

# Spatial lag model for spatial effects

## Parametrization

The `slm` model is defined as

$$\mathbf{x} = (I_n - \rho W)^{-1}(X\beta + \varepsilon)$$

where  $I_n$  is the identity matrix of dimension  $n \times n$ ,  $W$  is a spatial weights matrix,  $X$  is a matrix of covariates,  $\rho$  is a spatial autocorrelation parameter,  $\beta$  are coefficients of the covariates and  $\varepsilon$  is zero mean Gaussian noise with precision  $\tau$ .

## Hyperparameters

This model has two hyperparameters  $\theta = (\theta_1, \theta_2)$ . The precision parameter  $\tau$  is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ . The spatial autocorrelation parameter  $\rho$  is represented as

$$\rho^* = \frac{\rho - \rho_{\min}}{\rho_{\max} - \rho_{\min}}$$

and then

$$\theta_2 = \log(\rho^*/(1 - \rho^*))$$

and the prior is defined on  $\theta_2$ . Here,  $\rho_{\min}$  and  $\rho_{\max}$  are lower and upper limits of the legal range for  $\rho$ .

## Specification

The `slm` model is specified inside the `f()` function as

```
f(<whatever>, model="slm",  
  args.slm = list(rho.min = NULL,  
                  rho.max = NULL,  
                  X = NULL,  
                  W = NULL,  
                  Q.beta = NULL))
```

`args.slm` is used to define the `slm`-specific parameters in the model.

**rho.min** and **rho.max** define the range in which  $\rho$  can take values. Note that,  $\rho^*$  is in the interval  $(0, 1)$  and that it is re-scaled to the interval **(rho.min, rho.max)** when computing  $I_n - \rho W$ . Initial values on  $\rho$  need to be re-scaled to the  $(0, 1)$  interval.

**X** defines the matrix of covariates.

**W** defines the adjacency matrix.

**Q.beta** defines the precision of the vector of coefficients  $\beta$  in the model.

## Hyperparameter specification and default values

**hyper**

**theta1**

**hyperid** 34001  
**name** log precision  
**short.name** prec  
**initial** 4  
**fixed** FALSE  
**prior** loggamma  
**param** 1 5e-05  
**to.theta** function(x) log(x)  
**from.theta** function(x) exp(x)

**theta2**

**hyperid** 34002  
**name** rho  
**short.name** rho  
**initial** 0  
**fixed** FALSE  
**prior** normal  
**param** 0 10  
**to.theta** function(x) log(x/(1-x))  
**from.theta** function(x) 1/(1+exp(-x))

**constr** FALSE

**nrow.ncol** FALSE

**augmented** FALSE

**aug.factor** 1

**aug.constr**

**n.div.by**

**n.required** TRUE

**set.default.values** TRUE

**pdf** slm

**status** experimental

## Example

**##** Example using the Boston dataset from package spdep

```
require(INLA)
require(spdep)
data(boston)
```

```

## Index for the latent model
n <- nrow(boston.c)
boston.c$idx <- 1:n

## Define adjacency using a row-standardised matrix
lw <- nb2listw(boston.soi)
W <- as(as_dgRMatrix_listw(lw), "CsparseMatrix")

## Model definition
f1 <- log(CMEDV) ~ CRIM + ZN + INDUS + CHAS + I(NOX^2) + I(RM^2) + AGE +
  log(DIS) + log(RAD) + TAX + PTRATIO + B + log(LSTAT)
mmatrix <- model.matrix(f1, boston.c)

## Zero-variance for error term
zero.variance = list(prec=list(initial = 25, fixed=TRUE))

## Compute eigenvalues for SLM model, used to obtain rho.min and
## rho.max
e = eigenw(lw)
re.idx = which(abs(Im(e)) < 1e-6)
rho.max = 1/max(Re(e[re.idx]))
rho.min = 1/min(Re(e[re.idx]))
rho = mean(c(rho.min, rho.max))

## Precision matrix for beta coefficients' prior
betaprec <- .0001
Q.beta = Diagonal(n=ncol(mmatrix), betaprec)

## Priors on the hyperparameters
hyper = list(
  prec = list(
    prior = "loggamma",
    param = c(0.01, 0.01)),
  rho = list(
    initial=0,
    prior = "logitbeta",
    param = c(1,1)))

## Fit model
slmm1 <- inla( log(CMEDV) ~ -1 +
  f(idx, model="slm",
    args.slm=list(
      rho.min = rho.min,
      rho.max = rho.max,
      W=W,
      X=mmatrix,
      Q.beta=Q.beta),
    hyper=hyper),
  data=boston.c, family="gaussian",
  control.family = list(hyper=zero.variance),
  control.compute=list(dic=TRUE, cpo=TRUE)
)
summary(slmm1)

## Summary of the coefficients (at the end of the vector of random effects)
slmm1$summary.random$idx[n+1:ncol(mmatrix),]

```

```
## Re-scale rho to real scale
rhomarg <- inla.tmarginal(function(x){rho.min+x*(rho.max-rho.min)},
                          slmm1$marginals.hyperpar[[2]])
inla.zmarginal(rhomarg)

## Maximum likelihood estimate of model (used for comparison)
summary(m2 <- lagsarlm(f1, boston.c, lw))
```

## Notes

The estimates of  $\beta$  are included at the end of the vector of random effects. See the example for details on how to extract them.