Ideas of mathematical proof. Practical class Week 19

Note that solutions must be by the methods specified in the problems – even if you can see another solution, even if an easier one.

1.1. Use mathematical induction to prove that $5^n + 3$ is divisible by 4 for all positive integers n.

1.2. Use mathematical induction to prove that $11^n - 4^n$ is divisible by 7 for all positive integers n.

1.3. Use the method of undetermined coefficients to guess a formula for

$$1 + 3 + 5 + \cdots + (2n - 1)$$

as a quadratic expression in n, and then use mathematical induction to prove this formula. **(Pretend as if you did not know the formula for the arithmetical progression...)**

1.4. Use mathematical induction to prove that

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

for any positive integer n.

1.5. Use mathematical induction to prove that $n! > 3^n$ for any $n \geqslant 7$.

1.6. What is wrong in the following "proof" that all men are bald? We use induction on the number of hairs on the head, n. Indeed,

1°. If $n = 1$, then a man with just one hair is of course bald.

2°. Suppose the assertion is true for $n = k$, that is, having k hairs means the man is bald. Now, if he has $k+1$ hairs, it is just one more, surely, does not transform a man from being bald to non-bald. Thus, by the Axiom of Mathematical Induction, a man is bald if he has n hairs, for any n.

1.7. Prove by induction that for $x \neq 1$ we have

$$1 + x + x^2 + \cdots + x^{n-1} = \frac{1 - x^n}{1 - x}.$$

for any positive integer n.

1.8. Use mathematical induction to prove that

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{n}{2^n} = 2 - \frac{n+2}{2^n}$$

for all positive integers n.

1.9. Use mathematical induction to prove that $n^3 < 2^n$ for any $n \geqslant 10$.