3. PROBABILITY

SYNOPSIS

Important terminology, Classical definitionof probability, Addition theorem on probability

1. **Random Experiment :** A repetable experiment the results of which are known in advance but exact result will be known only after the experiment is completed is known as random experiment.
   Ex : Tossing a coin, rolling a die, drawing a pack of cards etc.

2. **Outcome and trial :** The result of a random experiment is known as an outcome and conduct of a random experiment is known as a trial.

3. **Simple event :** An event having only a single sample point is called a simple event.
   When a coin is tossed and we denote.
   $E_1\{H\}$ = the event of occurrence of head.
   E_2 $\{T\}$ = the event of occurrence of head.
   E_1,E_2 are simple events of occurrence of tail.

4. **Equally likely Events :** A set of events is said to be equally likely if there is no reason to expect one of them in preference to the others.

5. **Mutually Exclusive Events :** A set of events is said to be mutually exclusive if happening of one of them prevents the happening of any of the remaining events.

6. **Exhaustive Events :** The list of all possible outcomes is said to be exhaustive if necessarily one of them must happen.

7. **Classical Definition of Probability :** If there are ***'n'*** mutually exclusive, equally likely, exhaustive elementary events of an experiment and ***'m'*** of them are favourable to an event A then, the probability of A denoted by $P(A)$ is defined as ***m/n***.
   i) The limits of probability are 0 and 1.
   ii) If $P(A) = 0$ then, A is known as an impossible event.
   iii) If $P(A) = 1$ then, A is known as a certain event.

8. **Odds in favour and odds against an Event :** Suppose A is any event of an experiment. The odds in favour of event A is $P(A) : P(\overline{A})$, The odds against A is $P(\overline{A}) : P(A)$

9. If $P(A) : P(\overline{A}) = m : n$ then, $P(A) = \dfrac{m}{m+n}, P(\bar{A}) = \dfrac{n}{m+n}$

10. **Von Mises Statistical definition of Probability (or) Relative frequency approach to probability:**
    If a random experiment is conducted 'n' times and an event 'A' happens 'm' times then the ratio m/n is called the relative frequency of the event 'A'.
    The probability $P(A)$ of the event A is defined by $P(A) = \underset{n\to\alpha}{Lt}\dfrac{m}{n}$

11. Sample space :

The set of all possible outcomes of an experiment is called the sample space whenever the experiment is conducted and is denoted by S.

12. Event :

Any subset of the sample space 'S' is called an Event.

13. Axiomatic Approach to Probability :

Let S be finite sample space. A real valued function P from powerset of S into R is called probability function if

1) $P(A) \geq 0 \ \forall A \subseteq S$

2) $P(S) = 1, P(\phi) = 0$

3) $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \phi$

Here the image of A w.r.t P denoted by $P(A)$ is called probability of A

i) $P(A) + P(\overline{A}) = 1$

ii) If $A_1 \subseteq A_2$ then $P(A_1) \leq P(A_2)$ where A_1, A_2 are any two events.

14. Addition theorem on Probability :

i) If A, B are any two events in a sample space S, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

ii) If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

iii) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

15. If A and B are two events then

i) P(both A, B occurs) $= P(A \cap B)$,

ii) P(at least one of A, B occurs) $= P(A \cup B)$

iii) P(none of A, B occurs) $= P(\overline{A} \cap \overline{B})$

iv) P(exactly one of A, B occurs) $= P(A \cap \overline{B}) + P(B \cap \overline{A}) = P(A) + P(B) - 2P(A \cap B)$

(or) $P(A \cup B) - P(A \cap B)$

16. If A, B and C are three events then

i) P(at least one of A,B,C occurs) $= P(A \cup B \cup C)$

ii) P (at two A,B,C occur) $= P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C) + P(A \cap B \cap C)$

$= P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$

iii) P (exacty two of A,B,C occurs $= P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C)$

$= P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$

iv) P(exactly one of A,B,C occurs) $= P(A \cap \overline{B} \cap \overline{C}) + P(\overline{A} \cap B \cap \overline{C}) + P(\overline{A} \cap \overline{B} \cap C)$

$= P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(C \cap A) + 3P(A \cap B \cap C)$

Conditional probability, Multiplication theorem on probability, Total probability and Baye's theorem

17. *Conditional Probability :*

If A and B are two events in sample space and $P(A) \neq 0$. The probability of B after the event A has occured is called the conditional probability of B given A and is denoted by

$$P(B/A) = \frac{n(A \cap B)}{n(A)} = \frac{P(A \cap B)}{P(A)} \text{ Similarly } P(A/B)$$

$$= \frac{n(A \cap B)}{n(B)} = \frac{P(A \cap B)}{P(B)}$$

18. *Multiplication Theorem :*

If A and B are any two events in S then $P(A \cap B) = P(A)\ P(B/A)$ if $P(A) \neq 0$.

$= P(B)\ P(A/B)$ if $P(B) \neq 0$

19. *Independent events :*

Two events A and B of an experiment are said to be independent if occurence of A cannot influence the happening of the event B. i.e A, B are independent if i) $P(A/B) = P(A)$ and $P(B/A) = P(B)$.

i.e. $P(A \cap B) = P(A)\ .\ P(B)$

20. If A and B are independent $\Leftrightarrow$ (1) $\overline{A}$ & B are independent (2) A & $\overline{B}$ are independent (3) $\overline{A}$ & $\overline{B}$ are independent

21. i) A set of events A_1, A_2, A_3 .... A_n are said to be pairwise independent if

$P(A_i \cap A_j) = P(A_i)\ P(A_j)$ for all $i \neq j$

ii) A set of events A_1, A_2, A_3 .... A_n are said to be mutually independent if

$P(A_i \cap A_j) = P(A_i)P(A_j)$ for all $i \neq j$ and $P(A_1 \cap A_2 \cap \ldots\ldots \cap A_n) = P(A_1)P(A_2)\ldots\ldots P(A_n)$

22. *Total Probability theorem :*

Let S be the sample space and Let $E_1, E_2, \ldots.E_r$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or .....or E_n

then $P(A) = P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + \ldots\ldots + P(E_n).\ P(A/E_n)$

$$P(A) = \sum_{k=1}^{n} P(E_K)P(A/E_K), k = 1,2,3,\ldots\ldots n$$

23. *Baye's Theorem :*

Let S be the sample space and Let $E_1, E_2, \ldots.E_r$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or .....or E_n

then $$P\left(\frac{E_K}{A}\right) = \frac{P(E_K).P\left(\frac{A}{E_K}\right)}{\sum_{k=1}^{n} P(E_K).P\left(\frac{A}{E_K}\right)}, K = 1,2,3,4\ldots\ldots n$$

24. *Geometrical Probability :*

We observe that the definition of the probability of occurance of an event fails if the total number out comes of a trial in a random experiment is infinite

In such cases, the definition of probability is modified and extended to what is called geometrical probability or probability in continum. In such cases the proballity P of occurence of an event is given

by $P = \dfrac{\text{measure of the specified part of the region}}{\text{measure of the whole region}}$

Where measure stands for length, area or volume depending upon whether S is one, two or 3-dimension region.

Eg:- If any point in the closed interval $[\alpha,\beta]$ is a sample point and any point in the closed interval $[a,b] \subseteq [\alpha,\beta]$ be a favarable point for the event E. then the probability of the event E is

$$P(E) = \frac{\text{length of } [a,b]}{\text{length of } [\alpha,\beta]}$$

Some important results to be remembered :

1. Number of exhaustive cases of tossing n coins simultaneously (or tossing a coin n times) $=$ total outcomes $= 2^n$.
2. Number of exhaustive cases of throwing n dice simultaneously (or throwing one die n times) $=$ total outcomes $= 6^n$.
3. If a coin is tossed n times then the probability of getting n heads and $(n-x)$ tails $= \dfrac{{}^nC_x}{2^n}$
4. If there are n children in a family then probability that there are exactly x boys (girls) $= \dfrac{{}^nC_x}{2^n}$
5. If an unbiased coin is tossed n times, then probability of getting odd number of heads (tails)$=\dfrac{1}{2}$
6. If a coin is tossed n times, probability of getting atleast one head (tail) $= 1 - \dfrac{1}{2^n}$ or $\dfrac{2^n - 1}{2^n}$
7. When two fair dice are rolled once, then number of favourable cases to the sum of the digits on the faces may be given by

Sum	2	3	4	5	6	7	8	9	10	11	12
No. of Favourable cases	1	2	3	4	5	6	5	4	3	2	1

8. When three fair dice are rolled once, then number of favourable cases to the sum of the digits on the faces may be given by

Sum	3	4	5	6	7	8	9
No. of Favourable cases	${}^2C_2$	${}^3C_2$	${}^4C_2$	${}^5C_2$	${}^6C_2$	${}^7C_2$	${}^8C_2-3$

Sum	10	11	12	13	14	15	16	17	18
No. of Favourable cases	${}^9C_2-9$	${}^9C_2-9$	${}^8C_2-3$	${}^7C_2$	${}^6C_2$	${}^5C_2$	${}^4C_2$	${}^3C_2$	${}^2C_2$

9. When n fair dice are rolled once the number of favourable cases to get the sum r is coefficient of x^r in the multinomial expansion of $(x^1 + x^2 + x^3 + x^4 + x^5 + x^6)^n$

10. If $r\ (1 \le r \le 7)$ squares are selected at random from a chess board, then the probability that they lie on a diagonal $= \dfrac{4({}^7C_r + {}^6C_r + {}^5C_r + {}^4C_r + ... + {}^rC_r) + 2({}^8C_r)}{{}^{64}C_r}$

11. Out of n pairs of shoes if $r(<n)$ shoes are selected at random then the probability

i) there is no pair $= \dfrac{{}^nC_r \cdot 2^r}{{}^{2n}C_r}$

ii) there is atleast one pair $= 1 - \dfrac{{}^nC_r \cdot 2^r}{{}^{2n}C_r}$

iii) there are exactly k pairs $= \dfrac{{}^nC_k \cdot {}^{n-k}C_{r-2k}\, 2^{r-2k}}{{}^{2n}C_r}$

12. ***Dearrangement Problems***

If n letters corresponding to n envelopes with address on them are placed in the envelopes one in each, then

i) probability that all the letters are placed in right envelopes $= \dfrac{1}{n!}$

ii) probability that all letters are not in right envelopes (or atleast one letter is placed in wrongly addressed envelope) $= 1 - \dfrac{1}{n!}$

iii) Probability that all letters go into wrongly addressed envelopes $= \left[\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - ..... + \dfrac{(-1)^n}{n!}\right]$

iv) Probability that exactly r letters are in right envelops $= \dfrac{1}{r!}\left(\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - .... + (-1)^{n-r}\dfrac{1}{(n-r)!}\right)$

13. If A and B are two finite sets and if a mapping is selected at random from the set of mapping from A to B then, the probability that the mapping is a

i) one-one function $= \dfrac{^{n(B)}P_{n(A)}}{[n(B)]^{n(A)}}$

ii) many one function $= 1 - \dfrac{^{n(B)}P_{n(A)}}{[n(B)]^{n(A)}}$

iii) constant function $= \dfrac{n(B)}{[n(B)]^{n(A)}}$

iv) bijection $= \dfrac{[n(A)]!}{[n(B)]^{n(A)}}$ if $n(A) = n(B)$

v) on to function $= \dfrac{^{n}C_0 n^m - ^{n}C_1(n-1)^m + ^{n}C_2(n-2)^m - ^{n}C_3(n-3)^m + ... + (-1)^{n-1}\, ^{n}C_{n-1}(1)^m}{n^m}$ where $n(A) = m,\ n(B) = n,\ m \geq n$

14. Out of $(2n+1)$ tickets consecutively numbered three are selected at random. Then the probability that they are in A.P. $= \dfrac{3n}{4n^2-1}$

15. If n men among whom A and B stand in a row (line), then the probability that there will be exactly r persons between A and $B = \dfrac{2(n-r-1)}{n(n-1)}$

16. If n men among whom A and B sit along a circle then the probability that there will be exactly r persons between A and $B = \dfrac{2}{n-1}, \left(r \neq \dfrac{n-2}{2}\right)$. (in both directions)

$$= \frac{1}{n-1}, \left(r = \frac{n-2}{2}\right)$$

17. If n men among whom A and B sit along a circle then the probability that A and B sit together $= \dfrac{2}{n-1}$.

18. If n men among whom A and B sit along a circle then the probability that A and B never sit together $= \dfrac{n-3}{n-1}$.

19. If n men among whom A and B sit along a circle then the odds against A and B sitting together $= n-3 : 2$.

20. If n whole numbers are taken at random and multiplied together then the probability that the last digit of the product is

i) either 1, 3, 7 or 9 is $\dfrac{4^n}{10^n} = (2/5)^n$

ii) either 2, 4, 6 or 8 is $\dfrac{8^n - 4^n}{10^n} = \dfrac{4^n - 2^n}{5^n}$

iii) 5 is $\dfrac{5^n - 4^n}{10^n}$

iv) 0 is $\dfrac{10^n - 8^n - 5^n + 4^n}{10^n}$

21. If A has m shares in a lottery where there are m prizes and n ($>m$) blanks then, probability of A's winning $= 1 - \dfrac{{}^nC_m}{{}^{m+n}C_m}$

22. Using the vertices of a polygon having n sides a triangle is constructed at random. The probability that the triangle so formed is such that no side of the polygon is side of the triangle is $\dfrac{(n-4)(n-5)}{(n-1)(n-2)}$

23. Out of n persons sitting at a round table, three persons are selected at random then the probability that no two of them are consecutive is $\dfrac{(n-4)(n-5)}{(n-1)(n-2)}$.

24. E and F are two mutually exclusive events and the random experiment is repeated till E or F occurs. Then probability that E occurs before F is $\dfrac{P(E)}{[P(E)+P(F)]}$.

25. ***Two person game :*** (infinite G.P model) If p and q are the probabilities of success and failure of a game in which A and B play and if A starts the game then,

i) Probability of A's win $= \dfrac{p}{1-q^2} = \dfrac{1}{1+q}$

ii) Probability of B's win $= \dfrac{qp}{1-q^2} = \dfrac{q}{1+q}$

iii) The ratio of their success $= 1:q$

iv) If p_A and q_A are the probabilities of success and failures of A and p_B and q_B are the probabilities of success and failures of B and if A starts the game then, probability of A's win $= \dfrac{p_A}{1-q_A q_B}$

26. ***Three person game :***

If p and q are the probability of success and failure of a game in which A, B and C play in order, then

i) Probability of A's win $= \dfrac{p}{1-q^3} = \dfrac{1}{1+q+q^2}$

ii) Probability of B's win $= \dfrac{qp}{1-q^3} = \dfrac{q}{1+q+q^2}$

iii) Probability of C's win $= \dfrac{q^2p}{1-q^3} = \dfrac{q^2}{1+q+q^2}$

iv) The ratio of their success $= 1:q:q^2$

LECTURE SHEET

EXERCISE-I

Classical definition and Additonal theorem

LEVEL-I (MAIN)

Problems on Classical Definition :

1. The probability that a leap year contains 53 Mondays and 53 Tuesdays is

   1) $\frac{1}{7}$ 2) $\frac{2}{7}$ 3) $\frac{3}{7}$ 4) $\frac{5}{7}$

2. Six coins are tossed simultaneously. The probability of getting atleast 4 heads is

   1) $\frac{11}{64}$ 2) $\frac{11}{32}$ 3) $\frac{15}{44}$ 4) $\frac{21}{32}$

3. An unbiased coin is tossed five times. The odds in favour of getting atleast one tail is

   1) 1:31 2) 31:1 3) 31:32 4) 1:32

4. A coin whose faces are marked 3 and 5 is tossed 4 times. The odds against the sum of the numbers thrown being less than 15 are

   1) 11:5 2) 5:11 3) 11:16 4) 5:16

5. It two unbiased dice are rolled then the probability of getting a prime score is

   1) $\frac{5}{12}$ 2) $\frac{5}{6}$ 3) $\frac{5}{36}$ 4) $\frac{2}{5}$

6. Six faces of a die are marked with numbers 1, –1, 0, –2, 2, 3 and the die is thrown thrice. The probability that the sum of the numbers thrown is six, is

   1) $\frac{3}{216}$ 2) $\frac{6}{216}$ 3) $\frac{10}{216}$ 4) $\frac{18}{216}$

7. Two dice one green and the other red are rolled and seperate scores recorded. The probability that the scores on the dice differ by not more than 2 is :

   1) $\frac{2}{3}$ 2) $\frac{1}{2}$ 3) $\frac{5}{18}$ 4) $\frac{1}{6}$

8. A die is loaded such that 1 turning upwards is 2 times as often as 6 and 3 times as any other face (2 or 3 or 4 or 5). The probability that we get a face with 6 when we throw such a die is :

