(3) Replicated data

$$\text{prob}(y) = \sum_{n=N}^{\infty} Po(n;\lambda) \prod_{i=1}^{d} Bin(y_i; n, p)$$

$$N = \max(y)$$

Using recursive formulas:

$$Po(n;\lambda) \prod_{i=1}^{d} Bin(y_i; n, p)$$

$$= Po(n-1;\lambda) \prod_{i} Bin(y_i; n-1, p)$$

$$\cdot \frac{(1-p)^d \lambda}{n} \prod_{i=1}^{d} \frac{n}{n-y_i}$$

so its

$$\text{prob}(y) = Po(N;\lambda) \prod_{i=1}^{d} Bin(y_i; N, p)$$

$$\cdot fac$$

where facc is computed as

$fac = 1$

~~for~~ nn in nmax : (ymax+1)

$$fac = 1 + fac \cdot \frac{\lambda (1-p)^d}{nn} \prod_{i=1}^{d} \frac{nn}{nn - y_i}$$

(1)

Jan 17th-17

N-mix model : details

Model $\quad y \sim Bin(n, p)$

$\qquad\qquad n \sim Pois(\lambda)$

Normally, ~~to~~ $p = p(z)$, while, $\log(\lambda) = x^t\beta$

The likelihood is

$$prob(y) = \sum_{n=y}^{\infty} Pois(n; \lambda) \cdot Bin(y; n, p)$$

There is a nice recursive formula for this density, using that

$$Pois(n; \lambda) = Pois(n-1; \lambda) \frac{\lambda}{n}$$

$$Bin(y; n, p) = Bin(y; n-1, p) \frac{n}{n-y}(1-p)$$

So that

$$Pois(n; \lambda) Bin(y; n, p) = Pois(n-1; \lambda) Bin(y; n-1, p) \cdot \frac{\lambda}{n-y}(1-p)$$

Let $f_i = \dfrac{\lambda(1-p)}{i}$, $i = 1, 2, \ldots, n_{max}$

then

$$L = \sum_{n=y}^{n_{max}} \text{Pois}(n; \lambda) \cdot \text{Bin}(y; n, p)$$

$$= \text{Pois}(y; \lambda) \cdot \text{Bin}(y; y, p) \cdot \{$$

$$1 + f_1 + f_1 f_2 + f_1 f_2 f_3 + \cdots + f_1 \cdots f_{n_{max}} \}$$

$$= \text{Pois} \cdot \text{Bin} \{ \underbrace{1 + f_1(1 + f_2(1 + f_3(1 + f_4 (1 + \cdots))))}_{fac} \}$$

fac can ~~can~~ be computed..

```
fac = 1
for i in imax:1
        fac = 1 + fac · λ(1-p)/i
```

then

$$\underline{L = \text{Pois}(y; \lambda) \cdot \text{Bin}(y; y, p) \cdot fac.}$$

imax could be $\left\lfloor \frac{\lambda(1-p)}{\varepsilon} \right\rfloor$

so that $\frac{\lambda(1-p)}{i_{max}} \approx \varepsilon$