90% Confidence Level Upper Bound

Brief discussion of Feldman & Cousins

10-31-2024

Outline

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Parameter Estimataion

Poisson Distribution

- Expect an average of (real) $\mu \geq 0$ neutrinos per time

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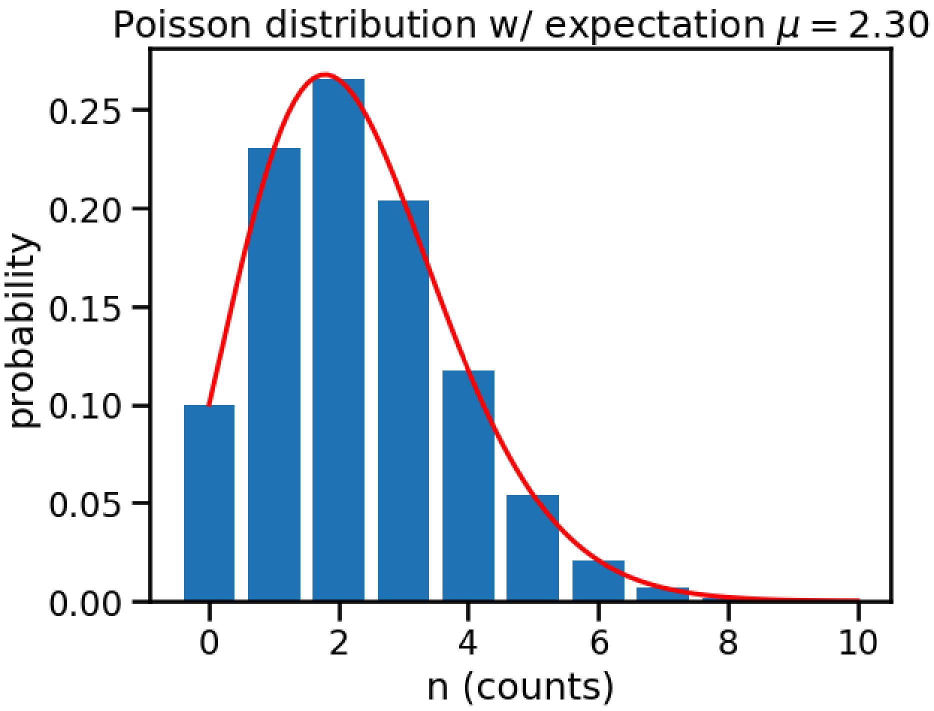
- Example:



Confidence Interval (CI)

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- Repeat experiment; get outcome $x_1 \longrightarrow$ construct $[\mu_l(x_1), \mu_u(x_1)]$
- More experiments; get a bunch of intervals. *i.e.* we get a set

$$C \equiv \{[\mu_l(x_0), \mu_u(x_0)], [\mu_l(x_1), \mu_u(x_1)], [\mu_l(x_2), \mu_u(x_2)]...\}$$

CI Definition

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- The members of C are called *confidence intervals*.

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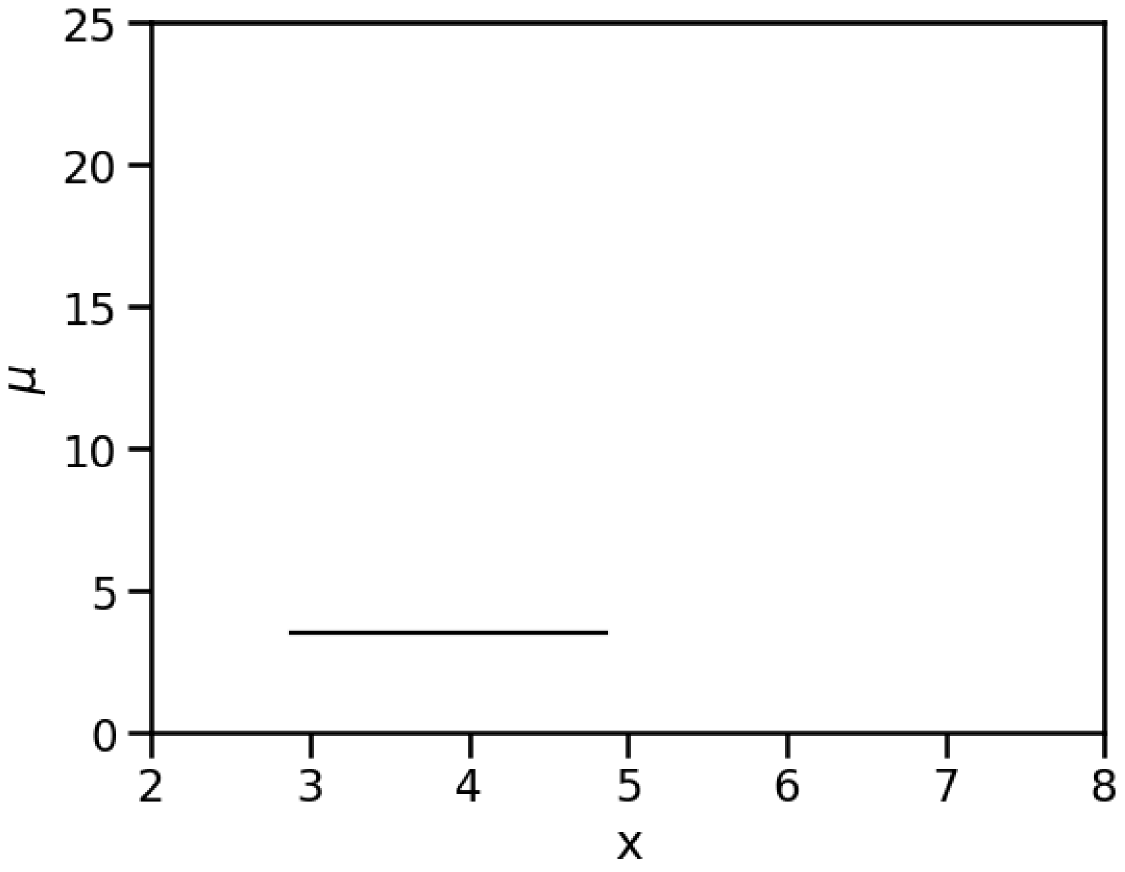

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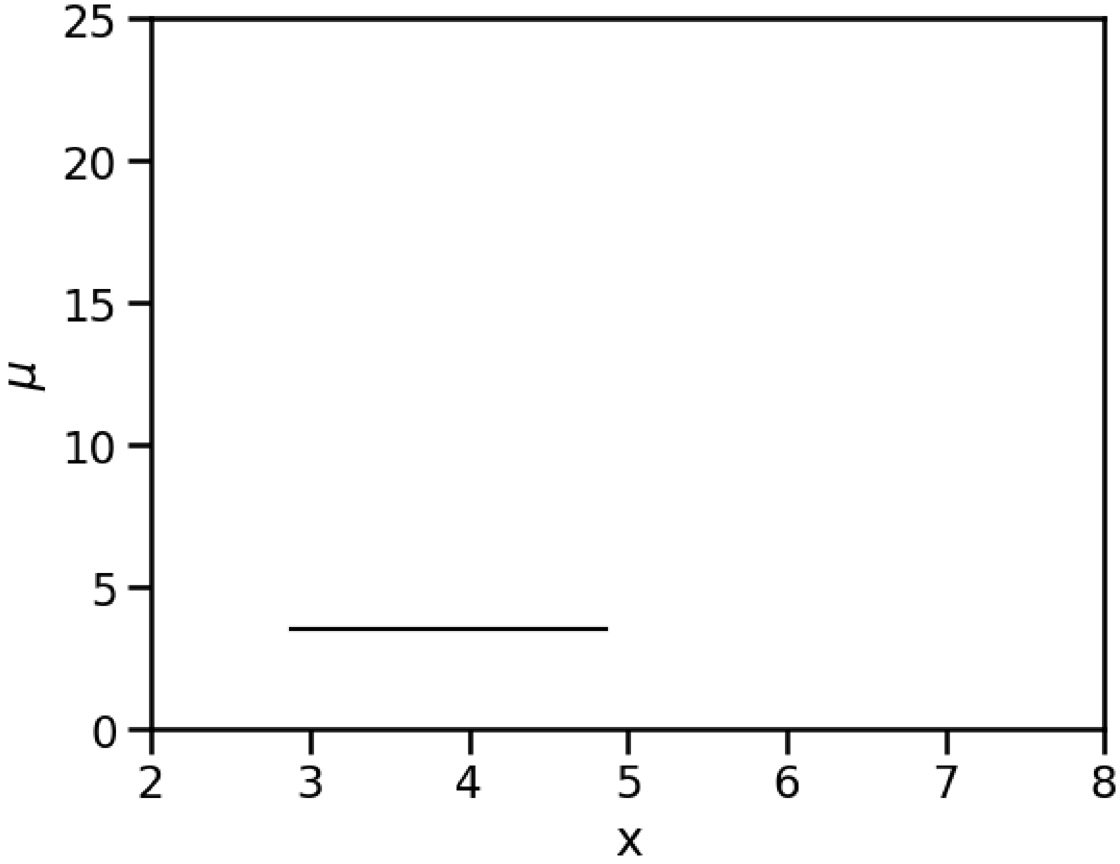

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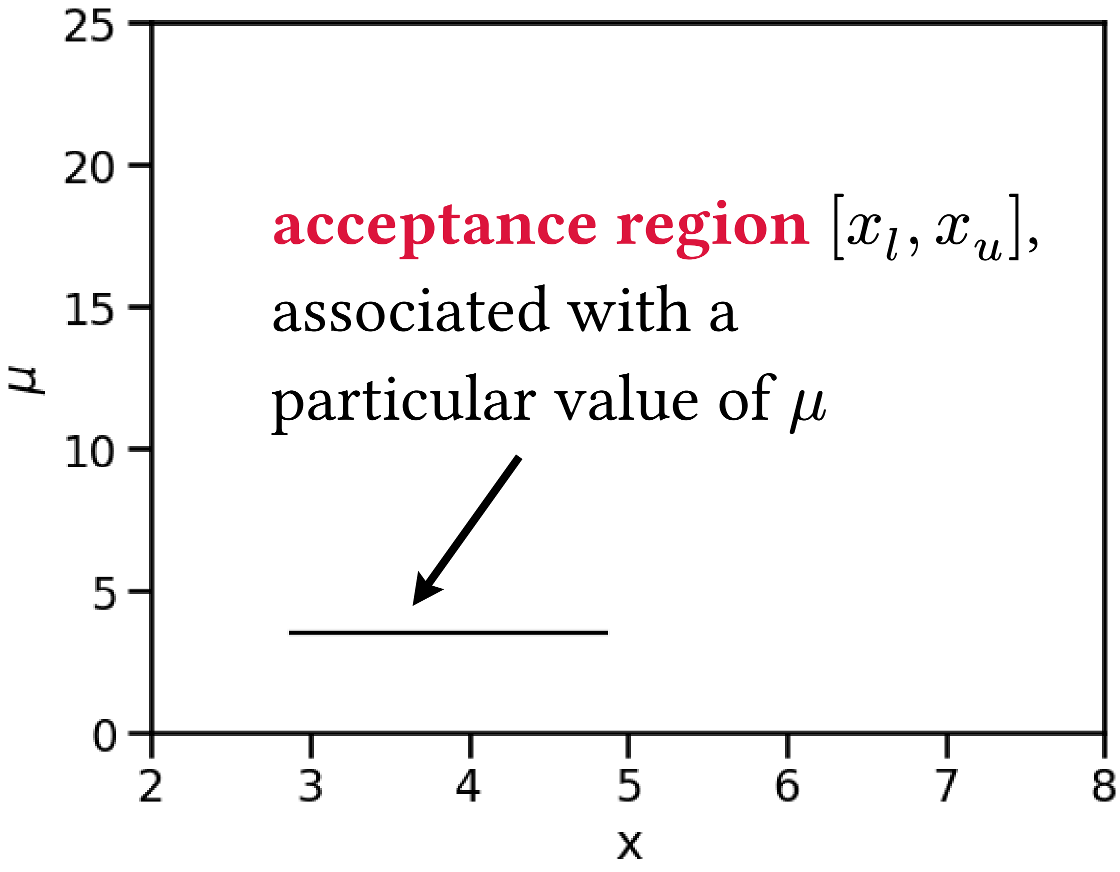