   1) $\frac{6}{17}$ 2) $\frac{2}{17}$ 3) $\frac{4}{17}$ 4) $\frac{3}{17}$

9. Three six faced dice are tossed together, then the probability that exactly two of the three numbers are equal is

   1) $\frac{165}{216}$ 2) $\frac{177}{216}$ 3) $\frac{51}{216}$ 4) $\frac{90}{216}$

10. Three faces of a fair die are yellow, two faces red and one face is blue. The die is tossed 3 times. The probability that the colours yellow, red, blue appear is (need not in order)

    1) $\frac{1}{36}$ 2) $\frac{1}{6}$ 3) $\frac{5}{6}$ 4) $\frac{1}{2}$

11. The coefficients b and c of the equation $x^2 + bx + c = 0$ are determined by throwing an ordinary die. The probability that the equation has equal roots is

1) $\frac{1}{18}$ 2) $\frac{13}{18}$ 3) $\frac{5}{18}$ 4) $\frac{1}{9}$

12. An unbiased die is rolled 4 times. Out of 4 face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5 is

1) $\frac{16}{81}$ 2) $\frac{1}{81}$ 3) $\frac{80}{81}$ 4) $\frac{65}{81}$

13. If 5 cards are drawn from a pack, then the probability of selecting the cards of which four of them have same face value is

1) $\frac{{}^{13}C_1 \times {}^{48}C_1}{{}^{52}C_5}$ 2) $\frac{{}^{13}C_4 \times {}^{39}C_1}{{}^{52}C_5}$ 3) $\frac{{}^{13}C_1 \times {}^{39}C_4}{{}^{52}C_5}$ 4) $\frac{{}^{13}C_1}{{}^{52}C_5}$

14. If 3 cards are drawn from a pack of 52 cards at random, then the probability of getting 2 cards from one suit and one card from another suit is

1) $\frac{{}^{4}C_2 \times {}^{13}C_2 \times {}^{13}C_1}{{}^{52}C_3}$ 2) $\frac{{}^{4}P_2 \times {}^{13}C_2 \times {}^{13}C_1}{{}^{52}C_3}$ 3) $\frac{{}^{13}C_2 \times {}^{13}C_1}{{}^{52}C_3}$ 4) $\frac{{}^{13}C_2 \times {}^{13}C_1}{{}^{52}C_3}$

15. A bag contains 50 tickets numbered 1, 2, 3...., 50 of which five are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_3 = 30$ is

1) $\frac{{}^{20}C_2}{{}^{50}C_5}$ 2) $\frac{{}^{29}C_2}{{}^{50}C_5}$ 3) $\frac{{}^{20}C_2 \times {}^{29}C_2}{{}^{50}C_5}$ 4) $\frac{{}^{20}C_2}{{}^{45}C_2}$

16. In a bag there are infinitely many red, white and black balls which are identical. If Ten balls are selected at random then the probability that selection includes atleast one ball from each colour is

1) $\frac{5}{11}$ 2) $\frac{6}{11}$ 3) $\frac{7}{11}$ 4) $\frac{4}{11}$

17. The letters of the word "QUESTION" are arranged in a row at random. The probability that there are exactly 2 letters between Q and S is

1) $\frac{5}{28}$ 2) $\frac{1}{7}$ 3) $\frac{3}{28}$ 4) $\frac{1}{14}$

18. The probability that in a random arrangement of letters of the word "COLLEGE" so that neither 2E's nor 2L's come together

1) $\frac{18}{21}$ 2) $\frac{20}{21}$ 3) $\frac{11}{21}$ 4) $\frac{17}{21}$

19. Five digited numbers are formed using {0,2,4, 5, 7} without repetetion. One number is selected at random. The probability that it is divisible by 5 is

1) $\frac{3}{16}$ 2) $\frac{5}{16}$ 3) $\frac{7}{16}$ 4) $\frac{9}{16}$

20. A 4 digit number made of digits 1, 2, 3, 4, 5 is written down at random without repetition. The probability that the number so formed is divisible by 6 is

1) 1/20 2) 1/10 3) 3/20 4) 3/10

21. Ten boys are arranged at random along a circle. The probability that 2 specified boys of those ten must be separated by exactly 3 boys in any direction is

1) $\frac{1}{9}$ 2) $\frac{2}{9}$ 3) $\frac{1}{3}$ 4) $\frac{5}{9}$

22. Out of 10 persons sitting at a round table, two persons are selected at random then the probability that they are not adjacent to each other is

1) $\frac{5}{12}$ 2) $\frac{7}{10}$ 3) $\frac{5}{7}$ 4) $\frac{7}{9}$

23. Two persons *A, B* have to speak at a function with 10 other persons. If the persons speak at random order, the probability that *A* speaks immediately before *B* is

1) $\frac{1}{12}$ 2) $\frac{1}{3}$ 3) $\frac{3}{8}$ 4) $\frac{5}{6}$

24. *S* is a set containing *n* elements. If two subsets *A* and *B* of *S* picked at random from the set of all subsets of *S*. Then the probability that *A* and *B* have no common element.

1) $\frac{1}{2^n}$ 2) $(2/3)^n$ 3) $(3/4)^n$ 4) $(4/5)^n$

25. If 6 letters are placed at random in 6 addressed envelopes. Then the odds in favour of arranging them such that no letter goes into correct envelope is

1) 53 : 91 2) 91 : 53 3) 97 : 64 4) 64 : 97

26. A bag contains $(2n+1)$ coins. It is known that 'n' of these have a head on both sides where as the remaining $(n+1)$ coins are fair. A coin is picked up at radom from the bag and tossed. If the probability that the toss results in a head is $\frac{31}{42}$, then value of 'n' is

1) 10 2) 8 3) 6 4) 25

27. If 4 different biscuits are distributed among 3 children then the probability of receiving atleast one biscuit by the 1st child is

1) $\frac{2^4}{3^4}$ 2) $1-\frac{2^4}{3^4}$ 3) $\frac{1}{3}$ 4) $\frac{1}{2}$

28. If 10 identical coins are distributed among 4 children at random. The probability of distributing so that each child gets atleast one coin is

1) $\frac{12}{143}$ 2) $\frac{42}{143}$ 3) $\frac{17}{143}$ 4) $\frac{101}{143}$

29. The probability that in a group of *n* people, atleast two of them will have the same date of birth of a non leap year (Assuming a year is having 365 days)

1) $1-\frac{{}^{365}P_n}{(365)^n}$ 2) $\frac{{}^{365}P_n}{(365)^n}$ 3) $\frac{1}{(365)^n}$ 4) $\frac{365\times 364}{(365)^n}$

30. Four persons entered the lift cabin on the ground floor of a 5-floor house (Assume ground floor as also one floor). Assume that each of them independently and with equal probability can leave the cabin at any floor beginning from the first. Find the probability for all the four persons to leave the cabin at different floors

1) 3/32 2) 1/256 3) 1/1024 4) 5/1024

31. If 4 squares are chosen at random on a chess board, the probability that they lie on a diagonal line is

1) $\dfrac{4\sum_{n=4}^{8} {}^{n}C_4}{{}^{64}C_4}$ 2) $\dfrac{2\sum_{n=4}^{8} {}^{n}C_4}{{}^{64}C_4}$ 3) $\dfrac{2\sum_{n=4}^{7} {}^{n}C_4 + {}^{8}C_4}{{}^{64}C_4}$ 4) $\dfrac{4\sum_{n=4}^{7} {}^{n}C_4 + 2({}^{8}C_4)}{{}^{64}C_4}$

32. Three of the six vertices of a regular hexagon are chosen at random. The Probability that the triangle with these vertices is equilateral is

1) $\frac{1}{5}$ 2) $\frac{2}{5}$ 3) $\frac{1}{10}$ 4) $\frac{1}{20}$

33. From first 20 natural numbers if two numbers are selcted at random then the probability of selecting them which are not consecutive is

1) $\frac{9}{10}$ 2) $\frac{19}{20}$ 3) $\frac{1}{10}$ 4) $\frac{1}{5}$

34. Using the vertices of a polygon having 12 sides a triangle is constructed at random. The probability that the triangle so formed is such that no side of the polygon is side of the triangle is

1) $\frac{18}{55}$ 2) $\frac{28}{55}$ 3) $\frac{17}{55}$ 4) $\frac{7}{55}$

35. In a set of lottery Tickets 7 carry prizes and 25 are blank. If three tickets are drawn then the probability to get a prize is

1) $\frac{{}^{7}C_3}{{}^{32}C_3}$ 2) $\frac{{}^{25}C_3}{{}^{32}C_3}$ 3) $1-\frac{{}^{25}C_3}{{}^{32}C_3}$ 4) cannot be decided

36. A boy forgets the last two digits of his friend's telephone number. He however remembers that they are different numbers. If he dials at random, the probability that he dials correctly is

1) $\frac{1}{100}$ 2) $\frac{1}{90}$ 3) $\frac{1}{10}$ 4) $\frac{8}{9}$

37. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is

1) $\frac{3}{5}$ 2) $\frac{1}{5}$ 3) $\frac{2}{5}$ 4) $\frac{4}{5}$

38. Out of (2n+1) tickets consecutively numbered, three are drawn at random. The chance that the numbers on them are in A.P. is

1) $\frac{n}{n^2-1}$ 2) $\frac{3n}{n^2-1}$ 3) $\frac{3n}{4n^2-1}$ 4) $\frac{3n}{4n^2+2n-1}$

39. Let x be a non-zero real number. A determinant is chosen from the set of all determinants of order 2 with entries x or $-x$ only. The probability that the value of the determinant is non-zero is

1) $\frac{3}{16}$ 2) $\frac{1}{4}$ 3) $\frac{1}{2}$ 4) $\frac{1}{3}$

40. Two players A and B each toss 10 coins. The probability that they show equal number of heads is

1) $\frac{{}^{20}C_{10}}{2^{20}}$ 2) $\frac{1}{2^{10}}$ 3) $\frac{1}{2^{20}}$ 4) $\frac{1}{2^{9}}$

41. From the set of numbers {2, 3, 4, ..., 30} a number is selected at random. If it is a composite number it is divided by 5 other wise it is divided by 3. The probability that the remainder is zero is

1) $\frac{6}{29}$ 2) $\frac{5}{29}$ 3) $\frac{4}{29}$ 4) $\frac{7}{29}$

42. Two friends A and B have equal number of sons. There are 3 cinema tickets which are to be distributed among the sons of A and B. The probability that all the tickets go to the sons of B is 1/20. The no. of sons each of them having is

1) 2 2) 4 3) 5 4) 3

43. If a is an integer and $a \in (-5,30]$ then the probability that the graph of the function $y = x^2 + 2(a+4)x - 5a + 64$ is strictly above the x-axis is

1) $\frac{1}{5}$ 2) $\frac{8}{25}$ 3) $\frac{8}{35}$ 4) $\frac{27}{35}$

44. A cubical die is loaded so that the probability of face K is proportional to K, K = 1, 2, 3, 4, 5, 6. It is rolled. Find the probability of getting an odd integer face

1) 1/7 2) 4/7 3) 3/7 4) 2/7

45. In constructing a problem on vectors, the three components of a vector are randomly choosen from the digits 0 to 5 with replacement. The probability that the magnitude of vector is 5 is

1) 1/6 2) 1/12 3) 1/24 4) 1/30

46. S is a sample space. $S = \{x \in \mathrm{N} : 1 < x \leq 100\}$ and $E = \{x : (x+1)(x-1) \in S\}$. Then $P(E) =$

1) $\frac{1}{10}$ 2) $\frac{2}{25}$ 3) $\frac{99}{100}$ 4) $\frac{1}{11}$

47. A natural number is chosen at random from the first 100 natural numbers. The probability that $x + \frac{100}{x} > 50$ is

1) $\frac{1}{10}$ 2) $\frac{11}{50}$ 3) $\frac{11}{20}$ 4) $\frac{9}{10}$

48. A point is taken at random from inside of the circumcircle of an equilateral triangle. The probability that it lies inside the circumcircle but outside the incircle is

1) 1/4 2) 3/4 3) 1/2 4) 1/3

Addition Theorem :

49. If $P(A \cup B) = \frac{3}{4}$ and $P(\bar{A}) = \frac{2}{3}$, then $P(\bar{A} \cap B) =$

1) 5/12 2) 7/12 3) 1/12 4) 1/2

50. If A,B,C are mutually exclusive and exhaustive events such that $P(A) = 2P(B) = 3P(C)$ then $P(B \cup C) =$

1) $\frac{6}{11}$ 2) $\frac{5}{11}$ 3) $\frac{4}{11}$ 4) $\frac{7}{11}$

51. Let A, B, C be three events such that $P(A)=0.3$, $P(B)=0.4$, $P(C)=0.8$, $P(A \cap B)=0.18$, $P(A \cap C) = 0.28$, $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) > 0.75$, then

1) $0.13 < P(B \cap C) \leq 0.38$ 2) $0.23 < P(B \cap C) < 0.78$

3) $0.48 < P(B \cap C) < 0.75$ 4) None

52. If $P(A) = 0.7$, $P(B) = 0.4$ then the interval in which $P(A \cap B)$ lies is

1) [0.1, 0.4] 2) [0.1, 0.6] 3) [0, 0.4] 4) [0, 0.8]

53. Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}, P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$ The set of possible values of x are in the interval

1) $\left[\frac{1}{2}, \frac{2}{3}\right]$ 2) $\left[\frac{1}{3}, \frac{13}{3}\right]$ 3) $[0, 1]$ 4) $\left[\frac{1}{3}, \frac{1}{2}\right]$

54. If A and B are two mutually exclusive events then

1) $P(A) \le P(\bar{B})$ 2) $P(A) > P(\bar{B})$ 3) $P(A) < P(B)$ 4) $P(A) \le P(B)$

55. One hundred students appeared for two examinations 60 passed in first, 50 passed the second and 30 passed both. The probability that a student selected at random has failed in both examinations is

1) $\frac{1}{5}$ 2) $\frac{4}{5}$ 3) $\frac{3}{5}$ 4) $\frac{1}{7}$

56. If one ticket is randomly selected from tickets numbered 1 to 30 then the probability that the number on the ticket is a multiple of 5 or 7

1) 1/3 2) 1/5 3) 5/12 4) 1/6

57. A card is drawn at random from a pack of cards. The probability that the card is either a face card (Jack, Queen, King) or a six is

1) 5/32 2) 4/13 3) 1/13 4) 1/4

58. The probability that a boy will get a scholarship is 0.9 and that a girl will get is 0.8 independently. The probability that at least one of them will get the scholarship is

1) 0.98 2) 0.89 3) 0.43 4) 0.34

59. If 3 dice are rolled then the probability of getting different numbers or sum 16 is

1) $\frac{7}{12}$ 2) $\frac{4}{9}$ 3) $\frac{2}{9}$ 4) $\frac{1}{9}$

60. In a class there are 10 men and 20 women. Out of them half the number of men and half the number of women have brown eyes. Out of them if a person is chosen at random the probability for the person chosen to be a man or a brown eyed person is :

1) 1/3 2) 1/15 3) 2/3 4) 2/5

61. An electric bulb will last 190 days or more with a probability 0.7 and it will last for atmost 200 days with a probability of 0.8. The probability that the bulb will last between 190 and 200 days is

1) 0.5 2) 0.56 3) 0.2 4) 0.3

62. A and B seek admission in I.I.T. The probability that A is selected is 0.5 and the probability that both A and B are selected is atmost 0.3. The probability of B getting selected is atmost is

1) 0.5 2) 0.6 3) 0.7 4) 0.8

63. One hundred tickets are numbered as 00, 01, 02, ..., 09, ..., 99 and one ticket is drawn at random from them. If A is the event of getting 9 as the sum of the numbers on the ticket and B is the event of getting 0 as the product of the numbers on the ticket then $P(A \cap B) =$

1) $\frac{1}{100}$ 2) $\frac{2}{100}$ 3) $\frac{10}{100}$ 4) $\frac{19}{100}$

64. Two numbers X and Y are choosen at random from the set {1,2, ... , 3n}. The probability that X^2-y^2 is divisible by '3' is

1) $\frac{5n-3}{3(3n-1)}$ 2) $\frac{5n+3}{3(3n-1)}$ 3) $\frac{5n-3}{3(3n+1)}$ 4) $\frac{5n}{3n+1}$

65. A determinant is choosen at random from the set of all determinants of order '2' with elemens 0 or 1 only. The probability that the determinant is positive is

1) $\frac{3}{16}$ 2) $\frac{3}{8}$ 3) $\frac{5}{8}$ 4) $\frac{7}{8}$

Numerical value type questions

66. Three numbers are chosen at random from the first 20 natural numbers. Then the probability that their product is odd is

67. The probability of an event A occuring is 0.5 and of B occuring is 0.3. If A and B are mutually exclusive events then the probability of neither A nor B occuring is

68. The odd against an event are 5 to 2 and the odds in favour of another disjoint event are 3 to 5. Then the probability that atleast one of the events will happen is

