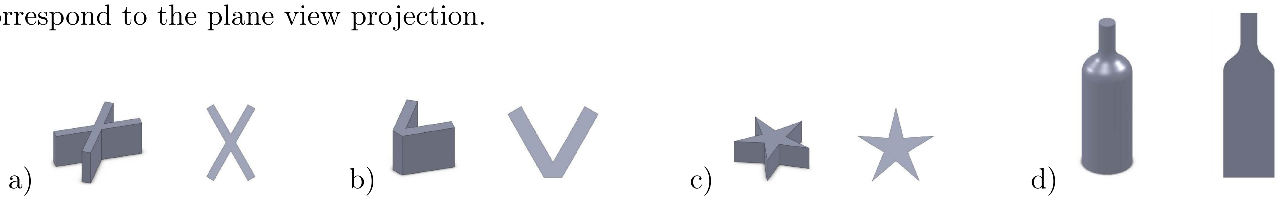
Rigid Body | Continuous Mass Distributions

1. **Moment of Inertia Tensor of a Rod**
A rod of mass $m = 1\,\text{kg}$ and negligible cross-section compared to its length $l = 1\,\text{m}$ is given. Align an axis $(\hat{z})$ with it.

   a) Calculate its moments of inertia.

   b) Show what happens with the products of inertia.

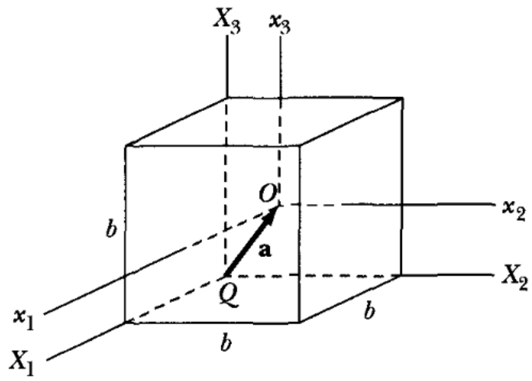
2. **Convenient Axes for Calculating Moments of Inertia**
Perspective views of various objects are drawn. On these, draw the axes intersecting at the most convenient point for calculating moments of inertia, that is, at the center of mass. Do the same with the two axes that correspond to the plane view projection.



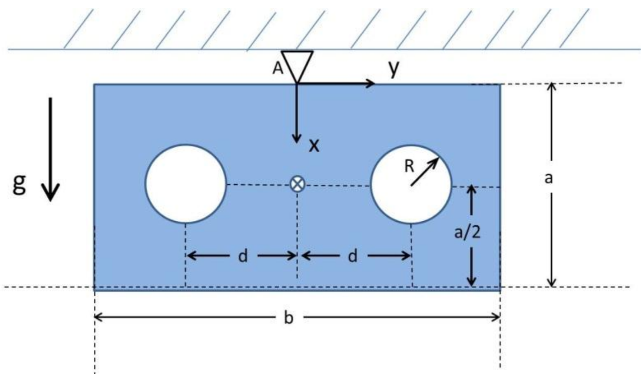
3. **Cube with Edge b** [Marion (e) ex. 11-3]

   (a) Calculate the inertia tensor from the coordinate system x_i with origin at the center of mass O.

   (b) Use the general form of Steiner's parallel axis theorem to calculate it in the X_i system with origin at vertex Q



4. **Perforated Plate**
In a plate of homogeneous density, two openings were cut symmetrically. Suspended from point A, it *oscillates* in the x, y plane. Therefore, it is relevant to know its moment of inertia I_{zz} from that point. Use the data available in a workshop: thickness e of the material, plane dimensions, and a measured mass m.
Follow this suggested sequence:



   (a) Calculate the metal density of the plate considering the missing area due to the perforations.

   (b) Calculate I_{zz} of one of the circular perforations as if it were made of this metal.

   (c) Calculate I_{zz} of an unperforated plate from its center of mass.

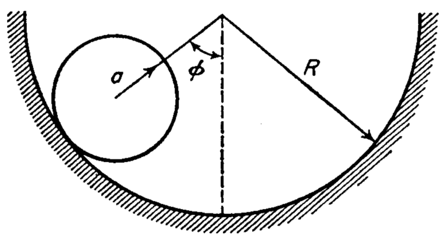
   (d) Transfer with Steiner's theorem the I_{zz} of both circular perforations to the center of the plate.

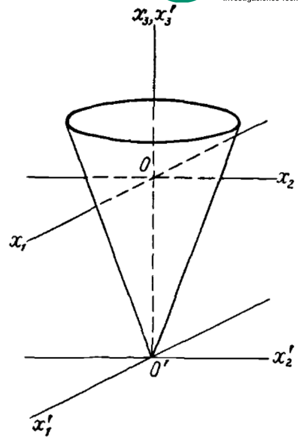
   (e) Subtract from the I_{zz} of the unperforated plate that of the circles to obtain that of the perforated plate.

   (f) Again with Steiner's theorem, transfer the I_{zz} of the perforated plate to pendulum point A.

Result: $I_{zz} = \frac{m\left(-12\pi R^4 - 6\pi R^2 a^2 - 24\pi R^2 d^2 + 4a^3 b + ab^3\right)}{12(-2\pi R^2 + ab)}$

5. **Cylinder in Semi-cylinder** [Landau §32 6]
Find the kinetic energy of a homogeneous cylinder of radius a rolling inside a cylindrical surface of radius R.
Result: $T = \frac{3m(R-a)^2\dot{\phi}^2}{4}$



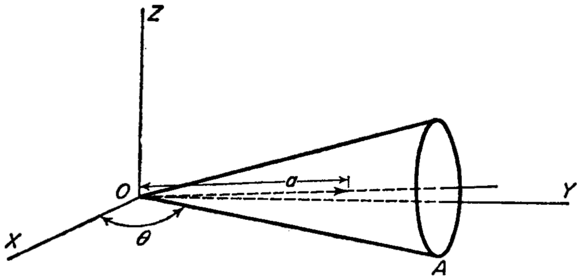


6. **Cone** [Landau §32 2e]
   This cone has a circular base of radius R and height h.

   (a) Calculate the position of the center of mass O from the vertex O'. Remember to choose integration limits based on the geometry. Result: $|\overline{OO'}| = \frac{3}{4}h$.

   (b) Calculate the moments of inertia from O'.
   Result: $I_{x_3'x_3'} = \frac{3}{10}mR^2 \qquad I_{x_1'x_1'} = I_{x_2'x_2'} = \frac{3m\left(R^2+4h^2\right)}{20}$

7. **Cone Rolling on a Plane** [Landau §32 7]
The instantaneous contact with the XY plane, $\overline{OA}$, forms angles θ with X and α with the cone's axis. The other known datum is the distance to the center of mass a.



   (a) Assuming known moments of inertia from the vertex in the axial direction I_3 and in the perpendicular directions $I_1 = I_2$, calculate the kinetic energy. Result: $T = \frac{1}{2}\cos^2(\alpha) I_1 \dot{\theta}^2 + \frac{1}{2}\frac{\cos^4(\alpha)}{\sin^2(\alpha)} I_3 \dot{\theta}^2 + \frac{1}{2}\cos^2(\alpha) m a^2 \dot{\theta}^2$

   (b) Express in the kinetic energy $I_{1,2,3}$, α and a as functions of the cone's base radius R and its height h.