t = L + r$

at $x = L, t = r$

Thus we have

$$E = \int_{L+r}^{r} \frac{c(L+r-t)dt}{4\pi\varepsilon_0 t^2}$$

$$= -\frac{c}{4\pi\varepsilon_0}\int\left(\frac{L+r}{t^2}dt - \frac{1}{t}dt\right)$$

$$= \frac{c(L+r)}{4\pi\varepsilon_0}\left[\frac{1}{t}\right]_{L+r}^{r} + \frac{c}{4\pi\varepsilon_0}[\log t]_{L+r}^{r}$$

$$= \frac{c}{4\pi\varepsilon_0}\left[\frac{L+r}{r} - 1\right] + \frac{c}{4\pi\varepsilon_0}\ln\left[\frac{r}{L+r}\right]$$

$$E = \frac{cL}{4\pi\varepsilon_0 r} - \frac{c}{4\pi\varepsilon_0}\ln\left(\frac{r+L}{r}\right) \quad .....(1.13)$$

1.29 ELECTRIC FIELD DUE TO A LARGE NON-UNIFORMLY CHARGED SLAB

Consider a region between xz plane and a plane parallel to xz plane at $y = t$ which is charged with a charge density which varies with y as $\rho = ay^n$ $(n > 0)$.

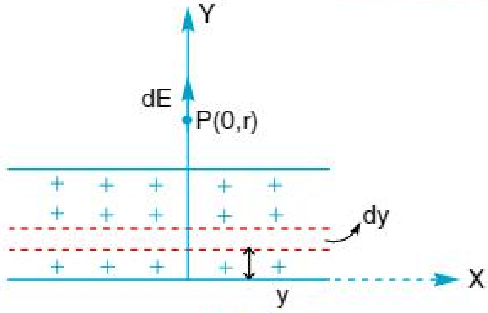


Fig 1.23

Let us calculate electric field strength due to this slab at P(0, r). To find this we consider an elemental sheet of width dy at a distance y from the xz plane. Electric field strength at P due to this sheet is given as

$$dE = \frac{\rho dy}{2\in_0} \quad \text{(for a sheet of thickness dy)}$$

Net electric field at P is given by

$$E_P = \int dE = \int_0^t \frac{ay^n}{2\in_0}dy = \frac{at^{n+1}}{2\in_0 (n+1)} \quad ...(1.14)$$

1.30 ELECTRICFIELD STRENGTH DUE TO A NON-UNIFORMLY CHARGED RING

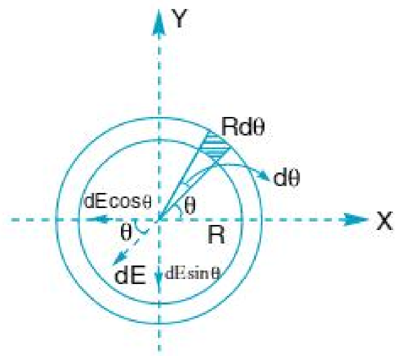


Fig 1.24

Consider a ring of nonuniform linear charge density $\lambda = \lambda_0 \cos\theta$. Where θ is polar angle with X-axis. Radius of the ring is R.

From the λ function we can say that first and fourth quadrants are positively charged and second and third quadrants are negatively charged.

Let us find the electric field strength at the centre of the ring. Consider an element on the ring of polar width $d\theta$ at an angle θ from x-axis. The charge on this element is given by $dq = \lambda R d\theta$

$= \lambda_0 \cos\theta \, R d\theta$

The electric field strength at centre of the ring due to this element can be given as

$$dE = \frac{1}{4\pi \epsilon_0} \frac{dq}{R^2}$$

To find net electric field at centre of ring we integrate the components of this electric field for the circumference of ring

dE can be resolved into two components

On integration, components $dE\sin\theta$ will cancel each other and $dE\cos\theta$ only has to be integrated. So, net electric field strength at centre will be given as $E_C = 2\int_{-\pi/2}^{+\pi/2} dE\cos\theta$

$$= 2\int_{-\pi/2}^{\pi/2} \frac{1}{4\pi\epsilon_0} \frac{\lambda_0 \cos^2\theta R d\theta}{R^2}$$

After evaluating this integration, we get

$$E_C = \frac{\lambda_0}{4\epsilon_0 R} \quad .... (1.15)$$

1.31 ELECTROSTATIC POTENTIAL ENERGY

We know that potential energy of a system of particles is defined only for conservative fields. We define potential energy in electric field as it is conservative. Electrostatic potential energy is defined in two ways

a) interaction energy of charged particles of a system

b) self energy of a charged body (we will discuss later)

When an electrostatic force acts between two or more charged particles within a system of particles, electric potential energy U can be assigned to the system. If configuration of a system is changed from an initial state to final state, work done by electrostatic force w on the particles is related as

$$\Delta U = U_f - U_i = -W$$

Here work done by electrostatic force is path independent. We usually take reference configuration of a system of charged particles while defining potential energy. For the charges at infinite seperation we can take initial potential energy $U_i = 0$. Let W_∞ be the work done by electrostatic forces while bringing the charges from infinite seperation to the desired seperation

then $U = -W_\infty$

1.32 ELECTROSTATIC INTERACTION ENERGY

The external work required to assemble the charged particles of a system by bringing them from infinity to a given configuration is known as electrostatic interaction energy of that system.

If charged particles are at infinite seperation, potential energy of that system is taken as zero as there will be no interaction between them. When these charges are brought closer to form a given configuration, external work is required and energy is supplied to the system. If the force between the charges is repulsive, work is done by external agent and final potential energy of the system will be positive. If the force between the charges is attractive, work will be done by the system and final potential energy of the system is negative.

1.33 INTERACTION ENERGY OF A SYSTEM OF TWO CHARGES

Consider a system of two point charges q_1 and q_2. Assume q_1 is fixed at position A. Charge q_2 is taken from B to P along ABP. Let $AB = r_1$ and $AP = r_2$.

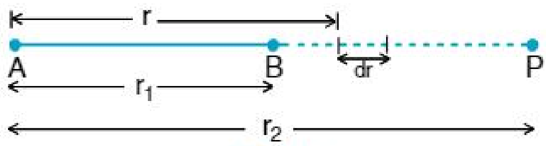


Fig 1.25

Consider a small displacement of the charge q_2 in which its distance from q_1 changes from r to $(r + dr)$.

The electrostatic force on the charge q_2 is given by $F = \frac{1}{4\pi \epsilon_0}\frac{q_1q_2}{r^2}$ along A to B.

The work done by this electric force for the small displacement dr is given by

$$dW = \overline{F}.\overline{dr} = Fdr\cos 0^0 = Fdr$$

$$\Rightarrow dW = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}dr$$

The total work done as the charge q_2 moves from B to P is $W = \int_{r_1}^{r_2} dW$

i.e., $W = \int_{r_1}^{r_2}\frac{1}{4\pi \epsilon_0}\frac{q_1q_2}{r^2}dr$

$$= \frac{q_1q_2}{4\pi \epsilon_0}\left\{\frac{1}{r_1} - \frac{1}{r_2}\right\}$$

As q_1 is fixed no work is done on q_1 by the electric force. We define change in electric potential energy of the system as negative of the work done by the electric forces as the configuration changes.

So, the change in potential energy is given by $U(r_2) - U(r_1) = -W = \frac{q_1q_2}{4\pi \epsilon_0}\left\{\frac{1}{r_1} - \frac{1}{r_2}\right\}$

We choose the potential energy of the two charge system to be zero when they have infinite separation. i.e., $U(\infty) = 0$. The potential energy when the seperation is r is

$$U(r) = U(r) - U(\infty)$$

$$U(r) = \frac{q_1q_2}{4\pi \epsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$U(r) = \frac{1}{4\pi \epsilon_0}\frac{q_1q_2}{r} \quad .... (1.16)$$

- The potential energy of the system of charges depends essentially on the seperation between the charges and is independent of their location in space.

- For a system of many charged particles q_1, q_2, q_3, ... q_n, the interaction potential energy is given by

$$U = \frac{1}{4\pi \epsilon_0}\left\{\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + ...\right\}$$

  Here r_{12} indicates seperation between q_1 and q_2. r_{23} indicates seperation between q_2 and q_3. r_{31} indicates seperation between q_1 and q_3 and so on.

  We have to consider all possible pairs while finding U. Here charges with their original signs must be used.

- Potential energy of a system of charges may be positive, negative or even zero.

- If the seperation between two point charges is increased, interaction potential energy may increase or decrease.

- Positive potential energy means that work can be obtained from the system. If two bodies having the same charges are free, they rush away from each other, releasing the stored potential energy as kinetic energy of the charges.

- Negative potential energy means that external agent would have to do work to seperate the charges.

- As electrostatic force is conservative, work done by external agency is given by

  $$W_{ext} = U(\text{final}) - U(\text{initial})$$

  Work done by the electric field is given by

  $$W_{field} = U(\text{initial}) - U(\text{final})$$

  $$\Rightarrow dU = -dW_{field} = dW_{ext}$$

1.34 ELECTRIC POTENTIAL

When a body is kept at a height above the earth, it is said to have gravitational potential energy. The potential energy of the body depends upon its position in the gravitational field (assuming zero on earth's surface). Electrical potential is analagous to gravitational potential. We know that every charge q has electric field which theoretically extends upto infinity. Now if a small positive test charge q_0 is kept in this electric field, the test charge will have electric potential energy. The potential energy of q and q_0 depends upon position of q_0 in the field of q (assuming fixed). Potential energy per unit charge has a unique value at any point in an electric field.

Assume a test charge q_0 moved from point A to point B in an electric field. (while all the other charges remain fixed). Due to the displacement of test charge, change in electric potential energy is $(U_B - U_A)$. Now potential energy difference per unit charge (q_0) is $\left(\frac{U_B - U_A}{q_0}\right)$. We define this difference as potential difference between points A and B.

$$\Rightarrow V_B - V_A = \frac{U_B - U_A}{q_0}$$

In converse we can say that if charge q_0 is taken through a potential difference $(V_B - V_A)$, the electric potential energy is increased by $(U_B - U_A) = q_0(V_B - V_A)$ so, electric potential difference between any two points i and f in an electric field is equal to the difference in potential energy per unit charge between those two points.

$$\Delta V = V_f - V_i = \frac{U_f - U_i}{q_0} = \frac{\Delta U}{q_0}$$

$$\Rightarrow \Delta V = V_f - V_i = \frac{-W}{q_0}$$

"Potential difference between two points is negative of the work done by the electrostatic force to move unit charge from one point to the other."

Potential difference can be positive, negative or zero depending on the signs and magnitudes of q_0 and W. We can define absolute electric potential at any point by choosing a reference. If we set $U_i = 0$ at infinity as our reference potential energy, then the electric potential must be zero there. Then we can define electric potential V at any point in an electric field as

$$V = \frac{-W_\infty}{q} \quad \text{or} \quad V = \frac{U - U(\infty)}{q_0} = \frac{U}{q_0}$$

$$\text{or} \quad V = \frac{-W_\infty}{q_0} \qquad \text{.... (1.17)}$$

Here W_∞ is the work done by the electric field on a charged particle as that particle moves from infinity to the given point. Potential V can be positive, negative or zero depending on the signs and magnitudes of q and W_∞.

"The electric potential at a point is equal to the work done per unit test charge by an external agent in moving the test charge from the reference point to that point without changing its kinetic energy". (Potential at reference point is taken as zero).

As potential energy is a scalar quantity, potential is also a scalar quantity. The S.I. unit of potential is Joule per coulomb which is known as volt.

