

2. SYSTEM OF CIRCLES

SYNOPSIS

Angle between two circles - orthogonality - circle passing through intersection of line and circle - circles through two fixed points - circles passing through a given point on a given line - miscellaneous problems

- If two circles $S = 0$ and $S^1 = 0$ intersect at P , then the angle between the tangents at P is called the angle between the circles
- Angle between two intersecting circles
 - with centres C_1, C_2 and radii r_1, r_2 is $\cos^{-1} \left[\frac{C_1 C_2^2 - r_1^2 - r_2^2}{2r_1 r_2} \right]$
 - with equations $x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$, $x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$ is $\cos^{-1} \left[\frac{c_1 + c_2 - 2(g_1 g_2 + f_1 f_2)}{2\sqrt{g_1^2 + f_1^2 - c_1} \sqrt{g_2^2 + f_2^2 - c_2}} \right]$
- If the angle between two circles is a right angle, then the circles are said to cut each other orthogonally.
- Two circles cut orthogonally
 - when C_1, C_2 are centres and r_1, r_2 are radii if $C_1 C_2^2 = r_1^2 + r_2^2$
 - when equations are $x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$, $x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$ if $2g_1 g_2 + 2f_1 f_2 = c_1 + c_2$
 - If d is the distance between the centres of two circles whose radii are r_1, r_2 then length of the common chord is $\frac{2r_1 r_2 \sin \theta}{\sqrt{r_1^2 + r_2^2 + 2r_1 r_2 \cos \theta}}$ where θ is angle between circles.

Radical axis-radical centre - coaxial system - limiting points - orthogonal coaxial system

- Radical axis :** The locus of the point whose powers with respect to the two given circles are equal is called radical axis (R.A) of these two circles.
- If $S=0, S^1=0$ are two circles in standard form, then their radical axis is $S-S^1=0$.
If $S=x^2+y^2+2g_1 x+2f_1 y+c_1=0$, $S^1=x^2+y^2+2g_2 x+2f_2 y+c_2=0$ then it is $2(g_1-g_2)x+2(f_1-f_2)y+(c_1-c_2)=0$
- Some important points on radical axis (R.A) :**
 - Radical axis is a straight line
 - If two circles intersect in A and B , then their common chord is radical axis. (i.e. \overline{AB} is R.A).
 - If two circles touch each other, then the common tangent at the point of contact is radical axis. R.A. of two equal circles is perpendicular bisectors of line segment joining their centres.
 - Radical axis bisects all the common tangents of two circles.
 - Radical axis of two circles is perpendicular to the line joining their centres.
 - If one circle lie in the other, then radical axis lies outside of both the circles.
 - If one circle lies outside the other, then radical axis lies in between both the circles.
 - The lengths of the tangents from any point on the radical axis to the two circles are equal.

- ix) The circle with centre on radical axis and length of tangent from it to the circles as radius cut the two circles orthogonally.
 - x) The centres of the circles which cuts two circles orthogonally lies on their radical axis.
 - xi) The number of radical axes of n circles, no three of their centres are collinear is nC_2 .
 - xii) If ' n ' circles are given, then the maximum number of radical axes taken two circles at a time is nC_2 and minimum number is zero. (i.e. when all the circles are concentric)
 - xiii) For three circles whose centres are non collinear, there will be three radical axes and they are concurrent.
 - xiv) For concentric circles there is no radical axis.
 - xv) Radical axis of three circles whose centres are non collinear are always concurrent.
4. **Radical centre :** If centres of three circles are non-collinear, then the point of concurrence of the three radical axes is called as radical centre.
5. **Some important points on radical centre (R.C.):**
- i) The power of radical centre with respect to the three circles is equal.
 - ii) The circle with radical centre as centre and length of tangent from radical centre to any of the circles as radius cuts the three circles orthogonally.
- Note :** Only when radical centre lies outside the three circles, we can have a circle cutting the three given circles orthogonally.
6. If $S = x^2 + y^2 + 2gx + 2fy + c = 0$, $S' = x^2 + y^2 + 2g'x + 2f'y + c' = 0$ are two circles, such that $S = 0$ bisects the circumference of $S' = 0$ then the centre of $S' = 0$ lies on common chord of $S = 0$, $S' = 0$ (i.e. $S - S' = 0$) i.e. $2(g - g')g' + 2(f - f')f' = (c - c')$

