Proof trees for transitions in Theorem 2

By Rule^n below we assume n applications of the transition rule Rule from Fig. 4. In case of n consecutive applications of rules $\mathsf{Par\text{-}l}$, $\mathsf{Par\text{-}r}$ we write Par^n. Notice that α-conversion is often used: in particular when the rule $\mathsf{Extrusion}$ is applied. We define $\chi_l(\vec{Y}, M)$ as the list of message terms obtained by the replacement of lth entry in $\vec{Y}$ with M. In *Case* 5, σ', θ' are the frames accumulated at the point of input of Y_l. In the proof trees presented below we use the following abbreviations

$$S \triangleq \nu c.\nu ch.\overline{card}\langle ch\rangle.C_{\text{fix}}(s, ch, c)$$

$$I \triangleq \nu c.!\nu ch.\overline{card}\langle ch\rangle.C_{\text{fix}}(s, ch, c)$$

$$\dfrac{\dfrac{pk_s \# out, s, !S \quad out =_E out}{\overline{out}\langle \mathtt{pk}(s)\rangle.!S \xrightarrow{\overline{out}(pk_s)} \left(\left\{{}^{\mathtt{pk}(s)}/_{pk_s}\right\}\right) \mid !S}\,\mathsf{Out} \qquad s \# out, pk_s}{FIX_{\text{spec}} \xrightarrow{\overline{out}(pk_s)} \nu s.\left(\left\{{}^{\mathtt{pk}(s)}/_{pk_s}\right\}\right) \mid !S}\,\mathsf{Res}$$

Case 1. Transition $FIX_{\text{spec}} \xrightarrow{\overline{out}(pk_s)} FIX^{\varnothing}_{\text{spec}}(\varnothing)$.

$$\dfrac{\dfrac{pk_s \# out, s, !I \quad out =_E out}{\overline{out}\langle \mathtt{pk}(s)\rangle.!I \xrightarrow{\overline{out}(pk_s)} \left(\left\{{}^{\mathtt{pk}(s)}/_{pk_s}\right\}\right) \mid !I}\,\mathsf{Out} \qquad s \# out, pk_s}{FIX_{\text{impl}} \xrightarrow{\overline{out}(pk_s)} \nu s.\left(\left\{{}^{\mathtt{pk}(s)}/_{pk_s}\right\}\right) \mid !I}\,\mathsf{Res}$$

Case 1. Transition $FIX_{\text{impl}} \xrightarrow{\overline{out}(pk_s)} FIX^{\varnothing,\varnothing}_{\text{impl}}(\varnothing)$.

$$\dfrac{u_{L+1} \# \mathit{card}, ch, C_{\text{fix}}(s, c_{L+1}, ch_{L+1}), \sigma \qquad \mathit{card}\sigma =_E \mathit{card}}{\overline{\mathit{card}}\langle ch_{L+1}\rangle.C_{\text{fix}}(s, c_{L+1}, ch_{L+1}) \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \sigma\circ\{^{ch_{L+1}}/_{u_{L+1}}\} \mid \mathcal{E}^{L+1}(ch_{L+1})}\ \mathsf{Out}$$

$$\dfrac{\cdots \qquad c_{L+1}, ch_{L+1} \# \mathit{card}, u_{L+1}, \sigma}{\sigma \mid S \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \nu c_{L+1}, ch_{L+1}.(\sigma\circ\{^{ch_{L+1}}/_{u_{L+1}}\} \mid \mathcal{E}^{L+1}(ch_{L+1}))}\ \mathsf{Extrusion}^2$$

$$\dfrac{\cdots \qquad c_{L+1}, ch_{L+1}, u_{L+1} \# S}{\sigma \mid !S \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \nu c_{L+1}, ch_{L+1}.(\sigma\circ\{^{ch_{L+1}}/_{u_{L+1}}\} \mid \mathcal{E}^{L+1}(ch_{L+1}) \mid !S)}\ \mathsf{Rep\text{-}act}$$