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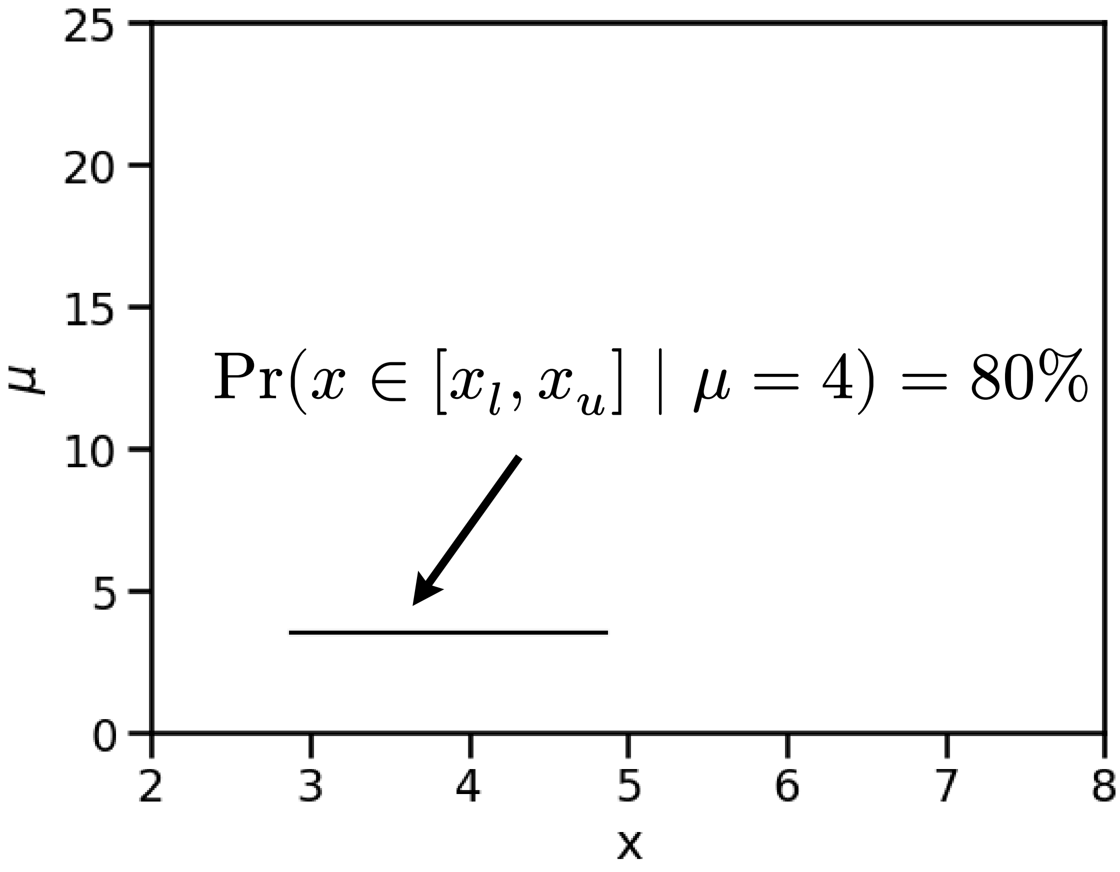

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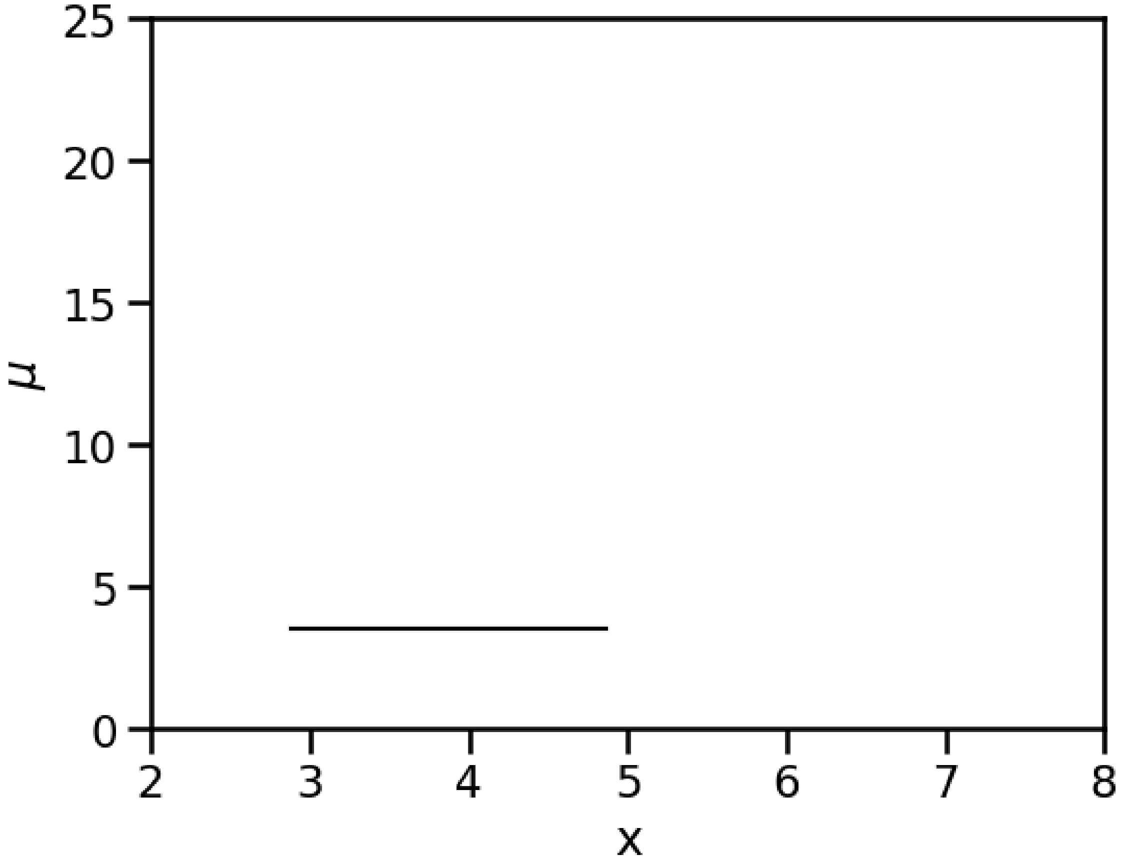

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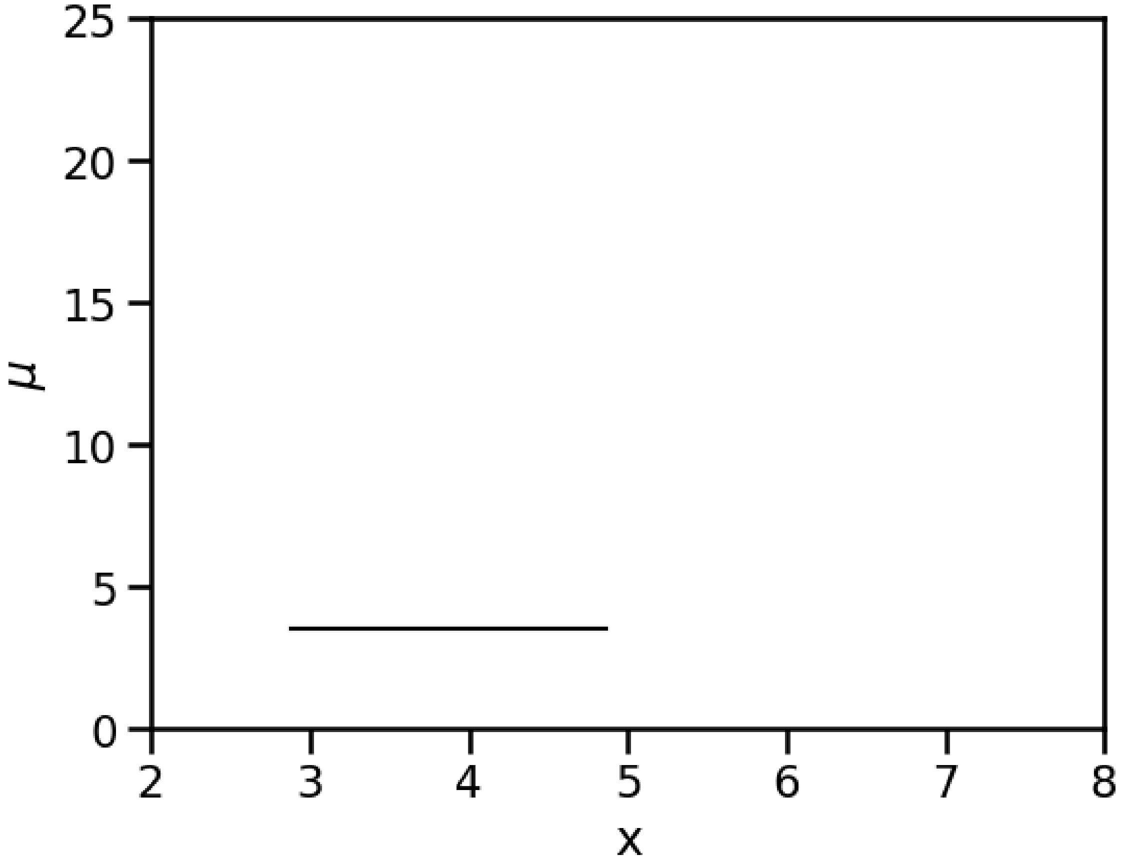

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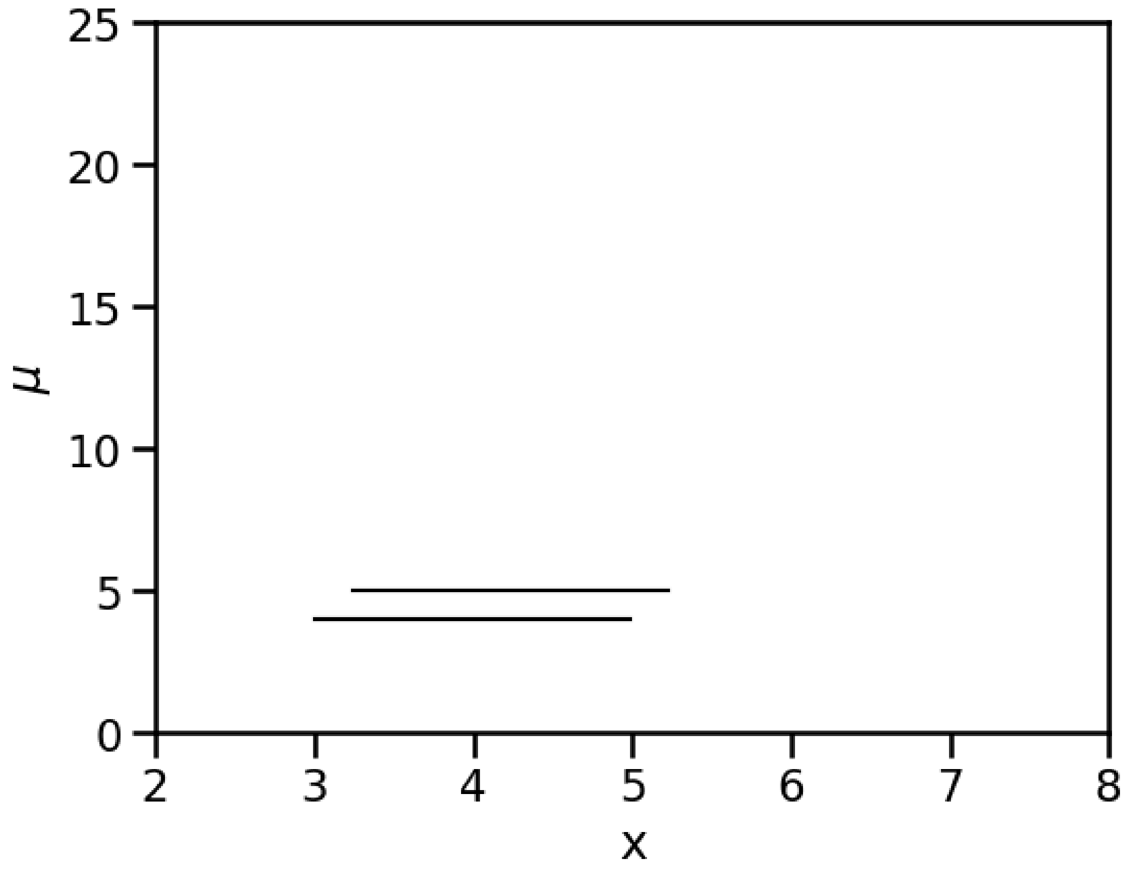

