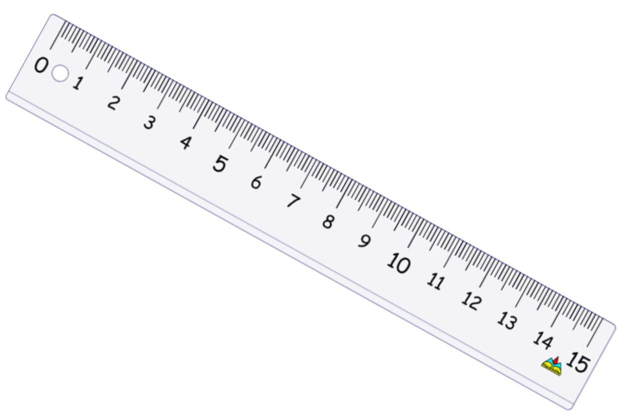
Geometrical Optics

Geometrical optics is mostly **Geometry**

[What is geometry?]

Maths + Technical drawing

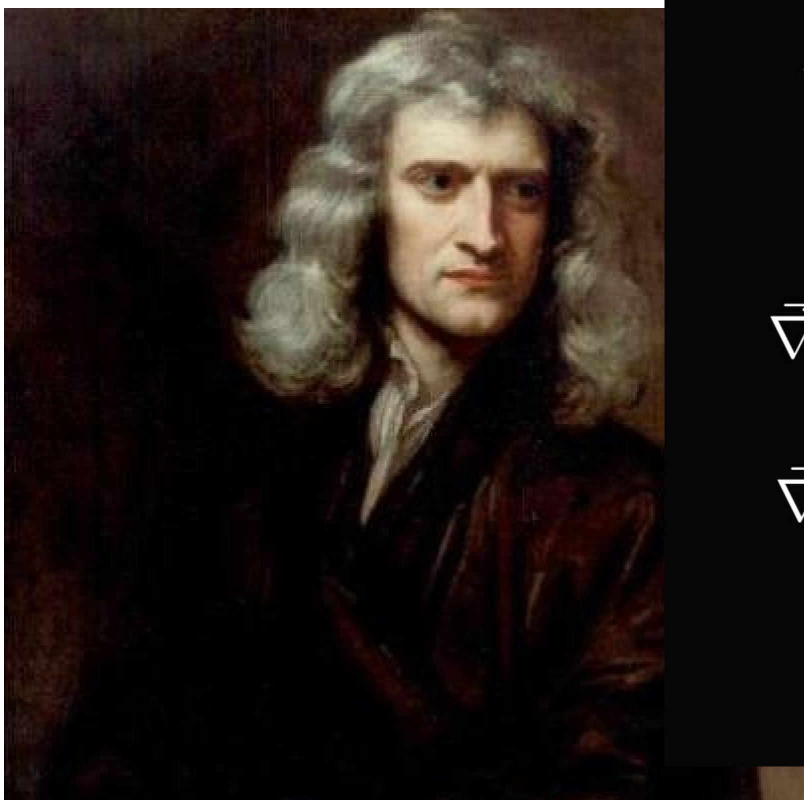
What do you need to have

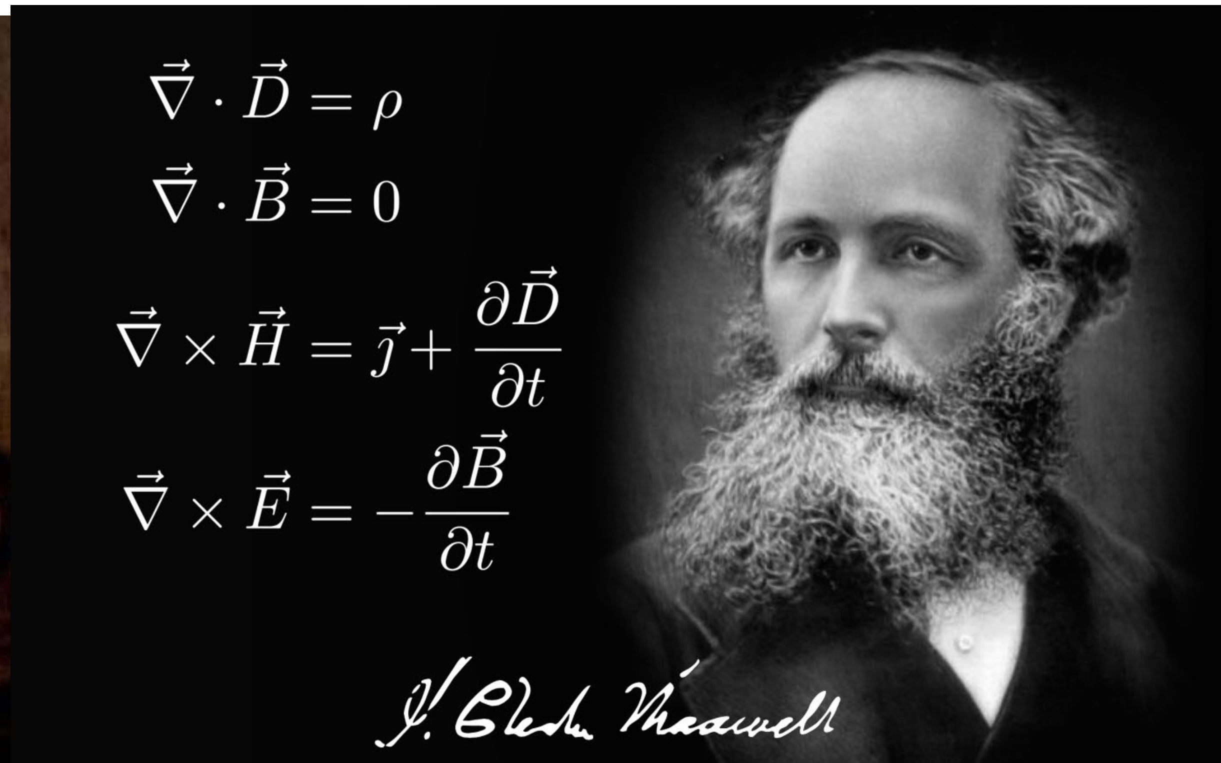


[Draw parallel lines]

≈350 years ago, in 1666 Newton made revolutionary inventions and discoveries in calculus, motion, optics and gravitation.

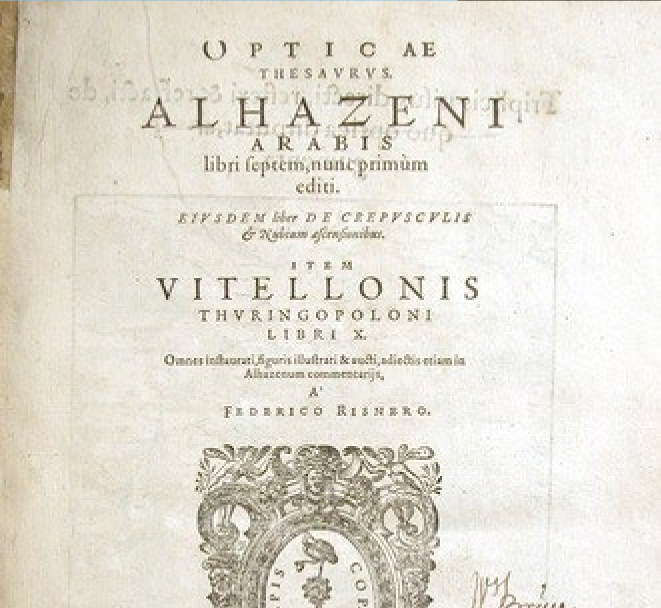
2015 was the International Year of Light.
≈150 years ago:
Maxwell's Equations 1861-1873





Sir Isaac Newton FRS
1643-1727

James Clerk Maxwell FRS FRSE
1831-1879



965-1040

In medieval Europe,
Ibn al-Haytham was honoured as the "Second Ptolemy".

2015 – 1000 years of his *Book of Optics (7 books)*.

Robert Grosseteste, bishop of Lincoln

1220 to 1235 he wrote:

De sphera. A text on astronomy.

De luce. On the "metaphysics of light."

De lineis, angulis et figuris.
Mathematical reasoning
in the natural sciences.

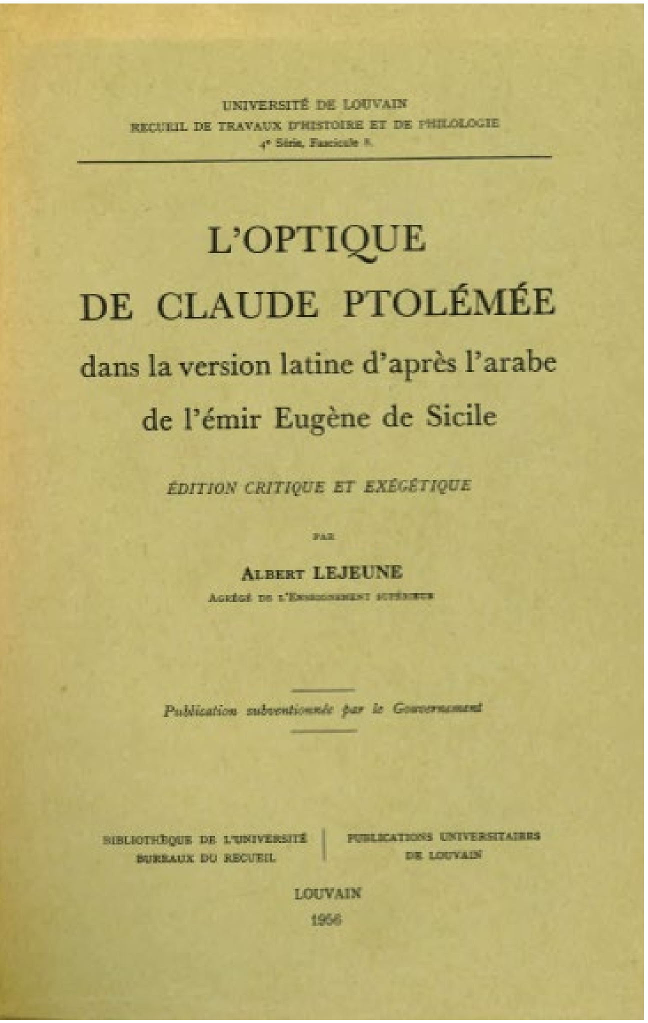
De iride. On the rainbow.

De Colore.

Robert Grosseteste, bishop of Lincoln

Colour is light incorporated in a transparent medium.
But, in fact, there are two different media: there are pure transparent media separated from earth materials or impure media mixed with them.
The light, however, is four-fold differentiated: there is the bright and the obscure light, and the intense or the tenuous light.

Ptolemy, 100-170, Alexandria



UNIVERSITÉ DE LOUVAIN
RECUEIL DE TRAVAUX D'HISTOIRE ET DE PHILOLOGIE
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L'OPTIQUE
DE CLAUDE PTOLÉMÉE
dans la version latine d'après l'arabe
de l'émir Eugène de Sicile

ÉDITION CRITIQUE ET EXÉGÉTIQUE

PAR

ALBERT LEJEUNE
AGRÉGÉ DE L'ENSEIGNEMENT SUPÉRIEUR

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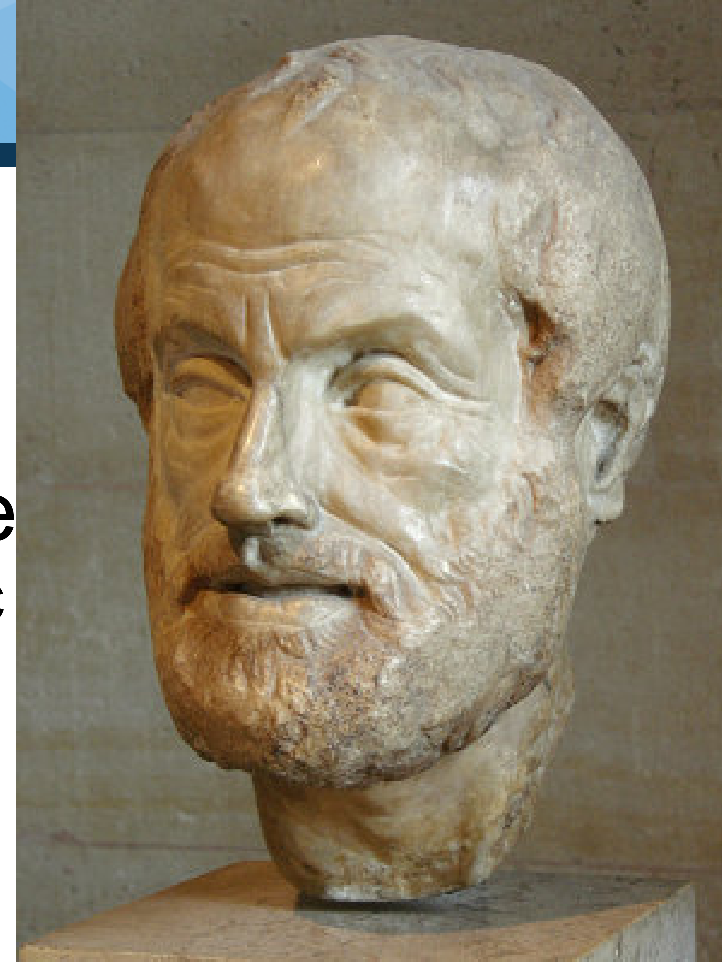
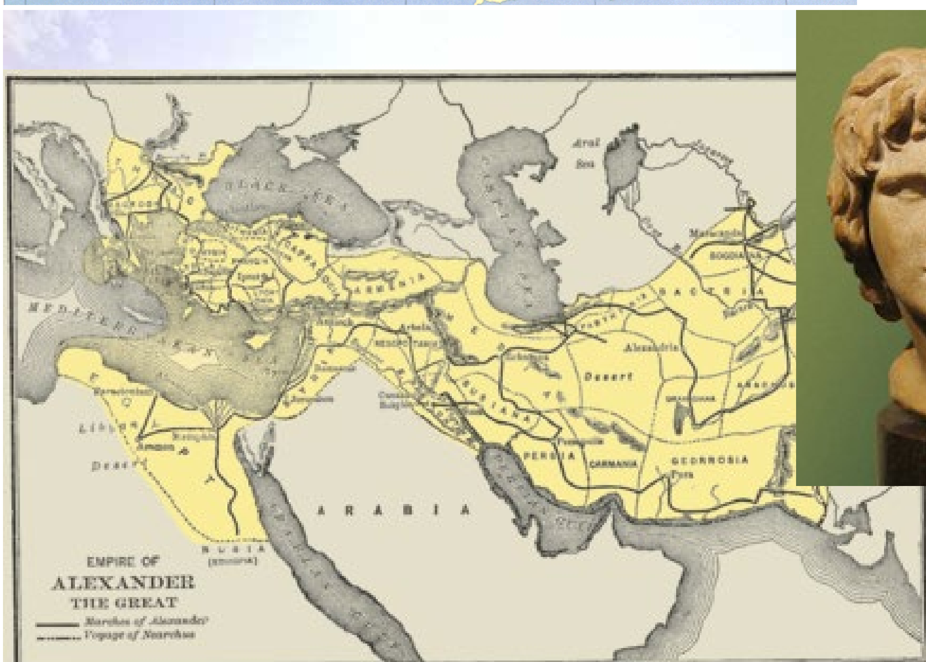
1956

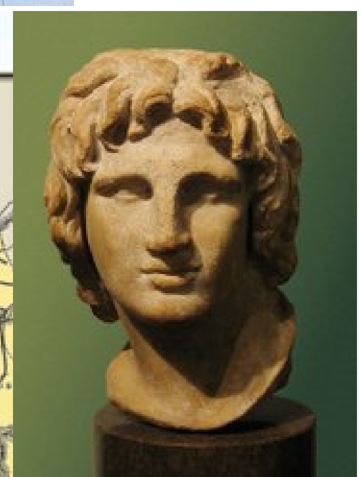
His *Optics* survives only in a poor Arabic translation and in about twenty manuscripts of a Latin version of the Arabic, which was translated by *c*. 1154. In it Ptolemy writes about properties of light, including reflection, refraction, and colour.

Aristotle

384 -322 BC





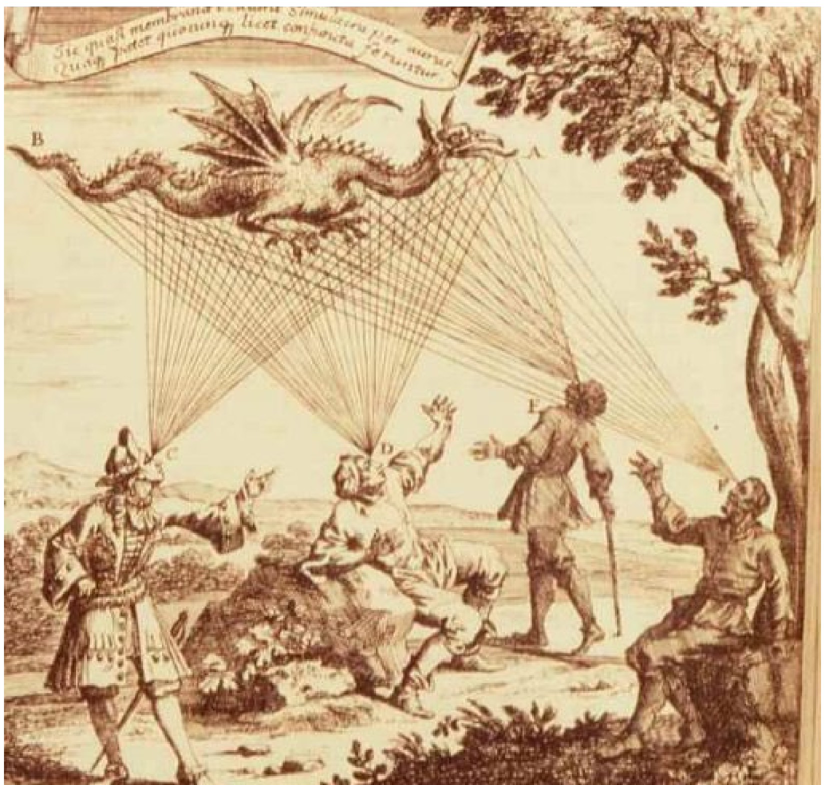


"In general, it is unreasonable to suppose that seeing occurs by something issuing from the eye."

Euclid's *Optics*

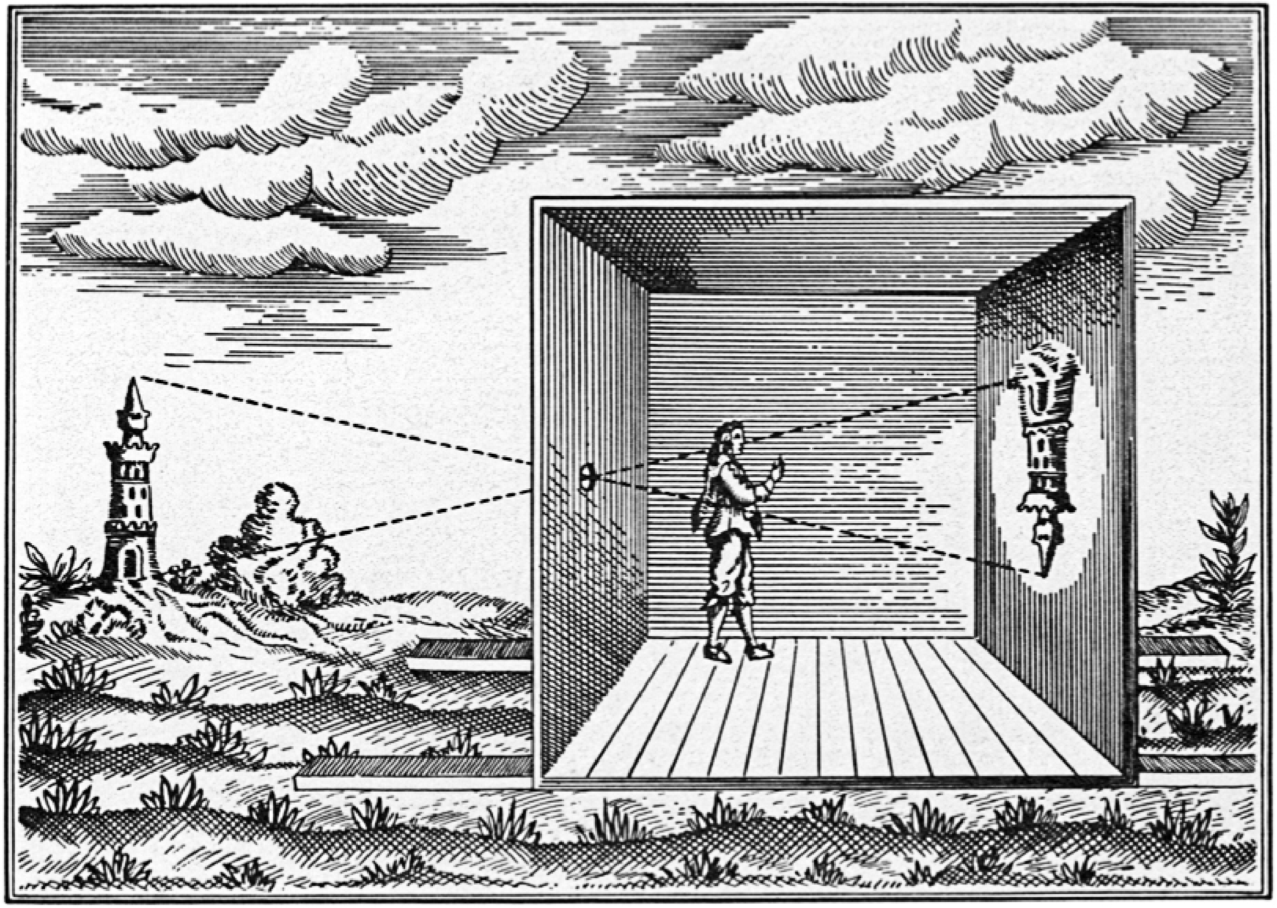
Visual rays proceed **from** the eyes onto objects.

Earliest surviving manuscript on *Optics.*



ca 4-3 century BCE
Alexandria
“father of geometry”

Camera obscura



First records 5-4 c BCE, China. Mentioned by Aristotle. Studies by Ibn Al-Haytham.

First clear description- L. da Vinci, 1502

Light and Optics

There are two historical models for the nature of light.

Newton was the chief architect of the particle theory of light.

- He believed the particles left the object and stimulated the sense of sight upon entering the eyes.

Christian Huygens argued that light might be some sort of a wave motion.

Dual Nature of Light

Light exhibits the characteristics of a wave in some situations and the characteristics of a particle in other situations.

Christian Huygens

1629 – 1695

Best known for contributions to fields of optics and dynamics

He thought light was a type of vibratory motion.

It spread out and produced the sensation of light when it hit the eye.

He deduced the laws of reflection and refraction.

He explained double refraction.

Photo Researchers, Inc.

Measurements of the Speed of Light

Since light travels at a very high speed, early attempts to measure its speed were unsuccessful.

- Remember $c = 3.00 \times 10^8$ m/s

Galileo tried by using two observers separated by about 10 km.

- The reaction time of the observers was more than the transit time of the light.

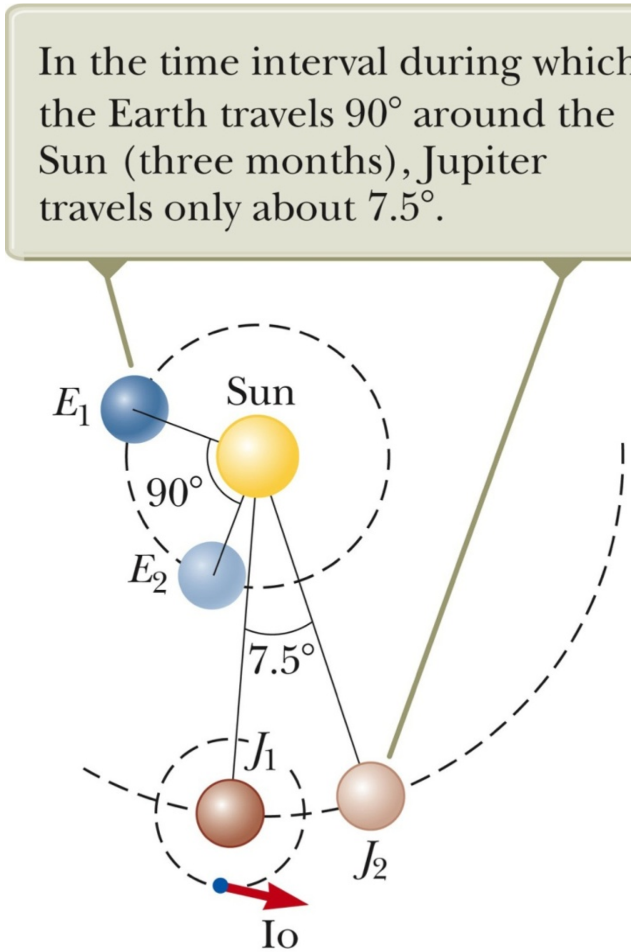
Measurement of the Speed of Light – Roemer's Method

In 1675 Ole Roemer used astronomical observations to estimate the speed of light.

He used the period of revolution of Io, a moon of Jupiter, as Jupiter revolved around the sun.

The angle through which Jupiter moves during a 90° movement of the Earth was calculated.

Using Roemer's data, Huygens estimated the lower limit of the speed of light to be 2.3 x 10^8 m/s.



Measurements of the Speed of Light – Fizeau's Method

This was the first successful method for measuring the speed of light by means of a purely terrestrial technique. It was developed in 1849 by Armand Fizeau.

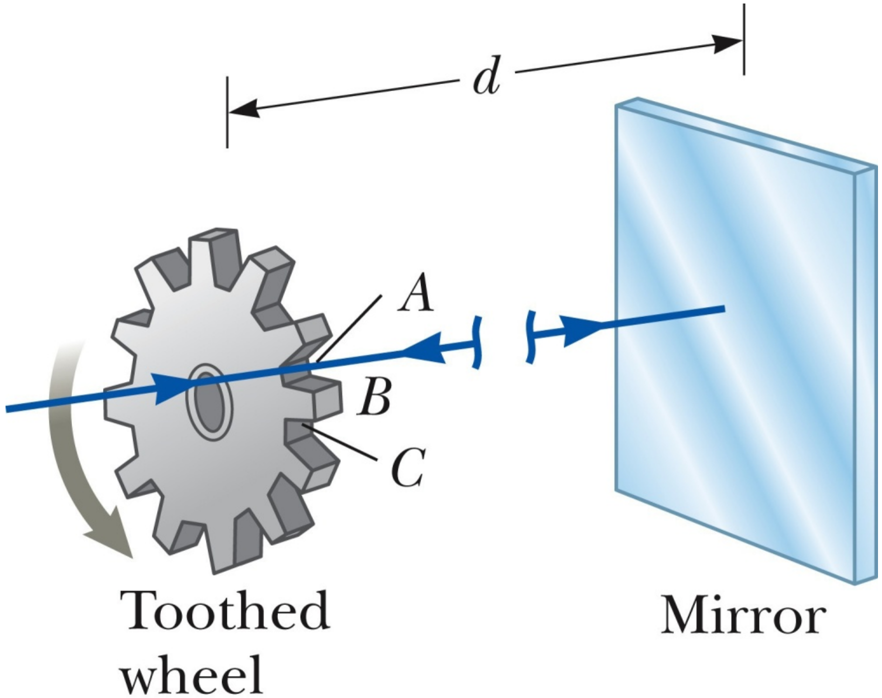
d is the distance between the wheel and the mirror.

