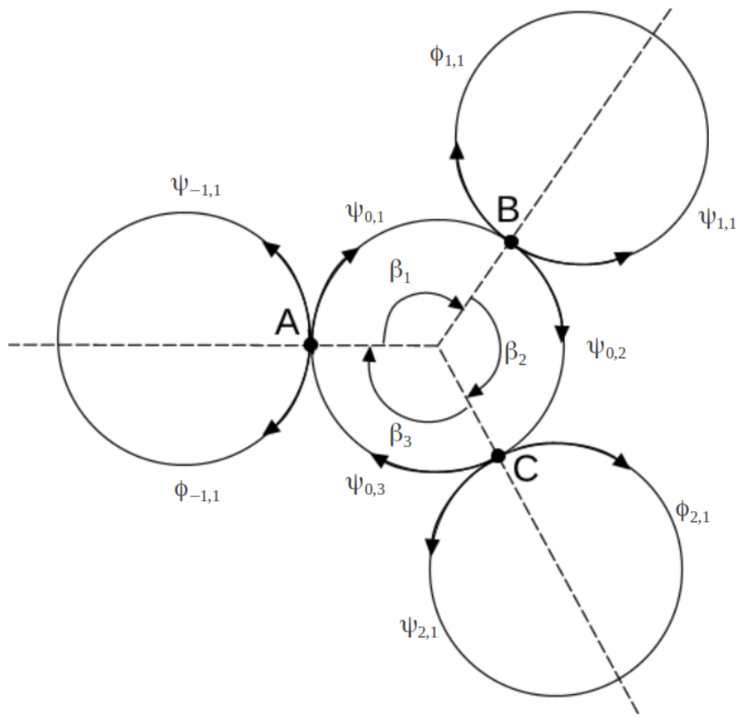

Рис. 1: Схема разветвления

$\beta_{1,2,3} \in (0, \pi)$,
$\sin(\beta_j) \neq 0$, $\cos(\beta_j) \neq 1$
Решения на базовом кольце:
$\psi_{0,1}(x) = e^{-i\Phi x}\left(A_1^+ e^{ikx} + A_1^- e^{-ikx}\right)$
$\psi_{0,2}(x) = e^{-i\Phi x}\left(A_2^+ e^{ikx} + A_2^- e^{-ikx}\right)$
$\psi_{0,3}(x) = e^{-i\Phi x}\left(A_3^+ e^{ikx} + A_3^- e^{-ikx}\right)$
Решения на ветвях:
$\psi_{j,m}(x) = e^{i\Phi x}\left(C_{j,m}^+ e^{ikx} + C_{j,m}^- e^{-ikx}\right)$
$\phi_{j,m}(x) = e^{-i\Phi x}\left(D_{j,m}^+ e^{ikx} + D_{j,m}^- e^{-ikx}\right)$
$\begin{pmatrix} D_{j,m}^+ \\ D_{j,m}^- \end{pmatrix} = S(M) \begin{pmatrix} C_{j,m}^+ \\ C_{j,m}^- \end{pmatrix}$
Условия дельта-соединения

$$
\begin{aligned}
&\psi_{0,1}(0)=\psi_{0,3}(\beta_3)=\psi_{-1,1}(0),\\
&i\hat{p}\psi_{-1,1}(0)+i\hat{p}\varphi_{-1,1}(0)-i\hat{p}\psi_{0,3}(\beta_3)+i\hat{p}\psi_{0,1}(0)=\alpha\cdot\psi_{-1,1}(0),\\
&\psi_{0,2}(0)=\psi_{0,1}(\beta_1)=\psi_{1,1}(0),\\
&i\hat{p}\psi_{1,1}(0)+i\hat{p}\varphi_{1,1}(0)-i\hat{p}\psi_{0,1}(\beta_1)+i\hat{p}\psi_{0,2}(0)=\alpha\cdot\psi_{1,1}(0),\\
&\psi_{0,3}(0)=\psi_{0,2}(\beta_2)=\psi_{2,1}(0),\\
&i\hat{p}\psi_{2,1}(0)+i\hat{p}\varphi_{2,1}(0)-i\hat{p}\psi_{0,2}(\beta_2)+i\hat{p}\psi_{0,3}(0)=\alpha\cdot\psi_{2,1}(0).
\end{aligned}
\tag{1}
$$

Вводим обозначения:

$$
\begin{aligned}
&\psi_{-1,1}(0)=u\neq 0,\\
&\alpha\psi_{-1,1}(0)-i\hat{p}\psi_{-1,1}(0)+i\hat{p}\varphi_{-1,1}(0)=v.
\end{aligned}
$$

