2. PERMUTATIONS & COMBINATIONS

PERMUTATIONS

SYNOPSIS

Fundamental Principle of Counting :

Multiplication Principle : If an operation can be performed in 'm' different ways ; following which a second operation can be performed in 'n' different ways, then the two operations in succession can be performed in $m \times n$ different ways.

Addition Principle : If an operation can be performed in 'm' different ways and another operation, which is independent of the first operation, can be performed in 'n' different ways, then either of the two operations can be performed in $m+n$ ways.

Note : **The above two principles can be extended for any finite number of operations.**

Permutations :

Each of the different arrangements which can be made by taking some or all of given number of things or objects at a time is called a Permutation.

Note : **Permutation of things means arrangement of things.**

Important Results on Permutations

1. $^nP_r = \dfrac{n!}{(n-r)!} = n(n-1)(n-2).....\{n-(r-1)\},\ \ 0 \le r \le n$ (^nP_r is always a positive integer)
2. $\dfrac{^nP_r}{^nP_{r-1}} = n-r+1$
3. $\dfrac{^nP_r}{^{n-1}P_{r-1}} = n$
4. $^nP_r = n \cdot {}^{n-1}P_{r-1} = n(n-1)\,{}^{n-2}P_{r-2} = n(n-1)(n-2)\,{}^{n-3}P_{r-3}.....etc.,$
5. $^nP_r + r \cdot {}^nP_{r-1} = {}^{(n+1)}P_r$
6. $^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$
7. Number of Permutations of 'n' different things taken all at a time is ^nP_n (where $^nP_n = n!$)
8. The number permutations of 'n' different things taken 'r' at a time when each thing may be repeated any number of times is 'n^r'.
9. The number of permutations of n things of which P are alike and the rest are different by taking all at a time is $\dfrac{n!}{P!}$
10. The number of permutations of 'n' things taken all at a time, out of which 'p' are alike and are of one type, 'q' are alike and are of second type and rest are all different is $\dfrac{n!}{p!q!}$.

Permutations under Restrictions :

1. Number of Permutations of 'n' different things taken 'r' at a time, when a particular thing is to be always included in each arrangement, is $r \cdot {}^{n-1}P_{r-1}$.

2. Number of Permutations of *'n'* different things taken *'r'* at a time, when *'s'* particular things are to be always included in each arrangement, is ${}^{r}P_{s}\cdot{}^{(n-s)}P_{r-s}$.
3. Number of Permutations of *'n'* different things taken *'r'* at a time, when a particular thing is never taken in each arrangement, is ${}^{n-1}P_{r}$.
4. Number of Permutations of *'n'* different things taken all at a time, when *'m'* specified things always come together, is $m!\ \times(n-m+1)!$.
5. Number of Permutations of *'n'* different things taken all at a time, when *'m'* specified things never come together, is $n!\ -(m!\ \times(n-m+1)!)$.
6. Number of permutations of *'n'* dissimilar things taken *'r'* at a time when atleast one thing is repeated, is $n^{r}-{}^{n}P_{r}$.
7. The number of permutations of *'n'* different things taken not more than *'r'* at a time when each thing may occur any number of times is $\dfrac{n(n^{r}-1)}{n-1}$.
8. The number of significant numbers consisting of *'r'* digits and formed out of *'n'* digits including zero, no digit being repeated in any number is ${}^{n}P_{r}-{}^{(n-1)}P_{(r-1)}$.

Note : The number of permutations of *'n'* different things where order of *'r'* things is not to be considered is $\dfrac{n!}{r!}$.

Circular Permutations :

1. Number of Circular Permutations of *'n'* different things taken *'r'* at a time, when clockwise and anticlockwise orders are taken as different, (in both directions) is $=\dfrac{{}^{n}P_{r}}{r}$ where $1\le r\le n$
2. Number of Circular arrangements (permutations) of *'n'* different things, taken all at a time is $(n-1)!$.
3. Number of Circular Permutations of *'n'* different things taken *'r'* at a time, when clockwise and anticlockwise orders are not different, (in one direction) is $=\dfrac{{}^{n}P_{r}}{2r}$
4. Number of Circular arrangements (Permutations) of *'n'* different things, when clock wise and anti clockwise arrangements are not different, is $\dfrac{1}{2}(n-1)!$

Some Important Results :

1. The sum of digits in unit places of all numbers formed with the help of n different non-zero digits $a_1, a_2, \ldots\ldots, a_n$ taken all at a time is $(n-1)!(a_1+a_2+\ldots\ldots+a_n)$
2. The sum of all possible numbers formed out of all the *'n'* digits without zero is $(n-1)!$ **(sum of all the digits) (1111 .... n times)**
3. The sum of all possible numbers formed out of all the *'n'* digits which includes zero is **[$(n-1)$! (sum of all digits) (111 ..... n times)] – [$(n-2)$! (sum of all digits) (111..... $(n-1)$ times]**
4. Sum of all the *'r'*- digit numbers formed by taking the given n digits (including 0) is **(sum of all the n digits) [${}^{n-1}P_{r-1}\times$(111.... r times)–${}^{n-2}P_{r-2}\times$(111 ....$(r-1)$ times)]**
5. Sum of all the *'r'*- digit numbers formed by taking the given n digits (excluding 0) is **(sum of all the n digits) [${}^{n-1}P_{r-1}\times$(111 .... r times)]**

Important Points to Remember :

1. The number of one one onto functions (bijections) that can be defined from set 'A' of 'n' elements to a set 'B' of 'n' elements is ^nP_n or $n!$
2. The number of mappings that can be defined from set A containing 'm' elements to set B containing 'n' elements is n^m.
3. The number of ways in which 'n' different things can be distributed into 'r' different groups where each group must have atleast one thing is equal to number of onto functions with $n(A) = m$, $n(B) = n$, is given by $n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - {}^nC_3(n-3)^m + \text{.........}$
4. The number of one one functions (injections) that can be defined from set A containing 'm' elements to set B containing 'n' elements is ^nP_m.

Note : The number of ways in which 'n' different things can be arranged into 'r' different groups is $^{(n+r-1)}P_n$.

LECTURE SHEET

Fundamental principles of counting, Results on Linear, Circular permutations & Rank of word

LEVEL-I (MAIN)

Single answer type questions

Problems on ^np_r :

1. If $^{(m+n)}P_2 = 90$ and $^{(m-n)}P_2 = 30$, then (m, n) is

   1) (7,3) 2) (1,2) 3) (9,1) 4) (8,2)

2. 1 + 1.1! + 2.2! + 3.3! +......+ n.n! is equal to

   1) $n!$ 2) $(n-1)!$ 3) $(n+1)!$ 4) n

3. If p denotes the number of permutations of $(x + 3)$ things taken all at a time, q denotes the number of permutations of $(x+1)$ things taken 11 at a time and r denotes the number of permutations of $(x-10)$ things taken all at a time such that p=182 qr, then the value of x is

   1) 15 2) 11 3) 12 4) 10

4. If $\frac{^nP_{r-1}}{a} = \frac{^nP_r}{b} = \frac{^nP_{r+1}}{c}$, then which of the following holds good ?

   1) $c^2 = a(b+c)$ 2) $a^2=c(a+b)$ 3) $b^2=a(b+c)$ 4) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$

Permutations of dissimilar things :

5. On a railway route there are 15 stations. The number of different kinds of II[nd] class tickets required in order that it may be possible to book a passenger from one station to another is

   1) 105 2) 210 3) 15! 4) 15! /2!

6. The number of different signals that can be made by 5 flags from 8 flags of different colours is

1) 6720 2) ${}^8C_5$ 3) 8^5 4) 5^8

(i) Formation of words :

7. The number of arrangements that can be formed out of 'LOGARITHM' so that

i) No two vowels come together is

1) $6!\ 7p_3$ 2) $6!\ 7!$ 3) $6!\ 3!$ 4) $7!\ 3!$

ii) All the vowels not come together is

1) $6!\ 7p_3$ 2) $9!-7!3!$ 3) $7!3!$ 4) $6!7!$

iii) No two consonants come together is

1) 0 2) $6!7!$ 3) $6!3!$ 4) $7!3!$

8. The letters of the word 'RANDOM' are arranged in all possible ways. The number of arrangements in which there are 2 letters between R and D is

1) 36 2) 48 3) 144 4) 72

(ii) Arrangement of Persons, things in a row :

9. The number of ways in which ten candidates $A_1, A_2, A_3, A_4, ..., A_{10}$ can be arranged in a verticle row (column)

i) if A_1 and A_2 are next to each other is

1) $9!\ 2!$ 2) $10!$ 3) $10!\ 2!$ 4) $9!$

ii) if A_1 is just above A_2 is

1) $9!\ 2!$ 2) $10!$ 3) $10!\ 2!$ 4) $9!$

iii) if A_1 is always above A_2 is

1) $\frac{10!}{3!}$ 2) $\frac{10!}{2!}$ 3) $9!\ 2!$ 4) $9!$

iv) if A_1 is always above A_2 and A_2 is above A_3 is

1) $\frac{10!}{3!}$ 2) $\frac{10!}{2!}$ 3) $8!\ 3!$ 4) $7!$

v) If A_1, A_2, A_3 are sit together in a specified order is

1) $\frac{10!}{3!}$ 2) $\frac{10!}{2!}$ 3) $8!$ 4) $7!$

10. The letters of the word 'HEXAGON' are arranged in all possible ways. If the order of the vowels is not to be considered then the number of possible arrangements is

1) 1680 2) 840 3) 420 4) $\frac{8!}{2!}$

11. Nine toys are to be packed in 9 boxes. If 5 of them are too big for 3 boxes, then the number of ways in which they can be packed is

1) ${}^6P_5 \cdot 4!$ 2) ${}^6P_5 \cdot 3!$ 3) $6!\ 3!\ 2!$ 4) $5!\ 3!$

(iii) Formation of Numbers :

12. The number of four digit even numbers that can be formed from 0, 1, 2, 3, 7 is

1) 156 2) 300 3) 42 4) 144

13. The number of natural numbers less than 1000 in which no two digits are repeated is :

1) 738 2) 792 3) 837 4) 720

14. The number of four digited numbers formed by 1, 2, 5, 6, 7 divisible by 25 is

1) 42 2) 12 3) 26 4) 46

15. The number of four digited numbers formed by 2, 4, 5, 7, 8 divisible by 4 is

1) 36 2) 46 3) 56 4) 66

16. The number of 5 digited numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 that are divisible by 6 when repetition is not allowed is

1) 84 2) 148 3) 180 4) 108

17. 5 digit number divisible by 9 are to be formed by using the digits 0, 1, 2, 3, 4, 7, 8 without repetition. The total number of such 5 digited numbers formed is

1) 216 2) 214 3) 212 4) 200

18. The number of numbers of 9 different non zero digits such that all the digits in the first four places are less than the digit in the middle and all the digits in the last four places are greater than that in the middle is

1) $2(4!)$ 2) $(4!)^2$ 3) $8!$ 4) $2(8!)$

19. The number of 5-digit numbers in which no two consecutive digits are identical is

1) $9^2 \times 8^3$ 2) 9×8^4 3) 9^5 4) 8^5

Permutations with repetition of things :

20. A letter lock consists of three rings each marked with 10 different letters. Maximum number of ways to make unsuccessful attempts to open the lock is

1) 899 2) 999 3) 479 4) 568

21. The number of four digit telephone numbers having atleast one of their digits repeated is (Note : 0 can be accepted in first place)

1) 5040 2) 4960 3) 2520 4) 2480

22. The number of odd numbers lying between 40000 and 70000 that can be made from the digits 0, 1, 2, 4, 5, 7 if digits can be repeated any number of times is

1) 1125 2) 1296 3) 766 4) 655

23. Number of 5 digited numbers using 0, 1, 2, 3, 4 divisible by 4 with repetition is

1) 800 2) 600 3) 400 4) 200

24. (i) Number of 4 digited numbers using 1, 2, 3, 4, 5, 6 divisible by 3 is (with repetition)

1) 234 2) 334 3) 432 4) 532

(ii) Number of 4 digited numbers using 0, 1, 2, 3, 4, 5 divisible by 6 with repetition is

1) 90 2) 100 3) 140 4) 180

25. A library has 6 copies of one book, 4 copies of each of two books, 6 copies of each of three books and single copies of 8 books. The number of arrangments of all the books is

1) $\frac{40!}{(2!)^4(3!)^6}$ 2) $\frac{40!}{6!\cdot(4!)^2(6!)^3}$ 3) $\frac{40!}{6!\cdot 4!\cdot 6!}$ 4) $\frac{40!}{6!\cdot(4!)^3(6!)^2}$

26. The total number of words that can be made by writing the letters of the word PARAMETER so that no vowel is in between two consonants is

1) 1800 2) 1440 3) 2160 4) 3600

27. The number of ways in which the letters of the word 'PROPORTION' be arranged without changing the relative positions of vowels and consonants is

1) $\frac{6!}{2!\,2!}\,\frac{5!}{4!}$ 2) $\frac{6!\,4!}{2!\,2!\,3!}$ 3) $\frac{10!}{3!\,2!\,2!\,3!}$ 4) $\frac{10!}{3!\,2!\,2!}$

28. The number of six digited numbers between 1,00,000 and 3,00,000 which are divisible by 4 and formed by rearranging digits of 112233 is

1) 12 2) 15 3) 18 4) 19

Circular Permutations :

29. The no. of ways in which 5 boys and 4 girls sit around a circular table so that no two girls sit together is

1) 5! 4! 2) 5! 3! 3) 5! 4) 4!

30. The number of ways of arranging 9 persons around a circle if there are two other persons between two particular persons is

1) $2 \times (7!)$ 2) $3 \times 7!$ 3) $9 \times {}^{8}P_{2}$ 4) $4 \times 7!$

31. The number of ways in which 6 gentlemen and 3 ladies be seated round a table so that every gentlemen may have a lady by his side is

1) 1440 2) 720 3) 240 4) 480

32. 20 persons are invited for a party. The different number of ways in which they can be seated on a circular table with two particular persons seated on either side of the host is

1) 20! 2) 2! 19! 3) 2! 18! 4) 18!