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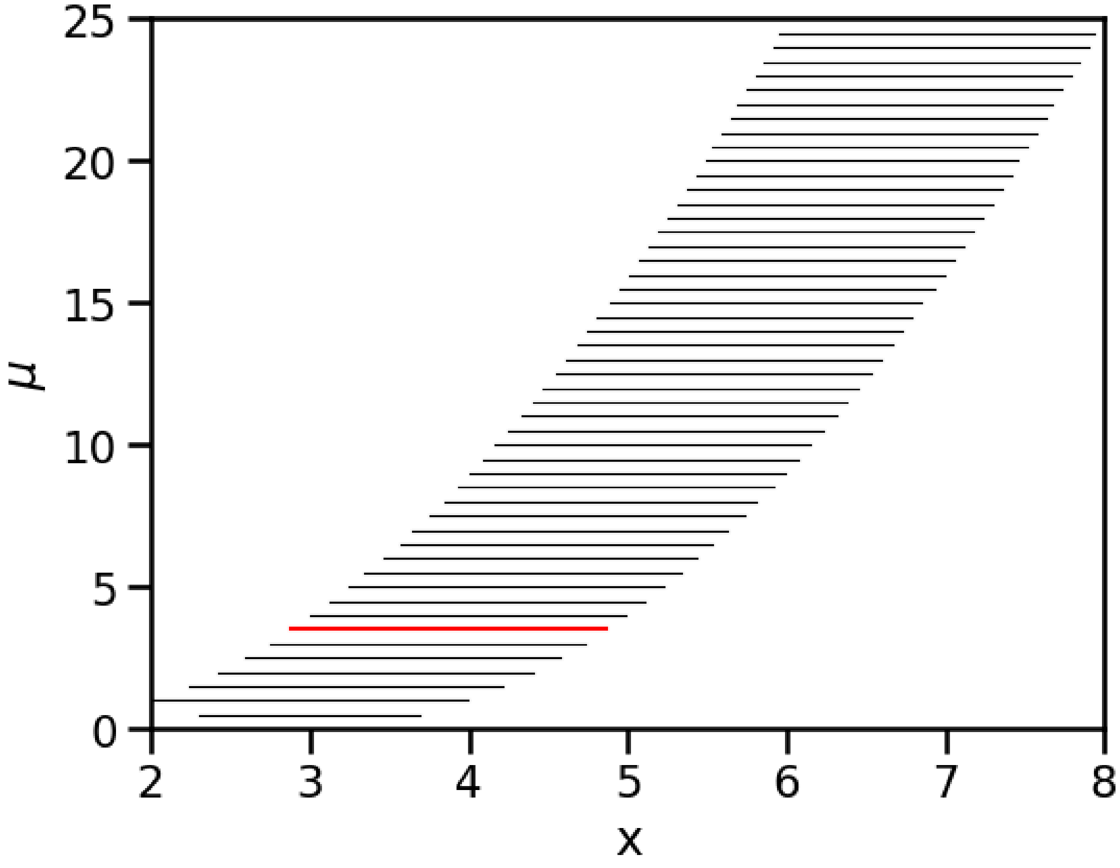

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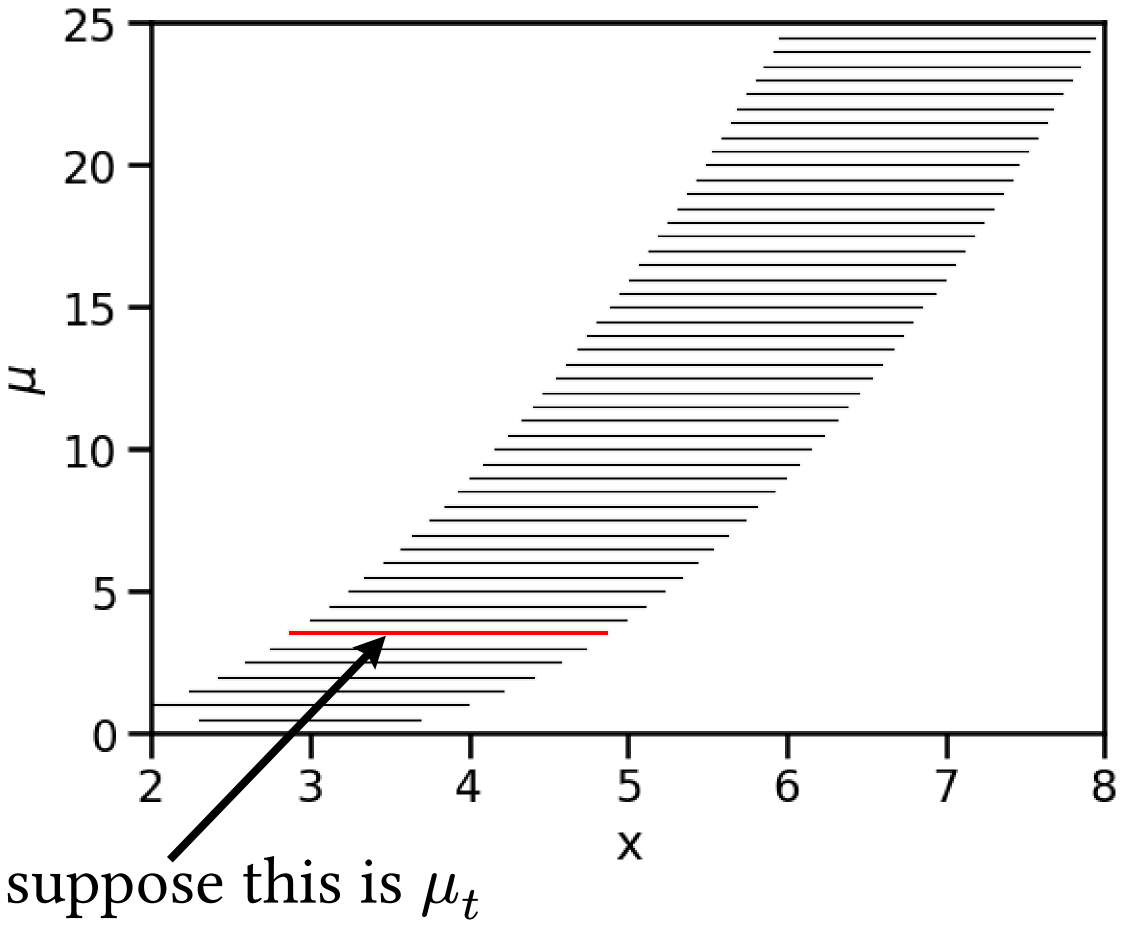

CI Construction: Confidence Belt

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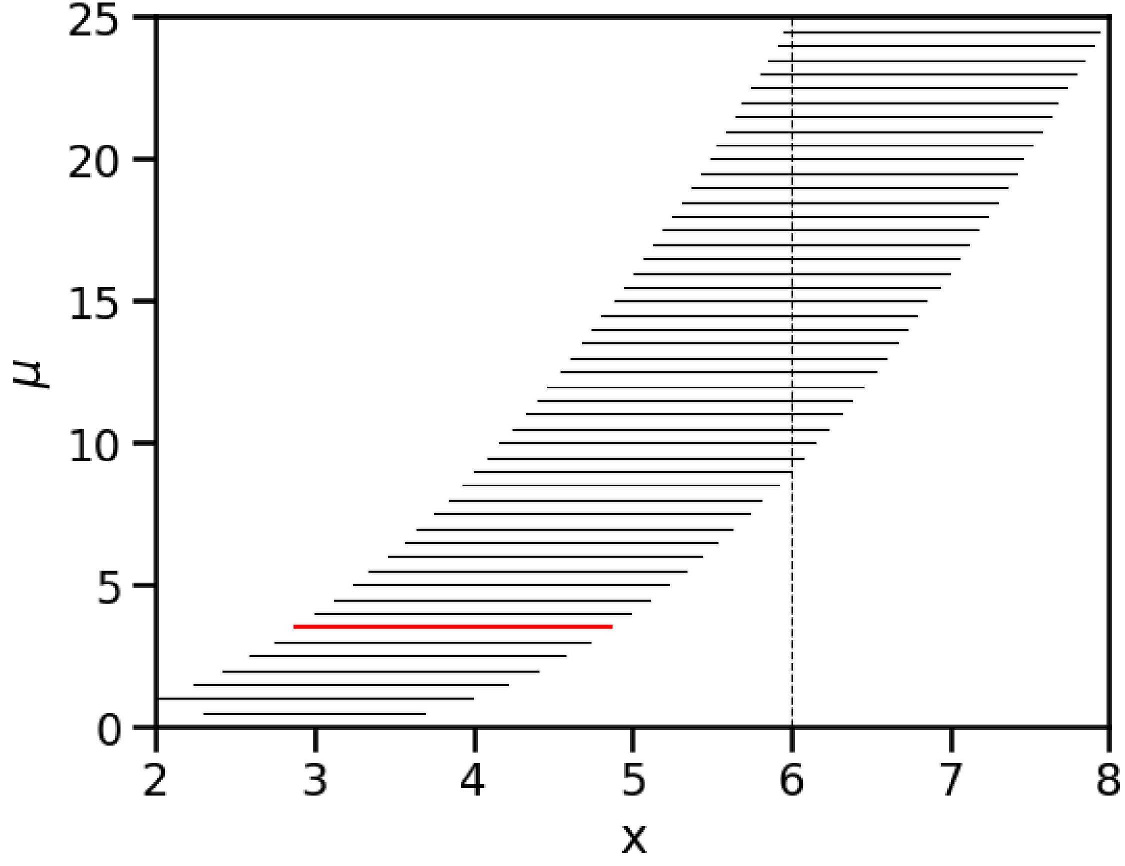
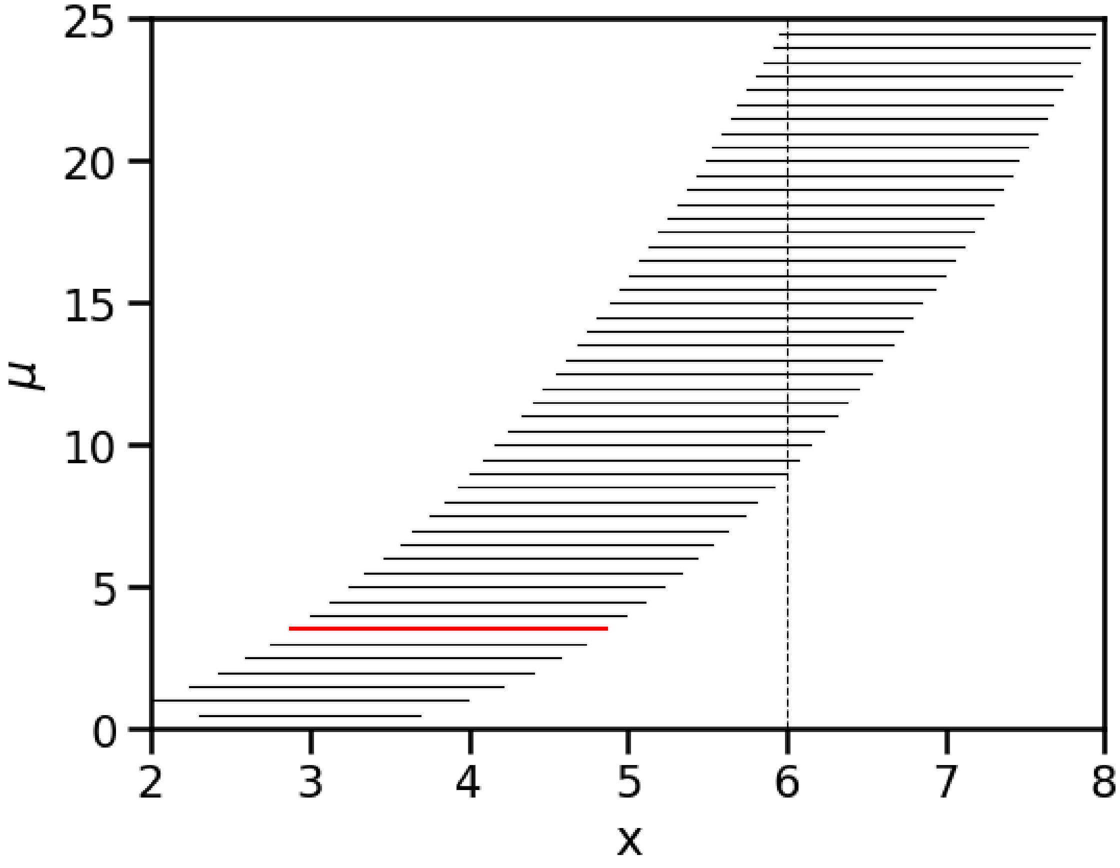


