MECHANICS PRACTICAL 6

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1. Newtonian gravitation

We consider an object m whose potential energy of gravitation with the a bigger mass M is $U_{grav}(r) = -\frac{\mathcal{G}M\,m}{r}$.

(a) **Escape velocity** - By using the conservation of mechanical energy, show that there is a minimum speed v_{esc} at which an object must be thrown from the surface of the Earth (of mass M_E) to escape its attraction.

(b) **Classical "Black Hole"** - Considering the result of question (a), imagine now that no object can go faster than the speed of light c. Determine then the distance from the centre of a star with mass M_s below which it becomes then impossible to escape by simply having a high enough initial speed.

2. Kepler's 2nd law

A point object M of mass m_1 interacts with another point object of mass m_2 located **and fixed** at the origin of coordinates O. The two masses interact only via the universal law of gravitation and the frame $(O, \hat{\imath}, \hat{\jmath})$ is considered galilean.

(a) Give the vector expression of the force $\vec{F}_{2\to 1}$ from the mass m_2 at the origin on mass m_1 at M.

(b) Recall the definition of the angular momentum L_O of point M with respect to point O.

(c) Rederive the general equation satisfied by the time variation $\dot{L}_O$ of the angular momentum L_O.

(d) Determine then the value of $\dot{L}_O$ for the point M subject to a gravitational force from a mass at O.

(e) Give an interpretation in terms of area swept by the segment joining points O and M as a function of time.

3. Merry-go-round

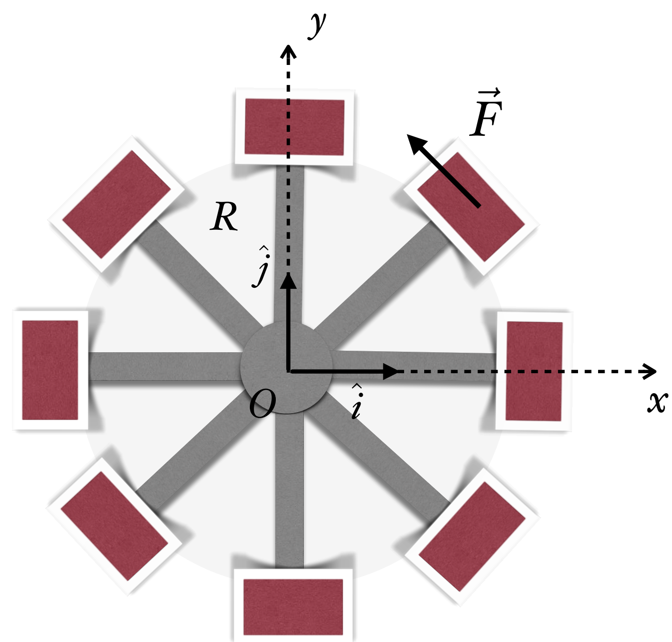

Figure 1. A spinning merry-go-round.

We consider a merry-go-round with 8 seats each at a distance $R = 1.5\,\mathrm{m}$ from the centre (see Fig. 1). The merry-go-round is initially unoccupied and each empty seat weighs $4.0\,\mathrm{kg}$. The problem is considered from a galilean frame.

(a) Determine the moment of inertia of the unoccupied merry-go-round with respect to the centre.

(b) At a time $t = 0$, a kid starts pushing on one of the seats of the merry-go-round with a constant force $\vec{F}$ perpendicular to the radial direction at all times and with magnitude $||\vec{F}|| = 50\,\mathrm{N}$. How long does it take for the merry-go-round to reach an angular velocity of 1 turn in 5 seconds?

(c) Two kids, one of mass $50\,\mathrm{kg}$ and another of mass $40\,\mathrm{kg}$ now sit on any two of the available seats. How long does it take now for the merry-go-round to reach an angular velocity of 1 turn in 5 seconds if the same force as that of question (b) is applied?

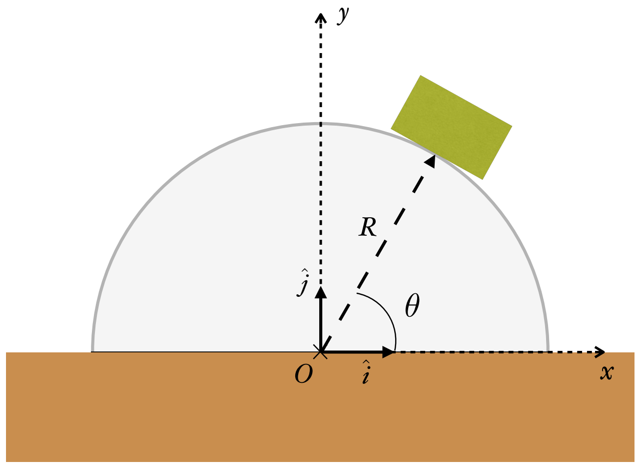

Figure 2. A block sliding on a semi-circular track of radius R.

4. A sliding block on a semi-circular track (More difficult)

We consider a block of mass m sliding without friction on a semi-circular track of radius R (see Fig. 2). The whole problem is studied in a terrestrial frame $(O, \hat{\imath}, \hat{\jmath})$ assumed galilean. At the initial time the block is at an angle $\theta = \pi/2$ and is given a velocity $\vec{v} = v_0\hat{\imath}$.

(a) Assuming the block is following the circular path of the track, determine the expression of the normal reaction force vector from the track on the block.
[hint1: you may need to use your knowledge on circular motion]
[hint2: you may need to use conservation of mechanical energy]

(b) We now consider that the interaction between the track and the block is purely that of excluded volume (i.e. the block cannot overlap with the track) and that there is no bonding mechanism between them. Determine the critical height y_c below which the block will actually leave the track.