Chapter - 1

ATOMIC PHYSICS

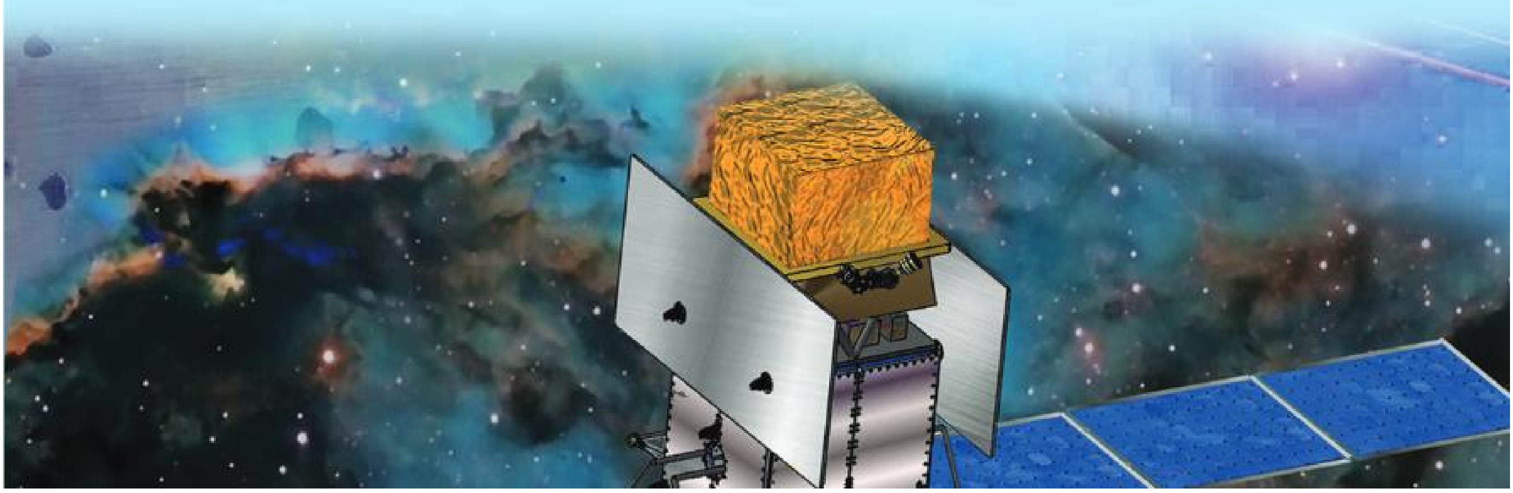
✤ Radiation pressure due to Electromagnetic Waves ✤

✤ Photoelectric Effect ✤

✤ de-Broglie Wave ✤

✤ Bohr's model of Hydrogen like atoms ✤

✤ X-Rays ✤



1.1 INTRODUCTION

Atoms are constituent particles of matter. In 1903 Dalton put forward his atomic theory according to which 'atom' is indivisible neutral particle. But Dalton's model of atom could not explain how electricity could interact with matter. Michael Faraday studied the passage of electricity through liquid solutions. His famous laws of electrolysis established that matter is electrical in nature and is composed of positively charged particles and negatively charged particles. All gases are insulators at normal atmospheric pressure. The experiments conducted by Masson, Heinrich Geissler using a discharge tube at low pressure revealed that even gases do conduct. In 1870 William Crookes did experiments about discharge of electricity through gases which led to the discovery of cathode rays. The cathode rays consist of streams of negatively charged particles i.e., electrons. The discovery of electrons produced in discharged tube paved the way for the new branch i.e., "Modern physics".

The laws that govern the characteristics of particles and waves are different in classical physics. For example, the working of jet plane obeys classical mechanics where as the phenomena of interference and diffraction which can be explained on wave nature do not find an explanation in Classical mechanics. We also observe that the energy that is carried by a wave is spread throughout the space in which the wavefronts travel but the energy that is carried by particle is confined to a small region of space. A particle or wave characteristic is not an inherent property but depends upon the type of experimental observation. Einstein was one of many who by developing quantum theory contributed to the development of a radical revision of our understanding of particle and wave character. French physicist Louis de Broglie proposed that under proper circumstances, particles can behave as waves while waves may behave as particles. Thus, dual nature : particle and wave, was established and another branch of physics, called Quantum mechanics, evolved. Quantum mechanics (or wave mechanics) involves concepts that are often strange which is not surprising since these concepts are used to describe phenomena observed on subatomic scale - which is much smaller than that accessible to normal human experience.

Maxwell's equations of electromagnetism and Hertz experiments on the generation and detection of electromagnetic waves in 1887 strongly established the wave nature of light. At the end of 19th century, experimental investigations on conduction of electricity (electric discharge) through gases at low pressure in a discharge tube led to many historic discoveries. The discovery of X-rays by Roentgen in 1895, and of electron by J.J. Thomson in 1897, were important milestones in the understanding of atomic structure.

1.2.1 DISCOVERY OF ELECTRON

J.J Thomson and Sir William Crookes studied the discharge of electricity through gases. The glass tube used to study the discharge of gases at low pressures is called discharge tube. It is a glass tube of 50 cm long and 4 cm in diameter as shown in the Fig 1.1.

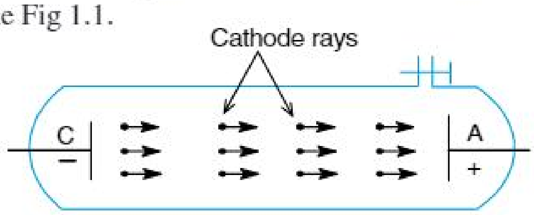


Fig 1.1

The pressure inside the discharge tube can be changed with the help of a vacuum pump. When a very high potential of the order of 15000 V to 20000 V is applied between the cathode (C) and anode (A) and when the pressure inside the discharge tube is decreased to about 0.01 mm of Hg, Crooke's dark space fills the whole length of the tube. At this stage a fluorescent glow appears on the walls of the tube, behind anode (A). This shows that some invisible radiations are coming from the cathode and travelling along the length of the tube towards anode. These invisible streams are called cathode rays. Later it was shown experimentally that they travel in straight lines and they possess momentum and energy. They ionise gases and affect photographic plates. They produce fluorescence. J.J Thomson observed that they are deflected by both electric and magnetic fields and the direction of their deflection is same as that of negatively charged particles. J.J Thomson determined the specific charge of these particles and also found that whatever may be the gas present in the discharge tube, the specific charge $\left(\frac{e}{m}\right)$ of cathode ray particles is same and they possess same properties and they are the constituents of every matter in nature. J.J Thomson called them "streams of negative corpuscles". Johnson Stoney suggested the name "electron". Thus the electron was discovered.

J.J. Thomson, suggested that they were fundamental, universal constituents of matter. For his epoch-making discovery of electron, through his theoretical and experimental investigations on conduction of electricity by gases, he was awarded the Nobel Prize in Physics in 1906. These electrons were found to travel with speeds ranging from about 0.1 to 0.2 times the speed of light (3×10^8 m/s). The presently accepted value of e/m is 1.76×10^{11} C/kg. Further, the value of e/m was found to be independent of the nature of the material used as the cathode (emitter), or the gas introduced in the discharge tube. This observation suggested the universality of the cathode ray particles.

In 1913, the American physicist R. A. Millikan performed the oil-drop experiment for the measurement of the charge on an electron. He found that the charge on an oil-droplet was always an integral multiple of an elementary charge, 1.602×10^{-19} C. Millikan's experiment established that electric charge is quantised. From the values of charge (e) and specific charge (e/m), the mass (m) of the electron could be determined.

1.2.2 PROPERTIES OF CATHODE RAYS

1) They travel along straight lines.
2) They cast sharp shadows of the objects in their path
3) They exert mechanical pressure
4) They are deflected by magnetic and electric fields.
5) When cathode rays strike a hard metallic surface, X-rays are produced.
6) Cathode rays ionise the gas through which they pass.
7) They cause fluorescence.
8) They can penetrate through thin sheets of matter
9) They have velocities upto $\frac{1}{10}$th of the velocity of light.

1.2.3 ELECTRON EMISSION

It is well known that the electrons in the outermost orbit of an atom are at maximum distance from the nucleus and hence most loosely bound to it. In case of metals, these loosely bound electrons can move freely inside the metals; but cannot come out of the metal surface by themselves. This is due to the reason that whenever an electron tries to leave the surface, the surface acquires a positive charge which pulls back the electron. To escape from the surface, an electron has to do a definite amount of work to overcome the force restraining it. To do this work, the electron must possess a certain minimum energy which can be imparted to it by an external source. This minimum energy is called the work function of the metal and is denoted by W.

The minimum energy required to liberate the most weakly bound surface electrons from a metal without giving them any velocity is called the work function of the metal.

Work function is measured in electron volt (eV), where $1\ eV = 1e \times 1\ V = (1.6\times10^{-19}C)(1\ V) = 1.6\times10^{-19}$ J.

This unit of energy is commonly used in atomic and nuclear physics. The work function W depends on the properties of the metal and the nature of its surface.

The minimum energy required for the electron emission from the metal surface can be supplied to the free electrons by any one of the following physical processes.

a) **Thermionic Emission:** The release of electrons from a metal as a result of its temperature, i.e., by heating is called thermionic emission. The electrons so emitted are called thermions or thermal electrons as thermal energy (i.e., heat) is supplied to obtain them.

b) **Field Emission:** It is a kind of electron emission in which a very strong electric field pulls the electrons out of the metal surface. High electric fields, of the order of 10^8 V/m, are necessary for the effect to be observed.

c) **Photoelectric Emission:** It is that kind of electron emission in which light of suitable frequency ejects the electrons from a metal surface.

Since these electrons are generated by light (photo), these are called photoelectrons.

Example-1.1

An electron at rest is accelerated through the potential difference applied across anode and cathode of 200 V. If the velocity of the electron is $8.4\times10^{6}\ ms^{-1}$, find its specific charge.

Solution : When electron is accelerated through a P. D. 'V', it gains a kinetic energy given by $\frac{1}{2}mv^2 = eV$

So specific charge $\frac{q}{m} = \frac{e}{m} = \frac{v^2}{2V} = 1.76\times10^{11}$ C/kg.

Example-1.2

An electron beam moving with a speed of $2.5\times10^7 ms^{-1}$ enters into a magnetic field directed perpendicular to its direction of motion. The magnetic induction of the field is $4\times10^{-3}\ Wb/m^2$. Find the intensity of an electric field to be applied so that the electron is undeflected due to the magnetic field.

Solution : When electron goes undeflected, force due to electric field is equal and opposite to force due to magnetic field i.e., $Ee = Bev$

$B = 4\times10^{-3}\ Wb/m^2$ and $v = 2.5\times10^7\ ms^{-1}$

$E = Bv = 4\times10^{-3}\times2.5\times10^7 = 10^5$ V/m

Example-1.3

A monoenergitic electron beam with the speed of $5.2\times10^6\ ms^{-1}$ enters a magnetic field of induction 3×10^{-4} T, directed normal to the beam. Find the radius of the circle traced by the beam.

(Give $e/m = 1.76\times10^{11} Ckg^{-1}$)

Solution : From the equation, $Bev = \frac{mv^2}{r}$

Radius of circle $r = \frac{mv}{Bq} = \frac{v}{B(e/m)}$

$$\therefore r = \frac{5.2\times10^6}{3\times10^{-4}(1.76\times10^{11})} = 0.1m$$

Example-1.4

In Millikan's oil drop experiment, an oil drop is held stationary by a potential difference of 400 V. If another drop of double the radius, but carrying the same charge, is to be held stationary, find the potential difference required ?

Solution : When oil drop is held stationary, $mg = Eq$

$$= \frac{4}{3}\pi r^3\rho g = \frac{V}{d}q,\ \text{i.e, } V \propto r^3$$

$$\text{(or) } \frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3 = \frac{V_2}{400} = \left(\frac{2r_1}{r_1}\right)^3$$

i.e., the potential difference required $V_2 = 3200$V.

1.3 PHOTOELECTRIC EFFECT

It has been observed that various materials have the ability to emit free electrons when irradiated by visible or ultraviolet light. This phenomenon of emission of electrons from a metal surface when exposed to electromagnetic radiation of sufficient frequency is called photoelectric effect. The emitted electrons are known as photoelectrons.

Hertz's observations

The phenomenon of photoelectric emission was discovered in 1887 by Heinrich Hertz (1857-1894), during his electromagnetic wave experiments. In his experimental investigation on the production of electromagnetic waves by means of a spark discharge, Hertz observed that high voltage sparks across the detector loop were enhanced when the emitter plate was illuminated by ultraviolet light from an arc lamp.

Light falling on the metal surface somehow facilitated the escape of free, charged particles which we now know as electrons. When light falls on a metal surface, some electrons near the surface absorb enough energy from the incident radiation to overcome the attraction of the positive ions in the material of the surface. After gaining sufficient energy from the incident light, the electrons escape from the surface of the metal into the surrounding space.

1.4.1 LENARD'S EXPERIMENTAL STUDY OF PHOTOELECTRIC EFFECT

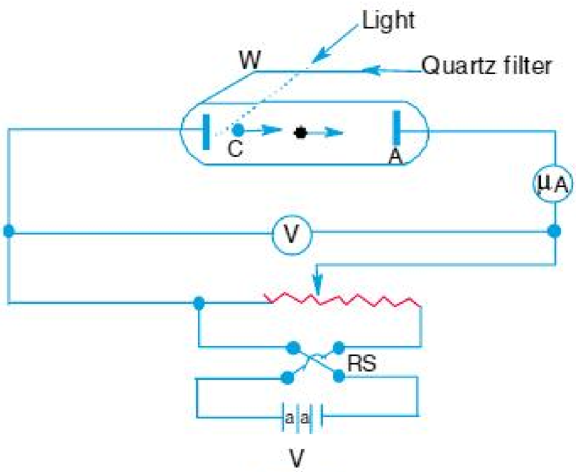


Fig 1.2

Description

Fig shows the experimental setup for studying the photoelectric effect. The arrangement consists of an evacuated glass or quartz tube enclosing a photosensitive cathode C and metallic anode A. A transparent window W is sealed onto the glass tube which can be covered with different filters to obtain the desired frequency. The anode A and cathode C are connected to a battery through a potential divider by which potential difference between anode and cathode can be changed. The reversing switch RS or commutator permits to make anode positive or negative w.r.t cathode. The potential difference between anode and cathode is measured by the voltmeter V while photoelectric current is indicated by the microammeter.

Working

1) When key K is open and monochromatic light is made incident on the cathode, then current is measured by the ammeter. i.e., even though applied voltage is zero current flows in the circuit.

These photoelectrons emitted from the cathode C move towards anode A. But less energetic electrons come to rest before reaching the anode. After some time the space between C and A contains a number of electrons making up **space charge**. This negative space charge repels the next coming electrons from the cathode. However some energetic electrons are able to reach the anode and causes photocurrent.

2) When anode is given positive potential w.r.t the cathode, electrons in the space charge are attracted towards the anode and so photocurrent increases. If potential of the anode is increased gradually the effect of space charge becomes negligible at some potential and then every electron that is emitted from the cathode will be able to reach the anode. The current then becomes constant even though voltage is increased and this current is called **saturation photocurrent**.

3) When anode is given negative potential w.r.t the cathode, the photo electrons will be repelled by the anode and some electrons will go back to cathode so current decreases. At some negative potential anode current becomes zero. This potential is called stopping potential.

The minimum negative potential (V_0) given to the collector with respect to the emitter for which 'photocurrent' becomes zero is called 'stopping potential'.

Stopping potential is related to maximum kinetic energy of photoelectrons, because at this potential even the most energetic electron just fails to reach the anode.

So work done by the stopping potential is equal to the maximum kinetic energy of the electrons.

$$(-e)(-V_0) = \frac{1}{2}mv_{max}^2 - 0$$

$$\therefore eV_0 = \frac{1}{2}mv_{max}^2$$

A graph is plotted with current on y-axis and applied voltage on x-axis. It is as shown in figure.

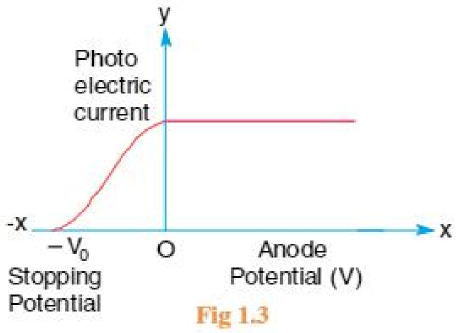


Fig 1.3

The dependence of photoelectric current on the following three factors is studied.

1) Effect of the intensity of Light

When the collecting electrode is positive relative to the plate and its potential is kept fixed, then for a given frequency and intensity of light, there is saturation photo current. This saturation photocurrent (i) is proportional to the intensity (I) of the light only as shown in Fig 1.4.

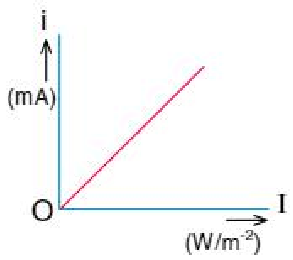


Fig 1.4

It is observed that

(a) One photon of incident light would emit one electron from the metal. But this does not imply that number of electrons emitted from the metal surface will be equal to the number of photons incident on it. Actually, the photons falling on the metal surface are involved in many other processes beside emission of electrons. Hence the ratio of number of electrons emitted to the number of photons incident on the surface is quite less than one. This is known as quantum efficiency of the surface. This can be increased by a special treatment of the surface.

> ***Note :*** In this chapter we consider number of photo electrons emitted equal to number of photons incident unless stated otherwise.

(b) There is no threshold intensity, but a bright light yields more photo electrons than dim one of the same frequency.

2) Effect of Potential Difference

For a given frequency and intensity of light, the photocurrent (i) first increases on increasing the positive potential of the collector with respect to the plate, and then becomes independent of the applied potential difference. The current is assumed to have reached its maximum value, called the saturation current. This is shown in Fig 1.5.

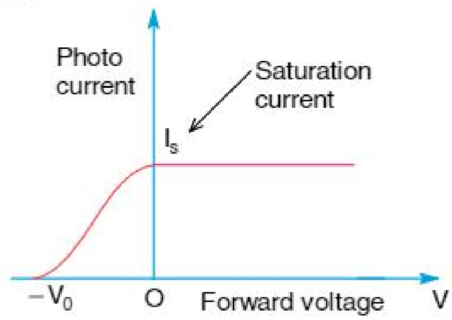


Fig 1.5

When the plate is made positive with respect to the collector, (i.e., when the applied potential difference is made negative), then the photocurrent starts decreasing linearly and finally becomes zero at a particular value of negative potential, called the stopping potential or cut off potential (V_0).

