

$$\ln[71] := \text{char}[k_, \varpi_, \theta_, \alpha_] =$$

$$\begin{aligned}
& -\frac{1}{4k} \left(\frac{1}{e^{\frac{i}{2}\pi\varpi} + e^{3\frac{i}{2}\pi\varpi}} \left(-2\frac{i}{2} \left(1 + e^{4\frac{i}{2}\pi\varpi} - 2e^{2\frac{i}{2}\pi(k+\varpi)} \right) k + e^{2\frac{i}{2}\pi\varpi} \left(-1 + e^{2\frac{i}{2}k\pi} \right) \alpha \right) \text{Csc}[k\pi] - \right. \\
& \quad \left. \text{Sec}[\pi\varpi] \left(4k \text{Cos}[k\pi] + \alpha \text{Sin}[k\pi] \right) \left(-1 + \sqrt{1 - \frac{16k^2 \text{Cos}[\pi\varpi]^2}{(4k \text{Cos}[k\pi] + \alpha \text{Sin}[k\pi])^2}} \right) \right) \\
& \left(\left(\left(-e^{\frac{i}{2}(\pi-\theta)(k-\varpi)} k - \frac{2e^{\frac{i}{2}(k(\pi-\theta)+(-3\pi+\theta)\varpi)} \left(-1 + e^{2\frac{i}{2}\pi(k+\varpi)} \right) k}{-1 + e^{2\frac{i}{2}k\pi}} - \left(e^{-\frac{i}{2}(k(\pi+\theta)+(\pi-\theta)\varpi)} \left(e^{2\frac{i}{2}k\pi} + \right. \right. \right. \right. \\
& \quad \left. \left. \left. e^{2\frac{i}{2}k(2\pi+\theta)} - 2e^{2\frac{i}{2}(k(\pi+\theta)+\pi\varpi)} \right) k \right) / \left(-1 + e^{2\frac{i}{2}k(\pi+\theta)} \right) + \frac{i}{2} e^{\frac{i}{2}(\pi-\theta)(k-\varpi)} \alpha \right) \right. \\
& \quad \left(2 \left(1 + e^{2\frac{i}{2}k\theta} - 2e^{2\frac{i}{2}k(\pi+\theta)} \right) \left(-1 + e^{2\frac{i}{2}\pi(k+\varpi)} \right) k + \frac{i}{2} e^{2\frac{i}{2}\pi\varpi} \left(-1 + e^{2\frac{i}{2}k\pi} \right) \left(-1 + \right. \right. \\
& \quad \left. \left. e^{2\frac{i}{2}k(\pi+\theta)} \right) \alpha \right) - \left(e^{-\frac{i}{2}(\pi-\theta)(k+\varpi)} k - \frac{2e^{-\frac{i}{2}(k\pi-k\theta+3\pi\varpi-\theta\varpi)} \left(-1 + e^{2\frac{i}{2}\pi(k+\varpi)} \right) k}{-1 + e^{2\frac{i}{2}k\pi}} - \right. \\
& \quad \left(e^{-\frac{i}{2}(k(\pi+\theta)+(\pi-\theta)\varpi)} \left(e^{2\frac{i}{2}k\theta} + e^{2\frac{i}{2}k(\pi+2\theta)} - 2e^{2\frac{i}{2}(k(\pi+\theta)+\pi\varpi)} \right) k \right) / \\
& \quad \left. \left(-1 + e^{2\frac{i}{2}k(\pi+\theta)} \right) + \frac{i}{2} e^{-\frac{i}{2}(\pi-\theta)(k+\varpi)} \alpha \right) \left(2 \left(-1 - e^{2\frac{i}{2}k\pi} + e^{4\frac{i}{2}k\pi} + e^{2\frac{i}{2}k(\pi+\theta)} + \right. \right. \\
& \quad \left. \left. e^{2\frac{i}{2}\pi\varpi} - e^{2\frac{i}{2}(2k\pi+k\theta+\pi\varpi)} \right) k + \frac{i}{2} e^{2\frac{i}{2}\pi\varpi} \left(-1 + e^{2\frac{i}{2}k\pi} \right) \left(-1 + e^{2\frac{i}{2}k(\pi+\theta)} \right) \alpha \right) \Bigg) \\
& \left(\frac{1}{e^{\frac{i}{2}\pi\varpi} + e^{3\frac{i}{2}\pi\varpi}} \left(-2\frac{i}{2} \left(1 + e^{4\frac{i}{2}\pi\varpi} - 2e^{2\frac{i}{2}\pi(k+\varpi)} \right) k + e^{2\frac{i}{2}\pi\varpi} \left(-1 + e^{2\frac{i}{2}k\pi} \right) \alpha \right) \text{Csc}[k\pi] - \right. \\
& \quad \left. \text{Sec}[\pi\varpi] \left(4k \text{Cos}[k\pi] + \alpha \text{Sin}[k\pi] \right) \left(-1 + \sqrt{1 - \frac{16k^2 \text{Cos}[\pi\varpi]^2}{(4k \text{Cos}[k\pi] + \alpha \text{Sin}[k\pi])^2}} \right) \right) \Bigg) / \\