LECTURE SHEET

EXERCISE-I

Angle between two circles - orthogonality - circle passing through intersection of line and circle - circles through two fixed points - circles passing through a given point on a given line - miscellaneous problems

1. The angle between two circles, each passing through the centre of the other is

1) $\frac{\pi}{6}$
2) $\frac{\pi}{2}$
3) $\frac{2\pi}{3}$
4) π
2. If the angle between the two equal circles with centres $(-2, 0)$, $(2, 3)$ is 120° then the radius of the circle is

1) 5
2) 3
3) 1
4) 2
3. The point $(3, -4)$ lies on both the circles $x^2 + y^2 - 2x + 8y + 13 = 0$ and $x^2 + y^2 - 4x + 6y + 11 = 0$. Then the angle between the circles is

1) 60°
2) 30°
3) 120°
4) 135°
4. If a circle passes through $(1, 2)$ and cuts $x^2 + y^2 = 4$, orthogonally then the locus of its centre is

1) $2x + 4y - 9 = 0$
2) $x + y - 3 = 0$
3) $x + y - 9 = 0$
4) $2x + 3y = 7$

5. The locus of centres of all circles which touch the line $x = 2a$ and cut the circle $x^2 + y^2 = a^2$ orthogonally is
 1) $y^2 + 4ax - 5a^2 = 0$ 2) $y^2 + 4ax + 5a^2 = 0$ 3) $y^2 = 4ax - 5a^2$ 4) $y^2 = 4ax + 5a^2$
6. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally then the locus of its centre is
 1) $2ax + 2by = a^2 + b^2 + k^2$ 2) $ax + by = a^2 + b^2 + k^2$
 3) $x^2 + y^2 + 2ax + 2by + k^2 = 0$ 4) $x^2 + y^2 - 2ax + 2by + k^2 = 0$
7. The condition that the circles which passes through the points $(0, a)$, $(0, -a)$ and touch the line $y = mx + c$ will cut orthogonally is
 1) $c^2 = a^2(1+m^2)$ 2) $c^2 = a^2(2+m^2)$ 3) $c^2 = a^2(3+m^2)$ 4) $c^2 = a^2(4+m^2)$
8. The point $(3, 1)$ is a point on a circle C with centre $(2, 3)$ and C is orthogonal to $x^2 + y^2 = 8$. The conjugate point of $(3, 1)$ w.r.t $x^2 + y^2 = 8$ which lies on C is
 1) $(5, 1)$ 2) $(5, 4)$ 3) $(1, 5)$ 4) $(0, 2)$
9. The points of intersection of two equal circles which cut orthogonally are $(2, 3)$ and $(5, 4)$. Then the radius of each circle is
 1) 1 2) $5\sqrt{2}$ 3) $\sqrt{5}$ 4) $\sqrt{2}$
10. A circle S passes through the point $(0, 1)$ and is orthogonal to circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then
 1) radius of S is 8 2) radius of S is 9 3) Centre of S is $(-7, 1)$ 4) centre of S is $(-8, 1)$
11. If the circle $x^2 + y^2 + 4x + 22y + 1 = 0$ bisects the circumference of circle $x^2 + y^2 - 2x + 8y - m = 0$ then $1 + m = \dots\dots$
 1) 60 2) 50 3) 46 4) 40
12. Suppose $ax + by + c = 0$ where a, b, c are in A.P. be a normal to a family of circles. The equation of the circle of the family which intersects the circle $x^2 + y^2 - 4x - 4y - 1 = 0$ orthogonally is
 1) $x^2 + y^2 - 2x + 4y - 3 = 0$ 2) $x^2 + y^2 + 2x - 4y - 3 = 0$
 3) $x^2 + y^2 - 2x + 4y - 5 = 0$ 4) $x^2 + y^2 - 2x - 4y + 3 = 0$

