"Ideas..." problems for tutorials week 26

I-26.1. Let $P(x, y)$ mean "x likes y", where x is a student of UoL, and y is a song by Beatles.

(1) Express in symbolic form using quantifiers the statement "Every student in UoL does not like some of Beatles songs".

(2) Apply the rules for negations of quantified statements to express the negation of the expression obtained in part (1) so that negation sign is not before quantifiers.

(3) Translate into the natural language the formula obtained in part (2).

I-26.2. Prove or disprove the following statements:

(1) "For positive integers $a, b, n \in \mathbb{N}$, if n is divisible by a and by b, then n is divisible by ab."

(2) "For positive integers $a, b, n \in \mathbb{N}$, if n is divisible by a and by b, and $ab < n$, then n is divisible by ab."

I-26.3. Prove that among any 5 points chosen in a rectangle $8 \times 6\,\text{cm}$ there is always a pair of points that are no more than $5\,\text{cm}$ apart.