& \left(16 \left(-1 + e^{2\frac{i}{2}k\pi} \right) \left(1 + e^{-2\frac{i}{2}\pi\varpi} \right) \left(e^{2\frac{i}{2}k\pi} - e^{2\frac{i}{2}k\theta} - e^{2\frac{i}{2}\pi\varpi} + e^{2\frac{i}{2}(k(\pi+\theta)+\pi\varpi)} \right) k^3 \right) - \\
& \left(\frac{i}{2} e^{-\frac{i}{2}k\pi} \left(- \left(-e^{\frac{i}{2}(\pi-\theta)(k-\varpi)} k - \frac{2e^{\frac{i}{2}(k(\pi-\theta)+(-3\pi+\theta)\varpi)} \left(-1 + e^{2\frac{i}{2}\pi(k+\varpi)} \right) k}{-1 + e^{2\frac{i}{2}k\pi}} - \right. \right. \right. \\
& \quad \left. \left(e^{-\frac{i}{2}(k(\pi+\theta)+(\pi-\theta)\varpi)} \left(e^{2\frac{i}{2}k\pi} + e^{2\frac{i}{2}k(2\pi+\theta)} - 2e^{2\frac{i}{2}(k(\pi+\theta)+\pi\varpi)} \right) k \right) / \right. \\
& \quad \left. \left(-1 + e^{2\frac{i}{2}k(\pi+\theta)} \right) + \frac{i}{2} e^{\frac{i}{2}(\pi-\theta)(k-\varpi)} \alpha \right) \left(2e^{2\frac{i}{2}k\pi} k + 2e^{2\frac{i}{2}k\theta} k - 2e^{2\frac{i}{2}k(\pi+\theta)} k - \right. \\
& \quad \left. 2e^{2\frac{i}{2}k(2\pi+\theta)} k + e^{2\frac{i}{2}(k(2\pi+\theta)+\pi\varpi)} (2k - \frac{i}{2}\alpha) - e^{2\frac{i}{2}\pi\varpi} (2k + \frac{i}{2}\alpha) + \frac{i}{2} e^{2\frac{i}{2}\pi(k+\varpi)} \alpha + \right. \\
& \quad \left. \frac{i}{2} e^{2\frac{i}{2}(k(\pi+\theta)+\pi\varpi)} \alpha \right) - \left(e^{-\frac{i}{2}(\pi-\theta)(k+\varpi)} k - \frac{2e^{-\frac{i}{2}(k\pi-k\theta+3\pi\varpi-\theta\varpi)} \left(-1 + e^{2\frac{i}{2}\pi(k+\varpi)} \right) k}{-1 + e^{2\frac{i}{2}k\pi}} - \right. \\
& \quad \left(e^{-\frac{i}{2}(k(\pi+\theta)+(\pi-\theta)\varpi)} \left(e^{2\frac{i}{2}k\theta} + e^{2\frac{i}{2}k(\pi+2\theta)} - 2e^{2\frac{i}{2}(k(\pi+\theta)+\pi\varpi)} \right) k \right) / \\
& \quad \left. \left(-1 + e^{2\frac{i}{2}k(\pi+\theta)} \right) + \frac{i}{2} e^{-\frac{i}{2}(\pi-\theta)(k+\varpi)} \alpha \right) \\
& \left(-4e^{2\frac{i}{2}k\pi} k + 2e^{4\frac{i}{2}k\pi} k + 2e^{2\frac{i}{2}k(2\pi+\theta)} k - e^{2\frac{i}{2}\pi(k+\varpi)} (2k + \frac{i}{2}\alpha) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(e^{2i(k(\pi+\theta)+\pi\varpi)} (2k + i\alpha) + e^{2i\pi\varpi} (4k + i\alpha) + i e^{2i(k(2\pi+\theta)+\pi\varpi)} \alpha \right) \right. \\
& \left. \sec[\pi\varpi] (\alpha + 4k \csc[k\pi] \sin[\pi\varpi] \sin[\pi(k+\varpi)]) \right) / \\
& \left(16 (-1 + e^{2ik\pi}) (1 + e^{-2i\pi\varpi}) (e^{2ik\pi} - e^{2ik\theta} - e^{2i\pi\varpi} + e^{2i(k(\pi+\theta)+\pi\varpi)}) k^3 \right) - \\
& \frac{1}{4k} i e^{-ik\pi} \sec[\pi\varpi] (\alpha + 4k \csc[k\pi] \sin[\pi\varpi] \sin[\pi(k+\varpi)]) \\
& \left(\left((-2 (1 + e^{2ik\theta} - 2 e^{2ik(\pi+\theta)}) (-1 + e^{2i\pi(k+\varpi)}) k - \right. \right. \\
& i e^{2i\pi\varpi} (-1 + e^{2ik\pi}) (-1 + e^{2ik(\pi+\theta)}) \alpha \left(e^{i(\pi-\theta)(k-\varpi)} - \right. \\
