

Practical 8

Problem 1. Consider the following transformations:

$$(i) \ T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3 \quad T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x - y \\ 2x - y \end{bmatrix}$$

$$(ii) \ T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3 \quad T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + 2y \\ y - 2z \\ x + y + z \end{bmatrix}$$

$$(iii) \ T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2 \quad T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - y + z \\ x + y - z \end{bmatrix}$$

Show that these transformations are linear.

Problem 2. Find a basis for the kernel and the range of the linear transformations (in Problem 1). What is their nullity and rank?

Problem 3. Which of the linear transformations (in Problem 1) are one-to-one and which are onto? Is any of them an isomorphism? Justify your answer.