Δt is the time for one round trip.

Then $c = 2d / \Delta t$

Fizeau found a value of

$c = 3.1 \times 10^8$ m/s.



The Ray Approximation in Ray Optics

Ray optics (sometimes called *geometric* optics) involves the study of the propagation of light.

It uses the assumption that light travels in a straight-line path in a uniform medium and changes its direction when it meets the surface of a different medium or if the optical properties of the medium are nonuniform.

The ray approximation is used to represent beams of light.

Reflection of Light

A ray of light, the *incident ray*, travels in a medium.

When it encounters a boundary with a second medium, part of the incident ray is reflected back into the first medium.

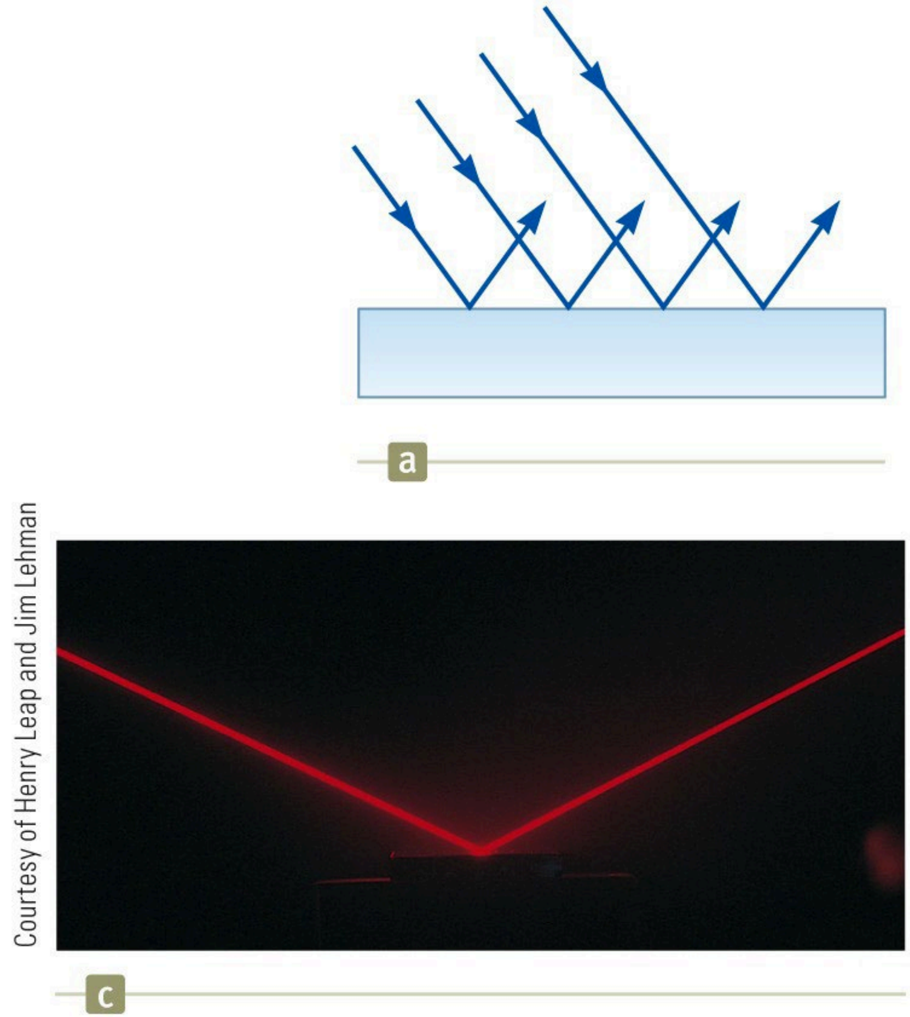
- This means it is directed backward into the first medium.

Specular Reflection

Specular reflection is reflection from a smooth surface.

The reflected rays are parallel to each other.

All reflection in this text is assumed to be specular.

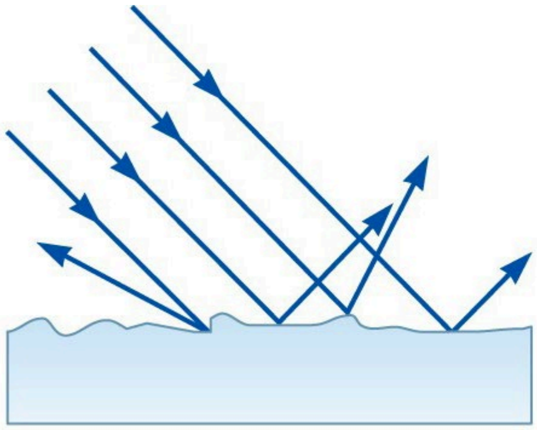


Diffuse Reflection

Diffuse reflection is reflection from a rough surface.

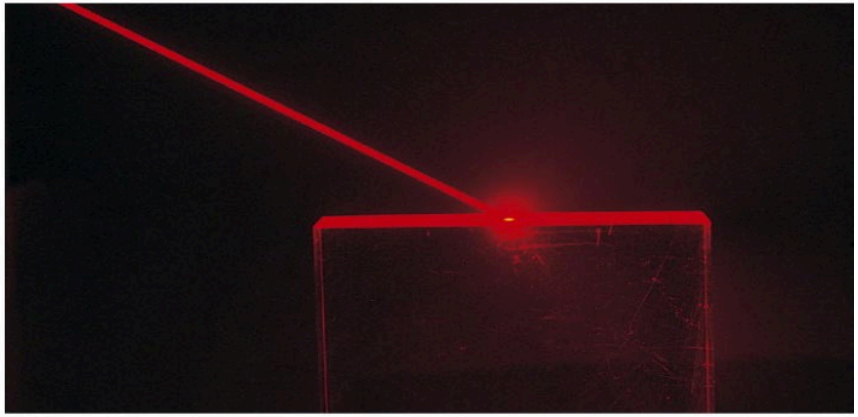
The reflected rays travel in a variety of directions.

A surface behaves as a smooth surface as long as the surface variations are much smaller than the wavelength of the light.



b

Courtesy of Henry Leap and Jim Lehman



d

Law of Reflection

The *normal* is a line perpendicular to the surface.

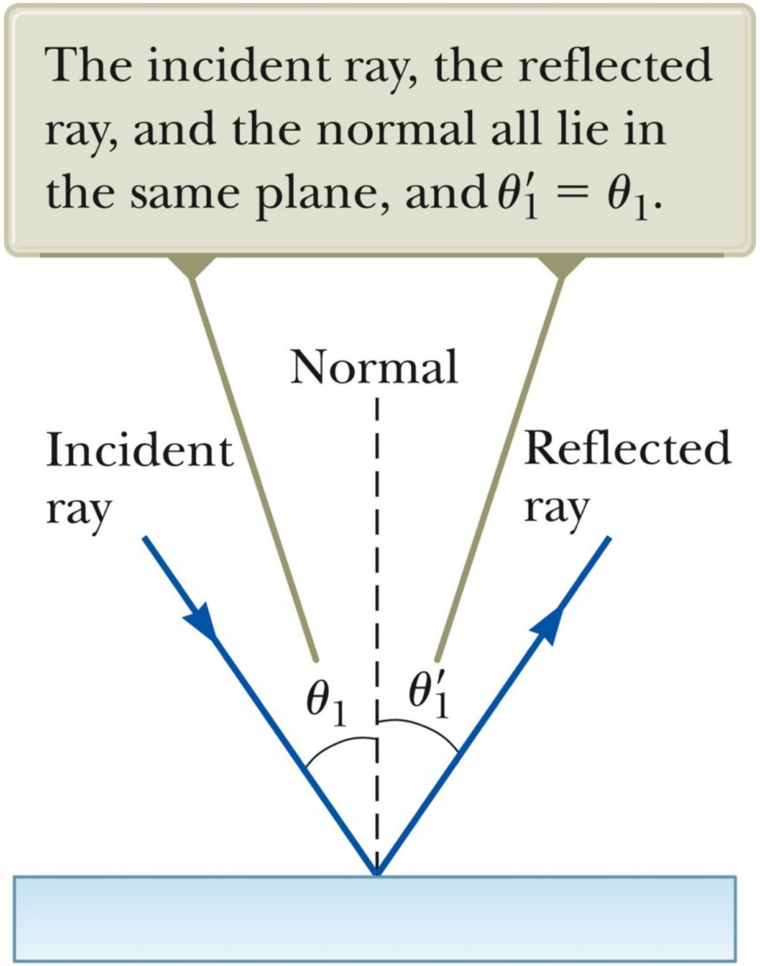
- It is at the point where the incident ray strikes the surface.

The incident ray makes an angle of θ_1 with the normal.

The reflected ray makes an angle of θ_1' with the normal.

$\theta_1' = \theta_1$ - Law of Reflection.

- The subscript 1 refers to parameters for the light in the first medium.
- If light travels in another medium, the subscript 2 will be associated with the new medium.



Refraction of Light

When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, part of the energy is reflected and part enters the second medium.

The ray that enters the second medium changes its direction of propagation at the boundary.

- This bending of the ray is called *refraction.*

Following the Reflected and Refracted Rays

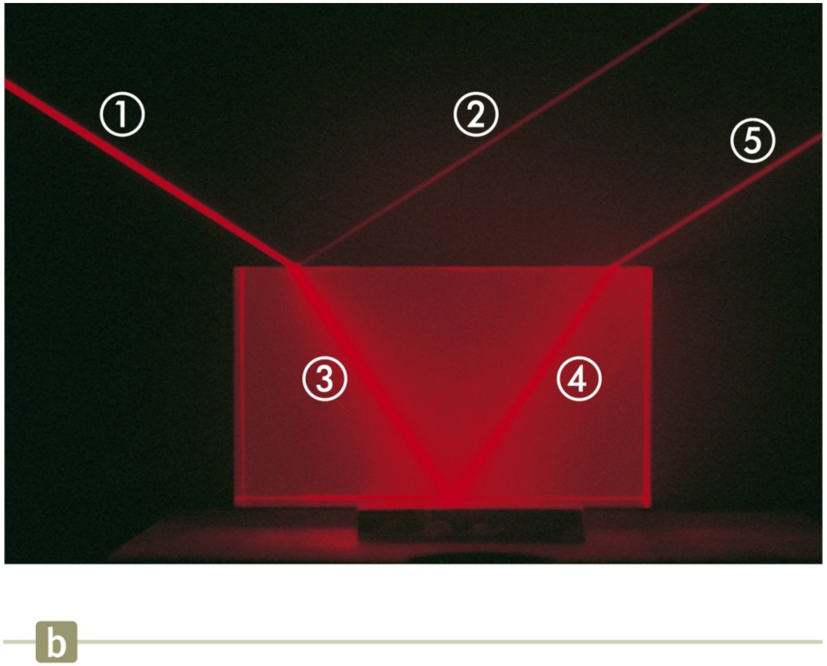
Ray ① is the incident ray.

Ray ② is the reflected ray.

Ray ③ is refracted into the lucite.

Ray ④ is internally reflected in the lucite.

Ray ⑤ is refracted as it enters the air from the lucite.



Courtesy of Henry Leap and Jim Lehman

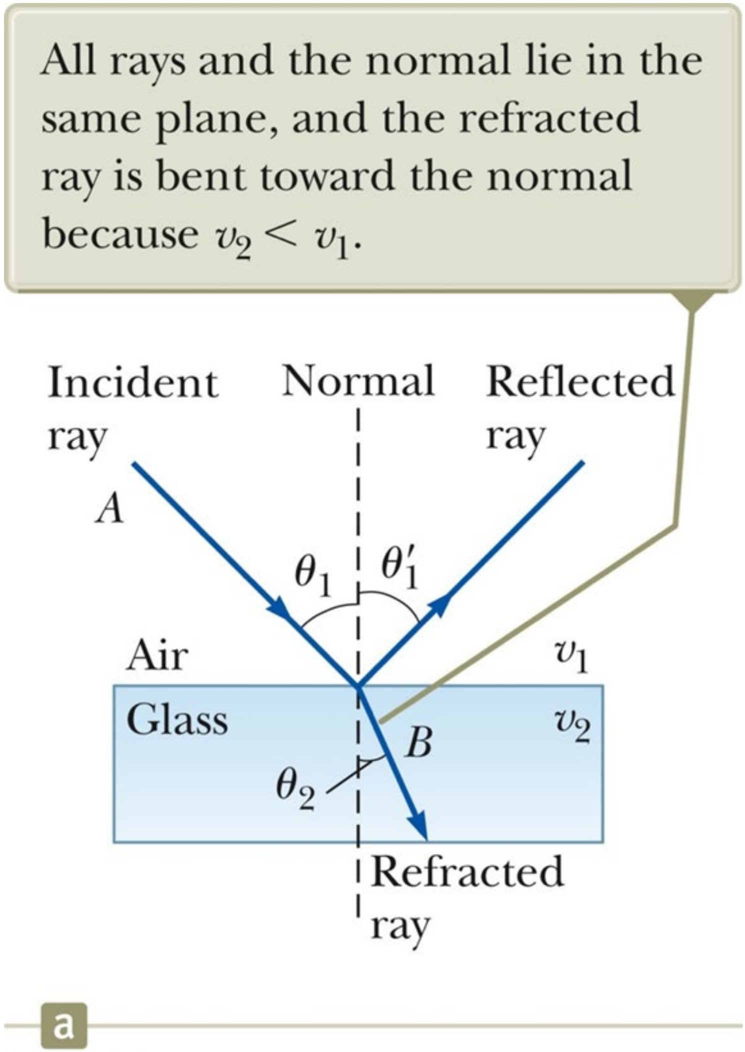
Refraction, cont.

The incident ray, the reflected ray, the refracted ray, and the normal all lie on the same plane.

The angle of refraction depends upon the material and the angle of incidence.

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}$$

- v_1 is the speed of the light in the first medium and v_2 is its speed in the second.



The Index of Refraction

The speed of light in any material is less than its speed in vacuum.

The **index of refraction**, n, of a medium can be defined as

$$n \equiv \frac{\text{speed of light in a vacuum}}{\text{speed of light in a medium}} \equiv \frac{c}{v}$$

For a vacuum, n = 1

- We assume n = 1 for air also

For other media, n > 1

n is a dimensionless number greater than unity.

- n is not necessarily an integer.

Some Indices of Refraction

Table 35.1 Indices of Refraction

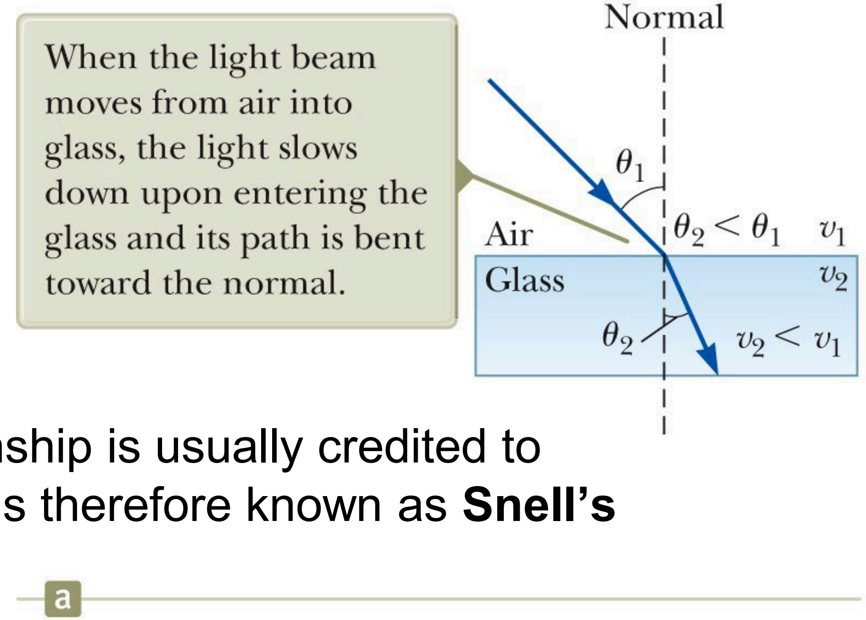
Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		*Liquids at 20°C*	
Cubic zirconia	2.20	Benzene	1.501
Diamond (C)	2.419	Carbon disulfide	1.628
Fluorite (CaF_2)	1.434	Carbon tetrachloride	1.461
Fused quartz (SiO_2)	1.458	Ethyl alcohol	1.361
Gallium phosphide	3.50	Glycerin	1.473
Glass, crown	1.52	Water	1.333
Glass, flint	1.66		
Ice (H_2O)	1.309	*Gases at 0°C, 1 atm*	
Polystyrene	1.49	Air	1.000 293
Sodium chloride (NaCl)	1.544	Carbon dioxide	1.000 45

Note: All values are for light having a wavelength of 589 nm in vacuum.

Snell's Law of Refraction

$n_1 \sin \theta_1 = n_2 \sin \theta_2$

- θ_1 is the angle of incidence
- θ_2 is the angle of refraction



The experimental discovery of this relationship is usually credited to Willebrord Snellius, ca 1621, Leiden, and is therefore known as **Snell's law of refraction.**

It was already formulated in 984 by Ibn Sahl in Baghdad.

Light may refract into a material where its speed is lower.

The angle of refraction is less than the angle of incidence.

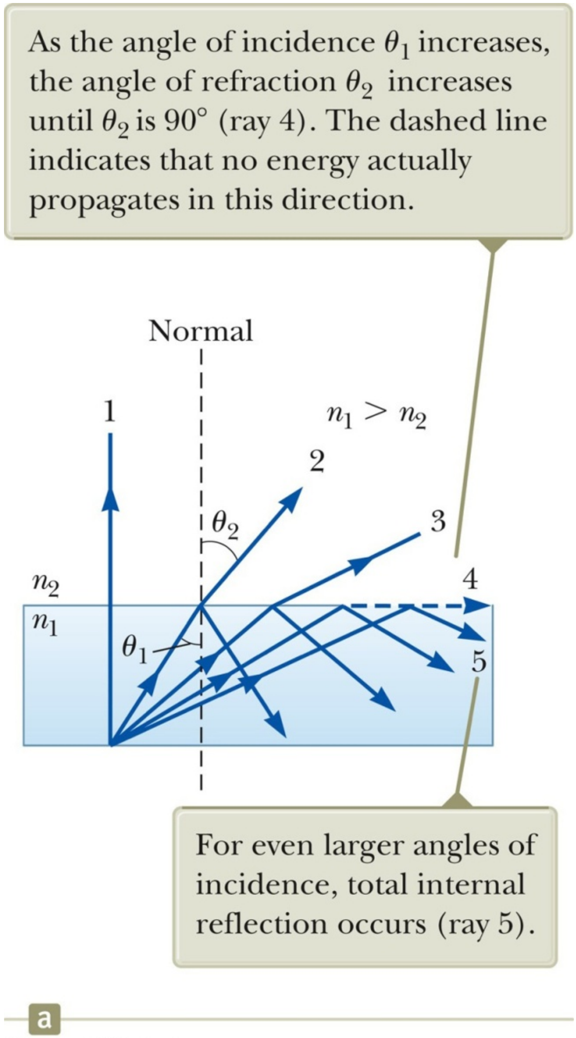
- The ray bends *toward* the normal.

Total Internal Reflection

A phenomenon called **total internal reflection** can occur when light is directed from a medium having a given index of refraction toward one having a lower index of refraction.

Possible directions of the beam are indicated by rays numbered 1 through 5.

The refracted rays are bent away from the normal since $n_1 > n_2$.



Critical Angle

There is a particular angle of incidence that will result in an angle of refraction of 90°.

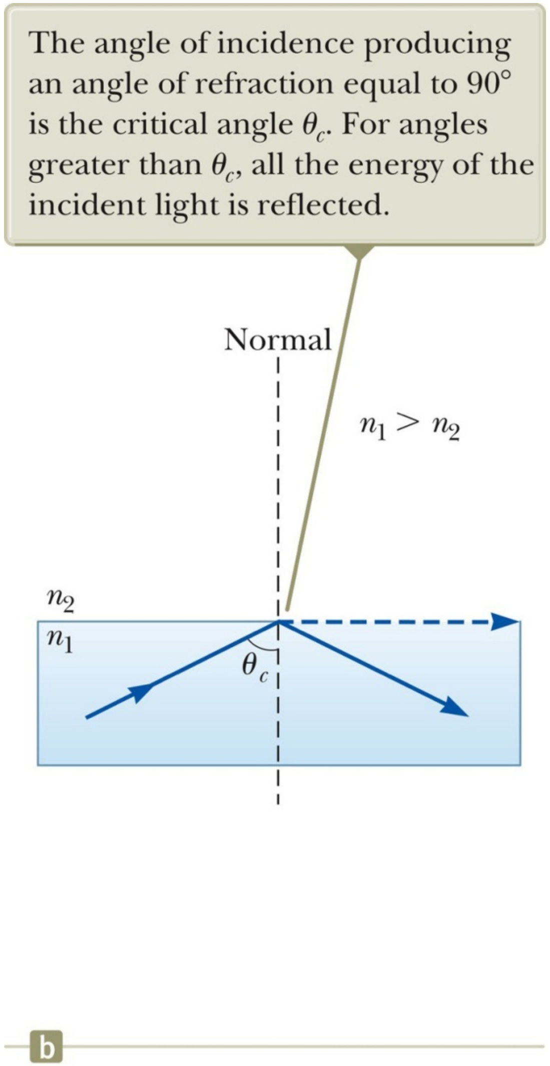
- This angle of incidence is called the *critical angle*, θ_C.

$$\sin \theta_C = \frac{n_2}{n_1} \quad (\text{for } n_1 > n_2)$$

For angles of incidence *greater* than the critical angle, the beam is entirely reflected at the boundary.

- This ray obeys the law of reflection at the boundary.

Total internal reflection occurs only when light is directed from a medium of a given index of refraction toward a medium of lower index of refraction.



Fiber Optics

An application of internal reflection

Plastic or glass rods are used to “pipe” light from one place to another.

Applications include:

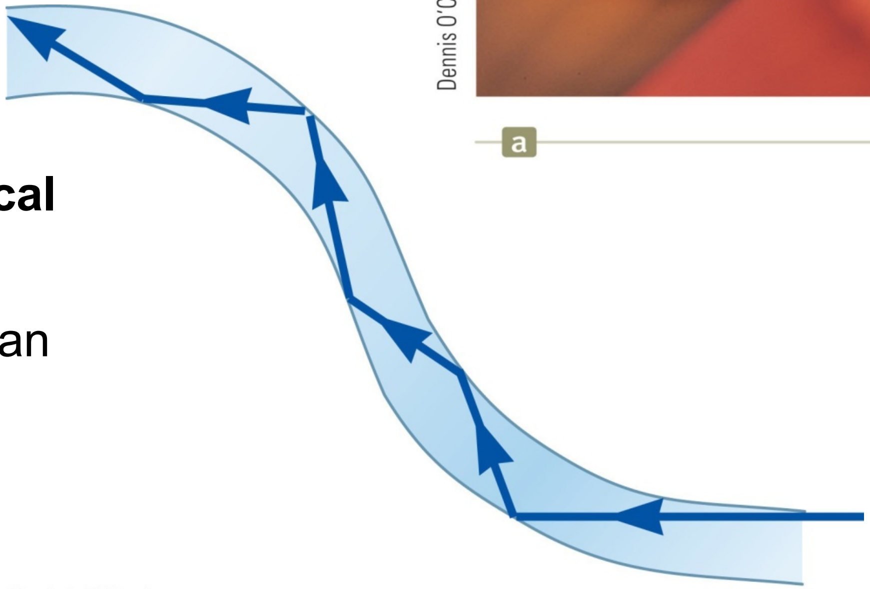
- Medical examination of internal organs
- Telecommunications

A flexible light pipe is called an **optical fiber.**

A bundle of parallel fibers (shown) can be used to construct an optical transmission line.

Dennis O'Clair/Getty Images

a



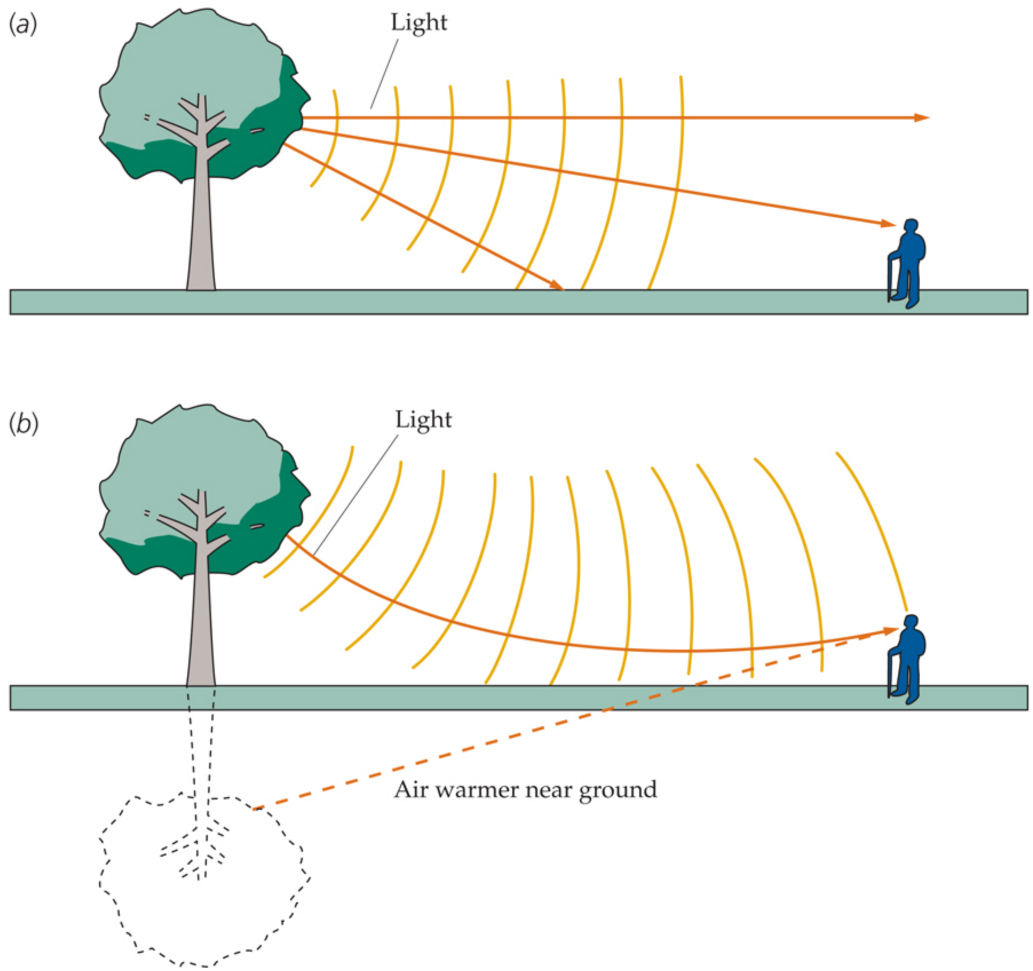
Fermat’s principle, 1662

Early version Ibn Al-Haytham.

