

# Normal Inverse Gaussian (NIG) model for Stochastic volatility

## Parametrization

The NIG likelihood for stochastic volatility models is defined as:

$$\pi(y|\eta) = \sigma\epsilon$$

where

$$\epsilon \sim \text{NIG}(\beta, \psi)$$

and  $\text{NIG}(\beta, \psi)$  is a standardised NIG distribution with density

$$\pi(\cdot|\beta, \psi) = \frac{\gamma\psi}{\pi} \sqrt{\frac{\beta^2 + \psi^2}{(\gamma x + \beta)^2 + \psi^2}} \exp\left(\psi^2 + \beta(\gamma x + \beta)\right) K_1\left(\sqrt{(\beta^2 + \psi^2)((\gamma x + \beta)^2 + \psi^2)}\right)$$

where  $\gamma^2 = 1 + \beta^2/\psi^2$ .

## Link-function

The squared scale parameter  $\sigma$  is linked to the linear predictor  $\eta$  as:

$$\sigma^2 = \exp(\eta)$$

## Hyperparameters

The skewness parameter  $\beta$  is represented as:

$$\theta_1 = \beta$$

and the shape parameter  $\psi$  as

$$\theta_2 = \log(\psi - 1)$$

as the prior is defined on  $\theta = (\theta_1, \theta_2)$

## Specification

- family = `stochvol.nig`
- Required argument:  $y$ .

## Hyperparameter spesification and default values

**hyper**

**theta1**

**hyperid** 84001

**name** skewness

**short.name** skew

**initial** 0

**fixed** FALSE

**prior** gaussian

**param** 0 10

**to.theta** function(x) x

**from.theta** function(x) x

```

theta2
  hyperid 84002
  name shape
  short.name shape
  initial 0
  fixed FALSE
  prior loggamma
  param 1 0.5
  to.theta function(x) log(x-1)
  from.theta function(x) 1+exp(x)

```

```
survival FALSE
```

```
discrete FALSE
```

```
link default log
```

```
pdf stochvolnig
```

## Example

In the following example we specify the likelihood for the stochastic volatility model to be NIG

```

#simulated data
n=500
phi=0.53
eta=rep(0.1,n)
for(i in 2:n)
  eta[i]=0.1+phi*(eta[i-1]-0.1)+rnorm(1,0,0.6)
y=exp(eta/2)*rnorm(n)
time=1:n
data=list(ret=y,time=time)

#fit the model
formula=ret~f(time,model="ar1")
result=inla(formula,family="stochvol.nig",data=data)

```

## Notes

None