UNIVERSITY OF LINCOLN

SCHOOL OF MATHEMATICS AND PHYSICS

MTH1005
PROBABILITY AND STATISTICS

Semester B

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SUMMARY OF LAST LECTURE

We have introduced the following concepts

- A short introduction to the graphical representation of data.
- Sample space, outcomes, events and probability distribution.
- Frequentist paradigm of probability

PROBABILITY

In this part we are going to learn the axioms of probability and connects them to counting problems –including combinatorial and permutations.

Learning outcomes:

- Sample spaces
- Outcomes
- Events
- Frequentist paradigm of probability
- The axioms of probability
- Calculate the probabilities of events occurring in simple sample spaces with equally likely outcomes.

BASIC DEFINITIONS

The theory of probability was developed with the aim to explain a game(gambling) governed by the law of chance.

From: Gonick & Smith The cartoon guide to Statistics

What is a probability?

Two possible definitions

Objective (or frequentist): *the probability is obtained by repeating random experiments and counting the number of times something occurs:*

$$P(A) = p = \lim_{N \to \infty} \frac{N_A}{N}$$

Subjective (Bayesian): A probability can describe many things, but it is not possible to repeat the process many times and sample how often the event occurs. In this case, the probability is interpreted as a reasonable expectation, representing a state of knowledge or quantifying a personal belief.

Bayesian methods are widely accepted and used in artificial intelligence in the field of machine learning, computer vision.

BASIC DEFINITIONS

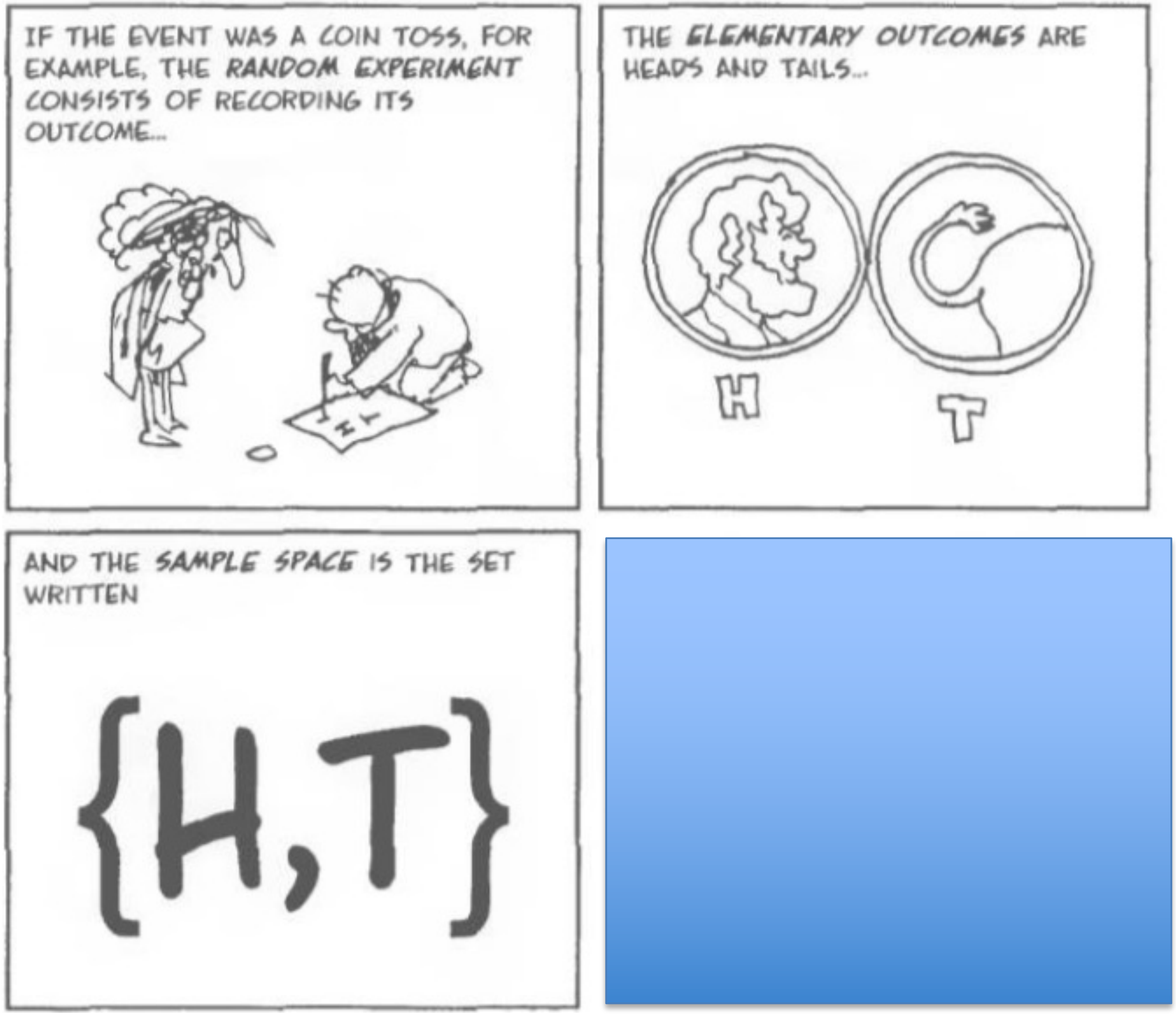
As a scientist, we call the game of the gambler a **random experiment**: in general, is a process of observing the outcome of a chance event (ex., the rolling of a dice, the radioactive decay, the fluctuation of the stock market).