33. Garlands are formed using 6 red roses and 6 yellow roses of different sizes. The number of arrangements in garland which have red roses and yellow roses come alternately is

1) $5! \times 6!$ 2) $6! \times 6!$ 3) $\frac{5!}{2!} \times 6!$ 4) $2(6! \times 6!)$

34. Six boys and six girls sit along a line alternately in x ways; and along a circle again alternatively in y ways, then

1) $x = y$ 2) $y = 12x$ 3) $x = 10y$ 4) $x = 12y$

Problems on finding rank :

35. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictoionary, then the word SACHIN appers at serial number is

1) 601 2) 600 3) 603 4) 602

36. If the letters of the word 'NAAGI' are arranged as in a dictionary then the rank of the given word is

1) 23 2) 84 3) 49 4) 48

37. All the numbers that can be formed using all the digits at a time from 35179 are arranged in the increasing order of magnitude. The rank of the number 35179 is

1) 25 2) 31 3) 65 4) 131

38. If the words formed by using the letters of the word 'AGAIN' are arranged in the form of dictionary then, 50th word is

1) NAAGI 2) NAAIG 3) NAGAI 4) NAIAG

Sum of the numbers :

39. The sum of all 4 digited numbers that can be formed using the digits 1,2,4,5,6 with out repetition is

1) 479952 2) 497952 3) 545958 4) 547598

40. The sum of 4 digited numbers formed by using the digits 0, 2, 4, 7, 8 without repetition is

1) 545958 2) 685784 3) 895452 4) 547598

41. The sum of all the numbers that can be formed by taking all digits 2,3,4,4,5 only is

1) 2399976 2) 21844 3) 630624 4) 181440

Problems on number of Functions and its applications :

42. The number of ways in which 10 letters can be posted in five letter boxes is

1) 10_{P_5} 2) 10_{C_5} 3) 5^{10} 4) 10^5

43. No. of ways in which n different prizes can be distributed among m - persons $(m < n)$ if each is entitled to receive atmost $(n - 1)$ prizes is

1) $n^m - n$ 2) m^n 3) mn 4) $m^n - m$

44. i) Number of ways in which 4 prizes can be distributed among 5 students if no student gets more than one prize is

1) 5^4 2) ${}^5P_4$ 3) 4^5 4) 620

ii) If a student is eligible for all the prizes is

1) 625 2) 620 3) 1024 4) 1020

iii) If no student gets all the prizes is

1) 625 2) 620 3) 1024 4) 1020

45. In an examination there are 3 multiple choice questions and each question has 4 choices. No. of ways in which a student can fail to get all answers correct is

1) 11 2) 12 3) 27 4) 63

46. The number of ways that 5 blue balls of different sizes and 5 red balls of different sizes can be arranged in a row so that no two balls of the same colour come together is

1) 5! 5! 2) $5!\ {}^6p_5$ 3) $2 \times 5!\ 6!$ 4) $2\ (5!)^2$

47. No. of diffferent matrices that can be formed with elements 0, 1, 2 or 3 each matrix having 4 elements is

1) 3×2^4 2) 2×4^4 3) 3×4^4 4) 4^4

48. Four dice are rolled then the number of possible out comes in which atleast one die shows 2 is

1) 1296 2) 625 3) 615 4) 671

49. The number of natural numbers smaller than 10^4 of which all digits are different, is

1) 5275 2) 10 !.9! 3) 5273 4) 5274

50. Let A be the set of all positive prime integers less than 30. The number of different rational numbers whose numerator and denominator belongs to A is

1) 90 2) 180 3) 91 4) 81

51. In a town the car plate numbers contain only three or four digits, not containing the digit 0. The maximum number of cars that can be numbered is

1) 6480 2) 7290 3) 5862 4) 3528

52. The number of 5 digited numbers that contain 7 exactly once is

1) $41(9^3)$ 2) $37(9^3)$ 3) $7(9^4)$ 4) $41(9^4)$

53. No. of different 6 digit numbers whose sum of digits to be odd is (repetitions are allowed)

1) 45×10^4 2) 45 3) 45×10^3 4) 10^3

54. Sum of four digit numbers formed with 2,3,4,5 using each digit any number of times is

1) $4 \times 1111 \times 64$ 2) $14 \times 1111 \times 64$ 3) $14 \times 1111 \times 16$ 4) $4 \times 1111 \times 16$

Numerical value type questions

55. 116 people participated in a knockout tennis tournament. The players are paired up in the first round, the winners of the first round are paired up in the second round, and so on till the final is played between two players. If after any round, there is odd number of players, one player is given a bye, i.e. he skips that round and plays the next round with the winners. The total number of matches played in the tournment is

56. The number of ordered pairs (m,n) where $m,n \in \{1,2,3,....50\}$, such that 6^m+9^n is a multiple of 5 is

57. I have tiled my square bathroom wall with congruent square tiles. All the tiles are red, except those along the two diagonals, which are all blue. If I used 121 blue tiles, then the number of red tiles I used are

58. A class has three teachers, Mr. X, Ms. Y and Mrs. Z and six students A, B, C, D, E, F. Number of ways in which they can be seated in a line of 9 chairs, if between any two teachers there are exactly two students is x. Then $\frac{x}{10}$ is

59. For how many positive integral values of n does n! end with precisely 25 zeroes ?

60. A man has three friends. The number of ways he can invite one friend everyday for dinner on six successive nights, so that no friend is invited more than three times, is

61. A four digit number is called a doublet if any of its digit is the same as only one neighbour. (For exampel, 1221 is doublets but 1222 is not), Number of such doublets are

62. Number of functions defined from $f:\{1,2,3,4,5,6\} \to \{7,8,9,10\}$ such that the sum $f(1) + f(2) + f(3) + f(4) + f(5) + f(6)$ is odd is

LEVEL-II (ADVANCED)

Single answer type questions

1. The number of different words ending and beginning with a consonant which can be made out of the letters of the word EQUATION is

a) 5200 b) 4320 c) 1295 d) 3000

2. The number of six letter words that can be formed using the letters of the word "ASSIST" in which S's alternate with other letters is

a) 12 b) 24 c) 18 d) 20

3. The number of 3 digit even numbers that can be formed by using the digits 0,1,2,3,4 and 5, is given by (repetition being not allowed)

a) 50 b) 52 c) 48 d) 60

4. How many numbers greater then 10 lacs be formed from 2,3,0,3,4,2,3

a) 420 b) 360 c) 400 d) 300

5. Number of 3-digits numbers in which the digit at hundreds place is greater than the other two digits is

a) 204 b) 240 c) 281 d) 285

6. A batsman can score 0, 2, 3 or 4 runs for each ball he receives the no. of ways of scoring a total of 20 runs in one over of 6 balls is

a) 90 b) 95 c) 45 d) 96

7. The number of 3×3 symmetric matrices using $-1, -1, -1, 1, 1, 1, 2, 2, 2$ is

a) 24 b) 36 c) 48 d) 52

8. Among the 8! Permutations of the digits 1,2,…8, consider those arrangements which have the following property. If we take any five consecutive positions, the product of digits in these positions is divisible by 5. Then number of such arrangements is

a) 7! b) $2 \times 7!$ c) $\frac{7!}{3!4!}$ d) $2 \times \frac{7!}{3!4!}$

9. The sum of the integers lying between 1 and 100 and divisible by 3 or 5 or 7 is

a) 2838 b) 3468 c) 2738 d) 3368

10. The number of three digit numbers with three distinct digits such that one of the digits is the arithmetic mean of the other two is

a) 120 b) 180 c) 112 d) 104

11. Messages are conveyed by arranging four white, one blue and three red flags on a pole. Flags of the same colour are alike. If a message is transmitted by the order in which the colours are arranged. The total number of messages that can be conveyed if exactly six flags are used is

a) 45 b) 65 c) 125 d) 185

12. The number of ways in which 4 marries couples can be seated such that four persons are no left side and four are on right side of a long table and no wife is infront of her husband and same gender persons do not sit opposite to each other, is

a) $144\underline{|4}$ b) $9\underline{|4}$ c) $70\underline{|4}$ d) None of these

13. Let there be three sketch pens of different colours. A regular pentagon with unit side length is to be drawn using 3 pens. Exactly one of the colours is to be used to draw a side of the pentagon and at least two colours are to be used to make the complete pentagon. The number of ways of doing it, is

a) 80 b) 48 c) 81 d) 240

14. X has 4 friends A,B,C,D. He wants to invite one or more them at dinner. If the invited person(s) & X are sitting around a circular table, then the number of arrangements in which C & D (if both are invited) are sitting together is

a) 64 b) 48 c) 36 d) 72

More than one correct answer type questions

15. Number of words that can be formed using all the letters of the word "REGULATIONS" such that *G* must come after *R, L* must come after *A* and *S* must come after N is

a) $\frac{11!}{(2!)^2}$ b) $\frac{11!}{(2!)^3}$ c) $\frac{11!}{2!}$ d) (990).7!

16. If $\alpha = x_1x_2x_3$ and $\beta = y_1y_2y_3$ be two three digits numbers, the number of pairs of α and β can be formed so that α can be subtracted form β without borrowing is

a) 2!10!10! b) $(45)(55)^2$ c) $3^2.5^3.11^2$ d) 136125

17. If a seven digit number made up of all distinct digits 8,7,6,4,2, x and y is divisible by 3 then

a) Maximum value of x–y is 9 b) Maximum value of x+y is 12

c) Minimum value xy is 0 d) Minimum value of x+y is 3

Linked comprehension type questions

Passage - I :

There are 12 seats in the first row of a theater of which 4 are to be occupied.

18. Find the number of ways of arranging 4 persons so that, no two persons sit side by side

a) 3023 b) 3024 c) 3025 d) 326

19. Find the number of ways of arranging 4 persons so that, there should be atleast 2 empty seats between any two persons

a) 360 b) 260 c) 560 d) 230

20. Find the number of ways of arranging 4 presons so that, each person has exactly one neighbour

a) 860 b) 862 c) 867 d) 864

Passage - II :

Consider the letters of the word *'MATHEMATICS'*.

21. Possible number of words taking all letters at a time such that at least one repeating letter is at odd position in each word is

a) $\frac{11!}{2!2!2!}-\frac{9!}{2!2!}$ b) $\frac{9!}{2!2!2!}$ c) $\frac{9!}{2!2!}$ d) $\frac{11!}{2!2!2!}$

22. Possible number of words taking all letters at a time such that in each word both M's are together and both T's are together but both A's are not together is

a) $\frac{11!}{2!2!2!}-\frac{10!}{2!2!}$ b) $7!\,{}^8C_2$ c) $\frac{6!4!}{2!2!}$ d) $\frac{9!}{2!2!2!}$

23. Possible number of words in which no two vowels are together is

a) $7!\,{}^8C_4\frac{4!}{2!}$ b) $\frac{7!}{2!}{}^8C_4\frac{4!}{2!}$ c) $\frac{7!}{2!2!}{}^8C_4\frac{4!}{2!}$ d) $\frac{7!}{2!2!2!}{}^8C_4\frac{4!}{2!}$

Matrix matching type questions

24. **COLUMN - I** **COLUMN - II**

A) Number of arrangements of the letters of the word ENGINEER so that all vowels are together but all Es are not together is p) ${}^{12}C_2/3$

B) Number of ways of arrangement the letters of the word ELEVEN so that no two vowels are together and no two consonants are together is k then 10 k equals q) 22

C) Number of ways of keeping 5 letters in 5 addressed envelopes such that no letter goes to its own envelope is p then p is divisible by r) 5!

D) Number of 6 digits numbers greater than 123000 using 1,2,2,3,3,3 is s) 59

25. Consider all possible permutations of the letters of the word MAST E R B L A S T E R S

COLUMN – I	COLUMN – II
A) The number of permutations containing the word RAAT is	p) $\frac{(7!)^2}{3!(2!)^4}$
B) The number of permutations in which S occurs in first place and R occurs in the last place is	q) $\frac{11!\times 4!}{3!\times(2!)^2}$
C) The number of permutations in which none of the letters S, T, R occur in first 7 positions is	r) $\frac{11!}{3!\times 2!}$
D) The number of permutations in which the letters A, S, R occur in even positions is using 1,2,2,3,3,3 is	s) $\frac{12!}{(2!)^4}$

Integer answer type questions

26. Number of ways in which 5A's and 6B's can be arranged in a row which reads the same backwards and forwards is N then the value of $\frac{N}{2}$ is

27. $ABCD$ is a convex quadrilateral. 3,4,5 and 6 points are marked on the sides AB, BC, CD and DA, respectively. The number of triangles with vertices on different sides is xyz then $x+y+z$ =

28. Let $A=\{x|x$ is a prime number and $x < 30\}$. The number of different rational numbers, whose numerator and denominator belong to A, is 13 x then x =

29. The number of 6-digit numbers that can be formed using the digits 1,3,5 so that 5 occurs twice in each number, is xyz then $x + y + z$ =

30. The number of numbers, greater than 400,000 that can be formed by using the digits 0,2,2,4,4,5 is $10x^2$ then x =

31. The number of three digit numbers having only two consecutive digits identical is abc then $a + b + c$ =

32. The number of triplets (x, y, z) of positive integers satisfying $2^x + 2^y + 2^z = 2336$ is

KEY SHEET (LECTURE SHEET)

LEVEL-I

1) **4** 2) **3** 3) **2** 4) **3** 5) **2** 6) **1** 7) i) **1** ii) **2** iii) **1**

8) **3** 9) i) **1** ii) **4** iii) **2** iv) **1** v) **3** 10) **2** 11) **1** 12) **3**

13) **1** 14) **2** 15) **1** 16) **4** 17) **1** 18) **2** 19) **3** 20) **2**

21) **2** 22) **2** 23) **1** 24) **3** 25) **2** 26) **1** 27) **2** 28) **2**

29) **1** 30) **1** 31) **1** 32) **3** 33) **3** 34) **4** 35) **1** 36) **3**

37) **2** 38) **2** 39) **1** 40) **1** 41) **1** 42) **3** 43) **4** 44) i) **2**
ii) **1** iii) **2** 45) **4** 46) **4** 47) **3** 48) **4** 49) **4** 50) **3**
51) **2** 52) **1** 53) **1** 54) **2** 55) **115** 56) **1250** 57) **3600**
58) **1296** 59) **25** 60) **510** 61) **2268** 62) **1024**

LEVEL-II

1) **b** 2) **a** 3) **b** 4) **b** 5) **d** 6) **d** 7) **b** 8) **b**
9) **c** 10) **c** 11) **d** 12) **a** 13) **b** 14) **b** 15) **bd** 16) **bc**
17) **abcd** 18) **b** 19) **a** 20) **d** 21) **d** 22) **b** 23) **c**
24) **A-r;B-r;C-p;D-s** 25) **A-r;B-s;C-p;D-p** 26) **5** 27) **9**
28) **7** 29) **6** 30) **3** 31) **9** 32) **6**