Линейная зависимость решений на ветвях:

$$
\begin{aligned}
&\psi_{1,1}(x)=\gamma_1\cdot\psi_{-1,1}(x),\quad x\in[0,\pi],\\
&\psi_{2,1}(x)=\gamma_2\cdot\psi_{-1,1}(x),\quad x\in[0,\pi],
\end{aligned}
$$

$$
\begin{aligned}
&\varphi_{1,1}(x)=\gamma_1\cdot\varphi_{-1,1}(x),\quad x\in[0,\pi],\\
&\varphi_{2,1}(x)=\gamma_2\cdot\varphi_{-1,1}(x),\quad x\in[0,\pi],
\end{aligned}
$$

где $\gamma_{1,2}\in\mathbb{C}\setminus\{0\}$.
С учетом новых обозначений получаем:

$$
\begin{aligned}
&\psi_{1,1}(0)=\gamma_1\cdot\psi_{-1,1}(0)=\gamma_1\cdot u\\
&\psi_{2,1}(0)=\gamma_2\cdot\psi_{-1,1}(0)=\gamma_2\cdot u\\
&\alpha\cdot\psi_{1,1}-i\hat{p}\psi_{1,1}(0)-i\hat{p}\varphi_{1,1}(0)=\gamma_1(\alpha\cdot\psi_{-1,1}-i\hat{p}\psi_{-1,1}(0)-i\hat{p}\varphi_{-1,1}(0))=\gamma_1\cdot v\\
&\alpha\cdot\psi_{2,1}-i\hat{p}\psi_{2,1}(0)-i\hat{p}\varphi_{2,1}(0)=\gamma_2(\alpha\cdot\psi_{-1,1}-i\hat{p}\psi_{-1,1}(0)-i\hat{p}\varphi_{-1,1}(0))=\gamma_2\cdot v
\end{aligned}
$$

В результате система уравнений (1) принимает более удобный вид:

$$
\begin{aligned}
&\psi_{0,1}(0)=\psi_{0,3}(\beta_3)=u,\\
&i\hat{p}\psi_{0,1}(0)-i\hat{p}\psi_{0,3}(\beta_3)=v,\\
&\psi_{0,2}(0)=\psi_{0,1}(\beta_1)=\gamma_1\cdot u,\\
&i\hat{p}\psi_{0,2}(0)-i\hat{p}\psi_{0,1}(\beta_1)=\gamma_1\cdot v,\\
&\psi_{0,3}(0)=\psi_{0,2}(\beta_2)=\gamma_2\cdot u\\
&i\hat{p}\psi_{0,3}(0)-i\hat{p}\psi_{0,2}(\beta_2)=\gamma_2\cdot v.
\end{aligned}
\tag{2}
$$

Действие оператора импульса на ветвях:

$i\hat{p}\psi_{j,m}(x) = ike^{i\Phi x}(C_{j,m}^{+}e^{ikx} - C_{j,m}^{-}e^{-ikx})$ - для дуг против часовой стрелки.

$i\hat{p}\varphi_{j,m}(x) = ike^{-i\Phi x}(D^{+}e^{ikx} - D^{-}e^{-ikx})$- для дуг по часовой стрелки.

Действие оператора импульса на базовом кольце (дуги по часовой стрелке):

$i\hat{p}\psi_{0,m}(x) = ike^{-i\Phi x}(A_m^{+}e^{ikx} - A_m^{-}e^{-ikx})$.

Применение

$$\psi_{0,1}(0) = \psi_{0,3}(\beta_3) = u,$$
$$ik(A_1^{+} - C_{0,1}^{-}) - ike^{-i\Phi\beta_3}(A_3^{+}e^{ik\beta_3} - A_3^{-}e^{-ik\beta_3}) = v,$$
$$\psi_{0,2}(0) = \psi_{0,1}(\beta_1) = \gamma_1 \cdot u,$$
$$ik(A_2^{+} - A_2^{-}) - ike^{-i\Phi\beta_1}(A_1^{+}e^{ik\beta_1} - A_1^{-}e^{-ik\beta_1}) = \gamma_1 \cdot v,$$
$$\psi_{0,3}(0) = \psi_{0,2}(\beta_2) = \gamma_2 \cdot u$$
$$ik(A_3^{+} - A_3^{-}) - ike^{-i\Phi\beta_2}(A_2^{+}e^{ik\beta_2} - A_2^{-}e^{-ik\beta_2}) = \gamma_2 \cdot v.$$

Решения на базовом кольце:

$\psi_{0,1}(x) = e^{-i\Phi x}\left(A_1^{+}e^{ikx} + A_1^{-}e^{-ikx}\right)$

$\psi_{0,2}(x) = e^{-i\Phi x}\left(A_2^{+}e^{ikx} + A_2^{-}e^{-ikx}\right)$

$\psi_{0,3}(x) = e^{-i\Phi x}\left(A_3^{+}e^{ikx} + A_3^{-}e^{-ikx}\right)$

1.1. $\psi_{0,1}(\beta_1) = \gamma_1 u = \gamma_1\psi_{0,1}(0)$,

$e^{-i\Phi\beta_1}\left(A_1^{+}e^{ik\beta_1} + A_1^{-}e^{-ik\beta_1}\right) = \gamma_1\left(A_1^{+} + A_1^{-}\right)$,

