Class Notes

Lecture 1.1: Base Conversion

Decimal Number (without fraction) to Other Base Number System

1. Divide the decimal number by the new base. If the target base is binary then divide it by 2. If the target base is octal or hexa-decimal divide the decimal number by 8 or 16.
2. The remainder from the division will be the rightmost digit (least significant digit) of the new base number.
3. Divide the quotient of the previous divide by the new base.
4. The remainder from Step 3 as the next digit of the new base number.
5. Repeat Steps 3 and 4, getting remainder from right to left, until the quotient becomes zero in Step 3.
6. The last remainder thus obtained will be the Most Significant Digit (LeftMost bit) of the new base number.

Suppose we want to convert $(19)_{10}$ to binary. Now we will divide 19 repeatedly by 2 as the base of binary is 2.

Division	Quotient	Remainder
19/2	9	1 (Rightmost bit)
9/2	4	1
4/2	2	0

2/2	1	0
1/2	0	1 (Leftmost bit)

So $19_{10} = 10011_2$

Now we want to convert $(1250)_{10}$ to hexadecimal. So we will divide 1250 repeatedly by 16 as the base of hexa is 16.

Division	Quotient	Remainder
1250/16	78	2(Rightmost bit)
78/16	4	14 (It will be E. In hex 14 is denoted as E)
4/16	0	4(LeftMost bit)

So $1250_{10} = 4E2_{16}$

Now we want to convert $(26)_{10}$ to base 5. So we will divide 26 repeatedly by 5 as the base is 5.

Division	Quotient	Remainder
26/5	5	1(Rightmost bit)
5/5	1	0
1/5	0	1((LeftMost bit)

So $26_{10} = 101_5$

Decimal Number (with fraction) to Other Base Number System

1. For the round part we have to follow the previous step.
2. For the fraction part we have to repeatedly multiply the quotient of the multiplication by the base value until the fraction part of the quotient of the multiplication becomes 0 and record the round part of the quotient of the multiplication.

Example 1: $12.125_{10} = (\ ??\ )_2$

Here $12_{10} = 1100_2$

$0.125_{10} = (?)_2$

Multiplication	Quotient	Round part of quotient	Fraction
0.125*2	0.25	0	0.25
0.25*2	0.5	0	0.5
0.5*2	1.0	1	0 (we will stop as fraction part of the quotient is 0)

$12.125_{10} = 1100.001_2$ [Notice the order of the digits for fraction. It is not 1100.100 but it is 1100.001].

Example 2: $12.125_{10} = (\ ??\ )_4$

Here $12_{10} = 30_4$

So $0.125_{10} = (?)_4$

Multiplication	Quotient	Round part of quotient	Fraction
0.125*4	0.5	0	0.5
0.5*4	2.0	2	0 (We will stop as fraction part of the quotient is 0)

$12.125_{10} = 30.02_4$ [Notice the order of the digits for fraction. It is not 30.20 but it is 30.02].

Lecture 1.2: Other Base Number System to Decimal Number

1. Determine the column (positional) value of each digit (this depends on the position of the digit and the base of the number system).
2. Multiply the obtained column values (in Step 1) by the digits in the corresponding columns.
3. Sum the products calculated in Step 2. The total is the equivalent value in decimal.

Example: $1001.011_2 = (?)_{10}$

Position:	-3	-2	-1	0	1	2	3
Binary:	1	0	0	1.	0	1	1

$$1001.011 = 1 * 2^3 + 0 * 2^2 + 0 * 2^1 + 1 * 2^0 + 0 * 2^{-1} + 1 * 2^{-2} + 1 * 2^{-3}$$
$$= 8 + 0 + 0 + 1 + 0 + (\tfrac{1}{4}) + (\tfrac{1}{8})$$
$$= 8 + 1 + 0.25 + 0.125$$
$$= 9.375$$

Example: $(412.32)_5 = (\ ?\ )_{10}$

$$(412.32)_5 = 4 * 5^2 + 1 * 5^1 + 2 * 5^0 + 3 * 5^{-1} + 2 * 5^{-2}$$
$$= 100 + 5 + 2 + 0.6 + 0.08$$
$$= (107.68)_{10}$$

Lecture 1.3: Addition

1. Add the rightmost digit of the numbers.
2. If the result is equal or greater than the base then divide the result by the base.
   Then write down the remainder of the division and forward the quotient of the division as carry.
   If the result is less than the base write down the result.
3. Add the other digits and process the same way like step 2.

