

# Weibull With Cure Fraction

## Background

$$\begin{aligned}Z_i &\sim \text{Bernoulli}(\rho) \\[Y_i|Z_i = 0] &\sim \text{Weibull}(\lambda_i, \alpha) \\[Y_i|Z_i = 1] &= \infty\end{aligned}$$

## Parametrisation

The Weibull is parametrized as `variant=0` of the `weibull` family.

$$f(y) = (1 - \rho)\alpha y^{\alpha-1}\lambda \exp(-\lambda y^\alpha), \quad 0 \leq y < \infty, \quad \alpha > 0, \quad \lambda > 0$$

$\alpha$ : shape parameter.

$\rho$ : the cure fraction parameter

## Link-function

The parameter  $\lambda$  is linked to the linear predictor as:

$$\lambda = \exp(\eta)$$

## Hyperparameters

The  $\alpha$  parameter is represented as

$$\theta_1 = \log \alpha$$

and  $\rho$  is transformed to

$$\theta_2 = \log[\rho/(1 - \rho)].$$

The priors are defined on  $\theta$ .

## Specification

Response variable  $y$  must be given using `inla.surv()`

## Hyperparameter spesification and default values

```
doc The Weibull-cure likelihood (survival)
hyper
  thetal
    hyperid 81001
    name log alpha
    short.name a
    initial 0.1
    fixed FALSE
    prior pc.alphaw
    param 5
    to.theta function(x, sc = 0.1) log(x)/sc
    from.theta function(x, sc = 0.1) exp(sc*x)
```

```

theta2
  hyperid 81002
  name logit probability
  short.name prob
  initial -1
  fixed FALSE
  prior gaussian
  param -1 0.2
  to.theta function(x) log(x/(1-x))
  from.theta function(x) exp(x)/(1+exp(x))

survival TRUE
discrete FALSE
link default log neglog
pdf weibullcure

```

## Example

In the following example we estimate the parameters in a simulated case

```

n = 1000
alpha = 2
beta = 2
rho = 0.5

x = runif(n)
censorTime = runif(n,0,2)
eta = 1+beta*x
lambda = exp(eta)
y = rweibull(n, shape= alpha, scale= lambda^(1/-alpha))
z = rbinom(n,size=1, prob=rho)

censoredEvent = (y > censorTime) | z
yObs = y
yObs[censoredEvent] = censorTime[censoredEvent]
event = as.numeric(!censoredEvent)
data = list(y=inla.surv(yObs, event), x=x)

model=inla(
  y ~ x,
  family ="weibullcure",
  data=data,
  control.family = list(hyper=list(
    'log alpha' = list(
      prior='loggamma', param=c(1,1)),
    'logit probability' = list(
      prior='logitbeta', param=c(1,1))))))

summary(model)

```

## Notes

- Weibull model can be used for right censored, left censored, interval censored data. If the observed times  $y$  are large/huge, then this can cause numerical overflow in the likelihood routine. If you encounter this problem, try to scale the observations, `time = time / max(time)` or similar.