Generalized Pareto distribution

Parametrisation

The generalized Pareto (GP) distribution with positive shape parameter has cumulative distribution function

$$F(y; \sigma, \xi) = 1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-1/\xi}, \quad y > 0,$$

for a continuous response y where

ξ: is the shape parameter, $\xi > 0$

σ: is the scale parameter, $\sigma > 0$

The limit for $\xi \downarrow 0$ is $F(y; \sigma, 0) = 1 - \exp(-x/\sigma)$.

Link function

The linear predictor η controls the α quantile of the GP

$$P(y \leq q_\alpha) = \alpha$$

and $q_\alpha = \exp(\eta)$. The scaling σ, is then a function of (q_α, ξ), as

$$\sigma = \frac{\xi \exp(\eta)}{(1-\alpha)^{-\xi} - 1}$$

Hyperparameters

The GP model has one hyperparameter. The shape $\xi > 0$ is represented as

$$\theta = \log \xi,$$

and the prior is defined on θ.

Specification

- family=`gp`
- Required arguments: y and the quantile α.

The quantile is given as `control.family=list(quantile=`α`)`.

Hyperparameter spesification and default values

hyper

theta

hyperid 101201
name shape
short.name xi
initial -2.30258509299405
fixed FALSE
prior loggamma
param 1 15
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

status experimental

survival FALSE

discrete TRUE

link default log

pdf genPareto

Example

```
rgp = function(n, sigma, eta, alpha, xi = 0.001)
{
    if (missing(sigma)) {
        stopifnot(!missing(eta) && !missing(alpha))
        sigma = exp(eta) * xi / ((1.0 - alpha)^(-xi) -1.0)
    }
    return (sigma / xi * (runif(n)^(-xi) -1.0))
}

n = 300
x = runif(n)-0.5
eta = 1+x
alpha = 0.99
xi = 0.3
y = rgp(n, eta = eta, alpha = alpha, xi=xi)

r = inla(y ~ 1+x,
         data = data.frame(y, x),
         family = "gp",
```

```
        control.family = list(quantile = alpha),
        control.predictor = list(compute=TRUE),
        verbose=TRUE)


rx = range(c(r$summary.fitted.values$mean, exp(eta)))
plot(r$summary.fitted.values$mean, exp(eta),
     xlim = rx, ylim = rx)
abline(a=0,b=1)
```

Notes