MECHANICS PRACTICAL 3

FABIEN PAILLUSSON

Reminder: For this practical, we recommend to use the step-by-step strategy seen during the lectures whenever required *i.e.* (1) draw a diagram of forces on the system considered, (2) list all forces acting on the system and give their expression when they are known, (3) invoke Newton's 2nd law, (4) get the components and (5) solve to get the desired answer. In this practical we consider friction forces to be negligible and we will use $g = 9.8\,\mathrm{m \cdot s^{-2}}$ for the downward acceleration on Earth.

1. Uniformly accelerated motion in 2D

We consider a point object with constant acceleration vector $\vec{a}(t) = a_x\,\hat{i} + a_y\,\hat{j}$ in a frame $(O,\,\hat{i},\,\hat{j})$ and initial velocity and position vectors $\vec{v}(0) = v_{x,0}\,\hat{i} + v_{y,0}\,\hat{j}$ and $\vec{r}(0) = x_0\,\hat{i} + y_0\,\hat{j}$. By using the method of integration, determine:

(a) The general expression of the velocity vector $\vec{v}(t)$.

(b) The general expression of the position vector $\vec{r}(t)$.

2. Projections and components

A point object M has a velocity vector $\vec{v}$ expressed as $\vec{v} = (\sqrt{3}\,\mathrm{m \cdot s^{-1}})\,\hat{i} + (-1\,\mathrm{m \cdot s^{-1}})\,\hat{j}$ in an orthonormal frame $R = (O,\,\hat{i},\,\hat{j})$. We now wish to consider a possible simpler orthonormal frame $R' = (O,\,\hat{i}',\,\hat{j}')$ tilted by an angle $\alpha = 30^{\,o}$ with respect to R such that $\hat{i}' = \cos\alpha\,\hat{i} - \sin\alpha\,\hat{j}$ and $\hat{j}' = \sin\alpha\,\hat{i} + \cos\alpha\,\hat{j}$.

(a) Determine the $\hat{i}'$ component of $\vec{v}$.

(b) Determine the $\hat{j}'$ component of $\vec{v}$.

(c) Conclude on whether or not the tilted frame R' is more adequate to represent $\vec{v}$.

3. Relativity of motion

Given a basis $(\hat{\imath}, \hat{\jmath})$, Bob, a runner, moves at velocity $\vec{v}(B|M) = (3\,\mathrm{m}\cdot\mathrm{s}^{-1})\,\hat{\imath}$ relative to Matt who sitting on a bench. In the mean time, Tim, a little boy, is playing on a merry-go-round and has a velocity $\vec{v}(T|M) = -(12\,\mathrm{m}\cdot\mathrm{s}^{-1})\sin[(6\,\mathrm{s}^{-1})t]\;\hat{\imath} + (12\,\mathrm{m}\cdot\mathrm{s}^{-1})\cos[(6\,\mathrm{s}^{-1})t]\;\hat{\jmath}$ relative to Matt.

(a) By using the law of composition of velocities, determine the relative velocities of Matt and Tim relative to Bob.

(b) By using the law of composition of velocities, determine the relative velocities of Matt and Bob relative to Tim.

4. Pushing two blocks

Two blocks B_1 and B_2 with respective masses $m_1 = 5$ kg and $m_2 = 20$ kg are being pushed by a constant force $\vec{F} = F$ applied $\hat{\imath}$ on B_1 (see Fig. 1). The two blocks move with the same acceleration $\vec{a} = 1\ \mathrm{m}\cdot\mathrm{s}^{-2}\,\hat{\imath}$. as seen from a galilean frame $(O, \hat{\imath}, \hat{\jmath})$.

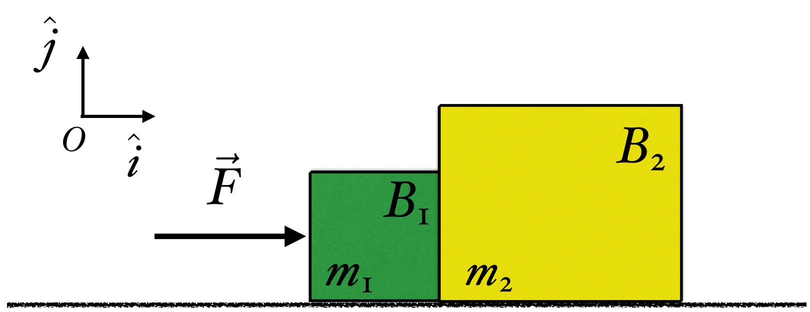

Figure 1. Two blocks being pushed.

(a) By considering the two blocks as a whole, determine the force $\vec{F}$ applied on B_1.

(b) By focusing on B_1 now, determine the force $\vec{F}_{B_2 \to B_1}$ exerted by B_2 on B_1.

(c) Determine the force $\vec{F}_{B_1 \to B_2}$ by the method of your choice.

5. Squatting on a weighing scale

In a galilean frame $(O, \hat{\imath}, \hat{\jmath})$ a man of mass $m = 70\,\mathrm{kg}$ is standing on a weighing scale. In what follows we assume that the reading returned by the scale is the magnitude of the force $\vec{F}_{man \to scale}$ divided by the downward acceleration on Earth g.

(a) If the man is standing still, determine what should be the reading on the scale.

(b) At $t = 0$, the man starts to squat whilst still on the scale. During this movement, its centre of mass is subject to an acceleration $\vec{a} = -2\,\mathrm{m} \cdot \mathrm{s}^{-2}\,\hat{\jmath}$. Determine the reading on the scale during this squatting movement.

6. The return of the dimensions

Using the bracket notation [.] give the dimension of either side of the following equations and conclude on whether or not they correspond to **complete** equations.

> Reminder: Here are the rules we have established so far:
> ⋆ General rules:
> - $[A^\alpha \times B^\beta] = [A]^\alpha \times [B]^\beta$ for any A and B
> - $[A + B] = [A] = [B]$ if $[A] = [B]$
>
> ⋆ Specific rules:
> - $[x] = [y] = [u] = L$ if $u =$ inches, cm, m, yards etc...
> - $[v_x] = [v_y] = L \times T^{-1}$
> - $[\theta] = [u_a] = 1$ if θ is an angle and $u_a =$ degrees, radians, seconds of arc etc..
> - $[t] = [u_t] = T$ if $u_t =$ seconds, hours, days, years etc...
> - $[m] = [u_m] = M$ if $u_m =$ kg, tonnes, onces, etc...
> - $[F] = [N] = M \times L \times T^{-2}$
> - $[n] = 1$ where $n \in \mathbb{R}$ like -1, 3.45674 or 5 for example.

(a) $\frac{y}{t^2} = \frac{F}{m}$

(b) $\frac{v_x^2}{x} = 12\,\mathrm{N}$

(c) $\frac{m \cdot a_x^2}{F} = (4\,\mathrm{m} \cdot \mathrm{s}^{-3})\, t$