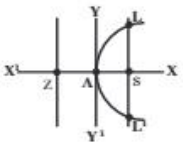
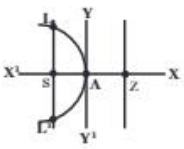
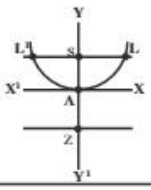
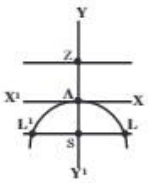
3. PARABOLA

SYNOPSIS

1. Let S be a given fixed point (focus) and let l be given fixed line (Directrix). Let SP and PM be the distances of a variable point P to the focus and directrix respectively and P moves such that $\frac{SP}{PM}=e$ (constant > 0) (eccentricity) then locus of P is called a conic or conic section.
2. If $e = 1$, the conic is called a parabola.
   If $e < 1$, the conic is called an ellipse.
   If $e > 1$, the conic is called a hyperbola.
3. The general equation of a conic is
   $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ (Second degree equation in x and y)
   i) If $\Delta \neq 0$, $h^2=ab$ then $S=0$ represents a parabola.
   ii) If $\Delta \neq 0$, $h^2<ab$ then S=0 represents an ellipse.
   iii) If $\Delta \neq 0$, $h^2>ab$ then $S=0$ represents a hyperbola.
   iv) If $\Delta \neq 0$, $h^2>ab$, a+b=0 then $S=0$ represents a rectangular hyperbola.
4. **Four standard forms of the parabola :($a > 0$)**

S.No.	Content	I	II	III	IV
	Equation	$y^2=4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
	Figure				
1.	Vertex (A)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
2.	Focus (S)	(a, 0)	(-a, 0)	(0, a)	(0,-a)
3.	Point of intersection of Axis and directrix (Z)	(-a, 0)	(a, 0)	(0, -a)	(0, a)
4.	End points of latus rectum (L, L^1)	(a, $\pm$2a)	(-a, $\pm$2a)	($\pm$2a, a)	($\pm$2a, -a)
5.	Eqn. of axis	y = 0	y = 0	x = 0	x = 0
6.	Eqn. of Directrix	x = -a	x = a	y = -a	y = a
7.	Eqn. of tangent at Vertex	x = 0	x = 0	y = 0	y = 0
8.	Eqn. of latus rectum	x = a	x = -a	y = a	y = -a
9.	Length of latus rectum (LL^1)	4a	4a	4a	4a
10.	Distance from Focus to Directrix(SZ)	2a	2a	2a	2a
11.	SA=AZ	a	a	a	a

5. i) If the axis of a parabola is parallel to x-axis, eqn. of the parabola will be of the form $(y-\beta)^2 = 4a(x-\alpha)$ (or) $(y-\beta)^2 = -4a(x-\alpha)$ (or) $x = ay^2 + by + c$.

   ii) If the axis of the parabola is parallel to y-axis, eqn. of the parabola will be of the form $(x-\alpha)^2 = 4a(y-\beta)$ (or) $(x-\alpha)^2 = -4a(y-\beta)$ (or) $y = ax^2 + bx + c$.

6. In the equation of the parabola $(y-\beta)^2=4a(x-\alpha)$

   i) Vertex = (α,β) ii) Focus = $(\alpha+a,\beta)$

   iii) Ends of latusrectum = $(\alpha+a,\beta\pm 2a)$ iv) Equation of axis is $y = \beta$

   v) Equation of directrix is $x = \alpha-a$ vi) Equation of latusrectum is $x = \alpha+a$

   vii) Length of latusrectum = $4a$

7. In the equation of the parabola $(x-\alpha)^2=4a(y-\beta)$

   i) Vertex = (α,β) ii) Focus = $(\alpha, \beta+a)$

   iii) Ends of latusrectum = $(\alpha\pm 2a,\ \beta+a)$ iv) Equation of axis is $x = \alpha$

   v) Equation of directrix is $y = \beta-a$ vi) Equation of latusrectum is $y = \beta+a$

   vii) Length of latusrectum = $4a$

8. The focal distance of the point $P(x_1, y_1)$ on the parabola

   i) $y^2 = 4ax$ is SP = $|x_1+a|$ ii) $x^2 = 4ay$ is SP = $|y_1+a|$

9. A chord of the parabola perpendicular to its axis is called **double ordinate** of the parabola.

10. A chord of the parabola which is passing through focus is called focal chord

11. The focal chord of the parabola which is perpendicular to axis is called latus rectum

12. Equation of Tangent to $y^2= 4ax$ at (x_1, y_1) is $S_1=0$.

13. The condition that the line $y = mx + c$ may be a tangent to the parabola $y^2 = 4ax$ is $c = \frac{a}{m}$ and the point of contact is $\left(\frac{a}{m^2},\frac{2a}{m}\right)$.

14. The condition for the line $lx+my+n=0$ to be a tangent to parabola $y^2= 4ax$ is $am^2 = ln$ and the point of contact is $\left(\frac{n}{l},-\frac{2am}{l}\right)$

15. i) The condition that the line $y = mx + c$ may be a tangent to $x^2 = 4ay$ is $c = -am^2$ and point of contact is $(2am, am^2)$

    ii) The condition that the line $lx+my+n=0$ may be a tangent to the parabola $x^2 = 4ay$ is $al^2 = mn$ and the point of contact is $\left(\frac{-2al}{m},\frac{n}{m}\right)$.

16. The equation of Tangent to $y^2 = 4ax$ in slope form is $y = mx + \frac{a}{m}$ (or) $m^2x - my + a = 0$.

17. If m_1 and m_2 are the slopes of tangents from an external point (x_1,y_1) to the parabola $y^2= 4ax$ then they are the roots of $m^2x_1-my_1+a = 0$ and hence $m_1+ m_2 = \frac{y_1}{x_1}$ and $m_1m_2 = \frac{a}{x_1}$

18. If θ is the acute angle between tangents drawn from (x_1, y_1) to parabola $S\equiv y^2-4ax = 0$ then Tan $\theta=\frac{\sqrt{S_{11}}}{|x_1+a|}$.

19. Equation of pair of tangents drawn from an external point (x_1, y_1) to parabola $y^2 = 4ax$ is $S_1^2 = S.S_{11}$

Parametric form - Tagents - normalchord of contact - no.of normals

20. Equation of normal to $y^2 = 4ax$ in slope form is $y = mx - 2am - am^3$ where m is slope of normal.
21. Equation of chord of contact of (x_1, y_1) to $y^2 = 4ax$ is $S_1 = 0$.
22. If (x_1, y_1) is midpoint of chord of $y^2 = 4ax$ then equation of chord is $S_1 = S_{11}$
23. For any curve $y = f(x)$, the slope of chord having (x_1, y_1) as middle point is $\left(\frac{dy}{dx}\right)_{at\ (x_1, y_1)}$.

Parametric form, Tangent normal :

24. For all values of t, the point $(at^2, 2at)$ lies on parabola $y^2 = 4ax$ and it is denoted by 't'.
    $\therefore$ equations $x = at^2, y = 2at$ are called parametric equations of $y^2 = 4ax$. ($x = 2at, y = at^2$ for $x^2 = 4ay$)
25. The focal distance of a point $P(at^2, 2at)$ on the parabola $y^2 = 4ax$ is $|at^2 + a|$.
26. If $(at^2, 2at)$ is one end of the double ordinate of $y^2 = 4ax$ then its length = $4at$.
27. Equation of chord joining t_1 and t_2 on parabola $y^2 = 4ax$ is $y(t_1 + t_2) = 2x + 2at_1t_2$
28. If t_1, t_2 are the ends of the focal chord of $y^2 = 4ax$, then $t_1t_2 = -1$
29. If $(at^2, 2at)$ is one end of focal chord of the parabola $y^2 = 4ax$ then its other end is $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$.
30. The point of intersection of tangents at t_1 and t_2 on $y^2 = 4ax$ is $(at_1t_2, a(t_1 + t_2))$.
31. Slope of the tangent at 't' on $y^2 = 4ax$ is $\frac{1}{t}$.
32. Equation of tangent at 't' to $y^2 = 4ax$ is $yt = x + at^2$.
33. Slope of the normal at 't' on $y^2 = 4ax$ is $-t$.
34. Equation of normal at 't' on $y^2 = 4ax$ is $y + xt = 2at + at^3$. Since this is a cubic equation in t, it has 3 roots in which at least one of them is real. Therefore from a given point, we can drawn atmost three normals to a parabola.

Results on Normal :

35. If t_1, t_2, t_3 are the feet of the normals drawn from a point (x_1, y_1) to the parabola $y^2 = 4ax$ then they are the roots of $at^3 + (2a - x_1)t - y_1 = 0$ and hence $t_1 + t_2 + t_3 = 0$, $t_1t_2 + t_2t_3 + t_3t_1 = \frac{2a - x_1}{a}$, $t_1t_2t_3 = \frac{y_1}{a}$.
36. The condition that the line $lx + my + n = 0$ to be a normal to parabola $y^2 = 4ax$ is $al^3 + 2alm^2 + m^2n = 0$.
37. If the normal at 't_1' on $y^2 = 4ax$ meets it again at 't_2' then $t_2 = -t_1 - \frac{2}{t_1}$
38. If the normals at t_1 and t_2 on the parabola $y^2 = 4ax$ meet again on parabola at t_3 then $t_1t_2 = 2$ and $t_1 + t_2 + t_3 = 0$.
39. If the normal chord at 't' on $y^2 = 4ax$ subtends a right angle at the vertex then $t^2 = 2$.
40. If the normal chord at 't' on $y^2 = 4ax$ subtends a right angle at the focus then $t^2 = 4$.
41. The angle between the normals drawn at the ends of latusrectum of the parabola $y^2 = 4ax$ is 90^0 and the point of intersection of these normals is $(3a, 0)$.

Criteron for the number of normal :

42. Write $H = \frac{2a - x_1}{3a}$, $G = -\frac{y_1}{a}$ and $\Delta = G^2 + 4H^3$ if $x_1 = 2a$ and $y = 0$ then the number of normals = 1.
    Assume either $x_1 \neq 2a$ or $y_1 \neq 0$

i) If $\Delta > 0$ then the number of normals is 1 ii) If $\Delta = 0$ then the number of normals is 2

iii) If $\Delta < 0$ then the number of normals is 3

Normal :

43. The point of intersection of normals drawn at t_1 and t_2 to $y^2 = 4ax$ is $[2a + a\{(t_1 + t_2)^2 - t_1t_2\}, -at_1t_2(t_1 + t_2)]$

Miscellaneous :

44. If (x_1, y_1), (x_2, y_2) are the extremities of a focal chord of the parabola $y^2 = 4ax$ then (1) $x_1x_2 = a^2$, (2) $y_1y_2 = -4a^2$.
45. The orthocentre of the triangle formed by the tangents at t_1, t_2, t_3 to the parabola $y^2 = 4ax$ is $[-a, a(t_1 + t_2 + t_3 + t_1t_2t_3)]$.
46. Orthocentre of the triangle formed by any three tangents to the parabola lies on the directrix of the parabola.

Miscellaneous problems on focal chords, tangent, normals chord with mid point - reflection property

Results on Tangent :

47. The tangent at end of focal chord of parabola is parallel to normal at the other end.
48. Equation of common tangent to two parabolas $y^2 = 4ax$ and $x^2 = 4by$ is $a^{1/3}x + b^{1/3}y + (ab)^{2/3} = 0$.
49. For the parabola $y^2 = 4a(x+a)$, equation of tangent in slope form is $y = m(x+a) + \frac{a}{m}$.
50. The Tangents and normals at the ends of latusrectum of parabola $y^2 = 4ax$ form a square whose side is $2\sqrt{2}\,a$ and area is $8a^2$
51. Locus of the point of the intersection of perpendicular tangents drawn to a parabola is its directrix.
52. The angle between the tangents drawn from a point on the directrix to the parabola is 90^0.
53. Tangents drawn at the ends of focal chord of a parabola are at right angles, they intersect on directrix.
54. The angle between the tangents drawn at the ends of latusrectum of the parabola $y^2 = 4ax$ is 90^0 and the point of intersection of these tangents is $(-a, 0)$.

Length of Chord :

55. If $P(x_1, y_1)$, $Q(x_2, y_2)$ are the ends of chord of a curve then its length $PQ = |x_1 - x_2|\sqrt{1 + m^2}$ where m is slope of $\overline{PQ}$.
56. If t_1 and t_2 are ends of chord of $y^2 = 4ax$ then its length $= a|t_1 - t_2|\sqrt{(t_1 + t_2)^2 + 4}$
57. The length of focal chord drawn at a point 't' on the parabola $y^2 = 4ax$ is $a\left(t + \frac{1}{t}\right)^2$
58. If a focal chord of a parabola $y^2 = 4ax$ makes an angle θ with its axis then its length $= 4a\,\mathrm{cosec}^2\theta$.
59. length of normal chord at t on $y^2 = 4ax$ is $\frac{4a(1 + t^2)^{3/2}}{t^2}$
60. Length of chord of contact of (x_1, y_1) w.r.t. to $y^2 = 4ax$ is $\frac{1}{a}\sqrt{S_{11}({y_1}^2 + 4a^2)}$
61. Length of the chord of $y^2 = 4ax$ having (x_1, y_1) as its mid point is $\frac{1}{a}\sqrt{(-S_{11})({y_1}^2 + 4a^2)}$

Area of the tangent :

62. Area of triangle formed by two tangents from (x_1, y_1) and its chord of contact is $\frac{(S_{11})^{3/2}}{2a}$

63. Area of triangle inscribed in parabola $y^2 = 4ax$ is $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ where y_1, y_2 and y_3 are ordinates of angular points

64. Area of the triangle formed by the tangents at three points whose ordinates are y_1, y_2, y_3 on $y^2 = 4ax$ is $\frac{1}{16a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$

Properties of Parabola :

65. The tangents at the extremities of a focal chord of parabola intersect at right angles on the directrix.

66. The tangent at any point on the parabola bisects the angle between the focal distance of the point and the paerpendicular on the directrix from the point.

67. If tangents at P and Q meet at T, then

    i) TP and TQ will subtends equal angle at focus S.

    ii) $ST^2 = SP.SQ$

    iii) The triangles STP and STQ are similar

68. If N is foot of perpendicular from focus S on tangent at point P to the parabola then N lies on tangent at vertex and $SN^2 = SA.SP$ (where A is vertex).

69. Circle drawn on focal chord as diameter touches the directrix.

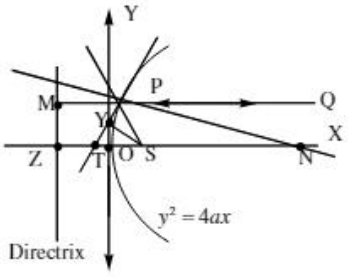
70. If the tangent at any point P of a parabola intersects the axis of parabola at any point T and M is the foot of perpendicular from point P on directrix and S is focus of the parabola, then the euadrilateral $SPMT$ is rhombus.

71. Reflection property of Parabola

    Tangent PT and normal PN are internal and external bisector of angle $\angle SPM$

    i.e., $\angle QPN = \angle SPN$ and $\angle SPT = \angle MPT$

    i.e., any ray of light coming parallel ot the axis of parabola then after reflection through parabola it passes through focus of hte parabola and conversely. i.e., ray after passing through focus becomes parallel to axis of parabola after reflection through parabola.



72. Orthocentre of any triangle formed by three tangents to a parabola lies on directrix.

73. It tangents at point Q and R intersect at P. If P_1, P_2, P_3 are length of perpendiculars from P, Q and R respectively on any tangents to parabola, then P_2, P_1, P_3 are in GP.

