

```

In[41]:= (**)
Psi[x_] = e^(-i * x) * {e^(i * k * x), e^(-i * k * x)}
Phi[x_] = e^(i * x) * {e^(i * k * x), e^(-i * k * x)}

PPsi[x_] = k * e^(-i * x) * {e^(i * k * x), -e^(-i * k * x)}
PPhi[x_] = k * e^(i * x) * {e^(i * k * x), -e^(-i * k * x)}

(*Решения на верхней и нижней дугах с к-нтами A, B *)
psi[x_, A_, B_] = Psi[x].{A, B}
phi[x_, A_, B_] = Phi[x].{A, B}

(*Действие оператора импульса на решения*)
pPsi[x_, A_, B_] = PPsi[x].{A, B}
pPhi[x_, A_, B_] = PPhi[x].{A, B}

(*Условие дельта-соединения в применении к одному из колец цепочки*)
psi[0, Cp, Cm] == phi[0, Dp, Dm]
psi[pi, Cp, Cm] == phi[pi, Dp, Dm]

Solve[psi[0, Cp, Cm] == phi[0, Dp, Dm] &&
      psi[pi, Cp, Cm] == phi[pi, Dp, Dm], {Dp, Dm}]

S0 = e^(-i * x) (e^(i * k * pi) - e^(-i * k * pi))^-1 *
      {{e^(i * k * pi) e^(-i * x) - e^(-i * k * pi) e^(i * x),
        e^(-i * k * pi) e^(-i * x) - e^(-i * k * pi) e^(i * x)},
       {e^(i * k * pi) e^(-i * x) - e^(i * k * pi) e^(i * x),
        e^(-i * k * pi) e^(-i * x) - e^(i * k * pi) e^(i * x)}};

Out[41]= {e^(i k x - i x), e^(-i k x - i x)}
Out[42]= {e^(i k x + i x), e^(-i k x + i x)}
Out[43]= {e^(i k x - i x) k, -e^(-i k x - i x) k}
Out[44]= {e^(i k x + i x) k, -e^(-i k x + i x) k}
Out[45]= B e^(-i k x - i x) + A e^(i k x - i x)
Out[46]= B e^(-i k x + i x) + A e^(i k x + i x)
Out[47]= -B e^(-i k x - i x) k + A e^(i k x - i x) k
Out[48]= -B e^(-i k x + i x) k + A e^(i k x + i x) k
Out[49]= Cm + Cp == Dm + Dp
Out[50]= Cm e^(-i k pi - i pi) + Cp e^(i k pi - i pi) == Dm e^(-i k pi + i pi) + Dp e^(i k pi + i pi)
Out[51]= {{Dp -> -((e^(-2 i pi) (-Cm - Cp e^(2 i k pi) + Cm e^(2 i pi) + Cp e^(2 i pi))) / (-1 + e^(2 i k pi))),
           Dm -> (e^(-2 i pi) (-Cm - Cp e^(2 i k pi) + Cm e^(2 i k pi + 2 i pi) + Cp e^(2 i k pi + 2 i pi))) / (-1 + e^(2 i k pi))}}

(*Запишем матрицу S исходя из результата решения ур-я*)

```

```
In[53]:= S = e-2 i π Φ * (-1 + e2 i k π)-1 *  
      {{e2 i π Φ - e2 i k π, e2 i π Φ - 1}, {e2 i k π + 2 i π Φ - e2 i k π, e2 i k π + 2 i π Φ - 1}}
```

```
Out[53]= {{ $\frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$ ,  $\frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$ },  
      {{ $\frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$ ,  $\frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$ }}}
```