1 volt = 1 joule per coulomb

The electric potential at a point in an electric field is 1 volt if one joule per coulomb of work is done in bringing a small positive test charge from infinity or reference point to the given point.

- In any situation involving potential energy we are free to define zero potential energy level in an arbitrary manner. In general we choose infinity as zero potential energy level.
- We take even the earth or ground also as reference at zero potential. If a charged conductor is connected to the earth or grounded, we take its potential as zero. The earth can be assumed to be electrically neutral. Addition of any amount of charge to the earth or removal of any amount of charge from the earth does not change the electrical status of the earth.
- If a negatively charged conductor is grounded, electrons flow from it to the earth and if a positively charged conductor is grounded, electrons flow from the earth to the conductor. In both cases potential of the conductor is finally zero.

Example-1.21

There is an infinite straight chain of alternating charges q and –q. The distance between neighbouring charges is equal to d. Find the interaction energy of each charge with all the other charges

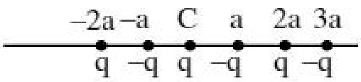
Solution :

The interaction energy of charge at C due to charges on one side (right or left) is

$$U=\frac{1}{4\pi\epsilon_0}\left\{\frac{q(-q)}{a}+\frac{q(q)}{2a}+\frac{q(-q)}{3a}+\frac{q(q)}{4a}+........\infty\right\}$$

$$=\frac{q^2}{4\pi\epsilon_0 a}\left\{-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}.........\right\}=\frac{-q^2\ell n\,2}{4\pi\epsilon_0 a}$$

So, total interaction energy is 2U



$$=\frac{-2q^2\,\ell n\,2}{4\pi\epsilon_0 a}$$

1.35 ELECTRIC POTENTIAL DUE TO A SINGLE POINT CHARGE

Consider a point charge + q fixed at a point O in free space. Let us find electric potential at P due to charge + q. Let r be the distance of P from O, i.e.. OP = r.

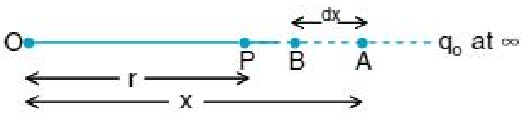


Fig 1.26

Consider point A at a distance x from + q. At that point electric field intensity is

$$E=\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}$$

If a unit positive test charge is moved from A to B, the amount of work done

$$dW=\bar{E}.d\bar{x}=-Edx \qquad (\because AB=dx).$$

Total amount of work done in bringing unit positive test charge from infinity to r is

$$W=\int_\infty^r dW=-\int_\infty^r E\,dx \Rightarrow W=-\int_\infty^r E\,dx$$

$$=-\int_\infty^r\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}dx=-\frac{q}{4\pi\epsilon_0}\int_\infty^r\frac{1}{x^2}dx$$

$\therefore$ Electric potential at P is given by

$$V=\frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad .... (1.18)$$

If q is positive, then potential at P is positive. On the other hand if q is negative, then potential at P is negative.

1.36 POTENTIAL AT A POINT DUE TO A GROUP OF POINT CHARGES

Electric potential obeys superposition principle. So, potential at any point P due to a group of point charges $q_1,q_2,.........q_n$ is equal to algebraic sum of the potentials due to $q_1,q_2,.........q_n$ at P.

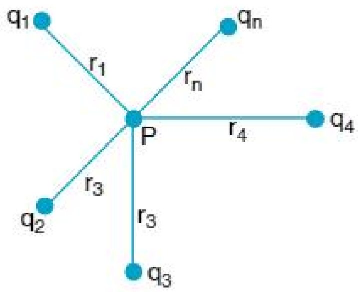


Fig 1.27

Let $V_1, V_2, V_3 .....V_n$ be the potentials at P due to $q_1,q_2.........q_n$

Now, total potential at P is

$$V=V_1+V_2+........+V_n$$

$$\Rightarrow V=\frac{1}{4\pi\epsilon_0}\left\{\frac{q_1}{r_1}+\frac{q_2}{r_2}+....+\frac{q_n}{r_n}\right\} \quad .... (1.19)$$

Electron–volt (eV)

The conventional unit of energy is joule. But this unit is very large for computing nuclear energies, electron energies in atomic physics. So, a smaller and convenient unit called electron-volt is used in such cases.

"The amount of energy gained by an electron when accelerated through a potential difference of one volt is known as electron-volt."

The energy gained by a charge q when accelerated through a potential difference of V is qV.

$\Rightarrow 1eV = 1.6 \times 10^{-19}$ J

- Electron volt is a unit of energy. It is not a unit of potential difference or voltage.
- Electron volt is not a standard SI unit of energy.
- If an electron is decelerated through a potential difference of one volt, amount of energy lost by it is 1eV.

1.37 LINE INTEGRAL OF ELECTRO STATIC FIELD

We can calculate the potential difference between two points i and f in an electrostatic field. To make the calculation, first we find the work done on a positive test charge by the field as it moves from i to f.

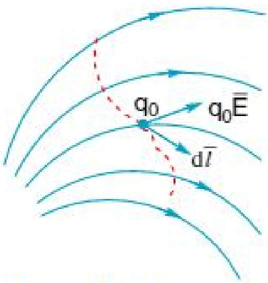


Fig 1.28 (Motion of test charge in electric field)

Consider an arbitrary electric field, represented by the field lines as shown. A positive test charge q_0 moves along the path shown with dotted lines from point i to point f. At any point on the path, an electrostatic force, $q_0\bar{E}$ acts on the charge as it moves through a differential displacement $d\bar{l}$. Work done for this displacement is

$$dW = \bar{F}.d\bar{l} \Rightarrow dW = q_0\bar{E}.d\bar{l}$$

Total work done on the test charge by the field as it moves from i to f is given by $W = \int_i^f q_0\bar{E}.d\bar{l}$

From the definition of potential difference, we can write

$$V_f - V_i = -\frac{W}{q_0}$$

$$V_f - V_i = -\int_i^f \bar{E}\cdot d\bar{l} \qquad .... (1.20)$$

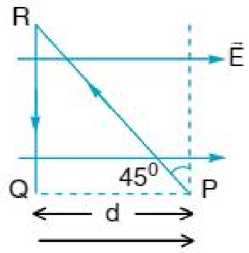
So, potential difference between any two points i and f in an electric field is equal to the negative of the line integral of $\bar{E}\cdot d\bar{l}$ from i to f.

- If we choose $V_i = 0$, then potential V at point f is given by $V = -\int_i^f \bar{E}.d\bar{l}$.
- As electrostatic force is conservative, all paths from i to f yield the same result, when the line integral is found.
- If A and B are two points in an electrostatic field,

$$V_B - V_A = -\int_A^B \bar{E}.d\bar{l} \text{ and } V_A - V_B = +\int_A^B \bar{E}.d\bar{l}$$

Example-1.22

A test charge q_0 is moved without acceleration from P to Q in a uniform electric field over the path shown in figure. The points P and Q are seperated by a distance d. Find the potential difference between P and Q.



Solution:

The test charge q_0 follows the path PRQ. For path PR.

$$V_R - V_P = -\int_P^R \bar{E}.d\bar{l}$$

$$= -\int_P^R E.dl\cos 135^0 = \int_P^R E.dl\cos 45^0$$

$$= E\cos 45^0 \int_P^R dl = E(PR)\cos 45^0$$

$$= E(PQ) = Ed \quad (\because PQ = PR\cos 45^0)$$

For path RQ

$$V_Q - V_R = -\int_R^Q \bar{E}.d\bar{l} = -\int_R^Q E.dl\cos 90^0 = 0$$

$$V_Q = V_R$$

No work is done in moving the charge at right angle to the electric field. So potential difference between P and Q is Ed.

1.38 CONSERVATIVE NATURE OF ELECTRIC FIELD

A conservative force is that force for which the work done around a closed path is zero.

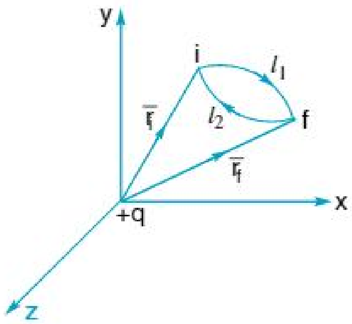


Fig 1.29

Consider a closed path i l_1f l_2i in the electric field of point charge +q. Let a small test charge $+q_0$ be moved over the closed path.

Then $$\frac{w_{i\to f}}{q_0} = -\int_i^f \bar{E}.d\bar{\ell} \qquad (\text{along } l_1)$$

$$= V_f - V_i$$

and $$\frac{w_{f-i}}{q_0} = -\int_f^i \bar{E}.d\bar{l} \qquad (\text{along } l_2)$$

$$= V_i - V_f$$

$$\Rightarrow \frac{w_{i-f}}{q_0} + \frac{w_{f-i}}{q_0} = (V_f - V_i) + (V_i - V_f) = 0$$

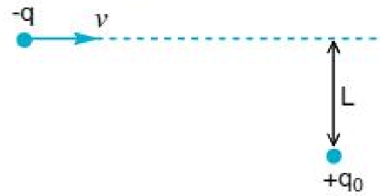
In other words work done in moving a unit positive test charge over a closed path in electrostatic field is zero. It means electrostatic field is a conservative field and electrostatic forces are conservative.

$$\Rightarrow \boxed{\oint \bar{E}.d\bar{\ell} = 0} \qquad \text{.... (1.21)}$$

The line integral of electrostatic field over a closed path in an electric field is zero.

Example-1.23

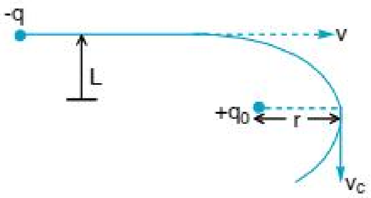
A charge $+q_0$ is fixed at a position in space. From a large distance another charged particle of charge $-q$ and mass m is thrown towards $+q_0$ with an impact parameter L as shown. The initial speed of the projected particle is v. Find the distance of closet approach of the two particles.



Solution :

As $-q$ moves towards $+q_0$ an attractive force acts on $-q$ towards $+q_0$. No torque acts on $-q$ relative to $+q_0$. In other words angular momentum of $-q$ must remain constant.

When $-q$ is closet to +q, it will be moving perpendicularly to the line joining the two charges as shown.



Let r be the closet seperation between the charges and v_c be the velocity of $-q$ at that instant.

From conservation of angular momentum, $mvL = mv_c r$

From conservation of mechanical energy

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_c^2 - \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$

On solving the above equation, we can get r.

Example-1.24

Determine the electric field strength vector if the potential of this field depends on (x, y, z) co-ordinates as $V = A(x^2 - y^2)$.

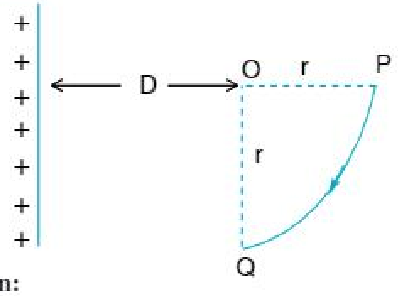
Solution :

$$E_x = -\frac{\partial V}{\partial x} = -2Ax \text{ and } E_y = -\frac{\partial V}{\partial y} = 2Ay$$

$$\bar{E} = E_x\hat{i} + E_y\hat{j} = 2A(-x\hat{i} + y\hat{j})$$

Example-1.25

A charge q_0 is moved from point P to point Q along the arc PQ with centre at O as shown in the figure near a long charged wire. The linear charge density of the wire is λ and it lies in the same plane. Find the work done in the process.