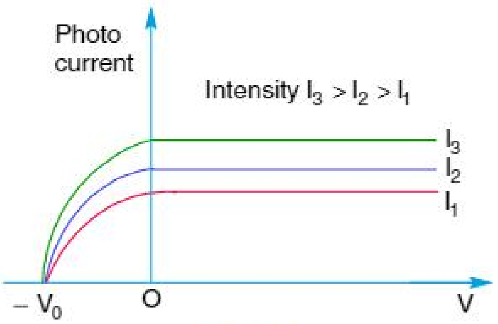


Fig 1.6

It is observed that

(a) The magnitude of saturation current depends on the intensity of light. Higher the intensity; larger the saturation current, as shown in Fig 1.6.

(b) At the stopping potential difference, the most energetic photoelectron is just prevented from reaching the collector.

(c) The value of stopping potential difference is independent of the light intensity.

3) Effect of Frequency of incident radiation

For a given intensity of light, rate of photo electron emission is constant. On increasing the frequency of light, kinetic energy of the photo electron increases and hence the stopping potential. Higher the frequency, larger the value of stopping potential, as shown in Fig 1.7.

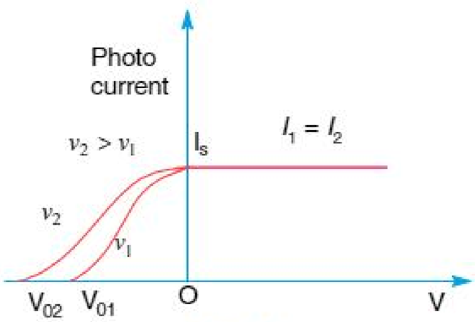


Fig 1.7

It is observed that

(a) The saturation current is independent of the frequency of radiation.

(b) The maximum kinetic energy of the emitted electrons depends on the frequency of light and the metal plate. It is independent of the intensity of light.

(c) For a metal plate there exists a minimum frequency called threshold frequency (ν_0) below which no electron is emitted irrespective of the intensity of light. Above that frequency, electrons are emitted with finite kinetic energy.

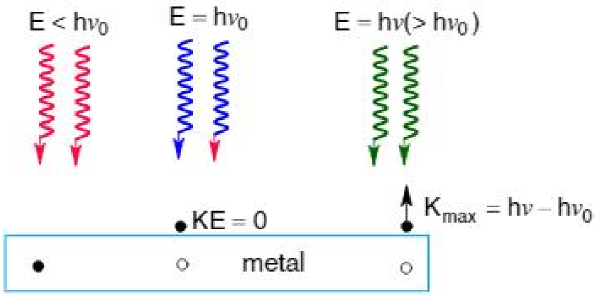


Fig 1.8

(d) Threshold frequency is a characteristic of the metal plate and at this frequency kinetic energy of the photo electron is zero. Above threshold frequency, photo electron will have energy from zero to a maximum value that increases linearly with increasing frequency as shown in the Fig 1.9.

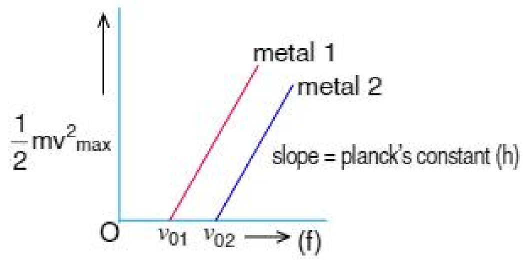


Fig 1.9

Note : Actually most of the electrons posses less kinetic energy than the maximum possible value because they lose a part of their kinetic energy due to collision with the atoms on their way to the surface. Thus the electrons with different energies are emitted from the metal. The electrons emitted from the surface of the metal have maximum kinetic energy because their energy is not lost by collisions.

1.4.2 PHOTOELECTRIC EFFECT AND WAVE THEORY OF LIGHT

By the end of the nineteenth century the wave nature of light was well established. The phenomena of interference, diffraction and polarisation were explained by the wave nature of light. According to this, light is an electromagnetic wave consisting of electric and magnetic fields with continuous distribution of energy over the region which the wave is extended. Let us now see if this wave picture of light can explain the observations on photoelectric emission given in the previous section.

According to the wave nature of light, the free electrons at the surface of the metal absorb the radiant energy continuously. Hence, greater the intensity, greater should be the energy absorbed by each electron. So, the maximum kinetic energy of the photoelectrons on the surface is then expected to increase with increase in intensity. Also, whatever may be the frequency of radiation, a sufficiently intense beam of radiation (over sufficient time) should be able to impart enough energy to the electrons, so that they exceed the minimum energy needed to escape from the metal surface. A threshold frequency, therefore, should not exist. These expectations of the wave theory directly contradict observations given as laws of photo electric emission.

Moreover, according to wave nature, the absorption of energy by electron takes place continuously over the entire wavelength range of the radiation. Since a large number of electrons absorb energy, the energy absorbed per electron per unit time will be small. Explicit calculations estimate that it can take hours or more for a single electron to pick up sufficient energy to overcome the work function and come out of the metal. This conclusion is again in striking contrast to law (iv) that the photoelectric emission is instantaneous. In short, the wave picture is unable to explain the most basic features of photoelectric emission.

1.4.3 LAWS OF PHOTOELECTRIC EMISSION

First Law: The number of photoelectrons emitted per second is proportional to the intensity of the incident radiation.

Second Law: The photoelectrons are emitted with a range of kinetic energies from zero up to a maximum which increases as the frequency of the radiation increases and is independent of the intensity of the radiation.

Third Law : For a given metal there is a certain minimum frequency of radiation, called the threshold frequency, below which no emission occurs irrespective of the intensity of the radiation. The corresponding wavelength of photon is called threshold wavelength. Photons having wavelength below this can only generate photoelectrons.

Fourth Law : There is no time lag (10^{-9}s) between the arrival of the incident radiation and the emission of photo electrons.

1.4.4 EINSTEIN'S EXPLANATION OF PHOTOELECTRIC EFFECT - EINSTEIN'S PHOTOELECTRIC EQUATION

Photoelectric effect cannot be explained by electromagnetic wave theory. It can be explained by quantum theory. Planck proposed quantum theory in which the energy is emitted and absorbed in the form of quanta of energy $h\nu$, where ν is the frequency of the radiation, explained photoelectric effect by assuming that light emitted by a source in the form of photons continues to travel through space in the form of photons and it will be incident on the metal surface in the form of photons.

When photons of frequency ν, each of energy $h\nu$, are incident on a metal surface they may collide with the valence electrons in the metallic bond called semi free electrons. During the collision process in a particular collision a photon may give up all its energy to an electron at the metal surface. Such an electron takes the entire energy of the photon and comes out of the metal surface doing a work W. The remaining energy i.e., ($h\nu-W$) appears as the maximum kinetic energy of the liberated electron called photoelectron.

Therefore the maximum kinetic energy of the photoelectron is

$K_{max} = h\nu - W$

If 'vmax' is the maximum velocity of the photoelectron and 'm' is its mass then

$$\frac{1}{2}mv^2{}_{max} = h\nu - W$$

In W is called work function. It is the minimum amount of energy required to eject an electron from a metal surface. It is different for different metals. Alkali metals have small work function and caesium has the least work function among all the alkali metals. Alkali metals can show photoelectric effect even with visible light.

The above equation is called Einstein's photoelectric equation and by this equation we can explain the laws of photoelectric emission.

For a given metal, work function W_0 is constant. If ν_0 is the threshold frequency, then the minimum energy of photon just sufficient to remove an electron from a metal surface is $h\nu_0$ which is equal to the work function (W).

$(W) = h\nu_0$

The threshold frequency, ν_0 is the minimum frequency of incident light for a metal surface, below which the light cannot eject electrons. It is the least for caesium.

Then $K_{max} = h\nu - W$ becomes

$$K_{max} = h\nu - h\nu_0 = h(\nu - \nu_0) = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right)$$

Where λ_0 and λ are the threshold wavelength and wavelength of incident radiation respectively and c is the velocity of light.

It can be seen that unless $\nu \geq \nu_0$ electrons cannot be liberated by a metal surface which is the first law of photoelectric emission.

The graph K_{max} versus the frequency of incident light is a straight line indicating that the maximum kinetic energy of the photoelectrons is a linear function of the frequency of incident light only. This explains the second law of photoelectric emission.

The kinetic energy of photoelectrons is maximum for the electrons liberated from the surface of the metal. If an interior electron is liberated from the metal due to the collision of a photon the electron has to do more work while coming out making its kinetic energy less than the maximum kinetic energy. Thus the kinetic energy of the photoelectrons is between 0 and K_{max}.

When the intensity of incident light increases, the number of photons incident on the metal surface increases. Since the intensity of light on a surface is the light energy incident per unit area per unit time which is equal to the product of the number of photons incident per unit area per unit time and the energy of the photon, $h\nu$. As the number of photons incident increases the collisions with electrons increase liberating greater number of photoelectrons. This increases the photoelectric current which is the third law of photoelectric emission.

If V_0 is the stopping potential, the work done to stop a photoelectron is equal to the maximum kinetic energy of the photoelectron.

$$eV_0 = \frac{1}{2}mv_{max}^2$$

Substituting in $K_{max} = h(\nu - \nu_0)$ we get

$$eV_0 = h(\nu - \nu_0)$$

$$V_0 = \frac{h}{e}(\nu - \nu_0)$$

This equation shows a linear relationship between the stopping potential (V_0) and the frequency (v) of incident radiation.

This is an important result. It predicts that V_0 versus ν curve is a straight line with slope = (h/e), independent of the nature of the material. During 1906-1916, Millikan performed a series of experiments on photoelectric effect, aimed at disproving Einstein's photoelectric equation. He measured the slope of the straight line obtained for sodium. Using the known value of e, he determined the value of Planck's constant h. This value was close to the value of Planck's constant ($=6.626\times10^{-34}$ Js) determined in an entirely different context. In this way, in 1916, Millikan proved the validity of Einstein's photoelectric equation, instead of disproving it.

The successful explanation of photoelectric effect using the hypothesis of light quanta and the experimental determination of values of h and W, in agreement with values obtained from other experiments, led to the acceptance of Einstein's picture of photoelectric effect. Millikan verified photoelectric equation with great precision, for a number of alkali metals over a wide range of radiation frequencies.

Example-1.5

What will be the minimum frequency of light source to get photocurrent, from a metal surface having work function 2 eV.

Solution :

Given, work function $(W) = 2\text{ eV}$

i.e, $h\nu_0 = 2\text{eV} = 2\times1.6\times10^{-19}\text{ J}$

Minimum frequency (or) threshold frequency

$$(\upsilon_0) = \frac{2\times1.6\times10^{-19}}{6.62\times10^{-34}}$$

$$v_0 = 4.8\times10^{14}\text{ Hz}$$

Example-1.6

The photoelectric work function of potassium is 2.3eV. If light having a wavelength of 2800Å falls on potassium, find (a) The kinetic energy in electron volts of the most energetic electrons ejected. (b) The stopping potential in volts

Solution :

Given, $W = 2.3\text{eV}$ $\lambda = 2800\text{Å}$

$$\therefore\ E(\text{in eV}) = \frac{12375}{\lambda(\text{in Å})} = \frac{12375}{2800} = 4.4\text{eV}$$

(a) $K_{max} = E - W = (4.4-2.3)\text{eV}$

$= 2.1\text{ eV}$

(b) $K_{max} = eV_0$

$\therefore\ 2.1\text{eV} = eV_0$ (or) $V_0 = 2.1$ volt

Example-1.7

When a beam of 10.6eV photons of intensity 2.0 W/m^2 falls on a platinum surface of area $1.0\times10^{-4}\text{m}^2$ and work function 5.6eV, 0.53% of the incident photons eject photo electrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take $1\text{eV} = 1.6\times10^{-19}$ J.

Solution :

Number of photoelectrons emitted per second

$$= \frac{(\text{Intensity})(\text{Area})}{(\text{Energy of each photon})}\times\frac{0.53}{100}$$

$$= \frac{(2.0)(1.0\times10^{-4})}{(10.6\times1.6\times10^{-19})}\times\frac{0.53}{100}$$

$= 6.25\times10^{11}$

Minimum kinetic energy of photo electrons,

Kmin = 0 and maximum kinetic energy is,

$K_{max} = E - W$

$= (10.6-5.6)\text{eV} = 5.0\text{ eV}$

1.4.5 PARTICLE NATURE OF LIGHT

Photoelectric effect clearly established that while interacting with matter, light behaved as if it were made of packets or quanta of energy, the energy of each light quantum of frequency v being hv.

Is the light quantum of energy to be associated with a particle? Einstein arrived at the important result, that the light quantum can also be associated with momentum (hv/c). A definite value of energy as well as momentum is a strong sign that the light quantum can be associated with a particle. This particle was later named photon. The particle-like behaviour of light was further confirmed, by the experiment of A.H. Compton (1892-1962) on scattering of X-rays from electrons.

The properties of photon are :

i) A photon travels in vacuum with a speed of light c.

ii) The rest mass of photon is zero.

iii) Whatever the intensity of radiation may be, all photons of light of a particular frequency ν have the same energy $h\nu$ and momentum $h\nu/c$ $(= h/\lambda)$.

iv) Photons are electrically neutral and are not deflected by electric and magnetic fields.

v) In a photon-particle collision, the total energy and total momentum are conserved.

But the number of photons in a collision may not be conserved. A photon may be absorbed (Photoelectric effect) or a new photon may be created (Compton effect).

vi) At low energies (less than about 0.5 meV), the process that removes photons from a beam is the photoelectric effect in which a photon is absorbed and transfers all of its energy to an electron.

vii) At intermediate energies, the process is Compton scattering, in which the photon transfers some of its energy.

viii) At high energies, the process is pair production in which an electron-positron pair is created. Since the rest mass energy of an electron-positron pair is 1.02 MeV, the γ -ray photon must have at least this much energy to produce an electron - positron pair.

ix) When a photon is emitted or absorbed by an atom, the orbital angular momentum of the atom changes. Since angular momentum must be conserved, the photon involved in the process must carry angular momentum. Hence A photon has energy, linear momentum and angular momentum.

1.4.6 PHOTO ELECTRIC CELLS

A device which converts light energy into electrical energy is known as a photo electric cell. Depending upon the different photo-effects employed, the photo-electric cells are mainly of three types

(a) Photo-emissive cell : This cell is based on the fact that electrons are emitted from a photo-sensitive surface by the action of light.

Fig 1.10 shows the photo-emissive cell. It consists of a photo-sensitive cathode C and anode A enclosed in an evacuated glass envelope G. A battery B maintains the anode A at positive potential with respect to the cathode. When light of frequency greater than threshold frequency is made to fall upon the cathode C, photo electrons are emitted from it.

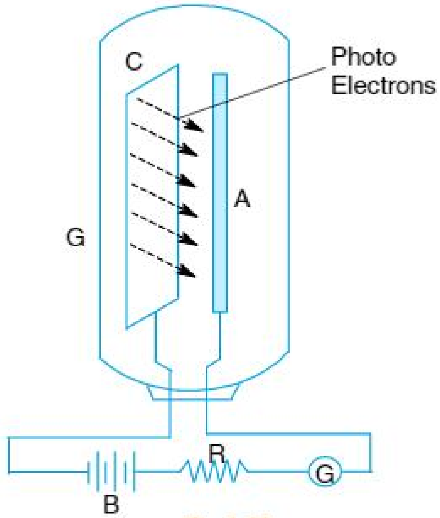


Fig 1.10

These photo-electrons are attracted by the positive anode to constitute current in the external circuit so long as the illumination is maintained. Sometimes, a small amount of inert gas like argon at low pressure (about a few mm) is introduced in the glass envelope. It is then called gas-filled photo-emissive cell. The presence of gas helps in obtaining more current for a given illumination due to ionization of the gas. Thus photo-emissive cells are of two types:

(I) In the vacuum type cell, current starts immediately after the light is incident and is proportional to the intensity of incident light. Hence this cell is used for photometry and in televisions.

(II) In gas-filled cells, the current is somewhat larger and is not proportional to the intensity of light. Hence, this type of cell is most suitable in cinematography and in the recording and reproduction of sound.

(b) Photo-conductive Cell : This cell is based on the fact that electrical resistance of certain semi-conductors e.g., selenium, lead sulphide etc, decreases when intensity of light falling on them increases.

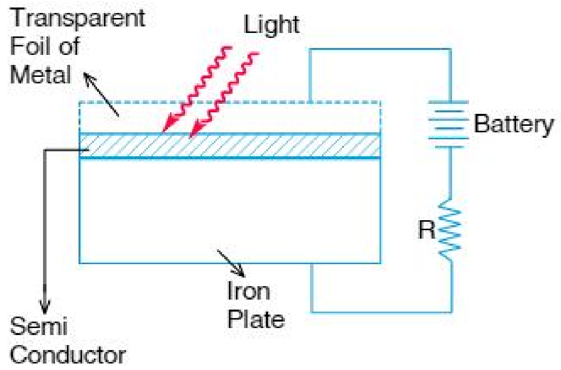


Fig 1.11

In this, a thin layer of semi-conductor is placed below a transparent film of some metal as shown in Fig 1.11. This combination is fixed over an iron plate. When light is allowed to fall on transparent film, the electrical resistance of the semi-conductor layer gets decreased i.e., its electrical conductance is increased. Hence a current starts flowing in the battery circuit connected between iron plate and the transparent metal film. This current changes with change in intensity of incident light with a time-lag . This is the main defect of this cell, due to which it is rarely used.

(c) Photo-Voltaic Cell: It is based on the fact that when the region of contact between two specially prepared conducting surfaces is illuminated, a flow of current takes place. No external battery is used to accelerate the emitted photoelectrons as the cell generates its own e.m.f.

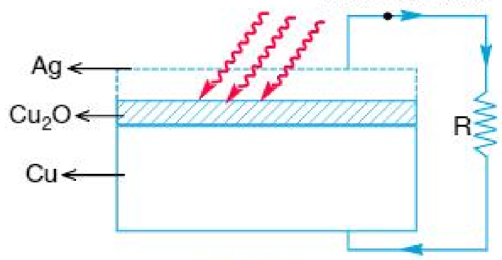


Fig 1.12

Fig 1.12 shows a photo-voltaic cell. It consists of a copper plate on which a layer of cuprous oxide is deposited. The active material cuprous oxide forms one electrode. The other electrode is a very thin film of gold or silver, so as to be transparent which is sprayed on the surface of cuprous oxide. When the light falls on the transparent electrode, it ejects electrons from the boundary between cuprous oxide layer and transparent layer. If a conducting path is provided by the external circuit, the current starts flowing in the circuit.

