Skew-Normal (version 1 and 2)

Parametrisation

The Skew-Normal distribution is

$$f(y) = 2\frac{\sqrt{w\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}w\tau\,(y-\mu)^2\right)\ \Phi(a\,a_{\max}\,[w\tau\,(y-\mu)])$$

for continuously responses y where $\Phi(\cdot)$ is the cumulative distribution function for a standard Normal, and

μ: is the the location parameter

τ: is the inverse scale

w: is a fixed weight, $w > 0$,

a: is the shape parameter

$a_{\mathbf{max}}$: is the (fixed) maximum value of the shape paramter (added for stability reasons). Default value is 5.

Link-function

The location parameter is linked to the linear predictor by

$$\mu = \eta$$

Hyperparameters

The inverse scale is represented as

$$\theta_1 = \log\tau$$

and the prior is defined on θ_1.

The shape parameter is

$$a = 2\frac{\exp(\theta_2)}{1 + \exp(\theta_2)} - 1$$

and the prior is defined on θ_2.

Specification

- family = `sn`

- Required arguments: y and w (keyword `scale`). The weights has default value 1.

- Optional control arguments: `sn.shape.max`. Default value is 5.0.

Hyperparameter spesification and default values

hyper

> **theta1**
>
> > **hyperid** 74001
> > **name** log inverse scale
> > **short.name** iscale

      initial 4

      fixed FALSE

      prior loggamma

      param 1 5e-05

   theta2

      hyperid 74002

      name logit skewness

      short.name skew

      initial 0

      fixed FALSE

      prior gaussian

      param 0 10

      to.theta `function(x, shape.max = 1) log((1+x/shape.max)/(1-x/shape.max))`

      from.theta `function(x, shape.max = 1) shape.max*(2*exp(x)/(1+exp(x))-1)`

survival FALSE

discrete FALSE

link default identity

pdf sn

Example

This is a simulated example requiring the package `sn`.

```
library(sn)
n = 1000
z = rnorm(n)
y = z + rsn(n, shape = 2)
formula = y ~ z
r = inla(formula, family = "sn", data = data.frame(z,y),
        control.family = list(sn.shape.max = 5.0))
summary(r)
```

Notes

An simpler approximation to $\Phi(\cdot)$ is used to improve the speed, which has maximum absolute error of 0.00197323; see the source code for further details.

Skew-Normal (version 2)

Parametrisation (version 2)

In the family "sn2" we offer an alternative parametersisation of the skew-normal with moment parameters, precision $w\tau$ (where w is a fixed weight or scale) and standarized skewness γ (where $|\gamma| < 1$ due to the skew-normal family[1]). In this parameterisation, the location parameter is linked to the linear predictor by

$$\mu = \eta$$

and μ equals the expected value $\xi + \omega\delta\sqrt{\frac{2}{\pi}}$, and $\delta = \alpha/\sqrt{1 + \alpha^2}$.

Hyperparameters

The precision τ is represented as

$$\theta_1 = \log\tau$$

and the prior is defined on θ_1. The (standarized) skewness γ is

$$\gamma = 2\frac{\exp(\theta_2)}{1 + \exp(\theta_2)} - 1$$

and the prior is defined on θ_2.

The function `INLA:::inla.sn.reparam` offer the mapping between the moments (mean, variance and skewness) and the parameters used in the skew-normal density in the format used in the package `sn`, which are (ξ, ω, α), where

$$f(x) = \frac{2}{\omega}\phi\left(\frac{x - \xi}{\omega}\right)\Phi\left(\alpha\left[\frac{x - \xi}{\omega}\right]\right)$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ is the density and cumulative distribution function for the standard Gaussian distribution.

Hyperparameter spesification and default values for `sn2`

hyper

    theta1

        hyperid 75001
        name log precision
        short.name prec
        initial 1
        fixed FALSE
        prior loggamma
        param 1 5e-05

    theta2

        hyperid 75002
        name logit skewness
        short.name skew
        initial 0
        fixed FALSE

---

[1] or to be presice, $|\gamma| < \frac{4-\pi}{2(\pi/2-1)^{3/2}} = 0.995271746\ldots$

> **prior** gaussian
> **param** 0 10
> **to.theta** function(x) log((1+x)/(1-x))
> **from.theta** function(x) (2*exp(x)/(1+exp(x))-1)

survival FALSE

discrete FALSE

link default identity

status experimental

pdf sn2

Example for sn2

This is a simulated example requiring the package **sn**.

```
library(sn)
n = 500
x = rnorm(n)
eta = 1/2 + 2*x
w = runif(n, min = 0.5, max = 2)
prec = 1 * w
skewness = 0.25
y = numeric(n)
for(i in 1:n) {
    param = INLA:::inla.sn.reparam(moments = c(eta[i], 1/prec[i], skewness))
    y[i] = rsn(1, xi=param$xi, omega = param$omega, alpha = param$alpha)
}
r = inla(y ~ 1 + x, family = "sn2", scale = w, data = data.frame(y, x, w),  verbose=T)
summary(r)
```

Notes for sn2

In this parametersiation there is no `sn.shape.max`.

An simpler approximation to $\Phi(\cdot)$ is used to improve the speed, which has maximum absolute error of 0.00197323; see the source code for further details.