1/ Given, $y = f(x) = \sqrt{1 + \frac{x^2}{22500}}$; $-250 \le x \le 150$

We can write from the volume formula,

$$\text{volume} = \int_{-250}^{150} [f(x)]^2 \, dx$$

$$= \pi \int_{-250}^{150} \left(1 + \frac{x^2}{22500}\right) dx$$

$$= \pi \left[x + \frac{x^3}{3 \times 22500}\right]_{-250}^{150}$$

$$= \pi \left[150 + \frac{150^3}{67500} + 250 + \frac{250}{67500}\right]$$

$$= 2190.92 \text{ unit}^3$$

Now, we know from the surface area equation,

$$\text{surface area} = 2\pi \int f(x) \sqrt{1 + [f'(x)]^2} \, dx$$

$$= 2\pi \int f(x) \sqrt{1 + \left(\frac{(2x/22500)}{2\sqrt{1 + \frac{x^2}{22500}}}\right)^2} \, dx$$

$$= 2\pi \int f(x) \sqrt{1 + \frac{x^2}{(22500 + x^2) \times 22500}} \, dx$$

$$= 2\pi \int f(x) \sqrt{\frac{22500^2 + 22501x^2}{(22500+x^2)\times 22500}}\, dx$$

$$= 2\pi \int \sqrt{\frac{22500+x^2}{22500} \times \frac{22500^2+22501x^2}{22500\times(22500+x^2)}}\, dx$$

$$= \frac{2\pi}{22500} \int \sqrt{(22500^2+22501x^2)}\, dx$$

Suppose, $x = \frac{22500\sqrt{22501}}{22501} \sinh u$

$$\Rightarrow dx = \frac{22500\sqrt{22501}}{22501} \cosh u\, du$$

$$\Rightarrow u = \sinh^{-1}\left(\frac{\sqrt{22501}}{22500} x\right)$$

$$\Rightarrow \sqrt{22501x^2+22500^2} = \sqrt{506250000\sinh^2 u + 506250000}$$

$$= 22500\sqrt{\sinh^2 u + 1}$$

$$= 22500 \cosh u.$$

Lastly, $\frac{2\pi}{22500} \int \sqrt{22501x^2 + 50650000}\, dx$

$$= \frac{\pi}{11250} \int \frac{506250000\sqrt{22501}\cosh^2(u).}{22501}\, du$$

$$= \frac{\pi}{11250} \int \frac{253125000\times\sqrt{22501}\times(\cosh 2u + 1)}{22501}\, du$$

$$= \frac{\pi}{22500}\left[\int \frac{506250000\times\sqrt{22501}}{22501}\cosh(2u)\,du + \int \frac{506250000\times\sqrt{22501}}{22501}\,du\right]$$

$$= \frac{\pi}{22500}\left[\frac{506250000\times\sqrt{22501}\,u}{22501} + \frac{50650000\sqrt{22501}}{22501}\int \cosh(2u)\,du\right]$$

$$= \frac{\pi}{22500}\left[\frac{506250000\times\sqrt{22501}\,u}{22501} + \frac{253125000\times\sqrt{22501}\sinh(2u}{22501}\right]$$

$$= \frac{\pi}{22500}\left[\frac{253125000\sqrt{22501}\times\sinh\left(2\sinh^{-1}\frac{22501u}{22500}\right)}{22501} + \frac{506250000\sinh^{-1}\left(\frac{22501\kappa}{22500}\right)}{\sqrt{22501}}\right]$$

$$= \frac{\pi}{22500}\left[22500u\sqrt{\frac{22501u^2}{506250000}+1} + \frac{506250000\times\sqrt{22501}\sinh^{-1}\left(\frac{\sqrt{22501}\,u}{22500}\right)}{22501}\right]$$

And,

$$\text{Volume} = \frac{2\pi}{22500} \int_{-250}^{150} \sqrt{22501x^2 + 506250000}\; dx$$

$$= \frac{\pi}{50627500} \Big[506250000\sqrt{22501}\, \sinh^{-1}\left(\frac{\sqrt{22501}}{150}\right) + 50627500\sqrt{45001} - 1406312500\sqrt{30601} - 506250000\sqrt{22501}\, \sinh^{-1}\left(\frac{\sqrt{22501}}{90}\right) \Big]$$

$$\approx 3213.311 \text{ unit}^3$$

Ans

2/ we know,

the formula for arc length is,

$$s = \int_a^b \sqrt{1+(y')^2}\, dx$$

And $y' = \frac{1}{2}\left(e^{\frac{x}{150}} - e^{-x/150}\right)$

$\Rightarrow (y')^2 = \frac{1}{4}\left(e^{x/75} - 2 + e^{-x/75}\right)$ [squaring]

$\Rightarrow 1+(y')^2 = \frac{1}{4}\left(e^{x/75} + 2 + e^{-x/75}\right)$ [Adding 1]

$= \left[\frac{1}{2}\left(e^{x/150} + e^{-x/150}\right)\right]^2$

Now, $s = \int_a^b \sqrt{1+(y')^2}\, dx$

$= \frac{1}{2}\int_{-100}^{100} \left(e^{x/150} + e^{-x/150}\right) dx$

$= 75\left[e^{x/150} - e^{-x/150}\right]_{-100}^{100}$

$= 150\left(e^{2/3} - e^{-2/3}\right)$ [Applying fundamental theorem of calculas]

≈ 215 feet.