Solution:

We know that electric field is conservative. So work done does not depend upon the path.

$$W_{P\to Q} = W_{P\to O} \quad (\because V_0 = V_Q)$$

$$W_{P\to Q} = -\int_{(r+D)}^{D} q\overline{E}.d\overline{r}$$

$$W = -q\int_{(r+D)}^{D} \frac{2\lambda}{4\pi\in_0}\frac{dr}{r} = \frac{\lambda q}{2\pi\in_0}\log_e\left(\frac{r+D}{D}\right).$$

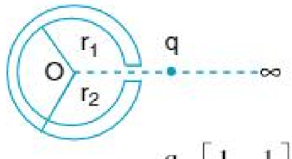
(Ref. Gauss's Law Ch. 2)

Example-1.26

A point charge q is isolated at the centre O of a spherical uncharged conducting layer provided with a small orifice. The inside and outside radii of the layer are equal to r_1 and r_2 respectively. Find the amount of work done to slowly transfer the charge q from the centre O, through the orifice and to infinity.

Solution :

Because of q, charges will induce on the outer and inner surface of the layer. Those are +q and –q respectively.



Potential at the centre $= \dfrac{q}{4\pi\varepsilon_0}\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right]$

Work done in transferring q from centre to infinity

$$\int V dq = \int \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]dq = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

* We can do this even by finding the difference in electric potential energy of the system.

Example-1.27

Find the potential difference between points A and B in an electric field $\overline{E} = (2\hat{i} + 3\hat{j} + 4\hat{k})NC^{-1}$ where $\overline{r}_A = (\hat{i} - 2\hat{j} + \hat{k})m$ and $\overline{r}_B = (2\hat{i} + \hat{j} - 2\hat{k})m$.

Solution :

We know that $dV = -\overline{E}.d\overline{r}$

$$V_{AB} = V_A - V_B = -\int_A^B \overline{E}.d\overline{r}$$

$$= -\int_{(2,1,-2)}^{(1,-2,1)} (2\hat{i} + 3\hat{j} + 4\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= -\int_{(2,1,-2)}^{(1,-2,1)} (2dx + 3dy + 4dz)$$

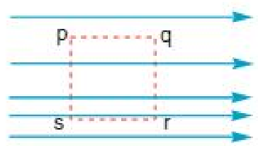
$$= -(2x + 3y + 4z)\Big|_{(2,1,-2)}^{(1,-2,1)} = -1 \text{ volt}$$

Example-1.28

An electric field is described by the field lines as shown. Explain about the nature of the field.

Solution :

Consider a closed path p q r s as shown in the given field



At pq field intensity is E_1 and at rs field intensity is E_2. From the given distribution of field lines $E_2 > E_1$. Field is perpendicular to qr or ps

Now $\oint \overline{E}.d\overline{\ell} = \int E_1(pq) + \int E_2(sr)$

$$= (E_1 - E_2)pq \neq 0$$

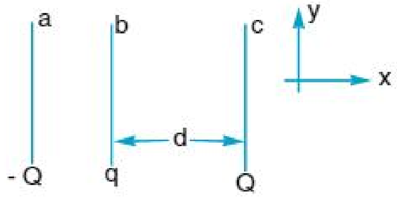
$$\left(\because \int \overline{E}.\,d\overline{\ell} = 0 \text{ for } qr \text{ and } ps\right)$$

Line integral of $\overline{E}.\,d\overline{\ell}$ along closed path is nonzero in this case. But we know that electric field is conservative and $\oint \overline{E}.d\overline{\ell} = 0$.

It means electric field as shown is not possible.

Example-1.29

Two fixed identical conducting plates a and b each of surface area A are charged to –Q and q respectively (Q > q > 0). A third identical plate c free to move is located on the other side of the plate with charge Q at a distance d. If c is released, it collides with plate b. Assume the collision as elastic and the time of collision is sufficient to redistribute charge among b and c.



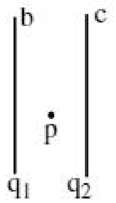
(a) Find the electric field acting on the plate c before collision. (b) Find the charges on b and c after the collision. (c) Find the velocity of plate c after the collision and at a distance d from the plate b if m is mass of the plate.

Solution :

(a) Electric field at C due to plates a and b are $\frac{-Q}{2\varepsilon_0 S}\hat{i}$ and $\frac{q}{2\varepsilon_0 S}\hat{i}$ respectively. So, before collision net electric field on C is $\frac{(Q-q)}{2\varepsilon_0 S}(-\hat{i})$

(b) When C collides with b, redistribution of charge takes place till the two plates acquire same potential. For a short duration the plates b and c from a single conductor and net electric field at a point p between them is zero. Let q_1 and q_2 be the changes on b and c after collision. Then net electric field at p will be

$$\overline{E} = \frac{-Q}{2\varepsilon_0 S}\hat{i} + \frac{q_1}{2\varepsilon_0 S}\hat{i} - \frac{q_2}{2\varepsilon_0 S}\hat{i}$$



But $\overline{E} = 0$ which means $-Q + q_1 - q_2 = 0$

and $Q = q_1 - q_2$ from conc. of charge,

$Q + q = q_1 + q_2$

$\Rightarrow q_1 = Q + q_2$ and $q_2 = q/2$

(c) Net electric field on plate C after collision is

$$\overline{E}_C = \frac{Q}{2\varepsilon_0 S}\hat{i} + \frac{q_1}{2\varepsilon_0 S}\hat{i} = \frac{q}{4\varepsilon_0 S}\hat{i} \ (\because -Q + q_1 = q/2)$$

Force acting on C till the collision takes place is

$$F_1 = \frac{(Q-q)Q}{2\varepsilon_0 S}$$

Force action on C after the collision is $F_2 = \frac{Q^2}{8\varepsilon_0 S}$

Total work done = Work done in travelling distance d before collision + workdone in travelling distance d after collision

$W = (F_1 + F_2)\,d$

$$= \left\{\frac{(Q-q)Q}{2\varepsilon_0 S} + \frac{q^2}{8\varepsilon_0 S}\right\}d = \frac{(Q-q/2)^2 d}{2\varepsilon_0 S}$$

If V is velocity C after collision and at a distance d from plate b, gain in kinetic energy $= \frac{1}{2}mv^2$ from work energy theorem, $W = \frac{1}{2}mv^2 \Rightarrow v = (Q - q/2)\sqrt{\frac{d}{\varepsilon_0 mS}}$

1.39 GRADIENT OF A SCALAR FIELD AND DEL OPERATOR

Let $\phi(x,y,z)$ be a scalar function with first derivatives continuous in a certain region. Then gradient of that function grad ϕ is a vector which has its component in any direction equal to the derivative of ϕ in that direction. At each point of the region grad ϕ has a magnitude equal to the maximum rate of change of ϕ with respect to space variables and extends in the direction of that change.

The gradient operation can be viewed formally as converting a scalar field in to a vector field. Also certain vector fields can be specified as the gradient of some ficititions scalar field. We have gravitational field, electrostatic field which can be expressed as grad ϕ where ϕ is corresponding scalar function.

Vector fields in general may be functions of space and time. The ∇ (pronounced as del) operator is a vector space function operator which is expressed in cartesian coordinates as

$$\nabla = \hat{i}\left(\frac{\partial}{\partial x}\right) + \hat{j}\left(\frac{\partial}{\partial y}\right) + \hat{k}\left(\frac{\partial}{\partial z}\right)$$

The use of partial derivatives is simply a method of mathematically holding time fixed and investigating the behaviour of a vector field spatially in the region at that particular instant of time. It is easily seen that grad ϕ can be expressed in terms of del operator as $\nabla\phi$.

- Curl of a vector field given by

  curl $\overline{A} = \nabla \times \overline{A}$

  where $\overline{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

  $$\text{Now } \nabla \times \overline{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \quad .... (1.22)$$

  A vector whose curl vanishes is called irrotational. Such a vector is incapable of causing rotation. Electrostatic field is one such field.

1.40 EQUIPOTENTIAL SURFACES

Equipotential surface in an electric field is a surface on which the potential is same at every point. In other words, the locus of all points which have the same electric potential is called equipotential surface.

An equipotential surface may be the surface of a material body or a surface drawn in an electric field. The important properties of equipotential surfaces are as given below.

(a) As the potential difference between any two points on the equipotential surface is zero, no work is done in taking a charge from one point to another.

(b) The electric field is always perpendicular to an equipotential surface. In other words electric field or lines of force are perpendicular to the equipotential surface.

(c) No two equipotential surfaces intersect. If they intersect like that, at the point of intersection field will have two different directions or at the same point there will be two different potentials which is impossible.

(d) The spacing between equipotential surfaces enables to identify regions of strong and weak fields $E = -\frac{dV}{dr}$. So $E \propto \frac{1}{dr}$ (if dV is constant).

(e) At any point on the equipotential surface component of electric field parallel to the surface is zero.

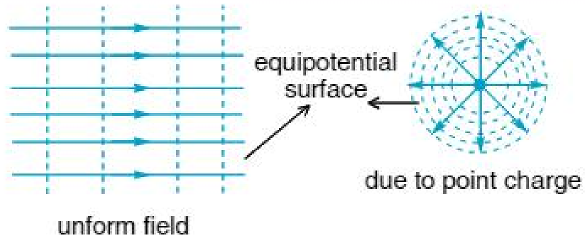


Fig 1.30 (a) Fig 1.30 (b)

In uniform field the lines of force are straight and parallel and equipotential surfaces are planes perpendicular to the lines of force as shown in figure (a).

The equipotential surfaces are a family of concentric spheres for a uniformly charged sphere for a point charge as shown in figure (b).

Equipotential surfaces in electrostatics are similar to wave fronts in optics. The wave front in optics is the locus of all points which are in the same phase. Light rays are normal to the wave fronts. On the other hand the equipotential surfaces are perpendicular to the lines of force.

- In case of non-uniform electric field, the field line are not straight, and in that case equipotential surfaces are curved but still perpendicular to the field.
- Electric potential and potential energy are always defined relative to a reference. In general we take zero reference at infinity. The potential at a point P in an electric field is V if potential at infinity is taken as zero. If potential at infinity is V_0, the potential at P is $(V-V_0)$.
- The potential difference is a property of two points and not of the charge q_0 being moved.

Field as the gradient of potential

How to calculate $\overline{E}$ when V is known?

Consider two surfaces of potentials V and V + dV in an electric field as shown in figure. Let a positive charge q_0 at point A on the surface V be moved to the point B on the surface V + dV along the displacement vector $\overline{dL}$ by any external agent.

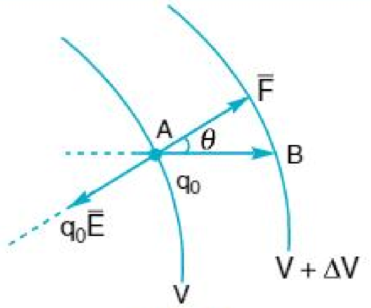


Fig 1.31

The force experienced by the test charge q_0 at A due to electric field will be q_0E. This force is in the direction of E which is at right angle to the surface V. In order to move the test charge q_0 without acceleration the external agent must apply an equal and opposite force F as shown in figure.