(\* Сверим полученную матрицу S с исходной, выведенной мной\*)

```
In[77]:=
```

(\*Точка контакта двух колец\*)

```
PPsi[0].{Cp1, Cm1} + PPhi[0].S.{Cp1, Cm1}
```

```
PPsi[π].{Cp0, Cm0} + PPhi[π].S.{Cp0, Cm0} - i * α * Psi[π].{Cp0, Cm0}
```

```
PPsi[0].{Cp1, Cm1} + PPhi[0].S.{Cp1, Cm1} ==
```

```
PPsi[π].{Cp0, Cm0} + PPhi[π].S.{Cp0, Cm0} - i * α * Psi[π].{Cp0, Cm0}
```

```
Psi[π].{Cp0, Cm0} == Psi[0].{Cp1, Cm1}
```

```
TracePrint[Solve[Psi[π].{Cp0, Cm0} == Psi[0].{Cp1, Cm1} &&
```

```
PPsi[0].{Cp1, Cm1} + PPhi[0].S.{Cp1, Cm1} == PPsi[π].{Cp0, Cm0} +
```

```
PPhi[π].S.{Cp0, Cm0} - i * α * Psi[π].{Cp0, Cm0}, {Cp1, Cm1}]]
```

```
Out[77]= -Cm1 k + Cp1 k + Cm1  $\left( \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right) +$   
Cp1  $\left( \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right)$ 
```

```
Out[78]= -Cm0 e-i k π - i π Φ k + Cp0 ei k π - i π Φ k +  
Cm0  $\left( \frac{e^{i k \pi - i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-i k \pi - i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right) +$   
Cp0  $\left( \frac{e^{i k \pi - i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \left( e^{-i k \pi - i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k \right) / (-1 + e^{2 i k \pi}) \right) -$   
(Cm0 e-i k π - i π Φ + Cp0 ei k π - i π Φ) i α
```

$$\begin{aligned}
\text{Out[79]} = & -\text{Cm1 } k + \text{Cp1 } k + \text{Cm1} \left( \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right) + \\
& \text{Cp1} \left( \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right) = -\text{Cm0 } e^{-i k \pi - i \pi \Phi} k + \\
& \text{Cp0 } e^{i k \pi - i \pi \Phi} k + \text{Cm0} \left( \frac{e^{i k \pi - i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-i k \pi - i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right) + \\
& \text{Cp0} \left( \frac{e^{i k \pi - i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - (e^{-i k \pi - i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k) / (-1 + e^{2 i k \pi}) \right) - \\
& (\text{Cm0 } e^{-i k \pi - i \pi \Phi} + \text{Cp0 } e^{i k \pi - i \pi \Phi}) i \alpha
\end{aligned}$$

$$\text{Out[80]} = \text{Cm0 } e^{-i k \pi - i \pi \Phi} + \text{Cp0 } e^{i k \pi - i \pi \Phi} = \text{Cm1} + \text{Cp1}$$

```
Solve[Psi[π].{Cp0, Cm0} == Psi[0].{Cp1, Cm1} && PPsi[0].{Cp1, Cm1} + PPhi[0].S.{Cp1, Cm1} ==
  PPsi[π].{Cp0, Cm0} + PPhi[π].S.{Cp0, Cm0} - i α Psi[π].{Cp0, Cm0}, {Cp1, Cm1}]
```

```
Solve
```

```
Psi[π].{Cp0, Cm0} == Psi[0].{Cp1, Cm1} && PPsi[0].{Cp1, Cm1} + PPhi[0].S.{Cp1, Cm1} ==
  PPsi[π].{Cp0, Cm0} + PPhi[π].S.{Cp0, Cm0} - i α Psi[π].{Cp0, Cm0}
```

```
And
```

```
Psi[π].{Cp0, Cm0} == Psi[0].{Cp1, Cm1}
```

```
Equal
```

```
Psi[π].{Cp0, Cm0}
```

```
Dot
```

```
Psi[π]
```

```
Psi
```

```
π
```

```
{e^{i k π - i π Φ}, e^{-i k π - i π Φ}}
```

```
List
```

```
e^{i k π - i π Φ}
```

```
Power
```

```
e
```

```
i k π - i π Φ
```

```
Plus
```

```
i k π
```

```
Times
```

```
i
```

```
k
```

```
π
```

```
- i π Φ
```

```
Times
```

```
-1
```

```
i
```

```

π
Φ
e-i k π - i π Φ
Power
e
-i k π - i π Φ
Plus
-i k π
Times
-1
i
k
π
-i π Φ
Times
-1
i
π
Φ
{Cp0, Cm0}
List
Cp0
Cm0
{ei k π - i π Φ, e-i k π - i π Φ}. {Cp0, Cm0}
Cm0 e-i k π - i π Φ + Cp0 ei k π - i π Φ
Psi[0]. {Cp1, Cm1}
Dot
Psi[0]
Psi
0
{ei k 0 - i 0 Φ, e-i k 0 - i 0 Φ}
List
ei k 0 - i 0 Φ
Power
e
i k 0 - i 0 Φ
Plus
i k 0
Times