$A_1^{+}e^{-i\Phi\beta_1}e^{ik\beta_1} + A_1^{-}e^{-i\Phi\beta_1}e^{-ik\beta_1} = \gamma_1\left(A_1^{+} + A_1^{-}\right)$,

$A_1^{+}\left(e^{-i\Phi\beta_1}e^{ik\beta_1} - \gamma_1\right) + A_1^{-}\left(e^{-i\Phi\beta_1}e^{-ik\beta_1}\gamma_1\right) = 0$,

$A_1^{-}\left(e^{-i\Phi\beta_1}e^{-ik\beta_1} - \gamma_1\right) = -A_1^{+}\left(e^{-i\Phi\beta_1}e^{ik\beta_1} - \gamma_1\right)$,

$A_1^{-} = -A_1^{+}\frac{e^{-i\Phi\beta_1}e^{ik\beta_1} - \gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1} - \gamma_1}$

1.2. $\psi_{0,2}(\beta_2) = \gamma_2 u = \frac{\gamma_2}{\gamma_1}\psi_{0,2}(0)$,

$A_2^{-} = -A_2^{+}\frac{e^{-i\Phi\beta_2}e^{ik\beta_2} - \frac{\gamma_2}{\gamma_1}}{e^{-i\Phi\beta_2}e^{-ik\beta_2} - \frac{\gamma_2}{\gamma_1}}$

1.3. $\psi_{0,3}(\beta_3) = u = \frac{1}{\gamma_2}\psi_{0,3}(0)$,

$A_3^{-} = -A_3^{+}\frac{e^{-i\Phi\beta_3}e^{ik\beta_3} - \frac{1}{\gamma_2}}{e^{-i\Phi\beta_3}e^{-ik\beta_3} - \frac{1}{\gamma_2}}$

2.1. $\psi_{0,1}(0)=u$
$A_1^+ + A_1^- = u$
$A_1^+ - A_1^+ \frac{e^{-i\Phi\beta_1}e^{ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1} = u$
$A_1^+ \left(1-\frac{e^{-i\Phi\beta_1}e^{ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1}\right) = u$
$A_1^+ \left(\frac{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1-e^{-i\Phi\beta_1}e^{ik\beta_1}+\gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1}\right) = u$
$A_1^+ \left(\frac{e^{-i\Phi\beta_1}e^{-ik\beta_1}-e^{-i\Phi\beta_1}e^{ik\beta_1}}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1}\right) = u$
$A_1^+ \frac{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}-e^{ik\beta_1}\right)}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1} = u$
$A_1^+ = u\frac{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}-e^{ik\beta_1}\right)}$
2.2. $\psi_{0,2}(0)=\gamma_1 u$
$A_2^+ + A_2^- = \gamma_1 u$
$A_2^+ - A_2^+ \frac{e^{-i\Phi\beta_2}e^{ik\beta_2}-\frac{\gamma_2}{\gamma_1}}{e^{-i\Phi\beta_2}e^{-ik\beta_2}-\frac{\gamma_2}{\gamma_1}} = \gamma_1 u$
$A_2^+ \frac{e^{-i\Phi\beta_2}\left(e^{-ik\beta_2}-e^{ik\beta_2}\right)}{e^{-i\Phi\beta_2}e^{-ik\beta_2}-\frac{\gamma_2}{\gamma_1}} = \gamma_1 u$
$A_2^+ = \gamma_1 u\frac{e^{-i\Phi\beta_2}e^{-ik\beta_2}-\frac{\gamma_2}{\gamma_1}}{e^{-i\Phi\beta_2}\left(e^{-ik\beta_2}-e^{ik\beta_2}\right)}$
2.3. $\psi_{0,3}(0)=\gamma_2 u$
$A_3^+ + A_3^- = \gamma_2 u$
$A_3^+ = \gamma_2 u\frac{e^{-i\Phi\beta_3}e^{-ik\beta_3}-\frac{1}{\gamma_2}}{e^{-i\Phi\beta_3}\left(e^{-ik\beta_3}-e^{ik\beta_3}\right)}$
Теперь коэффициенты компонент волновой фунции центрально кольца оказываются выраженными через значение компоненты $\psi_{0,1}(0)=u$ и два неизвестных множителя $\gamma_{1,2}$.
3.1. $i\hat{p}\psi_{0,1}(0)=ik(A_1^+ - A_1^-)$
$i\hat{p}\psi_{0,1}(0)=ik(A_1^+ + A_1^+\frac{e^{-i\Phi\beta_1}e^{ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1})$
$i\hat{p}\psi_{0,1}(0)=ik\frac{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1+e^{-i\Phi\beta_1}e^{ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1})$
$i\hat{p}\psi_{0,1}(0)=ikA_1^+\frac{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}+e^{ik\beta_1}\right)-2\gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1}$
$i\hat{p}\psi_{0,1}(0)=iku\frac{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}-e^{ik\beta_1}\right)}\cdot\frac{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}+e^{ik\beta_1}\right)-2\gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1}$
$i\hat{p}\psi_{0,1}(0)=iku\frac{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}+e^{ik\beta_1}\right)-2\gamma_1}{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}-e^{ik\beta_1}\right)}$
3.2. $i\hat{p}\psi_{0,2}(0)=ik\gamma_1 u\frac{e^{-i\Phi\beta_2}\left(e^{-ik\beta_2}+e^{ik\beta_2}\right)-2\frac{\gamma_2}{\gamma_1}}{e^{-i\Phi\beta_2}\left(e^{-ik\beta_2}-e^{ik\beta_2}\right)}$
3.3. $i\hat{p}\psi_{0,3}(0)=ik\gamma_2 u\frac{e^{-i\Phi\beta_3}\left(e^{-ik\beta_3}+e^{ik\beta_3}\right)-2\frac{1}{\gamma_2}}{e^{-i\Phi\beta_3}\left(e^{-ik\beta_3}-e^{ik\beta_3}\right)}$