& \frac{1}{2(1 + e^{-2i\pi\varpi}) k} \left(e^{i(\pi-\theta)(k-\varpi)} k + \frac{2 e^{i(k(\pi-\theta)+(-3\pi+\theta)\varpi)} (-1 + e^{2i\pi(k+\varpi)}) k}{-1 + e^{2ik\pi}} + \right. \\
& \left. \left(e^{-i(k(\pi+\theta)+(\pi-\theta)\varpi)} (e^{2ik\pi} + e^{2ik(2\pi+\theta)} - 2 e^{2i(k(\pi+\theta)+\pi\varpi)}) k \right) / \right. \\
& \left. \left. (-1 + e^{2ik(\pi+\theta)}) - i e^{i(\pi-\theta)(k-\varpi)} \alpha \right) \right) + \\
& (2 (-1 - e^{2ik\pi} + e^{4ik\pi} + e^{2ik(\pi+\theta)} + e^{2i\pi\varpi} - e^{2i(2k\pi+k\theta+\pi\varpi)}) k + \\
& i e^{2i\pi\varpi} (-1 + e^{2ik\pi}) (-1 + e^{2ik(\pi+\theta)}) \alpha \left(e^{-i(\pi-\theta)(k+\varpi)} - \right. \\
& \frac{1}{2(1 + e^{-2i\pi\varpi}) k} \left(-e^{-i(\pi-\theta)(k+\varpi)} k + \frac{2 e^{-i(k\pi-k\theta+3\pi\varpi-\theta\varpi)} (-1 + e^{2i\pi(k+\varpi)}) k}{-1 + e^{2ik\pi}} + \right. \\
& \left. \left(e^{-i(k(\pi+\theta)+(\pi-\theta)\varpi)} (e^{2ik\theta} + e^{2ik(\pi+2\theta)} - 2 e^{2i(k(\pi+\theta)+\pi\varpi)}) k \right) / \right. \\
& \left. \left. (-1 + e^{2ik(\pi+\theta)}) - i e^{-i(\pi-\theta)(k+\varpi)} \alpha \right) \right) \Bigg) \\
& \left(\frac{1}{e^{i\pi\varpi} + e^{3i\pi\varpi}} (-2i (1 + e^{4i\pi\varpi} - 2 e^{2i\pi(k+\varpi)}) k + e^{2i\pi\varpi} (-1 + e^{2ik\pi}) \alpha) \csc[k\pi] - \right. \\
& \sec[\pi\varpi] (4k \cos[k\pi] + \alpha \sin[k\pi]) \\
& \left. \left(-1 + \sqrt{1 - \frac{16k^2 \cos[\pi\varpi]^2}{(4k \cos[k\pi] + \alpha \sin[k\pi])^2}} \right) \right) / \\
& (8 (-1 + e^{2ik\pi}) (e^{2ik\pi} - e^{2ik\theta} - e^{2i\pi\varpi} + e^{2i(k(\pi+\theta)+\pi\varpi)}) k^2) - \\
& \left(i e^{-ik\pi} \left((2 e^{2ik\pi} k + 2 e^{2ik\theta} k - 2 e^{2ik(\pi+\theta)} k - 2 e^{2ik(2\pi+\theta)} k + e^{2i(k(2\pi+\theta)+\pi\varpi)} (2k - \right. \right. \\
& i \alpha) - e^{2i\pi\varpi} (2k + i\alpha) + i e^{2i\pi(k+\varpi)} \alpha + i e^{2i(k(\pi+\theta)+\pi\varpi)} \alpha) \left(e^{i(\pi-\theta)(k-\varpi)} - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2 \left(1 + e^{-2 \frac{i}{k} \pi \bar{\pi}}\right) k} \left(e^{\frac{i}{k} (\pi - \theta) (k - \bar{\pi})} k + \frac{2 e^{\frac{i}{k} (k (\pi - \theta) + (-3 \pi + \theta) \bar{\pi})} (-1 + e^{2 \frac{i}{k} \pi (k + \bar{\pi})}) k}{-1 + e^{2 \frac{i}{k} k \pi}} + \right. \\
& \left. \left(e^{-\frac{i}{k} (k (\pi + \theta) + (\pi - \theta) \bar{\pi})} \left(e^{2 \frac{i}{k} k \pi} + e^{2 \frac{i}{k} k (2 \pi + \theta)} - 2 e^{2 \frac{i}{k} (k (\pi + \theta) + \pi \bar{\pi})} \right) k \right) / \right. \\
& \left. \left(-1 + e^{2 \frac{i}{k} k (\pi + \theta)} \right) - \frac{i}{k} e^{\frac{i}{k} (\pi - \theta) (k - \bar{\pi})} \alpha \right) \Bigg) + \\
& \left(-4 e^{2 \frac{i}{k} k \pi} k + 2 e^{4 \frac{i}{k} k \pi} k + 2 e^{2 \frac{i}{k} k (2 \pi + \theta)} k - e^{2 \frac{i}{k} \pi (k + \bar{\pi})} (2 k + \frac{i}{k} \alpha) - e^{2 \frac{i}{k} (k (\pi + \theta) + \pi \bar{\pi})} \right. \\
