MTH1002 Calculus in-class test

Syllabus: All material covered in the **first six weeks of the course (practical classes and lectures)**.

Calculators are not allowed.

The test paper will include the following formulas:

1. **Table of values of trigonometric functions**

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	*

2. **Hyperbolic functions**

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x}$$

3. **Complex numbers**

$$\exp(i\theta) = \cos\theta + i\sin\theta$$

MTH1002 Calculus **Practice Test A**

Calculators are not allowed

1. Determine whether the following functions are odd, even or neither:

   (a) $f(y) = \sin(y^4 + y + 3)$

   (b) $g(v) = \dfrac{v^4 + 3v^2 + 5}{v^3 - v}$

[16 marks]

2. Calculate the following limits, showing all your working:

$$\lim_{u \to 3} \frac{u^2 + u - 12}{u^2 - 9} \qquad and \qquad \lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}.$$

[16 marks]

3. Write the complex number $z = 3\sqrt{2} - 3\sqrt{2}i$ in exponential form.

[12 marks]

4. Use the fact that $-\sqrt{3} + i = 2\exp(5\pi i/6)$ to calculate

$$\left(-\sqrt{3} + i\right)^6.$$

   Your answer should be purely real.

[12 marks]

5. Use **<u>logarithmic differentiation</u>** to find

$$\frac{d}{dt}\left(t^5 e^{-at}\sin(\omega t)\right),$$

   where a and ω are non-zero constants.

[16 marks]

6. Find the critical points of $x^{3/4}(2 - x)$.

[12 marks]

7. Find the absolute maximum and minimum of

$$f(x) = x^3 - 3x + 1$$

   on the interval $[0, 2]$.

[16 marks]

MTH1002 Calculus **Practice test B**

Calculators are not allowed

1. Determine whether the following functions are odd, even, or neither:

   (a) $f(x) = \ln(x^2 + 1)$ (b) $g(t) = t^3 + t + 1$

   [12 marks]

2. Write the complex number $z = -3\sqrt{3} + 3i$ in exponential form.

   [12 marks]

3. Find all the roots of $z^4 = i$, writing your answers in exponential form.

   [12 marks]

4. Calculate the following limits:

   $$\lim_{x\to\infty} \frac{x^4 + x - 1}{x^5 + x^2 + 2} \quad \text{and} \quad \lim_{t\to 0} \frac{\exp(at) - \exp(-at)}{t}$$

   where a is a non-zero constant.

   [16 marks]

5. Differentiate the following functions with respect to t using the standard rules of differentiation (e.g., the chain rule):

   $$f(t) = \sin(\omega t + \phi) \text{ and } g(t) = \ln\left(t^2 + \frac{1}{t^2}\right)$$

   where ω and ϕ are non-zero constants.

   [16 marks]

6. Find the critical points of $x^{2/3}(x - 1)$.

   [16 marks]

7. Find the absolute maximum and minimum values of the function

   $$f(x) = x^4 - 4x + 2$$

   on the interval $[0, 2]$.

   [16 marks]

MTH1002 Calculus **Practice test C**

Calculators are not allowed

1. Write down the domain and range of the functions

$$f(x) = \sqrt{1 - x^2} \qquad \text{and} \qquad g(x) = \frac{1}{(x-b)^4},$$

where b is a real, non-zero constant.

[16 marks]

2. Calculate the following limits:

$$\lim_{t \to 3} \frac{t^2 - 9}{t - 3} \text{ and } \lim_{u \to \infty} u^2 \exp(-u).$$

[20 marks]

3. Calculate, ***from first principles***, the derivative with respect to x of the functions

$$f(x) = x^2 + x \qquad \text{and} \qquad g(x) = \frac{1}{\sqrt{x}}.$$

[16 marks]

4. Write the complex number $z = 2 + 2i$ in polar and exponential form.

[12 marks]

5. Use the fact that $1 - i = \sqrt{2}\exp(7\pi i/4)$ to write $(1 - i)^5$ in standard form.

[12 marks]

6. Find the critical points of $x(3 - x)^{1/5}$.

[12 marks]

7. Find the intervals of increase and decrease of the function

$$f(x) = \frac{x^2}{x - 1}.$$

[12 marks]