

## Random walk model of order 2 (RW2)

### Parametrization

The random walk model of order 2 (RW2) for the Gaussian vector  $\mathbf{x} = (x_1, \dots, x_n)$  is constructed assuming independent second-order increments:

$$\Delta^2 x_i = x_i - 2x_{i+1} + x_{i+2} \sim \mathcal{N}(0, \tau^{-1})$$

The density for  $\mathbf{x}$  is derived from its  $n - 2$  second-order increments as

$$\pi(\mathbf{x}|\tau) \propto \tau^{(n-2)/2} \exp \left\{ -\frac{\tau}{2} \sum (\Delta^2 x_i)^2 \right\} \quad (1)$$

$$= \tau^{(n-2)/2} \exp \left\{ -\frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x} \right\} \quad (2)$$

where  $\mathbf{Q} = \tau \mathbf{R}$  and  $\mathbf{R}$  is the structure matrix reflecting the neighbourhood structure of the model.

It is also possible to define a *cyclic* version of the RW2 model.

### Hyperparameters

The precision parameter  $\tau$  is represented as

$$\theta = \log \tau$$

and the prior is defined on  $\theta$ .

### Specification

The RW2 model is specified inside the `f()` function as

```
f(<whatever>, model="rw2", values=<values>, cyclic=<TRUE,FALSE>,
```

The (optional) argument `values` is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See the example for RW1 for an application.

### Hyperparameter specification and default values

**hyper**

**theta**

**name** log precision

**short.name** prec

**prior** loggamma

**param** 1 5e-05

**initial** 4

**fixed** FALSE

**to.theta** function(x) log(x)

**from.theta** function(x) exp(x)

**constr** TRUE

**nrow.ncol** FALSE

**augmented** FALSE

```
aug.factor 1
aug.constr
n.div.by
n.required FALSE
set.default.values FALSE
pdf rw2
```

## Example

```
n=100
z=seq(0,6,length.out=n)
y=sin(z)+rnorm(n,mean=0,sd=0.5)
data=data.frame(y=y,z=z)

formula=y~f(z,model="rw2")
result=inla(formula,data=data,family="gaussian")
```

## Notes

- The RW2 is a intrinsic with rank deficiency 2.
- The RW2 model for irregular locations are supported although not described here.
- The  $\frac{n-r}{2} \log(|R|^*)$ -part (with  $r = 2$ ) of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it.