& \left. (2 k + \frac{i}{k} \alpha) + e^{2 \frac{i}{k} \pi \bar{\pi}} (4 k + \frac{i}{k} \alpha) + \frac{i}{k} e^{2 \frac{i}{k} (k (2 \pi + \theta) + \pi \bar{\pi})} \alpha \right) \left(e^{-\frac{i}{k} (\pi - \theta) (k + \bar{\pi})} - \right. \\
& \frac{1}{2 \left(1 + e^{-2 \frac{i}{k} \pi \bar{\pi}}\right) k} \left(-e^{-\frac{i}{k} (\pi - \theta) (k + \bar{\pi})} k + \frac{2 e^{-\frac{i}{k} (k \pi - k \theta + 3 \pi \bar{\pi} - \theta \bar{\pi})} (-1 + e^{2 \frac{i}{k} \pi (k + \bar{\pi})}) k}{-1 + e^{2 \frac{i}{k} k \pi}} + \right. \\
& \left. \left(e^{-\frac{i}{k} (k (\pi + \theta) + (\pi - \theta) \bar{\pi})} \left(e^{2 \frac{i}{k} k \theta} + e^{2 \frac{i}{k} k (\pi + 2 \theta)} - 2 e^{2 \frac{i}{k} (k (\pi + \theta) + \pi \bar{\pi})} \right) k \right) / \right. \\
& \left. \left(-1 + e^{2 \frac{i}{k} k (\pi + \theta)} \right) - \frac{i}{k} e^{-\frac{i}{k} (\pi - \theta) (k + \bar{\pi})} \alpha \right) \Bigg) \Bigg) \\
& \left. \text{Sec}[\pi \bar{\pi}] \left(\alpha + 4 k \text{Csc}[k \pi] \text{Sin}[\pi \bar{\pi}] \text{Sin}[\pi (k + \bar{\pi})] \right) \right) \Bigg) / \\
& \left(8 (-1 + e^{2 \frac{i}{k} k \pi}) \left(e^{2 \frac{i}{k} k \pi} - e^{2 \frac{i}{k} k \theta} - e^{2 \frac{i}{k} \pi \bar{\pi}} + e^{2 \frac{i}{k} (k (\pi + \theta) + \pi \bar{\pi})} \right) k^2 \right) \Bigg) \\
\text{Out[71]} = & -\frac{1}{4 k} \left(\left(5.55112 \times 10^{-17} - 0.707107 i \right) \left(\left(3.06162 \times 10^{-17} + 0.5 i \right) (-1 + e^{2 \frac{i}{k} k \pi}) - \right. \right. \\
& 2 i \left(\left(0. + 1.22465 \times 10^{-16} i \right) - 2 e^{2 \frac{i}{k} (0.25 + k) \pi} \right) k \Bigg) \text{Csc}[k \pi] - \\
& 1.41421 \left(-1 + \sqrt{1 - \frac{8. k^2}{(4 k \text{Cos}[k \pi] + 0.5 \text{Sin}[k \pi])^2}} \right) (4 k \text{Cos}[k \pi] + 0.5 \text{Sin}[k \pi]) \Bigg) \\
& \left(\left(0.03125 + 0.03125 i \right) \left(\left((-0.5 + 3.06162 \times 10^{-17} i) (-1 + e^{2 \frac{i}{k} k \pi}) \left(-1 + e^{\frac{7 i k \pi}{3}} \right) + \right. \right. \right. \\
& 2 \left(1 + e^{\frac{i k \pi}{3}} - 2 e^{\frac{7 i k \pi}{3}} \right) (-1 + e^{2 \frac{i}{k} (0.25 + k) \pi}) k \Bigg) \left((0. + 0.5 i) e^{\frac{5}{6} i (-0.25 + k) \pi} - \right. \\
& e^{\frac{5}{6} i (-0.25 + k) \pi} k - \frac{2 e^{\frac{i}{k} \left(-2.22529 + \frac{5 k \pi}{6} \right)} (-1 + e^{2 \frac{i}{k} (0.25 + k) \pi}) k}{-1 + e^{2 \frac{i}{k} k \pi}} - \frac{1}{-1 + e^{\frac{7 i k \pi}{3}}} \\
& e^{-i \left(0.654498 + \frac{7 k \pi}{6} \right)} \left(e^{2 \frac{i}{k} k \pi} + e^{\frac{13 i k \pi}{3}} - 2 e^{2 \frac{i}{k} \left(0.785398 + \frac{7 k \pi}{6} \right)} \right) k \Bigg) - \left((0. + 0.5 i) \right. \\
