Problems for Tutorials

Problem 1.

(a) If A is invertible, prove that a homogeneous system $A\mathbf{x} = \mathbf{0}$ has unique solution the zero solution $\mathbf{x} = \mathbf{0}$.

(b) If A is invertible, what is the rank of A? Justify your answer.

(c) Simplify the expression $(A^{-1}B^2C^{-2})^{-1}$.

(d) Prove that, if A satisfies the matrix equation $4A^3 - A^2 - 2I = O$, then A is invertible.

(e) Solve the matrix equation $A^{-1}X + B + 3A = O$ in terms of the matrix X.

Problem 2.

Find the inverse of the matrix:

(a)

$$A = \begin{bmatrix} 1 & -2 \\ 5 & 7 \end{bmatrix}$$

using the standard formula.

(b)

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

using the standard formula.

(c)

$$C = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

using Gauss–Jordan elimination.

By using its inverse C^{-1}, solve the system $C\mathbf{x} = \mathbf{u}$, where $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.