

Proof trees for transitions in Theorem 2

By Ruleⁿ below we assume n applications of the transition rule **Rule** from Fig. 4. In case of n consecutive applications of rules **Par-l**, **Par-r** we write **Parⁿ**. Notice that α -conversion is often used: in particular when the rule **Extrusion** is applied. We define $\chi_l(\vec{Y}, M)$ as the list of message terms obtained by the replacement of l th entry in \vec{Y} with M . In *Case 5*, σ' , θ' are the frames accumulated at the point of input of Y_l . In the proof trees presented below we use the following abbreviations

$$S \triangleq \nu c. \nu ch. \overline{card}(ch). C_{\text{upd}}(s, ch, c)$$

$$I \triangleq \nu c. !\nu ch. \overline{card}(ch). C_{\text{upd}}(s, ch, c)$$

$$\frac{\frac{pk_s \# out, s, !S \quad out =_E out}{\text{Out}} \quad \frac{\overline{out}(pk(s)).!S \xrightarrow{\overline{out}(pk_s)} \left(\left\{ \overline{pk}(s) /_{pk_s} \right\} \right) \mid !S \quad s \# out, pk_s}{\text{Res}}}{UPD_{\text{spec}} \xrightarrow{\overline{out}(pk_s)} \nu s. \left(\left\{ \overline{pk}(s) /_{pk_s} \right\} \right) \mid !S} \text{Res}$$

$$\text{Case 1. Transition } UPD_{\text{spec}} \xrightarrow{\overline{out}(pk_s)} UPD_{\text{spec}}^{\emptyset}(\emptyset).$$

$$\frac{\frac{pk_s \# out, s, !I \quad out =_E out}{\text{Out}} \quad \frac{\overline{out}(pk(s)).!I \xrightarrow{\overline{out}(pk_s)} \left(\left\{ \overline{pk}(s) /_{pk_s} \right\} \right) \mid !I \quad s \# out, pk_s}{\text{Res}}}{UPD_{\text{impl}} \xrightarrow{\overline{out}(pk_s)} \nu s. \left(\left\{ \overline{pk}(s) /_{pk_s} \right\} \right) \mid !I} \text{Res}$$

$$\text{Case 1. Transition } UPD_{\text{impl}} \xrightarrow{\overline{out}(pk_s)} UPD_{\text{impl}}^{\emptyset, \emptyset}(\emptyset).$$

$$\frac{\frac{\frac{u_{L+1} \# card, ch, C_{\text{upd}}(s, c_{L+1}, ch_{L+1}), \sigma}{card \sigma =_E card} \quad \text{Out} \quad \frac{\overline{card}(u_{L+1})}{\sigma \circ \left\{ \overline{ch}_{L+1} /_{u_{L+1}} \right\} \mid \mathcal{E}^{L+1}(ch_{L+1})} \quad \frac{c_{L+1}, ch_{L+1} \# card, u_{L+1}, \sigma}{\text{Extrusion}^2}}{\frac{\sigma \mid S}{\overline{card}(u_{L+1})} \quad \frac{c_{L+1}, ch_{L+1}, u_{L+1} \# S}{\text{Rep-act}}} \quad \frac{\nu c_{L+1}, ch_{L+1}. (\sigma \circ \left\{ \overline{ch}_{L+1} /_{u_{L+1}} \right\} \mid \mathcal{E}^{L+1}(ch_{L+1}))}{\sigma \mid !S} \quad \frac{c_{L+1}, ch_{L+1}, u_{L+1} \# C_i, i \leq L}{\text{Par}^L} \quad \frac{\nu c_{L+1}, ch_{L+1}. (\sigma \circ \left\{ \overline{ch}_{L+1} /_{u_{L+1}} \right\} \mid \mathcal{E}^{L+1}(ch_{L+1}) \mid !S)}{\sigma \mid C_1 \mid \dots \mid C_L \mid !S} \quad \frac{s, c_i, ch_i, a_k}{i \leq L, k \in \beta \cup \gamma \cup \delta \# card, u_{L+1}} \quad \frac{\nu c_{L+1}, ch_{L+1}. (\sigma \circ \left\{ \overline{ch}_{L+1} /_{u_{L+1}} \right\} \mid C_1 \mid \dots \mid C_L \mid \mathcal{E}^{L+1}(ch_{L+1}) \mid !S)}{UPD_{\text{spec}}^{\Psi}(\vec{Y}) \xrightarrow{\overline{card}(u_{L+1})} \nu s, c_1, \dots, c_L, c_{L+1}, ch_1, \dots, ch_L, ch_{L+1}, a_{l_1}, \dots, a_{l_K}. (\sigma \circ \left\{ \overline{ch}_{L+1} /_{u_{L+1}} \right\} \mid C_1 \mid \dots \mid C_L \mid \mathcal{E}^{L+1}(ch_{L+1}) \mid !S)} \text{Res}^{1+2L+K}$$