69. The probability that in a year of 22nd century chosen at random, there will be 53 Sundays, is

70. If three numbers are selected from the set of the first 20 natural numbers, the probability that they are in GP, is

71. Three dice are thrown.. The probability of getting a sum which is a perfect square, is

72. A quadratic equation is choosen from the set of all quadratic equations which are unchanged by squaring their roots. The chance that the chosen equation has equal roots, is

73. Three digit numbers are formed using the digits 0,1,2,3,4,5 without repetition of digits. If a number is chosen at random, then the probability that the digits either increase or decrease, is

74. If four vertices of a regular octagon are chsosen at random, then the probability that the quadrilatertal formed by them is a rectangle is

75. If a and b are randomly chosen from the set {1,2,3,4,5,6,7,8,9}, then the probability that the expression $ax^4+bx^3+(a+1)x^2+bx+1$ has positive values of all real values of x is

76. A word of at least 5 letteers is made at random from 3 volwels and 3 consonants, all the letters being different. The probability that no consonant falls betweeen any two vowels in the word is

77. 10 different books and 2 different pens are given to 3 boys so that each gets equal number of things. The probability that the same boy does not receive both the pens is

LEVEL-II (ADVANCED)

Single answer type questions

1. An elevator starts with 7 persons and stops at 9 floors. Then the probability that exactly two persons alight in one floor and the rest in different floors is

a) $\frac{320\times 7^2}{9^5}$ b) $\frac{21\times {}^8C_5}{9^7}$ c) $\frac{21\times {}^8P_5}{9^7}$ d) $\frac{160\times 7^2}{95}$

2. If three integers are chosen at random from the set {1, 2, 3,...., 25} then the probability that they are 3 consecutive numbers of an A.P is

a) $\frac{75}{574}$ b) $\frac{75}{2499}$ c) $\frac{36}{575}$ d) $\frac{36}{2479}$

3. If two numbers x,y are selected at random from the set {1, 2, 3... 65} then the probability that x^4-y^4 is divisible by 5 is

a) $\frac{33}{40}$ b) $\frac{27}{40}$ c) $\frac{29}{40}$ d) $\frac{31}{40}$

4. A person throws two dices, one the common cube and the other a regular tetrahedron, the number on the lower face being taken in the case of the tetrahedron. Then the chance that the sum of the numbers thrown is not less than 5 is

a) $\frac{3}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{2}{3}$

5. There are 17 railway stations in a railway track between A and B. The train stops at 4 intermediate stations at random. Then the probability that atleast two of these 4 stations are consecutive is

a) $\frac{197}{340}$ b) $\frac{227}{340}$ c) $\frac{143}{340}$ d) $\frac{113}{340}$

6. An ordinary cube has 3 blank faces, other 3 faces are marked with numbers 1,2,3. Then the probability of getting 12 in 5 throws is

a) $\frac{45}{6^5}$ b) $\frac{40}{6^5}$ c) $\frac{25}{6^5}$ d) $\frac{50}{6^5}$

7. A die is cast such that it is twice more likely to show an even number than to show an odd number. The dice is thrown 3 times. Then the probability that the sum on them is 14 is

a) $\frac{5}{72}$ b) $\frac{22}{243}$ c) $\frac{11}{243}$ d) $\frac{11}{72}$

8. Two dice are thrown until a 6 is obtained on at least one of them. The probability that a 6 appears at the kth throw for the first time, must be

a) $\left(\frac{5}{6}\right)^{k-1}\frac{11}{36}$ b) $\left(\frac{25}{36}\right)^{k-1}\frac{11}{36}$ c) $\left(\frac{25}{36}\right)^{k-1}\frac{1}{6}$ d) None of these

9. A purse contain 4 one rupee coins and 6 ten paise coins. If 5 coins are selected at random, then the probability that the sum on them exceeds Rs.2.25 is

a) $\frac{11}{42}$ b) $\frac{23}{42}$ c) $\frac{31}{42}$ d) $\frac{13}{42}$

10. Six persons stand at random in a queue for buying cinema ticket individually. Three of them have only a five rupee note each while each of the other three has a ten rupee note only. The booking clerk has an empty cash box. The the probability that the six persons will get tickets so that each paying rupees five is

a) $\frac{3}{4}$ b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) $\frac{3}{5}$

11. A 2n digit number starts with 2 and all its digits are prime, then the probability that the sum of all two consecutive digits of the number is prime is

a) 2×2^{-3n} b) 4×2^{-3n} c) 2^{-3n} d) None of these

12. Sum of digits of a randomly chosen five digit number is 41. The probability that such a number is divisible by 11 is

a) $\frac{2}{15}$ b) $\frac{11}{36}$ c) $\frac{3}{35}$ d) $\frac{6}{35}$

13. Shalini though of a two-digit number and divided the number by the sum of the digits of the number. She found that the remainder is 3. Jaya also through of a two-digit number and divided the number by the sum of the digits of the number. She also found that the remainder is 3. The probability that and jaya through of the same number is

a) $\frac{1}{15}$ b) $\frac{1}{14}$ c) $\frac{1}{7}$ d) $\frac{1}{9}$

14. A bag contains 10 balls numbered from 0 to 9. The balls are such that the person picking a ball out of bag is equally likely to pick anyone of them. A person picked a ball and replaced it in the bag after noting its number. He repeated this process 2 more times. What is the probability that the ball picked first is numbered higher than the ball picked second and the ball picked second is numbered higher than the ball picked third?

a) $\frac{28}{243}$ b) $\frac{1}{6}$ c) $\frac{4}{5}$ d) $\frac{3}{25}$

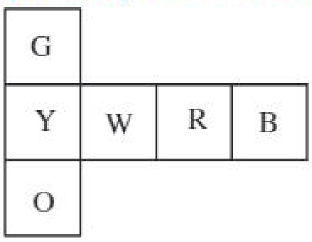
15. Mr. A forgot to write down a very important phone number. All he remembers is that it started with 713 and that the next set of 4 digit involved are 1,7 and 9 with one of these numbers appearing twice. He guesses a phone number and dials randomly. The odds in favour of dialing the correct telephone number, is

a) 1:35 b) 1:71 c) 1:23 d) 1:36

16. If a,b and c are three numbers(not necessarily different) chosen randomly and with replacement from the set $\{1,2,3,4,5\}$, the probability that $(ab + c)$ is even, is

a) $\frac{35}{125}$ b) $\frac{59}{125}$ c) $\frac{64}{125}$ d) $\frac{75}{125}$

17. A butterfly randomly lands on one of the six squares of the T-shaped figure shown and then randomly moves to an adjacent square. The probability that the butterfly ends up on the R square is



a) 1/4 b) 1/3 c) 1/5 d) 2/5

More than one correct answer type questions

18. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Which of the following relations are true ?

a) $p+m+c=\frac{19}{20}$ b) $p+m+c=\frac{27}{20}$ c) $pmc=\frac{1}{10}$ d) $pmc=\frac{1}{4}$

19. If A and B are two events that $P(A\cup B)\geq\frac{3}{4}$ and $\frac{1}{8}\leq P(A\cap B)\leq\frac{3}{8}$ then

a) $P(A)+P(B)\leq\frac{11}{8}$ b) $P(A).P(B)\leq\frac{3}{8}$

c) $P(A)+P(B)\geq\frac{7}{8}$ d) none of these

20. If A and B are two events such that $P(A)$ = 1/2 and $P(B)$ = 2/3, then

a) $P(A\cup B)\geq\frac{2}{3}$ b) $P(A\cap B^{1})\geq\frac{1}{3}$ c) $\frac{1}{6}\leq P(A\cap B)\leq\frac{1}{2}$ d) $\frac{1}{6}\leq P(A^{1}\cap B)\leq\frac{1}{2}$

21. Two numbers are chosen from $\{1,2,3,4,5,6,7,8\}$one after another without replacement. Then the probability that

a) the smaller value of two is less than 3 is 13/28

b) the bigger value of two is more than 5 is 9/14

c) product of two numbers is even is 11/14 d) none of these

22. If 3 number are chosen at random from $\{1,2,3,....,20\}$, then the probability that

a) They form A.P. $=\frac{3}{38}$ b) Their sum even $=\frac{1}{2}$

c) Their product is odd $=\frac{2}{19}$

d) They form A.P with odd common difference $=\frac{5}{114}$

23. Cards are drawn one by one in a pack of well shuffled cards replacement until all the cards are drawn then

a) Chance of getting a spade at the 13^{th} trail is $\frac{1}{13}$

b) Chance of getting a spade at the 13^{th} trail is $\frac{1}{4}$

c) Chance of getting last card as a spade is $\frac{1}{4}$

d) Chance of getting king at the 5^{th} trail and queen at the 10^{th} trail is $\frac{4}{663}$

Linked comprehension type questions

Passage - I :

A set A contains 10 elements A subset E of A is selected at random and after noting the elements they are replaced. Again a subset F of A is selected at random.

24. Probability that E and F have no common elements is

a) $\frac{{}^{20}C_{10}}{4^{10}}$ b) $\frac{1}{2^{10}}$ c) $\frac{10\times 3^8}{4^9}$ d) $\frac{3^{10}}{4^{10}}$

25. Probability that E and F have equal number of elements is

a) $\frac{{}^{20}C_{10}}{4^{10}}$ b) $\frac{10\times 3^8}{4^9}$ c) $\frac{1}{2^{10}}$ d) $\frac{3^{10}}{4^{10}}$

26. Probability that E and F have exactly 3 elements in common is

a) $\frac{3^{10}}{4^{10}}$ b) $\frac{10\times 3^8}{4^9}$ c) $\frac{1}{2^{10}}$ d) $\frac{{}^{20}C_{10}}{4^{10}}$

Passage - II :

There are 8 seats in the front row of a theatre in which 4 persons are to be seated. Then the probability of seating them so that

27. No 2 persons sit side by side, is

a) $\frac{1}{14}$ b) $\frac{3}{14}$ c) $\frac{1}{7}$ d) $\frac{3}{7}$

28. Each person has exactly one neighbour is

a) $\frac{1}{14}$ b) $\frac{3}{14}$ c) $\frac{1}{7}$ d) $\frac{3}{7}$

29. All of them sit together in a row avoiding the 2 end seats, is

a) $\frac{3}{14}$ b) $\frac{3}{70}$ c) $\frac{9}{70}$ d) $\frac{5}{14}$

Matrix matching type questions

30. In a tournament, there are sixteen players S_1, S_2, ..., S_{16} and divided into eight pairs at random. From each game a winner is decided on the basis of a game played between the two players of the pair. Assuming that all the players have equal strength

COLUMN - I	COLUMN - II
A) The probability that S_1 is one of the winners	p) $\frac{8}{15}$
B) The probability that exactly one of S_1 or S_2 is a winner	q) $\frac{1}{2}$
C) Both S_1 and S_2 are among eight winners	r) $\frac{7}{30}$
D) Both S_1 and S_2 are not among winners	s) $\frac{1}{240}$

31. A student has to match three histrorical events - Dandi March, Quite India Movement and Mahatma Gandhi's assassination with the years 1948, 1930 and 1942. The student has no knowledge of the correct answers and decides to match the events and years randomly. If X denotes the number of correct answer obtained by the student, then

COLUMN - I	COLUMN - II
A) $P(X=3)$	p) 1/2
B) $P(X=2)$	q) 0
C) $P(X=1)$	r) 1/6
D) $P(X=0)$	s) 1/3

Integer answer type questions

32. If 12 identical coins are distributed among three children at random. The probability of distributing so that each child gets atleast two coins is $\frac{k}{13}$ then k is

33. There are two red, two blue, two white and certain number (greater than 0) of green socks in a drawer. If two socks are taken at random from the drawer with out replacement, the probability that they are of the same colour is $\frac{1}{5}$, then the number of green socks are

34. Two non - negative integers are chosen at random from a set of non - negative integer with replacement. If the probability that sum of the squares divisible by 10 is P then $50P$ is

35. A rod of length ℓ is broken into 3 parts at random. Then the probability that a triangle can be formed from these parts is $\frac{p}{q}$ (where G.C.D of p & q is 1) then $p+q=$

36. 3 boys and 2 girls stand in a queue the probability that no.of boys ahead of every girl is at least one more than no.of girls ahead of her is of the form $\frac{p}{q}$ (where p & q are relatively prime) then $p+q=$

37. Consider the system of equation $ax+by=0$, $cx+dy=0$, where $a, b, c, d \in \{0,1,\}$, the probability that the system of equations has a unique solution is k then $8k$ is

EXERCISE-II

Conditional probability, Multiplication theorem, Total probability and Baye's theorem

LEVEL-I (MAIN)

Conditional Probability :

1. A and B are events such that $P(A)=0.3$, $P(B)=0.25$; $P(A \cap B)=0.2$ then $P(\bar{A}/\bar{B})=$

1) $\frac{11}{15}$ 2) $\frac{12}{15}$ 3) $\frac{13}{15}$ 4) $\frac{14}{15}$

2. Two dice are rolled and given that the sum is prime. The probability of getting sum more than 6 is

1) $\frac{7}{15}$ 2) $\frac{8}{15}$ 3) $\frac{1}{5}$ 4) $\frac{2}{5}$

3. Two cards are drawn from pack and given that they are of different colours. The probability of getting one king and one Queen is

1) $\frac{1}{169}$ 2) $\frac{2}{169}$ 3) $\frac{4}{169}$ 4) $\frac{5}{169}$

4. A box contains 10 mangoes out of which 4 are rotten. Two mangoes are taken out together. If one of them is found to be good, then the probability that both are good is

1) $\frac{5}{13}$ 2) $\frac{8}{13}$ 3) $\frac{1}{5}$ 4) $\frac{2}{3}$

5. A box contains 100 tickets, numbered 1, 2, ...100. Two tickets are chosen at random one after another with replacement. It is given that the maximum sum on the two chosen tickets is not more than 10. The minimum sum of them is 5 with probability

1) $\frac{11}{15}$ 2) $\frac{13}{15}$ 3) $\frac{13}{17}$ 4) $\frac{13}{19}$

6. For a biased die the probabilities for different faces to turn up are given below.

Face	1	2	3	4	5	6
Probability	0.1	0.32	0.21	0.15	0.05	0.17

The die is tossed and you are told that either face 1 or 2 has turned up. Then the probability that it is face 1 is

1) $\frac{5}{21}$ 2) $\frac{6}{23}$ 3) $\frac{5}{23}$ 4) $\frac{4}{23}$

7. In a class 40% students study mathematics 25% study chemistry and 15% both mathematics and chemistry. If a student is chosen at random the probability that he studies mathematics, If it is known that he studies chemistry is

1) $\frac{1}{8}$ 2) $\frac{3}{8}$ 3) $\frac{2}{5}$ 4) $\frac{3}{5}$

8. One ticket is selected at random from 100 tickets numbered 00, 01, 02, ... 99. Suppose A and B are the sum and product of the digits found on the ticket. Then $P(A=7/B=0)$is given by

1) $\frac{2}{13}$ 2) $\frac{2}{19}$ 3) $\frac{1}{50}$ 4) $\frac{17}{19}$

9. E_1, E_2 are events of a sample space such that $P(E_1)=\frac{1}{4}$, $P\left(\frac{E_2}{E_1}\right)=\frac{1}{2}$, $P\left(\frac{E_1}{E_2}\right)=\frac{1}{4}$ then $P\left(\frac{E_1}{E_2}\right)+P\left(\frac{E_1}{\overline{E_2}}\right)=$

1) 1/4 2) 1/3 3) 1/2 4) 3/4

Independent & dependent events :

10. E and F are independent events such that $0<P(E)<1$ and $0<P(F)<1$. Which of the following is a wrong statement

1) $P\left(\frac{E}{F}\right)+P\left(\frac{E^C}{F}\right)=1$ 2) E, F^C are independent

3) E^C and F^C are independent 4) E and F are mutually exlcusive

11. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$; $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$ where $\overline{A}$ stands for complement of event A. Then events A and B are

1) equally likely and mutually exclusive
2) equally likely but not independent
3) independent but not equally likely
4) mutually exclusive and independent

12. A bag contains 4 black, 5 white and 6 red balls. If 4 balls are drawn one by one with replacement the probability that none is red is

1) 81/625
2) 27/125
3) 81/125
4) 27/625

13. Let E and F are two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2 then

1) $P(E)=1/3,\ P(F)=1/5$
2) $P(E)=1/2,\ P(F)=1/6$
3) $P(E)=1/6,\ P(F)=1/2$
4) $P(E)=1/4,\ P(F)=1/3$

14. A and B are two independent events. The probability that both A and B occur, is 1/6 and the probability that none of them occur, is 1/3. The minimum value of probability of occurance of A is

1) 1/2
2) 1/3
3) 1/4
4) 1/6

15. The odds against A solving a problem are 8 to 6 and the odds in favour of B solving the same problem are 14 to 10. The probability of solving the problem if they both try independently is

