The Berkson Measurement Error (MEB) model

Parametrization

This is an implementation of the Berksom measurement error model for a fixed effect. The observed covariate is w but it is x that goes into the linear predictor

$$\eta = \ldots + \beta x + \ldots \ ,$$

where $x = w + u$. The error term u is Gaussian with prior $\mathcal{N}(0, \tau_u \mathbf{D})$[1], where τ_u is the observational precision of the error $\text{Prec}(u|x))$ with possible heteroscedasticy, encoded in the entries d_i of the diagonal matrix $\mathbf{D}$. The vector s contains the fixed scalings $s = (d_1, \ldots, d_n)$ (with n the number of data points).

Hyperparameters

This model has 2 hyperparameters, $\theta = (\theta_1, \theta_2)$. The hyperparameter specification is as follows:

$$\theta_1 = \beta$$

and the prior is defined on θ_1,

$$\theta_2 = \log(\tau_u)$$

and the prior is defined on θ_2.

Specification

The MEB is specified inside the `f()` function as

```
f(w, [<weights>,] model="meb", scale = <s>, values= <w>, hyper = <hyper>)
```

Here, `w` are the observed covariates, and the fixed scaling of the observational precision is given in argument `scale`. If the argument `scale` is not given, then s is set to 1.

Note that only the unique values of `w` are used, so if two or more elements of `w` are *identical*, then they refer to the *same* element in the covariate x. If data points with identical w values belong to different x values (e.g., different individuals), please add a *tiny* random value to w to make this difference obvious to the model.

Hyperparameter specification and default values

hyper

theta1

name beta
short.name b
prior gaussian
param 1 0.001
initial 1
fixed FALSE
to.theta `function(x) x`
from.theta `function(x) x`

theta2

[1] Note: The second argument in $\mathcal{N}(,)$ is the precision not the variance.

name prec.u
short.name prec
prior loggamma
param 1 1e-04
initial 6.90775527898214
fixed FALSE
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

status experimental

pdf meb

Example

```
n = 100
beta = 2
w = rnorm(n)
prec.u = 1
prec.y = 1
## heteroscedastic scaling
s = runif(n,min=0,max=1)
## true but unobserved covariate
x = w + rnorm(n, sd = 1/sqrt(s*prec.u))
y = 1 + beta*x + rnorm(n, sd = 1/sqrt(prec.y))

## prior parameters
prior.beta = c(0, 0.0001)
prior.prec.u = c(10, 9/prec.u)
prior.prec.y = c(10, 9/prec.y)


formula = y ~ f(w, model="meb", scale=s, values=w,
    hyper = list(
        beta = list(
            prior = "gaussian",
            param = prior.beta,
            fixed = FALSE
```

```
        ),
        prec.u = list(
            prior = "loggamma",
            param = prior.prec.u,
            initial = log(prec.u),
            fixed = FALSE
        )
    )
)

r = inla(formula, data = data.frame(y, w, s),
    family = "gaussian",
    control.family = list(
        hyper = list(
            prec = list(param = prior.prec.y,
                fixed = FALSE
            )
        )
    )
)
```

Notes

- `INLA` provides the posteriors of $\nu_i = \beta x_i$ and NOT x_i.
- The posteriors of ν_i come (default) in the order given by the sorted (from low to high) values of `w`. The entry `$ID` gives the mapping.
- The option `scale` defines the scaling in the same order as argument `values`. It is therefore adviced to also give argument `values` when `scale` is used to be sure that they are consistent.