COMPARISON OF FERRO, PARA AND DIA MAGNETIC MATERIALS

Ferromagnetism	Paramagnetism	Diamagnetism
1) Substances are strongly attracted by the magnet.	1) They are feebly attracted by the magnet.	1) They are repelled by the magnet.
2) These are strongly magnetised in the direction of applied magnetic field.	2) These are weakly magnetised in the direction of applied magneticfield.	2) They are weakly magnetised in a direction opposite to that of magnetic field.
3) When a ferromagnetic bar is suspended in an external magnetic field, it aligns itself parallel to the direction of the field.	3) When a paramagnetic bar is suspended in a uniform magneticfield, it comes to rest in the direction of the field.	3) When a diamagnetic bar is suspended in a uniform magneticfield, it comes to rest at right angles to the field.
4) Intensity of magnetisation (I) is very large, positive and varies non-linearly with the field.	4) 'I' is small, positive and varies linearly with field.	4) 'I' is small, negative and varies linearly with the field.
5) Magnetic Susceptibility is high, positive and temperature dependent $\psi = \frac{C}{T - T_c}$ (Curie–Weiss Law) These materials gets converted into Paramagnetic materials above Curie temperature.	5) ψ is small, positive and varies inversly with temperature $\psi = \frac{C}{T}$ These materials gets converted into diamagnetic materials above Curie temperature.	5) ψ is small, negative and independent of temperature. It means materials disobey Curie's Law.
6) Relative permeability (μ_r) is much greater than unity. $(\mu_r >> 1)$	6) μ_r is slightly greater than unity. $(\mu_r > 1)$	6) μ_r is slightly lesser than unity. $(\mu_r < 1)$
7) The magnetic lines of force are pulled in strongly by the substance in a magnetic field. $B >> B_0$, $B_0 =$ Magnetic Induction in vaccum.	7) The lines of force show a little more preference to pass through the substance than through vacuum. $B > B_0$	7) The lines of force passing through these substances are less than those in vacuum $< B_0$
8) These materials tend to move from weaker to stronger parts of a non-uniform magnetic field.	8) When it is kept in a non-uniform magnetic field, it moves from low to high field region.	8) When it is kept in a non-uniform magnetic field, it moves from high to low field region.
9) The atoms possess a permanent magnetic moment and due to the positive exchange interaction, the adjacent dipoles aligns in the same direction and this region behaves as a domain. <u>Ex</u>:- Fe, Co, Steel, Nickel and gadolinium etc.	9) The atoms possess permanent magnetic moment and they are called atomic dipoles. <u>Ex</u>:- A*l*, Mn, Pt, Oxygen etc.	9) The atoms of the diamagnetic substances do not have the resultant magnetic moment. <u>Ex</u>:- Cu, Ag, H_2O, Au, Sb, Bi, Hg and diamond etc.

1.1 INTRODUCTION

We can find magnetism as universal phenomenon in nature. Before the human evolution, earth has its magnetism. Magnetic oxide of iron, a naturally occurring iron ore, was found in the district of magnesia Asia minor in Greece. Hence this ore was called magnetite. The Greek philosopher, Thales of miletus, observed that a naturally occurring ore of iron attracted small pieces of iron, steel, cobalt, nickel etc., This phenomenon of attraction of iron ore was called magnetism.

When a small piece of the iron ore was suspended, it always aligned itself along the north-south direction due to directional property. Because of the directional property the substance was called "lodestone" or "leading stone". The Chinese and Europeans were known to have used lodestone as a navigational aid. William Gilbert showed that the earth was a huge magnet. It is due to earth's magnetism that a freely suspended magnetic needle always points in north-south direction. To day we find magnets and magnetic materials in ATM and Credit cards, audio head sets, computers etc., The properties of magnetic materials can be traced back to their atoms and electrons. But we start with a bar magnet in this topic.

Magnet

A piece of substance which possesses the property of attracting small pieces of iron towards it is called a magnet.

Properties of magnets

Magnets possess the following properties.

(i) Attractive property: Magnets attract pieces of iron, nickel, cobalt, etc. This property is known as attractive property.

(ii) Directive property: When a bar magnet is freely suspended, it always comes to rest along north-south direction. This property is known as directive property. The tip which points to the geographic north is called the north pole and the tip which points to the geographic south is called the south pole of the magnet.

(iii) Inductive property: If a magnetic material such as an iron bar is kept near a magnet, it behaves like a magnet i.e., the magnet induces magnetism into the iron bar. This property of the magnet, is known as inductive property.

(iv) Magnetic poles : When a magnet is dipped in iron fillings, the fillings are found to cling more to the magnet in the two regions near the ends. These are called magnetic poles.

"A magnetic pole is a point near the end of the magnet, where magnetism is concentrated."

Face to face length of the magnet is called geometric length while pole to pole length is called magnetic length of the magnet. The geometrical length of the bar magnet is always greater than its magnetic length.

$$\text{Magnetic length, } 2l = \frac{5}{6} \times \text{geometrical length}$$

Magnetic axis : The line joining the poles of a magnet is called magnetic axis.

Magnetic meridian : The vertical plane passing through the axis of a freely suspended magnet is called magnetic meridian.

(v) Law of magnetic poles : If the south pole of a magnet is brought near the north pole of another magnet they attract each other. On the other hand, if the north pole of a magnet is brought near the north pole of another magnet they repel each other. Therefore, unlike poles attract each other and like poles repel each other.

(vi) **Magnetic poles exist in pairs :** The magnetic poles exist only in pairs of opposite nature. It is not possible to obtain isolated magnetic pole. Isolated poles are only conceptual. Magnetic monopoles do not exist.

(vii) **Repulsion is the sure test of magnetism :** A magnetic pole not only attracts a magnetic material but also unlike pole of another magnet. But a magnetic pole will repel like pole of another magnet. So repulsion is the sure test of magnetism.

1.2 COULOMB'S INVERSE SQUARE LAW

The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them and acts along the line joining the poles.

Explanation

Suppose that two magnetic poles of strengths m_1 and m_2 are placed a distance "d" apart. If "F" is the force between the poles, then

Fig 1.1 Force between the poles

$$F \propto m_1 m_2 \text{ and } F \propto \frac{1}{d^2}$$

$$\therefore \quad F \propto \frac{m_1 m_2}{d^2} \text{ or } F = \frac{K m_1 m_2}{d^2}$$

Where 'K' is proportionality constant. It depends on the magnetic nature of medium in which the magnetic poles are placed.

The unit of pole strength in SI is Ampere metre (A–m). It is a scalar quantity.

If the poles are placed in any medium,

$$F = \frac{\mu}{4\pi} \frac{m_1 m_2}{d^2} \qquad \text{.... (1.1)}$$

Where 'μ' is called the permeability of the medium. The property of the medium which allows the lines of force through it is called permeability.

If the poles are placed in free space, then the equation (1.1) can be written as

$$F_o = \frac{\mu_o}{4\pi} \frac{m_1 m_2}{d^2} \qquad \text{.... (1.2)}$$

Where "μ_o" is called the permeability of free space (air or vacuum) and its value is $4\pi \times 10^{-7}$ henry/metre (or) NA^{-2}.

The absolute permeability of any medium as compared with the permeability of free space is known as the relative permeability (μ_r) of the medium.

Relative permeability,

$$\mu_r = \frac{\text{Absoslute permeability of the medium}}{\text{Permeability of free space}}$$

$$\text{i.e., } \mu_r = \frac{\mu}{\mu_o} \qquad \text{.... (1.3)}$$

This is a pure ratio and therefore it does not possess units.

From equation (1.3) we can write, $\mu = \mu_r \mu_o$ and hence the equation (1.1) can be written as

$$F = \frac{\mu_r \mu_o}{4\pi} \frac{m_1 m_2}{d^2} \qquad \text{.... (1.4)}$$

On comparing equations (1.2) and (1.4) ; we can write $\mu_r = 1$ for free space.

When m_1 and m_2 are in Am the equation (1.2) can be written as

$$F_o = \frac{4\pi \times 10^{-7}}{4\pi} \frac{m_1 m_2}{d^2} \quad \text{(in Newton)}$$

$$= 10^{-7} \frac{m_1 m_2}{d^2} \qquad \text{.... (1.5)}$$

The unit pole can be defined from the above expression.

If $m_1 = m_2 = 1$ Am and d=1m then $F_o = 10^{-7}$ N

Unit pole

The unit pole is that pole which experiences a repulsive force of 10^{-7} N when kept at a distance of 1 metre from a similar pole in air or vaccum.

Example-1.1

Two magnetic poles of strengths 40Am and 10 Am are separated by a distance of 20 cm in air. Find the force between them. If the distance is reduced to 10 cm, find the force.

Solution :

i) Pole strengths of magnetic poles,

$m_1 = 40Am$ and $m_2 = 10\,Am$

Distance between the poles, d = 20cm = 0.2m

Force between the magnetic poles, $\frac{\mu_0}{4\pi}\frac{m_1m_2}{d^2}$

$$= \frac{4\pi\times10^{-7}}{4\pi}\times\frac{40\times10}{(0.2)^2} = 10^{-3}N$$

ii) Distance between the poles, d^1 = 10 cm = 0.1m

Force between the magnetic poles,

$$F^1 = \frac{\mu_0}{4\pi}\frac{m_1m_2}{(d^1)^2}$$

$$= \frac{4\pi\times10^{-7}}{4\pi}\times\frac{40\times10}{(0.1)^2} = 4\times10^{-3}\,N.$$

Example-1.2

Two magnetic poles, one of which is three times as strong as the other, exert on each other, a force equal to 3×10^{-3} N when separated by a distance of 10 cm. Find the strength of each pole.

Solution:

Pole strengths, $m_1 = m$ and $m_2 = 3m$

Distance between poles, d = 10 cm = 0.1 m

Force on each pole, $F = 3\times10^{-3}$ N

Force between the poles, $F = \frac{\mu_0}{4\pi}\frac{m_1m_2}{d^2}$

$$\Rightarrow 3\times10^{-3} = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{m.3m}{(0.1)^2}$$

$$\Rightarrow 3m^2 = 300 \text{ (or) } m = 10$$

$\therefore$ The pole strengths are 10 A-m and 30 A-m.

1.3 MAGNETIC FIELD

Around a magnet at all points, the effect of magnet is felt. When another magnet or magnetic pole is brought into this space, a force acts on it. ***The space surrounding a magnet in which its effect is felt is called magnetic field region.*** If magnetic material like iron or nickel is placed in that space of the field a magnetic induction force acts on it. *Thus a magnetic field can also be defined as that which can exert a magnetic force and can produce magnetic induction in the matter placed in it.* **The magnetic induction at a point can also be defined as the force acting on unit pole placed at that point.**

1.4 MAGNETIC INDUCTION AT A POINT DUE TO MAGNETIC POLE

Consider a magnetic pole of strength 'm' at point 'O'. Consider a point 'P' at a distance 'r' from 'O'. To find the magnetic induction at the point 'P', imagine a unit north pole at P.

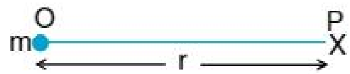


Fig 1.2 Magnetic induction due to magnetic pole

Force on unit north pole at $P = \frac{\mu_o}{4\pi}\frac{m\times1}{r^2}$ newton

Force on unit north pole at 'P' gives the intensity of magnetic induction at that point.

$\therefore$ Magnetic induction at P is

$$B = \frac{\mu_o}{4\pi}\frac{m}{r^2} \text{ newton/amp–metre (or) Tesla (T)} \quad ..... (1.6)$$

- The direction of magnetic induction is towards the pole if the pole is south and away from the pole if it is north.

Example-1.3

A N-pole of a very long magnetic needle is placed at a distance of 20 cm from a point 'P'. If the pole strength of the magnetic needle is 40 Am what is the magnetic induction at the point 'P' ?

Solution :

Since the magnetic needle is very long, the influence of S - pole can be neglected.

Pole strength of the needle, m = 40 Am.

Distance between the N-pole and the point P is $r = 20\text{ cm} = 20\times10^{-2}$ m

$\therefore$ Magnetic induction due to the N-pole at P is

$$B = \frac{\mu_o}{4\pi}\frac{m}{r^2} = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{40}{(20\times10^{-2})^2} = 10^{-4}T$$

1.5 MAGNETIC FIELD LINES

The magnetic field of magnet can be mapped by drawing magnetic lines of force. The concept of magnetic lines of force has been developed to visualise the nature (whether the field is uniform or not) and to estimate the intensity of the magnetic field at different points in a region surrounded by a magnet.

The path along which an isolated unit north pole would tend to move in a magnetic field is known as a magnetic line of force (or) simply magnetic "field line".

Since isolated magnetic pole does not exist, a small magnetic needle is used to plot the magnetic line of force.

The magnetic lines of force around a bar magnet can be traced using a magnetic needle as shown in the figure.

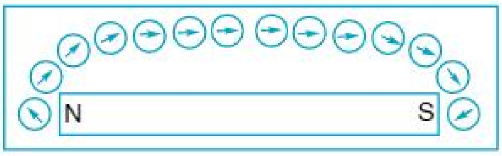


Fig 1.3 Magnetic line of force with magnetic needle

Characteristics of field lines

(i) Lines of force are directed away from a north pole and are directed towards a south pole. A line of force starts from a north pole and ends at a south pole if they are isolated poles.

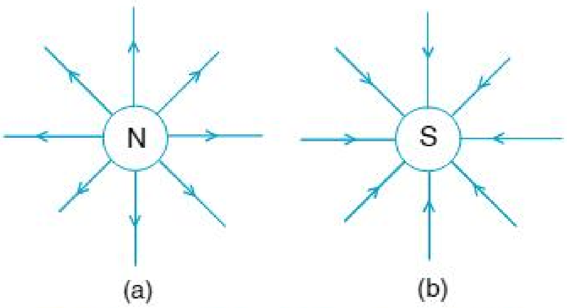


Fig 1.4 Magnetic lines of force due to isolated north & south poles

(ii) Tangent, at any point to the line of force gives the direction of magnetic field at that point.

(iii) Two lines of force never intersect each other. If the two lines of force intersect, at the intersecting point the field should have two directions which is impossible.

(iv) The lines of force tend to contract longitudinally or length wise i.e., they possess longitudinal strain. Due to this property the two unlike poles attract each other.

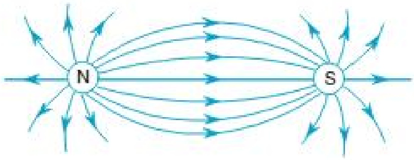


Fig 1.5 Magnetic lines of force between two unlike poles

(v) The lines of force tend to exert lateral pressure, i.e., they repel each other laterally. Due to this property the two similar poles repel each other.

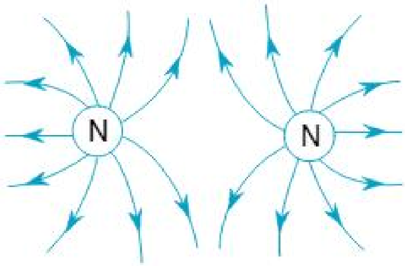


Fig 1.6 Magnetic lines of force between two like poles

(vi) If in any region in a combined field due to two magnets there are no lines of force, it follows that the resultant force in that region is zero. Such regions are called null or neutral points.

(vii) Lines of force in a field represent the strength of the field at a point in the field. Lines of force crowded themselves in regions where the field is intense and spread themselves out at places where the field is week.

(viii) Lines of force have a tendency to pass through magnetic substances. They prefer to pass through ferro magnetic materials.

(ix) If the magnetic lines of force are straight, parallel, and equally spaced, the magnetic field is said to be uniform.

The number of magnetic lines of induction passing normally through unit area round a point is equal to magnetic induction (B) at that point (or) magnetic flux density

If ϕ is the total number of lines of induction passing normally through a surface, then

$B = \frac{\phi}{A}$ where 'A' is the area of the surface.

Here ϕ is called magnetic flux and it is expressed in weber.

