Skew-Normal (version 1 and 2)

Parametrisation

The Skew-Normal distribution is

$$f(y) = 2\frac{\sqrt{w\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}w\tau\,(y-\mu)^2\right)\;\Phi(a\;a_{\max}\,[w\tau\,(y-\mu)])$$

for continuously responses y where $\Phi(\cdot)$ is the cumulative distribution function for a standard Normal, and

μ: is the the location parameter

τ: is the inverse scale

w: is a fixed weight, $w > 0$,

a: is the shape parameter

$a_{\max}$: is the (fixed) maximum value of the shape paramter (added for stability reasons). Default value is 5.

Link-function

The location parameter is linked to the linear predictor by

$$\mu = \eta$$

Hyperparameters

The inverse scale is represented as

$$\theta_1 = \log\tau$$

and the prior is defined on θ_1.

The shape parameter is

$$a = 2\frac{\exp(\theta_2)}{1+\exp(\theta_2)} - 1$$

and the prior is defined on θ_2.

Specification

- family = `sn`
- Required arguments: y and w (keyword `scale`). The weights has default value 1.
- Optional control arguments: `sn.shape.max`. Default value is 5.0.

Hyperparameter spesification and default values

hyper

theta1

name log inverse scale
short.name iscale
initial 4

fixed FALSE

prior loggamma

param 1 5e-05

theta2

name logit skewness

short.name skew

initial 0

fixed FALSE

prior gaussian

param 0 10

to.theta `function(x, shape.max = 1) log((1+x/shape.max)/(1-x/shape.max))`

from.theta `function(x, shape.max = 1) shape.max*(2*exp(x)/(1+exp(x))-1)`

survival FALSE

discrete FALSE

link default identity

pdf sn

Example

This is a simulated example requiring the package `sn`.

```
library(sn)
n = 1000
z = rnorm(n)
y = z + rsn(n, shape = 2)
formula = y ~ z
r = inla(formula, family = "sn", data = data.frame(z,y),
         control.family = list(sn.shape.max = 5.0))
summary(r)
```

Notes

An simpler approximation to $\Phi(\cdot)$ is used to improve the speed, which has maximum absolute error of 0.00197323; see the source code for further details.

Skew-Normal (version 2)

Parametrisation (version 2)

In the family "sn2" we offer an alternative parametersisation of the skew-normal with moment parameters, precision $w\tau$ (where w is a fixed weight or scale) and standarized skewness γ (where $|\gamma| < 1$ due to the skew-normal family[1]). In this parameterisation, the location parameter is linked to the linear predictor by

$$\mu = \eta$$

and μ equals the expected value $\xi + \omega\delta\sqrt{\frac{2}{\pi}}$, and $\delta = \alpha/\sqrt{1+\alpha^2}$.

Hyperparameters

The precision τ is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on θ_1. The (standarized) skewness γ is

$$\gamma = 2\frac{\exp(\theta_2)}{1+\exp(\theta_2)} - 1$$

and the prior is defined on θ_2.

The function `INLA:::inla.sn.reparam` offer the mapping between the moments (mean, variance and skewness) and the parameters used in the skew-normal density in the format used in the package `sn`, which are (ξ, ω, α), where

$$f(x) = \frac{2}{\omega}\phi\left(\frac{x-\xi}{\omega}\right)\Phi\left(\alpha\left[\frac{x-\xi}{\omega}\right]\right)$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ is the density and cumulative distribution function for the standard Gaussian distribution.

Hyperparameter spesification and default values for `sn2`

hyper

theta1

name log precision

short.name prec

initial 1

fixed FALSE

prior loggamma

param 1 5e-05

theta2

name logit skewness

short.name skew

initial 0

fixed FALSE

prior gaussian

param 0 10

[1] or to be presice, $|\gamma| < \frac{4-\pi}{2(\pi/2-1)^{3/2}} = 0.995271746\ldots$

to.theta `function(x) log((1+x)/(1-x))`

from.theta `function(x) (2*exp(x)/(1+exp(x))-1)`

survival FALSE

discrete FALSE

link default identity

status experimental

pdf sn2

Example for `sn2`

This is a simulated example requiring the package `sn`.

```
library(sn)
n = 500
x = rnorm(n)
eta = 1/2 + 2*x
w = runif(n, min = 0.5, max = 2)
prec = 1 * w
skewness = 0.25
y = numeric(n)
for(i in 1:n) {
    param = INLA:::inla.sn.reparam(moments = c(eta[i], 1/prec[i], skewness))
    y[i] = rsn(1, xi=param$xi, omega = param$omega, alpha = param$alpha)
}
r = inla(y ~ 1 + x, family = "sn2", scale = w, data = data.frame(y, x, w),  verbose=T)
summary(r)
```

Notes for `sn2`

In this parametersiation there is no `sn.shape.max`.

An simpler approximation to $\Phi(\cdot)$ is used to improve the speed, which has maximum absolute error of 0.00197323; see the source code for further details.