1.4.7 APPLICATIONS OF PHOTO-ELECTRIC CELLS

Photo-electric cells are being used in such varied fields of industry and research. Few important applications are mentioned here.

(i) The most important application of photo-electric cells is in the reproduction of sound in motion pictures.

(ii) In automatic switches for street lights : the photo electric cell is extensively used for switching ON or OFF street lights automatically. For this purpose, a photoelectric cell is inserted in the street light electric circuit. During night, the sun light ceases and the photo cell does not produce any current. This releases the relay and the street light circuit is switched on.

In the morning, as the sun rises, the light falls on the cell, producing a photoelectric current. This current is fed to the relay which breaks the street light circuit.

(iii) In Burglar's alarm : A pencil of invisible light (e.g. ultraviolet) is made to fall continuously on a photo-electric cell placed at a suitable place in the building. This gives photoelectric current which when amplified holds a relay. As soon as the thief enters unknowingly, he cuts light to the photocell which in turn releases the relay. This feeds current to an electric bell which starts ringing.

(iv) They are used to detect the opacity of solids

(v) These cells are used to open and close the doors automatically.

(vi) They are used to control temperature in furnace and chemical reactions.

(vii) The photoelectric cells are used for automatic counting of parts manufactured by a machine or of persons entering a room.

(viii) Photoelectric cells are also used in obtaining electric energy from sun-light during space-travel.

1.4.8 RADIATION PRESSURE OF ELECTROMAGNETIC WAVES

We know that light is an electromagnetic wave. An electromagnetic wave carries linear momentum, angular momentum as well as energy. Electromagnetic waves give rise to pressure when they are reflected or absorbed at the surface of a body.

Case (i) : We consider that the electromagnetic waves strike the surface at normal incidence and transport a total energy U to the surface in a time t. If the surface absorbs all the incident energy, the total momentum 'p' transported to the surface has a magnitude $p=\frac{U}{c}$. Therefore the pressure exerted on the surface is force per unit area F/A. From newton's 2^{nd} Law,

$$\text{Pressure}=\frac{F}{A}=\frac{1}{A}\left(\frac{\Delta p}{\Delta t}\right)=\frac{1}{Ac}\frac{dU}{dt}\left(\because p=\frac{U}{c}\right)$$

(or)

$$P=\frac{\frac{1}{A}\left(\frac{dU}{dt}\right)}{c}=\frac{I}{c}, \text{ where } I=\frac{1}{A}\left(\frac{dU}{dt}\right) \text{ is the}$$

intensity of the electromagnetic wave.

$\therefore$ Radiation pressure due to absorption $P_a=\frac{I}{c}$.

If S is the area of cross section of the photo beam then the force on the surface $F_a=\frac{I}{c}S$

If θ is the angle of incidence, the radiation pressure on the surface $P_a=\frac{I}{c}\cos\theta$. The force on the surface $F_a=\frac{I}{c}S\cos^2\theta$.

Case (ii) : If the surface is a perfect reflector and incidence is normal, then the momentum transported to the surface is $p=\frac{2U}{c}$. Hence the radiation pressure on a perfectly reflecting surface for normal incidence of the wave is $P_r=\frac{2I}{c}$

Force on the surface $F_r=\frac{2I}{c}S$.

If 'θ' is the angle of incidence, change in momentum at the perfectly reflecting surface is $2p\cos\theta$, Hence corresponding radiation pressure is $P_r=\frac{2I}{c}\cos\theta$. If S is the cross section area of the beam, then $F_r=\frac{2I}{c}S\cos^2\theta$

Case (iii) : If ρ is the reflection coefficient of the surface and θ is the angle of incidence, then the radiation pressure $P=\frac{I}{c}(1+\rho)\cos\theta$. The radiation force on the surface $F=\frac{I}{c}(1+\rho)S\cos^2\theta$

Note : If I is the intensity of radiation, then the number of photons of frequency f incident per second in given area S is $n=\frac{IS}{hf}$ $\left(\because I=\frac{\text{power}}{\text{area}}=\frac{N}{t}\frac{hf}{S}\right)$

Example-1.8

Intensity of incident radiation on a horizontal surface at sea level from sun is about 1 kW/m^2.

(a) Assuming that 50 percent of this intensity is reflected and 50 percent is absorbed, determine the radiation pressure on this horizontal surface.

(b) Find the ratio of this pressure to atmospheric pressure P_0 (about 1×10^5 Pa) at sea level.

Solution :

(a) The pressure exerted on the surface due to complete absorption is $P=\frac{I}{c}$ and due to reflection is $P=\frac{2I}{c}$.

Here pressure exerted by absorbed light $=\frac{1}{2}\left(\frac{I}{c}\right)$

Pressure exerted by reflected light $=\frac{1}{2}\left(\frac{2I}{c}\right)$

Total radiation pressure on the surface is

$$P_{rad}=\frac{\frac{3}{2}I}{c}=\frac{1.5\times10^{3}}{3\times10^{8}}=5\times10^{-6}\text{ Pa}$$

(b) $\frac{P_{rad}}{P_0}=\frac{5\times10^{-6}}{1\times10^{5}}=5\times10^{-11}$

Example-1.9

A plane light wave of intensity $I=0.20\text{W/cm}^2$ falls on a plane mirror surface with reflection coefficient $\rho=0.8$. The angle of incidence is 45°. In terms of corpuscular theory, find the magnitude of the normal pressure exerted on that surface.

Solution :

The pressure exerted on surface,

$$P=\frac{I}{c}(\rho+1)\cos\theta.$$

On substituting values,

we get $P=6\sqrt{2}\times10^{-6}\,\text{N/m}^2$.

1.5 DE BROGLIE WAVE AND WAVELENGTH

The phenomena of interference and diffraction of light confirm the wave theory of light. On the other hand the phenomena of photoelectric effect and Compton effect can be explained only in terms of the particle nature of light.

According to the wave nature of light, the radiant energy spreads out continuously in the form of wave. According to particle nature it behaves as a particle and is localized at a point in space. According to quantum theory, the radiant energy spreads out in discrete packets. Therefore it has been concluded that radiation behaves as wave and particle i.e., radiation has dual nature.

Whether a particle or wave description is best suited for understanding an experiment depends on the nature of the experiment. For example, in the familiar phenomenon of seeing an object with our eye, both descriptions are important.

The gathering and focussing mechanism of light by the eye-lens is well described in the wave picture. But its absorption by the rods and cones (of the retina) requires the photon picture of light.

A natural question arises: If radiation has a dual (wave-particle) nature, might not the particles of nature (the electrons, protons, etc.) also exhibit wave-like character? In 1924, the French physicist Louis Victor de Broglie (pronounced as de Broy) (1892-1987) put forward the bold hypothesis that moving particles of matter should display wave-like properties under suitable conditions.

De Broglie Hypothesis

de Broglie thought over the question "can electrons also have dual nature?" and put forward de Broglie hypothesis in his Ph.D. thesis. The hypothesis put forward by de Broglie is as given below.

1) The universe consists of matter and radiation only

2) Nature loves symmetry

3) If radiation has dual nature then matter should also have dual nature.

Particles like electrons, protons and neutrons can exhibit dual nature and behave like particles and waves. The waves associated with particles are called 'matter waves'. On the basis of theoretical considerations, de Broglie predicted that the wavelength of these waves is given by $\lambda=\frac{h}{p}$ where 'h' is the Planck's constant and p is the momentum of the particle.

Equation for a material particle is basically a hypothesis whose validity can be tested only by experiment. However, it is interesting to see that it is satisfied also by a photon. For a photon, as we have seen, $p=h\nu/c$

When a body is at rest, $v = 0$ and $P = 0$ and the wavelength of the matter waves becomes infinitely large indicating that matter waves do not exist for bodies at rest. If we consider macroscopic objects e.g., an object of mass 1 kg moving with a velocity of 1 m/s, its de Broglie wavelength, $\lambda = 6.63\times10^{-34}\,m$. It is impossible to see diffraction effects from such a small waves i.e., wave phenomena cannot be observed with such small waves. Matter waves are observable with microscopic particles like electrons. If an electron is accelerated through a voltage of 150 V, the wavelength of the matter waves associated with it is about $1A^0$ whose diffraction effects can be observed. Such an experiment was performed by Davisson and Germer. In 1929, de Broglie was awarded the Nobel prize in Physics for his discovery of the wave nature of electrons.

Note :

(i) If 'E' is the kinetic energy of the material particle, then $p = \sqrt{2mE}$, hence $\lambda = \dfrac{h}{\sqrt{2mE}}$

Example :

For neutron $\lambda = \dfrac{0.286}{\sqrt{E(eV)}}\,\mathring{A}$

(ii) If a charged particle carrying charge q is accelerated through a potential difference of V volt, then

$E = qV$, hence $\lambda = \dfrac{h}{\sqrt{2mqV}}$

Example :

For electron $\lambda = \dfrac{12.27}{\sqrt{V}}\,\mathring{A}$, for deuteron $\lambda = \dfrac{0.202}{\sqrt{V}}\,\mathring{A}$

for proton $\lambda = \dfrac{0.286}{\sqrt{V}}\,\mathring{A}$, for α particle $\lambda = \dfrac{0.101}{\sqrt{V}}\,\mathring{A}$

(iii) When the material particles like neutrons are in thermal equilibrium at absolute temperature T, then

$E = \dfrac{3}{2}kT = \dfrac{1}{2}mv^2$,

k is Boltzman's constant = 1.38×10^{-23} J/K,

Hence $\lambda = \dfrac{h}{\sqrt{3mkT}}$,

eg : for neutron $\lambda = \dfrac{30.8}{\sqrt{T}}\,\ddot{A}$

Example-1.10

Find the ratio of de Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27°C and 127°C respectively.

Solution :

Since, $\lambda = \dfrac{h}{m\upsilon} = \dfrac{h}{\sqrt{3mkT}}$

$$\frac{\lambda_H}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_H T_H}} = \sqrt{\frac{8}{3}}$$

Example-1.11

An α-particle and a proton are fired through the same magnetic field which is perpendicular to their velocity vectors. The α-particle and the proton move such that radius of curvature of their paths is same. Find the ratio of their de Broglie wavelengths.

Solution :

Since, $Bq\upsilon = \dfrac{m\upsilon^2}{r}$ or $m\upsilon = qBr$

The de Broglie wavelength $\lambda = \dfrac{h}{m\upsilon} = \dfrac{h}{qBr}$

$$\frac{\lambda_{\alpha-particle}}{\lambda_{proton}} = \frac{q_p r_p}{q_\alpha r_\alpha} = \frac{1}{2}\left(\frac{1}{1}\right) = \frac{1}{2}$$

1.5.1 HEISENBERG'S UNCERTAINTY PRINCIPLE

The matter wave (or) de Broglie wave associated with a moving particle should be considered as a wave packet instead of a continuous waves.

A wave packet is a localised wave consisting of continuous range of frequencies.

Except over a limited interval Δx, the various frequency components of the wave cancel each other. This interval represents the uncertainty in the position (x) of the particle. If we try to measure the position of the particle, it is sure to be found somewhere in the interval Δx.

The probability of finding the particle at any point is proportional to the intensity of the wave at that point.

The wave packet moves with a speed v where $v = p/m$ and $p = h/\lambda$, where 'p' is momentum of the particle and λ its de Broglie wavelength.

i) If the wave packet is long Fig 1.13(a) it contains a large number of waves. As such the chances of determining its wavelength and hence its momentum are better. But in this case, the particle may be anywhere within this long wave packet and as such its position is not precise.

ii) On the other hand, if the wave packet is narrow as shown in Fig 1.13(b), the position is determined more precisely as Δx is small. But now it is not easy to find its wavelength as it is much smaller than Δx.

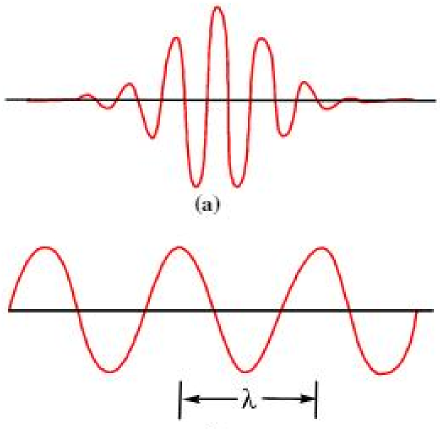


Fig 1.13

Thus, there is a reciprocal relationship between the inherent uncertainties in the position (Δx) of the particle and its momentum (Δp).

i.e., $\Delta x \propto 1/\Delta p$

i.e., the wave nature of the moving particle has put a lower limit on the product $\Delta x\, \Delta p$.

It is written as $\Delta x\, \Delta p \geq h/2\pi$

The above result was first discovered by Werner Heisenberg and is known after him as Heisenberg's uncertainty principle. It states that :

It is impossible to know simultaneously and with exactness both the position and the momentum of the fundamental particles constituting matter. The optimum precision in the knowledge of position and momentum is determined by $\Delta x \Delta p \geq h/2\pi$.

1.5.2 DAVISON AND GERMER EXPERIMENT

The wave nature of electron can be explained by Davisson and Germer experiment. They observed diffraction effects with beams of electrons scattered by crystals.

The experimental arrangement is shown in Fig 1.14. It consists of an electron gun with a tungsten filament 'F'. Electrons emitted by the filament are accelerated to a desired velocity by applying suitable voltage from power supply.

The electron beam is made to fall on the surface of nickel crystal. The electrons are scattered in all directions by the atoms of the crystal. The intensity of the electron beam scattered in a given direction is measured by the electron detector (collector).

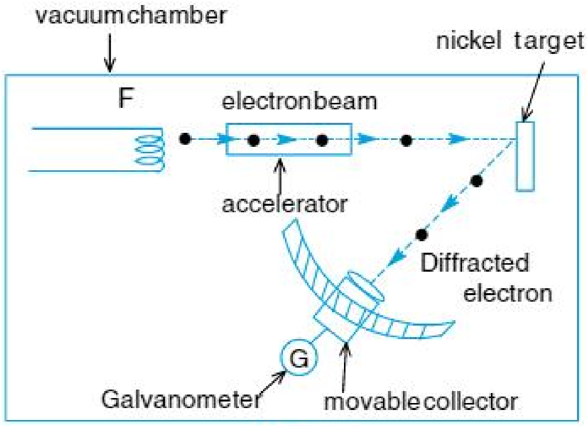


Fig 1.14

The detector can be moved on a circular scale and is connected to a sensitive galvanometer, which records the current. The deflection of the galvanometer is proportional to the intensity of the electron beam entering the collector. The apparatus is enclosed in an evacuated chamber.

By moving the detector on the circular scale at different positions, the intensity of the scattered electron beam is measured for different values of angle of scattering (θ) which is the angle between the incident and the scattered electron beams. The variation of intensity (I) of the scattered electron with the angle of scattering (θ) is obtained for different accelerating voltages. For the range of voltages 44V to 68 V, it was noticed that a strong peak appeared in the intensity (I) of the scattered electron for an accelerating voltage of 54V at a scattering angle $\theta = 50^{\circ}$. The appearance of the peak in a particular direction is due to the constructive inference of electrons scattered from different layers of the regularly spaced atoms of the crystals. The de-Broglie wavelength 'λ' associated with electrons, $\lambda = \frac{h}{p} = \frac{1.227}{\sqrt{V}}$ nm. For V = 54 volt, $\lambda = 0.167\text{nm}$.

Thus, there is an excellent agreement between the theoretical value and the experimentally obtained value of de Broglie wavelength. Davisson Germer experiment thus confirms the wave nature of electrons and the de Broglie relation. More recently, in 1989, the wave nature of a beam of electrons was experimentally demonstrated in a double-slit experiment, similar to that used for the wave nature of light. And, in an experiment in 1994, interference fringes were obtained with the beams of iodine molecules, which are about a million times more massive than electrons.

Thus the de Broglie hypothesis has become basic concept to the development of modern quantum mechanics. It has also led to the field of electron optics. The wave properties of electrons have been utilised in the design of electron microscope which is a great improvement, with higher resolution, over the optical microscope.

1.6 ATOMIC STRUCTURE

Atoms are the constituent particles of matter. In 1803 Dalton put forward his atomic theory according to which 'atom' is indivisible neutral particle. But Dalton's model of atom could not explain how electricity could interact with matter. Michael Faraday studied the passage of electricity through liquid solutions. His famous laws of electrolysis established that matter is electrical in nature and is composed of positively charged particles and negatively charged particles.

But what is the arrangement of the positive charge and the electrons inside the atom? In other words, what is the structure of an atom?

The first model of atom was proposed by J.J. Thomson in 1898.

Thomson's Model of an Atom: Sir J.J. Thomson studied the discharge phenomenon through gases at low pressures, and properties of cathode rays and positive rays. Then in 1907 he proposed what is now called Thomson's plum-pudding model of an atom.

According to this model, an atom consists of a sphere of radius of the order of 10^{-10} m. The particles responsible for the mass of the atom carry positive charge and are distributed uniformly over the sphere. The negatively charged particles, called electrons, are embedded within the atom. Every atom is electrically neutral so that the total positive charge on an atom is equal to the total negative charge on it.

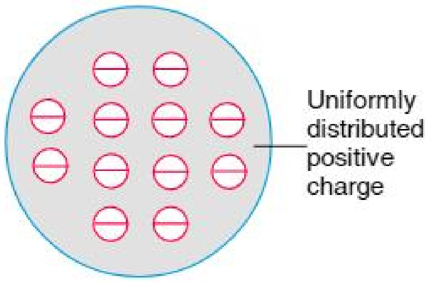


Fig 1.15

Thomson's model could not explain :

i) The emission of spectral lines from the atoms,

ii) the large angle of scattering of α-particles by thin metal foils.

Ernst Rutherford was engaged in experiments on α-particles emitted by some radioactive elements. In 1906, he proposed a classic experiment of scattering of these α-particles by atoms to investigate the atomic structure. This experiment was later performed around 1911 by Hans Geiger and Ernst Marsden, (who was 20 years-old student and had not yet earned his bacherlor's degree). The results of experiment led to Rutherford's planetory model of atom which is also called nuclear model of the atom.

According which:

i) The total positive charge of the atom and almost its entire mass is concentrated in a small region called nucleus of the atom.

ii) The nucleus is surrounded by electrons. As the atom is neutral, the total negative charge of the electrons is equal to the positive charge on the nucleus.

iii) The nuclear diameter is of the order of 10^{-14} m.

iv) For the stability of the atom, Rutherford assumed that the electrons are revolving at high speeds around the nucleus in closed circular orbits, so that the force of attraction between the nucleus and the electrons is balanced by the centrifugal force acting on the electrons.