$\therefore \phi = BA$ .... (1.7)

In vector form $\phi = \overline{B} \cdot \overline{A}$

- The C.G.S unit of magnetic flux is maxwell.
- 10^8 maxwell = 1 weber
- The SI unit of flux density is weber/m^2 (or) Tesla (T).
- A magnetic field having a magnetic induction of 1 $NA^{-1}m^{-1}$ has a flux density of 1wb m^{-2}.

1.6 UNIFORM AND NON-UNIFORM MAGNETIC FIELDS

A magnetic field can be represented by a set of magnetic lines of force. A magnetic field is of two types.

(i) Uniform magnetic field

A magnetic field is said to be uniform if it has same strength in magnitude and direction at all the points. In case of uniform magnetic field the lines of force are parallel to each other and equally spaced as shown in Fig 1.7(a).

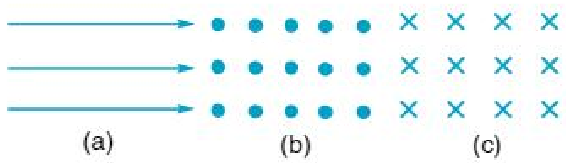


Fig 1.7 Representation of uniform magnetic field

- Fig 1.7 (b&c) represents uniform magnetic field at right angles to the plane of the paper directed outwards and inwards respectively.

  The magnetic field between two strong electromagnetic pole pieces is an example of uniform magnetic field.

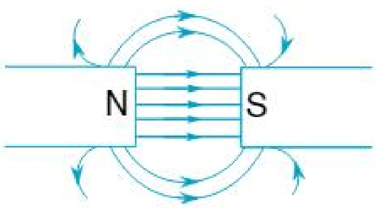


Fig 1.8 Magnetic field lines of force in uniform field

- The horizontal component of earth's magnetic field has constant magnitude and direction at a given place.

(ii) Non-uniform magnetic field

A magnetic field is said to be non-uniform if it has different field strength at different points in magnitude and direction.

- The magnetic field due to a bar magnet is an example to the non-uniform magnetic field.
- The non-uniform magnetic field between two like poles is as shown in the fig. (1.9)

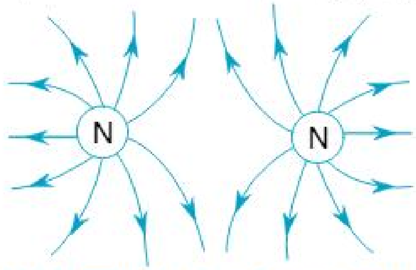


Fig 1.9 Magnetic field lines between like poles

Knowledge Plus 1.1

☺ Suppose we break apart a bar magnet the way we break a piece of chalk, can we isolate a single pole or monopole? Is there any formal way about this ?

☛ The simplest magnetic structure that can exist is a magnetic dipole. We know that magnetic monopoles do not exist, even if we break the magnet down to its individual atoms or its electrons and nuclei. Each fragment has a north pole and a south pole. Gauss's law for magnetic fields is a formal way of saying that magnetic monopoles does not exist. According to this law, the net magnetic flux ϕ_B through any closed Gaussian surface is zero $\phi_B = \oint \overline{B}.d\overline{s} = 0$.

We deal with Gauss's law for electric fields in electrostatics. Gauss's law for magnetic fields says that there can be no magnetic flux through the surface because there can be no net magnetic charge or individual magnetic poles enclosed by the surface.

1.7 MOMENT OF COUPLE ACTING ON A BAR MAGNET PLACED IN A UNIFORM MAGNETIC FIELD

Consider a uniform magnetic field of strength B. Let a bar magnet N S of length $2l$ and pole strength 'm' be suspended in it such that its axis makes an angle 'θ' with the field as shown in the figure (1.10).

The two poles N and S experience two equal and opposite forces 'mB'. These forces constitute a couple which tends to rotate the magnet in the direction of field.

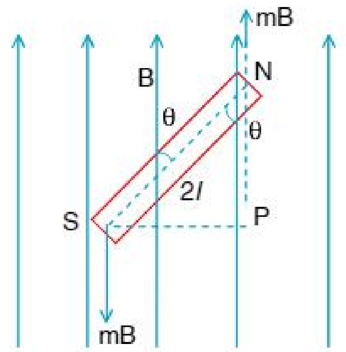


Fig 1.10 Couple acting on a bar magnet

The torque τ due to the couple or the moment of the couple is equal to the product of the force and the perpendicular distance between the two forces.

i.e., $\tau = mB \times AC$

$$= mB \times 2l \sin\theta \left[\begin{array}{l} \because \text{ In } \Delta NSP, \dfrac{NP}{NS} = \sin\theta \\ \text{or } NP = NS\sin\theta = 2l\sin\theta \end{array}\right]$$

$$= (m \times 2l)\, B \sin\theta \text{ or } \vec{\tau} = \vec{M} \times \vec{B} \qquad \text{.... (1.8)}$$

Here 'M' is the magnetic moment of the magnet.

- The torque on the magnet depends on the angle between the magnetic field and axis of the magnet.
- If $\theta = 0^o$ i.e., the magnet is parallel to the direction of the field, the torque on the magnet is equal to zero.
- If $\theta = 90^o$ i.e., the magnet is at right angles to direction of the field the torque on the magnet is maximum. $(\tau_{max} = MB)$.

Example-1.4

A bar magnet of length 10 cm and pole strength 2 Am is making an angle 60^o with a uniform field of induction 50T. Find the couple acting on it.

Solution :

Length of the bar magnet, $2l = 10$ cm $= 0.1$ m

pole strength, m = 2Am

magnetic induction, B = 50 T

Angle between the magnetic field and axis of the magnet, $\theta = 60^o$

$\therefore$ Couple acting on the bar magnet, $\tau = MB\sin\theta$

$\Rightarrow \tau = (2l \times m)B\sin\theta = (0.1 \times 2)50\sin 60^o$

$$= 0.2 \times 50 \times \frac{\sqrt{3}}{2} = 8.66 \text{ N–m}$$

Example-1.5

When a bar magnet is placed at 90^o to a uniform magnetic field, it is acted upon by a couple which is maximum. For the couple to be half of the maximum value, at what angle should the magnet be inclined to the magnetic field (B) ?

Solution :

We know that, $\tau = MB\sin\theta$

If $\theta = 90^\circ$ then $\tau_{max} = MB$ ..... (1)

$\dfrac{\tau_{max}}{2} = MB\sin\theta$ ..... (2)

From equations (1) and (2)

$$2 = \frac{1}{\sin\theta} \text{ or } \sin\theta = \frac{1}{2} \text{ or } \theta = 30^\circ$$

Example-1.6

If the moment of a magnet is 0.4 A–m^2 and force acting on each pole in a uniform magnetic field of induction 3.2×10^{-5} Wb/m^2 is 5.12×10^{-5} N, then find the distance between the poles of the magnet.

Solution :

Magnetic moment of the magnet, $M = 0.4$ A–m^2

Magnetic field of induction, $B = 3.2 \times 10^{-5}$ Wb/m^2

Force acting on the each pole of the bar magnet,

$F = 5.12 \times 10^{-5}$ N

Pole strength of the magnet,

$$m = \frac{F}{B} = \frac{5.12\times10^{-5}}{3.2\times10^{-5}} = 1.6 \text{ A} - \text{m}$$

Distance between the poles of the magnet (or) length of the magnet, $2l = \dfrac{M}{m} = \dfrac{0.4}{1.6} = 0.25$ m $= 25$ cm

Example-1.7

A magnetised needle of magnetic moment 4.8×10^{-2} Am^2 is placed at 30° with the direction of a uniform magnetic field of 3×10^{-2}T. If the needle is pivoted through its centre of mass and is free to rotate in the plane of the magnetic field, find the angular frequency of small oscillations. The moment of inertia of the needle about its axis of rotation is 2.25×10^{-5} Kg–m^2.

Solution :

$$\text{Angular frequency, } \omega = \sqrt{\frac{MB}{I}} = \sqrt{\frac{4.8\times10^{-2}\times3\times10^{-2}}{2.25\times10^{-5}}} = 8 \text{ rad s}^{-1}.$$

Example-1.8

A magnetic dipole is under the influence of two magnetic fields. The angle between the field direction is 60° and one of the fields has a magnitude of 1.2×10^{-2}T. If the dipole comes to stable equilibrium at an angle of 15° with this field, what is the magnitude of the other field?

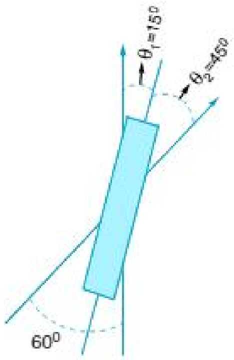
Solution :

Here $B_1 = 1.2 \times 10^{-2}$ T

Inclination of dipole with B_1 is $\theta_1 = 15^0$

Therefore, inclination of dipole with B_2 is

$$\theta_2 = 60^0 - 15^0 = 45^0$$



As the dipole is in equilibrium, therefore the torque on the dipole due to the two fields are qual and opposite. If M is magnetic dipole moment of the dipole, then

$$MB_1 \sin\theta_1 = MB_2 \sin\theta_2 \text{ or } B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2}$$

$$= \frac{1.2\times10^{-2}\times0.2588}{0.707} = 4.39\times10^{-3}\text{T}$$

1.8 OSCILLATIONS OF BAR MAGNET IN UNIFORM MAGNETIC FIELD

A magnet of magnetic moment M is suspended freely so as to rotate in a horizontal plane. Another magnet is brought near one of the poles of the suspended magnet and removed at once. If the suspended magnet is displacement through a small angle 'θ' from its equilibrium position, it is acted on by a couple,

$$\tau = M\, B_H \sin\theta \qquad \text{..... (1.9)}$$

Where B_H is the horizontal component of the earth's magnetic field. The magnet oscillates about its equilibrium position until it finally comes to rest.

If 'I' is the moment of inertia of the magnet about the axis of suspension, the restoring couple is $I\alpha$, where 'α' is the angular acceleration of the oscillating magnet.

At equilibrium position,

Deflecting couple = restoring couple

i.e., $MB\sin\theta = I\alpha$ (take $B_H = B$)

$$\text{or } \alpha = \frac{MB\sin\theta}{I} \qquad \text{..... (1.10)}$$

When 'θ' is very small, $\sin\theta \approx \theta$, then the equation (1.10) can be written as $\alpha = \frac{MB\theta}{I}$

As the angular acceleration 'α' is directly proportional to the angular displacement and is directed towards the equilibrium position, the magnet performs simple harmonic motion. We can write,

$$\frac{\theta}{\alpha} = \frac{I}{MB} \qquad \text{..... (1.11)}$$

Time period "T" for the simple harmonic motion is

$$T = 2\pi\sqrt{\frac{\theta}{\alpha}} \qquad \text{.... (1.12)}$$

Now from equations (1.11) and (1.12) we can write,

$$T = 2\pi\sqrt{\frac{I}{MB}} \qquad \text{.... (1.13)}$$

Example-1.9

A magnet vibrates in a magnetic field of strength 10^{-4} T. If the moment of the magnet is 10^{-1} A– m^2 and the moment of inertia about the axis of rotation is 10^{-5} kg m^2, then find the time period of oscillation in seconds?

Solution :

Period of oscillation,

$$T = 2\pi\sqrt{\frac{I}{MB}} = 2\pi\sqrt{\frac{10^{-5}}{10^{-1}\times10^{-4}}} = 2\pi \text{ sec}.$$

Example-1.10

A magnetic needle is pivoted through its centre of mass and is free to rotate in a plane containing uniform magnetic field 200×10^{-4} T. When it is displaced slightly from the equilibrium it makes 20 oscillations per second. If the moment of inertia of the needle about the axis of oscillation is 0.75×10^{-5} kg m^2, find the magnetic moment of the needle.

Solution :

Magnetic field induction,

$B = 200\times10^{-4}\,T = 2\times10^{-2}\,T$

Frequency $\nu = 2Hz$

Moment of inertia, $I = 0.75\times10^{-5}\,kgm^2$

Frequency of oscillation, $\nu = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$

On squaring we can write,

$$\nu^2 = \frac{1}{4\pi^2}\times\frac{MB}{I} \Rightarrow M = \frac{4\pi^2\times\nu^2\times I}{B}$$

$$= \frac{4(3.14)^2\times2^2\times0.75\times10^{-5}}{2\times10^{-2}} = 0.059$$

$\therefore$ Magnetic moment of the needle, $M = 0.06Am^2$

Example-1.11

Two bar magnets placed together in a vibration magnetometer take 3 seconds for 1 vibration. If one magnet is reversed, the combination takes 4 seconds for 1 vibration. Find the ratio of their magnetic moments.

Solution :

Given that, $T_1 = 3s$ and $T_2 = 4s$

$$\frac{M_1}{M_2} = \frac{T_2^2+T_1^2}{T_2^2-T_1^2} = \frac{4^2+3^2}{4^2-3^2} = \frac{16+9}{16-9} = \frac{25}{7}$$

or $\frac{M_1}{M_2} = 3.57$

Example-1.12

A bar magnet makes 40 oscillations per minute in a vibration magnetometer. An identical magnet is demagnetised completely and is placed over the magnet in the magnetometer. Calculate the time taken for 40 oscillations by this combination. Ignore induced magnetism.

Solution :

In the first case frequency of oscillation, $\nu = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$

In the second case frequency of oscillation,

$$\nu^1 = \frac{1}{2\pi}\sqrt{\frac{MB}{2I}} \Rightarrow \frac{\nu^1}{\nu} = \frac{1}{\sqrt{2}}, \frac{T^1}{T} = \sqrt{2} \text{ (or) } T^1 = \sqrt{2}T$$

(or) $40T^1 = \sqrt{2}\times40T$

(or) $t^1 = \sqrt{2}t = \sqrt{2}$ minute $= 1.414$ minute

Example-1.13

The time period of a thin bar magnet is T. It is cut into 'n' equal parts by cutting it normal to its length. What will be the time period of each of them in the same field?

Solution :

The expression for time period of a magnet,

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$\therefore T \propto \sqrt{\frac{I}{M}}$..............(1) [$\because$ B is same]

But $I = m\left(\frac{l^2+b^2}{12}\right)$, where m is mass

For a thin bar magnet breadth b <<l, So neglect b^2 in the above expression .

$\therefore I = \frac{ml^2}{12}$ or $I \propto ml^2$...............(2)

From eq's (1) and (2); $T \propto \sqrt{\frac{ml^2}{M}}$

$$\therefore \frac{T_2}{T_1} = \frac{l_2}{l_1}\sqrt{\frac{m_2}{m_1}\times\frac{M_1}{M_2}}$$

But $l_2 = \frac{l_1}{n}; m_2 = \frac{m_1}{n}$ and $M_2 = \frac{M_1}{n}$

$$\therefore \frac{T_2}{T_1} = \frac{1}{n}\sqrt{\frac{1}{n}\times n} = \frac{1}{n} \Rightarrow T_2 = \frac{T}{n}$$

Example-1.14

A vibration magnetometer consists of two identical bar magnets, placed one over the other, such that they are mutually perpendicular and bisect each other. The time period of oscillation in a horizontal magnetic field is 4 second. If one of the magnets is taken away, find the period of oscillation of the other in the same field.

Solution :

For a vibration magnetometer, we know that

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Let M be the magnetic moment and I be the moment of inertia of each magnet then,

$$M^{I} = \sqrt{M^2 + M^2} = \sqrt{2}M \text{ and } I^{I} = I + I = 2I$$

$$\therefore T^{I} = 2\pi\sqrt{\frac{2I}{\sqrt{2}MB}} = 2\pi \times \sqrt{\frac{\sqrt{2}I}{MB}} \quad .... (1)$$

When one of the magnet is taken away then,

$$M^{I} = M, I^{II} = I$$

$$\therefore T^{II} = 2\pi\sqrt{\frac{I}{MB}} \quad .... (2)$$

$$\frac{\text{eq }(2)}{\text{eq }(1)} \Rightarrow \frac{T^{II}}{T^{I}} = \frac{1}{(2)^{1/4}} \text{ or}$$

$$T^{II} = \frac{4}{(2)^{1/4}} = 3.36 \text{ sec}.$$

1.9 MAGNETIC MOMENT OF THE MAGNET

If $B = 1$ and $\theta = 90^{o}$ then $\tau = M$ ..... (1.14)

Magnetic moment of a magnet is the moment of couple acting on the magnet when placed at right angles to the direction of a uniform magnetic induction field of unit strength.

