Assignment 3 Questions

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Partial Derivatives

0 points possible (ungraded)

[SADT1] The ellipsoid $4x^2 + 2y^2 + z^2 = 16$ intersects the plane

$y = 2$ in an ellipse. Find parametric equations for the tangent line to this ellipse at the point $(1, 2, 2)$.

Submit

Chain Rule

0 points possible (ungraded)

[SADT13] If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ and hence show that:

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$$

Submit

Extrema

0 points possible (ungraded)

[SADT5] The temperature in space given by $T(x, y, z) = 200xyz^2$. Find the hottest temperature on a unit sphere centered at the origin.(DO NOT USE LAGRANGE MULTIPLIERS)

Submit

Optimization

0 points possible (ungraded)

[SADT9] Prove that in any triangle ABC there is a point P such that $PA^2 + PB^2 + PC^2$ is a minimum and that P is the intersection of the medians. (DO NOT USE LAGRANGE MULTIPLIERS)

Submit

[5ADT1] Solution:

At the plane $y=2$ the slope will be $\frac{\delta z}{\delta x}$.

So, $$\frac{\delta}{\delta x}(4x^2+2y^2+z^2) = \frac{\delta}{\delta x}16 = 0$$

$$\Rightarrow 8x + 2z\frac{\delta z}{\delta x} = 0$$

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$$\Rightarrow \frac{\delta z}{\delta x} = -\frac{4x}{z} \quad \left[\text{By solving for } \frac{\delta z}{\delta x}\right]$$

Here, $$\left.\frac{\delta z}{\delta x}\right|_{x=1,\, z=2} = -2$$

In the point $(1,2,2)$, the tangent line ~~wit~~ is passing through and the direction vector is $\langle 1, 0, -2 \rangle$.

Thus, the paramatic equations are,

$x = 1+t$, $y=2$ & $z = 2-2t$

Ans

[SADT 13] Solution:

Given, $x = r\cos\theta$ & $y = r\sin\theta$

$\theta.\ \frac{\delta x}{\delta \theta} = -r\sin\theta$ —— (4)

$\Rightarrow \frac{\delta y}{\delta \theta} = r\cos\theta$ —— (1)

$r.\ \frac{\delta x}{\delta r} = \cos\theta$ —— (2)

$\Rightarrow \frac{\delta y}{\delta r} = \sin\theta$ —— (3)

So, $z \to x,\ y \to r, \theta$

Using Chain Rule, $\frac{\delta z}{\delta r} = \frac{\delta z}{\delta x} \times \frac{\delta x}{\delta r} + \frac{\delta z}{\delta y} \times \frac{\delta y}{\delta r}$

$\Rightarrow \frac{\delta z}{\delta r} = \cos\theta \frac{\delta z}{\delta x} + \sin\theta \frac{\delta z}{\delta y}$

[Applying the value from 2 & 3]

—— (6)

Again,

Using Chain Rule,

$$\frac{\delta z}{\delta \theta} = \frac{\delta z}{\delta x} \times \frac{\delta x}{\delta \theta} + \frac{\delta z}{\delta y} \times \frac{\delta y}{\delta \theta}$$

$$\Rightarrow \frac{\delta z}{\delta \theta} = -r\sin\theta \frac{\delta z}{\delta x} + r\cos\theta \frac{\delta z}{\delta y} \quad \text{— (5)}$$

~~[Applying the value from 4 &~~

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[Applying the value from 4 & 1]

$$R.H.S = \left(\frac{\delta z}{\delta r}\right)^2 + \left(\frac{1}{r^2}\right)\left(\frac{\delta z}{\delta \theta}\right)^2$$

$$= \left(\cos\theta \frac{\delta z}{\delta x} + \sin\theta \frac{\delta z}{\delta y}\right)^2 + \left(\frac{1}{r^2}\right)\left(-r\sin\theta \frac{\delta z}{\delta x} + r\cos\theta \frac{\delta z}{\delta y}\right)^2$$

[Applying the value from 5 & 6]

$$= \cos^2\theta \left(\frac{\delta z}{\delta x}\right)^2 + 2\sin\theta \frac{\delta z}{\delta x} \times \cos\theta \frac{\delta z}{\delta y}$$

$$+ \sin^2\theta \left(\frac{\delta z}{\delta y}\right)^2 + \sin^2\theta \left(\frac{\delta z}{\delta x}\right)^2 - 2\sin\theta \frac{\delta z}{\delta x} \cos\theta \frac{\delta z}{\delta y}$$

$$+ \cos^2\theta \left(\frac{\delta z}{\delta y}\right)^2$$

$$= (\cos^2\theta + \sin^2\theta)\left(\frac{\delta z}{\delta x}\right)^2 + (\cos^2\theta + \sin^2\theta)\left(\frac{\delta z}{\delta y}\right)^2$$

$$= \left(\frac{\delta z}{\delta x}\right)^2 + \left(\frac{\delta x}{\delta y}\right)^2$$

$= L.H.S$

[Showed]

[SAQT 5] Solution:

Given, The temperature T at any point (x, y, z) in space in $T = 400\,xyz^2$

So, $x^2 + y^2 + z^2 = 1$.

