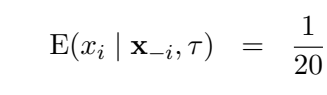
The RW2d-model

Parametrization

The 2-dimensional random walk model is defined on a regular grid. The full conditional distributions for the nodes in the interior of the grid are given by:

$$\mathrm{E}(x_i \mid \mathbf{x}_{-i}, \tau) = \frac{1}{20}\left(8 \; - 2 \; - 1 \; \right), \tag{1}$$



$$\mathrm{Prec}(x_i \mid \mathbf{x}_{-i}, \tau) = 20\tau. \tag{2}$$

Necessary corrections to to equations (1) and (2) near the boundary can be found using the stencils in [Terzopoulos, 1988]. For a detailed description of this model see [Rue and Held, 2005, Sec. 3.4.2].

Hyperparameters

The precision parameter τ is the only hyperparameter, $\theta = \tau$. It is represented internally as $\log \tau$ and the prior is assigned to $\log \tau$.

Specification

The `rw2d` model is specified insiede the `f()` function as:

```
f(<whatever>,model="rw2d",
  nrow=<n.of rows>,ncol=<n.of columns>,
  hyper= <hyper>, scale.model = FALSE)
```

The logical option `scale.model` determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is `FALSE` so that the model is not scaled.

Hyperparameter spesification and default values

hyper

theta

name log precision
short.name prec
initial 4
fixed FALSE
prior loggamma
param 1 5e-05
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

constr TRUE

nrow.ncol TRUE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values TRUE

pdf rw2d

Example

```
nrow=50
ncol=25
n = nrow*ncol
s.mat=matrix(NA,nrow=nrow,ncol=ncol)
j=1:ncol
for(i in 1:nrow)
    s.mat[i,j] = 0.1*(i+2*j)

## a covariate
z.mat=matrix(runif(nrow*ncol),nrow,ncol)

## noise
noise.mat=matrix(rnorm(nrow*ncol, sd=0.3),nrow,ncol)

## make simulated data
y.mat = s.mat  + 0.5*z.mat + noise.mat

## convert matrices to the internal representation in INLA
y = inla.matrix2vector(y.mat)
z = inla.matrix2vector(z.mat)
node = 1:n
formula= y ~ z + f(node, model="rw2d", nrow=nrow, ncol=ncol)
data=data.frame(y=y,z=z,node=node)
## fit the model
result=inla(formula, family="gaussian", data=data)

#plot the posterior mean for 'node' with the truth
dev.new()
INLA:::inla.display.matrix(s.mat)
dev.new()
INLA:::inla.display.matrix(INLA:::inla.vector2matrix(result$summary.random$node$mean,nrow,ncol
```

Notes

All indexes in the R-INLA library are one-dimensional so an appropriate mapping is required to get it into the ordering defined internally in `inla`; see `?inla.matrix2vector`, `?inla.vector2matrix`, `?inla.node2lattice` and `?inla.lattice2node`.

The $\frac{n-r}{2}\log(|R|^*)$-part (with $r = 3$) of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here, $Q = \tau R$.

References

[Rue and Held, 2005] Rue, H. and Held, L. (2005). *Gaussian Markov Random Fields: Theory and Applications*, volume 104 of *Monographs on Statistics and Applied Probability*. Chapman & Hall, London.

[Terzopoulos, 1988] Terzopoulos, D. (1988). The computation of visible-surface representations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 10(4):417–438.