

## Skew-Normal (version 1 and 2)

### Parametrisation

The Skew-Normal distribution is

$$f(y) = 2 \frac{\sqrt{w\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}w\tau(y-\mu)^2\right) \Phi(a a_{\max}[w\tau(y-\mu)])$$

for continuously responses  $y$  where  $\Phi(\cdot)$  is the cumulative distribution function for a standard Normal, and

$\mu$ : is the the location parameter

$\tau$ : is the inverse scale

$w$ : is a fixed weight,  $w > 0$ ,

$a$ : is the shape parameter

$a_{\max}$ : is the (fixed) maximum value of the shape paramter (added for stability reasons). Default value is 5.

### Link-function

The location parameter is linked to the linear predictor by

$$\mu = \eta$$

### Hyperparameters

The inverse scale is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ .

The shape parameter is

$$a = 2 \frac{\exp(\theta_2)}{1 + \exp(\theta_2)} - 1$$

and the prior is defined on  $\theta_2$ .

### Specification

- family = **sn**
- Required arguments:  $y$  and  $w$  (keyword **scale**). The weights has default value 1.
- Optional control arguments: **sn.shape.max**. Default value is 5.0.

### Hyperparameter spesification and default values

**hyper**

**theta1**

**hyperid** 74001

**name** log inverse scale

**short.name** iscale

```

    initial 4
    fixed FALSE
    prior loggamma
    param 1 5e-05
  theta2
    hyperid 74002
    name logit skewness
    short.name skew
    initial 0
    fixed FALSE
    prior gaussian
    param 0 10
    to.theta function(x, shape.max = 1) log((1+x/shape.max)/(1-x/shape.max))
    from.theta function(x, shape.max = 1) shape.max*(2*exp(x)/(1+exp(x))-1)

survival FALSE

discrete FALSE

link default identity

pdf sn

```

## Example

This is a simulated example requiring the package `sn`.

```

library(sn)
n = 1000
z = rnorm(n)
y = z + rsn(n, shape = 2)
formula = y ~ z
r = inla(formula, family = "sn", data = data.frame(z,y),
         control.family = list(sn.shape.max = 5.0))
summary(r)

```

## Notes

An simpler approximation to  $\Phi(\cdot)$  is used to improve the speed, which has maximum absolute error of 0.00197323; see the source code for further details.

## Skew-Normal (version 2)

### Parametrisation (version 2)

In the family “sn2” we offer an alternative parametersisation of the skew-normal with moment parameters, precision  $w\tau$  (where  $w$  is a fixed weight or scale) and standardized skewness  $\gamma$  (where  $|\gamma| < 1$  due to the skew-normal family<sup>1</sup>). In this parameterisation, the location parameter is linked to the linear predictor by

$$\mu = \eta$$

and  $\mu$  equals the expected value  $\xi + \omega\delta\sqrt{\frac{2}{\pi}}$ , and  $\delta = \alpha/\sqrt{1 + \alpha^2}$ .

### Hyperparameters

The precision  $\tau$  is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ . The (standardized) skewness  $\gamma$  is

$$\gamma = 2 \frac{\exp(\theta_2)}{1 + \exp(\theta_2)} - 1$$

and the prior is defined on  $\theta_2$ .

The function `INLA::inla.sn.reparam` offer the mapping between the moments (mean, variance and skewness) and the parameters used in the skew-normal density in the format used in the package `sn`, which are  $(\xi, \omega, \alpha)$ , where

$$f(x) = \frac{2}{\omega} \phi\left(\frac{x - \xi}{\omega}\right) \Phi\left(\alpha \left[\frac{x - \xi}{\omega}\right]\right)$$

where  $\phi(\cdot)$  and  $\Phi(\cdot)$  is the density and cumulative distribution function for the standard Gaussian distribution.

### Hyperparameter spesification and default values for sn2

**hyper**

**theta1**

**hyperid** 75001  
**name** log precision  
**short.name** prec  
**initial** 1  
**fixed** FALSE  
**prior** loggamma  
**param** 1 5e-05

**theta2**

**hyperid** 75002  
**name** logit skewness  
**short.name** skew  
**initial** 0  
**fixed** FALSE

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<sup>1</sup>or to be presice,  $|\gamma| < \frac{4-\pi}{2(\pi/2-1)^{3/2}} = 0.995271746\dots$

```

prior gaussian
param 0 10
to.theta function(x) log((1+x)/(1-x))
from.theta function(x) (2*exp(x)/(1+exp(x))-1)

survival FALSE

discrete FALSE

link default identity

status experimental

pdf sn2

```

### Example for sn2

This is a simulated example requiring the package `sn`.

```

library(sn)
n = 500
x = rnorm(n)
eta = 1/2 + 2*x
w = runif(n, min = 0.5, max = 2)
prec = 1 * w
skewness = 0.25
y = numeric(n)
for(i in 1:n) {
  param = INLA::inla.sn.reparam(moments = c(eta[i], 1/prec[i], skewness))
  y[i] = rsn(1, xi=param$xi, omega = param$omega, alpha = param$alpha)
}
r = inla(y ~ 1 + x, family = "sn2", scale = w, data = data.frame(y, x, w), verbose=T)
summary(r)

```

### Notes for sn2

In this parameterisation there is no `sn.shape.max`.

An simpler approximation to  $\Phi(\cdot)$  is used to improve the speed, which has maximum absolute error of 0.00197323; see the source code for further details.