

## Bym2 model for spatial effects

### Parametrization

This model is a reparameterisation of the BYM-model, which is a union of the besag model  $u^*$  and a iid model  $v^*$ , so that

$$x = \begin{pmatrix} v^* + u^* \\ u^* \end{pmatrix}$$

where both  $u^*$  and  $v^*$  has a precision (hyper-)parameter. The length of  $x$  is  $2n$  if the length of  $u^*$  (and  $v^*$ ) is  $n$ . The BYM2 model uses a different parameterisation of the hyperparameters where

$$x = \begin{pmatrix} \frac{1}{\sqrt{\tau}} (\sqrt{1-\phi} v + \sqrt{\phi} u) \\ u \end{pmatrix}$$

where both  $u$  and  $v$  are *standardised* to have (generalised) variance equal to one. The *marginal* precision is then  $\tau$  and the proportion of the marginal variance explained by the spatial effect ( $u$ ) is  $\phi$ .

### Hyperparameters

The hyperparameters are the marginal precision  $\tau$  and the mixing parameter  $\phi$ . The marginal precision  $\tau$  is represented as

$$\theta_1 = \log(\tau)$$

and the mixing parameter as

$$\theta_2 = \log\left(\frac{\phi}{1-\phi}\right)$$

and the prior is defined on  $\theta = (\theta_1, \theta_2)$ .

### Specification

The bym2 model is specified inside the `f()` function as

```
f(<whatever>, model="bym2", graph=<graph>,  
  hyper=<hyper>, adjust.for.con.comp = TRUE)
```

The neighbourhood structure of  $\mathbf{x}$  is passed to the program through the `graph` argument.

The option `adjust.for.con.comp` adjust the model if the graph has more than one connected compoment, and this adjustment can be disabled setting this option to `FALSE`. This means that `constr=TRUE` is interpreted as a sum-to-zero constraint on *each* connected component and the `rankdef` parameter is set accordingly.

### Hyperparameter spesification and default values

#### hyper

##### theta1

<b>name</b>	log precision
<b>short.name</b>	prec
<b>prior</b>	pc.prec
<b>param</b>	1 0.01
<b>initial</b>	4
<b>fixed</b>	FALSE

```

    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
  name logit phi
  short.name phi
  prior pc
  param 0.5 -1
  initial -3
  fixed FALSE
  to.theta function(x) log(x/(1-x))
  from.theta function(x) exp(x)/(1+exp(x))
constr TRUE
nrow.ncol FALSE
augmented TRUE
aug.factor 2
aug.constr 2
n.div.by
n.required TRUE
set.default.values TRUE
status experimental
pdf bym2

```

## Example

## Notes

The term  $\frac{1}{2} \log(|R|^*)$  of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here  $R$  is the precision matrix for the standardised Besag part of the model.

The generic PC-prior for  $\phi$  is available as `prior="pc"` and parameters `param="c(u, alpha)"`, where  $\text{Prob}(\phi \leq u) = \alpha$ . If  $\alpha < 0$  or  $\alpha > 1$ , then it is set to a value close to the minimum value of  $\alpha$  allowed. This prior depends on the graph and its computational cost is  $\mathcal{O}(n^3)$ .