UNIVERSITY OF LINCOLN
SCHOOL OF MATHEMATICS AND PHYSICS

MTH1005M PROBABILITY AND STATISTICS PRACTICAL 2

TOPICS: Calculation of probabilities of events occurring in simple sample spaces with equally likely outcomes.

Students are advised to solve by themselves the exercises unfinished during the Practical hour. Practical exercises are designed to help the students to be more confident in solving the Coursework(s) exercises.

Short review

AXIOMS OF PROBABILITY

Axiom 1 : The probability that an event will come from the sample space is unity, $P(S) = 1$

Axiom 2 : $P(A) \leq 1$ for all $A \subset S$

Axiom 3 : $P(A \cup B) = P(A) + P(B)$ if $A \cap B = 0$

SAMPLE SPACES WITH EQUALLY LIKELY OUTCOMES

Definition 1: It then follows from axiom 3 that for any event $E \subset S$ its probability will be given by the number of elementary outcomes in E

$$P(E) = \frac{\text{Number of outcomes in E}}{N} = \sum_{\omega \in E} P(\omega)$$

where N is the cardinality of S.

Definition 2: the complement of an event E (with respect to the sample set S) is

$$P(\bar{E}) = 1 - P(E)$$

Definition 3: E_1 and E_2 are mutually exclusive if

$$P(E_1 \cap E_2) = P(\emptyset) = 0$$

PROBALITY OF NOT INDEPENDENT EVENTS

Definition 4: Given two events A and B for which $A \cap B \neq 0$ then the probability for both the events is given by

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

PRINCIPLES OF COUNTING - COMBINATORIALS AND PERMUTATIONS

Permutations of N distinguishable objects: $N! = N(N-1)(N-2)\dots(2)(1)$. Note that $0! = 1! = 1$. The Stirling formula can be used to approximate this number as

$$ln(N!) \approx N ln(N) - N$$

The number of permutations of n objects taken r at a time is determined by the following formula:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Permutation with Repetitions: The number choices for N distinguishable objects in r groups such that in the group i contains N_i objects:

$$t = \frac{N!}{N_1! N_2! \dots N_r!}$$

Combinations. The number of way to combine n distinguishable objects from a group of $N \leq n$ distinguishable objects is

$$c(n,r) = \frac{N!}{(N-n)!n!}$$

Q1: A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face. (For example, a six is three times as probable as a two.)

- Write down a sample space for the experiment of rolling this die
- Work out the probability function for this die

- What is the probability of getting an even number in one throw?

Q2: Two dice are thrown. What is the probability that

- exactly one of them is a six.
- at least one of them is a six.
- at least one of them is greater than 3

You may want to draw a 6×6 table labelling the states to help you.

Q3: A coin is tossed 12 times.

- Find the probability that a coin turns up heads for the first time on the tenth, eleventh, or twelfth toss.

Q4: In a certain residential suburb, 60% of all households get Internet service from the local cable company, 80% get television service from that company, and 50% get both services from that company. If a household is randomly selected, what is the probability that it gets at least one of these two services from the company, and what is the probability that it gets exactly one of these services from the company?

Q5: An entrepreneur has 5 possible locations to base his business. If she awards each site a mark out of five (1,2,3,4,5) how many distinct scorecards are possible - if there are no restrictions in the assessment - the sites must be ranked in order of preference (no repetitions, in other words) - she is only interested in the top 2 ranked sites

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Q6: Give an answer the following questions:

- How many different ways can a chairperson and assistant chairperson be selected for a research project, if there are 7 students available?
- How many teams of two joint chairpeople, with equal powers, be selected for a research project, if there are 7 students available?
- How many distinct words can be formed using all the letters of the word 'supercalifragilisticexpialidocious'. Distinct words are orderings of letters that you could tell apart without having to label letters (and they don't have to be in the dictionary).

Q7: A coin is tossed twice. Consider the following events:

A. Heads on the first toss.

B. Heads on the second toss.

C. The two tosses come out the same.

- Show that A, B, C are pairwise independent but not independent.
- Show that C is independent of A and B but not of $A \cap B$.