Thus $F = -q_0E$ .... (1)

Further the workdone by the external agent to move the test charge from A to B along $\overline{dL}$ is given by

$$dw = F.\overline{dL}$$

$$dw = -q_0\overline{E}\,.\,\overline{dL}$$

$$\frac{dw}{q_0} = -\overline{E}\,.\,\overline{dL} \quad ....(2)$$

We know that (dw/q_0) is the potential difference dV between points A and B. Thus

$$dV = -\overline{E}\,.\,\overline{dL} \quad .... (3)$$

Let the co-ordinates of A and B be (x, y, z) and (x + dx, y + dy, z + dz) respectively. The potential difference dV between A and B can be expressed as

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$= \left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right).\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$= (\text{grad } V).dL = \nabla V.\overline{dL} \quad .... (4)$$

$\therefore$ ($\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} =$ grad V and $dx\hat{i}+dy\hat{j}+dz\hat{k}$ is the displacement vector dL between A and B)

Comparing the above equation (3) & (4)

we get $E = -\text{grad } V = -\nabla V$ .... (5)

If E_x, E_y, E_z be the components of $\overline{E}$ then, equation (5) can be written as

$$E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Thus we have

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z} \quad ....(1.23)$$

Thus the electric intensity at a point in the electric field is equal to the negative potential gradient in that direction.

Note : In case of polar co-ordinates the components are radial along radius vector and azimutal perpendicular to the radius vector. These can be calculated as

$$E_r = -\left(\frac{\partial V}{\partial r}\right)_\theta$$

$$E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial \theta}\right)_r \quad .... (1.24)$$

As electric field intensity is negative of potential gradient, the direction of electric field will be same as the direction along which potential decreases.

1.41 POTENTIAL DUE TO CHARGED SPHERICAL SHELL

i) Let us consider a charged spherical shell whose centre is "O" and with radius "R" as shown in figure. Let "σ" be the charge density on the surface and posses a total charge of q(say).

As the sphere considered is a conducting sphere the electric charge will be distributed uniformly, on the outer surface of the sphere and no charge resides inside.

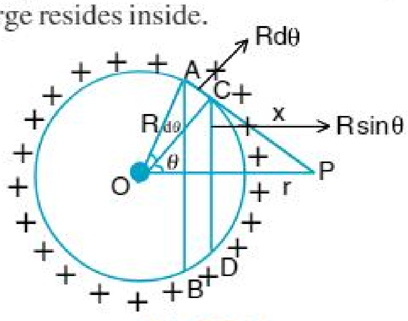


Fig 1.32 (a)

Let "P" be a point at a distance "r" from the centre of the spherical shell where the potential is to be calculated. Let us divide the sphere into no.of strips and let ABCD be one such strip. If the strip is cut perpendicular to the direction of OP, the slice obtained is a ring as shown in the figure below.

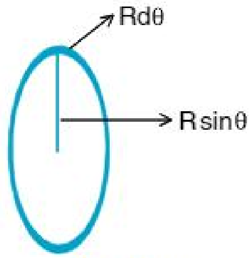


Fig 1.32 (b)

The radius of the slice is $R\sin\theta$. Here $d\theta$ is the angle subtanded by the edge of the slice at centre and θ is the inclination of the slice with the axis OP. Then the circumference of the ring is $2\pi R\sin\theta$. As width of AC is $Rd\theta$, the area of the ring is $2\pi(R\sin\theta)Rd\theta$.

$= 2\pi R^2 \sin\theta d\theta$

Charge dq on the ring

$= [2\pi R^2 \sin\theta d\theta]\sigma$ ....(1)

where σ is charge per unit area $= q/4\pi R^2$.

$\therefore dq = [2\pi R^2 \sin\theta d\theta]\dfrac{q}{4\pi R^2}$

$dq = \dfrac{q}{2}\sin\theta d\theta$.

Potential at "P" due to charge on the ring where $x = AP$ the distance of the point from the charge is

$dV = \dfrac{dq}{4\pi\varepsilon_0 x}$; $dV = \dfrac{q}{2}\dfrac{\sin\theta d\theta}{4\pi\varepsilon_0 x}$

$$dV = \frac{q\sin\theta d\theta}{8\pi\varepsilon_0 x} \quad .... (2)$$

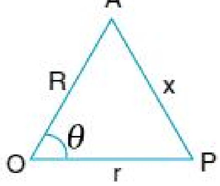


Fig 1.32(c)

But from Δle OAP

$x^2 = R^2 + r^2 - 2Rr\cos\theta$[cosine rule]

On differentiating we get

$2xdx = 2Rr\sin\theta d\theta$

$x = \dfrac{Rr\sin\theta d\theta}{dx}$

$\therefore \sin\theta d\theta = \dfrac{xdx}{Rr}$

$\therefore \Rightarrow dV = \dfrac{qxdx}{Rr8\pi\varepsilon_0 x}$

$$dV = \frac{qdx}{Rr8\pi\varepsilon_0} \quad .... (I)$$

a) Potential out side the sphere :

The limits of x extend from a distance of r–R to r + R for complete sphere.

So, total potential at P due to the entire sphere is

$$V = \int_{r-R}^{r+R} \frac{q}{Rr8\pi\varepsilon_0}dx = \frac{q}{Rr8\pi\varepsilon_0}[x]_{r-R}^{r+R}$$

$$= \frac{q}{Rr8\pi\varepsilon_0}[r + R - r + R]$$

$$= \frac{2q}{Rr8\pi\varepsilon_0} \Rightarrow V = \frac{q}{4\pi\varepsilon_0 r} \quad .... (1.25)$$

Hence the potential at a point out side a uniformly charged conducting spherical shell is same as if the charge is at the centre of the sphere.

b) Point 'P' on the sphere :

If the point 'P' lies on the sphere $R = r$. Hence potential on the surface is $q/4\pi\varepsilon_0 R$.

c) Point inside the sphere :

For a point inside the sphere the distribution of charge extends from x to x^1 i.e., w.r.t. P the limits will be R – r to R + r.

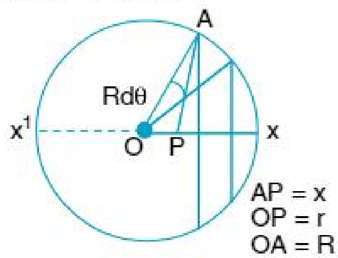


Fig 1.32 (d)

$$\therefore \Rightarrow dV = \frac{qdx}{8\pi\varepsilon_0 Rr}$$

$$\therefore \int dV = V = \int_{R-r}^{R+r} \frac{qdx}{8\pi\varepsilon_0 Rr} = \frac{q}{8\pi\varepsilon_0 Rr}[x]_{R-r}^{R+r}$$

$$= \frac{q}{8\pi\varepsilon_0 Rr}[R+r-R+r] = \frac{2qr}{8\pi\varepsilon_0 Rr}$$

$$V = \frac{q}{4\pi\varepsilon_0 R}.$$

Hence the potential at a point inside a hollow spherical shell is same as that on the surface.

The variation of potential with distance is as shown in figure.

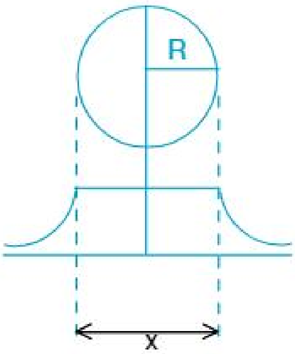


Fig 1.32 (e)

Example-1.30

A conducting spherical bubble of radius r and thickness $t(t << r)$ is charged to a potential V. Now it collapses to form a spherical droplet. Find the potential of the droplet.

Solution :

Here charge and mass are conserved. If R is the radius of the resulting drop formed and ρ is density of soap solution,

$$\frac{4}{3}\pi R^3\rho = 4\pi r^2 t\rho \Rightarrow R = (3r^2 t)^{1/3}$$

Now potential of the bubble is $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

or $q = 4\pi\epsilon_0 rV$

Now potential of resulting drop is

$$V^{1} = \frac{1}{4\pi\epsilon_0}\frac{q}{R} = \left(\frac{r}{3t}\right)^{1/3} V.$$

1.42 POTENTIAL DUE TO A LINE CHARGE OF INFINITE LENGTH

Let us consider a charge situated along line of infinite length. For convenience let us say that it is along x-axis.

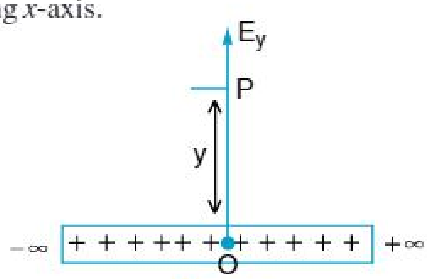


Fig 1.33

We know that the field at point P which is at a distance 'y' is given by.

$$E_y = \frac{\lambda}{2\pi\varepsilon_0 y} \qquad .... (1)$$

The electric potential at this point is given by

$$V_y = \int_{\infty}^{y} -Edy = -\int_{\infty}^{y} \frac{\lambda}{2\pi\varepsilon_0 y}dy$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{\infty}{y}\right) = \infty \qquad .... (2)$$

Thus the electric potential due to an infinite line charge is infinity everywhere.

Potential difference between positions with finite separation than is more useful absolute potential. Generally the potential is assumed to be zero at infinity but for convenience here we set V = 0 at some arbitary y_0 Now

$$V_y = \int_{y_0}^{y} Edy = \int_{y_0}^{y} -\frac{\lambda}{2\pi\varepsilon_0 y}dy \quad \left(\therefore E = -\frac{\partial V}{\partial y}\right)$$

$$= \int_{y}^{y_0} \frac{\lambda dy}{2\pi\varepsilon_0 y} = \frac{\lambda}{2\pi\varepsilon_0}[\log y]_{y}^{y_0}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}[\log_e y_0 - \log_e y]$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\log_e y + C \text{ where}$$

$$C = \frac{\lambda}{2\pi\varepsilon_0}\log_e y_0 \qquad .... (3)$$

Because y_0 is constant, we can replace $(\lambda / 2\pi\varepsilon_0)\log_e y_0$ by a constant C. From equation (3) we find the potential difference between two points at distances y_1 and y_2 hence

$$V_1 - V_2 = \left(-\frac{\lambda}{2\pi\varepsilon_0}\log_e y_1 + C\right) - \left(-\frac{\lambda}{2\pi\varepsilon_0}\log_e y_2 + C\right) = \frac{\lambda}{2\pi\varepsilon_0}\left(\log_e y_2 - \log_e y_1\right)$$

$$V_1 - V_2 = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{y_2}{y_1}\right) \quad ..... (1.26)$$

Equation (1.26) gives the potential differences between two points.

Potential due to infinite charged long wire

Consider infinitely long uniformly charged wire. To calculate the potential difference between two points A and B at a distance r_A and r_B from the wire respectively, we use

$$V_B - V_A = -\int_A^B \bar{E}.\overline{dL} = -\int_A^B EdL\cos\theta$$

where θ is the angle between E and dL. From Gauss's law.

$$E = \lambda / 2\pi\varepsilon_0 r \quad .... (2)$$

where λ is the charge per unit length of the wire.

From equation (1) & (2) we get

$$V_B - V_A = -\int_A^B \frac{\lambda}{2\pi\varepsilon_0 r} dL\cos\theta$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\int_A^B \frac{dL\cos\theta}{r}$$

But $dL\cos\theta = dr$ (component along field E

$$\therefore V_B - V_A = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_0}\left[\log r\right]_{r_A}^{r_B}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\log r_A - \log r_B\right]$$

$$V_B - V_A = \frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{r_A}{r_B}\right) \quad .... (1.27)$$

The absolute potential at any point can be calculated by taking reference point A at infinity i.e. $r_A = \infty$ Now

$$V_B = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{\infty}{r_B}\right) = \infty$$

Hence we get an infinite value for the potential at all points which are at finite distances from the wire. This is correct because we have assumed an infinite charge on the wire. However in practice we always deal with potential differences between points having a finite seperation and absolute potential has no real significance.