<u>Light travels along the path of least time.</u>

1607-1665

Best known for
Fermat's Last Theorem



Air of different temperature has different density and therefore different n.

Dispersion

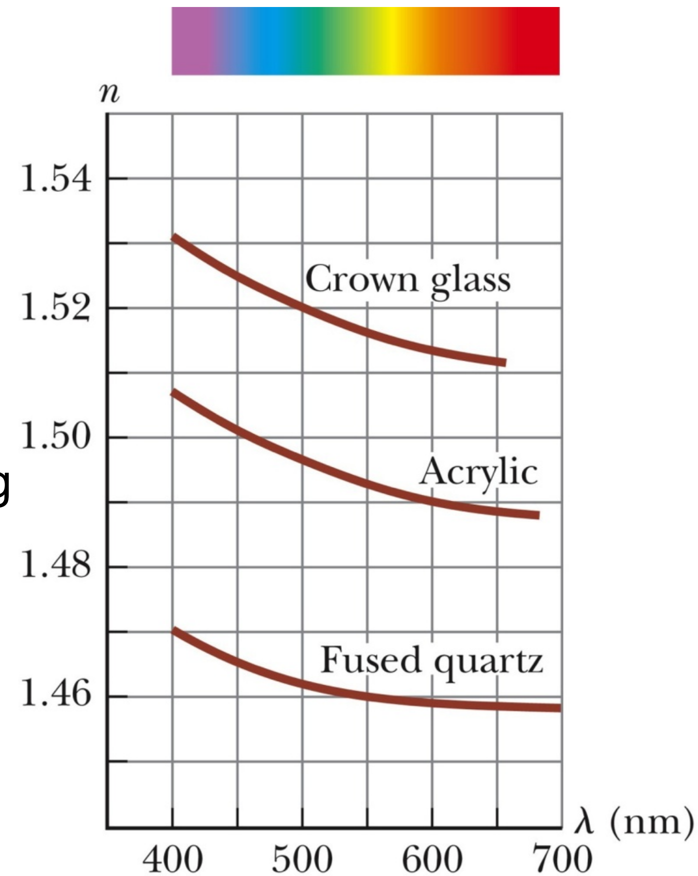
For a given material, the index of refraction varies with the wavelength of the light passing through the material.

This dependence of n on λ is called dispersion.

Snell's law indicates light of different wavelengths is bent at different angles when incident on a refracting material.

The index of refraction for a material generally decreases with increasing wavelength.

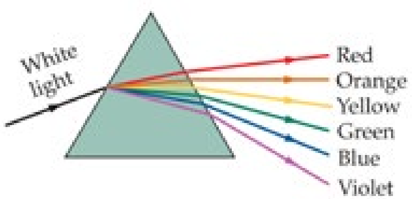
Violet light bends more than red light when passing into a refracting material.

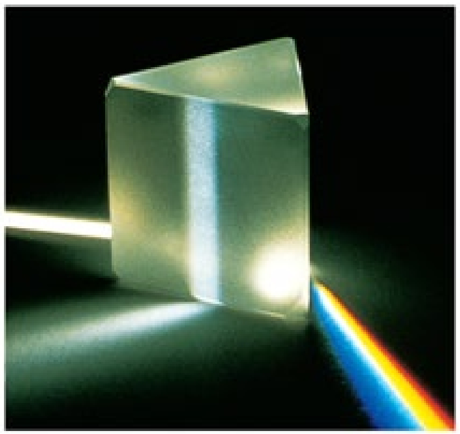


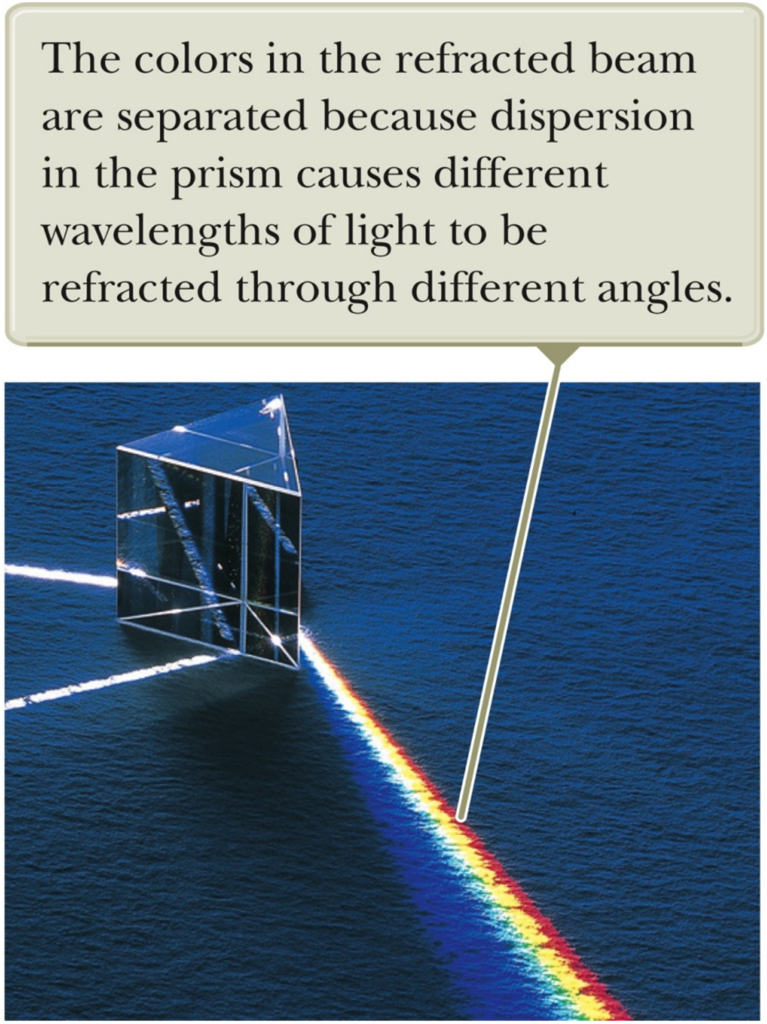
Refraction in a Prism

Since all the colors have different angles of deviation, white light will spread out into a *spectrum.*

- Violet deviates the most.
- Red deviates the least.
- The remaining colors are in between.







The Rainbow

A ray of light strikes a drop of water in the atmosphere.

It undergoes both reflection and refraction.

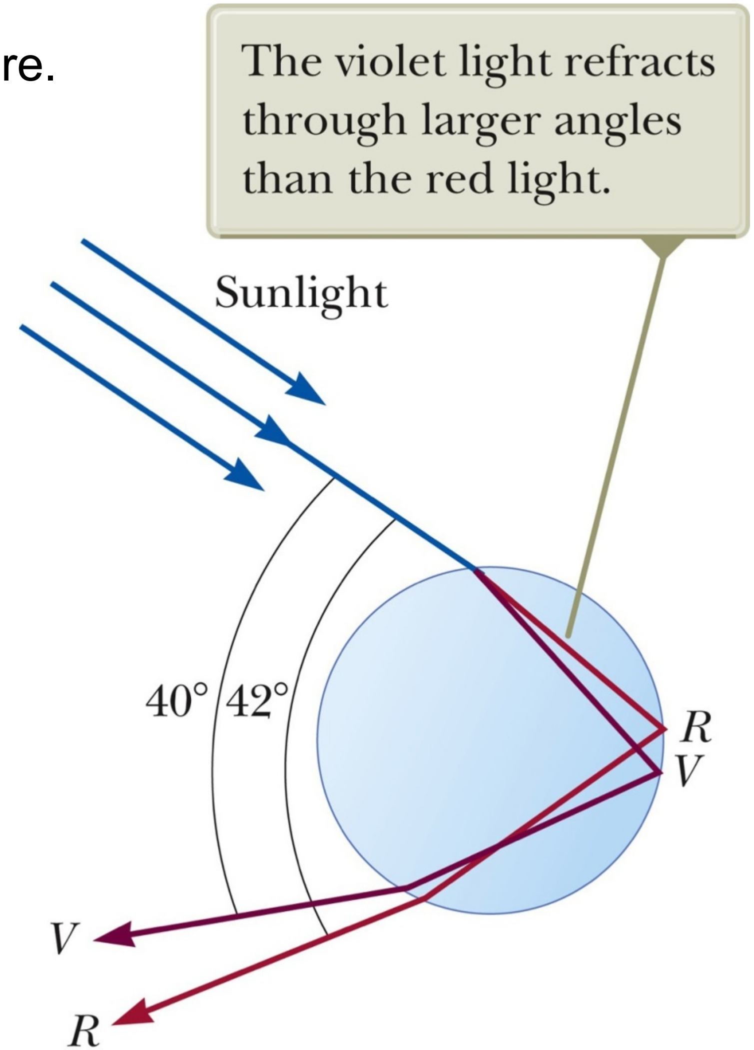
- First refraction at the front of the drop
  - Violet light will deviate the most.
  - Red light will deviate the least.

At the back surface the light is reflected.

It is refracted again as it returns to the front surface and moves into the air.

The rays leave the drop at various angles.

- The angle between the white light and the most intense violet ray is 40° .
- The angle between the white light and the most intense red ray is 42° .



Huygens's Principle

Huygens assumed that light is a form of wave motion rather than a stream of particles.

Huygens's Principle is a geometric construction for determining the position of a new wave at some point based on the knowledge of the wave front that preceded it.

All points on a given wave front are taken as point sources for the production of spherical secondary waves. These waves are called wavelets, they propagate outward through a medium with speeds characteristic of waves in that medium.

After some time has passed, the new position of the wave front is the surface tangent to the wavelets.

Huygens's Construction for Plane and Spherical Waves

- At $t = 0$, the wave front is indicated by the plane *AA.*'

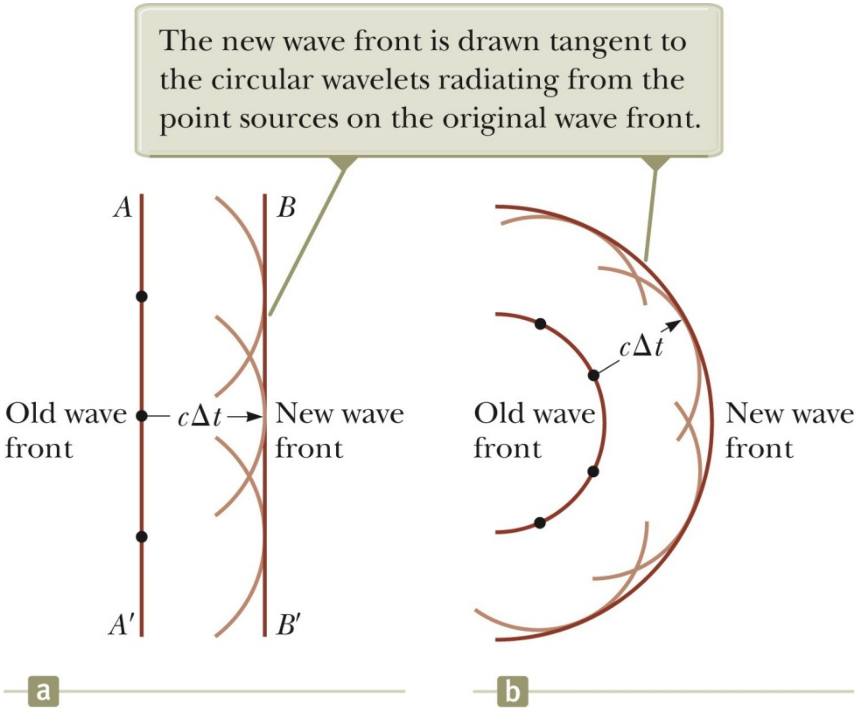
The points are representative sources for the wavelets.

After the wavelets have moved a distance $c\Delta t$, a new plane *BB*' can be drawn tangent to the wavefronts.

- The inner arc represents part of the spherical wave.

The points are representative points where wavelets are propagated.

The new wavefront is tangent at each point to the wavelet.



Mirrors and Lenses

Images Formed by Flat Mirrors

Simplest possible mirror

Light rays leave the source and are reflected from the mirror.

Point I is called the **image** of the object at point *O.*

The image is virtual. A flat mirror *always* produces a virtual image.

No light ray from the object can exist behind the mirror, so the light rays in front of the mirror only seem to be diverging from I.

There are an infinite number of choices of direction in which light rays could leave each point on the object.

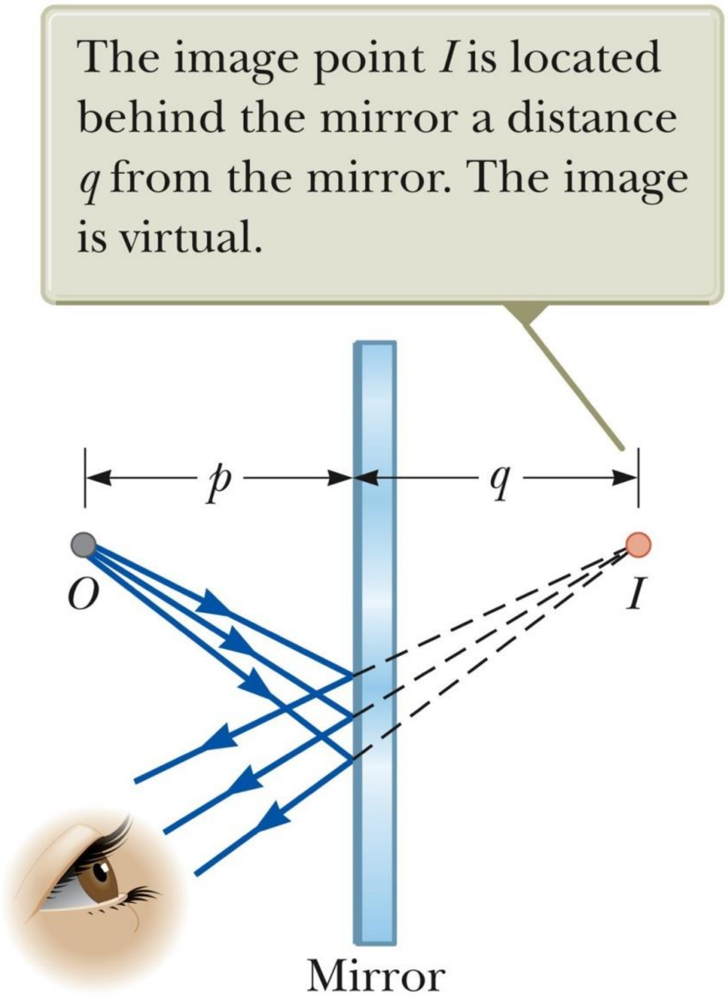


Image of Formation

Images can result when light rays encounter flat or curved surfaces between two media.

Images can be formed either by reflection or refraction due to these surfaces.

Mirrors and lenses can be designed to form images with desired characteristics.

Notation for Mirrors and Lenses

The **object distance** is the distance from the object to the mirror or lens.

- Denoted by p

The **image distance** is the distance from the image to the mirror or lens.

- Denoted by q

The **lateral magnification** of the mirror or lens is the ratio of the image height to the object height.

- Denoted by M

Images

Images are always located by extending diverging rays back to a point at which they intersect.

Images are located either at a point from which the rays of light *actually* diverge or at a point from which they *appear* to diverge.

A *real image* is formed when light rays pass through and diverge from the image point.

- Real images can be displayed on screens.

A *virtual image* is formed when light rays do not pass through the image point but only appear to diverge from that point.

- Virtual images cannot be displayed on screens.

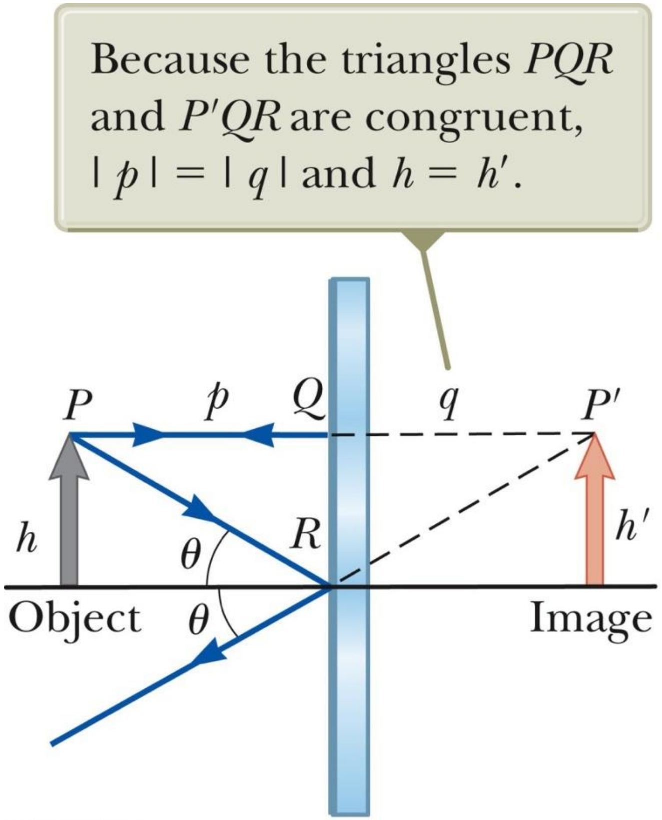
Images Formed by Flat Mirrors, Geometry

Two rays are needed to determine where an image is formed.

One ray starts at point *P*, travels to *Q* and reflects back on itself.

Another ray follows the path *PR* and reflects according to the law of reflection.

The triangles *PQR* and *P'QR* are congruent.



Images Formed by Flat Mirrors, final

To observe the image, the observer would trace back the two reflected rays to *P'.*

Point *P'* is the point where the rays appear to have originated.

The image formed by an object placed in front of a flat mirror is as far behind the mirror as the object is in front of the mirror.

- $|p| = |q|$

Reversals in a Flat Mirror

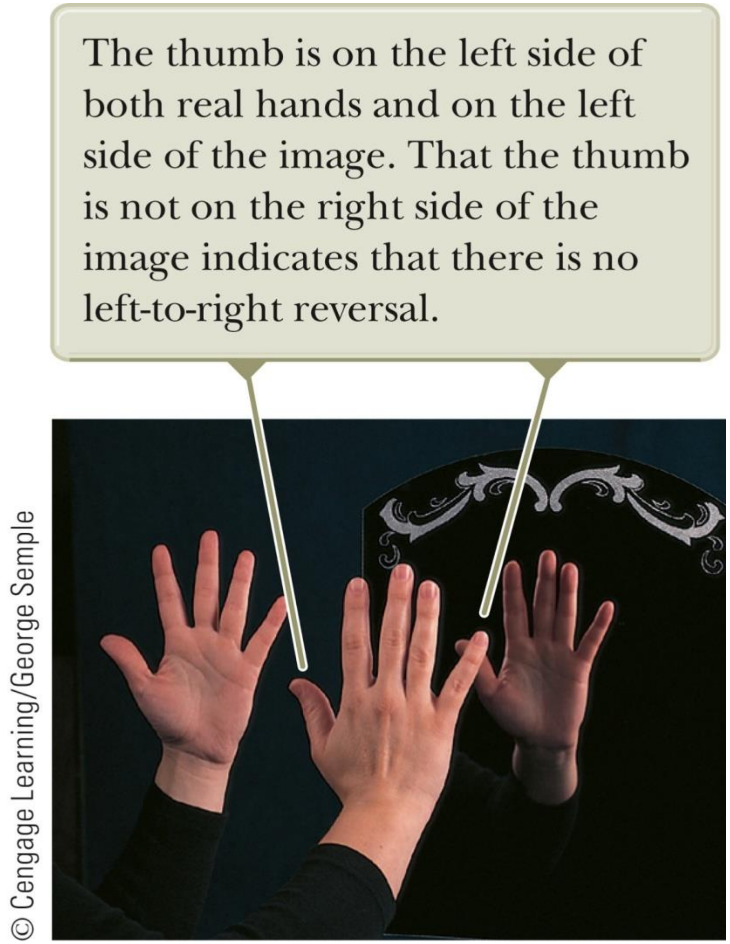
A flat mirror produces an image that has an *apparent* left-right reversal.

- For example, if you raise your right hand the image you see raises its left hand.

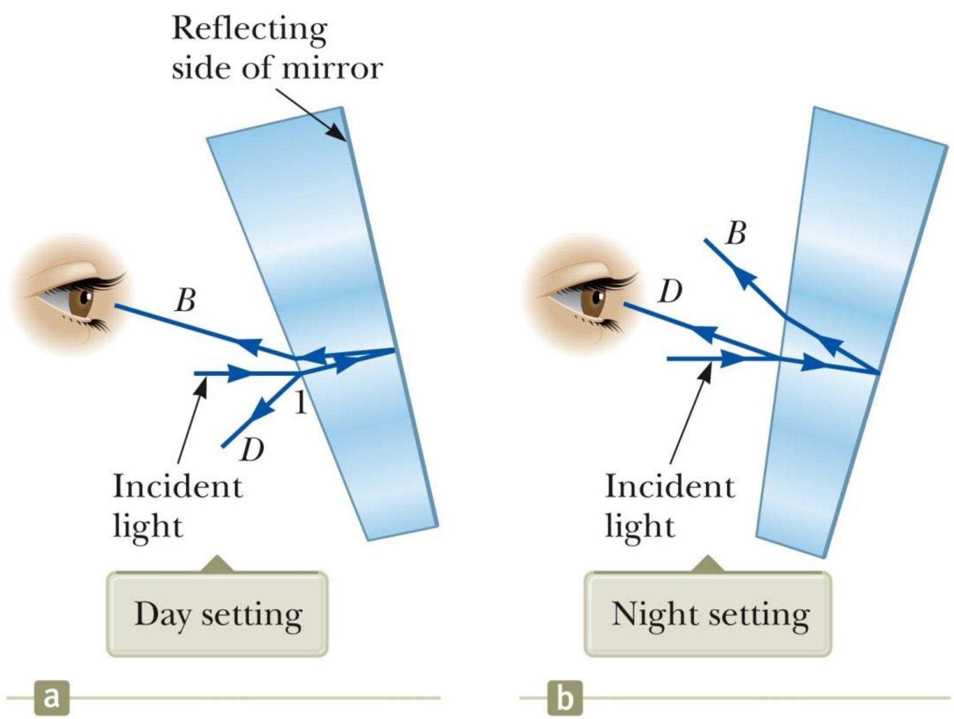
The reversal is not *actually* a left-right reversal.

The reversal is actually a *front-back* reversal.

- It is caused by the light rays going forward toward the mirror and then reflecting back from it.



Application – Day and Night Settings on Auto Mirrors



With the daytime setting, the bright beam (B) of reflected light is directed into the driver’s eyes.

With the nighttime setting, the dim beam (D) of reflected light is directed into the driver’s eyes, while the bright beam goes elsewhere.

Spherical Mirrors

A **spherical mirror** has the shape of a section of a sphere.

The mirror focuses incoming parallel rays to a point.

A *concave* spherical mirror has the silvered surface of the mirror on the inner, or concave, side of the curve.

A *convex* spherical mirror has the silvered surface of the mirror on the outer, or convex, side of the curve.

Concave Mirror, Notation

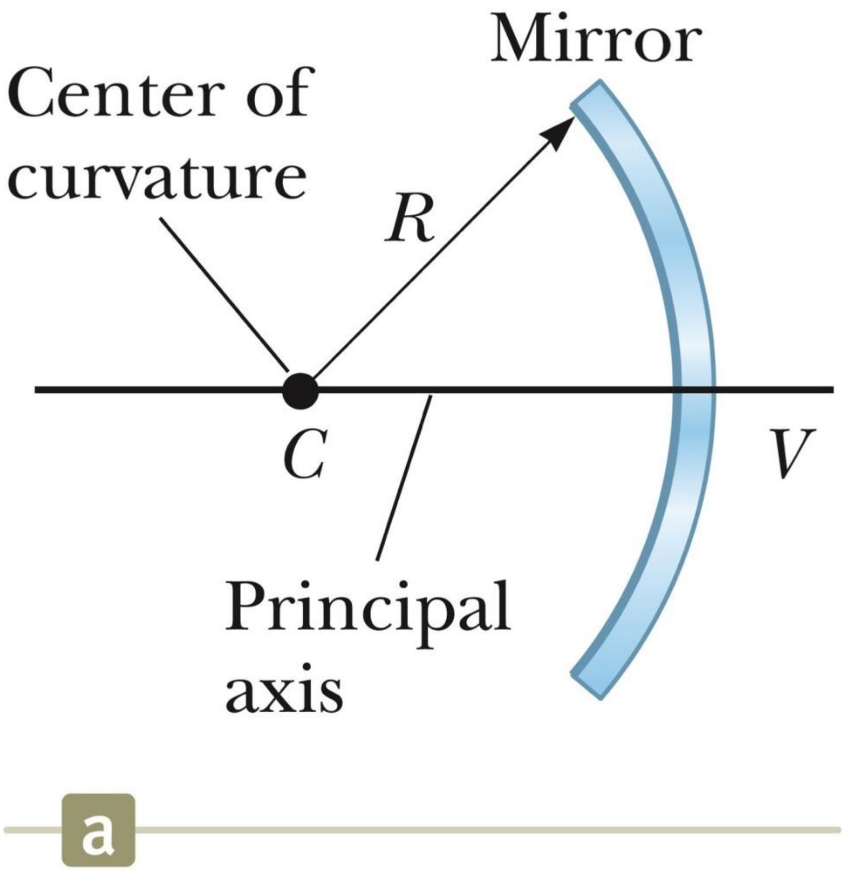
The mirror has a *radius of curvature* of *R.*

Its *center of curvature* is the point *C*

Point *V* is the center of the spherical segment.

A line drawn from *C* to *V* is called the *principal axis* of the mirror.

The blue band represents the structural support for the silvered surface.

