# MECHANICS PRACTICAL 5

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> Reminder: For this practical, we recommend to use the step-by-step strategy seen during the lectures whenever required *i.e.* (1) diagram, (2) list and intel on the forces acting on the system, (3) Newton's 2nd law, (4) components and (5) solve to get the desired answer. We will use $g = 9.8\ \mathrm{m \cdot s^{-2}}$ for the downward acceleration on Earth.

## 1. Conservative forces

Show that the following one-dimensional forces are conservative by finding a potential energy function they can be derived from:

(a) The one-dimensional weight along the vertical direction $W = -mg$.

(b) The Hookean spring force $F_{spring} = -k(x - \ell_0)$.

(c) The gravitational force $F_{grav} = -\frac{C}{x^2}$, where $C$ is a constant.

## 2. Non-conservative forces

Show that the following one-dimensional forces are non-conservative by showing that they fail to satisfy at least one of the definitions of the a conservative force:

(a) A one-dimensional solid kinematic friction force $F_{friction} = -F$ if $\dot{x} > 0$ and $F_{friction} = F$ if $\dot{x} < 0$.

(b) A one-dimensional drag force $F_{drag} = -\gamma v(t)$.

## 3. Block on an inclined plane: energy method

A block $B$ of mass $m = 5\ \mathrm{kg}$ initially at rest is let go on an inclined plane making an angle $\theta = 45^{\circ}$ with the horizontal (see Fig. 1) at a height $h = 2\ \mathrm{m}$ from the ground. The two frames $(O, \hat{i}, \hat{j})$ and $(O', \hat{i}\,', \hat{j}\,')$ are considered galilean.

(a) Assuming there is no friction on the block, determine the speed of the block when it reaches the ground by reasoning in term of energy.

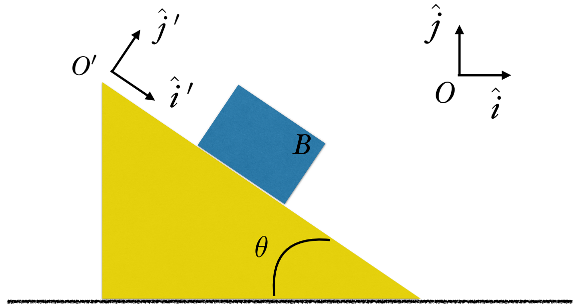


FIGURE 1. A block on an inclined plane.

(b) We now consider that the block is in fact subject to a kinematic solid friction with friction coefficient $\mu_k = 0.4$. Determine the speed of the block when it reaches the ground by taking this new information into account.

## 4. ELASTIC COLLISION IN 1D

We consider a particle 1 of mass $m_1$ and initial velocity $v_1$ incoming on a stationary particle 2 with mass $m_2$. The velocities after the collision are denoted $v_1'$ and $v_2'$ for particle 1 and 2 respectively.

(a) State the definition of an elastic collision.

(b) Which equations are satisfied for the elastic collision in the context of this question ($v_2 = 0$)?

(c) (Hard) Solve the equations of question (b) and show that

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1,$$
$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1.$$

(d) Use the solution to question (c) to determine what happens if (i) $m_1 = m_2$, (ii) $m_1 = 100m_2$ and (iii) $m_2 = 100m_1$.