The Kumaraswamy distribution

Parametrisation

The Kumaraswamy distribution is

$$f(y) = \alpha \beta y^{\alpha - 1} (1 - y^{\alpha})^{\beta - 1}$$

for 0 < y < 1 and $\alpha, \beta > 0$. The cumulative distribution function is

$$F(y) = 1 - (1 - y^{\alpha})^{\beta}$$
.

The parametrisation is given in terms of the quantile function

$$\kappa(q) = \left(1 - (1 - q)^{1/\beta}\right)^{1/\alpha}$$

and the precision parameter ϕ ,

$$\phi(q) = -\ln\left(1 - (1 - q)^{1/\beta}\right)$$

for fixed value of 0 < q < 1.

Link-function

The quantile κ to the linear predictor by

$$logit(\kappa) = \eta$$

using the default logit link-function.

Hyperparameters

The hyperparameter is

$$\theta = log(\phi)$$

and the prior is given for θ .

Specification

- family = qkumar
- Required arguments: y and the quantile q.

The quantile is given as control.family=list(quantile=q).

Hyperparameter spesification and default values

hyper

theta

hyperid 60001 name precision parameter short.name prec initial 0 fixed FALSE prior loggamma

```
param 1 0.001
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
survival FALSE
discrete FALSE
link default logit cauchit
\mathbf{pdf} qkumar
Example
rkumar = function(n, eta, phi, q=0.5)
{
    kappa = eta
    beta = log(1-q)/log(1-exp(-phi))
    alpha = log(1- (1-q)^(1/beta)) / log(kappa)
    u = runif(n)
    y = (1-u^(1/beta))^(1/alpha)
    return (y)
}
n = 100
q = 0.5
phi = 1
x = rnorm(n, sd = 1)
eta = inla.link.invlogit(1 + x)
y = rkumar(n, eta, phi, q)
r = inla(y ~1 + x,
    data = data.frame(y, x),
    family = "qkumar",
    control.family = list(quantile = q))
summary(r)
```

Notes

None.