Neutrino Cosmology

SSI 2010

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First Day: Macrophysics of the hot Big Bang

Second Day: Microphysics of the hot Big Bang

Third Day: New views with neutrinos

Opening Remarks

- What fundamental physics remains unknown?

  Are there more particles and interactions than we have seen in the lab?

  Is General Relativity the correct basis for understanding the universe?

  How are inner space and outer space connected?

- Fable of Yakov and the ~~goblin~~. demon.

$\boxed{\text{For Further Study}}$

- Mini-reviews in Review of Particle Physics.

- Books by Kolb and Turner,
  Dodelson,
  Bergstrom and Goobar,
  Ryden,
  and many others.

- Online lecture notes by Fields (Urbana);
  Google "Brian Fields Astronomy."

$\boxed{\text{First Day:}}$

A. Basic Concepts

B. Friedmann Equations

C. Friedmann Solutions

D. Towards A General Universe

A. Basic Concepts

- Contents and non-contents of the universe:

$$\Omega_{total} \simeq 1$$
$$\Omega_{\Lambda} \simeq 0.7$$
$$\Omega_{M} \simeq 0.3$$
$$(\Omega_{b} \simeq 0.04)$$
$$(\Omega_{\nu} \lesssim 0.01)$$
$$\Omega_{R} \simeq 10^{-4}$$

$$\Omega_{heavy\ el.} \sim 0$$
$$\Omega_{heavy\ SM} \sim 0$$
$$\Omega_{antimatter} \sim 0$$
$$\Omega_{life} \sim 0$$

$$\Omega_i = \frac{\rho_i}{\rho_c}, \quad \rho_c \simeq 5 \times 10^{-6} \text{ GeV cm}^{-3}$$

All defined now, as spatial averages.

Neutrinos connect with all of the above topics.

- Some key observations:

On large scales, universe appears homogeneous and isotropic from our position.

Hubble's Law, $v = H_0 r$, is based on observations that universe is expanding homologously from our position.

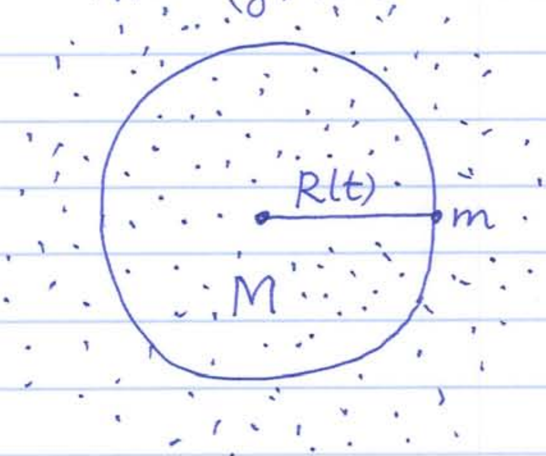
The cosmological principle says we're not special, and that all of space is expanding.

The age is finite, based on objects in the universe (~10 Gyr) and on the expansion rate ($H_0^{-1} \sim 14$ Gyr). The agreement suggests that the expansion rate was more rapid in the past.

This all suggests that the universe grew from a hot and dense beginning – the Big Bang – and that it has expanded since.

- Classical insights on expansion:

Consider an isotropically expanding sphere of homogeneous matter, with a test mass moving with the surface (outside stuff expanding too).



$$M = \frac{4}{3}\pi R^3 \cdot \rho$$

constant varying

all particles moving outward

sphere moves with them

$$m\ddot{R} = -\frac{GMm}{R^2} + (\underline{\text{zero}} \text{ from the outside})$$

$$\boxed{\left(\frac{\ddot{R}}{R}\right) = -\frac{4\pi G\rho}{3}}$$

decelerating

Nothing special about location, radius of sphere, so this should apply generally to the expansion of all of the universe.

More precisely, for the galaxies in the universe in the classical conception.

$$\dot{R} \times \left(\ddot{R} = -\frac{GM}{R^2} \right)$$

$$\frac{d}{dt} \left(\frac{1}{2}\dot{R}^2 = \frac{GM}{R} + \text{const.} \right)$$

$$\dot{R}^2 = \frac{2G\frac{4}{3}\pi R^3 \rho}{R} + \text{const.}$$

$$\boxed{H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\text{const.}}{R^2}}$$ expanding or contracting

$H \simeq$ constant for small changes

Special density $\rho_c = \frac{3H^2}{8\pi G}$ means barely unbound.

$H_0 = 100h \,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$, $h \simeq 0.7$

Recession speeds from $1+z = \frac{\lambda_{\text{received}}}{\lambda_{\text{emitted}}}$.

