5. DIFFERENTIAL EQUATIONS

SYNOPSIS

Definition : An equation involving one dependent variable, one or more independent variables and the differential coefficients of dependent variable with respect to independent variables is called a differential equation

Order : The order of a differential equation is the order of the highest order derivative appearing in the equation.

Degree : The degree of the differential equation is the power of the highest derivative involved in the differential equation, when the equation has been expressed in the form of a polynomial by eliminating radicals or fractional powers of the derivatives.

Note : The degree of the differential equation $y = \cos \dfrac{dy}{dx}$ and $x = y + \log \dfrac{dy}{dx}$ can not be determined and hence undefined because these equations cannot be expressed as a polynomial equation in $\dfrac{dy}{dx}$.

Types of Differential Equations :

i) **Ordinary Differential Equation :** A differential equation which involves only one independent variable and derivative with respect to it is an ordinary differential equation.

ii) **Partial Differential Equation :** A differential equation which involves two or more independent variables and partial derivatives with respect to either of these independent variables is called partial differential equation.

Formation of Differential Equation :

If an equation $f(x, y, c_1, c_2, c_3, \ldots, c_n) = 0$ has n arbitrary constants (parameters) then by differentiating n times succesively w.r.t x and then eliminate arbitrary constants, we get the differential equation $g(x, y, y^1, y^{11}, \ldots, y^{(n)}) = 0$

Note : The number of arbitrary constants in the general solution of the differential equation is equal to the order of the differential equation.

Solution of Differential Equation : A relation between the variables without the derivatives which satisfies a differential equation is said to be a solution of the equation.

General Solution : The solution which containing as many as arbitary constants as the order of the differential equation is called general solution.

Particular Solution : Solution obtained by giving particular values of the arbitrary constants in the general solution of the differential equation is called a particular solution.

Methods of solving first order and first degree differential equations.

i) **Variables Separable :** If a differential equation can be expressed in the form $\dfrac{dy}{dx} = \dfrac{f(x)}{g(y)}$ or $f(x)dx = g(y)dy$ then it is said to be of type "variables separable" and such equations can be solved by integrating on both sides. The solution is given by $\int f(x)dx = \int g(y)dy + c$ where c is any arbitrary constant.

ii) **Homogeneous Equations :** If the given differential equation is of the form $\dfrac{dy}{dx} = \dfrac{f(x,y)}{g(x,y)}$ where f and g are homogenous function in x, y of equal degree, then the equation is called homogeneous differential equation.

To solve $\dfrac{dy}{dx}=\dfrac{f(x,y)}{g(x,y)}$,

put $y = vx \Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$.

Separate the variables and integrate and replace by $v=\dfrac{y}{x}$ to get the solution :

To solve $\dfrac{dx}{dy}=\dfrac{f(x,y)}{g(x,y)}$,

put $x = vy \Rightarrow \dfrac{dx}{dy}=v+y\dfrac{dv}{dy}$

separate the variables and integrate and replace by $v=\dfrac{x}{y}$ to get the solution.

iii) Non-Homogeneous Equations : If the equation is of the form $\dfrac{dy}{dx}=\dfrac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$ where a_1, b_1, c_1 and a_2, b_2, c_2 are constants. Then the equation is called non-homogeneous.

Type - 1) If $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}$ then put $a_1x + b_1y = z$ to solve the equation.

Type - 2) If $\dfrac{a_1}{a_2}\neq\dfrac{b_1}{b_2}$ then put $x = X + h$, $y = Y + k$ then $\dfrac{dY}{dX}=\dfrac{a_1X+b_1Y}{a_2X+b_2Y}$ and hence it can be solved

On solving $a_1h + b_1k + c_1 = 0$ and $a_2h + b_2k + c_2 = 0$ we get h and k

Type - 3) If $a_2 + b_1 = 0$ then the solution is obtained by proper grouping of the terms.

iv) Linear Differential Equation : A differential equation is of the form $\dfrac{dy}{dx}+P(x)\,y=Q(x)$ is called a linear differential equation of first order in y where $P(x)$ and $Q(x)$ are functions of 'x' alone or constants.

Integrating factor (I.F) = $e^{\int p dx}$ and the solution is $y\cdot e^{\int p\,dx}=\int Q(x)e^{\int Pdx}\,dx+c$.

If the differential equation is of the form $\dfrac{dx}{dy}+P(y)x=Q(y)$ is called a linear differential equation of first order in 'x' where $P(y)$ and $Q(y)$ are functions of y alone or constants. I.F $=e^{\int p\,dy}$ and the solution is $x\cdot e^{\int p\,dy}=\int Q(y)\,e^{\int p\,dy}dy+c$

v) Bernoulli's Equation : An equation of the form $\dfrac{dy}{dx}+P(x)\cdot y=Q(x)\cdot y^n$ where $P(x)$ and $Q(x)$ are functions of x only is called a Bernoulli's equation.

Divide both sides with y^n we get $y^{-n}\dfrac{dy}{dx}+P\cdot y^{-n+1}=Q$ put $y^{-n+1}=v$

$$(-n+1)y^{-n}\frac{dy}{dx}=\frac{dv}{dx}\Rightarrow\frac{1}{-n+1}\frac{dv}{dx}+Pv=Q$$

$\dfrac{dv}{dx}+(1-n)Pv=(1-n)Q$ which is a linear differential equation in v and hence it can be solved.

Important Points :

a) $d(x+y) = dx + dy$

b) $d(xy) = ydx + xdy$

c) $d\left(\dfrac{x}{y}\right) = \dfrac{ydx - xdy}{y^2}$

d) $d(x^2 + y^2) = 2(xdx + ydy)$

e) $d\left(\dfrac{y}{x}\right) = \dfrac{xdy - ydx}{x^2}$

f) $d(\log(xy)) = \dfrac{ydx + xdy}{xy}$

g) $d\left(\log\dfrac{y}{x}\right) = \dfrac{xdy - ydx}{xy} = \dfrac{dy}{y} - \dfrac{dx}{x}$

h) $d\left(\log\dfrac{x}{y}\right) = \dfrac{ydx - xdy}{xy} = \dfrac{dx}{x} - \dfrac{dy}{y}$

i) $d\left(Tan^{-1}\dfrac{y}{x}\right) = \dfrac{xdy - ydx}{x^2 + y^2} = \dfrac{\dfrac{xdy - ydx}{x^2}}{1 + \dfrac{y^2}{x^2}} = \dfrac{d\left(\dfrac{y}{x}\right)}{1 + \dfrac{y^2}{x^2}}$

j) $d\left(\log(x^2 + y^2)\right) = 2\left(\dfrac{xdx + ydy}{x^2 + y^2}\right)$

k) $d(\sqrt{x^2 + y^2}) = \dfrac{xdx + ydy}{\sqrt{x^2 + y^2}}$

l) $d\left(-\dfrac{1}{xy}\right) = \dfrac{xdy + ydx}{x^2y^2}$

DIFFERENTIAL EQUATION OF DIFFERENT FAMILIES OF CURVES

Equation of a Curve	Differential Equation
1. $y = c_1 e^{\alpha x} + c_2 e^{\beta x}$ (c_1, c_2 are arbitrary constants)	1. $\dfrac{d^2y}{dx^2} - (\alpha + \beta)\dfrac{dy}{dx} + (\alpha\beta)y = 0$
2. $y = c_1 e^{\alpha x} + c_2 \cdot e^{\beta x} + c_3 e^{rx}$ (c_1, c_2, c_3 are arbitrary constants)	2. $\dfrac{d^3y}{dx^3} - (\alpha + \beta + \gamma)\dfrac{d^2y}{dx^2} + (\alpha\beta + \beta\gamma + \gamma\alpha)\dfrac{dy}{dx} - (\alpha\beta\gamma)y = 0$
3. $y = e^{\alpha x}(c_1 x + c_2)$ (c_1, c_2 are arbitrary constants)	3. $\dfrac{d^2y}{dx^2} - 2\alpha\dfrac{dy}{dx} + \alpha^2 y = 0$
4. $y = e^{\alpha x}(c_1 + c_2 x + c_3 x^2)$ (c_1, c_2, c_3 are arbitrary constants)	4. $\dfrac{d^3y}{dx^3} - 3\alpha\dfrac{d^2y}{dx^2} + 3\alpha^2\dfrac{dy}{dx} - \alpha^3 y = 0$
5. $y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$ (c_1, c_2 are arbitrary constants)	5. $\dfrac{d^2y}{dx^2} - 2\alpha\dfrac{dy}{dx} + (\alpha^2 + \beta^2)y = 0$

AIEEE SYNOPSIS :

1. Any curve which cuts every member of curve at right angle is called an orthogonal trajectory of the family.

   Procedure for finding the orthogonal trajectory :

   i) Let $f(x, y, c) = 0$ be the equation, where 'c' is an arbitrary parameter.

   ii) Differentiate the given equation w.r.t. x and then eliminate 'c'.

   iii) Replace $\frac{dy}{dx}$ by $\frac{-dx}{dy}$ in the equation obtained in (ii)

   iv) solve the D.E. in (iii).

LECTURE SHEET

EXERCISE-I

Order and degree of the D.E and Formation of D.E

LEVEL-I (MAIN)

Single answer type questions

1. The degree,order of the differential equation $y_2^{3/2} - y_1^{1/2} - 4 = 0$ is
   1) 6, 2   2) 3, 2   3) 2, 5   4) 4, 6

2. Order of the differential equation of the family of all concentric circles centred at (h, k) is
   1) 1   2) 2   3) 3   4) 4

3. The order, degree of the differential equation $x = 1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + ...$ is
   1) 3,1   2) 4,2   3) 1,1   4) not defined

4. The degree and order of the differential equation of the family of all parabolas whose axis is X-axis are respectively
   1) 2, 1   2) 1, 2   3) 3, 2   4) 2, -3

5. The differential equation whose solution is $Ax^2+By^2 = 1$, where A and B are arbitrary constants is of
   1) first order and second degree
   2) first order and first degree
   3) second order and first degree
   4) second order and second degree

6. The order, degree of the differential equation satisfying the relation
   $\sqrt{1+x^2} + \sqrt{1+y^2} = \lambda(x\sqrt{1+y^2} - y\sqrt{1+x^2})$ *is*
   1) 1,1   2) 2,1   3) 3,2   4) 0,1

Formation of Differential Equation :

7. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y=2$ is
   1) $(x-2)(y^1)^2 = 25 - (y-2)^2$ 2) $(y-2)(y^1)^2 = 25 - (y-2)^2$
   3) $(y-2)^2(y^1)^2 = 25 - (y-2)^2$ 4) $(x-2)^2(y^1)^2 = 25 - (y-2)^2$

8. The differential equation of all non vertical lines in a plane is

1) $\frac{d^2y}{dx^2}=0$ 2) $\frac{d^2x}{dy^2}=0$ 3) $\frac{dy}{dx}=0$ 4) $\frac{dx}{dy}=0$

9. The differential equations of hyperbolas with coordinate axis as asymptotes is

1) $x\frac{dy}{dx}-y=0$ 2) $x\frac{dy}{dx}+y=0$ 3) $\frac{dy}{dx}+xy=0$ 4) $x\frac{dy}{dx}=y^2$

10. The D.E. of the family of parabolas having vertices at the origin and foci on y-axis is

1) $\frac{dy}{dx}=\frac{2y}{x}$ 2) $\frac{dy}{dx}=\frac{y}{2x}$ 3) $\frac{dy}{dx}=\frac{y}{x}$ 4) $\frac{dy}{dx}=\frac{2y}{x^2}$

11. The differential equation whose solution is $y = Ax^5 + Bx^4$ is

1) $x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}+20y=0$

2) $x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}-20y=0$

3) $x^2\frac{d^2y}{dx^2}-8x\frac{dy}{dx}+20y=0$

4) $x^2\frac{d^2y}{dx^2}-8x\frac{dy}{dx}-20y=0$

12. $y = Ae^x + Be^{2x} + Ce^{3x}$ satisfies the differential equation

1) $y_3 - 6y_2 + 11y_1 - 6y = 0$

2) $y_3 + 6y_2 + 11y_1 + 6y = 0$

3) $y_3 + 6y_2 - 11y_1 + 6y = 0$

4) $y_3 - 6y_2 - 11y_1 + 6y = 0$

13. The D.E. of the family of straight lines $y = mx + \frac{a}{m}$ where m is the parameter is

1) $x\frac{dy}{dx} = a$

2) $(x-y)\frac{dy}{dx} = a$

3) $x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = a$

4) $x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + a = 0$

14. The differential equation of family of curves $r^2 = a^2\cos 2\theta$ where 'a' is an arbitrary constant is

1) $\frac{dr}{d\theta}=r\tan 2\theta$

2) $\frac{dr}{d\theta}=r\cot 2\theta$

3) $\frac{dr}{d\theta}\cos 2\theta + r\sin 2\theta = 0$

4) $\frac{dr}{d\theta}\sin 2\theta + r\cos 2\theta = 0$

15. The differential equation of the family $y = ae^x + bx\,e^x + cx^2\,e^x$ of curves, where a, b, c are arbitrary constants, is :

1) $y''' + 3y'' + 3y' + y = 0$

2) $y''' + 3y'' - 3y' - y = 0$

3) $y''' - 3y'' + 3y' - y = 0$

4) $y''' - 3y'' - 3y' + y = 0$

16. The differential equation of family of curves $x^2+y^2-2ay=0$ where 'a' is arbitrary constant is

1) $(x^2+y^2)\,y_1 = 2xy$ 2) $2(x^2+y^2)\,y_1 = xy$ 3) $(x^2-y^2)\,y_1 = 2xy$ 4) $2(x^2-y^2)\,y_1 = xy$

17. The differential equation of the family of parabolas with focus at origin and X–axis as axis is

1) $y\left(\frac{dy}{dx}\right)^2+4x\frac{dy}{dx}=4y$
2) $-y\left(\frac{dy}{dx}\right)^2=2x\frac{dy}{dx}-y$
3) $y\left(\frac{dy}{dx}\right)^2+y=2xy\frac{dy}{dx}$
4) $y\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}+y=0$

18. The differential equation of the system of curves given by $\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1$ (λ is arbitrary constant) is

1) $x^2-xy\frac{dy}{dx}=a^2$
2) $x^2-\frac{xy}{(\frac{dy}{dx})}=a^2$
3) $x^2+xy=a^2\frac{dy}{dx}$
4) $x^2-xy=a^2\frac{dy}{dx}$

Numerical value type questions

19. Differential equalion family of curves $y=C_1e^{2x}+C_2e^{-x}$ is $y_2+ly_1+my=0$ then $\frac{l}{m}=$ .....

20. Differential equation of family of curves $y=e^{-x}(C_1x+C_2)$ is $y_2+\lambda y_1+\mu=0$. Then minimum value of polynomial f in. Whose roots are λ and μ is ....

21. If $y=\sin(3\sin^{-1}x)$ and a, b, c, d are such that $a(1-x^2)\frac{d^2y}{dx^2}+bx\frac{dy}{dx}+cy=d$.

Then $a+b+c$ value is ......

