

# The Gamma-distribution

## Parametrisation

The Gamma-distribution has the following density

$$\pi(y) = \frac{b^a}{\Gamma(a)} y^{a-1} \exp(-by), \quad a > 0, \quad b > 0, \quad y > 0,$$

where  $E(y) = \mu = a/b$  and  $\text{Var}(y) = 1/\tau = a/b^2$ , where  $\tau$  is the precision and  $\mu$  is the mean. We will use the following parameterisation for the precision

$$\tau = (s\phi)/\mu^2$$

where  $\phi$  is the precision parameter (or  $1/\phi$  is the dispersion parameter) and  $s > 0$  is a fixed scaling, which gives this density

$$\pi(y) = \frac{1}{\Gamma(s\phi)} \left( \frac{(s\phi)}{\mu} \right)^{(s\phi)} y^{(s\phi)-1} \exp \left( -(s\phi) \frac{y}{\mu} \right)$$

## Link-function

The linear predictor  $\eta$  is linked to the mean  $\mu$  using a default log-link

$$\mu = \exp(\eta)$$

## Hyperparameter

The hyperparameter is the precision parameter  $\phi$ , which is represented as

$$\phi = \exp(\theta)$$

and the prior is defined on  $\theta$ .

## Specification

- family = `gamma`
- Required arguments:  $y$  and  $s$  (argument `scale`)

The scalings have default value 1.

## Hyperparameter spesification and default values

**doc** The Gamma likelihood

**hyper**

**theta**

**hyperid** 58001

**name** precision parameter

**short.name** prec

**initial** 4.60517018598809

**fixed** FALSE

**prior** loggamma

```
param 1 0.01
to.theta function(x) log(x)
from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default log quantile

pdf gamma
```

## Example

In the following example we estimate the parameters in a simulated example.

```
n = 1000
x = rnorm(n)
eta = 1 + x
mu = exp(eta)
prec.scale = runif(n, min = 0.5, max = 2)
prec.par = 1.2
a = prec.par * prec.scale
b = mu / (prec.par * prec.scale)
y = rgamma(n, shape = a, scale = b)
r = inla(y ~ 1 + x, data = data.frame(y, x),
        scale = prec.scale, family = "gamma")
```

## Notes

None.