PRACTICE SHEET

Formation of Numbers, Permutations of Similar (or) Dissimilar things, Problems on finding rank

LEVEL-I (MAIN)

Single answer type questions

1. There are 5 roads leading to a town from a village. The number of ways in which a villager can go to the town and return back is

1) 25 2) 20 3) 10 4) 5

2. The number of ways to arrange the letters of the word 'GARDEN' with vowels in alphabetical order is

1) 360 2) 240 3) 120 4) 480

3. The number of ways in which 5 boys and 5 girls can be arranged in a row so that

i) no two girls are together is :

1) $2(5!)^2$ 2) $5!\,{}^6P_5$ 3) $5!\;5!$ 4) $10!$

ii) the number of ways in which no two boys are together is

1) $2(5!)^2$ 2) $5!\,{}^6P_5$ 3) $(5!)^2$ 4) $10!$

iii) The number of ways in which they sit alternately is

1) $2(5!)^2$ 2) $5!\,{}^6P_5$ 3) $(5!)^2$ 4) $10!$

4. The number of ways of arranging 8 Eamcet Question papers so that best and worst never come together is

1) 30240 2) 21600 3) 5040 4) 4320

5. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that dictionary is always in the middle. Then the number of such arrangements is

1) at least 500 but less than 750   2) at least 750 but less than 1000

3) at least 1000   4) less than 500

6. Eight different letters of an alphabet are given. Words of four letters from these are formed. The number of such words with atleast one letter repeated is

1) $\binom{8}{4} - {}^{8}P_4$ 2) $8^4 + \binom{8}{4}$ 3) $8^4 - 8P_4$ 4) $8^4 - \binom{8}{4}$

7. If $n \in N$ and $300<n<3000$ and n is made of digits by taking from 0,1,2,3,4,5 then greatest possible number of values of n is : (repetition is allowed)

1) 539   2) 260   3) 320   4) 300

8. The number of positive integers which can be formed by any number of digits from 0, 1, 2, 3, 4, 5 but using each digit not more than once in each number is

1) 600   2) 1629   3) 1630   4) 601

9. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 when repetition is allowed is

1) 216   2) 375   3) 400   4) 720

10. No. of numbers greater than 1000 but less than 4000 that can be formed by using the digits 0, 1, 2, 3, 4 with repetition is

1) 375   2) 373   3) 374   4) 625

11. The number of different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even position is

1) 16   2) 36   3) 60   4) 180

12. Total no. of ways in which six '+' and four '–' signs can be arranged in a row so that no two '–' signs to be together, is

1) 35   2) 70   3) 6! x 4!   4) 24

13. There are $(n + 1)$ white similar balls and $(n + 1)$ black balls of different size. No. of ways the balls can be arranged in a row so that adjacent balls are of different colours is

1) $[(n+1)!]^2$ 2) $2[(2n)!]$ 3) $2[(n+1)!]$ 4) $2[(n+1)!]^2$

14. The number of ways of arranging the letters of the word 'SUCCESSFUL' so that

i) all S's will come together is

1) $\frac{8!}{2!\,2!}$ 2) $\frac{10!}{2!\,2!}$ 3) $\frac{9!}{3!}$ 4) $10!$

ii) all S's will not come together is

1) $\frac{10!}{3!\,2!\,2!} - \frac{8!}{2!\,2!}$ 2) $\frac{9!}{3!\,2!\,3!} - \frac{8!}{2!\,2!}$ 3) $\frac{9!}{2!\,2!} - \frac{8!}{3!}$ 4) $\frac{10!}{8!\,2!}$

iii) no two S's are together is

1) $\frac{7!}{2!\,2!}$ 2) $\frac{7!}{2!\,2!}\frac{^8P_3}{3!}$ 3) $\frac{7!}{2!\,2!}$ 4) $\frac{^8P_3}{3!}$

iv) S's and U's will come together is

1) $\frac{8!}{2!}$ 2) $\frac{9!}{2!}$ 3) $\frac{7!}{2!}$ 4) $\frac{9!}{2!\,2!}$

v) Two C's are together but no two S's are together is

1) $\frac{7!}{2!\ 2!}$ 2) $\frac{6!\,^7p_3}{2!\ 3!}$ 3) $\frac{9!}{2!\ 2!}$ 4) $\frac{10!}{2!\ 2!\ 3!}$

15. No. of arrangements of the letters of the word 'BANANA' in which two N's do not appear adjacently is

1) 40 2) 60 3) 80 4) 100

16. There are three copies each of 4 different books. The number of ways in which they be arranged on a shelf is

1) $\frac{12!}{(3!)^4}$ 2) $\frac{11}{(3!)^2}$ 3) $\frac{9}{(3!)^2}$ 4) $\frac{12!}{(3!)^5}$

17. The number of ways in which n things of which r are alike, can be arranged in a circular order, is

1) $\frac{(n-1)!}{r!}$ 2) $\frac{n!}{r!}$ 3) $\frac{n!}{(r-1)!}$ 4) $\frac{(n-1)!}{(r-1)!}$

18. Let A be a set of $n(\geq 3)$ distinct elements. The number of triplets (x, y, z) of the elements of A in which atleast two co-ordinates are equal is

1) $^nP_3$ 2) $n^3 - {^nP_3}$ 3) $3n^2-2n$ 4) $3n^2(n-1)$

19. Nine hundred distinct n-digit positive numbers are to be formed using only the digits 2, 5 and 7. The smallest value of n for which this is possible is

1) 8 2) 9 3) 6 4) 7

20. The number of quadratic expressions with the coefficients drawn from the set {0, 1, 2, 3} is

1) 27 2) 36 3) 48 4) 64

21. If 15 people sit around a round table then the unfavourable chance ratio of two particular persons sitting together is

1) 2 : 1 2) 6 : 1 3) 3 : 2 4) 1 : 6

22. The letters of the word 'VICTORY' are arranged in all possible ways, and the words thus obtained are arranged as in a dictionary. Then the rank of given word is

1) 3733 2) 5309 3) 5040 4) 3732

23. All the numbers that can be formed using 1, 2, 3, 4, 5 are arranged in the decreasing order. Rank of 35241 is

1) 51 2) 35 3) 24 4) 16

24. If the words formed by using the letters of the word HARFET are arranged in the form of dictionary, then 261th word

1) FATHER 2) FETHAR 3) FATHRE 4) FATERH

25. The sum of the digits in the unit place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is

1) 432 2) 108 3) 36 4) 18

LEVEL-II (ADVANCED)

Single answer type Questions

1. If $p = {}^{n+2}P_{n+2}; q = {}^{n}P_{11}, r = {}^{(n-11)}P_{n-11}$ and if $p = 182\,qr$ then the value of n is

a) 10 b) 12 c) 15 d) 18

2. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4: and then the men select the chairs from amongst the remaining. The number of possible arrangements is

a) ${}^{6}C_3 \times {}^{4}C_2$ b) ${}^{4}C_2 \times {}^{6}P_3$ c) ${}^{4}P_2 \times {}^{6}P_3$ d) ${}^{4}C_2 \times {}^{4}C_3$

3. The number of arrangements of the letters of the word FORTUNE such that the order of vowels is unaltered is x, the order of consonents is unaltered is y, the order of vowels is unaltered and the order of consonents is unaltered is z, then $\frac{x+y}{z}$ equals

a) 10 b) 20 c) 30 d) 40

4. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is

a) 300 b) 269 c) 271 d) 302

5. The number of ways in which 2 young men, 1 old and 1 lady can sit for a trip in a four-sitter car (two seats in the front and two in the back) so that the old man always sits in the back seat, it is being given that the two young men only can drive the car, is

a) 8 b) 12 c) 16 d) none of these

6. In a steamer there are stalls for 12 animals and there are cows, horses and calves (not less than 12 of each) ready to be shipped: the total number of ways in which the ship load can be made is

a) 3^{12} b) 12^3 c) ${}^{12}P_3$ d) ${}^{12}C_3$

7. Six identical coins are arranged in a row. The total number of ways in which the number of heads is equal to the number of tails, is

a) 9 b) 20 c) 40 d) 120

8. Two teams are to play a series of fine matches between them. A match ends in a win, loss or draw for a team. A number of people forecast the result of each match and no two people make the same forecast for series of matches. The smallest group of people in which one person forecasts correctly for all the matches will contains n people where 'n' is

a) 81 b) 243 c) 486 d) none of these

9. The number of ordered pairs (m, n) where $m, n, \in \{1, 2, 3, ...100\}$ such that $7^m + 7^n$ is divisible by 5 is

a) 250 b) 2500 c) 1500 d) 25200

10. When an ordinary dice is rolled four times, the number of ways such that the largest number appearing on the dice is not 4, is

a) 625 b) 175 c) 1121 d) 1040

11. 20 persons are to be seated around a circular table. Out of these 20, two are brothers. Then number of arrangements in which there will be atleast three persons between the brothers is

a) $18 \times 20!$ b) $36 \times 18!$ c) $13 \times 18!$ d) $13! \times 18!$

12. There are unlimited number of identical balls of four different colours. The number of arrangement of atmost 8 balls in a row that can be made by using them is

a) 87380 b) 65625 c) 84614 d) 70042

13. The number of ways in which 10 different toys can be distributed to 8 children so that each child gets at least one toy is

a) $1680 \times 8!$ b) $960 \times 8!$ c) $240 \times 8!$ d) $720 \times 8!$

More than one correct answer type questions

14. A letter lock consists of three rings marked with 15 different letters. If N denotes the number of ways in which it is possible to make unsuccessful attempts to open the lock, then

a) $482|N$
b) N is product of 3 distinct prime numbers
c) N is product of 4 distinct prime numbers
d) None of these.

15. The number of ways of arranging seven persons (having A,B,C and D among them) in a row so that A,B,C and D are always in order $A–B–C–D$ (not necessarily together) is

a) 210 b) 5040 c) $6 \times {}^7C_4$ d) ${}^7P_3$

16. A class has 30 students. The following prizes are to be awarded to the students of this class. First and second in Mathematics; first and second in physics first in chemistry and first in biology. If N denote the number of ways in which this can be done, then

a) $400|N$
b) $600|N$
c) $8100|N$
d) N is divisible by four distinct prime numbers

17. 10 persons stand in a line which include Ram and Shyam. Let x be the number of ways in which Ram is ahead of Shyam and y be the number of ways in which Shyam is ahead of Ram. Then

a) $x + y = 10!$ b) $x = 9\,!$ c) $x = \dfrac{10!}{2}$ d) $x \neq y$

Linked comprehension type questions

Passage I :

Four prizes are distributed among 5 students

18. If no student gets more than one prize, then the number of ways is

a) 5^4 b) ${}^5P_4$ c) 4^5 d) 620

19. If each student is eligible for all the prizes, then the number of ways is

a) 625 b) 620 c) 1024 d) 1020

20. If no student gets all the prizes, then the number of ways is

a) 625 b) 620 c) 1024 d) 1020

Passage - II :

Different words are being formed by arranging the letters of the word *"SUCCESS"*. On the basis of above information, answer the following question

21. The number of words in which the two *Cs* are together but no two *Ss* are together is

a) 120 b) 96 c) 24 d) 420

22. The number of words in which no two *Cs* and no two *Ss* are together is

a) 120 b) 96 c) 24 d) 420

23. The number of words in which the consonants appear in alphabetic order is

a) 42 b) 40 c) 420 d) 280

Matrix matching type questions

24. Consider all possible permutations of the letters of the word M A S T E R B L A S T E R S

COLUMN - I	COLUMN - II
A) The number of permutations containing the word RAAT is	p) $\frac{(7!)^2}{3!(2!)^4}$
B) The number of permutations in which S occurs in first place and R occurs in the last place is	q) $\frac{11!\times 4!}{3!\times(2!)^2}$
C) The number of permutations in which none of the letters S, T, R occur in first 7 positions is	r) $\frac{11!}{3!\times 2!}$
D) The number of permutations in which the letters A, S, R occur in even positions is using 1,2,2,3,3,3 is	s) $\frac{12!}{(2!)^4}$

25.

COLUMN - I	COLUMN - II
A) If n be the numbers between 500 and 4000 can be formed with the digits 2, 3, 4, 5, 6 when repetition is not allowed, then n is divisible by	p) 2
B) If n be the number of even numbers between 200 and 3000 can be formed with the digit 0,1,2,3,4 when repetition is not allowed, then n is divisible by	q) 3
C) If n the number of words that can be made by arranging the letters of the word ROORKEE that neither begin with R nor end with E, then n is divisible by	r) 5
D) The number of ways of arranging the letters of the word EAMCET is n then n divisible by	s) 11

Integer answer type questions

26. The number of arrangements that can be formed with the letters of the word ORDINATE, so that the vowels occupy odd places is k^2 then the greatest prime which can divide k is

27. The number of arrangements which can be made out of the letters of the word "ALGEBRA" so that no two vowels together is 360λ then $x =$

28. An eight digit number divisible by 9 is to be formed by using 8 digits out of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 without replacement. The number of ways in which this can be done is $\lambda^2 \underline{|7}$ then $\lambda =$

29. There are 3 candidates for a lecturership and one is to be selected by the votes of seven men. The number of ways can the votes be given is m^n then $n - m =$

30. The number of ways of arranging the letters of the word NALGONDA, such that the letters of the word GOD occur in that order (G before and O and O before D), is P then $\frac{P}{420}$

31. The number of ways in which we can make a garland with 5 flowers of one kind and 3 flowers of another kind, is

KEY SHEET (PRACTICE SHEET)

LEVEL-I

1) **1**	2) **1**	3) i) **2**	ii) **2**	iii) **1**	4) **1**	5) **3**	6) **3**
7) **1**	8) **3**	9) **4**	10) **3**	11) **3**	12) **1**	13) **3**	
14) i) **1**	ii) **1**	iii) **2**	iv) **3**	v) **2**	15) **1**	16) **1**	17) **1**	18) **2**
19) **4**	20) **3**	21) **2**	22) **1**	23) **1**	24) **1**	25) **2**	

LEVEL-II

1) **b**	2) **c**	3) **c**	4) **a**	5) **a**	6) **a**	7) **b**	8) **b**
9) **b**	10) **c**	11) **c**	12) **c**	13) **b**	14) **ab**	15) **ad**	16) **abcd**
17) **ac**	18) **b**	19) **a**	20) **b**	21) **c**	22) **b**	23) **a**	

24) **A-r;B-s;C-p;D-p** 25) **A-pq;B-q;C-pqrs;D-pqr**

26) **3**	27) **2**	28) **6**	29) **4**	30) **4**	31) **5**

COMBINATIONS

SYNOPSIS

Combination :

Each of the different groups or selections which can be made by taking some or all of a number of things (irrespective of order) is called a combination.