74. The tangent at one extremity of focal chord of parabola is parallel to normal at other extremity of vocal chord.

75. Circle drawn on focal radius as diameter touches tangent at vertex
76. Circle on any focal radii of point $P(t)$ of parabola $y^2 = 4ax$ interecpts a chord of length on normal at point P.
77. Circumcircle of triangle formed by three tangents to the parabola always passess through focus of parabola.

Common Tangents :

78. Equation of common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is $xa^{\frac{1}{3}} + yb^{\frac{1}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$.
79. The maximum number of common normals to $y^2 = 4ax$ and $x^2 = 4by$ may be 5 only.
80. The circle drawn on focal chord of a parabola as diameter touches the directrix.
81. The circle drawn on focal radius of a parabola as diameter touches the tangent at the vertex.
82. The length of sub tangent at any point $P(x_1, y_1)$ on $y^2 = 4ax$ is $2x_1$.
83. The length of sub normal at any point on a parabola is constant and is equal to semilatusrectum.

Some useful results on tangents :

84. The point of intersection of the tangents at t_1, t_2 on the parabola $y^2 = 4ax$ is $[at_1t_2, a(t_1 + t_2)]$.
85. If $l_1$and l_2 are the length segments of a focal chord of a parabola then its latus rectum is equal to $\frac{4l_1l_2}{l_1+l_2}$
86. The circum circle of the triangle formed by any three tangents to a parabola passes through the focus of the parabola.
87. P is a point on the parabola whose focus is S and PN is the perpendicular drawn from P to the directrix, then the tangent at P is the internal bisector of P and normal is the external bisector of P.

LECTURE SHEET

EXERCISE-I

General equation - varriours standard form - length of chord - tangent

LEVEL-I (MAIN)

1. The ends of latusrectum of a parabola are (–3, 1) and (1, 1) then equation of parabola is
   1) $(x + 1)^2 = 4y$ 2) $(x - 1)^2 = 4y$ 3) $(x + 1)^2 = 2y$ 4) $(x - 1)^2 = 2y$
2. The locus of the vertices of the family of the parabola's $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is
   1) $xy = \frac{35}{16}$ 2) $xy = \frac{64}{105}$ 3) $xy = \frac{105}{64}$ 4) $xy = \frac{3}{4}$
3. The curve described parametrically by $x = t^2+t+1$ and $y = t^2 - t + 1$ represents
   1) hyperbola 2) ellipse 3) parabola 4) rectangular hyperbola
4. Vertex is (4, 3) and diretctrix is $3x + 2y - 7 = 0$ then equation of latusrectum is
   1) $3x+2y-18 = 0$ 2) $3x+2y-29 = 0$ 3) $3x+2y-8 = 0$ 4) $3x+2y-31 = 0$
5. The parabola $(y+1)^2 = a(x-2)$ passes through the point $(1,-2)$ then the equation of its directrix is
   1) $4x + 1 = 0$ 2) $4x - 1 = 0$ 3) $4x + 9 = 0$ 4) $4x - 9 = 0$
6. If L and L^1 are ends of latusrectum of the parabola $9y^2 = 4x$ then the combined equation of OL and OL^1 is
   1) $4x^2 = 9y^2$ 2) $x^2 = 9y^2$ 3) $x^2 = 4y^2$ 4) $y^2 = 4x^2$

Double ordinates :

7. If an equilateral Δ is inscribed in a parabola $y^2 = 12x$ with one of the vertex being at the vertex of the parabola then its height is

1) $24\sqrt{3}$ 2) $16\sqrt{3}$ 3) 36 4) 24

8. If PQ is double ordinate of the parabola $y^2 = 4ax$ then locus of its point of trisection is

1) $9y^2 = 4ax$ 2) $9y^2 = 16ax$ 3) $3y^2 = 8ax$ 4) $9y^2 = 8ax$

9. An arch is in the shape of a parabola whose axis is vertically downwords and measures 80 mts across its bottom on the ground. Its highest point is 24 mts. The measure of the horizontal beam across its cross section at a height of 18 mts is

1) 50 2) 40 3) 45 4) 60

Tangents :

10. The line $x + y = k$ touches the parabola $y = x - x^2$, if $k =$

1) 0 2) –1 3) 1 4) 2

11. Equation of common tangent to $x^2 + y^2 = 2a^2$ and $y^2 = 8ax$ is

1) $y = \pm (x + a)$ 2) $y = \pm(x + 2a)$ 3) $y = \pm (x - 2a)$ 4) $y = \pm (x - a)$

12. Equation of line touching both parabolas $y^2 = 4x$ and $x^2 = -32y$ is

1) $x + 2y + 4 = 0$ 2) $2x + y - 4 = 0$ 3) $x - 2y - 4 = 0$ 4) $x - 2y + 4 = 0$

13. The points of contact of the tangents drawn from the point (4, 6) to the parabola $y^2 = 8x$

1) (2, 4), (18, 12) 2) (2, 4), (8, 8) 3) (8, 8) (18, 12) 4) (0, 0) $(1, 2\sqrt{2})$

14. Two straight lines are perpendicular to each other. One of them touches the parabola $y^2 = 4a(x+a)$ and the other touches $y^2 = 4b(x+b)$. Then the locus of point of intersection of two lines is

1) $x + a = 0$ 2) $x + b = 0$ 3) $x + a + b = 0$ 4) $x - a - b = 0$

15. The locus of foot of perpendicular from the focus upon any tangent to the parabola $y^2 = 4ax$ is

1) directrix 2) Tangent at vertex 3) $x = a$ 4) $y = a$

16. The locus of point of intersection of tangents drawn at the ends of chord of $y^2 = 4ax$ which subtends a right angle at vertex is

1) $x + a = 0$ 2) $x + 2a = 0$ 3) $x + 4a = 0$ 4) $y + a = 0$

17. Let 'O' be the origin and A be a point on the curve $y^2 = 4x$ then locus of the midpoint of OA is

1) $x^2 = 4y$ 2) $x^2 = 2y$ 3) $y^2 = 16x$ 4) $y^2 = 2x$

18. The locus of midpoint of focal chords of the parabola $y^2 = 4ax$ is

1) $y^2 = 2a(x+a)$ 2) $y^2 = 2a(x-a)$ 3) $y^2 = a(2x+a)$ 4) $y^2 = a(2x-a)$

Length of chord :

19. The length of the chord of the parabola $x^2 = 4ay$ passing through the vertex and having slope Tanα is

1) 4a Cosecα cotα 2) 4a tanα secα 3) 4a Cosα cotα 4) 4a sinα tanα

20. Length of chord of parabola $y^2 = 4ax$ whose equation is $y - \sqrt{2}x + 4\sqrt{2}a = 0$

1) $2\sqrt{11}\,a$ 2) $4\sqrt{2}\,a$ 3) $8\sqrt{2}\,a$ 4) $6\sqrt{3}\,a$

Numerical value type questions

21. The points of contact Q and R of tangents from the point $P(2, 3)$ on the parabola $y^2 = 4x$ is (α,β) and (γ,δ) then $\frac{\alpha+\beta+\gamma+\delta}{4} =$

22. If all chords of the parabola $y^2 = 4x + 4$ which subtends a right angle at point $(1,2\sqrt{2})$ passes through a fixed point (α,β), then the value of $|\alpha+\beta|$ is equal to _____

23. A line is drawn form $A(-2, 0)$ to intersect the curve $y^2 = 4x$ in P and Q in the first quadrant such that $\frac{1}{AP}+\frac{1}{AQ}<\frac{1}{4}$, then slope of the line is always greater than

24. If $y = mx+c$ is the normal at a point on the parabola $y^2=8x$ whose focal distance is 84 nits then c is _____

LEVEL-II (ADVANCED)

Single answer type questions

1. A parabola of latus-rectum l touches a fixed equal parabola. The axes of two parabolas are parallel. Then the locus of the vertex of the moving parabola is

   a) a parabola whose latus-rectum is $2l$.
   b) an ellipse
   c) a hyperbola
   d) a circle whose radius is $2l$

2. The point $(-2m, m + 1)$ is an interior point of the smaller region bounded by the circle $x^2 + y^2 = 4$ and the parabola $y^2 = 4x$. Then m belongs to the interval

   a) $-5-2\sqrt{6}<m<1$ b) $0<m<4$ c) $-1<m<\frac{3}{5}$ d) $-1<m<-5+2\sqrt{6}$

3. If the tangent at P on $y^2 = 4ax$ meets the tangent at the vertex in Q and S is the focus of the parabola, then $\angle SQP$ is equal to :

   a) $\pi/3$ b) $\pi/4$ c) $\pi/2$ d) $2\pi/3$

4. A movable parabola touches the x-axis and the y-axis at (1,0) and (0,1). Then the locus of the focus of the parabola is

   a) $2x^2-2x+2y^2-2y+1=0$
   b) $x^2-2x+2y^2-2y+1=0$
   c) $2x^2-2x+2y^2+2y+2=0$
   d) $2x^2+2x-2y^2-2y-2=0$

5. The number of points on the curve $y=\left|\left|1-e^x\right|-2\right|$ form which two perpendicular tangents can be drawn to the parabola $x^2 = -4y$ is equal to

   a) 2 b) 3 c) 4 d) 5

6. Consider two concentric circles C_1: $x^2 + y^2 - 1 = 0$ and C_2: $x^2 + y^2 - 4 = 0$. A parabola is drawn through the points where 'C_1' meets the x-axis and having arbitrary tangent of 'C_2' as its directrix. Then the locus of the focus of drawn parabola is

   a) $\frac{4}{3}x^2-y^2=3$ b) $\frac{3}{4}x^2-y^2=3$ c) $\frac{4}{3}x^2+y^2=3$ d) $\frac{3}{4}x^2+y^2=3$

7. The number of common chords of the parabolas $x = y^2 - 6y + 11$ and $y = x^2 - 6x + 11$ is

   a) 1 b) 2 c) 4 d) 6

8. A tangent of $y^2 = 12x$ makes an angle 60° with the line $y = 5x + 4$. then its equation.

a) $y=\left(\frac{5+\sqrt{3}}{1-5\sqrt{3}}\right)x+3\frac{(1-5\sqrt{3})}{5+\sqrt{3}}$ b) $y=\left(\frac{5-\sqrt{3}}{1-5\sqrt{3}}\right)x+3\frac{(1+5\sqrt{3})}{5-\sqrt{3}}$

c) $y=\left(\frac{5+\sqrt{3}}{1-5\sqrt{3}}\right)x+3\frac{(1-5\sqrt{3})}{5-\sqrt{3}}$ d) $y=\left(\frac{5-\sqrt{3}}{1+5\sqrt{3}}\right)x+3\frac{(1+5\sqrt{3})}{5-\sqrt{3}}$

9. CA, CB are tangents to a parabola and O is the orthocentre of the triangle ABC. Then the directrix bisects

a) OC b) OA c) OB d) AB

10. If two tangents drawn from a point P to the parabola $y^2 = 4x$ be such that the slope of one tangent is double the other then P lies on the curve

a) $9y = 2x^2$ b) $9x = 2y^2$ c) $2x = 9y^2$ d) $2x + 9y = 0$

11. Radius of the largest circle which passes through the focus of the parabola $y^2 = 4x$ and contained in it is

a) 8 b) 4 c) 2 d) 5

12. The equation $\sqrt{\{(x-3)^2+(y-1)^2\}}+\sqrt{\{(x+3)^2+(y-1)^2\}}=6$ represents

a) an ellipse b) a pair of straight lines

c) a circle d) a straight line joing the point (–3,1) to the point (3,1).

13. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola, then a point of intersection of the circle and the parabola is

a) $\left(\frac{p}{2},p\right)$ b) $\left(\frac{p}{2},-p\right)$ c) $\left(-\frac{p}{2},p\right)$ d) $\left(-\frac{p}{2},-p\right)$

14. In a parabola $y^2 = 4ax$ the angle θ that the latus rectum subtends at the vertex of the parabola is

a) dependent on the length of the latus rectum

b) independent of the latus rectum and lies between $\frac{5\pi}{6}$ and π

c) independent of the latus rectum and lies between $\frac{3\pi}{4}$ and $\frac{5\pi}{6}$

d) independent of the latus rectum and lies between $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$

15. All points on the curve $y^2 = 4a\ (x + a \sin x/a)$ at which the tangent is parallel to x–axis lies on

a) circle b) parabola c) an ellipse d) line

16. The set of values of m for which a chord of slope m of the circle $x^2 + y^2 = 16$, touches the parabola, $y^2 = 8x$

a) $(-\infty,-1)\cup(1,\infty)$ b) $(-\infty,\infty)$

c) $\left(-\infty,-\sqrt{\frac{\sqrt{2}-1}{2}}\right)\cup\left(\frac{\sqrt{\sqrt{2}-1}}{2},\infty\right)$ d) (–1, 1)

17. If a and c are the lengths of segments of any focal chord of the parabola $y^2 = 2bx (b > 0)$, then the roots of the equation $ax^2 + bx + c = 0$ are

a) real and distinct
b) real and equal
c) imaginary
d) none of these

18. The straight line joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is

a) $x^2 + 2y^2 - ax = 0$
b) $2x^2 + y^2 - 2ax = 0$
c) $2x^2 + 2y^2 - ay = 0$
d) $2x^2 + y^2 - 2ay = 0$

19. Angle between the tangents, drawn to the parabola $(x-3)^2 + (y+4)^2 = \frac{(3x-4y-6)^2}{25}$ at the extremities of chord $2x - 3y - 18 = 0$ is

a) $\frac{\pi}{3}$
b) $\frac{\pi}{4}$
c) $\frac{\pi}{6}$
d) $\frac{\pi}{2}$

More than one correct answer type questions

20. The equation of a parabola is $25[(x-2)^2 + (y+5)^2] = (3x + 4y - 1)^2$ For this parabola.

a) tangent at the vertex is $6x + 8y + 13 = 0$
b) focus = $(2, -5)$
c) directrix has the equation $3x + 4y - 1 = 0$
d) axis has the equation $4x - 3y - 23 = 0$

21. Let there be two parabolas with the same axis and focus of each being exterior to the other and the latus recta being $4a$ and $4b$. The locus of the middle points of the intercepts between the parabolas made on the lines parallel to the common axis is a

a) straight line if $a = b$ b) parabola if $a \neq b$
c) parabola for all a,b
d) none of these

22. If the parabola $y = (a-b)x^2 + (b-c)x + (c-a)$ touches the x–axis in the interval $(0, 1)$, then the line $ax + by + c = 0$

a) always passes through a fixed point
b) passes through $[-2, 1]$
c) always passes through origin
d) data insufficient

23. P is a point which moves in the x-y plane such that the point P is nearer to the centre of a square than any of the sides. The four vertices of the square are $(\pm a, \pm a)$. The region in which P will move is bounded by parts of parabolas of which one has the equation.

a) $y^2 = a^2 + 2ax$
b) $x^2 = a^2 + 2ay$
c) $y^2 + 2ax = a^2$
d) $x^2 + 2ay = a^2$

24. 

