CSE230: Discrete Mathematics

Assignment 2

Deadline: 17th July, 2022

1. The coefficient of y^p and y^{1+p} in the expansion of $(2 + 3y)^{19}$ are equal, find p.

2. Given that the coefficient of the r^{th} term from the beginning and the r^{th} term from the end of the expansion of $(3x + 5)^{15}$ are in the ratio $27: 125$. Find the value of r.

3. The fifth term in the expansion of $(2x + k)^6$ is equal to the fourth term in the expansion of $(2 + kx)^8$. Find the value of k.

4. What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x - 3y)^{200}$?

5. Given that $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 29$, where $k \in \{1, 2, 3, 4, 5, 6\}$ and a_k is a non-negative integer. What is the probability that
   a. $a_k > 1$
   b. $a_k > k$
   c. $a_1 \leq 5$

6. What is the coefficient of the term independent of x and y in the multinomial expansion $(5x + \frac{3}{y} + \frac{4}{x} + \frac{5}{7}y^2)$?

7. Two of 10 marbles, labeled from 1 to 10, are picked from a bag without replacement.
   a. Write down the sample space of this experiment using the set builder method.
   b. What is the probability that the number on the second ball is smaller than the number on the first?
   c. What is the probability that the number on one ball is even and the other is odd.

8. One card is randomly drawn from a deck of cards. Let,

   A: $The\ card\ is\ a\ face\ card\ (Jack,\ Queen\ or\ King)$
   B: $The\ card\ is\ a\ spade$
   C: $The\ card\ is\ red$

   Calculate the value of $P(AUBUC)$.

9. Two dice (black and white) are rolled. Test if the following events are independent:

   A: $Black\ die\ rolls\ a\ 1,\ 2,\ or\ 6$
   B: $White\ die\ rolls\ a\ 3,\ 4,\ 5\ or\ 7$
   C: $The\ sum\ of\ the\ rolls\ is\ 7$

10. A biased coin, in which the likeliness of landing on heads is three times more than tails, is tossed 100 times. Find the number of times we can expect the coin to land on heads.

11. Little Red Riding Hood carried her grandma's sewing basket which contained 8 balls of cotton: 4 green, 3 red and 1 yellow. Three cotton balls are randomly selected from the basket. Let the discrete random variables G, R and Y be the number of green, red and yellow balls selected respectively.
    a. Draw up the probability distribution table of R and Y.
    b. Find $E(Y)$ and $E(R)$
    c. Hence, without using the probability distribution of G, find $E(G)$.
    d. Find $Var(R)$

12. From each penalty Christiano Ronaldo takes, he has a 95% chance of scoring. What is the probability that he:
    a. Scores all in his next 10 penalty shots.
    b. Fails to score from exactly one of his next 7 shots.

13. 14% of the vehicles driven on a bridge are SUVs. Richard, a toll collector, counts the number of vehicles until the first SUV crosses him. What is the probability he counts:
    a. At most 3 vehicles
    b. At least 4 vehicles

14. In a sample space of 200 left handed batsmen (LHB) and 300 right handed batsmen (RHB), it is assumed that 0.5% of all LHB and 0.4% of all RHB have trouble playing in-swinger delivery. What is the probability that exactly 1 batsman in the sample space faces difficulty playing an in-swinger?