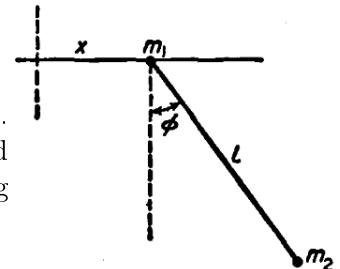


Exercises marked with (*) have extra difficulty, don't hesitate to ask for help.

1. Pendulum with free point of support [Landau §5 ex. 2]

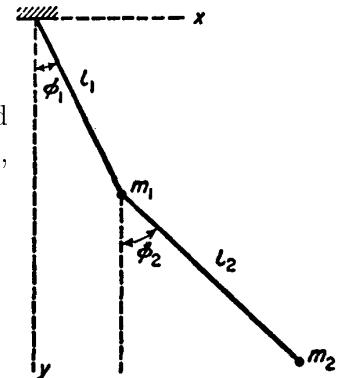
Particle of mass m_2 is hanging from a rigid bar of length ℓ and negligible mass. On the other end there is a device of mass m_1 linked to a horizontal bar, and it's free to move horizontally along the x axis. The device allows the hanging bar to span any angle φ with respect to the vertical axis.



- (a) After determining the generalized coordinates, write the position of each particle as a function of them.
- (b) Calculate the velocity of each particle.
- (c) Using these velocities, calculate the kinetic energy, T , and potential energy, V , for each particle.
- (d) Now calculate T and V using the functions with the masses and positions as inputs. Verify that you get the same result but in fewer steps.
- (e) Perform the necessary substitutions into the expressions for T and V found in the previous step, so that the particle with mass m_1 remains at rest. Verify that you recover the expressions of T and V of an ideal pendulum.

2. Double coplanar pendulum [Landau §5 ex. 1]

A rigid bar of length ℓ_1 and negligible mass has a particle of mass m_1 attached to one end. There is a second bar of negligible mass hanging from the first one, of length ℓ_2 , with a particle of mass m_2 attached to the other end too.



- (a) Write expressions for kinetic energy, T , and potential energy, V , as functions of the generalized coordinates suggested by the figure.

Result:

$$T_{\text{translational}} = \frac{\ell_1^2 m_1 \dot{\varphi}_1^2}{2} + \frac{m_2 (\ell_1^2 \dot{\varphi}_1^2 + 2\ell_1 \ell_2 \cos(\varphi_1 - \varphi_2) \dot{\varphi}_1 \dot{\varphi}_2 + \ell_2^2 \dot{\varphi}_2^2)}{2}$$

$$V_{\text{gravitational}} = -g (\ell_1 m_1 \cos(\varphi_1) + \ell_1 m_2 \cos(\varphi_1) + \ell_2 m_2 \cos(\varphi_2))$$

- (b) Use the `subs` function of SymPy to set $m_1 = 0$, $\varphi_1 = \varphi_2 = \varphi$ and $\ell_1 = \ell_2 = \frac{\ell}{2}$. Verify that you recover the expressions of T and V of an ideal pendulum.

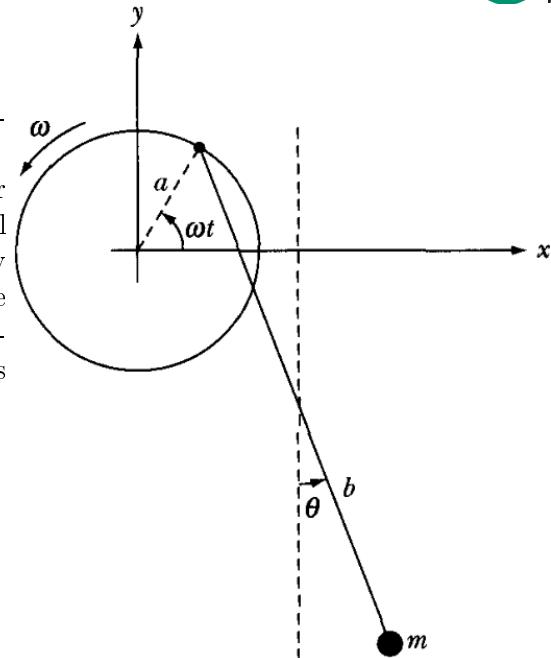
3. (*) Pendulum with rotating point of support [Marion (e) ex. 7.5] [Landau §5 ex. 3]

A particle of mass m is attached to the end of a rigid bar of length b . The point of support is linked to a vertical circle of radius a , and it rotates with constant frequency ω . It is assumed that all positions lie in the same plane and the mass of the bar is negligible. Calculate the kinetic energy, T , and potential V , of the particle of mass m .