In the figure a parabola is drawn to pass through the vertices B, C and D of the square $ABCD$. If $A(2,1)$ $C(2, 3)$. Then

a) Vertex is $(2, 3)$
b) Focus is $\left(2, \frac{11}{4}\right)$
c) $B\ (3, 2)$
d) $D\ (1, 2)$

25. The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts the parabola $x^2 = 4ay$ at the points $A_i(x_i, y_i)$. i = 1,2,3,4 then

a) $\sum x_i = 0$
b) $\sum y_i = -4(f + 2a)$
c) $\pi(x_i) = 16a^2c$
d) $\pi(y_i) = c^2$

26. The angle between the tangents drawn from the origin to the parabola $y^2 = 4a(x-a)$ is

a) 90^0 b) $\sin^{-1} x + \cos^{-1} x \ \forall x \in R$ c) 45^0

d) The angle between the bisectors of the co–ordinate axes

27. The locus of a point whose sum of the distances from the origin and the line $x = 2$ is 4 units is

a) $y^2 = -12(x-3)$ b) $y^2 = 12(3-x)$ c) $y^2 = 4(x+1)$ d) $x^2 = -12(y-3)$

28. The equation of the directrix of the parabola with vertex at the origin and having the axis along the x-axis and a common tangent of slope 2 with the circle $x^2 + y^2 = 5$ is _____.

a) $x = 10$ b) $x = 20$ c) $x = -10$ d) $x = -20$

29. The equation of a parabola is $y^2 = 4x$. $P(1, 3)$ and $Q(1, 1)$ are two points in the xy–plane then for the parabola

a) P is an exterior point b) Q is an interior point

c) P,Q are interior points d) $P.Q$ are exterior points

30. Let P be a point whose coordinates differ by unity and the point does not lie on any of the axes of reference. If the parabola $y^2 = 4x+1$ passes through P, then the ordinate of P may be

a) 3 b) –1 c) 5 d) 1

31. The points of contact Q and R of a tangent from the point $P(2, 3)$ to the parabola $y^2 = 4x$ are

a) $(-1, -2)$ $(-4, -4)$ b) $(1, 2), (4, 4)$ c) $(1, 4), (2, 4)$ d) $(1, 2), (-1, -4)$

32. The line $x + y + 2 = 0$ is a tangent to a parabola at point A, intersect the directrix at B and tangent at vertex at C respectively. The focus of parabola is $S(2, 0)$. Then

a) CS is perpendicular to AB b) $AC.\ BC = CS^2$

c) $AC.BC = 8$ d) $AC = BC$

33. A circle is drawn having centre at $C(0, 2)$ and passing through focus (S) of the parabola $y^2 = 8x$, if radius (CS) intersects the parabola at point P, then

a) distance of oint P from directrix is $(8 - 4\sqrt{2})$

b) distance of point C from point P is $(6\sqrt{2} - 8)$

c) Angle subtended by intercept made by circle on directrix at its centre is $\pi/2$'

d) point P is the mid-point of C and S

Linked comprehension type questions

Passage - I :

$y^2 = 4x$ and $y^2 = -8(x - a)$ intersect at points A and C. Points $O(0, 0)$, A, $B(a, 0)$, C are concyclic.

34. The length of common chord of parabolas is

a) $2\sqrt{6}$ b) $4\sqrt{3}$ c) $6\sqrt{5}$ d) $8\sqrt{2}$

35. The area of cyclic quadrilateral $OABC$ is

a) $24\sqrt{3}$ b) $48\sqrt{2}$ c) $12\sqrt{6}$ d) $18\sqrt{5}$

36. Tangents to parabola $y^2 = 4x$ at A and C intersect at point D and tangents to parabola $y^2 = -8(x-a)$ intersect at point E, then the area of quadrilateral $DAEC$ is

a) $96\sqrt{2}$ b) $48\sqrt{3}$ c) $54\sqrt{5}$ d) $36\sqrt{6}$

Passage - II :

If l, m are variable real number such that $5l^2 + 6m^2 - 4lm + 3l = 0$, then variable line $lx + my = 1$ always touches a fixed parabola, whose axis is parallel to x–axis.

37. Vertex of the parabola is

a) $\left(-\frac{5}{3},\frac{4}{3}\right)$ b) $\left(-\frac{7}{4},\frac{3}{4}\right)$ c) $\left(\frac{5}{6},-\frac{7}{6}\right)$ d) $\left(\frac{1}{2},-\frac{3}{4}\right)$

38. Focus of the parabola is

a) $\left(\frac{1}{6},-\frac{7}{6}\right)$ b) $\left(\frac{1}{3},\frac{4}{3}\right)$ c) $\left(\frac{3}{2},-\frac{3}{2}\right)$ d) $\left(-\frac{3}{4},\frac{3}{4}\right)$

39. Directrix of the parabola is

a) $6x + 7 = 0$ b) $4x + 11 = 0$ c) $3x + 11 = 0$ d) $x + 1 = 0$

Passage - III :

A parabola whose focus is $S(3,4)$ is touching the coordinate axes

40. The equation of the circle whose diameter is the portion of tangent at vertex of the parabola between the coordinate axis is

a) $x^2+y^2-3x-4y=0$ b) $x^2+y^2+6x+8y=0$

c) $x^2+y^2-6x-8y=0$ d) $x^2+y^2+3x+4y=0$

41. The equation of axis of the parabola is

a) $4x-3y+7=0$ b) $3x-4y=0$ c) $4x-3y=0$ d) $3x-4y+7=0$

42. If P, Q are ends of focal chord of the parabola then $\frac{1}{SP}+\frac{1}{SQ}=$

a) $\frac{12}{5}$ b) $\frac{2}{5}$ c) $\frac{6}{5}$ d) $\frac{5}{6}$

Matrix matching type questions

43. The equation of conics represented by the general equation of second degree $ax^2+2hxy+by^2+2gx+2fy+c=0$ and the discriminant of above equation is represented by $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

i.e., $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ and

i) If $\Delta = 0$, it is a degenerate

ii) If $\Delta \neq 0$ it is non - degenerate

COLUMN - I	COLUMN - II
A) The conic represented by the equation $\sqrt{\left(\frac{x}{a}\right)}+\sqrt{\left(\frac{y}{b}\right)}=1$ is $(a\neq 0, b\neq 0)$	p) degenerate
B) The conic represented by the equation $3x^2 + 10xy + 3y^2 - 15x - 21y + 18 = 0$ is	q) non-degenerate
C) The conic represented by the equation $8x^2 - 4xy + 5y^2 - 16x - 4y + 17 = 0$ is	r) a parabola
D) $2xy + 2x + 4y + 4 = 0$ represent	s) a pair of intersecting straight lines

44. If $y = x + 1$ is axis of parabola, $y + x = 4$ is tangent of same parabola at its vertex and $y = 2x + 3$ is one of its tangent, then

COLUMN - I	COLUMN - II
A) If equation of directrix of parabola is $ax + by - 29 = 0$, then $a + b = 0$	p) 9
B) If length of latus rectum of parabola is $\frac{a\sqrt{2}}{b}$ where a and b are relatively prime natural numbers, then $a + b =$	q) 18
C) Let extremities of latus rectum are (a_1, b_1) and (a_2, b_3), then $[a_1 + b_1 + a_2 + b_2] =$ (where [.] denote greatest integer function)	r) 23
D) If equation of parabola is $a(x - y + 1)^2 = b(x + y - 4)$ where a and b are relatively prime natural numbers then $a + b =$	s) 37

45.

COLUMN - I	COLUMN - II
A) Radius of the largest circle which passes through the focus of the parabola $y^2 = 4x$ and contained in it, is	p) 16
B) Two perpendicular tangents PA and PB are drawn to the parabola $y^2 = 16x$ then length AB may be	q) 5
C) The shortest distance between parabolas $y^2 = 4x$ and $y^2 = 2x - 6$ is d then $d^2 =$	r) 8
D) The harmonic mean of the segments of a focal chord of the parabola $y^2 = 8x$	s) 4
	t) 20

Integer answer type questions

46. If a and b be the segments of a focal chord and $2c$ the latusrectum of a parabola and $\frac{a^3+b^3}{c^3} > k$ then $k =$

47. The length of chord of circle $x^2 + y^2 = 4$ and $y^2 = 4(x - h)$ is maximum then $(-h) =$

48. Parabolas $y^2 = 4a(x-k)$ and $x^2 = 4a(y-l)$ are such that they touch each other. (k,l are variables) locus of point of contact is $xy = ma^2$ then m =

49. The angle between the two tangents drawn from the point (1, 4) to the parabola $y^2=12x$ is $\text{Tan}^{-1}(k)$ then $2k$ =

EXERCISE-II

Parametric form - Tagents - normalchord of contact - no.of normals

LEVEL-I (MAIN)

Parametric form : (Focal chord):

1. On the parabola $y^2 = 8x$ if one extremity of focal chord is $\left(\frac{1}{2}, -2\right)$ then its other extremity is

1) (2, 2) 2) $\left(\frac{1}{8}, -8\right)$ 3) $\left(8, \frac{1}{8}\right)$ 4) (8, 8)

2. If $P(-3, 2)$ is one end of focal chord PQ of the parabola $y^2 + 4x + 4y = 0$ then slope of the nomal at Q is

1) $-\frac{1}{2}$ 2) 2 3) $\frac{1}{2}$ 4) –2

Tangents :

3. If y_1, y_2 are the ordinates of two points P and Q on the parabola and y_3 is the ordinate of the point of intersection of tangents at P and Q, then

1) y_1,y_2,y_3 are in A.P. 2) y_1,y_3,y_2 are in A.P. 3) y_1,y_2,y_3 are in G.P. 4) y_1,y_3,y_2 are in G.P.

4. If the distances of two points P and Q on the parabola $y^2 = 4ax$ from the focus of a parabola are 4 and 9 respectively then the distance of the point of intersection of tangents at P and Q from the focus is

1) 8 2) 6 3) 5 4) 13

5. If a tangent to the parabola $y^2 = 4ax$ meets the x-axis in T and the tangent at the vertex A in P and the rectangle $TAPQ$ is completed then locus of Q is

1) $y^2 + ax = 0$ 2) $y^2 - ax = 0$ 3) $x^2 + axy = 0$ 4) $x^2 + ay = 0$

Normals :

6. If the normal at 'P' on $y^2 = 4ax$ cuts the axis of the parabola in G and if S is the focus then SG =

1) SP 2) 2SP 3) $\frac{1}{2}$SP 4) $\sqrt{SP}$

7. The normal at 'P' cuts the axis of the parabola $y^2= 4ax$ in G and S is the focus of the parabola. If $\triangle SPG$ is equilateral then each side is of length

1) a 2) $2a$ 3) $3a$ 4) $4a$

8. Number of normals drawn through the point (8, 4) to the parabola $y^2 = 2x$

1) 1 2) 2 3) 3 4) 0

9. The line $2x + y = k$ is a normal to $y^2 = 12x$ then k =

1) 12 2) –12 3) 36 4) –36

10. The locus of middle points of normal chords of the parabola $y^2 = 4ax$ is

1) $\frac{y^2}{2a}+\frac{4a^3}{y^2}=x-2a$ 2) $\frac{y^2}{2a}-\frac{4a^3}{y^2}=x-2a$ 3) $\frac{y^2}{2a}+\frac{4a^3}{y^2}=x+2a$ 4) $\frac{y^2}{2a}-\frac{4a^3}{y^2}=x-2a$

11. If the normal at (1, 2) on the parabola $y^2 = 4x$ meets the parabola again at the point $(t^2, 2t)$ then $t =$

1) 1 2) 3 3) –3 4) 1

Parametric form :

12. If the tangents at t_1 and t_2 on $y^2 = 4ax$ meet on the directrix then

1) $t_1 = t_2$ 2) $t_1 = -t_2$ 3) $t_1t_2 = 2$ 4) $t_1t_2 = -1$

13. If the normals at t_1 and t_2 on $y^2 = 4ax$ meet again on the parabola then $t_1t_2 =$

1) 1 2) –1 3) 2 4) –2

14. If the normals at (x_1, y_1) and (x_2, y_2) on $y^2 = 4ax$ meet again on parabola then $x_1x_2 + y_1y_2 =$

1) $4a^2$ 2) $8a^2$ 3) $12a^2$ 4) $10a^2$

General Problems :

15. Let M be the foot of the perpendicular from a point P on the parabola $y^2 = 8(x–3)$ onto its dirictrix and let S be the focus of the parabola. If Δ^{le} SPM is an equilateral triangle then $P =$

1) $(4\sqrt{3},8)$ 2) $(8,4\sqrt{3})$ 3) $(9,4\sqrt{3})$ 4) $(4\sqrt{3},9)$

16. $L = (1, 3)$ and $L^1 = (1, –1)$ are the ends of latus rectum of a parabola, then area of quadrilateral formed by tangents and normals at L and L^1 (in Square Units) is

1) 2 2) 4 3) 8 4) 16

17. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the point of intersection of the parabola's $y^2 = 4ax$ and $x^2 = 4ay$, then

1) $d^2+(2b+3c)^2 = 0$ 2) $d^2+(3b–2c)^2 = 0$ 3) $d^2+(2b–3c)^2 = 0$ 4) $d^2+(3b+2c)^2 = 0$

18. The point on the parabola $y = x^2 + 7x + 2$ which is closest to the line $y = 3x – 3$ is

1) (2, 8) 2) (2, –8) 3) (–2, 8) 4) (–2, –8)

19. The locus of centre of a circle which passes through the origin and cuts off a length of 4 units from the line $x = 3$ is :

1) $y^2+6x=0$ 2) $y^2+6x=13$ 3) $y^2+6x=10$ 4) $x^2+6y=13$

20. If A, B, C are 3 points on a parabola. Δ_1, Δ_2 are the areas of triangle is formed by the points A, B, C and the tangents at A, B, C. If Δ_1, Δ_2 are the roots of $px^2 + qx + r = 0$ then condition is

1) $9q^2 = 2pr$ 2) $9pr = 2q^2$ 3) $9p^2 = 2qr$ 4) $2p^2 = 9qr$

Numerical value type questions

21. If the normal subtends a right angle at the focus of the parabola $y^2 = 8x$ then its length is

22. If a normal chord of the parabola $y^2 = 4x$ makes an angle of 45° with the axis of the parabola then its length is

23. Shortest distance between the line $y = x$ and hte curve $y^2 = x - 2$ is _____

LEVEL-II (ADVANCED)

Single answer type questions

1. It two different tangents of $y^2 = 4x$ are the normals to $x^2 = 4by$ then

a) $|b| > \frac{1}{2\sqrt{2}}$ b) $|b| < \frac{1}{2\sqrt{2}}$ c) $|b| > \frac{1}{\sqrt{2}}$ d) $|b| < \frac{1}{\sqrt{2}}$

2. The set of points on the axis of the parabola $y^2 - 4x - 2y + 5 = 0$ from which all the three normals to the parabola are real is

a) $(k, 0)$; $k > 1$ b) $(k, 1)$; $k > 3$ c) $(k, 2)$; $k > 6$ d) $(k, 3)$; $k > 8$

3. Let α be the angle which a tangent to the parabola $y^2 = 4ax$ makes with its axis, the distance between the tangent and a parallel normal will be

a) $a\sin^2\alpha\cos^2\alpha$ b) $a\operatorname{cosec}\alpha\sec^2\alpha$ c) $a\tan^2\alpha$ d) $a\cos^2\alpha$

4. Normals are drawn from the point P with slopes m_1, m_2,m_3 to $y^2 = 4x$ and if locus of P such that $m_1.m_2 = \alpha$ is a part of the parabola itself, then α is equal to

a) 1 b) 2 c) 3 d) -2

Parametric :

5. The co-ordinates of a point R on the axis of the parabola $y^2 = 4ax$ such that if PQ is a variable chord through R, then $\frac{1}{PR^2} + \frac{1}{QR^2}$ is independent of the slope of the chord is.

