Chapter - 2

1st Order 1st Degree Ordinary Differential Equtions

2.2 - *Separable Variables*

(i) Solve the given differential equation by separation of variables:

$$1.\frac{dy}{dx}=\sin 5x,\quad 2.\ dx+e^{3x}dy=0,\quad 3.\ x\frac{dy}{dx}=4y,\quad 4.\ \frac{dy}{dx}+2xy=0,\quad 5.\ \frac{dy}{dx}=e^{3x+2y},$$

$$6.\ e^{x}y\frac{dy}{dx}=e^{-y}+e^{-2x-y},\quad 7.\ y\ln x\frac{dx}{dy}=\left(\frac{y+1}{x}\right)^2,$$

$$8.\ (e^{y}+1)^2e^{-y}dx+(e^{x}+1)^3e^{-x}dy=0.$$

(ii) Solve the given initial-value problem:

$$1.\ \frac{dx}{dt}=4(x^2+1),\quad x(\pi/4)=1;\quad 2.\frac{dy}{dx}=\frac{y^2-1}{x^2-1},\quad y(2)=2$$

$$3.x^2\frac{dy}{dx}=y-xy,\quad y(-1)=-1;\quad 4.\frac{dy}{dt}+2y=1,\quad y(0)=\frac{5}{2}.$$

2.3 - *Linear Equations*

(i) Find the general solution of the given differential equations:

$$1.\frac{dy}{dx}=5y,\quad 2.\frac{dy}{dx}+2y=0,\quad 3.\frac{dy}{dx}+y=e^{3x},\quad 4.(x^2-1)\frac{dy}{dx}+2y=(x+1)^2,$$

$$5.x\frac{dy}{dx}-y=x^2\sin x,\quad 6.x\frac{dy}{dx}+2y=3,\quad 7.x\frac{dy}{dx}+4y=x^3-x,$$

$$8.(1+x)\frac{dy}{dx}-xy=x+x^2,\quad 9.x^2y'+x(x+2)y=e^{x},\quad 10.\,ydx-4(x+y^6)dy=0,$$

$$11.(x+1)\frac{dy}{dx}+(x+2)y=2xe^{-x},\quad 12.x\frac{dy}{dx}+(3x+1)y=e^{-3x},\quad 13.\ 3\frac{dy}{dx}+12y=4.$$

(ii) Solve the given initial-value problem:

$$1.xy'+y=e^{x},\quad y(1)=2;\quad 2.y\frac{dx}{dy}-x=2y^2,\quad y(1)=5;$$

$$3.(x+1)\frac{dy}{dx}+y=\ln x,\quad y(1)=10;\quad 4.y'+(\tan x)y=\cos^2 x,\quad y(0)=-1.$$

2.4 - *Exact Equations*

(i) Determine whether the given differential equation is exact. If it is exact, solve it.

1.$(2x-1)\,dx+(3y+7)\,dy=0$, 2.$(2x+y)\,dx-(x+6y)\,dy=0$,

3.$2xy\,dx+(x^2-1)\,dy=0$, 4.$(x^2-y^2)dx+(x^2-2xy)dy=0$,

5. $(e^{2y}-y\cos xy)\,dx+(2xe^{2y}-x\cos xy+2y)\,dy=0$,

6.$(\sin y-y\sin x)\,dx+(\cos x+x\cos y-y)\,dy=0$,

7.$\left(1+\ln x+\dfrac{y}{x}\right)dx=(1-\ln x)\,dy$, 8. $x\dfrac{dy}{dx}=2xe^x-y+6x^2$,

9.$(x-y^3+y^2\sin x)\,dx=(3xy^2+2y\cos x)\,dy$,

10.$(y\ln x-e^{-xy})\,dx+\left(\dfrac{1}{y}+x\ln y\right)dy=0$, 11.$\left(x^2y^3-\dfrac{1}{1+9x^2}\right)\dfrac{dx}{dy}+x^3y^2=0$,

12. $(\tan x-\sin x\sin y)\,dx+\cos x\cos y\,dy=0$,

13.$(4t^3y-15t^2-y)\,dt+(t^4+3y^2-t)\,dy=0$.

(ii) Solve the given initial-value problem:

1.$(x+y)^2\,dx+(2xy+x^2-1)\,dy=0$, $y(1)=1$;

2.$(4y+2t-5)\,dt+(6y+4t-1)\,dy=0$, $y(-1)=2$;

3. $\dfrac{dy}{dx}=\dfrac{xy^2-\cos x\sin x}{y(1-x^2)}$, $y(0)=2$;

4. $(y^2\cos x-3x^2y-2x)\,dx+(2y\sin x-x^3+\ln y)\,dy=0$, $y(0)=e$.

2.5 - *Solutions by Substitution*

(i) Solve the given **homogeneous equation** by using an appropriate substitution:

1. $(x-y)dx + xdy = 0,$ 2. $(x+y)\,dx + x\,dy = 0,$ 3. $x\,dx + (y-2x)\,dy = 0,$

4. $y\,dx = 2(x+y)\,dy,$ 5. $(y^2+yx)\,dx - x^2\,dy = 0,$ 6. $\dfrac{dy}{dx} = \dfrac{y-x}{y+x},$

7. $(x^2+y^2)\,dx + (x^2-xy)\,dy = 0.$

(ii) Solve the given initial value problem:

$$xy^2\frac{dy}{dx} = y^3 - x^3, \quad y(1) = 2\,;$$

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Home Work : *Bernoulli's Equation*

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Problem : Solve the following *differential equation.*

1. $\dfrac{dy}{dx} = y(xy^3 - 1)$

2. $x\dfrac{dy}{dx} + y = x^2y^2$

3. $\dfrac{dy}{dx} - y = e^x y^2$

Comment : Each of the above differential Equations is a Bernoulli equation.