LEVEL-II (ADVANCED)

Single answer type questions

1. A normal at any point (x, y) to the curve $y=f(x)$ makes a triangle of unit area with the coordinate axes, then equation of the curve is

a) $y^2-x^2\left(\frac{dy}{dx}\right)^2=4\frac{dy}{dx}$
b) $x^2-y^2\left(\frac{dy}{dx}\right)^2=2\frac{dy}{dx}$
c) $x+y\frac{dy}{dx}=y$
d) $x^2+2xy\frac{dy}{dx}+y^2\left(\frac{dy}{dx}\right)^2=2\frac{dy}{dx}$

2. If $y=c_1e^{2x}+c_2e^{x}+c_3e^{-x}$ satisfies the differential equation $\frac{d^3y}{dx^3}+a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$ then $\frac{a^3+b^3+c^3}{abc}$ is equal to

a) $-\frac{1}{4}$
b) $-\frac{1}{2}$
c) 0
d) $\frac{1}{2}$

3. The order and degree of the differential equation $e^{\frac{d^3y}{dx^3}}-x\frac{d^2y}{dx^2}+y=0$ ....

a) 3, 1
b) 3, 2
c) 3, not defined
d) 2, 3

4. The differential equation having $y = (\sin^{-1}x)^2 + A\cos^{-1}x + B$ where A and B are arbitary constants is

a) $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ b) $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$

c) $(1+x^2)\frac{dy}{dx} + x\frac{d^2y}{dx^2} = 0$ d) $(1+x^2)\frac{d^2y}{dx^2} + 3y = 0$

5. If $y = \frac{x}{\log|cx|}$ where c is an arbitrary constant is the general solution of the differential equation $\frac{dy}{dx} = \left(\frac{y}{x}\right) + \phi\left(\frac{y}{x}\right)$ then the function $\phi\left(\frac{x}{y}\right)$ is ....

a) $\frac{x^2}{y^2}$ b) $-\frac{x^2}{y^2}$ c) $\frac{y^2}{x^2}$ d) $-\frac{y^2}{x^2}$

6. The degree of the differential equation satisfying $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ is

a) 1 b) 2 c) 3 d) 4

7. Tangent to a curve intersects y-axis at a point P. A line perpendicular to this tangent through P passes through another point (1, 0). The differential equation of the curve is

a) $y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$ b) $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

c) $y\frac{dx}{dy} + x = 1$ d) $x\frac{dy}{dx} + y = 5$

8. If $\phi(x) = \phi^1(x)$ and $\phi(1) = 2$ then $\phi(3) =$

a) e^2 b) $2e^2$ c) $3e^2$ d) $2e^3$

More than one correct answer type questions

9. The differential equation associated to the primitive $y = a + be^{5x} + ce^{-7x}$ is $y_3 + Ay_2 + By_1 = 0$ then

a) $A = 2$ b) $A = -2$ c) $B = 35$ d) -35

10. For the differential equation whose solution is $y = c_1\cos(x + c_2) - c_3e^{(-x+c_4)} + c_5\sin x$ where c_1, c_2, c_3, c_4, c_5 are arbitrary constants is of

a) order 3 b) order 5 c) degree 1 d) degree 3

11. For the differential equation whose solution is $(x - h)^2 + (y - k)^2 = a^2$ is (a is a constant)

a) order is 2 b) order is 3 c) degree 2 d) degree 3

12. The solution of the differential equation $[x \cot y + \log(\cos x)]\,dy + [\log \sin y - y \tan x]\,dx = 0$ is $(\sin y)^A (\cos x)^B = C$

a) $A = x$ b) $B = x$ c) $A = y$ d) $B = y$

Linked comprehension type questions

Passage - I :

A non - negative differentiable function $f(x)$ is defined on [0,1] with $f(1) = 1$. For each $a \in (0,1)$, the line $x = a$ divides the area bounded by $y = f(x)$ on the coordinate axes in two parts. The area bounded on the left (having y–axis as one boundary) is denoted by A on the other is denoted by B. It is known that $A-B = 2f(a) + 3a + b \ \forall \ a \in (0,1)$ where b is a constant independent of a

13. The function satisfies the DE

a) $\frac{dy}{dx} - 2y + 3 = 0$ b) $\frac{dy}{dx} - y + \frac{3}{2} = 0$ c) $\frac{dy}{dx} - 3y + \frac{5}{2} = 0$ d) none

14. The function $f(x)$ is

a) $f(x) = \frac{3}{2}(1 - e^{x-1})$ b) $f(x) = \frac{5}{2}(1 - e^{x-1})$ c) $f(x) = \frac{5}{3}(1 - e^{x-1})$ d) none

15. The value of b is

a) $\frac{5}{3e} - 3$ b) $\frac{5}{2e} - 3$ c) $\frac{3}{2e} - 3$ d) None

Matrix matching type questions

16.

COLUMN - I	COLUMN - II
A) All parabolas whose axis is x-axis	p) order 1
B) family of curves $y = a(x+a)^2$ where a is an arbitary constant	q) order 2
C) $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$	r) degree 1
D) family of curve $y^2 = 2c(x + \sqrt{c}), c > 0$	s) degree 3

17. If y = $\sin(3\sin^{-1}x)$ and a, b, c, d are numerical quantities such that $a(1-x^2)\frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = d$. Then

CLOUMN-I	COLUMN-II
A) The value of a is	p) -1
B) The value of b is	q) 0
C) The value of c is	r) 1
D) The value of $a+b$ is	s) 9

Integer answer type questions

18. If $y = e^{4x} + 2e^{-x}$ satisfies the equation $y_2 + Ay_1 + By = 0$ then $A-B$ is

19. If $y = 2x + c$ is a solution of $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3 - \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)$ then c =

20. The degree of the differential equation satisfying $y=c(x-c)^2$is

21. If the DE of all stright lines which are at a fixed distance of 10 unit from origin is $(y-xy_1)^2 = A(1+y_1)^2$, then $\sqrt{A-19}$

22. If y = $(\text{Sin}^{-1}x)^2 + A\cos^{-1}x+B$, where A and B are arbitrary constants, satisfies the DE $(1-x^2)\dfrac{d^2y}{dx^2} = x\dfrac{dy}{dx}+k$, where $k=$

EXERCISE-II

Variable Separables, Homogeneous and Non-Homogeneous Equations

LEVEL-I (MAIN)

Single answer type questions

1. If $dx + dy = (x+y)(dx-dy) \Rightarrow \log(x+y) =$

   1) $x+y+c$ 2) $x+2y+c$ 3) $x-y+c$ 4) $2x+y+c3$

2. The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{xy+y}{xy+x}$ is

   1) $x+y=\log\left(\dfrac{Cy}{x}\right)$ 2) $x+y=\log(Cxy)$ 3) $x-y=\log\left(\dfrac{Cx}{y}\right)$ 4) $y-x=\log\left(\dfrac{Cx}{y}\right)$

3. The solution of $(e^x+1)ydy + (y+1)dx = 0$ is

   1) $e^{x+y} = c(y+1)(e^x+1)$ 2) $e^{x+y} = c(y+1)(e^x-1)$
   3) $e^{x+y} = c(y-1)(e^x+1)$ 4) $e^{x+y} = c(y-1)(e^x-1)$

4. The solution of $xd(xy) = \left(\dfrac{f(xy)}{f^1(xy)}\right)dx$ is

   1) $f(xy) = c$ 2) $x f(xy) = c$ 3) $y f(xy) = c$ 4) $cx = f(xy)$

5. The solution of $\dfrac{dy}{dx} = \dfrac{x\log x^2 + x}{\sin y + y\cos y}$ is

   1) $y\sin y = x^2\log x+c$ 2) $y\sin y = x^2+c$
   3) $y\sin y = x^2+\log x+c$ 4) $y\sin y = x\log x+c$

6. The solution of $y-x\left(\dfrac{dy}{dx}\right) = a\left(y^2+\dfrac{dy}{dx}\right)$ is

   1) $(x+a)(y+a)=cy$ 2) $(x+a)(y-a)=cy$ 3) $(x+a)\left(y+\dfrac{1}{a}\right)=cy$ 4) $(x+a)(1-ay)=cy$

7. The solution of $y\,dx-xdy+3x^2y^2e^{x^3}dx = 0$ is

   1) $\dfrac{y}{x}+e^{x^3}=c$ 2) $\dfrac{y}{x}-e^{x^3}=c$ 3) $\dfrac{x}{y}+e^{x^3}=c$ 4) $\dfrac{x}{y}-e^{x^3}=c$

8. The solution of $\dfrac{dy}{dx} = 2xy-3y+2x-3$ is

   1) $e^{x^2}+3x = c(y+1)$ 2) $e^{x^2}-3x = c(2y+1)$ 3) $e^{x^2}-3x = c(y-1)$ 4) $e^{x^2-3x} = c(y+1)$

9. The solution of $\cos y \log(\sec x + \tan x)dx = \cos x \log(\sec y + \tan y)dy$ is

1) $[\log(\sec x + \operatorname{Tan} x)]^2 - [\log(\sec y + \operatorname{Tan} y)]^2 = c$

2) $[\log(\sec x + \operatorname{Tan} x)]^2 + [\log(\sec y + \operatorname{Tan} y)]^2 = c$

3) $[\log(\sec x - \operatorname{Tan} x)]^2 - [\log(\sec y - \operatorname{Tan} y)]^2 = c$

4) $[\log(\sec x - \operatorname{Tan} x)]^2 + [\log(\sec y - \operatorname{Tan} y)]^2 = c$

10. Solution of the differential equation $2y \sin x\frac{dy}{dx} = 2\sin x \cos x - y^2\cos x$ satisfying $y(\pi/2) = 1$ is given by

1) $y^2 = \sin x$ 2) $y = \sin^2 x$ 3) $y^2 = \cos x + 1$ 4) $y^2 \sin x = 4\cos^2 x$

11. The solution of the differential equation $(1+e^x)y\frac{dy}{dx} = e^x$ when $y = 1$ and $x = 0$ is

1) $e^{y^2/2} = \sqrt{e}(1+e^x)$ 2) $e^{y^2/2} = e(1+e^x)$

3) $2.e^{y^2/2} = \sqrt{e}(1+e^x)$ 4) $2.e^{y^2} = \sqrt{e}(1+e^x)$

12. The solution of $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is

1) $y = mx + c$ 2) $y = mx + a\sqrt{1+m^2}$

3) $y = mx$ 4) $y = a\sqrt{1+m^2}$

13. The solution of $(x^2 - y^2x^2)(dy/dx) + (y^2 + x^2y^2) = 0$ is

1) $x + \frac{1}{x} + y + \frac{1}{y} + c = 0$ 2) $x - \frac{1}{x} + y - \frac{1}{y} = c$

3) $x + \frac{1}{x} - y - \frac{1}{y} + c = 0$ 4) $x - \frac{1}{x} - y - \frac{1}{y} = c$

14. If $y = y(x)$ is the solution of the differential equation $\left(\frac{2+\sin x}{y+1}\right)\frac{dy}{dx} + \cos x = 0$ with $y(0)=1$, then $y\left(\frac{\pi}{2}\right) =$

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) 1 4) $\frac{4}{3}$

15. $\frac{dy}{dx} + 2x\tan(x-y) = 1 \Rightarrow \sin(x-y)$

1) Ae^{-x^2} 2) Ae^{2x} 3) Ae^{x^2} 4) Ae^{-2x}

16. The solution of $\cos y + (x \sin y - 1)\frac{dy}{dx} = 0$ is

1) $\tan y + \sec y = cx$ 2) $x \sec y = \tan y + c$

3) $x \sec y + \tan y = c$ 4) $\tan y - \sec y = cx$

17. The solution of the differential equation $\frac{dy}{dx} = \sin(x+y)\tan(x+y) - 1$:

1) $x + \operatorname{cosec}(x+y) = c$ 2) $\operatorname{cosec}(x+y) + \tan(x+y) = x + c$

3) $x + \tan(x+y) = c$ 4) $x + \sec(x+y) = c$

18. The solution of $\sin^{-1}\left(\frac{dy}{dx}\right) = y + x$ is

1) $\mathrm{Tan}\,(x+y) - \sec(x + y) = x + c$
2) $\mathrm{Tan}\left(\frac{x+y}{2}\right) = c$
3) $\mathrm{Tan}\,(x + y) = x + c$
4) $\mathrm{Tan}\,(x + y) - \sec(x + y) = c$

19. The solution of $\tan y\frac{dy}{dx} = \sin(x+y)+\sin(x-y)$ is :

1) $\sec y = 2\cos x + c$
2) $\sec y = -2\cos x + c$
3) $\tan y = -2\cos x + c$
4) $\sec^2 y = -2\cos x + c$

20. The solution of $(x^2y^3 + x^2)dx + (y^2x^3 + y^2)dy = 0$ is

1) $(x^3+1)(y^3+1) = c$
2) $(x^3-1)(y^3-1) = c$
3) $(x^3-1)(y^3+1) = c$
4) $(x^3+1)(y^3-1) = c$

21. The solution of $\frac{dy}{dx} = (4x+9y+1)^2$ is

1) $3(4x+9y+1) = \mathrm{Tan}(6x+c)$
2) $3(4x+9y+1) = 2\mathrm{Tan}(6x+c)$
3) $3(4x-9y-1) = 2\mathrm{Tan}(6x+c)$
4) $3(4x+9y-1) = \mathrm{Tan}(6x+c)$

22. A curve passes through the point (5, 3) and at any point (x, y) on it the product of its slope and the ordinate is equal to abscissa of the curve is

1) parabola 2) ellipse 3) hyperbola 4) circle

23. A curve C has the property that if the tangent drawn at any point 'P' on C meets the coordinate axes at A and B, and P is midpoint of AB. If the curve passes through the point (1, 1) then the equation of the curve is

1) $xy = 2$ 2) $xy = 3$ 3) $xy = 1$ 4) $xy = 4$

24. The normal line to a given curve at each point (x, y) on the curve passes through the point (3, 0). If the curve contains the point (3, 4) then its equation is

1) $x^2+y^2+6x-7=0$
2) $x^2+y^2-6x-7=0$
3) $x^2+y^2-6x-25=0$
4) $x^2+y^2+6x+25=0$

25. The solution of $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$ is

1) $y \cos x + x \cos y + xy = k$
2) $y \sin x + x \sin y + xy = k$
3) $\sin x + \sin y + xy = k$
4) $xy (\sin x + \sin y + 1) = k$

26. The solution of $\frac{dy}{dx} = \frac{px+q}{rx+s}$ represents a parabola when

1) $p = 0, q = 0$ 2) $r = 0, s = 0$ 3) $p = 0, q \neq 0$ 4) $r = 0, s \neq 0$

27. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is

1) $I - \frac{k(T-t)^2}{2}$ 2) e^{-kT} 3) $T^2 - \frac{I}{k}$ 4) $I - \frac{kT^2}{2}$

28. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ell n2)$ is equal to

1) 13 2) –2 3) 7 4) 5

29. The population p(t) at a time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population become zero is

1) 2 ln 18 2) ln 9 3) $\frac{1}{2}\ell n18$ 4) ln 18

30. The solution of $y(xy+1)dx + x(1+xy+x^2y^2)dy = 0$ is

1) $2x^2y^2 \log y - 2xy - 1 = cx^2y^2$ 2) $2x^2y^2 \log y + 2xy - 1 = cx^2y^2$
3) $2xy \log x + 2xy + 1 = cx^2y^2$ 4) $2xy \log y - 2xy - 1 = cx^2y^2$

[**Hint :** Put $xy = t$]

31. If $\frac{dy}{dx} = \frac{y + x\tan\frac{y}{x}}{x} \Rightarrow \sin\frac{y}{x} =$

1) cx^2 2) cx 3) cx^3 4) cx^4

32. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\varphi(y/x)}{\varphi'(y/x)}$ is

1) $x\varphi\left(\frac{y}{x}\right) = k$ 2) $\varphi\left(\frac{y}{x}\right) = kx$ 3) $y\varphi\left(\frac{y}{x}\right) = k$ 4) $\varphi\left(\frac{y}{x}\right) = ky$

33. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is

1) $y\log(x/y) = cx$ 2) $x\log(y/x) = cy$ 3) $\log(y/x) = cx$ 4) $\log(x/y) = cy$

34. The solution of $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is

1) $e^{y/x} = kx$ 2) $e^{y/x} = ky$ 3) $e^{-y/x} = kx$ 4) $e^{-y/x} = ky$

35. The solution of $y^2dx + (x^2 - xy + y^2)dy = 0$ is

1) $\tan^{-1}\left(\frac{x}{y}\right) + \log y + c = 0$ 2) $2\tan^{-1}\left(\frac{x}{y}\right) + \log x + c = 0$
3) $\log(y + \sqrt{x^2+y^2}) + \log y + c = 0$ 4) $\sinh^{-1}\left(\frac{x}{y}\right) + \log y + c = 0$

36. The solution of $\frac{dy}{dx} = \frac{2xy}{x^2+y^2}$ is

1) $x^2+y^2 = cy$ 2) $y^2-x^2 = cy$ 3) $y^2-x^2 = cx$ 4) $x^2-y^2 = \frac{c}{y}$

37. The solution of $x \text{ Cos}\left(\frac{y}{x}\right)(ydx + xdy) = y\text{Sin}\left(\frac{y}{x}\right)(xdy \pm ydx)$ is

1) $cxy \text{ Cos}\left(\frac{2y}{x}\right) = 1$ 2) $cxy \text{ Cos}\left(\frac{x}{y}\right) = 1$ 3) $cxy \text{ Cos}\left(\frac{y}{x}\right) = 1$ 4) $cxy \text{ Cos}\left(\frac{y}{x}\right) = 2$

38. The solution of $x \sin\left(\frac{y}{x}\right)\frac{dy}{dx} = y\sin\left(\frac{y}{x}\right) - x$ is

1) $e^{\frac{y}{x}} = cy$ 2) $e^{\cos y/x} = cx$ 3) $e^{\frac{x}{y}} = cx$ 4) $e^{\frac{2y}{x}} = cx$

39. The solution of $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ is

1) $e^{y-x} = c(x+y)$ 2) $e^{y-x} = c\,(x-y)$ 3) $e^{y+x} = c\,(x+y)$ 4) $e^{y-x} = c(2x+y)$

40. The solution of the differential equation $\frac{dy}{dx} = \frac{x-2y+1}{2x-4y}$

1) $(x-2y)^2 + 2x = C$ 2) $(x-2y)^2 + x = C$ 3) $(x-2y) + 2x^2 = C$ 4) $(x-2y) + x^2 = C$

41. To change $(3x + 4y + 5) - (2x + 3y + 4)\frac{dy}{dx} = 0$ into homogeneous equation, origin is shifted to (h, k) then $h + k =$

1) 3 2) 1 3) -2 4) -1

Numerical value type questions

42. If $y(x)$ is the solutions of the differential equation $(x+2)\frac{dy}{dx} = x^2+4x-9,\ x \neq -2$ and $y(0) = 0$. Then $y(-4) =$ .....