The **elementary outcomes** are all possible results of the random experiment.

The **sample space** is the set or collection of all the elementary outcome.

SAMPLE SPACES

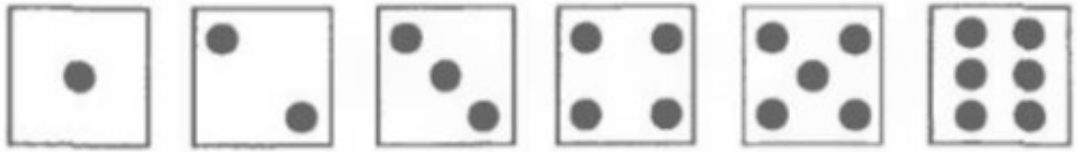
Example *What is the sample space of tossing a coin?*



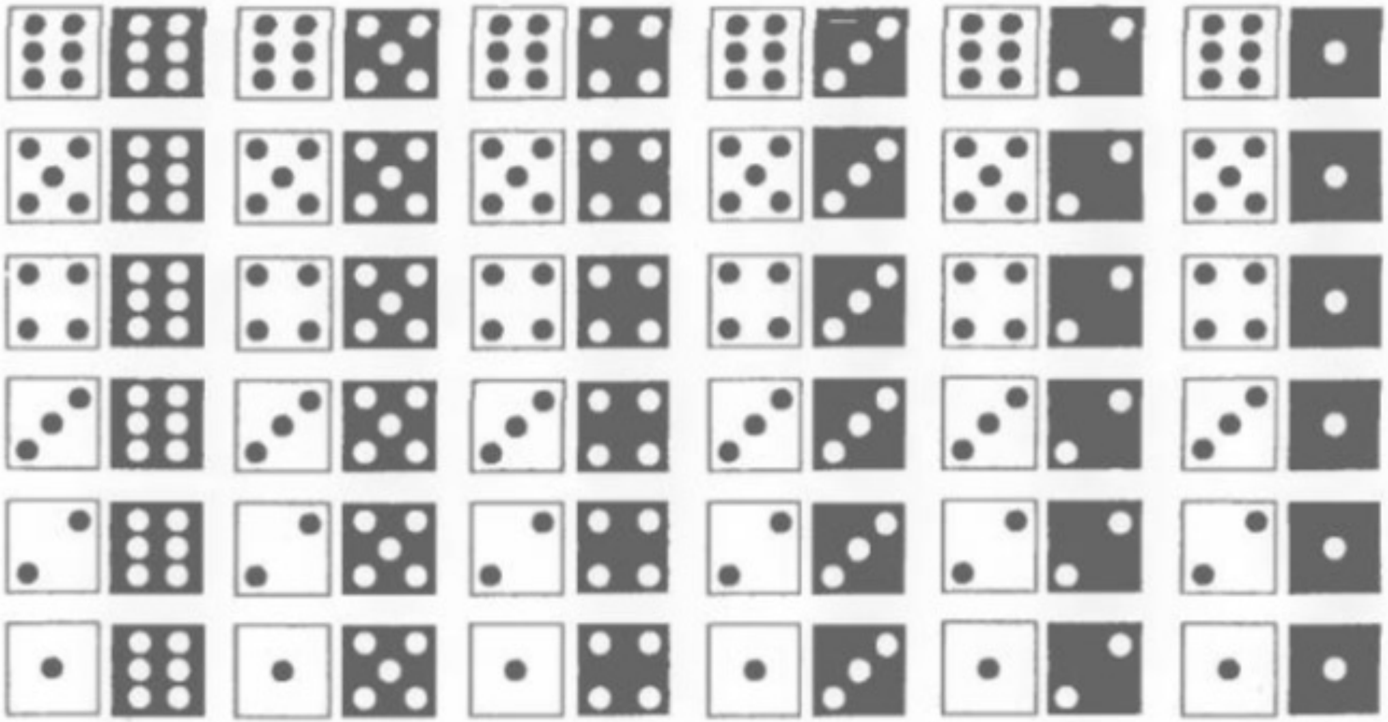
From: Gonick & Smith The cartoon guide to Statistics

BASIC DEFINITIONS

THE SAMPLE SPACE OF THE THROW OF A *SINGLE DIE* IS A LITTLE BIGGER.



AND FOR A *PAIR* OF DICE, THE SAMPLE SPACE LOOKS LIKE THIS (WE MAKE ONE DIE WHITE AND ONE BLACK TO TELL THEM APART):



SAMPLE SPACES

Note that sample spaces are not unique, sometimes there will be more than one way to break down and list the possible outcomes.

Example: An 'experiment' consists of flipping two coins? Give two possible sample spaces.

Define a single coin flip

$$F = \{H, T\}.$$

Then we have

$$S = \{F \times F\} = \{(H, H), (H, T), (T, H), (T, T)\}$$

SAMPLE SPACES

Alternatively, we could have three outcomes - 0 heads come up between the two flips, 1 head or 2 heads

$$S = \{0, 1, 2\}$$

Now our element {0} is identical to {T , T } previously.
But

$$\{1\} = \{(H, T), (T, H)\}$$

$$\{2\} = \{(H, H)\}$$

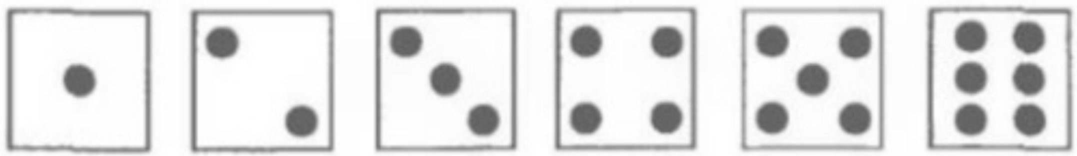
OUTCOMES

Definition: *Outcomes are the individual elementary elements of the sample space.*
(If the sample space is finite, or countably in finite, it will correspond to a discrete random variable.)