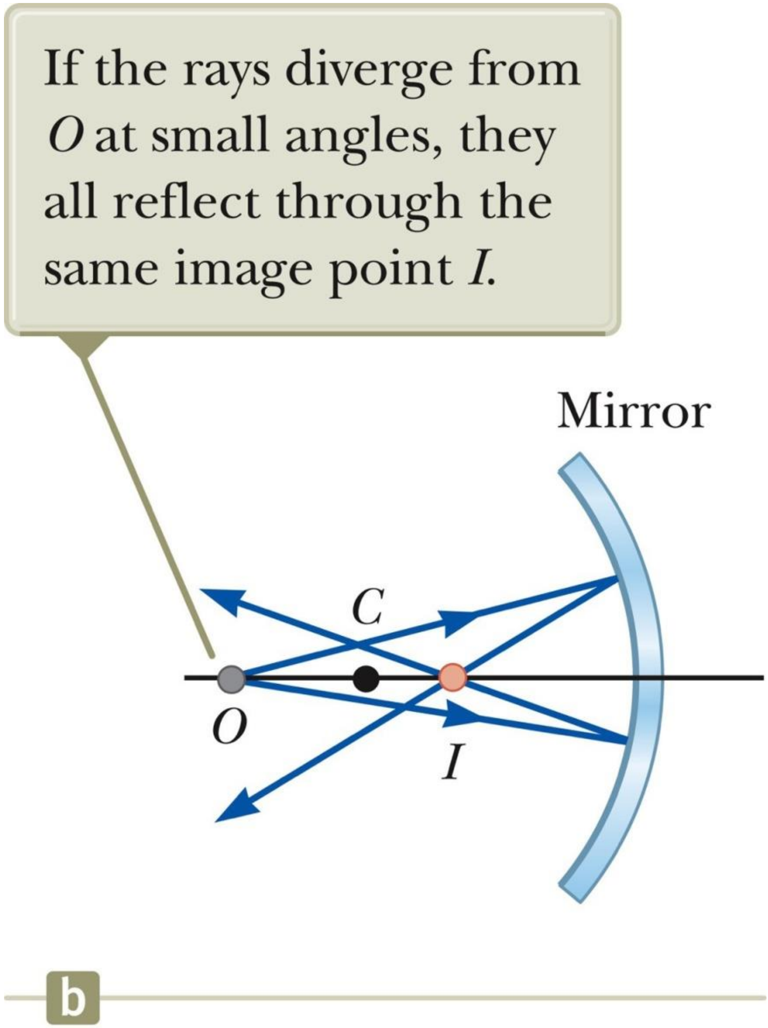


Paraxial Rays

We use only rays that diverge from the object and make a small angle with the principal axis.

Such rays are called **paraxial rays.**

All paraxial rays reflect through the image point.



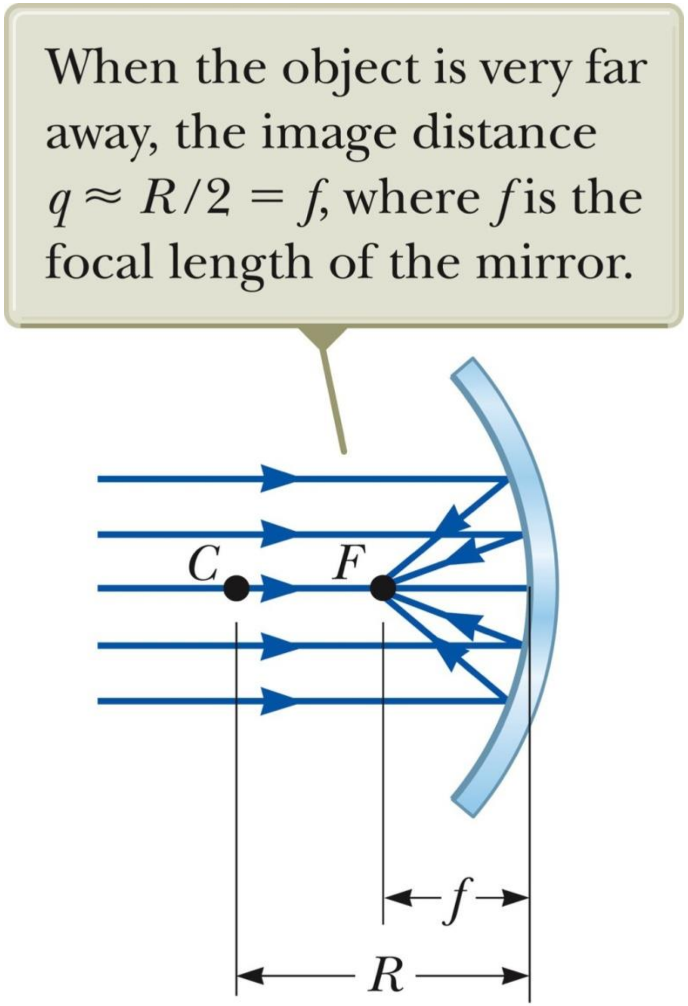
Focal Length

When the object is very far away, then p → ∞ and the incoming rays are essentially parallel.

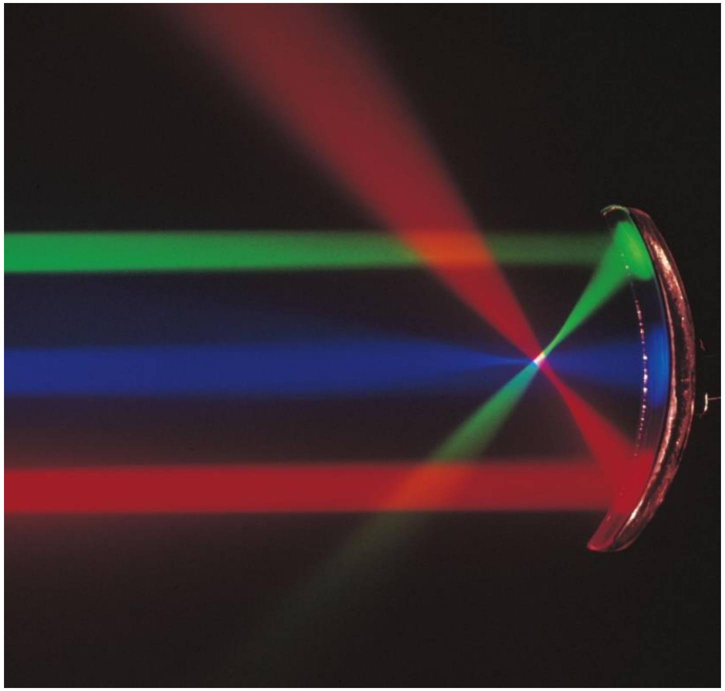
In this special case, the image point is called the **focal point.**

The distance from the mirror to the focal point is called the **focal length.**

- The focal length is ½ the radius of curvature.



Focal Point, cont.



The colored beams are traveling parallel to the principal axis.

The mirror reflects all three beams to the focal point.

The focal point is where all the beams intersect.

- The colors add to white.

Image Formed by a Concave Mirror

Distances are measured from V

Geometry can be used to determine the magnification of the image.

$$M = \frac{h'}{h} = -\frac{q}{p}$$

- *h*' is negative when the image is inverted with respect to the object.

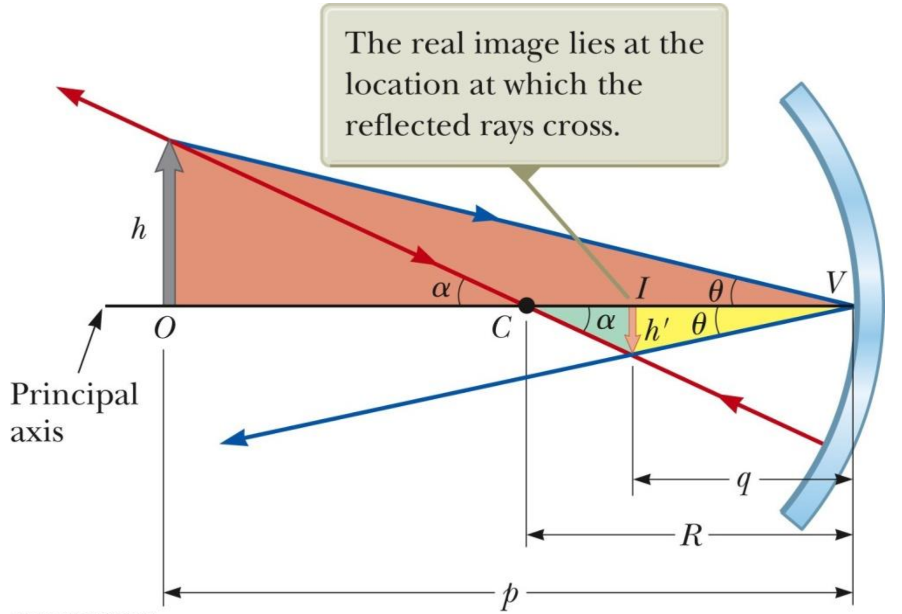


Image Formed by a Concave Mirror

Geometry also shows the relationship between the image and object distances.

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$

- This is called the **mirror equation.**

If p is much greater than R, then the image point is half-way between the center of curvature and the center point of the mirror.

- $p \rightarrow \infty$, then $1/p \approx 0$ and $q \approx R/2$

Focal Point and Focal Length, cont.

The focal point is dependent solely on the curvature of the mirror, not on the location of the object.

- It also does not depend on the material from which the mirror is made.

Since the focal length is related to the radius of curvature by f = R / 2, the mirror equation can be expressed as

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

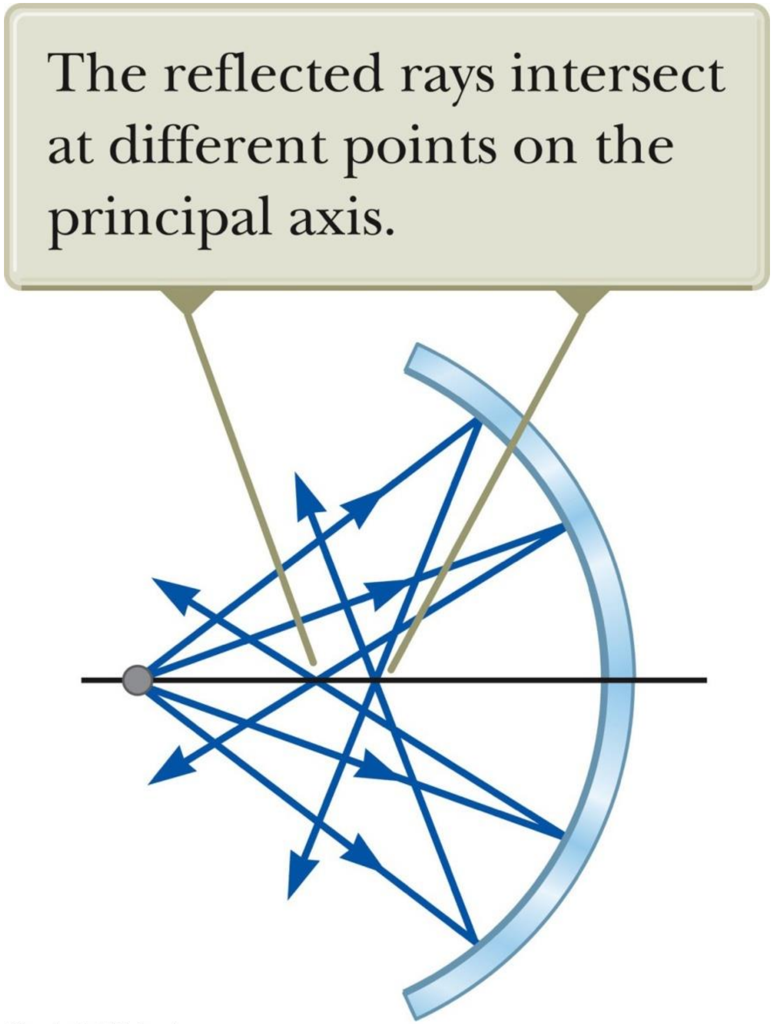
Spherical Aberration

Rays that are far from the principal axis converge to other points on the principal axis .

- The light rays make large angles with the principal axis.

This produces a blurred image.

The effect is called **spherical aberration.**



Convex Mirrors

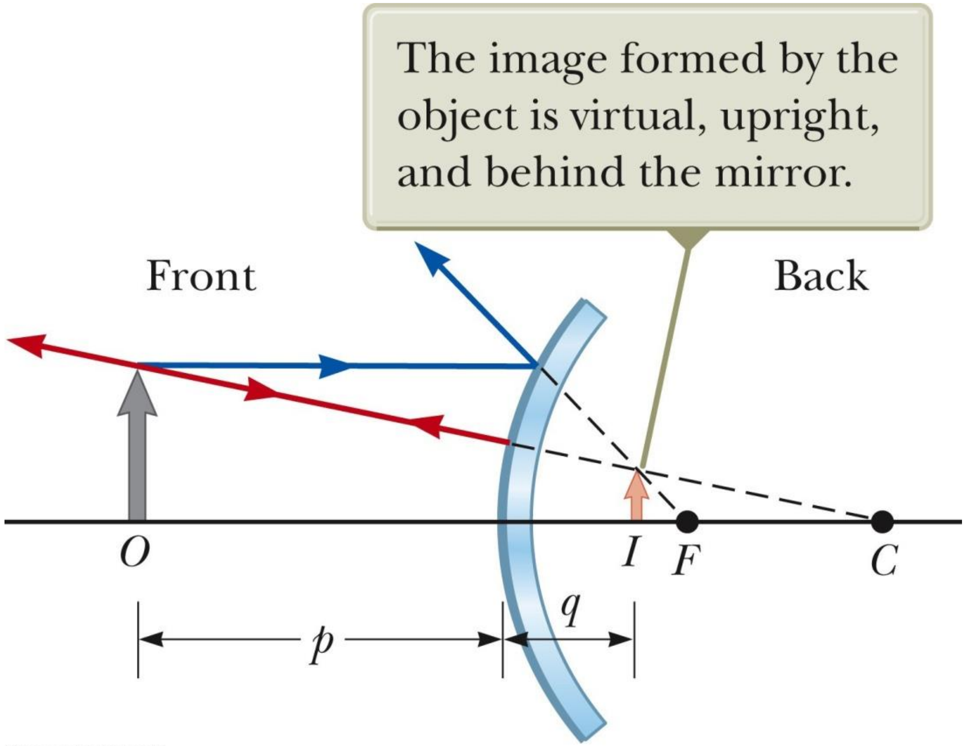
A **convex mirror** is sometimes called a *diverging mirror.*

- The light reflects from the outer, convex side.

The rays from any point on the object diverge after reflection as though they were coming from some point behind the mirror.

The image is virtual because the reflected rays only appear to originate at the image point.

Image Formed by a Convex Mirror



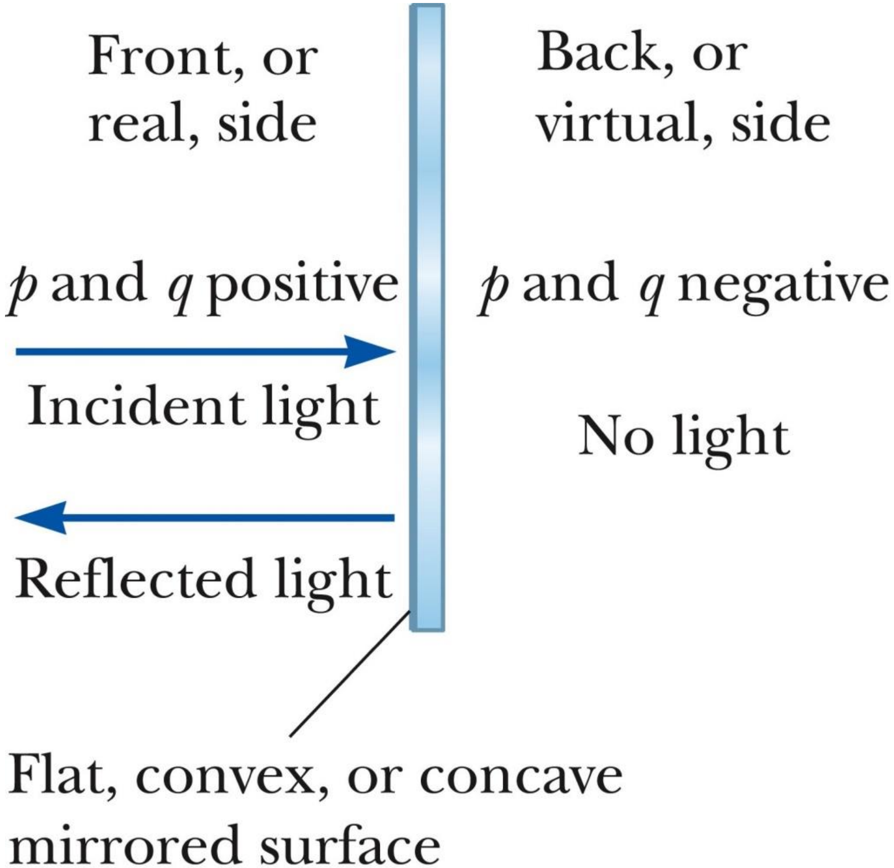
In general, the image formed by a convex mirror is upright, virtual, and smaller than the object.

Sign Conventions

These sign conventions apply to both concave and convex mirrors.

The equations used for the concave mirror also apply to the convex mirror.

Be sure to use proper sign choices when substituting values into the equations.



Sign Conventions, Summary Table

Table 36.1 Sign Conventions for Mirrors

Quantity	Positive When . . .	Negative When . . .
Object location (p)	object is in front of mirror (real object).	object is in back of mirror (virtual object).
Image location (q)	image is in front of mirror (real image).	image is in back of mirror (virtual image).
Image height (h')	image is upright.	image is inverted.
Focal length (f) and radius (R)	mirror is concave.	mirror is convex.
Magnification (M)	image is upright.	image is inverted.

Ray Diagrams

A *ray diagram* can be used to determine the position and size of an image.

They are graphical constructions which reveal the nature of the image.

They can also be used to check the parameters calculated from the mirror and magnification equations.

To draw a ray diagram, you need to know:

- The position of the object
- The locations of the focal point and the center of curvature.

Three rays are drawn.

- They all start from the same position on the object.

The intersection of any two of the rays at a point locates the image.

- The third ray serves as a check of the construction.

The Rays in a Ray Diagram – Concave Mirrors

Ray 1 is drawn from the top of the object parallel to the principal axis and is reflected through the focal point, *F.*

Ray 2 is drawn from the top of the object through the focal point and is reflected parallel to the principal axis.

Ray 3 is drawn through the center of curvature, C, and is reflected back on itself.

- Draw as if coming from the center C is p $< f$

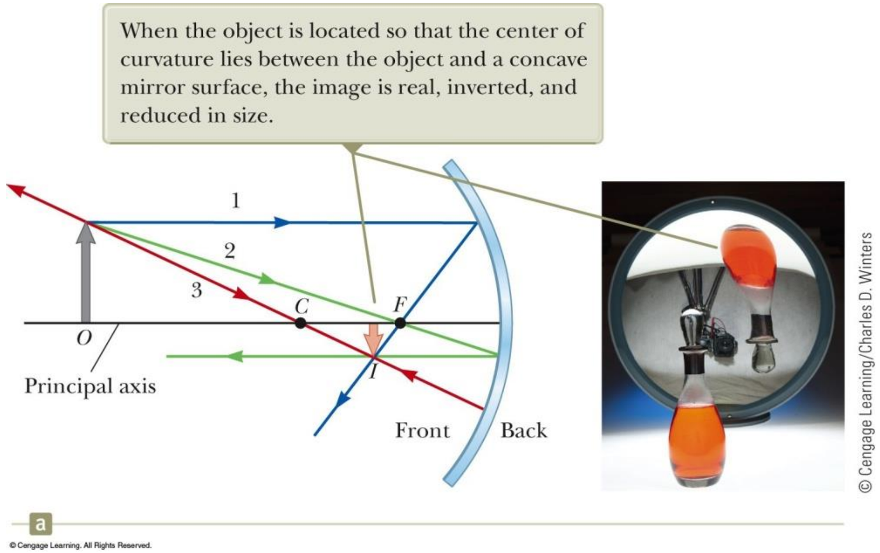
Notes About the Rays

A huge number of rays actually go in all directions from the object.

The three rays were chosen for their ease of construction.

The image point obtained by the ray diagram must agree with the value of q calculated from the mirror equation.

Ray Diagram for a Concave Mirror, p > R



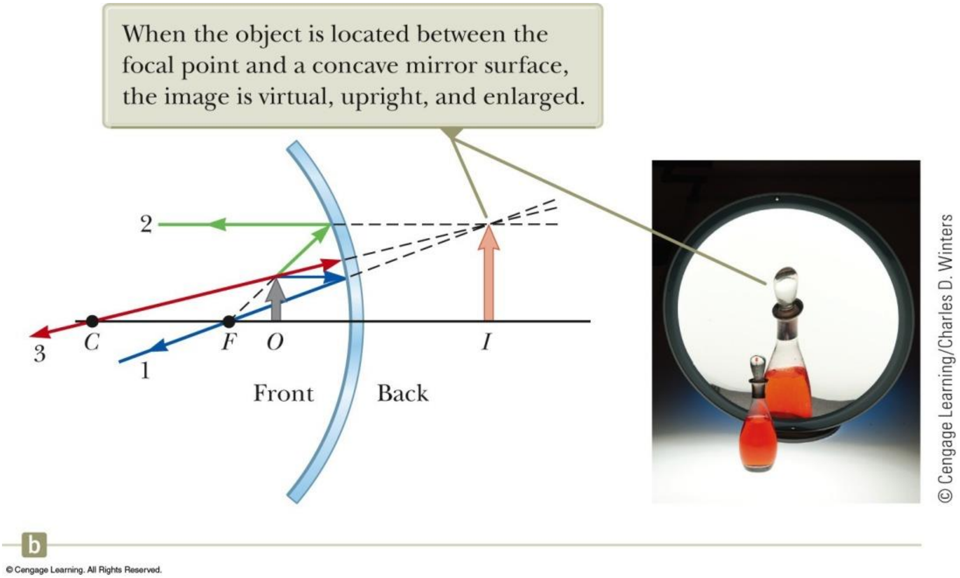
The center of curvature is between the object and the concave mirror surface.

The image is real.

The image is inverted.

The image is smaller than the object (reduced).

Ray Diagram for a Concave Mirror, $p < f$



The object is between the mirror surface and the focal point.

The image is virtual.

The image is upright.

The image is larger than the object (enlarged).

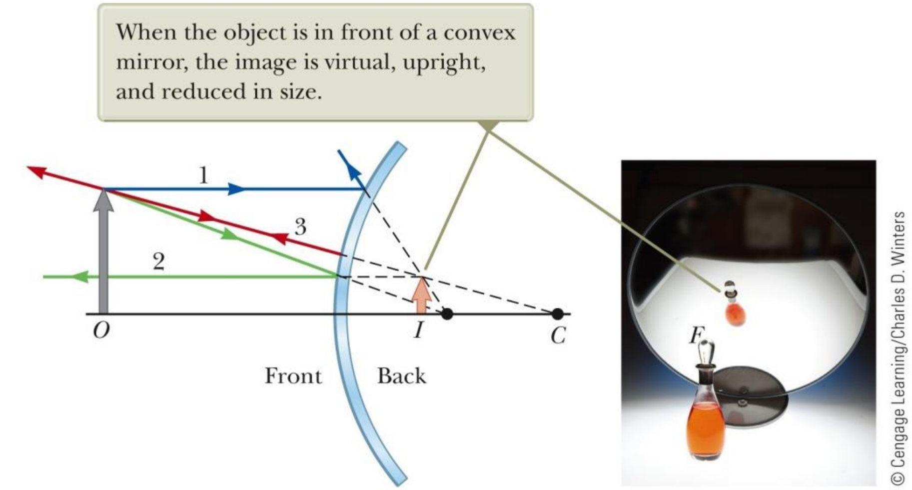
The Rays in a Ray Diagram – Convex Mirrors

Ray 1 is drawn from the top of the object parallel to the principal axis and is reflected away from the focal point, *F.*

Ray 2 is drawn from the top of the object toward the focal point and is reflected parallel to the principal axis.

Ray 3 is drawn through the center of curvature, C, on the back side of the mirror and is reflected back on itself.

Ray Diagram for a Convex Mirror



The object is in front of a convex mirror.

The image is virtual.

The image is upright.

The image is smaller than the object (reduced).

Notes on Images

With a concave mirror, the image may be either real or virtual.

- When the object is outside the focal point, the image is real.
- When the object is at the focal point, the image is infinitely far away.
- When the object is between the mirror and the focal point, the image is virtual.

With a convex mirror, the image is always virtual and upright.

- As the object distance decreases, the virtual image increases in size.

Images Formed by Refraction

Consider two transparent media having indices of refraction n_1 and $n_{2.}$

The boundary between the two media is a spherical surface of radius *R.*

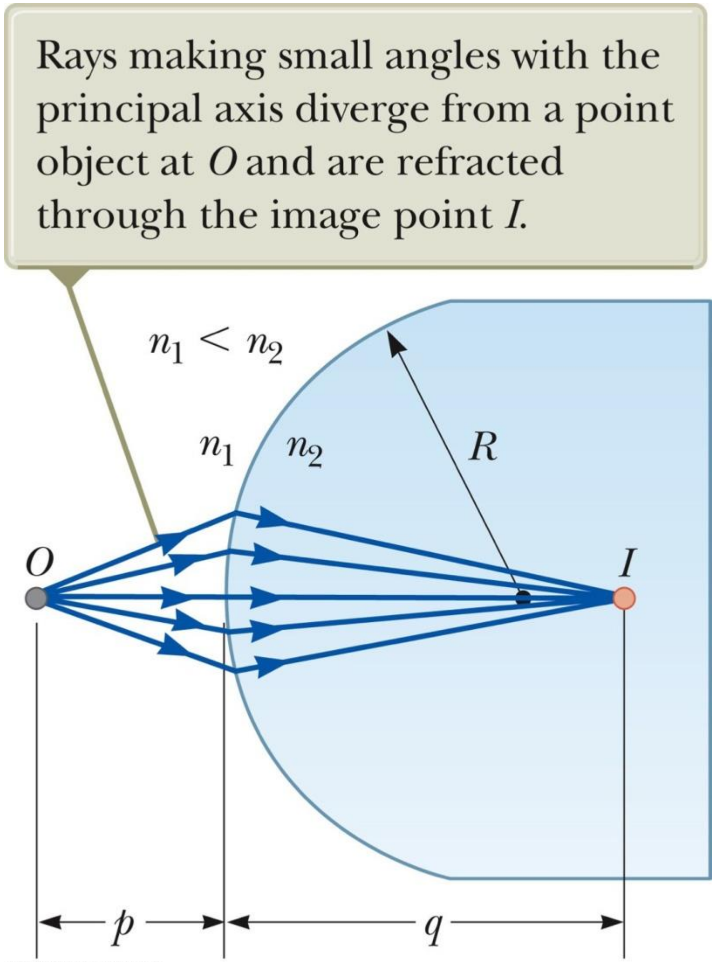
Rays originate from the object at point *O* in the medium with $n = n_{1.}$

The side of the surface in which the light rays originate is defined as the front side.

The other side is called the back side.

Real images are formed by refraction in the back of the surface.

- Because of this, the sign conventions for q and R for refracting surfaces are opposite those for reflecting surfaces.



