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# MTH1005 PROBABILITY AND STATISTICS

## PRACTICAL 4

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### Short review

**Definition 1:** Consider an experiment, with outcome set  $S$ , split into  $n$  mutually exclusive and exhaustive events  $E_1, E_2, E_3, \dots, E_n$ . A variable,  $X$  say, which can assume exactly  $n$  numerical values each of which corresponds to one and only one of the given events is called a random variable.

**Definition 2:** Let  $X$  be a discrete random variable that can take on only the values  $x_1, x_2, x_3, \dots, x_n$  with respective probabilities  $p_X(x_1), p_X(x_2), p_X(x_3), \dots, p(x_n)$ . Then, if

$$\sum_{x \in X} p_X(x) = 1$$

The function  $p(x) = P\{X = x\}$  is called the probability (mass) function of the variable  $X$ .

**Definition 3:** The cumulative distribution function of a discrete random variable,  $X$ , is

$$F_X(a) = P\{X \leq a\} = \sum_{x \leq a} p_X(x).$$

$X$  is a discrete random variable.

## Question 1

Let  $Z$  be the 'number around the base when a 4 sided die is rolled'.

- Write down a sample space for the experiment
- Define the values of  $Z$  and the events they correspond to.
- Write down the probability mass function of  $Z$ .

## Question 2

Fully define random variables corresponding to the experiments

- 'number of heads shown when two coins are thrown'.
- 'the number of successes out of 4 trials'.
- an urn contains 3 white and 2 blue balls. Two balls are selected without replacement, and £1 is paid out for each blue ball in the selection.

## Question 3

The *cumulative distribution functions (CDF)* of the for the variables in questions 1 and 2 is given by the expression  $F(a) = \sum_{\text{all } x \leq a} p_X(x)$ . How do the plots of the CDFs look like?

## Question 4

Let  $X$  be the discrete random variable '*number of tails shown when two coins are thrown*'. Define two more random variables:

$X_1$  = the number of tails shown on the first coin,

$X_2$  = the number of tails shown on the second coin.

Show that  $X$ ,  $X_1$  and  $X_2$  are random variables.

## Question 5

In a game 3 dice are rolled. The player bets £1. They get back £1 if they roll a single 5, £2 if 2 fives come up, and £3 if 3 fives come up (and their stake is returned). If no 5s come up they lose their £1 stake.

1. Calculate the value of the probability mass function.
2. Calculate the probability mass function for a game with 4 dice.

### Question 6

The probabilities that three men hit a target are respectively  $\frac{1}{6}$ ,  $\frac{1}{4}$ , and  $\frac{1}{3}$ . Each man shoots once at the target.

1. Find the probability  $p$  that exactly one of them hits the target,
2. If only one hit the target, what is the probability that it was the first man?