Assignment for MAT 120

Note:

- Answer all the following questions.
- Each problem carries **10 marks**.
- Submit soft copy (pdf file) of your assignment via Google form.
- Plagiarism will lead you 0 mark.
- Date of submission: 21 August 2022 (up to 11:59 pm).

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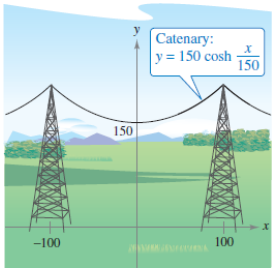
1. Cooling towers at nuclear power plants have a pinched chimney shape (which promotes cooling within the tower) formed by rotating the following hyperbola (that describes the shape of such a tower) $y = f(x) = \sqrt{1 + \frac{x^2}{22500}}$; $-250 \le x \le 150$ around x-axis. Determine the volume of the tower. Also find the surface area of the shape.

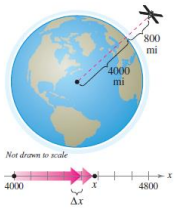
2. An electric cable whose equation is $y = 75\left(e^{\frac{x}{150}} - e^{-\frac{x}{150}}\right)$ is hung between two towers that are 200 feet apart. Find the arc length of the cable between the two towers.



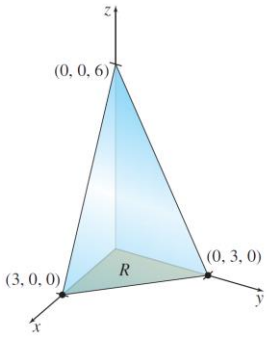
3. The total work done that is needed to propel a space module to space is given by

$$W = \int_a^b F(x)\,dx \qquad where, F(x) = \frac{240000000}{x^2}$$

- Find the work done in propelling the module to a height of 800 miles above the Earth ?
- How much work is required to propel the module into an unlimited distance away from Earth's surface?



4. The manufacturer of a mango juice drink has decided to try innovative packaging in order to optimize their sales. The mango juice drink is to be packaged in containers in the shape of tetrahedra described by the plane $z = f(x, y) = 6 - 2x - 2y$ in which three edges are perpendicular, two of the perpendicular edges is 3 inches long, and the third edge is 6 inches long. Find the volume of the container.



5. The differential equation Eq. (1) is known as Logistic equation which is applied in a research (see this link: https://doi.org/10.1016/j.idm.2020.08.006) to model the COVID-19 pandemic of Kuwait. The model is as follows:

$$\frac{dC}{dt} = rC\left(1 - \frac{C}{K}\right) \qquad (1)$$

where C is the cumulative number of infected cases, r is the infection rate, and K is the final epidemic size known as carrying capacity.

Prove that the solution of Eq. (1) is $C(t) = \dfrac{K}{1 + Ae^{-rt}}$,

where $A = \dfrac{K - C_0}{C_0}$ and C_0 is the initial number of infected cases.

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Infectious Disease Modelling

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Adequacy of Logistic models for describing the dynamics of COVID-19 pandemic

Abdallah Abusam [a, *], Razan Abusam [b], Bader Al-Anzi [c]

[a] Water Research Center, Kuwait Institute for Scientific Research, P. O. Box 24885, Safat, 13109, Kuwait
[b] Chemical Engineering Department, University of the West of Scotland, UK
[c] Department of Environment Technologies and Management, Kuwait University, Kuwait

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ABSTRACT

Logistic models have been widely used for modelling the ongoing COVID-19 pandemic. This study used the data for Kuwait to assess the adequacy of the two most commonly used logistic models (Verhulst and Richards models) for describing the dynamics COVID-19. Specifically, the study assessed the predictive performance of these two models and the practical identifiability of their parameters. Two model calibration approaches were adopted. In the first approach, all the data was used to fit the models as per the heuristic model fitting method. In the second approach, only the first half of the data was used for calibrating the models, while the other half was left for validating the models. Analysis of the obtained calibration and validation results have indicated that parameters of the two models cannot be identified with high certainty from COVID-19 data. Further, the models shown to have structural problems as they could not predict reasonably the validation data. Therefore, they should not be used for long-term predictions of COVID-19. Suggestion have been made for improving the performances of the models.