HNA Diff $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ w.r.to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Sol Let $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ —(i) & $z = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ —(ii)

we need to find $\frac{dy}{dz}$

$\therefore \frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz}$ — 10

Let $x = \tan\theta$

Eq^n(i) $\Rightarrow$

$$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$= \cos^{-1}\cos 2\theta$$

$$= 2\theta$$

$\therefore y = 2\tan^{-1}x$

$\frac{dy}{dx} = \frac{2}{1+x^2}$ — 15

Eq^n(ii) $\Rightarrow$

$$z = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$= \tan^{-1}\tan 2\theta$$

$$= 2\theta$$

$\therefore z = 2\tan^{-1}x$

$\frac{dz}{dx} = \frac{2}{1+x^2}$ — 15

$\therefore \frac{dy}{dz} = \frac{2}{1+x^2} \cdot \frac{1+x^2}{2}$

$= 1$ (10)

Ans

Total — 50

Scanned with CamScanner

HNA

Diff $x^{\sin^{-1}x}$ w.r.to $\sin^{-1}x$

Let $y = x^{\sin^{-1}x}$ —— (i) & $z = \sin^{-1}x$ —— (ii)

$$\frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz} \qquad \text{——} \quad 10$$

In on both sides of (i)

$$\ln y = \sin^{-1}x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \sin^{-1}x \cdot \frac{1}{x} + \frac{1}{\sqrt{1-x^2}} \ln x$$

$$\frac{dy}{dx} = y\left[\frac{\sin^{-1}x}{x} + \frac{\ln x}{\sqrt{1-x^2}}\right]$$

$$= x^{\sin^{-1}x}\left[\frac{\sin^{-1}x}{x} + \frac{\ln x}{\sqrt{1-x^2}}\right] \qquad \text{——} \quad 20$$

Eq (ii) ⇒

$$\frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{——} \quad 10$$

$$\frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz}$$

$$= x^{\sin^{-1}x}\left(\frac{\sin^{-1}x}{x} + \frac{\ln x}{\sqrt{1-x^2}}\right) \cdot \sqrt{1-x^2}$$

—— 10 Ans

2. let $u = \cos 3x$, $v = x^2$, Then by leibniz,

$$y''' = \sum_{i=0}^{3} \binom{3}{i} u^{3-i} v^{i}$$

$$= \sum_{i=0}^{3} \binom{3}{i} (\cos 3x)^{3-i} (x^2)^{i}$$

$u' = -3\sin 3x$, $u'' = -9\cos 3x$, $u''' = 27\sin 3x$

$v' = 2x$, $v'' = 2$, $v''' = 0$.

$\therefore$ Req^d ~~sol^n~~ 3rd derivative,

$$y''' = (27x^2 - 18)\sin 3x - 54x\cos 3x$$

4. $y = \dfrac{x^3}{x+2}$

let, $u = \dfrac{1}{x+2}$, $v = x^3$. Using Leibniz,

$$(uv)^n = \sum_{i=0}^{n} \binom{n}{i} u^{n-i} v^{i}$$

$$y''' = \sum_{i=0}^{3} \binom{3}{i} \left(\frac{1}{x+2}\right)^{3-i} (x^3)^{i}$$

$u' = -\dfrac{1}{(x+2)^2}$; $u'' = \dfrac{2}{(x+2)^3}$, $u''' = \dfrac{6}{(x+2)^4}$

$v' = 3x^2$; $v'' = 6x$; $v''' = 6$

Req^d 3^{rd} derivative at $x=1$,

$$y''' = \frac{48}{(x+2)^4}$$

$$\Rightarrow y'''(-1) = \frac{48}{(-1+2)^4}$$

$$= 48. \quad \underline{Ans}$$

1. $y = e^{2x} \ln x$

let, $u = e^{2x}$, $v = \ln x$

$u' = 2e^{2x}$, $u'' = 4e^{2x}$, $u''' = 8e^{2x}$

$v' = \frac{1}{x}$, $v'' = \frac{-1}{x^2}$, $v''' = \frac{2}{x^3}$

$\therefore$ The req^d 3^{rd} order derivative,

$$y''' = \sum_{i=0}^{3} \binom{3}{i} (e^{2x})^{(3-i)} (\ln x)^{i}$$

$$= 2e^{2x}\left(4\ln x + \frac{6}{x} - \frac{3}{x^2} + \frac{1}{x^3}\right) \quad \underline{\underline{Ans}}$$