4.1. $i\hat{p}\psi_{0,1}(\beta_1) = ike^{-i\Phi\beta_1}(A_1^+ e^{ik\beta_1} - A_1^- e^{-ik\beta_1})$

$i\hat{p}\psi_{0,1}(\beta_1) = ike^{-i\Phi\beta_1}(A_1^+ e^{ik\beta_1} + A_1^+ \frac{e^{-i\Phi\beta_1}e^{ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1} e^{-ik\beta_1})$

$i\hat{p}\psi_{0,1}(\beta_1) = ike^{-i\Phi\beta_1}A_1^+ \left(e^{ik\beta_1} + \frac{e^{-i\Phi\beta_1}e^{ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1} e^{-ik\beta_1}\right)$

$i\hat{p}\psi_{0,1}(\beta_1) = ike^{-i\Phi\beta_1}u\frac{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}-e^{ik\beta_1}\right)} \left(e^{ik\beta_1} + \frac{e^{-i\Phi\beta_1}e^{ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1} e^{-ik\beta_1}\right)$

$i\hat{p}\psi_{0,1}(\beta_1) = ike^{-i\Phi\beta_1}u\frac{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1}{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}-e^{ik\beta_1}\right)} \cdot \frac{e^{ik\beta_1}\left(e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1\right)+e^{-ik\beta_1}\left(e^{-i\Phi\beta_1}e^{ik\beta_1}-\gamma_1\right)}{e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1}$

$i\hat{p}\psi_{0,1}(\beta_1) = ike^{-i\Phi\beta_1}u\frac{e^{ik\beta_1}\left(e^{-i\Phi\beta_1}e^{-ik\beta_1}-\gamma_1\right)+e^{-ik\beta_1}\left(e^{-i\Phi\beta_1}e^{ik\beta_1}-\gamma_1\right)}{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}-e^{ik\beta_1}\right)}$

$i\hat{p}\psi_{0,1}(\beta_1) = ike^{-i\Phi\beta_1}u\frac{\left(e^{-i\Phi\beta_1}-e^{ik\beta_1}\gamma_1\right)+\left(e^{-i\Phi\beta_1}-e^{-ik\beta_1}\gamma_1\right)}{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}-e^{ik\beta_1}\right)}$

$i\hat{p}\psi_{0,1}(\beta_1) = ike^{-i\Phi\beta_1}u\frac{2e^{-i\Phi\beta_1}-\left(e^{ik\beta_1}+e^{-ik\beta_1}\right)\gamma_1}{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}-e^{ik\beta_1}\right)}$

4.2. $i\hat{p}\psi_{0,2}(\beta_2) = ike^{-i\Phi\beta_2}(A_2^+ e^{ik\beta_2} - A_2^- e^{-ik\beta_2})$

$i\hat{p}\psi_{0,2}(\beta_2) = ik\gamma_1 e^{-i\Phi\beta_2}u\frac{2e^{-i\Phi\beta_2}-\left(e^{ik\beta_2}+e^{-ik\beta_2}\right)\frac{\gamma_2}{\gamma_1}}{e^{-i\Phi\beta_2}\left(e^{-ik\beta_2}-e^{ik\beta_2}\right)}$

4.3. $i\hat{p}\psi_{0,3}(\beta_3) = ike^{-i\Phi\beta_3}(A_3^+ e^{ik\beta_3} - A_3^- e^{-ik\beta_3})$

$i\hat{p}\psi_{0,3}(\beta_3) = ik\gamma_2 e^{-i\Phi\beta_3}u\frac{2e^{-i\Phi\beta_3}-\left(e^{ik\beta_3}+e^{-ik\beta_3}\right)\frac{1}{\gamma_2}}{e^{-i\Phi\beta_3}\left(e^{-ik\beta_3}-e^{ik\beta_3}\right)}$

Используем условия дельта-соединения для производных в точках контакта базового кольца.

