Problems for Tutorials

Problem 1. Consider the following matrices

$$A_1 = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 2 & 0 & 1 & 3 & -8 \\ 0 & 0 & -1 & -1 & 3 \\ 0 & 0 & 4 & 7 & 7 \end{bmatrix}.$$

Find the matrices which are in row-echelon form and compute their rank. Which of those matrices are in reduced row-echelon form?

Problem 2. Show that (i) AA^T and (ii) $A + A^T$ are symmetric matrices.

Problem 3. Find x when:

$$\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -6 & 7 \\ 4 & 0 \end{bmatrix}$$