Example

A die is rolled once. We let X be the outcome of the experiment. The sample space for the experiment is the 6-element set

$$S = \{1, 2, 3, 4, 5, 6\},$$



and each outcome i , for *i= 1…6*, corresponds to the number of dots on the face that faces up.

EVENTS

Definition: an event is any subset, E , of the sample space, S. $E \subset S$.

(Recall that a set E is a subset of set S , if all of the members of E are also contained in S)

Example what is the event, E , corresponding to a die coming up with an even number?

E = {2, 4, 6}

Set builder notation

The **set builder notation** is a way of describing a set by specifying the properties that its elements must satisfy. It is also known as set comprehension or set notation. The basic structure of set builder notation is:

{variable | condition}

In this notation:

- "variable" represents the variable or variables that define the elements of the set.
- "|" (read as "such that") separates the variable from the condition.
- "condition" is the condition or set of conditions that the variable(s) must satisfy to be included in the set.

EVENTS

Example: *Consider a road race with the runners labelled 1 to 4. What is the event A that runner 1 won the race?*

Set builder notation

$$S = \{ (x1, x2, x3, x4) \mid xi = 1, 2, 3, 4 \text{ for } i = 1, 2, 3, 4, \; xi \neq xj \text{ for } i \neq j \}$$

In this notation:

- (x1, x2, x3, x4) represents each possible ordering of the elements (1, 2, 3, 4).
- "xi = 1, 2, 3, 4 for i = 1, 2, 3, 4" specifies that each element xi can take values from 1 to 4.
- "xi ≠ xj for i ≠ j" ensures that each element in the ordering is distinct, i.e., no two elements are the same.

This set builder notation constructs the set S by including all possible orderings of the elements (1, 2, 3, 4) that meet the specified conditions. The resulting set S would contain all permutations of the elements (1, 2, 3, 4) without repetitions.

EVENTS

Example: *Consider a road race with the runners labelled 1 to 4.*
What is the event A that runner 1 won the race?

To formulate event A, which represents the scenario where runner 1 won the race, in set builder notation, we can express it as follows:

$$A = 1234, 1243, 1324, 1342, 1423, 1432 = \text{all ordering of } (1234) \text{ beginning with } 1$$

A = { (x1, x2, x3, x4) | x1 = 1 and xi = 2, 3, 4 for i ≠ 1 }

In this notation:

- (x1, x2, x3, x4) represents each possible outcome or arrangement of the runners in the race.
- "x1 = 1" specifies that runner 1 finished first.
- "xi = 2, 3, 4 for i ≠ 1" ensures that the remaining runners (2, 3, 4) can finish in any order other than runner 1.

note that $A \subset S$

This set builder notation constructs event A by including all possible outcomes of the race where runner 1 finishes first, while the other runners (2, 3, 4) can finish in any order.

EVENTS

What is the event that runner 2 lost the race?

$$B = 1342, 1432, 3142, 3412, 4132, 4312 = \text{all ordering of } (1234) \text{ ending with } 2$$

1

B = { (x1, x2, x3, x4) | xi ≠ 2 for i ≠ 4 }

In this notation:

•(x1, x2, x3, x4) represents each possible outcome or arrangement of the runners in the race.

•"xi ≠ 2 for i ≠ 4" ensures that runner 2 finishes last. This means runner 2 cannot be in any position other than the last one.

1

note again that $B \subset S$

Note that for this scenario, we specifically mention that runner 2 cannot be in any position other than last (i.e., not in positions 1, 2, or 3) to fulfill the condition that runner 2 is the last to finish.

SET THEORY AND PROBABILITY

Our events are be considered as sets and therefore we can apply results from set theory.

"OR" or Unions $\cup$

The event E_A or E_B has occured is denoted $E_A \cup E_B$, called the union of E_A and E_B, and is composed of the outcomes that are in either E_A or E_B.

"AND" or Intersections $\cap$

The event that E_A and E_B has occured is denoted $E_A \cap E_B$, called the intersection of E_A and E_B, and is composed of the outcomes that are in both E_A and E_B.

SET THEORY AND PROBABILITY

Example - Four runners are racing.
What is the event, , that runner 1 was first and runner 2 was last?

Define E_1 the event that the first runner won the race $E_1 = \{1234, 1243, 1324, 1342, 1423, 1432\}$
and E_2 the event that the second runner lost the race $E_2 = \{1342, 1432, 3142, 3412, 4132, 4312\}$

A is given by the intersection of E_A and E_B, so outcomes that are in E_1 *and* E_2

$$\begin{aligned} E_1 \cap E_2 &= \{1234, 1243, 1324, 1342, 1423, 1432\} \cap \{1342, 1432, 3142, 3412, 4132, 4312\} \\ &= \{1342, 1432\} \end{aligned}$$