If the electrons in an atom were stationary, these would fall into the nucleus due to the electrostatic force of attraction between the electrons and the nucleus.

v) The existence of sufficient empty space within the atom explains why most of the α- particle go undeflected. Small angle of scattering is accounted for by the fact that the nucleus occupies only a fraction of the total volume of the atom.

Thus, the drawbacks of Thomson's model of atom were removed in the Rutherford's model.

1.7 RUTHERFORD'S α-PARTICLE SCATTERING EXPERIMENT

Rutherford was the first scientist to give a correct description of the distribution of positive and negative charges within the atom. The basis of Rutherford's atomic models was the α- particle scattering experiment. He bombarded a thin gold foil with highly energetic α- particles (emitted by a radio -active material) and studied the scattering of α- particles in order to investigate the structure of the atom. An α- particle is a positively charged particle having a mass equal to that of helium atom and positive charge in magnitude equal to twice the charge on an electron. The α- particle scattering experiment provided very useful informations about the structure of atom. The arrangement used in the experiment is shown in Fig 1.16. When a narrow beam of α- particles falls on a thin gold - foil (about 2.1×10^{-7} m thick), the α- particles are scattered in different directions which are detected with the help of α- particle detector. The whole arrangement was enclosed in a vacuum chamber to prevent the scattering of α- particles from air molecules.

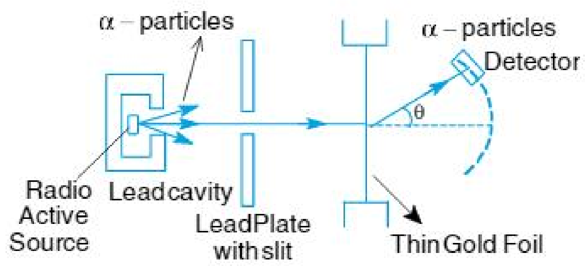


Fig 1.16

Experimental Observations

(a) Most of the α- particles were found to pass through the gold- foil without being deviated from their paths.

(b) Some α- particles were found to be deflected through small angles $\theta < 90°$.

(c) Few α- particles were found to be scattered at fairly large angles from their initial path $\theta > 90°$

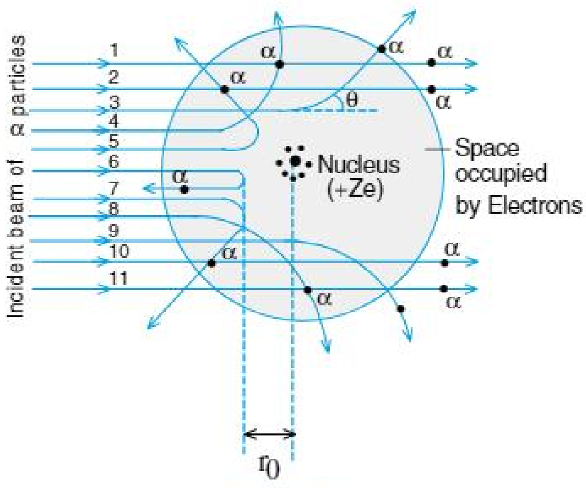


Fig 1.17 (a)

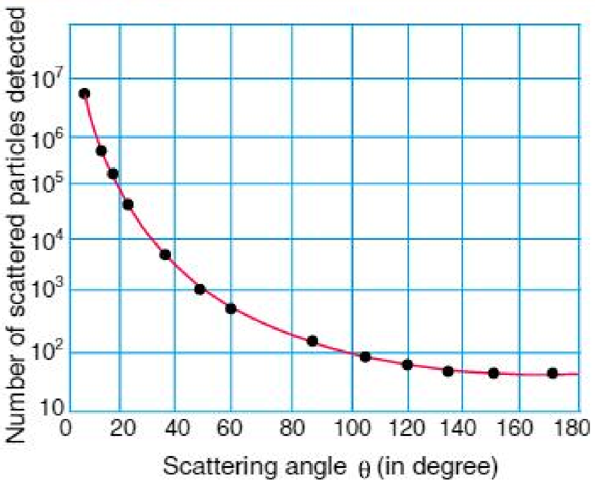


Fig 1.17 (b)

d) A very small number of α- particles about one in 8000 practically retraced their paths or suffered deflections of nearly 180° (e.g particle 6 in figure.)

(i) The observation (a) indicates that most of the portion of the atom is hollow inside.

(ii) Because α- particle is positively charged, from the observations (b) and (c), atom also has positive charge and the whole positive charge of the atom must be concentrated in a small space which is at the centre of the atom called nucleus. The remaining part of the atom and electrons are revolving around the nucleus in circular objects of all possible radii. The positive charge present in the nuclei of different metals is different. Higher the positive charge in the nucleus, larger will be the angle of scattering of α- particles.

Distance of Closest Approach

An α- particle which moves straight towards the nucleus in head on direction reaches the nucleus i.e, it moves close up to a distance r_0 as shown the Fig 1.17. As the α- particle approaches the nucleus, the electrostatic repulsive force due to the nucleus increases and kinetic energy of the alpha particle goes on converting into electrostatic potential energy. When whole of the kinetic energy is converted into electrostatic potential energy, the α- particle cannot further move towards the nucleus but returns back on its initial path i.e., α- particle is scattered through an angle of 180°. The distance of α- particle from the nucleus in this stage is called the distance of closest approach and is represented by r_0. Let m_α and v_α be the mass and velocity of the α- particle directed towards the centre of the nucleus. Then kinetic energy of the α- particle

$$K = \left(\frac{1}{2}\right) m_\alpha v^2{}_\alpha$$

Because the positive charge on the nucleus is Ze and that on the α -particle 2e, the electrostatic potential energy of the α- particle, when it is at a distance r_0 from the centre of the nucleus, is given by

$$V = \frac{1}{4\pi \in_0} \cdot \frac{(2e)(Ze)}{r_0}$$

Because at $r = r_0$ kinetic energy of the α- particle appears as its potential energy, we get

$$K = U(or)\ \frac{1}{4\pi \in_0} \cdot \frac{(2e)(Ze)}{r_0} = \frac{1}{2} m_\alpha v_\alpha^2$$

$$r_0 = \frac{1}{4\pi \in_0} \frac{4Ze^2}{m_\alpha {v_\alpha}^2}$$

Above equation shows that for a given nucleus, the value of r_0 depends upon the initial kinetic energy of the α- particle. As the initial kinetic energy of the alpha particle is increased more and more, α -particle will reach more and more close to the nucleus. At a certain very high value of initial kinetic energy, α- particle will reach extremely close to the nucleus. In this condition, the nucleus will no more be a point charge for the α- particle and coulomb's law will no more be applicable. Moreover, the nuclear force (which is a strong attractive force) now becomes effective. As a result of this, α- particle is now attracted into the nucleus and no more returns on its path. Thus the distance of closest approach of the α- particles corresponding to that maximum value of kinetic energy for which the particle is not scattered back, will be a measure of the radius of the nucleus.

1.7.1 IMPACT PARAMETER AND ANGLE OF SCATTERING

The trajectory of an α- particle depends on the impact parameter of collision.

Impact parameter is the perpendicular distance of the velocity vector of an α-particle from the centre of the nucleus (when the α-particle is far away from the atom).

It is denoted by 'b' as shown in the Fig 1.18.

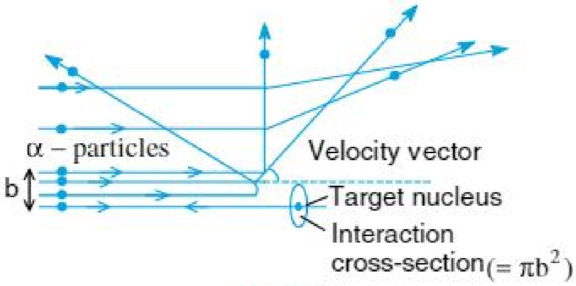


Fig 1.18

Angle of scattering is the angle between the direction of approach and the direction of the receding α-particle.

It is denoted by θ. From Rutherford's calculation $\cot(\theta/2)=\frac{2b}{r_0}$

Where $r_0=\frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K}$

[K = Kinetic Energy of α-particle]

Then,

$$\cot(\theta/2)=\frac{2b}{\frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K}}=\frac{4\pi\epsilon_0 K}{Ze^2}b$$

For a given nucleus (constant Ze) and an α-particle of given energy K, $\cot(\theta/2)\propto b$. So, a graph between *b* and $\cot(\theta/2)$ is a straight line. Thus, it is clear that θ increases as *b* decreases. It implies that only an α-particle passing close to the nucleus, suffers large angle deflection.

Example-1.12

Calculate the impact parameter of a 5 MeV α-particle scattered by 10^0 when it approaches a gold nucleus (Z = 79).

Solution:

As $\cot(\theta/2)=\frac{2b}{\frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K}}$

$$\Rightarrow b=\frac{1}{4\pi\varepsilon_0}\frac{Ze^2\cot(\theta/2)}{K}$$

where b is the impact parameter.

Since $Z=79,\ \theta=10^0$ and

$K=5\text{MeV}=5(1.6\times10^{-13}\text{J})=8\times10^{-13}\text{J}$

as $\cot(\theta/2)=\cot 5^0=1/\tan 5^0=1/0.0875=11.43$

$$b=\frac{(9\times10^9)(79)(1.6\times10^{-19})^2(11.43)}{8\times10^{-13}}\text{m}$$

$=2.6\times10^{-13}\text{m}$

1.8 BOHR'S MODEL OF HYDROGEN LIKE ATOMS

According to Rutherford electron can revolve in circular orbits of all possible radii. A charge while accelerating or decelerating loses energy by radiation. The electron moving around the nucleus continuously experience centripetal acceleration and therefore it must lose energy continuously. Due to this continuous loss of energy, the electron in Rutherford model should spiral towards the nucleus and fall into it when all its energy is lost due to radiation. Hence the atom must be able to emit continuous energy spectrum. But the observed spectrum from the atom is line spectrum. Rutherford was not able to explain this.

Later Bohr proposed a model for hydrogen atoms which is also applicable for some lighter atoms in which a single electron revolves in stationary orbits around a stationary nucleus of positive charge Ze.

With the help of his planetary model, he was able to explain quantitatively why particular wavelength appeared in the line spectrum of atomic hydrogens and how electromagnetic radiation originates in an atom.

On the basis of quantum theory, Bohr modified Rutherford's atomic model and had given following postulates to explain the observed facts.

(I) Electron can revolve round the nucleus only in certain allowed orbits called stationary orbits and the Coulomb's force of attraction between electron and the positively charged nucleus provides necessary centripetal force.

$$\frac{k(Ze)e}{r^2} = \frac{mV^2}{r}$$

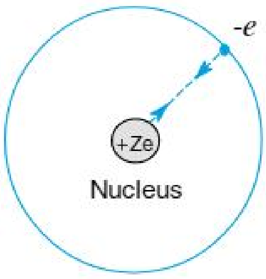


Fig 1.19

(II) Suppose m is the mass of electron, V is the velocity and 'r' is the radius of the orbit, then in stationary orbits the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$, where h is the Planck's constant.

The angular momentum

$$L = I\omega = mVr = n\frac{h}{2\pi},$$

where n = 1, 2, 3, 4, ... called principal quantum number.

(III) An electron in a stationary orbit has a definite amount of energy. It possesses kinetic energy because of its motion and potential energy on account of the attraction of the nucleus. Each allowed orbit is therefore associated with a certain quantity of energy called the energy of the orbit, which equals the total energy of the electron in it. In these allowed orbits electrons rotate without radiating energy.

(IV) Energy is radiated or absorbed when an electron jumps from one stationary orbit to another stationary orbit. This energy is equal to the energy difference between these two orbits and emitted or absorbed as one quantum of radiation of frequency ν given by Planck's equation $E_2 - E_1 = h\nu = \frac{hc}{\lambda}$. This is called Bohr's frequency condition.

Radius of Bohr's orbit : When mass of nucleus is large compared to revolving electron, the electron revolves around the nucleus in circular orbit.

According to first postulate

$$\frac{k(Ze)e}{r^2} = \frac{mV^2}{r} \quad \left(\text{where } k = \frac{1}{4\pi\epsilon_0}\right) \quad ....(1)$$

According to second postulate

$$mVr = n\frac{h}{2\pi} \quad \text{where } n = 1,2,3,4............$$

$$\text{(or)} \quad V = \frac{nh}{2\pi mr} \quad .... (2)$$

On solving the equations, radius of the orbit

$$r = \frac{n^2h^2}{4\pi^2 kZme^2}$$

$$\text{For } n^{th} \text{ orbit } r_n = \frac{h^2}{4\pi^2 ke^2}\cdot\left(\frac{n^2}{mZ}\right) \quad ....(3)$$

For hydrogen atom Z = 1, radius of the first orbit (n = 1) is given by

$$r_1 = 0.529\times10^{-10}\,m = 0.53\text{Å}$$

This value is called Bohr's radius and the orbit is called Bohr's orbit.

In general, the radius of the n^{th} orbit of a hydrogen like atom is given by

$$r_n = 0.53\left(\frac{n^2}{Z}\right)\text{Å} \quad \text{where } n = 1,2,3,....... \quad (4)$$

Velocity of the Electron in the orbit

The velocity of an electron in n^{th} orbit

$$V_n = \frac{nh}{2\pi m r_n}$$

$$\text{Hence } V_n = \frac{2\pi ke^2}{h}\cdot\left(\frac{Z}{n}\right)\left(\because r_n = \frac{n^2h^2}{4\pi^2 kZme^2}\right) \quad ....(5)$$

i.e., the velocity of electron in any orbit is independent of the mass of electron. On substituting the known values

$$\therefore V_n = \left(\frac{c}{137}\right)\cdot\frac{Z}{n}\ m/s \quad .... (6)$$

where 'c' is the speed of light in vacuum.

Time period of electron in the orbit

Angular velocity of electron in n^{th} orbit

$$\omega_n = \frac{V_n}{r_n} = \frac{\omega_0 Z^2}{n^3}, \text{ where } \omega_0 = \frac{8\pi^3 k^2 e^4 m}{h^3} \quad ...(7)$$

is the angular velocity of electron in Bohr's orbit.

$\therefore$ The time period of rotation of electron in

$$n^{th} \text{ orbit } T = \frac{2\pi}{\omega_n} = \frac{n^3}{2\pi\omega_0 Z^2} \quad .... (8)$$

i.e $T \propto \frac{n^3}{Z^2}$.

The time period of rotation increases as n increases and is independent of the mass of the electron.

Energy of the electron in the orbit

The kinetic energy of the electron revolving round the nucleus in n^{th} orbit is given by

$$K_n = \frac{1}{2}mV^2 = \frac{1}{2}m\left[\frac{2\pi ke^2}{h}.\frac{Z}{n}\right]^2$$

$$\text{(or) } K_n = \frac{2\pi^2 k^2 e^4}{h^2}.\left(\frac{mZ^2}{n^2}\right) \quad .... (9)$$

i.e., $K_n \, \alpha \, \frac{mZ^2}{n^2}$

If the reference level (zero potential energy level) is at infinity then the electrostatic potential energy is given by

$$U_n = -\frac{k(Ze)e}{r_n} = -kZe^2\left[\frac{4\pi^2 kmZe^2}{n^2h^2}\right]$$

$$\text{(or) } U_n = -\frac{4\pi^2 k^2 e^4}{h^2}\left(\frac{mZ^2}{n^2}\right) \quad .... (10)$$

Total energy of the electron in n^{th} orbit

$$E_n = K_n + U_n = -\frac{2\pi^2 k^2 mZ^2 e^4}{n^2h^2}$$

$$E_n = -\frac{2\pi^2 k^2 e^4}{h^2}\left(\frac{mZ^2}{n^2}\right) \quad .... (11)$$

The expression of total energy for hydrogen like atom may be simplified as

$$E_n = -13.6\frac{Z^2}{n^2} \text{ eV}, \; n = 1,2,3..... \quad ...(12)$$

where -13.6 eV is the total energy of the electron in the ground state of a hydrogen atom. From the above equations (9),(10) & (11) it is observed that double the kinetic energy with a negative sign is equal to the potential energy and kinetic energy with a negative sign is equal to the total energy.

i.e $U = -2K$ and $E = -K$ (or) $E = -K = U/2$

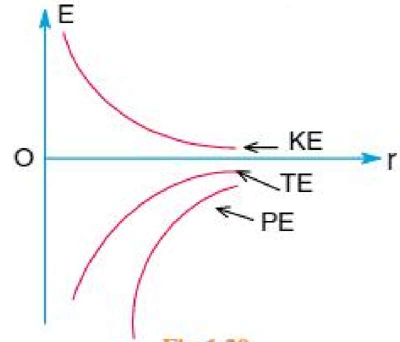


Fig 1.20

The relation $E = -K = \frac{U}{2}$ is true only when electrostatic force $F \alpha \frac{1}{r^2}$ and reference potential energy is zero at infinity.

The state n = 1 is called ground state and $n > 1$ states are called excited states. When electron goes from lower orbit to higher orbit, speed and hence kinetic energy decrease, but both potential energy and total energy increase. $E \alpha \frac{1}{n^2}$ tells us that the energy gap between the two successive levels decrease as the value of n increases. At infinity level the total energy of the atom becomes zero.

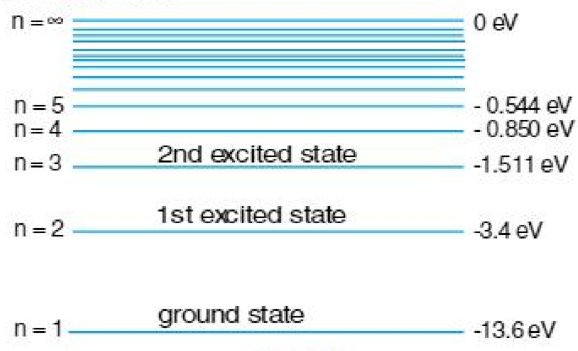


Fig 1.21

Energy level diagram of hydrogen atom (Z = 1) for normal and excited states is shown in Fig 1.21.

The energy level diagram of hydrogen like atom with atomic number Z for normal and excited states is shown in Fig 1.22.