CI Construction: Confidence Belt

- Make a measurement, get result $x_0 = 6$
- The probability of x_0 falling in the acceptance region (red) is 80%, by construction



CI Construction: Confidence Belt

- Make a measurement, get result $x_0 = 6$
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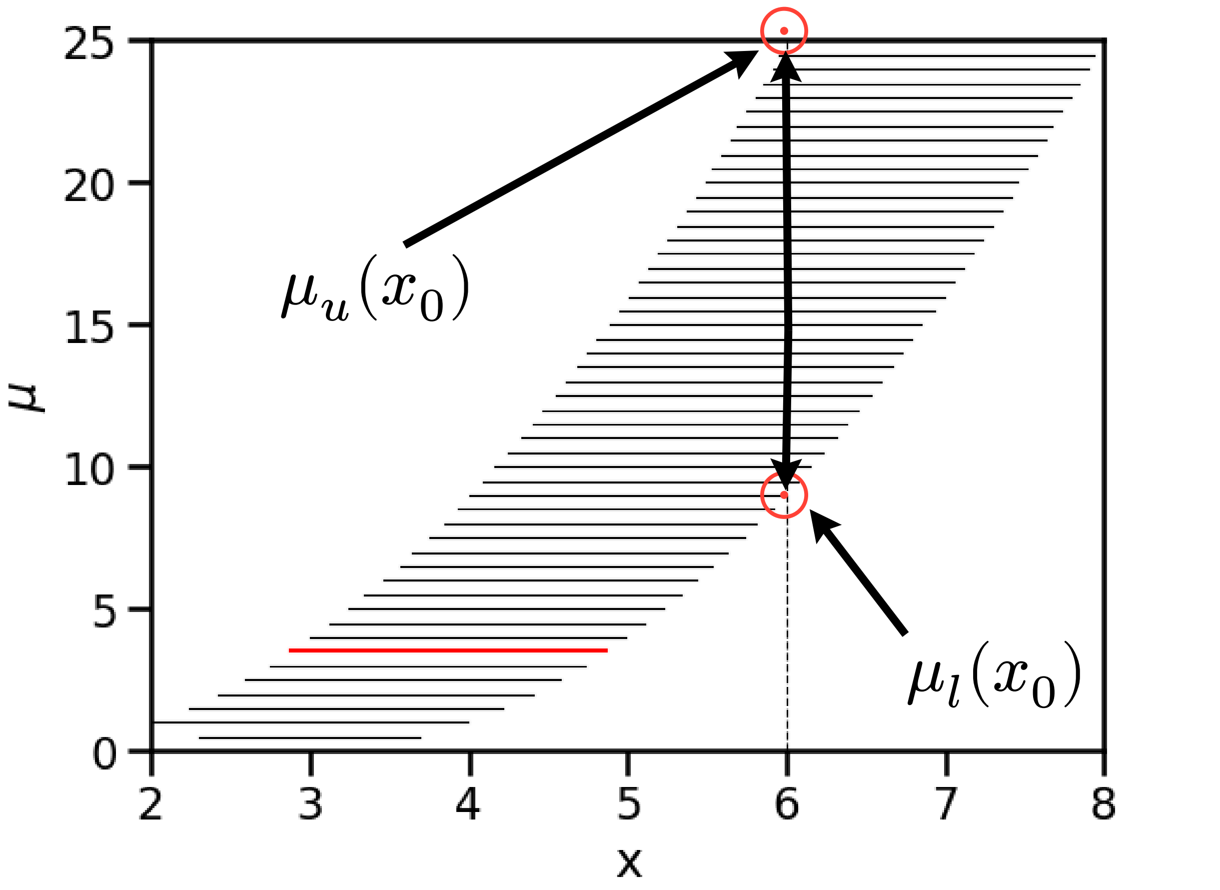
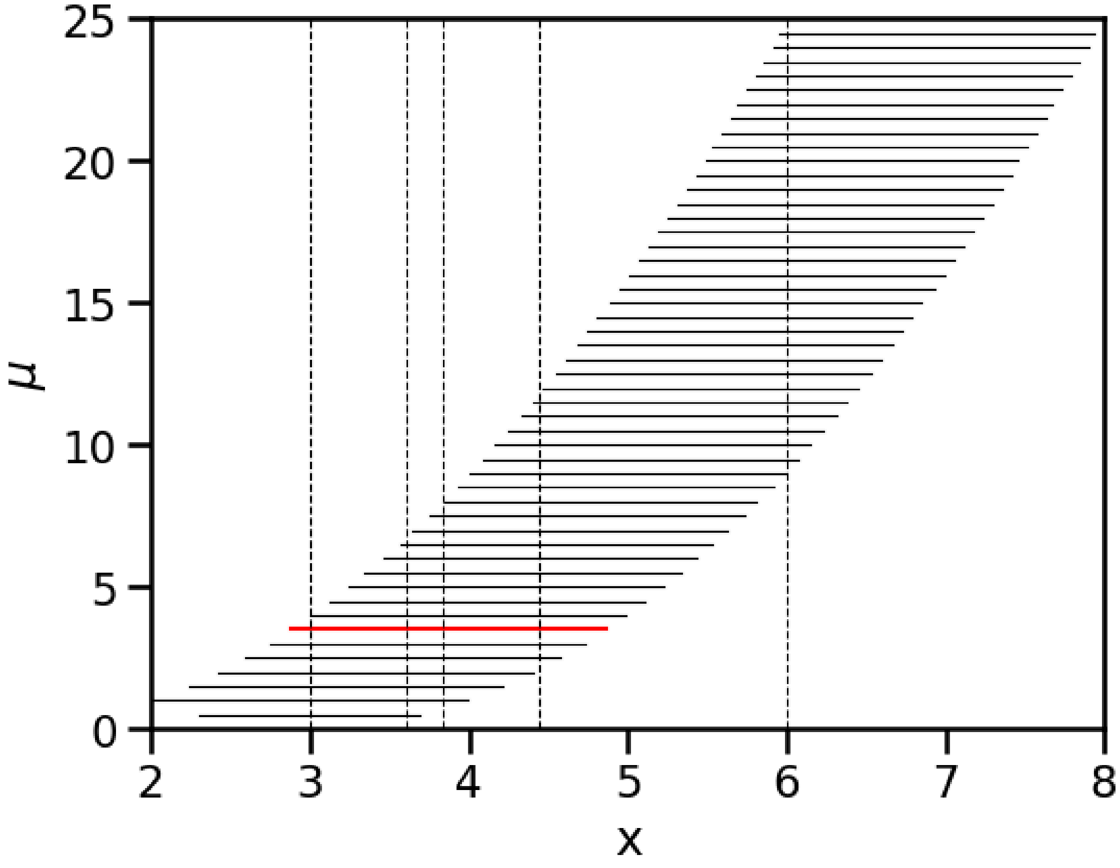
- The **confidence interval** $[\mu_l, \mu_u]$ from this experiment is the vertical intercept.



CI Construction: Confidence Belt

- Make some more measurements



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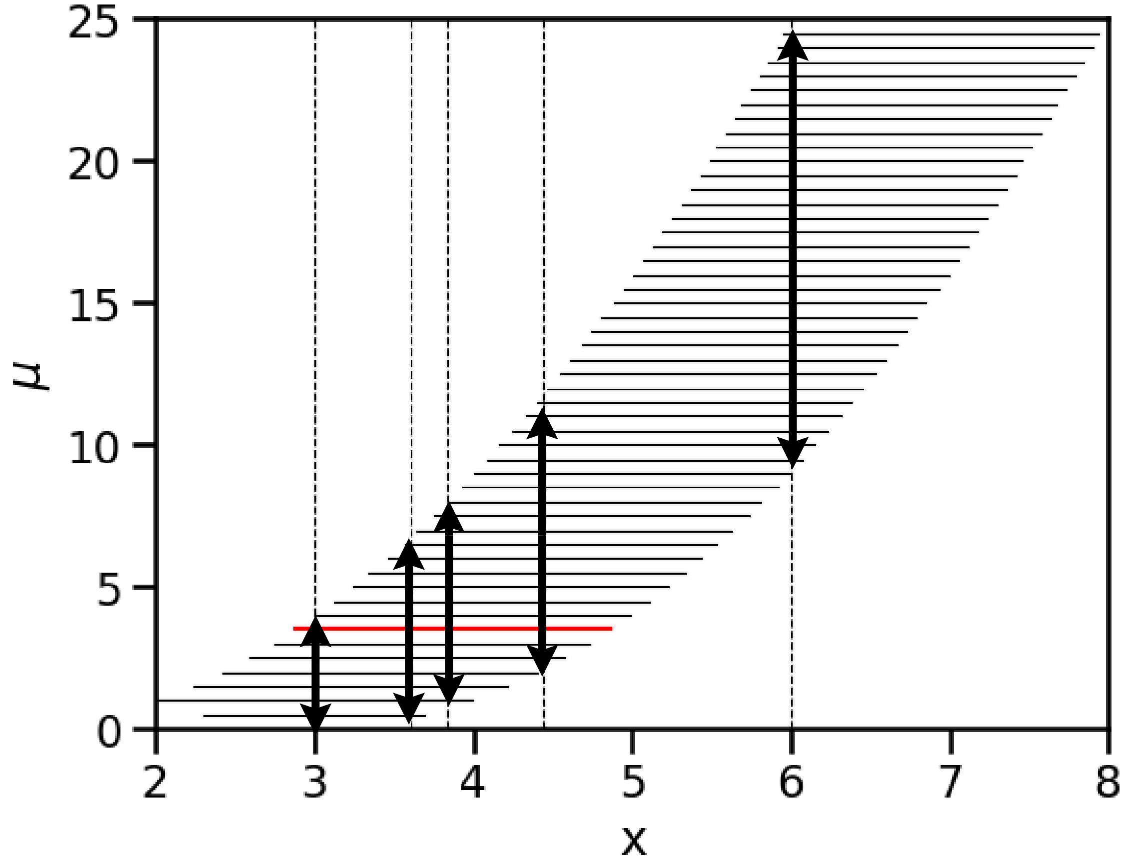
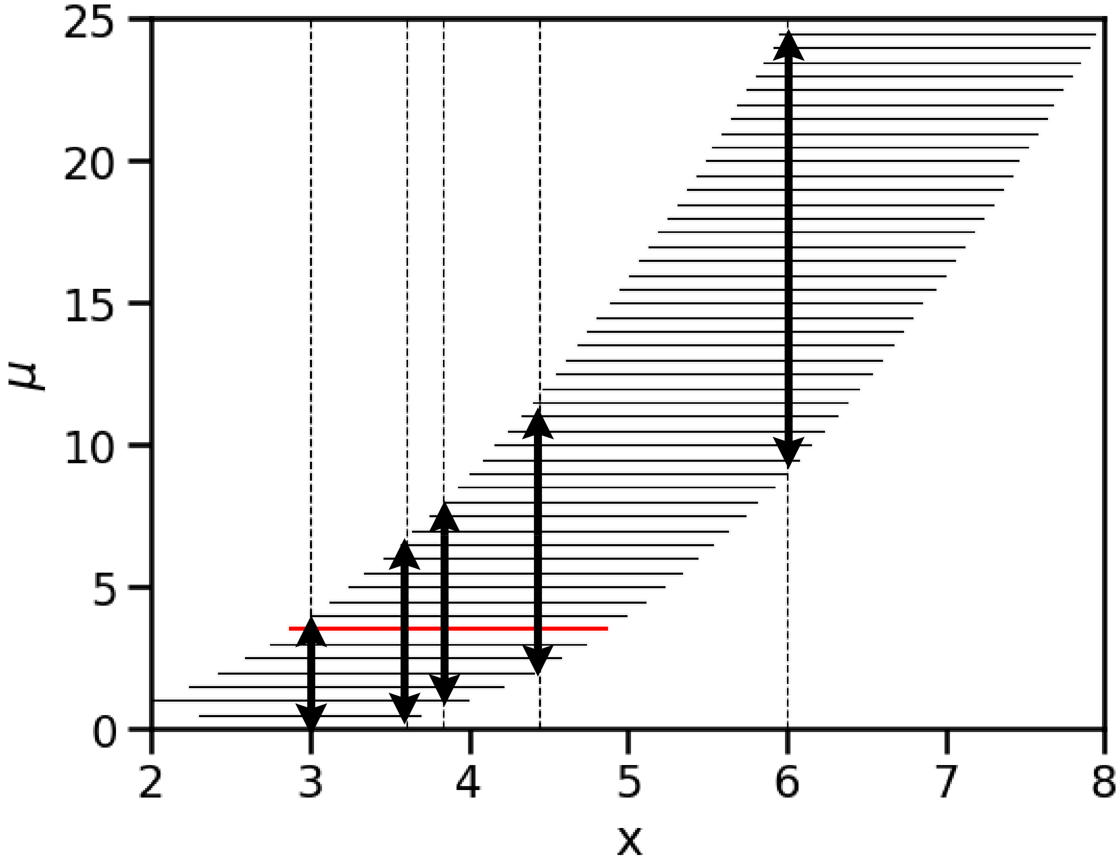