$$\dfrac{\cdots \qquad c_{L+1}, ch_{L+1}, u_{L+1} \# C_i, i \le L}{\sigma \mid C_1 \mid \cdots \mid C_L \mid !S \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \nu c_{L+1}, ch_{L+1}.(\sigma\circ\{^{ch_{L+1}}/_{u_{L+1}}\} \mid C_1 \mid \cdots \mid C_L \mid \mathcal{E}^{L+1}(ch_{L+1}) \mid !S)}\ \mathsf{Par}^L$$

$$\dfrac{\cdots \qquad s, c_i, ch_i, a_k \quad i \le L, k \in \beta\cup\gamma\cup\delta \# \mathit{card}, u_{L+1}}{\mathit{FIX}^{\Psi}_{\text{spec}}(\vec{Y}) \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \nu s, c_1, \cdots, c_L, c_{L+1}, ch_1, \cdots, ch_L, ch_{L+1}, a_{l_1}, \cdots, a_{l_K}.(\sigma\circ\{^{ch_{L+1}}/_{u_{L+1}}\} \mid C_1 \mid \cdots \mid C_L \mid \mathcal{E}^{L+1}(ch_{L+1}) \mid !S)}\ \mathsf{Res}^{1+2L+K}$$

Case 2. Transition $\mathit{FIX}^{\Psi}_{\text{spec}}(\vec{Y}) \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \mathit{FIX}^{\{\alpha\cup\{L+1\},\beta,\gamma,\delta\}}_{\text{spec}}((Y_1, \cdots, Y_L, \varnothing))$.

$$\dfrac{u_{L+1} \# \mathit{card}, ch_{L+1}, C_{\text{fix}}(s, c_d, ch_{L+1}), \theta \qquad \mathit{card}\theta =_E \mathit{card}}{\theta \mid \overline{\mathit{card}}\langle ch_{L+1}\rangle.C_{\text{fix}}(s, c_d, ch_{L+1}) \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \theta\circ\{^{ch_{L+1}}/_{u_{L+1}}\} \mid \mathcal{E}^{d}(ch_{L+1})}\ \mathsf{Out}$$

$$\dfrac{\cdots \qquad ch_{L+1} \# \mathit{card}, u_{L+1}, \theta}{\theta \mid \nu ch.\overline{\mathit{card}}\langle ch\rangle.C_{\text{fix}}(s, c_d, ch) \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \nu ch_{L+1}.(\theta\circ\{^{ch_{L+1}}/_{u_{L+1}}\} \mid \mathcal{E}^{d}(ch_{L+1}))}\ \mathsf{Extrusion}$$

$$\dfrac{\cdots \qquad ch_{L+1}, u_{L+1} \# \nu ch.\overline{\mathit{card}}\langle ch\rangle.C_{\text{fix}}(s, c_d, ch)}{\theta \mid !\nu ch.\overline{\mathit{card}}\langle ch\rangle.C_{\text{fix}}(s, c_d, ch) \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \nu ch_{L+1}.(\theta\circ\{^{ch_{L+1}}/_{u_{L+1}}\} \mid \mathcal{E}^{d}(ch_{L+1}) \mid !\nu ch.\overline{\mathit{card}}\langle ch\rangle.C_{\text{fix}}(s, c_d, ch))}\ \mathsf{Rep\text{-}act}$$

$$\dfrac{\cdots \qquad ch_{L+1}, u_{L+1} \# C^i_j, i \le D, j \le \max_{i\le D} L_i; \nu ch.\overline{\mathit{card}}\langle ch\rangle.C_{\text{fix}}(s, c_i, ch), i \le D, i \ne d; !I}{\theta \mid \cdots \mid !\nu ch.\overline{\mathit{card}}\langle ch\rangle.C_{\text{fix}}(s, c_d, ch) \mid \cdots \mid !I \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \nu ch_{L+1}.(\theta\circ\{^{ch_{L+1}}/_{u_{L+1}}\} \mid \cdots \mid \mathcal{E}^{d}(ch_{L+1}) \mid !\nu ch.\overline{\mathit{card}}\langle ch\rangle.C_{\text{fix}}(s, c_d, ch) \mid \cdots \mid !I)}\ \mathsf{Par}^{D+L}$$

