

# Zero-inflated models: Poisson and Binomial

## Parametrisation

There is support two types of zero-inflated models, which we name type 0 and type 1. These are defined for both the Binomial, the Poisson and the negative Binomial likelihood. For simplicity we will describe only the Poisson as the other two cases are similar.

### Type 0

The (type 0) likelihood is defined as

$$\text{Prob}(y \mid \dots) = p \times 1_{[y=0]} + (1 - p) \times \text{Poisson}(y \mid y > 0)$$

where  $p$  is a hyperparameter where

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and  $\theta$  is the internal representation of  $p$ ; meaning that the initial value and prior is given for  $\theta$ . This is model is called `zeroinflatedpoisson0` (and `zeroinflatedbinomial0`).

### Type 1

The (type 1) likelihood is defined as

$$\text{Prob}(y \mid \dots) = p \times 1_{[y=0]} + (1 - p) \times \text{Poisson}(y)$$

where  $p$  is a hyperparameter where

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and  $\theta$  is the internal representation of  $p$ ; meaning that the initial value and prior is given for  $\theta$ . This is model is called `zeroinflatedpoisson1` (and `zeroinflatedbinomial1`).

## Link-function

As for the Poisson, the Binomial and the negative Binomial.

## Hyperparameters

For Poisson and the Binomial, there is one hyperparameter; where

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and the prior and initial value is is given for  $\theta$ .

For the negative Binomial, there are two hyperparameters. The overdispersion parameter  $n$  is represented as

$$\theta_1 = \log(n)$$

and the prior is defined on  $\theta_1$ . The zero-inflation parameter  $p$ , is represented as

$$p = \frac{\exp(\theta_2)}{1 + \exp(\theta_2)}$$

and the prior and initial value is is given for  $\theta_2$ .

## Specification

- family = zeroinflatedbinomial0
- family = zeroinflatedbinomial1
- family = zeroinflatednbinomial0
- family = zeroinflatednbinomial1
- family = zeroinflatedpoisson0
- family = zeroinflatedpoisson1
- Required arguments: As for the Binomial, the negative Binomial and Poisson likelihood.

## Hyperparameter specification and default values

### Zeroinflated Binomial Type 0

#### hyper

##### theta

**name** logit probability  
**short.name** prob  
**initial** -1  
**fixed** FALSE  
**prior** gaussian  
**param** -1 0.2  
**to.theta** function(x) log(x/(1-x))  
**from.theta** function(x) exp(x)/(1+exp(x))

**survival** FALSE

**discrete** FALSE

**link** default logit probit cloglog

**pdf** zeroinflated

### Zeroinflated Binomial Type 1

#### hyper

##### theta

**name** logit probability  
**short.name** prob  
**initial** -1  
**fixed** FALSE  
**prior** gaussian  
**param** -1 0.2  
**to.theta** function(x) log(x/(1-x))  
**from.theta** function(x) exp(x)/(1+exp(x))

**survival** FALSE

**discrete** FALSE

**link** default logit probit cloglog

**pdf** zeroinflated

### **Zeroinflated NegBinomial Type 0**

**hyper**

**theta1**

**name** log size

**short.name** size

**initial** 2.30258509299405

**fixed** FALSE

**prior** loggamma

**param** 1 1

**to.theta** function(x) log(x)

**from.theta** function(x) exp(x)

**theta2**

**name** logit probability

**short.name** prob

**initial** -1

**fixed** FALSE

**prior** gaussian

**param** -1 0.2

**to.theta** function(x) log(x/(1-x))

**from.theta** function(x) exp(x)/(1+exp(x))

**survival** FALSE

**discrete** FALSE

**link** default log

**pdf** zeroinflated

### **Zeroinflated NegBinomial Type 1**

**hyper**

**theta1**

**name** log size

**short.name** size

**initial** 2.30258509299405

**fixed** FALSE

**prior** loggamma

**param** 1 1

**to.theta** function(x) log(x)

**from.theta** function(x) exp(x)

## **theta2**

**name** logit probability  
**short.name** prob  
**initial** -1  
**fixed** FALSE  
**prior** gaussian  
**param** -1 0.2  
**to.theta**  $\text{function}(x) \log(x/(1-x))$   
**from.theta**  $\text{function}(x) \exp(x)/(1+\exp(x))$

**survival** FALSE

**discrete** FALSE

**link** default log

**pdf** zeroinflated

## **Zeroinflated Poisson Type 0**

### **hyper**

#### **theta**

**name** logit probability  
**short.name** prob  
**initial** -1  
**fixed** FALSE  
**prior** gaussian  
**param** -1 0.2  
**to.theta**  $\text{function}(x) \log(x/(1-x))$   
**from.theta**  $\text{function}(x) \exp(x)/(1+\exp(x))$

**survival** FALSE

**discrete** FALSE

**link** default log

**pdf** zeroinflated

## **Zeroinflated Poisson Type 1**

### **hyper**

#### **theta**

**name** logit probability  
**short.name** prob  
**initial** -1  
**fixed** FALSE  
**prior** gaussian  
**param** -1 0.2

```

to.theta function(x) log(x/(1-x))
from.theta function(x) exp(x)/(1+exp(x))

```

**survival** FALSE

**discrete** FALSE

**link** default log

**pdf** zeroinflated

## Example

In the following example we estimate the parameters in a simulated example for both type 0 and type 1.

