

Set

1. List the members of these sets :
 - a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$
 - b) $\{x \mid x \text{ is a positive integer less than } 12\}$
 - c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$
2. Use set builder notation to give a description of each of these sets.
 - a) $\{0, 3, 6, 9, 12\}$
 - b) $\{-3, -2, -1, 0, 1, 2, 3\}$
 - c) $\{m, n, o, p\}$
3. Use a Venn diagram to illustrate the subset of even integers in the set of all integers whose absolute value does not exceed 15.
4. Is x an element of the set $S = \{\{x\}, \{x, \{x\}\}\}$? Explain your answer.
5. What is the cardinality of the set $S = \{a, b, \{a, b\}, c\}$? Determine $P(S)$.
6. Can you conclude that $A = B$ if A and B are two sets with the same power set?
7. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find
 - a. $A \times B$
 - b. $B \times A$
 - c. A^2

d. B^3

8. Let $A = \{a, b, c, d\}$, $B = \{m, n\}$ and $C = \{y, z\}$. Determine $A \times B \times C$ and $(A \times B) \times C$.

Explain why they are not the same.

9. Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find

a. $A \cap B \cap C$.

b. $A \cup B \cup C$.

c. $(A \cup B) \cap C$.

d. $(A \cap B) \cup C$.

10. Let $A = \{x : x \text{ is an even natural number and } 1 < x \leq 12\}$ and $B = \{x : x \text{ is a multiple of 3, } x \in \mathbb{N} \text{ and } x \leq 12\}$ be two sets. Find $A \cap B$.

11. Mother Dairy polls its customers on their favorite flavor: chocolate or vanilla? 103

customers said they liked chocolate, 98 customers said they like vanilla, while 27

customers said they liked both chocolate and vanilla. How many customers said they like only chocolate?

12. At a school of 500 students, there are 125 students enrolled in Algebra, 257 students who play sports and 52 students that are enrolled in Algebra and play sports. Create a Venn diagram to illustrate this information.

13. In a survey of 150 high school students it was found that: 80 students have laptops, 110 students have cell phones, 125 students have iPods, 62 students have both a laptop and a cell phone, 58 students have both a laptop and iPod, 98 students have both a cell phone and an iPod, 50 students have all three items

- a. How many students have just a cell phone?
- b. How many students have none of the mentioned items?
- c. How many students have an iPod and laptop but not a cellphone?

Function

1. Let $f(x) = x^2/3$. Find $f(S)$ if
 - a. $S = \{-2, -1, 0, 1, 2, 3\}$. b)
 - b. $S = \{0, 1, 2, 3, 4, 5\}$.
 - c. $S = \{1, 5, 7, 11\}$.
 - d. $S = \{2, 6, 10, 14\}$
2. Let f_1 and f_2 be functions from \mathbb{R} to \mathbb{R} such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. What are the functions $f_1 + f_2$ and $f_1 f_2$?
3. Determine whether each of these functions from \mathbb{Z} to \mathbb{Z} is one-to-one.
 - a. $f(n) = n - 1$
 - b. $f(n) = n^2 + 1$
 - c. $f(n) = n^3$
4. Give an example of a function from \mathbb{N} to \mathbb{N} that is
 - a. one-to-one but not onto.
 - b. onto but not one-to-one.
 - c. both onto and one-to-one (but different from the identity function).
 - d. neither one-to-one nor onto.
5. Find the inverse function of $f(x) = x^3 + 1$.
6. Draw the graph of the function $f(n) = 1 - n^2$ from \mathbb{Z} to \mathbb{Z} .

7. Find the ranges of the following functions.

a. $f(x) = x; x \in \mathbb{R}$

b. $f(x) = x^2; x \in \mathbb{R}$

c. $g(x) = \sin(x); x \in \mathbb{R}$

Combinatorics

1. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?
2. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?
3. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there in the committee. In how many ways can it be done?
4. In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?
5. In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?
6. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
7. In how many different ways can the letters of the word 'MATHEMATICS' be arranged such that the vowels must always come together?
8. There are 8 men and 10 women and you need to form a committee of 5 men and 6 women. In how many ways can the committee be formed?
9. How many 3-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?
10. In how many different ways can the letters of the word 'LEADING' be arranged such that the vowels should always come together?
11. A coin is tossed 3 times. Find out the number of possible outcomes.
12. A bag contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the bag, if at least one black ball is to be included in the draw?

13. How many 3 digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9 which are divisible by 5 and none of the digits is repeated?
14. There are 6 periods in each working day of a school. In how many ways can one organize 5 subjects such that each subject is allowed at least one period?
15. How many 6 digit telephone numbers can be formed if each number starts with 35 and no digit appears more than once?
16. An event manager has ten patterns of chairs and eight patterns of tables. In how many ways can he make a pair of table and chair?
17. 25 buses are running between two places P and Q. In how many ways can a person go from P to Q and return by a different bus?
18. A question paper has two parts P and Q, each containing 10 questions. If a student needs to choose 8 from part P and 4 from part Q, in how many ways can he do that?
19. In how many different ways can 5 girls and 5 boys form a circle such that the boys and the girls alternate?
20. In how many ways can 5 men draw water from 5 taps if no tap can be used more than once?