

SIMULATION | NUMERICAL SOLUTIONS FOR THE EULER-LAGRANGE EQUATION

In the following exercises, you will solve numerically the Euler-Lagrange equation for each generalized coordinate. Plotting these solutions, using the given initial conditions and within the given time ranges, you will be simulating the dynamics of these systems.

Use  $|\vec{g}| = 9,81 \text{ m s}^{-2}$  for the magnitude of the acceleration due to gravity.

Exercises marked with (\*) have extra difficulty, don't hesitate to ask for help.

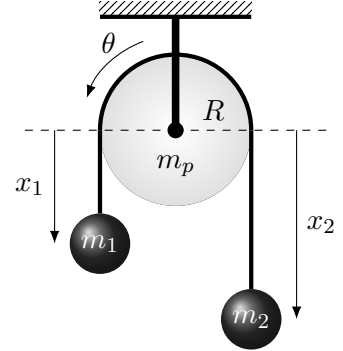
1. **Atwood machine**

Time from  $t = 0 \text{ s}$  to  $t = 10 \text{ s}$ . Parameters and initial conditions:

$\ell_{\text{rope}} > 150 \text{ m}$ ,  $R = 0,5 \text{ m}$ ,

$m_1 = 8 \text{ kg}$ ,  $m_2 = 1 \text{ kg}$ ,  $m_p = 4 \text{ kg}$ ,

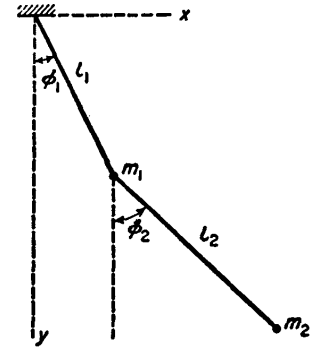
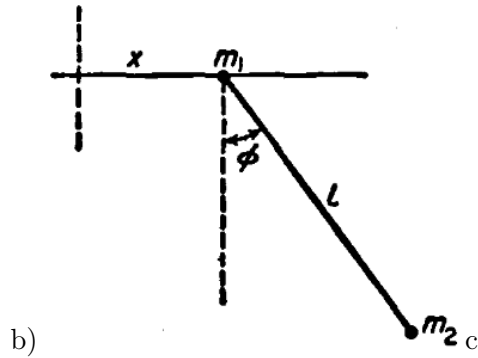
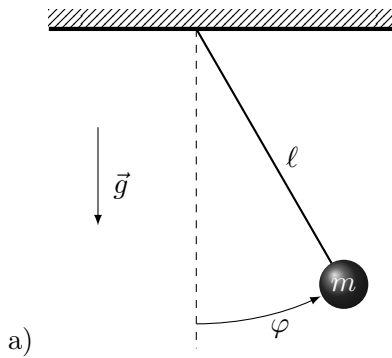
$x_1(t = 0) = 25 \text{ m}$ ,  $\dot{x}_1(t = 0) = -10 \text{ m s}^{-1}$ .



2. a) **Ideal pendulum** [Marion ex. 7.2]

b) **Pendulum with free support** [Landau §5 ex. 2]

c) **Double pendulum** [Landau §5 ex. 1]



Time from  $t = 0 \text{ s}$  to  $t = 10 \text{ s}$ . Parameters and initial conditions:

(a)  $m = 3 \text{ kg}$ ,  $\ell = 2 \text{ m}$ ,  $\varphi(t = 0) = \frac{\pi}{4}$ ,  $\dot{\varphi}(t = 0) = 0$ .

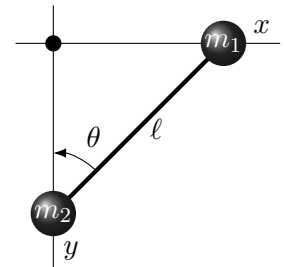
(b)  $m_1 = 3 \text{ kg}$ ,  $m_2 = 1 \text{ kg}$ ,  $\ell = 2 \text{ m}$ ,  $x(t = 0) = 1 \text{ m}$ ,  $\dot{x}(t = 0) = 0,5 \text{ m s}^{-1}$ ,  $\phi(t = 0) = \frac{\pi}{8}$ ,  $\dot{\phi}(t = 0) = 0$ .

(c)  $m_1 = 3 \text{ kg}$ ,  $m_2 = 1 \text{ kg}$ ,  $\ell_1 = 1 \text{ m}$ ,  $\ell_2 = 1 \text{ m}$ ,  
 $\phi_1(t = 0) = \frac{\pi}{8}$ ,  $\dot{\phi}_1(t = 0) = 0$ ,  $\phi_2(t = 0) = \frac{\pi}{4}$ ,  $\dot{\phi}_2(t = 0) = -\frac{\pi}{16} \text{ s}^{-1}$ .

3. **Pendulum of linked beads moving on rigid thin wires**

Time from  $t = 0 \text{ s}$  to  $t = 10 \text{ s}$ . Parameters and initial conditions:

$m_1 = m_2 = m = 2 \text{ kg}$ ,  $l = 2 \text{ m}$ ,  $\theta(t = 0) = \frac{\pi}{4}$ ,  $\dot{\theta}(t = 0) = 0$ .



4. (\*) **Compound Atwood machine** [Marion ex. 7.8]Time from  $t = 0$  s to  $t = 5$  s. Parameters and initial conditions:

$\ell_{\text{top}} = 15$  m,  $R_{\text{top pulley}} = 0,5$  m,  $\ell_{\text{bottom}} = 15$  m,  $R_{\text{bottom pulley}} = 0,5$  m,  
 $m_1 = 1$  kg,  $m_2 = 2$  kg,  $m_3 = 3$  kg,  $M_{\text{top pulley}} = 4$  kg,  $M_{\text{bottom pulley}} = 4$  kg,  
 $y(t = 0) = 1$  m,  $\dot{y}_1(t = 0) = 0$ ,  $y_2(t = 0) = 2$  m,  $\dot{y}_2(t = 0) = 0$