5.1. $i\hat{p}\psi_{0,1}(0)-i\hat{p}\psi_{0,3}(\beta_3)=v$

$iku\frac{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}+e^{ik\beta_1}\right)-2\gamma_1}{e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}-e^{ik\beta_1}\right)}-ik\gamma_2e^{-i\Phi\beta_3}u\frac{2e^{-i\Phi\beta_3}-\left(e^{ik\beta_3}+e^{-ik\beta_3}\right)\frac{1}{\gamma_2}}{e^{-i\Phi\beta_3}\left(e^{-ik\beta_3}-e^{ik\beta_3}\right)}=v$

$iku\frac{2\gamma_1-e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}+e^{ik\beta_1}\right)}{e^{-i\Phi\beta_1}\left(e^{ik\beta_1}-e^{-ik\beta_1}\right)}+ike^{-i\Phi\beta_3}u\frac{2\gamma_2e^{-i\Phi\beta_3}-\left(e^{ik\beta_3}+e^{-ik\beta_3}\right)}{e^{-i\Phi\beta_3}\left(e^{ik\beta_3}-e^{-ik\beta_3}\right)}=v$

$i\frac{2\gamma_1-e^{-i\Phi\beta_1}\left(e^{-ik\beta_1}+e^{ik\beta_1}\right)}{e^{-i\Phi\beta_1}\left(e^{ik\beta_1}-e^{-ik\beta_1}\right)}+ie^{-i\Phi\beta_3}\frac{2\gamma_2e^{-i\Phi\beta_3}-\left(e^{ik\beta_3}+e^{-ik\beta_3}\right)}{e^{-i\Phi\beta_3}\left(e^{ik\beta_3}-e^{-ik\beta_3}\right)}=\frac{v}{ku}$

$i\left(\frac{e^{+i\Phi\beta_1}2\gamma_1}{\left(e^{ik\beta_1}-e^{-ik\beta_1}\right)}-\frac{\left(e^{-ik\beta_1}+e^{ik\beta_1}\right)}{\left(e^{ik\beta_1}-e^{-ik\beta_1}\right)}\right)+i\left(e^{-i\Phi\beta_3}\frac{2\gamma_2}{\left(e^{ik\beta_3}-e^{-ik\beta_3}\right)}-\frac{\left(e^{ik\beta_3}+e^{-ik\beta_3}\right)}{\left(e^{ik\beta_3}-e^{-ik\beta_3}\right)}\right)=$
$\frac{v}{ku}$

$\left(\gamma_1\frac{e^{+i\Phi\beta_1}i2}{\left(e^{ik\beta_1}-e^{-ik\beta_1}\right)}-\frac{i\left(e^{-ik\beta_1}+e^{ik\beta_1}\right)}{\left(e^{ik\beta_1}-e^{-ik\beta_1}\right)}\right)+\left(\gamma_2e^{-i\Phi\beta_3}\frac{i2}{\left(e^{ik\beta_3}-e^{-ik\beta_3}\right)}-\frac{i\left(e^{ik\beta_3}+e^{-ik\beta_3}\right)}{\left(e^{ik\beta_3}-e^{-ik\beta_3}\right)}\right)=$
$\frac{v}{ku}$

$\left(\gamma_1e^{+i\Phi\beta_1}\frac{i2}{\left(e^{ik\beta_1}-e^{-ik\beta_1}\right)}-\frac{i\left(e^{-ik\beta_1}+e^{ik\beta_1}\right)}{\left(e^{ik\beta_1}-e^{-ik\beta_1}\right)}\right)+\left(\gamma_2e^{-i\Phi\beta_3}\frac{i2}{\left(e^{ik\beta_3}-e^{-ik\beta_3}\right)}-\frac{i\left(e^{ik\beta_3}+e^{-ik\beta_3}\right)}{\left(e^{ik\beta_3}-e^{-ik\beta_3}\right)}\right)=$
$\frac{v}{ku}$

$\left(\gamma_1e^{+i\Phi\beta_1}\frac{i2}{\left(e^{ik\beta_1}-e^{-ik\beta_1}\right)}-\frac{i\left(e^{-ik\beta_1}+e^{ik\beta_1}\right)}{\left(e^{ik\beta_1}-e^{-ik\beta_1}\right)}\right)+\left(\gamma_2e^{-i\Phi\beta_3}\frac{i2}{\left(e^{ik\beta_3}-e^{-ik\beta_3}\right)}-\frac{i\left(e^{ik\beta_3}+e^{-ik\beta_3}\right)}{\left(e^{ik\beta_3}-e^{-ik\beta_3}\right)}\right)=$
$\frac{v}{ku}$

$\gamma_1e^{+i\Phi\beta_1}\frac{1}{\sin(k\beta_1)}-\cot\left(k\beta_1\right)+\gamma_2e^{-i\Phi\beta_3}\frac{1}{\sin(k\beta_3)}-\cot\left(k\beta_3\right)=\frac{v}{ku}$