$$\text{Case 2. Transition } UPD_{\text{spec}}^{\Psi}(\vec{Y}) \xrightarrow{\overline{card}(u_{L+1})} UPD_{\text{spec}}^{\{\alpha \cup \{L+1\}, \beta, \gamma, \delta\}}((Y_1, \dots, Y_L, \emptyset)).$$

$$\begin{array}{c}
u_{L+1} \# \text{card}, ch_{L+1}, C_{\text{upd}}(s, c_d, ch_{L+1}), \theta \\
\hline
\text{card} \theta =_E \text{card} \quad \text{Out} \\
\theta \mid \overline{\text{card}}\langle ch_{L+1} \rangle . C_{\text{upd}}(s, c_d, ch_{L+1}) \\
\hline
\overline{\text{card}}(u_{L+1}) \rightarrow \quad ch_{L+1} \# \\
\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^d(ch_{L+1}) \right\} \mid \mathcal{E}^d(ch_{L+1}) \quad \text{card}, u_{L+1}, \theta \\
\hline
\theta \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_d, ch) \quad ch_{L+1}, u_{L+1} \# \\
\hline
\overline{\text{card}}(u_{L+1}) \rightarrow \quad \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_d, ch) \\
\nu ch_{L+1}. (\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^d(ch_{L+1}) \right\} \mid \mathcal{E}^d(ch_{L+1})) \quad \text{Rep-act} \quad ch_{L+1}, u_{L+1} \# \\
\theta \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_d, ch) \quad C_j^i, i \leq D, j \leq \max_{i \leq D} L_i; \\
\hline
\overline{\text{card}}(u_{L+1}) \rightarrow \quad \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_d, ch) \\
\nu ch_{L+1}. (\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^d(ch_{L+1}) \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_d, ch) \right\} \mid \mathcal{E}^d(ch_{L+1})) \quad C_{\text{upd}}(s, c_d, ch), \\
i \leq D, i \neq d; !I \\
\hline
\theta \mid \dots \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_d, ch) \mid \dots \mid !I \quad \text{Par}^{D+L} \\
\hline
\overline{\text{card}}(u_{L+1}) \rightarrow \quad \nu ch_{L+1}. (\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^d(ch_{L+1}) \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_d, ch) \mid \dots \mid !I \right\} \mid \mathcal{E}^d(ch_{L+1})) \\
s, c_i, ch_j, a_k, \\
i \leq D, j \leq L, k \in \beta \cup \gamma \cup \delta \# \\
\text{card}, u_{L+1} \\
\hline
UPD_{\text{impl}}^{\Psi, \Omega}(\vec{Y}) \xrightarrow{\overline{\text{card}}(u_{L+1})} \nu s, c_1, \dots, c_D, ch_1, \dots, ch_L, ch_{L+1}, a_{l_1}, \dots, a_{l_K}. (\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^d(ch_{L+1}) \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_d, ch) \mid \dots \mid !I \right\} \mid \mathcal{E}^d(ch_{L+1})) \quad \text{Res}^{1+D+L+K}
\end{array}$$

Case 2. Transition $UPD_{\text{impl}}^{\Psi, \Omega}(\vec{Y}) \xrightarrow{\overline{\text{card}}(u_{L+1})} UPD_{\text{impl}}^{\{\alpha \cup \{L+1\}, \beta, \gamma, \delta\}, \{\dots, \zeta^d \cup \{L+1\}, \dots\}}((Y_1, \dots, Y_L, \emptyset))$: card d starts new session.

