Problems for Tutorials: Week 21

Problem 1. Using Gauss–Jordan elimination:

(i) Determine whether the vector $\mathbf{u} = \begin{bmatrix} 9 \\ 0 \\ 4 \end{bmatrix}$ belongs to the span of the following vectors:

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \mathbf{z} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

(ii) Show that the vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ 2 \\ 4 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} -2 \\ 1 \\ 6 \end{bmatrix}$$

are linearly dependent and simplify $span(\mathbf{u}, \mathbf{v}, \mathbf{w})$ in terms of $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

Problem 2. For what k values will the systems:

(a)
$$\begin{cases} kx + 2y = 3 \\ 2x - 4y = -6 \end{cases}$$
, (b) $\begin{cases} x + ky = 1 \\ kx + y = 1 \end{cases}$

$$(b) \quad \left\{ \begin{array}{l} x + ky = 1 \\ kx + y = 1 \end{array} \right.$$

have: (i) infinitely many solutions,

- (ii) no solutions and
- (iii) a unique solution
- For (i) write the set of solutions.