$\gamma_1e^{+i\Phi\beta_1}\frac{1}{\sin(k\beta_1)}+\gamma_2e^{-i\Phi\beta_3}\frac{1}{\sin(k\beta_3)}=\frac{v}{ku}+\cot\left(k\beta_1\right)+\cot\left(k\beta_3\right)$

$\gamma_1R\left(k,\Phi,\beta_1\right)+\gamma_2R\left(k,-\Phi,\beta_3\right)=Q\left(\beta_1,\beta_3\right),$

$R\left(k,\Phi,\beta\right)=e^{i\Phi\beta}\frac{1}{\sin(k\beta)}$

$Q\left(x,y\right)=\frac{v}{ku}+\cot\left(kx\right)+\cot\left(ky\right)$

5.2. $i\hat{p}\psi_{0,2}(0)-i\hat{p}\psi_{0,1}(\beta_1)=\gamma_1\cdot v$

$\frac{\gamma_2}{\gamma_1}R\left(k,\Phi,\beta_2\right)+\frac{1}{\gamma_1}R\left(k,-\Phi,\beta_1\right)=Q\left(\beta_2,\beta_1\right)$

5.3. $i\hat{p}\psi_{0,3}(0)-i\hat{p}\psi_{0,2}(\beta_2)=\gamma_2\cdot v$

$\frac{1}{\gamma_2}R\left(k,\Phi,\beta_3\right)+\frac{\gamma_1}{\gamma_2}R\left(k,-\Phi,\beta_2\right)=Q\left(\beta_3,\beta_2\right)$

Получаем следующую систему уравнений

$\gamma_1R\left(k,\Phi,\beta_1\right)+\gamma_2R\left(k,-\Phi,\beta_3\right)=Q\left(\beta_1,\beta_3\right)$

$\frac{\gamma_2}{\gamma_1}R\left(k,\Phi,\beta_2\right)+\frac{1}{\gamma_1}R\left(k,-\Phi,\beta_1\right)=Q\left(\beta_2,\beta_1\right)$

$\frac{1}{\gamma_2}R\left(k,\Phi,\beta_3\right)+\frac{\gamma_1}{\gamma_2}R\left(k,-\Phi,\beta_2\right)=Q\left(\beta_3,\beta_2\right)$

