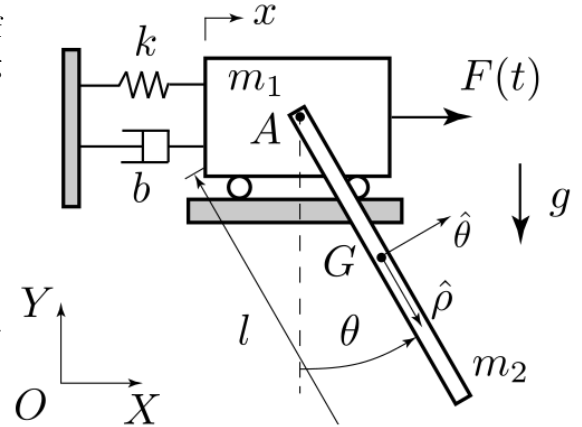


EXTERNAL FORCES IN THE LAGRANGIAN FORMULATION

1. Rod hanging from a cart

Find the dynamics equations for the system. The moment of inertia for a rod of mass m and length l about an axis passing through one end of the rod is $\frac{m}{12}l^2$.

- Write the Lagrangian.
- Write the nonconservative forces as generalized forces:
 - the external force $\vec{F}(t)$,
 - and the force exerted by the dumping system of constant b as a function of the cart's velocity, $-b\dot{x}\hat{x}$.
- Find the Euler-Lagrange equations.



2. Unbalanced torsion pendulum

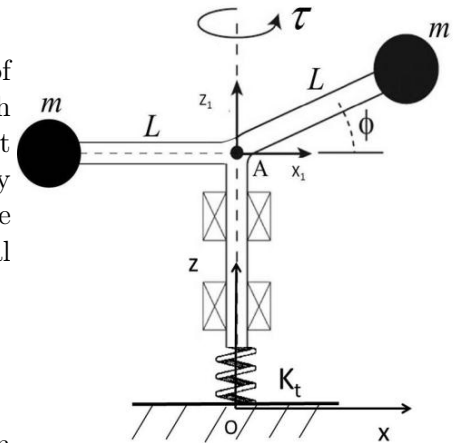
Two beads of identical masses m are attached to the ends of arms of negligible mass. One of these arms is inclined at a fixed angle ϕ with respect to the horizontal. There is no friction with the system that keeps the axis of rotation vertical. This axis is free to rotate at any angle θ and a torsion spring of constant K_t exerts an opposite torque each time $\theta \neq 0$. Additionally to this torque, there is an external torque that is a function of time: $\vec{\tau} = \tau(t)\hat{z}$.

Question: What is the unit for the generalized force?

- a) N b) $\frac{N}{m}$ c) N m d) Other

Solve for the angular acceleration using the Euler-Lagrange equation for the dynamics of this system. Result:

$$\ddot{\theta} = \frac{K_T\theta + \tau}{L^2m(\sin^2(\phi) - 2)}$$



3. Fixed cylinders

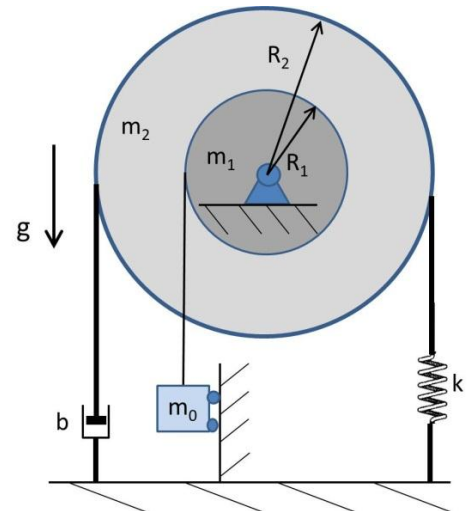
Two homogeneous cylinders of masses and radii m_1, m_2, R_1 and R_2 are welded together. This arrangement rotates without friction about an axis. A string of negligible mass is wound around the external cylinder and its ends connect a spring of constant k and a damper. This damper exerts a force opposite to the linear velocity,

$$\vec{F}_{\text{damper}} = -b\dot{r}\hat{r}.$$

A string of negligible mass is wrapped around the cylinder with the smaller radius, and a block of mass m_0 hangs from it.

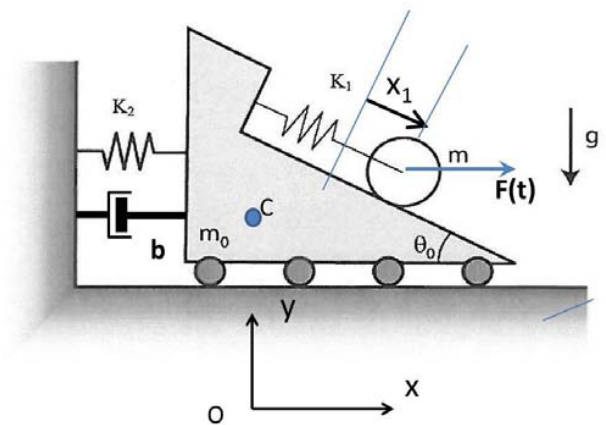
Solve for the angular acceleration using the Euler-Lagrange equation. Result:

$$\ddot{\theta} = \frac{2(R_1gm_0 - R_2^2b\dot{\theta} - R_2^2k\theta)}{2R_1^2m_0 + R_1^2m_1 + R_2^2m_2}$$



4. Oscillating inclined plane

A disk of radius R and mass m rotates over the surface of the cart of mass m_0 , inclined at an angle θ_0 . This disk won't leave the surface, even when the force $\vec{F} = F(t)\hat{x}$ is exerted upon it, due to a spring of constant K_1 that links its center with the cart. There is also a spring of constant K_2 attached to the wall and a damper, proportional to the velocity, of constant b . Both springs are initially at their natural lengths l_{10} and l_{20} . There is no friction between the cart and the floor.



Question: What is the expression for the generalized force corresponding to the virtual displacement δx due to \vec{F} ?

a) $F(t) \cos(\theta)$

b) $F(t)$

c) $F(t)\delta x$

d) 0

Find the Euler-Lagrange equations.