1) $\frac{16}{21}$
2) $\frac{5}{21}$
3) $\frac{4}{21}$
4) $\frac{1}{3}$

16. A speaks truth in 80% of the cases and B in 60% of the cases. The percentage of the cases of which they likely to contradict each other in stating the same fact is

1) 35%
2) 44%
3) 60%
4) 20%

17. Four positive integers are taken at random and are multiplied together. Then the probability that the product ends in an odd digit other than 5 is

1) $\frac{609}{625}$
2) $\frac{16}{625}$
3) $\frac{2}{5}$
4) $\frac{1}{5}$

18. Fifteen coupons are numbered 1, 2, ... 15 respectively seven coupons are selected at random one at time with replacement. The probability that the largest number appearing on a selected coupons is 9 is

1) $\left(\frac{9}{16}\right)^6$
2) $\left(\frac{8}{15}\right)^7$
3) $\left(\frac{3}{5}\right)^7$
4) $\frac{9^7-8^7}{15^7}$

19. A salesman has a 60% chance of making a sale to each customer. The behaviour of successive customers is independent. If two customers A and B enter. The probability that the salesman will make a sale to A or B is

1) 0.36
2) 0.84
3) 0.96
4) 0.74

20. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

1) $\frac{2}{9}$
2) $\frac{1}{9}$
3) $\frac{8}{9}$
4) $\frac{7}{9}$

21. The odds in favour of A winning a game of chess against B are 5 : 2. If 3 games are played then the odds in favour of A is winning atleast one game are

1) 335 : 8 2) 8 : 335 3) 335 : 343 4) none

22. Three persons A, B, C in order cut a pack of cards replacing them after each cut. The person who first cuts a spade shall win a prize. The probability that C wins the prize is

1) $\frac{16}{37}$ 2) $\frac{9}{37}$ 3) $\frac{12}{37}$ 4) $\frac{1}{37}$

23. In the above problem the ratio of the probabilities of their winning is

1) 16:12:9 2) 12:16:9 3) 9:12:16 4) 4:3:2

24. A man alternately tosses a coin and throws a die continuously. The probability of his getting a head on the coin before he gets 4 on the die is

1) $\frac{1}{2}$ 2) $\frac{6}{7}$ 3) $\frac{3}{4}$ 4) $\frac{2}{3}$

25. A and B throw a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, his chance of winning is

1) $\frac{5}{61}$ 2) $\frac{30}{61}$ 3) $\frac{35}{61}$ 4) $\frac{60}{61}$

Total Probability:

26. An urn A contains 8 black balls and 5 white balls. A second urn B contains 6 black and 7 white balls. The probability that a blind folded person in one draw shall obtain a white ball

1) 5/13 2) 7/13 3) 6/13 4) 5/26

27. One bag A contains 5 white and 3 black balls. Another bag B contains 6 white and 2 black balls. A card is drawn from pack of cards. If it is club card, a ball is drawn from bag A. If it is red card a ball is drawn from bag B. Otherwise he kept quiet. The probability of getting white ball is

1) 15/32 2) 17/32 3) 14/32 4) 19/32

28. One compartment of a purse contains three 25 paise coins and 2 one rupee coins and the other compartment contains two 25 ps. coins and 3 one rupee coins. The probability of drawing a rupee from the purse is

1) 1/5 2) 2/5 3) 3/5 4) 1/2

29. Three groups of children contain 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys respectively. One child is selected at random from each group. The chance that the selected group comprises of 1 girl and 2 boys is

1)13/32 2)16/32 3)19/32 4)3/12

30. Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn A. If one ball is now drawn from urn A, the probability that it is found to be red is

1) 32/55 2) 42/55 3) 36/55 4) none

Baye's Theorem:

31. A man is known to speak truth 2 out of 3 times. He throws a die and declares that it is one that appeared. The probability that it is actually one is

1) $\frac{2}{3}$ 2) $\frac{2}{5}$ 3) $\frac{2}{6}$ 4) $\frac{2}{7}$

32. A box contains 4 balls. 2 balls are drawn from it and are found to be white. The probability that all the balls in the bag are white is

1) $\frac{4}{5}$ 2) $\frac{3}{5}$ 3) $\frac{2}{5}$ 4) $\frac{1}{5}$

33. Box A contains 2 black and 3 red balls, while box B contains 3 black and 4 red balls. Out of these two boxes one is selected at random, and the probability of choosing box A is double that of box B. If a red ball is drawn from the selected box then the probability that it has come from box B is

1) 21/41 2) 10/31 3) 12/31 4) 13/41

34. A letter is known to have come either from 'TATANAGAR' or 'CALCUTTA'. On the envelope. Just two consecutive letters TA are visible. The probability that the letter comes from 'TATA NAGAR' is

1) $\frac{4}{11}$ 2) $\frac{7}{11}$ 3) $\frac{5}{11}$ 4) $\frac{6}{11}$

Miscellaneous:

35. There are 4 machines out of which 2 are defective. They are tested one by one at random till both the defective machines are identified. The probability that only 2 tests are needed for this is

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{6}$

36. A is one of 5 horses that entered the race and for it to be rided by one of the two jokeys P & Q and odds in favour of P rides it is 2 to 1. If P rides A, all the horses are likely to win. If Q rides A, A's chance of winning is tripled. The odds in favour of A's winning is

1) 1 : 3 2) 3 : 1 3) 1 : 2 4) 2 : 1

37. The probability that a teacher will conduct an unannounced test during any class meeting is 1/4. If a student of the class is absent twice, then the probability for the student to miss atleast one test is

1) $\frac{3}{16}$ 2) $\frac{6}{16}$ 3) $\frac{1}{16}$ 4) $\frac{7}{16}$

38. The key for a door is in the bunch of 10 keys. A man attempts to open the door by trying keys at random discarding the wrong key. the probability that the door is opened in the 5^{th} trial is

1) 0.1 2) 0.2 3) 0.5 4) 0.6

39. An urn contains nine balls of which threee are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

1) $\frac{2}{7}$ 2) $\frac{1}{21}$ 3) $\frac{2}{23}$ 4) $\frac{1}{3}$

40. The probability that the birthdays of six different persons will fall in exactly two calendar month is

1) $\frac{^{12}C_2(2^6-1)}{^{12}C_6}$ 2) $\frac{^{12}C_2(2^6-2)}{^{12}C_6}$ 3) $\frac{^{12}C_2(2^6-2)}{12^6}$ 4) $\frac{1}{12^6}$

41. The chance that Doctor A will diagonise disease X correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death after wrong diagnosis is 70%. A patient of Doctor A who had disease X died. The probability that his disease was diagonised correctly is

1) $\frac{5}{13}$ 2) $\frac{6}{13}$ 3) $\frac{2}{13}$ 4) $\frac{15}{13}$

42. A bag contains some white and some black balls, all combinations of balls being equally likely. The total number of balls in the bag is 10. If three balls are drawn at random without replacement and all of them are found to be black, the probability that the bag contians 1 white and 9 black balls is

1) $\frac{14}{55}$ 2) $\frac{12}{55}$ 3) $\frac{2}{11}$ 4) $\frac{8}{55}$

43. A number of six digits is written down at random. The probability that the sum of the digits of the number is even is

1) 1/2 2) 3/8 3) 3/7 4) 4/7

Numerical value type questions

44. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2 respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

45. A fair die is tossed twice. The probability of getting 4, 5 or 6 on the first toss and 1, 2, 3 or 4 on the second toss is

46. The probabilities of solving a problem by 3 students A,B,C independently are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$. The probability that the problem will be solved is

47. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is

48. A bag contains 5 red, 3 black balls and a second bag contains 4 red and 5 black balls. One of the bags is chosen at random and a draw of two balls is made from it. Find the chance that one is red and the other is black

49. If 4 people are chosen at random, then the probability that exactly two of them have the same birthday and another two of them have another same birthday of a week is

50. There are 15 cards. Of these 10 have the letter 'I' printed on them and the other 5 have the letter 'T' printed on them. If three cards are picked up at random one after the other and kept in the same order, the probability of making the word IIT is

LEVEL-II (ADVANCED)

1. A bag contains 7 red and 2 white balls and another bag contains 5 red and 4 white balls. Two balls are drawn, one from each bag. The probability that both the balls are white, is

a) $\frac{2}{9}$ b) $\frac{2}{3}$ c) $\frac{8}{81}$ d) $\frac{35}{81}$

2. $A_1, A_2, \ldots, A_n$ are n independent events with $P(A_j) = \frac{1}{1+j} (1 \le j \le n)$. The probability that none of $A_1, A_2, \ldots, A_n$ occur, is

a) $\frac{n!}{(n+1)!}$ b) $\frac{n}{(n+1)}$ c) $\frac{1}{(n+1)!}$ d) None of these

3. A three digit number is chosen at random. Then the probability that it is divisible by 9 given that it is divisible by 11 is

a) $\frac{8}{99}$ b) $\frac{1}{11}$ c) $\frac{10}{99}$ d) $\frac{1}{9}$

4. If 4 integers are selected at random form the first 23 natural numbers than the probability that their product is even if it is given that their sum is odd is

a) 1 b) $\frac{3}{4}$ c) $\frac{220}{1771}$ d) $\frac{1551}{1771}$

5. Urn A contains 6 red and 4 balck balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn. A. If one ball is now drawn at random from urn A, the probability that it is found out to be red is

a) $\frac{32}{55}$ b) $\frac{41}{55}$ c) $\frac{43}{55}$ d) $\frac{1}{2}$

6. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Then the probability that the ball drawn now is white, is

a) $\frac{n}{m+n}$ b) $\frac{m}{m+n}$ c) $\frac{2n}{m+n}$ d) None of these

7. A bag contains 5 balls, three red and two white. Balls are randomly removed one at a time without replacement until all the red balls are drawn or all the white balls are drawn. The probability that the last ball drawn is white, is

a) $\frac{3}{10}$ b) $\frac{5}{10}$ c) $\frac{6}{10}$ d) $\frac{7}{10}$

8. A person flips 4 fair coins and discards those which turn up tails. He again flips the remaining coin and then discards those which turn up tails. The probability that he discards at least 3 coins is

a) $\frac{135}{256}$ b) $\frac{189}{256}$ c) $\frac{27}{64}$ d) None of these

9. I alternatively toss a fair coin and a fair die until I, either toss a head or throw a '2'. If I toss the coin first, the probability that I throw a '2' before I toss a head, is

a) 1/7 b) 7/12 c) 5/12 d) 5/7

10. An experiment resulting in sample space as $S = [a, b, c, d, e, f]$ with $P(a)=\frac{1}{16}, P(b)=\frac{1}{16}, P(c)=\frac{2}{16}$, $P(d)=\frac{3}{16}, P(e)=\frac{4}{16}$ and $P(f)=\frac{5}{16}$, Let three events A,B and C are defined as $A=\{a,c,e\}$, $B=\{c,d,e,f\}$ and $C=\{b,c,f\}$. If $P(A/B) = p_1$, $P(B/C) = P_2$, $P(C/A^C) = P_3$ and $P(A^C/C)=p_4$, then the correct order sequence is

a) $p_1 < p_3 < p_2 < p_4$ b) $p_1 < p_4 < p_3 < p_2$ c) $p_1 < p_3 < p_4 < p_2$ d) $p_3 < p_1 < p_4 < p_2$

11. Bag A contains 7 white and 8 red balls. A ball is drawn at random if it is a white ball a pair of dice is thrown and if it is a red ball two numbers are taken at random from the set {2,3,4,5,6,7,8}. Then the probability that the sum of the numbers obtained is 9 is

a) $\frac{121}{945}$ b) $\frac{52}{720}$ c) $\frac{103}{945}$ d) $\frac{52}{945}$

12. A is one of 6 horses centered for a race, and is to be ridden by one of two jockeys B and C. It is 2 to 1 that B rides A, in which case all the horses are equally likely to win. If C rides A, his chance of winning is trebled. What are the odds against winning of A ?

a) 5 : 13 b) 5 : 18 c) 13 : 5 d) none of these

13. When a missile is fired from a ship, the probability that is intercepted is 1/3. The probability that the missile hits the target, given that the missile hits the target, given that it is not intercepted is 3/4. If three missiles are fired independently from the ship, the probability that all three hits the target, is

a) 1/12 b) 1/8 c) 3/8 d) 3/4

14. Two players A and B play a match which consists of a series of games (independent). Whoever first wins two games not necessarily consecutive, wins the match. The probability of A's winning, drawing or losing a game against B are $\frac{1}{2},\frac{1}{3},\frac{1}{6}$ respectively. It is known that A won the match at the end of 11^{th} game, the probability that B wins only one game is

a) 3/11 b) 8/11 c) 9/11 d) 10/11

15. A box contains 6 balls of unknown colours. 3 balls are drawn and they are found to be white. Then the probability that all 6 balls are white balls is

a) 7/16 b) 4/7 c) 1/35 d) 9/16

16. A box contains N coins, m of which are fair and the rest are baised. The probability of getting a head when a fair coin is tossed is $\frac{1}{2}$, while it is $\frac{2}{3}$ when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. Then the probability that the coin drawn is fair, when first toss head, second toss tail is

a) $\frac{9m}{8N+m}$ b) $\frac{9m}{8N-m}$ c) $\frac{9m}{8m-N}$ d) $\frac{9m}{8m+N}$

17. If 8 letters are placed in 8 addressed envelopes, then the probability that exactly 3 letters go into correct envelopes and none of the remaining letters go into correct envelop is

a) $\frac{11}{180}$ b) $\frac{^8C_3 \times 6!}{8!}$ c) $\frac{^8C_3 \times 76}{81}$ d) $\frac{^8C_3 \times 60}{81}$

18. Three letters are written to three different persons and address on the three envelopes are also written. Without looking at the addresse, the letters are kept in these envelopes. The probability that all the letters are not placed into their right envelopes is

a) 1/2 b) 1/3 c) 1/6 d) 5/6

More than one correct answer type questions

19. A bag initially contains 1 red and 2 blue balls. An experiment consisting of selecting a ball at random noting its colour and replacing it together with an additional ball of the same colour. If three such trails are made, then

a) Probability that at least one blue ball is drawn is 0.9

b) Probability that exactly one blue ball is drawn is 0.2

c) Probability that all the drawn balls are red given that all the drawn balls are of same colour is 0.2

d) Probability that atleast one red ball is drawn is 0.6

20. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F respectively and if $0 < (F) < 1$, then

a) $P(E/F)+P(\overline{E}/F)=1$ b) $P(E/F)+P(E/\overline{F})=1$

c) $P(\overline{E}/F)+P(E/\overline{F})=1$ d) $P(E/\overline{F})+P(\overline{E}/\overline{F})=1$

21. Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is $\frac{1}{5}$. The probability that bus B will be late is $\frac{7}{25}$. The probability that the bus B is late given that bus A is late is $\frac{9}{10}$. Then probability that

a) neither bus will be late on a particular day is $\frac{7}{10}$

b) the bus A is late given than bus B is late is $\frac{9}{14}$

c) at least one bus is late is $\frac{3}{10}$

d) at least one bus is in time is $\frac{4}{5}$

22. A consignment of 15 record players contains 4 defectives. The record players are selected at random one by one without replacement and examined.

a) probability of getting exactly 3 defective in the examination of 8 record players is $\frac{^4C_3 \times ^{11}C_5}{^{15}C_8}$

b) Probability that 9th one examined is the last defective, is $\frac{8}{195}$

c) Probability that 9th examined player is defective given that there were 3 defectives in the first 8 players examined is $\frac{1}{7}$

d) probability that 9th examined player is the last defective is $\frac{8}{197}$

Linked comprehension type questions

Passage - I :

A bag A contains 4 red and 5 black balls another bag B contains 6 red and 3 black balls and a third bag C contains 3 red and 6 black balls. A bag is selected and A ball is drawn at random from the bag. Let A,B,C denote the events of selecting the bags A,B,C respectively and E, F denote the events of getting red, black ball respectively. Then

23. $P(E/A \cup B) =$

a) 4/13 b) 1/2 c) 5/9 d) 14/27

24. $P(A/E) =$

a) 4/13 b) 4/7 c) 1/2 d) 5/9

25. $P(F) =$

a) 5/9 b) 4/7 c) 4/13 d) 14/27

Passage - II :

There are four boxes A_1, A_2, A_3, A_4. Box A_i has i cards and on each card a number is printed. The numbers are from 1 to i. A box is selected randomly, the probability of selection of box A_i is $\frac{i}{10}$ and then a card is drawn. Let E_i represents the event that a card with number 'i' is drawn.

