Neutrino Cosmology

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First Day : Macrophysics of the hot Big Bang

Second Day : Microphysics of the hot Big Bang

Third Day : New views with neutrinos

Second Day

A. Thermodynamics in the Expanding Universe

B. Weak (Hot) Relics : Neutrinos

C. Weak (Cold) Relics : WIMPs

D. Strong Relics : BBN

E. Electromagnetic Relics : CMB

F. Conclusions

A. Thermodynamics in the Expanding Universe

- Equilibrium distributions:

For particles of mass m in equilibrium in a~~n~~ homogeneous, isotropic, expanding universe.

$$\text{number density} = n = \frac{g}{2\pi^2}\int_m^\infty \left[\frac{\sqrt{E^2-m^2}}{\exp[(E-\mu)/T]\pm 1}\right] E\,dE$$

$$\text{energy density} = \rho = \frac{g}{2\pi^2}\int_m^\infty \left[\qquad '' \qquad\right] E^2\,dE$$

$$\text{isotropic pressure} = p = \frac{g}{2\pi^2}\int_m^\infty \left[\qquad '' \qquad\right] \frac{(E^2-m^2)}{3}\,dE$$

$$\text{entropy density} = s = \frac{\rho + p - \mu n}{T}$$

$+$ Fermi-Dirac
$-$ Bose-Einstein

mostly neglect μ below

These quantities are all in physical coordinates (not comoving).

For $m = 0$, $T \to T/R$ under expansion.

For $m \neq 0$, more complicated, but $\to$ ~ at rest

- Laws of thermodynamics:

First Law: $d(\rho R^3) = -p\,d(R^3)$

$$\boxed{\dot{\rho} = -3H(\rho + p)}$$

Second Law: entropy in a comoving volume element is conserved, (unless it changes; see g_*, $g_{*,s}$)

$$\boxed{\underbrace{S}_{\text{entropy in unit comoving volume}} = \underbrace{s}_{\text{entropy in physical volume}}\underbrace{R^3}_{\text{due to expansion}}}$$

Can multiply by R^3 by dividing by s.

$$\frac{n_B}{s} \sim \frac{n_B}{n_\gamma} \sim \frac{n_B}{T^3} \sim n_B R^3 \sim \text{constant}$$

($\sim 6 \times 10^{-10}$, tiny)

Equation of state: $\boxed{p = w\rho}$

$w \simeq 0$ matter
$w = 1/3$ radiation
$w = -1$ lambda

- ER limit ($\mu=0$), $T \gg m$ ("radiation")

$$n = \begin{cases} (\zeta(3)/\pi^2) g T^3 & \text{BE} \\ (3/4)(\zeta(3)/\pi^2) g T^3 & \text{FD} \end{cases} \qquad \zeta(3) = 1.20$$

$$\rho = \begin{cases} (\pi^2/30)\, g T^4 & \text{BE} \\ (7/8)(\pi^2/30)\, g T^4 & \text{FD} \end{cases}$$

$$p = \frac{\rho}{3}$$

$$s = \frac{4\rho}{3T}$$

T is only scale in problem; thats why it remains thermal on expansion.

- NR limit ($\mu=0$), $m \gg T$ ("matter")

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} \exp(-m/T)$$

$$\rho = mn$$

$$p = nT \ll \rho$$

$$s = \frac{mn}{T}$$

All of these suppressed by $\exp(-m/T)$.

- What about transitions that depend on particle properties?

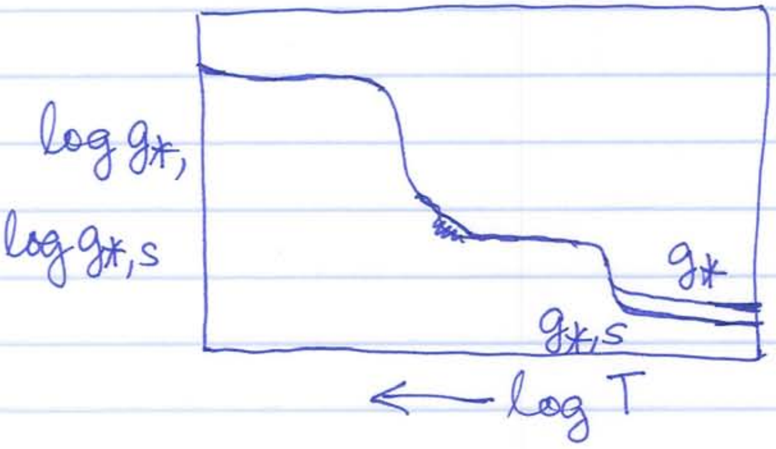
Mass?
Which particles contribute to ρ, etc., depends on which are ER.

$$\rho_{tot} = \frac{\pi^2}{30} g_* T^4 + \text{small from NR particles}$$

$$P_{tot} = \frac{1}{3} \rho_{tot} + \text{small from NR particles}$$

$$g_* = \sum_{\substack{i=\text{bosons}\\(\text{ER})}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{\substack{i=\text{fermions}\\(\text{NR})}} g_i \left(\frac{T_i}{T}\right)^4$$

Similar idea for s_{tot}, $g_{s,*}$.