Result:

$$T_{\text{translational}} = \frac{m(a^2\omega^2 - 2ab\omega \sin(\omega t - \theta)\dot{\theta} + b^2\dot{\theta}^2)}{2}$$

$$V_{\text{gravitational}} = gm(a \sin(\omega t) - b \cos(\theta))$$



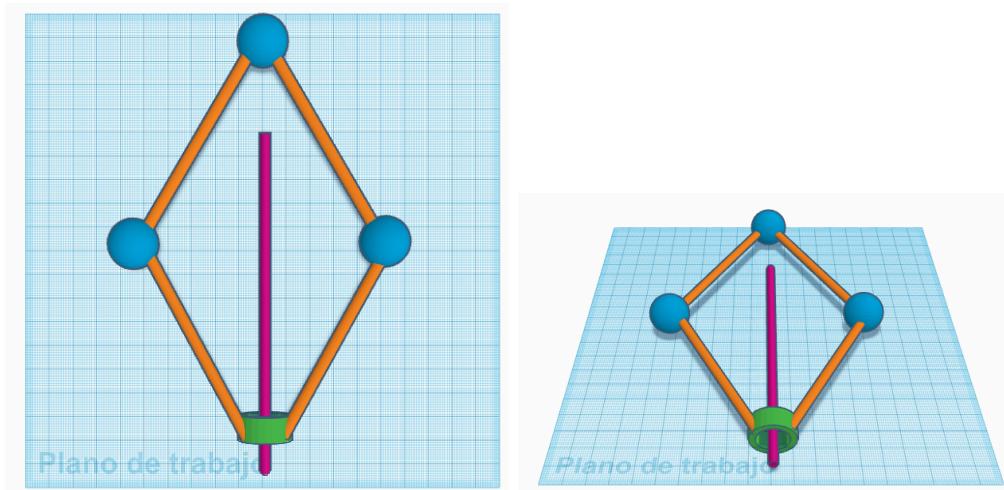
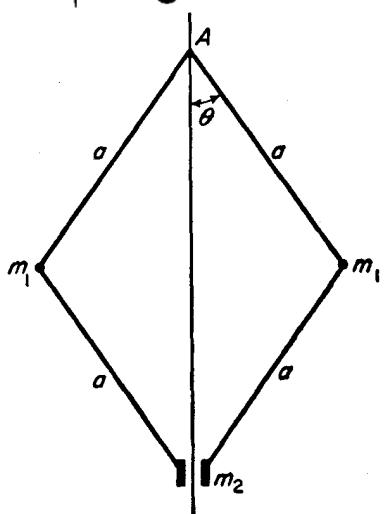
4. (*) Coupled weights rotating about a vertical axis [Landau §5 ex. 4]

Particle with mass m_2 moves on a vertical axis and the whole system rotates about this axis with a constant angular velocity Ω . This particle is linked to two particles of mass m_1 through bars of length a and negligible mass, and at the same time, these particles are linked to the fixed point A through identical bars, forming the variable angle θ with respect to the vertical axis. Calculate the kinetic energy of each of the three particles and find a compact expression of the kinetic energy of the whole system. Do the same for the potential energy.

Result:

$$T_{\text{translational}} = a^2 \left(m_1 (\Omega^2 \sin^2(\theta) + \dot{\theta}^2) + 2m_2 \sin^2(\theta)\dot{\theta}^2 \right)$$

$$V_{\text{gravitational}} = -2ag(m_1 + m_2) \cos(\theta)$$



In these figures, the top sphere is the fixed point A . Everything revolves around the pink axis with constant angular speed Ω . Therefore, the particles on each side, of mass m_1 , rotate entering and leaving the plane shown in the first image. This is equivalent to thinking that the light-blue plane rotates around the pink axis.

The piece of mass m_2 is a sliding hub that moves vertically without friction.