$\tilde{\alpha} = \frac{v}{u} = \frac{i\hat{p}\psi_{0,1}(0) - i\hat{p}\psi_{0,3}(\beta_3)}{u}$
$\tilde{\alpha} = \frac{ik\left(A_1^+ - A_1^-\right) - ike^{-i\Phi\beta_3}(A_3^+ e^{ik\beta_3} - A_3^- e^{-ik\beta_3})}{u}$
$\tilde{\alpha} = \frac{ik\left(u\frac{\gamma_1 e^{i\Phi\beta_1} - e^{-ik\beta_1}}{e^{ik\beta_1} - e^{-ik\beta_1}} + u\frac{\gamma_1 e^{i\Phi\beta_1} - e^{+ik\beta_1}}{e^{ik\beta_1} - e^{-ik\beta_1}}\right) - ike^{-i\Phi\beta_3}\left(u\frac{e^{i\Phi\beta_3} - \gamma_2 e^{-ik\beta_3}}{e^{ik\beta_3} - e^{-ik\beta_3}}e^{ik\beta_3} + u\frac{e^{i\Phi\beta_3} - \gamma_2 e^{+ik\beta_3}}{e^{ik\beta_3} - e^{-ik\beta_3}}e^{-ik\beta_3}\right)}{u}$
$\tilde{\alpha} = ik\left\{\frac{\gamma_1 e^{i\Phi\beta_1} - e^{-ik\beta_1} + \gamma_1 e^{i\Phi\beta_1} - e^{+ik\beta_1}}{e^{ik\beta_1} - e^{-ik\beta_1}} - e^{-i\Phi\beta_3}\frac{e^{i\Phi\beta_3}e^{ik\beta_3} - \gamma_2 + e^{i\Phi\beta_3}e^{-ik\beta_3} - \gamma_2}{e^{ik\beta_3} - e^{-ik\beta_3}}\right\}$
$\tilde{\alpha} = ik\left\{\frac{2\gamma_1 e^{i\Phi\beta_1} - e^{+ik\beta_1} - e^{-ik\beta_1}}{e^{ik\beta_1} - e^{-ik\beta_1}} - e^{-i\Phi\beta_3}\frac{e^{i\Phi\beta_3}e^{ik\beta_3} + e^{i\Phi\beta_3}e^{-ik\beta_3} - 2\gamma_2}{e^{ik\beta_3} - e^{-ik\beta_3}}\right\}$
$\tilde{\alpha} = ik\left\{\frac{2\gamma_1 e^{i\Phi\beta_1} - e^{+ik\beta_1} - e^{-ik\beta_1}}{e^{ik\beta_1} - e^{-ik\beta_1}} - \frac{e^{ik\beta_3} + e^{-ik\beta_3} - 2\gamma_2 e^{-i\Phi\beta_3}}{e^{ik\beta_3} - e^{-ik\beta_3}}\right\}$
$\tilde{\alpha} = ik\left\{\frac{2\gamma_1 e^{i\Phi\beta_1} - e^{+ik\beta_1} - e^{-ik\beta_1}}{e^{ik\beta_1} - e^{-ik\beta_1}} + \frac{2\gamma_2 e^{-i\Phi\beta_3} - e^{ik\beta_3} - e^{-ik\beta_3}}{e^{ik\beta_3} - e^{-ik\beta_3}}\right\}$
$\tilde{\alpha} = k\left\{\frac{i2\gamma_1 e^{i\Phi\beta_1}}{e^{ik\beta_1} - e^{-ik\beta_1}} - \frac{i\left(e^{+ik\beta_1} + e^{-ik\beta_1}\right)}{e^{ik\beta_1} - e^{-ik\beta_1}} + \frac{i2\gamma_2 e^{-i\Phi\beta_3}}{e^{ik\beta_3} - e^{-ik\beta_3}} - \frac{i\left(e^{ik\beta_3} + e^{-ik\beta_3}\right)}{e^{ik\beta_3} - e^{-ik\beta_3}}\right\}$
$\tilde{\alpha} = k\left\{\frac{\gamma_1 e^{i\Phi\beta_1}}{\sin(k\beta_1)} - \cot(k\beta_1) + \frac{\gamma_2 e^{-i\Phi\beta_3}}{\sin(k\beta_3)} - \cot(k\beta_3)\right\}$
$\frac{v}{ku} = \frac{\tilde{\alpha}}{k} = \frac{\gamma_1 e^{i\Phi\beta_1}}{\sin(k\beta_1)} - \cot(k\beta_1) + \frac{\gamma_2 e^{-i\Phi\beta_3}}{\sin(k\beta_3)} - \cot(k\beta_3) =$
$= \gamma_1 e^{i\Phi\beta_1}\frac{1}{\sin(k\beta_1)} - \cot(k\beta_1) + \gamma_2 e^{-i\Phi\beta_3}\frac{1}{\sin(k\beta_3)} - \cot(k\beta_3)$
$R(k, \Phi, \beta) = e^{i\Phi\beta}\frac{1}{\sin(k\beta)}$
$R_j = \frac{1}{\sin(k\beta_j)}$
$Q(x, y) = \frac{v}{ku} + \cot(kx) + \cot(ky)$
$Q_{m,n} = Q(\beta_m, \beta_n) = \frac{v}{ku} + \cot(k\beta_m) + \cot(k\beta_n)$
.
$Q_{m,n} = Q(\beta_m, \beta_n) = \frac{v}{ku} + \cot(k\beta_m) + \cot(k\beta_m) =$
$= \gamma_1 e^{i\Phi\beta_1}R_1 - \cot(k\beta_1) + \gamma_2 e^{-i\Phi\beta_3}R_3 - \cot(k\beta_3) + \cot(k\beta_m) + \cot(k\beta_m) =$
$= \gamma_1 e^{i\Phi\beta_1}R_1 + \gamma_2 e^{-i\Phi\beta_3}R_3 + \cot(k\beta_m) + \cot(k\beta_n) - \cot(k\beta_1) - \cot(k\beta_3)$
.
$Q_{m,n} = \gamma_1 e^{i\Phi\beta_1}R_1 + \gamma_2 e^{-i\Phi\beta_3}R_3 - \cot(k\beta_1) - \cot(k\beta_3) + \cot(k\beta_m) + \cot(k\beta_n)$
Перепишем предыдущую систему уравнений
$\gamma_1 R(k, \Phi, \beta_1) + \gamma_2 R(k, -\Phi, \beta_3) = Q(\beta_1, \beta_3)$
$\frac{\gamma_2}{\gamma_1}R(k, \Phi, \beta_2) + \frac{1}{\gamma_1}R(k, -\Phi, \beta_1) = Q(\beta_2, \beta_1)$
$\frac{1}{\gamma_2}R(k, \Phi, \beta_3) + \frac{\gamma_1}{\gamma_2}R(k, -\Phi, \beta_2) = Q(\beta_3, \beta_2)$
.
$\gamma_1 R(k, \Phi, \beta_1) + \gamma_2 R(k, -\Phi, \beta_3) = \gamma_1 e^{i\Phi\beta_1}R_1 + \gamma_2 e^{-i\Phi\beta_3}R_3 - \cot(k\beta_1) - \cot(k\beta_3) + \cot(k\beta_1) + \cot(k\beta_3)$
$\frac{\gamma_2}{\gamma_1}R(k, \Phi, \beta_2) + \frac{1}{\gamma_1}R(k, -\Phi, \beta_1) = \gamma_1 e^{i\Phi\beta_1}R_1 + \gamma_2 e^{-i\Phi\beta_3}R_3 - \cot(k\beta_1) - \cot(k\beta_3) + \cot(k\beta_2) + \cot(k\beta_1)$
$\frac{1}{\gamma_2}R(k, \Phi, \beta_3) + \frac{\gamma_1}{\gamma_2}R(k, -\Phi, \beta_2) = \gamma_1 e^{i\Phi\beta_1}R_1 + \gamma_2 e^{-i\Phi\beta_3}R_3 - \cot(k\beta_1) - \cot(k\beta_3) + \cot(k\beta_3) + \cot(k\beta_2)$
.
$\gamma_1 R(k, \Phi, \beta_1) + \gamma_2 R(k, -\Phi, \beta_3) = \gamma_1 e^{i\Phi\beta_1}R_1 + \gamma_2 e^{-i\Phi\beta_3}R_3$ - тождество?
$\frac{\gamma_2}{\gamma_1}R(k, \Phi, \beta_2) + \frac{1}{\gamma_1}R(k, -\Phi, \beta_1) = \gamma_1 e^{i\Phi\beta_1}R_1 + \gamma_2 e^{-i\Phi\beta_3}R_3 + \cot(k\beta_2) - \cot(k\beta_3)$
$\frac{1}{\gamma_2}R(k, \Phi, \beta_3) + \frac{\gamma_1}{\gamma_2}R(k, -\Phi, \beta_2) = \gamma_1 e^{i\Phi\beta_1}R_1 + \gamma_2 e^{-i\Phi\beta_3}R_3 + \cot(k\beta_2) - \cot(k\beta_1)$