1.43 POTENTIAL DUE TO UNIFORMLY CHARGED CIRCULAR DISC

Consider a uniformly charged circular disc with charge q as shown in figure. Let σ be the surface charge density. Here our aim is to calculate the potential V at any point P on the axis of the disc at a distance r from the centre O. For this purpose we divide the disc into a large number of circular strips. Further, we consider one such strip of radius 'y' and width dy. As the width of the strip is very small, each point of this strip can be assumed to be at equal distance.

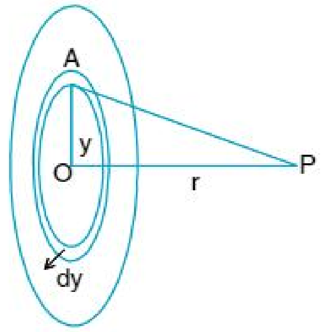


Fig 1.34

$\left\{AP = \sqrt{r^2 + y^2}\right\}$ from the point P. The charge dq contained by the strip will be $dq = \sigma$ (area of the strip).

$$= \sigma(2\pi y dy)$$

$\therefore$ The potential at P due to this charge element will be

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{AP} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi y dy)}{\sqrt{r^2+y^2}} \quad .... (1)$$

The potential at P due to the whole disc can be obtained by integrating equation (1) with in the limits o to R. Hence

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{ydy}{\sqrt{(r^2+y^2)}}$$

$$= \frac{\sigma}{2\varepsilon_0}\int_0^R (r^2+y^2)^{-1/2} ydy$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+r^2} - r\right] \quad .... (2)$$

At the centre of the disc r = 0 hence

$$V_0 = \sigma R/2\varepsilon_0 \quad .... (3)$$

In case when r >> R the quantity $\sqrt{r^2+R^2}$ can be approximated by binomial theorem as

$$\sqrt{r^2+R^2} = r(1+R^2/r^2)^{1/2}$$

$$= r\left[1+\frac{R^2}{2r^2}+....\right] = r+\frac{R^2}{2r}$$

$$\text{Now } V = \frac{\sigma}{2\varepsilon_0}\left[r+\frac{R^2}{2r}-r\right] = \frac{\sigma R^2}{4\varepsilon_0 r} = \frac{\pi R^2\sigma}{4\pi\varepsilon_0 r}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad ..... (1.28)$$

where q is the charge on the whole disc.

1.44 POTENTIAL ON THE RIM OF CHARGED DISC

Now we shall calculate the potential at a point P lying on the edge of the disc as shown in figure. We divide the disc into a large number of rings with P as the centre.

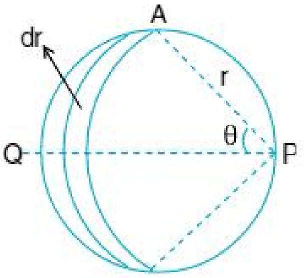


Fig 1.35

Further we consider the ring shown dotted with radius r and thickness dr centred at P. The length of the segment will be $2r\theta$ and its area will be $2r\theta dr$. Hence the charge dq on this segment is given by

$$dq = 2r\theta . dr\sigma$$

Potential at P due to this segment

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{2r\theta dr\sigma}{r}$$

In order to calculate the potential V at P due to whole disc we integrate dV with in the limits from 0 to 2R. Hence

$$V_P = \int \frac{1}{4\pi\varepsilon_0}\frac{2r\theta dr\sigma}{r} = \frac{\sigma}{2\pi\varepsilon_0}\int_0^{2R}\theta dr$$

From right angled triangle PAQ,

$r = 2R\cos\theta$ or $dr = -2R\sin\theta d\theta$.

Now when $r = 0, \cos\theta = 0$ hence $\theta = \frac{\pi}{2}$

$r = 2R, \cos\theta = 1$ hence $\theta = 0$

$$\therefore V_P = \frac{\sigma}{2\pi\varepsilon_0}\int_{\pi/2}^{0} -2R\theta\sin\theta d\theta$$

$$= \frac{\sigma R}{\pi\varepsilon_0}\left[\sin\theta - \theta\cos\theta\right]_0^{\pi/2}$$

$$V_P = \frac{\sigma R}{\pi\varepsilon_0} \quad .... (1.29)$$

Comparing the above equations we get that the potential at the centre is greater than the potential at the rim of the disc. So a uniformly charged disc is not an equipotential surface.

1.45 ELECTRIC DIPOLE

A system of two equal and opposite point charges fixed at a small distance constitutes an electric dipole. Electric dipole is analogous to bar magnet or magnetic dipole in magnetism. Every dipole has a characteristic property called dipole moment; which is similar to magnetic moment of a bar magnet. If $2l$ is the distance between the charges +q and –q, then electric dipole moment is $P = q2l$.

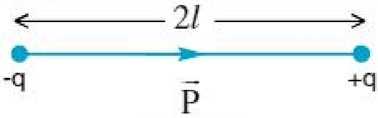


Fig 1.36

Dipole moment is a vector quantity and its direction is from negative charge to positive charge as shown.

1.46 ELECTRIC DIPOLE IN A UNIFORM ELECTRIC FIELD

Consider an electric dipole of dipole moment P placed at an angle θ to the direction of electric field. Force on each charge of the dipole is of magnitude qE. But the forces are equal, unlike and non collinear. The forces on the two unlike charges constitute a couple. The torque due to this couple is given by

$$\tau = qE \times 2l \sin\theta \text{ or } \tau = pE\sin\theta$$

in vector form $\bar{\tau} = \bar{p} \times \bar{E}$

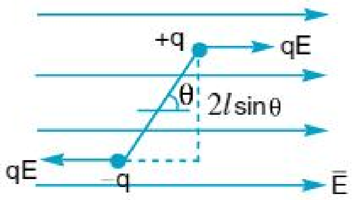


Fig 1.37

The torque on the dipole tends to align the dipole along the direction of electric field.

1.47 INTERACTION ENERGY OF DIPOLE IN AN ELECTRIC FIELD

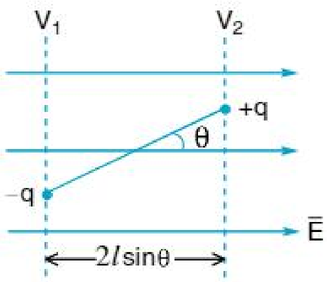


Fig 1.38

Consider a dipole of length $2l$ and dipole moment P in a uniform electric field as shown. Let θ be the angle between dipole moment and field direction. Here the two charges of the dipole are located on two equipotential surfaces having potentials V_1 and V_2. We can write

$$V_1 - V_2 = E2l\cos\theta$$

Now the interaction energy of the dipole with electric field is given by

$$U = -qV_1 + qV_2 = q(V_1 - V_2)$$

$$\Rightarrow U = -qE2l\cos\theta = -PE\cos\theta \quad (\because P = q2l)$$

In vector form $U = -\bar{P}.\bar{E}$

if $\theta = 0^o$; $\tau = 0$ and $U = -PE$

if $\theta = 90^o$; $\tau = PE$ and $U = 0$

if $\theta = 180^o$; $\tau = 0$ and $U = PE$

So, if $\bar{P} // \bar{E}$, potential energy is minimum and torque on the dipole is zero.

If $\bar{P} // -\bar{E}$, potential energy is maximum and again torque is zero.

The dipole will be in stable equilibrium if $\bar{P} // \bar{E}$ and in unstable equilibrium if $\bar{P} // -\bar{E}$.

1.48 WORK DONE IN ROTATING A DIPOLE IN ELECTRIC FIELD

Consider an electric dipole in uniform electric field. Let θ_1 be the angle made by the dipole moment with the direction of electric field. The interaction potential energy of the dipole with electric field is given by $U_i = -PE\cos\theta_1$

Let θ_2 be the angle made by the dipole moment with the field direction after rotating it,

The interaction potential energy of the dipole with electric field is $U_f = -PE\cos\theta_2$.

Work done in rotating the dipole to angle θ_2;

$$W = U_f - U_i$$

$$= PE(\cos\theta_1 - \cos\theta_2)$$

1.49 FORCE ON DIPOLE IN NON-UNIFORM ELECTRIC FIELD

Consider an electric dipole kept in a non uniform electric field at a point where the field intensity is E. We know that interaction energy of the dipole at this position is $U = -\bar{P}.\bar{E}$.

The force on the dipole due to electric field is given by $F = -\nabla U$ (Force = negative potential energy gradient).

If the electric field is along $\vec{r}$,

we can write

$$\overline{F} = -\frac{d}{dr}(\overline{P}.\overline{E})$$

If $\overline{P}$ and $\overline{E}$ are along the same direction we can write $\overline{F} = \frac{-d}{dr}(PE\cos\theta)$ or $F = -P\left(\frac{dE}{dr}\right)$.

1.50 DISTRIBUTED DIPOLE

We have discussed about electric dipole with two equal and unlike point charges separated by a small distance. But in some cases the two charges are not concentrated at its ends. (Like in water molecule) consider a situation as shown in the figure. Here three charges –2q, q and q are arranged as shown. It can be visualised as the combination of two dipoles each of dipole moment P = qd at an angle θ between them. As dipole moment is a vector the resultant dipole moment of the system is $P^{|} = 2P\cos\theta/2$.

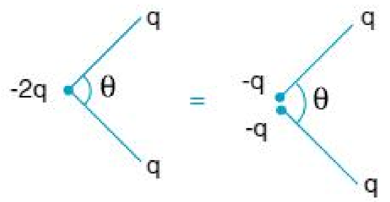


Fig 1.39(a)

Consider a half ring with a charge +q uniformly distributed and another equal negative charge –q placed at its centre. Here –q is point charge while +q is distributed on the ring. Such system is called distributed dipole.

Fig 1.39(b)

Consider a small polar element of angular width $d\theta$ at an angle θ with the vertical as shown.

Charge on this element is $dq = \left(\frac{q}{\pi}\right)d\theta$

At the location of –q, consider an element charge –dq which forms a dipole with the polar element on the semicircular r is.

The dipole moment of the element is

$$dP = dq(R) = \frac{q}{\pi}Rd\theta$$

The dipole moment of the system is given by integrating all such elemental dipole moments.

$$\Rightarrow P_{net} = 2\int_0^{\pi/2} dP\cos\theta + 2\int_0^{\pi/2} dP\sin\theta$$

Here $2\int_0^{\pi/2} dP\sin\theta = 0$

So $P_{net} = 2\int_0^{\pi/2} dP\cos\theta = \frac{2qR}{\pi}$

If $\theta = \phi$ $\quad P_{net} = 2\int_0^{\phi/2} dP\cos\theta$

$$= \frac{2qR}{\pi}\sin\frac{\phi}{2}$$

If the arrangement is a complete circle, $\frac{\phi}{2} = \pi$

$$\Rightarrow P_{net} = 0.$$

- The arrangement of two electric dipoles is known as quadrapole. Few quadrapoles are as shown in the figure.

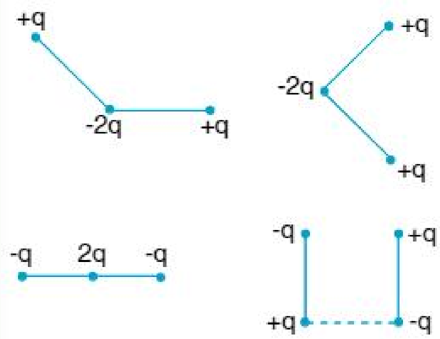


Fig 1.39(c)

1.51 FORCE BETWEEN TWO SHORT DIPOLES

Consider two short dipoles seperated by a distance r. There are two possibilities. First let us consider the dipoles placed parallel to each other.