26. $P(E_1)$ is equal to

a) 1/5 b) 1/10 c) 2/5 d) 1/4

27. $P(A_3/E_2)$ is equal to

a) 1/4 b) 1/3 c) 1/2 d) 2/3

28. Expectation of the number on the card is

a) 2 b) 2.5 c) 3.5 d) 3

Matrix matching type questions

29. A bag contains some white and some black balls, all combinations being equally likely. The total number of balls in the bag is 12. Four balls are drawn at random from the bag at random without replacement

COLUMN - I	COLUMN - II
A) Probability that all the four balls are black is equal to	p) $\frac{14}{33}$
B) If the bag contains 10 black and 2 white balls then the probability that all four balls are black is equal to	q) $\frac{1}{3}$
C) If all the four balls are black, then the probability that the bag contains 10 black balls is equal to	r) $\frac{70}{429}$
D) Probability that two balls are black and two are white is	s) $\frac{13}{165}$

30. The probabilities that a man makes a certain dangerous journey by car, motor cycle or on foot are 1/2,1/6 and 1/3 respectively. The probabilities of an accident when he uses these means of transport are 1/5,2/5 and 1/10 respectively.

COLUMN - I	COLUMN - II
A) The probability of an accident occurring in a single journey, is	p) $\frac{1}{6}$
B) If an accident is known to have happened, the probability that the man was travelling by car, is	q) $\frac{1}{3}$
C) If an accident is known to have happened, the probability that the man was travelling by motor cycle, is	r) $\frac{1}{2}$
D) If an accident is known to have happened, the probability that the man was travelling on foot, is	s) $\frac{1}{5}$

Integer answer type questions

31. There are two urns containing 4 red, 5black and 5 red balls, 6 black balls.One ball is drawn at random from the first and transferred to the second and the one ball is drawn from the second and transferred to the first. After this mutual transfer one ball is drawn at random from the first urn. Let P be the probability that it will be red ball$\frac{1}{16}$, then the least inter greater than $\left(\frac{243}{13}\right)P$ is

32. A girl speaks the truth in 75% cases and boy in 80% of the cases. Then the probability of the cases in which they are likely to contradict each other in stating the same fact is$\frac{K}{20}$. Then K is

33. A bag contains $(n+1)$ coins. It is known that one of these coins show heads on both sides, whereas the other coins are fair. One coin is selected at random and tossed. If the probability that toss results in head is $\frac{7}{12}$, then the value of n is

34. A bag contains some white and some black balls, all combination of balls being equally likely.The total number of balls in the bag is 10. If three balls are drawn at random without replacement and all of them are formed to be black. Let $P(E)$ be the probability that bag contains 1 white and 9 black balls. Then $\frac{55}{2}P(E) =$

35. Each of the n urns contains 4 white and 6 black balls. The $(n+1)$th urn contains 5 white and 5 black balls. Out of the $(n+1)$urns, an urn is chosen at random and two balls are drawn from it without replacement. Both the balls turn out to be black.If the probability that the $(n+1)$th urn was chosen to draw the balls is$\frac{1}{16}$, then the value of $\frac{n}{2}$ is

KEY SHEET (LECTURE SHEET)

EXERCISE-I

LEVEL-I

1) 1	2) 2	3) 2	4) 1	5) 1	6) 3	7) 1	8) 4
9) 4	10) 2	11) 1	12) 1	13) 1	14) 2	15) 3	16) 2
17) 1	18) 3	19) 3	20) 2	21) 2	22) 4	23) 1	24) 3
25) 1	26) 1	27) 2	28) 2	29) 1	30) 1	31) 4	32) 3
33) 1	34) 2	35) 3	36) 2	37) 3	38) 3	39) 3	40) 1
41) 1	42) 4	43) 1	44) 3	45) 3	46) 4	47) 3	48) 2
49) 1	50) 2	51) 1	52) 1	53) 4	54) 1	55) 1	56) 1
57) 2	58) 1	59) 1	60) 3	61) 1	62) 4	63) 2	64) 1
65) 1	66) 0.1	67) 0.2	68) 0.66	69) 5	70) 0.009	71) 0.15	72) 0.5
73) 0.3	74) 0.05	75) 0.39	76) 0.55	77) 0.22			

LEVEL-II

1) a	2) c	3) b	4) a	5) a	6) a	7) b	8) b
9) c	10) b	11) b	12) d	13) b	14) d	15) a	16) b
17) a	18) bc	19) ac	20) abcd	21) abc	22) ac	23) bcd	24) d
25) a	26) b	27) a	28) c	29) b	30) A-q;B-p;C-r;D-s		
31) A-r;B-q;C-p;D-s			32) 4	33) 4	34) 9	35) 5	36) 3
37) 3							

EXERCISE-II

LEVEL-I

1) 3	2) 2	3) 2	4) 1	5) 2	6) 1	7) 4	8) 2
9) 3	10) 4	11) 3	12) 1	13) 4	14) 2	15) 1	16) 2
17) 2	18) 4	19) 2	20) 2	21) 1	22) 2	23) 1	24) 2
25) 2	26) 3	27) 2	28) 4	29) 1	30) 1	31) 4	32) 2
33) 2	34) 2	35) 2	36) 3	37) 4	38) 1	39) 1	40) 3
41) 2	42) 1	43) 1	44) 0.14	45) 0.33	46) 0.6	47) 0.35	48) 0.55
49) 0.05	50) 0.16						

LEVEL-II

1) c	2) a	3) d	4) a	5) a	6) b	7) c	8) d
9) a	10) c	11) a	12) c	13) b	14) c	15) b	16) a
17) a	18) b	19) abcd	20) ad	21) abc	22) abc	23) c	24) a
25) d	26) c	27) b	28) a	29) A-q;B-p;C-r;D-q			
30) A-s;B-r;C-q;D-p			31) 9	32) 7	33) 5	34) 7	35) 5

PRACTICE SHEET

EXERCISE-I

Classical definition and Additonal theorem

LEVEL-I (MAIN)

Problems on Classical Definition :

1. The probability that the month February in a leap year contain 5 mondays or 5 Sundays

   1) $\frac{1}{7}$ 2) $\frac{2}{7}$ 3) $\frac{4}{7}$ 4) $\frac{3}{7}$

2. In a non-leap year, the probability of getting 53 Sundays or 53 Tuesdays or 53 Thursdays is

   1) $\frac{1}{7}$ 2) $\frac{2}{7}$ 3) $\frac{3}{7}$ 4) $\frac{4}{7}$

3. An unbiased coin is tossed to get 2 points for turning up a head and one point for the tail. If three unbiased coins are tossed simultaneously, then the probability of getting a total of odd number of points is

   1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{1}{8}$ 4) $\frac{3}{8}$

4. If two dice are rolled, then the probability of getting 5 on none of them is

   1) $\frac{1}{2}$ 2) $\frac{11}{36}$ 3) $\frac{25}{36}$ 4) $\frac{1}{3}$

5. Six faces of an unbiased die are numbered with 2, 3, 5, 7, 11 and 13. If two such dice are thrown, then the probability that the sum on the uppermost faces of the dice is an odd number is

   1) $\frac{5}{18}$ 2) $\frac{5}{36}$ 3) $\frac{13}{18}$ 4) $\frac{25}{36}$

6. If a and b are choosen randomly choosen when a die rolled twice then the probability that $\underset{x\to 0}{Lt}\left[\frac{a^x+b^x}{2}\right]^{2/x}=6$ is

   1) $\frac{1}{3}$ 2) $\frac{1}{4}$ 3) $\frac{1}{9}$ 4) $\frac{2}{9}$

7. A die is loaded so that the probability of a face i is proportional to i where i = 1, 2, 3, 4, 5, 6. The probability of an even number occuring when die is rolled is

   1) $\frac{11}{21}$ 2) $\frac{1}{21}$ 3) $\frac{4}{7}$ 4) $\frac{5}{7}$

8. If three dice are thrown, the probability that they show different numbers in A.P. is

   1) $\frac{1}{36}$ 2) $\frac{1}{18}$ 3) $\frac{2}{9}$ 4) $\frac{5}{18}$

9. Four dice are rolled, then the probability that at least one digit on the dice must be repeated is

   1) $\frac{1}{18}$ 2) $\frac{13}{18}$ 3) $\frac{5}{18}$ 4) $\frac{1}{9}$

10. An arbitrary cube has four blank faces, one face marked 2 and another marked 3. Then the probability of obtaining a total of exactly 12 in 5 throws is

1) $\frac{5}{1296}$ 2) $\frac{5}{1944}$ 3) $\frac{5}{2592}$ 4) $\frac{11}{1294}$

11. If m is a natural number such that $m \le 5$, then the probability that the quadratic equation $x^2 + mx + \frac{1}{2} + \frac{m}{2} = 0$ has real roots is

1) $\frac{1}{5}$ 2) $\frac{2}{5}$ 3) $\frac{3}{5}$ 4) $\frac{1}{5}$

12. If two cards are drawn from a well shuffled pack, the probability that atleast one of the two is heart is

1) $\frac{4}{13}$ 2) $\frac{11}{13}$ 3) $\frac{55}{221}$ 4) $\frac{15}{34}$

13. If a card is drawn at random from a packet of 100 cards numbered 1 to 100, the probability of drawing a number on the card that is a cube is

1) $\frac{3}{100}$ 2) $\frac{1}{25}$ 3) $\frac{9}{100}$ 4) $\frac{1}{10}$

14. In a bag there are 5 half rupee coins, 4 twenty paise coins and 4 ten paise coins. If two coins are drawn from the bag at random then the probability that the amount drawn to be minimum is

1) $\frac{9}{13}$ 2) $\frac{4}{13}$ 3) $\frac{2}{13}$ 4) $\frac{1}{13}$

15. At random all the letters of the word "ARTICLE" are arranged in all possible ways. The probability that the arrangement begins with vowel and ends with a consonant is

1) $\frac{1}{7}$ 2) $\frac{2}{7}$ 3) $\frac{3}{7}$ 4) $\frac{4}{7}$

16. A single letter is selected at random from the word PROBABILITY. The probability that it is a vowel is

1) $\frac{3}{11}$ 2) $\frac{4}{11}$ 3) $\frac{2}{11}$ 4) $\frac{1}{11}$

17. The letters of the word "ARTICLE" are arranged in all possible ways at random. The probability of arranging them so that the vowels must occur in a specified order (need not come together) is

1) $\frac{1}{2}$ 2) $\frac{1}{6}$ 3) $\frac{1}{8}$ 4) $\frac{1}{4}$

18. One number is selected at random from the four digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6, 7. Then the probability that it is an odd number is

1) $\frac{4}{7}$ 2) $\frac{2}{5}$ 3) $\frac{7}{16}$ 4) $\frac{1}{16}$

19. A five digited number without repetetion is formed by the digits 1, 2, 3, 4, 5, 6, 7, 8. The probability that the number has even digits at both ends is

1) $\frac{3}{14}$ 2) $\frac{3}{7}$ 3) $\frac{4}{7}$ 4) $\frac{5}{7}$

20. Using {1, 2, 3, 4, 5} four digited numbers are formed without repetetion at random. The probability that the number is divisible by 4 is

1) $\frac{1}{5}$ 2) $\frac{2}{5}$ 3) $\frac{3}{5}$ 4) $\frac{4}{5}$

21. A number n is choosen at random from {1, 2, 3, ..., 1000}. Then the probability that n is a number which leaves a remainder '1' when divided by 7 is

1) $\frac{71}{500}$ 2) $\frac{143}{1000}$ 3) $\frac{72}{500}$ 4) $\frac{71}{1000}$

22. If 5 boys and 4 girls are arranged in a row at random then the probability of arranging so that same sex do not come to gether is

1) $\frac{11}{126}$ 2) $\frac{1}{126}$ 3) $\frac{1}{125}$ 4) $\frac{7}{126}$

23. 5 boys and 5 girls sit in a row at random. The probability that the boys and girls sit alternatively is

1) $\frac{5}{14}$ 2) $\frac{3}{28}$ 3) $\frac{1}{126}$ 4) $\frac{1}{11}$

24. The odds against sitting of two particular persons together out of n persons seated round a circular table is :

1) $(n-3):2$ 2) $2:(n-3)$ 3) $(n-2):2$ 4) $2:(n-2)$

25. Seven persons sit in a row at random. The probability that three persons A, B, C sit together in a particular order is

1) $\frac{3!}{7!}$ 2) $\frac{4!}{7!}$ 3) $\frac{5!}{7!}$ 4) $\frac{3!\,5!}{7!}$

26. A set A contains 10 elements. A function from A to itself is formed. The probability that the function so formed is not on-to is

1) $\frac{\angle 10}{(10)^{10}}$ 2) $\frac{1}{10^9}$ 3) $\frac{\angle 10}{10^9}$ 4) $1-\frac{\angle 9}{(10^9)}$

27. There are 5 letters and 5 addressed envelops. If the letters are put at random in the envelops, the probability that atleast one letter may be placed in wrongly addressed envelope is

1) $\frac{119}{120}$ 2) $\frac{120}{343}$ 3) $\frac{1}{1155}$ 4) $\frac{139}{140}$

28. Four frightened pigeons go into their holes at random. The probability that no pigeon goes into its actual hole is

1) $\frac{5}{24}$ 2) $\frac{3}{8}$ 3) $\frac{7}{24}$ 4) $\frac{11}{24}$

29. There are 10 stations between A and B. A train is to stop at three of these 10 stations. The probability that no two of these stations are consecutive is

1) $\frac{7}{15}$ 2) $\frac{4}{15}$ 3) $\frac{8}{15}$ 4) $\frac{11}{15}$

30. If 5 different things are placed at random in 3 different boxes then the probability of placing them such that no box remains empty is

1) $\frac{31}{81}$ 2) $\frac{50}{81}$ 3) $\frac{40}{81}$ 4) $\frac{20}{81}$

31. If three people are chosen at random, then the probability that no two of them were born in the same date of the month of september is

1) $\frac{30}{49}$ 2) $\frac{203}{225}$ 3) $\frac{120}{343}$ 4) $\frac{6}{49}$

32. Two squares are chosen at random on a chess board. The probability that they have a side in common is

1) $\frac{1}{9}$ 2) $\frac{2}{7}$ 3) $\frac{1}{18}$ 4) $\frac{1}{3}$

33. Three squares of normal chess board are chosen. The probability of getting two squares of one colour and the other of different colour is

1) $\frac{16}{21}$ 2) $\frac{8}{21}$ 3) $\frac{8}{64\times 63\times 62}$ 4) $\frac{7}{21}$

34. There are 10 different pairs of shoes from which 4 shoes are picked at random. The probability that there is atleast one pair is

1) $\frac{99}{323}$ 2) $\frac{224}{323}$ 3) $\frac{2}{5}$ 4) $\frac{3}{5}$

35. In the above problem the probability that there is no pair is

1) $\frac{99}{323}$ 2) $\frac{224}{323}$ 3) $\frac{16}{53}$ 4) $\frac{17}{323}$

36. In the above problem the probability that there is exactly one pair is

1) $\frac{99}{323}$ 2) $\frac{224}{323}$ 3) $\frac{96}{323}$ 4) $\frac{95}{323}$

37. In the above problem the probability that there are two pairs is

1) $\frac{3}{323}$ 2) $\frac{95}{323}$ 3) $\frac{96}{323}$ 4) $\frac{320}{323}$

38. Four numbers are chosen at random from $\{1, 2, 3, ..., 40\}$. The probability that they are not consecutive is

1) $\frac{1}{2470}$ 2) $\frac{4}{7969}$ 3) $\frac{2469}{2470}$ 4) $\frac{7965}{7969}$

39. Out of 10 persons sitting at a round table, three persons are selected at random then the probability that no two of them are consecutive is

1) $\frac{7}{12}$ 2) $\frac{7}{10}$ 3) $\frac{5}{7}$ 4) $\frac{5}{12}$

40. Three electric lamps are fitted in a room. 3 bulbs are chosen at random from 10 bulbs having 6 good bulbs. The probability that the room is lighted is

1) $\frac{29}{30}$ 2) $\frac{49}{50}$ 3) $\frac{1}{10}$ 4) $\frac{43}{66}$

41. The numbers 1, 2, 3, ...,n are arranged in a random order. The probability that the digits 1, 2, 3, ..., k ($k < n$) appear together is

1) $\frac{(n-k)!}{n!}$ 2) $\frac{n-k+1}{{}^nC_k}$ 3) $\frac{n-k}{{}^nC_k}$ 4) $\frac{k!}{n!}$

42. 24 boys are divided randomly into two equal groups. The probability that two tallest boys are in the different groups is

1) $\frac{12}{23}$ 2) $\frac{1}{2}$ 3) $\frac{1}{4}$ 4) $\frac{{}^2C_2}{{}^{24}C_{12}}$

43. There are 2 locks on the door and the door keys are among the 6 different ones you carry in your pocket. In a hurry one key is dropped somewhere by you. The probability that you can still open the door is

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{2}{3}$ 4) $\frac{1}{4}$

44. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen is negative is

1) $\frac{5}{8}$ 2) $\frac{3}{8}$ 3) $\frac{1}{16}$ 4) $\frac{3}{16}$

45. A committee of five is to be chosen from a group of 8 people which included a married couple. The probability for the selected committee may contain or may not contain both of the married couple is

1) $\frac{13}{28}$ 2) $\frac{5}{14}$ 3) $\frac{1}{56}$ 4) $\frac{3}{56}$

46. A mapping is selected at random from the set of all mappings of the set A = {1, 2, 3,4} into itself. The probability that the mapping selected is a bijection.