For small z, $v \simeq cz \simeq H_0 r$ like Doppler shift

$$r \simeq \frac{c}{H_0} z \simeq 4000\,\text{Mpc} \cdot z$$

B. Friedmann Equations

- Many key questions raised (or ignored) in that classical approach, but some parts of it must be right.

Consider a new approach, where space itself is expanding, and General Relativity connects the properties of space to the contents of the universe.

- Robertson–Walker metric:

For a homogeneous, isotropic, expanding universe,

$$\boxed{ds^2 = dt^2 - \underbrace{R^2(t)}_{\text{scale factor}} \Bigg[\overbrace{\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)}^{\text{comoving coordinates}} \Bigg]}$$

curvature $k = -1, 0, +1$

(Here and throughout, $c = \hbar = k_B = 1$.)

Now redshift $1+z = \frac{\lambda_2}{\lambda_1} = \frac{R_2}{R_1}$, meaning that wavelengths expand along with space itself!

For a massless particle,

$$p = E \propto \frac{1}{R}.$$

For a massive particle,

$$p \propto \frac{1}{R}, \quad KE \propto \frac{1}{R^2}.$$

- Einstein's Equations:

$$\boxed{\underbrace{\mathcal{R}_{\mu\nu} - \frac{\mathcal{R}}{2} g_{\mu\nu}}_{\text{spacetime properties}} = \underbrace{8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}}_{\text{universe contents}}}$$

For homogeneous, isotropic, expanding universe, these simplify greatly.

Can show

$$\boxed{H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{k}{R^2}}$$

first ("energy") Friedmann equation

$$\boxed{\left(\frac{\ddot{R}}{R}\right) = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}}$$ second ("acceleration") Friedmann equation

ρ is already more general than before (now radiation in addition to matter), and we can generalize it further by defining $\Lambda = 8\pi G \rho_\Lambda$ and absorbing that.

Can relate two equations with $\boxed{\dot{\rho} = -3H(\rho + p),}$ which follows from the first law of thermodynamics, $d(\rho R^3) = -p\,d(R^3)$. A general component of the density has equation of state $\boxed{p = w\rho.}$

The scale factor $R = R(t)$ and the energy ~~phys~~ density $\rho = \rho(t)$ and isotropic pressure $p = p(t)$ (in physical, not comoving coordinates) all vary in general. $H_0 = H(t = \text{now})$, etc.

As far as we know, $\Omega_{tot} = 1$ and $k = 0$ (spatially flat universe).

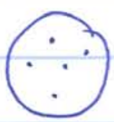
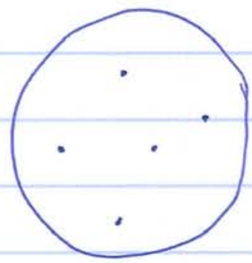
For $\rho_\Lambda = 0$, this $\overset{(k=0)}{\text{is}}$ the same as $\rho_{tot} = \boxed{\rho_c = \frac{3H_0^2}{8\pi G}}$ $\simeq 5 \times 10^{-6}$ GeV cm^{-3}, i.e., critically unbound.

For $\rho_\Lambda \neq 0$, connection between geometry and fate is more complicated.

C. Friedmann Solutions

- Consider single-component solutions (for $k=0$).
- Matter-dominated:

"matter" = non-relativistic and stays so (particles)

 expands to  no change in number

$\rho_M \propto R^{-3}$ for physical density

$p_M \sim 0$

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G \rho_M}{3} \qquad (k=0 \text{ means } \rho_M = \rho_c)$$

$$\frac{1}{R}\frac{dR}{dt} = \sqrt{\frac{8\pi G \rho_{M,0}}{3}} \left(\frac{R_0}{R}\right)^{3/2}$$

$$\frac{dR}{R} R^{3/2} = R_0^{3/2} \sqrt{H_0^2}\, dt$$

$$\int_0^R d\left(\frac{2}{3}R^{3/2}\right) = R_0^{3/2} \sqrt{H_0^2} \int_0^t dt$$

$$\boxed{\left(\frac{R}{R_0}\right) = \left(\frac{3}{2}H_0 t\right)^{2/3}}$$

$$H = H_0 \left(\frac{R_0}{R}\right)^{3/2} = H_0 \left(\frac{2}{3H_0 t}\right) = \frac{2}{3t}$$

$$\boxed{Ht = \frac{2}{3}} \qquad \boxed{H_0 t_0 = \frac{2}{3}} \qquad t_0 = t_{now} \neq 0 \,!$$

$$t_0 \simeq 14 \text{ Gyr}$$

- Radiation-dominated:

