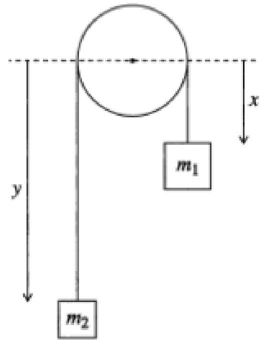
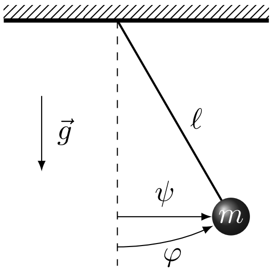
Simulation | Numerical solutions for the Euler-Lagrange equation

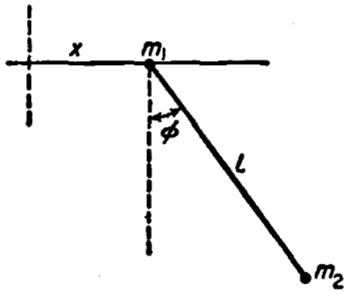
In the following exercises, you will solve numerically the Euler-Lagrange equation for each generalized coordinate. Plotting these solutions, using the given initial conditions and within the given time ranges, you will be simulating the dynamics of these systems.
Use $|\vec{g}| = 9{,}81\,\mathrm{m\,s^{-2}}$ for the magnitude of the acceleration due to gravity.
Exercises marked with (*) have extra difficulty, don't hesitate to ask for help.

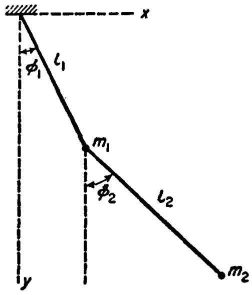


1. **Atwood machine**
   Time from $t = 0\,\mathrm{s}$ to $t = 10\,\mathrm{s}$. Parameters and initial conditions:
   $\ell_{\text{rope}} > 150\,\mathrm{m}$, $R_{\text{pulley 1}} = 0{,}5\,\mathrm{m}$,
   $m_1 = 8\,\mathrm{kg}$, $m_2 = 1\,\mathrm{kg}$, $M_{\text{pulley}} = 4\,\mathrm{kg}$,
   $x(t=0) = 25\,\mathrm{m}$, $\dot{x}(t=0) = -10\,\mathrm{m\,s^{-1}}$.

2. a) **Ideal pendulum** [Marion ex. 7.2]
   b) **Pendulum with free support** [Landau §5 ex. 2]
   c) **Double pendulum** [Landau §5 ex. 1]

a) 

b) 

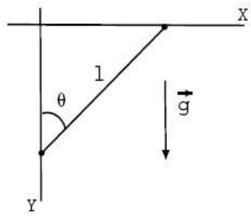
c) 

   Time from $t = 0\,\mathrm{s}$ to $t = 10\,\mathrm{s}$. Parameters and initial conditions:

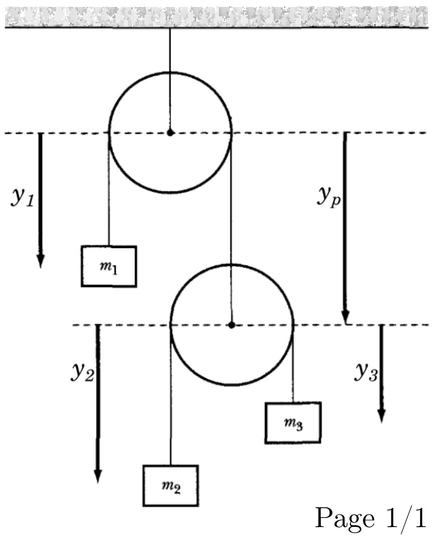
   (a) $m = 3\,\mathrm{kg}$, $\ell = 2\,\mathrm{m}$, $\varphi(t=0) = \frac{\pi}{4}$, $\dot{\varphi}(t=0) = 0$.

   (b) $m_1 = 3\,\mathrm{kg}$, $m_2 = 1\,\mathrm{kg}$, $\ell = 2\,\mathrm{m}$, $x(t=0) = 1\,\mathrm{m}$, $\dot{x}(t=0) = 0{,}5\,\mathrm{m\,s^{-1}}$, $\phi(t=0) = \frac{\pi}{8}$, $\dot{\phi}(t=0) = 0$.

   (c) $m_1 = 3\,\mathrm{kg}$, $m_2 = 1\,\mathrm{kg}$, $\ell_1 = 1\,\mathrm{m}$, $\ell_2 = 1\,\mathrm{m}$,
   $\phi_1(t=0) = \frac{\pi}{8}$, $\dot{\phi}_1(t=0) = 0$, $\phi_2(t=0) = \frac{\pi}{4}$, $\dot{\phi}_2(t=0) = -\frac{\pi}{16}\mathrm{s}^{-1}$.



3. **Pendulum of linked beads moving on rigid thin wires**
   Time from $t = 0\,\mathrm{s}$ to $t = 10\,\mathrm{s}$. Parameters and initial conditions:
   $m_1 = m_2 = m = 2\,\mathrm{kg}$, $l = 2\,\mathrm{m}$, $\theta(t=0) = \frac{\pi}{4}$, $\dot{\theta}(t=0) = 0$.



4. (*) **Compound Atwood machine** [Marion ex. 7.8]
   Time from $t = 0\,\mathrm{s}$ to $t = 5\,\mathrm{s}$. Parameters and initial conditions:
   $\ell_{\text{top}} = 15\,\mathrm{m}$, $R_{\text{top pulley}} = 0{,}5\,\mathrm{m}$, $\ell_{\text{bottom}} = 15\,\mathrm{m}$, $R_{\text{bottom pulley}} = 0{,}5\,\mathrm{m}$,
   $m_1 = 1\,\mathrm{kg}$, $m_2 = 2\,\mathrm{kg}$, $m_3 = 3\,\mathrm{kg}$, $M_{\text{top pulley}} = 4\,\mathrm{kg}$, $M_{\text{bottom pulley}} = 4\,\mathrm{kg}$,
   $y(t=0) = 1\,\mathrm{m}$, $\dot{y}_1(t=0) = 0$, $y_2(t=0) = 2\,\mathrm{m}$, $\dot{y}_2(t=0) = 0$