1) $\frac{1}{4^4}$ 2) $\frac{1}{4!}$ 3) $\frac{3!}{4^3}$ 4) $\frac{1}{4}$

Addition Theorem :

47. Suppose A and B are two events such that $P(A \cap B) = \frac{3}{25}$ and $P(B - A) = \frac{8}{25}$. Then $P(B)$=

1) $\frac{11}{25}$ 2) $\frac{3}{11}$ 3) $\frac{1}{11}$ 4) $\frac{9}{11}$

48. $P(A \cap B) = \frac{1}{4}$, $P(\bar{A}) = \frac{1}{3}$, $P(B) = \frac{1}{2}$, then $P(\bar{A} \cap \bar{B}) =$

1) $\frac{1}{12}$ 2) $\frac{11}{12}$ 3) $\frac{1}{4}$ 4) $\frac{3}{4}$

49. A and B are mutually exclusive events such that $P(A) = \frac{1}{2}$ $P(B)$ and $A \cup B = S$ then $P(A) =$

1) $\frac{2}{3}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{3}{4}$

50. One of the two events A and B must occur. If $P(A) = \frac{2}{3} P(B)$ the odds in favour of B is

1) 1 : 2 2) 2 : 1 3) 2 : 3 4) 3 : 2

51. The probabilities of two events A and B are 0.25 and 0.40 respectively. The probability that both A and B occur is 0.15. The probability that neither A nor B occur is

1) 0.35 2) 0.65 3) 0.5 4) 0.75

52. If a card is drawn from a pack of cards then the probability of selecting a club card or king card is

1) $\frac{17}{52}$ 2) $\frac{4}{13}$ 3) $\frac{14}{52}$ 4) $\frac{15}{52}$

53. If two dice are rolled then the probability of getting both even or sum 10 is

1) $\frac{4}{18}$ 2) $\frac{5}{18}$ 3) $\frac{7}{18}$ 4) $\frac{1}{3}$

54. In a class there are 60 boys and 40 girls. Among the boys as well as girls, half of them are Tamilians. If a student is selected at random then the probability of selecting a boy or Tamilian is

1) $\frac{3}{5}$ 2) $\frac{4}{5}$ 3) $\frac{2}{5}$ 4) $\frac{1}{5}$

Numerical value type questions

55. The probability that the 13th day of a randomly chosen month is a Friday, is

56. Two dice are rolled simultaneously. The probability that the numbers on them are different is

57. Two cards are drawn simultaneously from a pack of cards. Find the probability that none of them will be the ace of spade

58. If two balls are drawn from a bag containing 3 white, 4 black and 5 red balls then the probability that the drawn balls are of different colours is

59. A bag contains 6 white and 4 black balls. Two balls are drawn at random. The probability that they are of the same colour is

60. The probability that a number selected at random from the set of numbers {1, 2, 3 ..., 100} is a square is

61. Three squares of a chess board are chosen at random, the probability that two are of one colour and one of another is

62. From first twenty natural numbers, 2 numbers are selected at random, the probability that the selected numbers are such that their sum is even is

63. A determinent is chosen at random from the set of all 2×2 determinants with entries 0 or 1 only. The probability that a randomly chosen determinant has positive value is

64. In the above problem the probability of selecting a determinant having non zero determinant value is

LEVEL-II (ADVANCED)

Single answer type questions

1. If the numbers a, b and c are chosen form the set {1,2,3,4,5,6} with repetition, then the probability that the quadratic expression $ax^2 + bx + c$ is positive for any real value of x is

a) $\frac{43}{54}$ b) $\frac{11}{54}$ c) $\frac{22}{27}$ d) $\frac{173}{216}$

2. A set A has 22 elements a subset P of A is selected at random. After replacing the elements, again a subset Q of A is selected. The probability that P and Q have exactly 5 elements in common is

a) $\left(\frac{3}{4}\right)^{22}$ b) ${}^{22}C_5 \times \left(\frac{3}{4}\right)^{22}$ c) ${}^{22}C_{17} \cdot \frac{3^{17}}{4^{22}}$ d) ${}^{22}C_5 \cdot \frac{3^5}{2^{44}}$

3. When 4 dice are thrown, it is observed that the sum appearing on them is 13, then the probability that 4 appears on at least one of the dice is

a) $\frac{4}{7}$ b) $\frac{140}{6^4}$ c) $\frac{17}{35}$ d) $\frac{70}{6^3}$

4. An urn contains 'a' white and 'b' black balls. Balls are drawn one-by-one without replacement. Let P_n be the probability that nth drawn ball is balck. q_n be the probability that nth drawn ball is white. Then

a) $p_n < q_n$ for all $n > 1$ b) $p_n > q_n$ for all $n > 1$

c) $p_n = q_n$ for all n d) $p_n + q_n = 1$

5. A man has 8 relatives 5 women and 3 men and his wife has 8 relatives 4 women and 4 men. They dicided to invite 6relatives at random to a dinner party. They observed that among the invited guests there are 3 men and 3 women and 3 from his side and 3 from her side. Then the probability that all the 3 Iadies are from his side is

a) $\frac{1124}{{}^8C_3 \times {}^8C_3}$ b) $\frac{{}^5C_3 \times {}^4C_3}{{}^8C_3 \times {}^8C_3}$ c) $\frac{20}{1124}$ d) $\frac{10}{281}$

6. 52 cards are equally distributed among four players. The probability that ace of spade is held by first player must be

a) $\frac{1}{13}$ b) $\frac{1}{52}$ c) $\frac{1}{4}$ d) None of these

7. Six dice are thrown. The probability of getting a total of 34 must be

a) $\frac{13}{6^6}$ b) $\frac{15}{6^6}$ c) $\frac{21}{6^6}$ d) $\frac{21}{6^6}$

8. If two integers x,y are selected from the set {1,2,3,...15} at random then the probability that $|x - y|>7$ is

a) $\frac{4}{15}$ b) $\frac{2}{5}$ c) $\frac{41}{105}$ d) $\frac{31}{105}$

9. If a 7 digit number is formed at random using the digits 0,1,2,3,4,5 when repetition is allowed, then the probability that the number thus formed is divisible by 3 is

a) $\frac{1}{3}$ b) $\frac{5^7 + 5.5^4}{5 \times 6^6}$ c) $\frac{1}{6}$ d) $\frac{6^6 + 5.6^5}{5 \times 6^6}$

10. If 3 identical coins are distributed at random to 4 persons, the probability that one person among them receives exactly two coins is

a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

11. For a mixed doubles tennis game (one gent and one lady on each side) 4 persons are selected at random from ten married couples. Then the probability that the selection doesn't contain a couple (a husband and his wife) is

a) $\frac{28}{45}$ b) $\frac{168}{323}$ c) $\frac{224}{323}$ d) $\frac{27}{45}$

12. The probability that a randomly chosen 3 digit number has exactly 3 factors is

a) $\frac{2}{225}$ b) $\frac{7}{900}$ c) $\frac{7}{300}$ d) $\frac{3}{500}$

13. A box contains 5 different objects of best quality, 5 different objects of first quality and 6 different objects of second quality. Four articles are taken simultaneously at random from the box without regard their quality. If A is the event that atleast one of chosen article is of best quality and B is the event tht atleast one of chosen article is of second quality, then $P(A \cup B) =$

a) $\frac{263}{264}$ b) $\frac{163}{164}$ c) $\frac{363}{364}$ d) $\frac{463}{464}$

14. If the letters of the word MATHEMATICS are arranged at random. The probability that C comes before E, E comes before H, H before I and I before S is

a) $\frac{1}{75}$ b) $\frac{1}{24}$ c) $\frac{1}{120}$ d) $\frac{1}{720}$

15. In a convex hexagon two diagonals are drawn at random. The probability that the diagonals intersect at an interior point of hexagon is

a) $\frac{5}{12}$ b) $\frac{7}{12}$ c) $\frac{2}{5}$ d) $\frac{3}{5}$

16. Two numbers x and y are chosen at random without replacement from the first 30 natural numbers. The probability that $x^2 - y^2$ is divisible by 3 is

a) $\frac{3}{29}$ b) $\frac{3}{55}$ c) $\frac{3}{29}$ d) $\frac{47}{87}$

17. Let $W = 1$ be a complex cube root of unity. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $w^{r_1} + w^{r_2} + w^{r_3} = 0$ is

a) $\frac{1}{18}$ b) $\frac{1}{9}$ c) $\frac{2}{9}$ d) $\frac{1}{36}$

18. Each of 10 passengers boards any of the three buses randomly, which had no passenger initially. The probability that each bus has got atleast one passenger is

a) $\frac{{}^{10}P_3.3^7}{3^{10}}$ b) $1-\frac{{}^{10}C_3.3^7}{3^{10}}$ c) $1-\frac{2^{10}}{3^{10}}$ d) $\frac{3^{10}-3.2^{10}+3}{3^{10}}$

More than one correct answer type questions

19. If M and N are any two events, the probability that exactly one of them occur is

a) $P(M)+P(N)-2P(M\cap N)$ b) $P(M)+P(N)-P(M\cap N)$

c) $P(\overline{M})+P(\overline{N})-2P(\overline{M}\cap\overline{N})$ d) $P(M\cap\overline{N})+P(\overline{M}\cap N)$

20. For two given events A and B, $P(A\cap B)$ is

a) not less than $P(A) + P(B) - 1$ b) not greater than $P(A) + P(B)$

c) equal to $P(A) + P(B) - P(A\cap B)$ d) equal to $P(A) + P(B) + P(A\cap B)$

21. If A, B, C are 3 exhaustive events of a trial, then $P(A\cap\overline{B}\cap\overline{C})$ is equal to

a) $P(A)-P(B)-P(C)+P(B\cap C)$ b) $1-P(B)-P(C)+P(B\cap C)$

c) $1-P(\overline{A}\cap(B\cup C))$ d) $P(A)-P(A\cap B)-P(A\cap C)+P(A\cap B\cap C)$

22. The chance of happening of an event is the square of the chance of a second event but the odds against the first are cube of the odds against the second. If the chances of events are P_1, P_2 respectively, then

a) $P_1=\frac{1}{9}$ b) $P_1=\frac{1}{6}$ c) $P_2=\frac{1}{3}$ d) $P_2=\frac{1}{4}$

23. If A and B are exhaustive events in a sample space such that the probabilities of the events $A \cap B, A, B$ and $A \cup B$ are in A.P. If $P(A) = K$, where $0 < k \le 1$, then

a) $P(B) = \frac{K+1}{2}$

b) $P(A \cap B) = \frac{3K-1}{2}$

c) $P(A \cup B) = 1$

d) $P(A^1 \cup B^1) = \frac{3(1-K)}{2}$

24. A cube having all its sides painted is cut by two horizontal and four vertical plane so as to form 27 cubes all having the same dimensions, then which of the following are correct?

a) Probability that the cube selected has none of its faces painted is $\frac{1}{27}$

b) The probability that the cube selected has two faces painted is $\frac{4}{9}$

c) That number of cubes having at least one of its sides painted is 24

d) Probability of cube selected has three face painted is 1/3

Linked comprehension type questions

Passage - I :

Two persons A and B throw a pair of dice each simulteneously. Let X, Y be the scores (sum on the two dice) obtained by them individually. Then answer the following questions.

25. $P(X < Y) =$

a) $\frac{1}{6}$ b) $\frac{1}{108}$ c) $\frac{335}{1296}$ d) $\frac{575}{1296}$

26. $P(X = Y) =$

a) $\frac{335}{1296}$ b) $\frac{73}{648}$ c) $\frac{1}{108}$ d) $\frac{575}{1296}$

27. $P(X = 5 \text{ and } Y = 10) =$

a) $\frac{1}{108}$ b) $\frac{575}{1296}$ c) $\frac{1}{6}$ d) $\frac{73}{648}$

Matrix matching type questions

28. A coin has probability p of showing head when tossed. It is tossed n times. Let p_n denote the probability that no two (or more) consecutive heads occur, then

COLUMN - I	COLUMN - II
A) p_1	p) $1 - 2p^2 + p^3$
B) p_2	q) $1 - p^2$
C) p_3	r) 1
D) p_n	s) $(1-p)\, p_{n-1} + p(1-p)\, p_{n-2}$

29. $A_1, A_2, A_3, \ldots A_n$ be the n-vertices of a regular polygon of n-sides inscribed in a circle with centre O. Triangles are formed by joining the vertices of this regular polygon. From these triangles, if a triangle is choosen at random.

COLUMN - I	COLUMN - II
A) If $n = 45$ probability that the chosen triangle is equilateral	p) 0
B) If $n = 45$ probability that the chosen triangle is right angled triangle	q) $\frac{1}{946}$
C) If $n = 27$ probability that the chosen triangle is acute angled triangle	r) $\frac{1}{351}$
D) If $n = 27$ probability that the chosen triangle is right angled triangle	s) $\frac{7}{25}$

Integer answer type questions

30. Two different numbers are taken from the set $\{0,1,2,3,4,5,6,7,8,9,10\}$. The probability that their sum and positive difference are both multiple of 4 is $\frac{x}{55}$, then x equals.

31. If $P(A) = 0.6$ and the greatest value of $P(A \cap B) = 0.4$, then the greatest value of $P(B)$ is $\frac{K}{10}$. Then value of K is

32. A die is weighted such that the probability of rolling the face numbered n is proportional to $n^2(n=1,2,3,4,5,6)$. The die is rolled twice, yielding the numbers a and b. The propability that $a < b$ is p, then the value of $[2/p]$ is (where [.] represents the greatest integer function)

33. The papers of 4 students can be checked by any one of the 7 teachers. If the probability that all the 4 papers are checked by exactly 2 teachers is A, then the value of 49 A must be

34. Let $S = \{1,2,3,..n\}$, If X denotes the set of all subsets of S containing exactly two elements, then the value of $\sum_{A \in X} (\min A)$ is ${}^{n+1}C_\lambda$ then $\lambda =$_____

EXERCISE-II

Conditional probability, Multiplication theorem, Total probability and Baye's theorem

LEVEL-I (MAIN)

Single answer type questions

Conditional Probability :

1. If A and B are two events such that $P(A) = 0.3$, $P(B) = 0.6$ and $P(B/A) = 0.5$ then $P(A/B) =$

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{2}{3}$

2. Suppose E and F are two events of a random experiment. If the probability of occurence of E is $\frac{1}{5}$ and the probability of occurence of F given E is $\frac{1}{10}$. Then the probability of non - occurance of atleast one of the events E and F is

1) $\frac{1}{18}$ 2) $\frac{1}{2}$ 3) $\frac{49}{50}$ 4) $\frac{1}{50}$

3. Two coins are tossed. The probability of getting 2 tails if it is known that there is atleast one tail on the coins is

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{5}$

4. Two integers are selected at random from integers 1 to 11. If the sum is even then the probability that both numbers are odd is

1) $\frac{6}{11}$ 2) $\frac{3}{5}$ 3) $\frac{2}{5}$ 4) $\frac{4}{5}$

5. One ticket is selected at random from 50 tickets numbered 00,01,02,....49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals

1) $\frac{1}{4}$ 2) $\frac{5}{14}$ 3) $\frac{1}{50}$ 4) $\frac{1}{14}$

Independent & dependent events :

6. $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{2}$ then A and B are

1) independent 2) dependent 3) exclusive 4) cannot be decided

7. If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{5}$; A and B are independent events then $P\left(\frac{A}{A \cup B}\right) =$

1) $\frac{1}{6}$ 2) $\frac{3}{6}$ 3) $\frac{3}{4}$ 4) $\frac{5}{6}$

8. If A and B are independent events of random experiment such that $P(A \cap B) = \frac{1}{6}$ and $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$, then $P(A) =$

1) $\frac{1}{4}$ 2) $\frac{1}{5}$ 3) $\frac{1}{2}$ 4) $\frac{2}{3}$

9. If A_i ($i = 1, 2, 3, \ldots\ldots n$) are n independent events with $P(A_i) = \frac{1}{1+i}$ for each i, then the probability that none of A_i occurs is

1) $\frac{n-1}{n+1}$ 2) $\frac{n}{n+1}$ 3) $\frac{n}{n+2}$ 4) $\frac{1}{n+1}$

10. If the probability for A to fail in one exam is 0.2 and that for B is 0.3, then the probability that either A or B fails is

1) 0.14 2) 0.6 3) 0.44 4) 0.24

11. At a selection the probability of selection of A is $\frac{1}{7}$ and that of B is $\frac{1}{5}$. The probability that both of them would not be selected is

