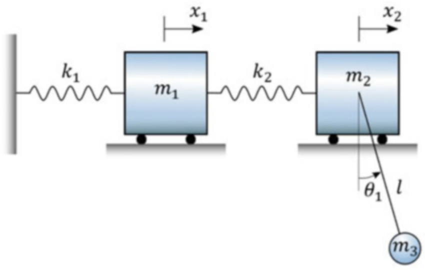
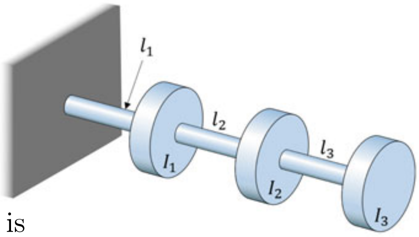
OSCILLATIONS IN DISCRETE SYSTEMS | MULTIPLE DEGREES OF FREEDOM

1. **Multiple carts** In the system shown in the figure $l = 0{,}5\,\mathrm{m}$, $k_1 = k_2 = k = 2 \times 10^3\,\mathrm{N\,m^{-1}}$ and $m_1 = m_2 = m_3 = m = 1\,\mathrm{kg}$. Assuming small oscillations around zero of the indicated coordinates:



   a) obtain the Euler-Lagrange equations,
   b) write them in matrix form (matrices M, K), and
   c) obtain the natural oscillation frequencies of the system.

2. **Compound torsional pendulum** The system in the figure consists of a shaft that passes through three disks having moments of inertia I_1, I_2 and I_3 all of equal magnitude $1 \times 10^3\,\mathrm{kg\,m^2}$. The steel shaft has a diameter $d = 0{,}01\,\mathrm{m}$ and its sections have lengths of $l_1 = l_2 = l_3 = 0{,}5\,\mathrm{m}$.



   Let us recall that for an angular coordinate θ the Euler-Lagrange equation is

$$\Gamma\dot{\theta} + \kappa\theta + I\ddot{\theta} = \tau,$$

   where Γ is the torsional friction, I the moment of inertia, τ the applied torque. κ is the torsional stiffness or torsion coefficient that responds to the restoring torque exerted by the piece when twisted through a unit angle, $\tau_{\text{restoring}} = -\kappa\theta$ and its magnitude is determined by

$$\kappa = \frac{GJ}{l},$$

   where l is the length of the piece, G the shear modulus specific to each material, and J is the torsion constant of the cross-sectional geometry transverse to the direction of $\vec{\tau}$. For a circular section J is equal to the second moment of area, or polar moment of inertia

$$J_{zz} = J_{xx} + J_{yy} = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}.$$

   According to the document Mechanical Properties of Structural Steels published by the U.S. National Institute of Standards and Technology, for the structural steel of the towers 1 and 2 of the World Trade Center in New York that disappeared in 2001,

$$\begin{aligned}
G &= g_0 + g_1 T + g_2 T^2 + g_3 T^3 + g_4 T^4 + g_5 T^5 \\
g_0 &= 80{,}005\,922\,\mathrm{GPa} \\
g_1 &= -0{,}018\,303\,811\,\mathrm{GPa\,{}^\circ C^{-1}} \\
g_2 &= -1{,}565\,028\,8 \times 10^{-5}\,\mathrm{GPa\,{}^\circ C^{-2}} \\
g_3 &= -1{,}516\,092\,1 \times 10^{-8}\,\mathrm{GPa\,{}^\circ C^{-3}} \\
g_4 &= -1{,}624\,291\,1 \times 10^{-11}\,\mathrm{GPa\,{}^\circ C^{-4}} \\
g_5 &= 7{,}727\,754\,3 \times 10^{-15}\,\mathrm{GPa\,{}^\circ C^{-5}}
\end{aligned}$$

   Disregarding rotational friction Γ:

   a) obtain the Euler-Lagrange equations,
   b) write them in matrix form (matrices M, K), and
   c) obtain the natural oscillation frequencies of the system.