Sign Conventions for Refracting Surfaces

Table 36.2 Sign Conventions for Refracting Surfaces

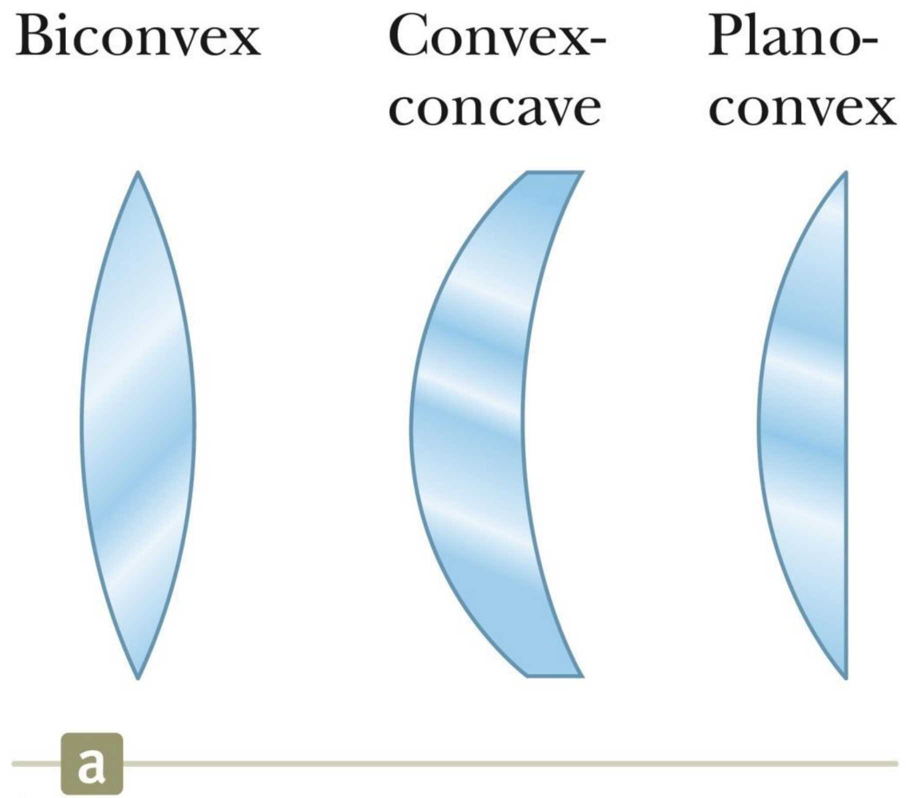
Quantity	Positive When . . .	Negative When . . .
Object location (p)	object is in front of surface (real object).	object is in back of surface (virtual object).
Image location (q)	image is in back of surface (real image).	image is in front of surface (virtual image).
Image height (h')	image is upright.	image is inverted.
Radius (R)	center of curvature is in back of surface.	center of curvature is in front of surface.

Thin Lens Shapes - *converging* lenses.

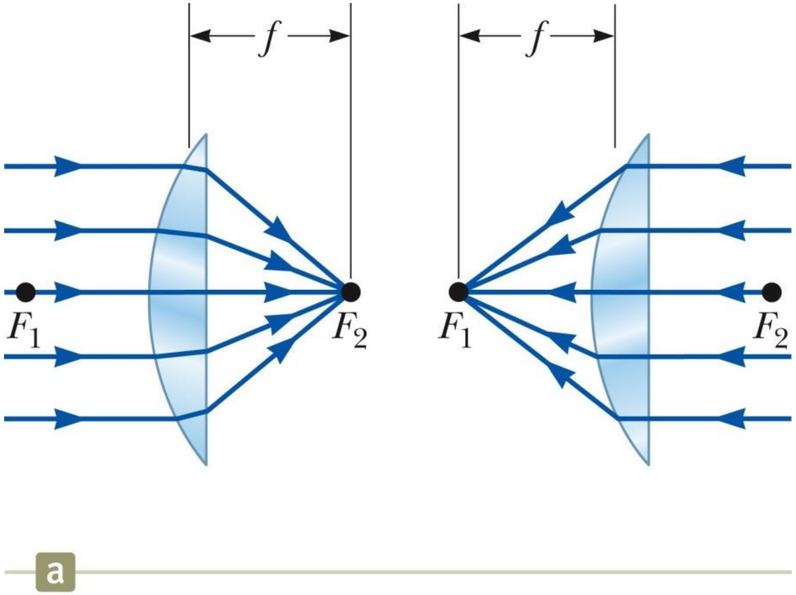
These are examples of *converging* lenses.

They have positive focal lengths.

They are thickest in the middle.



Focal Length of a Converging Lens



The parallel rays pass through the lens and converge at the focal point.

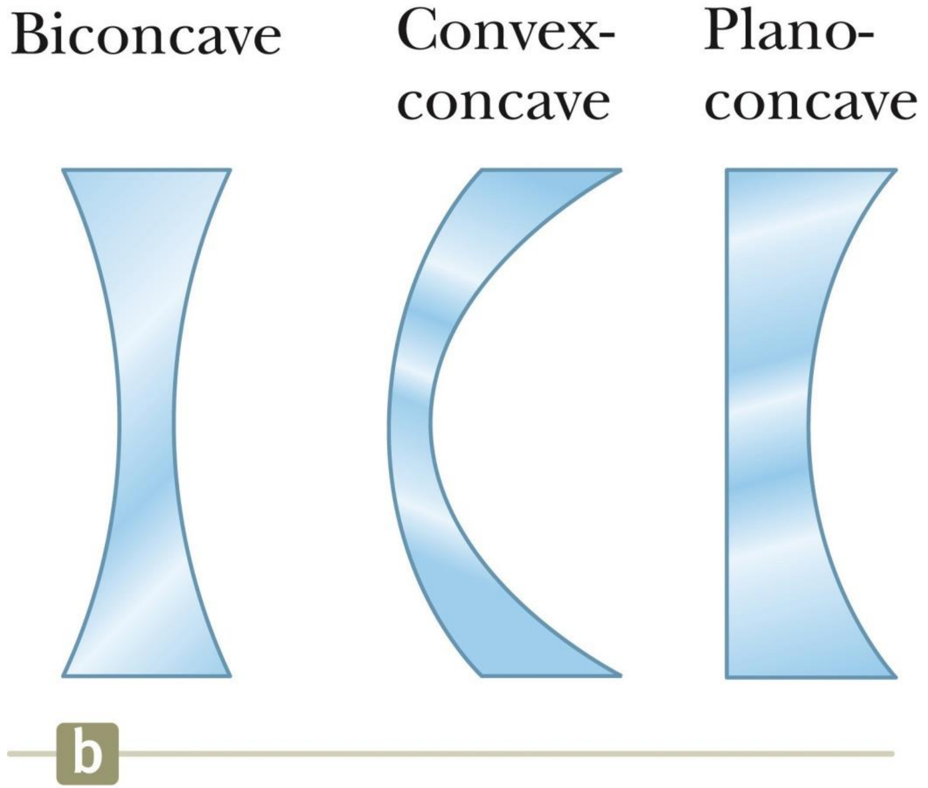
The parallel rays can come from the left or right of the lens.

More Thin Lens Shapes - *diverging* lenses.

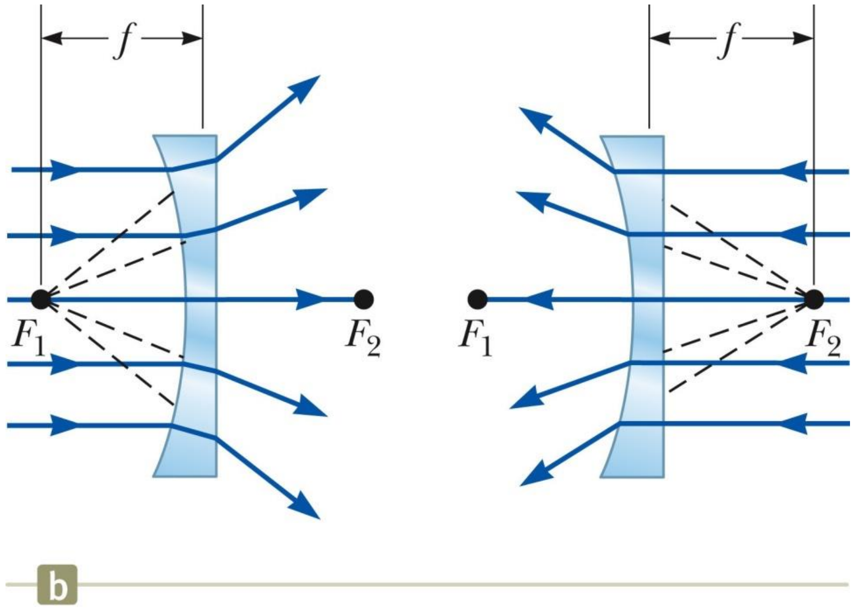
These are examples of *diverging* lenses.

They have negative focal lengths.

They are thickest at the edges.



Focal Length of a Diverging Lens



The parallel rays diverge after passing through the diverging lens.

The focal point is the point where the rays appear to have originated.

Images Formed by Lenses

Lenses are commonly used to form images by refraction.

Lenses are used in optical instruments.

- Cameras
- Telescopes
- Microscopes

Light passing through a lens experiences refraction at two surfaces.

The image formed by one refracting surface serves as the object for the second surface.

Lens-makers' Equation

$$(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f}$$

- This is called the lens-makers' equation.
  - It can be used to determine the values of R_1 and R_2 needed for a given index of refraction and a desired focal length f.

Image Formed by a Thin Lens

A thin lens is one whose thickness is small compared to the radii of curvature.

For a thin lens, the thickness of the lens can be neglected.

Thin Lens Equation

The relationship among the focal length, the object distance and the image distance is the same as for a mirror.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

Notes on Focal Length and Focal Point of a Thin Lens

Because light can travel in either direction through a lens, each lens has two focal points.

- One focal point is for light passing in one direction through the lens and one is for light traveling in the opposite direction.

However, there is only one focal length.

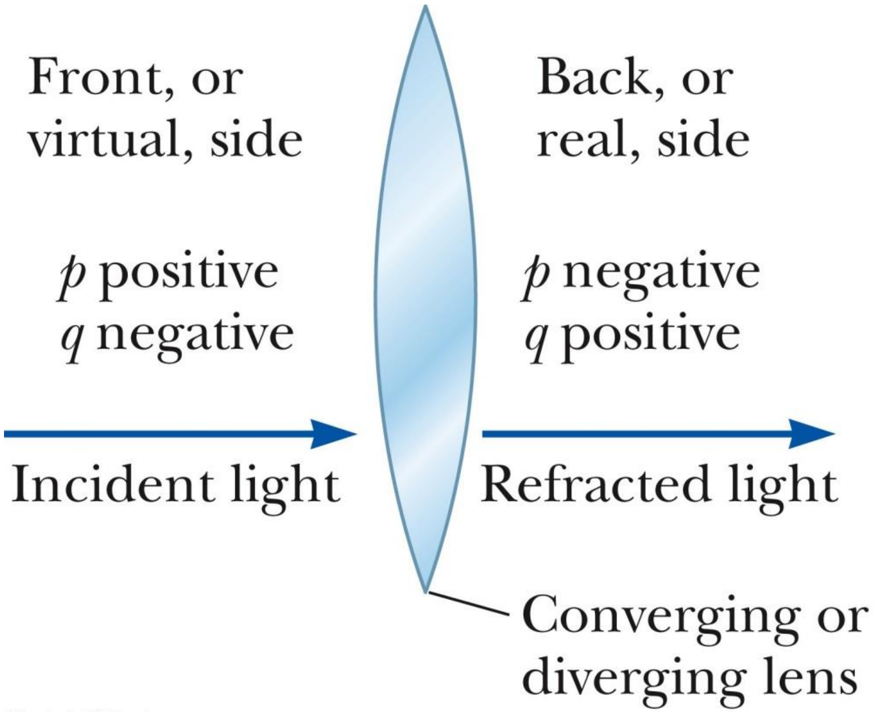
Each focal point is located the same distance from the lens.

Determining Signs for Thin Lenses

The front side of the thin lens is the side of the incident light.

The light is refracted into the back side of the lens.

This is also valid for a refracting surface.



Sign Conventions for Thin Lenses

Table 36.3 Sign Conventions for Thin Lenses

Quantity	Positive When . . .	Negative When . . .
Object location (p)	object is in front of lens (real object).	object is in back of lens (virtual object).
Image location (q)	image is in back of lens (real image).	image is in front of lens (virtual image).
Image height (h')	image is upright.	image is inverted.
R_1 and R_2	center of curvature is in back of lens.	center of curvature is in front of lens.
Focal length (f)	a converging lens.	a diverging lens.

Magnification of Images Through a Thin Lens

The lateral magnification of the image is

$$M = \frac{h'}{h} = -\frac{q}{p}$$

When *M* is positive, the image is upright and on the same side of the lens as the object.

When *M* is negative, the image is inverted and on the side of the lens opposite the object.

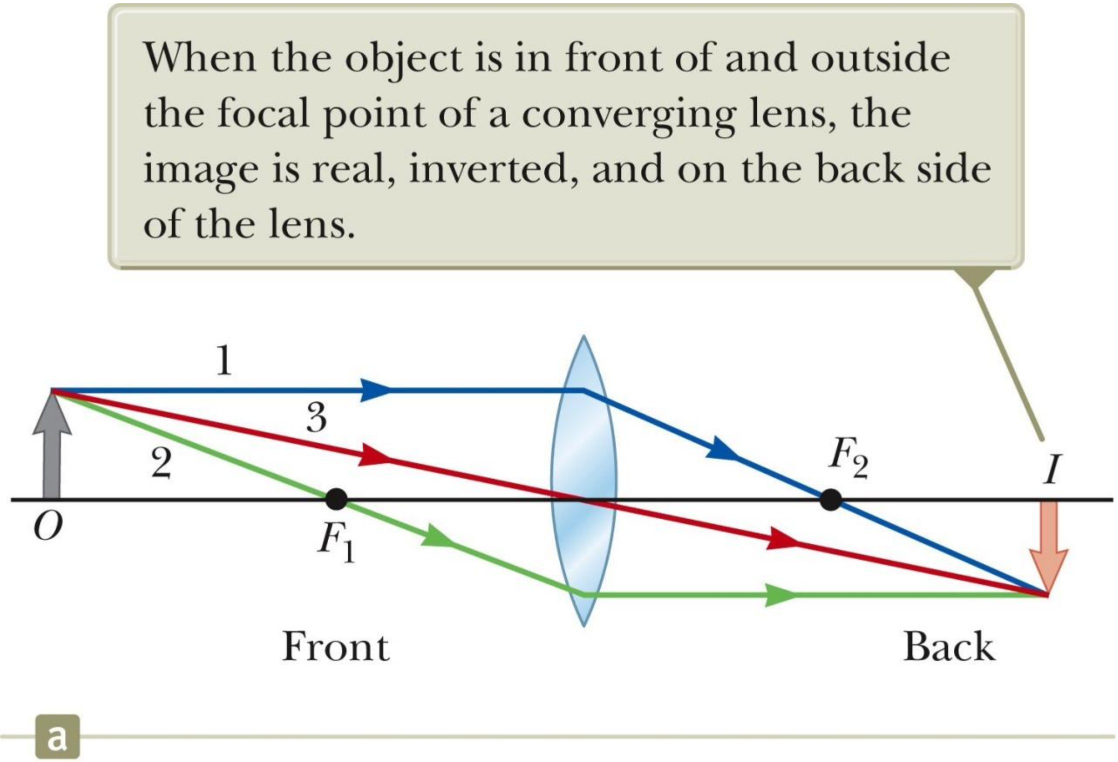
Ray Diagrams for Thin Lenses – Converging

Ray diagrams are convenient for locating the images formed by thin lenses or systems of lenses.

For a converging lens, the following three rays are drawn:

- Ray 1 is drawn parallel to the principal axis and then passes through the focal point on the back side of the lens.
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn through the focal point on the front of the lens (or as if coming from the focal point if $p < f$) and emerges from the lens parallel to the principal axis.

Ray Diagram for Converging Lens, $p > f$

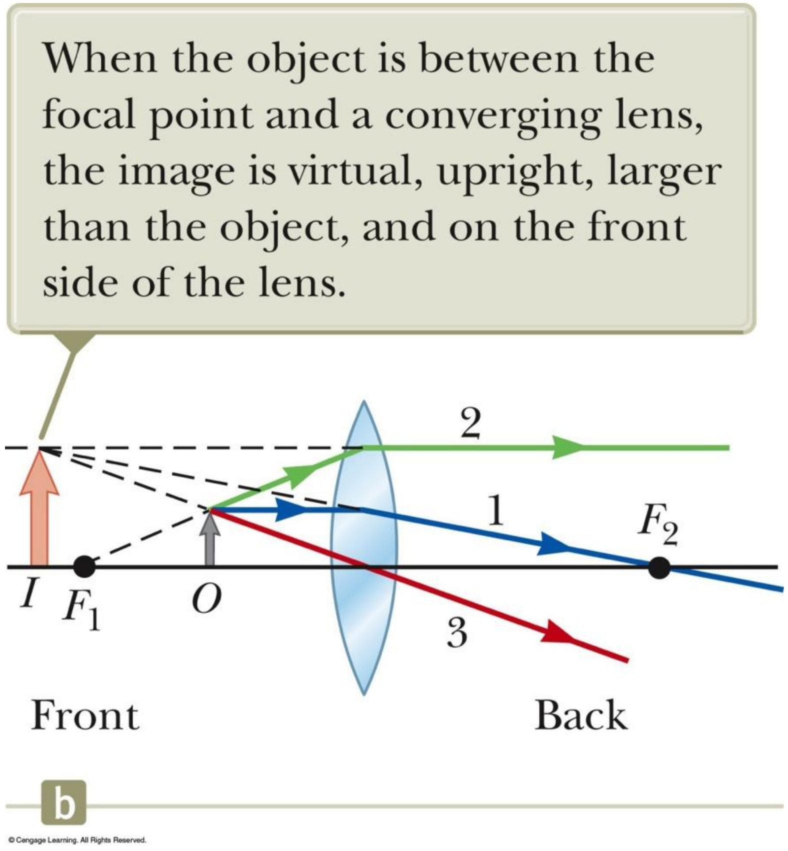


The image is real.

The image is inverted.

The image is on the back side of the lens.

Ray Diagram for Converging Lens, $p < f$



The image is virtual.

The image is upright.

The image is larger than the object.

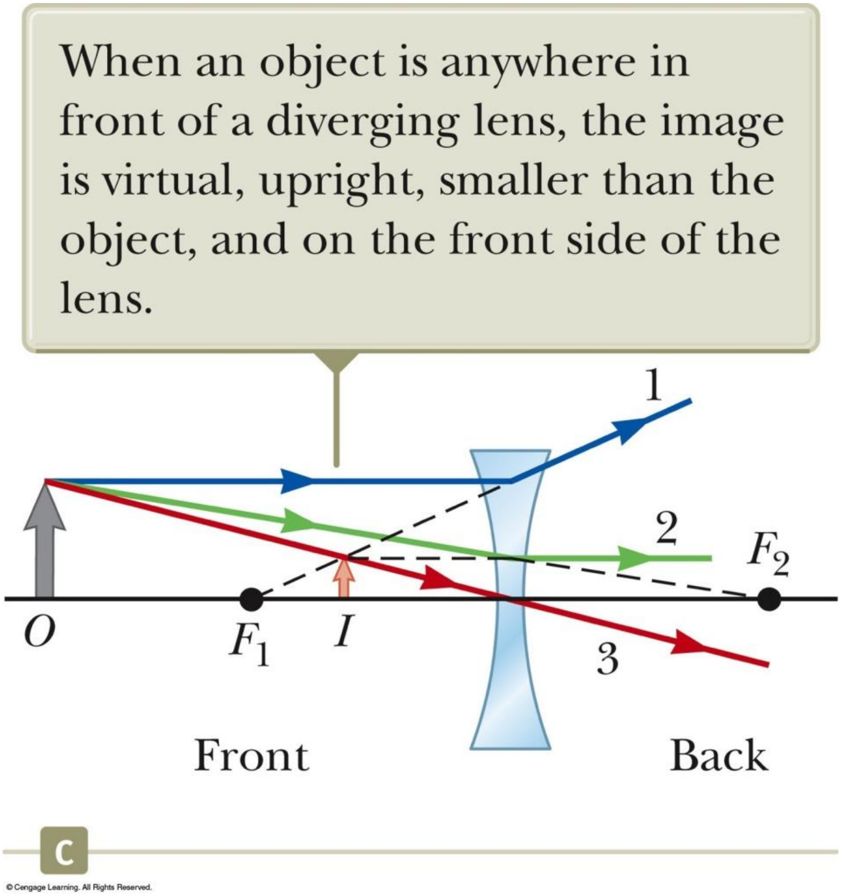
The image is on the front side of the lens.

Ray Diagrams for Thin Lenses – Diverging

For a diverging lens, the following three rays are drawn:

- Ray 1 is drawn parallel to the principal axis and emerges directed away from the focal point on the front side of the lens.
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn in the direction toward the focal point on the back side of the lens and emerges from the lens parallel to the principal axis.

Ray Diagram for Diverging Lens



The image is virtual.

The image is upright.

The image is smaller.

The image is on the front side of the lens.

Image Summary

For a converging lens, when the object distance is greater than the focal length, ($p > f$)

- The image is real and inverted.

For a converging lens, when the object is between the focal point and the lens, ($p < f$)

- The image is virtual and upright.

For a diverging lens, the image is always virtual and upright.

- This is regardless of where the object is placed.

Combinations of Thin Lenses

The image formed by the first lens is located as though the second lens were not present.

Then a ray diagram is drawn for the second lens.

The image of the first lens is treated as the object of the second lens.

The image formed by the second lens is the final image of the system.

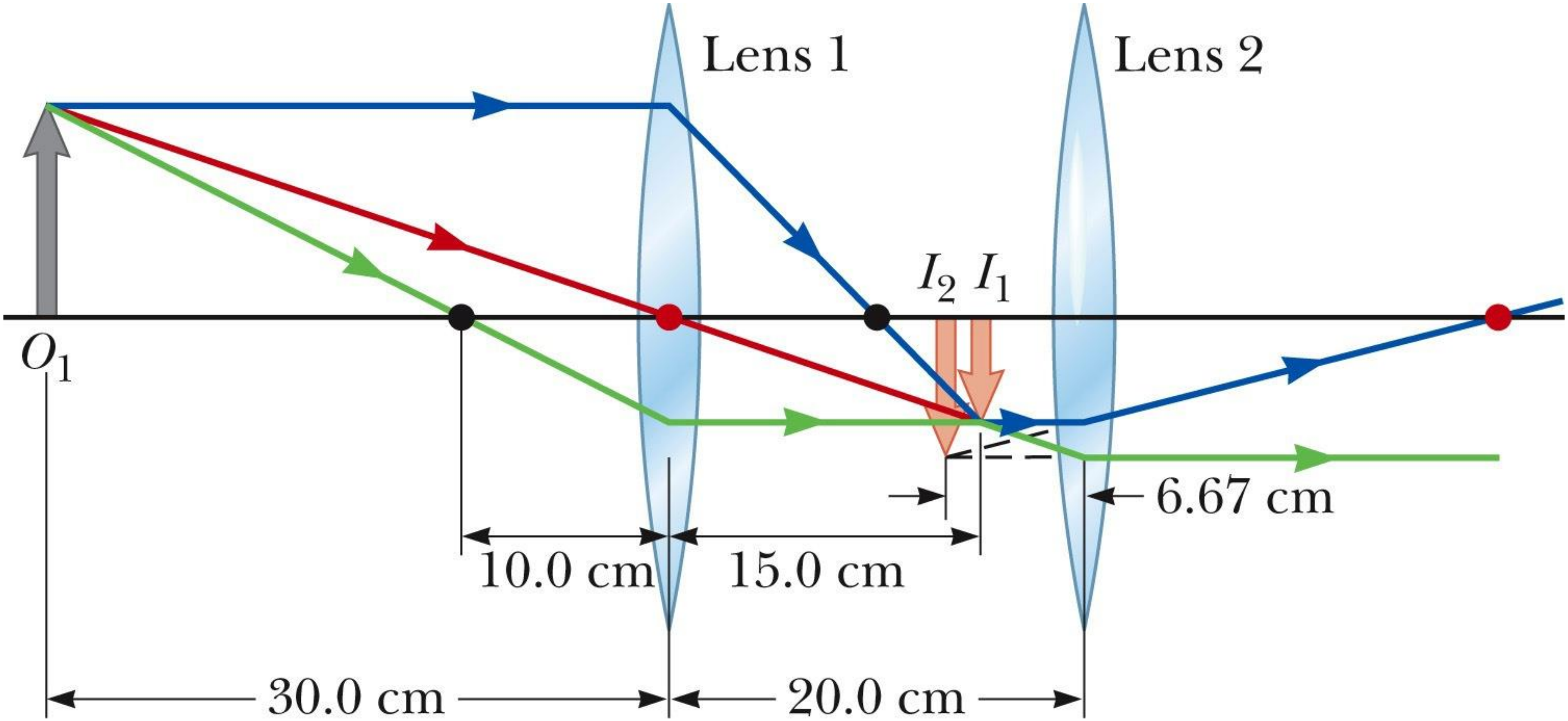
If the image formed by the first lens lies on the back side of the second lens, then the image is treated as a *virtual object* for the second lens.

- *p* will be negative

The same procedure can be extended to a system of three or more lenses.

The overall magnification is the product of the magnification of the separate lenses.

Combination of Thin Lenses, example



Combination of Thin Lenses, example

Find the location of the image formed by lens 1.

Find the magnification of the image due to lens 1.

Find the object distance for the second lens.

Find the location of the image formed by lens 2.

Find the magnification of the image due to lens 2.

Find the overall magnification of the system.

Lens Aberrations

Assumptions have been:

- Rays make small angles with the principal axis.
- The lenses are thin.

The rays from a point object do not focus at a single point.

- The result is a blurred image.
- This is a situation where the approximations used in the analysis do not hold.

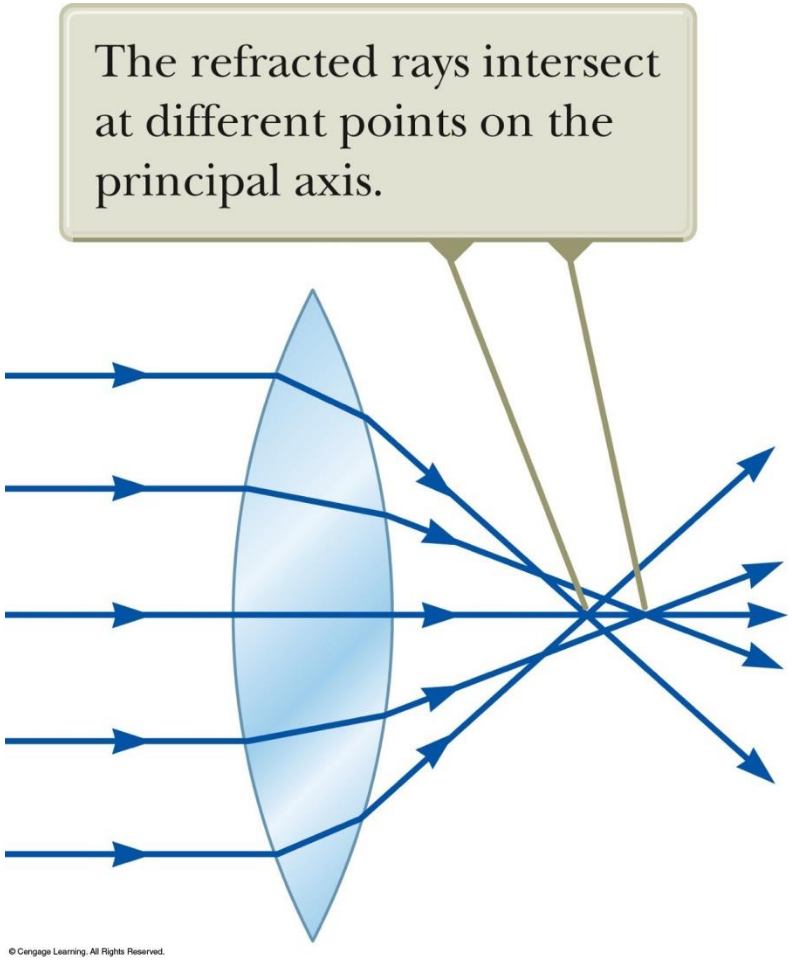
The departures of actual images from the ideal predicted by our model are called **aberrations.**

Spherical Aberration

This results from the focal points of light rays far from the principal axis being different from the focal points of rays passing near the axis.