$$\dfrac{\cdots \qquad s, c_i, ch_j, a_k, \quad i \le D, j \le L, k \in \beta\cup\gamma\cup\delta \# \mathit{card}, u_{L+1}}{\mathit{FIX}^{\Psi,\Omega}_{\text{impl}}(\vec{Y}) \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \nu s, c_1, \cdots, c_D, ch_1, \cdots, ch_L, ch_{L+1}, a_{l_1}, \cdots, a_{l_K}.(\theta\circ\{^{ch_{L+1}}/_{u_{L+1}}\} \mid \cdots \mid \mathcal{E}^{d}(ch_{L+1}) \mid !\nu ch.\overline{\mathit{card}}\langle ch\rangle.C_{\text{fix}}(s, c_d, ch) \mid \cdots \mid !I)}\ \mathsf{Res}^{1+D+L+K}$$

Case 2. Transition $\mathit{FIX}^{\Psi,\Omega}_{\text{impl}}(\vec{Y}) \xrightarrow{\overline{\mathit{card}}(u_{L+1})} \mathit{FIX}^{\{\alpha\cup\{L+1\},\beta,\gamma,\delta\},\{\cdots,\zeta^d\cup\{L+1\},\cdots\}}_{\text{impl}}((Y_1, \cdots, Y_L, \varnothing))$: card d starts new session.

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{l} u_{L+1} \mathrel{\#} card, ch_{L+1}, C_{\text{fix}}(s, c_{D+1}, ch_{L+1}), \theta \\ card\theta =_E card \end{array}}{\theta \mid \overline{card}\langle ch_{L+1}\rangle.C_{\text{fix}}(s, c_{D+1}, ch_{L+1}) \xrightarrow{\overline{card}(u_{L+1})} \theta \circ \left\{{}^{ch_{L+1}}/_{u_{L+1}}\right\} \mid \mathcal{E}^{D+1}(ch_{L+1})}\,\text{Out} \quad \begin{array}{l} ch_{L+1} \mathrel{\#} \\ card, \\ u_{L+1}, \theta \end{array}}{\theta \mid \nu ch.\overline{card}\langle ch\rangle.C_{\text{fix}}(s, c_{D+1}, ch) \xrightarrow{\overline{card}(u_{L+1})} \nu ch_{L+1}.(\theta \circ \left\{{}^{ch_{L+1}}/_{u_{L+1}}\right\} \mid \mathcal{E}^{D+1}(ch_{L+1})}\,\text{Extrusion} \quad \begin{array}{l} ch_{L+1}, u_{L+1} \mathrel{\#} \\ \nu ch.\overline{card}\langle ch\rangle. \\ C_{\text{fix}}(s, c_{D+1}, ch) \end{array}}{\theta \mid !\nu ch.\overline{card}\langle ch_{L+1}\rangle.C_{\text{fix}}(s, c_{D+1}, ch) \xrightarrow{\overline{card}(u_{L+1})} \nu ch_{L+1}.