Tutorials for Week 9: Change of basis – Gram-Schmidt orthogonalisation

Problem 1. Consider the bases $\mathcal{B} = \{\mathbf{e}_1, \mathbf{e}_2\}$ and $\mathcal{C} = \{\mathbf{u}_1, \mathbf{u}_2\}$ of $\mathbb{R}^2$, where

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} \quad .$$

Find the linear transformation which changes the standard basis $\mathcal{B}$ into the basis $\mathcal{C}$.

Problem 2. Consider the bases $\mathcal{B} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ and $\mathcal{C} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ of $\mathbb{R}^3$, where

$$\mathbf{u}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad .$$

i. Find the change-of-basis matrix from $\mathcal{B}$ to $\mathcal{C}$.
ii. Write the coordinates of the vector $\mathbf{v} = [1, 1, 1]^T$ with respect to the basis $\mathcal{C}$.

Problem 3. Find an orthonormal basis for the subspace

$$S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x - 3y + z = 0 \right\}$$

of $\mathbb{R}^3$. (Hint: Use Gram-Schmidt orthogonalisation)