At simplest level, can treat all ER particles as having same T.

Lifetime?
Which particles contribute depends on their ~~list~~ decay lifetime compared to expansion age.

In radiation-dominated era,

$$t \sim 1\text{s}\,(\text{MeV}/T)^2.$$

Most SM particles have short decay times relative to $t \sim 1\text{s}\,(\text{MeV}/m)$, meaning that they quickly disappear once they are no longer thermally produced. The neutron is an important exception.

Cross section?
Which particles matter depends on which interactions can be in equilibrium.

Once the cross section is large enough ~~for~~ to maintain equilibrium, making it larger changes nothing.

First estimate of decoupling is $\Gamma/H \lesssim 1$.
$\Gamma = n\sigma v$ = reaction rate per particle.
$H \sim \sqrt{G}/R^2 \sim T^2/M_{P\ell}$ in radiation era.

Note $G = \frac{\hbar c}{M_{P\ell}^2}$, $M_{P\ell} \simeq 10^{19}$ GeV.

Better calculation of decoupling uses the Boltzmann equation, $\dot{n} + 3Hn =$ "collision terms."

B. Weak (Hot) Relics : Neutrinos

- We know that neutrinos exist as particles; do they exist in the universe?

When do neutrinos decouple?

$\nu + \gamma$ $\quad \sigma \sim 0$, $n \sim$ big
$\nu + e$ $\quad \sigma \sim$ weak, $n \sim$ big
$\nu + N$ $\quad \sigma \sim$ weak (but bigger), $n \sim$ tiny

$\nu\bar{\nu} \longleftrightarrow e^+e^-$, $\nu e^\pm \longleftrightarrow \nu e^\pm$, $\bar{\nu} e^\pm \longleftrightarrow \bar{\nu} e^\pm$
(Plus also neutrino flavor mixing!)

$$\Gamma = n\sigma v \sim \underset{(e^\pm)}{T^3} \cdot G_F^2 T^2 \cdot 1 \sim G_F^2 T^5$$

note units

$$H \sim T^2/M_{P\ell}$$

$$H^2 = \frac{8\pi G\rho}{3} \sim \frac{1}{M_{P\ell}^2}\frac{1}{R^4} \sim \frac{1}{M_{P\ell}^2} T^4$$

$$\boxed{\frac{\Gamma}{H} \sim G_F^2 M_{P\ell} T^3 \lesssim 1}$$

G_F^2: $\sim (10^{-5}\,\mathrm{GeV}^{-2})^2$ $\qquad$ $M_{P\ell}$: $\sim 10^{19}\,\mathrm{GeV}$

$$\rightarrow \boxed{T \lesssim 1\,\mathrm{MeV} \quad \nu \text{ decoupled}}$$

$\Gamma \sim H \sim 10^{-25}\,\mathrm{GeV}$
$\Gamma \sim H \sim 0.1\,\mathrm{s}^{-1}$
at $T \sim 1\,\mathrm{MeV}$

$T \gtrsim 1\,\mathrm{MeV}$ neutrinos in equilibrium, $T \propto 1/R$

$T \lesssim 1\,\mathrm{MeV}$ neutrinos massless, $T \propto 1/R$, but not in equilibrium!

The number density today is robust since neutrinos were a hot relic; it is close to the CMB photon number density today since neutrinos are almost massless.

What if neutrino cross section were bigger? Same result; e.g., compare to CMB photon decoupling. What if it were smaller, e.g., for ν_R?

- Two other things happen at $T \sim 1\,\mathrm{MeV}$: neutron-proton conversions freeze out and electron-positron pairs annihilate.

n, p equilibrium set only by neutrinos. Thermal equilibrium maintained by stronger interactions. Neutrinos don't care about the baryons.