& e^{-\frac{5}{6} i (0.25 + k) \pi} + e^{-\frac{5}{6} i (0.25 + k) \pi} k - \frac{2 e^{-i \left(2.22529 + \frac{5 k \pi}{6} \right)} (-1 + e^{2 \frac{i}{k} (0.25 + k) \pi}) k}{-1 + e^{2 \frac{i}{k} k \pi}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{-1 + e^{\frac{7ik\pi}{3}}} e^{-i\left(0.654498 + \frac{7k\pi}{6}\right)} \left(e^{\frac{ik\pi}{3}} + e^{\frac{8ik\pi}{3}} - 2e^{2i\left(0.785398 + \frac{7k\pi}{6}\right)} \right) k \Bigg) \\
& \left((-0.5 + 3.06162 \times 10^{-17} i) (-1 + e^{2ik\pi}) \left(-1 + e^{\frac{7ik\pi}{3}} \right) + \right. \\
& \left. 2 \left((-1. + 1. i) - e^{2ik\pi} + e^{\frac{7ik\pi}{3}} + e^{4ik\pi} - e^{2i\left(0.785398 + \frac{13k\pi}{6}\right)} \right) k \right) \Bigg) \\
& \left((5.55112 \times 10^{-17} - 0.707107 i) \left((3.06162 \times 10^{-17} + 0.5 i) (-1 + e^{2ik\pi}) - \right. \right. \\
& \left. \left. 2i \left((0. + 1.22465 \times 10^{-16} i) - 2e^{2i(0.25+k)\pi} \right) k \right) \operatorname{Csc}[k\pi] - 1.41421 \right. \\
& \left. \left(-1 + \sqrt{1 - \frac{8 \cdot k^2}{(4k \operatorname{Cos}[k\pi] + 0.5 \operatorname{Sin}[k\pi])^2}} \right) (4k \operatorname{Cos}[k\pi] + 0.5 \operatorname{Sin}[k\pi]) \right) \Bigg) / \\
& \left((-1 + e^{2ik\pi}) \left((-6.12323 \times 10^{-17} - 1. i) - e^{\frac{ik\pi}{3}} + e^{2ik\pi} + e^{2i\left(0.785398 + \frac{7k\pi}{6}\right)} \right) k^3 \right) + \\
& \left((0.0441942 - 0.0441942 i) e^{-ik\pi} \left(\left((0. - 0.5 i) e^{\frac{5}{6}i(-0.25+k)\pi} + \right. \right. \right. \\
& \left. \left. e^{\frac{5}{6}i(-0.25+k)\pi} k + \frac{2e^{i\left(-2.22529 + \frac{5k\pi}{6}\right)} (-1 + e^{2i(0.25+k)\pi}) k}{-1 + e^{2ik\pi}} + \right. \right. \\
& \left. \left. \frac{1}{-1 + e^{\frac{7ik\pi}{3}}} e^{-i\left(0.654498 + \frac{7k\pi}{6}\right)} \left(e^{2ik\pi} + e^{\frac{13ik\pi}{3}} - 2e^{2i\left(0.785398 + \frac{7k\pi}{6}\right)} \right) k \right) \right. \\
& \left((0. + 0.5 i) e^{2i(0.25+k)\pi} + (0. + 0.5 i) e^{2i\left(0.785398 + \frac{7k\pi}{6}\right)} + 2e^{\frac{ik\pi}{3}} k + \right. \\
& \left. 2e^{2ik\pi} k - 2e^{\frac{7ik\pi}{3}} k - 2e^{\frac{13ik\pi}{3}} k + e^{2i\left(0.785398 + \frac{13k\pi}{6}\right)} ((0. - 0.5 i) + 2k) - \right. \\
& \left. (6.12323 \times 10^{-17} + 1. i) ((0. + 0.5 i) + 2k) \right) - \left((0. + 0.5 i) e^{-\frac{5}{6}i(0.25+k)\pi} + \right. \\
& \left. e^{-\frac{5}{6}i(0.25+k)\pi} k - \frac{2e^{-i\left(2.22529 + \frac{5k\pi}{6}\right)} (-1 + e^{2i(0.25+k)\pi}) k}{-1 + e^{2ik\pi}} - \right. \\
& \left. \frac{1}{-1 + e^{\frac{7ik\pi}{3}}} e^{-i\left(0.654498 + \frac{7k\pi}{6}\right)} \left(e^{\frac{ik\pi}{3}} + e^{\frac{8ik\pi}{3}} - 2e^{2i\left(0.785398 + \frac{7k\pi}{6}\right)} \right) k \right) \\
& \left((0. + 0.5 i) e^{2i\left(0.785398 + \frac{13k\pi}{6}\right)} - 4e^{2ik\pi} k + 2e^{4ik\pi} k + 2e^{\frac{13ik\pi}{3}} k - \right. \\
& \left. e^{2i(0.25+k)\pi} ((0. + 0.5 i) + 2k) - e^{2i\left(0.785398 + \frac{7k\pi}{6}\right)} ((0. + 0.5 i) + 2k) + \right. \\
