Generalized Poisson

The generalized Poisson distribution is given by

$$f(y|\lambda, w) = \frac{\lambda(\lambda + wy)^{y-1}}{y!} \exp(-(\lambda + wy))$$

for $y = 0, 1, 2, \ldots$ and where $\lambda > 0$ and $\max(-1, -\lambda/4) \leq w \leq 1$. The mean and variance of y are

$$\mu = \lambda(1 - w)^{-1} \qquad \text{and} \qquad \sigma^2 = \lambda(1 - w)^{-3} = \mu(1 - w)^{-2}.$$

Since the dispersion parameter w influence the mean as well as the variance, we will use the following parameterisation (ADD REFERENCES)

$$w = \frac{\varphi\mu^{p-1}}{1 + \varphi\mu^{p-1}},$$

for a fixed p, which gives the following density

$$f(y|\mu, \varphi, p) = \frac{\mu(\mu + \varphi\mu^{p-1}y)^{y-1}}{(1 + \varphi\mu^{p-1})^y y!} \exp\left(-\frac{\mu + \varphi\mu^{p-1}y}{1 + \varphi\mu^{p-1}}\right)$$

for $y = 0, 1, 2, \ldots$. We assume $\varphi \geq 0$.

Link-function

The mean and variance of y are given as

$$\mathrm{E}(y|.) = \mu \qquad \text{and} \qquad \mathrm{Var}(y|.) = \mu\left(1 + \varphi\mu^{p-1}\right)^2$$

and the mean is linked to the linear predictor by

$$\mu = E\exp(\eta)$$

Hyperparameters

The overdispersion parameter $\varphi \geq 0$ is represented as

$$\varphi = \exp(\theta)$$

The prior is defined on θ.

Specification

- `family="gpoisson"`
- `control.family = list(gpoisson.p = <p>)` defines the fixed parameter `p` (default 1).

Hyperparameter spesification and default values

hyper

theta

name overdispersion

short.name phi

initial -3

fixed FALSE

prior loggamma

param 1 1

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

survival FALSE

discrete TRUE

link default log

pdf gpoisson

status experimental

Example

In the following example we estimate the parameters in a simulated example with generalized Poisson responses.

```
dgpoisson = function(y, mu, phi, p)
{
    a = mu + phi * mu^(p-1.0) * y;
    b = 1. + phi * mu^(p-1.0);
    d = exp(log(mu) + (y-1.0)*log(a) -
            y*log(b) - lfactorial(y) -a/b)
    return (d)
}
rgpoisson = function(n, mu, phi, p)
{
    stopifnot(length(mu) == 1)
    s = sqrt(mu*(1+phi*mu^(p-1)))
    f = 10
    low = as.integer(max(0, mu - f*s))
    high = as.integer(mu + f*s)

    prob = dgpoisson(low:high, mu, phi, p)
    y = sample(low:high, n, replace=TRUE,
            prob = prob)

    return (y)
}

n = 1000
phi = 0
mu = 5
p = 1
y = rgpoisson(n, mu, phi, p)

r = inla(y ~ 1,  data = data.frame(y),
        family = "gpoisson",
        control.family = list(gpoisson.p = p))
```