Task 1

Newton's Divided Difference interpolation uses the formula

$$\begin{aligned} P(x) = & f(x_0) + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] + \dots \\ & \dots + (x - x_0)\cdots(x - x_{n-1})f[x_0, \dots, x_{n-1}, x_n], \end{aligned}$$

where

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

and, more generally,

$$f[x_0, x_1, \dots, x_{k-1}, x_k] = \frac{f[x_1, \dots, x_k] - f[x_0, \dots, x_{k-1}]}{x_k - x_0}.$$

Lagrange interpolation creates a curve that is forced to pass through particular points. It uses the formula

$$\begin{aligned} p(x) &= \frac{(x - x_1)\dots(x - x_n)}{(x_0 - x_1)\dots(x_0 - x_n)} y_0 + \frac{(x - x_1)\dots(x - x_n)}{(x_1 - x_0)\dots(x_1 - x_n)} y_1 + \dots \\ &= \sum_{i=0}^{n} \left(\prod_{\substack{0 \le j \le n \\ j \ne i}} \frac{x - x_j}{x_i - x_j} \right) y_i \end{aligned}$$

i) Determine the interpolating polynomial for the data $(0, 1), (1, 3), (3, 55)$ using Newton's Divided Difference interpolation.

ii) Determine the interpolating polynomial for the same data using Lagrange interpolation. Compare the results of the two interpolation methods.

iii) Write a program to carry out Lagrange interpolation on coordinate data.

Task 2

Bilinear interpolation involves simultaneously linearly interpolating in two directions. Given points $(x_1, y_1), (x_1, y_2), (x_2, y_1)$, and (x_2, y_2), we can first interpolate in the x direction at $y = y_1$ and at $y = y_2$.

$$\begin{aligned} f(x, y_1) &= \frac{x_2 - x}{x_2 - x_1} f(x_1, y_1) + \frac{x - x_1}{x_2 - x_1} f(x_2, y_1), \\ f(x, y_2) &= \frac{x_2 - x}{x_2 - x_1} f(x_1, y_2) + \frac{x - x_1}{x_2 - x_1} f(x_2, y_2), \end{aligned}$$

Then we can linearly interpolate in the y direction between $f(x, y_1)$ and $f(x, y_2)$.

$$
\begin{aligned}
f(x,y) &= \frac{y_2-y}{y_2-y_1} f(x,y_1) + \frac{y-y_1}{y_2-y_1} f(x,y_2) \\
f(x,y) &= \frac{y_2-y}{y_2-y_1}\left(\frac{x_2-x}{x_2-x_1} f(x_1,y_1) + \frac{x-x_1}{x_2-x_1} f(x_2,y_1)\right) \\
&\quad + \frac{y-y_1}{y_2-y_1}\left(\frac{x_2-x}{x_2-x_1} f(x_1,y_2) + \frac{x-x_1}{x_2-x_1} f(x_2,y_2)\right)
\end{aligned}
$$

i) Use bilinear interpolation to predict $T(5.25, 4.8)$ using the following data.

x	y	$T(x,y)$
2	2	60
2	6	55
9	1	57.5
9	6	70

ii) Generalise your program to conduct bilinear interpolation on different functions. Test your code using $f(x,y) = xy$. You can check your interpolation by evaluating the function at the point you interpolated to.

* If there is something up with this worksheet, please contact me (Ewan Dalgliesh) at 26192682@students.lincoln.ac.uk or will endeavour to fix any mistakes! (Apologies in advance, of course)