& \left. (6.12323 \times 10^{-17} + 1. i) ((0. + 0.5 i) + 4k) \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(0.5 + 2.82843 k \operatorname{Csc}[k \pi] \operatorname{Sin}[(0.25 + k) \pi] \right) \right) / \\
& \left((-1 + e^{2 i k \pi}) \left((-6.12323 \times 10^{-17} - 1. i) - e^{\frac{i k \pi}{3}} + e^{2 i k \pi} + e^{2 i \left(0.785398 + \frac{7 k \pi}{6} \right)} \right) k^3 \right) - \\
& \frac{1}{k} (0. + 0.353553 i) \\
& e^{-i k \pi} \\
& (0.5 + \\
& \quad 2.82843 \\
& \quad k \\
& \quad \operatorname{Csc}[k \pi] \\
& \quad \operatorname{Sin}[(0.25 + k) \pi]) \\
& \left(\left(\left((-0.5 + 3.06162 \times 10^{-17} i) (-1 + e^{2 i k \pi}) \left(-1 + e^{\frac{7 i k \pi}{3}} \right) + \right. \right. \right. \\
& \quad \left. \left. 2 \left((-1. + 1. i) - e^{2 i k \pi} + e^{\frac{7 i k \pi}{3}} + e^{4 i k \pi} - e^{2 i \left(0.785398 + \frac{13 k \pi}{6} \right)} \right) k \right) \right. \\
& \quad \left. \left(e^{-\frac{5}{6} i (0.25 + k) \pi} - \frac{1}{k} (0.25 + 0.25 i) \right) \left((0. - 0.5 i) e^{-\frac{5}{6} i (0.25 + k) \pi} - \right. \right. \\
& \quad \left. \left. e^{-\frac{5}{6} i (0.25 + k) \pi} k + \frac{2 e^{-i \left(2.22529 + \frac{5 k \pi}{6} \right)} (-1 + e^{2 i (0.25 + k) \pi}) k}{-1 + e^{2 i k \pi}} + 1 \right) / \right. \\
& \quad \left. \left(-1 + e^{\frac{7 i k \pi}{3}} \right) e^{-i \left(0.654498 + \frac{7 k \pi}{6} \right)} \left(e^{\frac{i k \pi}{3}} + e^{\frac{8 i k \pi}{3}} - 2 e^{2 i \left(0.785398 + \frac{7 k \pi}{6} \right)} \right) k \right) \right) + \\
& \left((0.5 - 3.06162 \times 10^{-17} i) (-1 + e^{2 i k \pi}) \left(-1 + e^{\frac{7 i k \pi}{3}} \right) - 2 \left(1 + e^{\frac{i k \pi}{3}} - 2 e^{\frac{7 i k \pi}{3}} \right) \right. \\
& \quad \left. (-1 + e^{2 i (0.25 + k) \pi}) k \right) \left(e^{\frac{5}{6} i (-0.25 + k) \pi} - \frac{1}{k} (0.25 + 0.25 i) \right) \left((0. - 0.5 i) \right. \\
& \quad \left. e^{\frac{5}{6} i (-0.25 + k) \pi} + e^{\frac{5}{6} i (-0.25 + k) \pi} k + \frac{2 e^{i \left(-2.22529 + \frac{5 k \pi}{6} \right)} (-1 + e^{2 i (0.25 + k) \pi}) k}{-1 + e^{2 i k \pi}} + \right. \\
& \quad \left. 1 \right) / \left(-1 + e^{\frac{7 i k \pi}{3}} \right) e^{-i \left(0.654498 + \frac{7 k \pi}{6} \right)} \left(e^{2 i k \pi} + e^{\frac{13 i k \pi}{3}} - 2 e^{2 i \left(0.785398 + \frac{7 k \pi}{6} \right)} \right) k \right) \right) \\
& \left((5.55112 \times 10^{-17} - 0.707107 i) \left((3.06162 \times 10^{-17} + 0.5 i) (-1 + e^{2 i k \pi}) - \right. \right. \\
& \quad \left. \left. 2 i \left((0. + 1.22465 \times 10^{-16} i) - 2 e^{2 i (0.25 + k) \pi} \right) k \right) \operatorname{Csc}[k \pi] - 1.41421 \right. \\
& \quad \left. \left(-1 + \sqrt{1 - \frac{8. k^2}{(4 k \operatorname{Cos}[k \pi] + 0.5 \operatorname{Sin}[k \pi])^2}} \right) (4 k \operatorname{Cos}[k \pi] + 0.5 \operatorname{Sin}[k \pi]) \right) \right) /
\end{aligned}$$