"radiation" = relativistic and stays so (particles)

 expands to  no change in number
wavelengths stretched

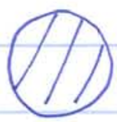
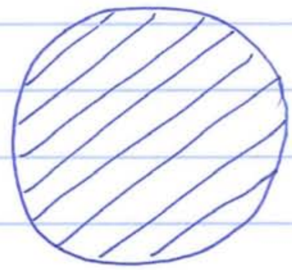
$$\rho_R \propto R^{-4}$$
$$p_R = \frac{1}{3}\rho_R$$

$$\boxed{\left(\frac{R}{R_0}\right) = \left(2H_0 t\right)^{1/2}}$$

$$\boxed{Ht = \frac{1}{2}} \qquad \boxed{H_0 t_0 = \frac{1}{2}}$$

- Lambda-dominated:

"lambda" = constant density and stays so (field)

 expands to 

$\rho_\Lambda \propto$ constant

$p_\Lambda = -\rho_\Lambda$ ("smooth tension")

$H^2 = H_0^2 = \frac{8\pi G \rho_\Lambda}{3}$ constant but $\dot{R}, \ddot{R} \to +\infty$

$$\frac{dR}{R\,dt} = H_0$$

$$\int_{R_i}^{R} \frac{dR}{R} = \int_{t_i \sim 0}^{t} H_0\, dt$$

$$\ln\left(\frac{R}{R_i}\right) = H_0 t$$

$$\boxed{R = R_i e^{+H_0 t}}$$

$$\boxed{H_0 t = \ln\left(\frac{R}{R_i}\right)} \qquad \boxed{H_0 t_0 = \ln\left(\frac{R_0}{R_i}\right)}$$

ln( ) could be huge, so $t_0 >> H_0^{-1}$

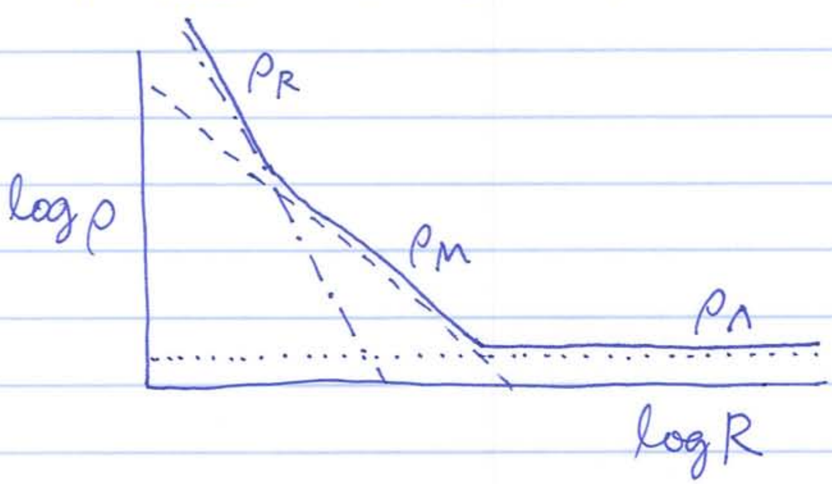
D. Towards A General Universe

- The energy density has components of varying importance. For $k=0$,

$$\boxed{\begin{aligned}\frac{H^2}{H_0^2} = \frac{\rho}{\rho_{c,0}} &= \Omega_{\Lambda,0} + \Omega_{M,0}\left(\frac{R_0}{R}\right)^3 + \Omega_{R,0}\left(\frac{R_0}{R}\right)^4 \\ &= \Omega_{\Lambda,0} + \Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4\end{aligned}}$$

$\sim 0.7 \quad \sim 0.3 \quad \sim 10^{-4}$

Radiation-dominated to matter-dominated at $z \sim 10^4$.
Matter-dominated to lamba-dominated at $z \sim 0.33$.



Energy density was much larger in the past, which means that we can't see too far back.

Integrating above gives $\boxed{t_0 \simeq 14\,\mathrm{Gyr}}$ as time to initial singularity of hot, dense Big Bang.

- Now a Colombo moment – just a few simple questions:

What are $\Omega_\Lambda, \Omega_M, \Omega_R$ made of?

Doesn't this depend on particle masses?
Can change from NR to ER by going to large enough z.

Doesn't this depend on particle lifetimes?
Can go to short times by going to large enough z.

Doesn't this depend on particle cross sections?
Can probe the smallest cross sections by going to large enough z.

The Friedmann equation is really about gravity, not microscopic particle properties.

What are we missing?