

# Besag model for spatial effects

## Parametrization

The besag model for random vector  $\mathbf{x} = (x_1, \dots, x_n)$  is defined as

$$x_i | x_j, i \neq j, \tau \sim \mathcal{N}\left(\frac{1}{n_i} \sum_{i \sim j} x_j, \frac{1}{n_i \tau}\right) \quad (1)$$

where  $n_i$  is the number of neighbours of node  $i$ ,  $i \sim j$  indicates that the two nodes  $i$  and  $j$  are neighbours.

## Hyperparameters

The precision parameter  $\tau$  is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ .

## Specification

The besag model is specified inside the `f()` function as

```
f(<whatever>, model="besag", graph=<graph>,  
  hyper=<hyper>, adjust.for.con.comp = TRUE,  
  scale.model = FALSE)
```

The neighbourhood structure of  $\mathbf{x}$  is passed to the program through the `graph` argument.

If the option `adjust.for.con.comp=TRUE` then the model is adjusted if the graph has more than one connected component. This adjustment can be disabled setting this option to `FALSE`. If `adjust.for.con.comp=TRUE` then `constr=TRUE` is interpreted as a sum-to-zero constraint on *each* connected component in the graph and the `rankdef` parameter is set to the number of connected components.

The logical option `scale.model` determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior specification much easier. Default is `FALSE` so that the model is not scaled.

## Hyperparameter specification and default values

### hyper

#### theta

```
name log precision  
short.name prec  
prior loggamma  
param 1 5e-05  
initial 4  
fixed FALSE  
to.theta function(x) log(x)  
from.theta function(x) exp(x)
```

**constr** TRUE

**nrow.ncol** FALSE

**augmented** FALSE

**aug.factor** 1

**aug.constr**

**n.div.by**

**n.required** TRUE

**set.default.values** TRUE

**pdf** besag

## Example

For examples of application of this model see the **Bym**, **Munich**, **Zambia** or **Scotland** examples in Volume I.

## Notes

The besag model intrinsic with rankdef 1.

The term  $\frac{1}{2} \log(|R|^*)$  of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here  $R$  is the precision matrix with a unit precision parameter.