43. The curve satisfying the differential equation $(x^2-y^2)\,dx + 2xy\,dy = 0$ and passes through point $(1, 1)$ is a circle of area ....

44. Eccentricity of the hyperbola satisfying the differential equation $2xy\frac{dx}{dy} = x^2 + y^2$ and passing through the point $(2, 1)$ is ....

LEVEL-II (ADVANCED)

Single answer type questions

1. The equation of the curve satisfying the differential equation $y_2(x^2 + 1) = 2xy_1$ and passing through the point $(0, 1)$ and having slope of tangent at $x = 0$ as 3 is

a) $y = x^2+3x+2$ b) $y^2 = x^2+3x+1$ c) $y = x^3+3x+1$ d) $y^2 = x^2-3x-1$

2. A curve $y = f(x)$ passes through the point $P(1, 1)$. The normal to the curve at point P is a $(y-1)+(x-1)=0$. If the slope of the tangent at any point on the curve is proportional to the ordinate at that point, then the equation to the curve is

a) $y=e^{ax}-1$ b) $y=e^{ax}+1$ c) $y=e^{ax}+a$ d) $y=e^{a(x-1)}$

3. A spherical rain drop evaporates at a rate proportional to its surface area at any instant t. The differential equation giving the rate of change of the radius of the raindrop is

a) $\frac{d^2r}{dt^2}+2r=0$ b) $\frac{d^2r}{dt^2}-3r=0$ c) $\frac{d^2r}{dt^2}=0$ d) $\frac{d^2r}{dt^2}-2r=0$

4. The solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

a) $-\left[\sqrt{1+x^2}+\frac{1}{2}\ln\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1+x^2}+1}\right|\right]=\sqrt{1+y^2}+c$

b) $\sqrt{1+x^2}+\frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1-x^2}+1}\right|=\sqrt{1-y^2}+c$

c) $\sqrt{1-x^2}+\frac{1}{2}\ln\left|\frac{\sqrt{1-x^2}+1}{\sqrt{1+x^2}-1}\right|=\sqrt{1-y^2}+c$

d) $\log\frac{\sqrt{1+x^2}+1}{\sqrt{1-x^2}-1}=\sqrt{1+x}+\sqrt{1-y^2}+c$

5. The slope of the tangent at (x, y) to a curve passing through $(1,\pi/4)$ is given by $\frac{y}{x}-\cos^2\left(\frac{y}{x}\right)$ then the equation of the curve is

a) $y=\tan^{-1}\left(In\left(\frac{e}{x}\right)\right)$

b) $y=x\tan^{-1}\left(In\left(\frac{x}{e}\right)\right)$

c) $y=x\tan^{-1}\left(In\left(\frac{e}{x}\right)\right)$

d) $y=\tan^{-1}\left(In\left(\frac{x}{e}\right)\right)$

6. If the solution of differential equation $\frac{dy}{dx}=\frac{2(y+2)^2}{(x+y-1)^2}$ is $e^{\alpha\tan^{-1}\frac{y+\alpha}{x-\beta}}=c.(y+\alpha)$ then $10\alpha+\beta=$

a) 23 b) 12 c) 13 d) 15

7. The solution of the differential equation $xy\frac{dy}{dx}=\frac{1+y^2}{1+x^2}(1+x+x^2)$ is

a) $(1+y^2)=cx\tan^{-1}x$ b) $\sqrt{1+y^2}=Cxe^{\tan^{-1}x}$ c) $\sqrt{1+y^2}=xTan^{-1}x+c$ d) $1+y^2=cxe^{Tan^{-1}x}$

8. The equation of the curve passing through (2, 1) which has constant sub-tangent

a) $K\log y=x-2$ b) $\log y=x+2$ c) $y=x-2+K$ d) $y=x+2+K$

9. The solution of $\frac{dy}{dx}=\frac{y-x+1}{y+x+5}$ is ....

a) $\log[(y+3)^2+(x+2)^2]+\mathrm{Tan}^{-1}\left(\frac{y+3}{y+2}\right)+C$

b) $\log[(y+3)^2+(x-2)^2]+\mathrm{Tan}^{-1}\left(\frac{y-3}{x-2}\right)=C$

c) $\log[(y+3)^2+(x+2)^2]+2\mathrm{Tan}^{-1}\left(\frac{y+3}{x+2}\right)=C$

d) $\log[(y+3)^2+(x+2)^2]-2\mathrm{Tan}^{-1}\left(\frac{y+3}{x+2}\right)=C$

10. The solution of the differential equation $\frac{dy}{dx}=\frac{3x^2y^4+2xy}{x^2-2x^3y^3}$ is ......

a) $\frac{y^2}{x}-x^3y^2=c$ b) $\frac{x^2}{y^2}+x^3y^2=c$ c) $\frac{x^2}{y}+x^3y^2=c$ d) $\frac{x^2}{3y}-2x^3y^2=c$

11. The solution of $xdx+ydy=\frac{xdy-ydx}{x^2+y^2}$ is

a) $\frac{1}{2}(x^2+y^2)=\tan^{-1}\left(\frac{y}{x}\right)+C$ b) $x^2+y^2=\tan^{-1}\left(\frac{y}{x}\right)+C$

c) $x^2+y^2=\cot^{-1}\left(\frac{y}{x}\right)+C$ d) $x^2-y^2=\tan^{-1}\left(\frac{y}{x}\right)+C$

12. The curve such that the intercept on the x–axis cut off between the origin and the tangent at a point is twice the abscissa and which passes through the point (1, 2) is ....

a) $x+y=3$ b) $xy=2$ c) $\frac{y}{x}=\frac{1}{2}$ d) $x-y=2$

13. The solution of the equation $\log\left|\frac{dy}{dx}\right|=ax+by$ is

a) $\frac{e^{by}}{b}=\frac{e^{ax}}{a}+C$ b) $\frac{e^{-by}}{-b}=\frac{e^{ax}}{a}+C$ c) $\frac{e^{-by}}{a}=\frac{e^{ax}}{b}+C$ d) $e^{ax}+e^{by}=C$

14. The solution of the differential equation $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y}$

a) $y\sin y=x^2\log x+c$ b) $y\cos y=x^2\log x+c$

c) $\sin y=x^3\log x+c$ d) $\cos y=x\log x^2+c$

15. Though any point (x, y) of a curve which passes through the origin, lines are drawn parallel to the coordinate axes. The curve, given that it divides the rectangle formed by two lines and the axes into two areas, one of which is twice the other represents a family of

a) Circles b) Parabolas c) hyperbolas d) Straight lines

16. A continuously differential function $y=f(x)$, $x\in(0,\pi)$ satisfying $y^1=1+y^2$, $y(0)=0=y(\pi)$ is

a) $\tan x$ b) $x(1-\pi)$

c) $(x-\pi)(1-e^x)$ d) not possible

17. If $xdx=y(x^2+y^2-1)dy$, then $x^2+y^2=$

a) ce^{x^2} b) ce^{y^2} c) ce^{-x^2} d) ce^{-y^2}

18. The equation of the curve for which the square of the ordinate is twice the rectangle contained by the abscissa and the intercept of the normal on x-axis and passing through (2,1) is

a) $x^2+y^2-x=0$ b) $4x^2+2y^2-9y=0$

c) $2x^2+4y^2-9x=0$ d) $4x^2+2y^2-9x=0$

19. The solution of $\frac{x^3dx + yx^2dy}{\sqrt{x^2+y^2}} = ydx - xdy$ is

a) $\sqrt{x^2+y^2} = cx$ b) $\sqrt{x^2+y^2} + \frac{y}{x} = c$ c) $\sqrt{x^2+y^2} + \frac{y}{x^2} = c$ d) $(x^2+y^2)^2 + xy^2 = c$

20. The solution of $\frac{y + \sin x \cos^2(xy)}{\cos^2(xy)}dx + \frac{xdy}{\cos^2(xy)} + \sin y dy = 0$ is given by

a) $\sin(xy) - \cos x - \cos y = c$ b) $\tan(xy) + \cos x + \cos y = c$

c) $\cos(xy) - (\sin x + \sin y) = c$ d) $\tan(xy) - (\cos x + \cos y) = c$

More than one correct answer type questions

21. The solution of $\frac{dy}{dx} + x = xe^{(n-1)y}$

a) $\frac{1}{n-1}\log\left(\frac{e^{(n-1)y}-1}{e^{(n-1)y}}\right) = x^2/2 + C$ b) $e^{(n-1)y} = Ce^{(n-1)y+(n-1)x^2/2} + 1$

c) $\log\left(\frac{e^{(n-1)y}-1}{(n-1)e^{(n-1)y}}\right) = x^2/2 + C$ d) $e^{(n-1)y} = Ce^{(n-1)y2/2+x} + 1$

22. The solution of $\frac{xdx + ydy}{xdy - ydx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2+y^2}}$ is

a) $\sqrt{x^2+y^2} = a(\sin(\tan^{-1} y/x) + C)$ b) $\sqrt{x^2+y^2} = a(\cos(\tan^{-1} y/x) + C)$

c) $\sqrt{x^2+y^2} = a(\tan(\sin^{-1} y/x) + \text{const})$ d) $x\tan\left(\text{const} + \sin^{-1}\frac{1}{a}\sqrt{x^2+y^2}\right) = y$

23. If $y = y(x)$ and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x$, $y(0) = 1$ then $y\left(\frac{\pi}{2}\right) =$ ....

a) $\frac{1}{3}$ b) $\sin\left(\frac{\pi}{2}\right) - \frac{2}{3}$ c) $\text{Tan}^2\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{2}\right)$ d) $\frac{7}{2}$

24. Which one of the following functions is/are homogeneous ?

a) $f(x,y) = \frac{x-y}{x^2+y^2}$ b) $f(x,y) = x^{\frac{1}{3}}y^{-\frac{2}{3}}\text{Tan}^{-1}\left(\frac{x}{y}\right)$

c) $f(x,y) = x(\log\sqrt{x^2+y^2} - \log y) + ye^{\frac{x}{y}}$

d) $f(x,y) = x\left[\log\frac{2x^2+y^2}{x} - \log(x+y)\right] + y^2\text{Tan}\left(\frac{x+2y}{3x-y}\right)$

25. Equation of the curve in which the subnormal is twice the square of the ordinate is given by

a) $\log y = 2x + \log c$ b) $y = ce^{2x}$ c) $\log y = 2x - \log c$ d) $y = ce^{3x} + c$

26. The solution of $\left(\frac{dy}{dx}\right)^2 + 2y\cot x\frac{dy}{dx} = y^2$ is

a) $x = 2\sin^{-1}\sqrt{\frac{c}{2y}}$ b) $x = 2\cos^{-1}\sqrt{\frac{c}{2y}}$ c) $y = \frac{c}{1-\cos x}$ d) $y - \frac{c}{1+\cos x} = 0$

27. Which of the following statements is/are true for a curve satisfying the DE $dy + 2xy^2dx = 0$ and $y(0) = 1$?

a) The curve is symmetric about y–axis
b) The curve is symmetric about x–axis
c) x–axis is an asymptote to the curve
d) Tangent at $x=0$ to the curve is parallel to x–axis

28. Tangent drawn at any point P of a curve, which passes through (1,1) cuts x,y–axes at A,B respectively. If $BP : AP = 3:1$, then

a) DE of the curve is $3x\frac{dy}{dx} + y = 0$
b) DE of the curve is $3x\frac{dy}{dx} - y = 0$
c) the curve passes through $\left(\frac{1}{8}, 2\right)$
d) The normal to the curve at (1,1) is $x+3y = 4$

29. If $x^m + y^m = cx^n$ is the solution of $(x^3 - 2y^3)dx + 3xy^2dy = 0$, then

a) $m - n = 1$ b) $m + n = 5$ c) $m = 3, n = 2$ d) $2m = 3n$

30. Let $y = f(x)$ be a curve passing through (e, e^e), which satisfy the DE $(2ny + xy\log_e x)\,dx - x\log x dy = 0$, $x > 0, y > 0$. If $g(x) = \underset{n\to\infty}{Lt} f(x)$ then $\int_{1/e}^{e} g(x)dx =$

a) e b) 1 c) 0 d) 2

Linked comprehension type questions

Passage - I :

A right cicular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant = k > 0). Suppose that r(t) is the radius of liquid cone at time t.

31. The time after which the cone is empty is

a) $H/2k$ b) H/k c) $H/3k$ d) $2H/k$

32. The radius of water cone at $t = 1$ is

a) $R\,[1 - k/H]$ b) $R\,[1 - H/k]$ c) $R\,[1 + H/k]$ d) $R\,[1 + k/H]$

33. The value of $\sum_{i=1}^{10} r(i)$ is equal to

a) $10\,R\left[2 - \frac{k}{H}\right]$ b) $5\,R\left[2 + \frac{11k}{H}\right]$ c) $5\,R\left[2 - \frac{11k}{H}\right]$ d) $4\,R\left[2 - \frac{11k}{H}\right]$

Passage - II:

A differential equation of the form $\frac{dy}{dx}=\frac{f(x,y)}{\phi(x,y)}$ where f(x, y) and ϕ(x,y) are homogeneous. This equation may also be reduced to the form $\frac{dy}{dx}=g\left(\frac{x}{y}\right)$ and is solved by putting $y=\vartheta x$ so that the dependent variable y is changed to another variable ϑ, where ϑ is some unknown function, the differential equation is transformed to an equation with variables separable

34. The solution of $ye^{-\frac{x}{y}}dx-(xe^{-\frac{x}{y}}+y^3)dy=0$ is

a) $e^{-\frac{x}{y}}+y^2=c$ b) $xe^{-\frac{x}{y}}+y=c$ c) $2e^{-\frac{x}{y}}+y^2=c$ d) $e^{-\frac{x}{y}}+2y^2=c$

35. The solution of $(x^2+xy)dy=(x^2+y^2)dx$ is

a) $\log x=\log(x-y)+\frac{y}{x}+c$ b) $\log x=2\log(x-y)+\frac{y}{x}+c$

c) $\log x=\log(x-y)+\frac{x}{y}+c$ d) $\log x=\log(x-y)+\frac{2x}{y}+c$

36. The x–intercept of the tangent to a curve is equal to the ordinate of the point of contact. The equation of the curve through the point (1, 1) is ... ...

a) $ye^{\frac{x}{y}}=e$ b) $xe^{\frac{x}{y}}=e$ c) $xe^{\frac{y}{x}}=e$ d) $ye^{\frac{y}{x}}=e$

Passage - III :

Equation of the form $\frac{dy}{dx}=\frac{ax+by+c}{Ax+By+C}$ $(aB\neq Ab, A+b\neq 0)$. Can be reduced to a homogeneous form by changing the variable *x, y* to *X, Y* by writing $x=X+h$, $y=Y+k$ where *h, k* are constants to be chosen so as to make the given equation homogeneous. We have $\frac{dy}{dx}=\frac{dY}{dX}$. Hence the given equation becomes $\frac{dY}{dX}=\frac{aX+bY+(ah+bk+C)}{AX+BY+(Ah+Bk+C}$.

