

$$1.68 \quad R_x = A_x + B_x$$

$$= (170 \text{ km}) \sin 68^\circ + (230 \text{ km}) \cos 48^\circ$$

$$= 311.5 \text{ km}$$

$$R_y = A_y + B_y$$

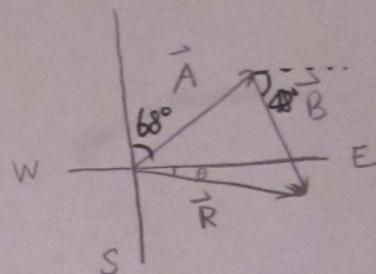
$$= (170 \text{ km}) \cos 68^\circ + (230 \text{ km}) \sin 48^\circ$$

$$= -107.2 \text{ km}$$

$$|R| = \sqrt{R_x^2 + R_y^2} = 330 \text{ km}$$

$$\tan \theta = \left| \frac{R_y}{R_x} \right| = 0.344$$

$\theta = 19^\circ$ South of east



$$1.74$$

$$\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0 \quad \vec{D} = -(\vec{A} + \vec{B} + \vec{C})$$

$$D_x = -[(147 \text{ km}) \cos 85^\circ + (106 \text{ km}) \cos 167^\circ + (166 \text{ km}) \sin 235^\circ]$$

$$= -34.3 \text{ km}$$

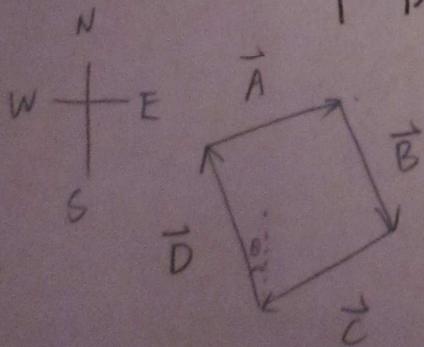
$$D_y = -[(147 \text{ km}) \sin 85^\circ + (106 \text{ km}) \sin 167^\circ + (166 \text{ km}) \cos 235^\circ]$$

$$= 185.7 \text{ km}$$

$$|D| = \sqrt{D_x^2 + D_y^2} = 189 \text{ km}$$

$$\tan \theta = \left| \frac{D_y}{D_x} \right| = 0.185$$

$\theta = 10.5^\circ$ west of north



1.94 Obtain a unit vector perpendicular to the two vectors given in Ex 1.53.

$$A = \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix}.$$

The idea is that: since any two vectors that do not lie along the same line span a two-dimensional plane, there is only one vector, up to a scalar multiple, perpendicular to that plane. We know that $\vec{A} \times \vec{B}$ is such a vector. (**Caution.** $\vec{A} \times \vec{B}$ is a vector, not a number!)

$$\begin{aligned}\vec{A} \times \vec{B} &= \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 4 \\ 3 & 1 & -3 \end{pmatrix} = \hat{i} \det \begin{pmatrix} 3 & 4 \\ 1 & -3 \end{pmatrix} - \hat{j} \det \begin{pmatrix} -2 & 4 \\ 3 & -3 \end{pmatrix} + \hat{k} \det \begin{pmatrix} -2 & 3 \\ 3 & 1 \end{pmatrix} \\ &= \hat{i}[3(-3) - 4 \cdot 1] - \hat{j}[(-2)(-3) - 4 \cdot 3] + \hat{k}[(-2)1 - 3 \cdot 3] \\ &= -13\hat{i} + 6\hat{j} - 11\hat{k}\end{aligned}$$

(It is always a good idea to check that the resulting cross product is really perpendicular to the original two vectors!) Finally, we have to normalize the vector $\vec{A} \times \vec{B}$ so that its magnitude is one, so we need to know the magnitude of $\vec{A} \times \vec{B}$.

$$|\vec{A} \times \vec{B}| = \sqrt{(-13)^2 + 6^2 + (-11)^2} = \sqrt{326}.$$

Therefore, the unit vector \hat{u} in the direction of $\vec{A} \times \vec{B}$ is
$$\boxed{-\frac{13}{\sqrt{326}}\hat{i} + \frac{6}{\sqrt{326}}\hat{j} - \frac{11}{\sqrt{326}}\hat{k}}.$$

1.96 $|A| = |B| = 3$. $A \times B = \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix}$. What is the angle between A and B ?

Recall that $|A \times B| = |A||B|\sin\theta$. $|A \times B| = \sqrt{2^2 + (-5)^2} = \sqrt{29}$ So $\sin\theta = \frac{\sqrt{29}}{9}$. This gives $\theta \approx 0.6414 \text{ rad} = 0.6414 \times \frac{360^\circ}{2\pi} \approx 36.75^\circ$.