



Figure 1:

Let now consider the straight semi-infinite chain of rings with delta-coupling imposed at each vertex. Wave function components on j -th ring are as follows:

$$\psi_j(x) = (C_j^+ e^{ikx} + C_j^- e^{-ikx}) e^{-i\Phi x}$$

$$\phi_j(x) = (D_j^+ e^{ikx} + D_j^- e^{-ikx}) e^{i\Phi x}$$

At the boundary points those components have following values for ψ_j

$$\psi_j(0) = C_j^+ + C_j^-,$$

$$\psi_j(\pi) = (C_j^+ e^{ik\pi} + C_j^- e^{-ik\pi}) e^{-i\Phi\pi},$$

and for ϕ_j

$$\phi_j(0) = C_j^+ + C_j^-,$$

$$\phi_j(\pi) = (C_j^+ e^{ik\pi} + C_j^- e^{-ik\pi}) e^{-i\Phi\pi}.$$

As delta-coupling demands the components values at the contact points of the ring must be equal to each other:

$$\psi_j(0) = \phi_j(0)$$

and

$$\psi_j(\pi) = \phi_j(\pi).$$

Solving this pair of equations we arrive at the relation between coefficients:

$$D_j^+ = (e^{ik\pi} - e^{-ik\pi})^{-1} e^{-i\Phi\pi} \left(C_j^+ (e^{i(k-\Phi)\pi} - e^{-i(k-\Phi)\pi}) + C_j^- e^{-ik\pi} (e^{-i\Phi\pi} - e^{i\Phi\pi}) \right),$$

$$D_j^- = (e^{ik\pi} - e^{-ik\pi})^{-1} e^{-i\Phi\pi} \left(C_j^+ e^{ik\pi} (e^{-i\Phi\pi} - e^{i\Phi\pi}) + C_j^- (e^{-i(k+\Phi)\pi} - e^{i(k+\Phi)\pi}) \right).$$