3. $y = x \sinh x$

let, $u = \sinh x$, $v = x$

$$y^{(4)} = \sum_{i=0}^{4} \binom{4}{i} (\sinh x)^{4-i} x^{i}$$

$u' = \cosh x$, $u'' = \sinh x$, $u''' = \cosh x$, $u^4 = \sinh x$

$\therefore$ req^d 4^{th} derivative,

$$y^4 = x\sinh x + 4\cosh x \quad \underline{Ans}$$

Tangents using Logarithmic Differentiation

0 points possible (ungraded)

(MJM) Using logarithmic differentiation find the gradient of the tangent to the curve represented by the equation $y = \frac{e^x \cos^3 x}{x^2+2x+1}$ at the point on the curve where $x = 0$. Hence, find the equation of the tangent to the curve at that point.

$$y = \frac{e^x \cos^3 x}{x^2+2x+1}$$

$$\ln y = \ln(e^x) + \ln(\cos^3 x) - \ln(x^2+2x+1)$$

$$\ln y = \ln(e^x) + 3\ln(\cos x) - \ln(x^2+2x+1)$$

[10 marks]

Differentiating both sides w.r.t x,

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{e^x}\cdot e^x + \frac{3}{\cos x}(-\sin x) - \frac{1}{x^2+2x+1}(2x+2)$$

[10 marks]

$$\frac{1}{y}\frac{dy}{dx} = 1 - 3\tan x - \frac{2x+2}{x^2+2x+1}$$

$$\frac{dy}{dx} = \frac{e^x \cos^3 x}{x^2+2x+1}\left(1 - 3\tan x - \frac{2x+2}{x^2+2x+1}\right)$$

[10 marks]

When $x = 0$, $\frac{dy}{dx} = \frac{1\cdot 1}{1}\left(1 - 3\cdot 0 - \frac{2}{1}\right) = -1$

$y = \frac{1\cdot 1}{1} = 1$

[10 marks]

$\therefore\ y - 1 = -1(x-0)$

$$\boxed{y = -x+1}$$

[10 marks]

Implicit differentiation and Tangents

0 points possible (ungraded)

(MJM) Find the equations of tangents for the implicit function $x^3y + \cos(x^2y) = x^2$ at the points on the curve where $y = 0$.

$$\frac{d}{dx}\left(x^3y + \cos(x^2y)\right) = \frac{d}{dx}(x^2)$$

$$\Rightarrow 3x^2y + x^3\frac{dy}{dx} - \sin(x^2y)\cdot\frac{d}{dx}(x^2y) = 2x$$

$$\Rightarrow 3x^2y + x^3\frac{dy}{dx} - \sin(x^2y)\cdot\left(2xy + x^2\frac{dy}{dx}\right) = 2x$$

[10 marks]

$$\Rightarrow 3x^2y + x^3\frac{dy}{dx} - 2xy\sin(x^2y) - x^2\sin(x^2y)\frac{dy}{dx} = 2x$$

$$\Rightarrow \frac{dy}{dx}\left(x^3 - x^2\sin(x^2y)\right) = 2x - 3x^2y + 2xy\sin(x^2y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 - 3xy + 2y\sin(x^2y)}{x^2 - x\sin(x^2y)}$$

[10 marks]

When $y = 0$, $\quad x^3\cdot 0 + \cos(x^2\cdot 0) = x^2$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1$$

[10 marks]

$(1, 0)$ and $(-1, 0)$

For $(1,0)$, $\frac{dy}{dx} = \frac{2+0+0}{1-0} = 2$

For $(-1,0)$, $\frac{dy}{dx} = \frac{2+0+0}{1-0} = 2$

[10 marks]