In such case, Force on each pole = $m \times 1$

Perpendicular distance between the two forces = NS = $2l$

$\therefore$ Torque acting on the magnet,

$$\tau = m \times 1 \times 2l \times \sin 90^{o}$$

$$\tau = m \times 2l \quad ..... (1.15)$$

From equations (1.14) and (1.15), we can write,

$M = m \times 2l$

Thus, the magnetic moment of a magnet is also defined as the product of its pole strength and the magnetic length of the magnet.

- Magnetic moment of a magnet is a vector quantity and is directed from south pole to north pole along the axis of magnet.

Fig 1.11 Direction of magnetic moment

- Since $\vec{M}$ and $\vec{B}$, both are vectors the torque acting on a magnet suspended in the magnetic field can be expressed as

$$\vec{\tau} = \vec{M} \times \vec{B} \quad ..... (1.16)$$

- The direction of $\overline{\tau}$ can be obtained by applying right hand thumb rule.
- The unit of magnetic moment in SI is joule/tesla (or) $Nm^3\ Wb^{-1}$ (or) Am^2

Example-1.15

The magnetic moment of a bar magnet of length 20cm is $3.6\times10^{-6} A-m^2$. The magnetic length is 90% of its geometric length. Then find the pole strength of the magnet.

Solution :

Magnetic moment of the bar magnet,

$M = 3.6 \times 10^{-6}\ A-m^2$

Given that magnetic length is 90% of its geometric length.

$$\therefore \text{ Magnetic length, } 2l = \frac{90}{100} \times 20 = 18\text{cm}$$

Pole strength of the magnet,

$$m = \frac{M}{2\ell} = \frac{3.6\times10^{-6}}{18\times10^{-2}} = 2\times10^{-5}\ A-m$$

Example-1.16

In a hydrogen atom, the electron revolves round the nucleus 6.8×10^{15} times per second in an orbit of radius $0.53\overset{0}{A}$. What is equivalent magnetic moment? (Given that $e = 1.6 \times 10^{-19}C$).

Solution :

The electron revolving in an orbit is equivalent to a current loop. The magnitude of current is given by,

$$I = \frac{e}{T} = e \times \frac{1}{T} = ef = (1.6\times10^{-19})(6.8\times10^{15})$$

$$= 1.1 \times 10^{-3}\ A$$

Magnetic moment of equivalent current loop is $M = nIA$

Here $n = 1$; $I = 1.1 \times 10^{-3}$ A and

area, $A = \pi(0.53\times10^{-10})^2$

$$\therefore M = (1)(1.1\times10^{-3})[\pi(0.53\times10^{-10})^2]$$

$$= 9.7\times10^{-24}\ Am^2$$

1.10 RESULTANT MAGNETIC MOMENT WHEN TWO BARMAGNETS ARE COMBINED

a)

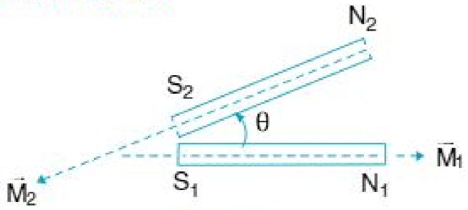


Fig 1.12 (a)

When two bar magnets of moments M_1 and M_2 are joined so that their like poles touch each other and their axes are at an angle 'θ' the resultant magnetic moment of the combination 'M^1' is given by

$$M^1 = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\theta}$$

b)

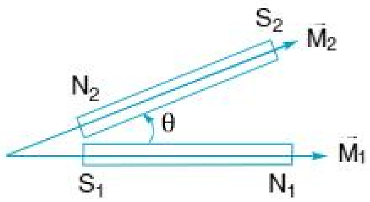


Fig 1.12 (b)

When two bar magnets of moments M_1 and M_2 are joined so that their unlike poles touch each other and their axes are at an angle 'θ' the resultant magnetic moment

$$M^1 = \sqrt{M_1^2 + M_2^2 - 2M_1M_2\cos\theta}$$

c)

Fig 1.12 (c)

When two bar magnets of moments M_1 and M_2 ($M_1 > M_2$) are placed coaxially with like poles in contact then resultant magnetic moment, $M^1 = M_1 - M_2$

d)

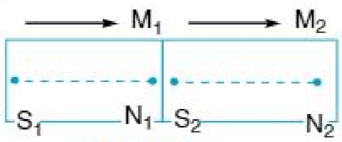


Fig 1.12 (d)

When two bar magnets of moments M_1 and M_2 ($M_1 > M_2$) are placed coaxially with unlike poles in contact, the resultant magnetic moment, $M^1 = M_1 + M_2$

e)

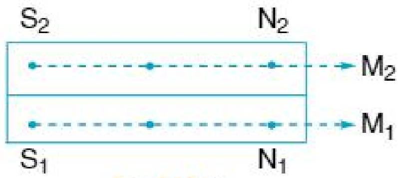


Fig 1.12 (e)

When two bar magnets of magnetic moments M_1 and M_2 are placed one over the other with like poles on the same side, then resultant magnetic moment, $M^1 = M_1 + M_2$

f)

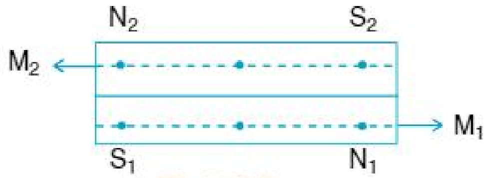


Fig 1.12 (f)

When two bar magnets of magnetic moments M_1 and M_2 are placed one over the other with unlike poles on the same side, then resultant magnetic moment, $M^1 = M_1 - M_2$ or $M_2 - M_1$.

g)

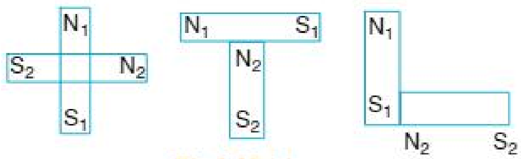


Fig 1.12 (g)

When two bar magnets of magnetic moments M_1 and M_2 are placed right angles to each other then resultant magnetic moment, $M^1 = \sqrt{M_1^2 + M_2^2}$.

h)

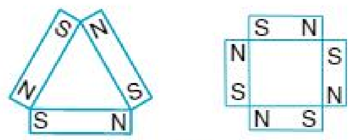


Fig 1.12 (h)

When identical magnets are arranged to form a closed figure like a triangle (or) square with unlike poles at each corner, then resultant magnetic moment, $M^1 = 0$.

Example-1.17

Two identical magnets are placed perpendicular to each other with their unlike poles in contact. If each magnet has a magnetic moment 'M', what is the magnetic moment of the combination?

Solution :

Magnetic moment is a vector.

It acts along South to North.

The resultant of two perpendicular vectors is obtained by using parallelogram law of vectors.

Here, $M_1 = M_2 = M$

$\therefore$ New magnetic moment

$= \sqrt{M^2 + M^2 + 2M.M\cos\theta} = \sqrt{M^2 + M^2 + 0}$

New magnetic moment $= \sqrt{2}\,M$

1.11 CHANGE IN MAGNETIC MOMENT DUE TO BENDING OF MAGNETS

a) When a thin bar magnet of magnetic moment M is bent into an arc of a circle subtending an angle θ at the centre of the circle, the magnetic moment becomes $M^1 = \frac{2M\sin\theta/2}{\theta}$ (θ must be in radians).

b) When a long thin magnet of magnetic moment 'M' is bent into a semi circle, the new magnetic moment $M^1 = \frac{2M}{\pi}$.

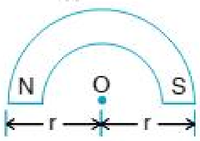


Fig 1.13 (a)

c) When a magnet in the form of an arc of a circle making an angle 'θ' at the centre having magnetic moment 'M' is straightened the new magnetic moment is given by

$$M^1 = \frac{M\theta}{2\sin\theta/2} \text{ (θ must be in radians)}$$

Note : If a semi circular magnet of moment 'M' is straightened, the new magnetic moment, $M^1 = \frac{\pi M}{2}$

d) When a bar magnet is bent its pole strength remains same but magnetic length decreases therefore magnetic moment decreases.

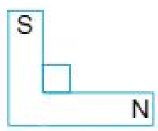


Fig 1.13 (b)

e) When a long thin magnet of magnetic moment 'M' is bent in the form of a quadrant of a circle, the new magnetic moment, $M^1 = \frac{2\sqrt{2}M}{\pi}$

When thin magnetic needle of magnetic moment (M) is bent at the middle, so that the two parts are perpendicular, its new magnetic moment $M^1 = \frac{M}{\sqrt{2}}$.

f) When a thin magnet of length '$2l$' and magnetic moment 'M' is bent at its midpoint such that the two equal parts making an angle 'θ' with each other then effective length becomes $2l\sin\theta/2$.

- i) When $\theta = 60°$ then effective length $= \frac{2l}{2}$ and new magnetic moment $M^1 = \frac{M}{2}$

  ii) When $\theta = 90°$ then effective length $= \frac{2l}{2}$ and new magnetic moment $M^1 = \frac{M}{\sqrt{2}}$.

  iii) When $\theta = 120°$ then effective length $= \frac{2l}{\sqrt{2}}$ and new magnetic moment, $M^1 = \frac{M\sqrt{3}}{2}$

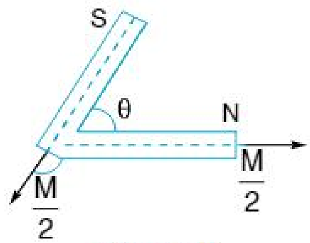


Fig 1.13 (c)

g) When a bar magnet of magnetic moment is bent in '⊔' shape with equal arm lengths, the new magnetic moment, $M^1 = \frac{M}{3}$.

h) When a bar magnet is twisted along its axis, the two poles of the bar magnet are comes closer. As a result the magnetic moment decreases.

Example-1.18

A bar magnet of magnetic moment M_1 is suspended by a wire in a magnetic field. The tip of the wire is rotated through 180^0, then the magnet rotated through 45^0. Under similar conditions the magnet of magnetic moment M_2 is rotated through 30^0. Then find the ratio of M_1 & M_2.

Solution :

We know that torque, $\tau = MB\sin\theta$

But $\tau = C\phi$.

$\therefore$ $C\phi = MB\sin\theta$

For first magnet,

$C \times 180 = M_1 B\sin 45^0$ ..... (1)

For second magnet,

$C \times 180 = M_2 B\sin 30^0$ ..... (2)

From eqs. (1) and (2)

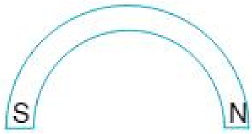
$M_1 B\sin 45^0 = M_2 B\sin 30^0 \Rightarrow \frac{M_1}{M_2} = \frac{\sin 30^0}{\sin 45^0}$

$\Rightarrow \frac{M_1}{M_2} = \frac{1/2}{1/\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\therefore M_1 : M_2 = 1 : \sqrt{2}$

Example-1.19

A bar magnet of magnetic moment M is bent into a semicircle. What is its new magnetic moment?



Solution :

Initial magnetic moment , $M = 2lm$

Where '$2l$' is the length of the magnet and 'm' is the pole strength.

When the magnet is bent in the form of a semi circle of radius 'r' the distance between the poles = 2r

But the length of the magnet, $2l = \pi r$

$\Rightarrow r = \frac{2l}{\pi}$

$\therefore$ Length of the new magnet = Distance between the poles = $2.\frac{2l}{\pi}$

New magnetic moment, $M^1 = 2 \times \frac{2l}{\pi} \times m = \frac{2M}{\pi}$.

1.12 WORK DONE IN DEFLECTING A MAGNET

Let a magnet of magnetic moment M be placed in a uniform magnetic field of induction B. Assume the axis of magnet makes an angle θ with the direction of B.

The moment of the couple acting on the magnet $= MB\sin\theta$.

The additional work done in rotating the magnet through an additional angle $d\theta$ is

$dW = MB\sin\theta(d\theta)$

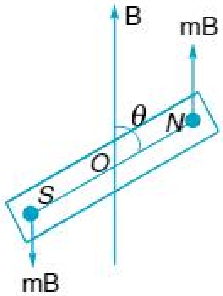


Fig 1.14 Work done in deflecting a magnet

Hence the work done in rotating the magnet from θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta(d\theta)$$

$$= MB\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= MB\left[\cos\theta_1 - \cos\theta_2\right]$$

$\therefore$ Work done or magnetic potential energy stored

$= MB\left[\cos\theta_1 - \cos\theta_2\right]$ .... (1.17)

Let $\theta_1 = 90°$ and $\theta_2 = \theta$. Then

$W = -MB\cos\theta$ 1.18

This workdone is stored in the form of potential energy 'U'

$\Rightarrow U = -MB\cos\theta$

and in vector form $U = -\vec{M}.\vec{B}$ 1.19

If $\theta = 0°$ then $U = -MB$

If $\theta = 90°$ then $U = 0$

If $\theta = 180°$ then $U = MB$

So potential energy of magnet is minimum when $\theta = 0°$ and hence it is in stable equilibrium.

For $\theta = 180°$ its potential energy is maximum and hence it is in unstable equilibrium.

We can get expression U in another way

$$U = \int \tau(\theta)\, d\theta = \int MB\sin\theta\, d\theta$$

$$= -MB\cos\theta = -\overline{M}\cdot\overline{B}$$

Example-1.20

A magnet is suspended at an angle 60^0 in an external magnetic field of 5×10^{-4} T. What is the work done by the magnetic field in bringing it in its direction? [The magnetic moment = 20 A-m^2]

Solution :

Work done by the magnetic field,

$W = -MB(\cos\theta_1 - \cos\theta_2)$

Here $\theta_1 = 60^0$ and $\theta_2 = 0$

$\therefore W = -20 \times 5 \times 10^{-4}\left[\cos 60^0 - \cos 0\right]$

$= -10^{-2}\left[\frac{1}{2} - 1\right] = 5 \times 10^{-3}\,J$.

1.13 MAGNETIC INDUCTION DUE TO A BAR MAGNET ON ITS AXIAL LINE

A line passing through the north and south poles of a magnet is known as the axial line of the magnet.

NS is a bar magnet of pole strength 'm', magnetic length '2l' and magnetic moment 'M'. 'P' is a point on the axial line of the magnet at a distance 'd' from the mid point 'O' of the magnet.

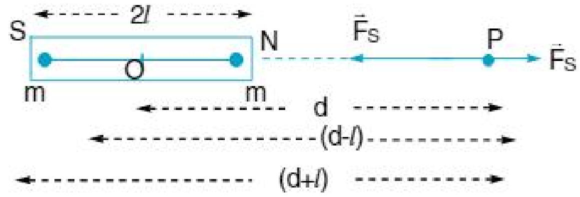


Fig 1.15 Intensity on the axial line

To determine the magnetic induction field strength at 'P', a unit north pole is imagined at P.

According to coulomb's law, force acting on unit north pole due to north pole of magnet is

$$F_N = \frac{\mu_o}{4\pi}\frac{m \times 1}{(d-l)^2} \text{ in the direction of NP.}$$

Similarly the force acting on unit north pole due to south pole of the magnet is

$$F_S = \frac{\mu_o}{4\pi}\frac{m \times 1}{(d+l)^2} \text{ in the direction of PS.}$$

These two forces are opposite and acts along the axial line. The resultant of these two forces gives the magnetic induction field strength at P.