1) $\frac{1}{35}$ 2) $\frac{24}{35}$ 3) $\frac{11}{35}$ 4) $\frac{1}{24}$

12. If 5 positive integers are taken at random and multiplied together. The probability that the last digit of the product is 2, 4, 6, 8 is

1) $\frac{4^5 - 2^5}{5^5}$ 2) $\frac{4^n + 3^n}{5^5}$ 3) $\frac{1}{5^5}$ 4) $\frac{1}{10^5}$

13. Mr. X is selected for interview for 3 posts. For the first post there are 5 candidates, for the second there are 4 and for the thrid there are 6. If the selection of each candidate is equally likely, find the chance that Mr. X will be selected for atleast one post

1) $\frac{1}{20}$ 2) $\frac{119}{120}$ 3) $\frac{1}{3}$ 4) $\frac{1}{2}$

14. A man throws a die until he gets a number bigger than 3. The probability that he gets 5 in the last throw in

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{2}{3}$ 4) $\frac{3}{5}$

Infinite G.P. Models:

15. Two persons A and B toss a die one after another. The person who throws 6 wins. If A starts then the probability of his winning is

1) $\frac{1}{2}$ 2) $\frac{5}{11}$ 3) $\frac{6}{11}$ 4) $\frac{10}{11}$

16. On a toss of two dice, A throws a total of 5. Then the probability that he will throw another 5 before he throws 7 is

1) $\frac{2}{45}$ 2) $\frac{2}{5}$ 3) $\frac{1}{81}$ 4) $\frac{1}{9}$

17. A biased coin with probability p, $0< p < 1$ of heads is tossed until a head appears for the first time. If the probaility that the number of tosses required is even is 2 / 5, then p equals

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{2}{5}$ 4) $\frac{3}{5}$

Total Probability:

18. An urn A contains 3 white and 5 black balls. Another urn B contains 6 white and 8 black balls. A ball is picked from A at random and then transferred to B. Then a ball is picked at random from B. The probability that it is a white ball is

1) $\frac{14}{40}$ 2) $\frac{15}{40}$ 3) $\frac{16}{40}$ 4) $\frac{17}{40}$

19. A bag contains 4 white and 2 black balls. Another bag contains 3 white and 5 black balls. If one ball is drawn from each bag, the probability that both are white

1) $\frac{2}{3}$ 2) $\frac{3}{8}$ 3) $\frac{1}{4}$ 4) $\frac{25}{48}$

20. A bag contains n coins of which five of them are counterfeit with heads on both sides and the rest are fair coins. If one coin is selected from the bag and tossed, the probability of getting head is 5/8 then $n =$

1) 16 2) 20 3) 24 4) 28

21. Les S be the sample space of the random experiment of throwing simultaneously two unbaised dice with six faces (numbered 1 to 6) and let $E_K = \{(a,b) \in S : ab = k\}$ for $k > 1$. If $P_k = P(E_k)$ for $k > 1$ then the correct, among the following is

1) $p_1 < p_{30} < p_4 < p_6$ 2) $p_{36} < p_6 < p_2 < p_4$

3) $p_1 < p_{11} < p_4 < p_6$ 4) $p_{36} < p_{11} < p_6 < p_4$

Miscellaneous:

22. India plays two hockey matches each with Pakistan and England. In any match, the probabilities of India getting points 0, 1, 2 are 0.4, 0.1, 0.5 respectively. Assuming that the outcomes are independent, the probability of India getting 7 points is

    1) 0.0125 2) 0.05 3) 0.250 4) 0.005

23. An article manufactured by a company consists of two parts A and B. In the process of manufacture 13 out of 104 parts of A and 5 out of 100 parts of B may be defective then the probability that the assembled product is not defective is

    1) $\frac{28}{160}$ 2) $\frac{33}{160}$ 3) $\frac{128}{160}$ 4) $\frac{133}{160}$

24. In a multiple choice question, there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks all the correct answers. The candidate decides to tick answers at random. If he is allowed upto 3 chances to answer the question, the probability that he will get marks in the question is

    1) $\frac{1}{15}$ 2) $\frac{2}{15}$ 3) $\frac{3}{15}$ 4) $\frac{4}{15}$

25. Four numbers are chosen at random (without replacement) from the set $\{1,2,3....,20\}$.

    Statement-1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

    Statement-2 : If the four chosen numbers from an *AP*, then the set of all possible values of common difference is $\{\pm1,\pm2,\pm3,\pm4,\pm5\}$.

    1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

    2) Statement-1 is true, Statement-2 is true; Statement-2 is not correct explanation for Statement-1

    3) Statement-1 is true, Statement-2 is false

    4) Statement-1 is false, Statement-2 is true

26. Three numbers are chosen at random without replacement from $\{1,2,3...,8\}$. The probability that their minimum is 3, given that their maximum is 6, is

    1) $\frac{3}{8}$ 2) $\frac{1}{5}$ 3) $\frac{1}{4}$ 4) $\frac{2}{5}$

27. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

    1) $\frac{17}{3^5}$ 2) $\frac{13}{3^5}$ 3) $\frac{11}{3^5}$ 4) $\frac{10}{3^5}$

Numerical value type questions

28. Two dice are thrown at a time and the sum of the numbers on them is 6. The probability of getting a number 4 on any one of the die is

29. A coin and six faced die, both unbiased, are thrown simultaneously. The probability of getting a head on the coin and an odd number on the die is

30. In the above problem, the probability that both are selected is

31. One bag A contains 4 white and 5 black balls another bag B contains 5 white and 6 black balls. One bag is selected at random and a ball is drawn from it. The probability that it is white is

LEVEL-II (ADVANCED)

Single answer type questions

1. The odds against an event E are 4 : 7 and the odds infavour of another event F independent of E are 6 : 7. Then the probability that exactly one of them occurs if it is given that at least one of them occurred is

a) $\frac{73}{115}$ b) $\frac{5}{7}$ c) $\frac{115}{143}$ d) $\frac{42}{115}$

2. A natural number x is chosen at random from the first 120 natural numbers and it is observed to be divisible by 8, then the probability that it is not divisible by 6 is

a) 1/3 b) 1/4 c) 3/4 d) 2/3

3. A student appears for tests I, II and III. The students is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in test I, II and III are p, q and 1/2 respectively. If the probability that the student is successful is ½, then

a) $p = q = 1$ b) $p = q = 1/2$ c) $p = 1, q = 0$ d) $p = 1, q = 1/2$

4. There are 'a' white and 'b' black balls in an Urn. A ball is drawn and its colour is noted then the ball is not replaced. If a second ball is drawn, the probability that both the balls are white is

a) $\frac{a(a-1)}{(a+b)^2}$ b) $\frac{a^2}{(a+b)^2}$ c) $\frac{a(a-1)}{(a+b)(a+b-1)}$ d) $\frac{a}{b}$

5. A fair coin is tossed until one of the two sides occurs twice in a row then the probability that the number of tosses required is even is

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{1}{6}$ d) $\frac{3}{4}$

6. For a student to qualify, he must pass at least two out of three exams. The probability that he will pass the 1st exam is p. If he fails in one of the exams, then the probability of his passing in the next exam is $p/2$, otherwise it remains the same. Then the probability that he will qualify, is

a) $2p^2-p$ b) $2p^2-2p$ c) $2p^2-p^3$ d) none of these

7. Three faces of a fair die are yellow, two faces red and one blue. The die is tossed three times. The probabiltiy that the colours. yellow, red and blue, appear in the first, second and the third tosses respectively is

a) 1/36 b) 5/36 c) 7/36 d) 1/2

8. A purse contains 2 six sided dice. One is a normal fair die, while the other has 2 ones, 2 threes and 2 fives. A die is picked up and rolled. Because of some secrect magnetic attraction of the unfair die, there is 75% chance of picking the unfair die and 25% chance of picking a fair fie. The die is rolled and shown up the face 3. The probability that a fair die was picked up, is

a) 1/7 b) 1/4 c) 1/6 d) 1/24

9. A bag contains 3 red and 3 green balls and a person draws out 3 at random. Then, he drops 3 blue balls into the bag and again draws out 3 at random. The chance that the 3 later balls being all of different colours, is

a) 15% b) 20% c) 27% d) 40%

10. *A* is one among the 8 horses in a race. *A* is to be ridden by one of the 3 jockeys *P, Q, R*. If *P* rides *A* all the horses are equally likely to win, if *Q* rides A his chances are doubled and if *R* rides *A* his chances are tripled. *A* die is thrown if 1 or 2 or 3 appears then *P* rides *A*, if 4 or 5 appears then *Q* rides *A* other-wise *R* rides *A*. Then the probability that *A* wins is

a) 1/12 b) 3/16 c) 5/24 d) 7/48

11. *A, B* and *C* are contesting in an election for the post of secreatary of a club which does not allow ladies to become members. The probabilities of *A, B* and *C* winning the election are $\frac{1}{3}, \frac{2}{9}$ and $\frac{4}{9}$, respectivley. The probabilities of introducing the clause of admitting lady members to the club by *A, B* and *C* are 0.6, 0.7 and 0.5, respectively. The probability that ladies will be taken as members in the club after the election is

a) $\frac{26}{45}$ b) $\frac{5}{9}$ c) $\frac{19}{45}$ d) none of these

12. It is known that in a bag there are five balls of different colours, out of which one is red. A person who speaks truth 3 times out of 4, draws a ball at random. The probability that he will say that the ball is red, is

a) 3/29 b) 3/4 c) 2/3 d) 1/5

13. A box has 10 coins. Five have heads on both sides. Three have tails on both sides and remaining two are fair. A coin is chosen at random and tossed, then the probability that head appears is

a) 4/5 b) 2/5 c) 3/5 d) 1/5

14. *A* and *B* are two independent witnesses (i.e., there is no collision between them) in a case. The probability that *A* will speak the truth is *x*, and the probability that *B* will speak the truth is *y*. *A* and *B* agree in a certain statement. Then the probability that this statement is true is

a) $\frac{xy}{1-x-y+2xy}$ b) $\frac{x}{1-x-y+2xy}$

c) $\frac{y}{1-x-y+2xy}$ d) $\frac{xy}{1+x+y-2xy}$

15. In a test of multiple choice questions with four choices, an examinee either guesses or recalls or computes the answer. The probability that he makes a guess is 1/3 and the probability that he recalls the answer is 1/6. The probability that his answer is correct, given that he recalls it is 1/8. The probability that he computes the answer to the question, given that he answered correctly is

a) $\frac{11}{24}$ b) $\frac{23}{24}$ c) $\frac{23}{29}$ d) $\frac{24}{29}$

16. Two players A and B throw alternately a pair of dice. A wins if he throws 8 before B throws 7 and B wins if he throws 7 before A throws 8. If A starts the game, then the ratio $P(A) : P(B) =$

a) 31 : 30 b) 30 : 31 c) 30 : 61 d) 31 : 61

17. In the new pattern of IIT-JEE paper there are multiple choice questions. There are 4 possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question then the probability that he was guessing is

a) $\frac{1}{37}$ b) $\frac{37}{40}$ c) $\frac{4}{37}$ d) $\frac{1}{4}$

More than one correct answer type questions

18. In a gambling between Mr. A and Mr. B a machine continues tossing a fair coin until either HT or TT on consecutive throws are obtained for the first time. If it is HT, Mr. A wins and if it is TT, Mr. B wins. Which of the following are true ?

a) Probability of winning Mr.A is $\frac{3}{4}$ b) Probability of winning Mr. B is $\frac{1}{4}$

c) Probability of winning Mr.A if first toss is head is 1

d) probability of winning Mr.A, if first toss is tail is $\frac{1}{2}$

19. Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is 1/5. The probability that bus B will be late is 7/25. The probability that the bus B is late given that bus A is late is 9/10 Then.

a) Probability that neither bus will be late on a particular day is 7/10

b) Probability that bus A is late given that bus B is late is 18/28

c) Probability that at least one bus is late is 3/10

d) Probability that at least one bus is in in time is 4/5

20. A rifleman is firing at a distant target and has only 10% chance of hitting it, then which of the following is/are correct?

a) The minimum number of rounds required in order to have more than 50% chance to hit the target atleast once is 7.

b) When two attempts are made, the probability that both are failures is $\frac{81}{100}$

c) When two attempts are made, the probability of hitting the target at least once is $\frac{21}{100}$

d) When two attempts are made, the probability to hit the target exactly one time is $\frac{18}{100}$

21. I posted a letter to my friend and donot receive a reply. It is known that one out of 10 letters do not reach the destination. It is certain that my friend would have replied if he received the letter. A denotes the event that my friend receives the letter and B that I get a reply then, which of the following is/are correct

a) $P(B)=\frac{81}{100}$ b) $P(A\cap B)=\frac{81}{100}$

c) $P(A/B^{1})=\frac{9}{19}$ d) $P(A\cup B)=\frac{9}{20}$

22. A bag initially contains 1 red and 2 blue balls. An experiment consisting of selecting a ball at random, noticing its colour and replacing it together with an additional ball of the same colour. If three such trails are made, then

a) probability that at least one blue ball is drawn is 0.9

b) probability that exactly one blue ball is drawn is 0.2

c) probability that all the drawn balls are red given that all the drawn balls are of same colour i9s 0.2

d) probability that at least one red ball is drawn is 0.6

Linked comprehension type questions

Passage - I:

A box contains w white, b black balls. Out of the box l balls are lost. A ball is drawn at random from the box and let $P(l)$ denote the probability of the event that the ball drawn is white. Then

23. $P(0)=$

a) $\frac{w}{b}$ b) $\frac{w}{b+w}$ c) $\frac{b}{w}$ d) 0

24. $P(1)=$

a) $\frac{w}{b}$ b) $\frac{w-1}{b+w-1}$ c) $\frac{w}{b+w}$ d) None of these

25. $P(2)=$

a) $\frac{w}{b+w}$ b) $\frac{w-2}{b+w-2}$ c) $\frac{w(w-2)}{(b+w)(b+w-1)}$ d) None of these

Passage - II :

6 letters are written by a person to his 6 friends. The address of each friend is written on 6 envelopes. Letters are put in the addressed envelopes at random then

26. The probability that no letters goes into correct envelope is

a) 0 b) $\frac{53}{144}$ c) $\frac{719}{920}$ d) $\frac{7}{90}$

27. The probability that exactly two letters go into correct envelopes is

a) $\frac{3}{16}$ b) $\frac{53}{144}$ c) $\frac{719}{920}$ d) $\frac{7}{90}$

28. The probability that at least two letters go into wrong envelopes is

a) 0 b) $\frac{3}{16}$ c) $\frac{53}{144}$ d) $\frac{719}{720}$

Matrix matching type questions

29. An urn contains r red balls and b black balls

COLUMN - I	COLUMN - II
A) If the probability of getting two red balls in first two draws (without replacement) is 1/2, then value of r can be	p) 10
B) If the probability of getting two red balls in first two draws (without replacement) is 1/2 and b is an even number, then r can be	q) 3
C) If the probability of getting exactly two red balls in four draws (with replacement) from the urn is 3/8 and b = 10, then r can be	r) 8
D) If the probability of getting exactly n red balls in 2n draws (with replacement) is equal to probability of getting exactly n black balls in 2n draws (with replacement), then the ratio r/b can be	s) 2

Integer answer type questions

30. The chance that a doctor will diagonse a certain disease correctly is 60%. The chance that a patient of the doctor will die by this treatment after correct diagonsis is 40% and the chances of death by wrong diagonsis is 70%. The chance that a patient of the doctor having the particular disease will survive is $\frac{2K}{25}$. Then $K =$

31. In a school, 5% of the boys and 1% of the girls have an IQ of more than 125. 70% of the students are girls. If a student is selected and has an IQ of more than 125, the chance that the student is a girl is $\frac{K}{22}$. Then K is

32. A,B,C and D cut a pack of 52 cards successively in the order given. The person who cuts a spade first receives 350/-. If the expectation of A is P, then $\frac{P}{16}$ is

33. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability P that none of events E_1, E_2, E_3 occurs satisfy the equations $(\alpha - 2\beta)P = \alpha\beta$ and $(\beta - 3\gamma)P = 2\beta\gamma$. All the given probabilities are assumed to lie in the internal (0,1).