Note : **Combination of things means selection of things**

The number of combinations of n dissimilar things taken 'r' at a time is denoted by ${}^nC_r$ where $0 \le r \le n$

Important results on Combinations :

1. ${}^nC_r = \binom{n}{r} = c(n,r) = \dfrac{n!}{(n-r)!r!} = \dfrac{{}^nP_r}{r!}$ (${}^nC_r$ is always a positive integer and ${}^nP_r$ is divisible by $r!$)
2. ${}^nC_{r-1} + {}^nC_r = {}^{(n+1)}C_r$
3. ${}^nC_r = {}^nC_{(n-r)}$
4. $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$
5. $\dfrac{{}^nC_r}{{}^{n-1}C_{r-1}} = \dfrac{n}{r}$
6. $\dfrac{{}^nC_{r-1}}{{}^{n-1}C_{r-1}} = \dfrac{n}{n-r+1}$
7. If ${}^nC_r = {}^nC_s$ then $r = s$ or $r + s = n$
8. If ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$ are in A.P. then $(n-2r)^2 = (n+2)$
9. If **'n' is even**, then greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.
10. If **'n' is odd**, then greatest value of ${}^nC_r$ is ${}^nC_{\frac{n-1}{2}}$ or ${}^nC_{\frac{n+1}{2}}$.
11. ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots\dots + {}^nC_n = 2^n$
12. ${}^nC_0 + {}^nC_2 + {}^nC_4 \dots\dots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots\dots = 2^{n-1}$

Combinations under Restrictions :

1. Number of combinations of 'n' different things taken 'r' at a time
   a) when 'p' particular things are always included is ${}^{n-p}C_{r-p}$.
   b) when 'p' particular things are never included is ${}^{n-p}C_r$.
   c) when 'p' particular things are not together in any selection is ${}^nC_r - {}^{n-p}C_{r-p}$.
2. a) Number of selections of 'r' consecutive things out of 'n' things in a row $= n - r + 1$
   b) Number of selections of 'r' consecutive things out of 'n' things along a circle $\begin{cases} n, \text{ when } r < n \\ 1, \text{ when } r = n \end{cases}$
3. The total number of combinations of 'n' different things taken any number (0 or more) at a time is 2^n
   i.e., ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots\dots + {}^nC_n = 2^n$
4. The total number of combinations of 'n' different things taken one or more at a time is $2^n - 1$
   i.e., ${}^nC_1 + {}^nC_2 + \dots\dots\dots + {}^nC_n = 2^n - 1$
5. Number of selections of 'r' things $(r \le n)$ out of 'n' identical things is 1

6. Number of selections of zero or more things out of *'n'* identical things = $n + 1$

7. Number of selections of one or more things out of *'n'* identical things = n

Number of Divisors :

1. Let 'n' ($\neq 1$) be a positive integer such that $n = a_1^p \cdot a_2^q \cdot a_3^r$ where a_1, a_2, a_3 are prime numbers and $p, q, r \in N$ then number of positive divisors of 'n' is $(p+1)(q+1)(r+1)$.
2. Number of divisors excluding 1 are $(p+1)(q+1)(r+1)-1$.
3. Number of divisors excluding number 'n' are $(p+1)(q+1)(r+1)-1$.
4. Number of proper divisors (non trival) of 'n' is $(p+1)(q+1)(r+1)-2$.
5. Number of improper (trivial) divisors of 'n' ($\neq 1$) is 2.
6. If N is a positive integer such that $N = P_1^{n_1} \times P_2^{n_2} \times .... \times P_k^{n_k}$ where $P_1, P_2, ...P_k$ are distinct primes and $n_1, n_2, ...n_k$ are positive integers then

   i) The number of divisors of N is $(n_1+1)(n_2+1)......(n_k+1)$

   ii) The sum of distinct positive integral divisors of N is $\left(\frac{P_1^{n_1+1}-1}{P_1-1}\right)\left(\frac{P_2^{n_2+1}-1}{P_2-1}\right).........\left(\frac{P_k^{n_k+1}-1}{P_k-1}\right)$.

7. a) If $N = 2^{n_1} \cdot P_2^{n_2} \cdot P_3^{n_3} .....P_k^{n_k}$

   (where $P_2, P_3, ...P_K$ are primes other than 2)

   i) The number of even divisors is $n_1(n_2+1)(n_2+1)(n_3+1).....(n_k+1)$

   ii) Number of odd divisors is $(n_2+1)(n_3+1)....(n_k+1)$

   b) The number of ways in which N can be resolved as a product of 2 factors

   i) $\frac{1}{2}(n_1+1)(n_2+1)\;......(n_k+1)$, if **N is not a perfect square.**

   ii) $\frac{1}{2}\{(n_1+1)(n_2+1)......(n_k+1)+1\}$, if **N is a perfect square.**

Distribution of Disimilar things into groups :

1. The number of ways in which $(m + n)$ different things can be divided into two different groups of *'m'* and *'n'* things respectively is $\frac{(m+n)!}{m!\ \ n!}.(m \neq n)$
2. Number of ways of dividing $m + n + p$ different things into three groups containing m, n and p things respectively is $\frac{(m+n+p)!}{m!\ \ n!\ \ p!}\ (m \neq n \neq p)$.
3. Number of ways of dividing '$2n$' different things into two groups, each containing 'n' things and the order of the groups is not important, is $\frac{(2n)!}{2!(n!)^2}$.
4. Number of ways of dividing '$2n$' different things in two groups, each containing 'n' things and the order of the groups is important, is $\frac{(2n)!}{(n!)^2}$.

5. Number of ways of dividing $'3m'$ different things in three groups, each containing $'m'$ things and the order of the groups is not important, is $\frac{(3m)!}{3!(m!)^3}$.

6. Number of ways of dividing $'3m'$ different things in three groups, each containing $'m'$ things and the order of the groups is important, is $\frac{(3m)!}{(m!)^3}$.

7. The number of ways of dividing kn things into 'k' equal groups is $\frac{(kn)!}{(n!)^k.k!}$.(when order of the groups is not important)

Distribution of Similar things into groups :

1. The number of ways in which 'n' identical things can be distributed into 'r' different groups is ${}^{(n+r-1)}C_{r-1}$.

2. The number of ways in which 'n' identical things can be distributed into 'r' different groups so that each group to receive atleast one thing is ${}^{(n-1)}C_{r-1}$.

3. The number of non negative integral solutions of the equation $x_1+x_2+x_3+......+x_r=n$ is ${}^{n+r-1}C_{r-1}$.

4. The number of positive integral solutions of the equation $x_1+x_2+......+x_r=n$ is ${}^{n-1}C_{r-1}$.

5. The number of ways of selection of 'r' things from 'n' different groups when each group may have identical things of 'r' or more things is ${}^{(n+r-1)}C_r$.

   Note : ${}^mC_r\,{}^nC_p+{}^mC_{r+1}\,{}^nC_{p-1}+{}^mC_{r+2}\,{}^nC_{p-2}+.........={}^{(m+n)}C_p$

Total number of combinations :

1. If out of $(p+q+r+t)$ things, $'p'$ are alike of one kind, $'q'$ are alike of second kind, $'r'$ are alike of third kind and $'t'$ are different, then the total number of selections is $(p+1)(q+1)(r+1)2^t-1$.

2. The number of ways of selecting some or all out of $p+q+r$ items where $'p'$ are alike of one kind, $'q'$ are alike of second kind and rest are alike of third kind is $[(p+1)(q+1)(r+1)]-1$

Selection of specified number of things from given things which are all not distinct.

1. If there are l like objects of one kind and m like objects of second kind then the number of ways of selecting 'r' objects out of these objects $(r \le l+m)$ is the coefficient of x^r in the expression of $(1+x+x^2+....+x^l)(1+x+x^2+....+x^m)$.

2. If there are 'l' like objects one kind and m like objects of second kind then the number of ways to select r objects from these $l+m$ objects such that atleast one object must be included from each kind $(2 \le r \le l+m)$ is the coefficient of x^r in $(x+x^2+...+x^l)(x+x^2+....+x^m)$.

3. If there are l objects fo one kind, m objects of second kind then the number of arrangements of r objects out of these $(l+m)$ objects is the coefficient of x^r in the expansion of $r!\left(1+\frac{x}{1!}+\frac{x^2}{2!}+....+\frac{x^l}{l!}\right)\left(1+\frac{x}{1!}+\frac{x^2}{2!}+....+\frac{x^m}{m!}\right)$

Geometrical Applications :

1. Given 'n' distinct points in the plane, no three of which are collinear, then (a) the number of line segments formed = ${}^nC_2$. (b) The number of things formed ${}^nC_3$ =
2. The number of lines formed out of 'n' points in a plane of which 'm' points are collinear is ${}^nC_2 - {}^mC_2 + 1$.
3. The number of tirangles formed out of 'n' points in a plane of which 'm' points are collinear is ${}^nC_3 - {}^mC_3$.
4. The number of diagonals of a polygon of 'n' sides is $\dfrac{n(n-3)}{2}$.

Number of Rectangles & Squares :

1. The number of parallelograms that can be formed from a set of 'n' parallel lines intersecting another set of 'm' parallel lines is ${}^mC_2 \cdot {}^nC_2$.
2. The no. of rectangles of any size in a square of $n \times n$ is $\sum_{r=1}^{n} r^3$ and number of squares of any size is $\sum_{r=1}^{n} r^2$.
3. Number of rectangles of any size in a rectangle of size $n \times p$ $(n < p)$ is ${}^nC_2 \cdot {}^pC_2$ and number of squares of any size is $\sum_{r=1}^{n}(n+1-r)(p+1-r)$.
4. n straight lines of which no two are parallel and no 3 are concurrent then no.of fresh lines obtained by joining the points of intersection of lines is $\dfrac{n(n-1)(n-2)(n-3)}{8}$

The inclusion and exclusion principle

Suppose A_1, A_2....A_n are finite sets, and

$A = A_1 \cup A_2 \cup A_3 \cup ......A_n$ then $n(A) = n(A_1 \cup A_2 \cup ..... \cup A_n)$

$$= \sum_{i=1}^{n} n(A_i) - \sum_{1 \le i < j \le n} n(A_i \cap A_j) + \sum_{1 \le i < j < k \le n} n(A_i \cap A_j \cap A_k).... + (-1)^{n+1} n(A_1 \cap A_2 \cap A_3 \cap .... \cap A_n)$$

Derangement (or) Maching Problem :

If 'n' things form an arrangement in a row, the number of ways in which they can be arranged so that none of them occupies its original place is $= n!\left[1 - \dfrac{1}{1!} + \dfrac{1}{2!} - \dfrac{1}{3!} + ... + (-1)^n \dfrac{1}{n!}\right]$

Exponent of Prime in n!

Exponent of Prime 'P' in $n!$ is defined as $E_P(n!) = \left[\dfrac{n}{P}\right] + \left[\dfrac{n}{P^2}\right] + ... + \left[\dfrac{n}{P^S}\right]$.

Where 'S' is the greatest positive integer such that $P^S \le n \le P^{S+1}$

LECTURE SHEET

Combinations under restrictions, Integral devisers of a number, Distribution of desimilar (similar) things into groups, Principle of Inclusion and Exclusion & Derangements

LEVEL-I (MAIN)

Single answer type questions

Problems on ^nc_r :

1. If ^nC_r denotes the number of combinations of n things taken r things at a time, then the expression $^nC_{r+1}+{}^nC_{r-1}+2\,{}^nC_r$ is

   1) $^{n+2}C_{r+1}$ 2) $^{n+1}C_r$ 3) $^{n+1}C_{r+1}$ 4) $^{n+2}C_r$

2. The value of $^{50}C_4+\sum_{r=1}^{6}{}^{56-r}c_3$ is

   1) $^{55}C_4$ 2) $^{55}C_3$ 3) $^{56}C_3$ 4) $^{56}C_4$

3. If $a_n=\sum_{r=0}^{n}\frac{1}{^nC_r}$ then $\sum_{r=0}^{n}\frac{r}{^nC_r}$ equal to

   1) $^{(n-1)}a_0$ 2) $n\cdot a_n$ 3) $\frac{1}{2}n.a_n$ 4) a_{n+1}

Problems on selection of dissimilar things :

4. In a football championship there were played 153 matches. Every two teams played one match with each other. The number of teams, participating in the championship is

   1) 14 2) 22 3) 18 4) 9

5. In a chess tournament where the participants were to play one game with another, two players fell ill having played 3 games each. If the total number of games played is 84, the number of participants at the beginning was, (game is not held between the two players who fell ill).