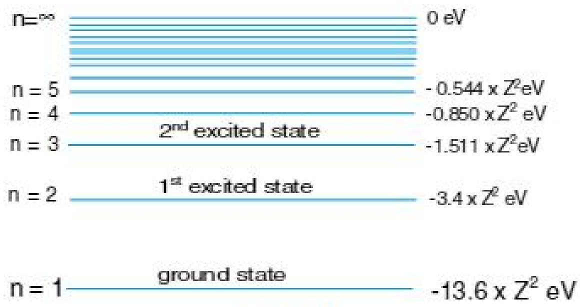


Fig 1.22

The total energy of the electron is negative implies that the atomic electron is bound to the nucleus. To remove the electron from its orbit against the nuclear pull, energy is required. The minimum energy required to remove an electron from the ground state of atom is called ionization energy and it is $13.6\,Z^2$ eV. In hydrogen atom, the ground state energy of electron is -13.6 eV, so 13.6 eV is the ionization energy of the Hydrogen atom.

In the given Fig 1.21 and 1.22, reference level (zero potential energy level) is assumed at infinity. If potential energy at infinity level is non - zero, then that non - zero value is added to both potential energy and total energy term without change of kinetic energy, because kinetic energy is independent of reference level. If the total energy is greater than zero, an electron would not follow closed orbit around the nucleus. i.e electron is free from the nucleus and hence does not constitute an atom.

1.8.1 DE BROGLIE'S EXPLANATION OF BOHR'S SECOND POSTULATE OF QUANTISATION

De Broglie tried to explain Bohr's criterion to select the allowed orbits in which angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$. According to his hypothesis, an electron revolving round nucleus is associated with certain wavelength 'λ' which depends on its momentum mv. It is given by $\lambda = \frac{h}{mv} = \frac{h}{p}$

In an allowed orbit, an electron can have an integral multiple of this wavelength. That is the n^{th} orbit consists of n complete De-Broglie wavelengths i.e., $2\pi r_n = n\lambda_n$, where r_n is the radius of n^{th} orbit and λ_n is the wavelength of n^{th} orbit $\lambda_n = \frac{2\pi r_n}{n}$ (or) $\lambda_n = \frac{2\pi}{n}(0.53\times n^2)$ Å (or) $\lambda_n = 2\pi r_1 n$ Å, where r_1 is radius of first orbit of H -atom.

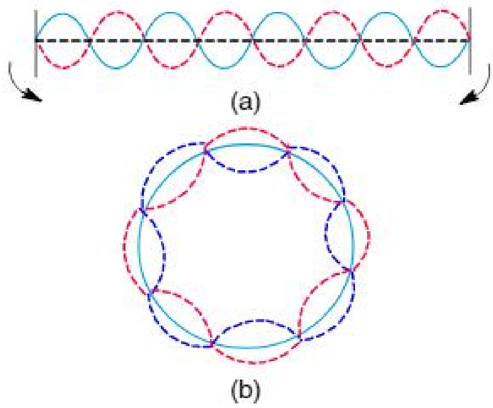


Fig 1.23

Fig1.23(a) shows the waves on a string having a wavelength related to the length of the string allowing them to interfere constructively as shown. If we imagine the string bent into a closed circle we get an idea of how electrons in circular orbits can interfere constructively as shown in Fig 1.23(b). If the wavelength does not fit in to the circumference, the electron interferes destructively and it cannot exist in such an orbit.

Example-1.13

The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$ where n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times that in the final state. What are the possible values of n_1 and n_2 ?

Solution :

Since, $T \propto n^3 \quad \therefore \frac{T_1}{T_2} = \frac{n_1^3}{n_2^3}$

As $T_1 = 8T_2$, $\left(\frac{n_1}{n_2}\right)^3 = 8$ (or) $n_1 = 2n_2$.

Thus the possible values of n_1 and n_2 are $n_1 = 2, n_2 = 1, n_1 = 4, n_2 = 2, n_1 = 6, n_2 = 3$; and so on ....

Example-1.14

Find the kinetic energy, potential energy and total energy in first and second orbits of hydrogen atom if potential energy in first orbit is taken to be zero.

Solution :

$E_1 = -13.60eV$; $K_1 = -E_1 = 13.60eV$;

$U_1 = 2E_1 = -27.20eV$;

$E_2 = -3.40eV$, $K_2 = 3.40eV$ and $U_2 = -6.80eV$

Now, $U_1 = 0$, i.e., potential energy has been increased by 27.20eV. So, we will increase U and E in all energy states by 27.20eV while kinetic energy will remain unchanged.

Hence K(eV), U(eV), E(eV) are (First) 13.60, 0, 13.60 eV, and (Second) 3.40, 20.40, 23.80eV respectively.

Example-1.15

A small particle of mass m moves in such a way that the potential energy $U = ar^2$ where a is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of n^{th} allowed orbit.

Solution :

The force at a distance r is, $F = -\frac{dU}{dr} = -2ar$ Suppose r be the radius of n^{th} orbit. Then the necessary centripetal force is provided by the above force.

Thus, $\frac{mv^2}{r} = 2ar$ ..... (i)

Further, the quantization of angular momentum gives,

$mvr = \frac{nh}{2\pi}$ ..... (ii)

Solving Eqs. (i) and (ii) for r, we get $r = \left(\frac{n^2h^2}{8am\pi^2}\right)^{1/4}$

1.9 ATOMIC SPECTRA

As mentioned earlier, each element has a characteristic spectrum of radiation, which it emits. When an atomic gas or vapour is excited at low pressure, usually by passing an electric current through it, the emitted radiation has a spectrum which contains certain specific wavelengths only. A spectrum of this kind is termed as emission line spectrum and it consists of bright lines on a dark background. The spectrum emitted by atomic hydrogen is shown in Fig 1.24. Study of emission line spectra of a material can therefore serve as a type of "fingerprint" for identification of the gas. When white light passes through a gas and we analyse the transmitted light using a spectrometer we find some dark lines in the spectrum. These dark lines correspond precisely to those wavelengths which were found in the emission line spectrum of the gas. This is called the absorption spectrum of the material of the gas.

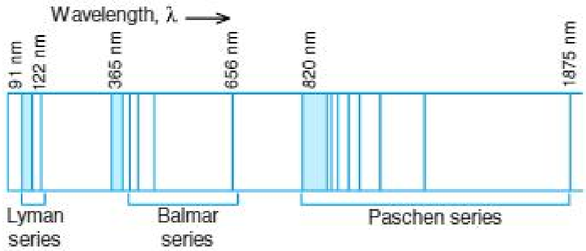


Fig 1.24

1.10 EMISSION OF RADIATION

When an electron jumps from higher energy level n_2 to a lower energy level n_1 in stationary atom, the difference in energy is radiated as a photon whose frequency ν is given by Planck's formula.

$$E_{n_2} - E_{n_1} = h\nu$$

$$\text{(or) } h\nu = E_2 - E_1 = 13.6Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \text{ e.V}$$

$$\left(\therefore E_n = -\frac{13.6Z^2}{n^2} \text{ e.V}\right)$$

Since $1eV = 1.6 \times 10^{-19}J$,

$$h\frac{c}{\lambda} = (2.18 \times 10^{-18})Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \text{ J (or)}$$

$$\text{wave number } \bar{\nu} = \frac{1}{\lambda} = R_\alpha Z^2.\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] m^{-1}$$

where R_α is called "Rydberg constant", when the nucleus is infinitely massive as compared to the revolving electron. In other words, the nucleus is considered to be stationary. The numerical value of R_α is $1.097 \times 10^7\ m^{-1}$.

Note : In case, the nucleus is not infinitely massive or stationary, the value of Rydberg constant is $R = \dfrac{R_\alpha}{1+\dfrac{m}{M}}$ where m is the mass of electron and M is the mass of nucleus. The value of Rydberg constant for an atom varies between $\dfrac{R_\alpha}{2}$ and R_α. That is $\dfrac{R_\alpha}{2} \le R \le R_\alpha$.

1.11 EMISSION SPECTRUM OF HYDROGEN ATOM

Electron in hydrogen atom, in its excited state hardly stays for 10^{-8} second. This is because, in the presence of conservative force system, particles always try to occupy stable equilibrium position and hence minimum potential energy, which is the case in ground state. Because of unstability, when an electron in excited state makes a transition to lower energy state, a photon is emitted. Collection of such emitted photon frequencies is called an emission spectrum. This is as shown in Fig 1.25.

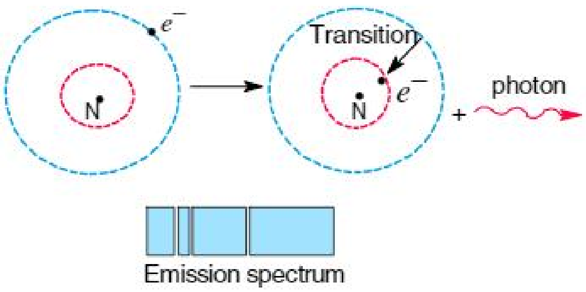


Fig 1.25

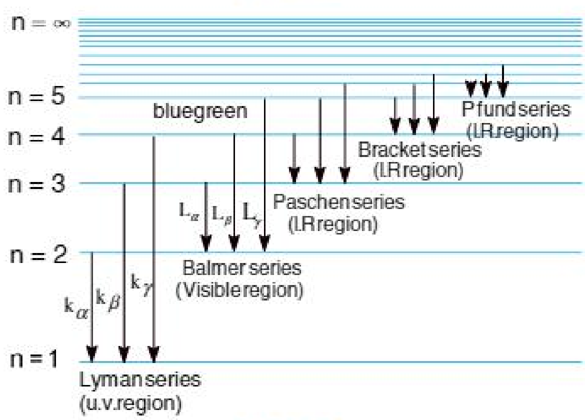


Fig 1.26

The spectral series of hydrogen atom as shown in Fig1.26, is explained below.

(a) Lyman Series

Lines corresponding to transition from outer energy levels $n_2 = 2,3,4,...\infty$ to first orbit ($n_1 = 1$) constitute Lyman series. The wave numbers of different lines are given by,

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^{\,2}}\right]$$

(i) Line corresponding to transition from $n_2 = 2$ to $n_1 = 1$ is first line; its wavelength is maximum.

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 1.1\times10^7\left[\frac{1}{1} - \frac{1}{4}\right]$$

$$\therefore \lambda_{max} = 1212Å$$

Similarly transition from $n_2 = \infty$ to $n_1 = 1$ gives line of minimum wavelength.

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 1.1\times10^7$$

$$\therefore \lambda_{min} = 912Å$$

(ii) Lyman series lies in ultraviolet region of electromagnetic spectrum.

(iii) Lyman series is obtained in emission as well as in absorption spectrum.

(b) Balmer Series

Lines corresponding to $n_2 = 3,4,5,....\infty$ to $n_1 = 2$ constitute Balmer series. The wave numbers of different lines are given by,

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$$

(i) Line corresponding to transition n_2=3 to n_1=2 is first line. Wavelength corresponding to this transition is maximum. Line corresponding to transition $n_2 = \infty$ to $n_1 = 2$ is last line; wavelength of last line is minimum.

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \therefore \lambda_{max} = 6568Å$$

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] \therefore \lambda_{min} = 3636Å$$

(ii) Balmer series lies in the visible region of electromagnetic spectrum. The wavelength of L_α line is 656.8 nm (red). The wavelength of L_β line is 486 nm (blue green). The wavelength of L_γ line is 434 nm (violet). The remaining lines of Balmer series are close to violet light wavelength. The speciality of these lines is that in going from one end to other, the brightness and the separation between them decreases gradually.

(iii) This series is obtained only in emission spectrum. Absorption lines corresponding to Balmer series do not exist, except extremely weakly, because very few electrons are normally in the state n = 2 and only a very few atoms are capable of having an electron knocked from the state n = 2 to higher states. Hence photons that correspond to these energies will not be strongly absorbed. In highly excited hydrogen gas there is a possibility of detecting absorption at Balmer-line wavelengths.

(c) Paschen Series

Lines corresponding to $n_2 = 4, 5, 6, ... \infty$ to $n_1 = 3$ constitute Paschen series.

The wave number of different lines are given by $\bar{v} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$

(i) Line corresponding to transition $n_2 = 4$ to $n_1 = 3$ is first line, having maximum wavelength. Line corresponding to transition $n_2 = \infty$ to $n_1 = 3$ is last line, having minimum wavelength

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] \therefore \lambda_{max} = 18747Å$$

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right] = 1.1 \times 10^7 \times \left[\frac{1}{9} - 0\right]$$

$$\therefore \lambda_{min} = 8202Å$$

(ii) Paschen series lies in the near infrared region of electromagnetic spectrum.

(iii) This series is obtained only in the emission spectrum.

(d) Bracket Series

The series corresponds to transitions from $n_2 = 5, 6, 7......., \infty$ to $n_1 = 4$.

The wave numbers are given by,

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]$$

(i) Line corresponding to transition from $n_2 = 5$ to $n_1 = 4$ has maximum wavelength and $n_2 = \infty$ to $n_1 = 4$ has minimum wavelength.

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{4^2} - \frac{1}{5^2}\right] \therefore \lambda_{max} = 40477Å$$

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right] \therefore \lambda_{min} = 14572Å$$

(ii) This series lies in the infrared region of electromagnetic spectrum.

(e) Pfund Series

This series corresponds to transitions from $n_2 = 6, 7, 8, ..., \infty$ to $n_1 = 5$.

The wave numbers are given by

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right]$$

(i) Line corresponding to transition from $n_2 = 6$ to $n_1 = 5$ has maximum wavelength and $n_2 = \infty$ to $n_1 = 4$ has minimum wavelength.

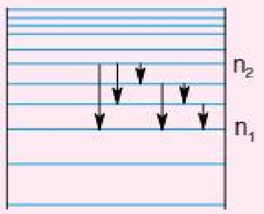
$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{5^2} - \frac{1}{6^2}\right] \therefore \lambda_{max} = 74563Å$$

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{5^2} - \frac{1}{\infty^2}\right] \therefore \lambda_{min} = 22768Å$$

(ii) This series lies in for infrared region of electromagnetic spectrum.

Note :

1) In an atom transition of electron may start from any higher energy level and end at any energy level below it. Hence in emission spectrum the total possible number of emission lines from some excited state n_2 to another energy state $n_1 (< n_2)$ is $\frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}$



Example : For $n_2 = 4$ and $n_1 = 1$, the number of possible lines are 6.

2) If ΔE is the energy difference between two given energy states, then done to transition between these two states wavelength of emitted photon is

$$\lambda(\text{Å}) = \frac{12375}{\Delta E(eV)}$$

3) If the atom is moving, the frequency of radiation emitted by it may be slightly different from that of radiation from an atom at rest. Depending on direction of motion of the atom and emission, the frequency may be slightly higher or lower or may be equal to that of radiations from the permanent rest atom.

Example-1.16

Consider a hydrogen-like atom whose energy in n^{th} excited state is given by $E_n = -\frac{13.6Z^2}{n^2}$. When this excited atom makes transition from excited state to ground state most energetic photons have energy $E_{max} = 52.224eV$ and least energetic photons have energy $E_{min} = 1.224eV$. Find the atomic number of atom and the state of excitation.

Solution :

Maximum energy is liberated for transition $E_n \to E_1$ and minimum energy for $E_n \to E_{n-1}$

Hence, $\frac{E_1}{n^2} - E_1 = 52.224eV$ ... (1)

and $\frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224eV$ ... (2)

Solving above equations simultaneously, we get $E_1 = -54.4eV$ and $n = 5$ Now

$E_1 = -\frac{13.6Z^2}{1^2} = -54.4eV$. Hence, $Z = 2$ i.e, gas is helium originally excited to $n = 5$ energy state.

Example-1.17

A hydrogen-like atom (atomic number Z) is in a higher excited state of quantum number n. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV respectively. Alternatively the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. Determine the values of n and Z (Ionisation energy of hydrogen atom = 13.6 eV)

Solution :

The electronic transitions in a hydrogen-like atom from a state n_2 to a lower state n_1 are given by

$$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right).$$

For the transition from a higher state n to the first excited state $n_1 = 2$, the total energy released is

$(10.2 + 17.0)$ eV or 27.2eV.

Thus $\Delta E = 27.2$ eV, $n_1 = 2$ and $n_2 = n$.

We have $27.2 = 13.6Z^2\left[\frac{1}{4} - \frac{1}{n^2}\right]$ .... (1)

For the eventual transition to the second excited state $n_1 = 3$, the total energy released is $(4.25 + 5.95)$ eV or 10.2eV.

Thus $10.2 = 13.6Z^2\left[\frac{1}{9} - \frac{1}{n^2}\right]$ .....(2)

Dividing the Eq. (1) by Eq. (2) we get

$\frac{27.2}{10.2} = \frac{9n^2 - 36}{4n^2 - 36}$. Solving we get $n^2 = 36$ or $n = 6$

Substituting $n = 6$ in any one of the above equations, we obtain $Z^2 = 9$ (or) $Z = 3$, Thus $n = 6$ and $Z = 3$.

1.12 EFFECT OF FINITE MASS OF NUCLEUS

In the atomic spectrum of hydrogen and like atoms, there will be a very small deviation with from the Bohr model results. The reason for this discrepancy is the assumption that the nucleus is infinitely massive compared with the mass of the electron so that it remains stationary during the rotation of the electron. In actual practice the nucleus is not infinitely massive, but it has a finite mass and hence both the electron and the nucleus rotate about the centre of mass of the system.

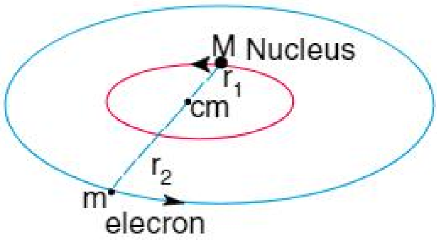


Fig 1.27

Let the nucleus (mass M) and electron (mass m) be rotating about their centre of mass with an angular velocity ω.

If 'r' be the separation between nucleus and electron, then $Mr_1 = mr_2$ and $r_1 + r_2 = r$ .... (1)

where r_1 and r_2 are distance of centre of mass from nucleus and electron respectively. Centripetal force to the electron is provided by the electrostatic force between nucleus and electron.

$$mr_2\omega^2 = k\frac{Ze^2}{r^2}, \text{ where } r_2 = \frac{M}{m+M}r$$

$$\text{(or) } \mu r^3\omega^2 = kZe^2, \quad .... (2)$$

where $\mu = \dfrac{mM}{m+M}$ is called reduced mass.

