MATH 314 Fall 2011
Section 001

Practice Midterm 1

This midterm covers:	
Chapter 1	1.1, 1.2, 1.3
Chapter 2	2.1, 2.2, 2.3
Chapter 3	3.1, 3.2, 3.3, 3.5, 3.6

No notes, books or calculators are to be used during the test.

The midterm will be in 106 Avery Hall from 6-8 pm on Thursday 09/29.

1. (a) Find the equation of the plane in $\mathbb{R}^3$ that passes through the origin and has normal vector

$$\vec{\mathbf{n}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

(b) Find a and b such that the plane in $\mathbb{R}^3$ of equation $ax + by + z = 2$ is parallel to the plane in part a).

(c) Find the vector equation of the line in $\mathbb{R}^3$ which passes through the point $P = (1, 1, 1)$ and is perpendicular to the plane in a).

2. Find all the values of k such that the linear system

$$\left\{\begin{array}{rcl} y + 2kz & = & 0 \\ x + 2y + 6z & = & 2 \\ kx + 2z & = & 1 \end{array}\right.$$

has

(a) no solutions.

(b) a unique solution.

(c) infinitely many solutions.

3. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 5 & -3 \\ 0 & 1 & -2 \\ -2 & -7 & 0 \end{bmatrix}$.

(b) Check that the matrix you found is really A^{-1}.

(c) Use part a) to write $\vec{\mathbf{b}} = \begin{bmatrix} -4 \\ -1 \\ 5 \end{bmatrix}$ as a linear combination of the vectors

$$\vec{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \vec{\mathbf{v}}_2 = \begin{bmatrix} 5 \\ 1 \\ -7 \end{bmatrix}, \vec{\mathbf{v}}_3 = \begin{bmatrix} -3 \\ -2 \\ 0 \end{bmatrix}.$$

4. Let $A = \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$. You may assume that $RREF(A) = \begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for the row space o A.

(b) Find a basis for the null space o A.

(c) Check that the rank nullity theorem holds for A.

5. (a) Define what a *subspace* is.

(b) Is $S = \{\begin{bmatrix} x \\ y \end{bmatrix}$ with $x \geq 0, y \leq 0\}$ a subspace of $\mathbb{R}^2$? Why or why not?

6. Are the following statements *true* or *false*. If you believe a statement is false give a counterexample. If you believe the statement is true state why it is true.

(a) Let A be a 3×4 matrix. Then the columns of A must be linearly dependent vectors.

(b) Let A be a 3×4 matrix. Then the nullity of A must be 1.

(c) If A, B, C are all $n \times n$ matrices and $AB = AC$ then $B = C$

(d) If A, B are $n \times n$ matrix and $AB = \mathbf{O}_{n \times n}$, then $A = \mathbf{O}_{n \times n}$ or $B = \mathbf{O}_{n \times n}$. (Here $\mathbf{O}_{n \times n}$ is the $n \times n$ zero matrix.)

7. Prove that if A is an invertible $n \times n$ matrix then

$$(A^T)^{-1} = (A^{-1})^T.$$

(Recall A^T means the matrix transpose and A^{-1} means the matrix inverse.)