Let h and k be chosen to satisfy the relation $ah+bk+c=0$ and $Ah+Bk+C=0$ $\frac{dY}{dX}=\frac{aX+bY}{AX+BY}$ can now be solved by substituting $Y=vX$

37. The solution of $\frac{dy}{dx}=\frac{x+y+1}{x+y-1}$ is

a) $e^{y-x}=C(x+y)$ b) $e^{y-x}=C(x-y)$

c) $e^{y+x}=C(x+y)$ d) $e^{y-x}=C(2x+y)$

38. The solution of $\frac{dy}{dx}=\frac{x-2y+1}{2x-4y}$ is

a) $(x-2y)^2=c$ b) $(x-2y)^2+2x=c$

c) $(x+2y)^2+2x=c$ d) $(x-2y)^2-4x=c$

39. The solution of $(2x + y + 3)dx = (2y + x + 1)\, dy$ is

a) $\left(x+y+\frac{4}{3}\right)(x-y+2)^3 = C$ b) $\left(x+y+\frac{4}{3}\right)(x+y+2) = C$

c) $(x+y+3)(x-y+z) = C$ d) $(x+y+7)(x-2y+6) = C$

Matrix matching type questions

40. **COLUMN - I** | **COLUMN - II**

A) General solution of $x^2y\, dx = (x^3 + y^3)\, dy$ is

B) General solution of $(xy - 2y^2)\, dx = (x^2 - 3xy)dy$ is

C) General solution of $(xy + x^2y^2)y\, dx + (xy - x^2y^2)x\, dy = 0$

D) General solution of differential equation $(x^2y^2 + xy + 1)y\, dx = (x^2y^2 - xy + 1)x\, dy$ is

p) $-\frac{1}{xy} + + \ln x - \ln y = c$

q) $\frac{x^3}{3y^3} = \ln(y) + c$

r) $\frac{x}{y} - 2\ln x + 3\ln|y| = c$

s) $\ln x - \ln y + \tan^{-1} xy = c$

41. Consider the non - zero function $f(x)$ defined as $f^2(x) = \int_0^x f(t)\frac{\sin t}{2+\cos t}dt, x \neq 0$ and $f(0) = 1$

COLUMN - I | **COLUMN - II**

A) $f(2\pi) =$

B) $f(x) - f(-x) =$

C) $f^1(x) + f^1(-x) =$

D) $f(\pi) - \frac{1}{2}\log 3$

p) 0

q) 1

r) –1

s) 2

Integer answer type questions

42. If $x\sin\left(\frac{y}{x}\right)dy = \left(y\sin\left(\frac{y}{x}\right) - x\right)dx$ and $y(1) = \frac{\pi}{2}$ then the value of $\frac{\cos\left(\frac{y}{e^{12}}\right)}{3} =$

43. If the solution of $\frac{x}{x^2+y^2}dy = \left(\frac{y}{x^2+y^2} - 1\right)dx$ satisfies $y(0) = 1$, then the value of $\frac{16}{\pi}y\left(\frac{\pi}{4}\right) =$

44. If solution of the differential equation $\frac{dy}{dx} + \frac{\cos x(3\cos y - 7\sin x - 3)}{\sin y(3\sin x - 7\cos y + 7)} = 0$ is $(\sin x + \cos y - 1)^{\lambda}(\sin x - \cos y + 1)^{\mu} = c$ where C is arbitrary constant then the value of $\lambda\mu$ is equal to

45. Solution of the differential equation $2y \sin x\frac{dy}{dx} = 2\sin x\cos x - y^2\cos x$ satisfying $y\left(\frac{\pi}{2}\right) = 1$ is $y^2 = f(x)$ then $f(\pi)$.

46. For the primitive integral equation $ydx + y^2\, dy = xdy$, $x \in R$, $y > 0$, $y(1) = 1$ then $y(-3) + 3 =$

47. The solution of primitive integral equation $(x^2 + y^2)\, dy = xy\, dx$ is $y = y(x)$. If $y(1) = 1$ and $y(x_0) = e$ then $x_0 - \sqrt{3}\, e =$

48. $y = ae^{-1/x} + b$ is a solution of $\dfrac{dy}{dx} = \dfrac{y}{x^2}$ then $b =$

49. A curve passing through the point (1,1) has the property that the perpendicualr distance of the origin from normal at any point P of the curve is equal to the distance of P from the x-axis is a circle with radius =

50. A normal is drawn at a point $P(x,y)$ of a curve. It meets x-axis at Q. If PQ is of constant length 'K' and such a curve passing through $(0, k)$ be $x^2 + py^2 = k^2$, then the value of 'p' is

EXERCISE-III

Linear Differential Equation, Bernoulli's Equation Orthogonal trajectories, Differential Equation of 1st Order and Higher degrees

LEVEL-I (MAIN)

Single answer type questions

1. The I.F of $(x + 2y^3)\dfrac{dy}{dx} = y^2$ is

1) $e^{1/y}$ 2) $e^{-1/y}$ 3) y 4) $-1/y$

2. An integrating factor of the differential equation $(1+x^2)\dfrac{dy}{dx} + xy = \dfrac{x^4}{(1+x^5)}(\sqrt{1-x^2})^3$ is

1) $\sqrt{1-x^2}$ 2) $\dfrac{x}{\sqrt{1-x^2}}$ 3) $\dfrac{x^2}{\sqrt{1-x^2}}$ 4) $\dfrac{1}{\sqrt{1-x^2}}$

3. An integrating factor of the equation $(1 + y + x^2y)\, dx + (x + x^3)dy = 0$ is

1) e^x 2) x^2 3) $\dfrac{1}{x}$ 4) x

4. The solution of the differential equation $\dfrac{dy}{dx} - 2y\tan 2x = e^x \sec 2x$ is

1) $y \sin 2x = e^x + c$ 2) $y \cos 2x = e^x + c$ 3) $y = e^x \cos 2x + c$ 4) $y \cos 2x + e^x = c$

5. The solution of $\dfrac{dy}{dx} + y \operatorname{Cot} x = 4x \operatorname{cosec} x$ given that $y = 0$; $x = \pi/2$ is

1) $y \operatorname{Sin} x = 2x^2 - \dfrac{\pi}{3}$ 2) $y \operatorname{Sin} x = x^2 - \dfrac{\pi^2}{2}$ 3) $y \operatorname{Sin} x = 2x^2 - \dfrac{\pi^2}{2}$ 4) $y \operatorname{Sin} x = 2x^2 + \dfrac{\pi^2}{2}$

6. The solution of $(1 + y^2) + (x - e^{\tan^{-1} y})\, y_1 = 0$ is

1) $x\, e^{\operatorname{Tan}^{-1} y} = \operatorname{Tan}^{-1} y + k$ 2) $x\, e^{2\operatorname{Tan}^{-1} y} = e^{\operatorname{Tan}^{-1} y} + k$

3) $(x-2) = Ke^{-\operatorname{Tan}^{-1} y}$ 4) $2x\, e^{\operatorname{Tan}^{-1} y} = e^{2\operatorname{Tan}^{-1} y} + k$

7. The solution of $(1+y^2)\,dx = (\mathrm{Tan}^{-1}y - x)\,dy$ is

1) $xe^{\mathrm{Tan}^{-1}y} = (\mathrm{Tan}^{-1}y-1)e^{\mathrm{Tan}^{-1}y}+c$ 2) $xe^{\mathrm{Tan}^{-1}y} = (\mathrm{Tan}^{-1}y-1)e^{\mathrm{Tan}^{-1}y}-x+c$

3) $x = (\mathrm{Tan}^{-1}y-1)e^{\mathrm{Tan}^{-1}y}-y+c$ 4) $x = (\mathrm{Tan}^{-1}y+1)e^{\mathrm{Tan}^{-1}y}-x+y+c$

8. The integrating factor of $x\,\mathrm{Cos}x\left(\frac{dy}{dx}\right)+(x\,\mathrm{Sin}x+\mathrm{Cos}x)\,y = 1$ is

1) x Cos x 2) x Sin x 3) x Sec x 4) x Cosec x

9. The solution of $\frac{dy}{dx}+\frac{3x^2y}{1+x^3}=\frac{1+x^2}{1+x^3}$ is

1) $y(1+x^3) = x+\frac{x^3}{3}+c$ 2) $y(1-x^3) = x+\frac{x^3}{3}+c$

3) $y(1+x^3) = x-\frac{x^3}{3}+c$ 4) $y(1-x^3) = x-\frac{x^3}{3}+c$

10. The solution of $x\,(x-1)\,\frac{dy}{dx}-(x-2)\,y = x^3\,(2x-1)$ is

1) $y\frac{(x-y)}{x^2} = x^2-x+c$ 2) $y\frac{(x-1)}{x^2} = x^2-x+c$

3) $y\frac{(x+1)}{x^2} = x^2-x+c$ 4) $\frac{y(x+y)}{x^2} = x^2+x+c$

11. The solution of $y^2\,dx + (3xy-1)dy = 0$ is

1) $xy^3 = y^2 + c$ 2) $xy^3=\frac{y^2}{2}+c$ 3) $xy^3=\frac{y^2}{3}+c$ 4) $xy^3=\frac{x^2}{2}+c$

12. The solution of $dx + x\,dy = e^{-y}\,\mathrm{Sec}^2y\,dy$ is

1) $x\,e^y = \mathrm{Sin}y + c$ 2) $x^2\,e^y = \mathrm{Tan}y + c$ 3) $x\,e^y = \mathrm{Tan}y - x + c$ 4) $x\,e^y = \mathrm{Tan}y + c$

13. The solution of $x\frac{dy}{dx}+y\log y = xy\,e^x$ is

1) $x\log y = (x+1)e^x+c$ 2) $\log y = (x-1)e^x+c$

3) $(x-1)\log y = xe^x+c$ 4) $x\log y = (x-1)\,e^x+c$

14. The solution of $\frac{dx}{dy}+\frac{x}{y}=x^2$ is

1) $\frac{1}{y}= cx - x\log x$ 2) $\frac{1}{x}= cy - y\log y$ 3) $\frac{1}{x}= cx + x\log y$ 4) $\frac{1}{y}= cx - y\log x$

15. Solution of the differential equation $\cos x\,dy = y(\sin x - y)dx,\ 0<x<\frac{\pi}{2}$ is

1) $y\sec x = \tan x + c$ 2) $y\tan x = \sec x + c$ 3) $\tan x = (\sec x + c)y$ 4) $\sec x = (\tan x + c)y$

16. Solution of $xy - \frac{dy}{dx} = y^3 e^{-x^2}$ is

1) $e^{-x^2} = y^2(2x-c)$ 2) $e^{x^2} = y^2(2x-c)$ 3) $y^2 = e^{x^2}(2x-c)$ 4) $y^2 = e^{-x^2}(2x-c)$

17. The general solution of $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ is

1) $e^y = e^x - 1 + c.e^{-ex}$ 2) $e^y = e^x + 1 + c.e^{-ex}$ 3) $e^y = e^x - 1 - c e^{-x}$ 4) $e^y = e^x - 2 + c.e^{-ex}$

18. The solution of $\frac{dy}{dx} - \frac{\text{Tan}y}{1+x} = (1+x)\, e^x \sec y$ is

1) $\frac{\sin y}{1+x} = e^x + c$ 2) $\frac{\cos x}{1+x} = e^x + c$ 3) $\frac{\cos x}{1-x} = e^x + c$ 4) $\frac{\cos y}{x+1} = e^y + c$

19. The solution of $2\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$ is

1) $\frac{1}{y} = \frac{1}{x} + c\sqrt{x}$ 2) $\frac{1}{y} = x + \frac{c}{\sqrt{x}}$ 3) $\frac{1}{y} = \frac{1}{x} + \frac{c}{\sqrt{x}}$ 4) $\frac{1}{y} + \frac{1}{x} = \frac{c}{\sqrt{x}}$

20. The orthogonal trajectories of the family of curves $a^{n-1}y = x^n$ are give by

1) $x^n + n^2 y$ = constant 2) $ny^2 + x^2$ = constant 3) $n^2x + y^n$ = constant 4) $n^2x - y^n$ = constant

21. The trajectories orthogonal to $x^2 + y^2 = 2ax$ is

1) $y = x^2$ 2) $y = c(x^2 + y^2)$ 3) $y = c(x^2 + 2y^2)$ 4) $y^2 = x$

Numerical value type questions

22. The orthogonal trajectories of the family of conics $y = cx^2$ are conic with eccentricity e = ...

23. If $y = y(x)$ be the solution of the differential equations $\sin x \frac{dy}{dx} + y \cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$ then $y(\pi/6) = \pi^2\lambda$. Then $\lambda =$ ....

24. If $y = y(x)$ is the solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$.

Then $y\left(\frac{1}{2}\right) =$ ....

25. If $\frac{dy}{dx} = \frac{2x - y + 1}{x + 2y + 1}$ and $y(1) = 1$. Then $y(0)$ = ...

26. Let $y = y(x)$ be the solution of the differential equation $(x^2+1)^2\frac{dy}{dx} + 2x(x^2+1)y = 1$ such that $y(0)=0$, If $\sqrt{a}\,y(1) = \frac{\pi}{32}$. Then value of a = ...