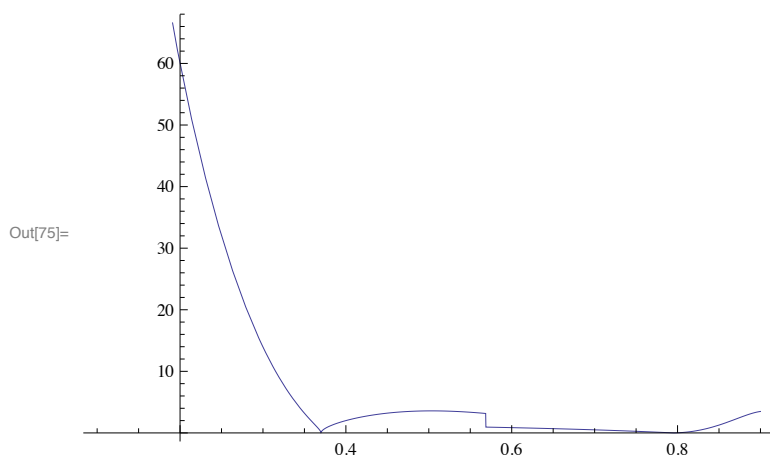
$$\begin{aligned}
& \left(8 \left(-1 + e^{2 i k \pi} \right) \left(\left(-6.12323 \times 10^{-17} - 1. i \right) - e^{\frac{i k \pi}{3}} + e^{2 i k \pi} + e^{2 i \left(0.785398 + \frac{7 k \pi}{6} \right)} \right) k^2 \right) - \\
& \left(\left(0. + 0.176777 i \right) e^{-i k \pi} \right. \\
& \left(\left(\left(0. + 0.5 i \right) e^{2 i \left(0.785398 + \frac{13 k \pi}{6} \right)} - 4 e^{2 i k \pi} k + 2 e^{4 i k \pi} k + 2 e^{\frac{13 i k \pi}{3}} k - \right. \right. \\
& e^{2 i \left(0.25 + k \right) \pi} \left(\left(0. + 0.5 i \right) + 2 k \right) - e^{2 i \left(0.785398 + \frac{7 k \pi}{6} \right)} \left(\left(0. + 0.5 i \right) + 2 k \right) + \\
& \left. \left(6.12323 \times 10^{-17} + 1. i \right) \left(\left(0. + 0.5 i \right) + 4 k \right) \right) \\
& \left(e^{-\frac{5}{6} i \left(0.25 + k \right) \pi} - \frac{1}{k} \left(0.25 + 0.25 i \right) \right) \left(\left(0. - 0.5 i \right) e^{-\frac{5}{6} i \left(0.25 + k \right) \pi} - \right. \\
& e^{-\frac{5}{6} i \left(0.25 + k \right) \pi} k + \frac{2 e^{-i \left(2.22529 + \frac{5 k \pi}{6} \right)} \left(-1 + e^{2 i \left(0.25 + k \right) \pi} \right) k}{-1 + e^{2 i k \pi}} + 1 / \\
& \left. \left(-1 + e^{\frac{7 i k \pi}{3}} \right) e^{-i \left(0.654498 + \frac{7 k \pi}{6} \right)} \left(e^{\frac{i k \pi}{3}} + e^{\frac{8 i k \pi}{3}} - 2 e^{2 i \left(0.785398 + \frac{7 k \pi}{6} \right)} \right) k \right) \right) + \\
& \left(\left(0. + 0.5 i \right) e^{2 i \left(0.25 + k \right) \pi} + \left(0. + 0.5 i \right) e^{2 i \left(0.785398 + \frac{7 k \pi}{6} \right)} + 2 e^{\frac{i k \pi}{3}} k + \right. \\
& 2 e^{2 i k \pi} k - 2 e^{\frac{7 i k \pi}{3}} k - 2 e^{\frac{13 i k \pi}{3}} k + e^{2 i \left(0.785398 + \frac{13 k \pi}{6} \right)} \left(\left(0. - 0.5 i \right) + 2 k \right) - \\
& \left. \left(6.12323 \times 10^{-17} + 1. i \right) \left(\left(0. + 0.5 i \right) + 2 k \right) \right) \\
& \left(e^{\frac{5}{6} i \left(-0.25 + k \right) \pi} - \frac{1}{k} \left(0.25 + 0.25 i \right) \right) \left(\left(0. - 0.5 i \right) e^{\frac{5}{6} i \left(-0.25 + k \right) \pi} + e^{\frac{5}{6} i \left(-0.25 + k \right) \pi} \right. \\
& k + \frac{2 e^{i \left(-2.22529 + \frac{5 k \pi}{6} \right)} \left(-1 + e^{2 i \left(0.25 + k \right) \pi} \right) k}{-1 + e^{2 i k \pi}} + 1 / \left(-1 + e^{\frac{7 i k \pi}{3}} \right) \\
& \left. e^{-i \left(0.654498 + \frac{7 k \pi}{6} \right)} \left(e^{2 i k \pi} + e^{\frac{13 i k \pi}{3}} - 2 e^{2 i \left(0.785398 + \frac{7 k \pi}{6} \right)} \right) k \right) \right) \right) \\
& \left. \left(0.5 + 2.82843 k \operatorname{Csc}[k \pi] \operatorname{Sin}[(0.25 + k) \pi] \right) \right) / \left(\left(-1 + e^{2 i k \pi} \right) \right. \\
& \left. \left(\left(-6.12323 \times 10^{-17} - 1. i \right) - e^{\frac{i k \pi}{3}} + e^{2 i k \pi} + e^{2 i \left(0.785398 + \frac{7 k \pi}{6} \right)} \right) k^2 \right) \right)
\end{aligned}$$

