Generalized Poisson

The generalized Poisson distribution is given by

$$f(y|\lambda, w) = \frac{\lambda(\lambda + wy)^{y-1}}{y!} \exp(-(\lambda + wy))$$

for $y = 0, 1, 2, \ldots$ and where $\lambda > 0$ and $\max(-1, -\lambda/4) \leq w \leq 1$. The mean and variance of y are

$$\mu = \lambda(1 - w)^{-1} \qquad \text{and} \qquad \sigma^2 = \lambda(1 - w)^{-3} = \mu(1 - w)^{-2}.$$

Since the dispersion parameter w influence the mean as well as the variance, we will use the following parameterisation (Consul and Jain (1973), Zamani and Ismail(2012))

$$w = \frac{\varphi\mu^{p-1}}{1 + \varphi\mu^{p-1}},$$

which gives the following density

$$f(y|\mu, \varphi, p) = \frac{\mu(\mu + \varphi\mu^{p-1}y)^{y-1}}{(1 + \varphi\mu^{p-1})^y y!} \exp\left(-\frac{\mu + \varphi\mu^{p-1}y}{1 + \varphi\mu^{p-1}}\right)$$

for $y = 0, 1, 2, \ldots$. We assume $\varphi \geq 0$.

Link-function

The mean and variance of y are given as

$$\mathrm{E}(y|.) = \mu \qquad \text{and} \qquad \mathrm{Var}(y|.) = \mu\left(1 + \varphi\mu^{p-1}\right)^2$$

and the mean is linked to the linear predictor by

$$\mu = E\exp(\eta)$$

Hyperparameters

The overdispersion parameter $\varphi \geq 0$ is represented as

$$\varphi = \exp(\theta_1)$$

The "shape" parameter p is represented as

$$p = \theta_2$$

Note that $\theta_2 = 1$ and `fixed = TRUE`, default. The prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

- `family="gpoisson"`

Hyperparameter spesification and default values

hyper

- **theta1**
  - **name** overdispersion
  - **short.name** phi
  - **initial** 0
  - **fixed** FALSE
  - **prior** loggamma
  - **param** 1 1
  - **to.theta** `function(x) log(x)`
  - **from.theta** `function(x) exp(x)`
- **theta2**
  - **name** p
  - **short.name** p
  - **initial** 1
  - **fixed** TRUE
  - **prior** normal
  - **param** 1 100
  - **to.theta** `function(x) x`
  - **from.theta** `function(x) x`

survival FALSE

discrete TRUE

link default log logoffset

pdf gpoisson

status experimental

Example

In the following example we estimate the parameters in a simulated example with generalized Poisson responses.

```
dgpoisson = function(y, mu, phi, p)
{
    a = mu + phi * mu^(p-1.0) * y;
    b = 1. + phi * mu^(p-1.0);
    d = exp(log(mu) + (y-1.0)*log(a) -
            y*log(b) - lfactorial(y) - a/b)
    return (d)
}
rgpoisson = function(n, mu, phi, p)
{
    stopifnot(length(mu) == 1)
    s = sqrt(mu*(1+phi*mu^(p-1))^2)
    f = 20
```

```
    low = as.integer(max(0, mu - f*s))
    high = as.integer(mu + f*s)
    prob = dgpoisson(low:high, mu, phi, p)
    y = sample(low:high, n, replace=TRUE,
            prob = prob)

    return (y)
}

n = 1000
phi = 1
p = 1
mu = exp(1 + 5*(1:n)/n)

y = numeric(n)
for(i in 1:n) {
    y[i] = rgpoisson(1, mu[i], phi, p)
}

idx = (1:n)/n
r = inla(y ~ 1 + idx,  data = data.frame(y, idx),
        family = "gpoisson")
```

Notes

The parameter p is default fixed to be 1. Allowing it to be estimated jointly with the overdispersion parameter, please note the following.

- The parameter p and the overdispersion parameter are strongly correlated when estimated jointly.
- You may want to chose an informative prior for p, as the shape of the likelihood might not be want you expect for "extreme" p.
- You may experience problems in the numerical optimization (fail to converge); a more informative prior (if available) for p will help with this issue.

References

- Consul, P. C. and Jain, G.C (1973) A generalization of Poisson distribution. Technometrics 15, 791-799.
- Zamani, H. and Ismail, N. (2011). Functional form for the generalized Poisson regression model. Communication in Statistics – Theory and Methods (IN PRESS).