(\theta \circ \left\{{}^{ch_{L+1}}/_{u_{L+1}}\right\} \mid \mathcal{E}^{D+1}(ch_{L+1}) \mid !\nu ch.\overline{card}\langle ch\rangle.C_{\text{fix}}(s, c_{D+1}, ch))}\,\text{Rep-act} \quad \begin{array}{l} c_{D+1} \mathrel{\#} \\ card, \\ u_{L+1}, \theta \end{array}}{\theta \mid I \xrightarrow{\overline{card}(u_{L+1})} \nu c_{D+1}, ch_{L+1}.(\theta \circ \left\{{}^{ch_{L+1}}/_{u_{L+1}}\right\} \mid \mathcal{E}^{D+1}(ch_{L+1}) \mid !\nu ch.\overline{card}\langle ch\rangle.C_{\text{fix}}(s, c_{D+1}, ch))}\,\text{Extrusion} \quad \begin{array}{l} c_{D+1}, \\ ch_{L+1}, \\ u_{L+1} \mathrel{\#} I \end{array}}{\theta \mid !I \xrightarrow{\overline{card}(u_{L+1})} \nu c_{D+1}, ch_{L+1}.(\theta \circ \left\{{}^{ch_{L+1}}/_{u_{L+1}}\right\} \mid \mathcal{E}^{D+1}(ch_{L+1}) \mid !\nu ch.\overline{card}\langle ch\rangle.C_{\text{fix}}(s, c_{D+1}, ch) \mid !I)}\,\text{Rep-act} \quad \begin{array}{l} c_{D+1}, ch_{L+1}, \\ u_{L+1} \mathrel{\#} C_j^i, \\ i \le D, j \le \max_{i \le D} L_i; \\ \nu ch.\overline{card}\langle ch\rangle. \\ C_{\text{fix}}(s, c_d, ch) \end{array}}{\theta \mid \cdots \mid !I \xrightarrow{\overline{card}(u_{L+1})} \nu c_{D+1}, ch_{L+1}.(\theta \circ \left\{{}^{ch_{L+1}}/_{u_{L+1}}\right\} \mid \cdots \mid \mathcal{E}^{D+1}(ch_{L+1}) \mid !\nu ch.\overline{card}\langle ch\rangle.C_{\text{fix}}(s, c_{D+1}, ch) \mid !I)}\,\text{Par}^{D+L} \quad \begin{array}{l} s, c_i, ch_j, a_k, \\ i \le D, j \le L, \\ k \in \beta \cup \gamma \cup \delta \mathrel{\#} \\ card, u_{L+1} \end{array}}{FIX^{\Psi,\Omega}_{\text{impl}}(\vec{Y}) \xrightarrow{\overline{card}(u_{L+1})} \nu s, c_1, \cdots, c_D, c_{D+1}, ch_1, \cdots, ch_L, ch_{L+1}, a_{l_1}, \cdots, a_{l_K}.(\theta \circ \left\{{}^{ch_{L+1}}/_{u_{L+1}}\right\} \mid \cdots \mid \mathcal{E}^{D+1}(ch_{L+1}) \mid !\nu ch.\overline{card}\langle ch\rangle.C_{\text{fix}}(s, c_{D+1}, ch) \mid !I)}\,\text{Res}^{1+D+L+K}
$$

