Fundamental theorem of invertible matrices, Rank, Vector Spaces

1. If A and B are *n x n* matrices of rank *n*, prove that the matrix AB has rank *n* (*hint:* use the fact that A and B are invertible).

2. Show that the set S= {$\mathbf{x} \in \mathbb{R}^3$: $x_1 \leq 0$ and $x_2 \geq 0$} is not a vector space by showing that at least one of the vector space axioms is not satisfied.

3. Show that the set of all upper triangular 2x2 matrices is a vector space.