According to Bohr's theory of angular momentum of the system.

$$L = I_1\omega + I_2\omega = \frac{nh}{2\pi} \text{ (or) } Mr_1^2\omega + mr_2^2\omega = \frac{nh}{2\pi}$$

$$\text{(or) } \mu r^2\omega = \frac{nh}{2\pi} \quad .... (3)$$

Solving the above equations 1, 2, 3

The radius of electron in n^{th} allowed orbit is

$$r_n = \frac{h^2}{4\pi^2 ke^2}\left(\frac{n^2}{\mu Z}\right) \quad .... (4)$$

Further potential energy of the system

$$U = -\frac{kZe^2}{r} \quad .... (5)$$

and kinetic energy

$$K = \frac{1}{2}(I_1 + I_2)\omega^2 = \frac{1}{2}\mu r^2\omega^2 = \frac{kZe^2}{2r})$$

$$\left(\because \omega^2 = \frac{kZe^2}{\mu r^3}\right)$$

$\therefore$ Total energy $E = K + U = -\dfrac{kZe^2}{2r}$

$\therefore$ The total energy of n^{th} allowed orbit

$$E_n = -\frac{\mu e^4 Z^2}{8\epsilon_0^2 n^2 h^2} = -\left(13.6\frac{\mu}{m}\right)\frac{Z^2}{n^2} \text{ eV} \quad ...(7)$$

These formulae can be obtained by replying m by μ is the formula for stationary nucleus. A system of this kind is equivalent to a single particle of mass μ revolving around the position of heavier particle. The wave number of emission spectrum is

$$\bar{v} = \frac{1}{\lambda} = \frac{R_\alpha}{1+\dfrac{m}{M}}Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where R_α is Rydberg constant when the nucleus is stationary.

Using the emission spectral lines we can compare the masses of isotopes. The wave number difference between the parent and companion faint lines enables us to estimate the mass of isotopes. Using this, the mass of deuterium is found to be double that of hydrogen. Because of greater nuclear mass, the spectral lines of deuterium are all shifted slightly towards shorter wavelength compared to hydrogen atom.

Example-1.18

An imaginary particle has a charge equal to that of an electron and mass 100 times that of the electron. It moves in a circular orbit around a nucleus of charge +4e. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system, find the wavelength of the radiation emitted when the particle jumps from fourth orbit to the second orbit.

Solution:

As we know, $E_n = -\left(13.6\dfrac{\mu}{m}\right)\dfrac{Z^2}{n^2}$ eV

$$E_4 = \frac{(-13.60)(4)^2}{(4)^2} \times 100 = -1360\text{eV} \text{ and}$$

$$E_2 = \frac{(-13.60)(4)^2}{(2)^2} \times 100 = -5440\text{eV}$$

$$\Delta E = E_4 - E_2 = 4080\text{eV}$$

$$\therefore \lambda(\text{in Å}) = \frac{12375}{\Delta E(\text{in eV})} = \frac{12375}{4080}\text{ Å}$$

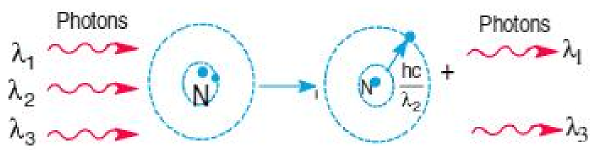
1.13 ATOMIC EXCITATIONS

In an atom just as the radiation (emission) of energy takes place in a discontinuous manner as quanta of energy, so also the absorption of energy involved in the excitation or ionization takes place in discrete quanta.

Energy can be supplied to the atoms by various processes e.g: by heat, by impact of electrons and other fundamental particles and by impact of photons.

(a) Absorption spectrum (Excitation by photon absorption)

When white light is passed through atomic hydrogen gas, certain frequencies are absent in the collected spectrum. The resulting spectrum consists of bright back ground with some dark lines as shown in Fig 1.28. This pattern of dark lines is called an absorption spectrum. The missing frequencies are same as observed in the corresponding emission spectrum of that atom.



Absorption spectrum

Fig 1.28

When a photon strikes an atom, the photon loses either all it's energy or none of it. When the energy of the colliding photon is exactly equal to the energy difference between lower state n_1 and some other excited state $n_2(> n_1)$, the photon can be absorbed, otherwise it must retain its initial energy. Since energy of orbits are quantized, an electron can take on only increments of energy corresponding to transition states.

In hydrogen atom photons with wavelength less than the Lyman series limit 91.2nm or energy greater than or equal to 10.2 eV can be absorbed. If photons have enough energy to excite electron to infinity, the electron becomes free from the nuclear bond and the remaining energy it retains as kinetic its energy. eg : If the energy of incident photon = 15eV, then the emitted electron has kinetic energy 1.4eV.

At ordinary temperature, practically all hydrogen atoms exist in the lowest possible energy state, n = 1. Hence only the absorption spectrum of the Lyman series are normally observed, which is not in the visible region. Hydrogen atom exists in the sun. Because of high temperature, the absorption spectrum of sun consists of Balmer series in addition to Lyman series.

(b) Excitations by Collision

The most convenient way of supply of energy to an atom is the bombardment by external particles such as electron, proton, neutron or hydrogen atoms with enough kinetic energy to excite the atom to its different quantized states. Two types of collision of the particles with the atom can be cited. If the particle energy is less than certain minimum value called critical energy, it can't excite the atoms. Then the collision is said to be elastic, whose characteristic property is that conservation of both kinetic energy of translation and of linear momentum without any conversion of external energy into internal energy and vice versa. If the particle energy is above or equal to critical energy the colliding particle loses considerable part of its kinetic energy which appears as internal change of energy in the atom. This leads to excitation and even ionization. Here momentum of the system is conserved but not kinetic energy. Such collisions are said to be inelastic collisions.

Let us consider a particle of mass 'm' moving with a velocity V which strikes the stationary hydrogen like atom of mass 'M' which is in ground state. It should be noted that the system loses maximum kinetic energy in a perfectly inelastic collision, in which cases. $mV + 0 = (M + m) V^1$, where V^1 is the velocity the particle and atom after collision. The maximum loss in kinetic energy of the system

$$\Delta E = \frac{1}{2}mV^2 - \frac{1}{2}(M+m)V^{1^2}$$

(or) $\Delta E=\frac{1}{2}mV^2-\frac{1}{2}(M+m)\left(\frac{mV}{M+m}\right)^2$

(or) $\Delta K=\frac{1}{2}\frac{mMV^2}{M+m}$

If ΔE is the difference in energy levels of state n = 1 and n = 2 in the atom,

(i) If $\Delta K<\Delta E$, the electron can't excite, the collisions is said to be elastic.

(ii) If $\Delta K=\Delta E$, the electron excite from state n = 1 to n = 2, the collision is said to be perfectly inelastic.

(iii) If $\Delta K>\Delta E$ the electron can excite to higher excited states or unbound states with some kinetic energy. The collision is said to be inelastic.

To know the minimum kinetic energy (critical energy) of colliding particle, to excite electron $\Delta K \geq \Delta E$

$$\therefore \frac{1}{2}\frac{mMV^2}{M+m}\geq \Delta E \text{ (or) } \frac{1}{2}mV^2 \geq \Delta E\left(\frac{M+m}{M}\right)$$

Hence, $K_{min}=\Delta E\left(\frac{M+m}{M}\right)$ (or)

$$K_{min}=\Delta E\left(1+\frac{m}{M}\right)$$

Here note that critical energy in a particle collision depends on whether the target atom is at rest or if not, on its direction of motion and speed.

Example-1.19

If two hydrogen atoms, each of mass M in ground state, are moving in opposite direction with same speed (V). To excite the electron in one of the atom, the possible loss in energy .

Solution :

$$\Delta E=2\left(\frac{1}{2}MV^2\right)=10.2eV \text{ (or) } \frac{1}{2}MV^2=5.1eV$$

$\therefore$ Hence the critical energy of each atom is 5.1 eV. If the energy of any one of the atoms is less than 5.1 eV, the collision is elastic, if energy is equal to or more than 5.1 eV, the collision is said to be inelastic.

Example-1.20

A doubly ionized lithium atom is hydrogen like with atomic number Z = 3. Find the wavelength of the radiation required to excite the electron in Li^{2+} from the first to the third Bohr orbit. Given the ionization energy of hydrogen atom as 13.6 eV.

Solution :

The energy of n^{th} orbit of a hydrogen-like atom is given as $E_n=-\frac{13.6Z^2}{n^2}$ Thus for Li^{2+} atom, as Z = 3, the electron energies of the first and third Bohr orbits are For $n=1$, $E_1=-122.4eV$, for $n=3$, $E_3=-13.6eV$.

Thus the energy required to transfer an electron from E_1 level to E_3 level is,

$$E=E_3-E_1=-13.6-(-122.4)=108.8eV$$

Therefore, the radiation needed to cause this transition should have photons of energy, $h\nu = 108.8$ eV

The wavelength of this radiation is

or $\lambda=\frac{hc}{108.8eV}=114.25$ Å

Example-1.21

A hydrogen atom in a state of binding energy 0.85 eV makes a transition to a state of excitation energy of 10.2 eV.

(i) What is the initial state of hydrogen atom?

(ii) What is the final state of hydrogen atom ?

(iii) What is the wavelength of the photon emitted ?

Solution :

(i) Let n_1 be initial state of electron.

Then, $E_1=-\frac{13.6}{n_1^2}eV$ Here $E_1=-0.85$ eV,

therefore $-0.85=-\frac{13.6}{n_1^2}$ or $n_1=4$

(ii) Let n_2 be the final excitation state of the electron.

Since excitation energy is always measured with respect to the ground state,

$\Delta E=13.6\left[1-\frac{1}{n_2^2}\right]$ here $\Delta E=10.2eV$, therefore,

$10.2=13.6\left[1-\frac{1}{n_2^2}\right]$ or $n_2=2$

Thus, the electron jumps from $n_1=4$ to $n_2=2$.

(iii) The wavelength of the photon emitted for a transition between $n_1=4$ to $n_2=2$, is given by

$$\frac{1}{\lambda}=R_\infty\left[\frac{1}{n_2^2}-\frac{1}{n_1^2}\right] \text{ (or) } \frac{1}{\lambda}=1.09\times10^7\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$$

$\lambda=4860$ Å

1.14 LIMITATIONS OF BOHR'S MODEL

1) It could not interpret the details of optical spectra of atoms containing more than one electron.

2) It involves the concept of orbit which could not be checked experimentally

3) It could be successfully applied only to single-electron atoms (e.g., H, He^{+}, Li^{2+}, etc.)

4) Bohr's model could not explain the binding of atoms into molecules.

5) No justification was given for the "principle of quantization of angular momentum".

6) Bohr's model could not explain the reason why atoms should combine to form chemical bonds and why the molecules become more stable on such combinations.

7) Bohr had assumed that an electron in the atom is located at definite distance from the nucleus and is revolving with a definite velocity around it. This is against the Heisenberg uncertainty principle. With the advancements in quantum mechanics, it became clear that there are no well defined orbits; rather there are clouds of negative charges.

1.15 X-RAYS

Roentgen discovered X-rays in 1895, when he was studying the phenomenon of discharge of electricity through rarefied gases. He observed that a photographic film wrapped in black paper became exposed when placed near a cathode ray tube due to same invisible radiation from the cathode ray tube. After careful study Roentgen concluded that when a beam of fast moving electrons strike a target of high atomic number, an invisible penetrating radiation is produced. These radiations are called X-rays, X-rays are electromagnetic radiation of very short wavelength ranging between nearly $0.1A^0$ to $100A^0$. Coolidge X-ray tube is used to produce X-rays.

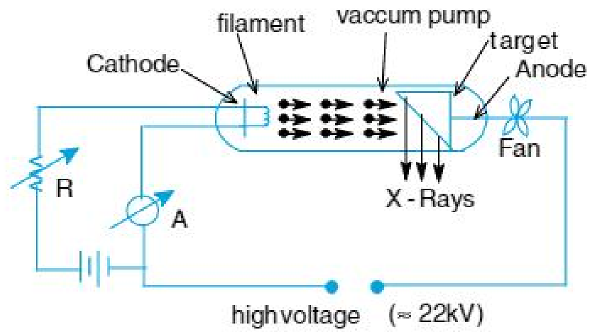


Fig 1.29

X-rays are produced in a highly evacuated tube called Coolidge tube, which contains an anode and a tungsten or molybdenum filament connected to a cathode as shown in Fig 1.29. Rontgen explained the production of these X-rays due to the bombardment of high velocity electrons with the target at anode. The filament at cathode is heated in vacuum to about $300^{\circ}C$ by passing electric current through the battery to emit the electrons and are accelerated to the anode by high potential difference The target is usually a thin sheet of high atomic number and melting point metal such as tungsten. The face of the tungsten is set at 45^0 to the incident electron stream and X-rays which are predominantly emitted at right angles to the beam pass through the side wall of the tube. During the operation of this tube, a huge quantity of heat is produced in the target. This heat is conducted through the anode to the cooling fan from where it is dissipated by radiation and convection. The intensity of X-rays emitted is directly proportional to the number of electrons striking the target. This in turn depends upon the filament temperature, which can be controlled by the heating current. Thus by changing the filament current with the help of rheostat R, thermionic emission and hence intensity of X-rays can be controlled. The quality and penetrating power of X-rays depends upon the energy of the electrons bombarding the target. This energy is determined by the potential difference between cathode and target. Higher the P.D between cathode and target, higher is the energy of bombarding electrons and hence larger is the penetrating power of X-rays.

X-rays of high penetrating power (high frequency) are called hard X-rays, while those having low penetrating power (low frequency) are called soft X-rays. X–Rays upto 4Å are called hard X–Rays while that of λ >4Å are called soft X–Rays.

> ***Note :*** The efficiency of X-ray tube
>
> $$\eta = \frac{\text{out put power}}{\text{input power}} \times 100$$
>
> Input power P = VI, where V is the potential difference applied across the tube and I is the anode current.

Origin of X-rays :

X-rays are produced by bombarding high speed electrons on some heavy element known as target. A big fraction of the kinetic energy of the majority of striking electrons is spent in undergoing collisions with the atoms of the target and consequently the temperature of the target material is considerably increased. It is found that nearly 99% of the incident energy of electrons is used up in heating the target. However, some fraction of kinetic energy of the bombarding electrons is used to produce X-rays in the following two ways.

(i) Braking (or) Bremsstrahlung (or) Continuous X-rays

The high speed electrons go into the interior of the atoms of the target material and are attracted by the positive charge on their nuclei. As an electron passes close to the positive nucleus of an atom in the target, the electron is deflected from its path as shown in Fig 1.30. This results in the deceleration of the electron. The loss in energy of the electron during deceleration is emitted in the form of X-rays. X-rays produced in this way are called braking or bremsstrahlung X-rays as they are produced due to the braking or slowing down of the bombarding electron by the atoms of the target.

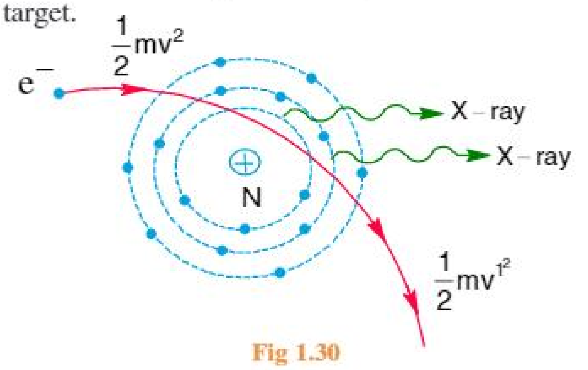


Fig 1.30

The wavelength of X–rays emitted depend upon the loss of velocity of the bombarding electron. If the electron is completely stopped, then X-rays of maximum frequency ν_{max} or minimum wavelength λ_{min} will be emitted. If 'v' is the velocity of the bombarding electron, then

$$\frac{1}{2}mv^2 = h\nu_{max}$$

where $h = 6.62\times10^{-34}\,J-s$ (Planck's constant). If the bombarding electron is accelerated through potential difference V_0, then $h\upsilon_{max} = eV_0$

$$\text{(or) } \upsilon_{max} = \left(\frac{e}{h}\right)V_0 \text{ (or) } \frac{c}{\lambda_{min}} = \left(\frac{e}{h}\right)V_0$$

$$\therefore \lambda_{min} = \frac{hc}{V_0 e} \text{ (or) } \lambda_{min} \approx \frac{12400}{V_0(\text{in eV})}\text{Å}$$

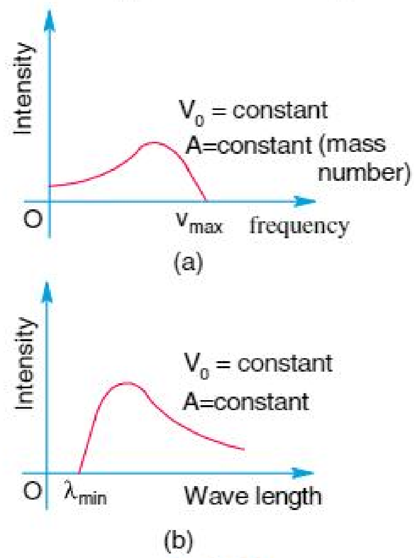


Fig 1.31

In general an electron loses energy in a number of impacts with target atoms and consequently a large number of wavelengths (or frequency) of X-rays are emitted. So, X-rays will be produced in continuous spectrum with wavelength $\lambda_{min}(>0)$ to ∞ (or) continuous spectrum with frequency 0 to ν_{max}. However, the minimum wavelength or cutoff wavelength 'λ_{min}' or maximum frequency ν_{max} of emitted X-rays corresponds to all the kinetic energy lost by the electron in a single impact.

When the potential difference increases cut off wavelength decreases and hence frequency increases. Also the peak of the intensity curve shift towards shorter wavelength or higher frequency side as shown in Fig 1.32.

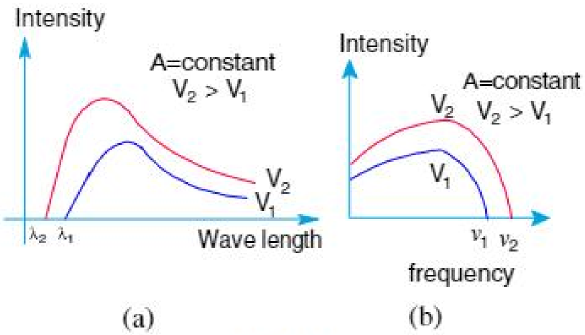


Fig 1.32

Braking X-rays are independent of target material but depend on the potential difference across cathode and anode of the X-ray tube as shown in Fig 1.33.

