

Skew-Normal (version 1 and 2)

Parametrisation

The Skew-Normal distribution is

$$f(y) = 2 \frac{\sqrt{w\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}w\tau(y-\mu)^2\right) \Phi(a a_{\max}[w\tau(y-\mu)])$$

for continuously responses y where $\Phi(\cdot)$ is the cumulative distribution function for a standard Normal, and

μ : is the the location parameter

τ : is the inverse scale

w : is a fixed weight, $w > 0$,

a : is the shape parameter

a_{\max} : is the (fixed) maximum value of the shape paramter (added for stability reasons). Default value is 5.

Link-function

The location parameter is linked to the linear predictor by

$$\mu = \eta$$

Hyperparameters

The inverse scale is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on θ_1 .

The shape parameter is

$$a = 2 \frac{\exp(\theta_2)}{1 + \exp(\theta_2)} - 1$$

and the prior is defined on θ_2 .

Specification

- family = **sn**
- Required arguments: y and w (keyword **scale**). The weights has default value 1.
- Optional control arguments: **sn.shape.max**. Default value is 5.0.

Hyperparameter spesification and default values

hyper

theta1

name log inverse scale

short.name iscale

initial 4

```

fixed FALSE
prior loggamma
param 1 5e-05
theta2
  name logit skewness
  short.name skew
  initial 0
  fixed FALSE
  prior gaussian
  param 0 10
  to.theta function(x, shape.max = 1) log((1+x/shape.max)/(1-x/shape.max))
  from.theta function(x, shape.max = 1) shape.max*(2*exp(x)/(1+exp(x))-1)
survival FALSE
discrete FALSE
link default identity
pdf sn

```

Example

This is a simulated example requiring the package `sn`.

```

library(sn)
n = 1000
z = rnorm(n)
y = z + rsn(n, shape = 2)
formula = y ~ z
r = inla(formula, family = "sn", data = data.frame(z,y),
         control.family = list(sn.shape.max = 5.0))
summary(r)

```

Notes

An simpler approximation to $\Phi(\cdot)$ is used to improve the speed, which has maximum absolute error of 0.00197323; see the source code for further details.

Skew-Normal (version 2)

Parametrisation (version 2)

In the family “sn2” we offer an alternative parametersisation of the skew-normal with moment parameters, precision $w\tau$ (where w is a fixed weight or scale) and standardized skewness γ (where $|\gamma| < 1$ due to the skew-normal family¹). In this parameterisation, the location parameter is linked to the linear predictor by

$$\mu = \eta$$

and μ equals the expected value $\xi + \omega\delta\sqrt{\frac{2}{\pi}}$, and $\delta = \alpha/\sqrt{1 + \alpha^2}$.

Hyperparameters

The precision τ is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on θ_1 . The (standardized) skewness γ is

$$\gamma = 2 \frac{\exp(\theta_2)}{1 + \exp(\theta_2)} - 1$$

and the prior is defined on θ_2 .

The function `INLA::inla.sn.reparam` offer the mapping between the moments (mean, variance and skewness) and the parameters used in the skew-normal density in the format used in the package `sn`, which are (ξ, ω, α) , where

$$f(x) = \frac{2}{\omega} \phi\left(\frac{x - \xi}{\omega}\right) \Phi\left(\alpha \left[\frac{x - \xi}{\omega}\right]\right)$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ is the density and cumulative distribution function for the standard Gaussian distribution.

Hyperparameter spesification and default values for sn2

hyper

theta1

name log precision
short.name prec
initial 1
fixed FALSE
prior loggamma
param 1 5e-05

theta2

name logit skewness
short.name skew
initial 0
fixed FALSE
prior gaussian
param 0 10

¹or to be presice, $|\gamma| < \frac{4-\pi}{2(\pi/2-1)^{3/2}} = 0.995271746\dots$

```

to.theta function(x) log((1+x)/(1-x))
from.theta function(x) (2*exp(x)/(1+exp(x))-1)

survival FALSE

discrete FALSE

link default identity

status experimental

pdf sn2

```

Example for sn2

This is a simulated example requiring the package `sn`.

```

library(sn)
n = 500
x = rnorm(n)
eta = 1/2 + 2*x
w = runif(n, min = 0.5, max = 2)
prec = 1 * w
skewness = 0.25
y = numeric(n)
for(i in 1:n) {
  param = INLA:::inla.sn.reparam(moments = c(eta[i], 1/prec[i], skewness))
  y[i] = rsn(1, xi=param$xi, omega = param$omega, alpha = param$alpha)
}
r = inla(y ~ 1 + x, family = "sn2", scale = w, data = data.frame(y, x, w), verbose=T)
summary(r)

```

Notes for sn2

In this parametersiation there is no `sn.shape.max`.

An simpler approximation to $\Phi(\cdot)$ is used to improve the speed, which has maximum absolute error of 0.00197323; see the source code for further details.