$$\begin{array}{c}
u_{L+1} \# \text{card}, ch_{L+1}, C_{\text{upd}}(s, c_{D+1}, ch_{L+1}), \theta \\
\hline
\text{card} \theta =_E \text{card} \quad \text{Out} \\
\theta \mid \overline{\text{card}}\langle ch_{L+1} \rangle . C_{\text{upd}}(s, c_{D+1}, ch_{L+1}) \\
\hline
\overline{\text{card}}(u_{L+1}) \rightarrow \quad ch_{L+1} \# \\
\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^{D+1}(ch_{L+1}) \right\} \mid \mathcal{E}^{D+1}(ch_{L+1}) \quad \text{card}, u_{L+1}, \theta \\
\hline
\theta \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_{D+1}, ch) \quad ch_{L+1}, u_{L+1} \# \\
\hline
\overline{\text{card}}(u_{L+1}) \rightarrow \quad \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_{D+1}, ch) \\
\nu ch_{L+1}. (\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^{D+1}(ch_{L+1}) \right\} \mid \mathcal{E}^{D+1}(ch_{L+1})) \quad \text{Rep-act} \\
\theta \mid \nu ch. \overline{\text{card}}\langle ch_{L+1} \rangle . C_{\text{upd}}(s, c_{D+1}, ch) \quad c_{D+1} \# \\
\hline
\overline{\text{card}}(u_{L+1}) \rightarrow \quad \nu ch_{L+1}. (\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^{D+1}(ch_{L+1}) \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_{D+1}, ch) \right\} \mid \mathcal{E}^{D+1}(ch_{L+1})) \quad \text{Extrusion} \\
\theta \mid I \quad \text{card}, u_{L+1}, \theta \\
\hline
\overline{\text{card}}(u_{L+1}) \rightarrow \quad \nu c_{D+1}, ch_{L+1}, u_{L+1} \# I \\
\nu c_{D+1}, ch_{L+1}. (\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^{D+1}(ch_{L+1}) \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_{D+1}, ch) \right\} \mid \mathcal{E}^{D+1}(ch_{L+1})) \\
\hline
\theta \mid !I \quad \text{Rep-act} \quad c_{D+1}, ch_{L+1}, \\
u_{L+1} \# C_j^i, \\
i \leq D, j \leq \max_{i \leq D} L_i; \\
\nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_d, ch) \\
\hline
\overline{\text{card}}(u_{L+1}) \rightarrow \quad \nu c_{D+1}, ch_{L+1}. (\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^{D+1}(ch_{L+1}) \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_{D+1}, ch) \right\} \mid \mathcal{E}^{D+1}(ch_{L+1})) \quad \text{Par}^{D+L} \\
\theta \mid \dots \mid !I \quad s, c_i, ch_j, a_k, \\
i \leq D, j \leq L, \\
k \in \beta \cup \gamma \cup \delta \# \\
\text{card}, u_{L+1} \\
\hline
UPD_{\text{impl}}^{\Psi, \Omega}(\vec{Y}) \xrightarrow{\overline{\text{card}}(u_{L+1})} \nu s, c_1, \dots, c_D, c_{D+1}, ch_1, \dots, ch_L, ch_{L+1}, a_{l_1}, \dots, a_{l_K}. (\theta \circ \left\{ \frac{ch_{L+1}}{u_{L+1}} \mid \mathcal{E}^{D+1}(ch_{L+1}) \mid \nu ch. \overline{\text{card}}\langle ch \rangle . C_{\text{upd}}(s, c_{D+1}, ch) \right\} \mid \mathcal{E}^{D+1}(ch_{L+1})) \quad \text{Res}^{1+D+L+K}
\end{array}$$

Case 2. Transition $UPD_{\text{impl}}^{\Psi, \Omega}(\vec{Y}) \xrightarrow{\overline{\text{card}}(u_{L+1})} UPD_{\text{impl}}^{\{\alpha \cup \{L+1\}, \beta, \gamma, \delta\}, \Omega \cup \{\{L+1\}\}}((Y_1, \dots, Y_L, \emptyset))$: a new card is created.