Examples

a)

$$\begin{array}{r} \mathbf{1110_2} \\ \mathbf{1011_2} \\ \underline{\mathbf{+\ 1011_2}} \end{array}$$

Step 1: Rightmost digit of the 3 numbers are 0,1 and 1.

0+1+1=2 which is equal to base 2.

So carry = 2 / 2 = 1

Resultant digit= 2 % 2 = 0

$$\begin{array}{rl} \mathbf{1} & \textbf{(carry from previous addition)} \\ \mathbf{1110_2} & \\ \mathbf{1011_2} & \\ \underline{\mathbf{+\ 1011_2}} & \\ \mathbf{0} & \textbf{(resultant bit)} \end{array}$$

Step 2:

1 + 1 + 1 + 1 = 4 > 2

So carry = 4 / 2= 2

Resultant digit = 4 % 2 = 0

$$\begin{array}{rl} \mathbf{2} & \textbf{(carry)} \\ \mathbf{1\ 1\ 1\ 0_2} & \\ \mathbf{1\ 0\ 1\ 1_2} & \\ \underline{\mathbf{+\ \ 1\ 0\ 1\ 1_2}} & \\ \mathbf{0\ 0} & \textbf{(resultant digit)} \end{array}$$

Step 3:

2 + 1 + 0 + 0 = 3 > 2 (2 is base)

So carry= 3 / 2 = 1 and resultant digit = 3 % 2 = 1

$$\begin{array}{r} \textbf{1 (carry)} \\ 1\,1\,1\,0_2 \\ 1\,0\,1\,1_2 \\ \underline{+\ 1\,0\,1\,1_2} \\ \textbf{1 0 0 (resultant digit)} \end{array}$$

Step 4:

1 + 1 + 1 + 1 = 4 > 2 (2 is base)

So carry= 4 / 2= 2 and resultant digit = 4 % 2 = 0

$$\begin{array}{r} \textbf{2 (carry)} \\ 1\,1\,1\,0_2 \\ 1\,0\,1\,1_2 \\ \underline{+\ 1\,0\,1\,1_2} \\ \textbf{0 1 0 0 (resultant digit)} \end{array}$$

As there is no other digit left so we have to convert 2 into binary which is 10 and write down this result.

$$\begin{array}{r} 1\,1\,1\,0_2 \\ 1\,0\,1\,1_2 \\ \underline{+\quad 1\,0\,1\,1_2} \\ 1\,0\,0\,1\,0\,0_2 \end{array}$$

b)

$$\begin{array}{r} (4\,3\,)_5 \\ \underline{+\quad (2\,1)_5} \end{array}$$

Step 1:

1 + 3 = 4 < 5 so no carry

$$\begin{array}{r} (4\,3\,)_5 \\ \underline{+\quad (2\,1)_5} \\ 4 \end{array}$$

<u>Step 2:</u>

4 + 2 = 6 > 5

Carry = 6 /5 = 1

Digit = 6 % 5 = 1

$$
\begin{array}{r}
(4\,3)_5 \\
+\quad \underline{(2\,1)_5} \\
1\;\;1\;\;4
\end{array}
$$

Lecture 1.4: Multiplication

Binary Multiplication

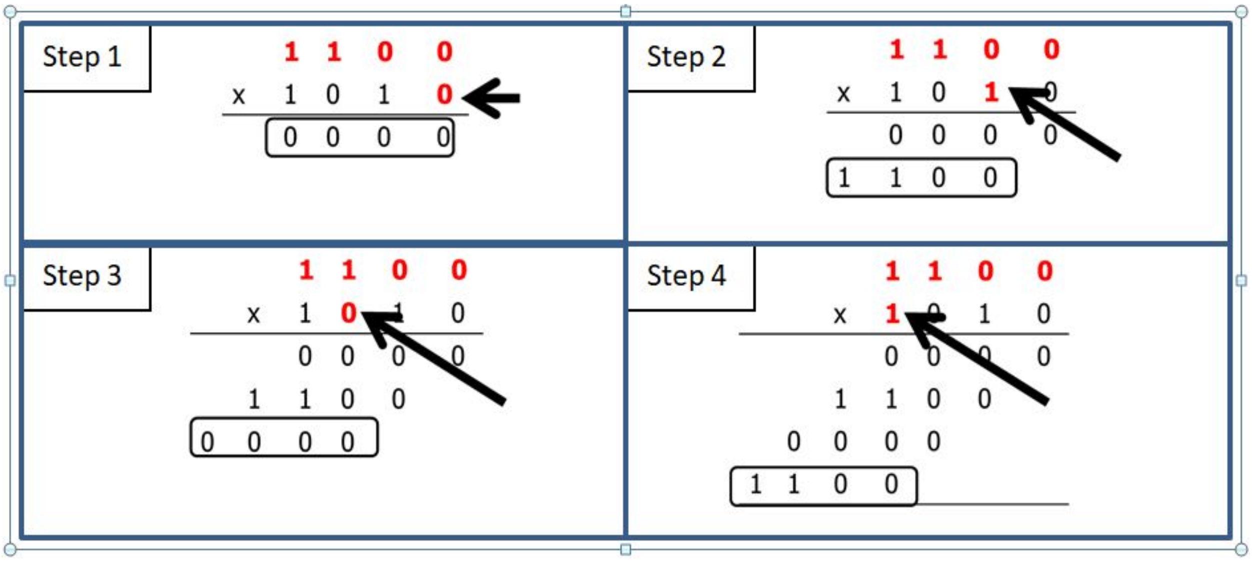
7. While multiplying two binary numbers, there can be 4 possible combinations. Multiplication of any bit with 0 results in 0. If both bits are 1, then the result is one.

