

# The quantile log-logistics distribution

## Parametrisation

The loglogistic distribution has cdf

$$F(y) = \frac{y^\beta}{\alpha^\beta + y^\beta}, \quad y \geq 0, \quad \alpha, \beta > 0$$

The quantile function is given by

$$\kappa(q) = F^{-1}(q) = \alpha \left( \frac{q}{1-q} \right)^{1/\beta}, \quad 0 < q < 1.$$

The median is  $\kappa(0.5)$  and equals  $\alpha$ . We will use a quantile parameterisation. Define for fixed  $q$ ,

$$\kappa = \alpha \left( \frac{q}{1-q} \right)^{1/\beta}$$

and

$$s = 1/\beta.$$

Then  $0 < \kappa < 1$  is the  $q$ -quantile, and  $s > 0$  is a scaling parameter. The cdf and the density are now

$$F(y) = \frac{q^* y^{1/s}}{\kappa^{1/s} + q^* y^{1/s}}$$

and

$$f(y) = \frac{q^* \kappa^{1/s} y^{1/s-1}}{s(\kappa^{1/s} + q^* y^{1/s})^2}$$

where  $q^* = q/(1-q)$ .

## Link-function

The quantile  $\kappa$  relates to the linear predictor,

$$\log(\kappa) = \eta$$

using the default log link-function.

## Hyperparameters

The hyperparameter is

$$\theta = \log(s)$$

and the prior is given for  $\theta$ .

## Specification

- family = `qloglogistic`
- Required arguments: Observations  $y$  and the value of  $q$ .

The quantile is given as `control.family=list(quantile=q)`.

## Hyperparameter spesification and default values

**hyper**

**theta**

```
hyperid 60011
name precision parameter
short.name prec
initial 2
fixed FALSE
prior loggamma
param 1 0.001
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

**survival** FALSE

**discrete** FALSE

**link** default log

**pdf** qloglogistic

## Example

```
rqloglogistic = function(eta, s, q=0.5)
{
  qs = q/(1-q)
  u = runif(length(eta))
  x = (u/((1-u)*qs))^s * exp(eta)
  return (x)
}

n = 30
q = .10
s = .1
x = rnorm(n, s=0.2)
eta = 1 + 2*x
y = rqloglogistic(eta=eta, s=s, q=q)
r = inla(y ~ 1 + x,
  data = data.frame(y, x),
  family = "qloglogistic",
  control.family = list(quantile = q))

summary(r)
```

## Notes