$$\begin{array}{c}
v_l \# u_l, a_l, \mathcal{F}^l(ch_l, a_l), \sigma \\
\hline
u_l \sigma =_E ch_l \quad \text{Out} \\
\sigma \mid \overline{ch_l}\langle \phi(a_l, \phi(c_l, \mathbf{g})) \rangle . \mathcal{F}^l(ch_l, a_l) \xrightarrow{\overline{u_l}(v_l)} \sigma \circ \left\{ \frac{\phi(a_l, \phi(c_l, \mathbf{g}))}{v_l} \mid \mathcal{F}^l(ch_l, a_l) \right\} \mid a_l \# u_l, v_l, \sigma \\
\hline
\sigma \mid \nu a. \overline{ch_l}\langle \phi(a, \phi(c_l, \mathbf{g})) \rangle . \mathcal{F}^l(ch_l, a) \xrightarrow{\overline{u_l}(v_l)} \nu a_l. (\sigma \circ \left\{ \frac{\phi(a_l, \phi(c_l, \mathbf{g}))}{v_l} \mid \mathcal{F}^l(ch_l, a_l) \right\} \mid \mathcal{F}^l(ch_l, a_l)) \quad \text{Extrusion} \\
a_l, v_l \# C_i, i \leq L, i \neq l; !S \\
\hline
\sigma \mid C_1 \mid \dots \mid \mathcal{E}^l(ch_l) \mid \dots \mid C_L \mid !S \xrightarrow{\overline{u_l}(v_l)} \nu a_l. (\sigma \circ \left\{ \frac{\phi(a_l, \phi(c_l, \mathbf{g}))}{v_l} \mid \mathcal{F}^l(ch_l, a_l) \right\} \mid \mathcal{F}^l(ch_l, a_l) \mid \dots \mid C_K \mid \dots \mid \mathcal{F}^l(ch_l, a_l) \mid \dots \mid !S) \quad \text{Par}^L \\
s, c_i, ch_i, a_k \\
i \leq L, k \in \beta \cup \gamma \cup \delta \# u_l, v_l \\
\hline
UPD_{\text{spec}}^{\Psi}(\vec{Y}) \xrightarrow{\overline{u_l}(v_l)} \nu s, c_1, \dots, c_L, ch_1, \dots, ch_L, a_{l_1}, \dots, a_{l_K}. (\sigma \circ \left\{ \frac{\phi(a_l, \phi(c_l, \mathbf{g}))}{v_l} \mid \mathcal{F}^l(ch_l, a_l) \right\} \mid \mathcal{F}^l(ch_l, a_l) \mid \dots \mid C_K \mid \dots \mid \mathcal{F}^l(ch_l, a_l) \mid \dots \mid !S) \quad \text{Res}^{1+2L+K}
\end{array}$$

Case 3. Transition $UPD_{\text{spec}}^{\Psi}(\vec{Y}) \xrightarrow{\overline{u_l}(v_l)} UPD_{\text{spec}}^{\{\alpha \setminus \{l\}, \beta \cup \{l\}, \gamma, \delta\}}(\vec{Y}), l \in \alpha$.

$$\begin{array}{c}
\frac{v_l \# u_l, a_l, \mathcal{F}^d(ch_l, a_l), \theta}{u_l \theta =_E ch_l} \text{Out} \\
\frac{\theta \mid \overline{ch_l}(\phi(a_l, \phi(c_d, \mathbf{g}))) \cdot \mathcal{F}^d(ch_l, a_l) \xrightarrow{\overline{u_l}(v_l)} \theta \circ \left\{ \phi(a_l, \phi(c_d, \mathbf{g})) \right\}_{|v_l|} \mid \mathcal{F}^d(ch_l, a_l) \quad a_l \# u_l, v_l, \theta}{\theta \mid \nu a. \overline{ch_l}(\phi(a, \phi(c_d, \mathbf{g}))) \cdot \mathcal{F}^d(ch_l, a) \xrightarrow{\overline{u_l}(v_l)} \nu a_l. (\theta \circ \left\{ \phi(a_l, \phi(c_d, \mathbf{g})) \right\}_{|v_l|} \mid \mathcal{F}^d(ch_l, a_l))} \text{Extrusion} \quad \begin{array}{l} a_l, v_l \# C_j^t, \\ i \leq D, j \leq \max_{i \leq D} L_i, \\ j \neq l; !I \end{array} \\
\frac{\theta \mid \dots \mid \mathcal{E}^d(ch_l) \mid \dots \mid !I \xrightarrow{\overline{u_l}(v_l)} \nu a_l. (\theta \circ \left\{ \phi(a_l, \phi(c_d, \mathbf{g})) \right\}_{|v_l|} \mid \dots \mid \mathcal{F}^d(ch_l, a_l) \mid \dots \mid !I)}{\text{Par}^{D+L} \quad \begin{array}{l} s, c_i, ch_j, a_k \\ i \leq D, j \leq L, k \in \beta \cup \gamma \cup \delta \# \\ u_l, v_l \end{array}} \text{Res}^{1+D+L+K} \\
UPD_{\text{impl}}^{\Psi, \Omega}(\vec{Y}) \xrightarrow{\overline{u_l}(v_l)} \nu s, c_1, \dots, c_D, ch_1, \dots, ch_L, a_{l_1}, \dots, a_{l_k}, a_l. (\theta \circ \left\{ \phi(a_l, \phi(c_d, \mathbf{g})) \right\}_{|v_l|} \mid \dots \mid \mathcal{F}^d(ch_l, a_l) \mid \dots \mid !I)
\end{array}$$