A	B	**AB**
0	0	**0**
0	1	**0**
1	0	**0**
1	1	**1**

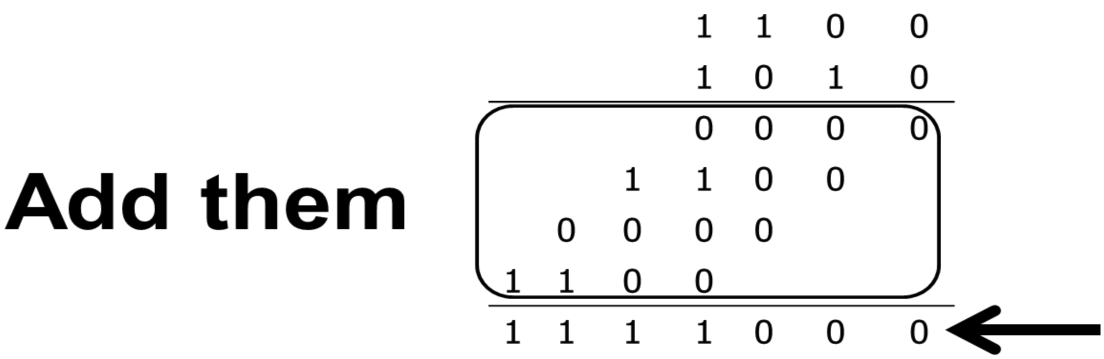
8. Multiplying two big binary numbers is very similar to decimal multiplication. Lets say, we want to multiply 1100 (12) with 1010 (10). Firstly we align the two numbers like normal subtraction.

	1	1	0	0
x	1	0	1	0

9. Next we start multiplying 1100 with 1010. One by one, we multiply 1100 with all the digits of 1010 starting from the rightmost side.



10. Finally we add the partial products and get the multiplication result.



Base-R Multiplication

- While multiplying two numbers with base other than 10, we need to keep track of two things:

  1. If the multiplicand is less than the base, then write it down directly in the result. For example, let's multiply $(5)_{13}$ with $(2)_{13}$. Their base is 13. The multiplicand will be 5*2=10 (A), which is less than 13. So our multiplication result will be $(A)_{13}$.

Steps

- 5x2 = 10 (A)
- Is 10 > base 13? - No
- Answer: $(A)_{13}$

2. If the multiplicand is greater or equal to our base, then first divide the multiplicand with the base. Then, write down the remainder in the result and the dividend in the carry. Let's multiply $(4)_5$ with $(2)_5$. The multiplicand is 4*2=8, which is greater than our base 5. So we divide 8 by 5, and get 1 as dividend and 3 as remainder. So our ultimate result will be $(13)_5$.

