# Bym model for spatial effects

#### Parametrization

This model is simply a union of the besag model u and a iid model v, so that

$$x = \begin{pmatrix} v + u \\ u \end{pmatrix}$$

Note that the length of x is 2n if the length of u (and v) is n. The benefite is that this allows to get the posterior marginals of the sum of the spatial and iid model; otherwise it offers no advantages.

## Hyperparameters

The hyperparameters are the precision  $\tau_1$  of the iid model (v) and the precision  $\tau_2$  of the besag model (u). The precision parameters are represented as

$$\theta = (\theta_1, \theta_2) = (\log \tau_1, \log \tau_2)$$

and the prior is defined on  $\theta$ .

## **Specification**

The bym model is specified inside the f() function as

```
f(<whatever>,model="bym",graph=<graph>,
  hyper=<hyper>, adjust.for.con.comp = TRUE,
  scale.model = FALSE)
```

The neighbourhood structure of x is passed to the program through the graph argument.

The option adjust.for.com.comp adjust the model if the graph has more than one connected component, and this adjustment can be disabled setting this option to FALSE. This means that constr=TRUE is interpreted as a sum-to-zero constraint on *each* connected component and the rankdef parameter is set accordingly.

The logical option scale.model determine if the besag-model-part of the model u should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is FALSE so that the model is not scaled.

### Hyperparameter spesification and default values

## hyper

### theta1

```
hyperid 10001
name log unstructured precision
short.name prec.unstruct
prior loggamma
param 1 5e-04
initial 4
fixed FALSE
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

```
hyperid 10002
name log spatial precision
short.name prec.spatial
prior loggamma
param 1 5e-04
initial 4
fixed FALSE
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

constr TRUE

nrow.ncol FALSE

augmented TRUE

aug.factor 2

aug.constr 2

n.div.by

n.required TRUE

set.default.values TRUE

 $\mathbf{pdf}$  bym

# Example

For examples of application of this model see the Bym example in Volume I.

# Details on the implementation

This gives some details of the implementation, which depends on the following variables

**nc1** Number of connected components in the graph with size 1. These nodes, *singletons*, have no neigbours.

**nc2** Number of connected components in the graph with size  $\geq 2$ .

scale.model The value of the logical flag, if the model should be scaled or not. (Default FALSE)

adjust.for.con.comp The value of the logical flag if the constr=TRUE option should be reinterpreted.

The case (scale.model==FALSE && adjust.for.con.comp == FALSE)

The option constr=TRUE is interpreted as a sum-to-zero constraint over the whole graph. Singletons are given a uniform distribution on  $(-\infty, \infty)$  before the constraint, which may give a singular posterior.

The case (scale.model==TRUE && adjust.for.con.comp == FALSE)

The option constr=TRUE is interpreted as a sum-to-zero constraint over the whole graph. Let  $Q = \tau R$  be the standard precision matrix from the besag-model with precision parameter  $\tau$ . Then R, except the singletons, are scaled so that the geometric mean of the marginal variances is 1, and R is modified so that singletons have a standard Normal distribution.

The case (scale.model==FALSE && adjust.for.con.comp == TRUE)

The option constr=TRUE is interpreted as one sum-to-zero constraint over each of the nc2 connected components of size  $\geq 2$ . Singletons are given a uniform distribution on  $(-\infty, \infty)$ , which may give a singular posterior.

The case (scale.model==TRUE && adjust.for.con.comp == TRUE)

The option constr=TRUE is interpreted as nc2 sum-to-zero constraints for each of the connected components of size  $\geq 2$ . Let  $Q = \tau R$  be the standard precision matrix from the besag-model with precision parameter  $\tau$ . Then R, are scaled so that the geometric mean of the marginal variances in each connected component of size  $\geq 2$  is 1, and modified so that singletons have a standard Normal distribution.

## Notes

The term  $\frac{1}{2}\log(|R|^*)$  of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here R is the precision matrix with a unit precision parameter for the Besag part of the model.