

# Asymmetric Laplace

## Parametrisation

The asymmetric Laplace distribution is

$$f(y) = \delta\tau(1 - \tau) \exp\{-\delta\rho_\tau(y - \mu)\}$$

for continuously responses  $y$  where  $\rho_\tau(u) = \{\tau - I(u < 0)\}u$  is the so-called check function in Koenker and Bassett (1978), and

$\mu$ : is the the location parameter ( $-\infty < \mu < \infty$ )

$\tau$ : is the fixed skewness parameter ( $0 < \tau < 1$ )

$\delta$ : is the inverse scale parameter ( $\delta > 0$ ).

## Scale mixtures of normal representation

The asymmetric Laplace random variable  $y$  can be represented as follows:

$$y = \mu + \xi w + \sigma \sqrt{w/\delta} z,$$

where  $\xi = \frac{1-2\tau}{\tau(1-\tau)}$  and  $\sigma^2 = \frac{2}{\tau(1-\tau)}$  are two scalars depending on  $\tau$ . The random variables  $w > 0$  and  $z$  are independent and have exponential distribution with mean  $\delta^{-1}$  and standard normal distribution, respectively. As a result,  $y$  has the following hierarchical structure:

$$y \mid w \sim N(\mu + \xi w, \sigma^2 \delta^{-1} w) \quad \text{and} \quad w \sim \text{Exp}(\delta).$$

## Approximating check function

The log likelihood of asymmetric Laplace distribution has zero second-order derivative everywhere. To implement INLA, we approximate the check function as follows:

$$\tilde{\rho}_{\tau,\gamma}(u) = \begin{cases} \gamma^{-1} \log(\cosh(\tau\gamma|u|)) & \text{if } u \geq 0 \\ \gamma^{-1} \log(\cosh((1-\tau)\gamma|u|)) & \text{if } u < 0, \end{cases}$$

where the parameter  $\gamma > 0$  is fixed and precision of the approximation increases as  $\gamma \rightarrow \infty$ .

## Link-function

The location parameter is linked to the linear predictor by

$$\mu = \eta$$

## Hyperparameters

The prior is defined on inverse scale  $\delta$ .

## Specification

- family = laplace

## Hyperparameter spesification and default values

## Example

## Notes

None.