Potential energy of dipole P_2 in the field of dipole P_1 is given by

$$U = -P_2E_1\cos 180^{o}$$

$$= \frac{-1}{4\pi\in_0}\frac{P_1P_2}{r^3}$$

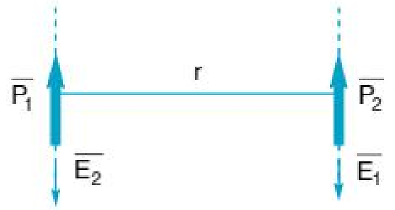


Fig 1.39(d)

Now force between them is given by

$$F = \frac{-dU}{dr}$$

$$= \frac{d}{dr}\left[\frac{-1}{4\pi\in_0}\frac{P_1P_2}{r^3}\right] \qquad = \frac{1}{4\pi\in_0}\frac{3P_1P_2}{r^4}$$

As the force is positive, it is repulsive. Similarly if $\overline{P_1} \parallel -\overline{P_2}$ the force is attractive.

Now let us consider the situation such that dipoles are on the same axis

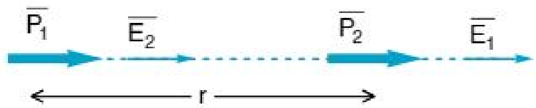


Fig 1.39(e)

In this case $U = -P_2E_1\cos 0^{o} = \dfrac{-2P_1P_2}{4\pi\in_0 r^3}$

The force between the dipoles is given by

$$F = -\frac{dU}{dr}$$

$$= -\frac{dU}{dr}\left[\frac{2P_1P_2}{4\pi\in_0 r^3}\right]$$

$$F = -\frac{1}{4\pi\in_0}\frac{6P_1P_2}{r^4} \qquad \text{.... (1.30)}$$

As the force is negative, it is attractive. Try the cases for $\overline{P_1} \perp \overline{P_2}$.

1.52 POTENTIAL DUE TO A DIPOLE

An electric dipole consists of two equal and opposite charges seperated by a very small distance. If 'q' is the charge and 2a the length of the dipole, electric dipole moment will be given by P = (2a)q.

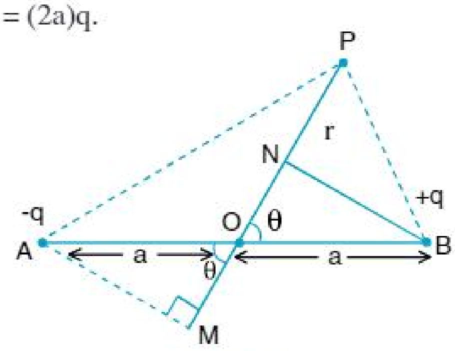


Fig 1.40

Let AB be a dipole whose centre is at 'O' & 'P' be the point where the potential due to dipole is to be determined. Let r,θ be the position co-ordinate of 'P' w.r.t the dipole as shown in figure. Let BN & AM be the perpendiculars drawn on to OP and line produced along PO. From geometry $ON = a\cos\theta = OM$. Hence the distance, BP from +q charge is $(r - a\cos\theta)$

[because PB = PN as AB is very small in comparison with *r*].

For similar reason

$AP = (r + a\cos\theta)[\therefore AP = PM]$.

Hence potential at P due to charge +q situated at B is $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(r - a\cos\theta)}$.

Similarly potential at P due to charge −q at A is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r + a\cos\theta)}.$$

Total potential at P

$$= \frac{q}{4\pi\varepsilon_0(r - a\cos\theta)} - \frac{q}{4\pi\varepsilon_0(r + a\cos\theta)}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right]$$

$$V = \frac{q(2a\cos\theta)}{4\pi\varepsilon_0(r^2 - a^2\cos^2\theta)}$$

But r >> a

$$\therefore (r^2 - a^2\cos^2\theta) \approx r^2$$

$$\therefore V = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2}. \qquad .... (1.31)$$

Hence potential varies inversely as the square of the distance from the dipole.

Special Cases

1) For a point on the axial line $\theta = 0^0$

$\therefore V_{axial} = P/4\pi\varepsilon_0 r^2$ for a dipole.

2) For a point on the equitorial line $\theta = 90^0$.

$\therefore V_{equitorial} = 0$

Equitorial line is a line where the potential is zero at any point, on it

1.53 ELECTRIC FIELD AT ANY POINT DUE TO A DIPOLE

We know that the electric field is the -ve gradient of potential. In polar form, if V is the potential at (r,θ) the electric field will have two components which are radial & transverse components represented by E_r & E_θ respectively.

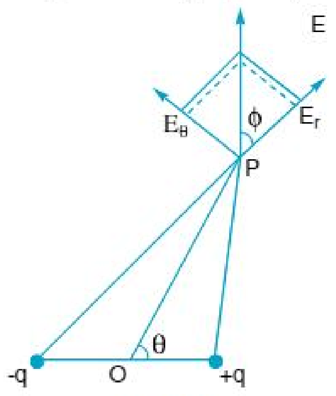


Fig 1.41

Then $E_r = -\left(\frac{\partial V}{\partial r}\right) = -\frac{P\cos\theta}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)$

$$E_r = \frac{2P\cos\theta}{4\pi\varepsilon_0 r^3} \quad .... (1) \qquad \left[\begin{array}{l} E_r = -\frac{\partial V}{\partial r} \\ E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial\theta}\right)\end{array}\right]$$

The transverse component of electric field

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta}$$

$$= -\frac{1}{r}\left(-\frac{P\sin\theta}{4\pi\varepsilon_0 r^2}\right)$$

$$E_\theta = \frac{P\sin\theta}{4\pi\varepsilon_0 r^3} \quad .... (2)$$

$$E = \sqrt{E_\theta^2 + E_R^2}$$

$$E = \sqrt{\frac{P^2\sin^2\theta}{(4\pi\varepsilon_0 r^3)^2} + \frac{4P^2\cos^2\theta}{(4\pi\varepsilon_0 r^3)^2}}$$

$$E = \frac{P}{4\pi\varepsilon_0}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\Rightarrow E = \frac{P}{4\pi\varepsilon_0 r^3}\sqrt{[1 + 3\cos^2\theta]} \qquad .... (1.32)$$

$$E_{axial} = \frac{2P}{4\pi\varepsilon_0 r^3}(as\,\theta = 0^0\,or\,180^0)$$

$$E_{equitorial} = E_{eq} = \frac{P}{4\pi\varepsilon_0 r^3}(as\,\theta = 90^0\text{ or }270^0)$$

The direction of E at any point is given by

$$\tan\phi = \frac{E_\theta}{E_r} = \frac{\frac{P\sin\theta}{4\pi\varepsilon_0 r^3}}{\frac{2P\cos\theta}{4\pi\varepsilon_0 r^3}} = \frac{1}{2}\tan\theta$$

$$\phi = \tan^{-1}[1/2\tan\theta]$$

Example-1.31

There is an electric field along x-direction magnitude of that electric field increases uniformly along the positive X-direction, at the rate of $10^4 NC^{-1}$ per metre. Find the force and torque experienced by a system having a total dipole moment equal to 10^{-6} cm in the negative X-direction.

Solution:

$$dF = F_1 \sim F_2 = qE_1 \sim qE_2$$
$$= 10^4 \times 10^{-6}$$
$$= 10^{-2}\ N$$

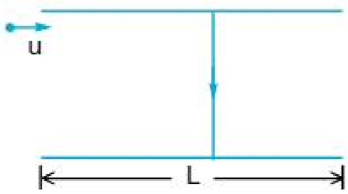
Direction of this force is along negative X-direction.

Here no torque acts on the system as $\overline{P} // \overline{E}$

(since $\overline{\tau} = \overline{P} \times \overline{E}$)

Example-1.32

A particle of mass m and change q enter the region between two charged plates. The length of the plate is L and there is a uniform electric field 'E' between those plates as shown. Charge q enters into the field with an initial speed U as shown. Find the vertical deflection of the particle at the far edge of the plate ?



Solution:

Here charged particle experiences force along vertical direction which is equal to Eq.

As a result its acceleration in the vertical direction is $\frac{Eq}{m}$.

Along the horizontal direction, its velocity doesnot change.

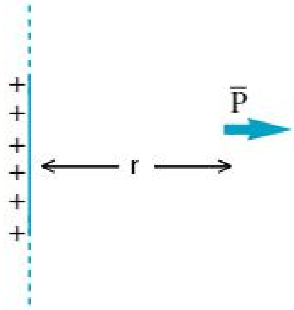
Time taken by the charge to reach the other end of the plate is $t = \frac{L}{U}$

During this time its vertical deflection is given by $y = \frac{1}{2}at^2$

$$y = \frac{1}{2}\frac{Eq}{m}\left(\frac{L}{U}\right)^2 = \frac{EqL^2}{2mU^2}$$

Example-1.33

An electric dipole of dipole moment p is kept at a distance r from an infinite long charged wire of linear charge density λ as shown. Find the force acting on the dipole.



Solution:

Field intensity at a distance r from the line of charge is $E = \frac{\lambda}{2\pi \epsilon_0 r}$

The force on the dipole is $F = p\frac{dE}{dr}$

$$= p\left[\frac{-\lambda}{2\pi \epsilon_0 r^2}\right] = -\frac{p\lambda}{2\pi \epsilon_0 r^2}$$

Here the net force on dipole due to the wire will be attractive.

1.54 SELF ENERGY OF A CHARGED SPHERICAL SHELL

When a spherical shell is charged work has to be done in bringing the charge from infinity (zero reference potential) on to that shell. Consider the shell with no charge in the beginning and after same time charge on it is q. If R is radius of the spherical shell, electric potential of the shell is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$.

Now a small charge dq is put on it (i.e., by bringing from infinity upto the shell) In this case work done is given by $dw = Vdq = \frac{1}{4\pi\varepsilon_0}\frac{qdq}{R}$.

If this process continues till charge on the shell is 'Q' total work done in this process is given by $\int dw$

$$\Rightarrow \int dw = \int_0^Q \frac{1}{4\pi\varepsilon_0}\frac{qdq}{R} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

But all this work done is stored in the from of electrostatics potential energy U, which is known as self energy

$$\Rightarrow \text{self energy } U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

1.55 SELF ENERGY OF A CHARGED SPHERICAL DISTRIBUTION

When a sphere (non conducting) is charged, charge distributes uniformly on it such that charge per unit volume 'ρ' remains constant. While charging it work has to be done in bringing charge from infinity onto that sphere. Here there will be electric field inside as well as outside the sphere. If Q is charge finally on that sphere volume charge density on that sphere is given by $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$.

Let R be the radius of the sphere and consider a point at a distance 'r' from the centre of the sphere (r < R).

Now at that point electric field is given by $\bar{E} = \dfrac{\rho}{3\varepsilon_0}\vec{r}$.

Now from $dv = -\bar{E}.d\bar{r}$ we can write $\int dv = -\int \bar{E}\,.\,d\bar{r}$ If V and V_s denote potentials at r = r and r = R (surface), we can write

$$\int_V^{V_s} dv = -\int_r^R \bar{E}\,.\,d\bar{r}$$

on simplification we get

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right]$$

Later if we use $\int dw = \int Vdq$, we get total work done as $\dfrac{3Q^2}{20\pi\varepsilon_0 R}$

This is self energy of a charged sphere of radius R (non conducting).