a) $(2a, 0)$ b) $(-2a, 0)$ c) $(0, 2a)$ d) $(0, -2a)$

6. A chord PP^1 of a parabola cuts the axis of the parabola at O, The feet of the perpendiculars from P and P^1 on the axis are M and M^1 respectively. V is the vertex then VM, VO, VM^1 are in.

a) AP b) GP c) HP d) none of these

7. If the normals at three points, P,Q,R of the parabola $y^2 = 4ax$ meet in a point O and S be its focus, then $|SP|.|SQ|.|SR|$ is equal to

a) a^2 b) $a(SO)^3$ c) $a(SO)^2$ d) none of these

8. P is the point 't' on the parabola $y^2 = 4ax$ and PQ is a focal chord, PT is the tangent at P and QN is the normal at Q. If the angle betwen PT and QN, be α and the distance between PT and QN be d then.

a) $0 < \alpha < 90^0$, $d=a$ b) $\alpha = 0^0$, $d = \frac{a}{t^2}(t^2+1)^{3/2}$

c) $d = 0, \alpha = 15^0$ d) $d = a\left(\sqrt{1+t^2} + \frac{1}{\sqrt{1+t^2}}\right)$, $\alpha = 35^0$

9. Normals AO, AA_1, AA_2 are drawn to parabola $y^2 = 8x$ from the point $A = (h, 0)$. If the triangle OA_1A_2 is equilateral, then possible value of 'h' is

a) 26 b) 24 c) 28 d) 8

10. The locus of the foot of perpendicular from the focus on any normal to the parabola $y^2 = 4ax$ is.
a) $y^2 = a(x-a)$ b) $y^2 = a(x+a)$ c) $y^2 = -a(x+a)$ d) $y^2 = -a(x-a)$

11. AB is a double ordinate of the parabola $y^2 = 4ax$. Tangents drawn to the parabola at A and B meets y-axis at A_1 and B_1 respectively. If the area of trapezium AA_1B_1B is equal to $24a^2$ then angle subtended by A_1B_1 at the focus of the parabola is equal to
a) $2\tan^{-1}(3)$ b) $\tan^{-1}(3)$ c) $2\tan^{-1}(2)$ d) $\tan^{-1}(2)$

12. The radius of circle touching parabola $y^2 = x$ at (1,1) and having dirictrix of $y^2 = x$ as its normal is
a) 5 b) $5\sqrt{5}$ c) $\frac{5\sqrt{5}}{4}$ d) $\frac{5\sqrt{5}}{8}$

13. The normals at points A and B on $y^2 = 4ax$ meet the parabola in a point C on the parabola, the locus of orthocentre of triangle ABC is the parabola
a) $y^2 = a(x + 6a)$ b) $y^2 = a(x + 6)$ c) $y^2 = (x + 6a)$ d) $y^2 = (x - 6a)$

14. Two tangents are drawn from a point to $y^2 = 4ax$ if these are normals to $x^2 = 4by$ then
a) $a^2 > b^2$ b) $a^2 > 4b^2$ c) $a^2 > 8b^2$ d) $4a^2 < b^2$

15. Angle between parabolas $y^2 = 4b(x - 2a + b)$ and $x^2 + 4a(y-2b-a) = 0$ at the common end of their latus rectum is
a) $\text{Tan}^{-1}(1)$ b) $Tan^{-1}1+\tan^{-1}\frac{1}{2}+Tan^{-1}\frac{1}{3}$ c) $Tan^{-1}\sqrt{3}$ d) $Tan^{-1}2+\tan^{-1}3$

16. If the ordinates of points P and Q on the parabola $y^2 = 12x$ are in the ratio 1: 2 then the locus of the point of intersection of the normals to the parabola at P and Q is
a) $343y^2 = 12(x-6)^3$ b) $343y^2 = 12(x+6)^3$ c) $343y^2 = -12(x-6)^3$ d) $343y^2 = -12(x+6)^3$

17. The locus of a point $P(h, k)$ such that the slopes of three normals drawn to the parabola $y^2 = 4ax$ from P be connected by the relation $\tan^{-1}m_1^2+\tan^{-1}m_2^2+\tan^{-1}m_3^2=\alpha$, is
a) $x^2\tan\alpha - y^2 + 2a(1-2\tan\alpha)x + a^2(3\tan\alpha - 4) = 0$
b) $x^2\tan\alpha + y^2 + 2a(1-2\tan\alpha)x + a^2(3\tan\alpha - 4) = 0$
c) $x^2\tan\alpha + y^2 + 3a(1+2\tan\alpha)x + a^3(3\tan\alpha - 4) = 0$
d) $x^2\tan\alpha + y^2 + 2a(1+2\tan\alpha)x + a^2(3\tan\alpha + 4) = 0$

18. A line bisecting the ordinate PN of a point $P(at^2, 2at)$, $t > 0$, on the parabola $y^2 = 4ax$ is drawn parallel to the axis to meet the curve at Q. If NQ meets the tangent at the vertex at the point T, then the coordinates fo T are
a) $(0,(4/3)at)$ b) $(0, 2at)$ c) $((1/4)at^2, at)$ d) $(0, at)$

19. Tangents to the parabola $y^2 = 4ax$. are drawn at the points whose abscissar are in the ratio k : 1. The locus of their point of intersection is $y^2 = f(k).\ ax$. Then minimum value of $f(k)$ is
a) 2 b) 4 c) $2\sqrt{2}$ d) 8

20. If the normal to a parabola $y^2 = 4ax$ at P meet the curve again in Q and if PQ and the normal at Q makes angle α and β respectively with the x-axis then $\{-\tan\alpha(\tan\alpha+\tan\beta)\}$
a) 1/3 b) 3 c) –2 d) 2

More than one correct answer type questions

21. If the normals to the parabola $y^2 = 4ax$ at the ends of the latus rectum meet the parabola again at the points P and Q then the points $P\&Q$ and the equation of PQ is ____

a) $P\ (9a, -6a)$ b) $Q(9a, 6a)$ c) $x-9a = 0$ d) $y - 6a = 0$

22. P is a point on the parabola whose ordinate equals its abscissa. A normal is drawn to the parabola at P to meet it again at Q. It S is the focus of the parabola. Slope of SP is m_1 and slope of SQ is m_2.

a) $P(4a, 4a)$ b) $Q(9a, -6a)$ c) $|SP.SQ| = 50a^2$ d) $m_1m_2 = -1$

23. The normal drawn to the parabola $y^2 = 4ax$ at the point $P(t_1)$ meets the curve again at $Q(t_2)$ then

a) $t_1 \geq 2\sqrt{2}$ b) $t_2 \geq 2\sqrt{2}$ c) $t_1 \leq -2\sqrt{2}$ d) $t_2 \leq -2\sqrt{2}$

24. If a tangent to the parabola $y^2 = 4ax$ meets the x–axis at T and intersect tangent at vertex A at P and the rectangle $TAPQ$ be completed. Then the locus of the point Q is

a) Another parabola at vertex (0, 0)

b) Whose equation is $y^2 + ax = 0$

c) Another parabola whose axis is negative direction of x–axis

d) Focus of the parabola is $\left(\frac{-a}{4}, 0\right)$

25. If P_1P_2 and Q_1Q_2, two focal chords of a parabola are at right angles, then :

a) area of the quadrilateral $P_1Q_1P_2Q_2$ is minimum when the chords are inclined at an angle $\frac{\pi}{4}$ to the axis of the parabola

b) minimum area is twice the area of the square on the latus rectum of the parabola

c) minimum $P_1Q_1P_2Q_2$ cannot be found

d) minimum area is thrice the area of the square on the latus rectum of the parabola

26. Whatever be the value of θ, the line $y = (x-1)\cos\theta - \cos 3\theta$ is a normal to the parabola of

a) $y^2 = 16x$ b) $x^2 = 16y$ c) $y^2 = 4x$ d) $x^2 = -4y$

27. Tangent is drawn at any point (x_1, y_1) other than vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point (x_2, y_2) then

a) x_1, a, x_2 are in G.P. b) $\frac{y_1}{2}, a, y_2$ are in G.P

c) $-4, \frac{y_1}{y_2}, \frac{x_1}{x_2}$ are in G.P d) $x_1x_2 + y_1y_2 = a^2$

Linked comprehension type questions

Passages - I :

A tangent is drawn at any point $P(t)$ on the parabola $y^2 = 8x$ and on it is taken a point $Q(\alpha, \beta)$ from which pair of tangents QA and QB are drawn to the circle $x^2 + y^2 = 8$. Using this information answer the following questions.

28. The locus of the point of concurrency of the chord of contact AB of the circle $x^2 + y^2 = 8$ is _____

a) $y^2 - 2x = 0$ b) $y^2 - x^2 = 4$ c) $y^2 + 4x = 0$ d) $y^2 - 2x^2 = 4$

29. The point from which perpendicular tangents can be drawn both to the given circle and the parabola is

a) $(4,\pm\sqrt{3})$ b) $(-1,\sqrt{2})$ c) $(-\sqrt{2},-\sqrt{2})$ d) $(-2,\pm 2\sqrt{3})$

30. The locus of circumcentre of ΔAQB if $t = 2$ is

a) $x-2y+2=0$ b) $x+2y-4=0$ c) $x-2y+4=0$ d) $x+2y+4=0$

Passage - II :

Consider the parabola $(y-2)^2 = 4(x-2)$ and $(x-2)^2 = 4(y-2)$. Let $S = 0$ be the largest circle touching the two parabola's internally and $S' = 0$ be the circle described on the common chord of the two parabolas as diameter. Then answer the following questions.

31. Centre of $S = 0$ is

a) $(2\sqrt{2},2\sqrt{2})$ b) (2, 2) c) (4, 4) d) $\left(2\sqrt{3},2\sqrt{3}\right)$

32. Radius of $S = 0$ is _____

a) $2(\sqrt{10}-\sqrt{8})$ b) $2(\sqrt{10}+\sqrt{8})$ c) $(\sqrt{10}+\sqrt{8})$ d) $(\sqrt{10}-\sqrt{8})$

33. If $x^2 + y^2 = k$ cuts $S = 0$ orthogonally then k = ___

a) 24 b) 12 c) 8 d) 4

Matrix matching type questions

34. $y^2 = 12x$ is a parabola and $P(9,\mu)$ be a point in the plane then match the following

COLUMN - I	COLUMN - II
A) Only one normal of $y^2=12x$ will go through P	p) $\lvert\mu\rvert = \frac{2}{\sqrt{3}}$
B) No normal of $y^2 = 12x$ will go through P	q) $\lvert\mu\rvert < \frac{2}{\sqrt{3}}$
C) Exactly two normals will go through P	r) Null set
D) Exactly three distinct normals will go through P	s) $\lvert\mu\rvert > \frac{2}{\sqrt{3}}$

35.

COLUMN - I	COLUMN - II
A) The x-coordinate of points on the axis of the parabola $y^2 - 4x - 2y + 5 = 0$ from which all the three normals to the parabola are real is	p) 4
B) The x-coordinate of points on the axis of the parabola $4y^2 - 32x + 4y + 65 = 0$ from which all the three normals to the parabola are real is	q) 5
C) The x-coordinate of points on the axis of the parabola $4y^2 - 16x - 4y + 41 = 0$ from which all the three normals to the parabola are real is	r) 6
D) The x-coordinate of point of the focus of the parabola $y^2 - 4y - 8x + 28 = 0$	s) 7

36. Normals are drawn at points P, Q and R lying on the parabola $y^2 = 4x$, which intersect at (3,0); then

COLUMN - I	COLUMN - II
A) Area of ΔPQR	p) 2
B) Radius of the circum circle of ΔPQR	q) 5/2
C) Distance of the vertex from the centroid of ΔPQR	r) 11/6
D) Distance of the centroid from the circumcentre of ΔPQR	s) 2/3

37. Let $A(t_1)$ and $B(t_2)$ be points on the parabola $y^2 = 4ax$ with vertex O. Then match $t_1t_2 + 2 =$

COLUMN – I	COLUMN – II
A) The tangents at A and B intersect on the directrix	p) 1
B) The normal at A and B intersect on the parabola	q) –2
C) $\angle AOB = \frac{\pi}{2}$	r) 2
D) AB is a focal chord	s) 4

38. Normals are drawn from point (4, 1) to the parabola $y^2 = 4x$. The tangents at the feet of normals to the parabola $y^2 = 4x$ from a triangle ABC

COLUMN - I	COLUMN - II
A) The distance of focus of parabola $y^2 = 4x$ from centroid of ΔABC is	p) 5/3
B) The distance of focus of parabola $y^2 = 4x$ from orthocentre of ΔABC is	q) $\frac{\sqrt{10}}{2}$
C) The distance of focus of parabola $y^2 = 4x$ from circumcenter of ΔABC is	r) $\frac{\sqrt{7}}{2}$
D) Area of ΔABC is	s) $\frac{\sqrt{5}}{2}$
	t) $\sqrt{5}$

Integer answer type questions

39. The length of the normals drawn from the point on the axis of the parabola $y^2 = 8x$ whose distance from the focus is

40. If the parabola $y^2 = 4ax$ and $y^2 = 4c(x - b)$ have a common normal other than x–axis, and $\frac{b}{a-c} > k$ then $k =$

41. Maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ can be equal to

42. A right angle triangle ABC is inscribed in a parabola $y^2 = 4x$, where A is vertex of parabola and $\angle BAC = \frac{\pi}{2}$. If $AB = \sqrt{5}$ and area of triangle ABC is k then $\frac{k}{10} =$

43. The focal chord of $y^2 = 16x$ touches the circle $(x–6)^2 + y^2 = 2$. Then the positive slope of the chord is

44. The normal at p cuts the axis of the parabola $y^2 = 4ax$ in Q and S is the focus of the parabola. If triangle SPQ is equilateral and each side length is ka then $k =$

KEY SHEET (LECTURE SHEET)

EXERCISE-I

LEVEL-I

1) **1** 2) **3** 3) **3** 4) **2** 5) **4** 6) **4** 7) **3** 8) **1**

9) **2** 10) **3** 11) **2** 12) **4** 13) **2** 14) **3** 15) **2** 16) **3**

17) **4** 18) **2** 19) **2** 20) **4** 21) **2.75** 22) **2.17** 23) **1.73** 24) **17.32**

LEVEL-II

1) **a** 2) **d** 3) **c** 4) **a** 5) **c** 6) **d** 7) **d** 8) **ad**

9) **a** 10) **b** 11) **b** 12) **d** 13) **ab** 14) **d** 15) **b** 16) **c**

17) **c** 18) **b** 19) **d** 20) **abcd** 21) **ab** 22) **ab** 23) **abcd**

24) **abcd** 25) **abcd** 26) **abd** 27) **bc** 28) **ac** 29) **ab** 30) **ad** 31) **ac**

32) **abc** 33) **abc** 34) **d** 35) **b** 36) **a** 37) **a** 38) **b** 39) **c**

40) **a** 41) **b** 42) **b** 43) **A-qr; B-s; C-q; D-s**

44) **A-q, B-r, C-p, D-s** 45) **A-s, B-p, C-q, D-s** 46) **2** 47) **1**

48) **4** 49) **1**

EXERCISE-II

LEVEL-I

1) **4** 2) **1** 3) **2** 4) **2** 5) **1** 6) **1** 7) **4** 8) **3**

9) **3** 10) **1** 11) **3** 12) **4** 13) **3** 14) **3** 15) **3** 16) **3**

17) **1** 18) **4** 19) **2** 20) **2** 21) **22.36** 22) **5.62** 23) **1.23**

LEVEL-II

1) **b** 2) **b** 3) **b** 4) **b** 5) **a** 6) **b** 7) **c** 8) **b**

9) **c** 10) **a** 11) **c** 12) **c** 13) **a** 14) **c** 15) **b** 16) **a**

17) **a** 18) **a** 19) **b** 20) **d** 21) **abc** 22) **abcd** 23) **bd** 24) **abcd**

