Practical 7

Problem 1. Find the eigenvalues and eigenvectors of the following matrix:

$$B = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}.$$

Show that its determinant equal to $\lambda_1 \lambda_2$.

Problem 2. Prove that any 2×2 matrix of the form:

$$A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

where $a \neq b, a, b$ in $\mathbb{R}$, has two real eigenvalues, which are: $a + b, a - b$.

Problem 3. Diagonalise the matrix:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Problem 4. Find B^n when

$$B = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}.$$

Problem 5. (a) Show that

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

is non-diagonalisable.
(b) The matrix

$$B = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$$

has two equal eigenvalues $\lambda_{1,2} = \lambda = 2$. What is the algebraic and what is the geometric multiplicity of λ ? Is B diagonalisable?