```

```

i
k
0
0 i k
0
-i 0  $\Phi$ 
Times
-1
i
0
 $\Phi$ 
0 i  $\Phi$ 
Times
0
i
 $\Phi$ 
0
0 + 0
0
 $e^0$ 
1
 $e^{-i k 0 - i 0 \Phi}$ 
Power
e
-i k 0 - i 0  $\Phi$ 
Plus
-i k 0
Times
-1
i
k
0
0 i k
Times
0
i
k
0
-i 0  $\Phi$ 

```

```

Times
-1
i
0
 $\Phi$ 
0 i  $\Phi$ 
Times
0
i
 $\Phi$ 
0
0 + 0
0
 $e^0$ 
1
{1, 1}
{Cp1, Cm1}
List
Cp1
Cm1
{1, 1}.{Cp1, Cm1}
Cm1 + Cp1
Cm0  $e^{-i k \pi - i \pi \Phi} + Cp0 e^{i k \pi - i \pi \Phi} == Cm1 + Cp1$ 
PPsi[0].{Cp1, Cm1} + PPhi[0].S.{Cp1, Cm1} ==
PPsi[ $\pi$ ].{Cp0, Cm0} + PPhi[ $\pi$ ].S.{Cp0, Cm0} - i  $\alpha$  Psi[ $\pi$ ].{Cp0, Cm0}
Equal
PPsi[0].{Cp1, Cm1} + PPhi[0].S.{Cp1, Cm1}
Plus
PPsi[0].{Cp1, Cm1}
Dot
PPsi[0]
PPsi
0
{ $e^{i k 0 - i 0 \Phi} k, -e^{-i k 0 - i 0 \Phi} k$ }
List
 $e^{i k 0 - i 0 \Phi} k$ 
Times
 $e^{i k 0 - i 0 \Phi}$ 
Power

```

```

e
i k 0 - i 0 \Phi
Plus
i k 0
Times
i
k
0
0 i k
0
-i 0 \Phi
Times
-1
i
0
\Phi
0 i \Phi
Times
0
i
\Phi
0
0 + 0
0
e^0
1
k
1 k
k
-e^{-i k 0 - i 0 \Phi} k
Times
-1
e^{-i k 0 - i 0 \Phi}
Power
e
-i k 0 - i 0 \Phi
Plus
-i k 0
Times

```

```

-1
i
k
0
0 i k
Times
0
i
k
0
-i 0  $\Phi$ 
Times
-1
i
0
 $\Phi$ 
0 i  $\Phi$ 
Times
0
i
 $\Phi$ 
0
0 + 0
0
 $e^0$ 
1
k
-k
-k
Times
-1
k
{k, -k}
{Cp1, Cm1}
List
Cp1
Cm1
{k, -k}. {Cp1, Cm1}
-Cm1 k + Cp1 k

