

# The Kumaraswamy distribution

## Parametrisation

The Kumaraswamy distribution is

$$f(y) = \alpha\beta y^{\alpha-1}(1-y^\alpha)^{\beta-1}$$

for  $0 < y < 1$  and  $\alpha, \beta > 0$ . The cumulative distribution function is

$$F(y) = 1 - (1 - y^\alpha)^\beta.$$

The parametrisation is given in terms of the quantile function

$$\kappa(q) = \left(1 - (1 - q)^{1/\beta}\right)^{1/\alpha}$$

and the precision parameter  $\phi$ ,

$$\phi(q) = -\ln \left(1 - (1 - q)^{1/\beta}\right)$$

for *fixed* value of  $0 < q < 1$ .

## Link-function

The quantile  $\kappa$  to the linear predictor by

$$\text{logit}(\kappa) = \eta$$

using the default logit link-function.

## Hyperparameters

The hyperparameter is

$$\theta_1 = \log(\phi)$$

and the prior is given for  $\theta_1$ .

For technical reasons, the fixed value of the quantile  $q$  is given as a second hyperparameter,

$$\theta_2 = q$$

for which its initial value must be given.

## Specification

- family = kumar
- Required arguments:  $y$

## Hyperparameter spesification and default values

hyper

theta1

hyperid 60001

name precision parameter

short.name prec

initial 0

```

    fixed FALSE
    prior loggamma
    param 1 0.001
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
    hyperid 60002
    name quantile
    short.name q
    initial 0.5
    fixed TRUE
    prior invalid
    param
    to.theta function(x) x
    from.theta function(x) x

```

**survival** FALSE

**discrete** FALSE

**link** default logit cauchit

**pdf** kumar

## Example

```

rkumar = function(n, eta, phi, q=0.5)
{
  kappa = eta
  beta = log(1-q)/log(1-exp(-phi))
  alpha = log(1- (1-q)^(1/beta)) / log(kappa)
  u = runif(n)
  y = (1-u^(1/beta))^(1/alpha)
  return (y)
}

n = 100
q = 0.5
phi = 1
x = rnorm(n, sd = 1)
eta = inla.link.invlogit(1 + x)
y = rkumar(n, eta, phi, q)
r = inla(y ~ 1 + x,
  data = data.frame(y, x),
  family = "kumar",
  control.family = list(
    hyper = list(q = list(initial = q))))
summary(r)

```

## Notes

None.