In[394]:=
```
(**)
Psi[x_] = e^(-i*Φ*x) * {e^(i*k*x), e^(-i*k*x)}
Phi[x_] = e^(i*Φ*x) * {e^(i*k*x), e^(-i*k*x)}

PPsi[x_] = k*e^(-i*Φ*x) * {e^(i*k*x), -e^(-i*k*x)}
PPhi[x_] = k*e^(i*Φ*x) * {e^(i*k*x), -e^(-i*k*x)}

(*Решения на верхней и нижней дугах с к-нтами A, B *)
psi[x_, A_, B_] = Psi[x].{A, B}
phi[x_, A_, B_] = Phi[x].{A, B}

(*Действие оператора импульса на решения*)
pPsi[x_, A_, B_] = PPsi[x].{A, B}
pPhi[x_, A_, B_] = PPhi[x].{A, B}

psi[0, Cp, Cm] == phi[0, Dp, Dm];
psi[π, Cp, Cm] == phi[π, Dp, Dm];

Solve[psi[0, Cp, Cm] == phi[0, Dp, Dm] &&
  psi[π, Cp, Cm] == phi[π, Dp, Dm], {Dp, Dm}]

S0 = e^(-i*Φ*π) (e^(i*k*π) - e^(-i*k*π))^-1 *
  {{e^(i*k*π) e^(-i*Φ*π) - e^(-i*k*π) e^(i*Φ*π),
    e^(-i*k*π) e^(-i*Φ*π) - e^(-i*k*π) e^(i*Φ*π)},
   {e^(i*k*π) e^(-i*Φ*π) - e^(i*k*π) e^(i*Φ*π),
    e^(-i*k*π) e^(-i*Φ*π) - e^(i*k*π) e^(i*Φ*π)}}
```

Out[394]= $\left\{e^{i k x - i x \Phi}, e^{-i k x - i x \Phi}\right\}$

Out[395]= $\left\{e^{i k x + i x \Phi}, e^{-i k x + i x \Phi}\right\}$

Out[396]= $\left\{e^{i k x - i x \Phi} k, -e^{-i k x - i x \Phi} k\right\}$

Out[397]= $\left\{e^{i k x + i x \Phi} k, -e^{-i k x + i x \Phi} k\right\}$

Out[398]= $B e^{-i k x - i x \Phi} + A e^{i k x - i x \Phi}$

Out[399]= $B e^{-i k x + i x \Phi} + A e^{i k x + i x \Phi}$

Out[400]= $-B e^{-i k x - i x \Phi} k + A e^{i k x - i x \Phi} k$

Out[401]= $-B e^{-i k x + i x \Phi} k + A e^{i k x + i x \Phi} k$

Out[404]= $$\left\{\left\{Dp \to -\frac{e^{-2 i \pi \Phi}\left(-Cm - Cp\, e^{2 i k \pi} + Cm\, e^{2 i \pi \Phi} + Cp\, e^{2 i \pi \Phi}\right)}{-1 + e^{2 i k \pi}},\right.\right.$$
$$\left.\left. Dm \to \frac{e^{-2 i \pi \Phi}\left(-Cm - Cp\, e^{2 i k \pi} + Cm\, e^{2 i k \pi + 2 i \pi \Phi} + Cp\, e^{2 i k \pi + 2 i \pi \Phi}\right)}{-1 + e^{2 i k \pi}}\right\}\right\}$$

Out[405]= $$\left\{\left\{\frac{e^{-i \pi \Phi}\left(e^{i k \pi - i \pi \Phi} - e^{-i k \pi + i \pi \Phi}\right)}{-e^{-i k \pi} + e^{i k \pi}}, \frac{e^{-i \pi \Phi}\left(e^{-i k \pi - i \pi \Phi} - e^{-i k \pi + i \pi \Phi}\right)}{-e^{-i k \pi} + e^{i k \pi}}\right\},\right.$$
$$\left.\left\{\frac{e^{-i \pi \Phi}\left(e^{i k \pi - i \pi \Phi} - e^{i k \pi + i \pi \Phi}\right)}{-e^{-i k \pi} + e^{i k \pi}}, \frac{e^{-i \pi \Phi}\left(e^{-i k \pi - i \pi \Phi} - e^{i k \pi + i \pi \Phi}\right)}{-e^{-i k \pi} + e^{i k \pi}}\right\}\right\}$$

$$\left\{\left\{Dp \to -\frac{e^{-2 i \pi \Phi}\left(-Cm - Cp\, e^{2 i k \pi} + Cm\, e^{2 i \pi \Phi} + Cp\, e^{2 i \pi \Phi}\right)}{-1 + e^{2 i k \pi}},\right.\right.$$
$$\left.\left. Dm \to \frac{e^{-2 i \pi \Phi}\left(-Cm - Cp\, e^{2 i k \pi} + Cm\, e^{2 i k \pi + 2 i \pi \Phi} + Cp\, e^{2 i k \pi + 2 i \pi \Phi}\right)}{-1 + e^{2 i k \pi}}\right\}\right\}$$

```
(*Запишем матрицу S исходя из результата решения ур-я*)
```

In[388]:= $S = \left(-1 + e^{2 i k \pi}\right)$ ^-1 * $\left\{\left\{e^{2 i \pi \Phi} - e^{2 i k \pi}, e^{2 i \pi \Phi} - 1\right\}, \left\{e^{2 i k \pi + 2 i \pi \Phi} - e^{2 i k \pi}, e^{2 i k \pi + 2 i \pi \Phi} - 1\right\}\right\}$

In[414]:=

```
(* Сверим полученную матрицу S с исходной, выведенной мной*)
```

In[425]:=

```
(*Точка контакта двух колец*)
PPsi[0].{Cp1, Cm1} + PPhi[0].S.{Cp1, Cm1};
PPsi[π].{Cp0, Cm0} + PPhi[π].S.{Cp0, Cm0} - i * α * Psi[π].{Cp0, Cm0};

PPsi[0].{Cp1, Cm1} + PPhi[0].S.{Cp1, Cm1} ==
  PPsi[π].{Cp0, Cm0} + PPhi[π].S.{Cp0, Cm0} - i * α * Psi[π].{Cp0, Cm0};

Psi[π].{Cp0, Cm0} == Psi[0].{Cp1, Cm1}
Solve[Psi[π].{Cp0, Cm0} == Psi[0].{Cp1, Cm1} &&
  PPsi[0].{Cp1, Cm1} + PPhi[0].S.{Cp1, Cm1} ==
   PPsi[π].{Cp0, Cm0} + PPhi[π].S.{Cp0, Cm0} - i * α * Psi[π].{Cp0, Cm0}, {Cp1, Cm1}]
```

Out[428]= $Cm0\, e^{-i k \pi - i \pi \Phi} + Cp0\, e^{i k \pi - i \pi \Phi} == Cm1 + Cp1$

Out[429]= $$\left\{\left\{Cp1 \to \frac{1}{2 k} e^{-i k \pi - i \pi \Phi}\left(2\, Cp0\, e^{2 i k \pi} k - Cm0\, i\, \alpha - Cp0\, e^{2 i k \pi}\, i\, \alpha\right),\right.\right.$$
$$\left.\left. Cm1 \to \frac{1}{2 k} e^{-i k \pi - i \pi \Phi}\left(2\, Cm0\, k + Cm0\, i\, \alpha + Cp0\, e^{2 i k \pi}\, i\, \alpha\right)\right\}\right\}$$