Case 2. Transition $FIX^{\Psi,\Omega}_{\text{impl}}(\vec{Y}) \xrightarrow{\overline{card}(u_{L+1})} FIX^{\{\alpha \cup \{L+1\}, \beta, \gamma, \delta\}, \Omega \cup \{\{L+1\}\}}_{\text{impl}}((Y_1, \cdots, Y_L, \emptyset))$: a new card is created.

$$
\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{l} v_l \mathrel{\#} u_l, a_l, \mathcal{F}^l(ch_l, a_l), \sigma \\ u_l\sigma =_E ch_l \end{array}}{\sigma \mid \overline{ch_l}\langle \phi(a_l, \phi(c_l, \mathrm{g}))\rangle.\mathcal{F}^l(ch_l, a_l) \xrightarrow{\overline{u_l}(v_l)} \sigma \circ \left\{{}^{\phi(a_l, \phi(c_l, \mathrm{g}))}/_{v_l}\right\} \mid \mathcal{F}^l(ch_l, a_l)}\,\text{Out} \quad a_l \mathrel{\#} u_l, v_l, \sigma}{\sigma \mid \nu a.\overline{ch_l}\langle \phi(a, \phi(c_l, \mathrm{g}))\rangle.\mathcal{F}^l(ch_l, a) \xrightarrow{\overline{u_l}(v_l)} \nu a_l.(\sigma \circ \left\{{}^{\phi(a_l, \phi(c_l, \mathrm{g}))}/_{v_l}\right\} \mid \mathcal{F}^l(ch_l, a_l))}\,\text{Extrusion} \quad a_l, v_l \mathrel{\#} C_i, i \le L, i \ne l; !S}{\sigma \mid C_1 \mid \cdots \mid \mathcal{E}^l(ch_l) \mid \cdots \mid C_L \mid !S \xrightarrow{\overline{u_l}(v_l)} \nu a_l.(\sigma \circ \left\{{}^{\phi(a_l, \phi(c_l, \mathrm{g}))}/_{v_l}\right\} \mid \cdots \mid C_K \mid \cdots \mid \mathcal{F}^l(ch_l, a_l) \mid \cdots \mid !S)}\,\text{Par}^L \quad \begin{array}{l} s, c_i, ch_i, a_k \\ i \le L, k \in \beta \cup \gamma \cup \delta \mathrel{\#} u_l, v_l \end{array}}{FIX^{\Psi}_{\text{spec}}(\vec{Y}) \xrightarrow{\overline{u_l}(v_l)} \nu s, c_1, \cdots, c_L, ch_1, \cdots, ch_L, a_{l_1}, \cdots, a_{l_K}, a_l.(\sigma \circ \left\{{}^{\phi(a_l, \phi(c_l, \mathrm{g}))}/_{v_l}\right\} \mid \cdots \mid C_K \mid \cdots \mid \mathcal{F}^l(ch_l, a_l) \mid \cdots \mid !S)}\,\text{Res}^{1+2L+K}
$$

Case 3. Transition $FIX^{\Psi}_{\text{spec}}(\vec{Y}) \xrightarrow{\overline{u_l}(v_l)} FIX^{\{\alpha \setminus \{l\}, \beta \cup \{l\}, \gamma, \delta\}}_{\text{spec}}(\vec{Y})$, $l \in \alpha$.

$$
\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{l} v_l \mathrel{\#} u_l, a_l, \mathcal{F}^d(ch_l, a_l), \theta \\ u_l\theta =_E ch_l \end{array}}{\theta \mid \overline{ch_l}\langle \phi(a_l, \phi(c_d, \mathrm{g}))\rangle.\mathcal{F}^d(ch_l, a_l) \xrightarrow{\overline{u_l}(v_l)} \theta \circ \left\{{}^{\phi(a_l, \phi(c_d, \mathrm{g}))}/_{v_l}\right\} \mid \mathcal{F}^d(ch_l, a_l)}\,\text{Out} \quad a_l \mathrel{\#} u_l, v_l, \theta}{\theta \mid \nu a.\overline{ch_l}\langle \phi(a, \phi(c_d, \mathrm{g}))\rangle.\mathcal{F}^d(ch_l, a) \xrightarrow{\overline{u_l}(v_l)} \nu a_l.(\theta \circ \left\{{}^{\phi(a_l, \phi(c_d, \mathrm{g}))}/_{v_l}\right\} \mid \mathcal{F}^d(ch_l, a_l))}\,\text{Extrusion} \quad \begin{array}{l} a_l, v_l \mathrel{\#} C_j^i, \\ i \le D, j \le \max_{i \le D} L_i, \\ j \ne l; !I \end{array}}{\theta \mid \cdots \mid \mathcal{E}^d(ch_l) \mid \cdots \mid !I \xrightarrow{\overline{u_l}(v_l)} \nu a_l.(\theta \circ \left\{{}^{\phi(a_l, \phi(c_d, \mathrm{g}))}/_{v_l}\right\} \mid \cdots \mid \mathcal{F}^d(ch_l, a_l) \mid \cdots \mid !I)}\,\text{Par}^{D+L} \quad \begin{array}{l} s, c_i, ch_j, a_k \\ i \le D, j \le L, k \in \beta \cup \gamma \cup \delta \mathrel{\#} \\ u_l, v_l \end{array}}{FIX^{\Psi,\Omega}_{\text{impl}}(\vec{Y}) \xrightarrow{\overline{u_l}(v_l)} \nu s, c_1, \cdots, c_D, ch_1, \cdots, ch_L, a_{l_1}, \cdots, a_{l_k}, a_l.(\theta \circ \left\{{}^{\phi(a_l, \phi(c_d, \mathrm{g}))}/_{v_l}\right\} \mid \cdots \mid \mathcal{F}^d(ch_l, a_l) \mid \cdots \mid !I)}\,\text{Res}^{1+D+L+K}
$$