LEVEL-II (ADVANCED)

Single answer type questions

1. Let $f(x)$ be continuously differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$, then $f(x)$ is

a) $\frac{1}{3x} + \frac{2x^2}{3}$ b) $-\frac{1}{3x} + \frac{4x^2}{3}$ c) $-\frac{1}{x} + \frac{2}{x^2}$ d) $\frac{1}{x}$

2. The equation of the curve which is such that the area of the rectangle constructed on the asscissa of any point and the intercept of the tangent at this point on y-axis is equal to 4 is

a) $\frac{y}{x} = \pm\frac{2}{x^2} + c$ b) $\frac{y}{x} = x^2 + c$ c) $y = x^3 + c$ d) $\frac{y}{x} = \pm 2x^2 + c$

3. The real value of n for which the substitution $y = u^n$ will transform the differential equation $2x^4 y \frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation is

a) 1/2 b) 1 c) 3/2 d) 2

4. The solution of $(x + 2y^3)dy = ydx$ is ....

a) $x = y^3 - cy^2$ b) $x = y^3 + xy + c$ c) $x = y^3 + cy$ d) $x = y^2 - cy$

5. The solution of the differential equation $x^2 \frac{dy}{dx} \cos\left(\frac{1}{x}\right) - y\sin\left(\frac{1}{x}\right) = -1$ where $y \to -1$ as $x \to \infty$ is

a) $y = \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$ b) $y = \frac{x+1}{x\sin\left(\frac{1}{x}\right)}$ c) $y = \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)$ d) $y = \frac{x+1}{x\cos\left(\frac{1}{x}\right)}$

6. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{xy[x^2 \sin y^2 + 1]}$ is

a) $x^2(\cos y^2 - \sin y^2 - 2ce^{\,y^2}) = 2$ b) $y^2(\cos x^2 - \sin y^2 - 2ce^{-y^2}) = 2$

c) $x^2(\cos y^2 - \sin y^2 - e^{-y^2}) = 4c$ d) $y^2(\cos x^2 + \sin y^2 + 2e^{-y^2}) = 2$

7. The solution of the differential equation $x(x^2 + 1)\frac{dy}{dx} = y(1 - x^2) + x^3 \log x$ is

a) $y\frac{(x^2+1)}{x} = \frac{1}{4}x^2 \log x + \frac{x^2}{2} + C$ b) $y^2\frac{(x^2-1)}{x} = \frac{1}{2}x^2 \log x - \frac{x^2}{4} + C$

c) $y\frac{(x^2+1)}{x} = \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C$ d) $y(x^2+1) = x^2 \log x + x^2 + C$

8. The equation to the curve which is such that the portion of x-axis cut off between the origin and the tangent at any point is proportional to the ordinate of that point is

a) $x = y\,(c - k \log y)$ b) $\log x = ky^2 + C$ c) $x^2 = y\,(c - k \log y)$ d) None

9. The DE $\phi(x)dy = y\{\phi^{|}(x) - y\}\ dx$ is changed in the form $df(x,y) = 0$. Then $f(x,y)$ is

a) $\frac{1}{2}\phi(x) + y$ b) $\frac{1}{y}\phi(x) - x$ c) $\frac{1}{y}\phi(x) + x$ d) $\frac{\phi(x)}{y}$

10. The orthgonal trajectory of $y^2 = 4ax$ is (a being the parameter)

a) $2x^2 - y^2 = 2c$ b) $2x^2 + y^2 = 2c$ c) $x^2 + y^2 = c$ d) $x^2 - y^2 = 2c$

11. Orthogonal trajectories of family of the curve $x^{2/3} + y^{2/3} = a^{2/3}$, where a is any arbitrary constant is

a) $x^{2/3} - y^{2/3} = c$ b) $x^{4/3} - y^{4/3} = c$ c) $x^{4/3} + y^{4/3} = c$ d) $x^{1/3} - y^{1/3} = c$

More than one correct answer type questions

12. The solution of $\left(\frac{dy}{dx}\right)(x^2y^3 + xy) = 1$ is

a) $1/x = 2 - y^2 + Ce^{-y^2/2}$

b) the solution of an equation which is reducible to linear equation

c) $2/x = 1 - y^2\, 2/x = 1 - y^2 + e^{-y/2}$

d) $\frac{1-2x}{x} = -y^2 + C\,e^{-y^2/2}$

13. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2 = a/x$ are

a) $9a(y+C)^2 = 4x^3$ b) $y + C = \frac{-2}{3\sqrt{a}}x^{3/2}$ c) $y + C = \frac{2}{3\sqrt{a}}x^{3/2}$ d) $9a(y-C)^2 = 4x^3$

14. The orthogonal trajectories of $xy = c$

a) $x^2 - y^2 = c$ b) $x + 3y = c$ c) $(x-y)(x+y) = c$ d) $e^{\log(x^2-y^2)} = c$

15. Solution of the equation $\frac{dy}{dx} + \frac{1}{x}\text{Tan } y = \frac{1}{x^2}\text{Tan} y \sin y$ is

a) $2x = \sin y(1 + 2cx^2)$ b) $2x = \sin y(1 + cx^2)$ c) $2x + \sin y(1 + cx^2) = 0$ d) $2x = \cos y(1 + cx^2)$

16. The solution of $\frac{dy}{dx}(x^2y^3 + xy) = 1$ is

a) $\frac{1}{x} = (2 - y^2) - ce^{-\frac{1}{2}y^2}$ b) $\frac{1}{x} = (2 + y^2) - ce^{y^2}$

c) $e^{y^2/2} = [(2 - y^2)(2x - xy^2) - cx]$ d) $e^{-y^2/2} = (2 + y^2)(2x + y^2) - cx$

17. The curve for which the area of the triangle formed by x–axis, the tangent line at any point P and the line OP is equal to a^2 is

a) $x = cy + \frac{a^2}{y}$ b) $y = x - cx^2$ c) $y = cx + \frac{a^2}{x}$ d) $y = cy - \frac{a^2}{y}$

18. A function y = $f(x)$ satisfying the DE $\frac{dy}{dx}\sin x - y\cos x + \frac{\sin^2 x}{x^2} = 0$ is such that $y \to 0$ as $x \to \infty$, then

a) $\underset{x\to 0}{Lt} f(x) = 1$ b) $\int_0^{\pi/2} f(x)dx < \frac{\pi}{2}$ c) $\int_0^{\pi/2} f(x)dx > 1$ d) $f(x)$ is even

Linked comprehension type questions

Passage - I:

$\frac{dy}{dx} + Py = Qy^n$ ....(1)

where P and Q are functions of x alone or constants Divide each term of equation (1) by y^n now we get $\frac{dy}{dx} + \frac{y}{x} = y^3$ ....(2)

Let $\frac{1}{y^{n-1}} = \vartheta$ so that $\frac{1}{y^n}\frac{dy}{dx} = \frac{1}{1-n}.\frac{d\vartheta}{dx}$ substituting in equation (2) we get $\frac{d\vartheta}{dx} + (1-n)P = Q(1-n)$.

This is a linear differential equation

19. The linear form of $(xy^2 - e^{\frac{1}{x^3}})dx - x^2y\, dy = 0$ is

a) $\frac{dt}{dx} - \frac{2}{x}t = \frac{-2}{x^2}e^{\frac{1}{x^3}}$ b) $\frac{dt}{dx} + \frac{2}{x}t = \frac{-2}{x^2}e^{\frac{1}{x^3}}$ c) $\frac{dt}{dx} + \frac{2}{x}t = \frac{2}{x^2}e^{\frac{1}{x^3}}$ d) $\frac{dt}{dx} - \frac{2}{x}t = \frac{2}{x^2}e^{\frac{1}{x^3}}$

20. The solution of $\frac{dy}{dx} + \frac{y}{x} = y^3$

a) $2xy^2 + y^2 = 1$ b) $2xy^2 + cx^2y^2 = 1$ c) $2xy^2 + x^2 = 1$ d) $2xy^2 + x^2 = 1$

21. The solution of $\frac{dy}{dx} = x^3y^3 - xy$

a) $\frac{1}{y^2} = x^2 + 1 - ce^{x^2}$ b) $y^2 = x^2 + 1 + ce^{x^2}$ c) $\frac{1}{y^2} = x^2 - 1 - ce^{x^2}$ d) $y^2 = x^2 - 1 + ce^{x^2}$

Passage - II :

Newton's law of cooling states that the rate of change of the temperature T of an object is proportional to the difference between T and the (constant) temperature τ of the surrounding medium. We can write it as $\frac{dT}{dt} = -k(T-\tau)$, k>0 constant. A cup of coffee is served at 185^0F in a room where the temperature is 65° F. Two minutes later the temperature of the coffee has dropped to 155°F ($\log_e \frac{3}{4}$ = 0.144, $\log_e 3$ = 1.09872)

22. The temperature of any object at $t = 2$ is

a) $\tau e^{-k} + T(0)e^{-2k}$ b) $\tau e^{k} + (T(0)+\tau)e^{-2k}$

c) $\tau + (T(0)-\tau)e^{-2k}$ d) $\tau + 2[T(0)-\tau]e^{-K}$

23. Time remained for coffee to have 105° F temperature is

a) 6 min b) 6.43 min c) 7.23 min d) 7.63 min

24. Temperature of the coffee at time t is given by

a) $65 + 120\, e^{-kt}$ b) $75 + 150\, e^{-kt}$ c) $65 + 140 e^{-kt}$ d) $75 + 140 e^{-kt}$

Matrix matching type questions

25. **COLUMN - I** **COLUMN - II**

A) If the function $y = e^{4x} + 2e^{-x}$ is a solution of the differential equation $\dfrac{\dfrac{d^3y}{dx^3} - 13\dfrac{dy}{dx}}{y} = k$ then the value of $k/3$ is p) 3

B) Number of straight lines which satisfy the differential equation $\dfrac{dy}{dx} + x\left(\dfrac{dy}{dx}\right)^2 - y = 0$ is q) 4

C) If real value of m for which the substitution $y = u^m$ will transform the differential equation $2x^4 y\dfrac{dy}{dx} + y^4 = 4x^6$ intohomogeneous then the value of $2m$ is r) 2

D) If the solution of differential equation $x^2\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} = 12y$ is $y = Ax^m + Bx^{-n}$ then $|m + n|$ is s) 1

26. **COLUMN -I** **COLUMN -II**

A) The slope of the tangent at (x, y) to a curve passing through a point (2, 1) is $\dfrac{x^2 + y^2}{2xy}$, then the equation of the curve is $2(x^2 - y^2) = kx$ then k .... p) the period of $\text{Tan}\left(\dfrac{\pi}{2}x\right)$

B) The solution of $\dfrac{dy}{dx} = \dfrac{2x - y}{2y - x}$ is $(y - x)(y + x)^k = C$ then $k =$ q) 2

C) The solution of $\dfrac{dy}{dx} + \dfrac{3}{x}y = \dfrac{1}{x^2}$ given that $y = 2$, $x = 1$ is $kx^3y = x^2 + 3$ then K is r) The maximum value of $3\cos\sqrt{1 + x + x^3}$

D) The solution of $x\dfrac{dy}{dx} - y = 2x^2 \operatorname{cosec} 2x$ is $\dfrac{y}{x} = \log(\operatorname{cosec} kx - \cot kx) + c$ then K is s) 3

Integer answer type questions

27. Let $y_1 = \dfrac{y\phi'(x) - y^2}{\phi(x)}$, where $\phi(x)$ is a specified function satisfying $\phi(1) = 1, \phi(4) = 1296$. If $y(1) = 1$ then $\dfrac{\sqrt{y(4)}}{3} =$

28. If a curve satisfying $xy_1 - 4y - x^2\sqrt{y} = 0$ passes through $\left(1, (\log 4)^2\right)$ then the value of $\dfrac{y(2)}{(\log 32)^2} =$

29. If $y(t)$ is a solution of $(1 + t)\dfrac{dy}{dt} - ty = 1$ and $y(0) = -1$ then $y(1) + \dfrac{3}{2} =$

30. The solution of $\left(\dfrac{dy}{dx}\right)^2 - x\dfrac{dy}{dx} + y = 0$ is $y = f(x)$ then $f(4) =$

31. Let $f : [1, \infty) \to [2, \infty)$ be a differential function such that $f(1) = 2$. If $6\int_1^x f(t)\,dt = 3x f(x) - x^3$ for all $x > 1$, then the value of $f(2)$ is

32. Let $y(x)$ be the general solution of $x(x-1)\dfrac{dy}{dx} - y = x^2(x-1)^2$. Then $4y(2) - y(-1)$ is equal to

KEY SHEET (LECTURE SHEET)

EXERCISE-I

LEVEL-I

01) **1**	02) **1**	03) **3**	04) **2**	05) **3**	06) **1**	07) **3**	08) **1**
09) **2**	10) **1**	11) **3**	12) **1**	13) **4**	14) **3**	15) **3**	16) **3**
17) **2**	18) **2**	19) **0.5**	20) **0.75**	21) **9**			

LEVEL-II

01) **d**	02) **a**	03) **c**	04) **b**	05) **d**	06) **a**	07) **a**	08) **b**
09) **a**	10) **a**	11) **a**	12) **a**	13) **b**	14) **a**	15) **c**	

16) **A-qr;B-ps;C-s;D-ps** 17) **A-r;B-p;C-p;D-q** 18) **1** 19) **6**

20) **3** 21) **9** 22) **2**

EXERCISE-II

LEVEL-I

01) **3**	02) **4**	03) **1**	04) **4**	05) **1**	06) **4**	07) **3**	08) **4**
09) **1**	10) **1**	11) **3**	12) **2**	13) **4**	14) **1**	15) **3**	16) **2**
17) **1**	18) **1**	19) **2**	20) **1**	21) **2**	22) **3**	23) **3**	24) **2**
25) **2**	26) **4**	27) **4**	28) **3**	29) **1**	30) **1**	31) **2**	32) **2**
33) **3**	34) **2**	35) **1**	36) **1**	37) **3**	38) **2**	39) **1**	40) **1**
41) **4**	42) **0**	43) **3.14**	44) **1.4142**				

LEVEL-II

01) c	02) d	03) c	04) a	05) c	06) a	07) b	08) a
09) c	10) c	11) a	12) b	13) b	14) a	15) b	16) a
17) b	18) d	19) b	20) d	21) ab	22) ad	23) abc	24) abc
25) abc	26) abcd	27) abd	28) ac	29) abcd	30) c	31) b	32) a
33) c	34) c	35) b	36) a	37) a	38) b	39) a	
40) A-q;B-r;C-p;D-s			41) A-q;B-p;C-p;D-q			42) 4	43) 4
44) 10	45) 0	46) 6	47) 0	48) 0	49) 1	50) 1	

EXERCISE-III

LEVEL-I

01) 1	02) 1	03) 4	04) 2	05) 3	06) 4	07) 1	08) 3
09) 1	10) 2	11) 2	12) 4	13) 4	14) 2	15) 4	16) 2
17) 1	18) 1	19) 3	20) 2	21) 2	22) 0.7071		
23) –0.8889		24) 3.0625		25) 1.736		26) 0.0625	

LEVEL-II

01) a	02) a	03) c	04) c	05) a	06) a	07) c	08) a
09) b	10) b	11) b	12) abd	13) abc	14) acd	15) ab	16) ac
17) ad	18) abcd	19) a	20) b	21) a	22) c	23) d	24) a
25) A-q;B-r;C-p;D-s			26) A-rs;B-rs;C-pq;D-pq			27) 6	28) 4
29) 1	30) 4	31) 6	32) 6				

PRACTICE SHEET

EXERCISE-I

Order and degree of the D.E and Formation of D.E

LEVEL-I (MAIN)

Single answer type questions

1. The degree and order of the differential equation $\left[1+2\left(\frac{dy}{dx}\right)^2\right]^{3/2} = 5\frac{d^2y}{dx^2}$ are

   1) 1,2   2) 2,2   3) 3,1   4) 4,3

2. The differential equation representing the family of curves $y^2 = 2c\,(x+\sqrt{c}\,)$, where $c > 0$, is a parameter is

   1) order 1, degree 2   2) order 1, degree 1   3) order 1, degree 3   4) order 2, degree 2

3. The order and degree of the differential equation of all circles in the first quadrant which touch the co-ordinate axes is

1) 1, 2 2) 2, 1 3) 3, 2 4) 4, 3

4. The differential equation by eliminating a,b from $xy = ae^x+be^{-x}$ is

1) $xy_2 + 2y_1 + xy =0$ 2) $xy_2 - 2y_1 + xy =0$ 3) $xy_2+2y_1 - xy = 0$ 4) $xy_2 - 2y_1 - xy = 0$

5. The differential equation which represents the family of curves $y = c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants, is

1) $y^{11}= y^1y$ 2) $yy^{11}= (y^1)^2$ 3) $yy^{11}= y^1$ 4) $y^1= y^2$

6. The subnormal at any point of a curve is of constant length '8'. Then the differential equation of the family of curves is

1) $\frac{dy}{dx} = 8y$ 2) $y\left(\frac{dy}{dx}\right)= 8$ 3) $y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 8$ 4) $y\sqrt{1+\left(\frac{dy}{dx}\right)^2} =8dy/dx$

7. The differential equation by elminating the arbitrary constants from the equation
$y = a \cos x + b \sin x + x\sin x$ is

1) $y_2 + y = 3\cos x$ 2) $y_2 + 2y = 2\cos x$ 3) $y_2 + y = 2\cos x$ 4) $y_2 - y = 2\cos x$

