Practical 2

Problem 1. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard vectors in $\mathbb{R}^3$. Simplify the sets:

$$\begin{gathered}(\text{i})\ span(\mathbf{e}_1, \mathbf{e}_1+\mathbf{e}_2, \mathbf{e}_1+\mathbf{e}_2+\mathbf{e}_3)\\ (\text{ii})\ span(\mathbf{e}_1, -2\mathbf{e}_2, \mathbf{e}_1+\mathbf{e}_2-\mathbf{e}_3)\\ (\text{iii})\ span(\mathbf{e}_1+\mathbf{e}_2, \mathbf{e}_2, \mathbf{e}_2-3\mathbf{e}_1)\end{gathered}$$

Problem 2. If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly independent vectors, determine whether $\mathbf{u}$ can be expressed as linear combination of $\mathbf{v}, \mathbf{w}$.

Problem 3. Find the span of the following vectors:

$$(i)\ \ \mathbf{u} = \begin{bmatrix}2\\0\\3\end{bmatrix}, \mathbf{v} = \begin{bmatrix}0\\1\\1\end{bmatrix}, \quad (ii)\ \ \mathbf{u} = \begin{bmatrix}0\\-2\\0\\5\end{bmatrix}, \mathbf{v} = \begin{bmatrix}3\\1\\1\\0\end{bmatrix}, \quad (iii)\ \ \mathbf{u} = \begin{bmatrix}1\\-1\\0\\2\end{bmatrix}, \mathbf{v} = \begin{bmatrix}0\\-1/2\\3/2\\1\end{bmatrix}, \mathbf{w} = \begin{bmatrix}-1\\0\\1\\0\end{bmatrix}$$

Problem 4. Find a value for c so that the following vectors:

$$(i)\ \ \mathbf{u} = \begin{bmatrix}1\\1-c\end{bmatrix}, \mathbf{v} = \begin{bmatrix}1\\0\end{bmatrix}, \quad (ii)\ \ \mathbf{u} = \begin{bmatrix}1\\c\\3\end{bmatrix}, \mathbf{v} = \begin{bmatrix}1/3\\-1/3\\1\end{bmatrix},$$

are linearly independent.

Problem 5. Check whether the following vectors in $\mathbb{R}^3$ are linearly independent :

$$\mathbf{u} = \begin{bmatrix}1\\1\\0\end{bmatrix}, \mathbf{v} = \begin{bmatrix}-1\\1\\0\end{bmatrix}, \mathbf{w} = \begin{bmatrix}0\\1\\0\end{bmatrix},$$

and find $span(\mathbf{u}, \mathbf{v}, \mathbf{w})$.

Problem 6. Find the dot product, the length and the angle between the vectors:

$$(i)\ \ \mathbf{u} = \begin{bmatrix}1\\-1\end{bmatrix}, \mathbf{v} = \begin{bmatrix}0\\3\end{bmatrix}, \quad (ii)\ \ \mathbf{u} = \begin{bmatrix}0\\1/2\\-1/2\end{bmatrix}, \mathbf{v} = \begin{bmatrix}1/3\\2/3\\2/3\end{bmatrix}, \quad (iii)\ \ \mathbf{u} = \begin{bmatrix}0\\-1/2\\2\\2\end{bmatrix}, \mathbf{v} = \begin{bmatrix}1\\-1\\1\\-1\end{bmatrix}$$

Verify the Cauchy–Schwarz inequality and the Triangle inequality.