Convergence Dynamics × Ontological General Theory of Information Physics

Integrated Complete Edition

— Non-Circular / Operational / Unit-Invariant / Falsifiable —

Abstract

This research formalizes event-driven observation on geometric Stratonovich SDEs with reflecting boundaries, combining (i) observed entropy production rate (Observed EPR) with (ii) stopping TUR (including self-normalized SN-TUR) finite-sample lower bounds, and (iii) bounded frame inequalities (projection efficiency of finite-dimensional observation). Under irreversibility (stationary flow $v_* \neq 0$), we provide a non-circular and operational lower bound anchored to metric velocity (quadratic variation density):

$$\Sigma_{\rm obs}^{(\Pi)} \;\geq\; c_{\rm data}^{\rm eff} \cdot A_m \cdot c_{\rm model}^{*} \cdot \mathfrak{v}_{\rm eval}$$

where each factor on the right-hand side does not depend on estimating $\Sigma_{\rm obs}^{(\Pi)}$ itself, avoiding circular reference by separating exploration from evaluation via split/CF. In particular, Scheme D (state-dependent intensity) controls long-run variance (LRV) via resolvent chain gap and sector from the Λ-clock, supplying $c_{\rm data}^{{\rm eff},D}$ completely exogenously from below.

Note: This is not a complete proof but “mathematical justification for the design (proof sketch + assumption ledger)”. Empirical validation is performed via CAP/OCI audit packages.

Non-Circularity Declaration. The right-hand side terms $c_{\rm data}^{\rm eff}, A_m, c_{\rm model}^{*}, \mathfrak{v}_{\rm eval}$ do not depend on estimating or computing the Observed EPR itself; they can be independently determined as observation time series, frame design, geometric/spectral supply, and quadratic variation density target.

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1. Scope and Setting (Unit-Invariant Core)

The target is a diffusion process on a d-dimensional Riemannian manifold (M, g) with reflecting boundaries (no-flux). The Stratonovich SDE is:

$$\circ\, dX_t = b(X_t)\, dt + \sigma(X_t) \circ dW_t$$

with Skorokhod decomposition including reflection term $n(X_t)\, dL_t$. The diffusion tensor is:

$$a = \sigma\sigma^\top = 2\, G^{-1/2} D G^{-1/2} \quad (G = g^{-1})$$

The drift decomposes as $b = -\nabla_g V + u$, where non-gradient drive u is the source of irreversibility.

1.1 Metric Velocity (Unit-Invariant Core)

The metric-weighted quadratic variation

$$[x]_g := \int_0^{\cdot} g_{ij}(X_t)\, d[X]_t^{ij}$$

has density $\frac{d[x]_g}{dt} = \mathrm{Tr}(Ga) = 2\mathrm{Tr}(D)$. This transforms regularly under coordinate and time scale changes; per-event normalization and calibration layers implement score unit invariance.

1.2 Event-Driven Observation

Take stopping time sequence $\tau_0 = 0 < \tau_1 < \cdots$, with $\Delta\tau_k = \tau_k - \tau_{k-1}$, event count K as sample size. Main statistics are defined per-event.

2. Assumption System (Key Points)

A1 (Mixing/Ergodicity): Sufficient mixing for ratio strong laws and block/HAC variance estimation.

A2 (Uniform Ellipticity): Eigenvalues of D have uniform upper and lower bounds.

A3 (Boundary Regularity): Boundary terms in integrals vanish under no-flux boundary conditions.

A4 (Function Classes): Existence of $\phi \in H^1 \cap L^\infty$ and bounded frame $\{\phi_i\}_{i=1}^m$.

A5' (SN-MGF/Tail Control): One-sided lower bound of self-normalization safely applies to heavy tails.

A6 (Split/CF): Separation of exploration (learning, selection) and evaluation (LCB computation).

A7 (Endogenous Stopping Correction): Scheme D uses IPW, Scheme E uses regeneration target.