25) **ab** 26) **ab** 27) **bcd** 28) **c** 29) **d** 30) **c** 31) **c**

32) **a** 33) **a** 34) **A-s; B-r; C-p; D-q** 35) **A-pqrs; B-s; C-qrs; D-q**

36) **A-p, B-q, C-s, D-r** 37) **A-p, B-s, C-q, D-p**

38) **A-p, B-t, C-q, D-s** 39) **8** 40) **2** 41) **5** 42) **2** 43) **1**

44) **4**

PRACTICE SHEET

EXERCISE-I

General equation - varriours standard form - length of chord - tangent

LEVEL-I (MAIN)

Equation of parabola :

1. The vertex and focus of a parabola are at a distance of h and k units on positive x–axis from origin. Then equation of parabola is

   1) $x^2 = 4(k-h)(y-k)$ 2) $y^2 = 4(k-h)(x-h)$

   3) $(x-h)^2 = 4(k-h)(y-k)$ 4) $(y-k)^2 = 4(k-h)(x-h)$

2. The equation $16x^2+y^2+8xy-74x-78y+212=0$ represents

   1) a circle 2) a parabola 3) an ellipse 4) hyperbola

Vertex, Focus, LR, Axes :

3. Focus of parabola $y = ax^2 + bx + c$ is

   1) $\left(\frac{-b}{2a}, \frac{b^2-4ac+1}{4a}\right)$ 2) $\left(\frac{-b}{2a}, \frac{4ac-b^2+1}{4a}\right)$ 3) $\left(\frac{-b}{2a}, \frac{b^2-4ac-1}{4a}\right)$ 4) $\left(\frac{b}{2a}, \frac{b^2+4ac+1}{4a}\right)$

4. The length of latus rectum of the parabola whose focus is $\left(\frac{u^2}{2g}\text{Sin}2\alpha, \frac{-u^2}{2g}\text{Cos}2\alpha\right)$ and directrix is $y = \frac{u^2}{2g}$ is

   1) $\frac{u^2}{g}\text{Cos}\alpha$ 2) $\frac{u^2}{g}\text{Cos}^2 2\alpha$ 3) $\frac{2u^2}{g}\text{Cos}2\alpha$ 4) $\frac{2u^2}{g}\text{Cos}^2\alpha$

Double ordinates :

5. The length of double ordinate of parabola $y^2 = 8x$ which subtends an angle $60°$ at vertex is

   1) $4\sqrt{3}$ 2) $8\sqrt{3}$ 3) $16\sqrt{3}$ 4) $32\sqrt{3}$

6. If a focal hord of $y^2 = 4ax$ makes an angle α, $\alpha \in (0, \pi/4]$ with the positive direction of x-axis. Then minimum length of this focal chord is

   1) $6a$ 2) $2a$ 3) $8a$ 4) None

Tangents :

7. Point of contact of $y = 1-x$ w.r.t. $y^2 - y + x = 0$ is

   1) (1, 1) 2) $\left(\frac{1}{2}, \frac{1}{2}\right)$ 3) (0, 1) 4) (1, 0)

8. The condition that the line $y = mx + c$ to be a tangent to the parabola $y^2 = 4a(x+a)$ is

   1) $c = a\left(m + \frac{1}{m}\right)$ 2) $c = a\left(m - \frac{1}{m}\right)$ 3) $c = \frac{a}{m}$ 4) $a = \left(m - \frac{1}{m}\right)$

9. The locus of point of intersection of tangents to $y^2 = 4ax$ which includes an angle α is
   1) $y^2 - 4ax = \text{Tan}^2\alpha$ 2) $y^2 - 4ax = (x+a)^2\,\text{Cot}^2\alpha$
   3) $y^2 - 4ax = (x+a)^2\,\text{Tan}^2\alpha$ 4) $y^2 - 4ax = (x-a)^2\,\text{Cot}^2\alpha$

10. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpedicular to the given tangent is
   1) $(-1, 1)$ 2) $(0, 2)$ 3) $(2, 4)$ 4) $(-2, 0)$

11. A chord of parabola $y^2 = 4ax$ subtends a right angle at the vertex. The tangents at the extremities of chord intersect on
   1) $x + a = 0$ 2) $x + 2a = 0$ 3) $x + 3a = 0$ 4) $x + 4a = 0$

12. The mid point of chord $2x+y-5 = 0$ of the parabola $y^2 = 4x$ is
   1) (2,1) 2) (1,3) 3) (3,–1) 4) $\left(\frac{5}{2},0\right)$

13. Maximum number of common chords of a parabola and a circle can be equal to
   1) 2 2) 4 3) 6 4) 8

LEVEL-II (ADVANCED)

Single answer type questions

1. If $x + a = 0$ is the directrix of the parabola $y^2 = 2y + ax + 2$ then the value of a is
   a) 2 b) 3 c) 4 d) 4/3

2. For the parabola $x = ay^2 + by$ the slope of the tangent at (1, 2) is 1/2 then the latusrectum of the parabola is
   a) 4 b) 3 c) 3/4 d) 4/3

3. The locus of the centriod of triangle formed by a tangent to $y^2 = 36x$ with co-ordinate axes is
   a) $y^2 = -9x$ b) $y^2 = -3x$ c) $y^2 = 3x$ d) $y^2 = 9x$

4. Equations of two tangents that can be drawn from (3, 4) to the parabola $y^2 = 4x$ are.
   a) $y = x+1,\ 3y = x+9$ b) $y = x-1,\ 3y = x-9$
   c) $y = -x-1,\ 3y = -x-9$ d) $-y = x+1,\ -3y = x+9$

5. The locus of point P such that if tangents are drawn from P to $y^2 = 4ax$, the chord of contact touches the circle $x^2 + y^2 = 4a^2$ is
   a) $x^2 - y^2 = 4a^2$ b) $x^2 + y^2 = 4a^2$ c) $y^2 = 4ax$ d) $y^2 = -4ax$

6. The locus of a point P such that if tangents are drawn from P to $y^2 = 4ax$, then the chord of contact subtends a right angle at the vertex is.
   a) $x + 4a = 0$ b) $x - 4a = 0$ c) $x - a = 0$ d) $x + a = 0$

7. If AB is a focal chord of $x^2 - 2x + y - 2 = 0$, whose focus is S. If $AS = P$ then $BS =$
   a) $\frac{P}{4}$ b) $\frac{P}{(4P+1)}$ c) $\frac{P}{4P-1}$ d) $\frac{P}{1-4P}$

8. If the tangent at the point $P(2, 4)$ to the parabola $y^2 = 8x$ meets the parabola $y^2 = 8x + 5$ at Q and R then the mid point of chord QR is

a) (2, 4) b) (4, 2) c) (1, 3) d) (3, 5)

9. $A = (-2, 0)$ and P is a point on the parabola $y^2 = 8x$. If Q bisects $\overline{AP}$ and the locus of Q is a parabola then its focus is

a) (0, 0) b) (1, 1) c) (5, 0) d) (4, 0)

10. If the angular bisectors of the coordinate axes cut the parabola $y^2 = 4ax$ at the points O, A, B then the area of ΔOAB is (O is the origin)

a) $32a^2$ b) $16a^2$ c) $64a^2$ d) $8a^2$

11. The abscissa of the orthocentre of the triangle formed by the lines $y = m_1x + \frac{b}{m_1}$, $y = m_2x + \frac{b}{m_2}$, $y = m_3x + \frac{b}{m_3}$ is

a) b b) $-b$ c) $2b$ d) $-2b$

12. The equation of common tangent at the point of contact of two parabolas $y^2 = x$ and $2y = 2x^2 - 5x + 1$ is

a) $x + y + 1 = 0$ b) $x + 2y + 1 = 0$ c) $x - 2y - 1 = 0$ d) $-x + 2y - 1 = 0$

More than one correct answer type questions

13. The equation of a tangent to the parabola $y^2 = 8x$ which makes an angle 45^0 with the line $y = 3x + 5$ is

a) $2x + y + 1 = 0$ b) $y = 2x + 1$ c) $x - 2y + 8 = 0$ d) $x + 2y - 8 = 0$

14. The ends of a line segment are $P(1, 3)$ and $Q(1, 1)$. R is a point on the line segment PQ such that $PR : QR = 1 : \lambda$. If R is an interior point of the parabola $y^2 = 4x$, then

a) $\lambda \in (0, 1)$ b) $\lambda \in \left(-\frac{3}{5}, 1\right)$ c) $\lambda \in \left(\frac{1}{2}, \frac{3}{5}\right)$ d) none of these

15. For the parabolas $y^2 = 4ax$, $x^2 = 4ay$.

a) Slope of common tangent is $m = 1$ b) Slope of common tangent is $m = -1$

c) Common tangent line is $x + y + a = 0$ d) Common tangent line is $x - y - a = 0$

16. Circle drawn having its diameter equal to focal distance of any point lying on the parabola $x^2 - 4x + 6y + 10 = 0$ will touch a fixed line which is

a) dirictrix b) $x - 2 = 0$ c) $y + 1 = 0$ d) the tangent at the vertex

17. Locus of trisection point of any double ordinate of $y^2 = 4ax$ is

a) $9y^2 = 4ax$ b) $x^2 + y^2 = 36a$ c) a circle d) again a parabola

18. Consider the equation of a parabola $y^2 + 4ax = 0$ where $a > 0$ which of the following are false?

a) Tangent at the vertex is $x = 0$ b) Directrix of the parabola is $x = 0$

c) Vertex of the parabola is at the origin d) Focus of the parabola is at $(a, 0)$

19. If the focus of the parabola $x^2 - ky + 3 = 0$ is (0.2) then the values of k are

a) 2 b) 3 c) 4 d) 6

20. $y^2 + 2y - x + 5 = 0$ represents a parabola, then the extremity of the latus rectum is

a) $\left(\frac{17}{4}, \frac{-1}{2}\right)$ b) (4,–1) c) (–1,4) d) $\left(\frac{17}{4}, \frac{-3}{2}\right)$

21. Let $y^2 = 4ax$ be a parabola and $x^2+y^2+2bx = 0$ be a circle. If parabola and circle touch each other externally then

a) $a > 0\ b > 0$ b) $a > 0\ b < 0$ c) $a < 0\ b > 0$ d) $a < 0\ \ b < 0$

Linked comprehension type questions

Passage - I :

We know that general equation of second degree i.e., $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents conic sections if $\Delta \neq 0$ where $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$. As a special case this represents a parabola if $\Delta \neq 0$ and $h^2 = ab$.

Alternatively a parabola is defined as the locus of a point which is equidistant from a point and fixed line. They are respectively called focus and directrix of the parabola.

22. The differential equation of all parabolas must be a differential equation of order

a) 2 b) 3 c) 4 d) 5

23. The equation $x^2 + \lambda xy + y^2 = \lambda$ represents a parabola for n integer values of λ then n must be

a) 0 b) 1 c) 2 d) 3

24. The equation $x^2 + 4xy + 4y^2 + 4x + 4y + \lambda = 0$ will represent a parabola

a) for all values of λ b) for all except for one value of λ

c) for no values of λ d) None of these

Passage - II :

$P_1 : y^2 = 4ax,\ P_2 : y^2 = -4ax, L : Y = X$

25. Area of the rectangle formed by joining the vertices of the latusrecta of the two parabolas P_1 and P_2 is

a) $2a^2$ sq.units b) $4a^2$ sq.units c) $8a^2$ sq.units d) $16a^2$ sq.units

26. Equation of the tangent at the point on the parabola P_1 where the line L meets the parabola is

a) $x - 2y + 4a = 0$ b) $x + 2y - 4a = 0$ c) $x + 2y - 8a = 0$ d) $x - 2y + 8a = 0$

27. The coordinates of other extremity of a focal chord of the parabola P_2, one of whose extremity is the point of intersection of L and P_2 is

a) $(-a, 2a)$ b) $(-a/4, a)$ c) $(-a/4, -a)$ d) $(-a, -2a)$

Matrix matching type questions

28. Consider the parabola $(x-1)^2+(y-2)^2=\frac{(12x-5y+3)^2}{169}$

COLUMN - I	COLUMN - II
A) Locus of point of intersection of perpendicular tangents is	p) $12x-5y-2=0$
B) Locus of foot of perpendicular from ocus upon any tangent is	q) $5x+12y-29=0$
C) Line along which minimum length of focal chord occurs	r) $12x-5y+3=0$
D) Line about which parabola is symmertric is	s) $24x-10y+1=0$

29.

COLUMN - I	COLUMN - II
A) Parabola $y^2=4x$ and the circle having its centre at (6, 5) intersects at right angle, at the point (a, a) then one value of a is equal to	p) 13
B) The angle between the tangents drawn to $(y-2)^2=4(x+3)$ at the points where it is intersected by the line $3x-y+8=0$ is $\frac{4\pi}{p}$, then p has the value equal to	q) 8
C) If the line $x-1=0$ is the directrix of the parabola $y^2-kx+8=0$, then one of the value of k is/are	r) $10\sqrt{5}$
D) Length of the normal of the parabola $y^2=8x$ at the point where abscissa and ordinate are equal is	s) 4 t) -8

Integer answer type questions

30. Circles are drawn with diameter being any focal chord of the parabola $y^2-4x-y-4=0$ will always touch a fixed line $kx+33=0$ then $\frac{k}{4}=$

31. The no of common tangents for the circle $x^2+y^2-4x+3=0$ and parabola $y^2=2x$ is

32. A circle is drawn to pass through the extermities of the latus rectum of parabola $y^2=8x$. Then the minimum radius of the circle is

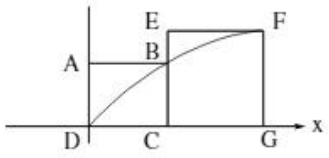
33. If $(2,-8)$ is at an end of a focal chord of the parabola $y^2=32x$. If the other end of the chord is (α,β), then $\frac{\alpha+\beta}{16}=$

EXERCISE-II

Parametric form - Tagents - normalchord of contact - no.of normals

LEVEL-I (MAIN)

1. ABCD and EFGC are square and the curve $y=k\sqrt{x}$ passes through origin D and points B and F the ratio $\frac{FG}{BC}$ is



1) $\frac{\sqrt{5}+1}{2}$ 2) $\frac{\sqrt{3}+1}{2}$ 3) $\frac{\sqrt{5}+1}{4}$ 4) $\frac{\sqrt{3}+1}{4}$

2. The area of triangle formed by the points t_1, t_2 and t_3 on $y^2 = 4ax$ is $k|(t_1-t_2)(t_2-t_3)(t_3-t_1)|$ then K =

1) $a^2/2$ 2) a^2 3) $2a^2$ 4) $4a^2$

3. The area of triangle formed by tangents at the parametric points t_1, t_2 and t_3 on $y^2 = 4ax$ is $k|(t_1-t_2)\ (t_2-t_3)\ (t_3-t_1)|$ then K =

1) $a^2/2$ 2) a^2 3) $2a^2$ 4) $4a^2$

4. The length of the perpendicular from the focus S of the parabola $y^2 = 4ax$ on the tangent at P is

1) $\sqrt{OS.SP}$ 2) OS.SP 3) OS+OP 4) $\sqrt{\frac{OS}{OP}}$

Normals :