$\therefore B = F_N - F_S$ in the direction of NP.

$$= \frac{\mu_o}{4\pi}\frac{m}{(d-l)^2} - \frac{\mu_o}{4\pi}\frac{m}{(d+l)^2}$$

$$= \frac{\mu_o m}{4\pi}\left[\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right]$$

$$= \frac{\mu_o m}{4\pi}\left[\frac{(d+l)^2 - (d-l)^2}{(d^2 - l^2)^2}\right]$$

$$= \frac{\mu_o m}{4\pi}\frac{4dl}{(d^2 - l^2)^2}$$

$$= \frac{\mu_o}{4\pi}\frac{2 \times 2lm \times d}{(d^2 - l^2)^2}$$

$$= \frac{\mu_o}{4\pi}\frac{2Md}{(d^2 - l^2)^2} \quad \text{..... (1.20)}$$

where $2lm = M$ is the magnetic moment.

The direction of the magnetic induction on the axial line is always in the same direction as the magnetic moment.

In case of short bar magnet, l<<d, hence $l^2/d^2 \simeq 0$ can be neglected.

$\therefore$ Magnetic induction at a point on the axial line of a short magnet is $B = \frac{\mu_o}{4\pi}\frac{2M}{d^3}$ ..... (1.21)

Example-1.21

The magnetic moment of a bar magnet is $45\,Am^2$ and its length is 10 cm. Calculate the magnetic field induction at a point 10 cm away from the centre of the magnet on the axial line.

Solution :

Length of the magnet, $2l = 10\text{ cm} = 10 \times 10^{-2}\text{ m}$

$\Rightarrow l = 5 \times 10^{-2}\text{ m}$

Distance of the point on axial line, d = 10 cm

$= 10 \times 10^{-2}\text{ m}$

Magnetic moment, $M = 45\,Am^2$

$\therefore$ Magnetic induction on the axial line of the bar magnet is $B = \frac{\mu_o}{4\pi}\frac{2Md}{(d^2 - l^2)^2}$

$$B = \frac{4\pi \times 10^{-7}}{4\pi}\frac{2 \times 45 \times 10 \times 10^{-2}}{[(10 \times 10^{-2})^2 - (5 \times 10^{-2})^2]^2}$$

$= 16 \times 10^{-3}\,T$.

1.14 MAGNETIC INDUCTION DUE TO A BAR MAGNET ON ITS EQUATORIAL LINE

A straight line passing through the midpoint of a bar magnet and perpendicular to its axial line is called the equatorial line of the magnet.

NS is a bar magnet of pole strength m, magnetic length 2l and magnetic moment M. P is a point on the equatorial line of the magnet at a distance d from its midpoint O. To determine the strength of field at P, a unit north pole is imagined to be placed at P.

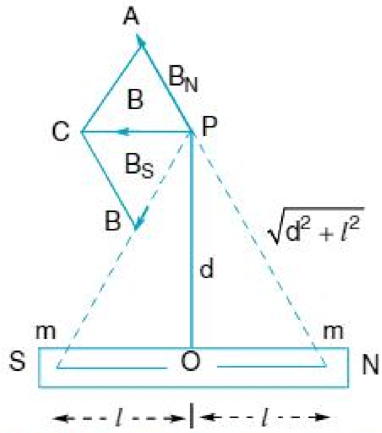


Fig 1.16 Intensity on equatorial line

The point P is equidistant from N and S and

$PN = PS = \sqrt{d^2+l^2}$

Magnetic induction at the point 'P' due to north pole of the magnet is

$$B_N = \frac{\mu_o}{4\pi}\frac{m\times 1}{(\sqrt{d^2+l^2})^2} = \frac{\mu_o}{4\pi}\frac{m}{(d^2+l^2)}$$

in the direction of PA

Similarly, magnetic induction at the point 'P' due to south pole of magnet is

$$B_s = \frac{\mu_0}{4\pi}\frac{m\times 1}{\left(\sqrt{d^2+l^2}\right)^2} = \frac{\mu_0}{4\pi}\frac{m}{(d^2+l^2)}$$

in the direction of PB

B_N and B_S can be vectorially represented along (PA and PB) the sides of the parallelogram PACB. Then PC the diagonal of the parallelogram gives the resultant field induction B. From the similar triangles PAC and NPS we have

$$\frac{PC}{NS} = \frac{PA}{NP} \text{ or } PC = \frac{PA}{NP}NS = \frac{B_N 2l}{\sqrt{d^2+l^2}}$$

$$\therefore\ PC = \frac{\mu_o}{4\pi}\frac{m.2l}{(d^2+l^2)\sqrt{d^2+l^2}}$$

$$\text{or } B = \frac{\mu_o}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} \qquad \text{.... (1.22)}$$

The direction of magnetic field induction on the equatorial line is always opposite to the direction of the magnetic moment.

In case of short bar magnet $l << d$ hence $\frac{l^2}{d^2}$ can be neglected.

$\therefore$ Magnetic induction at a point on the equatorial line of a short bar magnet is

$$B = \frac{\mu_o}{4\pi}\frac{M}{d^3} \qquad \text{..... (1.23)}$$

In the case of short magnet, the magnetic induction on the axial line is twice the magnetic induction of the field on the equatorial line due to the same magnet at the same distance.

Example-1.22

A bar magnet of length 0.1 m has a pole strength of 50 Am. Calculate the magnetic field at a distance of 0.2 m from its centre on its equatorial line.

Solution :

Length of the magnet, $2l = 0.1$ m or $l = 0.05$ m

Distance of the point on equatorial line, d = 0.2m

Pole strength, m = 50Am

Magnetic induction on the equatorial line of a bar magnet is,

$$B = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{m(2l)}{(d^2+l^2)^{3/2}}$$

$$B = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{50\times0.1}{[(0.2)^2+(0.05)^2]^{3/2}}$$

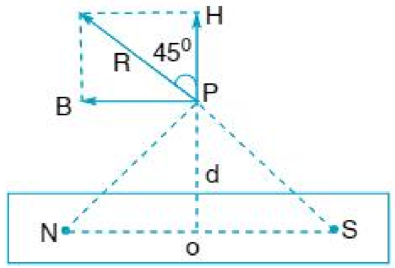
$$= 5.7\times10^{-5}\ T$$

Example-1.23

A short bar magnet of magnetic moment $5.25\times10^{-2} JT^{-1}$ is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet on the normal bisector is the resultant field inclined at 45^o with the earth's field. Magnitude of earth's field at that place is 0.42 G. [$1G=10^{-4}T$]

Solution :

Figure shows the conditions of the problem. Suppose P is the point on the normal bisector of the magnet where the resultant of B (due to magnet) and H is inclined at 45^0 with H. This is possible if magnitudes of B and H are the same.



$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$ ... for a short magnet

or $d^3 = \frac{\mu_0}{4\pi}\frac{M}{B} = 10^{-7} \times \frac{5.25\times10^{-2}}{0.42\times10^{-4}} = 125\times10^{-6}$

$\therefore\ d = (125\times10^{-6})^{1/3} m = 5\times10^{-2} m = 5 cm$

1.15 SUPERPOSITION OF MAGNETIC FIELDS

Earth's magnetic field is present every where and its horizontal component extends from south to north. When a magnet is placed any where its field gets super imposed over the earth's field, giving rise to resultant magnetic field. In this resultant magnetic field, there are certain points where the resultant magnetic induction field becomes zero. At these points. The horizontal component of earth's magnetic field exactly balances the field due to the magnet. These points are called null points *or* neutral points.

"The points in the magnetic field where the resultant magnetic induction field becomes zero are called null points.

or

The points in the region around a magnet, where its magnetic field is exactly neutralised by that due to earth's magnetic field are called null points".

a) North pole of the magnet pointing towards geographical north

When a magnet is placed in the magnetic meridian with its north pole facing geographic north. The field due to magnet (shown by continuous lines) and that of horizontal component of earth's magnetic field (shown by dotted lines) is shown in Fig (1.16).

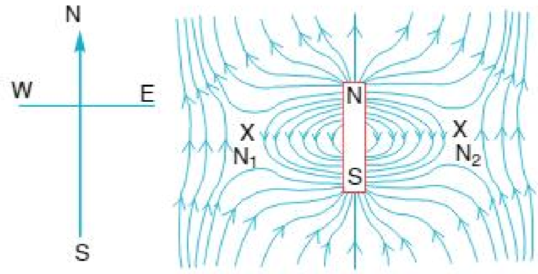


Fig 1.17(a) Magnetic lines of force when northpole of the magnet pointing towards geographic north

Results

i) Along the axial line on both sides. The two fields have same directions. The resultant magnetic field is increased.

ii) As we deviate from axial line, the two fields differ in directions.

iii) On the equatorial line the direction of the two fields are exactly opposite to each other.

iv) At N_1 and N_2 on the equatorial line. The magnetic induction field due to the magnet is exactly same as that of earth's horizontal component field. These points are called null points, If the average distance of N_1 and N_2 from the centre of the magnet is 'd' then

$$B_{magnet} = \frac{\mu_0}{4\pi}.\frac{M}{(d^2+l^2)^{3/2}}$$

$= B_H$ (earth's horizontal magnetic induction field)

$$\therefore \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = B_H \qquad ..... (1.24)$$

For short magnet $\frac{\mu_0}{4\pi}\frac{M}{d^3} = B_H$ ..... (1.25)

b) North pole of the magnet pointing towards geographic south

When a magnet is placed in the magnetic meridian with it's north pole facing geographic south, the field due to magnet (shown by continuous lines) and that of horizontal component of earth's magnetic field (shown by dotted lines) is shown in Fig 1.17(b).

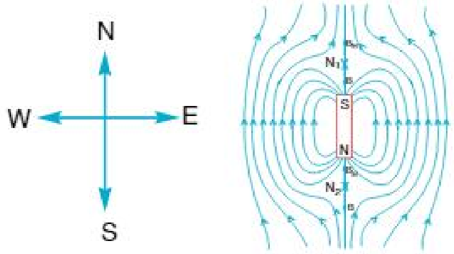


Fig 1.17(b) Magnetic lines of force when north pole of the magnet pointing towards geographic south

Results

i) The directions of the two fields (earth's horizontal component and the field due to the magnet) are exactly opposite to each other, on the axial line.

ii) As we deviate from the axial line, the two fields differ in direction.

iii) The directions of the two fields at all points on the equatorial line is the same.

iv) Along the axial line, the magnetic field due to magnet decreases in magnitude on moving away from the centre of the magnet. There will be points N_1 and N_2 situated at equal distances from the centre where the fields are exactly balanced by the earth's horizontal component field. These points are called null points.

At null points,

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2} = B_H \quad ..... (1.26)$$

(where B_H is earth's horizontal magnetic induction field)

For short magnet $\frac{\mu_0}{4\pi}.\frac{2M}{d^3} = B_H$ ..... (1.27)

c) Magnet placed perpendicular to the magnetic meridian

When a bar magnet is placed with its axial line perpendicular to the magnetic meridian with its north pole facing east of earth, the resultant magnetic field is shown in the Fig 1.17(c). Along a line making an angle of 45^0 with east - west line, there are two points (N_1 and N_2) where the resultant magnetic induction field is zero. Thus N_1 (on the N-W line) and N_2 (on the S-E line) are the null points.

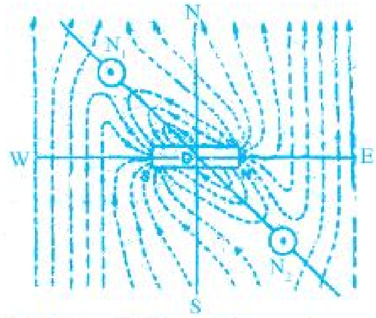


Fig 1.17(c) Magnetic lines of force when magnet placed perpendicular to the magnetic meridian

Example-1.24

Two like poles of strength 49×10^{-3}A–m and 9×10^{-3}A-m are separated by a distance of 10cm. Find the distance of the neutral point from the stronger pole where the magnetic induction due to the two poles will be zero.

Solution :

Let the distance of the neutral point from the weaker pole m_1 of the magnet be x then,

$$x = \frac{d}{\sqrt{\frac{m_2}{m_1}}+1} = \frac{10}{\sqrt{\frac{49\times10^{-3}}{9\times10^{-3}}}+1} = \frac{10}{\frac{7}{3}+1} = 3\text{cm}$$

Then the distance of the neutral point from stronger pole = 10cm – 3 cm = 7 cm

Example-1.25

A bar magnet is kept in the earth's magnetic field with its north pole pointing earth's north. The distance between the null points is 20 cm. If earth's horizontal magnetic field is 4×10^{-5}T, then find the magnetic moment of the magnet.

Solution :

The null points are formed on the equatorial line. At null point the resultant field is zero.

$$\therefore B_e = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

$$\Rightarrow 4\times10^{-5} = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{M}{10^{-3}}$$

$$\Rightarrow M = \frac{4\times10^{-5}\times10^{-3}}{10^{-7}} = 0.4\text{Am}^2$$

Example-1.26

A bar magnet of magnetic moment 0.4Am^2 is placed in the magnetic meridian with its north pole pointing north. A neutral point is obtained at a distance of 10 cm from the centre of the magnet. If the length of the magnet is also 10 cm, what is the value of horizontal component of earth's field at the place?

Solution :

Magnetic moment, $M = 0.4\ Am^2$

Distance of neutral point from centre of magnet,

$d = 10\ cm = 10 \times 10^{-2}\ m$

Length of the magnet,

$$2l = 10cm \Rightarrow \ l = 5cm = 5\times10^{-2}m$$

Horizontal component of earth's field = B_H

As north pole is pointing north, neutral points are obtained on equatorial line $B_H = B$

$$B_H = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}}$$

$$B_H = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{0.4}{\left[\left(10\times10^{-2}\right)^2+\left(5\times10^{-2}\right)^2\right]^{3/2}}$$

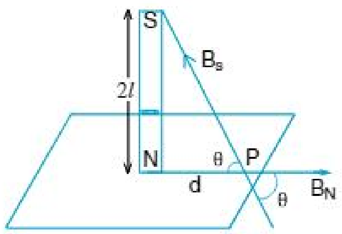
$= 2.86 \times 10^{-5}\ T$

Example-1.27

A 6 cm long bar magnet possessing magnetic dipole moment 0.3 A–m² is placed vertically on a horizontal wooden table. The north pole of the magnet touches the table. A neutral point is found on the table at a distance of 8 cm south of the magnet. Find the horizontal component of Earth's magnetic field.

Solution :

Magnetic field due to single pole of a magnet is given by



$$B = \frac{\mu_o}{4\pi}\frac{m}{d^2} \text{ where m is pole strength}$$

Field due to north pole,

$$B_N = \frac{\mu_o}{4\pi}\frac{m}{(d^2)} \text{ along NP}$$

Field due to south pole,

$$B_S = \frac{\mu_o}{4\pi}\frac{m}{(d^2+4l^2)} \text{ along PS}$$

Horizontal component of $B_S = B_S\cos\theta$

$$= \frac{\mu_o}{4\pi}\frac{m}{(d^2+4l^2)}\cdot\frac{d}{(d^2+4l^2)^{1/2}}$$

Resultant horizontal field

$$= \frac{\mu_o m}{4\pi}\left[\frac{1}{d^2}-\frac{d}{(d^2+4l^2)^{3/2}}\right]$$

$$\text{or, } B_H = \frac{\mu_o M}{4\pi(2l)}\left[\frac{1}{d^2}-\frac{d}{(d^2+4l^2)^{3/2}}\right]$$

(where magnetic moment $M = m \times 2l$)

On substituting numerical values, we get

$B_H = 3.81 \times 10^{-5}$ Tesla.

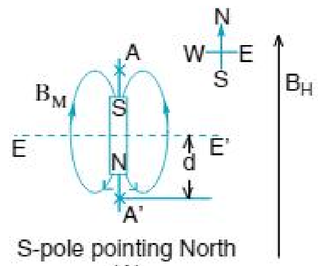
Example-1.28

A very small magnet is placed in the magnetic meridian with its S-pole pointing north. The null point is obtained 20 cm away from the centre of the magnet. What is the magnetic moment of the magnet if earth's field is 0.3×10^{-4} T?

