Bivariate meta-analysis of sensitivity and specificity

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The bivariate model is a model for meta-analysing diagnostic studies reporting pairs of sensitivity and specificity ([Reitsma et al., 2005]). Preserving the bivariate structure of the data, pairs of sensitivity (Se) and specificity (Sp) are jointly analysed. Any existing correlation between these two measures is taken into account via random effects. Covariates can be added to the bivariate model and have a separate effect on sensitivity and specificity.

Data are taken from a meta-analysis conducted by [Scheidler et al., 1997] to compare the utility of three types of diagnostic imaging - lymphangiography (LAG), computed tomography (CT) and magnetic resonance (MR) - to detect lymph node metastases in patients with cervical cancer. The dataset consists of a total of 46 studies: the first 17 for LAG, the following 19 for CT and the last 10 for MR. We analyse this data set using a generalised linear mixed model approach ([Chu and Cole, 2006]).

$$\mathrm{TN}^i|\mu_i \sim \mathrm{Bin}(\mathrm{TN}^i + \mathrm{FP}^i, \mathrm{Sp}^i), \qquad \mathrm{logit}(\mathrm{Sp}^i) = \boldsymbol{X}_i\boldsymbol{\alpha} + \mu_i, \tag{1}$$

$$\mathrm{TP}^i|\nu_i \sim \mathrm{Bin}(\mathrm{TP}^i + \mathrm{FN}^i, \mathrm{Se}^i), \qquad \mathrm{logit}(\mathrm{Se}^i) = \boldsymbol{Z}_i\boldsymbol{\beta} + \nu_i, \tag{2}$$

$$\begin{pmatrix}\mu_i\\ \nu_i\end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1/\tau_\mu & \rho/\sqrt{\tau_\mu\tau_\nu}\\ \rho/\sqrt{\tau_\mu\tau_\nu} & 1/\tau_\nu\end{pmatrix}\right], \tag{3}$$

where TN, FP, TP and FN represent the number of true negatives, false positives, true positives, and false negatives, respectively and $\boldsymbol{X}_i, \boldsymbol{Z}_i$ are (possibly overlapping) vectors of covariates related to $\mathrm{Sp} = \frac{\mathrm{TN}}{\mathrm{TN+FP}}$ and $\mathrm{Se} = \frac{\mathrm{TP}}{\mathrm{TP+FN}}$. The index i represents study i in the meta-analysis. Here, $\boldsymbol{X}_i\boldsymbol{\alpha} = \alpha_{\mathrm{LAG}} \cdot \mathrm{LAG}_i + \alpha_{\mathrm{CT}} \cdot \mathrm{CT}_i + \alpha_{\mathrm{MR}} \cdot \mathrm{MR}_i$ and $\boldsymbol{Z}_i\boldsymbol{\beta} = \beta_{\mathrm{LAG}} \cdot \mathrm{LAG}_i + \beta_{\mathrm{CT}} \cdot \mathrm{CT}_i + \beta_{\mathrm{MR}} \cdot \mathrm{MR}_i$ whereby

$$\mathrm{LAG}_i = \begin{cases}1 & \text{if} \quad i = 0,\dots,16\\ 0 & \text{else}\end{cases} \qquad \mathrm{CT}_i = \begin{cases}1 & \text{if} \quad i = 17,\dots,35\\ 0 & \text{else}\end{cases} \qquad \mathrm{MR}_i = \begin{cases}1 & \text{if} \quad i = 36,\dots,45\\ 0 & \text{else}\end{cases}$$

The model has three hyperparameters $\boldsymbol{\theta} = (\log\tau_\mu, \log\tau_\nu, \rho)$. The correlation parameter is constrained to $[-1, 1]$. We reparameterise the correlation parameter ρ using Fisher's z-transformation as

$$\rho^\star = \mathrm{logit}\left(\frac{\rho+1}{2}\right)$$

which assumes values over the whole real line and assign the following prior distribution to $\rho^\star$

$$\rho^\star \sim \mathcal{N}(0, 1/0.4)$$

The prior precision of 0.4 corresponds, roughly, to a uniform prior on $[-1, 1]$ for ρ. For the other hyperparameters we assign the following prior distributions

$$\log\tau_\mu \sim \mathrm{LogGamma}(0.25, 0.025)$$
$$\log\tau_\nu \sim \mathrm{LogGamma}(0.25, 0.025)$$

References

[Chu and Cole, 2006] Chu, H. and Cole, S. R. (2006). Bivariate meta-analysis of sensitivity and specificity with sparse data: a generalized linear mixed model approach. *Journal of Clinical Epidemiology*, 59:1331–1333.

[Reitsma et al., 2005] Reitsma, J. B., Glas, A. S., Rutjes, A. W. S., Scholten, R. J. P. M., Bossuyt, P. M., and Zwindermann, A. H. (2005). Bivariate analysis of sensitivity and specificity produces informative summary measures in diagnostic reviews. *Journal of Clinical Epidemiology*, 58:982–990.

[Scheidler et al., 1997] Scheidler, J., Hricak, H., Yu, K. K., Subak, L., and Segal, M. R. (1997). Radiological evaluation of lymph node metastases in patients with cervical cancer: a meta-analysis. *JAMA*, 278(13):1096–1101.