Coursework Assignment, Week 3

3rd academic week, 7 Oct 2024

Coursework (take-home) assignments to be submitted by 14 October at 23:59. Submit the solutions of this coursework assignment on `BlackBoard` via `TurnItIn` by uploading scanned or photographed copies of all your solutions bundled in **a single PDF file**. Write down **your name and your student ID number** on each sheet.

This whole Coursework Assignment is worth 100 marks.

Problem CW1 (10(a) + 15(b) = 25 marks)

Consider the code $C \subseteq \mathbb{F}_5^3$ consisting of the codewords

$$c_1 = 2\,4\,3, \quad c_2 = 3\,2\,1, \quad c_3 = 1\,0\,2.$$

a) Find the minimal distance $d_{\min}(C)$ of the code C, and determine how many errors it can detect and how many errors it can correct.

b) Find all words in $\mathbb{F}_5^3$ whose first symbol (i.e., coordinate) is equal to 0 and that have Hamming distance *exactly* 3 from *every* codeword of C (simultaneously).

Problem CW2 (10 marks)

Consider the code $C = \{0000, 1111, 0101, 1010\} \subseteq \mathbb{F}_2^4$ in a symmetric channel with symbol error probability $p = 0.05$. Suppose the codeword 0000 is sent.

What is, *in percentage*, the probability that we instead receive the word with exactly one erroneous symbol?

Problem CW3 (5(a) + 10(b) + 10(c) +10(d) = 35 marks)

A code

$$C = \{\downarrow\searrow\rightarrow p, \quad \rightarrow\downarrow\rightarrow p, \quad \downarrow\swarrow\leftarrow k, \quad p \rightarrow kp\}$$

is built to encode certain special moves of a prominent videogame, as a combination of inputs from a controller.

a) Draw a graph as follows: the vertices are the elements of C; and a pair of vertices is to be joined by an edge if and only if their Hamming distance is exactly one. Determine how many edges this graph has.

b) Determine the four parameters of C.

c) Determine how many errors C can detect and how many errors C can correct.

d) Calculate the volume of the solid sphere $S_2(\downarrow\searrow\rightarrow p)$ of radius 2 and centre $\downarrow\searrow\rightarrow p$ in the space of words of length four over the alphabet used to build C.

Problem CW4 (15(a) + 15(b) = 30 marks)

The following table lists all codewords of the original code proposed by R. Hamming:

0000000	1000110
0001111	1001001
0010011	1010101
0011100	1011010
0100101	1100011
0101010	1101100
0110110	1110000
0111001	1111111

a) Prove that Hamming's code is perfect.

b) Suppose that some codeword of Hamming's code was sent, but the word $w = 0\,0\,0\,0\,1\,1\,0$ was received. Assume that there were **two errors** in transmission. Explain, by means of examples, that it might not be possible to correctly decode w using the nearest neighbour decoding strategy.