Solution :

As magnetic dipole is placed with its S-pole pointing north and at neutral point the field of dipole (B_M) is cancelled by earth's field B_H, the neutral point will be on the axis of the dipole as shown in Fig. (A). And as for an axial point, $B_M = \frac{\mu_o}{4\pi}\frac{2M}{r^3}$, so for the neutral point,

$$\frac{\mu_o}{4\pi}\frac{2M}{r^3} = B_H \text{ i.e., } M = \frac{B_H\times r^3}{2\times10^{-7}} \quad \left[\text{as } \frac{\mu_o}{4\pi} = 10^{-7}\right]$$



S-pole pointing North

(A)

$$\text{So, } M = \frac{0.3\times10^{-4}\times(0.2)^3}{2}\times10^7 = 1.2\ \ A-m^2.$$

1.16 TANGENT LAW

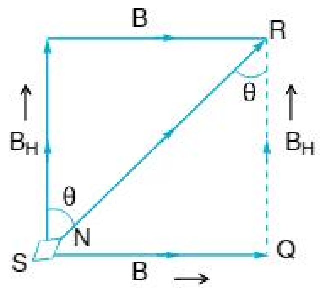


Fig 1.18 Magnetic needle in two uniform magnetic fields B & B_H

Consider a magnetic needle NS of length $2l$ suspended in two uniform fields B (magnetic induction due to external field) and B_H (horizontal component of earth's magnetic induction field) at right angles to each other. Under the action of two fields, the magnet comes to rest (equilibrium). Let 'θ' be the angle made by the magnet with the direction of B_H.

The couple (mB_H, mB_H) acting on the magnet tries to decreases the value of 'θ'. This is called restoring couple.

Torque acting on the magnet in anticlock wise direction $= 2mB_H 2l \sin\theta$ ..... (i)

The couple (mB,mB) acting on the magnet tries to increase the value of 'θ'. This is called deflecting couple.

Torque acting on the magnet in clock wise $= mB_H 2l \cos\theta$

In equilibrium, restoring couple acting on the magnet is equal to the deflecting couple.

$\therefore mB_H 2l \sin\theta = 2mB2l \cos\theta$

$$\frac{B_H}{B} = \frac{\cos\theta}{\sin\theta} \Rightarrow B = B_H \tan\theta \qquad ..... (1.28)$$

This is called tangent law.

1.17 CLASSIFICATION OF MAGNETIC MATERIALS

Curie and Faraday identified that all the materials present in the universe exhibit magnetism to some extent. These substances are classified as diamagnetic substances, paramagnetic substances and ferro magnetic substances. According to modern electron theory of magnetism, materials exhibit magnetic property due to the circulating electrons in the atoms. Each and every circulating electron constitutes a magnetic moment in a direction perpendicular to the plane of circulation. In an atom, an electron possesses spin motion and the orbital motion. Thus, the electron possesses two dipole moments. The net magnetic moment of electron is the vector sum of magnetic moments of orbital and spin motions. The magnitude and direction of this resultant magnetic moment is responsible for the magnetic behaviour of the materials. Now, we discuss about these materials in some detail.

1.18 DIAMAGNETISM

In some substances like antimony, bismuth copper, lead, gold, silver, zinc, quartz, mercury, alcohol, sodium chloride, water, hydrogen, air, argon, etc., the resultant magnetic moment due to all the electrons in the atom is zero. When diamagnetic materials are placed in an external magnetic field, the orbital motion of the electrons are affected. As a result weak resultant magnetic moment is induced in a direction opposite to the applied external field. The magnetic response of such type of materials is known as diamagnetism and the materials are called diamagnetic substances or diamagnets.

Super conductors may be considered as perfect diamagnets. At very low temperatures these metals exhibit perfect conductivity and perfect diamagnetism. The phenomenon of perfect diamagnetism in super conductors is called the meissner effect. Now a days the super conducting magnets are used for running magnetically leviated super fast trains.

Properties of diamagnetic substances

i) Diamagnetic substances are feebly repelled by strong magnets.

ii) When a diamagnetic rod is suspended freely in a uniform magnetic field, the rod aligns itself in a direction perpendicular to the direction of the magnetic field.

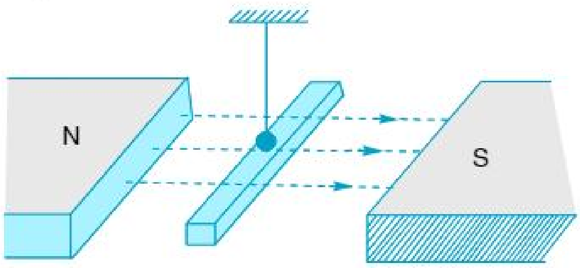


Fig 1.19(a) Freely suspended diamagnetic rod in magnetic field

iii) When a diamagnetic substance is placed in a magnetic field, it develops weak magnetisation in a direction opposite to the direction of the magnetising field.

iv) When a diamagnetic material is kept in an external magnetic field, the lines of force are repelled by the substance and tend to move away from the material as shown in the figure.

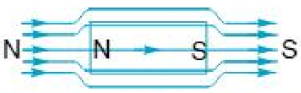


Fig 1.19(b) Line of force in diamagnetic substance in external magnetic field

v) The relative permeability of these materials is less than one. ($\mu_r < 1$)

vi) The diamagnetic substance loses its magnetism as soon as the magnetisation field is removed.

vii) The magnetic susceptibility of a diamagnetic substance has a small negative value.

viii) The magnetic susceptibility of a diamagnetic substance does not depend upon the temperature.

ix) Intensity of magnetisation I is very small, negative and is directly proportional to the magnetising field.

x) When a diamagnetic liquid contained in a watch glass is placed on two pole - pieces lying close to each other, it shows a depression in the middle and gets accumulated at the sides as shown in figure.

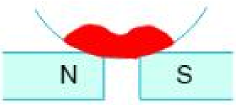


Fig 1.19(c) Diamagnetic liquid in a watch glass

When the pole pieces are sufficiently apart, the liquid now accumulates at the middle and shows depression at the sides.

1.19 PARAMAGNETISM

In some substances like aluminium, platinum, manganese, chromium, calcium, oxygen, platinum, alkali and alkaline earth metals etc., the resultant magnetic moment in atoms is not zero, but has a certain value. These are considered as tiny magnets known as "Atomic magnets". Due to thermal agitation these atomic magnets are randomly oriented, as a result the net magnetic moment is zero. In the presence of an external magnetic field, these atomic magnets align in the direction of applied field. Hence the substances possesses weak resultant magnetic moment. If the applied field is withdrawn, the material is completely demagnetized deorienting the atomic magnets due to thermal energy in the system. This behaviour is known as paramagnetism.

Properties of para magnetic substances

i) A paramagnetic substance is feebly attracted by a strong magnet.

ii) When a paramagnetic rod is suspended freely in a uniform magnetic field, it aligns itself in the direction of magnetic field.

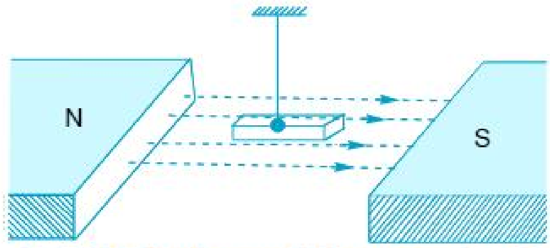


Fig 1.20(a) Freely suspended paramagnetic rod in a magnetic field

iii) A paramagnetic substance moves from weaker to stronger parts of the magnetic field.

iv) When a paramagnetic substance is kept in an external magnetic field, the lines of force are drawn more into the material as shown in figure.

Fig 1.20(b) Lines of force in paramagnetic substance in external magnetic field

v) The relative permeability of paramagnetic materials is greater than one.

vi) When external magnetic field is removed, the paramagnetic substance loses its magnetism.

vii) The magnetic susceptibility of a paramagnetic substance has a positive value.

viii) Intensity of magnetisation I is very small, positive and directly proportional to the magnetising field.

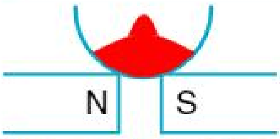


Fig 1.20(c) Paramagnetic liquid in a watch glass

ix) When a paramagnetic liquid contained in a watch glass is placed on two pole pieces lying close to each other, the liquid rises in the middle.

1.20 FERROMAGNETISM

In some substances like Iron, Cobalt, Nickel, Gadolinium, Dysprosium and some alloys like alnico (iron, aluminium, nickel, cobalt) possess magnetic moment in the atoms. An unpaired electron in one atom interacts strongly with the unpaired electron in the adjacent atom. Consequently, the magnetic moments get aligned in the same direction. This is called exchange interaction.

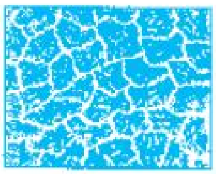


Fig 1.21

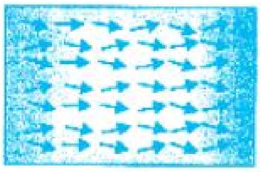


Fig 1.22

The atoms themselves exist together in tiny assemblies called domains. All the existing ferromagnetic materials have a characterstic "domain structure". These domains are of the size 10^{-4} m to 10^{-5} m and the domain contains about 10^{11} atoms. Initially, the magnetisation varies randomly from domain to domain and there is no bulk magnetisation. This is shown in Fig 1.21.

When an external field is applied, the domains orient themselves in the direction of magnetic field as shown in Fig 1.22. The result is that the net magnetic moment is in the direction of the applied field. Since the degree of alignment is very large even for a small external field, the magnetic field produced in the ferromagnetic material is often much greater than the external field. In some ferromagnetic materials the magnetisation persists when the external field is removed. Such materials are called hard magnetic materials or hard ferromagnets. Alnico and Naturally occurring lodestone are the examples for hard ferromagnets. Such materials form permanent magnets to be used among other things as a compass needle. On the other hand, there is a class of ferromagnetic materials in which the magnetisation disappears on removal of the external field. Soft iron is one such material. Such materials are called soft ferromagnetic materials. This nature of matter is known as ferromagnetism.

Properties of ferromagnetic substances

i) A ferromagnetic substance is strongly attracted by a magnet.

ii) When a ferromagnetic rod is freely suspended in a uniform magnetic field, it aligns itself parallel to the direction of the magnetic field as shown in the figure.

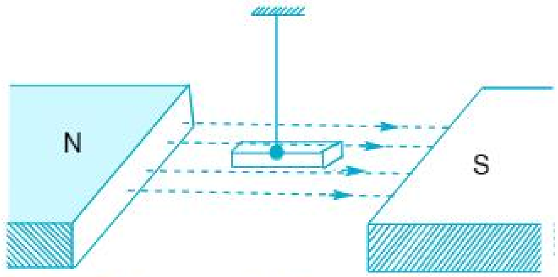


Fig 1.23 Freely suspended ferromagnetic rod in the magnetic field

iii) When a ferromagnetic substance is placed in a non - uniform magnetic field, it moves from weaker to stronger parts of the magnetic field.

iv) When a ferromagnetic substance is placed in a magnetic field, the magnetic lines of force tend to crowd into the specimen as shown in the figure.

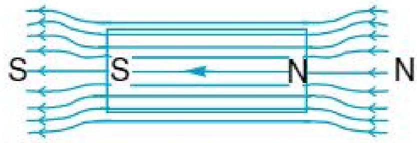


Fig 1.24 Lines of force in ferromagnetic substance in external magnetic field

v) The relative permeability of a ferromagnetic substance is very high.

vi) When a ferromagnetic substance is placed in a magnetic field, it is strongly magnetised in the direction of magnetic field and the magnetisation is retained ever after the magnetising field is removed.

vii) The magnetic susceptibility of ferromagnetics decreases with the increase of temperature.

viii) Ferromagnetics have large susceptibility.

ix) The intensity of magnetisation has a large positive value.

x) When a ferromagnetic liquid contained in a watch glass is placed on two closely placed magnetic pole pieces, the liquid rises in the middle.

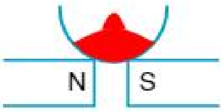


Fig 1.25 Ferromagnetic liquid in a watch glass

Knowledge Plus 1.2

☺ A frog placed in a magnetic field can float. Is it possible?

☛ A frog is diamagnetic just like any other animal. If the frog is placed in the diverging magnetic field at the top end of a vertical current-carrying solenoid, every atom in the frog will be repelled upward, away from the region of stronger magnetic field at the end of the solenoid. The frog moves upward into weaker and weaker magnetic field until the upward magnetic force balances the gravitational force on it. As a resultant it floats in air if we built a solenoid that was large enough. We could similarly levitate a person in mid air owing to the person's diamagnetism.

xi) The ferromagnetism decreases with the rise of temperature. It is maximum at absolute zero of temperature and drops to zero at a temperature called curie temperature. Above the curie temperature, the ferromagnetic material becomes paramagnetic. The curie temperatures for iron, nickel and cobalt are 770^0C, 365^0C and 1075^0C respectively.

1.21 PARAMETERS OF MAGNETIC MATERIALS

1) Intensity of Magnetising Field (H): It is defined as the ratio of magnetising field (B) to the permeability of free space.

$$H = \frac{B}{\mu_0}$$

Unit of H is amp/m. It is a vector.

2) Intensity of Magnetisation (I) : When a magnetic material is magnetised, the magnetic moment developed per unit volume is known as Intensity of magnetisation.

$$I = \frac{M}{V}$$

(or) The pole strength per unit area of cross-section is also called Intensity of Magnetisation.

$$I = \frac{m}{A}$$

Unit of I is A/m. It is a vector quantity whose direction is along the magnetic field.

3) Magnetic Susceptibility (ψ) : It is defined as the ratio of Intensity of magnetisation (I) to the magnetising field (H).

$$\psi = \frac{I}{H}$$

It is a measure of ease with which a material can be magnetised by a magnetising field.

‘ ’ is maximum for Soft Iron

4) Permeability (μ): The ratio of magnitude of total field inside the material to that of intensity of magnetising field is called magnetic permeability.

$$\mu = \frac{B}{H}$$

Unit of μ is Henry/m

Relation between I and H is given by

$$B = \mu_0 (H + I)$$

$$B = \mu_0 H(1 + I/H)$$

$$B = \mu_0 H (1 + \chi) \text{ (or) } \mu = \mu_0 (1 + \chi)$$

$$\mu_r = 1 + \chi \qquad [\mu_r = \mu / \mu_0]$$

Example-1.29

A magnetising field of $1600 Am^{-1}$ produces a magnetic flux of 2.4×10^{-5} weber in a bar of iron of cross section $0.2\ cm^2$. Calculate permeability and susceptibility of the bar.

Solution :

Magnetic induction, $B = \frac{\phi}{A} = \frac{2.4\times10^{-5}}{0.2\times10^{-4}}$

$= 1.2\ Wb/m^2$

i) Permeability, $\mu = \frac{B}{H} = \frac{1.2}{1600} = 7.5\times10^{-4}\,TA^{-1}m$

ii) As $\mu = \mu_0(1+\chi_m)$ then

Susceptibility, $\chi_m = \frac{\mu}{\mu_0} - 1 = \frac{7.5\times10^{-4}}{4\pi\times10^{-7}} - 1$

$= 596.1$

Example-1.30

An iron bar of length 10 cm and diameter 2 cm is placed in a magnetic field of intensity $1000Am^{-1}$ with its length parallel to the direction of the field. Determine the magnetic moment produced in the bar if permeability of its material is $6.3\times10^{-4}\ TmA^{-1}$.

