MTH1005 PROBABILITY AND STATISTICS TUTORIAL

QUESTION 1

In Glasgow, half of the days have some rain. The local weather forecaster is correct $\frac{2}{3}$ of the time, i.e., the probability that she has predicted rain on a rainy day, and the probability that she predicts no rain on a dry day, is both equal to $\frac{2}{3}$.
When rain is forecast, a Glaswegian lady takes her umbrella. When rain is not forecast, she takes it with probability $\frac{1}{3}$.

- Draw a tree diagram - start labeling by whether it rains, then what the forecaster will predict, and finally what the Glaswegian lady does.

- Check you know what types of probability you are labeling on the diagram.

- Calculate the probability that the lady has no umbrella, given that it rains.

- Calculate the probability that she brings her umbrella, given that it doesn't rain.

QUESTION 2

Microchips are made by three companies. 30% are supplied by the firm I, 50% by II and 20% by III.
The probabilities of $A = $ *'a defect in a chip'* are $P(A|I) = 0.03, P(A|II) = 0.04, P(A|III) = 0.01$.
If an unlabeled box of chips turns up, what are the probabilities that the box came from I, II, or III

(i) if a random test shows a defective chip?

(ii) if a random test shows a non-defective chip?

(iii) if the first chip was defective, and a second chip was then also tested and found defective?

QUESTION 3

Find the cumulative distribution function of the following probability density function.

$$f_X(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}, & 0 \le x \le 2 \\ 0, & x > 2 \end{cases}$$

QUESTION 4

Confirm that the cdf you found, $F_X(x)$

- is monotonically increasing;

- tends to zero as x tends to $-\infty$;

- tends to one as $x \to \infty$.

Check your solution by recovering the original $f_X(x)$ by differentiating the $F_X(x)$ you found.