```



```

PPhi[0].S.{Cp1, Cm1}

Dot
PPhi[0]
PPhi
0
{e^{i k 0+i 0 \Phi} k, -e^{-i k 0+i 0 \Phi} k}
List
e^{i k 0+i 0 \Phi} k
Times
e^{i k 0+i 0 \Phi}
Power
e
i k 0+i 0 \Phi
Plus
i k 0
Times
i
k
0
0 i k
0
i 0 \Phi
Times
i
0
\Phi
0 i \Phi
0
0 + 0
0
e^0
1
k
1 k
k
-e^{-i k 0+i 0 \Phi} k
Times
-1
e^{-i k 0+i 0 \Phi}

```

```

Power
e
-i k 0 + i 0  $\Phi$ 
Plus
-i k 0
Times
-1
i
k
0
0 i k
Times
0
i
k
0
i 0  $\Phi$ 
Times
i
0
 $\Phi$ 
0 i  $\Phi$ 
0
0 + 0
0
e0
1
k
-k
-k
Times
-1
k
{k, -k}
S
{ {  $\frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$ ,  $\frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$  },
{  $\frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$ ,  $\frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$  } }

```

```

{Cp1, Cm1}

List

Cp1

Cm1

{k, -k}.{ {  $\frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$ ,  $\frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$  },
 $\left\{ \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$ ,  $\frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi})}{-1 + e^{2 i k \pi}}$  } } . {Cp1, Cm1}

 $\left\{ \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}}$ ,
 $\frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right\} . \{Cp1, Cm1\}$ 

Dot

 $\left\{ \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}}$ ,
 $\frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right\}$ 

List

 $\frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}}$ 
 $\frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}}$ 

{Cp1, Cm1}

Cm1  $\left( \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right) +$ 
Cp1  $\left( \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right)$ 

(-Cm1 k + Cp1 k) +  $\left( Cm1 \left( \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right) + \right.$ 
 $\left. Cp1 \left( \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right) \right)$ 

-Cm1 k + Cp1 k + Cm1  $\left( \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right) +$ 
Cp1  $\left( \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} - \frac{e^{-2 i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}} \right)$ 

PPsi[ $\pi$ ].{Cp0, Cm0} + PPhi[ $\pi$ ].S.{Cp0, Cm0} - i  $\alpha$  Psi[ $\pi$ ].{Cp0, Cm0}

Plus

PPsi[ $\pi$ ].{Cp0, Cm0}

Dot

PPsi[ $\pi$ ]

```

PPsi

$\pi$

$\{e^{i k \pi - i \pi \Phi} k, -e^{-i k \pi - i \pi \Phi} k\}$

List

$e^{i k \pi - i \pi \Phi} k$

Times

$e^{i k \pi - i \pi \Phi}$

Power

e

$i k \pi - i \pi \Phi$

Plus

$i k \pi$

Times

i

k

$\pi$

$-i \pi \Phi$

Times

-1

i

$\pi$

$\Phi$

k

$-e^{-i k \pi - i \pi \Phi} k$

Times

-1

$e^{-i k \pi - i \pi \Phi}$

Power

e

$-i k \pi - i \pi \Phi$

Plus

$-i k \pi$

Times

-1

i

k

$\pi$

$-i \pi \Phi$

Times

```

-1
i
π
Φ
k
{Cp0, Cm0}
List
Cp0
Cm0
{ei k π - i π Φ k, -e-i k π - i π Φ k}. {Cp0, Cm0}
-Cm0 e-i k π - i π Φ k + Cp0 ei k π - i π Φ k
PPhi[π].S.{Cp0, Cm0}
Dot
PPhi[π]
PPhi
π
{ei k π + i π Φ k, -e-i k π + i π Φ k}
List
ei k π + i π Φ k
Times
ei k π + i π Φ
Power
e
i k π + i π Φ
Plus
i k π
Times
i
k
π
i π Φ
Times
i
π
Φ
k
-e-i k π + i π Φ k
Times
-1

