Ideas... problems for tutorials week 20

I-20.1. Use mathematical induction to prove that $2^n > 100n$ for any $n \geqslant 10$.

I-20.2. Where is a mistake in the following "proof" that all triangles have the same area?

"Obviously, it is sufficient to prove that, given any n triangles, they all have the same area. We use induction on n. Indeed,

1°. If we have one triangle, the assertion is of course true.

2°. Suppose the assertion is true for $n = k$. Consider any $k + 1$ triangles $T_1, \ldots, T_{k+1}$. Apply the induction hypothesis to the first k of them: $T_1, \ldots, T_k$ all have the same area. Then apply the induction hypothesis to the last k of them: $T_2, \ldots, T_{k+1}$ all have the same area. Then obviously all $k + 1$ triangles $T_1, \ldots, T_{k+1}$ have the same area: area of T_1 = area of T_2 $= \cdots =$ area of T_k = area of T_{k+1} .

Thus, by the Axiom of Mathematical Induction, the assertion is true for all n, and therefore all triangles have the same area."

I-20.3. How to tie a goat so as it can eat grass exactly within a semicircle? It is allowed to use several leashes, with several pegs. For example, one peg and leash mean that the goat eats within a circle. It is also allowed to string one rope tightly between two pegs, and to tie the goat with leash to a small ring sliding on the first rope. [*Hint*: use intersections of sets.]