Algebra – Practical session 4

Question: *Why do mathematicians mix up Halloween and Christmas?*
Answer: *Because* $31OCT = 25DEC$.

4.1. Compute the value of the constant term of the polynomial

$$(2x+1)(3x^2+2)(4x^3+3)(5x^4+4)(6x^5+5).$$

Also, compute the degree of this polynomial. Please justify your answers (that is, explain how you got them).

4.2. Compute the quotient and the remainder of dividing $2x^5+x^3-5x^2+2$ by $2x^2-1$.

Note: It is a good idea to always check your result of a polynomial division by multiplying the second polynomial by the quotient you found, and then adding the remainder you found.

4.3. Find the degree and compute the leading coefficient of the polynomial

$$(3x^2+x-1)(2x^3-2x^2-x+2)-(x^3+2x^2-4x-11)(6x^2-x+9).$$

Note: You need to compute only certain terms of the resulting polynomial to answer the question.

4.4. Use Ruffini's rule to check that the polynomial $4x^4-8x^3+6x^2+\dfrac{x}{2}-\dfrac{15}{2}$ is a multiple of $x-\dfrac{3}{2}$, and to find their quotient.

4.5. Write the expression $\dfrac{(x+3)(x^2-x-1)}{x^2+3x+4}$ in an equivalent form $f(x)+\dfrac{g(x)}{h(x)}$, where $f(x)$, $g(x)$, $h(x)$ are polynomials with $\deg(g)<\deg(h)$.

4.6. Use Ruffini's rule repeatedly to expand the polynomial $x^3-2x^2-30x-40$ in terms of powers of $x+3$. (That is, write the polynomial in the form $\cdots+c_2(x+3)^2+c_1(x+3)+c_0$, computing those coefficients.)

Note: Use Ruffini's rule as learnt in the lectures, not other means.

4.7. Factorise the following expressions as far as you can over $\mathbb{R}$, that is, using only real numbers:

$$a^2+b^2-c^2-2ab$$
$$8x^3+b^6$$
$$x^8-a^8$$

4.8. Use Ruffini's rule to check that $ix^4+(i-1)x^3-(4+i)x^2-(2+3i)x-6+3i$ is a multiple of $x+2-i$, and find their quotient.