$\bar{\nu}_e p \longleftrightarrow e^+ n, \; \nu_e n \longleftrightarrow e^- p, \; n \longrightarrow e^- p \bar{\nu}_e$

$$\Gamma = n\sigma v \sim \underset{(e^\pm,\nu)}{T^3} \cdot \underset{\sigma_{\nu N} > \sigma_{\nu e}}{G_F^2 T^2} \cdot \underset{NR}{(\lesssim 1)} \sim G_F^2 T^5$$

$$\boxed{\frac{\Gamma}{H} \sim G_F^2 M_{P\ell} T^3 \lesssim 1}$$

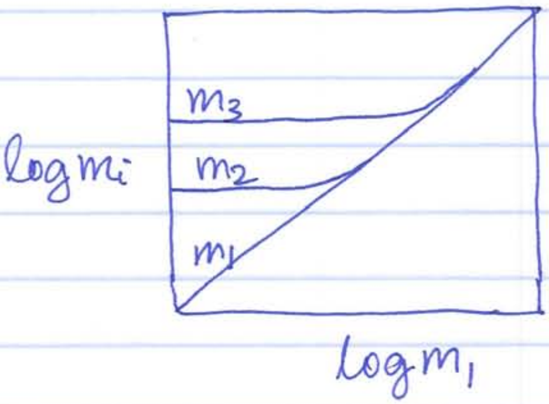
$\rightarrow T \lesssim 1\,\text{MeV}$ neutrinos stop $n \longleftrightarrow p$

Similar result but different calculation!

- What if the neutrinos are NR today?
  For $m_\nu \sim \sqrt{\delta m_\nu^2} \sim 10^{-3}\,\text{eV}$, can be.

For NR ν, $\rho_\nu = m_\nu n_\nu$

$\Omega_\nu h^2 \lesssim 1$ (don't exceed total) $\longrightarrow m_\nu \lesssim 100\,\text{eV}$
$\Omega_\nu h^2 \lesssim 0.1$ (don't exceed matter) $\longrightarrow m_\nu \lesssim 10\,\text{eV}$
detailed CMB, LSS constraints $\longrightarrow m_\nu \lesssim 1\,\text{eV}$



$m_3 = \sqrt{m_1^2 + \delta m_{sol}^2 + \delta m_{atm}^2}$

$m_2 = \sqrt{m_1^2 + \delta m_{sol}^2}$

$m_1 = m_1$

For normal hierarchy, active mixing only.

First shown in Beacom and Bell (2002).

C. Weak (Cold) Relics: WIMPs

- We think these might exist. Something must make $\Omega_M \simeq 0.3$, its interactions must be feeble, and it should be massive for LSS considerations.

Assume a self-annihilating species,

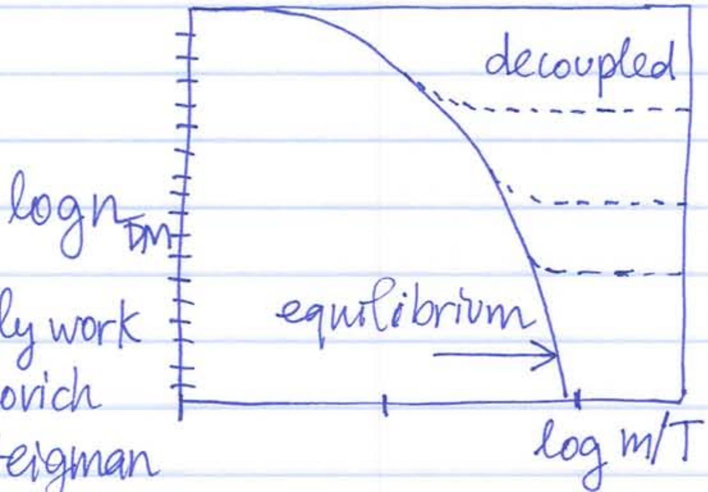
$$DM + DM \longleftrightarrow SM + SM,$$

with $M_{DM} >> M_{SM}$, so the reaction goes both ways at high T, and only to the right at low T.

We know $n_B/n_\gamma \sim 6 \times 10^{-10}$, so n_{DM}/n_γ tiny too. What would make it small? Maybe its abundance froze out when $T_f << M_{DM}$. Recall

$$\text{NR} \quad n_{DM} \sim (mT)^{3/2} e^{-m/T},$$

which is exponentially suppressed.



See early work by Zel'dovich and by Steigman

Density will flatten out (except for expansion) if equilibration breaks.

$m/T \sim 30$ gives $n \sim 10^{-10}$ drop from $m/T \sim 1$ value!

$\Gamma = n\sigma v \sim n_{DM} \langle \sigma_A v \rangle$ per WIMP.

