

## Bym2 model for spatial effects

### Parametrization

This model is a reparameterisation of the BYM-model, which is a union of the besag model  $u^*$  and a iid model  $v^*$ , so that

$$x = \begin{pmatrix} v^* + u^* \\ u^* \end{pmatrix}$$

where both  $u^*$  and  $v^*$  has a precision (hyper-)parameter. The length of  $x$  is  $2n$  if the length of  $u^*$  (and  $v^*$ ) is  $n$ . The BYM2 model uses a different parameterisation of the hyperparameters where

$$x = \begin{pmatrix} \frac{1}{\sqrt{\tau}} (\sqrt{1-\phi} v + \sqrt{\phi} u) \\ u \end{pmatrix}$$

where both  $u$  and  $v$  are *standardised* to have (generalised) variance equal to one. The *marginal* precision is then  $\tau$  and the proportion of the marginal variance explained by the spatial effect ( $u$ ) is  $\phi$ .

### Hyperparameters

The hyperparameters are the marginal precision  $\tau$  and the mixing parameter  $\phi$ . The marginal precision  $\tau$  is represented as

$$\theta_1 = \log(\tau)$$

and the mixing parameter as

$$\theta_2 = \log\left(\frac{\phi}{1-\phi}\right)$$

and the prior is defined on  $\theta = (\theta_1, \theta_2)$ .

### Specification

The bym2 model is specified inside the `f()` function as

```
f(<whatever>, model="bym2", graph=<graph>,  
  hyper=<hyper>, adjust.for.con.comp = TRUE)
```

The neighbourhood structure of  $\mathbf{x}$  is passed to the program through the `graph` argument.

The option `adjust.for.con.comp` adjust the model if the graph has more than one connected compoment, and this adjustment can be disabled setting this option to `FALSE`. This means that `constr=TRUE` is interpreted as a sum-to-zero constraint on *each* connected component and the `rankdef` parameter is set accordingly.

### Hyperparameter spesification and default values

**hyper**

**theta1**

**hyperid** 11001

**name** log precision

**short.name** prec

**prior** pc.prec

**param** 1 0.01

**initial** 4

```

    fixed FALSE
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
  theta2
    hyperid 11002
    name logit phi
    short.name phi
    prior pc
    param 0.5 0.5
    initial -3
    fixed FALSE
    to.theta function(x) log(x/(1-x))
    from.theta function(x) exp(x)/(1+exp(x))

  constr TRUE

  nrow.ncol FALSE

  augmented TRUE

  aug.factor 2

  aug.constr 2

  n.div.by

  n.required TRUE

  set.default.values TRUE

  status experimental

  pdf bym2

```

## Example

## Details on the implementation

This gives some details of the implementation, which depends on the following variables

**nc1** Number of connected components in the graph with size 1. These nodes, *singletons*, have no neighbours.

**nc2** Number of connected components in the graph with size  $\geq 2$ .

**scale.model** The value of the logical flag, if the model should be scaled or not. (Default FALSE)

**adjust.for.con.comp** The value of the logical flag if the **constr=TRUE** option should be reinterpreted.

**The case** (`scale.model==FALSE && adjust.for.con.comp == FALSE`)

The option **constr=TRUE** is interpreted as a sum-to-zero constraint over the whole graph. Singletons are given a uniform distribution on  $(-\infty, \infty)$  before the constraint.

**The case** (`scale.model==TRUE && adjust.for.con.comp == FALSE`)

The option **constr=TRUE** is interpreted as a sum-to-zero constraint over the whole graph. Let  $Q = \tau R$  be the standard precision matrix from the **besag**-model with precision parameter  $\tau$ . Then  $R$ , except the singletons, are scaled so that the geometric mean of the marginal variances is 1, and  $R$  is modified so that singletons have a standard Normal distribution.

**The case** (`scale.model==FALSE && adjust.for.con.comp == TRUE`)

The option **constr=TRUE** is interpreted as one sum-to-zero constraint over each of the **nc2** connected components of size  $\geq 2$ . Singletons are given a uniform distribution on  $(-\infty, \infty)$ .

**The case** (`scale.model==TRUE && adjust.for.con.comp == TRUE`)

The option **constr=TRUE** is interpreted as **nc2** sum-to-zero constraints for each of the connected components of size  $\geq 2$ . Let  $Q = \tau R$  be the standard precision matrix from the **besag**-model with precision parameter  $\tau$ . Then  $R$ , are scaled so that the geometric mean of the marginal variances in each connected component of size  $\geq 2$  is 1, and modified so that singletons have a standard Normal distribution.

## Notes

The term  $\frac{1}{2} \log(|R|^*)$  of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here  $R$  is the precision matrix for the standardised Besag part of the model.

The generic PC-prior for  $\phi$  is available as `prior="pc"` and parameters `param="c(u, alpha)"`, where  $\text{Prob}(\phi \leq u) = \alpha$ . If  $\alpha < 0$  or  $\alpha > 1$ , then it is set to a value close to the minimum value of  $\alpha$  allowed. This prior depends on the graph and its computational cost is  $\mathcal{O}(n^3)$ .