Then $\frac{probability\ of\ occurence\ E_1}{probability\ of\ occurence\ of\ E_3} =$

KEY SHEET (PRACTICE SHEET)

EXERCISE-I

LEVEL-I

1) 2	2) 3	3) 1	4) 3	5) 1	6) 3	7) 3	8) 2
9) 2	10) 1	11) 3	12) 4	13) 2	14) 4	15) 2	16) 2
17) 2	18) 1	19) 1	20) 1	21) 2	22) 2	23) 3	24) 1
25) 3	26) 4	27) 1	28) 2	29) 1	30) 2	31) 2	32) 3
33) 1	34) 1	35) 2	36) 3	37) 1	38) 3	39) 4	40) 1
41) 2	42) 1	43) 3	44) 4	45) 1	46) 3	47) 1	48) 1
49) 2	50) 4	51) 3	52) 2	53) 2	54) 2	55) 0.14	56) 0.83
57) 0.96	58) 0.71	59 0.47	60) 0.1	61) 0.76	62) 0.47	63) 0.19	64) 0.38

LEVEL-II

1) d	2) c	3) a	4) d	5) d	6) c	7) a	8) a
9) a	10) c	11) a	12) b	13) c	14) c	15) a	16) d
17) d	18) d	19) ad	20) ab	21) bd	22) ac	23) abcd	24) ab
25) d	26) b	27) a	28) A-r;B-q;C-p;D-s			29) A-q;B-p;C-s;D-p	
30) 6	31) 8	32) 5	33) 6	34) 3			

EXERCISE-II

LEVEL-I

1) 3	2) 3	3) 1	4) 2	5) 4	6) 1	7) 4	8) 3
9) 4	10) 3	11) 2	12) 1	13) 4	14) 2	15) 3	16) 2
17) 1	18) 4	19) 3	20) 2	21) 1	22) 2	23) 4	24) 3
25) 3	26) 2	27) 3	28) 0.4	29) 0.25	30) 0.03	31) 0.45	

LEVEL-II

1) a	2) d	3) c	4) c	5) b	6) c	7) a	8) a
9) c	10) c	11) a	12) d	13) c	14) a	15) d	16) b
17) a	18) abcd	19) abc	20) abd	21) abc	22) abcd	23) b	24) c
25) a	26) b	27) a	28) b	29) A-qr;B-r;C-p;D-pqrs			30) 6
31) 7	32) 8	33) 8					

ADDITIONAL EXERCISE

LECTURE SHEET (ADVANCED)

Single answer type questions

1. A die is thrown six times. It is being known that each time a different digit is shown. The probability that a sum of 12 will be obtained in the first three throws is

a) $\frac{5}{24}$ b) $\frac{25}{216}$ c) $\frac{3}{20}$ d) $\frac{3}{12}$

2. 'A' is targeting to shoot B. 'B','C' are targeting to shoot 'A' Probability of hitting the target by A,B,C are respectively $\frac{2}{3},\frac{1}{2}$ and $\frac{1}{3}$. If each of them shoot once simultaneously and if 'A' is hit, then the probability that 'B' hits the target and 'C' does not is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{3}{4}$

3. A letter chosen at random from each of the letters of the words MATHEMATICS and STATISTICS. The probability of getting same letter is

a) $\frac{1}{10}$ b) $\frac{6}{55}$ c) $\frac{7}{55}$ d) $\frac{1}{4}$

4. Three numbers a,b,c are chosen at random from the set of natural numbers. The probability that $a^2+b^2+c^2$ is divisible by 7 is

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{5}$ d) $\frac{1}{7}$

5. Three are 10 empty seats in a row. Three persons A,B,C wish to occupy. Given that no. two of them sit adjacent the chance that B sits in the middle between A and C is

a) $\frac{1}{7}$ b) $\frac{1}{14}$ c) $\frac{2}{7}$ d) None

6. A card is drawn from a pack. If it is spade a number x is selected from set $A=\{1,2,3,4\}$ otherwise it is selected from set $B=\{3,4,5,6\}$. Given that $\cos^{-1}(\cos x)$ is irrational, the chance that $x \in B$ equals

a) $\frac{3}{10}$ b) $\frac{9}{10}$ c) $\frac{1}{2}$ d) $\frac{1}{3}$

7. In a test students either guesses or copies or knowns the answer to a multiple choice questions with four choices in which exactly one choice is correct. The probability that he makes a guess is $\frac{1}{3}$; The probability that he copies the answer is $\frac{1}{6}$. The Probability that his answer is correct given that he copied it is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that the correctly answered it is

a) $\frac{29}{35}$ b) $\frac{24}{29}$ c) $\frac{1}{7}$ d) $\frac{1}{9}$

8. The probability that a randomly chosen positive divisor of 10^{99} is an integer multiple of 10^{88} is

a) $\left(\frac{3}{25}\right)^2$ b) $\left(\frac{3}{5}\right)^2$ c) $\left(\frac{8}{9}\right)^2$ d) $\left(\frac{8}{9}\right)^4$

9. A single which can be green or red with probability 4/5 and 1/5 respectively by station A and then transmitted to station B. The probability of each station receiving the signal correctly (3/4). If the signal received at station B is green, Then the probability that the original signal was green, is

a) $\frac{1}{5}$ b) $\frac{6}{7}$ c) $\frac{20}{23}$ d) $\frac{9}{23}$

10. In a regular polygon of 12 sides, three vertices are selected to form a triangle. $P(A)$ denotes the probability of getting an acute angle $P(B)$ denotes the probability of getting an obtuse angled triangle $P(C)$ denotes the probability of getting a right angled triangle

i) $P(A)=\frac{2}{11}$ ii) $P(B)=\frac{5}{11}$ iii) $P(C)=\frac{4}{11}$. Then

a) i is correct b) i,ii are correct c) i,ii,iii are correct d) i,ii,iii are wrong

11. An unbiased die, with faces numbered 1, 2,3,4,5,6, is thrown n times, $(n \geq 3)$ and the list of n numbers showing up is noted. What is the probability that, among the numbers 1; 2,3,4,5,6 only three numbers appear in this list?

a) $\frac{{}^6C_3\{3^n+3(2^n)+3\}}{6^n}$ b) $\frac{{}^6C_3\{3^n-3(2^n)+3\}}{6^n}$ c) $\frac{{}^6C_3\{3^n+3(2^n)-3\}}{6^n}$ d) None

12. A man parks his car along the existing n cars standing in a row, his car not being parked at an end. On his return he finds that exactly m of the n cars are still there. The probability that both the cars parked on two sides of his car, have left is

a) $\frac{(n-m)(n-m+1)}{(n-1)(n-2)}$ b) $\frac{(n-m-1)(n-m-2)}{(n-1)(n-2)}$ c) $\frac{(n-m)(n-m-1)}{n(n-1)}$ d) None

More than one correct answer type questions

13. Let a matrix 'A' be selected from set of all matrices of type $n \times n$, each of whose elements is -1 or 0 or 1. Then the probability that

a) "Trace of A is n" is $\left(\frac{1}{3}\right)^n$

b) "Matrix A is symmetric" is $\left[\left(\frac{1}{3}\right)^{\frac{n^2-n}{2}}\right]$

c) "Matrix A is skew-symmetric" is $\left[\left(\frac{1}{3}\right)^{\frac{n^2+n}{2}}\right]$

d) "Matrix A is a diagonal matrix" is $\left(\frac{1}{3}\right)^{n^2-n}$

(Which of the above is/are correct)

14. A player throws an ordinary die with faces numbered 1 to 6. Whenever he throws 1, he gets an additional throw. The probability of obtaining a total score n is

a) $\frac{1}{5}\left(\frac{6^4-1}{6^4}\right)$, if n = 5 b) $\frac{1}{5}\left(\frac{6^5-1}{6^5}\right)$, if n = 5 c) $\frac{1}{30}\left(\frac{6^5-1}{6^5}\right)$, if n = 7 d) $\frac{1}{30}\left(\frac{6^6-1}{6^6}\right)$, if n=7

15. Three unbiased dice are rolled and the numbers on them are noted as a,b,c. The probability that the planes $ax+by+cz=0$, $bx+cy+az=0$, $cx+ay+bz=0$

a) Intersect along a line is 0

b) Intersect at only one point is $\frac{105}{108}$

c) Form a triangular prism is 0

d) Form a triangular prism is $\frac{3}{108}$

Linked comprehension type questions

Passage - I :

Three bags A,B and C are given, each containing 6 marbles. The first bag A has 5 black marbles and 1 white. The second bag B has 4 black marbles and 2 white marbles. The third bag C has 3 black marbles and 3 white marbles marbles. Two marbles are drawn randomly one from each of two different bags (we do not known which bags) and found to be one white and the other black. Let P denote the probability that a marble drawn from the remaining bag is white.

16. If one white and one black marble has been drawn, the probability that bags A and B were selected, is

a) $\frac{6}{25}$ b) $\frac{7}{25}$ c) $\frac{8}{25}$ d) $\frac{9}{25}$

17. If $P=\frac{m}{n}$ (as a reduced fraction), then the value of $(m + n)$ equals

a) 25 b) 33 c) 42 d) 47

Integer answer type questions

18. If P is chosen at random in [0, 5] and the probability that the equation $x^2+px+\frac{p+2}{4}=0$ has real roots is $\frac{\lambda}{10}$ then λ =

19. There are 3 different pairs of shoes in a basket. Now 3 persons come and pick the shoes randomly so that each gets 2. Let p be the probability that no one is able to wear shoes. Then value of $\frac{13p}{4-p}$ is

20. A and B play a game of tennis. The condition of game is as follows. If one score 2 consecutive points after a deuce he wins(If lose of a point is followed by win of a point is called deuce) the chance of server to win a point is $\frac{2}{3}$. If the game is at deuce and A is serving then the probility that A will win the match is $\frac{p}{q}$ (where p & q are relatively prime) then $2p + q$ =

21. Let A,B,C are 3 pair wise and mutually independent events each having probability p and if $P(\overline{C}/A\cap(B\cap C))=\frac{x-p}{y-p}$ then $x^2 + y^2$ =

22. An artilary target may be either at point I with probability $\frac{8}{9}$ or at point II with probability $\frac{1}{9}$. We have 21 shells each of which can be fired either at point I or II. Each shell may hit the target independent of other with probability $\frac{1}{2}$. The no.of shells must be fired at point I to hit the target with maximum probability is n then $\frac{n}{6}$=

PRACTICE SHEET (ADVANCED)

Single answer type questions

1. In a set of games it is 3 to 2 in favour of the winner of the previous game. Then the probability that a person who has won the first game shall win at least 3 out of the next 4 games is

a) $\frac{3^5}{5^4}$ b) $\left(\frac{2}{5}\right)^5$ c) $\left(\frac{3}{5}\right)^5$ d) $\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2$

2. There are 4 defective items in a lot of 15 items. The items are selected one by one at random withourt replacement till the last defective item is drawn. The probability that the 10th item examined is the last defective item is

a) $\frac{1}{65}$ b) $\frac{2}{65}$ c) $\frac{3}{65}$ d) $\frac{4}{65}$

3. The sum of two positive Quantities is equal to 2n. The probability that their product is not less than $\frac{3}{4}$ times their greatest product is

a) 0 b) $\frac{3}{4}$ c) $\frac{1}{4}$ d) $\frac{1}{2}$

4. If 3 numbers are chosen at random with replacement from the first 120 natural numbers then the probability that their AM is 45 is

a) $\frac{9316}{120^3}$ b) $\frac{8956}{120^3}$ c) $\frac{8638}{120^3}$ d) $\frac{9846}{120^3}$

5. A binary operation is selected at random from the set of all binary operations on a set with 10 elements then the probability that it is commutative is

a) $\frac{1}{10^{60}}$ b) $\frac{1}{10^{55}}$ c) $\frac{1}{10^{50}}$ d) $\frac{1}{10^{45}}$

6. Two real numbers x, y are chosen from the interval [0,8] then the probability that $y^2 > 2x$ is

a) $\frac{1}{3}$ b) $\frac{11}{32}$ c) $\frac{11}{64}$ d) $\frac{2}{3}$

7. Two real numbers x and y are chosen at random and are such that $|x|<3$, $|y|<5$. The probability of fraction $\frac{x}{y}$ being positive is

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

8. A man firing at a distant target has 10% chance of hitting the target in one shot. The number of times he must fire at the target to have about 50% chance of hitting the target, is

a) 11 b) 9 c) 7 d) 5

9. A tosses 2 coins and B tosses 3 coins simultaneously. The game is won by the person who throws greater number of heads. In case of tie the game is continued under identical conditions until some one wins the game, then the probability that B wins the game is

a) $\frac{6}{25}$ b) $\frac{19}{25}$ c) $\frac{3}{11}$ d) $\frac{8}{11}$

10. In a tournment, team x plays with each of the other 6 teams once. For each match the probabilities of a win, draw and loss are equal. Then the probability that the team x has more wins than losses is

a) $\frac{98}{243}$ b) $\frac{37}{243}$ c) $\frac{64}{243}$ d) $\frac{68}{243}$

11. If $\{x, y\}$ is a subset of $s = \{1,2,3...,100\}$, then the probability that xy is a multiple of 3 is

a) $\frac{43}{100}$ b) $\frac{61}{150}$ c) $\frac{83}{150}$ d) $\frac{87}{150}$

More than one correct answer type questions

12. If the probability of choosing an integer 'k' out of 2m integers 1,2,3....$2n$ is inversely proportional to k^4 ($1<k<2m$). If x_1 is the probability that choosen number is odd and x_2 is the probability that choosen number is even, then

a) $x_1 > \frac{1}{2}$ b) $x_1 > \frac{2}{3}$ c) $x_2 < \frac{1}{2}$ d) $x_2 < \frac{2}{3}$

13. In a precision bombing attack, there is a 50% chance that any one bomb will strike the target two direct hits are required to destroy the target completely. The number of bombs which should be dropped to give a 99% chance to better of completely destroying the target can be

a) 12 b) 11 c) 10 d) 13

Linked comprehension type questions

Passage - I :

In Meghalaya, its rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability $\frac{1}{2}$, and given that it is not rainy, there will be heavy traffic with probability $\frac{1}{4}$. If its rainy and there is heavy traffic, I arrive late for work with probability $\frac{1}{2}$. On a day when it's not raining and there is no heavy traffic 1 arrive late for work with probability $\frac{1}{8}$. In other situations (rainy and no heavy traffic, not rainy and heavy traffic) the probability of I being is 0.25. You pick a random day.

14. What is the probability that I am late?

a) $\frac{7}{48}$ b) $\frac{11}{48}$ c) $\frac{13}{24}$ d) $\frac{11}{24}$

15. Given that I arrived late at work, what is the probability that it rained on that day?

a) $\frac{6}{11}$ b) $\frac{2}{3}$ c) $\frac{4}{11}$ d) $\frac{5}{11}$

Matrix matching type questions

16. Supoose n (> 6) people are asked successively in a random order and exactly 3 out of n people know the answer. Let p_r denotes the probability that rth person asked is the first to know the answer, then

COLUMN - I	COLUMN - II
A) probability first four do not know the answer	p) $\frac{3(n-3)}{n(n-1)}$
B) p_2	q) $\frac{(n-4)(n-5)(n-6)}{n(n-1)(n-2)}$
C) p_{n-2}	r) $\frac{1}{{}^nC_3}$
D) p_r	s) $\frac{3(n-r)(n-r-1)}{n(n-1)(n-2)}$

Integer answer type questions

17. A bag contains a white and b black balls. Two players A and B alternatively draw a ball from the bag, replacing the ball each time after the draw till one of them draws a white ball and wins the game. A begins the game .If the probability of A winning the game is three times that of B, then 28 $\frac{a}{b}=$

18. If two loaded dice each have the property that 2 or 4 is three times as likely to appear as 1,3,5, or 6 on each roll. When two such dice are rolled, the probability of obtaining a total of 7 is p, then the value of $\left[\frac{1}{P}\right]$ is, where $[x]$ represents the greatest integer less than or equal to x

19. A point $P(x,y)$ is selected at random inside the square with boundaries $x = 0, y = 0$ and $x = 4, y = 4$. The probability that P is inside the parabola $y^2 = x$ is $\frac{k}{9}$ then k is

20. A artillery target may be either at point I with probability $\frac{8}{9}$ or at point II with probability $\frac{1}{9}$. We have 21 shells each of which can be fired either at point I or II. Each shall may hit the target independently of the other shall with probability 1/2. The number of shells must be fired at point I to hit the target with maximum probability is x, then $\frac{x}{2}$ is

21. A die is weighted such that the probability of rolling the face numbered n is proportional to n^2 $(n = 1,2,3,4,5,6)$. The die is rolled twice, yielding the numbers a and b. The probability that $a<b$ is P. Then the value of $\left[\frac{2}{P}\right]$ is where [.] represent the greatest integer function.

KEY SHEET (ADDITIONAL EXERCISE)

LECTURE SHEET (ADVANCED)

1) **c**	2) **a**	3) **c**	4) **d**	5) **b**	6) **b**	7) **b**	8) **a**	9) **c**	10) **a**
11) **b**	12) **c**	13) **ac**	14) **ac**	15) **abc**	16) **b**	17) **b**	18) **6**	19) **2**	20) **4**
21) **5**	22) **2**								

PRACTICE SHEET (ADVANCED)

1) **a**	2) **d**	3) **b**	4) **c**	5) **d**	6) **d**	7) **c**	8) **c**	9) **d**	10) **a**
11) **c**	12) **ab**	13) **abd**	14) **b**	15) **a**	16) **A-q;B-p;C-r;D-s**			17) **2**	18) **7**
19) **3**	20) **6**	21) **5**							