8. The differenatial equation by eliminating the arbitrary constants from the equation $xy = ae^x+be^{-x}+x^2$ is

1) $xy_2 + 2y_1- xy = 2 - x^2$ 2) $xy_2 + 2y_1 + xy = 2$
3) $xy_2 - 2y_1 - xy = 2$ 4) $xy_2 - 2y_1 + xy = 2$

9. The differential equation of all circles passing through the origin and having their centres on the X-axis is

1) $x^2 = y^2 + xy\frac{dy}{dx}$ 2) $x^2 = y^2 + 3xy\frac{dy}{dx}$ 3) $y^2 = x^2 + 2xy\frac{dy}{dx}$ 4) $y^2 = x^2-2xy\frac{dy}{dx}$

10. The D.E. of the family of parabolas with vertex at (0,–1) and having axis along the y axis is

1) $xy^1 -2y-2=0$ 2) $xy^1 +y+1=0$ 3) $xy^1 -y-1=0$ 4) $xy^1 +2xy+1=0$

LEVEL-II (ADVANCED)

Single answer type questions

1. The differential equation of family of lines concurrent at the origin is ....

a) $xdy - ydx = 0$ b) $xdy + ydx = 0$ c) $x^2dy - y^2dx = 0$ d) $xdy + y^2dx = 0$

2. The differential equation of all non horizontal lines in a plane is

a) $\frac{d^2y}{dx^2} = 0$ b) $\frac{d^2x}{dy^2} = 0$ c) $\frac{dy}{dx} = 0$ d) $\frac{dx}{dy} = 0$

More than one correct answer type questions

3. Let $y = (A + Bx)e^{3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + m\frac{dy}{dx} + ny = 0,\ m,n \in I$ then

a) $m = -6$ b) $n = -6$ c) $m = 9$ d) $n = 9$

4. The solution of the differential equation $\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$ and $y(0) = 0$ is $y = -\frac{\sin Ax}{9} + e^x + \frac{x^4}{B} + \frac{x}{3} - 1$

a) $A = 3$ b) $A = 4$ c) $B = 12$ d) $B = 10$

Linked comprehension type questions

Passage - I :

The differential equation corresponding to $y = c_1e^{m_1x} + c_2e^{m_2x} + c_3e^{m_3x}$ where c_1, c_2, c_3 are arbitrary constants and m_1, m_2, m_3 are roots of the equation $m^3 - 7m + 6 = 0$ is $a\frac{d^3y}{dx^3} + b\frac{d^2y}{dx^2} + c\frac{dy}{dx} + d = 0$, where *a, b, c, d* are constants

5. The value of a and b respectively

a) 0, 1 b) 1,0 c) –1, 0 d) 0, –1

6. The value of c is

a) 6 b) –7 c) 2 d) –1

7. The value of d is

a) 6 b) –7 c) 2 d) –1

Matrix matching type questions

8. Match the following relations with their differentiated relations $\left(y^1 = \frac{dy}{dx},\ y^{11} = \frac{d^2y}{dx^2}\right)$

COLUMN - I	COLUMN - II
A) $y = e^x \sin x$	p) $2(y_1)^2 = (y-1)y_2$
B) $y = \frac{x-3}{x+4}$	q) $yy_2 + y^2 + 1 = 0$
C) $y = \sqrt{2x - x^2}$	r) $y_2 - 2y_1 + 2y = 0$
D) $y = \frac{3-x}{x+4}$	s) $2(y_1)^2 = (y+1)y_2$

Integer answer type questions

9. The differential equation whose solution represents the family $y = ae^{3x} + be^{5x}$ is $y_2 - 8y_1 + \lambda y = 0$ then the value of λ must be

10. The differential equation whose solution is $Ax^2 + By^2 = 1$ where A and B are arbitray constants is of degree

11. The differential of family of lines situated at a constant distance P from the origin whose degree is

12. The coinciding solution of two equations $y^1 = y^2 + 2x - x^4$ and $y^1 = -y^2 - y + 2x + x^2 + x^4$ is $y = x^n$ then $n + 2$ is

EXERCISE-II

Variable Separables, Homogeneous and Non-Homogeneous Equations

LEVEL-I (MAIN)

Single answer type questions

1. The solution of the equation $\frac{dy}{dx} = e^{-2x}$ is

1) $\frac{e^{-2x}}{4} = y$ 2) $\frac{e^{-2x}}{-2} + c = y$ 3) $\frac{1}{4}e^{-2x} + cx^2 + d = y$ 4) $\frac{1}{4}e^{-2x} + cx + d = y$

2. The solution of $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$ is

1) $x^{2/3} + y^{2/3} = c$ 2) $y^{2/3} - x^{2/3} = c$ 3) $x^{1/3} + y^{1/3} = c$ 4) $y^{1/3} - x^{1/3} = c$

3. The solution of $\frac{dy}{dx} = (1+y^2)(1+x^2)^{-1}$ is

1) $y - x = c(1 + xy)$ 2) $y + x = c(1 + xy)$

3) $y + x = c(1 - xy)$ 4) $y - x = c(1 - xy)$

4. The solution of the differential equation $y\,dx + (x + x^2y)\,dy = 0$ is

1) $\frac{-1}{xy} = c$ 2) $\log y = cx$ 3) $\frac{1}{xy} + \log y = c$ 4) $\frac{-1}{xy} + \log y = c$

5. The solution of $x\,dx + y dy = x^2 y dy - xy^2\,dx$ is

1) $x^2 - 1 = c(1+y^3)$ 2) $x^2 + 1 = c(1-y^2)$ 3) $x^2 - 1 = c(1+y^2)$ 4) $x^2 + 1 = c(1-y^3)$

6. The solution of $(x+y)^2 \frac{dy}{dx} = a^2$ is

1) $y = \tan^{-1}\left(\frac{x+y}{a}\right) + c$ 2) $ay = \tan^{-1}(x+y) + c$

3) $y = a \tan^{-1}\left(\frac{x+y}{a}\right) + c$ 4) $x = a \tan^{-1}\left(\frac{x+y}{a}\right) + c$

7. The value of 'b' such that the solution of $x = by\frac{dy}{dx}$ represents a family of circles with centre origin is

1) $b = 1$ 2) $b = -1$ 3) for all value of b

4) for any value of b the solution does not represents the circle

8. A curve passes through the point (5, 3) and at any point (x, y) on it the product of its slope and the ordinate is equal to abscissa of the curve is

1) parabola 2) ellipse 3) hyperbola 4) circle

9. The normal to a curve at $P(x,y)$ meets the x-axis at G. If the distance of G from the origin is twice the abscisa of P, then the curve is a

1) Ellipse 2) Parabola 3) Circle 4) Hyperbola

10. A particle moves in a line with velocity given by $\frac{ds}{dt} = s + 1$. The time taken by the particle to cover a distance of 9 metre is

1) 1 2) $\log_e 10$ 3) $2\log_e 10$ 4) 10

11. The solution of $\frac{dy}{dx} = (3x + y + 4)^2$ is

1) $3x + y - 4 = \sqrt{3}\text{Tan}(\sqrt{3}x + c)$ 2) $3x + y + 4 = \sqrt{3}\text{Tan}(x + c)$

3) $3x + y + 4 = \sqrt{3}\text{ Tan }(\sqrt{3}x + c)$ 4) $3x + y + 4 = \sqrt{3}\text{Tan}(2x + c)$

12. The equation of the curve not passing through origin and having portion of the tangent included between the coordinate axes is bisected the point of contact is

1) a parabola 2) an ellipse or a straight line

3) a circle or an ellipse 4) a hyperbola

13. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dp}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then new level of production of items is

1) 2500 2) 3000 3) 3500 4) 4500

14. A family of curves has the differential equation $xy\frac{dy}{dx} = 2y^2 - x^2$. Then the family of curves is :

1) $y^2 = cx^2 + x^3$ 2) $y^2 = cx^4 + x^3$ 3) $y^2 = x + cx^4$ 4) $y^2 = x^2 + cx^4$

15. The solution of $\frac{dy}{dx} = \frac{(x+y)^2}{2x^2}$ is

1) $\text{Tan}^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log(cx)$ 2) $\text{Tan}^{-1}\left(\frac{x}{y}\right) = \frac{1}{2}\log(cx)$ 3) $\text{Tan}^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\ cy$ 4) $\text{Tan}^{-1}\left(\frac{y}{x}\right) = \frac{1}{3}\log\ cx$

16. The solution of $(x^2+y^2)\,dx = 2xy\,dy$ is

1) $c(x^2-y^2) = x$ 2) $c(x^2+y^2) = x$ 3) $c(x^2-y^2) = y$ 4) $c(x^2+y^2) = y$

LEVEL-II (ADVANCED)

Single answer type questions

1. The solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 4x - 2y - 2$ given that $y = 1$ when $x = 1$

a) $2e^{2y+2} = e^{4x} + e^4$ b) $e^{2y+2} = e^{4x} + e^2$ c) $2e^{2y} = e^{4x} + e^4$ d) $e^{2y} = e^{4x+3} + e^3$

2. The solution of $xdy = \left[y + x\dfrac{f(y/x)}{f^1(y/x)}\right]dx$ is

a) $|f(y/x)| = c|x|,\ c > 0$

b) $|f(y/x)| = \left|\dfrac{y}{x}\right| + c, c > 0$

c) $|f(y/x)| = \left|\dfrac{y}{x} + x\right| + c$

d) $|f(y/x)| = \left|\dfrac{x}{y} + c\right|, c > 0$

3. The general solution of the differential equation $\dfrac{dy}{dx} + \sin\left(\dfrac{x+y}{2}\right) = \sin\left(\dfrac{x-y}{2}\right)$ is ....

a) $\log \text{Tan}\left(\dfrac{y}{2}\right) = c - 2\sin x$

b) $\log \text{Tan}\left(\dfrac{y}{4}\right) = c - 2\sin(x/2)$

c) $\log \text{Tan}\left(\dfrac{y}{2} + \dfrac{\pi}{4}\right) = c - 2\sin x$

d) $\log\left(\dfrac{y}{4} + \dfrac{\pi}{4}\right) = c - 2\sin\left(\dfrac{x}{2}\right)$

4. The equation of the curves through the point (1, 0) and whose slope is $\dfrac{y-1}{x^2 + x}$ is ....

a) $(y - 1)(x + 1) + 2x = 0$

b) $2x(y - 1) + x + 1 = 0$

c) $x(y - 1)(x + 1) + 2 = 0$

d) $3x(y + 1) + x + 1 = 0$

5. The normal to a curve at $P(x,y)$ meets the x–axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a

a) a parabola
b) circle
c) hyperbola
d) ellipse

6. The solution of the differential equation $\sin\dfrac{dy}{dx} = a$ with $y(0)=1$ is

a) $\sin^{-1}\left(\dfrac{y-1}{x}\right) = a$
b) $\sin\left(\dfrac{y-1}{x}\right) = a$
c) $\sin\left(\dfrac{1-y}{1+x}\right) = a$
d) $\sin\left(\dfrac{y}{x+1}\right) = a$

7. A particle starts at origin and moves along x–axis in such a way that its velocity at the point $(x, 0)$ is given by the formula $\dfrac{dx}{dt} = \cos^2 \pi x$ then the particle never reaches the point on

a) $x = \dfrac{1}{4}$
b) $x = \dfrac{3}{4}$
c) $x = \dfrac{1}{2}$
d) x=1

8. The curve for which the normal at any point (x, y) and the line joining origin to that point forms an isosceles triangle with x–axis as base is

a) an ellipse

b) a rectangular hyperbola

c) a circle

d) a parabola

9. The equation of a curve passing through (1,0) for which the product of the abscissa of a point P and the intercept made by a normal at P on x-axis equals twice the square of the radius vector of the point P is

a) $x^2 + y^2 = x^4$
b) $x^2 + y^2 = 4x^4$
c) $x^2 + y^2 = 4x^4$
d) $x^2 - y^2 = x^5$

10. The solution of the DE $(x\cos x - \sin x)\,dx = \frac{x}{y}\sin x\,dy$ is

a) $\sin x = \ln|xy| + c$ b) $\ln\left|\frac{\sin x}{x}\right| = y + c$ c) $\left|\frac{\sin x}{xy}\right| = c$ d) none

More than one correct answer type questions

11. A normal is drawn at a point $P(x, y)$ of a curve, It meets the x-axis at Q. If PQ is of constant length k. such a curve passing through $(0, k)$ is

a) a circle with centre (0, 0)
b) $x^2 + y^2 = k^2$
c) $(1+k)x^2 + y^2 = k^2$
d) $x^2 + (1+k^2)y^2 = k^2$

12. Solution of $\left(x\frac{dy}{dx} - y\right)\tan^{-1}\frac{y}{x} = x$, and $y(1)=0$ is $\sqrt{x^2+y^2} = e^{A\tan^{-1}B}$

a) $A = B$ b) $A = \frac{-y}{x}$ c) $A = \frac{y}{x}$ d) $A + B = 0$

13. The solution of the D.E $\dfrac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = \dfrac{x\sin^2(x^2+y^2)}{y^3}$ *is*

a) $-\cot(x^2 + y^2) = \left(\frac{x}{y}\right)^2 + c$
b) $\frac{y^2}{x^2 + yc} = -\tan(x^2 + y^2)$
c) $-\cot(x^2 + y^2) = \left(\frac{y}{x}\right)^2 + c$
d) $\frac{y^2 + x^2c}{x^2} = -\tan(x^2 + y^2)$

14. The tangent at any point P on $y = f(x)$ meets x,y axes at A and B. If $PA : PB = 2:1$ then equation of the curve is

a) $|x|y = c$ b) $x|y| = c$ c) $|x|y^2 = c$ d) $x = cy^2$

Matrix matching type questions

15. If $y(xy + 1)dx + x(1 + xy + x^2y^2)dy = 0$ is a differential equation the solution is $Ax^2y^2\log y + B\,xy + C = Kx^2y^2$

COLUMN - I	COLUMN - II
A) -2	p) A
B) 1	q) $2C - B$
C) 0	r) $A + B$
D) 2	s) B
	t) $A + C$

16. Let $y = f(x)$ be the solution of $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$, $y(1) = \frac{\pi}{2}$, then

COLUMN - I	COLUMN - II
A) f is defined on	p) $\left[\frac{-1}{2}, \frac{1}{2}\right]$
B) The range of f contains	q) $[-1, 1]$
C) f is continuous on	r) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
D) $f < 2$ on	s) $[0, 1]$

Integer answer type questions

17. If the curve satisfying $ydx + (x + x^2y)\,dy = 0$ passes through (1, 1) then $\left(\dfrac{-4\log y(4) + \dfrac{1}{y(4)}}{2}\right)$ is equal to

18. The solution of $x\frac{dy}{dx} = y(\log y - \log x + 1)$ is $y = f(x)$ then $f(0) =$

19. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with radius

20. A curve $y = f(x)$ passes through (1, 1) and tangent at $P(x, y)$ cuts the x-axis and y-axis at A and B respectively such that $BP : AP = 3 : 1$ then $f\left(\frac{1}{2}\right) =$

21. The equation of the curve satisfying the differential equation $y_2(x^2+1) = 2xy_1$ Passing through the point (0, 1) and having slope of tangent at $x = 0$ as 3 then the minimum value of y_1 is

22. If $y_1 - x\tan(y-x) = 1$, $y(o) = \frac{\pi}{2}$ then the value of $\sin(y(4) - 4)e^{-8} =$

23. The curve represented by the DE $xdy - ydx = y\,dy$ intersects y –axis at (0,1) and the line $y = e$ at (a,b) then the value of $a+b$ is

EXERCISE-III

Linear Differential Equation, Bernoulli's Equation Orthogonal trajectories, Differential Equation of 1st Order and Higher degrees

LEVEL-I (MAIN)

Single answer type questions

1. The solution of $\frac{dy}{dx} + y = e^x$ is

1) $2y = e^{2x} + c$ 2) $2y\,e^x = e^x + c$ 3) $2y\,e^x = e^{2x} + c$ 4) $2y\,e^{2x} = 2.e^x + c$

2. $y + x^2 = \frac{dy}{dx}$ has the solution of

1) $y+x^2+2x+2 = c\,e^x$ 2) $y+x+2x^2+2 = c\,e^x$

3) $y+x+x^2+2 = c\,e^x$ 4) $y^2+x+x^2+2 = c\,e^{2x}$

3. The solution of $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ is

1) $3x(1+y^2) = 4y^3+c$ 2) $3y(1+x^2) = 4x^3+c$

3) $3x(1-y^2) = 4y^3+c$ 4) $3y(1+y^2) = 4x^3+ c$

4. The solution of $\frac{dy}{dx}-y\,\mathrm{Tan}x = e^x \sec x$ is

1) $ye^x = \cos x+c$ 2) $y\cos x = e^x+c$ 3) $y\sin x = e^x+c$ 4) $ye^x = \sin x+c$

5. The solution of $y\,dx - x\,dy + \log x\,dx = 0$ is

1) $y - \log x - 1 = cx$ 2) $x + \log y + 1 = cx$

3) $y + \log x + 1 = cx$ 4) $y + \log x - 1 = cx$

6. Observe the following statements

I) $dy + 2xy\,dx = 2e^{-x^2}dx \Rightarrow ye^{x^2} = 2x + c$

II) $ye^{x^2} + 2x = c \Rightarrow dx = \left|2e^{-x^2} - 2xy\right| dy$

Which of the following is a correct statement?