❖❖❖ EXERCISE-II ❖❖❖

Radical axis-radical centre - coaxial system - limiting points - orthogonal coaxial system

Mains level straight objective type

1. The locus of the centre of the circle which cuts the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 4x + 6y + 4 = 0$ orthogonally is
 1) $8x + 12y - 5 = 0$ 2) $8x - 12y + 5 = 0$ 3) $4x - 6y + 5 = 0$ 4) $4x - 6y + 3 = 0$
2. The distance of the point $(1, -2)$ from the common chord of the circles $x^2 + y^2 - 5x + 4y - 2 = 0$, $x^2 + y^2 - 2x + 8y + 3 = 0$
 1) 2 2) 4 3) $1/2$ 4) 0

3. From the point $P(2, 3)$ tangents PA, PB are drawn to the circle $x^2 + y^2 - 6x - 8y - 1 = 0$. The equation to the line joining the mid points of PA and PB is
- 1) $x + y + 7 = 0$ 2) $x - y - 7 = 0$ 3) $x + y - 7 = 0$ 4) $x - y + 7 = 0$
4. The common chord of $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ subtends at the origin an angle equal to
- 1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$
5. $x^2 + y^2 + 2\lambda x + 5 = 0$ and $x^2 + y^2 + 2\lambda y + 5 = 0$ are two circles. P is a point on the line $x - y = 0$. If PA and PB are the lengths of the tangents from P to the two circles and $PA = 3$, then $PB =$
- 1) 1 2) 3 3) 8 4) 5
6. The radical centre of the circles $x^2 + y^2 - x + 3y - 3 = 0$, $x^2 + y^2 - 2x + 2y - 2 = 0$, $x^2 + y^2 + 2x + 3y - 9 = 0$
- 1) $(2, -1)$ 2) $(2, 3)$ 3) $(-2, -1)$ 4) $(-2, -3)$
7. If A, B, C are the centres of three circles touching mutually externally then the radical centre of the circles for $\triangle ABC$ is
- 1) centroid 2) orthocentre 3) circum centre 4) incentre
8. ABC is a triangle. The radical centre of the circles with AB, BC, CA as the diameters is $(-6, 5)$. If $A(3, 2), B(2, 1)$ then $C =$
- 1) $(1, 1)$ 2) $(1, 2)$ 3) $(2, 3)$ 4) $(1, -2)$
9. A, B, C are the centres of the three circles C_1, C_2, C_3 such that C_1, C_2 touch each other externally and they both touch C_3 from inside then the radical centre of the circles is for triangle ABC
- 1) incentre 2) excentre opposite to C
3) excentre opposite to B 4) excentre opposite to A
10. B and C are two points on the circle $x^2 + y^2 = a^2$ point $A(b, c)$ lies on that circle such that $AB = AC = d$, then the equation of the line \vec{BC} is
- 1) $bx + ay = a^2 - d^2$ 2) $bx + ay = d^2 - a^2$
3) $2(bx + cy) = 2a^2 - d^2$ 4) $2(bx + ay) = 2a^2 - d^2$
11. If P and Q are the points of intersection of the circle $x^2 + y^2 + 3x + 7y - 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P, Q and $(1, 1)$ for
- 1) all except one value of p 2) all except two values of p
3) exactly one value of p 4) all values of p
12. If $C_1(1, 3)$ and $C_2(4, 3)$ are the centres of two circles whose radical axis is y -axis. If the radius of the 1st circle is $2\sqrt{2}$ units, then the radius of the second circle is
- 1) $\sqrt{23}$ 2) 3 3) 4 4) $2\sqrt{2}$
13. The circles having radii 1, 2, 3 touch each other externally. Then the radius of the circle which cuts the three circles orthogonally is
- 1) 1 2) $\frac{3}{2}$ 3) 2 4) 3

