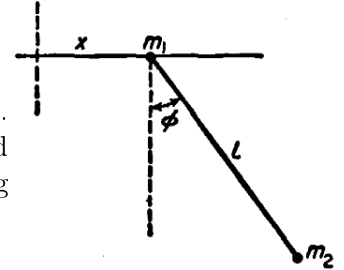


Exercises marked with (\*) have extra difficulty, don't hesitate to ask for help.

1. **Pendulum with free point of support** [Landau §5 ex. 2]

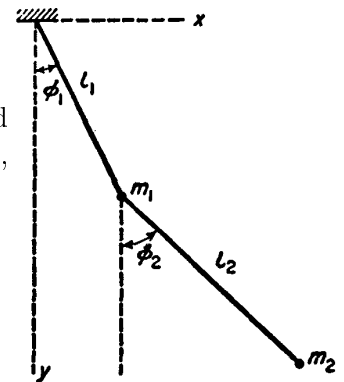
Particle of mass  $m_2$  is hanging from a rigid bar of length  $\ell$  and negligible mass. On the other end there is a device of mass  $m_1$  linked to a horizontal bar, and it's free to move horizontally along the  $x$  axis. The device allows the hanging bar to span any angle  $\varphi$  with respect to the vertical axis.



- After determining the generalized coordinates, write the position of each particle as a function of them.
- Calculate the velocity of each particle.
- Using these velocities, calculate the kinetic energy,  $T$ , and potential energy,  $V$ , for each particle.
- Now calculate  $T$  and  $V$  using the functions with the masses and positions as inputs. Verify that you get the same result but in fewer steps.
- Perform the necessary substitutions into the expressions for  $T$  and  $V$  found in the previous step, so that the particle with mass  $m_1$  remains at rest. Verify that you recover the expressions of  $T$  and  $V$  of an ideal pendulum.

2. **Double coplanar pendulum** [Landau §5 ex. 1]

A rigid bar of length  $\ell_1$  and negligible mass has a particle of mass  $m_1$  attached to one end. There is a second bar of negligible mass hanging from the first one, of length  $\ell_2$ , with a particle of mass  $m_2$  attached to the other end too.



- Write expressions for kinetic energy,  $T$ , and potential energy,  $V$ , as functions of the generalized coordinates suggested by the figure.

Result:

$$T_{\text{translational}} = \frac{\ell_1^2 m_1 \dot{\varphi}_1^2}{2} + \frac{m_2 (\ell_1^2 \dot{\varphi}_1^2 + 2\ell_1 \ell_2 \cos(\varphi_1 - \varphi_2) \dot{\varphi}_1 \dot{\varphi}_2 + \ell_2^2 \dot{\varphi}_2^2)}{2}$$

$$V_{\text{gravitational}} = -g(\ell_1 m_1 \cos(\varphi_1) + \ell_1 m_2 \cos(\varphi_1) + \ell_2 m_2 \cos(\varphi_2))$$

- Use the `subs` function of `SymPy` to set  $m_1 = 0$ ,  $\varphi_1 = \varphi_2 = \varphi$  and  $\ell_1 = \ell_2 = \frac{\ell}{2}$ . Verify that you recover the expressions of  $T$  and  $V$  of an ideal pendulum.

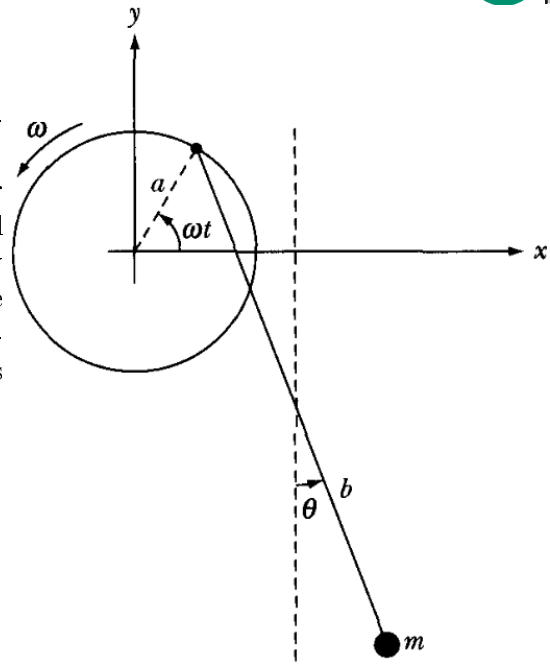
3. (\*) **Pendulum with rotating point of support** [Marion (e) ex. 7.5] [Landau §5 ex. 3]

A particle of mass  $m$  is attached to the end of a rigid bar of length  $b$ . The point of support is linked to a vertical circle of radius  $a$ , and it rotates with constant frequency  $\omega$ . It is assumed that all positions lie in the same plane and the mass of the bar is negligible. Calculate the kinetic energy,  $T$ , and potential  $V$ , of the particle of mass  $m$ .