In[72]:= **$\mathfrak{x} = 0.25$** Out[72]= **0.25**In[73]:= **$\theta = \pi / 6$** Out[73]= **$\frac{\pi}{6}$**

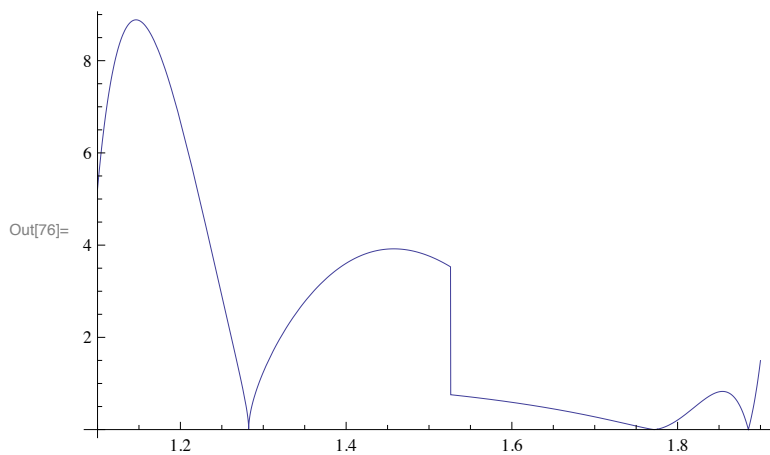
In[74]:= $\alpha = 0.5$

Out[74]= 0.5

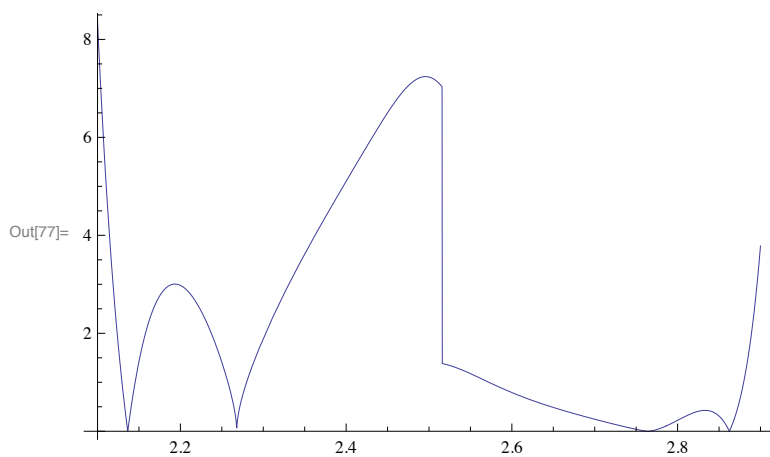
In[75]:= `Plot[Abs[char[x, Ξ , θ , α]], {x, 0.1, 0.9}]`



In[76]:= `Plot[Abs[char[x, Ξ , θ , α]], {x, 1.1, 1.9}]`

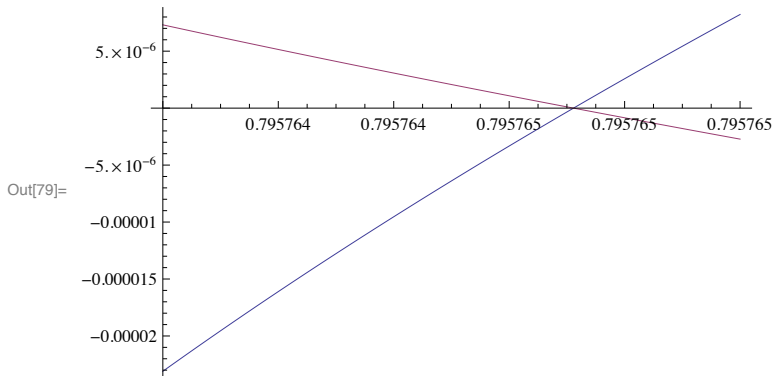


In[77]:= `Plot[Abs[char[x, Ξ , θ , α]], {x, 2.1, 2.9}]`



In[78]:= (* Далее будем строить вместе графики обеих частей нашей комплексной функции и смотреть за сближением их корней при изменении параметра *)

```
In[79]:= Plot[{Re[char[x,  $\xi$ ,  $\theta$ ,  $\alpha$ ]], Im[char[x,  $\xi$ ,  $\theta$ ,  $\alpha$ ]]}, {x, 0.795764, 0.795765}]
```



```
In[80]:= Abs[char[0.795765,  $\xi$ ,  $\theta$ ,  $\alpha$ ]]
```

```
Out[80]:=  $8.65703 \times 10^{-6}$ 
```

```
In[83]:= FindMinimum[Abs[char[x,  $\xi$ ,  $\theta$ ,  $\alpha$ ]], {x, 0.4}]
```

FindMinimum::lstol :

The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>

```
Out[83]:= {0.0000139959, {x → 0.370026}}
```

```
In[84]:= FindMinimum[Abs[char[x,  $\xi$ ,  $\theta$ ,  $\alpha$ ]], {x, 0.8}]
```

FindMinimum::lstol :

The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>

```
Out[84]:= { $1.06554 \times 10^{-8}$ , {x → 0.795765}}
```

```
In[85]:= FindMinimum[Abs[char[x,  $\xi$ ,  $\theta$ ,  $\alpha$ ]], {x, 1.3}]
```

FindMinimum::lstol :

The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>

```
Out[85]:= { $6.92878 \times 10^{-11}$ , {x → 0.795765}}
```

```
In[86]:= FindMinimum[Abs[char[x,  $\xi$ ,  $\theta$ ,  $\alpha$ ]], {x, 1.8}]
```

FindMinimum::lstol :

The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>

```
Out[86]:= { $3.04674 \times 10^{-15}$ , {x → 1.77163}}
```

```
In[87]:= FindMinimum[Abs[char[x,  $\xi$ ,  $\theta$ ,  $\alpha$ ]], {x, 2.1}]
```

FindMinimum::lstol :

The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>

```
Out[87]:= { $1.05588 \times 10^{-6}$ , {x → 3.14924}}
```



```
In[88]:= FindMinimum[Abs[char[x,  $\mathfrak{x}$ ,  $\theta$ ,  $\alpha$ ]], {x, 2.3}]
```

FindMinimum::lstol :

The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>

```
Out[88]= {1.59345  $\times 10^{-6}$ , {x  $\rightarrow$  2.13648}}
```

```
In[89]:= FindMinimum[Abs[char[x,  $\mathfrak{x}$ ,  $\theta$ ,  $\alpha$ ]], {x, 2.8}]
```

FindMinimum::lstol :

The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances. >>

```
Out[89]= {1.36818  $\times 10^{-9}$ , {x  $\rightarrow$  2.76406}}
```