### Poisson

```

## type 0
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
p = 0.2
E = sample(c(1,5,10,15), size=n, replace=TRUE)
lambda = E*exp(eta)

## first sample y|y>0
y = rpois(n, lambda = lambda)
is.zero = (y == 0)
while(sum(is.zero) > 0)
{
  y[is.zero] = rpois(sum(is.zero), lambda[is.zero])
  is.zero = (y == 0)
}
## then set some of these to zero
y[ rbinom(n, size=1, prob=p) == 1 ] = 0

data = list(y=y,z=z)
formula = y ~ 1+z
result0 = inla(formula, family = "zeroinflatedpoisson0", data = data, E=E)
summary(result0)

## type 1
y = rpois(n, lambda = lambda)
y[ rbinom(n, size=1, prob=p) == 1 ] = 0
data = list(y=y,z=z)
formula = y ~ 1+z
result1 = inla(formula, family = "zeroinflatedpoisson1", data = data, E=E)
summary(result1)

```

## Binomial

```
## type 0
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
p = 0.2
Ntrials = sample(c(1,5,10,15), size=n, replace=TRUE)
prob = exp(eta)/(1 + exp(eta))

y = rbinom(n, size = Ntrials, prob = prob)
is.zero = (y == 0)
while(sum(is.zero) > 0)
{
  y[is.zero] = rbinom(sum(is.zero), size = Ntrials[is.zero], prob = prob[is.zero])
  is.zero = (y == 0)
}
y[ rbinom(n, size=1, prob=p) == 1 ] = 0
data = list(y=y,z=z)
formula = y ~ 1+z
result0 = inla(formula, family = "zeroinflatedbinomial0", data = data, Ntrials = Ntrials)
summary(result0)

## type 1
y = rbinom(n, size = Ntrials, prob = prob)
y[ rbinom(n, size=1, prob=p) == 1 ] = 0
data = list(y=y,z=z)
formula = y ~ 1+z
result1 = inla(formula, family = "zeroinflatedbinomial1", data = data, Ntrials=Ntrials)
summary(result1)
```

## Notes

None.

## Extentions

There are some extentions available which currently is only implemented for the cases where its needed/requested.

**Type 2** Is like Type 1 but where (for the Poisson)

$$p = 1 - \left( \frac{E \exp(x)}{1 + E \exp(x)} \right)^\alpha$$

where  $\alpha > 0$  is the hyperparameter instead of  $p$  (and  $E \exp(x)$  is the mean). Available for Poisson as `zeroinflatedpoisson2`, for binomial as `zeroinflatedbinomial2` and for the negative binomial as `zeroinflatednbinomial2`.

The internal representation is  $\theta = \log(\alpha)$  and prior is defined on  $\log(\alpha)$ .

## **Zeroinflated Poisson Type 2**

**hyper**

**theta**

**name** log alpha  
**short.name** a  
**initial** 0.693147180559945  
**fixed** FALSE  
**prior** gaussian  
**param** 0.693147180559945 1  
**to.theta** function(x) log(x)  
**from.theta** function(x) exp(x)

**survival** FALSE

**discrete** FALSE

**link** default log

**pdf** zeroinflated

## **Zeroinflated Binomial Type 2**

**hyper**

**theta**

**name** alpha  
**short.name** alpha  
**initial** -1  
**fixed** FALSE  
**prior** gaussian  
**param** -1 0.2  
**to.theta** function(x) log(x)  
**from.theta** function(x) exp(x)

**survival** FALSE

**discrete** FALSE

**link** default logit probit cloglog

**pdf** zeroinflated

## **Zeroinflated Negative Binomial Type 2**

**hyper**

**theta1**

**name** log size  
**short.name** size  
**initial** 2.30258509299405  
**fixed** FALSE

```

    prior loggamma
    param 1 1
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
    name log alpha
    short.name a
    initial 0.693147180559945
    fixed FALSE
    prior gaussian
    param 2 1
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
survival FALSE
discrete FALSE
link default log
pdf zeroinflated

```