

## Model for seasonal variation

### Parametrization

A model for seasonal variation with periodicity  $m$  for the random vector  $\mathbf{x} = (x_1, \dots, x_n)$ ,  $n > m$  is obtained assuming that the sums  $x_i + x_{i+1} + \dots + x_{i+m-1}$  are independent Gaussian with precision  $\tau$ .

The density for  $\mathbf{x}$  is derived from the  $n - m + 1$  increments as

$$\pi(\mathbf{x}|\tau) \propto \tau^{\frac{(n-m+1)}{2}} \exp \left\{ -\frac{\tau}{2} \sum (x_i + x_{i+1} + \dots + x_{i+m-1})^2 \right\} \quad (1)$$

$$= \tau^{\frac{(n-m+1)}{2}} \exp \left\{ -\frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x} \right\} \quad (2)$$

where  $\mathbf{Q} = \tau \mathbf{R}$  and  $\mathbf{R}$  is the structure matrix reflecting the neighbourhood structure of the model.

### Hyperparameters

The precision parameter  $\tau$  is represented as

$$\theta = \log \tau$$

and the prior is defined on  $\theta$ .

### Specification

The seasonal model is specified inside the `f()` function as

```
f(<whatever>, model="seasonal", season.length=<season.length>,  
    hyper = <hyper>)
```

### Hyperparameter specification and default values

**hyper**

**theta**

**name** log precision

**short.name** prec

**prior** loggamma

**param** 1 5e-05

**initial** 4

**fixed** FALSE

**to.theta** function(x) log(x)

**from.theta** function(x) exp(x)

**constr** FALSE

**nrow.ncol** FALSE

**augmented** FALSE

**aug.factor** 1

**aug.constr**

**n.div.by**

**n.required** FALSE

**set.default.values** FALSE

**pdf** seasonal

## Example

```
n=203
```

```
n.seas=12
```

```
trend=seq(1:n)
```

```
seasonal=rep(1:n.seas, ceiling(n/n.seas))[1:n]
```

```
a=1
```

```
b=0.5
```

```
y = rnorm(n,a+b*trend,1)+rnorm(n,0.2*seasonal,1)
```

```
data=data.frame(y=y,trend=trend,seasonal=seasonal)
```

```
formula = y~f(trend,model="rw2")+f(seasonal,model="seasonal",  
                                     season.length=n.seas)
```

```
result=inla(formula,family="gaussian",data=data)
```

## Notes

The seasonal is intrinsic with rank deficiency  $m - 1$ .