BRAC University
CSE230 : Discrete Mathematics

Deadline : 2nd December, 11:59 pm

Submission Form : https://forms.gle/KQDSZjS8ztQN4mox7

[Write your answers in your copy, create a pdf and submit it in the link above.]

[Each question carries one mark]

Submission Guideline:

- Your pdf file must contain your name, id and section. If you miss any of these three, your script will be discarded.
- The pdf file naming format is "Sec_ID_Name". For example, the pdf named "7_12345678_Rosen.pdf" is for a student named Rosen from section 7 whose id is 12345678.
- The form will automatically be closed on the above-mentioned deadline. It will **not** be extended even for valid reasons.

Suppose, 'rolling a regular 6 sided die and a biased 4 sided die together' is called a TRIAL. The biased die has 1, 2, 2 and 4 labeled on its four faces. X is an RV defined as the sum of the numbers that appear on top of the dice after one trial. Now answer Q01 to Q04.

Q01: Find the probability distribution of X. (Different Values of X and their corresponding probabilities)

Q02: Find the number of TRIALs to expect a total = 46.

Q03: What is the expected number of TRIALs for $7 \geq x \geq 5$ where $x \epsilon X$

Q04: What is the probability that exactly 3 out of 8 TRIALs will be successful if success in a TRIAL is defined as receiving 2 from exactly one of the two dice.

Q05: A random variable X is defined to be the taking values from this sample space, S = {2, 3, 5}
X has the probability distribution, P(X = k) = 0.1k
Determine the value of E(X) and V(X).

Q06: Richard goes for his Cricket practice sessions everyday. During a drill his coach throws a ball to him thrice. A random variable Y is defined to be the number of catches dropped by a Richard during that drill. If E(Y) = 1.
What's the probability of Richard catching a ball thrown to him?

Q07: How many numbers are there in the set {1,2,3,...,200} which are divisible by either 7 or 9 but not both?

Q08: How many numbers are there in the set {1,2,3,...,200} which are divisible by neither 3 nor 4 nor 12?

Q09: 6 couples A,B,C,D,E,F (=12 persons) want to sit around a circular table so that each husband can stay next to his wife. Furthermore, the couples A,E are close, and want to sit next to each other. In how many ways can they sit?

Q10: A fair dice is rolled 3 times. Let X denote the sum of the dice rolls, and Y denote the sum of the squares of the dice rolls. For example, if the outcome of the roll is (1,3,4), then X=8, Y=1+9+16=26. Find E(X) and E(Y).

Q11: Brac University Building 8 has a 15-story structure. Mr. Raju works on the 11th floor of this building. Except for stopping on a floor where the button has been pressed, the single elevator travels endlessly between floors 1,2,3,..., 14,15,14,...,3,2,1. Assume that the amount of time required to load and unload passengers is minimal compared to the actual trip time.

Mr. Raju always wonders when he is about to leave his office at 6 PM, most of the time, the lift goes up before stopping at his floor. One day, Raju encounters you in the lift and shared his thoughts with you. Since you are taking Discrete Mathematics course, he expects you to solve the mystery for him. Provide a valid explanation in terms of probability to Mr. Raju.

Q12: 8 friends decide to meet up after a long time in a cafe. The seating arrangement consisted of a circular table. Among them, Tamim has a bestfriend who didnot yet join the meet up. After half an hour Tamim's bestfriend show up along with another friend. Tamim's bestfriend wants to seat in between Tamim and his other friend with whom he came to the hangout. In how many ways can they be seated?

Q13: Suppose that 8% of the patients tested in a clinic are infected with Covid-19. Furthermore, suppose that when a blood test for Covid-19 is given, 95% of the patients tested positive given they are infected with Covid-19 and that 3% of the patients tested positive given they are not infected with Covid-19. What is the probability that a patient is not infected with it given they have been tested negative?

Q14: What is the coefficient of the term independent of x and y in the multinomial expansion $(5x + \frac{3}{y} + \frac{4}{x} + \frac{5}{7}y)^8$?

Q15: The 2022 FIFA World Cup is taking place in Qatar where 32 teams are participating. These teams are separated into 8 groups of 4 teams. All the teams of the same group will play against each other. What is the total number of games that will be played in this group level (that is, before the round of 16)?