CI Construction: Confidence Belt

- Make some more measurements
- Get some more confidence intervals.
- Have a set

$$C = \{\mathrm{CI}_1, \mathrm{CI}_2, \mathrm{CI}_3, \mathrm{CI}_4, \mathrm{CI}_5\}$$

- 80% of this set would cover the true value, μ_t.



Standard CIs

Acceptance region?

- Recall acceptance region:

$$\Pr(n \in [n_1, n_2] \mid \mu_{\text{fixed}}) = 90\%$$

[1] See Feldman & Cousins Section II.B

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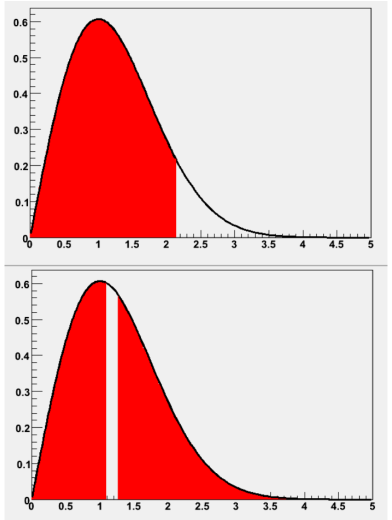

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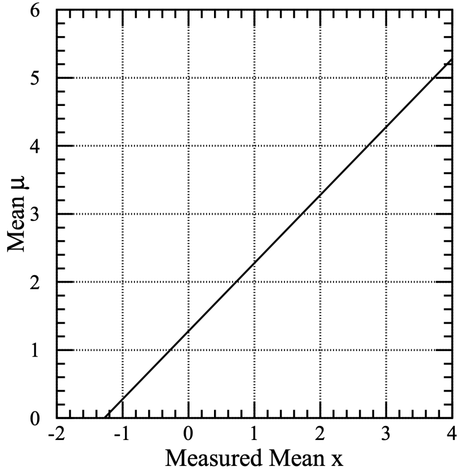

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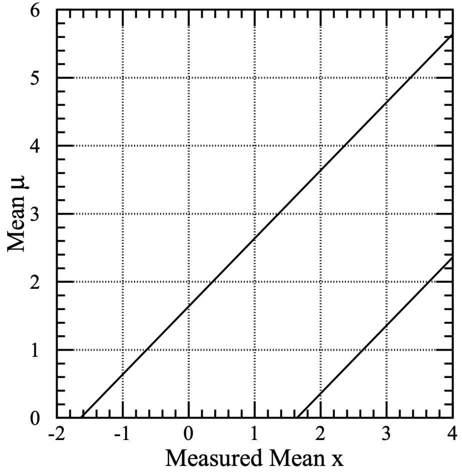
- Another common choice leads to the two-sided **central interval**.



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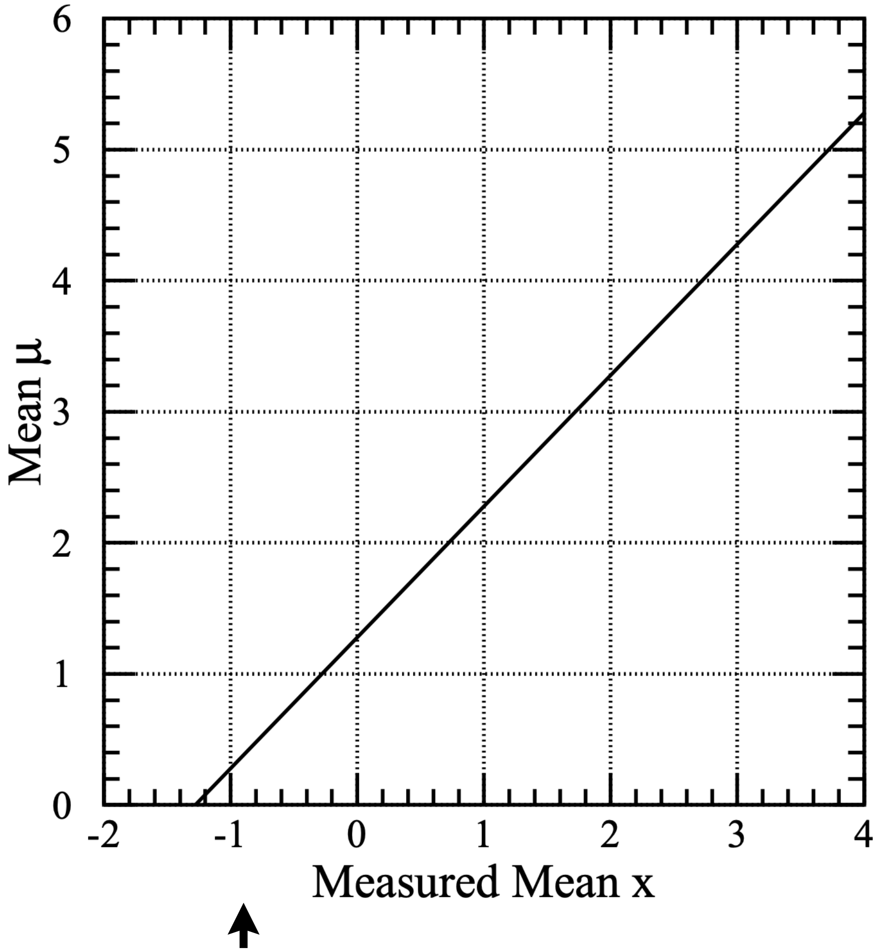
Example CIs