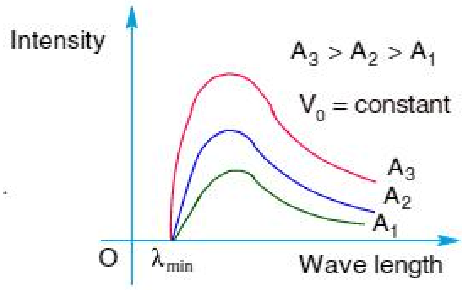


Fig 1.33

(ii) Characteristic spectrum or line spectrum of X–rays

The bombarding electrons may knock out the electrons from the inner shells of atoms of the target, thus emitting X-rays. If the bombarding electron has sufficient energy, it may penetrate into the atom of the target and knock out the tightly bound electron from its inner shells, example : K, L, M shells etc as shown in Fig 1.34 (a). This gives rise to vacancy in the shell which is immediately filled by a jump of outer orbit electron. Since the energy of the electron in the outer orbit is more, this jump results in the release of excess energy in the form of X-rays as shown in Fig 1.34 (b).

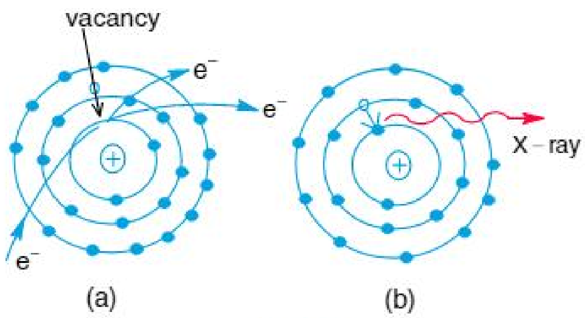


Fig 1.34

X-rays produced in this way are called characteristic X-rays. It is because radiations emitted in the process is the characteristic of target atom considered and its energy depends upon the energy of the electrons in the shells of that atom.

For example, if the bombarding electron has knocked out one electron from K-shell, let us suppose that this vacancy is filled by the electron jump from L-shell. The energy loss by electron during this transition is emitted as X-rays.

Characteristic X-rays produce line spectrum. This is because, these X-rays are the results of electron jump from higher to lower orbits and hence have discrete energies. If the transitions are to the K-shell, the X-rays are labelled as $K_\alpha, K_\beta, K_\gamma$.... If the transition are to the L-shell they are labelled as $L_\alpha, L_\beta, L_\gamma$..... as shown in the Fig 1.35.

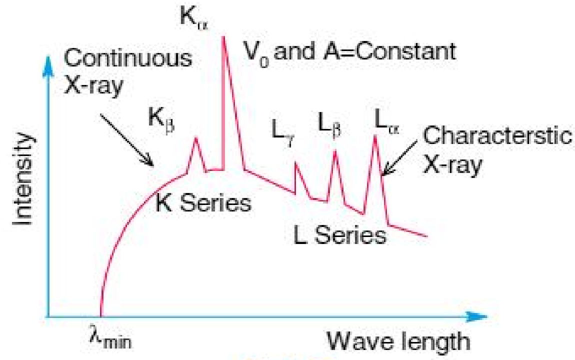


Fig 1.35

Thus X-ray spectrum consists of a continuous spectrum with a line spectrum superimposed on it. The intensity of X-ray depends upon the target material and the voltage across the tube. It is approximately proportional to the voltage across the tube and mass number of the target nuclei.

Characteristic X-rays depend upon the material of the target but not on the potential difference applied between cathode and target. This is shown in Fig 1.36.

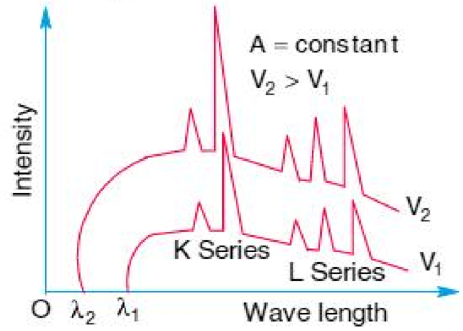


Fig 1.36

When the mass number of target increases, the peak of the series shifts towards the shorter wave lengths as shown in Fig 1.37

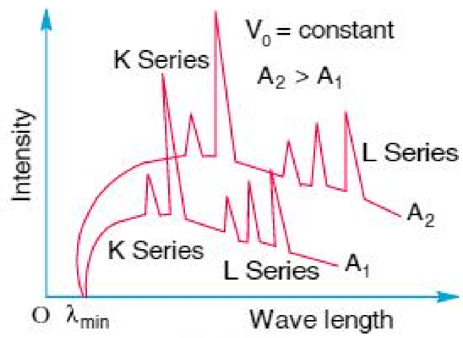


Fig 1.37

Example-1.22

A potential difference of 20 kV is applied across a X-ray tube. Find the minimum wavelength of X-rays generated

Solution :

$$\lambda_{min} = \frac{hc}{20\times10^3\times1.6\times10^{-19}} \text{ (or) } \lambda_{min} = 0.62\text{Å}$$

Example-1.23

The K_α X-ray emission line of tungsten occurs at $\lambda = 0.021$ nm. What is the energy difference between K and L levels in this atom ?

Solution : The energy of the X–ray is

$$\Delta E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^8}{0.021\times10^{-9}} = 59 \text{ keV}$$

This is the required energy difference

1.15.1 MOSELEY'S LAW

The characteristic X-ray spectrum of K and L shells of different elements was analysed by Moseley and found that K and L series radiation of each element was distinct from that of any other element. For a given series, α-line always has a longer wavelength than β-line. The wavelength of a spectral line decreases as the atomic number of the target is increased. On the basis of these observations, Moseley stated that the square root of frequency of any line in the characteristic X-ray spectrum is proportional to the atomic number of the element. This is known as Moseley's law. Mathematically $\sqrt{v}\,\alpha(z-b)$ (or) $\sqrt{v} = m(z-b)$

where Z is the atomic number of the element and m gives the slope of $\sqrt{v}-Z$ line which depends on type of transition. For a given pair of principle quantum numbers, m is same for all the elements. b is called screening constant and it depends on transition series. Hence b is different for different series. For K-series b = 1, for L - series b = 7.4, and M-series b = 9.

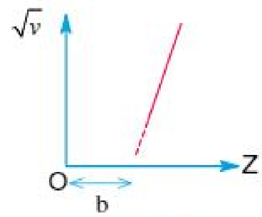


Fig 1.38

Moseley observed a graph between $\sqrt{\nu} - Z$ as shown in Fig 1.38. In general according to Moseley, the wavelength of all series of transition is given by

$$\frac{1}{\lambda} = R_{\infty} Z_{eff}^{\;2}\left[\frac{1}{n_1^{\;2}} - \frac{1}{n_2^{\;2}}\right], \text{for } (n_2 > n_1)$$

(or)

$$\frac{1}{\lambda} = R_{\infty}(Z-b)^2\left[\frac{1}{n_1^{\;2}} - \frac{1}{n_2^{\;2}}\right], \text{for } (n_2 > n_1)$$

For K-series $b = 1$, $n_1 = 1$

$$\frac{1}{\lambda} = R_{\infty}(Z-1)^2\left[1 - \frac{1}{n^2}\right]$$

where $n = 2, 3, 4......$

Here we restrict only to K_{α} -line.

$$\frac{1}{\lambda} = R_{\infty}(Z-1)^2\left[1 - \frac{1}{2^2}\right] \text{(or)} \frac{1}{\lambda} = \frac{3}{4}R_{\infty}(Z-1)^2$$

$$\frac{c}{\lambda} = \frac{3}{4}cR_{\infty}(Z-1)^2$$

$$\nu = m^2(Z-1), \left(\text{where } m = \sqrt{\frac{3}{4}R_{\infty}c}\right)$$

$\therefore$ The slope of K_{α} line in $\sqrt{\nu} - Z$ curve is

$$\sqrt{\frac{3}{4}R_{\infty}c}$$

For different series, $\sqrt{\nu} - Z$ graphs are as shown in Fig 1.39.

It show that $K_{\beta} > K_{\alpha} > L_{\beta} > L_{\alpha}$, which can be verified of calculation.

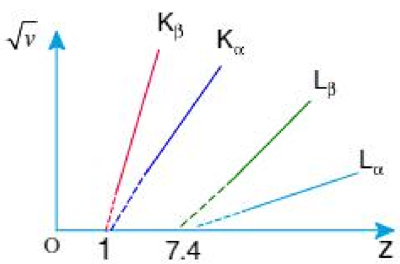


Fig 1.39

Significance of Moseley's Law

i) According to this law, the physical and chemical properties of an element are determined by its atomic number and not by its atomic weight. Hence the atoms in the periodic table must be arranged according to their atomic numbers and not according to their atomic weights. He found that cobalt comes before nickel and argon before potassium.

ii) Moseley's law helped to perfect the periodic table by determining the atomic numbers of rare earth elements.

iii) The law helped in the discovery of new elements like Masurium (43), Illinium (61) etc.

iv) It gave proof for the validity of Bohr's theory.

Example-1.24

If the short series limit of the Balmer series for hydrogen is 3646Å, calculate the atomic number of the element which gives X-ray wavelength down to 1.0Å. Identify the element.

Solution :

The short limit of the Balmer series is given by

$$\bar{\nu} = 1/\lambda = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = R/4$$

$$\therefore\ R = 4/\lambda = (4/3646)\times 10^{10} m^{-1} \quad ......(1)$$

Further the wavelengths of the K_{α} series are given by the relation

$$\bar{\nu} = \frac{1}{\lambda} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

The maximum wave number corresponds to $n = \infty$

Therefore, we have $\bar{\nu} = \frac{1}{\lambda} = R(Z-1)^2 \quad ......(2)$

from (1) and (2)

$(Z-1) = \sqrt{911.5} \cong 30.2 \quad$ or $Z = 31.2 \cong 31$

Thus the atomic number of the element concerned is 31. The element having atomic number Z = 31 is Gallium.

1.15.2 PROPERTIES OF X-RAYS

1) Wavelength of X-rays lies between that of γ-rays and ultraviolet rays.

2) Penetrating power or hardness $\propto \frac{1}{\lambda} \propto V$, where V is accelerating potential.

3) In a Coolidge tube both a.c. and d.c potential can be applied, but X-rays are produced only during positive half cycles.

4) X-rays are not affected by electric and magnetic fields;

5) X-rays obey all the laws of light propagation, exhibit reflection, refraction, diffraction, interference and polarization, etc.

6) Frequency of characteristic X - rays does not depend on accelerating voltage.

7) The slope of $\sqrt{\upsilon} - Z$ curve is equal to constant m. The value of m is maximum for K series.

8) X-rays penetrate through different depths into different substances.

9) X-rays ionise the gases they pass through

10) X-rays produce photoelectric effect, and Compton effect.

1.16 BRAGG'S LAW

Bragg observed that X-rays can be reflected by the cleavage planes of the crystal when they are incident on their surfaces nearly at glancing angle. The planes parallel to a surface along which a crystal can be readily split are called bragg's planes.

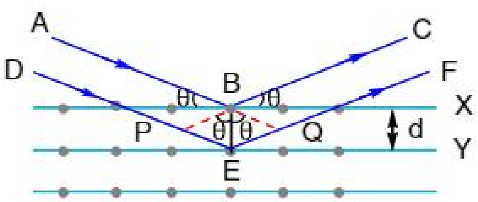


Fig 1.40

Consider a monochromatic beam of X-rays of wavelength λ incident on a crystal and after reflection from the Bragg's planes X and Y go along BC and EF respectively as shown. Let d be the distance between the Bragg's planes. Let θ be the glancing angle.

The path difference between the waves = PE + EQ

We can find PE = EQ = $d\sin\theta$

Now path difference between the two waves $= 2d\sin\theta = n\lambda$

Where n = 1,2,3,....

This is known as Bragg's law or Bragg's equation.

For n = 1, we get first order spectrum and for n = 2. We get second order spectrum.

If λ of X-rays is known, we can find d and vice versa. Bragg's law is useful to study crystal structures.

Example-1.25

In the Auger process, an atom makes a transition to a lower state without emitting a photon. The excess energy is transformed to an outer electron which may be ejected by the atom. (This is called Auger electron). Assuming the nucleus to be massive, calculate the kinetic energy of an n = 4 Auger electron emitted by chromium by absorbing the energy from a n = 2 to n = 1 transition.

Solution :

Chromium has a single valence electron as per its configuration. So we can use Bohr's model to describe its energy states.

Energy corresponding to n^{th} order $= E_n = \frac{-13.6}{n^2} Z^2 (ev)$

As chromium is massive its recoil linear momentum may be neglected. So, entire transition energy may be assumed as transformed to the Auger electron.

Energy required to eject an electron in the state n = 4 is given by $E_2 = \frac{13.6 Z^2}{4^2} = 0.85 Z^2 eV$

So, kinetic energy of the Auger electron will be KE $= E_1 - E_2 = 9.35\, Z^2 = 9.35(24)^2 = 5385.6$ eV.

EXERCISE

LONG ANSWER QUESTIONS

1. Define photoelectric effect? Write the laws of photoelectric effect.
2. Explain continuous and characteristic X - rays spectra. Discuss their origin.

SHORT ANSWER QUESTIONS

1. Define the terms work function and threshold frequency? Explain the relation between them.
2. Write the laws of photoelectric effect.

3. What is photoelectric cell ? Give two applications of photoelectric cell ?
4. Write briefly de Broglie hypothesis.
5. Describe the origin of X - rays spectra.
6. What is Moseley's law ? Discuss briefly its importance.

VERY SHORT ANSWER QUESTIONS

1. When proton, deutron and α - particle are accelerated through same potential difference, what is the ratio of their velocities?
2. Among the particles - Proton, α-particle and β - particle which has lowest specific charge?
3. Can a particle have a charge of 4.0×10^{-19}C?
4. What is work function of a metal?
5. A radiation of wavelength 6000Å is incident on a metal surface whose work function is 4 eV. Do you observe the phenomenon of photoelectric effect?
6. Can photoelectric phenomenon take place with radiation of all frequencies?
7. Generally, why alkali metals are preferred for photoelectric emission?
8. It is found that orange light does not eject photoelectrons from a metal. Can we use red light to emit photoelectrons from that metal?
9. Can photoelectric effect take place for a gaseous target rather than a solid?
10. If it is difficult to remove a free electron from aluminium than sodium, which metal has greater work function?
11. Is photon a wave or a particle? What is its rest mass?
12. What is the equation of wavelength associated with a moving particle?
13. What are de Broglie waves?
14. If a proton and an electron have same de Broglie wavelength, which has more momentum and kinetic energy?
15. Is the de-Broglie wavelength of a photon of an electromagnetic radiation equal to the wavelength of the radiation?
16. Wave nature of matter is not apparent in our daily observation, Why?
17. Explain why all emission lines are not observed in absorption?
18. If radius of an orbit electron is increased does its total energy increase or decrease? Does its kinetic energy increase or decrease?
19. What is the energy of shortest wavelength photon that can be emitted by the hydrogen atom?
20. What happens to an electron with E = 0 and $n \rightarrow \infty$?
21. In analyzing the absorption spectrum of hydrogen atom at room temperature, one finds absorption lines corresponding to wave lengths in the Lyman series but not to those in Balmer series. Explain.
22. If the electron in the hydrogen atom starts at energy level with principal quantum number n, how many possible lines could be observed?
23. What is the maximum possible kinetic energy of a beam of electrons such that collisions with hydrogen atom are elastic?
24. According to Bohr's postulate $\Delta E = h\nu$. Can this equation be exactly true, taking into account conservation of linear momentum?
25. Consider a ground state hydrogen atom.

    (a) What happens if a photon of frequency $\left(\frac{E_3 - E_1}{h}\right) < \nu < \frac{(E_4 - E_1)}{h}$ is incident?

    (b) What happens if an electron beam of $E_3 - E_1 < E_4 - E_1$ is used?
26. Which process can be thought of as inverse photo electric effect? Can this process be explained classically or with the help of quantum theory or can it be explained by both on the basis of classical as well as of quantum theory?
27. What is the momentum of X-ray photon of wave-length 0.2 Å?
28. Can characteristic X-rays be emitted by hydrogen atom?
29. To identify the element characteristic X-rays emitted by that element are used. Can continuous X-rays be used for this purpose?
30. Can hard X-rays be produced by increasing operating voltage?
31. Can X - rays produce photoelectric effect ? Calculate the work function of a metal producing photo-electrons with X - rays of wavelength 1A°.
32. Can X-rays produce photoelectric effect from Zinc?
33. State Moseley's Law. What is its importance?
34. Instead of plotting $\sqrt{\nu}$ versus Z curve, if Moseley had plotted ν versus Z, what would have been the nature of the curve?
35. On increasing the voltage across an X-ray tube discuss the effects on (a) the minimum wavelength X-ray photon, (b) the minimum frequency X-ray photon, (c) characteristic X-rays observed.

PROBLEMS

LEVEL - I

1. An electron beam moving with a speed of $2.5\times10^7 ms^{-1}$ enters a magnetic field directed perpendicular to its direction of motion. The magnetic induction of the field is 4×10^{-3} wb/m^2. Find the intensity of the electric field applied so that the electron is undeflected due to the magnetic field.
[Ans: 10^5 N/C]

2. A mono energetic electron beam with the speed of $5.2\times10^6 ms^{-1}$ enters a magnetic field of induction 3×10^{-4}T, directed normal to the beam. Find the radius of the circle traced by the beam. (given $e/m = 1.76\times10^{11}$ C kg^{-1}) [Ans: 0.098 m]

3. Calculate the force experienced by a moving electron which is entering into a condenser having its plates 0.1m apart and potential difference of 300 V. The direction of the electric field is perpendicular to the motion of the electron. [Ans: 4.8×10^{-16} N]

4. A charged particle is accelerated through a potential difference of 100V passes through crossed electric and magnetic fields without any deflection. If $E = 15\times10^6$ V/m and $B = 5\times10^3$T, find the specific charge of the particle [Ans: 4.5×10^4 C/Kg]

5. What will be the minimum frequency of light source to get photocurrent, from a metal surface having work function 2eV ? [Ans: 4.8×10^{-14} Hz]

6. The threshold wavelength for emission of photoelectrons from a metal surface is 6×10^{-7} m. What is the work function of the material of the metal surface ? [Ans: 3.33×10^{-19} J]

7. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6eV fall on it is 4eV. Find the stopping potential in volts.
[Ans: 4V]

8. Find the maximum kinetic energy of the photoelectrons ejected when light of wavelength 350 nm is incident on a cesium surface. Work function of cesium = 1.9 eV. [Ans: 1.6 eV]

9. What will be the maximum kinetic energy of the photo-electrons ejected from magnesium (for which the work function W = 3.7 eV) when irradiated by ultraviolet light of frequency 1.5×10^{15} sec^{-1}.
[Ans: 2.8 eV]

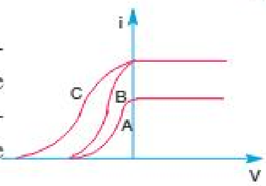
10. The work function of a photoelectric material is 4.0 eV. (a) What is the threshold wavelength? (b) Find the wavelength of light for which the stopping potential is 2 V. Ans: (a) 310 nm ; (b) 190 nm]

11. Photoelectrons are ejected from the surface of a metal having work function 4.5 eV. Find the impulse transmitted to the surface of the metal when the electron flies off due to collision of light quanta of energy 4.9eV. [Ans: 3.43×10^{-25} kg ms^{-1}]

12. Light of wavelength 2000 Å is incident on a metal surface of work function 3.0 eV. Find the minimum and maximum kinetic energy of the photoelectrons.
[Ans: Zero, 3.19 eV]

13. Let K_1 be the maximum kinetic energy of photoelectrons emitted by light of wavelength λ_1 and K_2 corresponding to wavelength λ_2. If $\lambda_1 = 2\lambda_2$ then find $K_2 - K_1$. (h is planck's constant and c is speed of electromagnetic wave) [Ans : $\frac{hc}{\lambda_1}$]

14. The figure shows the variation of photo-current with anode potential for a photo-sensitive surface for the different radiations.



