UNIVERSITY OF LINCOLN
SCHOOL OF MATHEMATICS AND PHYSICS

MTH1005M PROBABILITY AND STATISTICS PRACTICAL 5

Review

- **The probability density function** $f_Y(y)$ of a continuous random variable Y is a function whose integral in a given subset $[a,b]$ gives the likelihood of the subset in the sample space of Y :

$$P\{a \leq Y \leq b\} = \int_a^b f_Y(y)dy = F_Y(b) - F_Y(a)$$

  To be a valid probability density function of a random variable, we must have $f_Y(y) \leq 0$ for all y, and

$$\int_{-\infty}^{\infty} f_Y(y)dy = 1 \tag{0.1}$$

- **The cumulative distribution function** for the continuous random variable, Y, is defined as

$$F_Y(a) = P\{Y \leq a\} = \int_{-\infty}^{a} f_Y(y)dy.$$

- **The expectation value or mean** of a discrete variable, X is given by

$$E[X] = \mu_X = \sum_{x \in R_x} xp(x).$$

and of a continuous random variable Y by

$$E[Y] = \mu_Y = \int_{-\infty}^{\infty} y f_Y(y) dy$$

- **The statistical moments of probability distribution functions** are specific quantitative measure of the shape of a function. The zeroth moment is the total probability (i.e. one, Equation 0.1), the first moment, $E[Y]$, correspond to the mean, and it is given by

  the second central moment of a continuous random variable is given by

  $$E[Y^2] = \int_{-\infty}^{\infty} y^2 f_Y(y) dy$$

  the third standardized moment is the skewness, and the fourth standardized moment is the kurtosis.

- The **variance of the random variable** X is defined by

  $$\sigma_X^2 = Var(X) = E[(X - \mu)^2]$$

  and it can be also written as

  $$\sigma_X^2 = Var(X) = E[X^2] - (E[X])^2.$$

Question 1

Let V be a random variable with probability mass function

$$p_V(v) = \begin{cases} \frac{125}{216} & \text{at } v = -1, \\ \frac{75}{216} & \text{at } v = 1, \\ \frac{15}{216} & \text{at } v = 2, \\ \frac{1}{216} & \text{at } v = 3, \\ 0 & \text{otherwise} \end{cases}$$

calculate the expectation (mean, μ_V), variance (σ_V^2) and standard deviation (σ_V) of V.

Question 2

Let X be the discrete random variable 'number of tails shown when two coins are thrown'. Define two more random variables X_1 = the number of tails shown on the first coin and X_2 the number of tails shown on the second coin.

- Show that X, X_1 and X_2 are random variables.
- Calculate the values of the expectation and the variance of the random variables.

Question 3

A variable Z has a probability density function

$$f_Z(z) = \begin{cases} Ce^{-0.0001z} & z \geq 0\,, \\ 0 & z < 0 \end{cases}$$

- calculate C to make $f_Z(z)$ a correct probability density function.
- Compute the expectation value of Z

Question 4

Calculate the expectation and variance of

$$f_T(t) = \begin{cases} \frac{2}{15} - \frac{2t}{225} & 0 \leq \mathrm{t} \leq 15\,, \\ 0 & \text{otherwise} \end{cases}$$

Question 5

A variable Y has a probability density function

$$f_Y(y) = \begin{cases} \frac{1}{4}(2y+3) & 0 \leq \mathrm{y} \leq 1\,, \\ 0 & \text{otherwise} \end{cases}$$

- compute the expectation value of Y and aY
- compute the expectation value of Y^2 and $(aY)^2$
- compute the variance of Y and aY
- compute the variance of $(aY + b)$
- would this result apply to other random variables?