Equation of tangent at $(1,0)$,

$$y - 0 = 2(x-1)$$
$$\Rightarrow \boxed{y = 2x - 2}$$

Equation of tangent at $(-1,0)$,

$$y - 0 = 2(x+1)$$
$$\Rightarrow \boxed{y = 2x + 2}$$

[10 marks]

Parametric Differentiation and Tangents

0 points possible (ungraded)

(MJM) Find the tangent line equation for the parametric curve represented by the equations $x = 2\cos(t + \frac{\pi}{3})$ and $y = 4\sin t$ at the point $t = -\frac{\pi}{6}$. Give your answer in the form $y = a - bx$ where a and b are real numbers in their exact form.

$$\frac{dx}{dt} = -2\sin\left(t + \frac{\pi}{3}\right) \qquad \frac{dy}{dt} = 4\cos t$$

[10 marks]

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4\cos t}{-2\sin\left(t + \frac{\pi}{3}\right)}$$

[10 marks]

When $t = -\frac{\pi}{6}$, $\frac{dy}{dx} = \frac{4\cos\left(-\frac{\pi}{6}\right)}{-2\sin\left(-\frac{\pi}{6} + \frac{\pi}{3}\right)}$

$$= \frac{4 \cdot \frac{\sqrt{3}}{2}}{-2 \cdot \frac{1}{2}} = -2\sqrt{3}$$

[10 marks]

When $t = -\frac{\pi}{6}$,

$$x = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$$

$$y = 4\sin\left(\frac{\pi}{6}\right) = 2$$

[10 marks]

Equation of tangent at $(\sqrt{3}, 2)$,

$$y - 2 = -2\sqrt{3}(x - \sqrt{3})$$

$$\Rightarrow y - 2 = -2\sqrt{3}x + 6$$

$$\Rightarrow \boxed{y = 4 - 2\sqrt{3}x}$$

[10 marks]

Parametric Differentiation and Tangents

0 points possible (ungraded)

(MJM) Find the equations of the tangent lines to the curve whose parametric equations are $x = 2t^2 - 4t + 1$, $y = 3t^4 - 12t^2 + 6$; at the point $(1, 6)$.

$$\frac{dx}{dt} = 4t - 4 \qquad \frac{dy}{dt} = 12t^3 - 24t$$

[10 marks]

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{12t^3 - 24t}{4t - 4}$$

$$= \frac{12t(t^2-2)}{4(t-1)} = \frac{3t(t^2-2)}{t-1}$$

[10 marks]

When $x = 1$, $2t^2 - 4t + 1 = 1$
$\Rightarrow 2t(t-2) = 0$
$t = 0$, $t = 2$

When $y = 6$, $3t^4 - 12t^2 + 6 = 6$
$\Rightarrow 3t^2(t^2 - 4) = 0$
$t = 0$, $t = 2$, $t = -2$

[10 marks]

$\therefore$ For $t = 0$, $\frac{dy}{dx} = 0$. Equation of tangent: $y - 6 = 0 \Rightarrow \boxed{y = 6}$

[10 marks]

For $t = 2$, $\frac{dy}{dx} = \frac{6(4-2)}{1} = 12$. Equation of tangent: $y - 6 = 12(x - 1)$
$\Rightarrow \boxed{y = 12x - 6}$

[10 marks]

Find the local linear approximation of the function $f(x) = \ln x$, at $x=1$

Solution:

$$f(x) = \ln x$$
$$f'(x) = \frac{1}{x}$$
$$f'(1) = 1/1$$
$$f'(1) = 1 \quad \text{......(20 marks)}$$

$$L(x) = f(a) + f'(a)(x-a)$$
$$= f(1) + 1(x-1)$$
$$= 0 + x - 1$$
$$= x - 1. \quad \text{......(30 marks)}$$

3. Find the local linear approximation $L(x)$ of the function $f(x) = x^2+3$ at $x=1$

Solution: $f(x) = x^2+3$

$f'(x) = 2x$

$f'(1) = 2$

$f(1) = 1^2+3 = 4$ .... (20)

$L(x) = f(a) + f'(a)(x-a)$

$= f(1) + f'(1)(x-1)$

$= 4 + 2(x-1)$

$= 4+2x-2 = 2x+2$ ....(30 marks)

4. Use an appropriate linear approximation to estimate the value of $\ln(1.01)$. Compare your approximation to the result produced directly by a calculating utility.