14. Three circles are such that each touch the other two externally. The common tangents are concurrent at P . The length of the tangent to each circle is p . The ratio of the product of their radii to sum of their radii is
 1) $p/2$ 2) $p^2/2$ 3) p 4) p^2
15. $x^2+y^2 = a^2$ and $(x-c)^2+y^2 = b^2$ are two intersecting circles. If a, b, c are the sides BC, CA, AB of $\triangle ABC$. If p_1, p_2, p_3 are the altitudes through A, B, C respectively then the length of the common chord is
 1) $2p_1$ 2) $2p_2$ 3) $2p_3$ 4) p_1
16. If the circle $S = x^2 + y^2 - 16 = 0$ intersects another circle $S' = 0$ of radius 5 in a such a manner that the common chord is of maximum length and has a slope equal to $\frac{3}{4}$ then the centre of $S' = 0$ is
 1) $\left(\frac{9}{5}, \frac{-12}{5}\right)$ or $\left(\frac{-9}{5}, \frac{12}{5}\right)$ 2) $\left(\frac{9}{5}, \frac{12}{5}\right)$ or $\left(\frac{-9}{5}, \frac{-12}{5}\right)$
 3) $\left(\frac{9}{7}, \frac{-12}{7}\right)$ or $\left(\frac{-9}{7}, \frac{12}{7}\right)$ 4) $\left(\frac{9}{7}, \frac{12}{7}\right)$ or $\left(\frac{-9}{7}, \frac{-12}{7}\right)$
17. The length of the common chord of two circles of radii r_1 and r_2 which intersect at right angles is
 1) $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$ 2) $\frac{2r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$ 3) $\frac{r_1 + r_2}{\sqrt{r_1^2 + r_2^2}}$ 4) $\frac{r_1 r_2}{r_1^2 + r_2^2}$
18. The length of the common chord of the circles $x^2+y^2+ax+by+c=0$ and $x^2+y^2+bx+ay+c=0$ is
 1) $\sqrt{\frac{(a+b)^2 - 8c}{2}}$ 2) $\sqrt{\frac{(a-b)^2 - 8c}{2}}$ 3) $\sqrt{\frac{(a-b)^2 + 8c}{2}}$ 4) $\sqrt{\frac{(a+b)^2 + 8c}{2}}$
19. If the circle $x^2+y^2+2gx+2fy+c=0$ bisects the circumference of the circle $x^2+y^2+2g'x+2f'y+c'=0$ then the length of the common chord of the circles is
 1) $2\sqrt{g^2 + f^2 - c}$ 2) $2\sqrt{g'^2 + f'^2 - c'}$ 3) $2\sqrt{g^2 + f^2 + c}$ 4) $2\sqrt{g'^2 + f'^2 + c'}$
20. The equation of the circle with the chord $2x=y$ of the circle $x^2+y^2=10x$ as its diameter is
 1) $x^2+y^2=2x+4y$ 2) $x^2+y^2=2x+4y+5$ 3) $x^2+y^2=x+2y$ 4) $x^2+y^2=2x+y$
21. The equation of the circle which passes through the origin has its centre on the line $x+y=4$ and cuts orthogonally the circle $x^2+y^2-4x+2y+4=0$
 1) $x^2+y^2-4x-4y=0$ 2) $x^2+y^2-2x-6y=0$ 3) $x^2+y^2-6x-2y=0$ 4) $x^2+y^2+4x-12y=0$
22. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts each of the circle
 $x^2 + y^2 - 4 = 0$,
 $x^2 + y^2 - 6x - 8y + 4 = 0$ and
 $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter, then
 1) $c = -5$ 2) $g + f = c - 2$ 3) $g^2 + f^2 - c = 17$ 4) $gf = 7$
23. The equation of the circle passing through the point of intersection of the circle $x^2 + y^2 = 4$ and the line $2x + y = 1$ and having minimum possible radius is
 1) $5x^2 + 5y^2 + 18x + 6y - 5 = 0$ 2) $5x^2 + 5y^2 + 9x + 8y - 15 = 0$
 3) $5x^2 + 5y^2 + 4x + 9y - 5 = 0$ 4) $5x^2 + 5y^2 - 4x - 2y - 18 = 0$