PROBABILITY THEORY: A BIT OF HISTORICAL BACKGROUND

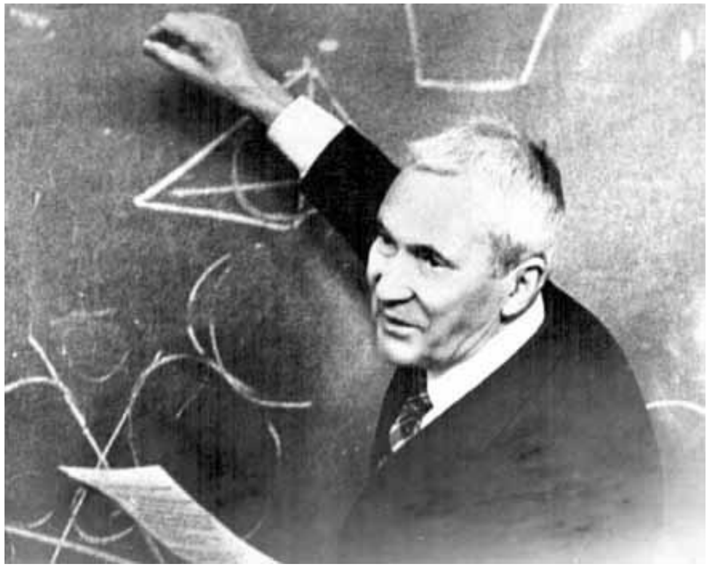
Christiaan Huygens
(1629-1695)

Rev. Thomas Bayes
(1701-1761)

Pierre-Simon Laplace
(1749-1827)

Andrey Markov
(1856-1922)

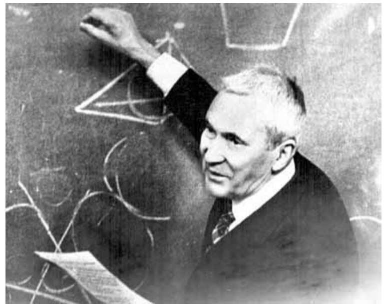
Modern probability theory is based on an axiomatic description of the properties of probability formulated by the Russian mathematician A. N. Kolmogorov (1903-1987).



http://enacademic.com/dic.nsf/enwiki/54484

AXIOMS OF PROBABILITY

We suppose that for each event of an experiment with sample space there is a number , which is in accord with the following three axioms:



Axiom 1 - The probability that an event will come from the sample space is unity

$$P(S) = 1$$

Axiom 2

$$P(A) \leq 1 \text{ for all } A \subset S$$

Axiom 3

$$P(A \cup B) = P(A) + P(B) \text{ if } A \cap B = \emptyset$$

We call a function P(A) the probability of an event if these axioms hold.

AXIOMS OF PROBABILITY

WE NEED TO MAKE SOME IMPORTANT CONSIDERATIONS

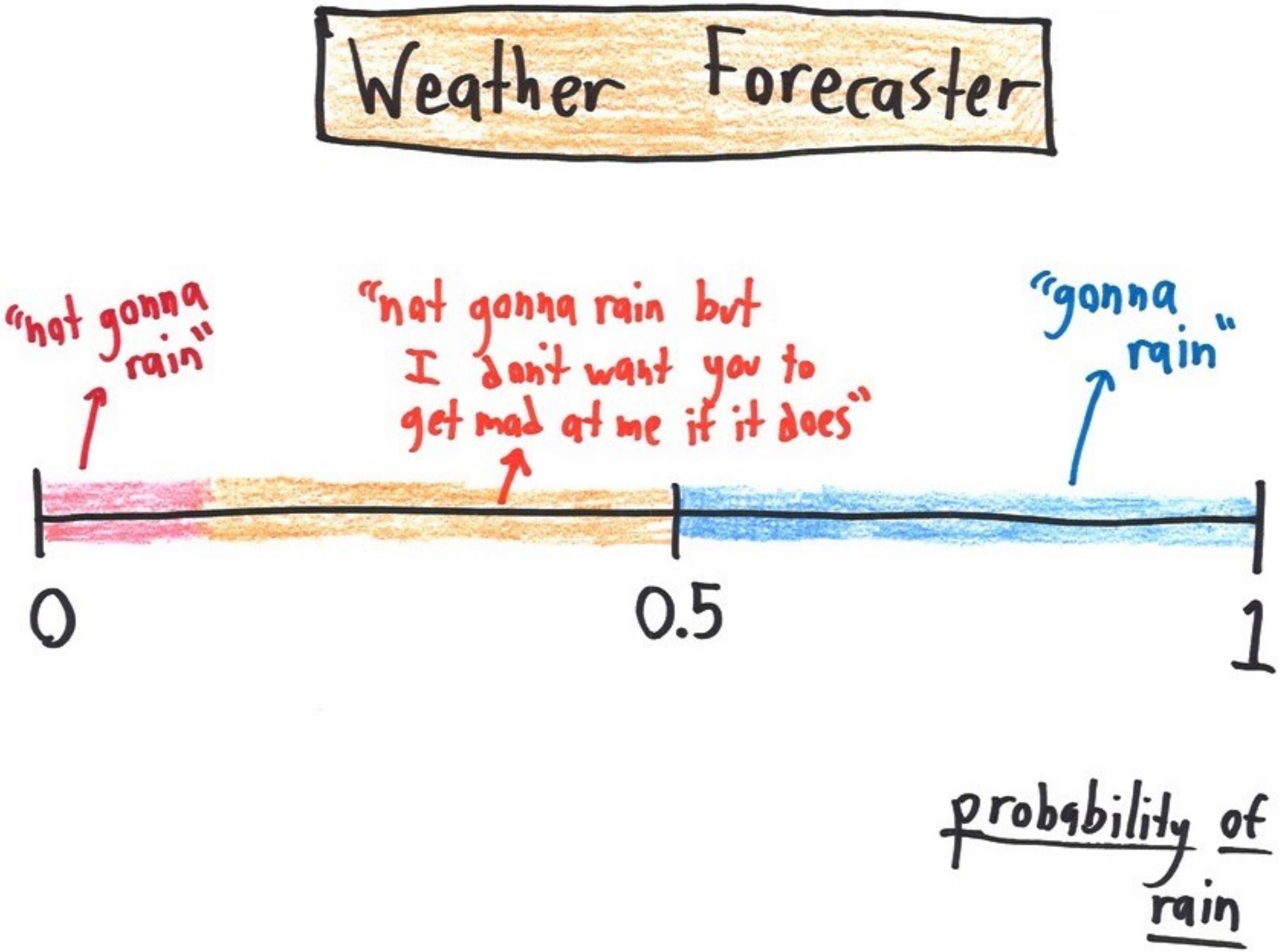
1.Using intuition and physical insight: Probability theory often involves making judgments about uncertain events based on available information, intuition, and physical insight.