Solution :

Here $H = 1000\ Am^{-1}$; $\mu = 6.3\times10^{-4}TmA^{-1}$,

$l = 10cm; d = 2cm$

Radius of the iron bar = 1 cm = 10^{-2}m

we known that, $\mu = \mu_0(1+\chi_m)$

$\Rightarrow \chi_m = \frac{\mu}{\mu_0} - 1 = \frac{6.3\times10^{-4}}{4\pi\times10^{-7}} - 1$

$= 500.6$

Intensity of magnetisation,

$I = \chi H = 500.6\times1000$

$= 5\times10^5\ Am^{-1}$

Magnetic moment,

$M = IV = 5\times10^5\times\pi\times(10^{-2})^2\times0.1 = 5\pi Am^2$

1.22 ELEMENTS OF EARTHS MAGNETISM

As is known today the earth is enveloped by a magnetic field, such that the lines of magnetic induction run from magnetic south pole to magnetic north pole. This can be imagined as the field due to a short bar magnet at the earth's centre aligned with its north's pole towards earth's magnetic south pole. The geographic axis does not coincide with the magnetic axis of the earth. The exact cause of terrestrial magnetism yet remains undiscovered.

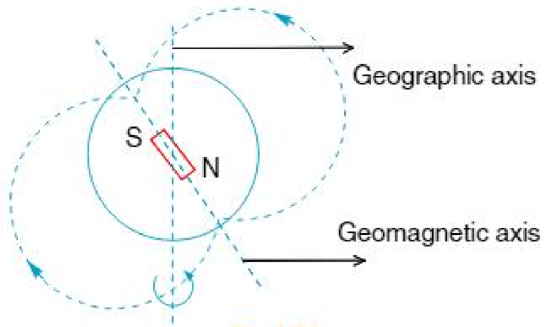


Fig 1.26

The magnetic field of earth, at a place can be completely characterised by three parameters given as

a) magnetic declination
b) magnetic dip or inclination
c) horizontal component of earth's magnetic field.

1.23 MAGNETIC DECLINATION

"It is defined as the angle between the magnetic meridian and geographical meridian measured in the horizontal plane"

The magnetic meridian at a place is vertical plane passing through the earth's magnetic poles. Similarly the geographical meridian at a place is vertical plane passing through the earth's geographical poles. The angle of declination ϕ varies from place to place on the earth's surface. The declination is helpful in steering the ships in the right direction with the help of mariner's compass. Magnetic declination arises M due to the non coincidence of earth's magnetic axis with geographical axis. The curve joining different places on the earth's surface having the same magnetic declinations on a magnetic map is known as an "isogonic line". That particular line which corresponds to 0^0 declination is known as "agonic line"

1.24 MAGNETIC DIP OR INCLINATION (δ)

It is defined as the angle made by the resultant magnetic field of the earth at a place with the horizontal.

In general, the earth's magnetic field at a place is inclined to the horizontal (except at the magnetic equator).

Dip or inclination δ at a place can be measured using an instrument called dip circle. When a magnetic needle is so mounted that it is free to rotate in a vertical plane, it is called dip needle. When the plane of rotation of the dip needle is in the magnetic meridian, the needle will orient itself in the direction of R, total intensity of earth's magnetic field. The angle δ between the needle and the horizontal is angle of dip at that place. The value of dip (δ) is different at different places on the surface of the earth. At the magnetic poles of the earth, R is perpendicular to the surface of the earth. So, there the value of dip is 90^0. At poles dip needle becomes vertical (even freely suspended magnet). At the magnetic equator, value of dip is 0^0. So dip needle becomes horizontal. The curves joining places on earth surface having the same dip are known as "isoclinic lines". The line joining places of 0^0 dip is known as aclinic line.

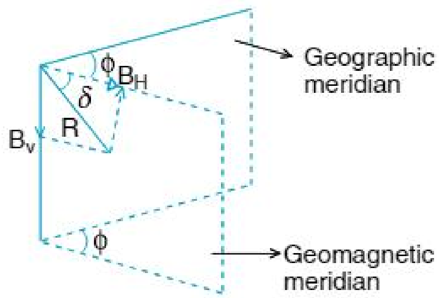


Fig 1.27

1.25 HORIZONTAL COMPONENT OF EARTH'S MAGNETIC FIELD (B_H)

It is the component of earth's total magnetic field along horizontal direction in the magnetic meridian. It is denoted by B_H.

From the figure, we can find $B_H = R\cos\delta$ and $B_V = R\sin\delta$ where B_H and B_V are horizontal and vertical component of earth's magnetic field.

Now we can write $R = \sqrt{B_H^2 + B_V^2}$

and $\tan\delta = \dfrac{B_V}{B_H}$

Let δ_1 be the apparent angle of dip measured in any vertical plane not necessarily the magnetic meridian. Now the plane of the needle is turned through 90^0 and this time apparent dip is δ_2. Let θ be the angle between the plane of the needle initially and the magnetic meridian plane. Here the vertical component of earth's magnetic field remain same, i.e., B_V is same. But the horizontal components are $B_H\cos\theta$ and $B_H\sin\theta$.

$$\cot\delta_1 = \frac{B_H\cos\theta}{B_V} \text{ and } \cot\delta_2 = \frac{B_H\sin\theta}{B_V}$$

On squaring and adding, we get

$$\cot^2\delta_1 + \cot^2\delta_2 = \left(\frac{B_H}{B_V}\right)^2 = \cot^2\delta$$

Here δ is the true dip at that place

$$\boxed{\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2}$$

1.26 MAGNETIC HYSTERISIS

When a ferromagnetic substance is subjected to a cycle of magnetisation, it is found that flux density B in the material lags behind the applied magnetising field H. This phenomenon is known as hysterisis.

"The phenomenon of lagging of flux density B behind the magnetising field H in a ferromagnetic substance subjected to a cycle of magnetisation is known as hysterisis"

If a piece of ferromagnetic material is subjected to one cycle of magnetisation, the resultant B-H curve is a closed loop abcdefa . This loop is known as hysterisis loop. Here it can be observed that B always lags behind H. At point b, the value of H is zero but B has a finite positive value ob. Similarly at point e, value of H is zero and B has a finite negative value oe.

As H is increased by increasing magnetising current, B increases along oa and reaches its saturation value B_{max} at a. At this stage all domains of the material are aligned in the direction of H.

If now H is gradually decreased by decreasing the magnetising current, the curve follows ab instead of ao. At point b, it is observed that H = 0 but $B \neq 0$. The value of $B = B_r = ob$ is called residual flux density known as " retentivity" or remanence. To reduce B in the material to zero, it is necessary to apply H in the reverse direction. This can be done by reversing the magnetising current. As H is gradually increased in the reverse direction, the curve follows bc. At point c, it is observed that B = 0 and $H = -H_C$. Here value of H needed to wipe out residual magnetism is called coercive force H_C. Now H is further increased in the reverse direction until point d is reached where the sample is saturated in the reverse direction ($-B_{max}$). If now H is reduced to zero, point e is reached and the sample again retains magnetic flux density $-B_r$. The remaining part of the loop is obtained by increasing current to produce H in the original direction.

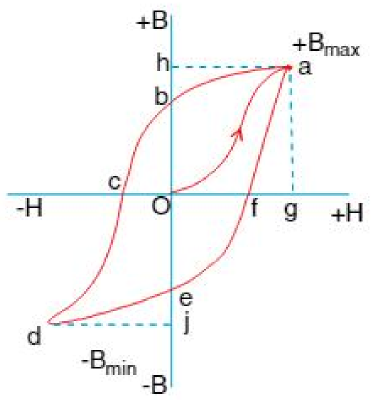


Fig 1.28 Hysterisis loop

When a ferromagnetic sample is subjected to a cycle of magnetisation, the domains of the sample resist to be magnetised. Energy is thus expended in the substance to overcome this. This loss of energy will be in the form of heat and is called hystresis loss. Due to hysterisis loss temperature of the substance increases.

The shape and size of hysterisis loop area depends upon the nature of the material. The smaller the hysterisis loop area of the material, the smaller is the loss. For silicon steel, the area of hysterisis loop is small. For this reason silicon steel is widely used for making cores of transformers and rotating machine points which may be subjected to rapid reversal of magnetisation. If we compare the loops for soft iron and steel, area of the loop is larger for steel than for soft iron. Soft iron can be easily magnetised but it can retain only small magnetisation if field is removed. The retentivity and coercivity are larger for steel than for soft iron. So, steel is quite suitable for making permanent magnets. As hysterisis loss is more for steel, it is not suitable for the construction of electrical machines.

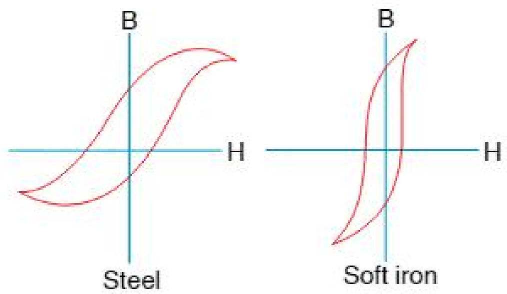


Fig 1.29 (a) Fig 1.29 (b)

- For soft iron residual magnetism and coercivity are fairly low. So soft iron is suitable for making cores of electromagnets, transformers, moving coil galvanometers etc.,

1.27 MAGNETIC INTENSITY AND MAGNETISATION

A circulating electron in an atom results in magnetic moment. In any material, these atomic magnetic moments add up vectorially and produce net magnetic moment. Net magnetic moment per unit volume is known as magnetisation $\bar{I}$. It is a vector with dimension $L^{-1}A$ and is measured in Am^{-1}.

The magnetic field at any point inside a long solenoid is given by $\bar{B}_0 = \mu_0 ni$. If the interior of the solenoid is filled with a material having non zero magnetisation, the field inside the solenoid will be greater than $\overline{B}_0$. Now, the net field in the interior of the solenoid is $\overline{B}=\overline{B}_0+\overline{B}_m$

Here $\overline{B}_m$ is field contributed by core material. Here $\overline{B}_m$ is proportional to the magnetisation $\overline{I}$ of the material. $\overline{B}_m=\mu_0\overline{I}$

$\overline{H}$ is a another vector field known as magnetic intensity. It is given by $\overline{H}=\dfrac{\overline{B}}{\mu_0}-\overline{I}$

$\overline{H}$ and $\overline{M}$ have same dimensions and units. Now total magnetic field is given by $\overline{B}=\mu_0\left(\overline{H}+\overline{I}\right)$

Here $\overline{H}$ is due to external factors such as the currents in the solenoid. $\overline{I}$ is due to the specific nature of the magnetic material. Here $\overline{I}$ can be influenced by external factors. It can be expressed as $\overline{I}=\chi\overline{H}$. Where χ is a dimensionless quality known as magnetic susceptibility. How a magnetic material responds to an external field will be given by χ. It is small and positive for paramagnetics. It is small and negative for diamagnetics. Now we can write $\overline{B}=\mu\overline{H}(1+\chi)$

We know that $\overline{B}=\mu\overline{H}=\mu_0\mu_r\overline{H}$

$\Rightarrow 1+\chi=\mu_r$ which is relative magnetic permeability of the substance. μ_r in magnetism is analog to dielectric constant in electrostatics

1.28 CURIE'S LAW

The magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T.

$$\Rightarrow \overline{I}=C\frac{\overline{B}_0}{T}$$

It may be written as $\chi=C\dfrac{\mu_0}{T}$ also or $\chi\times\dfrac{1}{T}$ which is known as curie's law here C is called Cureie's constant. For a paramagnetic material both χ and μ depend not only on the material but also on the temperature. As the field is increased or temperature is lowered, magnetisation increases until it reaches satuation when all the dipoles are perfectly aligned with the field.

The ferromagnetic property also depends on temperature. At sufficiently high temperature, a ferromagnetic becomes paramagnetic material. It is like a phase transition with domain structure disintegrating with temperature. The temperature at which a ferromagnet converts into paramagnet is called Curie temperature T_C. The susceptibility above T_C is explained by $\chi=\dfrac{C}{T-T_C}$.

1.29 ELECTRO MAGNETS AND PERMANENT MAGNETS

Permanent magnets are those substances which retain the ferromagnetic property at room temperature for a long time there are different methods of perparing permanent magnets. If an iron rod is kept in the north-south direction and hammered repeatedly for a long time, it can be made into a permanent magnet of a steel rod is struck with one end of a bar magnet a large number of times, always in the same sense, it can be connected into permanent magnet. The most efficient way to prepare a permanent magnet is to keep a rod of ferromagnetic material in a solenoid and pass a current. Here magnetic field of the solenoid magnetises the rod.

From the hysteresis curve we can select suitable materials for permanent magnets. The material should have high sensitivity so that the magnet is strong and high coercivity so that the magnetisation is not erased by surrounding magnetic fields, fluctuation in temperature or minor mechanical damages. Besides this the material should have high permeability also. Steel is most suitable to prepare permanent magnets. It has a slightly smaller retentivity than soft iron though coercivity of soft iron is much smaller. There are special alloys suitable for permanent magnets like alnico, cobalt steel and ticonal.

Ferromagnetic materials which have high permeability and low retentivity are most suitable for core of electromangnets soft iron is most suitable for core of electromagnets. We know that a current carrying solenoid produces magnetic field. If a soft iron rod is kept along the axis of the solenoid, magnetic field can be increased to thousand fold. It the current is surtched off, the magnetic field also will turn off due to the low retentivity of soft iron. So a soft iron core in solenoid acts as an electromagnet.

In some special application materials will be subjected through an ac cycle of magnetisation for long time. The best examples are transformer cores and telephone diaphrams. The hysterisis curve for such materials must be narrow. As a result, energy dissipated and heatily will be small. In addition to this that material must have high resistivity to lower coldy current loses. Electromagnets are used in electric bells, land speakers and telephone diaphrams. Huge electromagnets are used in canes to lift bulk quantities of iron and steel.

1.30 GAUSS'S LAW IN MAGNETISM

We have studied about Gauss's law in electrostatics. The total flux linked with a Gaussian surface is proportional to the change enclosed by that surface. Net outward flux emerging out from the surface is positive and the flux entering into the surface is negative. This situation is different for magnetic fields. The main difference is magnetic field lines form closed loops where as electrostatic field lines do not. If we consider a closed Gaussian surface in magnetic field, the number of field lines leaving the surface is balanced by the number of field line entering it. So net magnetic flux is zero for any closed surface.

Consider a small vector area element $\Delta\bar{s}$ of a closed surface as in fig. The magnetic flux through $\Delta\bar{s}$ is given by $\Delta\phi_B = \bar{B}.\Delta\bar{s}$, where $\vec{B}$ is the field at $\Delta\bar{s}$. The given surface may be divided into many small area elements and calculate the individual flux through each. The net flux through that surface is

$$\phi_B = \sum_{all} \Delta\phi_B = \sum_{all} \bar{B}.\Delta\bar{s} = 0$$

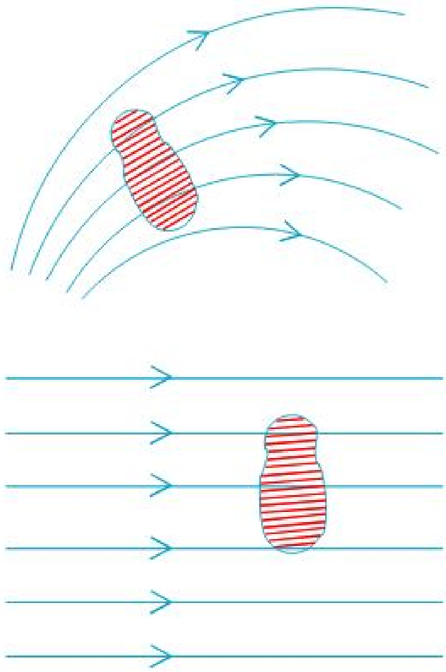


Fig 1.30

Here $\sum_{all}$ indicates all area elements.

In electrostatics $\sum_{all} \bar{E}.\Delta\bar{s} = \frac{q}{\epsilon_0}$

Here q is the net electric charge enclosed by the surface.

From Gauss's law of magnetism it is evident that isolated magnetic poles or monopoles do not exist. All magnetic phenomena can be explained in terms of dipoles or current loops.

So Gauss's law for magnetism can be stated as "The net magnetic flux through any closed surface is zero".

