Random walk model of order 2 (RW2)

Parametrization

The random walk model of order 2 (RW2) for the Gaussian vector $\mathbf{x} = (x_1, \ldots, x_n)$ is constructed assuming independent second-orderincrements:

$$\Delta^2 x_i = x_i - 2\ x_{i+1} + x_{i+2} \sim \mathcal{N}(0, \tau^{-1})$$

The density for $\mathbf{x}$ is derived from its $n-2$ second-order increments as

$$\begin{aligned} \pi(\mathbf{x}|\tau) &\propto \tau^{(n-2)/2} \exp\left\{-\frac{\tau}{2}\sum(\Delta^2 x_i)^2\right\} &(1)\\ &= \tau^{(n-2)/2} \exp\left\{-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}\right\} &(2)\end{aligned}$$

where $\mathbf{Q} = \tau\mathbf{R}$ and $\mathbf{R}$ is the structure matrix reflecting the neighbourhood structure of the model.

It is also possible to define a *cyclic* version of the RW2 model.

Hyperparameters

The precision parameter τ is represented as

$$\theta = \log\tau$$

and the prior is defined on θ.

Specification

The RW2 model is specified inside the `f()` function as

```
f(<whatever>, model="rw2", values=<values>,
  cyclic=FALSE, scale.model = FALSE)
```

The (optional) argument `values` is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See the example for RW1 for an application.

The logical option `scale.model` determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is `FALSE` so that the model is not scaled.

Hyperparameter spesification and default values

hyper

theta

name log precision
short.name prec
prior loggamma
param 1 5e-05
initial 4
fixed FALSE
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

constr TRUE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

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Example

```
n=100
z=seq(0,6,length.out=n)
y=sin(z)+rnorm(n,mean=0,sd=0.5)
data=data.frame(y=y,z=z)

formula=y~f(z,model="rw2")
result=inla(formula,data=data,family="gaussian")
```

Notes

- The RW2 is a intrinsic with rank deficiency 2.
- The RW2 model for irregular locations are supported although not described here.
- The $\frac{n-r}{2}\log(|R|^*)$-part (with $r = 2$) of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it.