$H \sim T^2 / M_{Pl}$

$\Gamma / H \sim (mT)^{3/2} e^{-m/T} \langle \sigma_A v \rangle M_{Pl} / T^2 \sim 1$

$$e^{m/T} \sim (m/T)^{1/2} \cdot m M_{Pl} \langle \sigma_A v \rangle$$

Hence freezeout at $\frac{m}{T} \sim \ln \left[(\#) M M_{Pl} \langle \sigma_A v \rangle \right]$.

$$\frac{n_{DM}}{s} \sim \frac{(mT)^{3/2} e^{-m/T}}{T^3} \sim \left(\frac{m}{T} \right)^{3/2} e^{-m/T}$$

$$\frac{n_{DM}}{s} \sim (\#) \frac{1}{m M_{Pl} \langle \sigma_A v \rangle}$$

$$\rho_{DM,0} = M n_{DM,0} = M \left(\frac{n_{DM}}{s} \right) s_0 \sim M n_{DM} \left(\frac{T_0}{T_f} \right)^3$$

$$\boxed{\rho_{DM,0} \sim \frac{T_0^{\,3}}{M_{Pl} \langle \sigma_A v \rangle}}$$

independent of m

$T_0 = T_{CMB,0}$

The "WIMP miracle" is that a weak cross section will give $\rho_{DM,0} \sim 10^{-6}$ GeV cm^{-3} for the spatially-averaged density (higher in halos). Use $\langle \sigma_A v \rangle \simeq 3 \times 10^{-26}$ cm^3 s^{-1} and convert units carefully.

D. Strong Relics : BBN

- What happens to neutrons?

As noted above, weak $n \leftrightarrow p$ scatterings stop at $T \sim 1\,\mathrm{MeV}$ ($t \sim 1\,\mathrm{s} << \tau_n \sim 10^3\,\mathrm{s}$).

Can show $\frac{n_n}{n_p} \sim e^{-\Delta m/T} \sim \frac{1}{6}$ at this point.

If weak interactions were stronger, could drive this exponentially smaller; freezeout details matter.

$np \to d\gamma$ releases 2.2 MeV.
Might think that this and its inverse freeze out at $T \sim 2\,\mathrm{MeV}$. The tiny $n_B/n_\gamma \sim 6 \times 10^{-10}$ means that $\gamma d \to pn$ is effective down to $T \sim 0.1\,\mathrm{MeV}$ ($t \sim 100\,\mathrm{s}$).

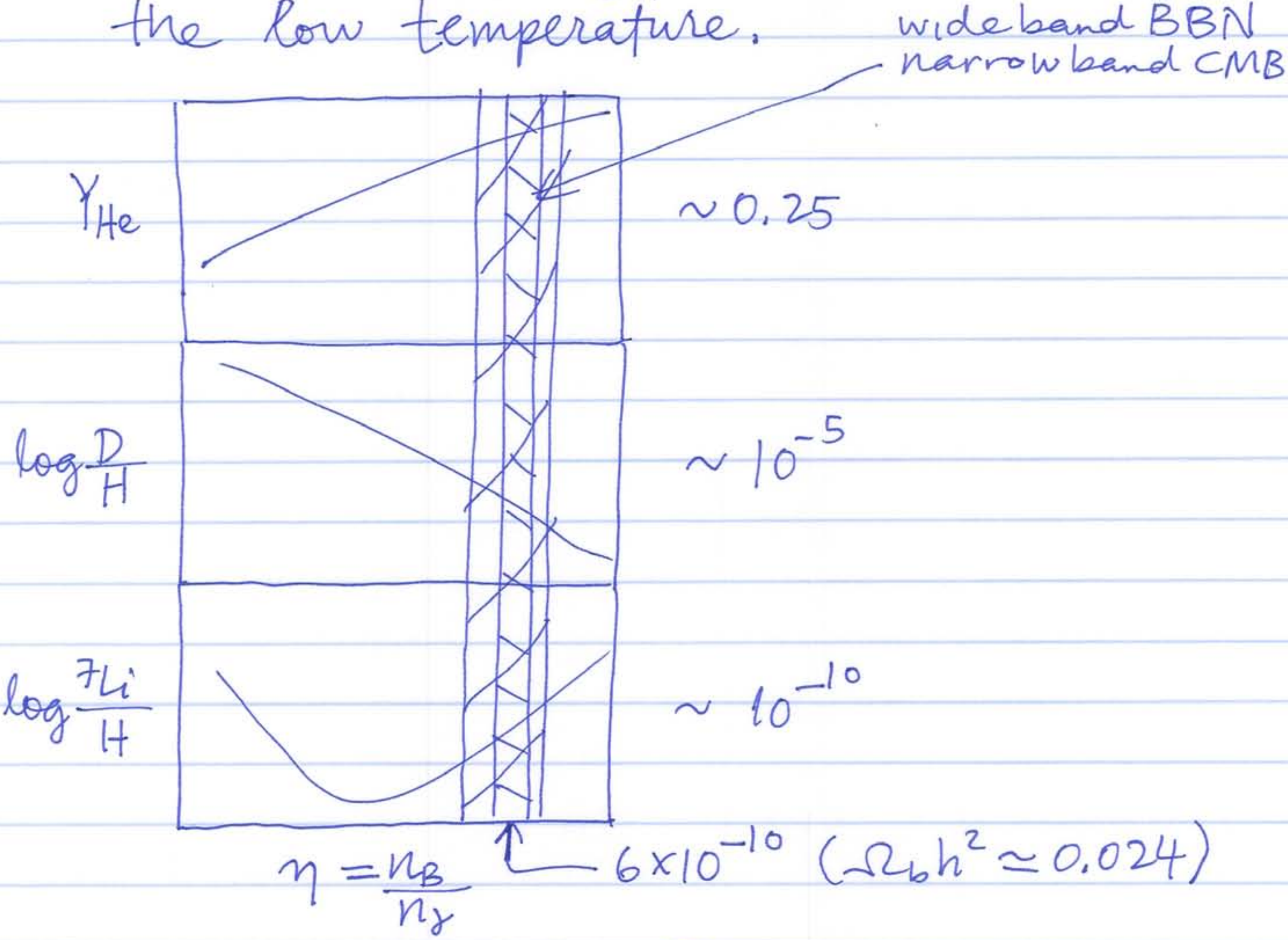
- Subsequent nuclear reactions flow to the tightly-bound ^{4}He (nucleons get in, but they don't get out). Almost all neutrons end up there. By this time $n_n/n_p \sim 1/7$ due to decays.