At a Glance

1. Charges are responsible for the electric force just as masses are responsible for gravitational force.
2. Charges are of two types- positive and negative.
3. Like charges repel and unlike charges attract each other.
4. The conductivity of a conductor is due to presence of free electrons in it.
5. Induction precedes attraction.
6. Coulomb's increase square law gives the electric force between stationary, point charges.
7. Electrostatic forces are of very large magnitude compared with gravitational forces. $F_e : F_g = 10^{36}$ for two protons and it is 10^{42} for two electrons and it is 10^{39} for a proton and neutron.
8. The electrostatic force between charges in medium dependent where as gravitational force between the bodies is independent of medium through both obey invex square law.
9. The electrostatic force between any two charges is not affected by the presence of other charges.
10. Electric field lines help us to visualise the nature of electric field in a region.
11. Electric field is the region where the influence of charge or charges is felt.
12. A tangent to the line of force gives the direction of electric field at a point. If an isolated positive test charge is released at a point on the line of force it tries of more off along the tangent.
13. Electrorlate lines of force are not closed curves but magnetic field lines can form closed curves (e.g bar magnet).
14. For points outside a charged conducting spherical shell, the entire charge on the surface of the shell behaves as though it were concentrated at the centre.
15. When a conductor is charged, charge resides on the outer surface and concentrates more at sharp points.
16. The force experienced by an isolated unit positive test charge in an electric field is known as intensity of electric field E at that point. E is a vector.
17. The amount of work done in brining an isolate unit positive test charge from infinity or reference point to a given point is known as electric potential V at the point. V is a scalar.

18. The electric field intensity is negative potential gradient i.e., $\overline{E}=\frac{-dV}{dr}$. Here negative sign indicates that direction of $\overline{E}$ is same as the direction along which electric potential decreases.
19. We apply principle of superposition to get resultant $\overline{E}$ or V at any point in the combined field due to various charges for charge distribution.
20. Electrostatic field is conservative. So line integral of $\overline{E}$ along closed loop in it is zero $\phi\overline{E}.d\overline{l}=0$. Work done in moving a charge in the electric field is independent of the path followed.
21. The total amount of work done in keeping point charges for a given configuration by bringing all those from infinity is known as elecrorlate potential energy or interaction energy of the system.
22. Interaction energy may be positive, negative or zero.
23. Two equal and unlike point charges separated by a very small distance constitute electric dipole.

EXERCISE

LONG ANSWER QUESTIONS

1. State and explain coulombs law in electrostatics. From this law, define the unit of charge and intensity of the electric field?
2. Define electric potential and potential difference. Define VOLT Derive an expression for the electro-static potential energy of a system of charged particles?
3. Define electric field, field strength, potential and potential difference, between two points?

SHORT ANSWER QUESTIONS

1. State and explain's law in electricity and define permitivity, relative permitivity and unit charge?
2. Derive an expression for intensity of electric field E at a point?
3. Define (a) Field intensity (b) Potential difference between two points and derive the relation between them?
4. State and explain the principle of superposition in electrostatics concerning the force due to multiple charges.
5. What is electric dipole find electric potential at a point on the axis of a dipole.
6. What is electric dipole on the equatorial live of dipole.

VERY SHORT ANSWER QUESTIONS

1. Usually it is the negative charge that is transferred when two bodies are rubbed together. Can you explain. Why?

A. The electrons are very light and loosely bound to the atoms than the positive charges.

2. A metallic sphere is charged negatively. Will its mass increase, decrease or remain the same?

A. When the sphere is negatively charged, it means electrons have been added to it. Since electrons have infinite mass, the mass of the negatively charged sphere will increase.

3. Electrostatic experiments cannot be conducted successfully on humid days. Explain.

A. The humid air becomes conducting. Therefore, the static charge on the apparatus leaks off to the air. For this reason, electrostatic experiments do not work well on humid days.

4. Both mass and charge are scalars and hence got the additive property. However, in adding charges it is not enough to just add the amounts of charges. Why?

A. Unlike mass charges are of two different kinds, positive and negative.

5. What is meant by the statement "charge is quantized".

A. The minimum charge that may be transferred from one body to the other body is equal tot he charge of an electron (1.6×10^{-19}C).

The charge is available in the multiples of charge on electron.

Hence the charge is said to be quantised.

6. What are the most significant differences between electrostatic forces and gravitational forces?

A. i) Gravitational forces are always attractive but electrostatic forces may be attractive or repulsive depending upon the signs of the charges.

ii) The gravitational constant (G) is independent of nature of the medium. However, electrical constant $k(=1/4\pi\varepsilon_0 K)$ depends upon the nature of the medium.

iii) Electrostatic forces are extremely large as compared to the gravitational forces. For example, electrostatic force of attraction between an electron and a proton is about 10^{39} times stronger than the gravitational force between them.

7. **What similarities do electrostatic forces have to gravitational forces?**

A. (i) Both obey inverse square law.

(ii) Both are central forces, i.e., forces act along the lines joining the centres of the bodies.

(iii) Both are conservative forces, i.e., work done by them does not depend upon the path followed.

(iv) Both involve a property of the interacting particles–the mass in one case and the charge in the other.

8. **Can the relative permittivity of a medium be less than 1?**

A. No. Air or vacuum has minimum relative permittivity $(K = \varepsilon_0\varepsilon_0 = 1)$. The relative permittivity of all other media is greater than 1.

9. **Vehicles carrying inflammable materials usually have chains that hang down and drag on the ground. Why?**

A. When a vehicle is in motion, its tyres rub against the road and get charged due to friction. Further, due to friction of air, the body of the vehicle also get charged. If the accumulated charge becomes excessive, sparking may occur and the inflammable material may catch fire. Since the chain ropes are touching the ground, the charge leaks to the earth. Hence, the danger of fire is avoided.

10. **Although ordinary rubber is insulator, the rubber tyres of air crafts are made slightly conducting. Why?**

A. During the take off and landing, the friction between tyres and the run-way causes electrification of tyres. If the tyres are non-conducting, excessive charge will accumulate on the tyres which may cause sparking. If the material of the tyres is slightly conducting, the accumulated charge can flow to earth, thus eliminating any danger of fire.

11. **An uncharged body if kept in contact for some time with a charged body gets repelled. Why?**

A. When charged body is in contact with a body for sometime, the charge is shared by both of them, due to the force of repulsion between the like charge, the body is repelled.

12. **Electrostatic field lines of forces do not form closed loops. If they form closed loops then the work done in moving a charge along a closed path will not be zero. From the above two statements can you guess the nature of electrostatic force?**

A. It is a conservative force.

13. **The electric lines of force do not intersect. Why?**

A. If they intersect, at the point of intersection there should be electrical field in two directions, which is not possible.

So they do not intersect.

14. **Consider two charges + q and –q placed at B and C of an equilateral triangle ABC. For this system, the total charge is zero. But the electric field (intensity) at A which is equidistant from B and C is not zero. Why?**

A. Charges are scalars, but the electrical intensities are vectors and add vectorially.

15. **Repulsion is the sure test of electrostatics than attraction. Why?**

A. If a positively charged body is brought near a negatively charged body or an uncharged body, there exist a force of attraction.

So the attraction is due to oppositvely charged body or chargeless body.

If a positively charged body is brought near a positively charged body there exist a force of repulsion. Thus repulsion is the sure test of elecrification.

16. **What are conductors and insulators?**

A. **Conductors:** Substances which allow electricity to pass through them are known as conductors. All metals, human body and the earth are good conductors of electricity. The charge carries in metal conductors are free electrons.

Insulators: Substances which do not allow electricity to pass through them are known insulators. Glass, rubber, plastic etc., are insulators.

17. **Can there be electric potential at a point with zero electric intensity?**

A. Yes. There can exist potential at a point where the electrical intensity is zero.

Ex-1: In the case of charged spherical conductor, the intensity of the electric field is zero inside that conductor but the potential inside the spherical conductor is constant.

Ex-2: Consider the similar charges of equal magnitude separated by a distance. At the mid-point the electric field is zero.

But there exists some potential as the potential due to the charge are of same polarity.

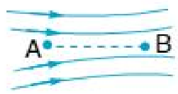
18. What is the difference between electric lines of force and magnetic lines of induction?

A. The electric lines of force always leave or end on the surface of a charged body.

Magnetic lines of induction are closed curves starting from north pole and ending on the south pole of the magnet.

Within the magnet they run from south to north.

19. Fig shows electric lines of force emerging from a charged body. Is electric field intensity more at A or B?



A. An electric field is represented by electric lines of force. If they are closely spaced, the electric field intensity is more at that place. since field lines are closely spaced at A and widely spaced at A, $E_B > E_A$.

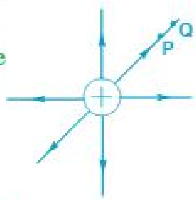
20. A charged particle is free to move in an electric field. Will it always move along electric lines of force?

A. If the charged particle is initially at rest in an electric field, it will move along the tangent to the electric line of force. However, if the charged particle has initial velocity and makes some angle with the line of force, then the resultant path will not be along the line of force.

21. Fig shows the electric field lines due to a point positive charge. P and Q are two point in the electric field.

i) What is the sign pf $V_P - V_Q$?

ii) What is the sign of the potential energy difference of a small negative charge between Q and P?



iii) What is the sign of work done by the field in moving a small positive charge from Q to P ?

iv) What is the sign of work done by external agent in moving a positive charge from Q to P ?

A. (i) Potential at a point $V \propto 1/r$. Therefore, $V_P > V_Q$. Hence. sign of $V_P - V_Q$ is positive.

(ii) If a negative charge is placed at Q, it will move towards P.

Since, a charge moves from higher potential energy to lower potential energy, $U_Q > U_P$. Therefore, $U_Q - U_P$ is positive.

(iii) If a small positive charge is placed at P, work done by the field in moving it to Q is positive. Hence, work done by the field in moving small positive charge from Q to P is negative.

(iv) Since in moving the positive charge from Q to P, external agent should do work against the field, the work done is positive.

22. Electric field intensity in a given region is zero. Can we conclude that electric potential must be zero?

A. Not necessarily, $E = -dV/dr$ or $0 = -dV/dr$. This means that electric potential is constant. The constant value can be zero or non-zero.

23. Two nearby points are at the same potential. What is the intensity of electric field in this region?

A. $E = -dV/dr$ where dV is change in potential for displacement dr. Since there is no change in potential, $dV/dr = 0$. Hence, electric intensity in the region is zero.

