3. INTEGRAL AS A LIMIT OF SUMMATION

SYNOPSIS

Important points to remember :

1) $\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=0}^{n-1} f\left(\frac{r}{n}\right)=\int_0^1 f(x)dx$

2) $\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)=\int_0^1 f(x)dx$

3) $\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=0}^{a(n-1)} f\left(\frac{r}{n}\right)=\int_0^a f(x)dx$

4) $\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{an} f\left(\frac{r}{n}\right)=\int_0^a f(x)dx$

LECTURE SHEET

EXERCISE

1. $\underset{n\to\infty}{Lt}\left[\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}.....+\frac{1}{nb}\right]=$

1) $\log\left(\frac{b}{a}\right)$ 2) $\log\left(\frac{a}{b}\right)$ 3) $\log a$ 4) $\log b$

2. $\underset{n\to\infty}{Lt}\left\{\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+.....+\frac{1}{n}\right\}=$

1) $\frac{\pi}{4}+\frac{1}{4}\log 2$ 2) $\frac{\pi}{2}+\frac{1}{4}\log 2$ 3) $\frac{\pi}{2}+\frac{1}{2}\log 2$ 4) $\frac{\pi}{4}+\frac{1}{2}\log 2$

3. $\underset{n\to\infty}{Lt}\frac{1}{n}\left[Sin^2\frac{\pi}{2n}+Sin^2\frac{2\pi}{2n}+.....+Sin^2\frac{n\pi}{2n}\right]=$

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{2}{3}$ 4) $\frac{3}{2}$

4. $\underset{n\to\infty}{Lt}\left(\frac{\sqrt{1}+2\sqrt{2}+3\sqrt{3}+.....+n\sqrt{n}}{n^{5/2}}\right)=$

1) 1 2) $\frac{5}{2}$ 3) 0 4) $\frac{2}{5}$

5. $\underset{n\to\infty}{Lt}\left[\frac{\sqrt{n^2-1^2}}{n^2}+\frac{\sqrt{n^2-2^2}}{n^2}+\frac{\sqrt{n^2-3^2}}{n^2}.+.....n\ terms\right]=$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

6. $\underset{n\to\infty}{Lt}\left(\frac{1}{\sqrt{2n-1^2}}+\frac{1}{\sqrt{4n-2^2}}+\frac{1}{\sqrt{6n-3^2}}+.....+\frac{1}{n}\right)=$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{3}$

7. $\underset{n\to\infty}{Lt}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+n}}+\frac{1}{\sqrt{n^2+2n}}+\frac{1}{\sqrt{n^2+3n}}+.....\frac{1}{\sqrt{n^2+n(n-1)}}\right\}=$

1) $2\sqrt{2}$ 2) 2 3) $2-2\sqrt{2}$ 4) $2\sqrt{2}-2$

8. If $S_n=\left\{\frac{1}{1+\sqrt{n}}+\frac{1}{2+\sqrt{2n}}+\frac{1}{3+\sqrt{3n}}+.....+\frac{1}{n+\sqrt{n^2}}\right\}$ then $\underset{n\to\infty}{Lt}S_n=$

1) 2log 2 2) log2 3) 3log 2 4) $\frac{1}{2}\log 2$

9. $\underset{n\to\infty}{Lt}\frac{3}{n}\left[1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+\sqrt{\frac{n}{n+9}}+....+\sqrt{\frac{n}{4n-3}}\right]=$

1) 1 2) 2 3) 3 4) 4

10. $\underset{n\to\infty}{Lt}\left(\frac{1}{n}+\frac{1}{n+1}+.....+\frac{1}{3n}\right)=$

1) log 2 2) log 3 3) log 4 4) log 5

11. $\underset{n\to\infty}{Lt}\frac{1}{n}\left(\frac{1}{n+1}+\frac{2}{n+2}.....+\frac{3}{4}\right)=$

1) 1– log 2 2) 2 – log 3 3) 3 + log 4 4) 3 – log 4

12. $\underset{n\to\infty}{Lt}\left[\frac{1}{n^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}+.....+\frac{n^2}{n^3}\right]=$

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{2}{3}$ 4) $\frac{3}{2}$

13. $\underset{n\to\infty}{Lt}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+.....+\frac{1}{n}\sec^2 1\right]=$

1) $\frac{1}{2}\sec 1$ 2) $\frac{1}{2}\operatorname{cosec}1$ 3) $\tan 1$ 4) $\frac{1}{2}\tan 1$

14. $\underset{n\to\infty}{Lt}\frac{\pi}{2n}\left(1+\cos\frac{\pi}{2n}+....+\cos\frac{(n-1)\pi}{2n}\right)=$

