A time series with seasonal component: the drivers data

This example is taken from [Rue and Held, 2005, Sec 4.4.2].

The data consist in monthly counts of car drivers in Great Britain killed or seriously injured in car accidents from January 1969 to December 1984. The time series has $n_d = 192$ data points and exhibits a strong seasonal pattern. One of our goals is to predict the pattern of counts in the 12 month following the last observation.

We assume the squared root of the counts y_t to be conditionally independent Gaussian random variables:

$$y_t|\eta_t, \lambda_y \sim \mathcal{N}(\eta_t, 1/\lambda_y), \quad t = 0, \ldots, n_d - 1$$

The conditional mean η_t is then a sum of a smooth trend and a seasonal component:

$$\eta_t = \text{season}_t + \text{trend}_t, \quad t = 0, \ldots, n_\eta - 1 \tag{1}$$

We assume the vector $\textbf{season} = (\text{season}_0, \ldots, \text{season}_{n_\eta - 1})$ to follow the seasonal model in (3.58) of [Rue and Held, 2005], with length 12 and unknown precision $\lambda_{\textbf{season}}$, and the vector $\textbf{trend} = (\text{trend}_0, \ldots, \text{trend}_{n_\eta - 1})$ to follow a RW2 with unknown precision $\lambda_{\textbf{trend}}$.

Note that we have that $n_\eta = n_d + 12 = 204$, since no observations y_t are available for $t = n_d, n_d+1, \ldots, n_d+11$. For prediction we are interested in the posterior marginals of $(\eta_{n_d}, \ldots, \eta_{n_d+11})$.

There are three hyperparameters in the model $\boldsymbol{\theta} = (\log \lambda_y, \log \lambda_{\textbf{season}}, \log \lambda_{\textbf{trend}})$ for which we choose the following prior distributions:

$$\begin{aligned}
\lambda_y &\sim \text{LogGamma}(4, 4) \\
\lambda_{\textbf{season}} &\sim \text{LogGamma}(1, 0.1) \\
\lambda_{\textbf{trend}} &\sim \text{LogGamma}(1, 0.0005)
\end{aligned}$$

See [Rue and Held, 2005] for more details.

References

[Rue and Held, 2005] Rue, H. and Held, L. (2005). *Gaussian Markov Random Fields: Theory and Applications*, volume 104 of *Monographs on Statistics and Applied Probability*. Chapman & Hall, London.