MTH1002 Calculus Coursework

Show all your working   ***A table of values of the trig functions is overleaf***

1. Determine whether the following functions are odd, even or neither:

   (a) $f(x) = \sin(3x^4 - x^2 + 2)$

   [4 marks]

   (b) $g(v) = \dfrac{v^5 - v^3 + 1}{v^4 + v^2 + v}$

   [4 marks]

2. Using only the definitions $\sinh x = (e^x - e^{-x})/2$ and $\cosh x = (e^x + e^{-x})/2$ and no other identities, derive the following:

   (a) $2 \sinh x \cosh x = \sinh 2x$

   [4 marks]

   (b) $\cosh x \cosh y + \sinh x \sinh y = \cosh(x + y)$

   [8 marks]

3. For the pair of complex numbers $z_1 = 3 + i$ and $z_2 = 7 - i$, calculate

   (a) $z_1 z_2$

   [4 marks]

   (b) $|z_1|/|z_2|$

   [4 marks]

   (c) z_1/z_2

   [6 marks]

   (d) $\overline{z_2}/\overline{z_1}$

   [6 marks]

4. Plot each of the following complex numbers on an Argand diagram, and express each number in polar form:

   (a) $2\sqrt{2} - 2\sqrt{2}i$

   [6 marks]

   (b) $-2\sqrt{3} + 2i$

   [6 marks]

5. Find all the roots of the following equations, writing your results in exponential form:

   (a) $z^3 = 4(-1 + \sqrt{3}i)$

   [8 marks]

   (b) $z^4 = \dfrac{1}{i}$

   [8 marks]

6. Evaluate the following limits:

   (a) $\displaystyle \lim_{x \to 3} \frac{x - 3}{x^2 + x - 12}$

   [6 marks]

   (b) $\displaystyle \lim_{v \to \infty} \frac{1 - v^2 + 3v^3 - 4v^4}{9v^4 + 2v^3 - 5}$

   [8 marks]

   (c) $\displaystyle \lim_{y \to 0} \frac{e^{ay} - 1 - ay}{y^2}$ where $a \neq 0$ is a constant

   [8 marks]

   (d) $\displaystyle \lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x - 1} \right)$

   [10 marks]

	0	**π/6**	**π/4**	**π/3**	**π/2**
sin	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	*