# Asymmetric Laplace

#### Parametrisation

The asymmetric Laplace distribution is

$$f(y) = \delta \tau (1 - \tau) \exp\{-\delta \rho_{\tau}(y - \mu)\}\$$

for continuously responses y where  $\rho_{\tau}(u) = \{\tau - I(u < 0)\}u$  is the so-called check function in Koenker and Bassett (1978), and

 $\mu$ : is the the location parameter  $(-\infty < \mu < \infty)$ 

 $\tau$ : is the fixed skewness parameter (0 <  $\tau$  < 1)

 $\delta$ : is the inverse scale parameter ( $\delta > 0$ ).

### Scale mixtures of normal representation

The asymmetric Laplace random variable y can be represented as follows:

$$y = \mu + \xi w + \sigma \sqrt{w/\delta} z,$$

where  $\xi = \frac{1-2\tau}{\tau(1-\tau)}$  and  $\sigma^2 = \frac{2}{\tau(1-\tau)}$  are two scalars depending on  $\tau$ . The random variables w>0 and z are independent and have exponential distribution with mean  $\delta^{-1}$  and standard normal distribution, respectively. As a result, y has the following hierarchical structure:

$$y \mid w \sim N\left(\mu + \xi w, \sigma^2 \delta^{-1} w\right)$$
 and  $w \sim \text{Exp}(\delta)$ .

## Approximating check function

The log likelihood of asymmetric Laplace distribution has zero second-order derivative everywhere. To implement INLA, we approximate the check function as follows:

$$\tilde{\rho}_{\tau,\gamma}(u) = \begin{cases} \gamma^{-1} \log(\cosh(\tau \gamma |u|)) & \text{if } u \ge 0\\ \gamma^{-1} \log(\cosh((1-\tau)\gamma |u|)) & \text{if } u < 0, \end{cases}$$

where the parameter  $\gamma > 0$  is fixed and precision of the approximation increases as  $\gamma \to \infty$ .

### **Link-function**

The location parameter is linked to the linear predictor by

$$\mu = \eta$$

#### Hyperparameters

The prior is defined on inverse scale  $\delta$ .

### Specification

• family = laplace

#### Hyperparameter spesification and default values

# Example

Notes

None.