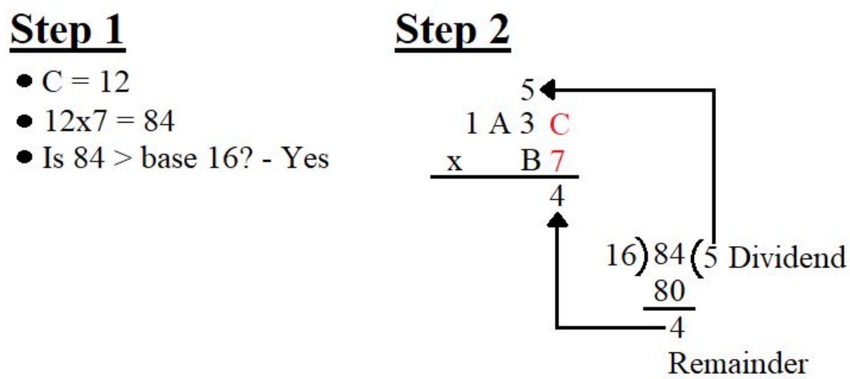
Steps

a. 4x2 = 8
b. Is 8 > base 5? - Yes
c. $5 \overline{)\,8\,}(1$ Dividend
$\quad\underline{5}$
$\quad 3$ Remainder
d.
$$\begin{array}{r} 4 \\ \times\ 2 \\ \hline 1\,3 \end{array}$$
1 Dividend, 3 Remainder

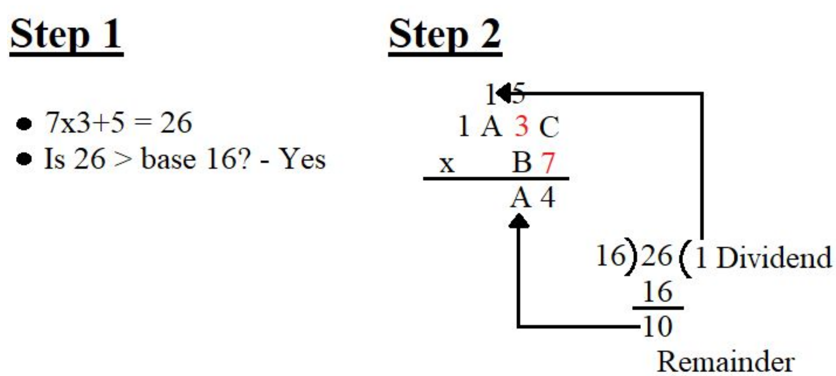
- These two rules will be applicable while multiplying big numbers as well. Let's say we multiply 1A3C with B7, both of their bases are 16.
- We start by multiplying 1A3C with the rightmost number of B7.

$$\begin{array}{rcccc} & 1 & A & 3 & C \\ \times & & & B & 7 \\ \hline \end{array}$$

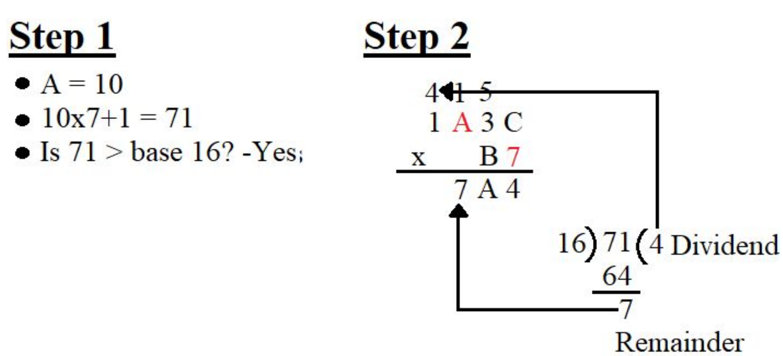
- First we multiply C with 7.



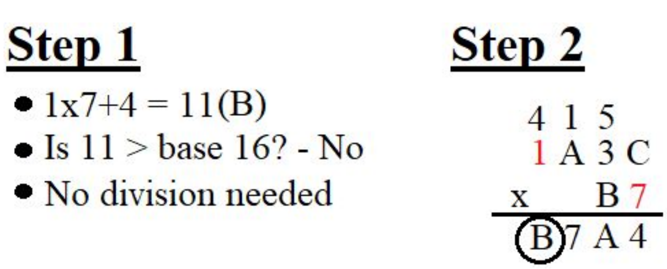
- Next, we multiply 3 with 7 and add it with the carry 5.



- Now, multiply A (10) by 7 and add it with the carry 1.



- Multiply 1 with 7 and add it with the carry 4.



- Similarly, multiply 1A3C with B

```
    1 A 3 C
        B 7
  ---------
    B 7 A 4
  1 2 0 9 4      Similarly, multiply 1A3C with B
  ---------
```

- Add the partial products and note down the answer.

```
    1 A 3 C
        B 7
  -----------
    B 7 A 4
                 Add the partial products
  1 2 0 9 4
  -----------
  1 2 C 0 E 4
```

Answer: $(12C0E4)_{16}$

Lecture 1.5: Subtraction

Binary subtraction

- We can subtract two binary numbers in four combinations.