Case 3. Transition $FIX^{\Psi,\Omega}_{\text{impl}}(\vec{Y}) \xrightarrow{\overline{u_l}(v_l)} FIX^{\alpha \setminus \{l\}, \beta \cup \{l\}, \gamma, \delta\}, \Omega}_{\text{impl}}(\vec{Y})$, $l \in \alpha$.

$$\dfrac{\dfrac{\dfrac{u_l\sigma =_E ch_l}{\sigma \mid ch_l(y).\mathcal{G}^l(ch_l,a_l,y) \xrightarrow{u_l\,Y_l} \sigma \mid \mathcal{G}^l(ch_l,a_l,Y_l\sigma)}\,\textsf{Inp}}{\sigma \mid C_1 \mid \cdots \mid \mathcal{F}^l(ch_l,a_l) \mid \cdots \mid C_L \mid !S \xrightarrow{u_l\,Y_l} \sigma \mid \cdots \mid \mathcal{G}^l(ch_l,a_l,Y_l\sigma) \mid \cdots \mid !S}\,\textsf{Par}^L \quad \begin{array}{l} s,c_i,ch_i,a_k\\ i\le L,k\in\beta\cup\gamma\cup\delta \# u_l,Y_l\end{array}}{FIX^{\Psi}_{\text{spec}}(\vec{Y}) \xrightarrow{u_l\,Y_l} \nu s,c_1,\cdots,c_L,ch_1,\cdots,ch_L,a_{l_1},\cdots,a_{l_k}.\{\sigma \mid \cdots \mid \mathcal{G}^l(ch_l,a_l,Y_l\sigma) \mid \cdots \mid !S\}}\,\textsf{Res}^{1+2L+K}$$

Case 4. Transition $FIX^{\Psi}_{\text{spec}}(\vec{Y}) \xrightarrow{u_l\,Y_l} FIX^{\{\alpha,\beta\setminus\{l\},\gamma\cup\{l\},\delta\}}_{\text{spec}}(\chi_l(\vec{Y},Y_l))$, $l\in\beta$.

$$\dfrac{\dfrac{\dfrac{u_l\theta =_E ch_l}{\theta \mid ch_l(y).\mathcal{G}^d(ch_l,a_l,y) \xrightarrow{u_l\,Y_l} \theta \mid \mathcal{G}^d(ch_l,a_l,Y_l\sigma)}\,\textsf{Inp}}{\theta \mid \cdots \mid \mathcal{F}^d(ch_l,a_l) \mid \cdots \mid !I \xrightarrow{u_l\,Y_l} \theta \mid \cdots \mid \mathcal{G}^d(ch_l,a_l,Y_l\sigma) \mid \cdots \mid !I}\,\textsf{Par}^{D+L} \quad \begin{array}{l} s,c_i,ch_j,a_k\\ i\le D,j\le L,k\in\beta\cup\gamma\cup\delta \#\\ u_l,Y_l\end{array}}{FIX^{\Psi,\Omega}_{\text{impl}}(\vec{Y}) \xrightarrow{u_l\,Y_l} \nu s,c_1,\cdots,c_D,ch_1,\cdots,ch_L,a_{l_1},\cdots,a_{l_K}.\{\theta \mid \cdots \mid \mathcal{G}^d(ch_l,a_l,Y_l\sigma) \mid \cdots \mid !I\}}\,\textsf{Res}^{1+D+L+K}$$

Case 4. Transition $FIX^{\Psi,\Omega}_{\text{impl}}(\vec{Y}) \xrightarrow{u_l\,Y_l} FIX^{\{\alpha,\beta\setminus\{l\},\gamma\cup\{l\},\delta\},\Omega}_{\text{impl}}(\chi_l(\vec{Y},Y_l))$ if there is a card at the stage $\mathcal{F}$.