Case 3. Transition $UPD_{\text{impl}}^{\Psi, \Omega}(\vec{Y}) \xrightarrow{\overline{u_l}(v_l)} UPD_{\text{impl}}^{\alpha \setminus \{l\}, \beta \cup \{l\}, \gamma, \delta\}, \Omega(\vec{Y})$, $l \in \alpha$.

$$\begin{array}{c}
\frac{u_l \sigma =_E ch_l}{\sigma \mid ch_l(y) \cdot \mathcal{G}^l(ch_l, a_l, y) \xrightarrow{u_l Y_l} \sigma \mid \mathcal{G}^l(ch_l, a_l, Y_l \sigma)} \text{Inp} \\
\frac{\sigma \mid C_1 \mid \dots \mid \mathcal{F}^l(ch_l, a_l) \mid \dots \mid C_L \mid !S \xrightarrow{u_l Y_l} \sigma \mid \dots \mid \mathcal{G}^l(ch_l, a_l, Y_l \sigma) \mid \dots \mid !S}{\text{Par}^L \quad \begin{array}{l} s, c_i, ch_i, a_k \\ i \leq L, k \in \beta \cup \gamma \cup \delta \# u_l, Y_l \end{array}} \text{Res}^{1+2L+K} \\
UPD_{\text{spec}}^{\Psi}(\vec{Y}) \xrightarrow{u_l Y_l} \nu s, c_1, \dots, c_L, ch_1, \dots, ch_L, a_{l_1}, \dots, a_{l_k}. \{\sigma \mid \dots \mid \mathcal{G}^l(ch_l, a_l, Y_l \sigma) \mid \dots \mid !S\}
\end{array}$$

Case 4. Transition $UPD_{\text{spec}}^{\Psi}(\vec{Y}) \xrightarrow{u_l Y_l} UPD_{\text{spec}}^{\{\alpha, \beta \setminus \{l\}, \gamma \cup \{l\}, \delta\}, \Omega}(\chi_l(\vec{Y}, Y_l))$, $l \in \beta$.

$$\begin{array}{c}
\frac{u_l \theta =_E ch_l}{\theta \mid ch_l(y) \cdot \mathcal{G}^d(ch_l, a_l, y) \xrightarrow{u_l Y_l} \theta \mid \mathcal{G}^d(ch_l, a_l, Y_l \sigma)} \text{Inp} \\
\frac{\theta \mid \dots \mid \mathcal{F}^d(ch_l, a_l) \mid \dots \mid !I \xrightarrow{u_l Y_l} \theta \mid \dots \mid \mathcal{G}^d(ch_l, a_l, Y_l \sigma) \mid \dots \mid !I}{\text{Par}^{D+L} \quad \begin{array}{l} s, c_i, ch_j, a_k \\ i \leq D, j \leq L, k \in \beta \cup \gamma \cup \delta \# \\ u_l, Y_l \end{array}} \text{Res}^{1+D+L+K} \\
UPD_{\text{impl}}^{\Psi, \Omega}(\vec{Y}) \xrightarrow{u_l Y_l} \nu s, c_1, \dots, c_D, ch_1, \dots, ch_L, a_{l_1}, \dots, a_{l_k}. \{\theta \mid \dots \mid \mathcal{G}^d(ch_l, a_l, Y_l \sigma) \mid \dots \mid !I\}
\end{array}$$

