Algebra – Practical session 6

Various problem sheets for the practicals contain more problems than you can be reasonably expected to solve during a one-hour practical. The first few problems in a problem sheet tend to be the most essential. Bring the rest home for further practice, or for final revision in preparation for the exam.

6.1. Use the Rational Root Test to factorise the polynomial $x^3 + 4x^2 + 5x + 6$ into a product of irreducible factors in $\mathbb{Q}[x]$. Show that each of the factors which you find is actually irreducible.

6.2. Find the complex roots of the biquadratic polynomial $3x^4 - 7x^2 - 6$, and write its complete factorisations in $\mathbb{Q}[x]$, in $\mathbb{R}[x]$, and in $\mathbb{C}[x]$.

6.3. Consider the self-reciprocal polynomial $6x^4 - 35x^3 + 62x^2 - 35x + 6$.

(i) Using the method described in the lectures, factorise it into a product of linear factors in $\mathbb{C}[x]$.

(ii) What is the full factorisation of the polynomial in $\mathbb{Q}[x]$?

(iii) Can you actually write that last factorisation in $\mathbb{Z}[x]$, meaning that you use only integer coefficients?

6.4. Factorise the polynomial

$$x^4 + (2 - \sqrt{3})x^2 - 2\sqrt{3}$$

into a product of linear factors (that is, factors of degree one) with complex coefficients.

Now factorise the same polynomial into a product of irreducible factors over $\mathbb{R}$.

Hint: For this problem you are allowed to guess and avoid using the solving formula for quadratic equations, which may get you into trouble with *double radicals* $\sqrt{a \pm \sqrt{b}}$.

6.5. Compute the square roots of the complex number $\frac{5}{4} + 3i$, expressing them using only square roots of real numbers (that is, expressing their real and complex part using only algebraic operations on real numbers).

Questions for further practice

6.6. Consider the polynomial

$$f(x) = 2x^4 + 3x^3 + 2x^2 + 6x - 4$$

with rational coefficients. Find the complete factorisation of $f(x)$ into a product of irreducible factors over $\mathbb{Q}$. (Start with applying the Rational Root Test.)

6.7. Consider the polynomial

$$f(x) = 4x^5 - 4x^4 + 9x^3 - 16x^2 + 10x - 2$$

with rational coefficients. Find the complete factorisation of $f(x)$ into a product of irreducible factors over $\mathbb{Q}$.

6.8. Using any of the methods and ideas we have seen, find all complex roots of the following polynomial:

$$x^6 - 10x^4 + 31x^2 - 30.$$