```

```

e-i k π+i π Φ

Power

e

-i k π+i π Φ

Plus

-i k π

Times

-1

i

k

π

i π Φ

Times

i

π

Φ

k

S

{ {  $\frac{e^{-2 i \pi \Phi} \left(-e^{2 i k \pi}+e^{2 i \pi \Phi}\right)}{-1+e^{2 i k \pi}}, \frac{e^{-2 i \pi \Phi} \left(-1+e^{2 i \pi \Phi}\right)}{-1+e^{2 i k \pi}}\right\},$ 
 $\left\{\frac{e^{-2 i \pi \Phi} \left(-e^{2 i k \pi}+e^{2 i k \pi+2 i \pi \Phi}\right)}{-1+e^{2 i k \pi}}, \frac{e^{-2 i \pi \Phi} \left(-1+e^{2 i k \pi+2 i \pi \Phi}\right)}{-1+e^{2 i k \pi}}\right\}\right\}$ 

{Cp0, Cm0}

List

Cp0

Cm0

{ei k π+i π Φ k, -e-i k π+i π Φ k}. { {  $\frac{e^{-2 i \pi \Phi} \left(-e^{2 i k \pi}+e^{2 i \pi \Phi}\right)}{-1+e^{2 i k \pi}}, \frac{e^{-2 i \pi \Phi} \left(-1+e^{2 i \pi \Phi}\right)}{-1+e^{2 i k \pi}}\right\},$ 
 $\left\{\frac{e^{-2 i \pi \Phi} \left(-e^{2 i k \pi}+e^{2 i k \pi+2 i \pi \Phi}\right)}{-1+e^{2 i k \pi}}, \frac{e^{-2 i \pi \Phi} \left(-1+e^{2 i k \pi+2 i \pi \Phi}\right)}{-1+e^{2 i k \pi}}\right\}\right\}. \{Cp0, Cm0\}$ 
 $\left\{\frac{e^{i k \pi-i \pi \Phi} \left(-e^{2 i k \pi}+e^{2 i \pi \Phi}\right) k}{-1+e^{2 i k \pi}}-\frac{e^{-i k \pi-i \pi \Phi} \left(-e^{2 i k \pi}+e^{2 i k \pi+2 i \pi \Phi}\right) k}{-1+e^{2 i k \pi}},$ 
 $\frac{e^{i k \pi-i \pi \Phi} \left(-1+e^{2 i \pi \Phi}\right) k}{-1+e^{2 i k \pi}}-\frac{e^{-i k \pi-i \pi \Phi} \left(-1+e^{2 i k \pi+2 i \pi \Phi}\right) k}{-1+e^{2 i k \pi}}\right\}. \{Cp0, Cm0\}$ 

Dot

{  $\frac{e^{i k \pi-i \pi \Phi} \left(-e^{2 i k \pi}+e^{2 i \pi \Phi}\right) k}{-1+e^{2 i k \pi}}-\frac{e^{-i k \pi-i \pi \Phi} \left(-e^{2 i k \pi}+e^{2 i k \pi+2 i \pi \Phi}\right) k}{-1+e^{2 i k \pi}},$ 
 $\frac{e^{i k \pi-i \pi \Phi} \left(-1+e^{2 i \pi \Phi}\right) k}{-1+e^{2 i k \pi}}-\frac{e^{-i k \pi-i \pi \Phi} \left(-1+e^{2 i k \pi+2 i \pi \Phi}\right) k}{-1+e^{2 i k \pi}}\right\}$ 

List

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$$\frac{e^{i k \pi - i \pi \Phi} \left( -e^{2 i k \pi} + e^{2 i \pi \Phi} \right) k}{-1 + e^{2 i k \pi}} - \frac{e^{-i k \pi - i \pi \Phi} \left( -e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi} \right) k}{-1 + e^{2 i k \pi}}$$

$$\frac{e^{i k \pi - i \pi \Phi} \left( -1 + e^{2 i \pi \Phi} \right) k}{-1 + e^{2 i k \pi}} - \frac{e^{-i k \pi - i \pi \Phi} \left( -1 + e^{2 i k \pi + 2 i \pi \Phi} \right) k}{-1 + e^{2 i k \pi}}$$

{Cp0, Cm0}

$$\text{Cm0} \left( \frac{e^{i k \pi - i \pi \Phi} \left( -1 + e^{2 i \pi \Phi} \right) k}{-1 + e^{2 i k \pi}} - \frac{e^{-i k \pi - i \pi \Phi} \left( -1 + e^{2 i k \pi + 2 i \pi \Phi} \right) k}{-1 + e^{2 i k \pi}} \right) +$$

$$\text{Cp0} \left( \frac{e^{i k \pi - i \pi \Phi} \left( -e^{2 i k \pi} + e^{2 i \pi \Phi} \right) k}{-1 + e^{2 i k \pi}} - \frac{e^{-i k \pi - i \pi \Phi} \left( -e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi} \right) k}{-1 + e^{2 i k \pi}} \right)$$

-(i α Psi[π].{Cp0, Cm0})

Times

-1

i α Psi[π].{Cp0, Cm0}

Times

i

α

Psi[π].{Cp0, Cm0}

Dot

Psi[π]

Psi

π

{e^{i k π - i π Φ}, e^{-i k π - i π Φ}}

List

e^{i k π - i π Φ}

Power

e

i k π - i π Φ

Plus

i k π

Times

i

k

π

-i π Φ

Times

-1

i

π

Φ