   1) 13 2) 14 3) 15 4) 10

6. From 0 to 9, four digited numbers can be formed such that the digits are in ascending order is

   1) $^{10}P_4$ 2) $^{10}C_4$ 3) $^{10}P_4-{}^9P_3$ 4) $^{10}C_4-{}^9C_3$

7. A committee of 6 is chosen from 10 men and 7 women so as to contain atleast 3 men and 2 women. If 2 particular women refuse to serve on the same committee, the number of ways of forming the committee is

   1) 7700 2) 8610 3) 7800 4) 810

8. 12 persons are seated at a round table. Number of ways of selecting 2 persons not adjacent to each other is

   1) 10 2) 11 3) 54 4) 48

9. A double decker bus can accomodate 75 passengers 35 in the lower deck 40 in the upper deck. The number of ways the passengers can be accomodated if 5 want to sit only in lower deck and 8 want to sit only in upper deck is

   1) $^{62}C_{27}$ 2) $^{62}C_{35}$ 3) $^{62}C_{30}$ 4) $^{75}C_{40}$

10. A boat is to be manned by 9 crew with 4 on the stroke side, 4 on the row side and one to steer. There are 11 men of which 2 can stroke only and 1 can row only while 3 can steer only. The number of ways the crew can be arranged for the boat is

1) 10 2) 30 3) 44 4) 17280

Problems on divisors:

11. The sum of the even divisors of 168 is

1) 448 2) 460 3) 42 4) 122

12. No. of odd proper divisors of $3^p.6^m.21^n$ is

1) $(p+m+n+1)(n+1)-1$ 2) $(p+m+n+1)(n+1)$

3) $(p+m+n)(n+1)$ 4) $(p+m+n)(n-1)$

13. If 20% of three element subsets of the set $\{a_1, a_2, a_3, ..., a_n\}$ are three element subsets with an element a_1, then n is

1) 15 2) 16 3) 17 4) 18

14. A student is allowed to select atmost 'n' books from a collection of $(2n + 1)$ books. If the total no. of ways in which he can select a book is 63, then n is

1) 1 2) 2 3) 3 4) 4

15. There are $2n$ things out of which 'n' are alike and 'n' are different, the number of ways of selecting 'n' things is

1) ${}^{2n}C_n$ 2) 2^n-1 3) 2^n 4) n

Distribution of Disimilar things into groups:

16. At an election 3 wards of a town are canvassed by 2, 3, 4 men respectively. The number of ways in which 12 men volunteers can be allotted to different wards is

1) ${}^{12}C_3\dfrac{9!}{2!\,3!\,4!}$ 2) ${}^{12}C_9\dfrac{9!}{3!\,4!}$ 3) ${}^{12}C_9\dfrac{9!}{(3!)^4}$ 4) ${}^{12}C_9\dfrac{9!}{(3!)^3}$

17. Total no. of ways in which '$2n$' persons can be divided into 'n' couples is

1) $\dfrac{(2n)!}{n(2!)^n}$ 2) $\dfrac{(2n)!}{n!(2!)^n}$ 3) $\dfrac{(2n)!}{n(2!)}$ 4) $\dfrac{(2n)!}{n}$

18. The number of ways can a collection of 30 books be divided into two groups of 10 and 20 so that the first group always contains a particular book is

1) ${}^{29}C_{29}$ 2) ${}^{29}C_{20}$ 3) ${}^{29}C_{10}$ 4) ${}^{29}C_9\times{}^{29}C_{20}$

Distribution of Similar things into groups :

19. The number of ways that 30 mangoes can be distributed among 5 boys if each boy is eligible for any number of mangoes is

1) ${}^{34}C_4$ 2) ${}^{35}C_4$ 3) ${}^{36}C_4$ 4) ${}^{33}C_4$

20. No. of ways in which 3 sovereings be given away to 4 applicants and any applicant may have either 0, 1, 2 and 3 sovereings is

1) 18 2) 16 3) 19 4) 20

21. The number of ways in which an examiner can assign 30 marks to 8 questions given not less than 2 marks to any question is

1) ${}^{21}C_7$ 2) ${}^{20}C_7$ 3) ${}^{30}C_8$ 4) 1412

22. The number of positive integral solutions of $abc = 30$ is

1) 27 2) 8 3) 30 4) 45

Total number of combinations:

23. In a book - stall, there are 4 copies of one book, 5 copies of another and single copy of 5 other different books. Then the no. of ways that a person can purchase books is

1) 340 2) 535 3) 959 4) 1002

24. A question paper has 5 questions. Each question has an alternative. The number of ways in which a student can attempt atleast one question is

1) $2^5 - 1$ 2) $3^5 - 1$ 3) $3^4 - 1$ 4) $2^4 - 1$

25. In a crossword puzzle 10 words are to be guessed of which 5 words have each an alternative solution also. No. of possible solutions is

1) 8 2) 16 3) 32 4) 64

26. The maximum number of persons in a country in which no two persons have an identical set of teeth assuming that there is no person without a tooth is

1) 2^{32} 2) 32! 3) $2^{32}-1$ 4) 32!–1

27. Given 5 different green dyes, 4 different blue dyes, 3 different red dyes. The number of combinations of dyes atleast one green and one blue dye is

1) 3870 2) $(2^5 - 1)(2^4-1)(2^3-1)$

3) $(2^5-1)(2^5-1)2^3.12!$ 4) 3720

Geometrical Applications:

28. There are 10 straight lines in a plane no two of which are parallel and no three are concurrent. The points of intersection are joined, then the number of fresh lines formed are

1) 630 2) 615 3) 730 4) 600

29. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these points as vertices is

1) 205 2) 210 3) 315 4) 216

30. Given 5 line segments of lengths 2, 3, 4, 5, 6 units. Then the number of triangles that can be formed by joining these lines is

1) ${}^5C_3$ 2) ${}^5C_3 - 3$ 3) ${}^5C_3 - 2$ 4) ${}^5C_3 - 1$

31. Let T_n denotes the number of triangles which can be formed from the vertices of regular polygon of n sides. If $T_{n+1} - T_n = 21$ then n is

1) 5 2) 4 3) 6 4) 7

32. A regular polygon of n sides is constructed. No. of ways 3 vertices be selected so that no two vertices are consecutive is

1) ${}^nC_3 - n\cdot(n-4)$ 2) ${}^nC_3 - n - n(n-4)$

3) ${}^nC_3 + n - n(n-4)$ 4) ${}^nC_3 - n\cdot n\cdot\ ({}^{(n-4)}C_2)$

33. There are 10 points in a plane of which no 3 points are collinear and 4 points are concyclic. No. of different circles that can be drawn through atleast 3 points of these given points is

1) $({}^8C_3 - {}^6C_3)+1$ 2) $({}^{10}C_3 - {}^4C_3)+1$ 3) $({}^6C_3 - {}^4C_3)+1$ 4) $({}^4C_3 - {}^2C_3)+1$

34. A regular polygon has 23 vertices and consequently 23 sides. The number of additional lines need be drawn so that every pair of vertices will be connected is

1) 253 2) 230 3) 256 4) 276

Dearrangement Problems :

35. The number of ways that all the letters of the word SWORD can be arranged such that no letter is in its original position is

1) 44 2) 32 3) 28 4) 20

36. $f: A \to A$, $A=\{a_1, a_2, a_3, a_4, a_5\}$, then the number of one one functions so that $f(x_i) \neq x_i,\ x_i \in A$ is

1) 44 2) 88 3) 22 4) 20

37. There are 5 different coloured balls and 5 boxes of the same colour as that of the balls. The number of ways that exactly 2 balls will go to the boxes of their own colour is

1) 20 2) 32 3) 28 4) 44

38. Ajay writes letters to his five friends and addresses the corresponding. The number of ways can the letters be placed in the envelopes so that atleast two of them are in the wrong envelopes is

1) 120 2) 125 3) 119 4) 130

Miscellaneous problems

39. If $P = n(n^2-1^2)\ (n^2-2^2)\ (n^2-3^2)......(n^2-r^2)$, $n>r$, $n\in N$, then P is divisible by

1) (2r+2)! 2) (2n+2)! 3) (2n+1)! 4) (2r+1)!

40. A guard of 12 men is formed from a group of n soldiers. It is found that 2 particular soldiers A and B are 3 times as often together on guard as 3 particular soldiers C, D & E. Then n is equal to

1) 31 2) 32 3) 41 4) 42

41. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is

1) 5040 2) 6210 3) 385 4) 1110

42. A is a set containing n elements. A subset P of A is chosen. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen.

i) The number of ways of choosing P and Q so that $P \cap Q = \phi$ is

1) 2^n 2) 2^{2n} 3) 3^n 4) 3^{2n}

ii) Number of ways of choosing P and Q so that $P \cap Q$ contains exactly one element is

1) 3^n 2) 3^{n-1} 3) $n.3^{n-1}$ 4) $n.3^n$

iii) P and Q have equal number of elements is

1) ${}^{2n}C_n$ 2) $(2n)!$ 3) ${}^nC_n$ 4) $n\,!$

43. If there are 5 periods in each working day of a school, then the number of ways that you can arrange four subjects during the working days is

1) 125 2) 180 3) 220 4) 240

44. The number of 6-digit numbers that can be made with the digits 1, 2, 3 and 4 and having exactly two pairs of digits, is :

1) 480 2) 540 3) 1080 4) 1240

45. The number of four letter words can be formed from the letters of the word INFINITY so that 3 are alike, one is different is

1) ${}^8C_4$ 2) 4 3) 16 4) 8

46. The number of ways in which a mixed double game can be arranged from amongst 9 married couples if no husband and wife play in the same game is

1) 1512 2) 3024 3) 9! 4) 216

47. The number of 3 digited numbers 'abc' such that $b < c$ is

1) 450 2) 405 3) 400 4) 410

48. A box contains 5 pairs of shoes. If 4 shoes are selected, then the number of ways in which exactly one pair of shoes obtained is

1) 120 2) 140 3) 160 4) 180

49. A gentlemen hosts a party of $(m+n)$ guests and places 'm' at one round table and the remaining 'n' at the other round table. Number of ways the guests can be arranged is

1) $\frac{(m+n)!}{m \cdot n}$ 2) $\frac{(m+n)}{m \cdot n}$ 3) $\frac{(m+n)}{m! \cdot n!}$ 4) $\frac{(m+n)}{m^n}$

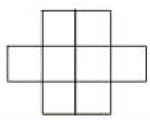
50. At an election, three wards of a town are canvassesed by 3, 4 and 5 men respectively. If 20 men volunteer, then the number ways can they be alloted to the different wards is

1) ${}^{20}C_3$ 2) ${}^{17}C_4$ 3) ${}^{13}C_5$ 4) ${}^{20}C_3.{}^{17}C_4.{}^{13}C_5$

51. The number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs exactly twice

1) ${}^7P_2.2^5$ 2) ${}^7C_2.2^5$ 3) ${}^7C_2.5^2$ 4) ${}^7C_2.10$

52. Six x's have to be placed in the squares of given figure such that each row contains atleast one x. Then the number of ways in which this can be done is



1) 28 2) 26 3) 6! 4) 8! 6!

53. The number of ways to fill each of the four cells of the table with a distinct natural numbers such that the sum of the numbers is 10 and the sums of the numbers placed diagonally are equal is :

1) 2!x2! 2) $(4!)^2$ 3) 2(4!) 4) 8

54. For any integer $n \geq 1$, the number of positive divisotrs of n is denoted by $d(n)$. Then for a prime p, $d(d(d(p^7)))$ is

1) 1 2) 2 3) 3 4) p

55. The number of ways of dividing 15 men and 15 women into 15 couples each consisting a man and a woman is

1) 1240 2) 1840 3) 1820 4) 15 !

56. The number of different sums can be formed with the following coins; a rupee, a fifty paise, a twenty five paise, a ten paise, a five paise is

1) 31 2) 32 3) 5! 4) 44

57. A question paper contains 6 questions each having an alternative. The number of ways that an examinee can answer one or more questions is

1) 243 2) 242 3) 729 4) 728

58. In a polygon, no three diagonals are concurrent. If the total no. of point of intersection of diagonals in polygon is 70, then no. of diagonals of polygon is

1) 4 2) 20 3) 12 4) 16

59. No. of ways of selecting two squares having common side in a chess board is (1 unit size squares)

1) 112 2) 124 3) 64 4) 80

60. No. of squares in a chess board is

1) 204 2) 220 4) 242 4) 300

61. In a plane there are two families of lines $y=x+c$, $y=-x+c$ where $c \in \{0,1,2,3,4\}$ the number of squares of diagonals of length 2 units formed by the lines is

1) 25 2) 16 3) 9 4) 3

62. There are 12 points in a plane of which no three points are collinear and 5 points are concyclic. The number of different circles that can be drawn through atleast 3 points of these points is

1) 120 2) 220 3) 211 4) 144

63. The maximum number of points of intersection of 4 circles and 4 straight lines is

1) 25 2) 50 3) 56 4) 72

64. The number of words that can be made by arranging the letter of the word "ROORKEE" that neither begin with R nor end with E is

1) 270 2) 330 3) 510 4) 8!

65. The number of permutations from the letters A to G so that neither the set BEG nor CAD appears is

1) $\frac{7!}{(3!)^2}$ 2) 4806 3) $\frac{7!}{(3!)^3}$ 4) 2346

66. All possible two factor products are formed from the numbers 1, 2, ...100. The number of factors out of the total obtained which are multiple of 3 is

1) 2211 2) 4950 3) 2739 4) 1540

67. A tea party is arranged for 16 people along two sides of a long table with 8 chairs on each side. Four men wish to sit on one particular side and two on the other side. Then the number of ways that they can be seated is

1) ${}^{10}C_4$ 2) 8! 3) ${}^{10}C_4.8!$ 4) ${}^{10}C_4.(8!)^2$

68. 5 balls of different colours are to be placed in 3 boxes of different sizes. Each box can hold all 5 balls. No. of different ways the balls can be placed so that no box remain empty is

1) 50 2) 150 3) 300 4) 350

69. The number of words which can be formed out of letters a, b, c, d, e, f taken 3 together, each containing one vowel atleast is

1) ${}^4P_1 \cdot {}^4P_2$ 2) 96 3) ${}^6P_3$ 4) 120

Numerical value type questions

70. There are two sets of parallel lines, their equation being $x\cos\alpha + y\sin\alpha = p$ and $x\sin\alpha - y\cos\alpha = p$; $p = 1,2,3,....n$ and $\alpha \in (0, \pi/2)$. If the number of rectangles formed by these two sets of lines is 225, then the value of n is equal to

71. The number of increasing function from $f: A \to B$ where $A \in \{a_1, a_2, a_3, a_4, a_5, a_6\}, B \in \{1,2,3,....9\}$ such that $a_{i+1} > a_i \; \forall i \in N$ and $a_i \neq i$ is

72. The number of homogeneous products of degree 3 from 4 variables is equal to

73. The number of ways of partitioning the set {a,b,c,d} into one or more non empty subset is

74. Let y be an element of the set $A = \{1,2,3,5,6,10,15,30\}$ and x_1, x_2, x_3 be integers such that $x_1x_2x_3 = y$, then the number of positive integral solutions of $x_1x_2x_3 = y$ is

75. There are 3 rows containing 2 seats in each row. The number of ways in which 3 persons can be seated such that no row remains empty is p then $\frac{p}{16} =$

LEVEL-II (ADVANCED)

