UNIVERSITY OF LINCOLN
SCHOOL OF MATHEMATICS AND PHYSICS

MTH1005M PROBABILITY AND STATISTICS PRACTICAL 3

TOPICS: Calculation of probabilities of events occurring in simple sample spaces with equally likely outcomes, conditional probabilities, Bayes' formula.

Students are advised to solve by themselves the exercises unfinished during the Practical hour. Practical exercises are designed to help the students to be more confident in solving the Coursework(s) exercises.

Short review

PRINCIPLES OF COUNTING - COMBINATORIALS AND PERMUTATIONS

Permutations of N distinguishable objects: $N! = N(N-1)(N-2)\ldots(2)(1)$. Note that $0! = 1! = 1$. The Stirling formula can be used to approximate this number as

$$ln(N!) \approx N ln(N) - N$$

The number of permutations of n objects taken r at a time is determined by the following formula:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Permutation with Repetitions: The number choices for N distinguishable objects in r

groups such that in the group i contains N_i objects:

$$t = \frac{N!}{N_1!N_2!\ldots N_r!}$$

Combinations. The number of way to combine n distinguishable objects from a group of $N \leq n$ distinguishable objects is

$$c(n,r) = \binom{N}{k} = \frac{N!}{(N-n)!n!}$$

CONDITIONAL PROBABILITY

Definition : the conditional probability of the event E given the event F is

$$P(E|F) = \frac{P(E\cap F)}{P(F)}$$

combined with the standard properties of probabilities this implies

$$P(\bar{E}|F) = 1 - \frac{P(E\cap F)}{P(F)}$$

if E and F are mutually exclusive, then $P(E|F) = 0$ because $E \cap F = \emptyset$.

MULTIPLICATIVE RULE OF PROBABILITIES

The probability that E and F occur is the probability that F occurs multiplied by the probability that E occurs given that F has occurred.

$$P(E\cap F) = P(E|F)P(F)$$

and for many events

$$P(E_1 \cap E_2 \cap E_3 \cap \ldots E_n) = P(E_1)P(E_2|E_1)P(E_3|E_1\cap E_2)\ldots P(E_n|E_1\cap \ldots E_{n-1})$$

LAW OF TOTAL PROBABILITY

Given a set of exaustive event $E_i \in S$. The total probability for the event $A \cap E_i$ is given by

$$P(A) = \sum^{n} P(A\cap E_i) = \sum_{i=1}^{n} P(A|E_i)P(E_i)$$

BAYES' FORMULA

$$P(E_i|A) = \frac{P(A\cap E_i}{P(A)} = \frac{P(A|E_i)P(E_i)}{\sum_{i=1}^{n} P(A|E_i)P(E_i)}$$

with

- $P(E_i|A)$: the posterior probability.

- $P(A|E_i)P(E_i)$: likehood.
- $P(E_i)$: prior probability.
- $P(A)$: total probability (or marginal probability).

Question 1

Two fair six sided dice are thrown

- the first die comes down a 6, what is the chance the total is greater than 10?
- it is known that at least one of the dice shows > 3. Find the probability that at least one of them is a six.

Question 2

Three letters are chosen at random from all the letters of the word AEGEAN. Find the probability that

- the first letter is a consonant,
- either the second or the third letter is a vowel,
- a letter E is not included.

Question 3

In a certain college, 4% of the men and 1% of the women are taller than 6 feet. Furthermore, 60% of the students are women. Now if a student is selected at random and is taller than 6 feet, what is the probability that the student is a woman?

Question 4

Six married couples are standing in a room.

1. If 2 people are chosen at random, find the probability p that (a) they are married, (b) one is male and one is female.
2. If 4 people are chosen at random, find the probability p that (a) 2 married couples are chosen, (b) no married couple is among the 4, (c) exactly one married couple is among the 4.

Question 5

In Glasgow, half of the days have some rain. The local weather forecaster is correct $\frac{2}{3}$ of the time, i.e., the probability that she has predicted rain on a rainy day, and the probability that she predicts no rain on a dry day, are both equal to $\frac{2}{3}$.
When rain is forecast, a Glaswegian lady takes her umbrella. When rain is not forecast, she takes it with probability $\frac{1}{3}$.

- Draw a tree diagram - start labelling by whether it rains, then what the forecaster will predict and finally what the Glaswegian lady does.
- Check you know what types of probability you are labelling on the diagram.
- Calculate the probability that the lady has no umbrella, given that it rains.
- Calculate the probability that she brings her umbrella, given that it doesn't rain.

Question 6

One bag contains two dice:

- a fair one that gives equiprobable outcomes of 1 to 6.
- a "rigged" one: throwing it always gets 6.

A player has rolled one of the dice and, without examining it, rolled it (once) to get a 6. What is the probability that he used the loaded die?

Question 7

Q6: Microchips are made by three companies. 30% are supplied by firm I, 50% by II and 20% by III.
The probabilities of $A =$ *'a defect in a chip'* are $P(A|I) = 0.03$, $P(A|II) = 0.04$, $P(A|III) = 0.01$.
If an unlabeled box of chips turns up, what are the probabilities that the box came from I, II or III

(i) if a random test shows a defective chip?

(ii) if a random test shows a non-defective chip?

(iii) if the first chip was defective, and a second chip was then also tested and found defective?