Random walk model of order 1 (RW1)

Parametrization

The random walk model of order 1 (RW1) for the Gaussian vector $\mathbf{x} = (x_1, \ldots, x_n)$ is constructed assuming independent increments:

$$\Delta x_i = x_i - x_{i+1} \sim \mathcal{N}(0, \tau^{-1})$$

The density for $\mathbf{x}$ is derived from its $n-1$ increments as

$$\begin{aligned} \pi(\mathbf{x}|\tau) &\propto \tau^{(n-1)/2} \exp\left\{-\frac{\tau}{2}\sum(\Delta x_i)^2\right\} \\ &= \tau^{(n-1)/2} \exp\left\{-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}\right\} \end{aligned}$$

where $\mathbf{Q} = \tau\mathbf{R}$ and $\mathbf{R}$ is the structure matrix reflecting the neighbourhood structure of the model.

It is also possible to define a *cyclic* version of the RW1 model, in this case the graph is modified so that last node x_n is neighbour of x_{n-1} and x_1.

Hyperparameters

The precision parameter τ is represented as

$$\theta = \log \tau$$

and the prior is defined on θ.

Specification

The RW1 model is specified inside the `f()` function as

```
f(<whatever>, model="rw1", values=<values>, cyclic=<TRUE|FALSE>,
              hyper = <hyper>)
```

The (optional) argument `values` is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See next example for an application.

Hyperparameter spesification and default values

hyper

theta

name log precision
short.name prec
prior loggamma
param 1 5e-05
initial 4
fixed FALSE
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

constr TRUE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

pdf rw1

Example

```
n=100
z=seq(0,6,length.out=n)
y=sin(z)+rnorm(n,mean=0,sd=0.5)
data=data.frame(y=y,z=z)

formula=y~f(z,model="rw1",
            hyper = list(prec = list(prior="loggamma",param=c(1,0.01))))
result=inla(formula,data=data,family="gaussian")

#here we estimate the effect only for some of the values in z
formula1=y~f(z,model="rw1",
             hyper = list(prec = list(prior="loggamma",param=c(1,0.01))),
             values=z[seq(1,length(z),2)])
result1=inla(formula1,data=data,family="gaussian")
```

Notes

- The RW1 is intrinsic with rank deficiency 1.
- The RW1 model for irregular locations are supported although not described here.
- The $\frac{n-r}{2}\log(|R|^*)$-part (with $r=1$) of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it.