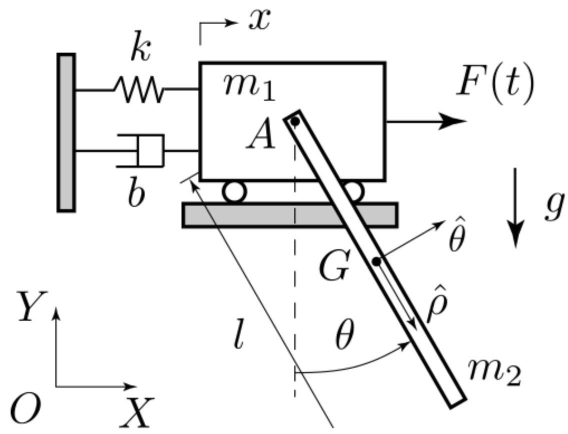
External forces in the Lagrangian formulation

1. **Rod hanging from a cart**
   Find the dynamics equations for the system. The moment of inertia for a rod of mass m and length l about an axis passing through one end of the rod is $\frac{m}{12}l^2$.



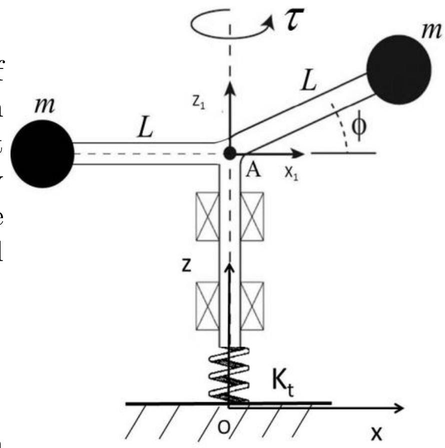
   (a) Write the Lagrangian.

   (b) Write the nonconservative forces as generalized forces:

   - the external force $\vec{F}(t)$,
   - and the force exerted by the dumping system of constant b as a function of the cart's velocity, $-b\dot{x}\hat{x}$.

   (c) Find the Euler-Lagrange equations.

2. **Unbalanced torsion pendulum**
   Two beads of identical masses m are attached to the ends of arms of negligible mass. One of these arms is inclined at a fixed angle ϕ with respect to the horizontal. There is no friction with the system that keeps the axis of rotation vertical. This axis is free to rotate at any angle θ and a torsion spring of constant K_t exerts an opposite torque each time $\theta \neq 0$. Additionally to this torque, there is an external torque that is a function of time: $\vec{\tau} = \tau(t)\hat{z}$.
   Question: What is the unit for the generalized force?

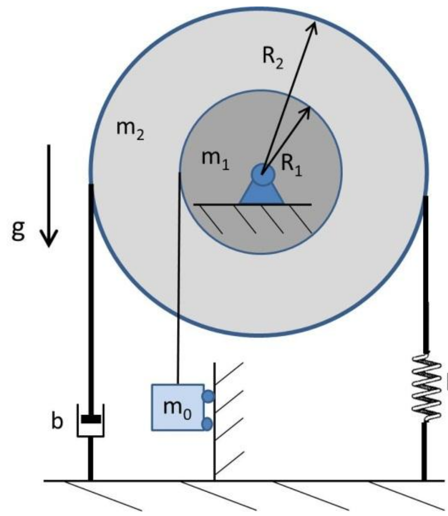


   a) N b) $\frac{\mathrm{N}}{\mathrm{m}}$ c) N m d) Other

   Solve for the angular acceleration using the Euler-Lagrange equation for the dynamics of this system. Result:

$$\ddot{\theta} = \frac{K_T\theta + \tau}{L^2 m\left(\sin^2(\phi) - 2\right)}$$

3. **Fixed cylinders**
   Two homogeneous cylinders of masses and radii m_1,m_2, R_1 and R_2 are welded together. This arrangement rotates without friction about an axis. A string of negligible mass is wounded around the external cylinder and its ends connect a spring of constant k and a damper. This damper exerts a force opposite to the linear velocity,

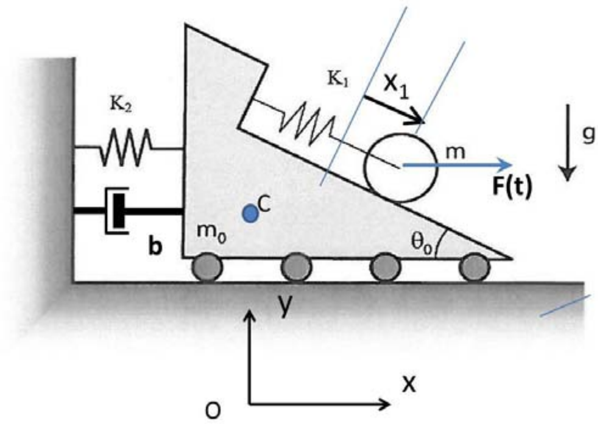


$$\vec{F}_{\text{damper}} = -b\dot{\vec{r}}.$$

   A string of negligible mass is wrapped around the cylinder with the smaller radius, and a block of mass m_o hangs from it.
   Solve for the angular acceleration using the Euler-Lagrange equation. Result:

$$\ddot{\theta} = \frac{2\left(R_1 g m_0 - R_2^2 b\dot{\theta} - R_2^2 k\theta\right)}{2R_1^2 m_0 + R_1^2 m_1 + R_2^2 m_2}$$

4. **Oscillating inclined plane**
A disk of radius R and mass m rotates over the surface of the cart of mass m_0, inclined at an angle θ_0. This disk won't leave the surface, even when the force $\vec{F} = F(t)\hat{x}$ is exerted upon it, due to a spring of constant K_1 that links its center with the cart. There is also a spring of constant K_2 attached to the wall and a damper, proportional to the velocity, of constant b. Both springs are initially at their natural lengths l_{10} and l_{20}. There is no friction between the cart and the floor.



Question: What is the expression for the generalized force corresponding to the virtual displacement δx due to $\vec{F}$?

a) $F(t)\cos(\theta)$ b) $F(t)$ c) $F(t)\delta x$ d) 0

Find the Euler-Lagrange equations.

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