MATH 314 Fall 2011

Practice Final Exam

The final exam covers:	
Chapter 5	5.2, 5.3, 5.4
Chapter 6	6.1, 6.2
Chapter 7	7.1
all the basic concepts from earlier chapters	

Things you need to know:

(a) Concepts:

- linear independence, spanning set, basis, dimension, coordinates
- rank, nullity, the four fundamental spaces
- inner product, norm, distance
- orthogonal vectors, orthogonal basis, orthonormal basis

(b) Procedures:

- solving equations by Gaussian elimination/Gauss-Jordan/inverse matrix
- finding bases for the four fundamental spaces
- finding othogonal/orthonormal bases by Gramm-Schmidt
- deciding if a set is linearly dependent/independent/basis
- computing coordinates with respect to a basis/orthogonal basis

You are allowed to bring 1 sheet of notes to the exam.

The final is in 110 Avery Hall, 10-12 Thursday 12/14.

1. Fill in the following definitions:

(a) Let V be a vector space. A **basis** of V is a set $\mathcal{B}$ such that

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(b) Let W be a subspace of $\mathbb{R}^n$. The **orthogonal complement** $W^\perp$ is

(c) Let A be a matrix. The **nullspace** $Null(A)$ is

(d) Let V be a vector space. An **inner product** $\langle , \rangle$ on V has the following properties:

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Note: Throughout $\mathcal{P}_2$ denotes the set of polynomials of the form ax^2+bx+c with a, b, c any real numbers and $M_{2,2}$ denotes the set of all 2×2 matrices.

2. Give examples of:

(a) Four different vector spaces each having dimension 10.

(b) A set of linearly independent polynomials in $\mathcal{P}_2$ which is not a basis.

(c) A basis of $\mathcal{P}_2$ obtained by extending the set in part (b).

(d) A set of matrices that span $M_{2,2}$ but do not form a basis.

(e) A subspace W of $M_{2,2}$ of such that the dimension of W is 2.

3. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ -3 & 1 & -1 \\ 5 & 2 & 4 \\ 0 & -2 & -1 \\ 5 & 3 & 5 \end{bmatrix}$.

(a) Find bases for $Row(A)$ and $Null(A)$.

(b) Find bases for $Col(A)$ and $Null(A^T)$.

(c) Find the rank of A and the nullity of A. Check the rank theorem.

(d) Find the rank and the nullity of A^T. Check the rank theorem for A^T.

4. Consider $W = \mathit{Span}\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} \right\}$ as a subspace of $\mathbb{R}^3$.

(a) Find $dim(W)$ and say what W is as a geometric object.

(b) Find and **orthonormal** basis for W.

(c) Find the projection of the vector $\vec{\mathbf{v}} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ onto W.

(d) Find the distance from the point $(1, 2, 0)$ to the subspace W.

(e) Describe $W^{\perp}$ (the orthogonal complement of W) by giving a basis for $W^{\perp}$ and describing what it is as a geometric object.

(f) Write the linear equation for which the solution set is W.

5. For this problem we work in the vector space $\mathcal{P}_2$ with the inner product $\langle P(x), Q(x)\rangle = \int_0^1 P(x)Q(x)dx$.

(a) Show that the polynomials $P(x) = 1 + x, Q(x) = 1 - x, R(x) = x^2$ are linearly independent.

(b) Show that $Span(P, Q, R) = \mathcal{P}_2$. (Use the same P, Q, R as in part (a)).

(c) Explain why $\{P, Q, R\}$ is a basis of $\mathcal{P}_2$.

(d) Find an orthogonal basis of $\mathcal{P}_2$ with respect to the inner product described in the beginning of the problem.

(e) Find the coordinates of the polynomial $T(x) = x^2 + x + 1$ with respect to the orthogonal basis in (d).

6. True or false? If true give a proof if false say why it fails to be true.

(a) $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ with } a + c = b + d \right\}$ is a subspace of $\mathcal{M}_{2\times 2}$ with the usual addition and scalar multiplication.

(b) $W = \{A \in \mathcal{M}_{2\times 2} \text{ with } det(A) = 1\}$ is a subspace of $\mathcal{M}_{2\times 2}$ with the usual addition and scalar multiplication.

(c) $\mathcal{B} = \{1 + x, 2 - x + x^2, 3x - 2x^2, -1 + 3x + x^2\}$ is a basis for $\mathcal{P}_2$.

(d) If V and W are subspaces of $\mathbb{R}^3$ then

$$V \cap W = \text{ the set of vectors that are both in } V \text{ and in } W$$

is a subspace of $\mathbb{R}^3$.

(e) If V and W are subspaces of $\mathbb{R}^3$ then

$$V + W = \{\vec{\mathbf{v}} + \vec{\mathbf{w}} \text{ with } \vec{\mathbf{v}} \in V, \vec{\mathbf{w}} \in W\}$$

is a subspace of $\mathbb{R}^3$.