"Ideas..." problems for tutorials week 23

I-23.1. How does the graph of a function $f(x)$ change under the following transformations (where a is a constant)? That is, how to obtain the graph of $F(x)$ from the graph of $f(x)$, where

(1) $F(x) = f(x) + a$;

(2) $F(x) = f(x + a)$;

(3) $F(x) = f(ax)$.

I-23.2. Assume that the mapping $f : [0; 1) \to [1; \infty)$, $f(x) = \dfrac{1}{\sqrt{1 - x^2}}$, is a bijection. Apply transformations in Question 1 to the mapping f to produce a bijection $F : [2; 4) \to [0, \infty)$.

I-23.3. Prove that the set W of all finite sequences ('words') composed of two letters A, B is countable.

[*Hints*: One method is to use a theorem in the lectures by which it is sufficient to produce an injective mapping to $\mathbb{N}$. Another method is to describe a systematic enumeration of all such words, that is, to show that the set W can be represented as a sequence.]