

## Poisson

### Parametrisation

The Poisson distribution is

$$\text{Prob}(y) = \frac{\lambda^y}{y!} \exp(-\lambda)$$

for responses  $y = 0, 1, 2, \dots$ , where

$\lambda$ : the expected value.

### Link-function

The mean and variance of  $y$  are given as

$$\mu = \lambda \quad \text{and} \quad \sigma^2 = \lambda$$

and the mean is linked to the linear predictor by

$$\lambda(\eta) = E \exp(\eta)$$

where  $E > 0$  is a known constant (or  $\log(E)$  is the offset of  $\eta$ ).

### Hyperparameters

None.

### Specification

- family = poisson
- Required arguments:  $y$  and  $E$

### Example

In the following example we estimate the parameters in a simulated example with Poisson responses.

```
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
E = sample(1:10, n, replace=TRUE)
lambda = E*exp(eta)
y = rpois(n, lambda = lambda)

data = list(y=y,z=z)
formula = y ~ 1+z
result = inla(formula, family = "poisson", data = data, E=E)
summary(result)
```

### Notes

This likelihood also accept  $E = 0$  and in this case  $\log(E)$  is *defined* to be 0. Non-integer values of  $y \geq 0$  is accepted although the normalising constant of the likelihood is then wrong (but its a constant only).