MECHANICS PRACTICAL 2

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1. Dimensions strike back

Using the bracket notation [.] give the dimension of either side of the following equations and conclude on whether or not they correspond to **complete** equations. Note: in the equations below, x and y are components of the position vector, t is a time interval and v_x and v_y are the components of the velocity vector.

<u>Reminder:</u> Here are the rules we have established so far:

- $[x] = [y] = L$
- $[u] = L$ if $u =$ inches, cm, m, yards etc...
- $[v_x] = [v_y] = L \times T^{-1}$
- $[\theta] = 1$ if θ is an angle
- $[u_a] = 1$ if $u_a =$ degrees, radians, seconds of arc etc..
- $[t] = T$
- $[u_t] = T$ if $u_t =$ seconds, hours, days, years etc...
- $[n] = 1$ where n can be any real number like -1, 3.45674 or 5 for example.
- $[A \times B] = [A] \times [B]$ for any A and B
- $[A/B] = [A]/[B]$ for any A and B
- $[A + B] = [A] = [B]$ if $[A] = [B]$

(a) $\sqrt{x^2 + y^2} = 2\,\mathrm{m}$

(b) $\frac{v_x^2}{y} = (63\,\mathrm{km} \cdot \mathrm{h}^{-3})\, t$

(c) $y^{-1/2}\sqrt{v_x^3} = (12\,\mathrm{cm})\, t^{3/2}$

(d) $\frac{\sqrt{v_x^2}}{v_y} = 3$

(e) $1\,\text{light year} = (1.17\,\mathrm{rad})\, x$

(f) $\theta^2 v_x^2 = \frac{y^2}{t^2}$

2. Kinematics in 1D

Find the velocity and the relative position (in 1D) by integrating the following accelerations given that $v_x(t=0) = 0$ and $x(t=0) = 0$. Optional question: try to guess which type of physical conditions these accelerations correspond to.

(a) $a_x(t) = a_0$, with a_0 a constant quantity.

(b) $a_x(t) = \gamma\, t$, with γ a constant quantity.

(c) $a_x(t) = \frac{v_\infty}{\tau} e^{-t/\tau}$, with v_∞ and τ being constants.

(d) $a_x(t) = a_0 \cos(\omega t)$, with a_0 and ω being constants.

3. Basics of kinematics in 2D

Give the instantaneous velocity and acceleration vectors in the orthonormal frame $(O, \hat{\imath}, \hat{\jmath})$ given the following position vectors in that frame:

(a) $\vec{r}(t) = (10\ \mathrm{m \cdot s^{-1}})\, t\ \hat{\imath} + (-2.5\ \mathrm{m \cdot s^{-2}})\, t^2\ \hat{\jmath}$

(b) $\vec{r}(t) = R\cos(\omega t)\,\hat{\imath} + R\sin(\omega t)\,\hat{\jmath}$

(c) $\vec{r}(t) = R(\omega t - \sin(\omega t))\,\hat{\imath} + R(\omega t - \cos(\omega t)\,\hat{\jmath}$

4. A story of averages

(a) State the definition of the average velocity in 1D as seen from the lectures.

(b) From the answer of question (a), show that the average velocity $v_{x,avg}$ can also be written as:

$$v_{x,avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v_x(t)\, dt$$

(c) Using the equation given in question (b) show that if the 1D acceleration is constant then we have that the average velocity is equal to the arithmetic average:

$$v_{x,avg} = \frac{1}{2}(v_x(t_1) + v_x(t_2))$$