```

e-i k π - i π Φ

Power

e

-i k π - i π Φ

Plus

-i k π

Times

-1

i

k

π

-i π Φ

Times

-1

i

π

Φ

{Cp0, Cm0}

List

Cp0

Cm0

{ei k π - i π Φ, e-i k π - i π Φ}. {Cp0, Cm0}

Cm0 e-i k π - i π Φ + Cp0 ei k π - i π Φ

i α (Cm0 e-i k π - i π Φ + Cp0 ei k π - i π Φ)

(Cm0 e-i k π - i π Φ + Cp0 ei k π - i π Φ) i α

- ((Cm0 e-i k π - i π Φ + Cp0 ei k π - i π Φ) i α)

- (Cm0 e-i k π - i π Φ + Cp0 ei k π - i π Φ) i α

(-Cm0 e-i k π - i π Φ k + Cp0 ei k π - i π Φ k) +

(Cm0 (  $\frac{e^{i k \pi - i \pi \Phi} (-1 + e^{2 i k \pi}) k}{-1 + e^{2 i k \pi}}$  -  $\frac{e^{-i k \pi - i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}}$  ) +

Cp0 (  $\frac{e^{i k \pi - i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}}$  - (e-i k π - i π Φ (-e2 i k π + e2 i k π + 2 i π Φ) k) / (-1 + e2 i k π))) -

(Cm0 e-i k π - i π Φ + Cp0 ei k π - i π Φ) i α

-Cm0 e-i k π - i π Φ k + Cp0 ei k π - i π Φ k +

Cm0 (  $\frac{e^{i k \pi - i \pi \Phi} (-1 + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}}$  -  $\frac{e^{-i k \pi - i \pi \Phi} (-1 + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}}$  ) +

Cp0 (  $\frac{e^{i k \pi - i \pi \Phi} (-e^{2 i k \pi} + e^{2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}}$  -  $\frac{e^{-i k \pi - i \pi \Phi} (-e^{2 i k \pi} + e^{2 i k \pi + 2 i \pi \Phi}) k}{-1 + e^{2 i k \pi}}$  ) -

