Ideas of mathematical proof. Practical class week 24

Note that solutions must be by the methods specified in the problems – even if you can see another solution, even if an easier one.

6.1. Let the following statements be denoted as:

$R =$ It is raining.
$S =$ The sun is shining.
$B =$ There is a rainbow.

Translate into logical formulae the following statements:

(a) There is a rainbow only if it is raining but the sun is shining.
(b) There is no rainbow although it is raining and the sun is shining.

Translate into natural language the following statements:

(c) $(\neg B \wedge R) \Rightarrow \neg S$.

(d) $S \vee (R \wedge B)$.

(It does not matter which of these statements are actually true or make sense.)

6.2. Use truth tables to prove the logical equivalences

(a) $\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$;

(b) $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$.

6.3. Use the properties of logical operations to simplify the expression

$$\neg P \wedge \neg(Q \wedge \neg P).$$

6.4. Prove by contradiction:

(a) if k is an integer such that k^2 is not divisible by 3, then k is not divisible by 3;

(b) $\sqrt{5} \notin \mathbb{Q}$.

6.5. Use the Cantor–Bernstein–Schröder theorem to show that $|A| = |B|$ by producing injective mappings $A \to B$ and $B \to A$, where A is the square $A = [0, 1] \times [0, 1]$ and B is the disc $B = \{(x, y) \mid x^2 + y^2 \leqslant 1\}$. [*Hint*: use a geometric construction by uniformly 'shrinking' one object and then placing within the other.]