Case 4. Transition $UPD_{\text{impl}}^{\Psi, \Omega}(\vec{Y}) \xrightarrow{u_l Y_l} UPD_{\text{impl}}^{\{\alpha, \beta \setminus \{l\}, \gamma \cup \{l\}, \delta\}, \Omega}(\chi_l(\vec{Y}, Y_l))$ if there is a card at the stage \mathcal{F} .

$$\begin{array}{c}
\frac{w_l \# u_l, m^l(a_l, Y_l \sigma'), \sigma}{u_l \theta =_E ch_l} \text{Out} \\
\frac{\sigma \mid \overline{ch_l}(m^l(a_l, Y_l \sigma')) \xrightarrow{\overline{u_l}(w_l)} \sigma \circ \left\{ m^l(a_l, Y_l \sigma') \right\}_{|w_l|} \mid \mathcal{H}^l}{\sigma \mid C_1 \mid \dots \mid \mathcal{G}^l(ch_l, a_l, Y_l \sigma') \mid \dots \mid C_L \mid !S \xrightarrow{\overline{u_l}(w_l)} \sigma \circ \left\{ m^l(a_l, Y_l \sigma') \right\}_{|w_l|} \mid \dots \mid \mathcal{H}^l \mid \dots \mid !S} \text{Par}^L \quad \begin{array}{l} s, c_i, ch_i, a_k \\ i \leq L, k \in \beta \cup \gamma \cup \delta \# u_l, w_l \end{array} \\
UPD_{\text{spec}}^{\Psi}(\vec{Y}) \xrightarrow{\overline{u_l}(w_l)} \nu s, c_1, \dots, c_L, ch_1, \dots, ch_L, a_{l_1}, \dots, a_{l_k}. \{\sigma \circ \left\{ m^l(a_l, Y_l \sigma') \right\}_{|w_l|} \mid \dots \mid \mathcal{H}^l \mid \dots \mid !S\}
\end{array}$$

Case 5. Transition $UPD_{\text{spec}}^{\Psi}(\vec{Y}) \xrightarrow{\overline{u_l}(w_l)} UPD_{\text{spec}}^{\{\alpha, \beta, \gamma \setminus \{l\}, \delta \cup \{l\}\}, \Omega}(\vec{Y})$, $l \in \gamma$.

$$\begin{array}{c}
\frac{w_l \# u_l, m^d(a_l, Y_l \theta'), \theta}{u_l \theta =_E ch_l} \text{Out} \\
\frac{\theta \mid \overline{ch_l}(m^d(a_l, Y_l \theta')) \xrightarrow{\overline{u_l}(w_l)} \theta \circ \left\{ m^d(a_l, Y_l \theta') \right\}_{|w_l|} \mid \mathcal{H}^d}{\theta \mid \dots \mid \mathcal{G}^d(ch_l, a_l, Y_l \theta') \mid \dots \mid !I \xrightarrow{\overline{u_l}(w_l)} \theta \circ \left\{ m^d(a_l, Y_l \theta') \right\}_{|w_l|} \mid \dots \mid \mathcal{H}^d \mid \dots \mid !I} \text{Par}^{D+L} \quad \begin{array}{l} s, c_i, ch_j, a_k \\ i \leq D, j \leq L, k \in \beta \cup \gamma \cup \delta \# \\ u_l, w_l \end{array} \\
UPD_{\text{impl}}^{\Psi, \Omega}(\vec{Y}) \xrightarrow{\overline{u_l}(w_l)} \nu s, c_1, \dots, c_D, ch_1, \dots, ch_L, a_{l_1}, \dots, a_{l_k}. \{\theta \circ \left\{ m^d(a_l, Y_l \theta') \right\}_{|w_l|} \mid \dots \mid \mathcal{H}^d \mid \dots \mid !I\}
\end{array}$$

Case 5. Transition $UPD_{\text{impl}}^{\Psi, \Omega}(\vec{Y}) \xrightarrow{\overline{u_l}(w_l)} UPD_{\text{impl}}^{\{\alpha, \beta, \gamma \setminus \{l\}, \delta \cup \{l\}\}, \Omega}(\vec{Y})$, $l \in \gamma$.