The Beta-distribution

Parametrisation

The Beta-distribution has the following density

$$\pi(y) = \frac{1}{B(a,b)} y^{a-1}(1-y)^{b-1}, \qquad 0 < y < 1, \quad a > 0, \quad b > 0$$

where $B(a,b)$ is the Beta-function

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and $\Gamma(x)$ is the Gamma-function. The (re-)parameterisation used is

$$\mu = \frac{a}{a+b}, \qquad 0 < \mu < 1$$

and

$$\phi = a + b, \qquad \phi > 0,$$

as it makes

$$\mathrm{E}(y) = \mu \quad \text{and} \quad \mathrm{Var}(y) = \frac{\mu(1-\mu)}{1+\phi}.$$

The parameter ϕ is know as the *precision parameter*, since for fixed μ, the larger ϕ the smaller the variance of y. The parameters $\{a, b\}$ are given as $\{\mu, \phi\}$ as follows,

$$a = \mu\phi \quad \text{and} \quad b = -\mu\phi + \phi.$$

Link-function

The linear predictor η is linked to the mean μ using a default logit-link

$$\mu = \frac{\exp(\eta)}{1 + \exp(\eta)}.$$

Hyperparameter

The hyperparameter is the precision parameter ϕ, which is represented as

$$\phi = \exp(\theta)$$

and the prior is defined on θ.

Specification

- family = `beta`
- Required arguments: y.

Hyperparameter spesification and default values

hyper

theta

hyperid 61001
name precision parameter

short.name phi

initial 2.30258509299405

fixed FALSE

prior loggamma

param 1 0.1

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

survival FALSE

discrete FALSE

link default logit cauchit probit cloglog loglog

pdf beta

Example

In the following example we estimate the parameters in a simulated example.

```
## the precision parameter in the beta distribution
phi = 5

## generate simulated data
n = 1000
z = rnorm(n, sd=0.2)
eta = 1 + z
mu = exp(eta)/(1+exp(eta))
a = mu * phi
b = -mu * phi + phi
y = rbeta(n, a, b)

## estimate the model
formula = y ~ 1 + z
r = inla(formula, data = data.frame(y, z), family = "beta")
summary(r)
```

Notes

None.