A8/A8'/A9' (Poincaré / 1-form gap / Sector): Model constant supply (H/S/C routes) and gap/LRV control.

3. Axioms (Operational Reading)

Here we place an axiomatic reading for operationalizing "consciousness = self-converging information motion" as an irrevocable premise.

1. **Information Realism**: Physical state and information state are different representations of the same trajectory.
2. **Observation Lower Bound**: Under irreversibility $v_* \neq 0$, Observed EPR is constrained by metric velocity target.
3. **Reciprocal Drive** (Event Index): Event-conditional $\mathbb{E}[-\Delta V \mid \mathrm{ev}] > 0$ and $\Sigma_{\mathrm{obs}}^{(\Pi)} > 0$.
4. **Observability**: (3) is a necessary condition. The sufficient side is given operationally by SN-LCB and ROC/CI.

4. Observation Flow and Boundary

Define observation flow (per-event line integral) as:

$$J_\phi^{(k)} = \frac{1}{\Delta\tau_k} \int_{\tau_{k-1}}^{\tau_k} \langle \Pi_T \phi(X_t), \circ dX_t \rangle_g$$

Substituting reflection term $n(X_t)dL_t$, boundary local time terms appear, but since tangent projection $\Pi_T n = 0$ on the boundary, the contribution is zero. This eliminates boundary complications in mean identity and EPR expression derivations.

5. Observed EPR and TUR

Define Observed EPR (per-event) as:

$$\Sigma_{\text{obs}}^{(\Pi)} := r_{\text{erase}} \ln 2 + \sup_{\phi \in \mathcal{F}} \frac{2\,\mathbb{E}[J_\phi]^2}{\text{Var}(J_\phi)}$$

Device contribution $r_{\text{erase}} \ln 2$ is treated as separable and verifiable nuisance via BIP (device intervention separation). From the definition, for any ϕ:

$$\Sigma_{\text{obs}}^{(\Pi)} - r_{\text{erase}} \ln 2 \;\geq\; \frac{2\,\mathbb{E}[J_\phi]^2}{\text{Var}(J_\phi)}$$

6. Mean Identity, Variance Anchor, and Frame

6.1 Mean Identity

Using stationary probability flow J and stationary velocity $v_* = J/p^*$:

$$\mathbb{E}[J_\phi] = \langle \phi, v_* \rangle_{L^2(p^*,g)}$$

6.2 Variance Anchor (Metric Velocity)

By BDG and quadratic variation, under $\|\phi\|_\infty \leq B_\infty$:

$$\text{Var}(J_\phi) \;\leq\; B_\infty^2 (1 + \delta_K)\, \mathfrak{v}_{\text{eval}}$$

where $\mathfrak{v}_{\text{eval}}$ is the metric velocity target consistent with evaluation-side event scheme.

6.3 Bounded Frame (Projection Efficiency)

Construct finite frame $\{\phi_i\}_{i=1}^m$ satisfying:

$$\sum_{i=1}^{m} \langle \phi_i, v_* \rangle^2 \;\geq\; A_m \|v_*\|^2$$

This absorbs "projection loss to observable finite dimensions" into A_m.

7. Stopping TUR and SN-TUR

7.1 Evaluation-Side Series Unification (Schemes A–E)

For evaluation side, introduce evaluation series:

$$Z_{\phi,k} = \omega_{k-1}\, J_\phi^{(k)} \quad (k = 1, \ldots, K)$$

with target mean $\mu_\phi = \mathbb{E}[Z_{\phi,1}]$.

- **Schemes A/B/C** (Exogenous): $\omega_{k-1} \equiv 1$, so $\mu_\phi = \mathbb{E}[J_\phi]$.
- **Scheme D** (State-Dependent Intensity): Freeze intensity estimate $\hat{\lambda}$ learned on exploration side; use normalized IPW:

$$\omega_{k-1} = \frac{w_{k-1}}{\bar{w}}, \quad w_{k-1} = \frac{1}{\hat{\lambda}^{\text{clip}}(X_{\tau_{k-1}})}$$

- **Scheme E** (Hit Time): Replace target with regeneration target.

