

# Fractional Gaussian Noise (FGN)

## Parametrization

The (stationary) FGN (Gaussian) process has correlation function at lag  $k$

$$\rho(k) = |k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}$$

where  $H$  is the Hurst parameter or self-similarity parameter, which we assume to be

$$1/2 \leq H < 1.$$

so the process has long range properties for  $H > 1/2$ . The locations of the process is fixed to  $1, 2, \dots, n$ , where  $n$  is the dimension of the finite representation of the FGN process.

## Hyperparameters

The marginal precision,  $\tau$ , of the process is represented as

$$\tau = \exp(\theta_1)$$

The Hurst parameter  $H$  is represented as

$$H = \frac{1}{2} + \frac{1}{2} \frac{\exp(\theta_2)}{1 + \exp(\theta_2)}$$

and the prior is defined on  $\theta = (\theta_1, \theta_2)$ .

## Specification

The FGN model is specified as

```
f(<whatever>, model="fgn", order=<order>, hyper = <hyper>)
```

The parameter `order` gives the order of the Markov approximation. Currently, only `order=3` is implemented.

## Hyperparameter specification and default values

### hyper

#### theta1

```
hyperid 13101
name log precision
short.name prec
prior pc.prec
param 3 0.01
initial 1
fixed FALSE
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

#### theta2

```
hyperid 13102
name logit H
```

```

    short.name H
    prior pcfnh
    param 0.9 0.1
    initial 2
    fixed FALSE
    to.theta function(x) log((2*x-1)/(2*(1-x)))
    from.theta function(x) 0.5 + 0.5*exp(x)/(1+exp(x))

constr FALSE

nrow.ncol FALSE

augmented TRUE

aug.factor 4

aug.constr 1

n.div.by

n.required FALSE

set.default.values TRUE

order.default 3

order.defined 3

pdf fgn

```

## Example

```

library(FGN)
n = 1000
H = 0.77
y = SimulateFGN(n, H)
y = y - mean(y)
r = inla(y ~ -1 + f(idx, model="fgn"),
        data = data.frame(y, idx=1:n),
        control.family = list(hyper = list(prec = list(initial = 12, fixed=TRUE))))
print(c(MLE=FitFGN(y, demean=TRUE)$H,
        Post.mean=r$summary.hyperpar[2,"mean"],
        Post.mode=r$summary.hyperpar[2,"mode"]))

```

## Notes

In the example above, then the `f(idx,model="fgn")` object will expand into a Gaussian of length  $(\text{order} + 1) * n$ . The first  $n$  elements is the FGN model (which is of interest), then there are `order` vector of AR1 processes each of length  $n$ , and the sum of these AR1 processes is used to represent the FGN.

The PC-prior for  $H$  take two arguments  $(U, \alpha)$  where  $\text{Prob}(U < H < 1) = \alpha$ .