MATH 314 Fall 2011 Section 001

Practice Midterm 2

This midterm covers:	
Chapter 4	4.1, 4.2, 4.3, 4.4, 4.6
Chapter 5	5.1

Things you need to know:

(a) Concepts:

- linear independence, spanning set, basis
- characteristic equation, eigenvalue, eigenvector, eigenspace
- orthogonal vectors, orthogonal basis, orthonormal basis
- coordinates with respect to a given basis

(b) Procedures:

- computing determinants: using the quick formulas (2x2 and 3x3 only), by expansion along a row or column, by row operations
- properties of determinants
- computing cross products
- finding eigenvalues, eigenvectors, eigenspaces, multiplicities
- diagonalizing a matrix (finding P and D)
- computing coordinates with respect to an orthogonal basis
- computing orthogonal projections

No notes, books or calculators are to be used during the test. The midterm will be in 106 Avery Hall from 6-8 pm

on Thursday 11/03.

1. Compute the following:

(a)
$$\begin{vmatrix} 1 & 1 & 0 & 3 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 \end{vmatrix} =$$

(b)
$$\begin{vmatrix} 1 & 1 & 0 & 3 \\ 2 & 2 & 0 & 6 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 \end{vmatrix} =$$

(c)
$$\begin{vmatrix} a & b & 0 & \dots & 0 \\ c & d & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & & \ddots & \\ 0 & 0 & 0 & \dots & 1 \end{vmatrix} =$$

(all entries off the diagonal are 0 and all entries on the diagonal are 1 except in the leftmost 2×2 block)

(d) Given
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$
, compute:

- (a) $det(A), det(5A), det(A^{-1}), det(A^{10})$
- (b) all the minors of A
- (c) all the cofactors of A
- (d) the adjoint matrix of A
- (e) the inverse matrix A^{-1} using the adjoint formula

2. Let
$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$
.

(a) Find the characteristic equation of A.

- (b) Find the eigenvalues of A.
- (c) Find the eigenspaces of A.

(d) Find for each eigenvalue the algebraic and geometric multiplicity. Is A diagonalizable?

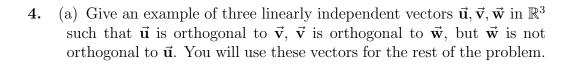
3. Let A be an unknown 3x3 matrix that has eigenvalues $\lambda_1 = 3$ and $\lambda_2 = 1$ and corresponding eigenspaces

$$E_{\lambda_1} = Span \left\{ \begin{bmatrix} 1\\1\\0 \end{bmatrix}, \begin{bmatrix} 2\\0\\1 \end{bmatrix} \right\}, E_{\lambda_2} = Span \left\{ \begin{bmatrix} 3\\1\\0 \end{bmatrix} \right\}$$

- (a) What are the algebraic and geometric multiplicities of λ_1, λ_2 ? Explain.
- (b) Find an invertible matrix P and a diagonal matrix D such that

$$P^{-1}AP = D.$$

- (c) Compute A^{100} .
- (d) Find a probability vector $\vec{\mathbf{x}}$ such that $A\vec{\mathbf{x}} = \vec{\mathbf{x}}$.
- (e) Can A be a stochastic matrix?



(b) Let
$$S = Span\{\vec{\mathbf{u}}, \vec{\mathbf{v}}\}$$
. Compute $proj_S(\vec{\mathbf{w}})$ and $perp_S(\vec{\mathbf{w}})$.

(c) Find an orthogonal basis $\mathcal B$ of $\mathbb R^3$ that contains the vectors $\vec{\mathbf u}$ and $\vec{\mathbf v}$.

(d) Find the coordinates of $\vec{\mathbf{w}}$ with respect to the basis \mathcal{B} described above.

5. The purpose of this problem is to prove that eigenspaces are subspaces.
(a) Let A be an $n \times n$ matrix. Define what an eigenvalue λ of A is and what an eigenvector $\vec{\mathbf{v}}$ corresponding to the eigenvalue λ is.
(b) Let $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ be eigenvectors corresponding to the <i>same</i> eigenvalue λ . Prove that $\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2$ is an eigenvector corresponding to the eigenvalue λ as well.
(c) Let $\vec{\mathbf{v}}$ be an eigenvectors corresponding to the eigenvalue λ and let c be a scalar. Prove that $c\vec{\mathbf{v}}$ is an eigenvector corresponding to the eigenvalue λ .
(d) Define the eigenspace E_{λ} and explain how the facts above prove it is a subspace of \mathbb{R}^n .