(Cm0 e-i k π - i π Φ + Cp0 ei k π - i π Φ) i α

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$$\begin{aligned}
& -\text{Cm1 } k + \text{Cp1 } k + \text{Cm1} \left( \frac{e^{-2i\pi\Phi} (-1 + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-2i\pi\Phi} (-1 + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) + \\
& \text{Cp1} \left( \frac{e^{-2i\pi\Phi} (-e^{2ik\pi} + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-2i\pi\Phi} (-e^{2ik\pi} + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) == -\text{Cm0 } e^{-ik\pi-i\pi\Phi} k + \\
& \text{Cp0 } e^{ik\pi-i\pi\Phi} k + \text{Cm0} \left( \frac{e^{ik\pi-i\pi\Phi} (-1 + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-ik\pi-i\pi\Phi} (-1 + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) + \\
& \text{Cp0} \left( \frac{e^{ik\pi-i\pi\Phi} (-e^{2ik\pi} + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-ik\pi-i\pi\Phi} (-e^{2ik\pi} + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) - \\
& (\text{Cm0 } e^{-ik\pi-i\pi\Phi} + \text{Cp0 } e^{ik\pi-i\pi\Phi}) i \alpha
\end{aligned}$$

$$\text{Cm0 } e^{-ik\pi-i\pi\Phi} + \text{Cp0 } e^{ik\pi-i\pi\Phi} == \text{Cm1} + \text{Cp1} \&\&$$

$$\begin{aligned}
& -\text{Cm1 } k + \text{Cp1 } k + \text{Cm1} \left( \frac{e^{-2i\pi\Phi} (-1 + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-2i\pi\Phi} (-1 + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) + \\
& \text{Cp1} \left( \frac{e^{-2i\pi\Phi} (-e^{2ik\pi} + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-2i\pi\Phi} (-e^{2ik\pi} + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) == -\text{Cm0 } e^{-ik\pi-i\pi\Phi} k + \\
& \text{Cp0 } e^{ik\pi-i\pi\Phi} k + \text{Cm0} \left( \frac{e^{ik\pi-i\pi\Phi} (-1 + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-ik\pi-i\pi\Phi} (-1 + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) + \\
& \text{Cp0} \left( \frac{e^{ik\pi-i\pi\Phi} (-e^{2ik\pi} + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \left( e^{-ik\pi-i\pi\Phi} (-e^{2ik\pi} + e^{2ik\pi+2i\pi\Phi}) k \right) / (-1 + e^{2ik\pi}) \right) - \\
& (\text{Cm0 } e^{-ik\pi-i\pi\Phi} + \text{Cp0 } e^{ik\pi-i\pi\Phi}) i \alpha
\end{aligned}$$

And

$$\text{Cm0 } e^{-ik\pi-i\pi\Phi} + \text{Cp0 } e^{ik\pi-i\pi\Phi} == \text{Cm1} + \text{Cp1}$$

$$\begin{aligned}
& -\text{Cm1 } k + \text{Cp1 } k + \text{Cm1} \left( \frac{e^{-2i\pi\Phi} (-1 + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-2i\pi\Phi} (-1 + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) + \\
& \text{Cp1} \left( \frac{e^{-2i\pi\Phi} (-e^{2ik\pi} + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-2i\pi\Phi} (-e^{2ik\pi} + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) == -\text{Cm0 } e^{-ik\pi-i\pi\Phi} k + \\
& \text{Cp0 } e^{ik\pi-i\pi\Phi} k + \text{Cm0} \left( \frac{e^{ik\pi-i\pi\Phi} (-1 + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-ik\pi-i\pi\Phi} (-1 + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) + \\
& \text{Cp0} \left( \frac{e^{ik\pi-i\pi\Phi} (-e^{2ik\pi} + e^{2i\pi\Phi}) k}{-1 + e^{2ik\pi}} - \frac{e^{-ik\pi-i\pi\Phi} (-e^{2ik\pi} + e^{2ik\pi+2i\pi\Phi}) k}{-1 + e^{2ik\pi}} \right) - \\
& (\text{Cm0 } e^{-ik\pi-i\pi\Phi} + \text{Cp0 } e^{ik\pi-i\pi\Phi}) i \alpha
\end{aligned}$$