$$Y_{He} = Y_P \simeq \frac{4 M_p n_{He}}{M_p(n_p + n_n)} \simeq \frac{4(n_n/2)}{n_p + n_n} \simeq \frac{2(n_n/n_p)}{1 + (n_n/n_p)}$$

$$\boxed{Y_P \simeq \frac{2(1/7)}{1+(1/7)} \simeq \frac{2}{8} \simeq 0.25.}$$

- Also produce trace amounts of D, $^3H (\rightarrow {}^3He)$, ^3He, ^4He, $^7Be (\rightarrow {}^7Li)$, ^7Li.

Not more due to gaps at A = 5, 8 and the low temperature.



- What do we learn?

Where nuclei began.

$\min(T_{universe}) \sim 1$ MeV (very opaque)

$\Omega_b >> \Omega_{stars}$, but $\Omega_b << \Omega_m, \Omega_{tot}$

$N_\nu \simeq 3 \pm 1$, consistent with lab (more $N_\nu \rightarrow$ more Y_p)

E. Electromagnetic Relics: CMB

- What happens to free electrons and nuclei?

End of radiation-dominated era at $z \sim R^{-1} \sim 10^4$, $T \sim$ few MeV.

Can make H atoms freeze out once there are not too many ionizing photons. Might guess when $T \sim 13.6$ eV, but recall $n_B/n_\gamma \simeq 6\times10^{-10}$.

- Can show that freezeout occurs when $z \sim R^{-1} \sim 10^3$, $T \sim 0.3$ MeV.

It's complicated since recombination (first combination), freezein of a residual ionization fraction $\sim 10^{-3}$, and ~~the~~ decoupling (last scattering of $\gamma e \to \gamma e$) all occur at $z \sim 10^3$.

- What do we learn?

Where atoms began.

γ-Visible universe restricted to $z \sim 10^3$, $T \sim$ eV.

Ω_b from BBN, CMB agree!

N_ν from BBN, CMB agree!

CMB details (anisotropies) crucial for testing m_ν.

F. Conclusions

- Expansion of universe depends on gravity and amounts and types of stuff, but not directly on microphysical particle properties.

  This followed from the <u>perfection</u> of the universe — homogeneity, isotropy, equilibrium.

- Early universe was arbitrarily hotter, denser, and younger, activating the properties of all known and unknown particles.

✱ Demon : "But you don't get to see!"

- The cooling of the universe leaves relic clues due to its <u>imperfections</u>.

  ✱ Yakov : "The universe is the poor man's particle accelerator!"

- Truly profound that the macro–micro interplay works so well and that we learn so much.

- Best evidence for new physics:

  Ω_Λ, Ω_m from cosmos — can we probe in lab?

  m_ν, hierarchy problem from lab — can we probe in cosmos?