In many cases, there isn't a strict mathematical formula that determines the probabilities of individual outcomes. Instead, we rely on our understanding of the situation, relevant knowledge, and sometimes even gut feelings to estimate probabilities.

2. No general way to determine probabilities: While there are well-defined mathematical principles and methods in probability theory, there isn't a one-size-fits-all approach to determining probabilities for every problem.

The complexity and uniqueness of each situation may require different techniques or considerations. This is why probability theory encompasses various tools and approaches, allowing flexibility in analyzing different scenarios.

What is a probability?



https://mathwithbaddrawings.com/2015/09/23/what-does-probability-mean-in-your-profession/

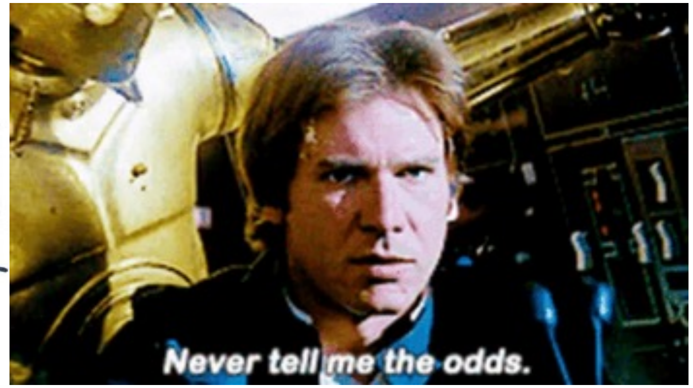
What is a probability?

Millennium Falcon Captain

~~NEVER TELL ME THE ODDS~~

0 1

probability of successfully navigating an asteroid field



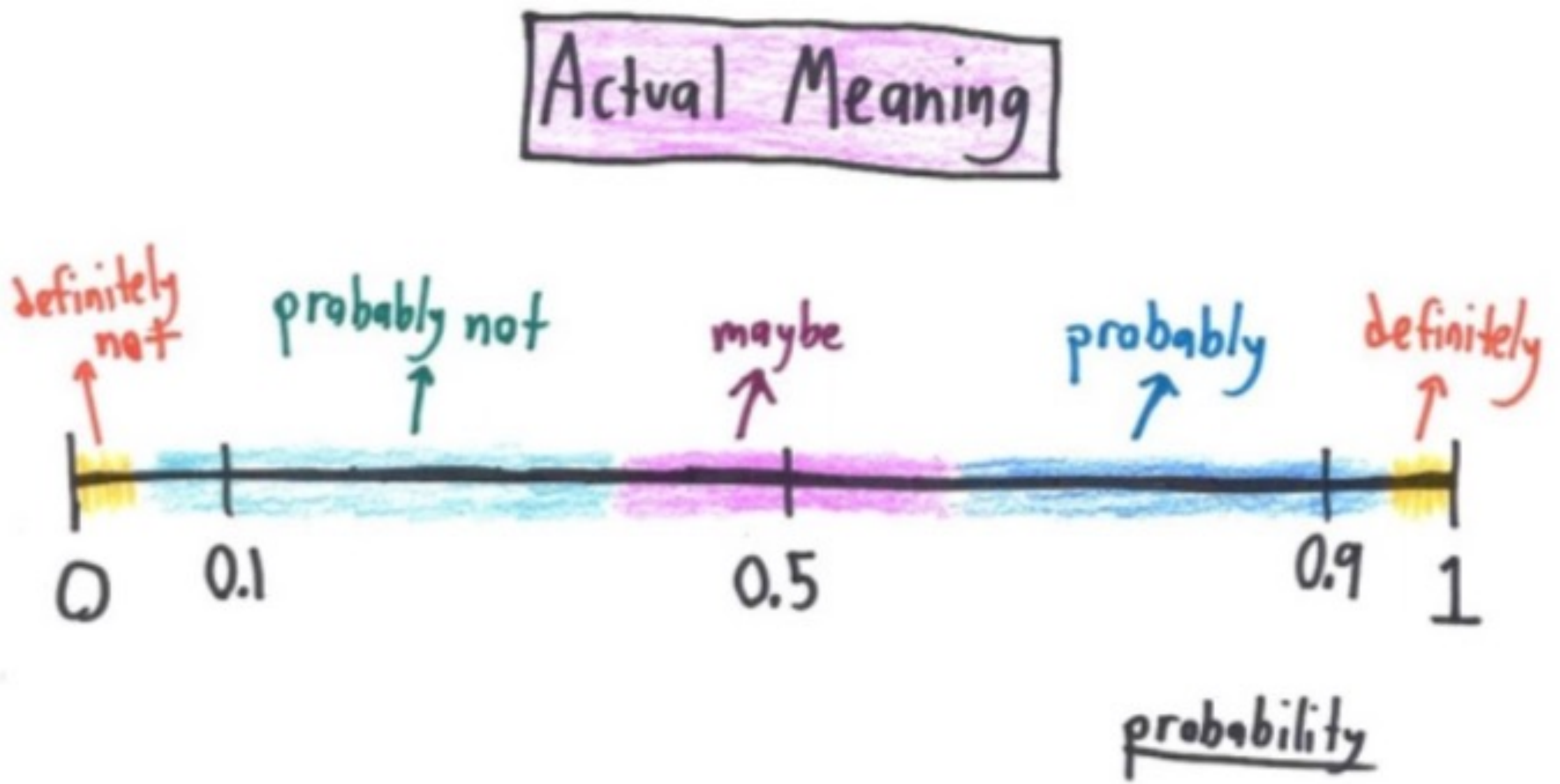
https://giphy.com/gifs/millennium-9mKMpXfvrwSl2

