Practical 3

Problem 1. Find the solution set which corresponds to the following row echelon forms:

$$(i)\ \left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right], \quad (ii)\ \left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 0 \\ 0 & 3 & -1 & -1 & 0 \end{array}\right]$$

Problem 2. Solve the system with 3 equations and 4 unknowns using Gauss–Jordan elimination:

$$\begin{cases} x_1 + 2x_2 - 2x_3 + 2x_4 = 1 \\ 2x_1 - 4x_2 + 4x_3 + 6x_4 = 0 \\ x_1 + 2x_2 - 2x_3 + 3x_4 = 0 \end{cases}$$

and write its set of solutions.

Problem 3. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

vectors in $\mathbb{R}^3$. Using Gauss-Jordan elimination:

(i) Show that the vectors are linearly dependent.

(ii) Determine whether the vector $\mathbf{u} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ belongs to the span of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

Problem 4. Consider the following matrices

$$B_1 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \\ 0 & 0 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, B_4 = \begin{bmatrix} 1 & 0 & 0 & -2 & 5 \\ 0 & 1 & 0 & 9 & -2 \\ 0 & 0 & 1 & 4 & 3 \end{bmatrix}$$

Find the matrices which are in row-echelon form. Which of those matrices are in reduced row-echelon form?