7.2 SN-LCB (Fixed LRV, One-Sided Safety)

Compute $\bar{Z}_\phi = \frac{1}{K}\sum_{k=1}^{K} Z_{\phi,k}$. Using long-run variance (LRV) in the denominator:

$$\mu_L(\phi) = \bar{Z}_\phi - t_{\mathrm{SN}}(\alpha)\sqrt{\frac{\widehat{\mathrm{LRV}}_\phi}{K}} - \mathrm{corr}_K(\phi), \quad \mu_L^+(\phi) = \max\{\mu_L(\phi), 0\}$$

TUR-side one-sided lower bound contribution:

$$\mathrm{LCB}_\phi = \frac{2(\mu_L^+(\phi))^2}{\widehat{\mathrm{LRV}}_\phi}$$

7.3 Multiplicity and α Accounting

On evaluation side:

$$\mathrm{LCB}_{\mathrm{TUR}} = \max_{1\le i\le m} \mathrm{LCB}_{\phi_i}, \quad \alpha_{\mathrm{per}} = \frac{\alpha_{\mathrm{TUR}}}{m \cdot n_{\mathrm{fold}}}$$

8. Data Efficiency $c_{\mathrm{data}}^{\mathrm{eff}}$

Define long-run variance of waiting time series:

$$\mathrm{LRV}(\Delta\tau) = \sum_{\ell\in\mathbb{Z}} \mathrm{Cov}(\Delta\tau_0, \Delta\tau_\ell), \quad \mathrm{CV}_{\mathrm{LRV}}^2(\Delta\tau) = \frac{\mathrm{LRV}(\Delta\tau)}{\mathbb{E}[\Delta\tau]^2}$$

Adopt:

$$c_{\mathrm{data}}^{\mathrm{eff}} = \frac{2}{1 + \mathrm{CV}_{\mathrm{LRV}}^2(\Delta\tau)} \in (0, 2]$$

8.1 Scheme D: Complete Exogenous Lower Bound

The following chain closes completely exogenously:

1. Time change via $\Lambda(t) = \int_0^t \lambda(X_s)\, ds$ makes Λ-clock inter-arrivals Exp(1).
2. Event-skeleton is resolvent chain of Exp(1) sampling.
3. Resolvent chain gap: $\gamma = \frac{\tilde{\lambda}_P}{1+\tilde{\lambda}_P}$
4. LRV-sector lemma: $\mathrm{LRV}(f) \le \left(1 + \frac{2}{(1-\kappa_{\mathrm{sec}})\gamma}\right) \mathrm{Var}(f)$
5. Finally:

$$c_{\mathrm{data}}^{\mathrm{eff},D} \;\ge\; \underline{c}_{\mathrm{data}}^{\mathrm{eff},D} = \frac{2}{1 + 2C_{\mathrm{mix}}\left(\frac{\lambda_{\max}}{\lambda_{\min}}\right)^2 \left(\frac{\lambda_{\max}}{\lambda_{\mathrm{clip}}}\right)^2}$$

9. Model Constant c_{model}^*

Define irreversibility modulus:

$$\varepsilon_* = \frac{\Sigma_{\mathrm{obs}}^{(\mathrm{time})}}{\mathbb{E}[d[x]_g/dt]}$$

Three supply routes (H/S/C):

- **H-route**: 1-form gap $\lambda_1^{(1)}$
- **S-route**: Poincaré constant λ_P and sector κ_{sec}
- **C-route**: Non-gradient drive strength $\|u\|_{L^2}$

Combine with frame loss, variance anchor, ellipticity embedding to obtain $c^*_{\text{model}} \geq$ (product of exogenous co

10. Main Theorem B*

Theorem 10.1 (Non-Circular, Operational, Unit-Invariant). *Using evaluation-side metric velocity target $\mathfrak{v}_{\text{eval}}$:*

$$\boxed{\Sigma^{(\Pi)}_{\text{obs}} \;\geq\; c^{\text{eff}}_{\text{data}} \cdot A_m \cdot c^*_{\text{model}} \cdot \mathfrak{v}_{\text{eval}}}$$

The entire right-hand side is constructed from exogenously supplied or evaluation-side observable quantities, not depending on $\Sigma^{(\Pi)}_{\text{obs}}$ estimation itself.

