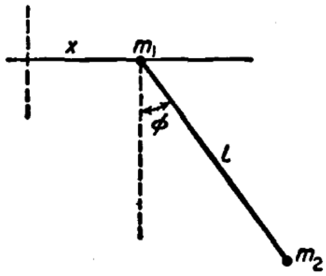
Exercises marked with (*) have extra difficulty, don't hesitate to ask for help.

1. **Pendulum with free point of support** [Landau §5 ex. 2]
Particle of mass m_2 is hanging from a rigid bar of length ℓ and negligible mass. On the other end there is a device of mass m_1 linked to a horizontal bar, and it's free to move horizontally along the x axis. The device allows the hanging bar to span any angle φ with respect to the vertical axis.



   (a) After determining the generalized coordinates, write the position of each particle as a function of them.

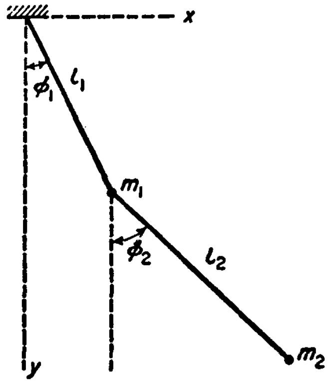
   (b) Calculate the velocity of each particle.

   (c) Using these velocities, calculate the kinetic energy, T, and potential energy, V, for each particle.

   (d) Now calculate T and V using the functions with the masses and positions as inputs. Verify that you get the same result but in fewer steps.

   (e) Perform the necessary substitutions into the expressions for T and V found in the previous step, so that the particle with mass m_1 remains at rest. Verify that you recover the expressions of T and V of an ideal pendulum.

2. **Double coplanar pendulum** [Landau §5 ex. 1]
A ridig bar of lentgh ℓ_1 and negligible mass has a particle of mass m_1 attached to one end. There is a second bar of negligible mass hanging from the first one, of length ℓ_2, with a particle of mass m_2 attached to the other end too.

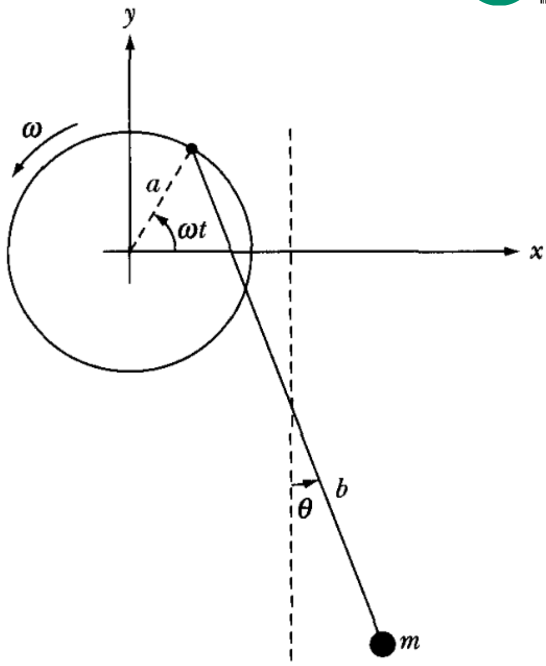


   (a) Write expressions for kinetic energy, T, and potential energy, V, as functions of the generalized coordinates suggested by the figure.

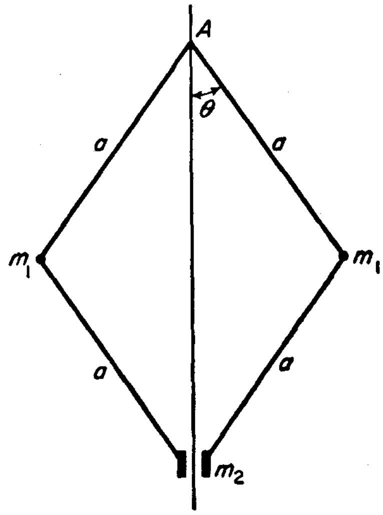
   Result:
   $T_{\text{translational}} = \frac{\ell_1^2 m_1 \dot{\varphi}_1^2}{2} + \frac{m_2\left(\ell_1^2\dot{\varphi}_1^2 + 2\ell_1\ell_2\cos\left(\varphi_1-\varphi_2\right)\dot{\varphi}_1\dot{\varphi}_2 + \ell_2^2\dot{\varphi}_2^2\right)}{2}$
   $V_{\text{gravitational}} = -g\left(\ell_1 m_1\cos\left(\varphi_1\right) + \ell_1 m_2\cos\left(\varphi_1\right) + \ell_2 m_2\cos\left(\varphi_2\right)\right)$