AXIOMS OF PROBABILITY

3. Utilizing all available information: When estimating probabilities, it's crucial to consider all available information relevant to the problem at hand.
This includes data, background knowledge, context, and any other pertinent factors. By incorporating all available information, we can make more informed judgments about the likelihood of different outcomes.

4. Comparing with reality: Probability theory serves as a framework for reasoning about uncertainty, but its ultimate goal often lies in representing and understanding real-world phenomena.
Therefore, it's essential to compare the results of probability analyses with real-world observations and outcomes. This helps validate the accuracy of the probability distribution and its usefulness in modeling the real world.

What is a probability?



https://mathwithbaddrawings.com/2015/09/23/what-does-probability-mean-in-your-profession/

SAMPLE SPACES WITH EQUALLY LIKELY OUTCOMES

In many situations, especially when conducting experiments with well-defined and uniform conditions, we can reasonably assume that each possible outcome of the experiment has an equal likelihood of occurring.

For example, consider the **rolling of a fair six-sided die**. Since the die is unbiased and each face is identical, it's intuitive to assume that each number from 1 to 6 has an equal chance of appearing.

This assumption can be formalized mathematically when dealing with a finite sample space, denoted as S, containing N distinct outcomes.

In such cases, it is often natural and convenient to assume that the probability of any single outcome occurring, denoted as $P(\{i\})$ where i ranges from 1 to N representing each outcome, is equal to a common probability value, denoted as p.

SAMPLE SPACES WITH EQUALLY LIKELY OUTCOMES

Mathematically, this assumption translates to:

$P(\{1\}) = P(\{2\}) = \dots = P(\{N\}) = p,$

where each individual outcome {i} in the sample space has the same probability p.

This assumption simplifies the calculation of probabilities in situations where all outcomes are equally likely, allowing for more straightforward analyses and predictions within the framework of probability theory.

SAMPLE SPACES WITH EQUALLY LIKELY OUTCOMES

Now, it follows from axiom 1 of probability theory that the probability of the entire sample space, denoted as S, is equal to 1.

By axiom 3, the probability of a union of mutually exclusive events equals the sum of their probabilities.

Since each elementary outcome {1}, {2}, ..., {N} in the sample space S is assumed to have the same probability p, the probability of the entire sample space can be expressed as the sum of the probabilities of all elementary outcomes:

$1 = P(S) = P(\{1\}) + P(\{2\}) + \ldots + P(\{N\}) = Np.$

This equation follows from axiom 2 and the assumption that all elementary outcomes are equally likely.

Axiom 1 - The probability that an event will come from the sample space is unity

$$P(S) = 1$$

Axiom 2

$$P(A) \leq 1 \text{ for all } A \subset S$$

Axiom 3

$$P(A \cup B) = P(A) + P(B) \text{ if } A \cap B = \emptyset$$

SAMPLE SPACES WITH EQUALLY LIKELY OUTCOMES

Furthermore, axiom 3 of probability theory states that for any event E that is a subset of the sample space S, its probability is determined by the ratio of the number of outcomes in E to the total number of outcomes in the sample space S. Mathematically, this can be expressed as:

$$P(E) = \frac{Number\ of\ outcomes\ in\ E}{Total\ number\ of\ outcomes\ in\ S}$$

This means that to calculate the probability of any event, we simply count the number of elementary outcomes that constitute the event, and then divide this number by the total number of elementary outcomes in the sample space.

This intuitive principle allows us to determine probabilities by considering the relative frequency of occurrence of the desired outcomes compared to the total number of possible outcomes in the experiment

Axiom 1 - The probability that an event will come from the sample space is unity

$$P(S) = 1$$

Axiom 2

$$P(A) \leq 1 \text{ for all } A \subset S$$

Axiom 3

$$P(A \cup B) = P(A) + P(B) \text{ if } A \cap B = \emptyset$$

SAMPLE SPACES WITH EQUALLY LIKELY OUTCOMES

In the more general case where each elementary outcome may not have the same probability of occurring, denoted as $P(\omega)$, we need to assign specific probabilities to each elementary outcome to ensure that the total probability of the sample space S is equal to 1, satisfying axiom 2 of probability theory.

This implies that the sum of the probabilities of all elementary outcomes must equal 1:

$$1 = P(S) = P(\{1\}) + P(\{2\}) + \dots + P(\{N\}).$$

Each individual probability $P(\omega)$ contributes to the total probability of the sample space S.

$P(\omega)$ is called a *Probability Function* of the experiment.