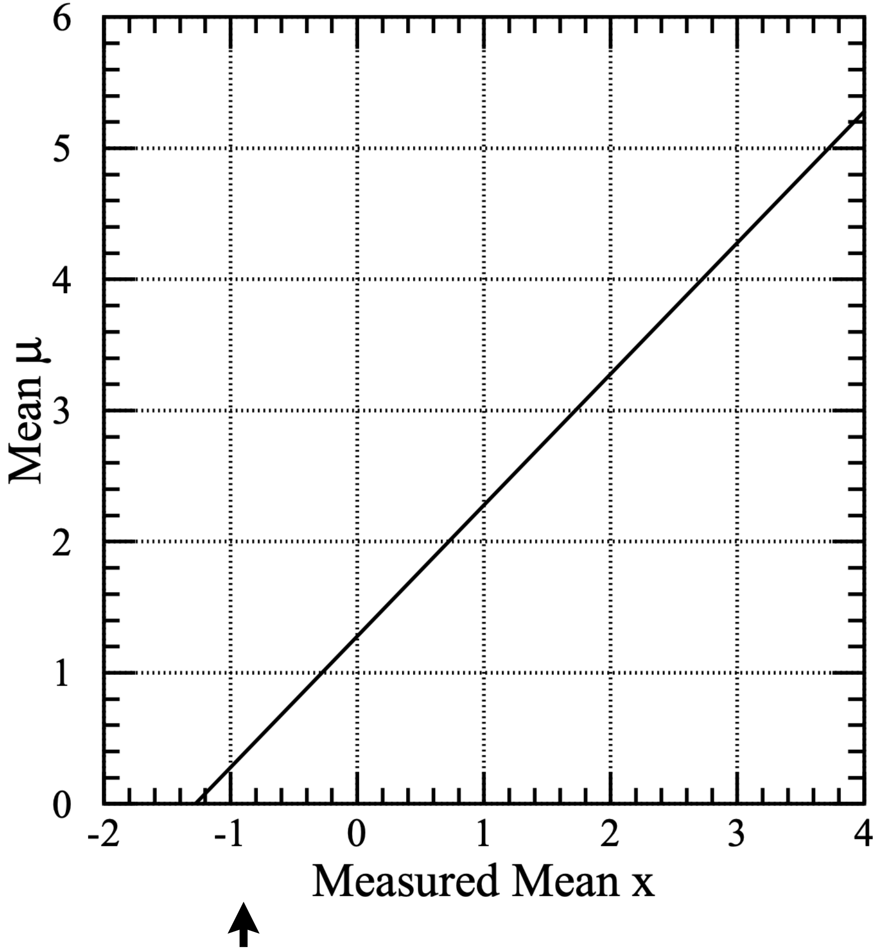


standard 90% upper limit

Gaussian likelihood with known $\sigma = 1$

$$\Pr(x \mid \mu) = \frac{1}{\sqrt{2\pi}} \cdot e^{-(x-\mu)^2/2}$$

Example CIs



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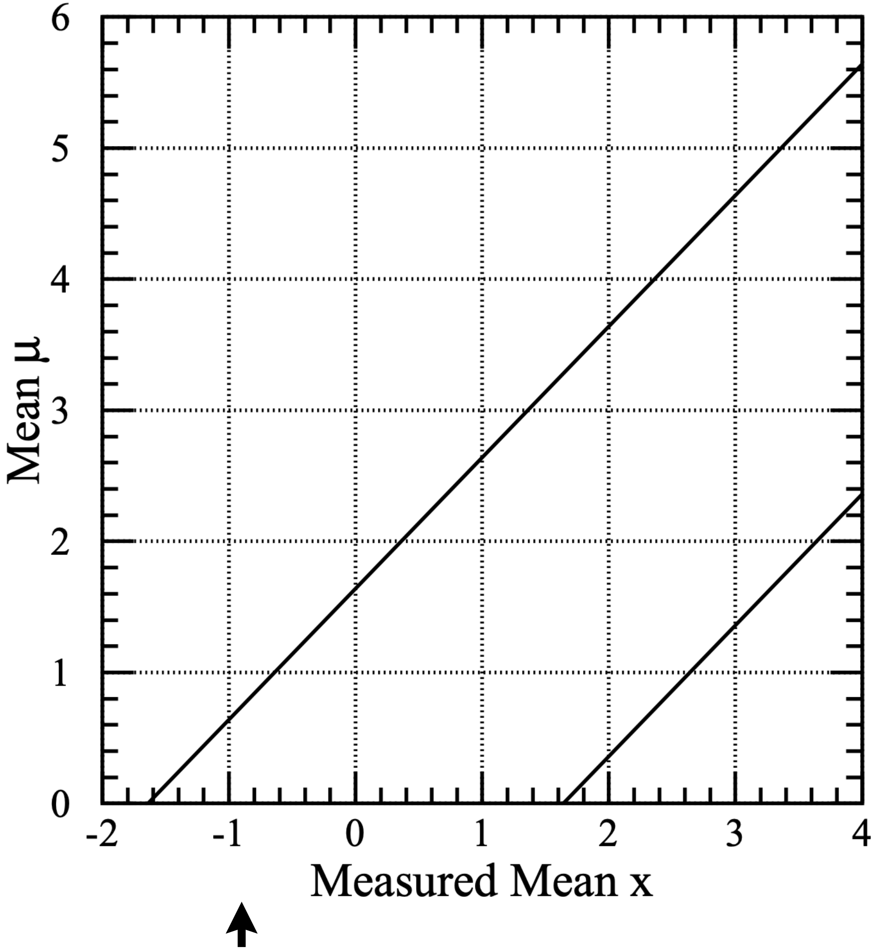
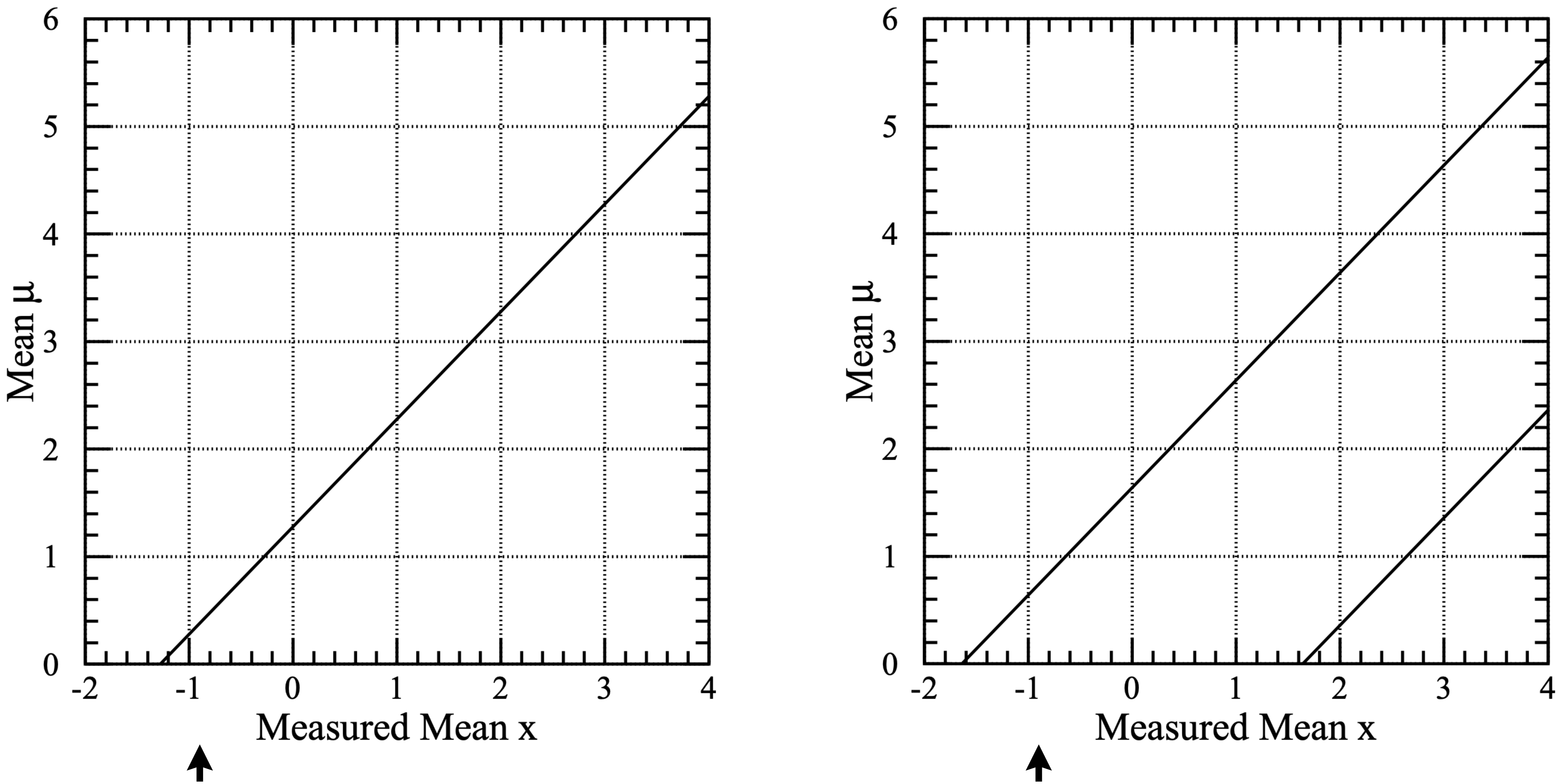


standard 90% central CI

Feldman & Cousins CI

Question: which one to use?



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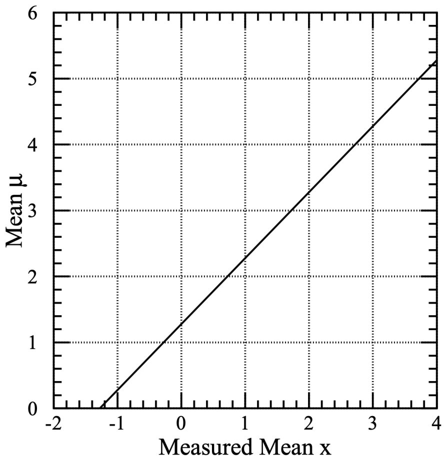
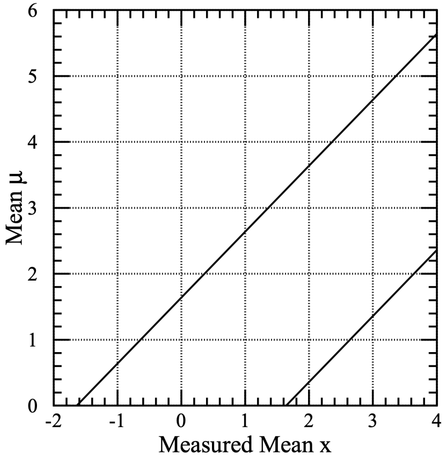
Problem: “flip-flop”





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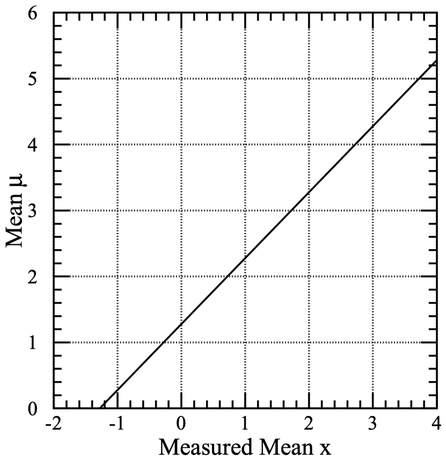
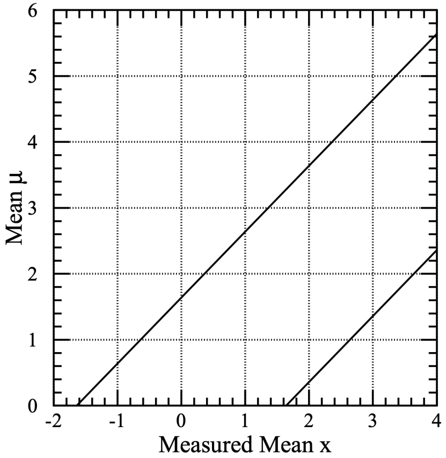
Problem: “flip-flop”





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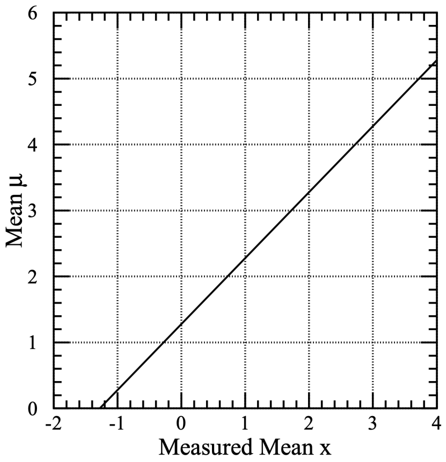
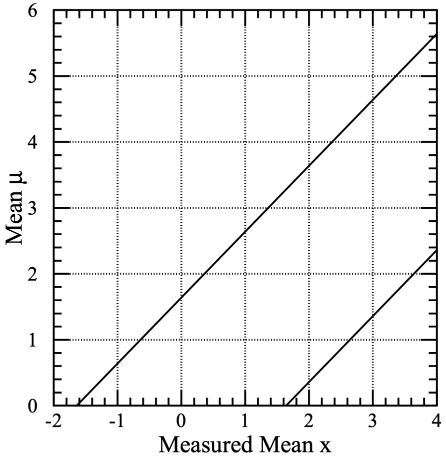
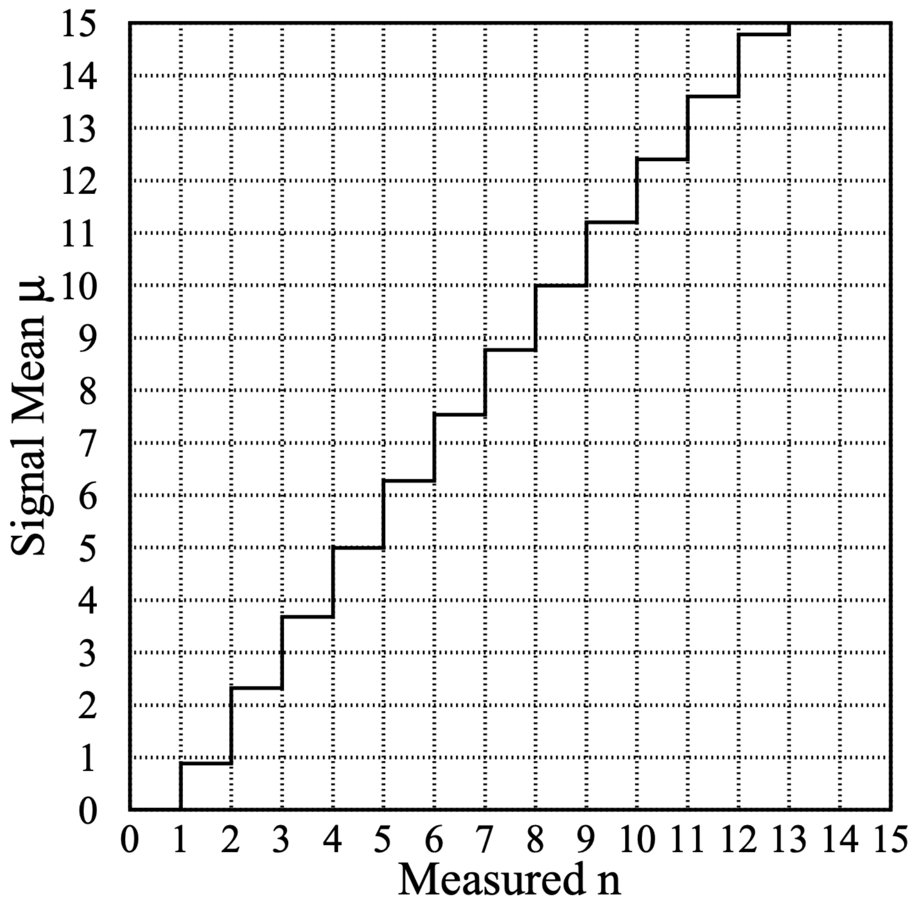




- One **cannot** pick the type of interval after-the-fact. The Choice is made before we perform the experiment. Otherwise we would be "flip-flopping".
- It turns out that flip-flopping leads to invalid intervals
- Feldman & Cousins's approach removes the possibility of (or motivation to commit) flip-flopping.

