Negative Binomial

Parametrisation

The negative Binomial distribution is

$$Prob(y) = \frac{\Gamma(y+n)}{\Gamma(n)\Gamma(y+1)} p^n (1-p)^y$$

for responses $y = 0, 1, 2, \ldots$, where

n: number of successful trials (size), or dispersion parameter. Must be strictly positive, need not be integer.

p: probability of success in each trial.

Link-function

The mean and variance of y are given as

$$\mu = n \frac{1-p}{p}$$
 and $\sigma^2 = \mu(1 + \frac{\mu}{n})$

and the mean is linked to the linear predictor by

$$\mu = E \exp(\eta)$$

where the hyperparameter n (size) plays the role of an overdispersion parameter. E represents knows constant and $\log(E)$ is the offset of η .

Hyperparameters

The default parameterization (variant=0) is that the overdispersion parameter n (size) is represented as

$$\theta = \log(n)$$

and the prior is defined on θ .

An alternative parameterization (variant=1) is that the overdispersion parameter n (size) is represented as

$$\theta = \log(n) - \log(E)$$

and the prior is defined on θ .

Specification

- family = nbinomial
- Required arguments: y and E (default E = 1).
- Chose variant with either control.family = list(variant=0) (default) or control.family = list(variant=1)

Hyperparameter spesification and default values

```
hyper
```

```
name size
short.name size
initial 2.30258509299405
fixed FALSE
prior loggamma
param 1 1
to.theta function(x) log(x)
from.theta function(x) exp(x)
survival FALSE
discrete TRUE
link default log logoffset
pdf nbinomial
```

Example

In the following example we estimate the parameters in a simulated example with negative binomial responses and assign the hyperparameter θ a Gaussian prior with mean 0 and precision 0.01

```
n = 1000
x = rnorm(n, sd = 0.2)
eta = 1 + x
E = runif(n, min = 0, max=10)
mu = E * exp(eta)
size = 3
y = rnbinom(n, size=size, mu=mu)
r = inla(y ~1 + x, data = data.frame(y, x, E),
    family = "nbinomial", E=E)
mu = E * exp(eta)
size = E*3
y = rnbinom(n, size=size, mu=mu)
rr = inla(y ~ 1 + x, data = data.frame(y, x, E),
    family = "nbinomial",
    control.family = list(variant = 1),
    E=E)
```

Notes

As $n \to \infty$, the negative Binomial converges to the Poisson distribution. For numerical reasons, if n is too large:

$$\frac{\mu}{n} < 10^{-4}$$

then the Poisson limit is used.