1.31 ANALOGY BETWEEN ELECTRO-STATICS AND MAGNETISM

parameter	electrostatic	magnetism
1. Dipole moment	$\overline{P} = q2\overline{l}$	$\overline{M} = m2\overline{l}$
2. Field due to short		
a) on axial line	$\overline{E} = \frac{1}{4\pi \epsilon_0}\frac{2\overline{P}}{r^3}$	$\overline{B} = \frac{\mu_0}{4\pi}\frac{2\overline{M}}{r^3}$
b) on equatorial line	$\overline{E} = \frac{1}{4\pi \epsilon_0}\frac{-\overline{P}}{r^3}$	$\overline{B} = \frac{\mu_0}{4\pi}\frac{-\overline{M}}{r^3}$
3. Torque on dipole in filed	$\overline{\tau} = \overline{P}\times\overline{E}$	$\overline{\tau} = \overline{M}\times\overline{B}$
4. Potential energy	$U = -\overline{P}.\overline{E}$	$U = -\overline{M}.\overline{B}$

Example-1.31

A compass needle of magnetic moment 60A-m^2, pointing towards geographical north at a certain place where the horizontal component of earth's magnetic field is $40\mu\,Wb/m^2$ experiences a torque of 1.2×10^{-3} Nm. Find the declination at that place.

Solution :

If θ is the declination of the place, then the torque acting on the needle is $\tau = M\,B_H \sin\theta$

$$\Rightarrow \sin\theta = \frac{\tau}{M B_H} = \frac{1.2\times10^{-3}}{60\times40\times10^{-6}} = \frac{1}{2}$$

$$\Rightarrow\therefore\ \theta = 30^0$$

Example-1.32

The horizontal component of earth's magnetic induction at a place is 0.32×10^{-4} T. The angle of dip at the point is 60^0, then find value of vertical component ?

Solution :

Vertical component,

$$B = B_H \tan\theta = 0.32\times10^{-4}\times\sqrt{3} = 0.55\times10^{-4}\,T$$

Example-1.33

A magnet is suspended so that it may oscillate in the horizontal plane. It performs 20 oscillations per minute at a place where the angle of dip is 30^0 and 15 oscillations per minute, where the angle of dip is 60^0. Compare the earth's total magnetic field at these two places.

Solution :

we know that frequency,

$$v = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}} \text{ or } v \propto \sqrt{B_H} \text{ or } v \propto \sqrt{B\cos\delta}$$

$$\text{or } B\cos\delta \propto v^2 \text{ or } B \propto \frac{v^2}{\cos\delta}$$

$$\frac{B_1}{B_2} = \frac{v_1^2}{\cos\delta_1}\times\frac{\cos\delta_2}{v_2^2} = \frac{20\times20\times\cos60^0}{\cos30^0\times15\times15} = \frac{16}{9\sqrt{3}}$$

Example-1.34

A magnet suspended at 30^0 with magnetic meridian makes an angle of 45^0 with the horizontal. What shall be the actual value of the angle of dip ?

Solution :

Here, $\theta = 30^0$

Apparent value of dip, $\delta_1 = 45^0$

Actual value of dip $\delta = ?$

If H is horizontal component of earth's magnetic field in magnetic meridian, then $\tan\delta = \frac{V}{H}$

Let H_1 be component of H at 30^0 to magnetic meridian, then $\tan\delta_1 = \frac{V}{H_1} = \frac{V}{H\cos\theta} = \frac{\tan\delta}{\cos\theta}$

$$\text{or } \tan\delta = \tan\delta_1\times\cos\theta = \tan45^0\times\cos30^0$$

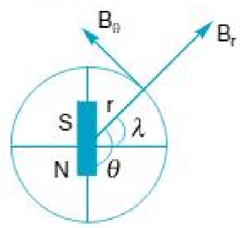
$$= 1\times\frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.866$$

$$\therefore \delta = \tan^{-1}(0.866) = 40.9^0$$

Example-1.35

Consider the earth as a short magnet with its centre coinciding with the centre of earth. Show that the angle of dip ϕ is related to magnetic latitude λ through the relation $\tan\phi = 2\tan\lambda$.

Solution :



Consider the situation as shown in the figure. For dipole, at position (r,θ) we have

$$B_r = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3} \text{ and } B_\theta = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3} \quad .... (1)$$

and as $\tan\phi = \frac{B_V}{B_H} = -\frac{B_r}{B_\theta}$, so in the light of eq (1),

$\tan\phi = -2\cot\theta$

But from figure $\theta = 90 + \lambda$

So, $\tan\phi = -2\cot(90+\lambda)$, i.e., $\tan\phi = 2\tan\lambda$.

Example-1.36

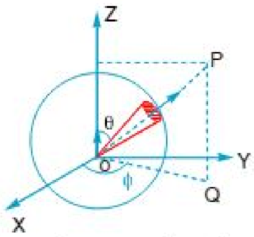
Verify Gauss's law for magnetic field of a point dipole of dipole moment $\vec{m}$ at the origin for the surface which is a sphere of radius R. Magnetic field at p with position vector $\hat{i}$ is $\frac{\mu_0}{4\pi}\left(\frac{3m\cos\theta}{r^3}\right)\hat{i}$.

Solution :

Let us use spherical polar coordinates. Here $\bar{r}$ is radial distance from origin O. Here $0 \le r \le \infty$. Here θ is zenith angle which is made by OP with Z axis. $0 \le \theta \le \pi$

Here ϕ is azimuthal angle which line OQ makes with X axis. Where OQ is projection of OP on XY plane $0 \le \phi \le 2\pi$

Consider a Gaussian surface of radius r arrow a point dipole of dipole moment $\vec{m}$ pointinf along z axis $\Rightarrow \vec{m} = m\hat{k}$



Let $d\bar{s}$ be the surface area of an element.

Magnetic field at a point on that element is

$$\bar{B}_r = \frac{\mu_0}{4\pi}\left(\frac{3m\cos\theta}{r^3}\right)\hat{r};\ d\bar{s} = (r^2\sin\theta,\ d\theta,\ d\phi)\hat{r}$$

$$\oint \bar{B}\cdot d\bar{s} = \oint \bar{B}_r \cdot d\bar{s} = 0$$

Hence Gauss's law is proved.

Example-1.37

A paramagnetic gas consists of atom each with a dipole moment of 1.5×10^{-23} J/T. The temperature of the gas is 27°C and number of atom per unit volume is $2 \times 10^{26}/m^3$. What is the maximum possible magnetisation of the sample when placed in an external field 3T?

Solution :

Dipole moment of each atom = 1.5×10^{-23} J/T

Temperature of the gas = 300 K

Number density n = 2×10^{26} /m^3

Magnetic field B = 3T

The magnetisation of the sample is maximum when all the dipoles are alined parallel to the direction of magnetic field

so, maximum magnetisation

$$= (2\times 10^{26}) \times (1.5 \times 10^{-23}) = 3 \times 10^3\ \text{JA}^{-1}$$

In fact the actual value of magnetisation is much less than this maximum magnetisation an account of thermal agitation.

At a Glance

1. The magnetic induction due to a pole of pole strength 'm' at a distance 'd' from it is given by $B = \frac{\mu_0}{4\pi}\frac{m}{d^2}$ where μ_0 is the permeability of free space.
2. The path in which unit N-pole would move in a magnetic field is known as a 'magnetic field line' or 'line of force'. The tangent to the magnetic field line at a point gives the direction of the field induction B at that point.
3. When a bar magnet of moment 'M' is placed in a uniform magnetic field 'B' at an angle 'θ' to the direction of the field, a torque, $\tau = MB\sin\theta$ acts on it.
4. The magnetic induction on the axial line of a short bar magnet of moment 'M', at a distance 'd' from its centre is given by $B_A = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$
5. The magnetic induction on the equatorial line of a short bar magnet of magnetic moment 'M' at a distance 'd' from its centre is given by $B_E = \frac{\mu_0}{4\pi}\frac{M}{d^3}$
6. Tangent law in magnetism states that, if a magnetic needle is suspended in a magnetic field of induction B, acting at right angles to the magnetic meridian, the magnetic needle comes to rest at an angle 'θ' to the magnetic meridian such that $B = B_H\tan\theta$
7. A bar magnet suspended in magnetic meridian can be displaced and made to execute angular simple harmonic motion with a time period $T = 2\pi\sqrt{\frac{I}{MB_H}}$, where 'I' is the moment of inertia of the magnet.

   A vibration magnetometer works on the above principle and can be used to determine 'M' and 'B_H'.
8. The relative permeability of diamagnetic materials is less than one ($\mu_r < 1$) and a magnetic moment is induced in them in a direction opposite to the external field.
9. The relative permeability of paramagnetic material is greater than one ($\mu_r > 1$) and a magnetic moment is induced in them parallel to the external field.

10. The relative permeability of ferromagnetic materials is very high ($\mu_r \simeq 10$ to $10{,}000$) and strong magnetic regions called 'domains' are always present in them in which a resultant magnetic moment exists even in the absence of external field.

EXERCISE

LONG ANSWER QUESTIONS

1. State inverse square law and explain how can it be verified by Gauss method.
2. Describe the principle of working of deflection magnetometer. Explain how the magnetic moments of two short bar magnets can be compared in tan A position by (i) Equal distance method, (ii) Null method.
3. Define Magnetic moment of a bar magnet. Explain the method of comparing the magnetic moments of two short bar magnets in tan B position in (i) Equal distance method, (ii) Null method.
4. Explain the principle of working of a vibration magnetometer. How do you determine the magnetic moment ('M') of a bar magnet and the horizontal component of the earth's magnetic field (B_H) using vibration magnetometer and deflection magnetometer.

SHORT ANSWER QUESTIONS

1. Derive the equation for the couple acting on a bar magnet in a uniform magnetic field and hence deduce the definition of "magnetic moment".
2. Derive an expression for the magnetic induction at a point on the axial line of a bar magnet.
3. Derive an expression for the magnetic induction at a point on the equatorial line of a bar magnet.
4. State and explain "Tangent law" in magnetism.
5. Explain tanA & tanB positions of a deflection magnetometer.
6. Derive an expression for the period of oscillation of a bar magnet in vibration magnetometer
7. How do you distinguish Dia, Para and Ferro-magnetism in matter?

VERY SHORT ANSWER QUESTIONS

1. The force between two magnetic poles separated by a distance 'd' in air is F. At what distance between them the force becomes doubled.

A. According coulomb's law we can write,

$$F = \frac{\mu_o}{4\pi}\frac{m_1 m_2}{d^2}$$

i.e., $F \propto \frac{1}{d^2} \Rightarrow \frac{F_1}{F_2} = \frac{d_2^2}{d_1^2}$

$$\Rightarrow d_2^2 = \frac{F_1}{F_2}d_1^2 = \frac{F}{2F}d^2 = \frac{d^2}{2} \text{ or } d_2 = \frac{d}{\sqrt{2}}.$$

2. Can the force on unit N - pole between two isolated equal like poles be zero at any point?

A. Yes, at the midpoint of the line joining the two poles.

3. Can the force on unit N- pole between two isolated unlike poles be zero at any point?

A. No. At every point on the line joining them the resultant force in non-zero.

4. Do you find two magnetic field lines intersecting ? Why?

A. No, Because at the point of intersection the magnetic field will have two directions which is not possible.

5. Distinguish between uniform and non-uniform magnetic fields. Give examples?

A. **Uniform magnetic field:** If the strength of the magnetic induction field is same in magnitude and direction at all points in it, it is said to be uniform.

1) Magnetic field between two strong electromagnetic pole pieces.
2) Horizontal component of earth's magnetic field in a given place.

Non-uniform magnetic field:

If the strength of the magnetic induction field at all points in it differ in magnitude and direction it is said to be non-uniform.

eg.: Magnetic field due to a bar magnet.

6. When is the couple acting on a bar magnet in a uniform magnetic field (i) maximum (ii) minimum.

A. i) If the magnet is at right angles to direction of the field the couple acting on a bar magnet is maximum.

ii) If the magnet is parallel to the direction of the field, the couple acting on the bar magnet is minimum.

7. **Define the magnetic moment of a bar magnet. What is its direction?**

A. Magnetic moment of a magnet is the moment of couple acting on the magnet when placed at right angles to the direction of a uniform magnetic induction field of unit strength. Magnetic moment is a vector quantity and is directed from south pole to north pole along the axis of the two magnet.

8. **What are the units of the following physical quantities?**

i) magnetic moment ii) magnetic permeability

A. i) The unit of magnetic moment is Am^2.

ii) The unit of magnetic permeability is Hm^{-1} or NA^{-2}.

9. **What is the magnetic moment of a semi circular magnet of radius 'r' and pole strength 'm' ?**

A. Magnetic moment =
distance between poles × pole strength =
$m \times 2r = 2mr$

10. **What is the magnetic moment of each piece when a bar magnet of moment 'M' is cut into two equal parts (i) along its axis and (ii) along its equatorial line.**

A. Magnetic moment of the magnet, $M = 2lm$

i) When the magnet is cut along its axis, length remains same but pole strength becomes m/2.

Now the magnetic moment of each piece

$$= 2l \times \frac{m}{2} = \frac{M}{2}$$

ii) When the magnet is cut along its equatorial line,

pole strength remains same but length becomes $\frac{2l}{2}$

Now the magnetic moment of each piece = $\frac{2l}{2} \times m = \frac{M}{2}$

11. **The magnetic moment of a bar magnet is M. If it is cut into two pieces in the ratio 1:2 perpendicular to its length, what is the ratio of their magnetic moments.**

A. Given that, $l_1 : l_2 = 1 : 2$

New magnetic moment of first piece, $M_1 = \frac{M}{3}$

New magnetic moment of second piece, $M_2 = \frac{2M}{3}$

∴ Ratio of magnetic moments, $\frac{M_1}{M_2} = \frac{1}{2}$.

12. **Define magnetic flux density (magnetic induction). The magnetic induction of a point is B. What is the magnetic flux through an area A?**

A. **Magnetic flux density:** The magnetic flux passing normally through unit area at a point is called magnetic flux density at that point. If ϕ be the magnetic flux passing through the area 'A' normally,

magnetic flux density, $B = \frac{\phi}{A}\ Wb/m^2$ (or Tesla)

∴ magnetic flux, $\phi = \vec{B} \cdot \vec{A}$ Wb

13. **Give the expressions for the magnetic induction due to a bar magnet on (i) axial line (ii) the equatorial line?**

A. i) Magnetic induction on the axial line of a bar magnet is,

$$B = \frac{\mu_o}{4\pi} \frac{2Md}{(d^2 - l^2)^2}$$

In case of short bar magnet the magnetic induction on the axial line is $B = \frac{\mu_o}{4\pi} \frac{2M}{d^3}$.

ii) Magnetic induction on the equatorial line of a bar magnet is, $B = \frac{\mu_o}{4\pi} \frac{M}{(d^2 + l^2)^{3/2}}$

In case of short bar magnet the magnetic induction on the equatorial line is, $B = \frac{\mu_o}{4\pi} \frac{M}{d^3}$

14. **Where do you find the null points in the combined field due to a bar magnet and the earth, when**

(i) the N-pole of the magnet is kept towards the north of the earth,

(ii) the S-Pole of the magnet is kept towards the north of the earth?

A. i) On the equatorial line of the magnet.

ii) On the axial line of the magnet.

15. **What is the magnetic induction at the mid-point of the straight line joining the two poles of a horse shoe magnet, separated by a distance 'd'? (The pole strength of each pole is 'm'.)**

A. Magnetic induction $B = \frac{\mu_0 2m}{4\pi d^2}$, towards the S-pole and away from N - pole.

16. **Two magnets of magnetic moments M_1 and M_2 are joined at their centres to form a cross (perpendicular to each other). The combination is suspended in a uniform magnetic field directed vertically upwards. It stands in equilibrium when either of them makes an angle of 45^0 with the vertical field. What is the ratio of their magnetic moments?**

A. $M_1 : M_2 = 1 : 1$

17. **On what factors does the period of oscillation of a bar magnet in a uniform magnetic field depend.**

A. 1) Moment of inertia of the rectangular bar magnet (I)

2) Magnetic moment of the bar magnet (M).