For a camera, a small aperture allows a greater percentage of the rays to be paraxial.

For a mirror, parabolic shapes can be used to correct for spherical aberration.

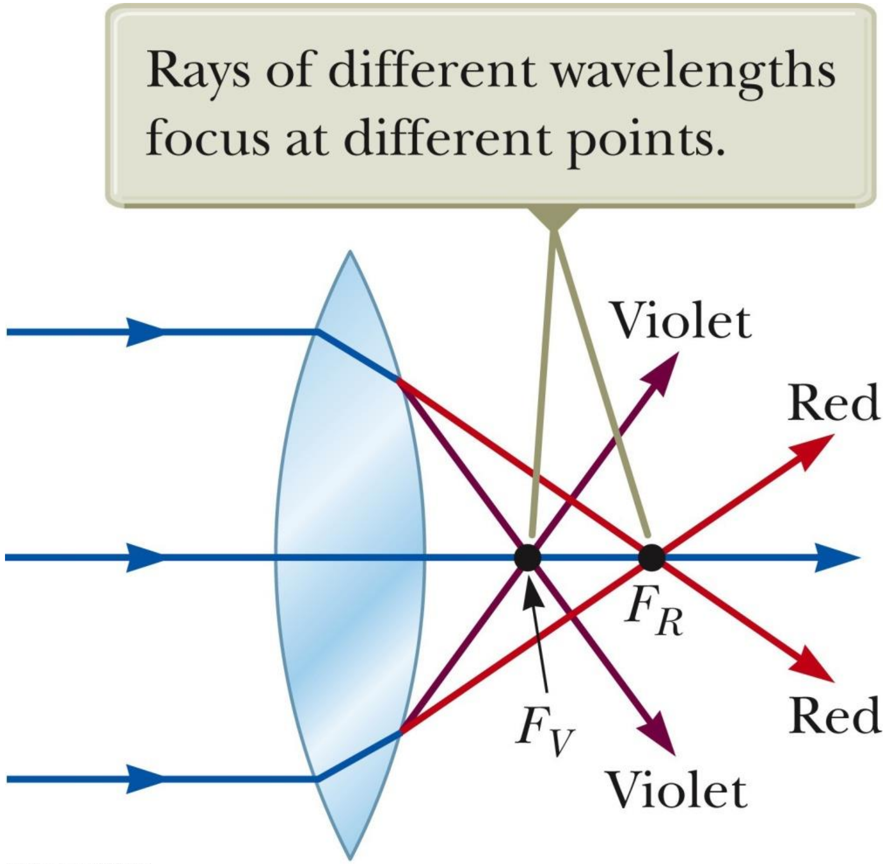


Chromatic Aberration

Different wavelengths of light refracted by a lens focus at different points.

- Violet rays are refracted more than red rays.
- The focal length for red light is greater than the focal length for violet light.

Chromatic aberration can be minimized by the use of a combination of converging and diverging lenses made of different materials.



Fresnel Lens

Refraction occurs only at the surfaces of the lens.

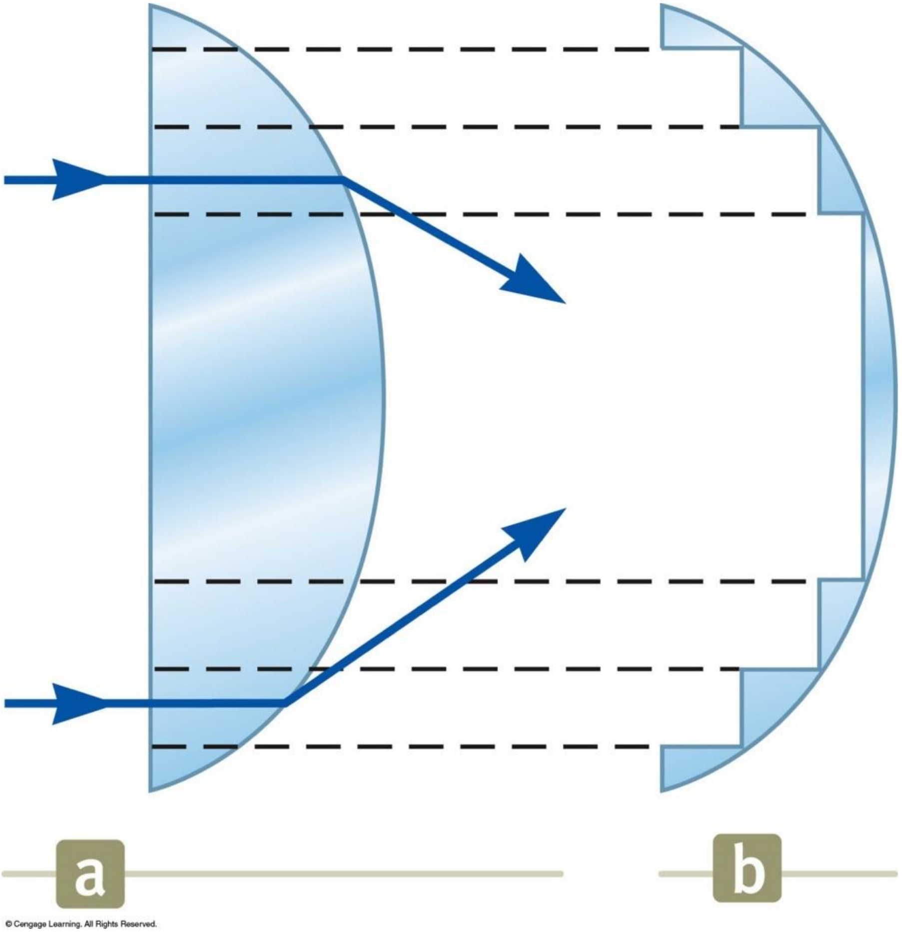
A *Fresnel lens* is designed to take advantage of this fact.

It produces a powerful lens without great thickness.

Only the surface curvature is important in the refracting qualities of the lens.

The material in the middle of the Fresnel lens is removed.

Because the edges of the curved segments cause some distortion, Fresnel lenses are usually used only in situations where image quality is less important than reduction of weight.



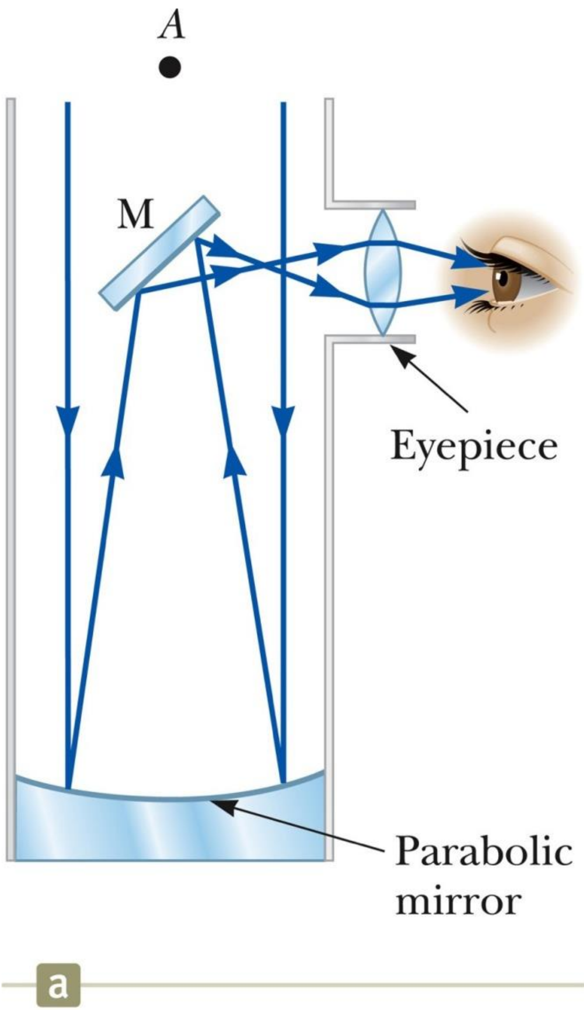
Reflecting Telescope, Newtonian Focus

The incoming rays are reflected from the mirror and converge toward point *A.*

- At *A*, an image would be formed.

A small flat mirror, M, reflects the light toward an opening in the side and it passes into an eyepiece.

- This occurs before the image is formed at *A.*



Summary

- Flat mirrors
- Spherical mirrors
- Thin lenses
- Combination of lenses
- Lens aberrations

Wave Optics

Wave Optics

Wave optics is a study concerned with phenomena that cannot be adequately explained by geometric (ray) optics.

These phenomena include:

- Interference
- Diffraction

Interference

In *constructive interference* the amplitude of the resultant wave is greater than that of either individual wave.

In *destructive interference* the amplitude of the resultant wave is less than that of either individual wave.

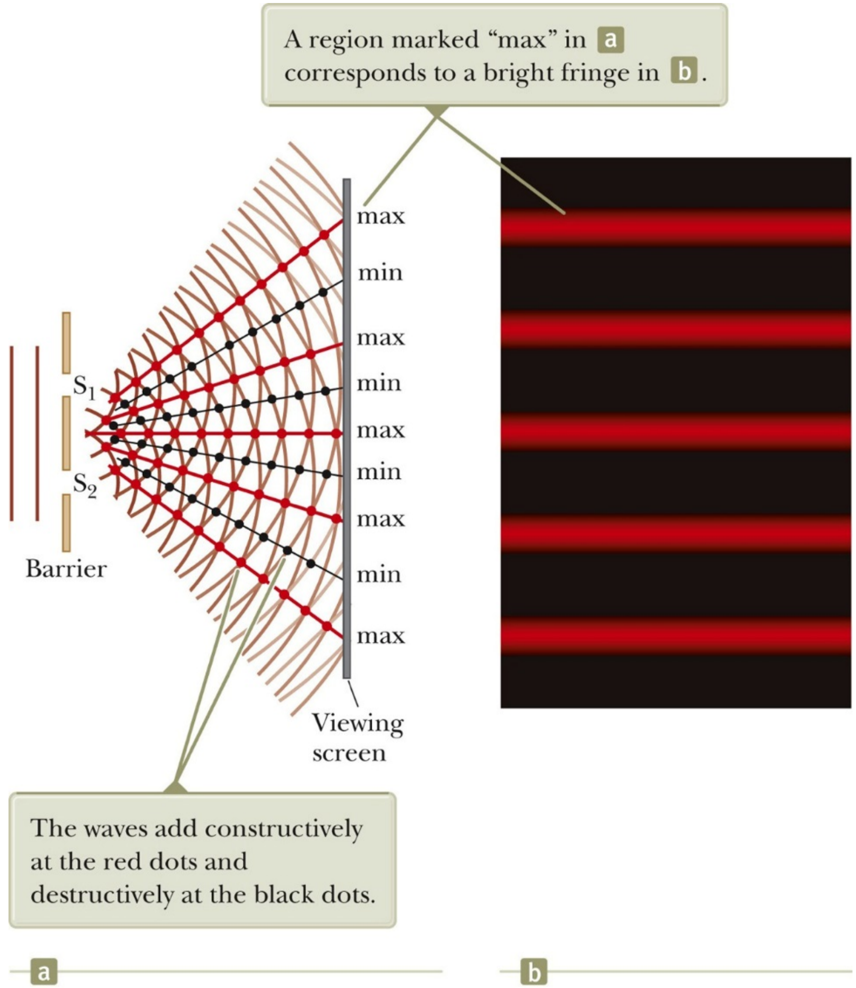
All interference associated with light waves arises when the electromagnetic fields that constitute the individual waves combine.

Young's Double-Slit Experiment: Schematic

Thomas Young first demonstrated interference in light waves from two sources in 1801.

The narrow slits S_1 and S_2 act as sources of waves.

The waves emerging from the slits originate from the same wave front and therefore are always in phase.



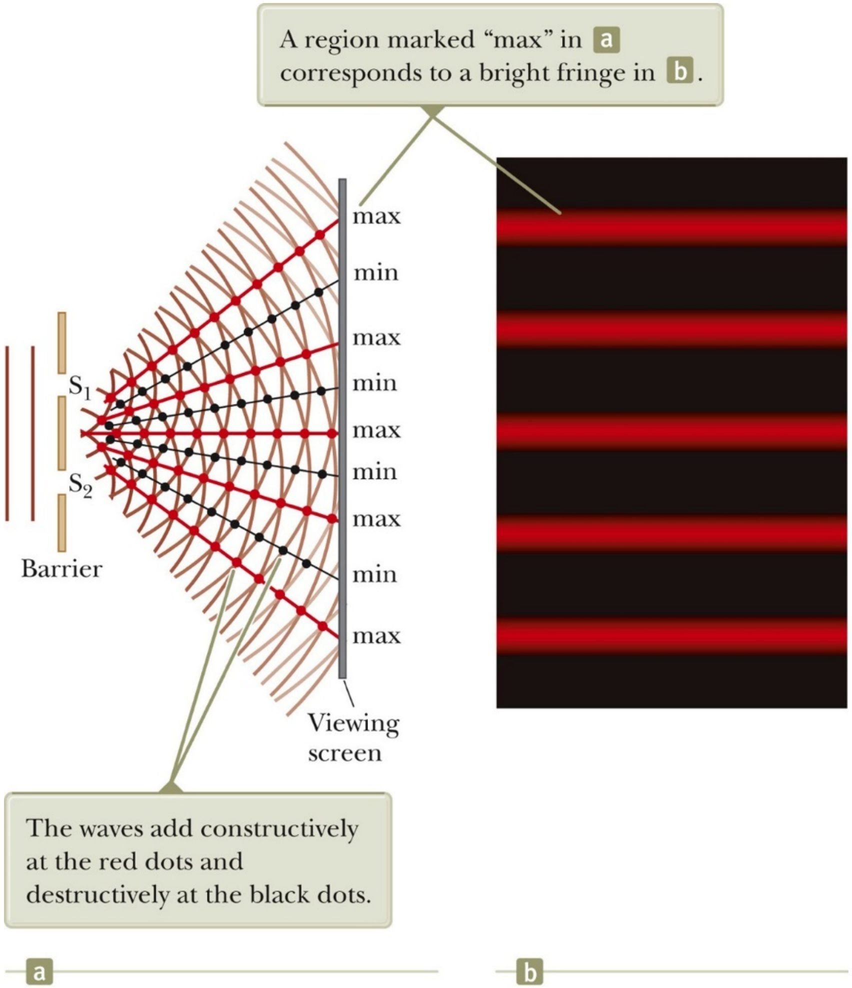
Resulting Interference Pattern

The light from the two slits forms a visible pattern on a screen.

The pattern consists of a series of bright and dark parallel bands called *fringes.*

Constructive interference occurs where a bright fringe occurs.

Destructive interference results in a dark fringe.



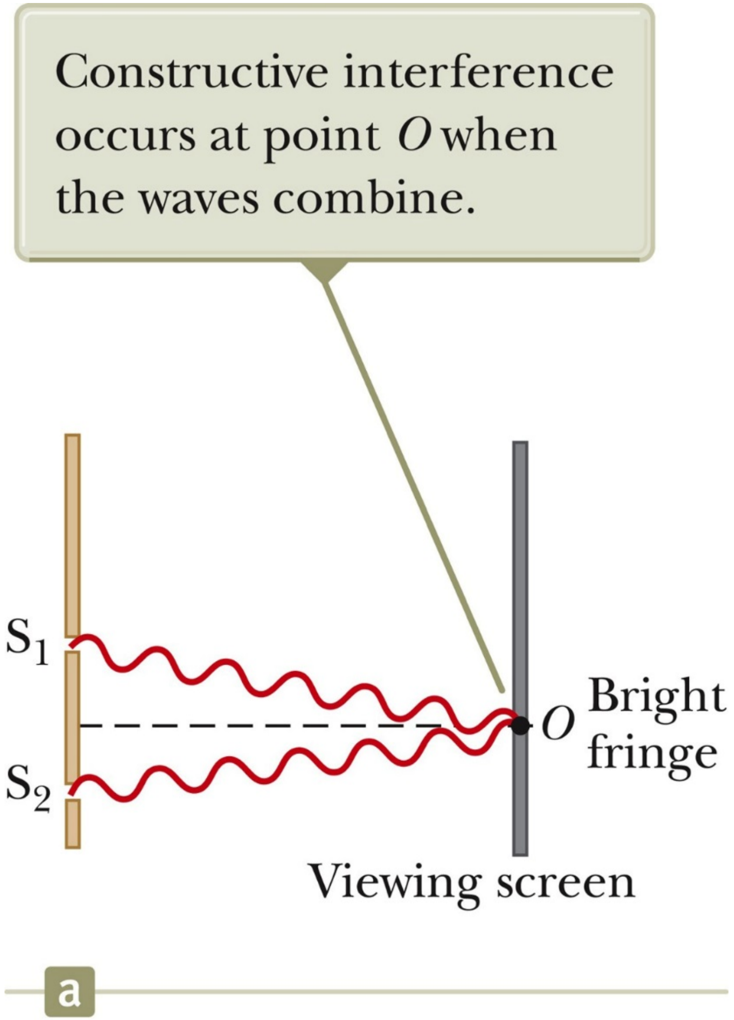
Interference Patterns

Constructive interference occurs at point O.

The two waves travel the same distance.

- Therefore, they arrive in phase

As a result, constructive interference occurs at this point and a bright fringe is observed.



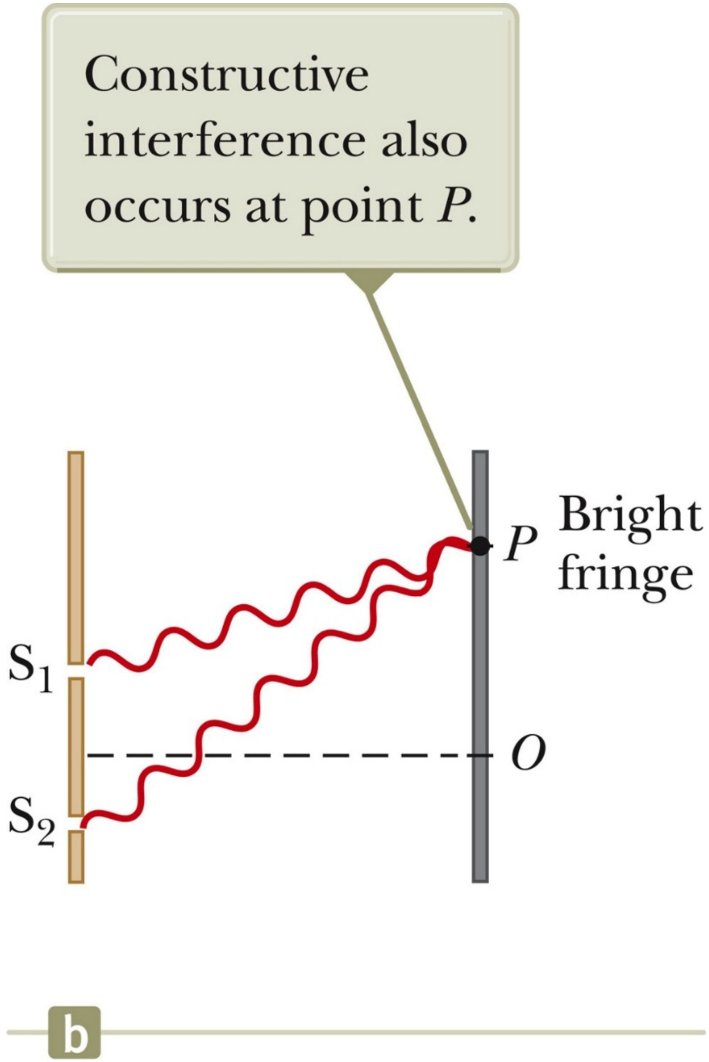
Interference Patterns, 2

The lower wave has to travel farther than the upper wave to reach point *P.*

The lower wave travels one wavelength farther.

- Therefore, the waves arrive in phase

A second bright fringe occurs at this position.



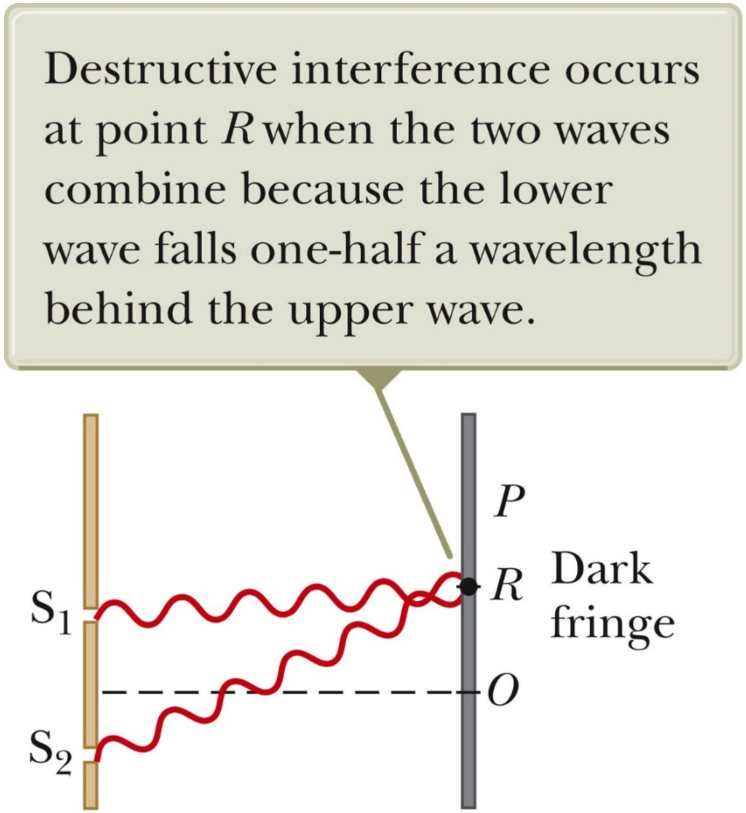
Interference Patterns, 3

The upper wave travels one-half of a wavelength farther than the lower wave to reach point *R.*

The trough of the upper wave overlaps the crest of the lower wave.

This is destructive interference.

- A dark fringe occurs.



Conditions for Interference

To observe interference in light waves, the following two conditions must be met:

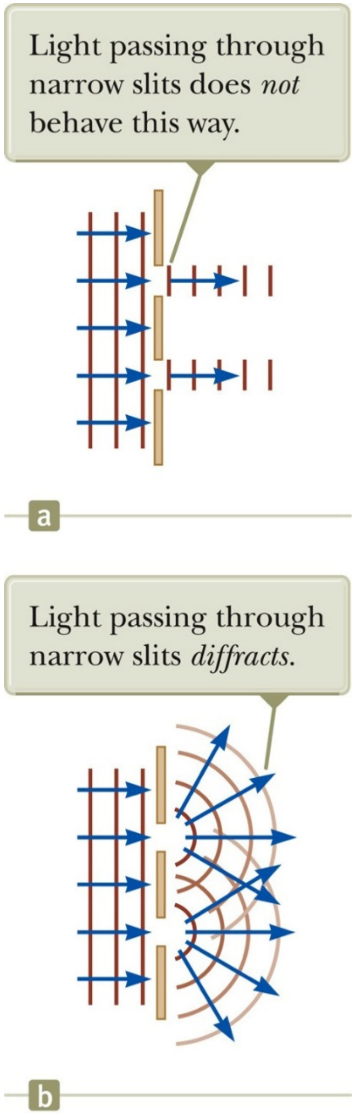
- The sources must be **coherent.**
  - They must maintain a constant phase with respect to each other.
- The sources should be **monochromatic.**
  - Monochromatic means they have a single wavelength.

Diffraction

If the light traveled in a straight line after passing through the slits, no interference pattern would be observed.

From Huygens's principle we know the waves spread out from the slits.

This divergence of light from its initial line of travel is called **diffraction.**

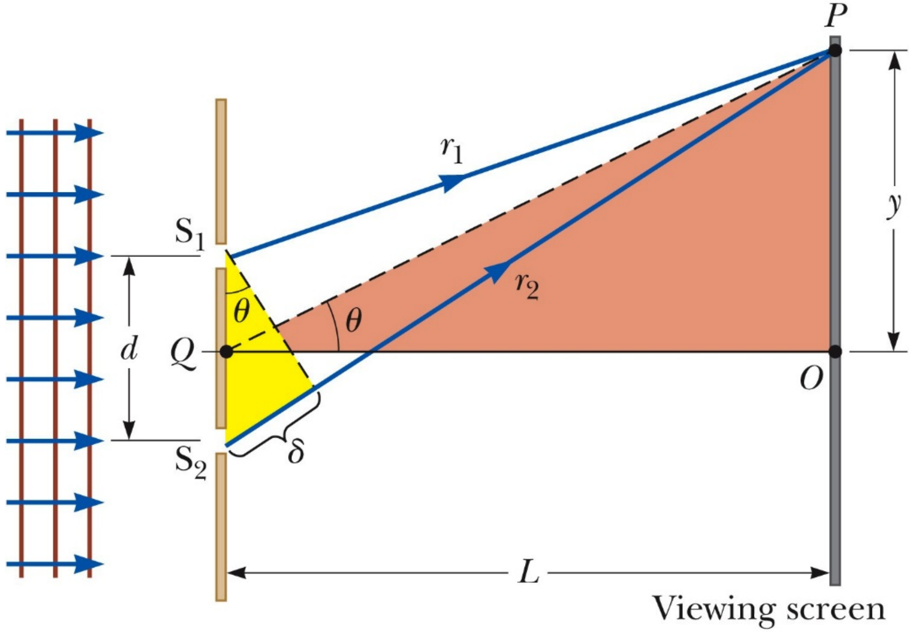


Young's Double-Slit Experiment: Geometry

The path difference, δ, is found from geometry.

$\delta = r_2 - r_1 = d \sin \theta$

- This assumes the paths are parallel.
- Not exactly true, but a very good approximation if *L* is much greater than *d*



Interference Equations

For a bright fringe produced by constructive interference, the path difference must be either zero or some integer multiple of the wavelength.

$\delta = d \sin \theta_{bright} = m\lambda$

- $m = 0, \pm 1, \pm 2, \ldots$
- m is called the order number
  - When m = 0, it is the zeroth-order maximum
  - When m = $\pm$1, it is called the first-order maximum

When destructive interference occurs, a dark fringe is observed.

