Gaussian

Parametrisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{s\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}s\tau\left(y-\mu\right)^2\right)$$

for continuously responses y where

μ: is the the mean

τ: is the precision

s: is a fixed scaling, $s > 0$.

Link-function

The mean and variance of y are given as

$$\mu \quad \text{and} \qquad \sigma^2 = \frac{1}{s\tau}$$

and the mean is linked to the linear predictor by

$$\mu = \eta$$

Hyperparameters

The default behaviour is to represent the precision $\tau = \kappa_1$ where

$$\theta_1 = \log \kappa_1$$

and the prior is defined on θ_1.

The more general formulation have a second (fixed) hyperparameter θ_2 which determines a fixed offset $1/\kappa_2$, $\theta_2 = \log \kappa_2$, for the variance (scaling not included) of the response. In this case,

$$1/\tau = 1/\kappa_1 + 1/\kappa_2$$

or

$$\tau = \frac{1}{1/\kappa_1 + 1/\kappa_2}$$

In the case where $1/\kappa_2$ is zero, then $\tau = \kappa_1$ and we are back to the default behaviour. We use the convension that $1/\kappa_2$ is zero if $1/\kappa_2 <$`.Machine$double.eps`,which is $\theta_2 \geq 36.05$ for common machines.

Specification

- family = `gaussian`
- Required arguments: y and s (argument `scale`)

The scalings have default value 1.

Hyperparameter spesification and default values

doc The Gaussian likelihoood

hyper

theta1

- **hyperid** 65001
- **name** log precision
- **short.name** prec
- **initial** 4
- **fixed** FALSE
- **prior** loggamma
- **param** 1 5e-05
- **to.theta** `function(x) log(x)`
- **from.theta** `function(x) exp(x)`

theta2

- **hyperid** 65002
- **name** log precision offset
- **short.name** precoffset
- **initial** 72.0873067782343
- **fixed** TRUE
- **prior** none
- **param**
- **to.theta** `function(x) log(x)`
- **from.theta** `function(x) exp(x)`

survival FALSE

discrete FALSE

link default identity logit cauchit log logoffset

pdf gaussian

Example

The first example estimate the parameters in a simulated example with Gaussian responses, giving τ a Gamma-prior with parameters $(1, 0.01)$ and initial value (for the optimisations) of $\exp(2.0)$. The second example shows the use of an fixed offset in the variance.

```
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
tau = 100
scale = exp(rnorm(n))
prec = scale*tau
y = rnorm(n, mean = eta, sd = 1/sqrt(prec))
```

```
data = list(y=y, z=z)
formula = y ~ 1+z
result = inla(formula, family = "gaussian", data = data,
        control.family = list(hyper = list(
                                    prec = list(
                                            prior = "loggamma",
                                            param = c(1.0,0.01),
                                            initial = 2))),
              scale=scale, keep=TRUE)
summary(result)

## with an offset in the variance
var0 = 1.0 ## fixed offset
var1 = 2.0
v = var0 + var1
s = sqrt(v)
x = rnorm(n)
y = 1 + x + rnorm(n, sd = s)
rr = inla(y ~ x,
         data = data.frame(y, x),
         control.family = list(
             hyper = list(precoffset = list(initial = log(1/var0)))),
         verbose = TRUE)
summary(rr)
plot(rr$internal.marginals.hyperpar[[1]], type = "l", lwd=3)
abline(v = log(1.0/var1), lwd=3, col = "blue")
```

Notes

An error is given if θ_2 is not fixed.