24. Tangents are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles. $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0$, λ being the variable. The locus of the point of intersection of these tangents is
- 1) $2x - y + 10 = 0$ 2) $x - 2y - 10 = 0$ 3) $x - 2y + 10 = 0$ 4) $2x + y - 10 = 0$
25. If C_1 , C_2 and C_3 belong to a family of circles through the points (x_1, y_1) and (x_2, y_2) the ratio of lengths of the tangent from any point on C_1 to the circles C_2 and C_3 is
- 1) constant 2) 1 : 2 3) 2 : 1 4) 3 : 4
26. If θ is the angle of intersection of two circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ then the length of the common chord of two circles is
- 1) $\frac{ab}{\sqrt{a^2 + b^2 - 2ab\cos\theta}}$ 2) $\frac{2ab}{\sqrt{a^2 + b^2 - 2ab\cos\theta}}$
- 3) $\frac{2ab\sin\theta}{\sqrt{a^2 + b^2 - 2ab\sin\theta}}$ 4) $\frac{2ab\cos\theta}{\sqrt{a^2 + b^2 - 2ab\sin\theta}}$

PRACTICE SHEET

EXERCISE-I

Angle between two circles - orthogonality - circle passing through intersection of line and circle - circles through two fixed points - circles passing through a given point on a given line - miscellaneous problems

1. Angle between the circles $x^2 + y^2 - 4x - 6y - 3 = 0$, $x^2 + y^2 + 8x - 4y + 11 = 0$ is
- 1) $\frac{\pi}{3}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{2}$ 4) $\frac{5\pi}{3}$
2. The circle $x^2 + y^2 + 4x + 6y + 3 = 0$ and $2(x^2 + y^2) + 6x + 4y + c = 0$ will cut orthogonally if $c =$
- 1) 4 2) 18 3) 12 4) 16
3. If the circles of same radius 'a' and centres at (2, 3) and (5, 6) cut orthogonally, then $a =$
- 1) 3 2) 4 3) 6 4) 10
4. The equation of the circle which pass through the origin and cuts orthogonally each of the circles $x^2 + y^2 - 6x + 8 = 0$ and $x^2 + y^2 - 2x - 2y = 7$ is
- 1) $3x^2 + 3y^2 - 8x - 13y = 0$ 2) $3x^2 + 3y^2 - 8x + 29y = 0$
- 3) $3x^2 + 3y^2 + 8x + 29y = 0$ 4) $3x^2 + 3y^2 - 8x - 29y = 0$
5. The circle through the two points $(-2, 5)$, $(0, 0)$ and intersecting $x^2 + y^2 - 4x + 3y - 1 = 0$ orthogonally is
- 1) $2x^2 + 2y^2 - 11x - 16y = 0$ 2) $x^2 + y^2 - 4x - 4y = 0$
- 3) $x^2 + y^2 + 2x - 5y = 0$ 4) $2x^2 + 2y^2 + 2x - 5y + 1 = 0$
6. A circle passes through origin and has its centre on $y = x$. If it cuts $x^2 + y^2 - 4x - 6y + 10 = 0$ orthogonally then the equation of the circle is
- 1) $x^2 + y^2 - x - y = 0$ 2) $x^2 + y^2 - 6x - 4y = 0$
- 3) $x^2 + y^2 - 2x - 2y = 0$ 4) $x^2 + y^2 + 2x + 2y = 0$

7. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally then the locus of its centre is
- 1) $2ax + 2by + a^2 + b^2 + 4 = 0$ 2) $2ax - 2by - (a^2 + b^2 + 4) = 0$
 3) $2ax - 2by + (a^2 + b^2 + 4) = 0$ 4) $2ax + 2by - (a^2 + b^2 + 4) = 0$
8. If r and r^1 are the radii of the circles $S = 0$ and $S^1 = 0$ respectively then the circles $\frac{S}{r} \pm \frac{S^1}{r^1} = 0$ intersect at an angle of
- 1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{6}$
9. $(1, 2)$ is a point on the circle $x^2 + y^2 + 2x - 6y + 5 = 0$ which is orthogonal to $x^2 + y^2 = 5$. The conjugate point of $(1, 2)$ w.r.t the circle $x^2 + y^2 = 5$ and which lies on the first circle is
- 1) $(7, -1)$ 2) $(9, -2)$ 3) $(-3, 4)$ 4) $(0, 5)$
10. The points $A(2, 3)$ and $B(-7, -12)$ are conjugate points w.r.t to the circle $x^2 + y^2 - 6x - 8y - 1 = 0$. The centre of the circle passing through A and B and orthogonal to given circle is
- 1) $(-5, -9)$ 2) $(-9, -15)$ 3) $\left(-\frac{5}{2}, -\frac{9}{2}\right)$ 4) $\left(\frac{1}{2}, \frac{3}{2}\right)$

❖❖❖ EXERCISE-II ❖❖❖

Radical axis-radical centre - coaxial system - limiting points - orthogonal coaxial system

1. The common tangent at the point of contact of the two circles $x^2 + y^2 - 2x - 4y - 20 = 0$, $x^2 + y^2 + 6x + 2y - 90 = 0$ is
- 1) $4x + 3y + 35 = 0$ 2) $3x + 4y + 35 = 0$ 3) $4x + 3y - 35 = 0$ 4) $4x - 2y - 110 = 0$
2. The equation of the circle which cuts the three circles $x^2 + y^2 - 4x - 6y + 4 = 0$, $x^2 + y^2 - 2x - 8y + 4 = 0$, $x^2 + y^2 - 6x - 6y + 4 = 0$ orthogonally is
- 1) $x^2 + y^2 = 4$ 2) $x^2 + y^2 = 2$ 3) $x^2 + y^2 = 1$ 4) $x^2 + y^2 = 8$
3. The slope of the radical axis of the circles $x^2 + y^2 + 3x + 4y - 5 = 0$ and $x^2 + y^2 - 5x + 5y + 6 = 0$ is
- 1) 1 2) 2 3) 5 4) 8
4. If the circles $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + bx + ay + c = 0$ touch each other then $(a+b)^2 = \lambda c$ where λ is
- 1) 2 2) 4 3) 6 4) 8
5. The radical centre of the circles $x^2 + y^2 - x + 3y - 3 = 0$, $x^2 + y^2 - 2x + 2y - 2 = 0$, $x^2 + y^2 + 2x + 3y - 9 = 0$
- 1) $(2, -1)$ 2) $(2, 3)$ 3) $(-2, -1)$ 4) $(-2, -3)$
6. The radical centre of the three circles described on the three sides of a triangle as diameter is.....of the triangle
- 1) centroid 2) orthocentre 3) circum centre 4) incentre
7. A, B, C are the centres of three circles of equal radii which do not touch externally pairwise whose centers are non-collinear. The radical centre of the circles for triangle ABC is
- 1) circumcentre 2) centroid 3) orthocentre 4) incentre