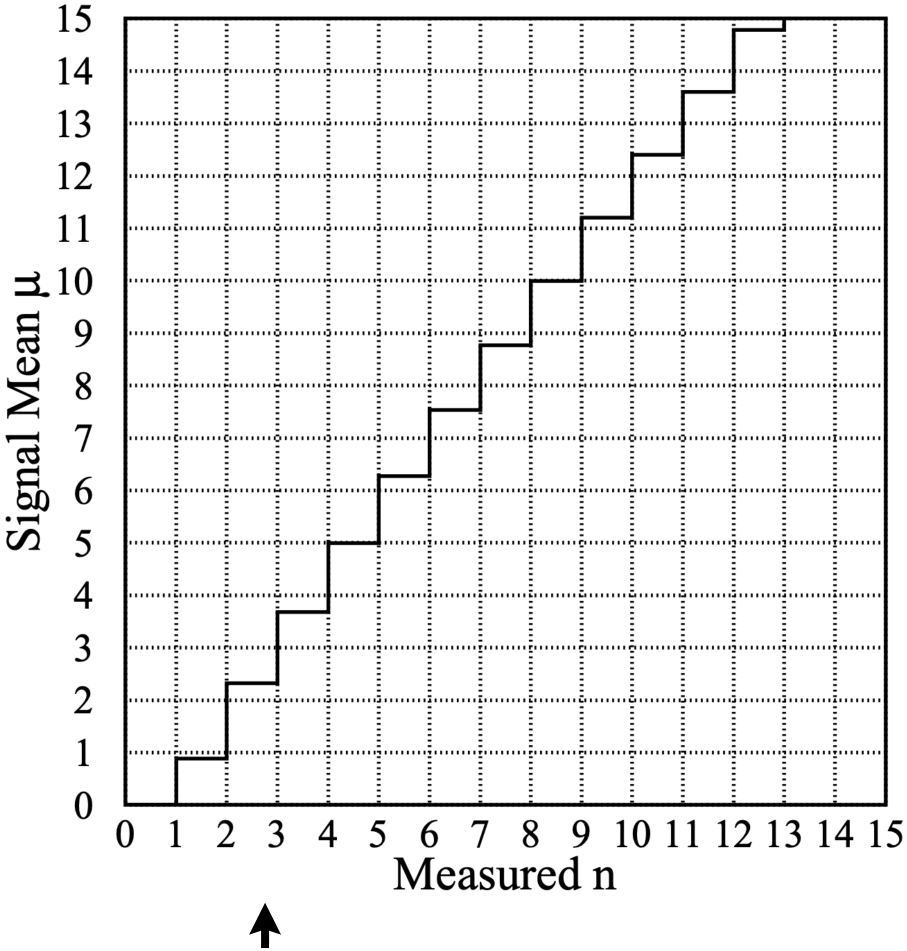
Standard Poisson (90% Upper) CI



Poisson likelihood with background

$$\mathcal{L} \equiv \Pr(n \mid \mu) = \frac{(\mu + b)^n e^{-(\mu+b)}}{n!}$$

Comparison: look at small n



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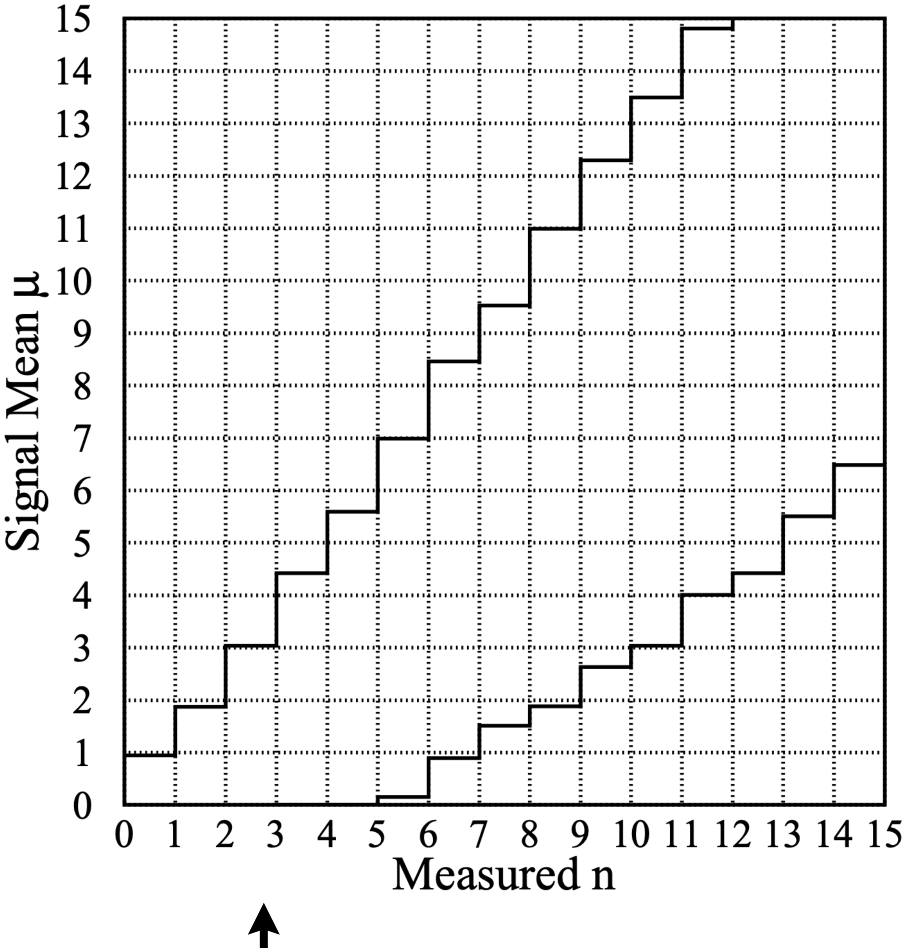
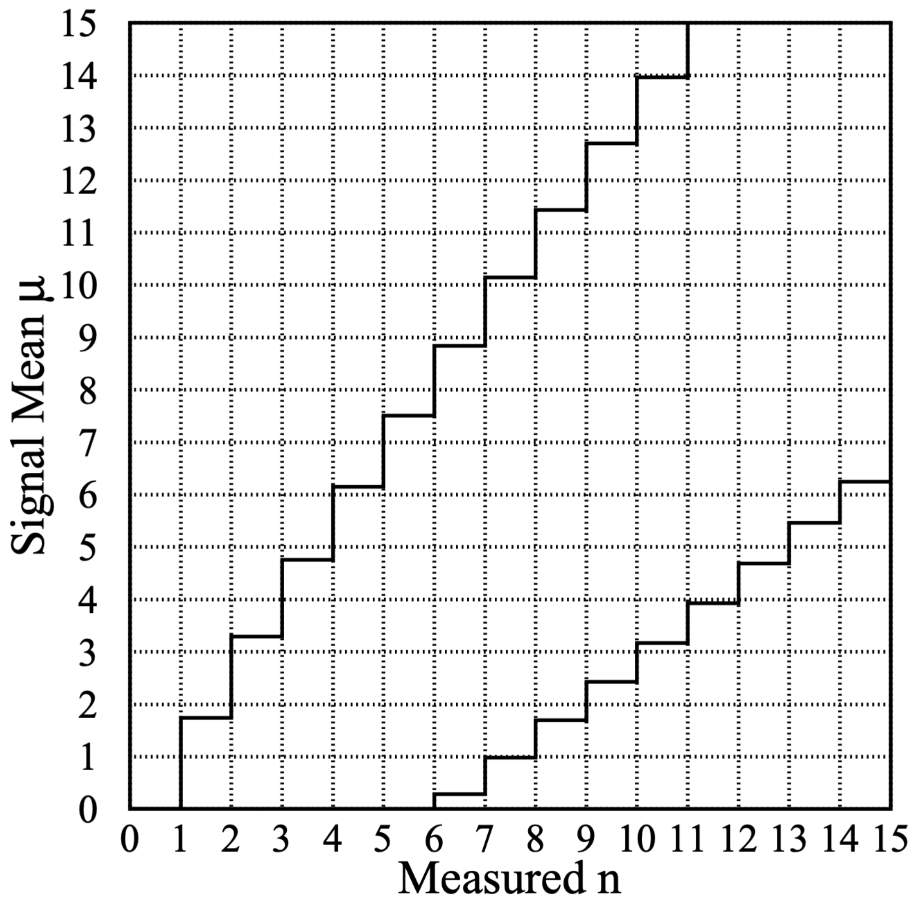
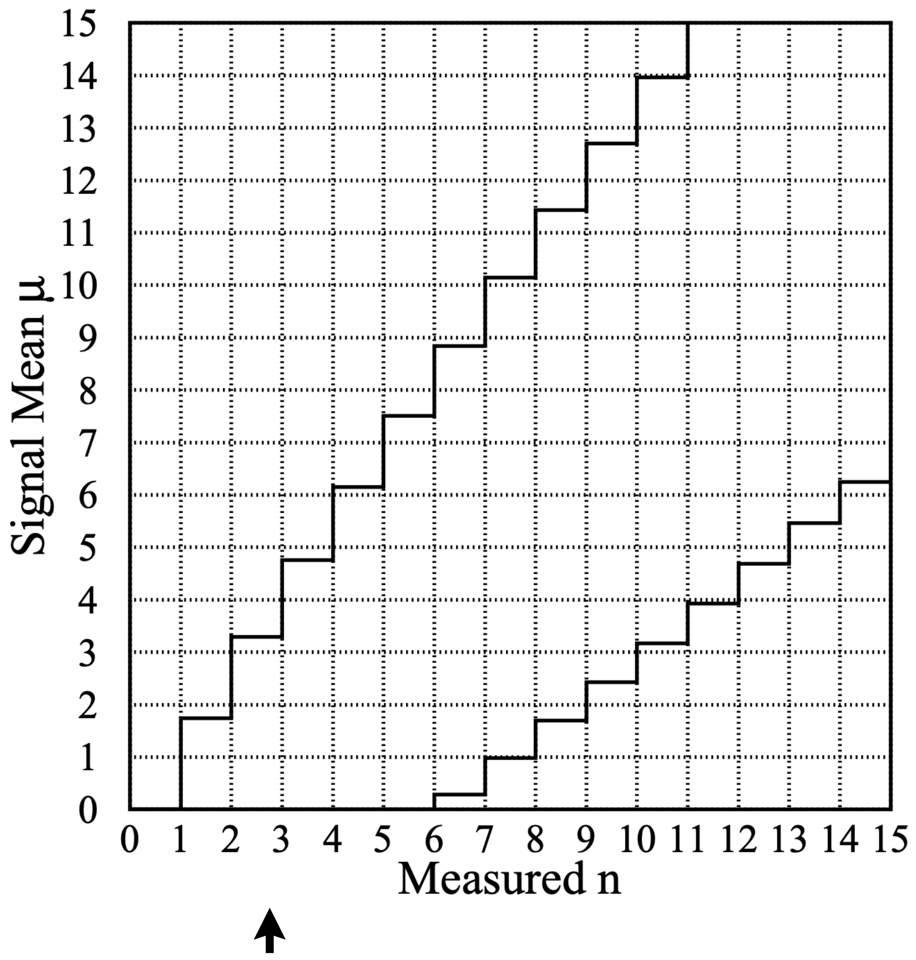