Ans

3| Given, $\int_a^b F(x)\,dx$, $F(x) = \frac{240000000}{x^2}$

$a = 4000$, $b = 4000 + 800 = 4800$

Now, $W = \int_{4000}^{4800} 240000000\, x^{-2}\, dx$

$= \left[240000000 \, \frac{x^{-2+1}}{-2+1}\right]_{4000}^{4800}$

$= 240000000 \left(-\frac{1}{4800} + \frac{1}{4000}\right)$

$= -50000 + 60000$

$= 1000$ ~~d~~ mile tones

$\therefore$ Workdone $= 1000$.

$\approx 1.164 \times 10^{11}$ foot pounds

In S.I unit, using converzsation factor of 1 foot-pound ≈ 1.35582 Joules

Thereforce, the work done in propelling module ~~of~~ to a height of 800 miles above earth is , $W = 1.578 \times 10^{11}$ Joules Ans

For unlimited distance, Let $b = \infty$

So, $W = 240000000 \left(-\frac{1}{\infty} + \frac{1}{4000} \right)$

$W = 60000$ units

Therefore, 60000 units work required to propel the module into an unlimited distance.

Ans

41 ~~Plane~~ From the question,

Plane through 3 units are $\frac{x}{5} + \frac{y}{2} + \frac{z}{10}$

$= 1$

$\Rightarrow \frac{2x+5y+z}{10} = 1$

$\therefore z = 10 - 2x - 5y$

Now, $0 < z < 10 - 2x - 5y$.

So, $2x + 5y = 10$

$\Rightarrow y = \frac{10-2x}{5}$

$\Rightarrow 0 < y < \frac{10-2x}{5}$ where, $0 < x < 5$

And, $R.V = \int_0^5 \int_0^{\frac{10-2x}{5}} 10 - 2x - 5y \, dy \cdot dx$

$$= \int_0^5 (10-2x) \cdot \frac{(10-2x)}{5} - \frac{5}{2}\left(\frac{(10-2x)^2}{25}\right) dx$$

$$= \int_0^5 \frac{1}{5}(10-2x)^2 - \frac{1}{10}(10-2x)^2 \, dx$$

$$= \frac{1}{10}\int_0^5 (10-2x)^2 \, dx$$

$$= \left[\frac{1}{10} \times \frac{1}{3}(-1) \times \frac{(10-2x)^3}{2}\right]_0^5$$

$$= \frac{5 \times \cancel{21} \times 10}{\cancel{63}\,3}$$

$$= \frac{50}{3}$$ Ans

5/ Given, $\frac{dc}{dt} = rc\left(1 - \frac{c}{K}\right) \quad \cdots \text{①}$

$\Rightarrow \frac{dc}{dt} = \frac{r}{K} c (K - c)$

$\Rightarrow \frac{dc}{c(K-c)} = \frac{r}{K} dt \quad \left[\because \frac{1}{c(K-c)} = \frac{1}{K}\left[\frac{1}{c} + \frac{1}{K-c}\right]\right]$

$\Rightarrow \frac{dc}{c(K-c)} = \frac{1}{K}\left(\frac{1}{c} + \frac{1}{K-c}\right) dc = \frac{r}{K} dt$

$\Rightarrow \frac{1}{K} \cancel{\frac{dc}{c(K-c)}} \left(\frac{dc}{c} + \frac{dc}{K-c}\right) = \frac{r}{K} dt$

Now, integratting both sides,

$\Rightarrow \frac{1}{K}\int \frac{dc}{c} + \frac{1}{K}\int \frac{dc}{K-c} = \frac{r}{K} \int dt$

$\Rightarrow \frac{1}{K} \ln(c) - \frac{1}{K} \ln(K-c) = \frac{r}{K} t$

$\Rightarrow \frac{1}{K} \ln\left(\frac{c}{K-c}\right) = \frac{r}{K} + t\,t$

$\left[\because \ln(A) - \ln(B) = \ln\left(\frac{A}{B}\right)\right.$

$\Rightarrow \frac{1}{K} \ln\left(\frac{c(t)}{K - c(t)}\right) \Rightarrow \frac{rt}{K} + c' \quad \cdots \text{(ii)}$

At, $t = 0$, $c(t) = c(0) = c_0$

$$\Rightarrow \frac{1}{K} \ln\left(\frac{c_0}{K-c_0}\right) = 0 + c'$$

$$\Rightarrow c' = \frac{1}{K} \ln\left(\frac{c_0}{K-c_0}\right)$$

puttin c' value in (11),

$$\frac{1}{K} \ln\left(\frac{C(t)}{K-C(t)}\right) = \frac{rt}{K} + \frac{1}{K} \ln\left(\frac{c_0}{K-c_0}\right)$$

$$\Rightarrow \ln\left(\frac{C(t)}{K-C(t)}\right) - \ln\left(\frac{c_0}{K-c_0}\right) = rt$$

$$\Rightarrow \ln\left(\frac{C(t)\,(K-c_0)}{(K-C(t)) \times c_0}\right) = rt \quad \left[\because \ln(A) - \ln(B) = \ln\left(\frac{A}{B}\right)\right]$$

Now, let, $\frac{K-c_0}{c_0} = A$

$$\Rightarrow \ln\left(\frac{C(t)\,A}{K-C(t)}\right) = rt$$

$$\Rightarrow \frac{C(t)\,A}{K-C(t)} = e^{rt}$$

$$\Rightarrow A \cdot C(t) = \{K - C(t)\} e^{rt}$$

$$\Rightarrow C(t)(A + e^{rt}) = Ke^{rt}$$

$$\Rightarrow C(t) = \frac{Ke^{rt}}{A + e^{rt}}$$

$$\Rightarrow C(t) = \frac{K}{Ae^{-rt} + 1}$$

$$\therefore C(t) = \frac{K}{1 + Ae^{-rt}} \quad \text{where } A = \frac{K-c_0}{c_0}$$

Ans