

## Generalized Poisson

The generalized Poisson distribution is given by

$$f(y|\lambda, w) = \frac{\lambda(\lambda + wy)^{y-1}}{y!} \exp(-(\lambda + wy))$$

for  $y = 0, 1, 2, \dots$  and where  $\lambda > 0$  and  $\max(-1, -\lambda/4) \leq w \leq 1$ . The mean and variance of  $y$  are

$$\mu = \lambda(1 - w)^{-1} \quad \text{and} \quad \sigma^2 = \lambda(1 - w)^{-3} = \mu(1 - w)^{-2}.$$

Since the dispersion parameter  $w$  influence the mean as well as the variance, we will use the following parameterisation (Consul and Jain (1973), Zamani and Ismail(2012))

$$w = \frac{\varphi\mu^{p-1}}{1 + \varphi\mu^{p-1}},$$

which gives the following density

$$f(y|\mu, \varphi, p) = \frac{\mu(\mu + \varphi\mu^{p-1}y)^{y-1}}{(1 + \varphi\mu^{p-1})^y y!} \exp\left(-\frac{\mu + \varphi\mu^{p-1}y}{1 + \varphi\mu^{p-1}}\right)$$

for  $y = 0, 1, 2, \dots$ . We assume  $\varphi \geq 0$ .

### Link-function

The mean and variance of  $y$  are given as

$$E(y|.) = \mu \quad \text{and} \quad \text{Var}(y|.) = \mu (1 + \varphi\mu^{p-1})^2$$

and the mean is linked to the linear predictor by

$$\mu = E \exp(\eta)$$

### Hyperparameters

The overdispersion parameter  $\varphi \geq 0$  is represented as

$$\varphi = \exp(\theta_1)$$

The “shape” parameter  $p$  is represented as

$$p = \theta_2$$

Note that  $\theta_2 = 1$  and `fixed = TRUE`, default. The prior is defined on  $\theta = (\theta_1, \theta_2)$ .

### Specification

- `family="gpoisson"`

## Hyperparameter specification and default values

**hyper**

**theta1**

**name** overdispersion  
**short.name** phi  
**initial** 0  
**fixed** FALSE  
**prior** loggamma  
**param** 1 1  
**to.theta** function(x) log(x)  
**from.theta** function(x) exp(x)

**theta2**

**name** p  
**short.name** p  
**initial** 1  
**fixed** TRUE  
**prior** normal  
**param** 1 100  
**to.theta** function(x) x  
**from.theta** function(x) x

**survival** FALSE

**discrete** TRUE

**link** default log

**pdf** gpoisson

**status** experimental

## Example

In the following example we estimate the parameters in a simulated example with generalized Poisson responses.

```
dgpoisson = function(y, mu, phi, p)
{
  a = mu + phi * mu^(p-1.0) * y;
  b = 1. + phi * mu^(p-1.0);
  d = exp(log(mu) + (y-1.0)*log(a) -
          y*log(b) - lfactorial(y) - a/b)
  return (d)
}
rgpoisson = function(n, mu, phi, p)
{
  stopifnot(length(mu) == 1)
  s = sqrt(mu*(1+phi*mu^(p-1))^2)
  f = 20
```

```

    low = as.integer(max(0, mu - f*s))
    high = as.integer(mu + f*s)
    prob = dgpoisson(low:high, mu, phi, p)
    y = sample(low:high, n, replace=TRUE,
               prob = prob)

    return (y)
}

n = 1000
phi = 1
p = 1
mu = exp(1 + 5*(1:n)/n)

y = numeric(n)
for(i in 1:n) {
  y[i] = rgpoisson(1, mu[i], phi, p)
}

idx = (1:n)/n
r = inla(y ~ 1 + idx, data = data.frame(y, idx),
         family = "gpoisson")

```

## Notes

The parameter  $p$  is default fixed to be 1. Allowing it to be estimated jointly with the overdispersion parameter, please note the following.

- The parameter  $p$  and the overdispersion parameter are strongly correlated when estimated jointly.
- You may want to chose an informative prior for  $p$ , as the shape of the likelihood might not be want you expect for “extreme”  $p$ .
- You may experience problems in the numerical optimization (fail to converge); a more informative prior (if available) for  $p$  will help with this issue.

## References

- Consul, P. C. and Jain, G.C (1973) A generalization of Poisson distribution. *Technometrics* 15, 791-799.
- Zamani, H. and Ismail, N. (2011). Functional form for the generalized Poisson regression model. *Communication in Statistics – Theory and Methods* (IN PRESS).