SAMPLE SPACES WITH EQUALLY LIKELY OUTCOMES

From axiom 3 of probability theory, for any event E that is a subset of the sample space S, its probability will be given by the sum of the probabilities of the elementary outcomes in E:

$$P(E) = \sum_{\omega \in E} P(\omega)$$

This expression represents the sum of the probabilities of all elementary outcomes ω that belong to the event E.

In essence, this formulation indicates that to determine probabilities in scenarios where each elementary outcome may have a different likelihood of occurring, we must consider the specific probabilities assigned to each outcome.

This requires us to calculate the total probability of events by summing the probabilities of their constituent elementary outcomes. Thus, determining probabilities involves not only counting the number of ways in which a given event can occur but also weighting those outcomes by their assigned probabilities.

SAMPLE SPACES WITH EQUALLY LIKELY OUTCOMES

Example. An experiment consists of throwing a coin twice. As we saw we can assign the sample space in two ways:

either

$$S_1 = \{HH, HT, TH, TT\}$$

with

$$P(HH) = \frac{1}{4}, P(HT) = \frac{1}{4}, P(TH) = \frac{1}{4}, P(TT) = \frac{1}{4}$$

or

$$S_2 = \{0, 1, 2\}.$$

In the latter case, we take the number of times a head appears as our outcomes.

SAMPLE SPACES WITH NOT EQUALLY LIKELY OUTCOMES

In the case of the probabilities of the individual outcomes are ***not* equal** –

there are twice as many ways of getting the outcome one head ({H,T},{T,H})

as there are of getting zero ({T,T}

or two heads ({H,H}).

So, it makes sense to have

$$P(0) = \frac{1}{4}, P(1) = \frac{1}{2}, P(2) = \frac{1}{4}$$

OTHER EXAMPLES

A six sides die is rolled. In this case, the sample space is given by S={1,2,3,4,5,6}

For a fair die, each outcome can be argued to be equally likely. We can deduce the probability of the outcomes as

$$1 = P(S) = P(\{1\}) + P(\{2\}) + \cdots + P(\{6\}) = 6p$$

which implies that $p = \frac{1}{6}$, as you'd expect.

Define two events,

- A, "the roll is less than 5", so $A = \{1, 2, 3, 4\}$
- B, "the roll is more than 2", so $B = \{3, 4, 5, 6\}$

The number of elements in *A* is *n(A) =n(B)=4*

So $P(A) = n(A)p = 4\frac{1}{6}$ *and*

$$P(B) = n(B)p = 4\frac{1}{6}$$

ANOTHER USEFUL EXAMPLE

What is the probability of $A \cup B$?

In words what is the probability that either

”The dice roll is less than 5

OR

“The die roll is more than 2”

ANOTHER USEFUL EXAMPLE

The answer is not $n(A) + n(B)$ because the two sets have an intersection

$$A \cap B(A\ AND\ B) = \{1,2,3,4\} \cap \{3,4,5,6\}=\{3,4\}$$

One of the basic rules of sets is we don't need to keep duplicate members - so

$$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6\}=\{1,2,3,4,3,5,6\}=\{1,2,3,4,5,6\}=S$$

We see in this case that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

which for probabilities proportional to number of elements also gives

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

SUMMARY

- The probability of any event can be decided once we have the *probability function*, $P(\omega)$, for our experiment.

- The probability function is the function $P(\omega)$, where the ω are the elementary outcomes of the experiment.
- It has domain S.
- It obeys the axioms of probability.
- The actual probabilities are assigned based on some model; they are outside the idealised mathematical model.
- We will demonstrate that the interpretation of $P(E)$ as the relative frequency of the event E when the experiment is repeated many times is consistent with the axioms.
- Belief based interpretations can also satisfy these requirements.

NOTE

Notations are not totally standard in probability theory. I'll use what I think is the closest to standard and should allow you to read most works on probability.

For example, in some textbook, you can find that the sample space is called Ω, the individual outcomes ω_i and the probability function $m(\omega)$.

OTHER USEFUL RELATIONSHIPS AND DEFINITIONS

Definition 1: Complement of an event E (with respect to the sample set S) is

$$P(\bar{E}) = 1 - P(E)$$

Definition 2: E_1 and E_2 are mutually exclusive if

$$P(E_1 \cap E_2) = P\{0\} = 0$$

Definition 3: A set of events $E_1, E_2, \ldots E_n$ of some experiment are said to be *exhaustive* if

$$E_1 \cup E_2 \cup E_3 \cup \cdots E_n = S$$

Definition 4: If E_1 and E_2 are events of the same experiment

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

PRINCIPLES OF COUNTING - COMBINATORIALS AND PERMUTATIONS

Suppose two experiments are carried out

- experiment 1 can result in any of m outcomes
- for each outcome of experiment 1 experiment 2 can have *n* outcomes
- together there are *mn* outcomes.

We can write all possible outcomes as an ordered pair, *(m, n)* .
The *mn* possible outcomes can be tabulated

(1,1)	(1,2)	$\cdots$	(1,n)
(2,1)	(2,2)	$\cdots$	(2,n)
$\vdots$	$\vdots$	$\vdots$	$\vdots$
(m,1)	(m,2)	$\cdots$	(m,n)

PRINCIPLES OF COUNTING - COMBINATORIALS AND PERMUTATIONS

Example: 2 balls are randomly drawn from a bowl that contains 6 black and 5 white balls.

- what is the chance that we draw one black and one white ball?

We want to apply

$$P(E) = \frac{\text{Number of outcomes in } E}{N}$$

First calculate the total number of possible outcomes …

First calculate N the total number of possible outcomes:

- ***experiment 1***, the first ball can be picked in *11* different ways
  $m = 11$.