{Cp1, Cm1}

List

Cp1

Cm1



$$\left\{ \left\{ \begin{aligned} \text{Cp1} &\rightarrow \frac{1}{2k} e^{-i k \pi - i \pi \Phi} \left( 2 \text{Cp0} e^{2 i k \pi} k - \text{Cm0} i \alpha - \text{Cp0} e^{2 i k \pi} i \alpha \right), \\ \text{Cm1} &\rightarrow \frac{1}{2k} e^{-i k \pi - i \pi \Phi} \left( 2 \text{Cm0} k + \text{Cm0} i \alpha + \text{Cp0} e^{2 i k \pi} i \alpha \right) \end{aligned} \right\} \right\}$$

List

$$\left\{ \begin{aligned} \text{Cp1} &\rightarrow \frac{1}{2k} e^{-i k \pi - i \pi \Phi} \left( 2 \text{Cp0} e^{2 i k \pi} k - \text{Cm0} i \alpha - \text{Cp0} e^{2 i k \pi} i \alpha \right), \\ \text{Cm1} &\rightarrow \frac{1}{2k} e^{-i k \pi - i \pi \Phi} \left( 2 \text{Cm0} k + \text{Cm0} i \alpha + \text{Cp0} e^{2 i k \pi} i \alpha \right) \end{aligned} \right\}$$

List

$$\begin{aligned} \text{Cp1} &\rightarrow \frac{1}{2k} e^{-i k \pi - i \pi \Phi} \left( 2 \text{Cp0} e^{2 i k \pi} k - \text{Cm0} i \alpha - \text{Cp0} e^{2 i k \pi} i \alpha \right) \\ \text{Cm1} &\rightarrow \frac{1}{2k} e^{-i k \pi - i \pi \Phi} \left( 2 \text{Cm0} k + \text{Cm0} i \alpha + \text{Cp0} e^{2 i k \pi} i \alpha \right) \end{aligned}$$

$$\text{Out[81]=} \left\{ \left\{ \begin{aligned} \text{Cp1} &\rightarrow \frac{1}{2k} e^{-i k \pi - i \pi \Phi} \left( 2 \text{Cp0} e^{2 i k \pi} k - \text{Cm0} i \alpha - \text{Cp0} e^{2 i k \pi} i \alpha \right), \\ \text{Cm1} &\rightarrow \frac{1}{2k} e^{-i k \pi - i \pi \Phi} \left( 2 \text{Cm0} k + \text{Cm0} i \alpha + \text{Cp0} e^{2 i k \pi} i \alpha \right) \end{aligned} \right\} \right\}$$

In[75]:= (\*Выпишем получившуюся трансфер-матрицу\*)

$$\begin{aligned} \mathbf{M}[\mathbf{k}_-, \Phi_-] &= e^{-i k \pi - i \pi \Phi} / (2k) * \left\{ \left\{ 2 e^{2 i k \pi} k - e^{2 i k \pi} i \alpha, (i \alpha) \right\}, \left\{ e^{2 i k \pi} i \alpha, 2k + i \alpha \right\} \right\}; \\ \mathbf{M}[\mathbf{k}_-, \Phi_-] &= e^{-i k \pi - i \pi \Phi} / (2k) * \left\{ \left\{ e^{2 i k \pi} (2k - i \alpha), (i \alpha) \right\}, \left\{ e^{2 i k \pi} i \alpha, 2k + i \alpha \right\} \right\} \end{aligned}$$

$$\text{Out[76]=} \left\{ \left\{ \frac{e^{i k \pi - i \pi \Phi} (2k - i \alpha)}{2k}, \frac{e^{-i k \pi - i \pi \Phi} i \alpha}{2k} \right\}, \left\{ \frac{e^{i k \pi - i \pi \Phi} i \alpha}{2k}, \frac{e^{-i k \pi - i \pi \Phi} (2k + i \alpha)}{2k} \right\} \right\}$$