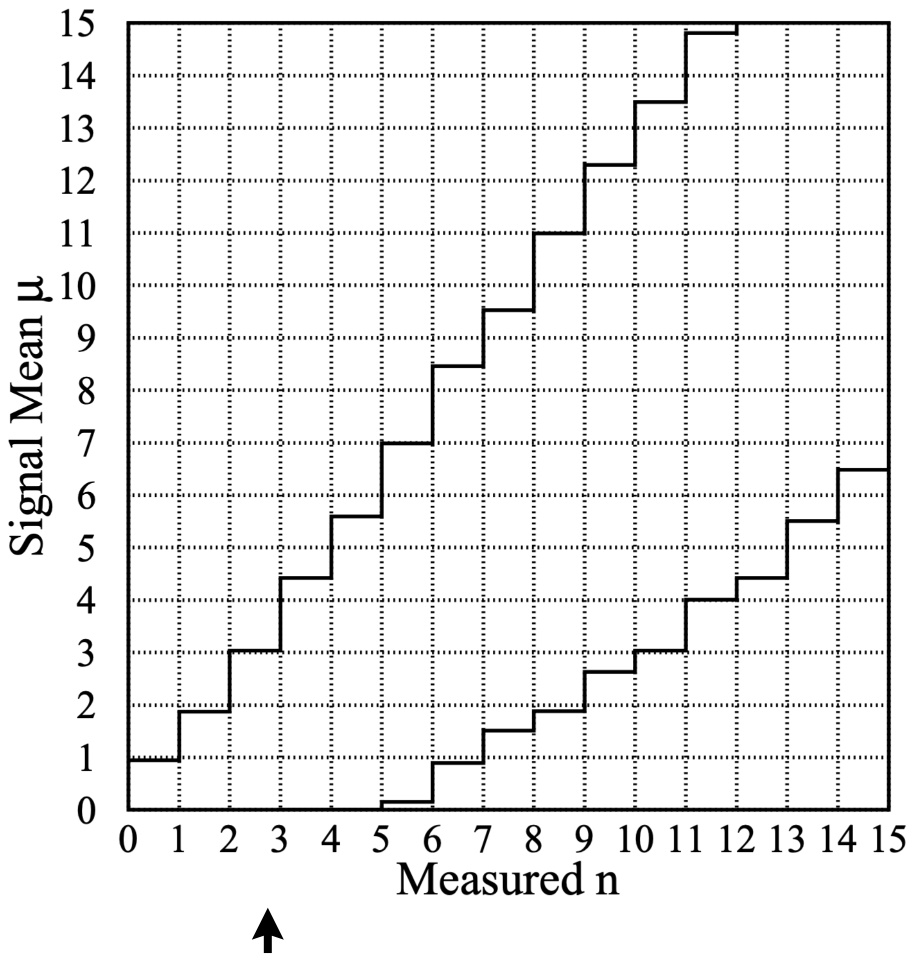
Feldman & Cousins

Standard Poisson (90% central) CI



Comparison: look at large n



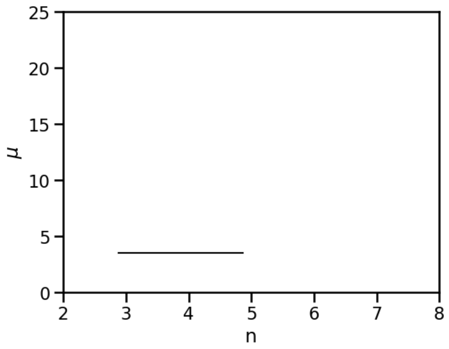


standard 90% central CI

Feldman & Cousins

F&C CI construction

Maximum Likelihood



- Recall acceptance region:

$$\Pr(n \in \boxed{[n_1, n_2]} \mid \mu_{\text{fixed}}) = 90\%$$

- Complete freedom in choosing how to construct the acceptance regions.
- Consider likelihood: Poisson with background b:

$$\mathcal{L} \equiv \Pr(n \mid \mu) = \frac{(\mu + b)^n e^{-(\mu+b)}}{n!}$$

- F&C propose to compute a likelihood ratio R
  - This needs a "best fit" $\boxed{\mu_{\text{best}} \equiv \max(0, n - b)}$

Maximum Likelihood

Derivation

- Likelihood is a Poisson in this case.

$$\mathcal{L} \equiv \Pr(n \mid \mu) = \frac{(\mu + b)^n e^{-(\mu+b)}}{n!}$$

- Find maximum (fixing n, vary μ):

$$\left.\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\mu}\right|_{\mu=\mu_{\text{best}}} = 0$$

- Result: “best fit” $\mu = \mu_{\text{best}} = n - b$
- Require physical $\mu \geq 0 \Rightarrow \mu_{\text{best}} = \max(0, n - b)$

Likelihood Ratio

- Do this for representative values of μ; say we start with $\mu = 0.5$
  - As an example background, $b = 3$
  - $\Rightarrow \mu_{\text{best}} \equiv \max(0, n - b) = \max(0, n - 3)$
- Procedure:
  -
  -
  -
  -

n	$\Pr(n\|\mu)$	μ_{best}	$\Pr(n\|\mu_{\text{best}})$	R	rank
0					

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  -
  -
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  -

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0	0.03	0			

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  - For $n = 0$, compute $\Pr(n \mid \mu = \mu_{\text{best}})$
  -

n	$\Pr(n\|\mu)$	μ_{best}	$\Pr(n\|\mu_{\text{best}})$	R	rank
0	0.03	0	0.05		

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  - For $n = 0$, compute $\Pr(n \mid \mu = \mu_{\text{best}})$
  - Divide likelihoods to get R.

n	$\Pr(n\|\mu)$	μ_{best}	$\Pr(n\|\mu_{\text{best}})$	R	rank
0	0.03	0	0.05	0.607	

Likelihood Ratio

- As a reminder, this is still just for $\mu = 0.5$ (and example $b = 3$)

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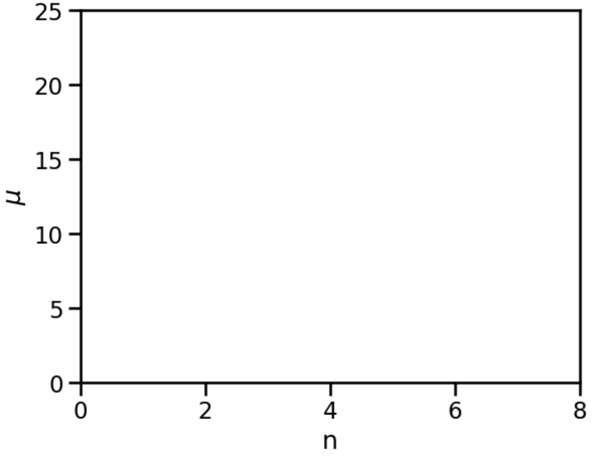
- We will see that the acceptance region for $\mu = 0.5$ is this:



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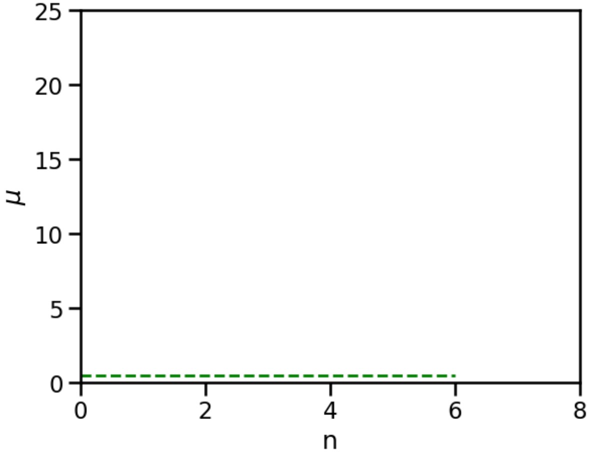

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- To construct the region, make a new row for $n = 1$

n	$\Pr(n\|\mu)$	μ_{best}	$\Pr(n\|\mu_{\text{best}})$	R	rank
0	0.030	0	0.050	0.607	
1	0.106	0	0.149	0.708	

Example acceptance region for $\mu = 0.5$

And then for a bunch of other n.

Example acceptance region for $\mu = 0.5$

n	$\Pr(n\|\mu)$	μ_{best}	$\Pr(n\|\mu_{\text{best}})$	R	rank
0	0.030	0	0.050	0.607	
1	0.106	0	0.149	0.708	
2	0.185	0	0.224	0.826	
3	0.216	0	0.224	0.963	
4	0.189	1	0.195	0.966	
5	0.132	2	0.175	0.753	
6	0.077	3	0.161	0.480	
7	0.039	4	0.149	0.259	

Example acceptance region for $\mu = 0.5$

n	$\Pr(n\|\mu)$	μ_{best}	$\Pr(n\|\mu_{\text{best}})$	R	rank
0	0.030	0	0.050	0.607	6
1	0.106	0	0.149	0.708	5
2	0.185	0	0.224	0.826	3
3	0.216	0	0.224	0.963	2
4	0.189	1	0.195	0.966	1
5	0.132	2	0.175	0.753	4
6	0.077	3	0.161	0.480	7
7	0.039	4	0.149	0.259	

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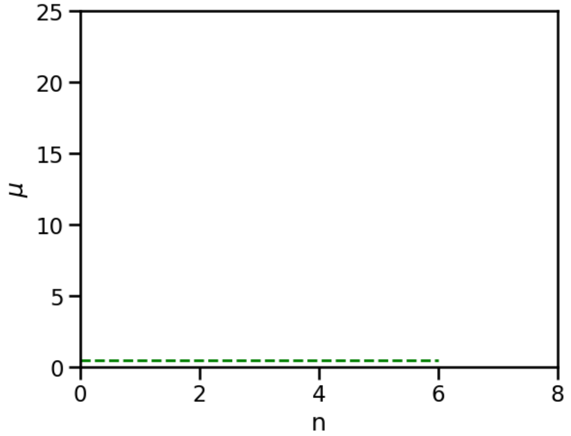

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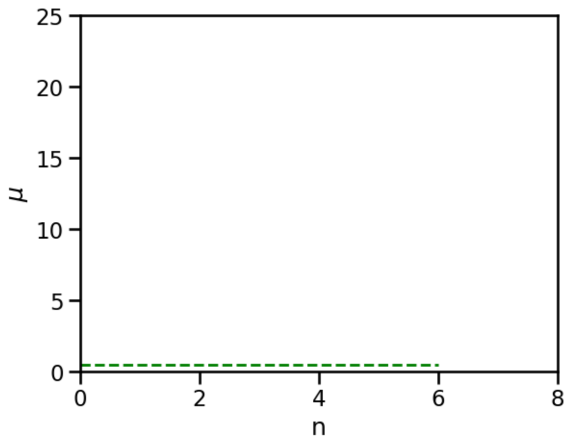
- Acceptance region for $\mu = 0.5$ is therefore $n \in [0, 6]$
- Next, construct the acceptance region for other μ as well.



Bayesian

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Posterior, encodes possible values of μ_t as a probability dist.

The "truth" μ_t is **not** fixed from the Bayesian perspective

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- If we specify a prior, we get the posterior.
- "uniform prior" $\Pr(\mu) = 1 \implies \Pr(\mu \mid n) = \Pr(n \mid \mu)$
- Suppose we measure $n = 0$ event, then the posterior is

$$\Pr(\mu \mid n = 0) = \Pr(n = 0|\mu) = \frac{e^{-\mu} \cdot \mu^n}{n!} = e^{-\mu}$$

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- so we “estemate with 90% confidence that $\mu \leq 2.3$” base on a non-detection.