# Task 1

**Newton's root-finding method** uses the following iterative formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

and requires one initial guess to start running. Use this to find all the roots of:

i) $x^2 = 612$

ii) $x^3 = \cos x$

# Task 2

**The secant method** uses the following discrete formula:

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

and requires two initial guesses to start running. Use this to find all the roots of the same equations:

i) $x^2 = 612$

ii) $x^3 = \cos x$

How many iterations does each method need until the approximate root gets arbitrarily close to the actual root? You can implement a check to see the size of the change from one guess to the next.

# Task 3

**Newton's method for optimisation** uses the second-order Taylor expansion of $f(x)$ around a point $x_k$ for some small step size $x_k \to x_{k+1}$:

$$f(x_{k+1}) = f(x_k) + f'(x_k)(x_{k+1} - x_k) + \frac{1}{2} f''(x_k)(x_{k+1} - x_k)^2$$

Since we are wanting to find **extrema**, we want to find roots of the first derivative at $x_{k+1}$. That is, we need to differentiate with respect to $x_{k+1}$, set the resulting expression to zero, then solve for $x_{k+1}$ to get the iterative formula.

**Derive** the iterative formula for this optimisation method, then use it to find the extrema of the following functions:

i) $f(x) = x^2 - \cos x$

ii) $f(x) = x^4 - 14x^3 + 60x^2 - 70x$

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If there is something up with this worksheet, please contact me (Ewan Dalgliesh - 26192682) and I will endeavour to fix any mistakes! (Apologies in advance, of course)