Single answer type Questions

1. The number of words that can be formed by taking 4 letters at a time out of the letters of the word MATHEMATICS is

a) 2500 b) 2550 c) 2454 d) 3000

2. The number of divisors of $2^2.3^3.5^3.7^5$ of the form $4n + 1, n \in N$ is

a) 46 b) 47 c) 96 d) 94

3. The number of divisors of the number $N = 2^3.3^5.5^7.7^9$ which are perfect square is

a) 120 b) 140 c) 100 d) 5

4. The number of ways in which the number 300300 can be split into 2 factors which are relatively prime is

a) 16 b) 64 c) 32 d) 18

5. If r,s,t are prime numbers, 'p' and 'q' are two numbers whose LCM is $r^2s^4t^2$ then number of possible pairs of (p,q) are

a) 225 b) 254 c) 256 d) 248

6. Poor Dolly's T.V. has only 4 channels; all of them quite boring, hence it is surprising that she desires to switch (change) channel after every one minute. Then the number of ways in which she can the channels so that she is back to her original channel for the first time after 4 minutes is

a) 10 b) 11 c) 12 d) 8

7. A candidate is required to answer 7 out of 15 questions which are divided into three groups A, B, C each containing 4,5,6 questions respectively. He is required to select at least 2 questions from each group. He can make up his choice in

a) 1200 b) 2700 c) 2000 d) 1800

8. The number of functions defined from $f:\{1,2,3,4,5,6\}\to\{7,8,9,10\}$ such that the sum $f(1)+f(2)+f(3)+f(4)+f(5)+f(6)$ is odd, is

a) 2^{10} b) 2^{11} c) 2^{12} d) $2^{12}-1$

9. The number of ways in which 6 different balls can be put in two boxes of different sizes so that no box is empty is

a) 64 b) 62 c) 60 d) 30

10. In an examination the maximum marks for each of the three papers are 50 each. Maximum marks for the fourth paper are 100. The number of ways in which the candidate can score 60% marks in aggregate is

a) 110256 b) 110456 c) 110556 d) 110356

11. The number of non-negative integral solutions to the system of equations $x+y+z+u+t = 20$ and $x+y+z = 5$ is

a) 336 b) 346 c) 246 d) 216

12. The number of positive integral solutions of the inequality $3x+y+z\le 30$, is

a) 1115 b) 1215 c) 1315 d) 1296

13. If a, b, c are three natural numbers in A.P. such that $a+b+c=21$ then the possible number of values of a, b, c is

a) 13 b) 14 c) 15 d) 16

14. The number of ways in which the six faces of a cube be painted with six different colours is

a) 6 b) 6! c) ${}^6C_2$ d) 30

More than one correct answer type questions

15. Using the elements –3, –2, –1 0, 1, 2, 3

a) The number of 3 × 3 matrices having trace 0 is $37(7^6)$

b) The number of 3 × 3 matrices is 7^3

c) The number of 3 × 3 skew symmetric matrices is 7^3

d) The number of 3 × 3 symmetric matrices is 7^6

16. The number of ways in which three numbers in A.P. can be selected from 1,2,3,.......,n is

a) $\frac{n(n-2)}{4}$, when n is even b) $\frac{1}{4}(n-1)^2$, when n is odd

c) $\frac{n(n-2)}{2}$, when n is even d) $\frac{n(n-1)}{4}$

17. A father with 8 children takes them 3 at a time to the zoological garden, as often as he can without taking the same 3 children together more than once. Then,

a) number of times he will go is 56 b) number of times each child will go is 21

c) number of times a particular child will not go is 35 d) all of these

18. If N is the number of positive integral solutions of $x_1x_2x_3x_4$=770. Then,

a) N is divisible by 4 distinct primes b) N is a perfect square

c) N is a perfect 4th power d) N is a perfect 8th power

19. The number of non-negative integral solutions of $x_1+x_2+x_3+x_4 \le n$ (where n is a positive integer) is

a) ${}^{n+4}C_n$ b) ${}^{n+4}C_4$ c) ${}^{n+3}C_3$ d) ${}^{n+3}C_n$

20. The number of ways of distributing 10 different books among 4 students S_1,S_2, S_3, S_4 such that S_1 and S_2 get 2 books and S_3 and S_4 get 3 books each is

a) 12600 b) 25200 c) ${}^{10}C_4$ d) $\frac{10!}{2!2!3!3}$

Linked comprehension type questions

Passage - I :

There are five different boxes and seven different balls. All the seven balls are to be distributed in the five boxes placed in a row so that any box can receive any number of balls.

21. In how many ways can these balls be distributed so that no box is empty?

a) 71 b) 16800 c) 1775 d) 2160

22. Suppose, all the balls are identical, then in how many ways can all these balls be distributed into these boxes?

a) 110 b) 220 c) 330 d) 1440

23. In how many ways can these balls be distributed so that box 2 and box 4 contain only 1 and 2 balls, respectively?

a) 5522 b) 8505 c) 2305 d) 2160

Passage - II :

Let p be a prime number and n be a positive integer, then exponent of p in $n!$ is denoted by $E_p(n!)$ and is given by $E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + ... + \left[\frac{n}{p^k}\right]$ where $p^k < n < p^{k+1}$ *and* $[x]$ denotes the integral part of x. If we isolate the power of each prime contained in any number N, then N can be written as $N = 2^{\alpha_1}.3^{\alpha_2}.5^{\alpha_3}.7^{\alpha_4}......$ where α_i are whole numbers.

24. The exponent of 7 in ${}^{100}C_{50}$ is

a) 0 b) 1 c) 2 d) 3

25. The number of zeros at the end of 108! is

a) 10 b) 13 c) 25 d) 26

26. The last non zero digit in 20! must be equal to

a) 2 b) 4 c) 6 d) 8

Matrix matching type questions

27. **COLUMN - I** **COLUMN - II**

A) Number of diagonals for an octagon — p) 30

B) Number of ways in which six different toys can be given to two children so that all toys do not go the same child — q) 18

C) The number of even numbers of 4 digits that can be formed with 2,4,6,7 with no repetition of digit — r) 20

D) Number of squares of size 5×5 or more on a chess board. — s) 62

28. **COLUMN - I** **COLUMN - II**

A) The number of ways one or more balls can be selected out of 10 white, 9 green and 7 blue balls is — p) 4851

B) The number of factors (excluding 1and the number it self) of $a^7b^4c^3$ def. where a,b,c,d,e,f are prime numbers — q) 879

C) The number of natural numbers less than 10^4 in the decimal notation in which all the digits are different — r) 1278

D) The number of ordered triplets of positive integers (x,y,z) which satisfy the equation $x+y+z=100$ is — s) 5274

Integer answer type questions

29. If N is the number of ways in which a person can walk up a stair-way which has 7 steps if he can take 1 or 2 steps up the stairs at a time, then the value of $\frac{N}{3}$ is

30. A man has 3 friends. If N is number of ways he can invite one friend everyday for dinner on 6 successive nights so that no friend is invited more than 3 times then the value of $\frac{N}{170}$ is

31. If N is the number of ways in which 3 distinct numbers can be selected from the set $\{3^1, 3^2, 3^3, ...3^{10}\}$ so that they form a G.P then the value of $\frac{N}{5}$ is

32. Consider a set of 8 vectors $V = \left\{a\hat{i} + b\hat{j} + c\hat{k}; a,b,c \in \{-1,1\}\right\}$ the number of ways of choosing 3 non coplanar vectors from V is 2^p then the value of p is

33. Let $f: A \to B$ be any function where A is the set containing the positive integral solutions of the in equality $Sin^{-1}(Sin 2) > x^2 - 3x$ and B is the set of all divisors of 30. If $f(i) \le f(j) \forall i < j$ then the number of mapping from A to B is 15K then the value of K is

KEY SHEET (LECTURE SHEET)

LEVEL-I

1) **1**	2) **4**	3) **3**	4) **3**	5) **3**	6) **4**	7) **3**	8) **3**
9) **3**	10) **4**	11) **1**	12) **1**	13) **1**	14) **3**	15) **3**	16) **1**
17) **2**	18) **2**	19) **1**	20) **4**	21) **1**	22) **1**	23) **3**	24) **2**
25) **3**	26) **3**	27) **4**	28) **1**	29) **1**	30) **2**	31) **4**	32) **2**
33) **2**	34) **2**	35) **1**	36) **1**	37) **1**	38) **3**	39) **4**	40) **2**
41) **3**	42) i) **3**	ii) **3** iii) **1**	43) **4**	44) **3**	45) **3**	46) **1**	47) **2**
48) **1**	49) **1**	50) **4**	51) **2**	52) **2**	53) **4**	54) **3**	55) **4**
56) **1**	57) **4**	58) **2**	59) **1**	60) **1**	61) **3**	62) **3**	63) **2**
64) **2**	65) **2**	66) **3**	67) **4**	68) **2**	69) **2**	70) **6**	71) **28**
72) **20**	73) **15**	74) **64**	75) **3**				

LEVEL-II

1) **c**	2) **b**	3) **a**	4) **c**	5) **a**	6) **c**	7) **b**	8) **b**
9) **b**	10) **c**	11) **a**	12) **b**	13) **a**	14) **d**	15) **abcd**	16) **ab**
17) **abcd**	18) **bcd**	19) **ab**	20) **bd**	21) **b**	22) **c**	23) **d**	24) **a**
25) **c**	26) **b**	27) **A-r;B-s;C-q;D-p**			28) **A-q;B-r;C-s;D-p**		
29) **7**	30) **3**	31) **4**	32) **5**	33) **8**			

PRACTICE SHEET

LEVEL-I (MAIN)

Single answer type questions

Problems on ^nc_r :

1. If $^nP_r = 720,\ ^nC_r = 120$ then (n, r) is
   1) (7, 4) 2) (6, 2) 3) (8, 4) 4) (10, 3)

2. If $^nC_4$, $^nC_5$, $^nC_6$ are in A.P., then the value of n is
   1) 11 2) 17 3) 8 4) 14 or 7

Problems on selection of dissimilar things :

3. From a group of persons the number of ways of selecting 5 persons is equal to that of 8 persons. No. of ways of selecting 2 persons is
   1) 78 2) 92 3) 89 4) 41

4. The total number of 4 digit numbers in which the digits are in descending order is
   1) $^{10}C_4 \times 4!$ 2) $^{10}C_4$ 3) $\dfrac{10!}{4!}$ 4) 1200

5. The number of ways in which 5 players be chosen from 12 players so as to include one particular player is
   1) $^{12}C_4$ 2) $^{11}C_5$ 3) $^{12}C_5$ 4) $^{11}C_4$

6. The number of ways in which a team of eleven players can be selected from 22 players always including 2 of them and excluding 4 of them is
   1) $^{16}C_{11}$ 2) $^{16}C_5$ 3) $^{16}C_9$ 4) $^{20}C_9$

7. A committee of 12 is to be formed from 9 women and 8 men in which atleast 5 women have to be included. No. of committees in which the men are in majority, is
   1) 6002 2) 1008 3) 2702 4) 3002

8. A student is to answer 10 out of 13 questions in an examination such that he must choose atleast 4 from the first 5 questions. The number of choices available to him is
   1) 196 2) 280 3) 346 4) 140

9. A boat's crew consists of 8 men, 3 of whom can only row on one side, 2 only on the other. The number of ways the crew can be arranged is
   1) 1728 2) 14×9! 3) $^{12}C_8$ 4) $^9C_5$

Problems on divisors, Distribution of similar things in to groups :

10. No. of non-trivial divisors of 1512 is
    1) 10 2) 20 3) 30 4) 40

11. Number of even divisors of 1600 is
    1) 21 2) 18 3) 3 4) 6

12. The number of ways in which 1800 can be divided into two factors is
    1) 17 2) 18 3) 36 4) 34

13. The number of all three element subsets of set $\{a_1, a_2, a_3, \ldots\ldots, a_n\}$ which contain a_3 is

1) ${}^{n}C_3$ 2) ${}^{n+1}C_3$ 3) ${}^{n-1}C_2$ 4) 2^n

14. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is

1) 5 2) ${}^{8}C_3$ 3) 3^8 4) 21

15. Number of ways of allotting 30 marks to 10 questions so that each question to get atleast two marks is

1) ${}^{19}C_6$ 2) ${}^{19}C_{10}$ 3) ${}^{30}C_{10}$ 4) ${}^{30}C_{15}$

16. Total number of ways of selecting six coins out of 20 one rupee coins, 10 fifty paise coins and 7 twenty five paise coins only is

1) ${}^{8}C_6$ 2) ${}^{12}C_6$ 3) ${}^{24}C_6$ 4) ${}^{32}C_6$

Geometrical Applications:

17. If a polygon of n sides has 275 diagonals, then n is equal to

1) 25 2) 35 3) 20 4) 15

18. The polygon in which no. of diagonals is twice the no. of sides is

1) Hexagon 2) Heptagon (or) Septagon

3) Octagon 4) Decagon

19. In a plane there are 10 points, no three are in same straight line except 4 points which are collinear, then

i) the number of straight lines are

1) 39 2) 41 3) 45 4) 40

ii) the number of triangles formed are

1) 120 2) 20 3) 116 4) 119

20. Let L_1 and L_2 be two lines intersecting at P. If A_1, B_1, C_1 are points on L_1; A_2, B_2, C_2, D_2, E_2 are points on L_2 and if none of these coincides with P, then the number of triangles formed by these 8 points is

1) 56 2) 55 3) 46 4) 45

21. There are 10 parallel lines intersected by a family of 5 parallel lines. The number of parallelograms thus formed in the net work is

1) 225 2) 450 3) 730 4) 600

22. The number of triangles whose vertices are at vertices of an octagon but none of whose sides happen to come from the sides of the octagon is

1) 24 2) 52 3) 48 4) 16

Miscellaneous problems :

23. Two women and some men participated in a chess tournament in which very every paticipant played two games with each of the other partcipants. It the number of games than the men played between themselves exceeds the number of games that the men played with the women by 66, then the number of men who participated in the tournament lies in the interval

1) [8, 9] 2) [10, 12) 3) (11, 13] 4) (14, 17)

24. If n is an integer between 0 and 25, then the minimum value of $n!\ (25-n)!$ is

1) $(12!)$ 2) $12!\ 13!$ 3) $(13!)^2$ 4) $25!$

25. Let $n = 1! + 4! + 7! + ... + 400!$. Then ten's digit of n is

1) 1 2) 6 3) 2 4) 7

26. Let $a_n = \frac{10^n}{n!}$ for $n = 1,2,3,$ .... Then the greatest value of n for which a_n is the greatest is

1) 11 2) 20 3) 10 4) 8

27. In a club election the number of contestants is one more than the number of maximum candidates for which a voter can vote. If the total number of ways in which a voter can vote be 62, then the number of candidates is

1) 5 2) 6 3) 7 4) 8

28. Total number of 6 digited numbers in which all the odd digits and only odd digits appear is

1) $\frac{5}{2}\angle 6$ 2) $\angle 6$ 3) $\frac{1}{2}\angle 6$ 4) $\frac{5}{2}\angle 5$

29. A three digit number n is such that the last two digits of it are equal and different from the first. The number of such n's is