5. The equation of normal to $y^2 = 4ax$ at the point of contact of a tangent $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ is

1) $y = mx - 2am - am^3$ 2) $m^3y = m^2x - 2am^2 - a$

3) $m^3y = 2am^2 - m^2x + a$ 4) $m^3x = m^2y + 2am^2 - a$

6. If t_1, t_2, t_3 are the feet of normals drawn from (x_1, y_1) to the parabola $y^2 = 4ax$ then the value of $t_1t_2t_3$ =

1) 0 2) $\frac{y_1}{a}$ 3) $\frac{2a - x_1}{a}$ 4) $\frac{x_1 - 2a}{a}$

7. The feet of the normals to $y^2 = 4ax$ from the point $(6a, 0)$ are

1) (0, 0) 2) $(4a, -4a)$

3) $(4a, -4a)$ 4) $(0,0),(4a, 4a),(4a,-4a)$

8. The number of normals drawn to the parabola $y^2 = 4x$ from the point (1, 0) is

1) 0 2) 1 3) 2 4) 3

9. If two of the three feet of normals drawn from a point to the parabola $y^2 = 4x$ be (1, 2) (1, -2) then third foot is

1) $(2,2\sqrt{2})$ 2) $(2,-2\sqrt{2})$ 3) (0, 0) 4) (1, 1)

10. On the parabola $y^2 = 4x$ the normal at (1, 2) meets the parabola again at the point

1) (–6, 9) 2) (–9, –6) 3) (9, –6) 4) (–6, –9)

11. If the normals at P and Q meet again on $y^2 = 4ax$ at R then centroid of PQR lies on

1) axis 2) latus rectum 3) directrix 4) parabola

Lengths of Normal chord :

12. If a normal subtends a right angle at the vertex of a parabola $y^2 = 4ax$ then its length is

1) $2\sqrt{3}a$ 2) $4\sqrt{3}a$ 3) $6\sqrt{3}a$ 4) $8\sqrt{3}a$

13. The normal at P (8, 8) to the parabola $y^2 = 8x$ cuts it again at Q then PQ =

1) 10 2) $10\sqrt{5}$ 3) $5\sqrt{10}$ 4) 50

Parametric form :

14. The point of intersection of tangents at t_1 and t_2 to the parabola $y^2 = 12x$ is

1) $(2t_1t_2, 2(t_1-t_2))$ 2) $(3t_1t_2, 3(t_1-t_2))$ 3) $(3t_1t_2, 3(t_1+t_2))$ 4) $(2t_1t_2, 2(t_1+t_2))$

15. If the tangents at t_1 and t_2 on $y^2 = 4ax$ meets on its axis then

1) $t_1 = t_2$ 2) $t_1 = -t_2$ 3) $t_1t_2 = 2$ 4) $t_1t_2 = -1$

16. The area of triangle formed by tangents and the chord of contact from (3, 4) to $y^2 = 2x$ is

1) $10\sqrt{10}$ 2) $2\sqrt{10}$ 3) $\frac{10\sqrt{10}}{3}$ 4) $\frac{5\sqrt{10}}{3}$

17. $L = (1, 3)$, $L^1 (1, -1)$ are the ends of latusrectum of a parabola. A is the vertex of the parabola then area of ΔALL^1 in sq. units is

1) 2 2) 4 3) 8 4) 1

18. The point on $y^2 = 4ax$ nearest to the focus has its abscissa equal to

1) $-a$ 2) a 3) $a/2$ 4) 0

19. No. of points of intersection of circle $x^2 + y^2 + 2x = 0$ with $y^2 = 4x$ is

1) 1 2) 2 3) 3 4) 4

20. If a circle cuts a parabola in four points then the sum of ordinates of four points is

1) 1 2) –1 3) 0 4) 2

LEVEL-II (ADVANCED)

1. The condition that the parabolas $y^2 = 4c(x-d)$ and $y^2 = 4ax$ have a common normal other than x-axis $(a > c > 0)$ is

a) $2a < 2c + d$ b) $2a < 2c + d$ c) $2d < 2a + c$ d) $2d < 2c + a$

2. The maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ is equal to

a) 3 b) 4 c) 6 d) 5

Parametric :

3. If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, then $PA.PB$ is equal to [where $P \equiv (\sqrt{3}, 0)$]

a) $\frac{4(\sqrt{3}+2)}{3}$ b) $\frac{4(2-\sqrt{3})}{3}$ c) $\frac{4\sqrt{3}}{2}$ d) $\frac{2(\sqrt{3}+2)}{3}$

4. Minimum distance between the curve $y^2 = 4x$, $x^2 + y^2 - 12x + 31 = 0$ is

a) $\sqrt{21}$ b) $\sqrt{26} - \sqrt{5}$ c) $\sqrt{5}$ d) $\sqrt{28} - \sqrt{5}$

5. AB, AC are tangents to a parabola $y^2 = 4ax$ and p_1, p_2, p_3 are the lengths of the perpendiculars from A, B, C on any tangent to the curve, then p_2, p_1, p_3 are in

a) AP b) GP c) HP d) none of these

6. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissae are in AP, then the slopes of the tangents at the three conormal points are in

a) AP b) GP c) HP d) none of these

7. The locus of a point O when the three normals drawn from it are such that the sum of the three angles made by with them the axis is constant (say α) is.

a) $y=(x-a)\tan\alpha$ b) $y=(x-a)\cot\alpha$ c) $y=(x-a)$ d) $x=y-a$

8. AB is a chord of the parabola $y^2 = 4ax$ such that the normals at A and B intersect at the point C $(at^2, 2at)$, then the equation of AB is

a) $2x+ty+4a=0$ b) $2x-ty+4a=0$ c) $2x-ty-4a=0$ d) $2x+ty-4a=0$

9. The tangents at three points A, B, C on the parabola $y^2 = 4x$ taken in pairs intersect at the points P, Q and R. If Δ,Δ' be the areas of the triangle ABC and PQR respectively then

a) $\Delta=\Delta'$ b) $\Delta'=2\Delta$ c) $4\Delta=\Delta'$ d) $2\Delta'=\Delta$

10. AB is a chord of the parabola $y^2 = 4ax$ such that the normals at A and B intersect at the point C $(at^2, 2at)$. then the length of AB is

a) $a\sqrt{(t^2-8)(t^2+4)}$ b) $a\sqrt{(t^2+8)(t^2+4)}$

c) $a\sqrt{(t^2-8)(t^2-4)}$ d) $a\sqrt{(t^2+8)(t^2-4)}$

11. If A, B, C are 3 points on a parabola Δ_1,Δ_2 are the area of triangle formed by the points A, B, C and tangents at A,B,C. If Δ_1,Δ_2 are the roots of $px^2 + qx + r = 0$, then the condition is

a) $9q^2=2pr$ b) $9pr=2q^2$ c) $9p^2=2qr$ d) $2p^2=9qr$

12. AP is perpendicular to PB. When A is vetex of parabola $y^2 = 4x$ and P on the parabola. B is on the axis of parabola, then the locus of centriod if ΔPAB is

a) $9y^2=2(3x-4)$ b) $9y^2=2(3x+4)$ c) $9x^2=2(3y-4)$ d) $9x^2=2(3y+4)$

13. For the parabola $y^2 = 8x$ tangent and normal are drawn at $P(2, 4)$ which meet the axis of the parabola in A and B. Then the length of the diameter of the circle through A, P, B is

a) 2 b) 4 c) 8 d) 6

14. If tangents are drawn to the parabola $y^2 = 4ax$ at points whose abscissa are in the ratio $m^2 : 1$, then the locus of their point of intersection is the curve $(m > 0)$

a) $y^2=(m^{1/2}-m^{-1/2})^2ax$ b) $y^2=(m^{1/2}+m^{-1/2})^2ax$

c) $y^2=(m^{1/2}+m^{-1/2})^2x$ d) $y^2=(m^{1/2}-m^{-1/2})^2x$

15. If the tangents & normals at the extremities of a focal chord of a parabola $y^2 = 4ax$ intersect at (x_1, y_1) and (x_2, y_2) respectively, then

a) $x_1=x_2$ b) $x_1=y_2$ c) $y_1=y_2$ d) $x_2=y_1$

16. The equation of common tangent at the point of contact of two parabolas $y^2 = x$ and $2y = 2x^2 - 5x + 1$ is

a) $x + y + 1 = 0$ b) $x + 2y + 1 = 0$ c) $x - 2y - 1 = 0$ d) $-x + 2y - 1 = 0$

17. Let PQ be a chord of the parabola $y^2 = 4x$. A circle drawn with PQ as a diameter passes through the vertex V of the parabola. If area of $\Delta PVQ = 20$ sq units, then coordinates of P are

a) $(-16, -8)$ b) $(-16, 8)$ c) $(16, -8)$ d) $(16, 8)$

18. The ends of a line segment are $P(1, 3)$ and $Q(1, 1)$. R is a point on the line segment PQ such that $PR : QR = 1 : \lambda$. If R is an interior point of the parabola $y^2 = 4x$, then

a) $\lambda \in (0, 1)$ b) $\lambda \in \left(-\frac{3}{5}, 1\right)$ c) $\lambda \in \left(\frac{1}{2}, \frac{3}{5}\right)$ d) none of these

19. Tangents drawn to parabola $y^2 = 4ax$ at the points A and B intersect at C then,

a) ordinates of A, C, B are in AP b) Ordinates of A, B, C are in AP

c) abscissae of A, C, B are in GP d) absissae of A, B, C are in GP

20. Let P be a point whose coordinates differ by unity and the point does not lie on any of the axes of reference. If the parabola $y^2 = 4x + 1$ passes through P, then the ordinate of P may be

a) 3 b) -1 c) 5 d) 1

21. A quadrilateral is inscribed in a parabola, then

a) quadrilateral may be cyclic

b) diagonals of the quadrilateral may be equal.

c) all possible pairs of adjacent sides may be perpendicular.

d) quadrilateral can never be cyclic.

22. The locus of the midpoint of the focal distance of a variable point moving on the parabola, $y^2 = 4ax$ is a parabola whose

a) latus rectum is half the latus rectum of the original parabola

b) vertex is $\left(\frac{a}{2}, 0\right)$

c) directrix is y-axis

d) focus has the co-ordinates $(a, 0)$

23. P is a point on the parabola $y^2 = 4ax$, $y^2 = 4x$ and Q is a point on the line $2x + y + 4 = 0$. If the line $x - y + 1 = 0$ is the perpendicular bisector of PQ, then the co-ordinates of P can be:

a) $(1, -2)$ b) $(4, 4)$ c) $(9, -6)$ d) $(16, 8)$

Linked comprehension type questions

Passage - I :

If P is a point moving on a parabola $y^2 = 4ax$ and Q is point moving on the circle $x^2 + y^2 - 24ay + 128a^2 = 0$. The points P and Q will be closest when they lie along a normal to the parabola $y^2 = 4ax$ passing through the centre of the circle.

24. If the normal at $(at^2, 2at)$ of the parabola passes through the centre of the circle then the value of t must be

a) 1 b) 2 c) 3 d) 4

25. The shortest distance between P and Q must be

a) $a(\sqrt{2}-1)$ b) $2a(\sqrt{5}-1)$ c) $4a(\sqrt{5}-1)$ d) $4a(\sqrt{5}+1)$

26. When P and Q are closest then P must be the point

a) $(a, 2a)$ b) $(2a, 2\sqrt{2}a)$ c) $(4a, 4a)$ d) (a, a)

Passage - II :

If $P(x_1, y_1)$ is a point on C : $y^2 = 4ax$, where S is the focus of the parabola and tangent at P makes an angle of measure α with SP. If the angle between the parabola and the line $y = y_1$ be of measure β, then

27. Which one of the following is true?

a) $\alpha > \beta$ b) $\alpha < \beta$ c) $\alpha = \beta$ d) $2\alpha = \beta$

28. Area of the region bounded by the curve C and the line $x = a$, is

a) $4a^2$ sq.units b) $\frac{4}{3}a^2$ *sq.units* c) $\frac{16}{3}a^2$ *sq.units* d) $\frac{8}{3}a^2$ *sq.units*

29. If $a = 2$, then the point on the parabola 'C' at a minimum distance from $A(10, 4)$ is

a) (8, 8) b) (4, 4) c) (4, 8) d) (8, 4)

Matrix matching type questions

30.

COLUMN - I	COLUMN - II
A) If $kx + y = 0$ is a normal to the parabola $y^2 = 4x$ then $k =$	p) 0
B) If $y = 2x + k$ is a normal to the parabola $y^2 = 4x$ then $k =$	q) –1
C) If $y = x\sqrt{2} + 4\sqrt{2}$ is a normal to the parabola $y^2 = 4kx$ then $k =$	r) –12
D) If $y = x\sqrt{2} + k$ is a normal to the parabola $y^2 = 4x$ then $k =$	s) $-4\sqrt{2}$

31. A triangle ABC is inscribed in the parabola $y^2 = 4x$ with A at the vertex of the parabola and its orthocenter at the focus.

COLUMN - I	COLUMN - II
A) The distance of the chord BC from A	p) 2
B) The distance of the centroid from A	q) 10/3
C) The distance of the circumcentre from A	r) 9/2
D) The inradius of ΔABC	s) 5

32. AB is a chord of the parabola $y^2 = 4x$ such that the normals at A and B intersect at the point $C(9, 6)$.

COLUMN – I	COLUMN – II
A) The length AB	p) 20
B) The area of ΔABC	q) $\frac{4}{\sqrt{13}}$
C) The distance of the origin from the line through AB	r) $\sqrt{13}$
D) The area bounded by the coordinate axes and the line through AB	s) 4/3

33. Normals are drawn from point (4, 1) to the parabola $y^2 = 4x$. The tangents at the feet of normals to the parabola $y^2 = 4x$ from a triangle ABC

COLUMN - I	COLUMN - II
A) The distance of focus of parabola $y^2 = 4x$ from centroid of ΔABC is	p) 5/3
B) The distance of focus of parabola $y^2 = 4x$ from orthocentre of ΔABC is	q) $\frac{\sqrt{10}}{2}$
C) The distance of focus of parabola $y^2 = 4x$ from circumcenter of ΔABC is	r) $\frac{\sqrt{7}}{2}$
D) Area of ΔABC is	s) $\frac{\sqrt{5}}{2}$
	t) $\sqrt{5}$

Integer answer type questions

34. P is a point on the parabola $y^2 = 4x$ and Q is a point on the line $2x + y + 4 = 0$. If the line $x - y + 1 = 0$ is the perpendicular bisector of PQ, the x coordinate of P

35. Normals AO, AA_1, AA_2 are drawn to parabola $y^2 = 8x$ from the point $A(h, 0)$. If triangle OA_1A_2 is equilateral then possible value of $\frac{h}{7} =$

36. If two different tangents of $y^2 = 4x$ are the normals to $x^2 = 4ky$ and $8k^2 < \alpha$, then $\alpha =$

37. The tangent at a *pt.* P on $y^2 = 4ax$ meets the Axis at T. If A is the vertex, S is the focus PN is the ordinate of P, Then AT/AN is

38. The number of points on $y^2 = 4ax$ at which the normal chords drawn subtend a right angle at the vertex is

39. M is the foot of the perpendicular from a point P on the parabola $y^2 = 8(x–3)$ to its directrix and S is the focus of the parabola and SPM is an equilateral triangle. Then the length of each side of the triangle is

40. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) be three points one parabola $y^2 = 4ax$ and the normals at these points meet in a point, then $\frac{x_1 - x_2}{y_3} + \frac{x_2 - x_3}{y_1} + \frac{x_3 - x_1}{y_2}$ is equal to

KEY SHEET (PRACTICE SHEET)

EXERCISE- I

LEVEL-I

1) **2** 2) **2** 3) **2** 4) **4** 5) **3** 6) **3** 7) **3** 8) **1**
9) **3** 10) **4** 11) **4** 12) **3** 13) **3**

LEVEL-II

1) **a** 2) **d** 3) **b** 4) **a** 5) **a** 6) **a** 7) **c** 8) **a**
9) **a** 10) **b** 11) **b** 12) **c** 13) **ac** 14) **abc** 15) **bc** 16) **cd**
17) **ad** 18) **bd** 19) **ad** 20) **ad** 21) **ad** 22) **c** 23) **c** 24) **a**
25) **c** 26) **a** 27) **b** 28) **A-r, B-s, C-p, D-q**
29) **A-s, B-q, C-s, D-r** 30) **c** 31) **c** 32) **4** 33) **4**

EXERCISE-II

LEVEL-I

1) **1** 2) **2** 3) **1** 4) **1** 5) **3** 6) **2** 7) **4** 8) **2**
9) **3** 10) **3** 11) **2** 12) **2** 13) **3** 14) **2** 15) **2** 16) **1**
17) **1** 18) **4** 19) **1** 20) **3**

LEVEL-II

1) **a** 2) **d** 3) **a** 4) **c** 5) **b** 6) **a** 7) **a** 8) **a**
9) **d** 10) **a** 11) **b** 12) **a** 13) **c** 14) **b** 15) **c** 16) **c**
17) **cd** 18) **abc** 19) **ac** 20) **ad** 21) **ab** 22) **abcd** 23) **ac** 24) **b**
25) **c** 26) **c** 27) **c** 28) **d** 29) **a** 30) **A-p, B-r, C-q, D-s**
31) **A-s, B-q, C-r, D-p** 32) **A-r, B-p, C-q, D-s**
33) **A-p, B-t, C-q, D-s** 34) **9** 35) **4** 36) **1** 37) **1** 38) **2**
39) **8** 40) **0**

ADDITIONAL EXERCISE

LEVEL-I (MAIN)

1. The circle $x^2 + y^2 + 2\lambda x = 0$ and $y^2 = 4x$ touch each other exterally then $\lambda \in$
   1) $(-\infty, 0)$ 2) $(1, \infty)$ 3) $(-\infty, 1)$ 4) $(0, \infty)$

2. The orthocentre of a triangle formed by tangents to the parabola $y^2 = 4ax$ at t_1, t_2, t_3 lies on
   1) Axis 2) Directrix 3) Tangent at the vertex 4) none

3. If M is the foot of the perpendicular from a point P on a parabola to its directix and SPM is an equilateral triangle, where S is the focus, then SP =

1) a 2) $2a$ 3) $3a$ 4) $4a$

4. The tangent at the point $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x+b)$ at Q and R, then the midpoint of QR is

1) (x_1, y_1) 2) (x_1+b, y_1) 3) (x_1+b, y_1+b) 4) $(x_1- b, y_1- b)$

5. If the normal to $y^2 = 4ax$ at t_1 cuts the parabola again at t_2 then___

1) $t_2^2 \le 8$ 2) $t_2^2 \ge 8$ 3) $-8 \le t_2 \le 8$ 4) $t_2^2 < 8$

6. Point of concurrence of the normals drawn at (2, 8), (128, 64), (162, –72) to the parabola $y^2 = 32x$ is

1) (2, 8) 2) (128, 64) 3) (162, –72) 4) (162, 72)

7. If the two normals to $y^2 = 8x$ at (2, 4) and (18, 12) intersect at $P(x_1, y_1)$ then foot of the 3rd normal through P is

1) (32, 16) 2) (32, –16) 3) (–16, 32) 4) (2, 4)

8. P and Q are two points on the parabola $(y - 2)^2 = 4(x - 3)$. The normals and tangents at P and Q form a square then point of intersection of tangents at P, Q is

1) (–1, 0) 2) (–4, 2) 3) (2, 2) 4) (3, 2)

9. The normals at two points P and Q on the parabola $y^2= 4ax$ meet at (x_1,y_1) on the parabola. Then PQ^2=

1) $(x_1 + 4a)(x_1 + 8a)$ 2) $(x_1 + 4a)(x_1 - 8a)$ 3) $(x_1 - 4a)(x_1 + 8a)$ 4) $(x_1 - 4a)(x_1 - 8a)$

10. The circle with focal radii as diameter touches

1) Directrix 2) tangent at the vertex 3) latus rectum 4) cannot say

11. The circle with focal chord as diameter touches

1) Directrix 2) tangent at the vertex 3) latus rectum 4) cannot say

12. The line $\frac{x}{p}+\frac{y}{q}=1$ is a normal to $y^2 = 4ax$, then $a(2p^2 + q^2)$

1) q^3 2) p^3 3) p^2q 4) pq^2

13. If PQ is a chord of $y^2 = 4ax$ which has fixed direction. The locus of the point of Intersection of the normals at P and Q is

1) a parabola
2) a tangent to $y^2 = 4ax$
3) a normal to $y^2 = 4ax$
4) The direction of $y^2 = 4ax$

14. Shortest distance between the two parabolas $y^2 = x - 2$, $x^2 = y - 2$ is

1) $\frac{1}{4\sqrt{2}}$ 2) $\frac{5}{4\sqrt{2}}$ 3) $\frac{7}{2\sqrt{2}}$ 4) $\frac{6}{7\sqrt{2}}$

15. Through the vertex O of the parabola $y^2 = 4ax$ a perpendicular is drawn to any tangent meeting it at P and the parabola at Q. Then $OP.OQ$ =

1) a^2 2) $2a^2$ 3) $3a^2$ 4) $4a^2$

16. Point of contact of the line $kx + y - 4 = 0$ w.r.t. the parabola $y = x - x^2$ is

1) $(-2, 2)$ 2) $(2, -2)$ 3) $(-2, 6)$ 4) $(2, -6)$

17. Tangents at P and Q on $y^2 = 4x$ meet at T. If S is the focus and $SP = a$, $ST = b$, $SQ = c$ then roots of $ax^2 + bx + c = 0$ are

1) real and different 2) real and equal 3) Irrational 4) complex

18. If (x_1, y_1), (x_2, y_2), (x_3, y_3) are feet of the three normals drawn from a point to the parabola $y^2 = 4ax$ then $\sum \frac{x_1 - x_2}{y_3} =$

1) $4a$ 2) $2a$ 3) 1 4) 0

19. If the normals at P and Q meet again on the parabola $y^2 = 4ax$ then the chord joining P and Q passes through a fixed point

1) $(-a, 0)$ 2) $(-2a, 0)$ 3) $(-3a, 0)$ 4) $(-4a, 0)$

20. The tangent at 't' on the parabola $y^2 = 4ax$ is parallel to a normal chord then distance between them is

1) $\frac{a\sqrt{1+t^2}}{t^2}$ 2) $\frac{a(1+t^2)^{3/2}}{t^2}$ 3) $\frac{a^2(1+t^2)^{3/2}}{t}$ 4) $\frac{a(1+t^2)^{3/2}}{t}$

21. A chord PQ is a normal to the parabola $y^2 = 4ax$ at P and PQ subtends right angle at the vertex. then $\frac{SQ}{SP} =$

1) 1 2) 2 3) 3 4) 4

22. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is

1) $\frac{2\sqrt{3}}{8}$ 2) $\frac{3\sqrt{2}}{5}$ 3) $\frac{\sqrt{3}}{4}$ 4) $\frac{3\sqrt{2}}{8}$

23. Length of the Normal chord $y = x\sqrt{2} - 4a\sqrt{2}$ of $y^2 = 4ax$ is

1) $8\sqrt{2}a$ 2) $5\sqrt{5}a$ 3) $6\sqrt{3}a$ 4) $4\sqrt{3}a$

LEVEL-II

LECTURE SHEET (ADVANCED)

Single answer type questions

1. Parabola $y^2 = 4x$ and the circle having its centre at (6, 5) intersect at right angle. Then the possible points of intersection of these two curves are ____

a) (4, 4) (6, 9) b) (4, 4) (9, 6) c) (1, 2) (4, 4) d) (1, 2) (9, 6)

2. A telegraphic wire suspended between two poles of height 15 mt is in the shape of a parabola. The distance between the poles is 20 mt and maximum sag of the cable wire is 4mt. Then the height of the cable at a distance of 5mt from one end is

a) 12 b) 11 c) 10 d) 13

3. If the tangent at the point $P(2, 4)$ to the parabola $y^2 = 8x$ meets the parabola $y^2 = 8x + 5$ at Q and R then the mid point of chord QR is

a) (2, 4) b) (4, 2) c) (1, 3) d) (3, 5)

4. Minimum area of circle which touches the parabolas $y = x^2 + 1$ and $x = y^2 + 1$ is

a) $\frac{9\pi}{16}$ sq.unit b) $\frac{9\pi}{32}$ sq.unit c) $\frac{9\pi}{8}$ sq.unit d) $\frac{9\pi}{4}$ sq.unit

5. $\min\left[(x_1 - x_2)^2 + \left(5 + \sqrt{1 - x_1^2} - \sqrt{4x_2}\right)^2\right] \forall x_1, x_2 \in R$ is

a) $4\sqrt{5}+1$ b) $4\sqrt{5}-1$ c) $\sqrt{5}+1$ d) $\sqrt{5}-1$

6. Vertex A of a parabola $y^2 = 4ax$ is joined to any point P on it and line PQ is drawn at right angle to AP to meet the axis at Q. Then the projection of PQ on the axis is always equal to

a) $3a$ b) $2a$ c) $3a$ d) $4a$

7. A pair of tangents are drawn from a point P to the parabola $y^2 = 4ax$. If these tangents meet the coordinate axes in concyclic points, then the locus of P.

a) $x = a$ b) $x + a = 0$ c) $y = a$ d) $y + a = 0$

8. A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $(y - 2)^2 = 4(x + 1)$. After reflection, the ray must pass through the point

a) (–2, 0) b) (–1, 2) c) (0, 2) d) (2, 0)

9. $A = (-2, 0)$ and P is a point on the parabola $y^2 = 8x$. If Q bisects $\overline{AP}$ and the locus of Q is a parabola then its focus is

a) (0, 0) b) (1, 1) c) (5, 0) d) (4, 0)

10. If the tangents & normals at the extremities of a focal chord of a parabola $y^2 = 4ax$ intersect at (x_1, y_1) and (x_2, y_2) respectively, then

a) $x_1 = x_2$ b) $x_1 = y_2$ c) $y_1 = y_2$ d) $x_2 = y_1$

More than one correct answer type questions

11. The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts the parabola $x^2 = 4ay$ at the points $A_i(x_i y_i)$. i = 1,2,3,4 then

a) $\sum x_i = 0$ b) $\sum y_i = -4(f + 2a)$ c) $\pi(x_i) = 16a^2c$ d) $\pi(y_i) = c^2$

12. Whatever be the value of θ, the line $y = (x - 1)\cos\theta - \cos 3\theta$ is a normal to the parabola of

a) $y^2 = 16x$ b) $x^2 = 16y$ c) $y^2 = 4x$ d) $x^2 = -4y$

13. Let there be two parabolas with the same axis, focus of each being exterior to the other and the latus recta being $4a$ and $4b$. The locus fo the middle points of the intercepts between the parabolas made on the lines parallel to the common axis is a :

a) straight line if $a = b$ b) parabola if $a \neq b$

c) parabola $\forall a, b \in R$ d) none of these

14. The angle between the tangents drawn from the origin to the parabola $y^2 = 4a(x-a)$ is

a) 90^0 b) $\sin^{-1} x + \cos^{-1} x \; \forall x \in R$

c) 45^0 d) The angle between the bisectors of the co–ordinate axes

15. A quadrilateral is inscribed in a parabola, then

a) quadrilateral may be cyclic

b) diagonals of the quadrilateral may be equal.

c) all possible pairs of adjacent sides may be perpendicular.

d) quadrilateral can never be cyclic.

16. The line $x + y + 2 = 0$ is a tangent to a parabola at point A, intersect the directrix at B and tangent at vertex at C respectively. The focus of parabola is $S(2, 0)$. Then

a) CS is perpendicular to AB b) $AC.\ BC = CS^2$

c) $AC.BC = 8$ d) $AC = BC$

Linked comprehension type questions

Passage - I :

If a source of light is placed at the fixed point of a parabola and if the parabola is a reflecting surface, then the ray will bounce back in a line parallel to the axis of the parabola.

17. A ray of light is coming along the line $y = 2$ from the positive direction of x-axis and strikes a concave mirror whose intersection with the xy–plane is a parabola $y^2 = 8x$, then the equation of the reflected ray is

a) $2x+5y=4$ b) $3x+2y=6$ c) $4x+3y=8$ d) $5x+4y=10$

18. A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $y^2 + 10y - 4x + 17 = 0$. After reflection, the ray must pass through the point

a) (–2,–5) b) (–1,–5) c) (–3,–5) d) (–4,–5)

19. A ray of light is coming along the line $x = \lambda, (\lambda < 0)$ from the positive direction of y-axis and strikes a concave mirror $x^2 = 4(y-1)$, the ray is reflected from it is $x + y = 2$, if this reflected ray again strikes the same mirror, then the equation of new reflected ray is

a) $y+1=2\sqrt{2}$ b) $x+1=2\sqrt{2}$ c) $y+2=2\sqrt{2}$ d) $x+2=2\sqrt{2}$

Passage - II :

The limiting value of expression $\dfrac{4x^2-+2y^2-6xy}{6x^2+\sqrt{2}y-8xy}$ is A as point (x, y) on curve $x^2 + y^2 = 1$ approaches the position $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ where A is such that $(5A, 0)$ is a point as focus of parabola S having axis parallel to x-axis vertex at origin

20. The two common tangents can be drawn to both circle and parabola from external point whose coordinates are

a) $\left(\dfrac{-4}{\sqrt{15}-1}, 0\right)$ b) $\left(\dfrac{-4}{\sqrt{17}+1}, 0\right)$ c) $\left(\dfrac{-4}{\sqrt{17}-1}, 0\right)$ d) $\left(\dfrac{-4}{\sqrt{15}+1}, 0\right)$

21. Locus of midpoints of chords of parabola, which subtend a right angle at vertex of parabola is

a) $y^2 - 4x + 32 = 0$ b) $y^2 + 4x - 32 = 0$ c) $y^2 - 32x + 4 = 0$ d) $y^2 + 32x - 4 = 0$

22. Equation of the parabola with the given data is ———

a) $y^2 = 8x$ b) $x^2 = 8y$ c) $y^2 = 4x$ d) $x^2 = 4y$

Passage - III :

Consider the inequality, $9^x - a,\ 3^x - a + 3 \le 0$ where $'a'$ is a real parameter

23. The given inequality has atleast one negative solution for $a \in$

a) $(-\infty, 2)$ b) $(3,\infty)$ c) $(-2,\infty)$ d) $(2, 3)$

24. The given inequality has atleast one positive solution for $a \in$

a) $(-\infty,-2)$ b) $[3,\infty)$ c) $(2,\infty)$ d) $[-2,\infty)$

25. The given inequality has atleast one real solution for $a \in$

a) $(-\infty,3)$ b) $[2,\infty)$ c) $(3,\infty)$ d) $[-2,\infty)$

Matrix matching type questions

26. AB is a chord of the parabola $y^2 = 4x$ such that the normals at A and B intersect at the point $C(9, 6)$.

COLUMN – I	COLUMN – II
A) The length AB	p) 20
B) The area of ΔABC	q) $\frac{4}{\sqrt{13}}$
C) The distance of the origin from the line through AB	r) $\sqrt{13}$
D) The area bounded by the coordinate axes and the line through AB	s) 4/3