1) –1 2) 1 3) $-\frac{1}{2}$ 4) $\frac{1}{2}$

15. $\underset{n\to\infty}{Lt}\left[\frac{1}{\sqrt{n^2+1^2}}+\frac{1}{\sqrt{n^2+2^2}}+.....+\frac{1}{\sqrt{n^2+n^2}}\right]=$

1) $\log(\sqrt{2})$ 2) $\log(1+\sqrt{2})$ 3) $\sqrt{2}$ 4) $2\sqrt{2}$

16. $\underset{n\to\infty}{Lt}\left[\frac{1}{\sqrt{n^2-1^2}}+\frac{1}{\sqrt{n^2-2^2}}+.....+\frac{1}{\sqrt{2n-1}}\right]=$

1) π 2) 2π 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

17. $\underset{n\to\infty}{Lt}\left[\frac{\sqrt{n+1}+\sqrt{n+2}.....+\sqrt{n+n}}{n\sqrt{n}}\right]=$

1) $\frac{2(2\sqrt{2}-1)}{3}$ 2) $\frac{2\sqrt{2}-1}{3}$ 3) $\frac{2\sqrt{2}+1}{3}$ 4) $\frac{2(2\sqrt{2})+1}{3}$

18. $\underset{n\to\infty}{Lt}\left(\frac{1^4}{1^5+n^5}+\frac{2^4}{2^5+n^5}+\frac{3^4}{3^5+n^5}+--+\frac{n^4}{n^5+n^5}\right)=$

1) $\frac{1}{4}\log 2$ 2) $\frac{1}{3}\log 2$ 3) $\log 2$ 4) $\frac{1}{5}\log 2$

19. $\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\frac{1}{n}\sqrt{\frac{n+r}{n-r}}=$

1) $\frac{\pi}{2}$ 2) 2π 3) $\frac{\pi}{2}-1$ 4) $\frac{\pi}{2}+1$

20. $\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\frac{1}{\sqrt{4n^2-r^2}}=$

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

21. $\underset{n\to\infty}{Lt}\left\{\sum_{r=1}^{n}\frac{1}{n}e^{r/n}\right\}=$

1) $e+1$ 2) $e-1$ 3) $1-e$ 4) e

22. $\underset{n\to\infty}{Lt}\frac{1}{n}\left\{f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+.....+f(2)\right\}=$

1) $\int_0^1 f\left(\frac{1}{x}\right)dx$ 2) $\int_0^1 f(x)\,dx$ 3) $\int_0^1 f(2x)\,dx$ 4) $\int_0^2 f(x)\,dx$

23. $\underset{n\to\infty}{Lt}\left(\frac{1^m+2^m+3^m+.....+n^m}{n^{m+1}}\right)=$

1) $\frac{1}{m+1}$ 2) $\frac{1}{m+2}$ 3) $\frac{1}{m}$ 4) $\frac{1}{m+3}$

24. $\underset{n\to\infty}{Lt}\left(\frac{1+2^4+3^4+.....+n^4}{n^5}\right)-\underset{n\to\infty}{Lt}\left(\frac{1+2^3+3^3+.....+n^3}{n^5}\right)=$

1) 0 2) $\frac{1}{4}$ 3) $\frac{1}{5}$ 4) $\frac{1}{30}$

25. $\underset{n\to\infty}{Lt}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)....\left(1+\frac{n}{n}\right)\right]^{1/n}=$

1) $2e$ 2) $2/e$ 3) $4/e$ 4) $4e$

26. $\underset{n\to\infty}{Lt}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)....\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}=$

1) $\frac{2}{e^2}e^{\pi/2}$ 2) $\frac{e^2}{2}e^{\pi/2}$ 3) $\frac{2}{e^{\pi/2}}e^2$ 4) $\frac{e^{\pi/2}}{2e^2}$

27. $\underset{n\to\infty}{Lt}\left(\frac{(2n)!}{n!n^n}\right)^{\frac{1}{n}}=$

1) $\frac{2}{e}$ 2) $\frac{3}{e}$ 3) $\frac{4}{e}$ 4) $\frac{1}{e}$

28. $\underset{n\to\infty}{Lt}\left(\frac{n!}{n^n}\right)^{\frac{1}{n}}=$

1) e 2) $2e$ 3) $\frac{1}{2e}$ 4) $\frac{1}{e}$

KEY SHEET (LECTURE SHEET)

EXERCISE

1) **1**	2) **4**	3) **1**	4) **4**	5) **1**	6) **2**	7) **4**	8) **1**	9) **2**	10) **2**
11) **4**	12) **2**	13) **4**	14) **2**	15) **2**	16) **3**	17) **1**	18) **4**	19) **4**	20) **4**
21) **2**	22) **4**	23) **1**	24) **3**	25) **3**	26) **1**	27) **3**	28) **4**		