8. A line 'l' meets the circle $x^2+y^2=61$, in A, B and P (-5, 6) is such that $PA = PB = 10$. Then the equation of 'l' is
 1) $5x+6y+11=0$ 2) $5x-6y-11=0$ 3) $5x-6y+11=0$ 4) $5x-6y+12=0$
9. The centres of the circles are (a,c) and (b,c) and their radical axis is y-axis. The radius of one of the circles is r. The radius of the other circle is
 1) $r^2 - a^2 + b^2$ 2) $2(r^2 - a^2 + b^2)$ 3) $\sqrt{r^2 - a^2 + b^2}$ 4) $2\sqrt{r^2 - a^2 + b^2}$
10. If the circles $x^2+y^2+2ax+cy+a=0$ and $x^2+y^2-3ax+dy-1=0$ intersect in two distinct points P and Q then the line $5x+by-a=0$ passes through P and Q for
 1) exactly one value of 'a' 2) no value of 'a'
 3) infinitely many values of 'a' 4) exactly two values of 'a'
11. If A(2, -1), B(3, 1), C(1, -2) then the radical centre of the circles with AB, BC, CA as diameters is
 1) (1, -2) 2) (2, 1) 3) (11, -7) 4) (4, 7)
12. The polars of a point w.r.t. P two given circles meet in Q. The radical axis of the circles divide PQ in the ratio
 1) 1 : 1 2) 1 : 2 3) 2 : 1 4) 2 : 3
13. The polar of P(3, 5) with respect to circles $x^2+y^2-16x+36=0$ and $x^2+y^2+16x+36=0$ intersect at Q. Then the circle with PQ as diameter passes through
 1) (± 4 , 0) 2) (± 6 , 0) 3) (± 8 , 0) 4) (± 16 , 0)
14. Given points are P = (1, -2), Q = (7, 6). O is the origin. The length of the common chord of the circles with OP and OQ as diameters is
 1) 1 2) 2 3) 4 4) 6
15. The length of the common chord of the circles of radii 15 and 20 whose centres are 25 units apart is
 1) 24 2) 25 3) 15 4) 20
16. The length of the common chord of the two circles $(x-a)^2+(y-b)^2=c^2$, $(x-b)^2+(y-a)^2=c^2$ is
 1) $\sqrt{4c^2+2(a-b)^2}$ 2) $\sqrt{4c^2+2(a+b)^2}$ 3) $\sqrt{4c^2-2(a-b)^2}$ 4) $\sqrt{c^2-(a-b)^2}$
17. If the circle $x^2+y^2+4x+22y+c=0$ bisects the circumference of the circle $x^2+y^2-2x+8y-d=0$ then $c+d =$
 1) 50 2) 25 3) 60 4) 30
18. The circle on the chord $x\cos\alpha + y\sin\alpha = p$ of the circle $x^2+y^2=r^2$ as diameter is
 1) $x^2+y^2-r^2-2p(x\cos\alpha + y\sin\alpha - p)=0$ 2) $x^2+y^2-r^2+2p(x\cos\alpha - p)=0$
 3) $x^2+y^2-r^2-p(x\cos\alpha + y\sin\alpha - p)=0$ 4) $x^2+y^2-r^2+2p(x\cos\alpha + y\sin\alpha + p)=0$
19. If the circle $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x + 8y - d = 0$ (where c and d are greater than zero). Then maximum value of cd is
 1) 25 2) 125 3) 425 4) 625

❖❖❖ KEY SHEET ❖❖❖

LECTURE SHEET

EXERCISE- I

- 1) 3 2) 1 3) 4 4) 1 5) 1 6) 1 7) 2 8) 3 9) 3 10) 3
11) 2 12) 1

EXERCISE- II

- 1) 2 2) 4 3) 1 4) 4 5) 2 6) 1 7) 4 8) 4 9) 2 10) 3
11) 1 12) 1 13) 1 14) 3 15) 4 16) 1 17) 2 18) 1 19) 2 20) 1
21) 1 22) 3 23) 4 24) 1 25) 1 26) 3

PRACTICE SHEET

EXERCISE- I

- 1) 1 2) 2 3) 1 4) 2 5) 1 6) 3 7) 4 8) 3 9) 3 10) 3

EXERCISE- II

- 1) 3 2) 1 3) 4 4) 4 5) 1 6) 3 7) 4 8) 3 9) 3 10) 1
11) 3 12) 1 13) 2 14) 2 15) 1 16) 3 17) 1 18) 1 19) 4