Let I_a, I_b and I_c be the intensities and f_a, f_b and f_c be the frequencies for the curves a, b and c respectively. Compare the given parameters.
[Ans : $f_c > f_b = f_a$ and $I_c = I_b > I_a$]

15. For a certain metal the threshold frequency is v_0. If light of frequency $2v_0$ is incident on it the electron comes out with a maximum velocity of 4×10^6 m/s. If light of frequency of $5v_0$ is incident on it, find the maximum velocity of the photo electron.
[Ans: 8×10^6 m/s]

16. An electron and a photon have same wavelength. If p is the momentum of electron and E the energy of photon, find the magnitude of p/E in SI units
[Ans: 3.33×10^{-9} sm^{-1}]

17. A particle of mass 3m at rest decays into two particles of masses m and 2m having non-zero velocities. Find the ratio of the de-Broglie wavelengths of the particles $\left(\frac{\lambda_1}{\lambda_2}\right)$: [Ans: 1 : 1]

18. Find the momentum of an electron having wavelength 2Å ($h = 6.62\times10^{-34}$Js) [Ans: 0.243×10^{-11} m]

19. Calculate the wavelength of de Broglie waves associated with a beam of protons of kinetic energy 5×10^2 eV. (Mass of each proton $= 1.67\times10^{-27}$ kg, $h = 6.62 \times 10^{-34}$ Js) [Ans: 1.28×10^{-12} m]

20. Find the de-Broglie wavelengths of (a) 46 gm golf ball with a velocity of 30 m/s. (b) an electron with a velocity of 10^7 m/s.
[Ans: (a) 4.8×10^{-34}m ; (b) 7.3×10^{-11}m]

21. The de Broglie wavelength of the thermal neutron at 927°C is λ. Find its wavelength at 327°C
[Ans: $\lambda\sqrt{2}$]

22. An α-particle of energy 5MeV is scattered through 180° by a fixed uranium nucleus. Find the order of distance of closest approach. [Ans: 10^{-12} cm]

23. The distance of closest approach of an alpha particle fired at nucleus with momentum p is r_0. Find the distance of closest approach when the alpha particle is fired at same nucleus with momentum 2p
[Ans: $\frac{r_0}{4}$]

24. Find the binding energy of a hydrogen atom in the state n = 2. [Ans: 3.4 eV]

25. Find the radius and energy of a He^+ ion in the states (a) n = 1, (b) n = 4, (c) n = 10.
[Ans: (a) 0.265 A, –54.4eV; (b) 4.24 A, – 3.4 eV ; (c) 26.5 A, –0.544 eV]

26. A hydrogen atom emits ultraviolet radiation of wavelength 102.5 nm. What are the quantum numbers of the states involved in the transition?
[Ans: 1 and 3]

27. Find the maximum Coulomb force that can act on the electron due to the nucleus in hydrogen atom.
[Ans: 8.2×10^{-8} N]

28. Find the longest wavelength present in the Balmer series of hydrogen. [Ans: 656 nm]

29. A hydrogen atom in state n = 6 makes two successive transitions and reaches the ground state. In the first transition a photon of 1.13 eV is emitted. (a) Find the energy of the photon emitted in the second transition. (b) What is the value of n in the intermediate state ?
[Ans: 12.1 eV, 3]

30. For molybdenum the wavelength of the K_α line is 0.71 nm and of the K_β line it is 0.63 nm. Use this information to find wavelength of the L_α line.
[Ans: 0.559 nm]

31. The binding energy of an electron in the ground state of He atom is equal to $E_0 = 24.6$eV. Find the energy required to remove both electrons from the atom.
[Ans: 79 eV]

32. Magnetic moment due to the motion of the electron in n^{th} energy state of hydrogen atom is proportional to n^x. Find the value of x. [Ans: 1]

33. Find the ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom (both in ground state) [Ans: 8 : 1]

34. In a hydrogen atom, the electron is in n^{th} excited state. It comes down to first excited state by emitting ten different wavelengths. Find the value of n :
[Ans: 6]

35. According to Bohr's theory of hydrogen atom, find the product of the binding energy of the electron in the n^{th} orbit and its radius in the n^{th} orbit.
[Ans: 7.2eV - Å]

36. The angular momentum of an electron in the hydrogen atom is $\frac{3h}{2\pi}$. Here h is Planck's constant. Find the kinetic energy of this electron.
[Ans: 1.51 eV]

37. When an electron in the hydrogen atom in ground state absorbs a photon of energy 12.1 eV. Find the change its angular momentum [Ans: $\frac{h}{\pi}$]

38. In a hydrogen atom, the binding energy of the electron in the ground state is E_0. Find the frequency of revolution of the electron in the n^{th} orbit :
[Ans: $\frac{2E_0}{nh}$]

39. Which electronic transition in Li^{2+} ion would emit radiation of same wavelength as the wavelength of second Balmar line of H-atom ? [Ans: $12 \rightarrow 6$]

40. Difference between n^{th} and $(n + 1)^{th}$ Bohr's radius of 'H' atom is equal to it's $(n - 1)^{th}$ Bohr's radius. Find the value of n. [Ans: 4]

41. A hydrogen atom is in an excited state of principle quantum number n. It emits a photon of wavelength λ when returns to the ground state. Find the value of n.
[Ans: $\sqrt{\frac{\lambda R}{\lambda R - 1}}$]

42. An electron moving in a circular orbit around the nucleus has a total (kinetic + potential) energy E_0. Find its potential energy [Ans: $2E_0$]

43. As per Bohr model, find the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom (Z = 3). [Ans: 122.4 eV]

44. Heat at the rate of 200 W is produced in an X-ray tube operating at 20 kV. Find the current in the circuit. Assume that only a small fraction of the kinetic energy of electrons is converted into X-rays.
[Ans: 10 mA]

45. Find the energy, the frequency and the momentum of an X-ray photon of wavelength 0.10 nm.
[Ans: 12.4 ke V, 3×10^{18}Hz, 6.62×10^{-24}kg - m / s]

46. Find the cutoff wavelength for the continuous X-rays coming from an X-ray tube operating at 30 kV. [Ans: 41.4 pm]

47. The X-ray coming from a Coolidge tube has a cutoff wavelength of 80 pm. Find the kinetic energy of the electrons hitting the target. [Ans: 15.5 ke V]

48. The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 mA. Then find the number of electrons striking the target per second [Ans: 2×10^{16}]

49. K_α wavelength emitted by an atom of atomic number Z = 11 is λ. Find the atomic number for an atom that emits K_α radiation with wavelength 4λ. [Ans: Z = 6]

50. From what material is the anode of an X-ray tube made, if the K_α - line wavelength of the characteristic spectrum is 0.76 Å? [Ans: Z = 41]

51. Characteristic X-ray of frequency 4.2×10^8 Hz are produced when transitions from L shell take place in a certain target material. Use Moseley's law and determine the atomic number of the target material. Given Rydberg constant $R=1.1\times10^7$ m^{-1}.
[Ans: Z = 42]

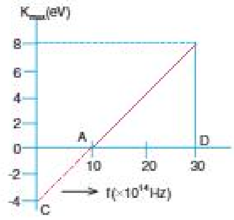
LEVEL - II

1. (a) A proton is moving at a speed much less than the speed of light. It has kinetic energy K_1 and momentum p_1. If the momentum of the proton is doubled, so that $p_2 = 2p_1$, how is its new kinetic energy K_2 related to K_1? (b) A photon with energy E_1 has momentum p_1. If another photon has momentum p_2 that is twice p_1, how is the energy E_2 of the second photon related to E_1? [Ans: (a) $K_2 = 4K_1$; (b) $E_2 = 2E_1$]

2. An atom absorbs a photon of wavelength 500 nm and emits another photon of wavelength 700 nm. Find the net energy absorbed by the atom in the process. [Ans: 1.1×10^{-19} J]

3. When a metal is illuminated with light of frequency *f* the maximum kinetic energy of the photoelectrons is 1.2 eV. When the frequency is increased by 50% the maximum kinetic energy increases to 3.6 eV. What is the threshold energy for this metal.
[Ans: 3.6 eV]

4. A graph regarding photo-electric effect is shown between the kinetic energy of electrons and the frequency of the incident light. On the basis of data as shown in the graph, calculate:



(a) Threshold frequency,
(b) Work function,
(c) Planck's constant
[Ans:(a) 10^{15} Hz ; (b) 4eV ; (c) 6.4×10^{-34} J–s]

5. The electric field associated with a light wave is given by $E=E_0\sin(1.57\times10^7\text{m}^{-1})(x-ct)]$. Find the stopping potential when this light is used in an experiment on photoelectric effect with a surface having work function 1.9 eV. [Ans: 1.2 V]

6. The electric field at a point associated with a light wave is E = (100 V/m) sin [$(3.0\times10^{15}\ \text{s}^{-1})t$]. sin[$6.0\times10^{15}\text{s}^{-1})t$. If this light falls on a metal surface having a work function of 2.0 eV, what will be the maximum kinetic energy of the photoelectrons ?
[Ans: 3.93 eV]

7. 1.5 mW of 400 nm light is directed at a photoelectric cell. If 0.1 percent of the incident photons produce photo-electrons, find the current in the cell.
[Ans: 0.48μA]

8. Radiation of wavelength $\frac{\lambda_0}{3}$ (λ_0 being the threshold wavelength) is incident on a photosensitive sphere of radius R. Find the charge developed on the sphere when electrons cease to be emitted.
[Ans: $\frac{8\pi\varepsilon_0 Rhc}{e\lambda_0}$]

9. The energy of a photon is equal to the kinetic energy of a proton. The energy of the photon is E. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. Then the ratio $\frac{\lambda_1}{\lambda_2}$ is proportional to E^X. Find the value of x. [Ans: 1/2]

10. A parallel beam of monochromatic light of wave-length 500 nm is incident normally on a perfectly absorbing surface. The power through any cross-section of the beam is 10 W. Find (a) the number of photons absorbed per second by the surface and (b) the force exerted by the light beam on the surface
[Ans: (a) 2.52×10^{19} (b) 3.33×10^{-8}N]

11. A beam of white light is incident normally on a plane surface absorbing 70% of the light and reflecting the rest. If the incident beam carries 10 W of power, find the force exerted by it on the surface. **[Ans : 4.3×10^{-8}N]**

12. A small plate of a metal is placed at a distance of 2 m from a monochromatic light source of wavelength 4.8×10^{-7} m and power 1.0 watt. The light falls normally on the plate. Find the number of photons striking the metal plate per square metre per second. **[Ans: 4.82×10^{16} /m^2-s]**

13. A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is 1.0×10^{19}. Calculate the force exerted by the light beam on the mirror. **[Ans: 10^{-8} N]**

14. In an experiment on photoelectric effect, light of wavelength 400 nm is incident on a cesium plate at the rate of 5.0 W. The potential of the collector plate is made sufficiently positive with respect to the emitter so that the current reaches its saturation value. Assuming that on the average one out of every 10^6 photons is able to eject a photoelectron, find the photocurrent in the circuit. **[Ans: 1.6μA]**

15. The stopping potential for the photoelectrons emitted from a metal surface of work function 1.7 eV is 10.4 V. Find the wavelength of the radiation used. Also identity the energy levels in hydrogen atom, which will emit this wavelength. **[Ans:1022 Å, n = 3 to n = 1]**

16. Hydrogen atoms absorb radiations of wavelength λ_0 and consequently emit radiations of 6 different wave lengths of which two wavelengths are shorter than λ_0. Find the initial state and final excited state values of n. **[Ans: 2, 4]**

17. A gas of hydrogen-like ions is prepared in a particular excited state A. It emits photons having wavelength equal to the wavelength of the first line of the Lyman series together with photons of five other wavelengths. Identify the gas and find the principal quantum number of the state A. **[Ans: He^+, 4]**

18. Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms falls on helium ions in n = 1 and n = 2 states. What are the possible transitions of helium ions as they absorb energy from the radiation? **[Ans: n = 2 to n = 3 and n = 2 to n = 4]**

19. The ground state and first excited state energies of hydrogen atom are –13.6eV and –3.4eV respectively. If potential energy in ground state is taken to be zero, find (a) potential energy of the first excited state (b) total energy of the first excited state (c) kinetic energy of the first excited state (d) total energy of the ground state
[Ans: (a) 20.4 eV, (b) 23.8 eV, (c) 3.4eV, (d) 13.6eV]

20. Find an expression for the magnetic dipole moment and magnetic field induction at the centre of Bohr's hypothetical hydrogen atom in the n^{th} orbit of the electron in terms of universal constants.
[Ans: $\dfrac{neh}{4\pi m}, \dfrac{\mu_0 \pi m^2 e^7}{8 \epsilon_0 h^5 n^5}$]

21. Find the quantum number n corresponding to n^{th} excited state of He^+ ion if on transition to the ground state the ion emits two photons in succession with wavelengths 108.5 nm and 30.4 nm. The ionization energy of the hydrogen atom is 13.6 eV. **[Ans: n = 5]**

22. Work function of metal A is equal to the ionization energy of hydrogen atom in first excited state. Work function of metal B is equal to the ionization energy of He^+ ion in second orbit. Photons of same energy E are incident on both A and B. Maximum kinetic energy of photoelectrons emitted from A is twice that of photoelectrons emitted from B. Find the value of E (in eV) **[Ans: 23.8eV]**

23. The hydrogen atom in its ground state is excited by means of monochromatic radiation. Its resulting spectrum has six different lines. These radiations are incident on a metal plate. It is observed that only two of them are responsible for photoelectric effect. If the ratio of maximum kinetic energy of photoelectrons in the two cases is 5, then find the work function of the metal. **[Ans: 11.93 eV]**

24. Suppose in an imaginary world the angular momentum is quantized to be even integral multiples of $h/2\pi$. What is the longest possible wavelength emitted by hydrogen atoms in visible range in such a world according to Bohr's model? **[Ans: 487 nm]**

25. Consider a hypothetical atom with single electron. In this atom, when an electron de-excites from energy level n = x to n = 2, wavelength (λ) of the radiation emitted is given by $\lambda = \dfrac{Ax^2}{x^2-4}$ (where A is a constant). Find the least and most energetic photons emitted during such transitions. **[Ans : 1.8A, A]**

26. In a hypothetical system a particle of mass m and charge –3q is moving around a very heavy particle having charge q. Assuming Bohr's model to be true to this system, find the orbital velocity of mass m when it is nearest to heavy particle (h = planck's constant)

[Ans: $\frac{3q^2}{2\varepsilon_0 h}$]

27. Suppose a monochromatic X-ray beam of wavelength 100 pm is sent through a Young's double slit and the interference pattern is observed on a photographic plate placed 40 cm away from the slit. What should be the separation between the slits so that the successive maxima on the screen are separated by a distance of 0.1 mm. [Ans: 4×10^{-7} m]

28. A particle of mass 'm' is moving on a circular path under potential U = A + B log(r). If Bohr's quantum condition holds, find the minimum permissible value of the radius.

[Ans: $\frac{h}{2\pi\sqrt{mB}}$]

29. Assume that the de-Broglie wavelength associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electron in eV and the least value of d for which the standing wave of the type described above can form.

[Ans: 150.8 eV, 0.5 Å]

30. Consider an excited hydrogen atom in state n moving with a velocity v(v < <c). It emits a photon in the direction of its motion and changes its state to lower state m. Apply momentum and energy conservation principles to calculate the frequency ν of the emitted radiation. Compare this with the frequency ν_0 emitted if the atom were at rest.

[Ans: $\nu = \nu_0\left(1 + \frac{v}{c}\right)$]

31. An electron of a stationary hydrogen atom (mass m) passes from the fifth energy level to the ground level. Find the velocity of hydrogen atom acquired as a result of photon emission [Ans: $\frac{24hR}{25m}$]

32. When a photon is emitted from an atom, the atom recoils. The kinetic energy of recoil and the energy of the photon come from the difference in energies between the states involved in the transition. Suppose, a hydrogen atom changes its state from n = 3 to n = 2. Calculate the fractional change in the wavelength of light emitted, due to the recoil. [Ans: 10^{-9}]

33 A beam of monochromatic light of wavelength λ ejects photoelectrons from a cesium surface (work function = 1.9eV). These photoelectrons are made to collide with hydrogen atoms in ground state. Find the maximum value of λ for which (a) hydrogen atoms may be ionized, (b) hydrogen atoms may get excited from the ground state to the first excited state and (c) the excited hydrogen atoms may emit visible light. [Ans: (a) 80 nm ; (b) 102 nm ; (c) 89 nm]

34. A neutron moving with a speed v makes a head on collision with a hydrogen atom in ground state kept at rest. Find the minimum kinetic energy of neutron for which inelastic collision will take place : (assume that mass of proton is nearly equal to the mass of neutron) [Ans: 20.4 eV]

35. Two hydrogen atoms in the ground state are moving in opposite direction with the same speed. Find the minimum kinetic energy of each hydrogen atom for the collision to be inelastics so that both atoms are excited. [Ans: 10.2eV]

36. If proton had a radius R and the charge was uniformly distributed, calculate using Bohr's theory, the ground state energy of a H-atom when (a) R = $0.1A^0$ and (b) R = $10A^0$. [Ans : (a) –13.6 V. (b) –3.59 eV]

37. A particle A with a mass m_1 is moving with velocity V hits a particle B of mass m_2 at rest. If the collision is treated as elastic, find the change in de Broglie Wavelength of the particle A.

[Ans : $\Delta\lambda = \frac{h}{m_1 v}\left(\frac{m_1 + m_2}{m_1 - m_2} - 1\right)$]