$\frac{\gamma_2}{\gamma_1}R_2^+ + \frac{1}{\gamma_1}R_1^- = \gamma_1 R_1^+ + \gamma_2 R_3^- + \cot(k\beta_2) - \cot(k\beta_3)$
$\frac{1}{\gamma_2}R_3^+ + \frac{\gamma_1}{\gamma_2}R_2^- = \gamma_1 R_1^+ + \gamma_2 R_3^- + \cot(k\beta_2) - \cot(k\beta_1)$
.
$\gamma_2 R_2^+ + R_1^- = \gamma_1^2 R_1^+ + \gamma_1\gamma_2 R_3^- + \gamma_1(\cot(k\beta_2) - \cot(k\beta_3))$
$R_3^+ + \gamma_1 R_2^- = \gamma_1\gamma_2 R_1^+ + \gamma_2^2 R_3^- + \gamma_2(\cot(k\beta_2) - \cot(k\beta_1))$
.
$\gamma_1^2 R_1^+ + \gamma_1\gamma_2 R_3^- + \gamma_1(\cot(k\beta_2) - \cot(k\beta_3)) - \gamma_2 R_2^+ - R_1^- = 0$
$\gamma_2^2 R_3^- + \gamma_1\gamma_2 R_1^+ + \gamma_2(\cot(k\beta_2) - \cot(k\beta_1)) - \gamma_1 R_2^- - R_3^+ = 0$
Получили два уравнения кривых 2-го порядка
.
Рассмотрим первое уравнение системы
$\gamma_1^2 R_1^+ + \gamma_1\gamma_2 R_3^- + \gamma_1(\cot(k\beta_2) - \cot(k\beta_3)) - \gamma_2 R_2^+ - R_1^- = 0$
$a_{11}x^2 + a_{22}y^2 + 2a_{12}xy + 2a_{13}x + 2a_{23}y + a_{33} = 0$
$a_{11} = R_1^+$
$a_{22} = 0$
$a_{12} = \frac{1}{2}R_3^-$
$a_{13} = \frac{1}{2}(\cot(k\beta_2) - \cot(k\beta_3))$
$a_{23} = -\frac{1}{2}R_2^+$
$a_{33} = -R_1^-$
.
$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}^2$$
$$D = -\left(\frac{1}{2}R_3^-\right)^2 = -\frac{1}{4}\left(R_3^-\right)^2 = -\frac{1}{4}\left(e^{i\Phi\beta_3}\frac{1}{\sin(k\beta_3)}\right)^2 = -\frac{1}{4}e^{2i\Phi\beta_3}\frac{1}{\sin^2(k\beta_3)}$$
$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} = -a_{11}a_{23}^2 - a_{12}^2 a_{23} = -a_{23}\left(a_{11}a_{23} - a_{12}^2\right) =$$
$$= -\frac{1}{2}R_2^+\left(-\frac{1}{2}R_1^+R_2^+ - \left(\frac{1}{2}R_3^-\right)^2\right) = \frac{1}{4}R_2^+\left(R_1^+R_2^+ + \frac{1}{2}\left(R_3^-\right)^2\right)$$
$$I = \operatorname{tr}\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} = a_{11} + a_{22} = R_1^+$$