Practical 4

Problem 1. Find $A + B$ and $A \cdot B$, when

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & -2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 3 & -2 & 3 \end{bmatrix}$$

Do the matrices A, B commute ($AB = BA$)?

Problem 2. Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Find conditions for a, b, c, d which make A:
(a) A diagonal matrix
(b) A symmetric matrix
(c) An upper triangular matrix
(d) A skew-symmetric matrix ($A^T = -A$)

Problem 3. Let $A\mathbf{x} = \mathbf{0}$ be a homogeneous system with 3 equations and 3 unknowns. Find the rank of the matrix A, when the system has:
(a) a unique solution.
(b) infinite solutions of the form $x = 2z, y = -z$, z in $\mathbb{R}$.

Problem 4. Determine the rank of following matrices

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 1/2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

and of their matrix powers A^2 and B^2.

Problem 5. (a) Is $(A - B)(A + B) = A^2 - B^2$ true for any $n \times n$ A, B matrices? Justify your answer.
(b) Show that AA^T is a symmetric matrix.

Problem 6. Let $\mathbf{u}, \mathbf{v}$ be two linearly independent vectors in $\mathbb{R}^2$ and A the matrix with columns the vectors $\mathbf{u}, \mathbf{v}$. Which is the rank of A?