1) Both I and II are true 2) Neither I nor II is true

3) I is true; II is false 4) I is false; II is true

7. The solution of $x\log x\frac{dy}{dx}+ y = 1$ is

1) $\log x = \frac{c}{(y-1)}$ 2) $y\log x\frac{dy}{dx}+y = 1$ 3) $xy = \log(\log x) +c$ 4) $\frac{x}{y}\log y = c$

8. Solve $x\,dy - y\,dx = a(x^2 + y^2)\,dy$

1) $\tan^{-1}\left(\frac{x}{y}\right) = ax+c$ 2) $\tan^{-1}\left(\frac{x}{y}\right) = ay+c$

3) $\tan^{-1}\left(\frac{y}{x}\right) = ay+c$ 4) $\tan^{-1}\left(\frac{y}{x}\right) = ax+c$

9. The solution of $x\frac{dy}{dx}+ y = y^2 \log x$ is

1) $\frac{1}{xy} = \frac{\log x}{x}+\frac{1}{x}+c$ 2) $xy = \frac{(\log x)^2}{2}+c$ 3) $xy = \frac{(\log x)^3}{3}+c$ 4) $\frac{1}{xy} = \frac{\log x}{x}-\frac{1}{x}+c$

10. The solution of $x^2 y - x^3 \dfrac{dy}{dx} = y^4 \cos x$, given that $x = \pi$, $y = \pi$ is

1) $y^3 (1 + 3 \sin x) = x^3$
2) $x^3 (1 + \sin x) = y^3$
3) $y^3 (1 - \sin x) = x^3$
4) $x^3 (1 - \sin x) = y^3$

LEVEL-II (ADVANCED)

Single answer type questions

1. The equation of the curve passing through the origin if the middle point of the segment of its normal from any point of the curve to the x-axis lies on the parabola $2y^2 = x$ is

a) $y^2 = 2x + 1 - e^{2x}$ b) $y^2 = 2x + 5 - e^{2x}$ c) $y^2 = e^{2x} - 3x + 3$ d) $y^2 = 2xy + 5e^{2x}$

2. Integrating factor of $\dfrac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$

a) e^{-x^2} b) e^{2x} c) e^{x^3} d) e^{x^2}

3. The differential equation of the curve for which the initial ordinate of any tangent is equal to the corresponding subnormal

a) is linear
b) is homogeneous of 2nd degree
c) has separable variable
d) is of second order

4. Which of the following transformation reduces the $DE \dfrac{dz}{dx} + \dfrac{z}{x}\log z = \dfrac{z}{x^2}(\log z)^2$ into the form $\dfrac{du}{dx} + P(x)\, u = Q(x)$?

a) $u = \log z$ b) $u = e^z$ c) $u = (\log z)^{-1}$ d) $u = (\log z)^2$

More than one correct answer type questions

5. The solution of the differential equation $x\cos x\left(\dfrac{dy}{dx}\right) + y(x\sin x + \cos x) = 1$ is

a) $xy = \sin x + c\cos x$
b) $xy\sec x = \tan x + c$
c) $xy + \sin x + c\cos x = 0$
d) $xy = \tan x + \cot x + c$

6. The solution of the differential equation $(1+y^2)dx = (\mathrm{Tan}^{-1}y - x)dy$ is

a) $xe^{\mathrm{Tan}^{-1}y} = (1 - \mathrm{Tan}^{-1}y)e^{\mathrm{Tan}^{-1}y} + c$
b) $xe^{\mathrm{Tan}^{-1}y} = (\mathrm{Tan}^{-1}y - 1)e^{\mathrm{Tan}^{-1}y} + c$
c) $x = \mathrm{Tan}^{-1}y - 1 + ce^{-\mathrm{Tan}^{-1}y}$
d) $xe^{\cot^{-1}y} = (\mathrm{Tan}^{-1}y - 1) + c$

7. The graph of the function $y = f(x)$ passing through the point (0, 1) and satisfying the differential equation $\dfrac{dy}{dx} + y \cos x = \cos x$ is such that

a) It is a constant function
b) It is periodic
c) It is neither an even or odd function
d) It is continuous and differentiable for all x

8. The solutions of the DE $\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}(e^x + e^{-x}) + 1 = 0$ are given by

a) $y+e^{-x}=c$ b) $y-e^{-x}=c$ c) $y+e^{x}=c$ d) $y-e^{x}=c$

9. Solve the differential equation $x^2\frac{dy}{dx} - 3xy - 2x^2 = 4x^4$

a) $y = 4x^3\log|x| - x + cx^3$ b) $y = 4x\log x + x + cx^2$

c) $y = x^3\log x + cx$ d) $y = x\log x + cx^3$

Matrix matching type questions

10. **COLUMN -I** **COLUMN -II**

COLUMN -I	COLUMN -II
A) The solution of $(1+x+y)\frac{dy}{dx} = 1$	p) $\frac{2}{1+\cos 2x}$
B) Integrating factor of differential equation $\cos x\frac{dy}{dx} + y\sin x = 1$ is	q) $\dfrac{1+\mathrm{Tan}^2\frac{x}{2}}{1-\mathrm{Tan}^2\frac{x}{2}}$
C) Integrating factor of $\cos^2 x\frac{dy}{dx} + y\sin 2x = 5$	r) $x = -(y+2) + ce^y$
D) The solution of $dy - \sin x\sin y\,dx = 0$ is $e^{\frac{1}{K}}.\mathrm{Tan}\frac{y}{2} = c$ then K is	s) $\sec x$

Integer answer type questions

11. The solution of $(1+y^2)+(x-e^{Tan^{-1}y})\frac{dy}{dx} = 0$ is $2x = e^{f(y)} + kf(y)$ then $f(0) =$

12. If $x^2y - x^3\frac{dy}{dx} = y^4\cos x \Rightarrow \frac{x^3}{y^3} = 3f(x)+c$ then $f\left(\frac{\pi}{2}\right) =$

13. A curve $y = f(x)$ passes through (2, 0) and the slope at (x, y) as $\frac{(x+1)^2+(y-3)}{x+1}$. then $f(3) =$

14. If $(1+x^2)\,y_1 = x(1-y)$, $y(o) = \frac{4}{3}$ then $y(\sqrt{8}) - \frac{1}{9}$ is

15. Let f be a real valued differentiable function on R such that $f(1) = 1$. If the y–intercept of the tangent at any point $P(x,y)$ on the curve $y = f(x)$ is equal to cube of the abscissa of P, then the value of $f(-3)$ is equal to

KEY SHEET (PRACTICE SHEET)

EXERCISE-I

LEVEL-I

01) 2 02) 3 03) 1 04) 3 05) 2 06) 2 07) 3 08) 1
09) 3 10) 1

LEVEL-II

01) a 02) b 03) a 04) a 05) b 06) b 07) a
08) A-r;B-p;C-q;D-s 09) 15 10) 1 11) 2 12) 4

EXERCISE-II

LEVEL-I

01) 2 02) 2 03) 1 04) 4 05) 3 06) 3 07) 2 08) 3
09) 4 10) 2 11) 3 12) 4 13) 3 14) 4 15) 1 16) 1

LEVEL-II

01) a 02) a 03) b 04) a 05) c 06) b 07) c 08) b
09) a 10) c 11) ab 12) ac 13) ab 14) cd 15) A-s;B-t;C-qr;D-p
16) A-pqs;B-pqrs;C-pqs;D-pqs 17) 2 18) 0 19) 1 20) 8
21) 3 22) 1 23) 0

EXERCISE-III

LEVEL-I

01) 3 02) 1 03) 2 04) 2 05) 3 06) 3 07) 1 08) 3
09) 1 10) 1

LEVEL-II

01) a 02) d 03) a 04) c 05) ab 06) bc 07) abd 08) ad
09) a 10) A-r;B-qs;C-p;D-qs 11) 0 12) 1 13) 3 14) 1
15) 9

ADDITIONAL EXERCISE

LEVEL-I (MAIN)

Single answer type questions

1. Order and degree of the differential equation $\frac{dy}{dx}+2\frac{dx}{dy}=7$ is

   1) 1, 1 2) 1, 2 3) 2, 1 4) 2, 2

2. The degree and order of the differential equation of all tangent lines to the parabola $x^2 = 4y$ is

   1) 2, 1 2) 2, 2 3) 1, 3 4) 1, 4

3. The degree of differential equation of which $y^2 = 4a(x+a)$ is a solution is

   1) 1, 2 2) 2, 3 3) 3, 1 4) 4, 4

4. If $y=e^{4x}+2e^{-x}$ satisfies the differential equation $y_3+Ay_1+By=0$ then

1) A=12, B = 13   2) A = 13, B =12   3) A =–12, B=–1   4) A= –13, B = –12

5. The D.E whose solution is $y = ax^2+bx+c$ is

1) $y_3= y$ 2) $y_3= x$ 3) $y_3= 0$ 4) $y_2= 0$

6. If the solution of the differential equation $\frac{1}{\sin^{-1}x}\left(\frac{dy}{dx}\right)=1$ is $y = x\sin^{-1}x + f(x) + c$ then $f(x)$ is

1) $1-x^2$ 2) $\sqrt{1-x^2}$ 3) $1+x^2$ 4) $\sqrt{1+x^2}$

7. If the population of the country doubles in 50 years, the number of years it triples under the assumption that the rate of increase is proportional to the number of inhabitants

1) 50 years   2) 60 years   3) 80 years   4) 90 years

LEVEL-II

LECTURE SHEET (ADVANCED)

Single answer type questions

1. The solution of the D.E $yy_1 = x\left[\frac{y^2}{x^2}+\frac{f(y^2/x^2)}{f^1(y^2/x^2)}\right]$ is

a) $f\left(\frac{y^2}{x^2}\right)=cx^2$ b) $x^2 f\left(\frac{y^2}{x^2}\right)=c^2y^2$ c) $x^2 f(y^2/x^2)=c$ d) $f\left(\frac{y^2}{x^2}\right)=cy/x$

2. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satifies the differential equation, $y(1+xy)dx=xdy$, then $f\left(-\frac{1}{2}\right)$ is equal to

a) $-\frac{2}{5}$ b) $-\frac{4}{5}$ c) $\frac{2}{5}$ d) $\frac{4}{5}$

3. The solution of $2y\cos y^2\frac{dy}{dx}-\frac{2}{x+1}\sin y^2=(x+1)^3$ is

a) $\sin y^2=(x+1)^2\left[(x+1)^2+c\right]$ b) $\sin y^2=(x+1)^2\left[\frac{(x+1)^2}{2}+c\right]$

c) $\sin y^2=(x+1)^2\left[\frac{(x+1)^3}{3}+c\right]$ d) $\sin y^2=(x+1)^2\left[\frac{(x+1)^4}{4}+c\right]$

4. General solution of differential equation $\frac{dy}{dx}+yg'(x)=g(x)g'(x)$ where $g(x)$ is a function x.

a) $g(x)+\log(1+y+g(x)) = c$ b) $g(x)+\log(1+y-g(x))= c$

c) $g(x)-\log(1+y-g(x))= c$ d) $g(x)+\log(y+g(x)) = c$

5. The function $f(\theta)=\frac{d}{d\theta}\int_0^\theta \frac{dx}{1-\cos\theta\cos x}$ satisfies the differential equation.

a) $\frac{df}{d\theta}+2f(\theta)\cot\theta=0$ b) $\frac{df}{d\theta}-2f(\theta)\cot\theta=0$

c) $\frac{df}{d\theta}+2f(\theta)=0$ d) $\frac{df}{d\theta}-2f(\theta)=0$

6. Solution of the differential equation $\cos x dy=y(\sin x-y)dx$, $0<x<\frac{\pi}{2}$ is

a) $y \sec x=\tan x+c$ b) $y \tan x=\sec x+c$ c) $\tan x=(\sec x+c)y$ d) $\sec x=(\tan x+c)y$

7. At present a firm is manufacturing 2000 items. It is estimated that the rate of change of production P wrt additional number of workers x is given by $\frac{dp}{dx}=100-12\sqrt{x}$. If the firm employees 25 more workers, then the new level of production of items is

a) 2500 b) 3000 c) 3500 d) 4000

8. The order and degree of the differential equation $\frac{d^{2y}}{dx^2}+\left(\frac{dy}{dx}\right)^{1/4}+x^{1/5}=0$ Respectively, are

a) 2 and 4 b) 2 and 2 c) 2 and 3 d) 3 and 3

9. The equation of a curve passing through origin if the slop of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abscissa and ordinate of the point.

a) $e^{2x}=\left|\frac{1-x+y}{1-x-y}\right|$ b) $e^{-2x}=\left|\frac{1+x-y}{1+x-y}\right|$ c) $e^{2x}=\left|\frac{1+x-y}{1-x+y}\right|$ d) none

10. The real value of m for which the substitution $y=u^m$ will trasform the differential equation $2x^4y\frac{dy}{dx}+y^4=4x^6$ into a homogeneous equation is

a) $m=0$ b) $m=1$ c) $m=\frac{3}{2}$ d) $m=\frac{2}{3}$

11. If the solution of the differential equation $\frac{dy}{dx}=\frac{1}{x\cos y+\sin 2y}$ is $x=ce^{\sin y}-k(1+\sin y)$ then $k=$

a) 1 b) 2 c) 3 d) 4

12. Solution of the differential eqution $f(x)\frac{dy}{dx}=f^2(x)+f(x)y+f^1(x).y$ is

a) $y=f(x)+ce^x$ b) $y=-f(x)+ce^x$ c) $y=-f(x)+ce^xf(x)$ d) $y=cf(x)+e^x$

13. A sperical rain drop evaporates at arate proportional to its surface ara. If originally its radius is 3mm and 1hr later it reduces to 2mm, that at any time t the radius of the rain drop satisfices the relation.

a) R(t+2)=6 b) t(R–1)=5 c) R+t=9 d) Rt=8

More than one correct answer type questions

14. Solution of the equation $\frac{dy}{dx}+\frac{1}{x}\tan y=\frac{1}{x^2}\tan y\sin y$ is

a) $2x$ = siny$(1 + 2x^2)$
b) $2x$ = siny$(1 + cx^2)$
c) $2x$ + siny$(1 + cx^2)$=0
d) x + 2siny$(1 + 2cx^2) = 0$

15. If $y(x)$ satisfies the differential equation y'–ytan$x$=2$x$sec$x$ and y(0) = 0, then

a) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$
b) $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$
c) $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$
d) $y\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

16. If the equation of a curve $y = y(x)$ satisfies the differential equation $x\int_0^x y(t)dt=(x+1)\int_0^x ty(t)dt,\ x>0$ and $y(1) = e$, then $y\left(\frac{1}{2}\right)$ is equal to

a) 2
b) 4
c) 6
d) 8

Linked Comprehension Type Questions

Passage - I :

One of the important application of the differential equation is to find out the family of curves for which some conditions involving the derivatives are given. For this one can proceed by making use of the following facts.

