Algebra – Practical session 3

3.1. (a) Convert the number $(110210)_3$ (written here in base 3) to decimal notation.
(b) Convert the integer 13539 (written here in decimal notation) to base 7.

3.2. Find a fraction m/n, with m and n integers, such that

$$\frac{m}{n} = 0.6\overline{351} = 0.6\dot{3}5\dot{1}$$

Simplify the resulting fraction using the Euclidean algorithm. (You may use a pocket calculator for this part.)

3.3. Recognise that the following is an infinite geometric series (with complex terms) and compute its sum:

$$1 + \frac{i}{2} - \frac{1}{4} - \frac{i}{8} + \frac{1}{16} + \frac{i}{32} - \frac{1}{64} - \frac{i}{128} + \cdots$$

Draw a few partial sums in the complex plane. (That means mark the sums of the first $1, 2, 3, \ldots$ terms as points in the complex plane. If you join each partial sum to the next one by a vector, those vectors will be the individual terms that you add to make up the sum.)

3.4. Convert the number $(21.201)_3$, written here in base 3, to decimal notation.

3.5. Convert the decimal number 8.57 to base 6, correct to 5 digits after the point.

3.6. (a) Express the periodic binary number $(1.\dot{0}01\dot{1})_2$ as a fraction of integers (written as decimals).
Hint: You may adapt to binary the rule used to convert periodic decimal numbers to fractions.
(b) Write 6/5 in binary (as a periodic binary number).
Hint: One way is converting 6 and 5 to binary, and then doing long division working in binary. Another way is expressing the fraction as a decimal first, and then converting that to binary.

3.7*. Write the number π in base 3, with 10 digits after the point. Use a pocket calculator and start with typing in the approximation 3.1415926 for π.

3.8*. The positive numbers $a_1, a_2, \ldots, a_{12}$ form a geometric progression, with $a_1 = \sqrt{2}/2$ and $a_3 = \sqrt{2}$. Compute $a_1 + a_2 + a_3 + \cdots + a_{12}$, expressing the result in the simplest possible way. (Also, use the theory, do not just add together all the terms of the sum.)