Jan 17th-17

N-mix model : details

Model $\quad y \sim Bin(n, p)$

$\quad\quad\quad\;\; n \sim Pois(\lambda)$

Normally, ~~for~~ $p = p(z)$, while, $\log(\lambda) = x^T\beta$

The likelihood is

$$prob(y) = \sum_{n=y}^{\infty} Pois(n; \lambda) \cdot Bin(y; n, p)$$

There is a nice recursive formula for this density, using that

$$Pois(n; \lambda) = Pois(n-1; \lambda) \frac{\lambda}{n}$$

$$Bin(y; n, p) = Bin(y; n-1, p) \frac{n}{n-y} (1-p)$$

So that

$$Pois(n; \lambda) Bin(y; n, p) = Pois(n-1; \lambda) Bin(y; n-1, p) \cdot \frac{\lambda}{n-y}(1-p)$$

Let $f_i = \frac{\lambda(1-p)}{i}$, $i = 1, 2, \ldots, n_{max}$

then

②

$$L = \sum_{n=y}^{n_{max}} \mathrm{Pois}(n;\lambda)\cdot \mathrm{Bin}(y;n,p)$$

$$= \mathrm{Pois}(y;\lambda)\cdot \mathrm{Bin}(y;y,p)\cdot \Big\{ 1 + f_1 + f_1 f_2 + f_1 f_2 f_3 + \dots + f_1 \cdots f_{n_{max}} \Big\}$$

$$= \mathrm{Pois}\cdot \mathrm{Bin}\ \underbrace{\Big\{ 1 + f_1(1 + f_2(1 + f_3(1 + f_4(1 + \dots)))) \Big\}}_{fac.}$$

~~can~~ fac can be computed..

```
fac = 1
for i in imax:1
    fac = 1 + fac * λ(1-p)/i
```

then

$$\underline{L = \mathrm{Pois}(y;\lambda)\cdot \mathrm{Bin}(y;y,p)\cdot fac.}$$

imax could be $\left\lfloor \frac{\lambda(1-p)}{\varepsilon} \right\rfloor$

so that $\frac{\lambda(1-p)}{imax} \approx \varepsilon$

```r
p = runif(1)
lambda = sample(1:20, 1) + runif(1)
N = rpois(1,  lambda)
y = rbinom(1, N, p)
nmax = floor(y + lambda*(1-p)/0.01)  # estimate of a large 'n'

L = 0
for(n in y:nmax)
    L = L + dpois(n, lambda) * dbinom(y, n, p)
L.direct = L

L = dpois(y, lambda) * dbinom(y, y, p)
ff = lambda * (1-p)
fac = 1
for(i in nmax:1)
    fac = 1 + fac * ff / i
L.recursive = L * fac

print(c(N=N, nmax = nmax, p=p, lambda=lambda, y=y,
        L.recursive = L.recursive,  abs.err = abs(L.direct - L.recursive)))
```