- ***experiment 2***, for each of the results of experiment 1, we can pick the next ball in one of 10 ways
  $n = 10$.

- so total number of outcomes possible is $\boldsymbol{mn = 110}$

PRINCIPLES OF COUNTING - COMBINATORIALS AND PERMUTATIONS

Shall we write it in set builder notation

$$B = \{(x_1, x_2) | x_1 = 1,2, \dots 11, x_2 = 1,2, \dots 10, x_2 \neq x_1\}$$

and so, if we were to list all the ordered pairs in a table, they would be 11*10

$$N = n(B) = |B| = 11 * 10 = 110$$

PRINCIPLES OF COUNTING - COMBINATORIALS AND PERMUTATIONS

Now we have the denominator we need to calculate the number of ways of getting one black and one white ball. Either

- the first ball is black, which can occur in 6 ways (m), for each way there are 5 ways the second ball can be white (n)

- the first ball is white (5 ways, m) and the second ball is black (6 ways, n)

Either of the two ways (or, union of the two events) leads to an event with one black and one white ball.

Therefore, by the axiom 3, the total number of ways to get one black and one white ball is

$$5*6+6*5=60.$$

Shall we write it in set builder notation.

We number the balls so that 1 to 6 are white and 7 to 11 are black.

The first ball is x_1 and second x_2

$$T = \{(x_1, x_2) | x_1 = 1,2,3,4,5,6, x_2 = 7,8,9,10,11\}$$

$$U = \{(x_1, x_2) | x_1 = 7,8,9,10,11, x_2 = 1,2,3,4,5,6\}$$

Therefore

$$P(E) = \frac{\text{Number of outcomes in } E}{N} = \frac{5*6+6*5}{110} = \frac{6}{11}$$

FACTORIAL

If there are more than two experiments to be performed, then the above rule generalises to $n_1 \cdot n_2 \cdot \cdots \cdot n_r$ *possible outcomes of r experiments.*

So continuing our previous example, how many ways are there to select all the balls from the bowl?

In this case there are $11 \cdot 10 \cdot 9 \cdot \ldots \cdot 2 \cdot 1 = 11!$

Lets calculate this just to see how big these numbers get!

i	i!
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800

FACTORIAL: STIRLING APPROXIMATION

A useful approximation to the factorial is the Stirling approximation

$$\ln n! = n ln\, n - n + O(\ln n)$$

The big *O* notation indicate the error (difference with the correct value) that can be quantify with a constant $\frac{1}{2}\ln(2\pi n)$, yielding to the more precise formula

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

N	N!	Stirling approximation	Error %
1	1	1.00	0.227445%
2	2	2.00	0.032602%
3	6	6.00	0.009986%
4	24	24.00	0.004266%
5	120	120.00	0.002198%
6	720	720.01	0.001276%
7	5040	5040.04	0.000805%
8	40320	40320.22	0.000540%
9	362880	362881.38	0.000380%
10	3628800	3628810.05	0.000277%
11	39916800	39916883.11	0.000208%
12	479001600	479002368.48	0.000160%
13	6227020800	6227028659.89	0.000126%
14	87178291200	87178379323.32	0.000101%

PERMUTATIONS

Simple Definition: A full set of permutations is all the ways of arranging some distinguishable objects. Each permutation swaps between two of these ways of arranging them.

Rigorous Definition: A permutation is a one-to-one mapping of a set onto itself whilst changing the ordering of the elements.

For instance if A = {a, b, c} a possible permutation would be (using Cauchy's two-line notation)

$$\sigma = \begin{pmatrix} a & b & c \\ c & b & a \end{pmatrix}$$

where the permutation sends a to c , b to c, and c to a .

PERMUTATIONS

The number of permutations of n objects can be found from the counting rules to be **n!** , in much the same way as for the ball selection example above.

EXAMPLE

Find all the permutations of A = {a, b, c}

$$B = \{(1,2,3),(1,3,2),(2,1,3),(2,3,1),(3,1,2),(3,2,1)\}$$

and

$$n(B) = 3!$$

COMBINATIONS

Definition: We define the number of combinations of *r* objects taken from a set of size *n* , $\binom{n}{r}$, for $r \leq n$, by

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

$\binom{n}{r}$ is the number of combinations of *n* objects taken *r* at a time, also referred to as **the binomial coefficient.**

The binomial theorem states that

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}$$

We'll encounter it again when we will talk about the binomial and Poisson distributions.

COMBINATIONS

EXAMPLE

We need to select student representatives for the school - a committee of 5 people is selected from 8 physicists and 30 mathematicians

a) If we decide we need to have 2 physicists and 3 mathematicians on the committee, how many possible sets of representatives are there?

we have two 'experiments' here

- First, we choose 2 physicists out of 8 - i.e., $\binom{8}{2}$ = 28
- secondly for each choice of the physicists we can select 3 mathematicians out of 30 - i.e. . $\binom{30}{3}$=4060

So, in total 28*4060 = 113680 possible committees of this composition...!

COMBINATIONS

b) what is the probability of getting a committee with 4 mathematicians and 1 physicist?

In this case, we need to consider all the students (30+8=38)

The total number of ways of selecting the committee is $\binom{38}{5}$ = 501942

the number of ways of getting 4 mathematicians and 1 physicist is basically the same as part a) above

$$\binom{30}{4} \cdot \binom{8}{1}$$

So, the probability is

$$P = \frac{\binom{30}{4} \cdot \binom{8}{1}}{\binom{38}{5}} = 0.44$$