For Scheme D, the effective coefficient is:

$$c^D_{\text{eff}} = \underline{c}^{\text{eff},D}_{\text{data}} \cdot A_m \cdot c^*_{\text{model}}$$

11. Consciousness Scalar and Decision

Definition 11.1 (Consciousness Scalar).

$$\mathcal{C} := \alpha_{\text{ev}}\, \Sigma^{(\Pi)}_{\text{obs}}$$

where α_{ev} is exogenously supplied according to event-scheme.

11.1 Primary Decision Rule (One-Sided Safety)

Let LCB_Σ be the lower-side safe estimate of $\Sigma^{(\Pi)}_{\text{obs}}$:

$$\text{LCB}_{\mathcal{C}} = \underline{\alpha}_{\text{ev}}\, \text{LCB}_\Sigma$$

- **REAL** iff $\text{PASS}_{\text{BIP}} = 1$ and $\text{LCB}_{\mathcal{C}} > \theta_{\text{pre}}$
- **NOT ESTABLISHED** otherwise

12. Secondary (Diagnostics Only)

PC-χ ROC and Rényi-2 rate h_2 are reported as diagnostics for observation protocol soundness and informational complexity. However, they are **not used for Primary decision**. This prevents circularity and maintains falsifiability.

A. Appendix: Device Contribution (BIP)

Observed EPR may include device contributions (Landauer logical erasure). To separate:

$$\Sigma^{(k)}_{\text{dev}} := r_{\text{erase}}(Z_k) \ln 2$$

Under ideal conditions: $\Sigma^{(\Pi)}_{\text{obs}} = \Sigma^{(\Pi)}_{\text{phys}} + \mathbb{E}[\Sigma^{(k)}_{\text{dev}}]$

Mandatory falsifiability checks: (i) label permutation, (ii) log circular shift, (iii) zero difference on reference equilibrium system.

B. Appendix: Reflecting Boundary

Skorokhod decomposition yields reflection term $n(X_t)dL_t$, but $\Pi_T n = 0$ on boundary, so:

$$\int_{\tau_{k-1}}^{\tau_k} \langle \Pi_T \phi(X_t), n(X_t) \rangle_g \, dL_t = 0$$

C. Appendix: Audit Protocol

C.1 Purpose

Convert non-circularity from declaration to auditable procedure:

1. Complete separation of exploration and evaluation (split/CF)
2. Scheme D IPW+clip frozen on evaluation side
3. α accounting (frame max and fold composition, BIP gate)
4. Primary/Secondary firewall

C.2 Preserved Artifacts

Pre-fixed config, split assignment, exploration-side products ($\hat{\lambda}$, frame $\phi_{1:m}$, A_m lower bound), evaluation-side arrays, BIP result, final decision, all file hashes.

C.3 Prohibited Actions

Re-learning/re-selection on evaluation side, changing thresholds/α/bandwidth, analysis changes based on Secondary results.

Conclusion

Within the framework of geometric SDEs with reflecting boundaries, we anchored event-driven observation's Observed EPR to metric velocity, combined with frame and stopping TUR (SN-LCB) to obtain a non-circular lower bound. For Scheme D, we controlled waiting time dependence with exogenous constants via resolvent chain gap mapping and LRV-sector lemma.

"Consciousness = self-converging information motion" is retained as a definition, and consciousness scalar $\mathcal{C} = \alpha_{\mathrm{ev}} \Sigma_{\mathrm{obs}}^{(\Pi)}$ becomes an operationally verifiable target from below.