Mid-Term Test

Problem 1. Find the solution set S of the following homogeneous system using Gauss–Jordan elimination

$$\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 0 \\ \quad\quad\;\; x_2 + x_3 + x_4 = 0 \end{cases}$$

Show that S is a vector subspace of $\mathbb{R}^4$ and determine its basis and its dimension.

Problem 2. Let A, P be $n \times n$ matrices.

a) If A satisfies the matrix equation

$$A^3 + A - 4I = O$$

show that A is invertible and find its inverse.

b) Using the determinant properties, prove that $\det(P^{-1}AP) = \det A$.

Problem 3. Find a basis for each of the vector subspaces $null(A)$ and $col(A)$ where

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 1 \\ 1 & 3 & 0 \end{bmatrix}.$$

Determine their dimension and verify the Rank Theorem.

Problem 4. Given that the eigenvalues of the following matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

are $\lambda_1 = \lambda_2 = 1, \lambda_3 = 0$, show that it is diagonalisable.

Attempt all problems. Each problem counts for 25 marks.