1) 64 2) 72 3) 81 4) 900

30. They are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

1) 36 2) 66 3) 108 4) 3

31. The number of subsets of the set $A = \{a_1, a_2, ..., a_n\}$ which contain even number of elements is

1) 2^{n-1} 2) 2^n-1 3) 2^{n-2} 4) 2^n

32. From a company of 20 soldiers any 5 are placed on guard, each batch to watch 5 hours. The total number of hours watched by different batches selected is

1) ${}^{20}C_5$ 2) ${}^{20}P_5$ 3) ${}^{20}C_5 \times 5$ 4) ${}^{20}P_5 \times 5$

33. A binary sequence is an array of 0's and 1's.The nunmber of n-digit binary sequences" which contain even number of 0s' is

1) 2^{n-1} 2) 2^n-1 3) $2^{n-1}-1$ 4) 2^n

34. There are two bags each of which contains 'n' balls. A man has to select an equal number of balls from both the bags. The number of ways in which a man can choose at least one ball from each bag is

1) ${}^{2n}C_n$ 2) $({}^{n}C_n)^2$ 3) ${}^{2n}C_1$ 4) ${}^{2n}C_n - 1$

35. The number of ways in which 52 cards be formed into 4 groups of 13 cards each is

1) $\frac{52!}{(13!)^4 4!}$ 2) $\frac{52!}{(13!)^5}$ 3) $\frac{52!}{13!}$ 4) ${}^{52}C_{13}$

36. The number of ways in which 13 gold coins can be distributed among three persons such that each one gets at least 2 gold coins is

1) 36 2) 24 3) 12 4) 6

37. The maximum number of points of intersection of 8 circles is

1) 16 2) 24 3) 28 4) 56

38. The number of rectangles that can be found on a chess board is

1) ${}^8C_2.{}^8C_2$ 2) ${}^{64}C_4$ 3) ${}^9C_2.{}^9C_2$ 4) ${}^8C_2$

39. p points are chosen on each of the three copolanar lime. The maximum number of triangles formed with vertices at these points is

1) p^3+3p^2 2) $\frac{1}{2}(p^3+p)$ 3) $\frac{p^3}{2}(5p\text{-}3)$ 4) $p^2(4p-3)$

40. In a packet there are m different books, n different pens and p different pencils. No. of selections of atleast one article from the packet is

1) $(m+1)(n+1)(p+1)-1$ 2) $2^{m+n+p}-1$ 3) 2^{m+n+p} 4) $(m+1)(n+p+1)$

41. If N denotes the set of all positive integers and if $f: N \to N$ is defined by $f(n)$= the sum of positive divisors of n, then $f(2^k.3)$, where k is a positive integer, is

1) $2^{k+1}-1$ 2) $2(2^{k+1}-1)$ 3) $3(2^{k+1}-1)$ 4) $4(2^{k+1}-1)$

42. The set $S=\{1,2,3,......12\}$ is to be partitioned into three sets A, B, C of equal size. Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = C \cap A = \phi$. The number of ways to partition is

1) $\frac{12!}{(4!)^3}$ 2) $\frac{12!}{3!(3!)4}$ 3) $\frac{12!}{(4!)^3 3!}$ 4) $\frac{12!}{(3!)^4}$

43. The number of ways of selecting 10 balls out of an unlimited number of white, red, green and blue balls is

1) 236 2) 256 3) 276 4) 286

LEVEL-II (ADVANCED)

Single answer type questions

1. For $2 \le r \le n$, $\binom{n}{r}+2\binom{n}{r-1}+\binom{n}{r-2}=$

a) $\binom{n+1}{r-1}$ b) $\binom{n+1}{r+1}$ c) $\binom{n-2}{r}$ d) $\binom{n+2}{r}$

2. Out of 16 players of a cricket team, 4 are bowlers and 2 are wicket keepers. A team of 11 players is to be chosen so as to contain at least 3 bowlers and at least 1 wicket-keeper. The number of ways in which the team be selected is

a) 2400 b) 2472 c) 2500 d) 960

3. If $a,b,c \in N$, the number of points having position vector $a\hat{i}+b\hat{j}+c\hat{k}$ such that $6 \le a+b+c \le 10$ is

a) 110 b) 116 c) 120 d) 227

4. The number of proper divisors of $\underline{|15}$ is

a) 4032 b) 4030 c) 4031 d) None

5. Which of the following statements is/are false?

a) $10^9 - {}^{10}c_1.9^9 + {}^{10}c_2.8^9 - +...{}^{10}c_9.1^9$ is a positive integer.

b) the number of surjective mapping from '5' element set to 4 element set is coefficient of x^5 in $\underline{|5}\left(e^5-1\right)^4$.

c) when two identical ordinary dice are rolled simultaneously the number of possible outcomes is 21

d) the number of ways of painting 6 faces ofa cube with 6 different colours is $\underline{|6}$

6. The number of ways of selecting 3 squares of the same size from a chess board where squares of the same size are considered to be identical is

a) 4 b) 6 c) 7 d) 9

7. The total number of integral solutions for x, y, z such that $xyz = 24$, is

a) 30 b) 60 c) 90 d) 120

8. The number of 5 digit number of the form abcba in which $a < b$ is

a) 320 b) 340 c) 360 d) 380

9. In an examination there are three multiple choice questions and each question has 4 choices of answers in which only one correct. the total number of ways in which an examinee can fail to get all answers correct is

a) 11 b) 12 c) 27 d) 63

10. Number of ways in which Rs. 18 can be distributed amongst four persons such that no body receives less than Rs. 3 is

a) 42 b) 24 c) 4! d) 84

11. If a,b,c,d are odd natural numbers such that $a + b + c + d = 20$, then the number of values of the ordered quadruplet (a,b,c,d) is

a) 165 b) 310 c) 295 d) 398

12. An ordinary cubical dice having six faces marked with alphabets A,B,C,D,E and F is thrown n times and the list of n alphabets showing up are noted, then the total number of ways in which among the alphabets A,B,C,D,E and F only three of them always appear in the list is

a) ${}^6C_3(3^n-{}^3C_2(2^n-2)-3)$ b) ${}^6C_3(3^n+{}^3C_2(2^n-2)-3)$

c) ${}^6C_3(3^n-{}^3C_2(2^n-2)+3)$ d) none of these

13. The number of different seven digit numbers that can be written using only the three digits 1,2 and 3 with the condition that the digit 2 occures twice in each number is

a) ${}^7P_2.2^5$ b) ${}^7C_2.2^5$ c) ${}^7C_2.5^2$ d) none of these

14. The number of functions f from the set $A=\{0,1,2\}$ into the set $A=\{0,1,2,3,4,5,6,7\}$ such that $f(i) \le f(j)$ for $i < j$ and $i,\ j \in A$ is

a) ${}^8C_3$ b) ${}^8C_3+2({}^8C_2)$ c) ${}^{10}C_3$ d) ${}^8C_3+{}^{10}C_3$

15. $A = \{a_1, a_2, a_3, a_4, a_5\}$, $B\{b_1, b_2, b_3, b_4, b_5\}$ The number of one-one functions from A to B such that $f(a_i) \neq b_i$ for $i = 1, 2, 3, 4, 5$ and $f(a_1) = b_2$

a) 11 b) 14 c) 24 d) 18

16. The number of arrangements of the letters of the word ORACLE such that neither O follows A, nor A follows E, nor E follows O is ___

a) 432 b) 426 c) 438 d) None

17. A fruit basket contains 4 oranges, 5 apples and 6 mangoes. The number of ways a person make selection of fruits from among the fruits in the basket is

a) 2100 b) 209 c) 208 d) 210

18. Given 5 different green dyes, four different blue dyes and three different red dyes. The number of combinations of dyes which can be chosen taking at least one green and one blue dye is

a) 3600 b) 3720 c) 3800 d) 3660

19. There is a rectangular sheet of dimension $(2m-1)\times(2n-1)$, (where $m > 0$, $n > 0$). It has been divided into square of unit area by drawing lines perpendicular to the sides. Find number of rectangles having sides of odd unit length.

a) $(m+n+1)^2$ b) $mn(m+1)(n+1)$ c) 4^{m+n-2} d) m^2n^2

20. $PQRS$ is a quadrilateral having 3,4,5,6 points on PQ, QR, RS and SP respectively. The number of triangles with vertices on different sides is

a) 220 b) 270 c) 282 d) 342

More than one correct answer type questions

21. If ${}^{n}C_{r-1} = (k^2 - 8)\,({}^{n+1}C_r)$, then k belongs to

a) $[-3, -2\sqrt{2}\,]$ b) $[-3, -2\sqrt{2}\,)$ c) $[2\sqrt{2}, 3]$ d) $(2\sqrt{2}, 3]$

22. If $10! = 2^p 3^q 5^r 7^s$ then

a) $p + q = 12$ b) $qr = 8$ c) $p+q+r+s = 15$ d) $r - s = 3$

23. A forecast is to be made of the results of 5 cricket matches, each of which can be a win or a draw or a loss for a team A. Let

$\alpha =$ number of forecasts with exactly 1 error; $\beta =$ number of forecasts with exactly 3 errors

$\gamma =$ number of forecasts with all five errors

then

a) $2\beta = 5\gamma$ b) $8\alpha = \beta$ c) $16\alpha = 5\gamma$ d) $\alpha + \beta > 2\gamma$

24. The number ways of distributing 'n' distinct games among 'n' children so that exactly one child does not get any game, is

a) ${}^{n}C_{n-1}n^{n-1}$ b) ${}^{n}C_2\underline{\left| n \right.}$ c) $\dfrac{\underline{\left| n \right.}}{\underline{\left| 2 \right.}(\underline{\left| 1 \right.})^{n-2}\underline{\left| n-2 \right.}} \times \underline{\left| n-1 \right.}$

d) ${}^{n}c_1((n-1)^n - {}^{n-1}c_1(n-2)^n + c_2(n-3)^n + (-1)^{n-2}\,{}^{n-1}c_{n-2})$

Linked comprehension type questions

Passage - I :

Inclusion and exclusion principle is useful in many problems involving permutations and combinations. With usual notations

$$n(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 \cap \overline{A}_4 \ldots\ldots \cap \overline{A}_n) = n(s) - \sum n(A_i) + \sum_{i} \sum_{<\, j} n(A_i \cap A_j) - \sum_{i<} \sum_{j<} \sum_{k} n(A_i \cap A_j \cap A_k) + \ldots\ldots$$

is called inclusion and exclusion principle

25. Total number of words that can be formed using all letters of the word 'BRIJESH' that neither begins with 'I' nor ends with 'B' is equal to

a) 3600 b) 3720 c) 4800 d) 4920

26. The number of permutations from the letters A to G so that neither the set "BEG" nor "CAD" appear

a) 3600 b) 3720 c) 4806 d) 4906

27. If a number is selected at random from {1, 2, 3....50} then the number of numbers which are divisible by none of 2, 3, 5 is

a) 2 b) 4 c) 12 d) 20

Passage - II :

Let $f(n)$ denotes the numebr of different ways the positive integer n can be expressed as the sum of 1's and 2's. For example $f(4) = 5$

i.e, 4=1+1+1+1=1+1+2 =1+2+1=2+1+1=2+2

28. The value of $f(6)$ is

a) 10 b) 13 c) 16 d) 19

29. The value of $f(f(6))$ is

a) 356 b) 377 c) 389 d) 427

30. The number of solutions of the equation $f(n) = n$, where $n \in N$ is

a) 1 b) 2 c) 3 d) 4

Matrix matching type questions

31.

COLUMN - I	COLUMN - II
A) 300 boys and 100 girls were selected in JEE - 2007 from class-room programme. It was decided to take a picture of the whole students in such a way that all students to stand in a row, with boys standing in decreasing order according to their height (assuming all has distinct height) from left to right and girls standing in increasing order according to their height (assuming all has distinct height) from left to right. In how may ways this can be done (The boys need not stand together and girls need to stands together)	P) ${}^{400}C_{300}$
B) 54 maths faculty were called for meeting. After long conversation, it was decided to split them into 5 groups of 8 and 2 groups of 7 to continue meeting. Number of ways of doing it is	q) ${}^{20}C_{10} - 2$
C) A, B are two disjoint sets, each containing 10 elements. If $P \subset A$, $Q \subset B$, $P \neq \phi$ and $Q \neq \phi$ then the number of ordered pairs (P, Q) so that P, Q contain same numer of elements is	r) 54.2^{53}
D) How many committees with a chairman can be chosen from a set of 54 persons ?	s) $\dfrac{54!}{2!(8!)^5(7!)^2.5!}$

Integer answer type questions

32. If N is the number of rational numbers between 0 and 1 whose digits after the decimal point are non zero and are in the decreasing order then the sum of digits of the number N is

33. Let n = 2005. The least positive integer k for which $k(n^2)(n^2-1^2)(n^2-2^2)(n^2-3^2)..........(n^2-(n-1)^2) = r!$ for some positive integer r is

34. A student is allowed to select at most n books from a collection of $(2n+1)$ books. If the total number of ways in which he can select books is 63, the value of n is

35. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2,5,and 7. The smallest value of n for which this is possible is

KEY SHEET (PRACTICE SHEET)

LEVEL-I

1) 4	2) 4	3) 1	4) 2	5) 4	6) 3	7) 2	8) 1
9) 1	10) 3	11) 2	12) 2	13) 3	14) 4	15) 2	16) 1
17) 1	18) 2	19) i) 4	ii) 3	20) 4	21) 2	22) 4	23) 2
24) 2	25) 2	26) 3	27) 2	28) 1	29) 3	30) 3	31) 1
32) 3	33) 3	34) 4	35) 1	36) 1	37) 4	38) 3	39) 4
40) 2	41) 4	42) 1	43) 4				

LEVEL-II

1) d	2) b	3) a	4) b	5) a	6) c	7) c	8) c
9) d	10) d	11) a	12) a	13) b	14) c	15) a	16) a
17) b	18) b	19) d	20) d	21) bd	22) abc	23) abcd	24) bd
25) b	26) c	27) b	28) b	29) b	30) c	31) A-p;B-s;C-q;D-r	
32) 7	33) 2	34) 3	35) 7				

ADDITIONAL EXERCISE

LECTURE SHEET (ADVANCED)

