MECHANICS PRACTICAL 4

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> Reminder: For this practical, we recommend to use the step-by-step strategy seen during the lectures whenever required *i.e.* (1) diagram, (2) list and intel on the forces acting on the system, (3) Newton's 2nd law, (4) components and (5) solve to get the desired answer. We will use $g = 9.8\,\mathrm{m}\cdot\mathrm{s}^{-2}$ for the downward acceleration on Earth.

1. Ideal projectile motion

A particle is shot with an initial velocity $\vec{v}_0 = (130\,\mathrm{m}\cdot\mathrm{s}^{-1})\,\hat{\imath} + (75\,\mathrm{m}\cdot\mathrm{s}^{-1})\,\hat{\jmath}$ from the origin of a galilean frame $(O, \hat{\imath}, \hat{\jmath})$ on Earth.

(a) Determine the maximum altitude reached by the projectile during its travel.

(b) What is the total distance travelled by the projectile in the x-direction when it has reached the ground?

2. Block on an inclined plane and solid friction

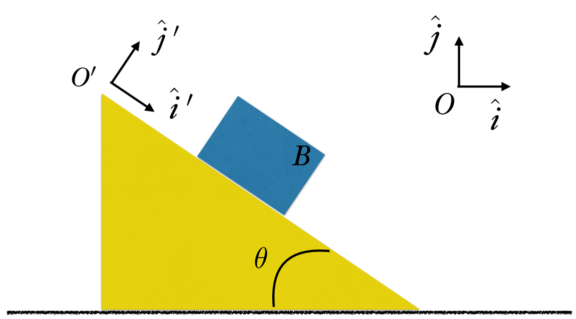

Figure 1. A block on an inclined plane.

A 1 kilogram block B is initially at rest on an inclined plane with an adjustable angle θ with the horizontal (see Fig. 1). The two frames $(O, \hat{\imath}, \hat{\jmath})$ and $(O', \hat{\imath}\,', \hat{\jmath}\,')$ are considered galilean.

(a) For small angle values, the block is subject to a solid static friction force which prevents it from moving down the plane. It is noticed that as long as $\theta \leq 30\,^{\circ}$, the block doesn't move. Use this information to determine the value of the static friction coefficient μ_s between the plane and the block.

(b) We now consider that the inclined plane has an angle $\theta = 45\,^{\circ}$ with the horizontal. The block slides down the incline because of its weight but is subject to a kinetic static friction force with $\mu_k = 0.5$. Determine the acceleration vector of the block in the basis $(\hat{\imath}', \hat{\jmath}')$.

(c) Determine now the acceleration vector of the block in the basis $(\hat{\imath}', \hat{\jmath}')$ in absence of solid friction.

(d) Express the found acceleration in question (b) in the basis $(\hat{\imath}, \hat{\jmath})$.

3. A sky-diving ant

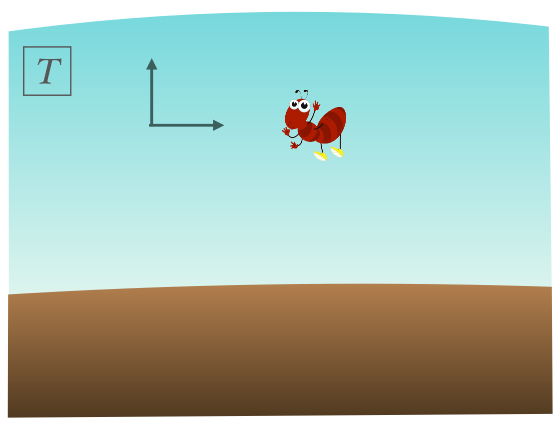

FIGURE 2. A sky diving ant.

A $m = 1$ mg ant is falling vertically through air while being subject to its own weight and a linear fluid friction force due to air resistance with drag coefficient γ.

(a) Show that the y-component of the velocity vector of the ant satisfies the following first order ordinary differential:

$$\frac{dv_y(t)}{dt} = -\frac{\gamma}{m} v_y(t) - g$$

(b) We seek a solution to the equation of motion of the form $v_y(t) = v_\infty \left(1 - e^{-t/\tau}\right)$. Show that such a functional form satisfies the above differential equation if $v_\infty = -\frac{mg}{\gamma}$ and $\tau = \frac{m}{\gamma}$

(c) Calculate the value of the velocity as time tends to infinity knowing that $\gamma = 7.5 \cdot 10^{-7}\ \mathrm{kg} \cdot \mathrm{s}^{-1}$.

4. Figuring the dimension of an unknown quantity

Reminder: Here are the rules we have established so far:

⋆ General rules:

- $[A^\alpha \times B^\beta] = [A]^\alpha \times [B]^\beta$ for any A and B
- $[A + B] = [A] = [B]$ if $[A] = [B]$

⋆ Specific rules:

- $[x] = [y] = [u] = L$ if u = inches, cm, m, yards etc...
- $[v_x] = [v_y] = L \times T^{-1}$
- $[\theta] = [u_a] = 1$ if θ is an angle and u_a = degrees, radians, seconds of arc etc..
- $[t] = [u_t] = T$ if u_t = seconds, hours, days, years etc...
- $[m] = [u_m] = M$ if u_m = kg, tonnes, onces, etc...
- $[F] = [N] = M \times L \times T^{-2}$
- $[n] = 1$ where $n \in \mathbb{R}$ like -1, 3.45674 or 5 for example.

Consider the following equations to be **complete**.

(a) Given $F_{drag} = -\gamma\, v_x$, determine the dimension of γ.

(b) Given $|F_{friction}| = \mu_k |F_{normal}|$, determine the dimension of μ_k.

(c) Given $F_{spring} = -k\, x$, determine the dimension of k.