NMix

Parametrisation

The N-Mixture distribution is a Poisson mixture of the Binomials, as

$$\text{Prob}(y) = \sum_{n=y}^{\infty} \binom{n}{y} p^n (1-p)^{n-y} \times \frac{\lambda^n}{n!} \exp(-\lambda)$$

for responses $y = 0, 1, 2, \ldots, n$, where n is Poisson number of trials, and p is probability of success. Replicated reponses $y_1, y_2, \ldots, y_r$, are iid from the Binomial, given (a common) n from the Poisson,

$$\text{Prob}(y_1, \ldots, y_r) = \sum_{n=\max\{y_1,\ldots,y_r\}}^{\infty} \left\{ \prod_{i=1}^{r} \binom{n}{y_i} p^n (1-p)^{n-y_i} \right\} \times \frac{\lambda^n}{n!} \exp(-\lambda)$$

Link-function

The probability p is linked to the linear predictor by

$$p(\eta) = \frac{\exp(\eta)}{1 + \exp(\eta)}$$

for the default logit link, while λ depends on fixed covariates

$$\log(\lambda) = \sum_{j=1}^{m} \beta_j x_j$$

with one vector of covariates for each observation. m can be maximum 5 and minimum 1.

Hyperparameters

The parameters $\beta_1, \beta_2, \ldots, \beta_m$

Hyperparameter spesification and default values

hyper

theta1

hyperid 101101
name beta1
short.name beta1
initial 2.30258509299405
fixed FALSE
prior normal
param 0 0.5
to.theta `function(x) x`
from.theta `function(x) x`

theta2

hyperid 101102
name beta2
short.name beta2

initial 0
fixed FALSE
prior normal
param 0 1
to.theta `function(x) x`
from.theta `function(x) x`

theta3

hyperid 101103
name beta3
short.name beta3
initial 0
fixed FALSE
prior normal
param 0 1
to.theta `function(x) x`
from.theta `function(x) x`

theta4

hyperid 101104
name beta4
short.name beta4
initial 0
fixed FALSE
prior normal
param 0 1
to.theta `function(x) x`
from.theta `function(x) x`

theta5

hyperid 101105
name beta5
short.name beta5
initial 0
fixed FALSE
prior normal
param 0 1
to.theta `function(x) x`
from.theta `function(x) x`

status experimental

survival FALSE

discrete TRUE

link default logit probit

pdf nmix

Specification

- family = `nmix`
- Required arguments: the response Y and covariates X as `inla.mdata(Y, X [, additional.covariates])`

The response is a matrix where each row are replicates, where responses that are `NA`'s are ignored. The covariates is one or many vectors, matrices or data.frames. Each row of the covariates $(x_{i1}, x_{i2}, \ldots, x_{im})$ defines the covariates used for the i'th response(s) (the i'th row of `Y`). Note that $\beta_{m+1}, \ldots, \beta_5$ are fixed to zero.

Example

In the following example we estimate the parameters in a simulated example with binomial responses.

```
nrep = 10
n = 50
y = matrix(NA, n, nrep)
x = c()
xx = c()
h = 0.01

for(i in 1:n) {
    local.x = runif(1) - 0.5
    lambda = exp(2 + local.x)
    local.xx = runif(1) - 0.5
    intercept = 1
    eta = intercept + local.xx
    p = exp(eta)/(exp(eta) + 1)
    nr = sample(1:nrep, 1) ## sample number of replications
    N = rpois(1, lambda)
    y[i, 1:nr]= rbinom(nr, size = N, prob = p)
    x = c(x, local.x)
    xx = c(xx, local.xx)
}

Y = inla.mdata(y, 1, x)
r = inla(Y ~ 1 + xx,
         data = list(Y=Y, xx=xx, off=rep(intercept, n)),
         family = "nmix",
         control.fixed = list(prec.intercept=1,  prec=1))
```

Notes