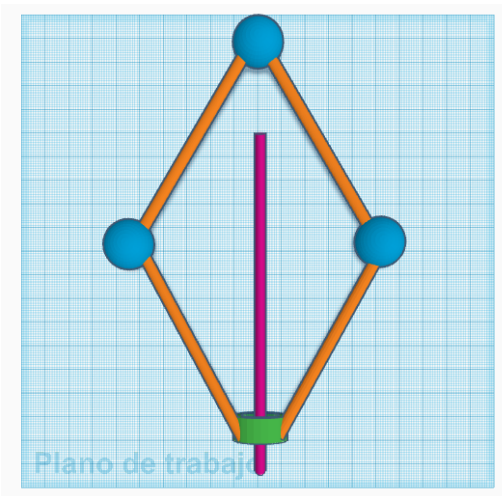
   (b) Use the `subs` function of `SymPy` to set $m_1 = 0$, $\varphi_1 = \varphi_2 = \varphi$ and $\ell_1 = \ell_2 = \frac{\ell}{2}$. Verify that you recover the expressions of T and V of an ideal pendulum.

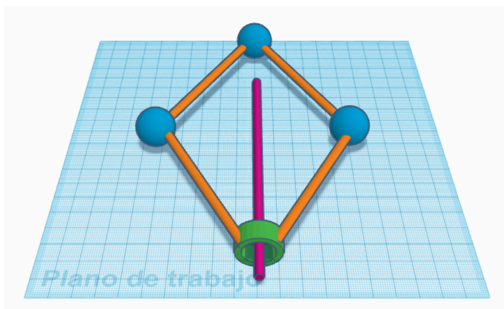
3. (*) **Pendulum with rotating point of support** [Marion (e) ex. 7.5] [Landau §5 ex. 3]
A particle of mass m is attached to the end of a rigid bar of length b. The point of support is linked to a vertical circle of radius a and it rotates with constant frequency ω. It is assumed that all positions lie in the same plane and the mass of the bar is negligible. Calculate the kinetic energy, T, and potential V, of the particle of mass m.
Result:
$$T_{\text{translational}} = \frac{m\left(a^2\omega^2 - 2ab\omega\sin(\omega t - \theta)\dot{\theta} + b^2\dot{\theta}^2\right)}{2}$$
$$V_{\text{gravitational}} = gm\left(a\sin(\omega t) - b\cos(\theta)\right)$$



4. (*) **Coupled weights rotating about a vertical axis** [Landau §5 ex. 4]
Particle with mass m_2 moves on a vertical axis and the whole system rotates about this axis with a constant angular velocity Ω. This particle is linked to two particles of mass m_1 through bars of length a and negligible mass, and at the same time, these particles are linked to the fixed point A through identical bars, forming the variable angle θ with respect to the vertical axis. Calculate the kinetic energy of each of the three particles and find a compact expression of the kinetic energy of the whole system. Do the same for the potential energy.
Result:
$$T_{\text{translational}} = a^2\left(m_1\left(\Omega^2\sin^2(\theta) + \dot{\theta}^2\right) + 2m_2\sin^2(\theta)\dot{\theta}^2\right)$$
$$V_{\text{gravitational}} = -2ag\left(m_1 + m_2\right)\cos(\theta)$$

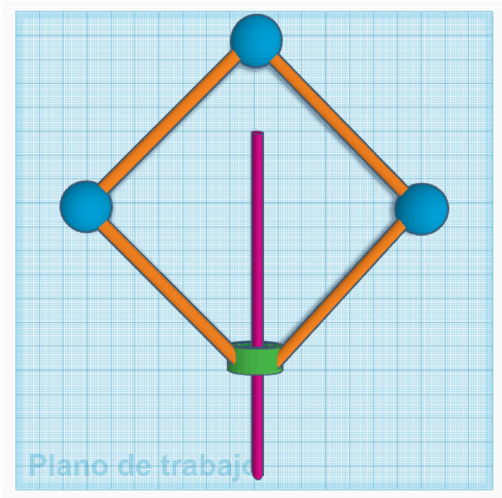


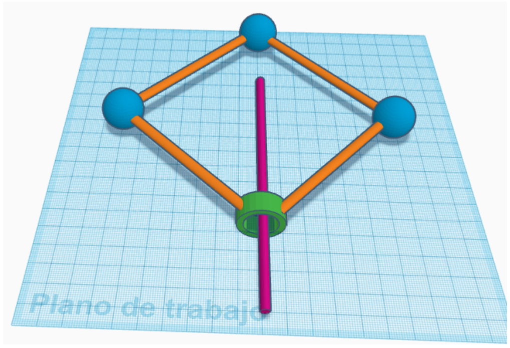




In these figures, the top sphere is the fixed point A. Everything revolves around the pink axis with constant angular speed Ω. Therefore, the particles on each side, of mass m_1, rotate entering and leaving the plane shown in the first image. This is equivalent to thinking that the lightblue plane rotates around the pink axis.

The piece of mass m_2 is a sliding hub that moves vertically without friction.


Plano de trabajo


Plano de trabajo