3) Uniform magnetic field (B).

18. **How do you determine the magnetic nature of a material, given in the form of a rod?**

A. The given rod is placed near a magnet.

If it is strongly attracted then it is Ferromagnetic.

If it is feebly attracted then it is paramagnetic.

If it is repelled by the magnet then it is diamagnetic.

19. **Classify the following substances into Dia, Para and Ferro magnetic materials.**

(i) Manganese, (ii) Bismuth, (iii) Cobalt, (iv) Oxygen, (v) Copper and (vi) Aluminium.

A. Bismuth - Diamagnetic

Cobalt - Ferromagnetic

Oxygen - paramagnetic

Copper - Diamagnetic

Aluminium - Paramagnetic

20. **The relative permeability of silicon is 0.999837 and that of palladium is 1.000692. What do you infer about the magnetic nature of silicon and palladium?**

A. Silicon - diamagnetic

Palladium - paramagnetic

21. **What happens to the length of a ferromagnetic rod when it is magnetised?**

A. The length slightly increases because of the alignment of the domains in the direction of the external field.

22. **What happens to the magnetism of an iron bar magnet when it is melted?**

A. When an iron bar magnet is melted its temperature will be above Curie temperature (770^0C). Hence it loses its magnetism.

23. **A red hot steel needle is suspended along the North - South direction and cooled. What happens**

A. A weak magnetic moment is induced parallel to the external field and the needle becomes a weak magnet.

PROBLEMS

LEVEL - I

1. The distance between a north pole of strength 6×10^{-3} Am and a south pole of strength 8×10^{-3} Am is 10 cm. The poles are separated in air. Find the force between them. **[Ans: 48×10^{-11} N]**

2. Two poles separated by 10 cm experiences a force of 5 mN. Find the force between them when the distance is doubled and pole strengths are doubled. **[Ans: 5 mN]**

3. What is the magnetic induction due to a magnet of pole strength 20A-m and length 20 cm at a distance of 0.5m from its centre on the axial line? **[Ans: 69.44×10^{-7} T]**

4. Find the force experienced by a pole of strength 100 Am at a distance of 20 cm from a short bar magnet of length 5 cm and pole strength of 200 Am on its axial line. **[Ans: 0.025 N]**

5. Find the magnetic induction at a distance of 20 cm on the equatorial line of a short bar magnet with a magnetic moment $60Am^2$. **[Ans: 0.75×10^{-3} T]**

6. What is the magnetic induction due to a magnet of pole strength 20A-m and length 20cm at a distance of 0.2m from its centre on the equatorial line? **[Ans: 357.8×10^{-7} T]**

7. What is the magnetic moment of a semi circular magnet of radius 'r' and pole strength 'm'? **[Ans: 2mr]**

8. The magnetic moment of a bar magnet of length 0.2m is $1Am^2$. If it is cut into two equal pieces along its axis, what is the magnetic moment of each piece. **[Ans: $0.5Am^2$]**

9. Two magnets have their lengths in the ratio 1:3 and their pole strengths in the ratio 3:1. Find the ratio of their magnetic moments. **[Ans: 1:1]**

10. If the maximum couple acting on a magnet in a field of induction 0.2T is 10Nm, what is its magnetic moment? **[Ans: $50Am^2$]**

11. The maximum torque on the magnet of length 2×10^{-1} m in a uniform magnetic field having induction 2×10^{-1} T is 10Nm. Calculate its pole strength. **[Ans: 250 Am]**

12. At what angle with the magnetic meridian will a magnetic needle rest if it is subjected to a magnetic field of induction 0.6×10^{-4}T perpendicular to the magnetic meridian? ($B_H = 0.2 \times 10^{-4}$T) **[Ans: $71^0 34^1$]**

13. A bar magnet of moment of inertia 1×10^{-2}Kgm^2 vibrates in a magnetic field of induction 0.36×10^{-4} Tesla. The time period of vibration is 10 s. Find the magnetic moment of the bar magnet. **[Ans: 110 Am^2]**

14. A long bar magnet of time period T, used in vibration magnetometer is cut into four equal parts cutting it perpendicular to both length and breadth. Find the time period of one small part. **[Ans: T/2]**

15. Two bar magnets of the same length and breadth but having magnetic moments M and 2M are joined together pole for pole and suspended by a string. The period of oscillation of this assembly in a uniform magnetic field B is 3sec. What will be the period of oscillation if the polarities of one of the magnets is changed and the combination is again made to oscillate in the same field? **[Ans: $3\sqrt{3}$ sec]**

16. The permeability of substance is 6.28×10^{-4} wb/Am. Find its relative permeability and susceptibility? **[Ans: 500, 499]**

17. The magnetic moment of magnet of mass 75 gm is 9×10^{-7} A-m^2. If the density of the material of magnet is 7.5×10^3 kg m^{-3}, then find intensity of magnetisation ? **[Ans: 0.09Am^{-1}]**

18. A magnetic field strength (H) 3×10^3Am^{-1} produces a magnetic field of induction (B) of $12\pi T$ in an iron rod. Find the relative permeability of iron? **[Ans: 10^4]**

19. A ship is sailing due east according to Mariner's compass. If the declination of that place is 18^0 east of north, what is the actual direction of the ship? **[Ans: 18^0 south of east]**

20. A bar magnet of magnetic moment 10 A m^2 has a cross sectional area of 2.5×10^{-4}m^2. If the intensity of magnetisation of the magnet is 10^6 Am^{-1}, find the length of the magnet? **[Ans: 4 cm]**

21. A rod of cross sectional area 10cm^2 is placed with its length parallel to a magnetic field of intensity 1000 Am^{-1}. The flux through the rod is 10^4 Wb. Find the permeability of the material of the rod. **[Ans: 10^4 Wb/Am]**

LEVEL - II

1. The force between the two poles is reduced to 'x' newtons when their separation is increased n times. It is increased by 'y' newtons when their separation is made 1/nth of their original value. What is the relation between x and y ? **[Ans: $y= (n^4 - n^2)$ x]**

2. The field intensities of two points on the axial line of a bar magnet at distances of 10×10^{-2} m and 15×10^{-2} m are in the ratio 5.8:1. Find the distance between the two poles of the magnet. **[Ans: 12×10^{-2} m]**

3. The magnetic induction on the equatorial line of a short magnet at distance d from the centre of the magnet is B. At what distance on the equatorial line of the magnet the magnetic induction would become 4B? **[Ans: $d/2^{2/3}$]**

4. A short bar magnet produces magnetic fields of equal induction at two points one on the axial line and the other on the equatorial line. What is the ratio of their distances. **[Ans: $2^{1/3}$: 1]**

5. Calculate the moment of couple required to keep a bar magnet of magnetic moment 2×10^2 Am^2 in a uniform field of induction 0.36 T at an angle of 30^o with the direction of the field. **[Ans: 36 N–m]**

6. A bar magnet of length 0.2m placed in a uniform magnetic field of induction 15 Wbm^{-2}, with its axis at 30^o to the field experience a couple of 7.5 N–m. Find the pole strength of the bar magnet. **[Ans: 5 Am]**

7. A bar magnet of magnetic moment 2.5 Am^2 is free to rotate about a vertical axis through its centre. The magnet is released from the east-west direction. Find the kinetic energy of the magnet as it aligns itself in the north-south direction ($B_H = 0.3\times10^{-4}$T). **[Ans: 75µJ]**

8. A magnet of magnetic moment 5Am^2 is freely suspended in a uniform magnetic field of strength 2T. Find the work done in rotating the magnet through an angle of 60^0. **[Ans: 5J]**

9. A very long magnet of pole strength 4A-m is placed vertically with its one pole on the table. At what distance from the pole, will there be a neutral point on the table? [$B_H = 4 \times 10^{-5}$ Wbm^{-2}] **[Ans: 0.1m]**

10. A short magnet is placed with its magnetic axis on the magnetic meridian, with its N-pole facing north. A neutral point is found on the perpendicular bisector at 10cm from the centre of the magnet. If $B_H = 0.4 \times 10^{-4}$ T, then find the magnetic moment of the magnet ? **[Ans: 0.4Am^2]**

11. A magnet is suspended so as to swing horizontally makes 50 oscillations/min at a place where dip is 30^0, and 40 vibrations where dip is 45^0. Compare the earth's total fields at the two places. **[Ans: $25:8\sqrt{6}$]**

12. A magnet makes 10 oscillations per minute at a place where the angle of dip is 45^0 and the resultant earth's field is 0.4 gauss. Calculate the number of oscillations made per second by the same magnet at another place where the angle of dip is 60^0 and the resultant earth's field is 0.5 gauss. **[Ans: 0.157s^{-1}]**

13. A magnetic needle is free to rotate in a vertical plane which makes an angle of 60^0 with the magnetic meridian. If the needle stays in a direction making an angle of $\tan^{-1}\frac{2}{\sqrt{3}}$ with the horizontal, what would be the dip at that place? **[Ans: 30^0]**

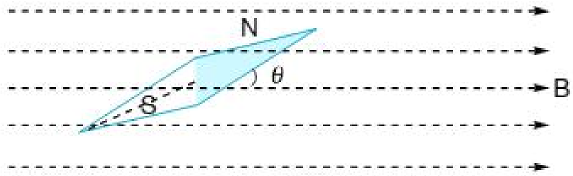
14. The work done in rotating a magnet from the direction of uniform field to the opposite direction of the field is W. Find work done in rotating the magnet from the field direction to half the maximum couple position. **[Ans: $\frac{W}{4}(2-\sqrt{3})$]**

15. A freely suspended short magnet makes 20 oscillations per minute in the earth's horizontal magnetic field of 40 μ T. Another short magnet of moment 1.6 A m^2 is now placed at 20 cm east pointing its north pole towards north. Find the new number of oscillations per minute. **[Ans: 14.14]**

16. The magnetic needle of a vibration magnetometer completes 10 oscillations in 92s. When a small magnet is placed in the magnetic meridian 10 cm due north of the needle, it completes 15 oscillations in 69s. Find the magnetic moment of the magnet ($B_H = 0.3$ G) **[Ans : 0.45Am^2 or 0.15Am^2]**

ADDITIONAL EXERCISE

1. In Fig. the magnetic needle has magnetic moment 6.7×10^{-2} Am^2 and moment of inertia $I = 7.5 \times 10^{-6}$ kg m^2. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field?



2. A short bar magnet placed with its axis at 30° with an external field of 800 G experiences a torque of 0.016 Nm.
   (a) What is the magnetic moment of the magnet?
   (b) What is the work done in moving it from its most stable to most unstable position?
   (c) The bar magnet is replaced by a solenoid of cross-sectional area 2×10^{-4} m^2 and 1000 turns, but of the same magnetic moment. Determine the current flowing through the solenoid.

3. (a) What happens if a bar magnet is cut into two pieces:
   (i) transverse to its length, (ii) along its length?
   (b) A magnetised needle in a uniform magnetic field experiences a torque but no net force. An iron nail near a bar magnet, however, experiences a force of attraction in addition to a torque. Why?
   (c) Must every magnetic configuration have a north pole and a south pole? What about the field due to a toroid?
   (d) Two identical looking iron bars A and B are given, one of which is definitely known to be magnetised. (We do not know which one.) How would one ascertain whether or not both are magnetised? If only one is magnetised, how does one ascertain which one? [Use nothing else but the bars A and B.]

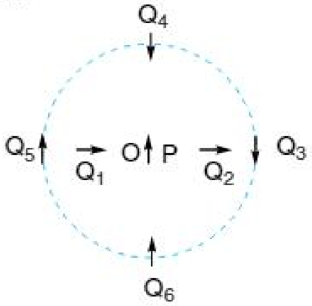
4. What is the magnitude of the equatorial and axial fields due to a bar magnet of length 8.0 cm at a distance of 50 cm from its mid-point? The magnetic moment of the bar magnet is 0.40 A m^2, the same as in Q.2

5. Figure shows a small magnetised needle P placed at a point O. The arrow shows the direction of its magnetic moment. The other arrows show different positions (and orientations of the magnetic moment) of another identical magnetised needle Q.

(a) In which configuration the system is not in equilibrium?

(b) In which configuration is the system in (i) stable, and (ii) unstable equilibrium?

(c) Which configuration corresponds to the lowest potential energy among all the configurations shown?



6. (a) Magnetic field lines show the direction (at every point) along which a small magnetised needle aligns (at the point). Do the magnetic field lines also represent the *lines of force* on a moving charged particle at every point?

(b) Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why?

(c) If magnetic monopoles existed, how would the Gauss's law of magnetism be modified?

(d) Does a bar magnet exert a torque on itself due to its own field? Does one element of a current-carrying wire exert a force on another element of the *same wire*?

(e) Magnetic field arises due to charges in motion. Can a system have magnetic moments even though its net charge is zero?

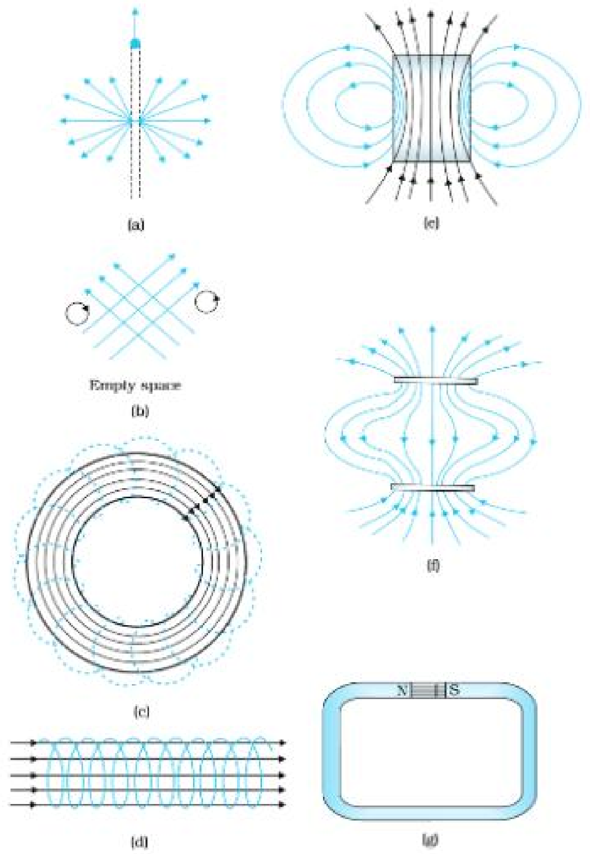
7. The earth's magnetic field at the equator is approximately 0.4 G. Estimate the earth's dipole moment.

8. In the magnetic meridian of a certain place, the horizontal component of the earth's magnetic field is 0.26G and the dip angle is 60º. What is the magnetic field of the earth at this location?

9. A solenoid has a core of a material with relative permeability 400. The windings of the solenoid are insulated from the core and carry a current of 2A. If the number of turns is 1000 per metre, calculate (a) H, (b) M, (c) B and (d) the magnetising current Im.

10. A domain in ferromagnetic iron is in the form of a cube of side length 1 μ m. Estimate the number of iron atoms in the domain and the maximum possible dipole moment and magnetisation of the domain. The molecular mass of iron is 55 g/mole and its density is 7.9 g/cm^3. Assume that each iron atom has a dipole moment of 9.27×10^{-24} A m^2.

11. Many of the diagrams given in Fig. show magnetic field lines (thick lines in the figure) wrongly. Point out what is wrong with them. Some of them may describe electrostatic field lines correctly. Point out which ones.



Chapter - 1

MAGNETISM

✤ Magnets–properties of magnets ✤

✤ Coulomb's inverse square law ✤ Magnetic field ✤

✤ Magnetic field due to a bar magnet ✤ Bar magnet in magnetic field ✤

✤ Superposition of magnetic fields, Tangent law ✤

✤ Elements of earth's magnetism, magnetic hysterisis ✤