27. If $y = x + 1$ is axis of parabola, $y + x = 4$ is tangent of same parabola at its vertex and $y = 2x + 3$ is one of its tangent, then

COLUMN - I	COLUMN - II
A) If equation of directrix of parabola is $ax + by - 29 = 0$, then $a + b = 0$	p) 9
B) If length of latus rectum of parabola is $\frac{a\sqrt{2}}{b}$ where a and b are relatively prime natural numbers, then $a + b =$	q) 18
C) Let extremities of latus rectum are (a_1, b_1) and (a_2, b_3), then $[a_1 + b_1 + a_2 + b_2] =$ (where [.] denote greatest integer function)	r) 23
D) If equation of parabola is $a(x - y + 1)^2 = b(x + y - 4)$ where a and b are relatively prime natural numbers then $a + b =$	s) 37

28. **COLUMN - I** **COLUMN - II**

A) Parabola $y^2 = 4x$ and the circle having its centre at (6, 5) intersects at right angle, at the point (a, a) then one value of a is equal to — p) 13

B) The angle between the tangents drawn to $(y-2)^2 = 4(x + 3)$ at the points where it is intersected by the line $3x - y + 8 = 0$ is $4p/p$, then p has the value equal to — q) 8

C) If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the value of k is/are — r) $10\sqrt{5}$

D) Length of the normal of the parabola $y^2 = 8x$ at the point where abscissa and ordinate are equal is — s) 4

Integer answer type questions

29. Minimum distance between the curves $y^2 = x-1$ and $x^2 = y-1$ is equal d, then $2\sqrt{2}.d =$

30. The mirror image of any point on the directrix of the parabola $y^2 = 4(x+1)$ in the line mirror $x + 2y - 3 = 0$ lies on $3x - 4y + 4k = 0$. Then $k =$

31. 'O' is the vertex of parabola $y^2 = 4x$ and L is the upper end of latus rectum. If LH is drawn perpendicular to OL meeting in H, then length of double ordinate through H is $\sqrt{N}$, then $\frac{N}{10} =$ _____

32. If the normals at the points P, Q, R on the parabola $y^2 = 4ax$ meet in the point (h, k). If the centroid and orthocentre of the triangle PQR is (x_1, y_1) and (x_2, y_2), then the value of $3x_1 - 2x_2$ is .....

33. The radius of circle whic pases through the focus of parabola $x^2 = 4y$ and touches it at point (6, 9) is $k\sqrt{10}$, then $k =$

34. M is the foot of the perpendicular from a point P on the parabola $y^2 = 8(x-3)$ to its directrix and S is the focus of the parabola and SPM is an equilateral triangle. Then the length of each side of the triangle is

35. If the normals at the points P, Q, R on the parabola $y^2 = 4ax$ meet in the point (h, k). If the centroid and orthocentre of the triangle PQR is (x_1, y_1) and (x_2, y_2), then find the value of $\frac{3x_1 - 2x_2}{a}$ is____

36. The number of parabolas passing through the three points (1, 3), (6, 13), (–5, –9) is_____

37. An equilateral triangle ABC is inscribed in the parabola $y = x^2$ and one of the side of the equilateral triangle has the gradient 2. If the sum of x-coordinates of the vertices of the triangle is a rational in the form $\frac{p}{q}$ where p and q are coprime, then the value of $\frac{(p+q)}{7}$ is .....

PRACTICE SHEET (ADVANCED)

Single answer type questions

1. The locus of the vertices of the equilateral triangle ABC such that its sides touch the parabola $y^2 = 4ax$, is

a) $y^2 = (x+a)^2 + 4ax$ b) $y^2 = 3(x+a)^2 + 4ax$ c) $y^2 = 3(x+a)^2 - ax$ d) $y^2 = 2ax$

2. A ray of light along $y = 4$ is reflected at a point on the parabola $y^2 = 4x$ the slope of the reflected ray

a) 1 b) $\frac{2}{3}$ c) $\frac{3}{4}$ d) $\frac{4}{3}$

3. If AB is a focal chord of $x^2 - 2x + y - 2 = 0$, whose focus is S. If $AS = P$ then $BS =$

a) $\frac{P}{4}$ b) $\frac{P}{(4P+1)}$ c) $\frac{P}{4P-1}$ d) $\frac{P}{1-4P}$

4. $ABCD$ is a square. Equation of AB is $y = 2x-17$. The other two vertices. C and D lie on the parabola $y = x^2$. The maximum possible area of the square $ABCD$ in square units is

a) 160 b) 1280 c) 640 d) 320

5. For the parabola $y^2 = 8x$ tangent and normal are drawn at $P(2, 4)$ which meet the axis of the parabola in A and B. Then the length of the diameter of the circle through A, P, B is

a) 2 b) 4 c) 8 d) 6

6. Tangents to the parabola $y^2 = 4ax$. are drawn at the points whose abscissar are in the ratio $k : 1$. The locus of their point of intersection is $y^2 = f(k).\ ax$. Then minimum value of $f(k)$ is

a) 2 b) 4 c) $2\sqrt{2}$ d) 8

7. If the 4th term in the expansion of $\left(px+\frac{1}{x}\right)^n, n \in N$ is $\frac{5}{2}$ and three normals to the parabola $y^2 = x$ are drawn through a point $(q, 0)$, then

a) $q = p$ b) $q > p$ c) $q < p$ d) $pq = 1$

8. If b and c are the lengths of the segments of any focal chord of a parabola $y^2 = 4ax$ then the length of the semi latus rectum is

a) $\frac{bc}{b+c}$ b) $\sqrt{bc}$ c) $\frac{b+c}{c}$ d) $\frac{2bc}{b+c}$

9. LL^1 is the latus rectum of $y^2 = 4ax$ whose vertex is A. P is a variable point on the are LAL^1. Then minimum projection of LL^1 on the tangent at P is

a) 2a b) $2\sqrt{2}a$ c) $4a$ d) a

10. If the angular bisectors of the coordinate axes cut the parabola $y^2 = 4ax$ at the points O, A, B then the area of ΔOAB is (O is the origin)

a) $32a^2$ b) $16a^2$ c) $64a^2$ d) $8a^2$

11. The straight line joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is

a) $x^2 + 2y^2 - ax = 0$ b) $2x^2 + y^2 - 2ax = 0$ c) $2x^2 + 2y^2 - ay = 0$ d) $2x^2 + y^2 - 2ay = 0$

More than one correct answer type questions

12. The equation of the directrix of the parabola with vertex at the origin and having the axis along the x-axis and a common tangent of slope 2 with the circle $x^2 + y^2 = 5$ is _____.

a) $x = 10$ b) $x = 20$ c) $x = -10$ d) $x = -20$

13. Variable circle is described to passes through the point (1, 0) and tangent to the curve $y = \tan(\tan^{-1}x)$. The locus of the centre of the circle is a parabola whose

a) length of the latus rectum is $2\sqrt{2}$ b) axis of symmetry has the equation $x + y = 1$

c) vertex has the co-ordinates (3/4, 1/4) d) none of these

14. P is a point on the parabola $y^2 = 4ax$, $y^2 = 4x$ and Q is a point on the line $2x + y + 4 = 0$. If the line $x - y + 1 = 0$ is the perpendicular bisector of PQ, then the co-ordinates of P can be:
a) (1, –2) b) (4, 4) c) (9, –6) d) (16, 8)

15. Let P be a point whose coordinates differ by unity and the point does not lie on any of the axes of reference. If the parabola $y^2 = 4x+1$ passes through P, then the ordinate of P may be
a) 3 b) –1 c) 5 d) 1

16. Tangent is drawn at any point (x_1, y_1) other than vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point (x_2, y_2) then
a) x_1, a, x_2 are in G.P. b) $\frac{y_1}{2}, a, y_2$ are in G.P c) $-4, \frac{y_1}{y_2}, \frac{x_1}{x_2}$ are in G.P d) $x_1x_2 + y_1y_2 = a^2$

Linked comprehension type questions

Passage - I :

Tangents are drawn to the parabole $y^2 = 4x$ from the point $P(6, 5)$ to touch the parabola Q and R C_1 is a circle which touches the parbola at R. Both the circles C_1 and C_2 posses through the focus of the parabola

17. Area of ΔPQR equals
a) 1/2 b) 1 c) 2 d) 1/4

18. Radius of the circle C_2 is
a) $5\sqrt{5}$ b) $5\sqrt{10}$ c) $10\sqrt{2}$ d) $\sqrt{210}$

19. The common chord of the circles C_1 and C_2 passes through the
a) Incentre of ΔPQR b) Circumcentre of ΔPQR
c) Controid of ΔPQR d) Ortho centre of ΔPQR

Passage - II :

If $P(x_1, y_1)$ is a point on C : $y^2 = 4ax$, where S is the focus of the parabola and tangent at P makes an angle of measure α with SP. If the angle between the parabola and the line $y = y_1$ be of measure β, then

20. Which one of the following is true?
a) $\alpha > \beta$ b) $\alpha < \beta$ c) $\alpha = \beta$ d) $2\alpha = \beta$

21. Area of the region bounded by the curve C and the line $x = a$, is
a) $4a^2$ sq.units b) $\frac{4}{3}a^2$ *sq.units* c) $\frac{16}{3}a^2$ *sq.units* d) $\frac{8}{3}a^2$ *sq.units*

22. If $a = 2$, then the point on the parabola 'C' at a minimum distance from $A(10, 4)$ is
a) (8, 8) b) (4, 4) c) (4, 8) d) (8, 4)

Passage - III :

The equation of the curve represented by $C \equiv 9x^2 - 24xy + 16y^2 - 20x - 15y - 60 = 0$, then

23. The locus of the curve C given in above statement is
a) a circle b) a pair of straight lines c) a parabola d) an ellipse

24. The equation of the axis of the curve C is

a) $x = 4y$ b) $3x = y$ c) $3x = -4y$ d) $3x = 4y$

25. The equation of directrix of the curve C is

a) $16x + 9y = 53$ b) $16x + 12y + 53 = 0$ c) $16 + 2y = 53$ d) $16x + 9y - 53 = 0$

Matrix matching type questions

26. Normals are drawn from point (4, 1) to the parabola $y^2 = 4x$. The tangents at the feet of normals to the parabola $y^2 = 4x$ from a triangle ABC

COLUMN - I	COLUMN - II
A) The distance of focus of parabola $y^2 = 4x$ from centroid of ΔABC is	p) 5/3
B) The distance of focus of parabola $y^2 = 4x$ from orthocentre of ΔABC is	q) $\frac{\sqrt{10}}{2}$
C) The distance of focus of parabola $y^2 = 4x$ from circumcenter of ΔABC is	r) $\frac{\sqrt{7}}{2}$
D) Area of ΔABC is	s) $\frac{\sqrt{5}}{2}$
	t) $\sqrt{5}$

27.

COLUMN - I	COLUMN - II
A) Radius of the largest circle which passes through the focus of the parabola $y^2 = 4x$ and contained in it, is	p) 16
B) Two perpendicular tangents PA and PB are drawn to the parabola $y^2 = 16x$ then length AB may be	q) 5
C) The shortest distance between parabolas $y^2 = 4x$ and $y^2 = 2x-6$ is d then $d^2 =$	r) 8
D) The harmonic mean of the segments of a focal chord of the parabola $y^2 = 8x$	s) 4

Integer answer type questions

28. An arch is in the shape of a parabola whose axis is vertically downward and measures 80 mts. across its bottom on the ground. Its highest point is 24 mts. the measure of horizontal beam across section at a height of 18 mts is l then $\frac{l}{10} =$

29. Tangents to the parabola at the extremities of a common chord AB of the circle $x^2 + y^2 = 5$ and the parabola $y^2 = 4x$ intersect at the point T. A square $ABCD$ is constructed on this chord lying inside the parabola then $[(TC)^2 + (TD)^2]^2$ is k then $\frac{k}{80^2}$ is ____

30. Let $L_1 : x + y = 0$ and $L_2 : x - y = 0$ are tangent to a parabola whose focus is $S(1, 2)$. If the length of latus rectum of the parabola can be expressed as $\frac{m}{\sqrt{n}}$ (where m and n are coprime them value of $\frac{m+n}{11}$ is

31. A series of chords is drawn to the parabola $y^2 = 4ax$, at point Q. If the tangents at P and Q meet at point R, if the area of triangle PQR is $\frac{4a^2(1+m^2)^3}{m^\lambda}$. Then $\lambda =$ .....

32. Vertex of the parabola whose parametric equations are $x = t^2 - t + 1$, $y = t^2 + t + 1$, $t \in R$ is (α,β) then $\alpha + \beta =$

33. If the locus of centres of a family of circles passing through the vertex of the parabola $y^2 = 4ax$ and cutting the parabola orthogonally at the other point of intersection is $2y^2(2y^2 + x^2 - 12ax) = ax(kx - 4a)^2$, then find the value of k

34. The two parabolas $y = x^2$ and $y^2 = x$ cut at $O(0, 0)$ and $A(1, 1)$ and $P(x_1, x_2)$ and $Q(x_2, y_2)$ $(0 \le x_1, x_2 \le 1)$ be any point on the parabola $y = x^2$ and $y^2 = x$ respectively given that PQ is perpendicular to chord OA. Then the maximum value of PQ must be $\sqrt{k}$ where 'k' is

KEY SHEET (ADDITIONAL EXERCISE)

LEVEL-I (MAIN)

1) **1**	2) **2**	3) **4**	4) **1**	5) **2**	6) **3**	7) **2**	8) **3**	9) **2**	10) **2**
11) **1**	12) **2**	13) **3**	14) **3**	15) **4**	16) **2**	17) **4**	18) **4**	19) **2**	20) **2**
21) **3**	22) **4**	23) **3**							

LEVEL-II

LECTURE SHEET (ADVANCED)

1) **b**	2) **a**	3) **a**	4) **b**	5) **b**	6) **d**	7) **a**	8) **c**	9) **a**	10) **c**
11) **abcd**	12) **ab**	13) **ab**	14) **abd**	15) **ab**	16) **abc**	17) **c**	18) **b**	19) **d**	20) **c**
21) **a**	22) **a**	23) **d**	24) **c**	25) **b**	26) **A - r, B - p, C - q, D - s**				
27) **A-q, B-r, C-p, D-s**			28) **A-s, B-q, C-s, D-r**			29) **3**	30) **4**	31) **8**	32) **8**
33) **5**	34) **8**	35) **8**	36) **0**	37) **2**					

PRACTICE SHEET (ADVANCED)

1) **a**	2) **d**	3) **c**	4) **b**	5) **c**	6) **b**	7) **b**	8) **d**	9) **b**	10) **b**
11) **b**	12) **ac**	13) **bc**	14) **ac**	15) **ad**	16) **bcd**	17) **a**	18) **b**	19) **c**	20) **c**
21) **d**	22) **a**	23) **c**	24) **d**	25) **b**	26) **A-p, B-t, C-q, D-s**				
27) **A-s, B-p, C-q, D-s**			28) **4**	29) **1**	30) **1**	31) **4**	32) **2**	33) **3**	34) **2**