Probabilities for Weibull cure model

Event time is ∞ with probability ρ, and has the following Weibull distribution otherwise:

$$f(t; \mu_i, \nu) = \mu_i \nu t^{\nu-1} \exp\left[-\mu_i t^\nu\right].$$

Given that a subject reached time L without cancer incidence, the density function for their cancer incidence time T is

$$pr(T = t | T > L) = \left[(1-\rho)f(t; \mu_i, \nu)\right] \bigg/ \left[\rho + (1-\rho)\int_L^\infty f(u; \mu_i, \nu)du\right]$$

$$= \frac{\mu_i \nu t^{\nu-1} \exp\left[-\mu_i t^\nu\right]}{\left[\rho/(1-\rho) + \exp(-\mu_i L^\nu)\right]}.$$

The probability that an individual has a cancer time greater than r (right censoring) given that they are cancer-free at age L is

$$pr(T > t | T > L) = \frac{\rho + (1-\rho)\int_t^\infty f(u; \mu_i, \nu)du}{\rho + (1-\rho)\int_L^\infty f(u; \mu_i, \nu)du}$$

$$= \frac{\rho + (1-\rho)\exp(-\mu_i t^\nu)}{\rho + (1-\rho)\exp(-\mu_i L^\nu)}.$$

Left censoring.

$$pr(T < t | T > L) = \frac{(1-\rho)\int_L^t f(u; \mu_i, \nu)du}{\rho + (1-\rho)\int_L^\infty f(u; \mu_i, \nu)du}$$

$$= \frac{(1-\rho)\exp[-\mu_i(t^\nu - L^\nu)]}{1 - (1-\rho)\exp(-\mu_i L^\nu)}.$$

interval censoring

$$pr(t_1 < T < t_2 | T > L) = \frac{(1-\rho)\int_{\max(L,T_1)}^{t_2} f(u; \mu_i, \nu)du}{\rho + (1-\rho)\int_L^\infty f(u; \mu_i, \nu)du}$$

$$= \frac{(1-\rho)\exp[-\mu_i(t_2^\nu - \max(L, t_1)^\nu)]}{1 - (1-\rho)\exp(-\mu_i L^\nu)}.$$