Ideas of mathematical proof. Practical class week 22

4.1. For which of the following pairs $A, B \subseteq \mathbb{R}$ does the relation $f = \{(x, y) \mid x = |y|\} \subseteq A \times B$ define a mapping $f : A \to B$?
Give reasons to your answers.

(1) $A = \mathbb{R}, \ \ B = [0, \infty)$.

(2) $A = [0, \infty), \ \ B = \mathbb{R}$.

(3) $A = [0, \infty), \ \ B = (-\infty, 0]$.

4.2. Given the mappings

$f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{R}, \ \ f((x, y)) = x + y$, and

$g : \mathbb{R} \to \mathbb{R}, \ \ g(a) = a^2 + 1$,

determine which of the composites $f \circ g, \ \ g \circ f$ is defined and write the resulting mapping in standard form.

4.3. For each of the following mappings, determine whether it is
(a) injective, (b) surjective.

(1) $f : \mathbb{Z} \to \mathbb{N}, \ \ f(k) = k^2 + 2$.

(2) $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}, \ \ f((x, y)) = x + y$.

(3) $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}, \ \ f((m, n)) = 2^{m-1} \cdot (2n - 1)$.

4.4. Let $A = \mathscr{P}(\{u, v, w\})$ be the set of all subsets of $\{u, v, w\}$ and let $f : A \to A$, $f(X) = X \cap \{u, v\}$. Draw the diagram of the Cartesian product $A \times A$ and indicate f as a subset of $A \times A$. What is the image of f?

4.5. Find the image of the mapping $f : \mathbb{R} \to \mathbb{R}, \ \ f(x) = \dfrac{1}{x^2 + 5}$.

4.6. Show that $f : [3, \infty) \to (0, 1], \ \ f(x) = \dfrac{2}{x - 1}$ is a bijection and find the inverse mapping f^{-1} (also indicate the domain and image of f^{-1}).

4.7. Let $f : \mathbb{R} \to \mathbb{R}, \ \ f(x) = 2x + 1$. Define recursively the mappings $F_1 = f$ and $F_{k+1} = f \circ F_k$ for all $k \in \mathbb{N}$. Compute several first mappings, conjecture an expression for F_n and prove it by induction.