

CSE230: Discrete Mathematics
Assignment 1
Deadline: 26th June, 2022

1. Determine the number of ways of reaching (10,13) from (7,7) in a 2D space (xy plane) provided each path can only be traversed one unit to the right or one unit upward.
2. A coin is flipped 12 times. How many possible outcomes contain:
 - a. No heads
 - b. Exactly 8 heads
 - c. At least 8 heads
 - d. The same number of heads and tails
 - e. Smaller number of heads than tails
3. Oxchill, a burger chain, allows its customers to order their burgers with or without the following add-ons: honey mustard sauce, mayo, lettuce, pickle, bacon and extra patty. In how many ways can a customer order a burger from Oxchill.
4. A password to a vault must have at least 8 characters where each character can be an English alphabet (case sensitive) or a digit. A hacker observes a user inserting the password and based on the time taken to insert the password, the hacker is rest assured that the password is no longer than 11 characters. If the hacker can try a million permutations per second, how long will it take to try all possible passwords?
5. 10 cars are to be parked in an open field containing 20 parking spaces set out in two rows of 10. How many different patterns of unoccupied spaces are available if:
 - a. The cars are parked in any of the 20 spaces.
 - b. The cars are parked in the same row.
 - c. The same number of cars parked in each row.
 - d. Two more cars parked in one row than in the other.

6. $f(x) = 5x + \frac{3}{x}; x \neq 0$ $g(x) = 12x$

Given that $(f \circ g)(x) = (g \circ g)(x)$, find the value of x

7. A bookshelf has 12 computer science textbooks. The lecturer is concerned about the coverage of the topics of A* Search (A), Binary Search Tree (B) and C++ (C) on those specific books. After scrutinizing the books, it was found that

$$\begin{array}{lll} |A| = 8 & |B| = 13 & |C| = 13 \\ |A \cap B| = 5 & |A \cap C| = 3 & |B \cap C| = 6 \end{array} \quad |A \cap B \cap C| = 2$$

- a. How many of the books contain material on exactly one of the topics?

- b. How many do not have any of the topics?
 - c. How many contain material on only A* Search and Binary Search Tree?
8. Find the range and the domain of the following functions:
- a. $f: R \rightarrow R, f(x) = \frac{1}{x^2-2}$
 - b. $f: R \rightarrow R, f(x) = 3 - \log(1 + x^2)$
 - c. $f: R \rightarrow R, f(x) = \frac{1}{\log(1-x^2)}$
 - d. $f: R \rightarrow R, f(x) = \sqrt[3]{49 + x^2}$
9. $A_n = [-2n, 3n]$ where $n \in \mathbb{Z}^+$, find the value of:
- a. $\cap_{n=1}^7 A_n$
 - b. $\cup_{n=1}^7 A_n$