PROBLEMS

LEVEL - I

1. Calculate the magnitude of force between two like charges each of magnitude 4C seperated by distance 3 m. **[Ans:** 16×10^9 N]

2. Two charges 4μC and 1μC are seperated by 16m. Where do you place a third charge so that it doesn't experience any force. **[Ans:** 10.6 m from 4μC]

3. Calculate the ratio of electric and gravitational force between two electrons. **[Ans:** $\approx \frac{4}{9} \times 10^{45}$]

4. Three charges, 4μC each, are kept at the vertices of an equilateral triangle of side 9 cm. Find magnitude of force on any charge. **[Ans:** $\frac{16\sqrt{3}}{9}$ N]

5. A charge 4μC is placed in an electric field of magnitude 8 N/C. Find the force on it. **[Ans:** 32×10^{-6} N]

6. Three charges 4μC each are placed at the vertices of an equilateral triangle of side 9 cm. Find the electric potential at the centroid of that triangle. **[Ans:** $12\sqrt{3} \times 10^5$ V]

7. A charge 2C is moved between two points where potentials are 1V and 8V respectively. What is the work done in moving the charge. **[Ans:** 14 Joules]

8. Electric potential at origin is zero. Field in that space is $10\hat{i}+10\hat{j}NC^{-1}$. Find electric potential at (1, 1).
[Ans: –20 volts]

9. Infinite charges 'q' each are placed at x = 1, 2, 4, ... Find the electric potential at the origin.
[Ans: $\frac{q}{2\pi \epsilon_0}$]

10. Two point charges 4μF and –4μF are seperated by 8cm. Find the electric potential between them at a distance of 2cm from the positive charge.
[Ans: 12×10^5 Volts]

11. A charge 'q' is held at rest at a point. Another identical charge of mass 'm' is projected towards this charge from infinity with a velocity 'v'. What is the shortest distance between these charges.
[Ans: $d = \frac{1}{4\pi \epsilon_0} \frac{2q^2}{mv^2}$]

12. A charge q is released with a velocity 1×10^6 m/s from a large distance from a fixed positive charge Q. What is the closest distance of approach? The mass of the charge q is m.
[Ans: $\frac{18Qq}{10^3 m}$]

13. A metal sphere A of radius 'a' is charged to potential V. What will be its potential if it is enclosed by a spherical conducting shell B of radius 'b' and the two are connected by a wire.
[Ans: $\frac{a}{b}V$]

14. A charged particle P has a mass of 10^{-16}kg and carries a charge of 4.9×10^{-19}C. Calculate the intensity of the electric field to be applied on it in vertically upward direction, so as to keep it at rest.
[Ans: 2×10^3 N/C]

15. Two balls with charges 5μC and 10μC are at a distance of 1 m from each other. In order to reduce the distance between them to 0.5 m what amount of work should be done?
[Ans: 0.45 J]

16. A constant electric field of intensity 36N/C exists along the z-axis. If P and Q be two points whose coordinates are (10 cm, 0, –20 cm) and (0, –10 cm, 30 cm) respectively, then find the potential difference $V_P - V_Q$.
[Ans: 18V]

17. The electric potential V as a function of distance x is given by $V = (5x^2 + 10x - 9)$ Volt. Where x is in metre. Find the electric field at a point x = 1m
[Ans: 20 Vm^{-1}]

18. Four identical charges, each equal to –Q, are placed at the four corners of a square and a charge q is placed at its centre. If the system is in equilibrium, find the value of q
[Ans: $\frac{Q}{4}(1+2\sqrt{2})$]

LEVEL - II

1. A metal sphere with its centre at A and radius R has a charge 2q on it. The field at a point B outside the sphere is E. If another metal sphere of radius 3R and having a charge –3q is placed with its centre at point B, find out the resultant electric field at a point mid way between A and B.
[Ans: 10 E]

2. An electron with a velocity of $2.4 \times 10^6 ms^{-1}$ flies into a uniform electric field of intensity 135 Vm^{-1}. It moves along a field line until it comes to rest. What is the distance travelled by the electron before coming to rest within the field.
[Ans: 0.12 m]

3. Two similar metal spheres are suspended by silk threads from the same point. When the spheres are given equal charges of 2μC, the distance between them becomes 6cm. If length of each thread is 5 cm, find the mass of any one sphere ($g = 10 m/s^2$)
[Ans: $\frac{4}{3}$ kg]

4. A body of mass 2 gm is projected horizontally from the top of a tower of height 20m with a velocity 10 m/s. The charge on the body is 2C. Electric field is applied vertically downwards and of intensity 10^{-2} N/C. Find the time taken by the body to touch the ground ($g = 10 m/s^2$).
[Ans: 1.414 sec]

5. A body of mass 10 gm and having charge 2C is attached to a spring which is suspended from the ceiling. It vibrates with a time period 1 sec. If an electric field of intensity 100 N/C is now applied in the downward direction, find the time period.
[Ans: 1 sec]

6. Four charges Q, q, Q and q are placed at the corners A, B, C and D of a square ABCD. If the resultant electric force on the charge at the corner C is zero, find the value of Q/q.
[Ans: $-2\sqrt{2}$]

7. A charge Q is fixed on the X–Y plane at point (0, a). Find the electric field strength component along the X–axis, at any point (x, 0).
[Ans: $\frac{1}{4\pi \epsilon_0} \frac{Qx}{(a^2+x^2)^{3/2}}$]

8. Two charges 4q and q are fixed at points (0, 9) and (12, 0) respectively on the X–Y plane. Find the coordinates of the point where the electric field strength is zero. **[Ans: (8, 3)]**

9. Four point charges q, q, –q and –q are held fixed at the corners of a square ABCD with diagonals of length $2l$. Determine the field intensity at a point distant x from the plane of the square on its axis.

[Ans: $\frac{1}{4\pi\epsilon_0}\frac{2\sqrt{2}q\ell}{(\ell^2+x^2)^{3/2}}$ **]**

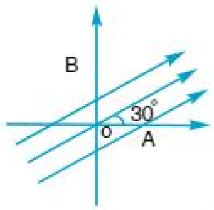
10. An arc of radius R and having a total charge Q has its charge uniformly distributed. The arc subtends an angle 2θ at the centre O of the corresponding circle. Find the electric field intensity at O.

[Ans: $\frac{1}{2\pi\epsilon_0}\left(\frac{Q}{R^2\theta}\right)\sin\theta/2$ **]**

11. Two point charges 4μC and 9μC are separated by 50 cm. Find the potential at the point between them where the field is zero. **[Ans:** 4.5×10^5 **V]**

12. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20V$. (i.e., plate 2 is at a higher potential). The plates are separated by d = 0.1 m and can be treated infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2? **[Ans:** 2.65×10^6 **m/s]**

13. A field of $100Vm^{-1}$ is directed at 30° to positive x – axis. Find V_{BA} if OA = 2m and OB = 4m



[Ans: $100\left(\sqrt{3}-2\right)$ **V]**

14. A hollow sphere of radius 2R is charged to V volts and another smaller sphere of radius R is charged to V/2 volts. Now the smaller sphere is placed inside the bigger sphere without changing the net charge on each sphere. Find the potential difference between the two spheres.

[Ans: $\frac{V}{4}$ **]**

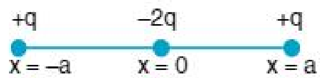
15. Two charges, Q each, are at a distance d apart. They are released. What is the velocity of each charged body of mass m when the distance between them is 2d.

[Ans: $v=\frac{Q}{\sqrt{8\pi\epsilon_0 dm}}$ **]**

16. Two particles of mass m and 2m carry a charge +q each. Initially the heavier particle is at rest on a smooth horizontal plane and the other is projected along the plane directly towards the first from a distance d with speed u. Find their closest distance of approach.

[Ans: $\frac{3q^2d}{3q^2+4\pi\epsilon_0 mu^2d}$ **]**

17. Two dipoles that are back to back form a linear quadrapole



i) Calculate E_x for points on the X-axis such that x >>a

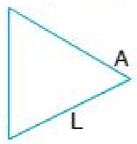
ii) Calculate E_y for points on the Y-axis such that y>>a

[Ans: $\frac{6a^2q}{4\pi\epsilon_0 x^4},\frac{3qa^2}{4\pi\epsilon_0 y^4}$ **]**

18. Two charged particles of charge –2q and +q have masses m and 2m respectively. They are kept in uniform electric field and allowed to move for the same time, find the ratio of their kinetic energies.

[Ans: 8:1]

19. A cone made of insulating material has a total charge Q spread uniformly over its sloping surface. Calculate the energy required to bring up a small test charge q from infinity to the apex A of the cone. The cone has a slope length L



[Ans: $\frac{Qq}{2\pi\epsilon_0 L}$ **]**

20. Electric potential V in space as a function of cartesian co-ordinates is given by $V=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$. Then find electric field intensity at (1, 1, 1)

[Ans: $(\hat{i}+\hat{j}+\hat{k})$ **]**

21. A solid dielectric (K = 1) sphere of radius R is charged uniformly by a total charge Q. At what distance from the centre will the electrostatic potential, be the average of that at the centre and at the surface.

[Ans: 0.707R]

22. A uniform rod of length l and mass m is given a charge Q and is suspended vertically by means of a hinge at the top end. A horizontal electric field E is switched on, in the direction in which the rod can sway freely. Find the angle made by the rod with the vertical in equilibrium.

[Ans: $\tan^{-1}\left(\frac{QE}{mg}\right)$]

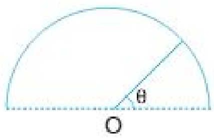
23. A uniformly charged wire, carrying charge q, is laid in the form of a semicircle of radius R. Find the electric field generated by the semicircle at its centre.

[Ans: $E = \frac{Q}{2\pi^2 \epsilon_0 R^2}$]

24. A total charge Q is distributed uniformly along a metallic ring of radius R. Two small elemental lengths each dx are removed at an angular separation 2θ, relative to the centre O. Find the electric field intensity at O.

[Ans: $\frac{1}{4\pi \epsilon_0}\left[\frac{Qdx\cos\theta}{\pi R^3}\right]$]

25. Figure shows a semicircular ring of radius R with linear charge distribution λ given by $\lambda = \lambda_0 \sin\theta$. Find the electric field intensity at the centre O of the ring.



[Ans: $\frac{\lambda_0}{8 \epsilon_0 R}$]

26. A hollow copper sphere is placed in front of a point charge Q such that the latter is at a distance r from the centre O of the former. What is the electric field at the centre O, due to the charges induced in the sphere?

[Ans: $\frac{1}{4\pi \epsilon_0}\frac{Q}{r^2}$]

27. If two electric charges q and –2q are placed at distance 6a apart, find the locus of points in the plane of the charges, where the field potential is zero taking q as origin and the line joining the charges as x-axis.

[Ans: $x^2 + y^2 + 4ax - 12a^2 = 0$]

28. In a region the electric potential is given by V = 2x + 2y –3z. Obtain the expression for electric field 0.

[Ans: $(-2\hat{i} - 2\hat{j} + 3\hat{k})$]

29. What will be the change in electric potential energy of a positive test charge q_0 when it is displaced in a uniform electric field $E = \bar{E}_0 j$ from $y_i = a$ to $y_f = 2a$ along the y-axis?

[Ans: $-q_0E_0a$]

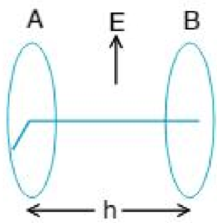
30. Two protons are at a distance of 0.53×10^{-10} m. Calculate the potential energy of the system in eV.

[Ans: 27.2 eV.]

31. The electric potential (V) in a certain region of space depends only on x–coordinate of point as $V = -\alpha x^3 + \beta$ (α and β constants). Find the volume charge density (ρ) of this region of space

[Ans: $6\alpha \epsilon_o x$]

32.



Two circular rings A and B each of radius 30 cm are placed coaxially with their axes horizontal in a uniform electric field $E = 10^5\ NC^{-1}$ directed vertically up. Distance between the centres of these rings is h = 40 cm. Ring A has a positive charge $q_1 = 10\mu C$, while ring B has a negative charge of magnitude $20\mu C$. A particle of mass m = 100 g and carrying a positive charge $10\mu C$ is released from rest at t = 0 at the centre of the ring A. Find the velocity of the particle when it has moved through 40 cm? ($g=10ms^{-2}$)

[Ans: $6\sqrt{2}\ ms^{-1}$]