Result:

$$T_{\text{translational}} = \frac{m \left( a^2 \omega^2 - 2ab\omega \sin(\omega t - \theta) \dot{\theta} + b^2 \dot{\theta}^2 \right)}{2}$$

$$V_{\text{gravitational}} = gm(a \sin(\omega t) - b \cos(\theta))$$



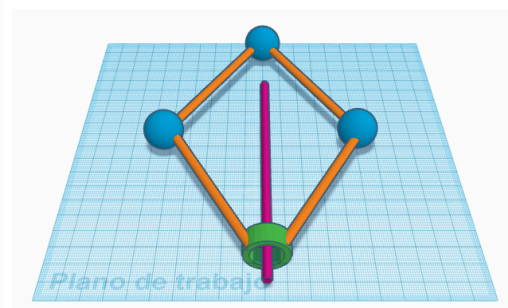
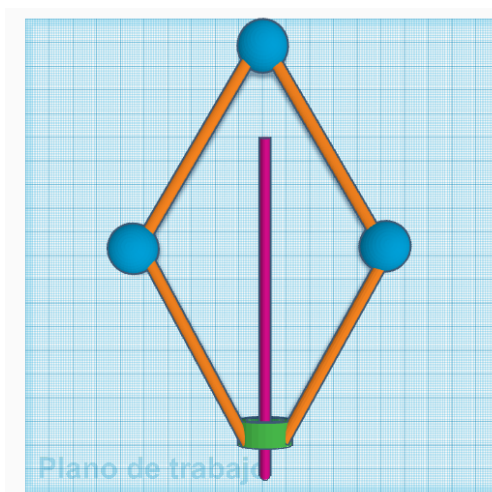
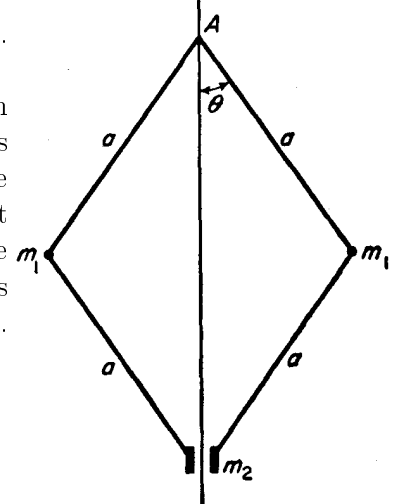
4. (\*) **Coupled weights rotating about a vertical axis** [Landau §5 ex. 4]

Particle with mass  $m_2$  moves on a vertical axis and the whole system rotates about this axis with a constant angular velocity  $\Omega$ . This particle is linked to two particles of mass  $m_1$  through bars of length  $a$  and negligible mass, and at the same time, these particles are linked to the fixed point  $A$  through identical bars, forming the variable angle  $\theta$  with respect to the vertical axis. Calculate the kinetic energy of each of the three particles and find a compact expression of the kinetic energy of the whole system. Do the same for the potential energy.

Result:

$$T_{\text{translational}} = a^2 \left( m_1 \left( \Omega^2 \sin^2(\theta) + \dot{\theta}^2 \right) + 2m_2 \sin^2(\theta) \dot{\theta}^2 \right)$$

$$V_{\text{gravitational}} = -2ag(m_1 + m_2) \cos(\theta)$$



In these figures, the top sphere is the fixed point  $A$ . Everything revolves around the pink axis with constant angular speed  $\Omega$ . Therefore, the particles on each side, of mass  $m_1$ , rotate entering and leaving the plane shown in the first image. This is equivalent to thinking that the light-blue plane rotates around the pink axis.

The piece of mass  $m_2$  is a sliding hub that moves vertically without friction.