This needs a path difference of an odd half wavelength.

$\delta = d \sin \theta_{dark} = (m + \tfrac{1}{2})\lambda$

- $m = 0, \pm 1, \pm 2, \ldots$

Interference Equations, cont.

The positions of the fringes can be measured vertically from the zeroth-order maximum.

Using the large triangle in figure,

- $y_{bright} = L \tan \theta_{bright}$
- $y_{dark} = L \tan \theta_{dark}$

Interference Equations, final

Assumptions in a Young's Double Slit Experiment:

- L >> d
- d >> λ

Approximation:

- θ is small and therefore the small angle approximation tan θ ~ sin θ can be used

$y = L \tan\theta \approx L \sin\theta$

For small angles,

$$y_{\text{bright}} = L\frac{m\lambda}{d} \text{ and } y_{\text{dark}} = L\frac{(m+{}^{1}\!/_{2})\lambda}{d}$$

Uses for Young's Double-Slit Experiment

Young's double-slit experiment provides a method for measuring wavelength of the light.

This experiment gave the wave model of light a great deal of credibility.

- It was inconceivable that particles of light could cancel each other in a way that would explain the dark fringes.

Multiple Slits, Graph

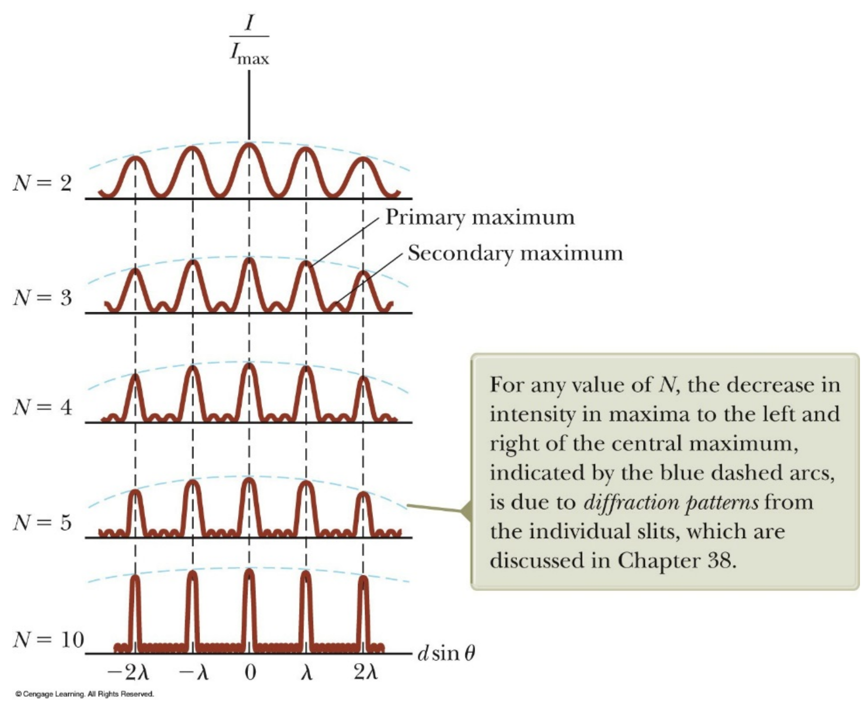
With more than two slits, the pattern contains primary and secondary maxima.

For N slits, the intensity of the primary maxima is N^2 times greater than that due to a single slit.

As the number of slits increases, the primary maxima increase in intensity and become narrower.

- The secondary maxima decrease in intensity relative to the primary maxima.

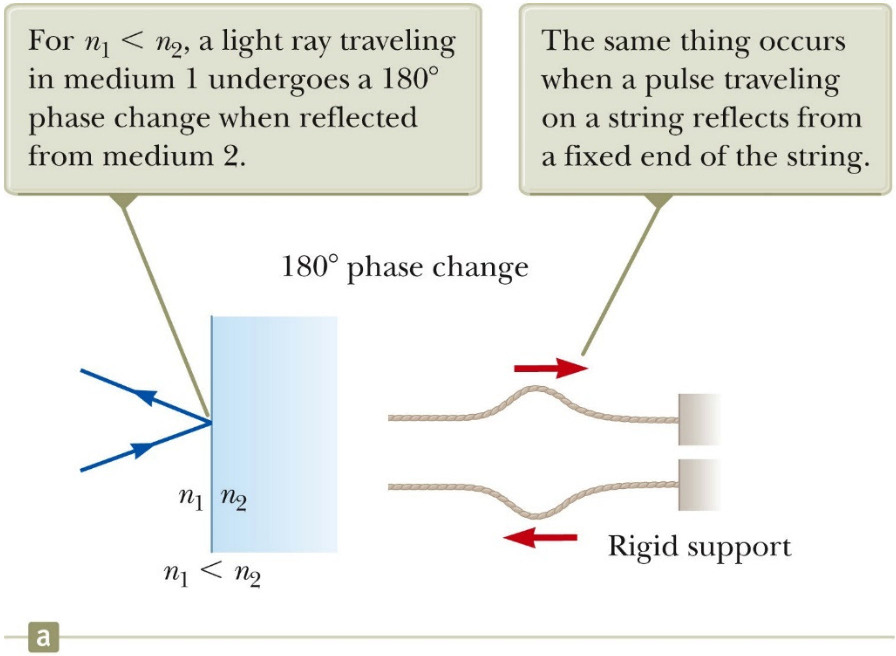
The number of secondary maxima is N – 2, where N is the number of slits.



Phase Changes Due To Reflection

An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling.

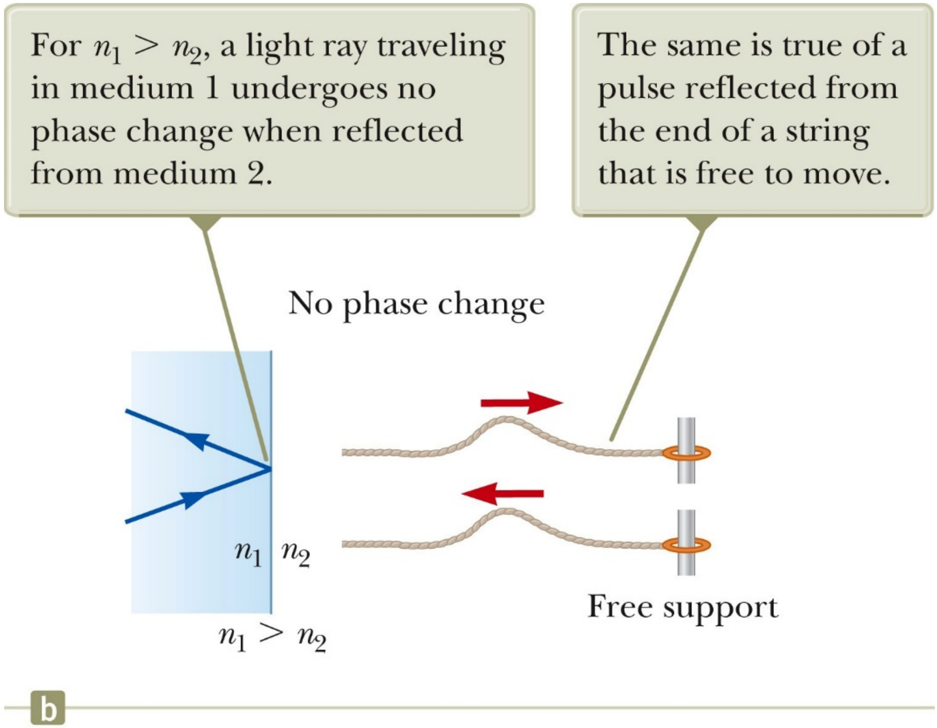
- Analogous to a pulse on a string reflected from a rigid support



Phase Changes Due To Reflection, cont.

There is no phase change when the wave is reflected from a boundary leading to a medium of lower index of refraction.

- Analogous to a pulse on a string reflecting from a free support



Interference in Thin Films

Interference effects are commonly observed in thin films.

- Examples include soap bubbles and oil on water

The various colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film.

Facts to remember:

- An electromagnetic wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change on reflection when $n_2 > n_1$.
  - There is no phase change in the reflected wave if $n_2 < n_1$.
- The wavelength of light λ_n in a medium with index of refraction n is $\lambda_n = \lambda/n$ where λ is the wavelength of light in vacuum.

Interference in Thin Films, 2

Assume the light rays are traveling in air nearly normal to the two surfaces of the film.

Ray 1 undergoes a phase change of 180° with respect to the incident ray.

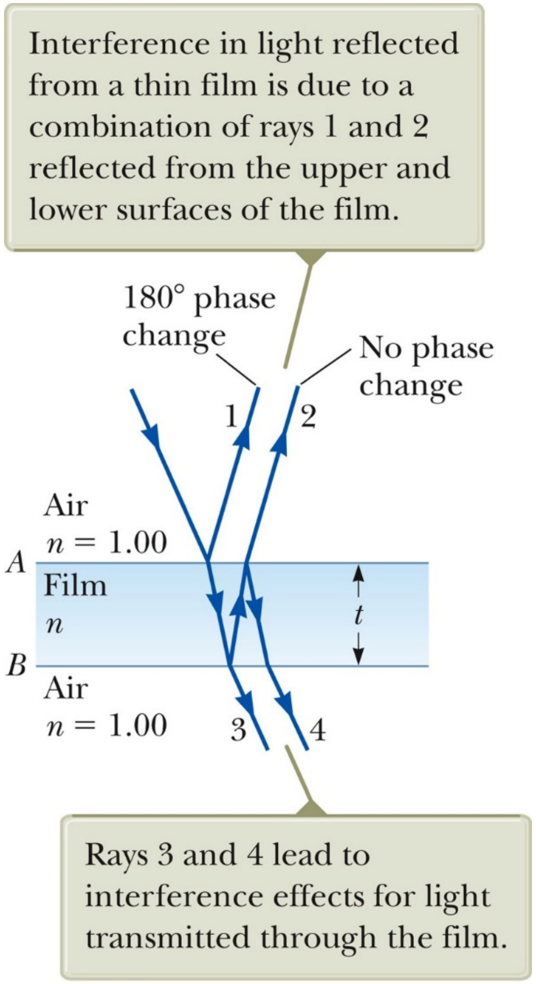
Ray 2, which is reflected from the lower surface, undergoes no phase change with respect to the incident wave. Ray 2 also travels an additional distance of 2t before the waves recombine.

For constructive interference

- $2nt = (m + ½)\lambda$ $(m = 0, 1, 2 \ldots)$
  - This takes into account both the difference in optical path length for the two rays and the 180° phase change.

For destructive interference

- $2nt = m\lambda$ $(m = 0, 1, 2 \ldots)$



Interference in Thin Film, Soap Bubble Example

Newton’s Rings

Another method for viewing interference is to place a plano-convex lens on top of a flat glass surface.

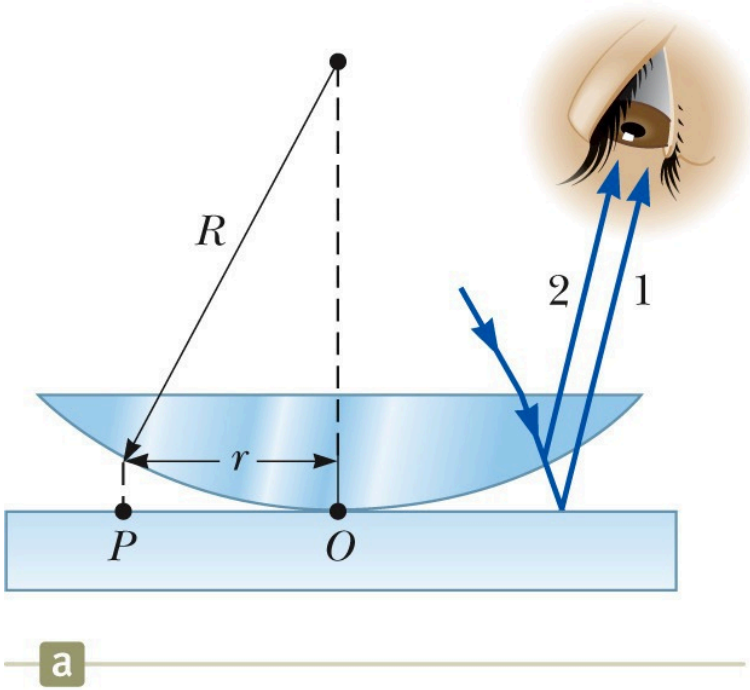
The air film between the glass surfaces varies in thickness from zero at the point of contact to some thickness t.

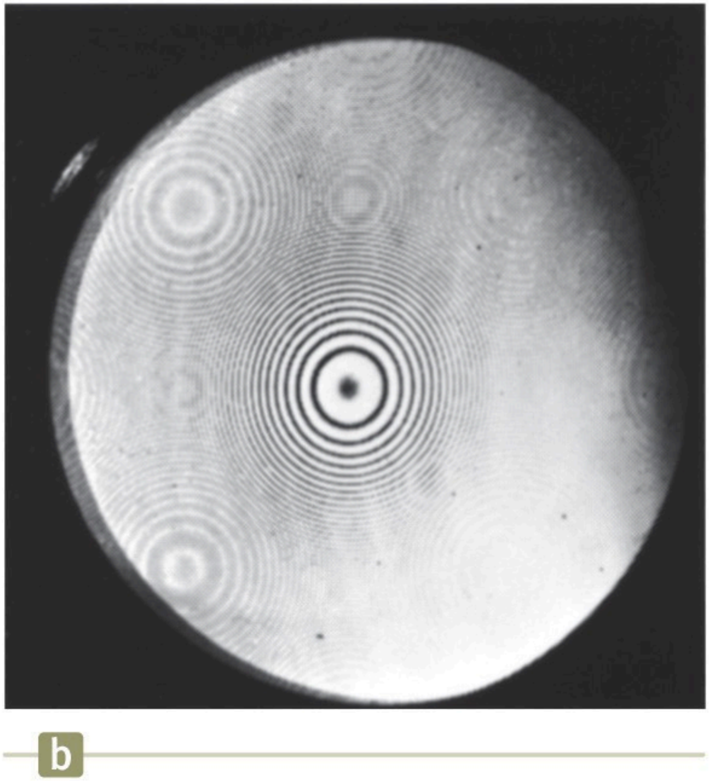
A pattern of light and dark rings is observed.

- These rings are called Newton’s rings.
- The particle model of light could not explain the origin of the rings.

Newton’s rings can be used to test optical lenses.

Newton's Rings, Set-Up and Pattern





Diffraction Pattern

A single slit placed between a distant light source and a screen produces a **diffraction pattern.**

- It will have a broad, intense central band
  - Called the **central maximum**
- The central band will be flanked by a series of narrower, less intense secondary bands.
  - Called **side maxima** or **secondary maxima**
- The central band will also be flanked by a series of dark bands.
  - Called **minima**

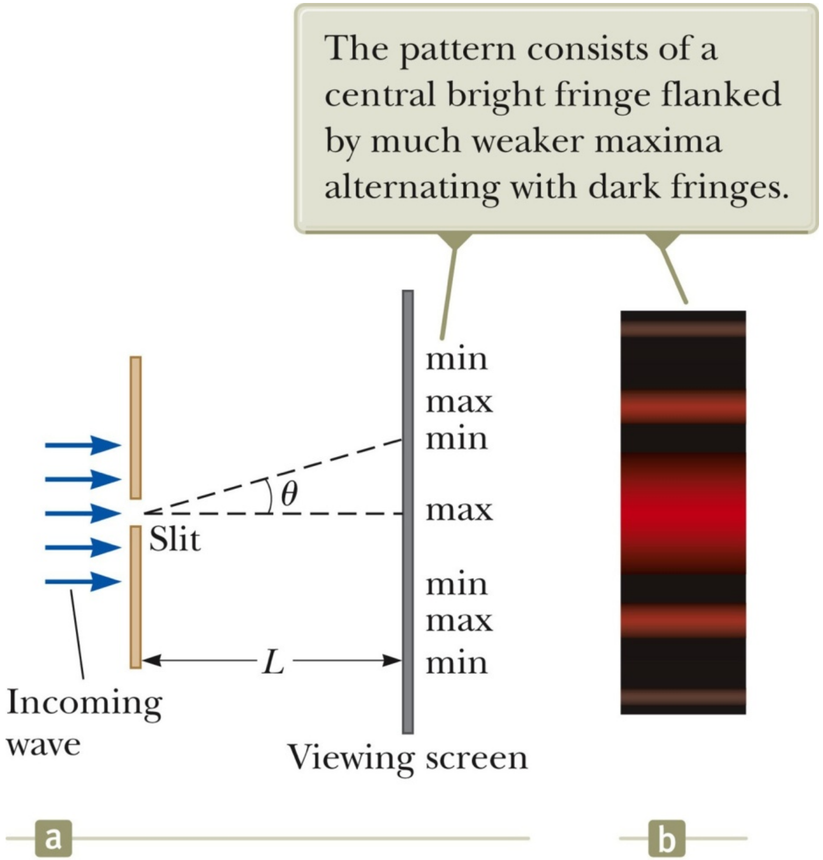
Fraunhofer Diffraction Pattern

A **Fraunhofer diffraction pattern** occurs when the rays leave the diffracting object in parallel directions.

- Screen very far from the slit

A bright fringe is seen along the axis ($\theta = 0$).

Alternating bright and dark fringes are seen on each side.



Single-Slit Diffraction

The finite width of slits is the basis for understanding Fraunhofer diffraction.

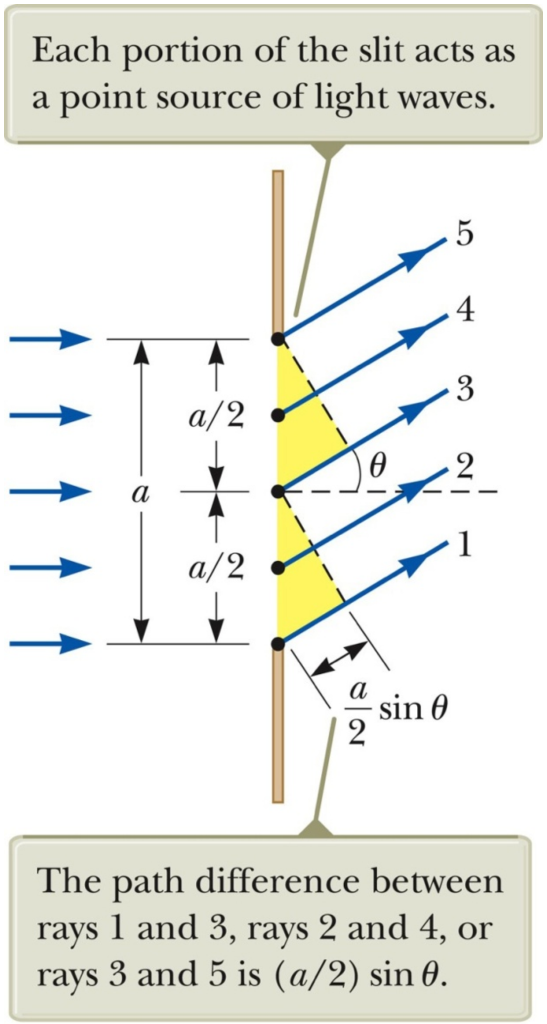
According to Huygens's principle, each portion of the slit acts as a source of light waves.

Therefore, light from one portion of the slit can interfere with light from another portion.

The resultant light intensity on a viewing screen depends on the direction *θ.*

The diffraction pattern is actually an interference pattern.

- The different sources of light are different portions of the single slit.



Single-Slit Diffraction, Analysis

All the waves are in phase as they leave the slit.

Wave 1 travels farther than wave 3 by an amount equal to the path difference.

- $(a/2) \sin \theta$

If this path difference is exactly half of a wavelength, the two waves cancel each other and destructive interference results.

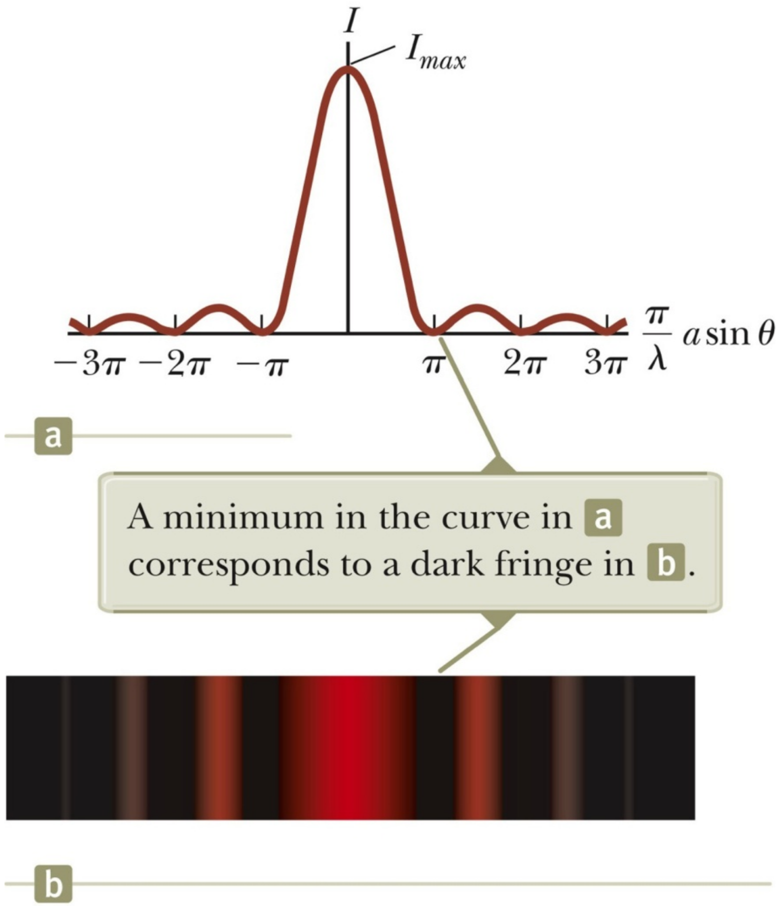
In general, destructive interference occurs for a single slit of width a when $\sin \theta_{dark} = m\lambda / a$.

- $m = \pm 1, \pm 2, \pm 3, \ldots$

Intensity

Most of the light intensity is concentrated in the central maximum.

The graph shows a plot of light intensity vs. (π / λ) a sin θ.

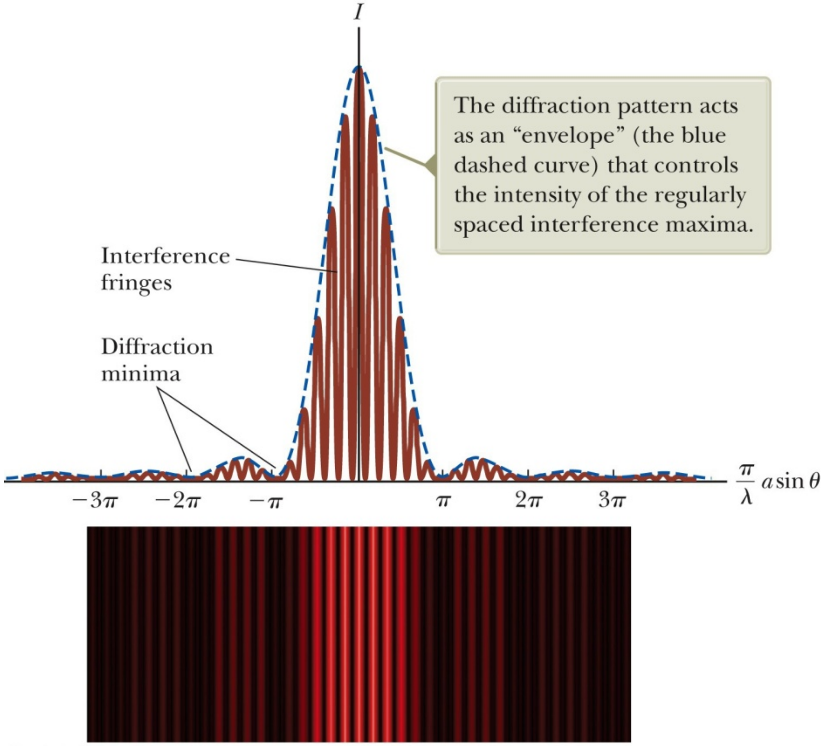


Intensity of Two-Slit Diffraction Patterns

When more than one slit is present, consideration must be made of

- The diffraction patterns due to individual slits
- The interference due to the wave coming from different slits

The single-slit diffraction pattern will act as an "envelope" for a two-slit interference pattern.



Two-Slit Diffraction Patterns, Maxima and Minima

To find which interference maximum coincides with the first diffraction minimum.

$$\frac{d \sin \theta}{a \sin \theta} = \frac{m\lambda}{\lambda} \rightarrow \frac{d}{a} = m$$

- The conditions for the m^{th} interference maximum
  - d sin θ = m λ
- The conditions for the first diffraction minimum
  - a sin θ = λ

Resolution

The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light.

If two sources are far enough apart to keep their central maxima from overlapping, their images can be distinguished.

- The images are said to be resolved.

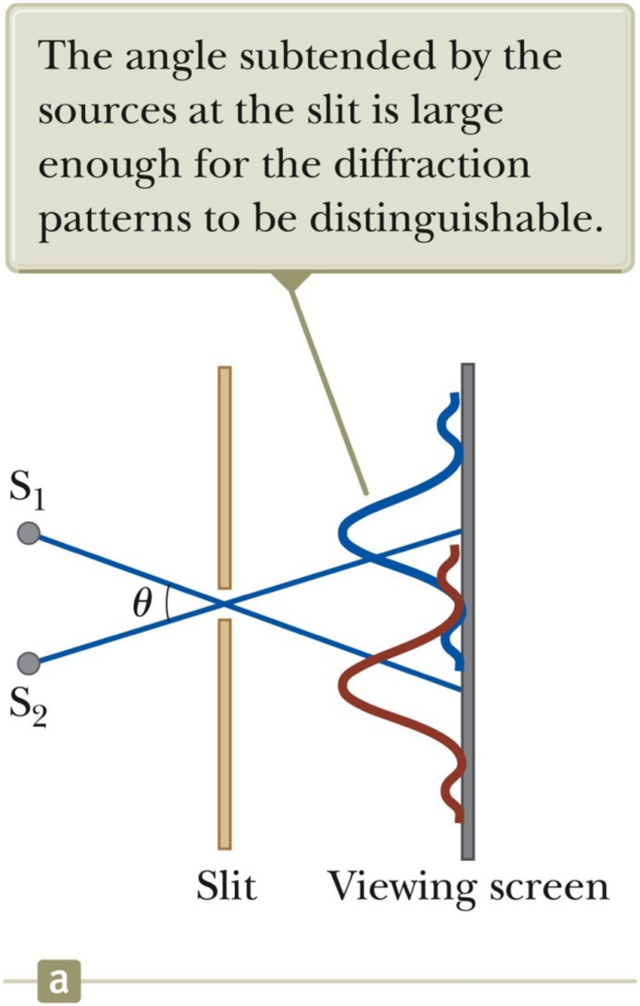
If the two sources are close together, the two central maxima overlap and the images are not resolved.

