The Kumaraswamy distribution

Parametrisation

The Kumaraswamy distribution is

$$f(y) = \alpha\beta y^{\alpha-1}(1-y^{\alpha})^{\beta-1}$$

for $0 < y < 1$ and $\alpha, \beta > 0$. The cummulative distribution function is

$$F(y) = 1 - (1-y^{\alpha})^{\beta}.$$

The parametrisation is given in terms of the quantile function

$$\kappa(q) = \left(1 - (1-q)^{1/\beta}\right)^{1/\alpha}$$

and the precision parameter ϕ,

$$\phi(q) = -\ln\left(1 - (1-q)^{1/\beta}\right)$$

for *fixed* value of $0 < q < 1$.

Link-function

The quantile κ to the linear predictor by

$$\text{logit}(\kappa) = \eta$$

using the default logit link-function.

Hyperparameters

The hyperparameter is

$$\theta_1 = log(\phi)$$

and the prior is given for θ_1.

For technical reasons, the fixed value of the quantile q is given as a second hyperparameter,

$$\theta_2 = q$$

for which its initial value must be given.

Specification

- family = `kumar`
- Required arguments: y

Hyperparameter spesification and default values

hyper

theta1

name precision parameter
short.name prec
initial 0
fixed FALSE

prior loggamma

param 1 0.001

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta2

name quantile

short.name q

initial 0.5

fixed TRUE

prior invalid

param

to.theta `function(x) x`

from.theta `function(x) x`

survival FALSE

discrete FALSE

link default logit

pdf kumar

Example

```
rkumar = function(n, eta, phi, q=0.5)
{
    kappa = eta
    beta = log(1-q)/log(1-exp(-phi))
    alpha = log(1- (1-q)^(1/beta)) / log(kappa)
    u = runif(n)
    y = (1-u^(1/beta))^(1/alpha)
    return (y)
}

n = 100
q = 0.5
phi = 1
x = rnorm(n, sd = 1)
eta = inla.link.invlogit(1 + x)
y = rkumar(n, eta, phi, q)
r = inla(y ~ 1 + x,
    data = data.frame(y, x),
    family = "kumar",
    control.family = list(
        hyper = list(q = list(initial = q))))
summary(r)
```

Notes

None.