A	B	**A - B**
0	0	**0**
0	1	**???**
1	0	**1**
1	1	**0**

- There is only one special rule for binary subtraction, which is “borrow”. Borrow is used when we try to subtract 1 from 0 (0-1). In this case, we need to borrow a number from the adjacent leftmost digit. The number we can borrow in binary is 2, which is equivalent to the base 2.

- For example, let's subtract 1 from 2 in binary. The binary for 1 is 01 and binary for 2 is 10.

$$\begin{array}{r} {\scriptstyle 0\;2} \\ \cancel{1}0 \\ -\ 01 \\ \hline 01 \end{array}$$

  - Here, since we need to perform 0-1, we borrow 2 from the leftmost bit. Next, we subtract 2-1 and note down the result

  - After borrowing 2, the left bit gets reduced by 1. So it becomes 0.

- Now let’s subtract $(001)_2$ from $(100)_2$

  - While we subtract 0-1, we need to borrow 2 from its adjacent left bit.

$$\begin{array}{r} 100 \\ -\ 001 \\ \hline \end{array}$$

- However, the left bit itself is 0, so it borrows 2 from its adjacent bit. Hence it becomes 2. The adjacent bit gets reduced by 1.

$$\begin{array}{r} 100 \\ -\ 001 \\ \hline \end{array} \Rightarrow \begin{array}{r} {\scriptstyle 0\ 2\ \ } \\ \cancel{1}00 \\ -\ 001 \\ \hline \end{array}$$

- Now the rightmost bit can borrow 2 from its adjacent left bit. The adjacent left bit gets reduced by 1.

$$\begin{array}{r} {\scriptstyle 1\ \ } \\ {\scriptstyle 0\ \cancel{2}\ 2} \\ \cancel{1}00 \\ -\ 001 \\ \hline \end{array}$$

- Finally we perform the subtraction.

$$\begin{array}{r} {\scriptstyle 1\ \ } \\ {\scriptstyle 0\ \cancel{2}\ 2} \\ \cancel{1}00 \\ -\ 001 \\ \hline 011 \end{array}$$

Base-R subtraction

- Just like binary subtraction, we need the concept of borrow in other bases as well.
    - The amount we can borrow is equivalent to the base. For example, if our base is 5, we can borrow 5 from the adjacent left bit. If we subtract two numbers of base r, then we can borrow r.
    - The bit/number we borrow from is always reduced by 1.
- Let's subtract $(35)_6$ from $(54)_6$. Note that the base is 6.

$$\begin{array}{r} (54)_6 \\ -\ (35)_6 \\ \hline \end{array}$$

- Since 4 is less than 5, so we need to borrow 6 from its adjacent left bit. Note that after borrowing, the left bit gets reduced by 1. At the same time, 4 becomes 4+6 = 10 after borrowing.

$$\begin{array}{r} {\scriptstyle 4\ 10} \\ (\cancel{5}\cancel{4})_6 \\ -\ (35)_6 \\ \hline \end{array}$$

- Now we perform the subtraction.

$$\begin{array}{r} {\scriptstyle 4\ 10} \\ (\cancel{5}\cancel{4})_6 \\ -\ (35)_6 \\ \hline (15)_6 \end{array}$$

● Subtract $(1B3)_{16}$ from $(4A6)_{16}$

$$\begin{array}{r} (4\ A\ 6)_{16} \\ -\ (1\ B\ 3)_{16} \\ \hline \end{array}$$

- Since 6 > 3, no borrowing needed.

$$\begin{array}{r} (4\ A\ 6)_{16} \\ -\ (1\ B\ 3)_{16} \\ \hline 3\phantom{)_{16}} \end{array}$$

- Since 10 (A) < 11 (B), we need to borrow from the adjacent left bit. After borrowing, 10 becomes 10+16 = 26 and 4 becomes 4-1 = 3. Then we perform the subtraction.

$$\begin{array}{r} {\scriptstyle 3\ \ 26} \\ (\cancel{4}\ \cancel{A}\ 6)_{16} \\ -(1\ B\ 3)_{16} \\ \hline 3\phantom{)_{16}} \end{array} \Rightarrow \begin{array}{r} {\scriptstyle 3\ \ 26} \\ (\cancel{4}\ \cancel{A}\ 6)_{16} \\ -(1\ B\ 3)_{16} \\ \hline F\ 3\phantom{)_{16}} \end{array}$$

- Since 3 > 1, no borrowing needed.

$$
\begin{array}{r}
\overset{3}{\cancel{4}}\ \overset{26}{\cancel{A}}\ 6)_{16} \\
-(1\ B\ 3)_{16} \\
\hline
(2\ F\ 3)_{16}
\end{array}
$$