Single answer type questions

1. How many words can be made with the letters of the word 'GENIUS' if each word neither begins with 'G' nor ends in 'S', is

a) 540 b) 504 c) 720 d) 702

2. Six persons A,B,C,D,E and F are to be seated at a circular table. The number of ways this can be done if A must have either B or C on his right and B must have either C or D on his right, is

a) 12 b) 18 c) 24 d) 36

3. The number of word of four letters containing equal number of vowels and consonants, where repetition is allowed, is

a) 210×243 b) 105×243 c) $(105)^2$ d) 150×21^2

4. In the decimal system of numeration of six digit numbers in which the sum of the digits is divisible by 5 is

a) 180000 b) 5×10^5 c) 540000 d) None of these

5. The number of rational numbers lying in the interval (2002,2003) all of whose digits after the decimal point are non-zero and are in decreasing order is

a) $\sum_{i=1}^{9} {}^9P_i$ b) 2^9 c) 2^9-1 d) $2^{10}-1$

6. A delegation of four friends are to be selected from a group of 12 friends. The number of ways the delegation be selected if two particular friends refused to be together and two other particular friends wish to be together only in the delegation.

a) 226 b) 114 c) 156 d) 170

7. The number of positive integral divisors of 1200 which are multiples of '6' is

a) 6 b) 12 c) 8 d) 24

8. The number of four letter words that can be formed from the letters of the word "IIT JEE"

a) 96 b) 102 c) 84 d) 66

More than one correct answer type questions

9. A box contains two white, three black and four red balls. The number of ways in which we can select three balls from the box, if at least one black ball is to be included in the selection, is

a) 2^6 b) 2^5 c) 2^7-2^6 d) none of these

10. The number of ways to select 2 numbers x and y from {0,1,2,3,4} such that the sum of the squares of the selected numbers is divisible by 5 are (repitition of digits is allowed)

a) ${}^9C_1$ b) ${}^9C_8$ c) 9 d) 7

11. Consider seven digit number $x_1, x_2, ...x_7$, where $x_1, x_2, ...x_7 \neq 0$ having the property that x_4 is the greatest digit and digits towards the left and right of x_4 are in decreasing order. Then total number of such numbers in which all digits are distinct is

a) ${}^9C_7.{}^6C_3$ b) ${}^9C_6.{}^5C_3$ c) ${}^{10}C_7.{}^6C_3$ d) ${}^9C_2.{}^6C_3$

12. Sanjay has 10 friends among whom two are married to each other. She wishes to invite 5 of the them for a party. If the married couple refuse to attend separately, then the number of different ways in which she can invite five friends is

a) ${}^8C_5$ b) $2\times{}^8C_3$ c) ${}^{10}C_5-2\times{}^8C_4$ d) none of these

13. The number of ways in which we can arrange the 2n letters $x_1,x_2,......,x_n$, $y_1,y_2,......,y_n$ in a line so that the suffixes of letters x and those of the letters y are respectively in ascending order of magnitude is

a) $({}^nC_0)^2+({}^nC_1)^2+.......+({}^nC_n)^2$ b) ${}^{2n}C_n$

c) $2^n\,[1.3.5....(2n-1)]/n!$ d) ${}^{2n}C_n-1$

14. A is a set containing n elements. A subset P_1 of A is chosen. The set A is reconstructed by replacing the elements of P_1. Next a subset P_2 of A is chosen and again the set is reconstructed by replacing the elements of P_2. In this way $m(>1)$ subsets $P_1, P_2, ...P_m$ so that $P_i \cap P_j = \phi$ for $i \neq j$, is

a) $(m+1)^n$ b) $2^m - {}^mC_n$ c) $\sum_{k=0}^{n} {}^mC_k m^k$ d) $(2m-1)^n$

15. Let $a_n = \frac{10^n}{n!}$ for $n \geq 1$. Then a_n take the greatest value when n is equal to

a) 9 b) 10 c) 11 d) 12

16. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$. Among all the functions from A to B, the number of functions f such that

a) $f(i) < f(i)$ whenever $i < j$, is 35 b) $f(i) \leq f(j)$ whenever $i < j$, is 84

c) $f(i) > f(j)$ whenever $i < j$ is 35 d) none of these

Linked comprehension type questions

Passage - I :

Suppose a lot of n objects contains n_1 objects of one kind, n_2 objects of second kind, n_3 objects of third kind,....., n_k objects of kth kind. Such that $n_1 + n_2 + n_3 + ... n_k = n$, then the number of possible arrangements/permutations of r objects out of this lot is the coefficient of x^r in the expansion of $r!\prod\left(\sum_{\lambda=0}^{n_1}\frac{x^\lambda}{\lambda!}\right)$ on the basis of above infromation, answer the following questions:

17. The number of permutations of the letters of the word INDIA taken three at a time must be

a) 27 b) 30 c) 33 d) 57

18. If $n_1 = n_2 = n_3 = ... n_k = 1$, then the number of permutations of r objects must be

a) ${}^nP_r$ b) ${}^nC_r$ c) ${}^kP_r$ d) ${}^kC_r$

19. The number of permutations of the letters of the word INEFFECTIVE taken four at a time must be

a) 2214 b) 1422 c) 5425 d) 2454

Passage - II :

10 digit numbers are formed by using all the digits 0,1,2,3,4,5,6,7,8 and 9 such that they are divisible by 11111.

20. The digit in the tens place, in the smallest of such numbers, is

a) 9 b) 8 c) 7 d) 6

21. The digit in the units place, in the greatest of such numbers, is

a) 4 b) 3 c) 2 d) 1

Integer answer type questions

22. The number of ways of distrbuting 8 identical balls in 3 boxes, so that no box contains more than 3 balls is

23. Consider $S=\{1,2,3,4,.......10\}$. Then sum of all products of numbers by taking two or more from S is $(11!-k)$ than $\left[\frac{k}{11}\right]$ where [ ] is $G.I.F$ is

24. The number of 5-digited numbers that can be made the sum of whose digits is even is $2^a\ 3^b\ 5^c$ then $a + b + c$ equals.

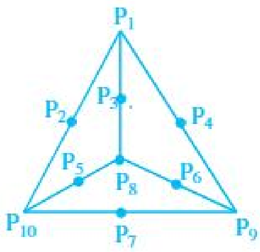
25. If a, b, c are positive two digit numbers 2 and 0 are digits in the units place of b and c and $D=\begin{vmatrix} a & 5 & 2 \\ b & 0 & 1 \\ c & 1 & 0 \end{vmatrix}$ such that $D + 2$ is divided by 10, Then the digit in the units place of a is

26. Let $f(n)=\sum_{r=0}^{n}\sum_{k=r}^{n}\binom{k}{r}$. The total number of divisors of $f(9)$ is.

PRACTICE SHEET (ADVANCED)

Single answer type questions

1. As shown in the diagram, points $p_1, p_2, p_3.....p_{10}$ are either the vertices or the midpoints of the edges of a tetrahedron respectively. Then the number of groups of four points



(p_1, p_i, p_j, p_k) $(1 < i < j < k \le 10)$ on the same plane is.

a) 30 b) 33 c) 36 d) 39

2. The number of polynomials of the form $x^3 + ax^2 + bx + c$ that are divisible by $x^2 + 1$, where $a,b,c \in \{1,2,3,.......10\}$, are

a) 10^3 b) 10 c) 9 d) 90

3. The number of points having position vector $a\hat{i}+b\hat{j}+c\hat{k}$ where $a, b, c \in \{1,2,3,4,5\}$ such that $2^a + 3^b + 5^c$ is divisible by 4 is

a) 70 b) 140 c) 210 d) 280

4. The number of ways of selecting 3-element subsets of the set $\{1,2,3,....,25\}$ such that the elements form a G.P. with integer common ratio is ___

a) 10 b) 11 c) 12 d) 14

5. The total number of ways in which a begger can be given at least one rupee from four 25paise coins, three 50 paise coins and 2 onerupee coins, is

a) 54 b) 50 c) 52 d) none of these

6. There are 20 boys seated around a circle. The number of way to select 4 boys such that there must be odd number of boys left over between any two consecutively selected boys, is equal to

a) 360 b) 420 c) 840 d) 1680

7. If 'x' be the number of ways in which 7 walls in a row can be coloured using 5 different given colours (no wall is supposed to be coloured with more than one colour and all 5 colours are to be used) and 'y' be the number of ways that 7 students may group themselves into 5 groups to use 5 copies of the same book in a college library, then $\frac{x}{y}$ is equal to

a) 24 b) 30 c) 60 d) 120

8. Six cards six envelopes are numbers 1,2,3,4,5,6 and cards are to be placed in the envelope so that each envelope contain exactly one card and no card is placed in the envelope baring the same number more over the number one is placed always in the envelope numbered two. Then the number of ways it can be done is

a) 53 b) 106 c) 159 d) 265

More than one correct answer type questions

9. All the five digit numbers in which each successive digit exceeds its predecessor are arranged in the increasing order. The 105th number does not contain the digit.

a) 1 b) 3 c) 4 d) 5

10. The number of triangles formed by the vertices of a decagon such that. each triangle and decagon have.

a) exactly one side is in common, is 60

b) exactly two sides is in common, is '10'

c) exactly three sides is in common, is zero

d) at least one side is in common, is 70

11. Let $f(n)$ be the number of regions in which 'n' coplanar circles can divide the plane. If it is known that each pair of circles intersect in two different points and no three of them have common point of intersection, then

a) $f(20) = 382$ b) $f(n)$ is always an even number

c) $f^{-1}(92) = 10$ d) $f(n)$ can be odd

12. If a,b are chosen from first five natural numbers then the no.of ordered pair (a,b) such that

a) $ax^2 + bx + 1 = 0$ has non real roots is 13

b) $ax^4 + bx^3 + (a + 1)x^2 + bx + 1$ is positive for all $x \in R$ is 13

c) $ax^3 + bx^2 + x + 1$ is increasing $\forall x \in R$ is 12

d) $ax^3 + 3bx^2 + 3x + 3$ has exactly one local minimum and one local maximum is 18

13. Kumar has ten friends among whom two are married to each other. He wishes to invite five of them for a party. If the married couple refuse to attend separately, then the no. of different ways in which he can invite five friends is

a) ${}^8C_5$ b) $2.{}^8C_3$ c) ${}^{10}C_5 - 2.{}^8C_4$ d) ${}^{10}C_5$

14. The number of selections of four letters taken from the word COLLEGE must be

a) 22 b) 18

c) coeff. of x^4 in the expansion of $(1 + x + x^2)^2(1 + x)^3$

d) coeff. of in the expansion of $(1 + x + x^2)^3(1 + x)^2$

15. Let be n numbers each of which is either 1 or –1.
If $a_1a_2a_3a_4 + a_2a_3a_4a_5 + ...+ a_na_1a_2a_3 = 0$, then

a) n must be even b) n must be even but not divisible by 4

c) n must be divisible by 4 d) n has to be odd.

16. The number of shortest paths in which one can reach from the point (0,0,0) to the point (3,7,11) in 3-dimentional space where the movement is possible only along the +ve x-axis, +ve y-axis and +ve z-axis or parallel to them and change of axes is permitted only at integral points (An integral point is one which has its coordinates as integers) is

a) ${}^{21}c_3\ {}^{18}c_7$ b) ${}^{21}c_{11}\ {}^{10}c_3$

c) $\dfrac{\underline{|14}}{(2)^2\underline{|4}\underline{|5}}$ if the travel is through (1,2,4) d)

Linked comprehension type questions

Passage - I :

For $n \in N$, let $\sigma(n)$ denote the sum of all divisors of n. A number is said to be perfect is $\sigma(n) = 2n$.

17. If $n \in N$ and $\{d_1, d_2, ...d_k\}$ is the set of all its divisors, then $\sum_{j=1}^{k}\frac{1}{d_j}$ is equal to

a) $\sigma(n)$ b) $n\ \sigma(n)$ c) $\sigma(n)/n$ d) none of these

18. If p is a prime, then $(p - 1)\sigma(p^n)$ equals

a) p^{n+1} b) $p^{n+1}-1$ c) $p^{n+1}+1$ d) none of these

19. If p, q are two distinct primes, then $\sigma(pq)$ equals

a) $\sigma(p)+\sigma(p)$ b) $\sigma(p)\ \sigma(p)$ c) $q\sigma(p)+p\sigma(p)$ d) none of these

Integer answer type questions

20. The number of positive integer pairs (x, y) such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{2007}, x<y$ is

21. Let $x=\{1,2,3,4\}$ the no.of one – one functions $f : x \to x$ satisfying $f(f(i))=i, \forall 1 \le i \le 4$ is $2K$ then $K =$

22. Let $A(1,2)$, $B(4,6)$, $C(9,10)$ are 3 points in xy – plane starting from A line segments of unit length are drawn either right wards (or) upwards only in each step until C is reached. Then the no.of ways of connecting A and C through B is afour digited number then its units place digit is

23. If p,q are randomly chosen from the set $\{1,2,3,4,5,6\}$ with replacement, then the number of ways in which the expression $x^4 + px^3 + (q + 1)x^2 + px + q$ will have positive values for all real x is two digited number then its units place digit is

24. If n is number of ways in which 15 identical blankets can be distributed among six beggars such that everyone gets at least one blanket and two particular beggars get equal blankets and another three particular beggars get equal blankets, then the value of n/2 is

25. The number of monotonically increasing functions from the set $\{1,2,3,...6\}$ to itself with the property that $f(x) \geq x \forall x \in \{1,2,3,...6\}$ is equal to $\frac{2}{k} \cdot {}^{11}c_5$ where k is equal to ______

26. 'A' & 'B' are two students among 100 students appearing for a competitive exam. If all 100 students are given ranks the number of ways in which 'A' will be better ranked than B is $50\underline{|k}$ where $\frac{k}{11}$ is equal to ______

KEY SHEET (ADDITIONAL EXERCISE)

LECTURE SHEET (ADVANCED)

1) **b**	2) **b**	3) **d**	4) **a**	5) **c**	6) **a**	7) **b**	8) **b**	9) **ac**	10) **abc**
11) **ad**	12) **bc**	13) **abc**	14) **ac**	15) **ab**	16) **abc**	17) **c**	18) **a**	19) **b**	20) **d**
21) **a**	22) **3**	23) **5**	24) **9**	25) **6**	26) **8**				

PRACTICE SHEET (ADVANCED)

1) **b**	2) **b**	3) **a**	4) **a**	5) **a**	6) **b**	7) **d**	8) **a**	9) **abcd**	
10) **abcd**	11) **abc**	12) **abcd**	13) **bc**	14) **bc**	15) **ac**	16) **abc**	17) **c**	18) **b**	19) **b**
20) **7**	21) **5**	22) **0**	23) **7**	24) **6**	25) **7**	26) **9**			