Equation of teh tangent at a point (x, y) to the curve $y=f(x)$ is given by $Y-y=\frac{dy}{dx}(X-x)$

At the X-axis, Y = 0 and $X=x-\frac{y}{dy/dx}$

At the Y-axis, X = 0 and $Y=y-\frac{dy}{dx}$

Similar things, can be obtained for normals by writing its equation as $(Y-y)==\frac{1}{\frac{dy}{dx}}(X-x)$

17. If $y = f(x)$ be a curve such that length of x intercept of tangent is equal to the distance of the point of tangenc from x-axis $f(x)$ is given by

a) $c=ye^{x/y}$
b) $c=ye^{x^2/y}$
c) $c=ye^2e^{x/y}$
d) $c=ye^2e^{y/x}$

18. The equation of curve $y = f(x)$, such that length of sub normal is euql to the abscissae of tangency is a

a) $y^2 = 4ax$
b) $x^2+y^2 = c$
c) $x^2-y^2 = c$
d) 2 and 3

19. The equation of curve passing through point (1, 0) and the slope of whose tangent at (x, y) is $\frac{x^2+y^2}{2xy}$, is

a) $x^2 - y^2 = x$ b) $x^2 - y^2 = y$ c) $x^2 + y^2 = x$ d) $x^2 + y^2 = y$

Passage - II :

Let $f(x)$ satisfies the equation $f(x) = (e^{-x} + e^{x})\cos x - 2x - \int_0^x (x-t)f'(t)dt$

20. y satisfies the differential equation

a) $\frac{dy}{dx} + y = e^x(\cos x - \sin x) - e^{-x}(\cos x + \sin x)$

b) $\frac{dy}{dx} - y = e^x(osx - \sin x) + e^{-x}(\cos x + \sin x)$

c) $\frac{dy}{dx} + y = e^x(\cos x - \sin x) + e^{-x}(\cos x - \sin x)$

d) $\frac{dy}{dx} - y = e^x(\cos x - \sin x) + e^{-x}(\cos x - \sin x)$

21. The value of $f'(0)+f'(0)$ equals

a) –1 b) 2 c) 1 d) 0

22. $f(x)$ as a function of x equals

a) $e^{-x}(\cos x - \sin x) + \frac{e^x}{5}(3\cos x + \sin x) + \frac{2}{5}e^{-x}$

b) $e^{-x}(\cos x + \sin x) + \frac{e^x}{5}(3\cos x - \sin x) - \frac{2}{5}e^{-x}$

c) $e^{-x}(\cos x - \sin x) + \frac{e^x}{5}(3\cos x - \sin x) + \frac{2}{5}e^{-x}$

d) $e^{-x}(\cos x + \sin x) + \frac{e^x}{5}(3\cos x - \sin x) - \frac{2}{5}e^{x}$

Passage - III :

The tangent is drawn at the point (x_i, y_i) on the curve $y = f(x)$, which intersects the x-axis at $(x_{i+1}, 0)$. now again tangent is drawn at (x_{i+1}, y_{i+1}) on the curve which intersects the x–axis at $(x_{i+2}, 0)$ and process is repeated n times i.e., i=1, 2, 3.....n.

23. $x_1, x_2, x_3........x_n$ form an arithmetic progression with common difference equal to $\log_2 e$ and curve passes through (0, 2) then the curve passes through

a) (1, 4) b) (5, 1/30) c) (2, 1/2) d) None of these

24. If x_1, x_2, x_3.......... x_n form a geometric progression with common ration equal to 2 and the curve passes through (1, 2) then the curve is

a) Circle   b) Parabola   c) Ellipse   d) Hyperbola

Matrix matching type questions

25. Match the List I with List II and select the correct answer using the codes given below the lists:

COLUMN -I (Differential Equation)	COLUMN - II (Solution)
A) $\dfrac{x+y\dfrac{dy}{dx}}{y-x\dfrac{dy}{dx}}=x^2+2y^2+\dfrac{y^4}{x^2}$	p) $x^2+2xy=c$
B) $\dfrac{dy}{dx}=e^{x-y}+x^2+x^2e^{-y}$	q) $y=\dfrac{1}{2}\log(1+x)^2+c$
C) $xdx+(x+y)dx=0$	r) $e^y=e^x+\dfrac{x^3}{3}+c$
D) $\dfrac{dy}{dx}=\dfrac{x}{1+x^2}$	s) $\dfrac{2y}{x}-\dfrac{1}{x^2+y^2}=c$

26. Match the following family of curves with

COLUMN -I	COLUMN - II
A) $y=cx+c^2$	p) $y\left(1-\left(\dfrac{dy}{dx}\right)^2\right)=2x\dfrac{dy}{dx}$
B) $y=ae^{2x}+be^{3x}$	q) $x\dfrac{d^2y}{dx}+2\dfrac{dy}{dx}-xy+x^2-2=0$
C) $y^2=4a(x+a)$	r) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+6y=0$
D) $xy=ae^x+be+be^{-x}+x^2$	s) $y=x\dfrac{dy}{dx}+\left(\dfrac{dy}{dx}\right)^2$

Integer answer type questions

27. The fuel charges for running a train are proportional to the square of the speed generated in km per hour and cost Rs. 100 per hour at 20km/hr. If the fixed charge amount to Rs. 10, 000per hour, the most economical speed in km per hour is equal to K, then $\dfrac{K}{50}=$

28. If $f:R-\{1\}\rightarrow$ and f is differentiable function satisfies $f(x+f(y)+xf(y)) = y = f(x)+xf(x)\forall x$; $y\in R-\{-1\}$ then find the value of 2010 $[1+f(2009)]$ = ————.

PRACTICE SHEET (ADVANCED)

Single answer type questions

1. If the function $y = e^{4x} + 2e^{-x}$ is a solution of the differential equation $\dfrac{\frac{d^3y}{dx^3} - 13\frac{dy}{dx}}{y} = k$ then the value of k is

a) 4 b) 6 c) 9 d) 12

2. If the dependent variable y is changed to z by the substitution y = tan z then the diffeential equation $\dfrac{d^2y}{dx^2} = 1 + \dfrac{2(1+y)}{1+y^2}\left(\dfrac{dy}{dx}\right)^2$ is changed to $\dfrac{d^2z}{dx^2} = \cos^2 z + k\left(\dfrac{dz}{dx}\right)^2$, then the value of k equals.

a) –1 b) 0 c) 1 d) 2

3. The general solution of the differential equation $\left(\dfrac{1}{x^2} + \dfrac{3y^2}{x^4}\right)dx - \dfrac{2y}{x^3}dy = 0$, is

a) $x^3 + y^3 = cx^2$ b) $x^2 + y^2 = cx^3$ c) $x^2 + y^3 = cx^3$ d) $x^3 + y^2 = cx^2$

4. $\dfrac{dy}{dx} = y + \int_0^1 y\,dx$ given y =1 when x = 0 the solution of the differential equation is

a) $\dfrac{e^x - 1}{e-3}$ b) $f(x) = \dfrac{2e^x - 1}{e^x - 1}$ c) $f(x) = \dfrac{e^x - e}{e-1}$ d) $f(x) = \dfrac{(2e^x - e + 1)}{3-e}$

5. Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank it is known that the rate at which the water level drops is proprotional to the square root of water depth y, where the constant of proprotionality k > 0 depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and $k = \dfrac{1}{15}$ then the time to drain the tank if the water is 4 m deep to start with, is

a) 30 min b) 45 min c) 60 min d) 80 min

6. The differential equation of all straight lines which are at a constant distance from the origin is

a) $(y-xy_1)^2 = p^2(1+y_1^2)$ b) $(x+xy_1)^2\ p^2(1-y_1^2)$

c) $(y+xy_1)^2 = p^2(1-y_1^2)$ d) $(y+xy_1)^2 = p^2(1-y_1^2)$

7. If f(x), g(x) and h(x) are continuous, thrice differentiable functions and each satisfy the differential equation $\dfrac{d^2y}{dx^2} + y = 0$ then $\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f_1(x) & g_1(x) & h_1(x) \\ f_2(x) & g_2(x) & h_2(x) \end{vmatrix}$ (here $f_k(x)$, $g_k(x)$) and $h_k(x)$ are k^{th} derivatives of $f(x)$, $g(x)$ and $h(x)$ respectively

a) 0 b) $f(x)\ g(x)\ h(x)$ c) $f_1(x)\ g_1(x)\ h_1(x)$ d) 1

More than one correct answer type questions

8. Solution of the differential equation : $\dfrac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=\dfrac{x\sin^2(x^2+y^2)}{y^3}$

a) $-\cot(x^2+y^2)=\left(\dfrac{x}{y}\right)^2+c$

b) $\dfrac{y^2}{x^2+y^2c}=-\tan(x^2+y^2)$

c) $-\cot(x^2+y^2)=\left(\dfrac{y}{x}\right)^2+c$

d) $\dfrac{y^2+x^2c}{x^2}=-\tan^2(x^2-y^2)$

9. Let $f(x)$ be a non-zero function, whose all successive derivaties existg and are non-zero. If $f(x)$, $f'(x)$ and $f''(x)$ are in G.P. $f(0)=f(0)=1$

a) $f(x)>0\forall x\in R$ b) $f'(x)>0\forall x\in R$ c) $f''(0)=1$ d) $f(x)\le 1\forall x\in R$

10. A function y = $f(x)$ satisfying the differential equation $\dfrac{dy}{dx}\cdot\sin x-y\cos x=\dfrac{\sin^2 x}{x^2}=0$ is such that, $y\to 0$ as $x\to\infty$ then :

a) $\underset{x\to 0}{Limit}\ f(x)=1$

b) $\int_0^{\pi/2} f(x)dx$ is less than $\dfrac{\pi}{2}$

c) $\int_0^{\pi/2} f(x)$ is greater than unity

d) $f(x)$ is an even function

11. Given a function 'g' which has a derivative $g'(x)$ for every real x and satisifes $g'(0)=2$ and $g(x+y)=e^y g(x)=e^x g(y)$ for all x and y

a) $g(x)$ is increasing for all

b) range of $g(x)$ is $\left(\dfrac{-2}{e},\infty\right)$

c) $g''(x)>0\ \forall x$

d) $\lim\limits_{x\to 0}\dfrac{g(x)}{x}=1$

Linked comprehension type questions

Passage - I :

Let $y=f(x)$ and $y=g(x)$ be the pair of curves such that

(i) the tangents at point with equal abscissa intersect on y-axis.

(ii) the normal drawn at points with equal abscissa intersect on x-axis and

(iii) one curve passes through (1, 1) and the other one passes through (2, 3) then

12. The curve $g(x)$ if given by

a) $x-\dfrac{1}{x}$ b) $x+\dfrac{2}{x}$ c) $x^2-\dfrac{1}{x^2}$ d) $x^2+\dfrac{1}{x^2}$

13. The number of positive integral solutions for $f(x)=g(x)$, are

a) 4 b) 5 c) 6 d) 0

14. The value of $\int_1^2\left(g(x)-f(x)\right)dx$, is

a) 2 b) 3 c) 4 d) 5

Passage -II :

If any differential equation be in the form

$f\left(f_1(x,y)\right)d\left((f_1(x,y)\right)+\phi\left(f_2(x,y)\right)d\left(f_2(x,y)\right)+....=0$ **then we can perform term by term integration. For e.g.,** $\int \sin xyd\,(xy)+\int\left(\frac{x}{y}\right)d\left(\frac{x}{y}\right) = -\cos xy+\frac{1}{2}\left(\frac{x}{y}\right)^2+c$

15. The solution of the differential equation $f^{11}(x)=f^{1}(x)\Rightarrow\frac{f^{11}(x)}{f^{1}(x)}=1$ is

a) $\frac{x^2}{4}+\frac{1}{2}\left(\frac{d}{y}\right)^2=c$

b) $\frac{x^4}{4}+\frac{1}{3}\left(\frac{x}{y}\right)^3=c$

c) $\frac{x^4}{4}-\frac{1}{2}\left(\frac{x}{y}\right)^2=c$

d) $1\frac{x^2}{4}-\frac{1}{2}\left(\frac{x}{y}\right)^2=c$

16. Solution of diffeential equation $(2x\cos y + y^2\cos x)dx + (2y\sin x - x^2\sin y)dy = 0$ is

a) $x^2\cos y+y^2\sin x=c$

b) $x\cos y-y\sin x=c$

c) $x^2\cos^2 y+y^2\sin^2 x=c$

d) none of these

17. Solution of differential equation $\frac{x^2}{\frac{1}{x^2}+y^2}(xdy+ydx)+y^2\,(xdy-ydx)=0$ is

a) $\tan^{-1}xy-\frac{1}{2}\left(\frac{y}{x}\right)^2=c$

b) $\tan^{-1}xy+\frac{1}{2}\left(\frac{y}{x}\right)^2=c$

c) $\tan^{-1}xy-\frac{1}{2}\left(\frac{y}{x}\right)^3=c$

d) none of these

Matrix matching type questions

18. **COLUMN - I** / **COLUMN - II**

A) The solution of the differential equation $(1+x^2y^2)ydx+(x^2y^2-1)xdy=0$ is

B) The solution of the differential equation $2x^3ydy+(1-y^2)(x^2y^2+y^2-1)\,dx=0$

C) The solution of the differential equation $\frac{X+\frac{X^3}{3!}+\frac{X^5}{5!}+.........}{1+\frac{X^2}{2!}+\frac{X^4}{4!}+.........}=\frac{dx-dy}{dx+dy}$ is

D) The solution of $\mathrm{In}\left(\frac{dy}{dx}\right)=3x+4y$, is

p) $2ye^{2x}=ce^{2x}-1$

q) $4e^{3x}+3.e^{-4y}=C$

r) $x^2+y^2=2\,l\,n\frac{y}{x}+c$

s) $x^2y^2=(cx-1)(1-y^2)$

19. Match the following

LIST - I	LIST - II
A) A straight line with slope 2	1) $y_3 = 0$
B) Parabola whose axis is parallel to y–axis	2) $xdy+ydx = 0$
C) Rectangular hyperbola whose asymptotes are $xy = 0$	3) $y \log y= xy_1$

	A	B	C	D
a)	1	3	4	2
b)	4	3	2	1
c)	1	2	3	4
d)	4	1	2	3

Integer answer type questions

20. Let $y(x)$ be the general solution of $x(x-1)\dfrac{dy}{dx}-y=x^2(x-1)^2$. Then $4y(2)-y(-1)$ equals.

21. Let f be a real-valued differentiable function on r (the set of all real numbers) such that $f(1) = 1$. If the y–intercept of the tangent at any point P(x,y) on the curve$y = f(x)$ is equal to the cube of the abscissa of P, then the value of $f(-3)$ is equal to

KEY SHEET (ADDITIONAL EXERCISE)

LEVEL-I (MAIN)

01) **4** 02) **1** 03) **1** 04) **4** 05) **3** 06) **2** 07) **3**

LEVEL-II

LECTURE SHEET (ADVANCED)

01) **d** 02) **b** 03) **b** 04) **b** 05) **a** 06) **c** 07) **c** 08) **a** 09) **c** 10) **c**

11) **b** 12) **c** 13) **a** 14) **ab** 15) **ad** 16) **d** 17) **a** 18) **d** 19) **a** 20) **a**

21) **d** 22) **3** 23) **3** 24) **4** 25) **A-r;B-s;C-p;D-q** 26) **A-s;B-r;C-p;D-q**

27) **4** 28) **1**

PRACTICE SHEET (ADVANCED)

01) **d** 02) **d** 03) **b** 04) **d** 05) **c** 06) **a** 07) **a** 08) **ab** 09) **abc** 10) **abcd**

11) **abc** 12) **b** 13) **d** 14) **b** 15) **b** 16) **a** 17) **c** 18) **A-r;B-s;C-p;D-q**

19) **4** 20) **6** 21) **9**