Also, $2x = 400\,yz^2$

$2y = 400\,xz^2$

$2z = 800\,xyz$

Now, $\frac{z}{2x} = \frac{x}{z}$

$\Rightarrow z^2 = 2x^2$

~~Similarly, $2y^2 =$~~

Similarly, $2y^2 = z^2$

And $2z^2 = 1$

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Thus, $z = \pm 1/\sqrt{2}$, $y = \pm 1/2$ & $x = \pm 1/2$

So, the hottests tempareature is,

$$T = 200 \times 1/2 \times 1/2 \times (1/\sqrt{2})^2$$

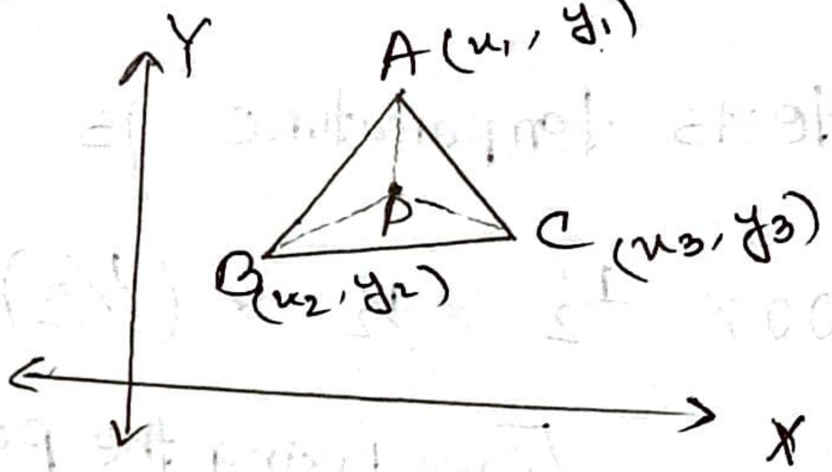
[Only taking the positive values]

$$\Rightarrow T = 50 \times \frac{1}{2}$$

$$\therefore T = 25$$

Ans

[SADT 9] Solution:



Let, P, A, B & C have the coordinates $P(x, y)$, $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$.

If, P is is a point inside ΔABC then: $PA^2 + PB^2 + PC^2$ is minimum.

Let, $PA^2 = (x_1 - x)^2 + (y_1 - y)^2$

$PB^2 = (x_2 - x)^2 + (y_2 - y)^2$

$PC^2 = (x_3 - x)^2 + (y_3 - y)^2$

So, $PA^2 + PB^2 + PC^2 = f(x, y)$

Now, $\frac{\delta f}{\delta x} = 2(x_1 - x) + 2(x_2 - x) + 2(x_3 - x)$

$= 0$

$\therefore x_1 - x + x_2 - x + x_3 - x = 0$

$\Rightarrow x_1 + x_2 + x_3 - 3x = 0$

$\Rightarrow 3x = x_1 + x_2 + x_3$

$\therefore x = \frac{x_1 + x_2 + x_3}{3}$

$$\frac{\delta f}{\delta y} = 2(y_1 - y) + 2(y_2 - y) + 2(y_3 - y) = 0$$

$$\therefore y_1 - y + y_2 - y + y_3 - y = 0$$

$$\Rightarrow y_1 + y_2 + y_3 - 3y = 0$$

$$\Rightarrow 3y = y_1 + y_2 + y_3$$

$$\therefore y = \frac{y_1 + y_2 + y_3}{3}$$

Thus, $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$

is the intersection on the point P.

$$f_{xx} = 2 + 2 + 2 = 6$$

$$f_{yy} = 2 + 2 + 2 = 6$$

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And $f_{xy} = 0$

So, $f_{xx} \cdot f_{yy} - (f_{xy})^2 = 36 - 0 = 36 > 0$

$\therefore$ The point P is a minimum.

[Proved]