Random walk model of order 1 (RW1)

Parametrization

The random walk model of order 1 (RW1) for the Gaussian vector $\mathbf{x} = (x_1, \ldots, x_n)$ is constructed assuming independent increments:

$$\Delta x_i = x_i - x_{i+1} \sim \mathcal{N}(0, \tau^{-1})$$

The density for $\mathbf{x}$ is derived from its $n-1$ increments as

$$\begin{aligned} \pi(\mathbf{x}|\tau) &\propto \tau^{(n-1)/2} \exp\left\{-\frac{\tau}{2}\sum(\Delta x_i)^2\right\} \\ &= \tau^{(n-1)/2} \exp\left\{-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}\right\} \end{aligned}$$

where $\mathbf{Q} = \tau\mathbf{R}$ and $\mathbf{R}$ is the structure matrix reflecting the neighbourhood structure of the model.

It is also possible to define a *cyclic* version of the RW1 model, in this case the graph is modified so that last node x_n is neighbour of x_{n-1} and x_1.

Hyperparameters

The precision parameter τ is represented as

$$\theta = \log \tau$$

and the prior is defined on θ.

Specification

The RW1 model is specified inside the `f()` function as

```
f(<whatever>, model="rw1", values=<values>, cyclic=<TRUE|FALSE>,
             hyper = <hyper>, scale.model = FALSE)
```

The (optional) argument `values` is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See next example for an application.

The logical option `scale.model` determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is `FALSE` so that the model is not scaled.

Current recommended prior

If you do not know which prior to use, the current recommendation is

```
u = 1
f(<whatever>, model="rw1", scale.model = TRUE
              hyper = list(theta = list(prior="pc.prec", param=c(u,0.01))))
inla.doc("pc.prec")
```

where `u` should be set to a value appropriate for your case:

Gaussian likelihood (no link) Set `u` to be the empirical standard deviation of your data

Poisson likelihood and log link Set `u` to 1

Binomial and logit link Set `u` to 0.5

Binomial and probit link Set `u` to 0.33

Increasing `u` gives a weaker prior, decreasing `u` gives a stronger prior.

Hyperparameter spesification and default values

hyper

theta

hyperid 4001

name log precision

short.name prec

prior loggamma

param 1 5e-05

initial 4

fixed FALSE

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

constr TRUE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

min.diff 1e-05

pdf rw1

Example

```
n=100
z=seq(0,6,length.out=n)
y=sin(z)+rnorm(n,mean=0,sd=0.5)
data=data.frame(y=y,z=z)

formula=y~f(z,model="rw1",
            hyper = list(prec = list(prior="loggamma",param=c(1,0.01))))
result=inla(formula,data=data,family="gaussian")

#here we estimate the effect only for some of the values in z
formula1=y~f(z,model="rw1",
             hyper = list(prec = list(prior="loggamma",param=c(1,0.01))))
result1=inla(formula1,data=data,family="gaussian")
```

Notes

- The RW1 is intrinsic with rank deficiency 1.
- The RW1 model for irregular locations are supported although not described here.
- The term $\frac{1}{2}\log(|R|^*)$ of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it.