

## Beta-binomial

### Parametrisation

The Beta-binomial distribution arise from a hierarchical model where the probability  $p$  is Beta-distributed<sup>1</sup>

$$\pi(p) = \frac{1}{B(\alpha, \beta)} p^{\alpha-1} (1-p)^{\beta-1}, \quad \alpha > 0, \beta > 0$$

and the response  $y$  is Binomial

$$\pi(y | p) = \binom{n}{y} p^y (1-p)^{(n-y)}, \quad y = 0, 1, \dots, n.$$

The marginal distribution for  $y$  is the Beta-binomial,

$$\pi(y) = \binom{n}{y} \frac{B(y + \alpha, n - y + \beta)}{B(\alpha, \beta)}.$$

### Link-function

The mean and variance of  $y$  are given as

$$\mu = n \frac{\alpha}{\alpha + \beta} = n \mu_p, \quad \text{and} \quad \sigma^2 = n \mu_p (1 - \mu_p) (1 + (n - 1) \rho),$$

where  $\mu_p = \frac{\alpha}{\alpha + \beta}$  is the mean of the probability  $p$  from the Beta-distribution, and  $\rho = \frac{1}{\alpha + \beta + 1}$  ( $0 < \rho < 1$ ) is the pairwise correlation between the  $n$  Bernoulli draws and is the over-dispersion parameter.

The mean probability  $\mu_p$  is linked to the linear predictor by

$$\mu_p = \frac{\exp(\eta)}{1 + \exp(\eta)}$$

using the default logit-link.

### Hyperparameter

The hyperparameter is the over-dispersion parameter  $\rho$ , which is represented as

$$\rho = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and the prior is defined on  $\theta$ .

### Specification

- family = `betabinomial`
- Required arguments:  $y$  and `Ntrials` =  $n$  (default `Ntrials` = 1).

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<sup>1</sup>The Beta-function is  $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ , where  $\Gamma(x)$  is the Gamma-function.

## Hyperparameter specification and default values

**hyper**

**theta**

**name** overdispersion

**short.name** rho

**initial** 0

**fixed** FALSE

**prior** gaussian

**param** 0 0.4

**to.theta** function(x) log(x/(1-x))

**from.theta** function(x) exp(x)/(1+exp(x))

**survival** FALSE

**discrete** TRUE

**link** default logit probit cloglog

**pdf** betabinomial

## Example

In the following example we estimate the parameters in a simulated example.

```
## overdispersion parameter in the betabinomial
rho = 0.7

n = 1000
z = rnorm(n, sd=0.2)
Ntrials = sample(1:10, n, replace=TRUE)
eta = 1 + z
p.eta = exp(eta)/(1+exp(eta))
a = p.eta * (1-rho)/rho
b = (p.eta * rho - p.eta - rho + 1)/rho
p = rbeta(n, a, b)
y = rbinom(n, Ntrials, p)

formula = y ~ 1 + z
data = data.frame(y, z)
r = inla(formula, data = data,
         family = "betabinomial", Ntrials=Ntrials)
summary(r)
```

## Notes

None.