Solution: $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$

$x = 1.01 \Rightarrow f'(a) = 1$

$a = 1 \Rightarrow f(a) = \ln 1 = 0$

.... (20)

$f(x) \approx f(a) + f'(a)(x-a)$

$\approx 0 + 1(1.01-1)$

$= 1(0.01)$

≈ 0.01

$\ln 1.01 \approx 0.01$ ..... (20)

By the calculator approximation $\ln 1.01 \approx 0.009$ ... (10 marks)

Find the local linear approximation of the function $f(x) = 5-x^2$ at $x=2$

<u>solution:</u> $f(x) = 5-x^2$

$f'(x) = -2x$

$f'(2) = -4$

$f(2) = 5-2^2 = 1$ ..... (20 marks)

$L(x) = f(a) + f'(a)(x-a)$

$= f(2) + f'(2)(x-2)$

$= 1-4(x-2)$

$= 1-4x+8$

$= 9-4x$ .. (30 marks)

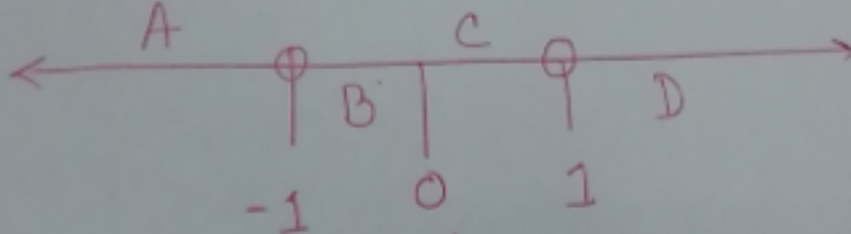
Find relative extrema of $f(x) = 3x^5 - 5x^3$

$\therefore \quad f'(x) = 15x^4 - 15x^2$

$\quad\quad f''(x) = 60x^3 - 30x$

<u>C.P:</u> $f'(x) = 0$

$\Rightarrow x = 0, \pm 1$



$f''(0) = 0$, inconclusive

$f''(1) = 30 > 0$, $f(x)$ will attain min value

Min. Value of $f(x)$, $f(1) = -2$

$f''(-1) = -30 < 0$, $f(x)$ — max. value

Max. Value of $f(x)$, $f(-1) = 2$

Value of $f(x) = \frac{x+3}{x-}$

$f''(-1) = -30 < 0$. $f(x)$ — max. value

Max. Value of $f(x)$, $f(-1) = 2$

Extreme Value of $f(x) = \frac{x+3}{x-2}$.

$$\therefore f'(x) = \frac{1}{(x+2)^2}$$

~~The 1st~~ There is no point where

$$f'(x) = 0$$

$\therefore$ The function has no extreme value.

$f(-1) = -30 < 0$, max ...

$\therefore f(1) = -2$, minimum value

$f(-1) = 2$, max value

1

Surface Area $(A) = 2\pi r h + \pi r^2$

$\pi r^2 h = 1$

$h = \frac{1}{\pi r^2}$

$\Rightarrow A = \frac{2}{r} + \pi r^2$

$\Rightarrow A' = -\frac{2}{r} + 2\pi r$

if $A' = 0$, $r = 0.6828$

and $h = 0.47$ meters

Ques:- A company Manufactures .. ✓ . 1 Cubic M.
What should be ...... in their production?

Set 3

Ques:- Airlines requires ... as its width. what is the largest Air flight?

(1) $l+w+h=62$

$\Rightarrow l = 62 - w - h$

$\therefore$ volume $V = (62-2w)w \cdot w$

$= 124w - 6w^2$

$V' = 124w - 6w^2$ (Taking $V'=0$)

$\Rightarrow w = 0$ or $62/3$
NA

$\therefore w = l = h = \frac{62}{3} = 20.66$.

$(x^3+2x^2)e^{2x}$