Resolved Images, Example

The images are far enough apart to keep their central maxima from overlapping.

The angle subtended by the sources at the slit is large enough for the diffraction patterns to be distinguishable.

The images are resolved.

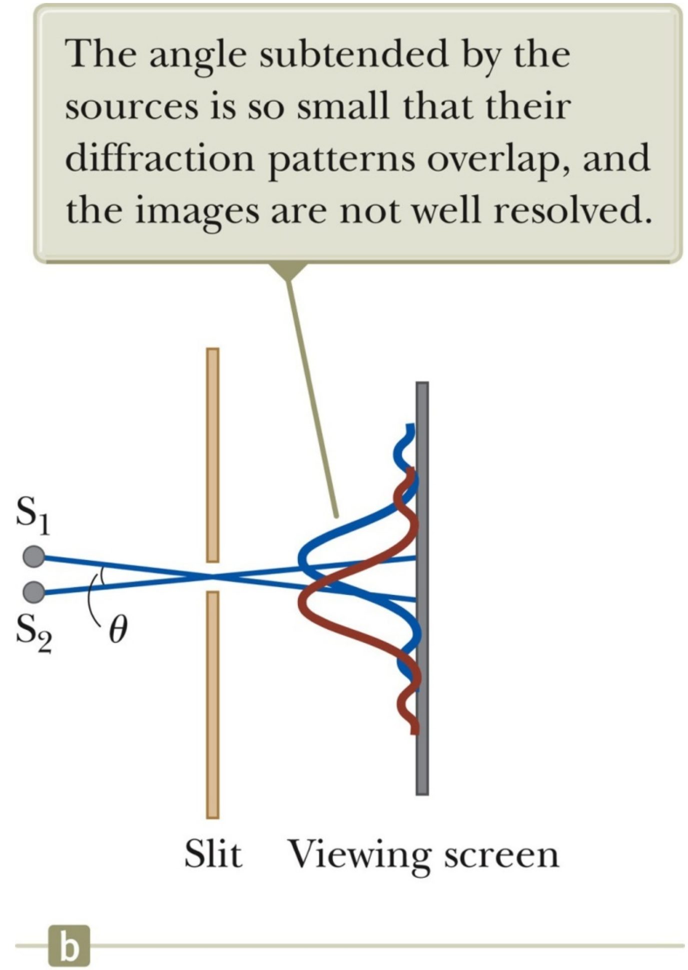


Images Not Resolved, Example

The sources are so close together that their central maxima do overlap.

The angle subtended by the sources is so small that their diffraction patterns overlap.

The images are not resolved.



Resolution, Rayleigh’s Criterion

When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved.

This limiting condition of resolution is called **Rayleigh’s criterion.**

The angle of separation, θ_{min}, is the angle subtended by the sources for which the images are just resolved.

Since $\lambda << a$ in most situations, $\sin\theta$ is very small and $\sin\theta \approx \theta$.

Therefore, the limiting angle (in rad) of resolution for a slit of width a is

$$\theta_{\min} = \frac{\lambda}{a}$$

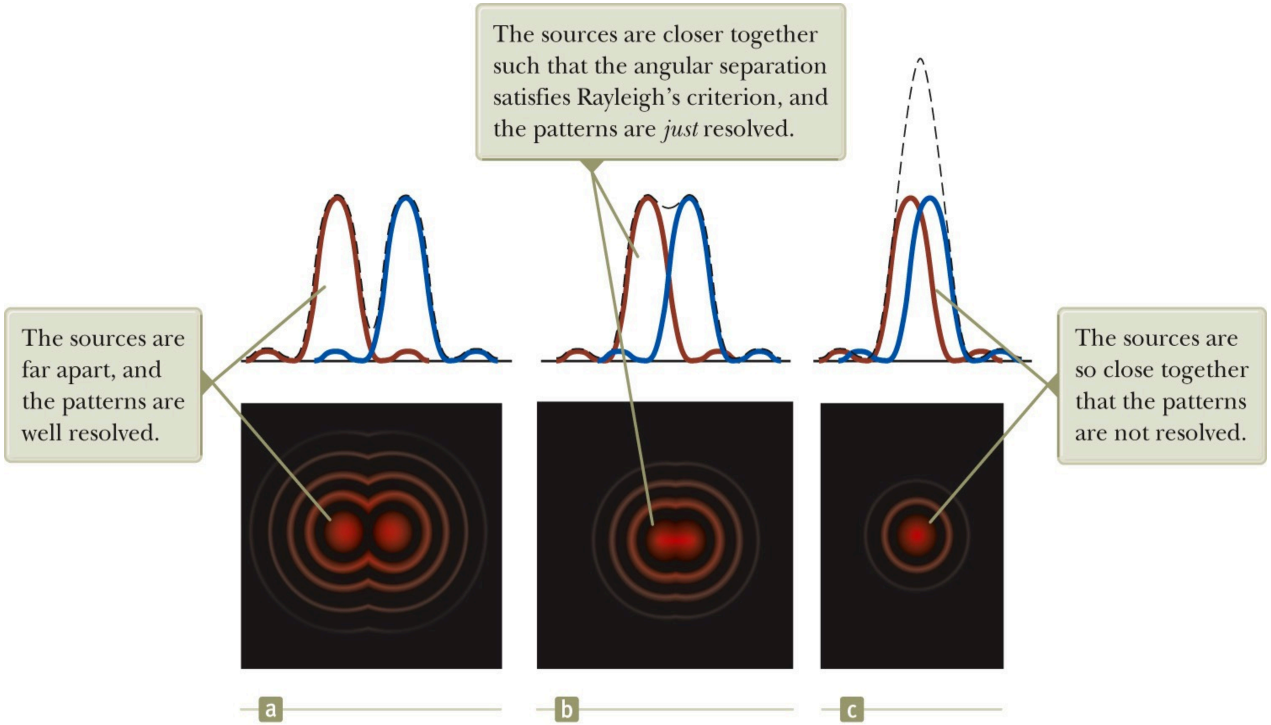
Circular Apertures

The diffraction pattern of a circular aperture consists of a central bright disk surrounded by progressively fainter bright and dark rings.

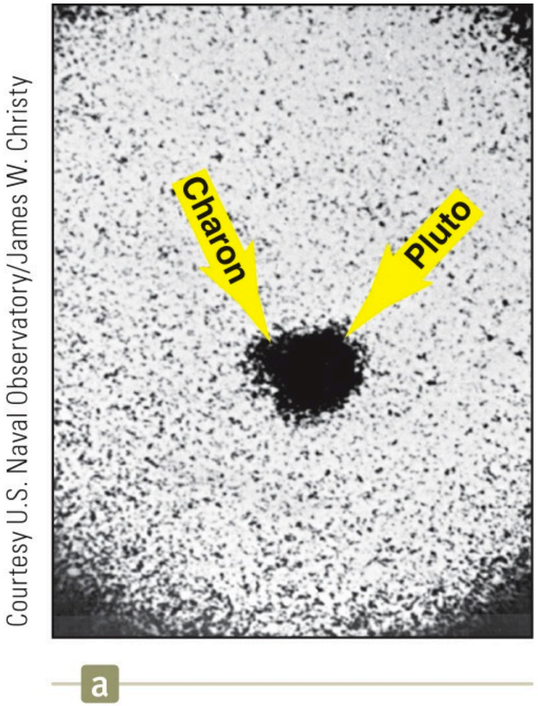
The limiting angle of resolution of the circular aperture is

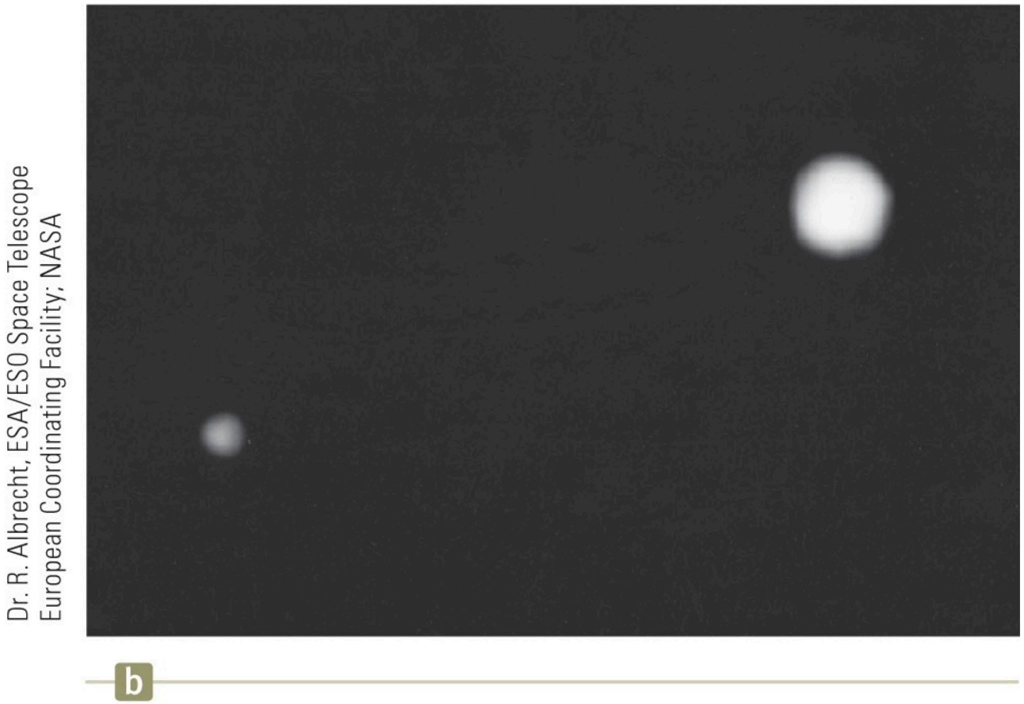
$$\theta_{\min} = 1.22\,\frac{\lambda}{D}$$

- D is the diameter of the aperture.



Resolution, Example





Pluto and its moon, Charon

Left: Earth-based telescope is blurred

Right: Hubble Space Telescope clearly resolves the two objects

Diffraction Grating

The diffracting grating consists of a large number of equally spaced parallel slits.

- A typical grating contains several thousand lines per centimeter.

The intensity of the pattern on the screen is the result of the combined effects of interference and diffraction.

- Each slit produces diffraction, and the diffracted beams interfere with one another to form the final pattern.

Diffraction Grating, Types

A *transmission* grating can be made by cutting parallel grooves on a glass plate.

- The spaces between the grooves are transparent to the light and so act as separate slits.

A *reflection* grating can be made by cutting parallel grooves on the surface of a reflective material.

- The spaces between the grooves act as parallel sources of reflected light, like the slits in a transmission grating.

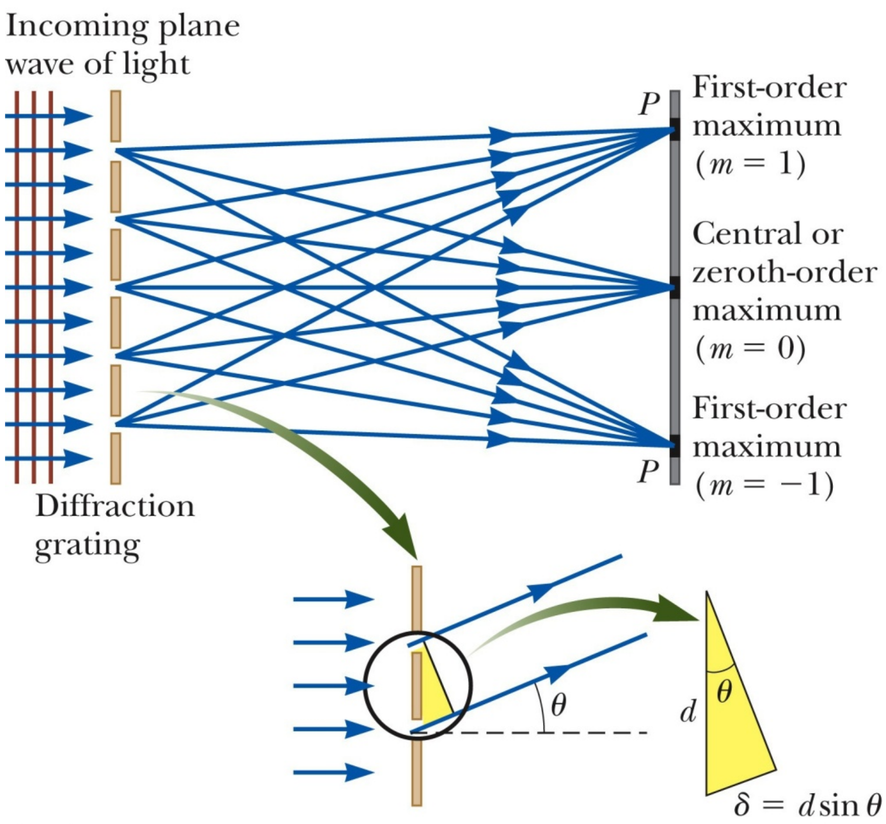
Diffraction Grating, cont.

The condition for *maxima* is

- $d \sin \theta_{bright} = m\lambda$
  - $m = 0, \pm 1, \pm 2, \ldots$

The integer m is the *order number* of the diffraction pattern.

If the incident radiation contains several wavelengths, each wavelength deviates through a specific angle.



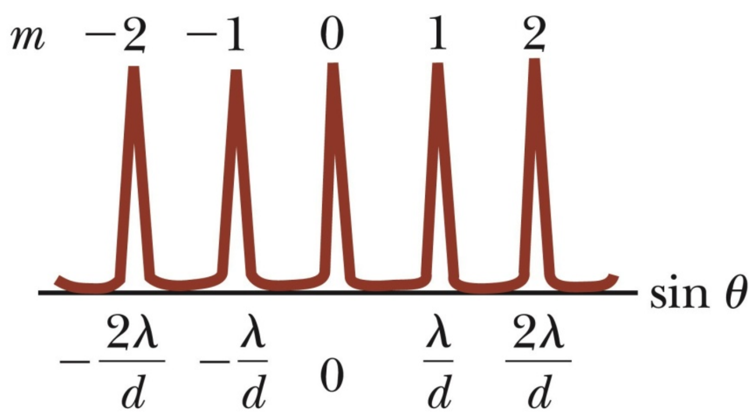
Diffraction Grating, Intensity

All the wavelengths are seen at $m = 0$.

- This is called the zeroth-order maximum.

The first-order maximum corresponds to $m = 1$.

Note the sharpness of the principle maxima and the broad range of the dark areas.



Characteristics of the intensity pattern:

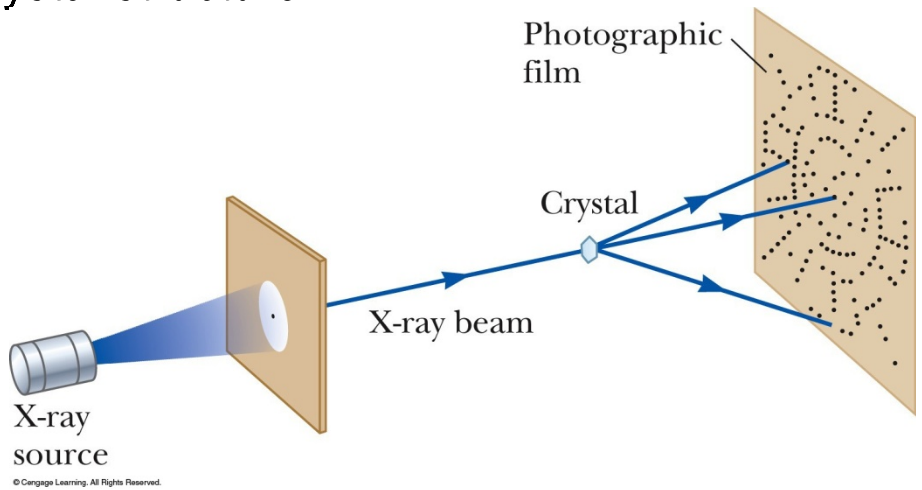
- The sharp peaks are in contrast to the broad, bright fringes characteristic of the two-slit interference pattern.
- Because the principle maxima are so sharp, they are much brighter than two-slit interference patterns.

Diffraction of X-Rays by Crystals

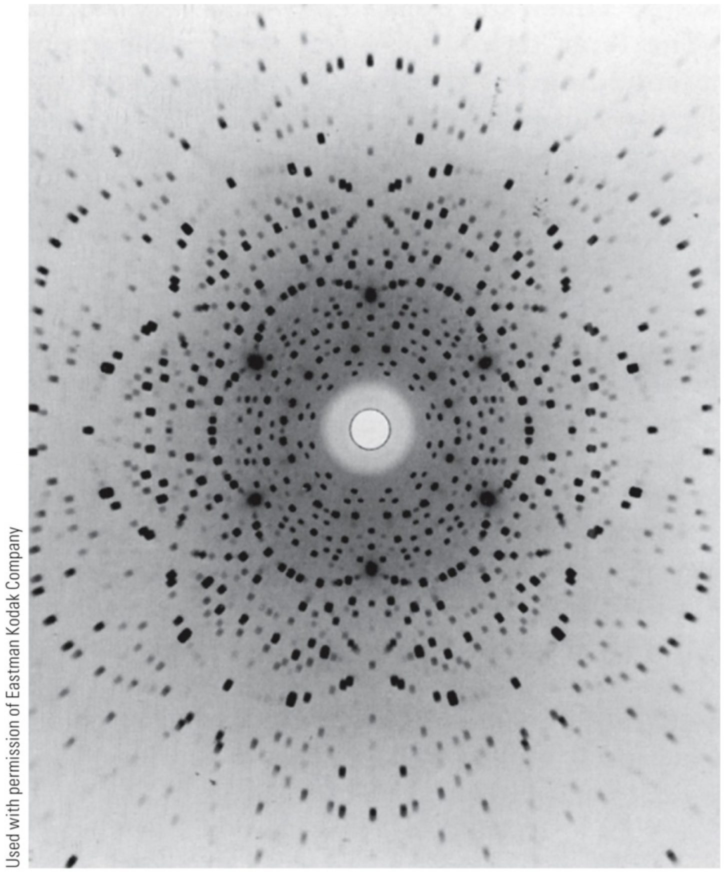
X-rays are electromagnetic waves of very short wavelength.

Max von Laue suggested that the regular array of atoms in a crystal could act as a three-dimensional diffraction grating for x-rays.

The diffraction patterns from crystals are complex because of the three-dimensional nature of the crystal structure.



Laue Pattern for Beryl



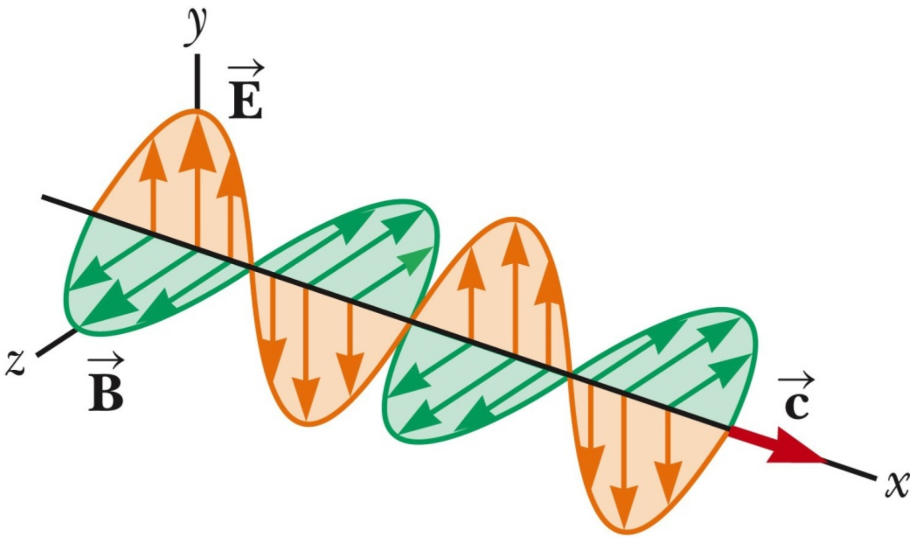
Polarization of Light Waves

The *direction of polarization* of each individual wave is defined to be the direction in which the electric field is vibrating.

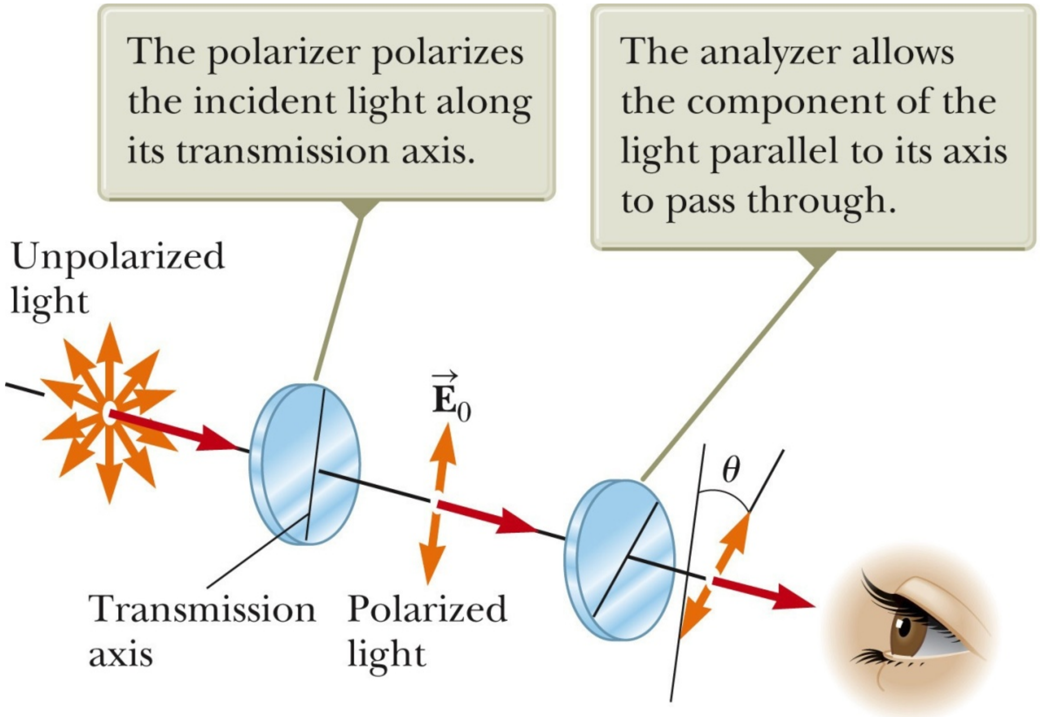
In this example, the direction of polarization is along the *y*-axis.

All individual electromagnetic waves traveling in the *x* direction have an electric field vector parallel to the *yz* plane.

This vector could be at any possible angle with respect to the *y* axis.



Polarization by Selective Absorption



The most common technique for polarizing light.

Uses a material that transmits waves whose electric field vectors lie in the plane parallel to a certain direction and absorbs waves whose electric field vectors are in all other directions.

Selective Absorption, cont.

E. H. Land discovered a material that polarizes light through selective absorption.

- He called the material *Polaroid.*
- The molecules readily absorb light whose electric field vector is parallel to their lengths and allow light through whose electric field vector is perpendicular to their lengths.

It is common to refer to the direction perpendicular to the molecular chains as the *transmission axis.*

In an *ideal* polarizer,

- All light with the electric field parallel to the transmission axis is transmitted.
- All light with the electric field perpendicular to the transmission axis is absorbed.

Intensity of a Polarized Beam

The intensity of the polarized beam transmitted through the second polarizing sheet (the analyzer) varies as

- $I = I_{max} \cos^2 \theta$
  - I_{max} is the intensity of the polarized wave incident on the analyzer.
  - This is known as **Malus' law** and applies to any two polarizing materials whose transmission axes are at an angle of θ to each other.

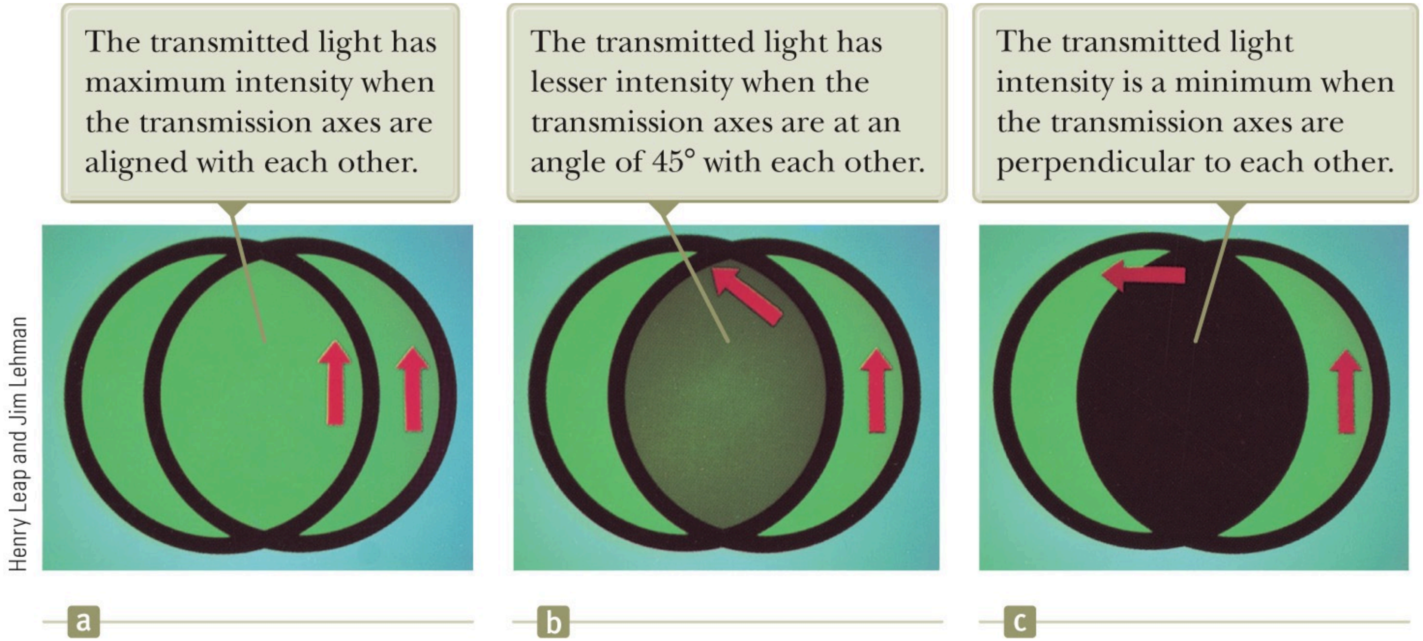
The intensity of the transmitted beam is a maximum when the transmission axes are parallel.

- $\theta = 0$ or 180^{o}

The intensity is zero when the transmission axes are perpendicular to each other.

- This would cause complete absorption.

Intensity of Polarized Light, Examples



On the left, the transmission axes are aligned and maximum intensity occurs.

In the middle, the axes are at 45^{o} to each other and less intensity occurs.

On the right, the transmission axes are perpendicular and the light intensity is a minimum.