$$\dfrac{\dfrac{\dfrac{\begin{array}{l} w_l \# u_l, m^l(a_l,Y_l\sigma'),\sigma\\ u_l\theta =_E ch_l\end{array}}{\sigma \mid \overline{ch_l}\langle m^l(a_l,Y_l\sigma')\rangle \xrightarrow{\overline{u_l}(w_l)} \sigma\circ\left\{{}^{m^l(a_l,Y_l\sigma')}/_{w_l}\right\} \mid \mathcal{H}^l}\,\textsf{Out}}{\sigma \mid C_1 \mid \cdots \mid \mathcal{G}^l(ch_l,a_l,Y_l\sigma') \mid \cdots \mid C_L \mid !S \xrightarrow{\overline{u_l}(w_l)} \sigma\circ\left\{{}^{m^l(a_l,Y_l\sigma')}/_{w_l}\right\} \mid \cdots \mid \mathcal{H}^l \mid \cdots \mid !S}\,\textsf{Par}^L \quad \begin{array}{l} s,c_i,ch_i,a_k\\ i\le L,k\in\beta\cup\gamma\cup\delta \# u_l,w_l\end{array}}{FIX^{\Psi}_{\text{spec}}(\vec{Y}) \xrightarrow{\overline{u_l}(w_l)} \nu s,c_1,\cdots,c_L,ch_1,\cdots,ch_L,a_{l_1},\cdots,a_{l_K}.\{\sigma\circ\left\{{}^{m^l(a_l,Y_l\sigma')}/_{w_l}\right\} \mid \cdots \mid \mathcal{H}^l \mid \cdots \mid !S\}}\,\textsf{Res}^{1+2L+K}$$

Case 5. Transition $FIX^{\Psi}_{\text{spec}}(\vec{Y}) \xrightarrow{\overline{u_l}(w_l)} FIX^{\{\alpha,\beta,\gamma\setminus\{l\},\delta\cup\{l\}\}}_{\text{spec}}(\vec{Y})$, $l\in\gamma$.

$$\dfrac{\dfrac{\dfrac{\begin{array}{l} w_l \# u_l, m^d(a_l,Y_l\theta'),\theta\\ u_l\theta =_E ch_l\end{array}}{\theta \mid \overline{ch_l}\langle m^d(a_l,Y_l\theta')\rangle \xrightarrow{\overline{u_l}(w_l)} \theta\circ\left\{{}^{m^d(a_l,Y_l\theta')}/_{w_l}\right\} \mid \mathcal{H}^d}\,\textsf{Out}}{\theta \mid \cdots \mid \mathcal{G}^d(ch_l,a_l,Y_l\theta') \mid \cdots \mid !I \xrightarrow{\overline{u_l}(w_l)} \theta\circ\left\{{}^{m^d(a_l,Y_l\theta')}/_{w_l}\right\} \mid \cdots \mid \mathcal{H}^d \mid \cdots \mid !I}\,\textsf{Par}^{D+L} \quad \begin{array}{l} s,c_i,ch_j,a_k\\ i <\le D,j\le L,k\in\beta\cup\gamma\cup\delta \#\\ u_l,w_l\end{array}}{FIX^{\Psi,\Omega}_{\text{impl}}(\vec{Y}) \xrightarrow{\overline{u_l}(w_l)} \nu s,c_1,\cdots,c_D,ch_1,\cdots,ch_L,a_{l_1},\cdots,a_{l_K}.\{\theta\circ\left\{{}^{m^d(a_l,Y_l\theta')}/_{w_l}\right\} \mid \cdots \mid \mathcal{H}^d \mid \cdots \mid !I\}}\,\textsf{Res}^{1+D+L+K}$$

Case 5. Transition $FIX^{\Psi,\Omega}_{\text{impl}}(\vec{Y}) \xrightarrow{\overline{u_l}(w_l)} FIX^{\{\alpha,\beta,\gamma\setminus\{l\},\delta\cup\{l\}\},\Omega}_{\text{impl}}(\vec{Y})$, $l\in\gamma$.