Probabilities for Weibull cure model

Event time is ∞ with probability ρ, and has the following Weibull distribution otherwise:

$$f(t;\mu_i,\nu) = \mu_i \nu t^{\nu-1} \exp\left[-\mu_i t^{\nu}\right].$$

Given that a subject reached time L without cancer incidence, the density function for their cancer incidence time T is

$$\begin{aligned} pr(T=t|T>L) =& [(1-\rho) f(t;\mu_i,\nu)] \Big/ \left[\rho + (1-\rho)\int_L^{\infty} f(u;\mu_i,\nu)du\right] \\ =& \frac{\mu_i \nu t^{\nu-1} \exp\left[-\mu_i t^{\nu}\right]}{[\rho/(1-\rho) + \exp(-\mu_i L^{\nu})]}. \end{aligned}$$

The probability that an individual has a cancer time greater than r (right censoring) given that they are cancer-free at age L is

$$\begin{aligned} pr(T>t|T>L) =& \frac{\rho + (1-\rho)\int_t^{\infty} f(u;\mu_i,\nu)du}{\rho + (1-\rho)\int_L^{\infty} f(u;\mu_i,\nu)du} \\ =& \frac{\rho + (1-\rho)\exp(-\mu_i t^{\nu})}{\rho + (1-\rho)\exp(-\mu_i L^{\nu})}. \end{aligned}$$

Left censoring.

$$\begin{aligned} pr(T<t|T>L) =& \frac{(1-\rho)\int_L^{t} f(u;\mu_i,\nu)du}{\rho + (1-\rho)\int_L^{\infty} f(u;\mu_i,\nu)du} \\ =& \frac{(1-\rho)\exp[-\mu_i(t^{\nu} - L^{\nu})]}{1-(1-\rho)\exp(-\mu_i L^{\nu})}. \end{aligned}$$

interval censoring

$$\begin{aligned} pr(t_1<T<t_2|T>L) =& \frac{(1-\rho)\int_{\max(L,T_1)}^{t_2} f(u;\mu_i,\nu)du}{\rho + (1-\rho)\int_L^{\infty} f(u;\mu_i,\nu)du} \\ =& \frac{(1-\rho)\exp[-\mu_i(t_2^{\nu} - \max(L,t_1)^{\nu})]}{1-(1-\rho)\exp(-\mu_i L^{\nu})}. \end{aligned}$$