Indian Institute of Technology, Kharagpur

Lecture 21

Image Enhancement
Frequency Domain Processing

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Enhancement in frequency domain

Spatial Domain $\Rightarrow$ Convolution

$$f(x,y) * h(x,y)$$

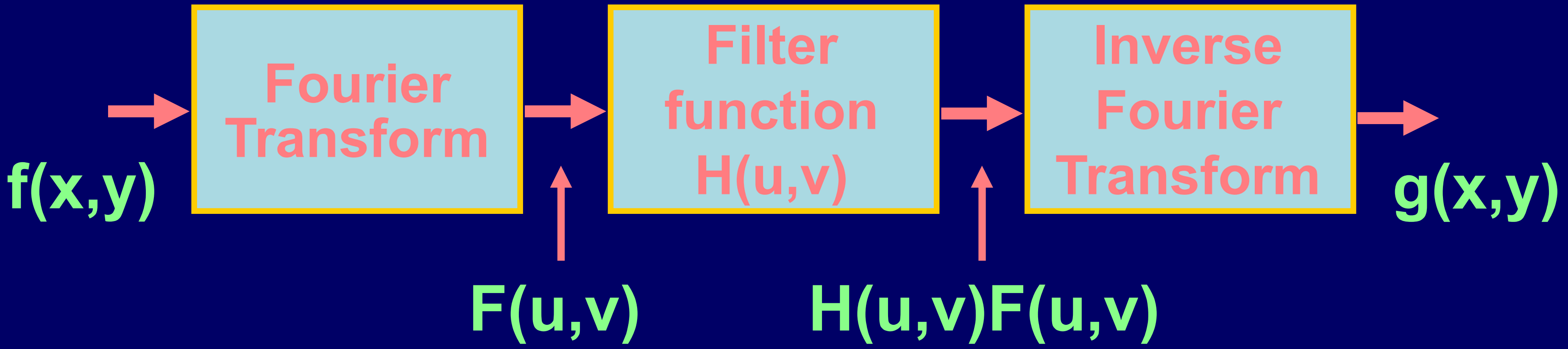
image Filter mask

Convolution Theorem $\Rightarrow$

$$f(x,y) * h(x,y) \Leftrightarrow F(u,v)H(u,v) \longrightarrow \text{Current interest}$$

$$f(x,y)\,h(x,y) \Leftrightarrow F(u,v) * H(u,v)$$

Frequency Domain Enhancement



1-D case

Consider filters based on Gaussian function

- Shapes are easily specified.
- Forward and Inverse Fourier transforms are real Gaussian functions.

H(u) denote a Gaussian filter in frequency domain

$$H(u) = A\, e^{-u^2/2\sigma^2}$$

$\sigma \rightarrow$ standard deviation of Gaussian curve

Corresponding filter in spatial domain

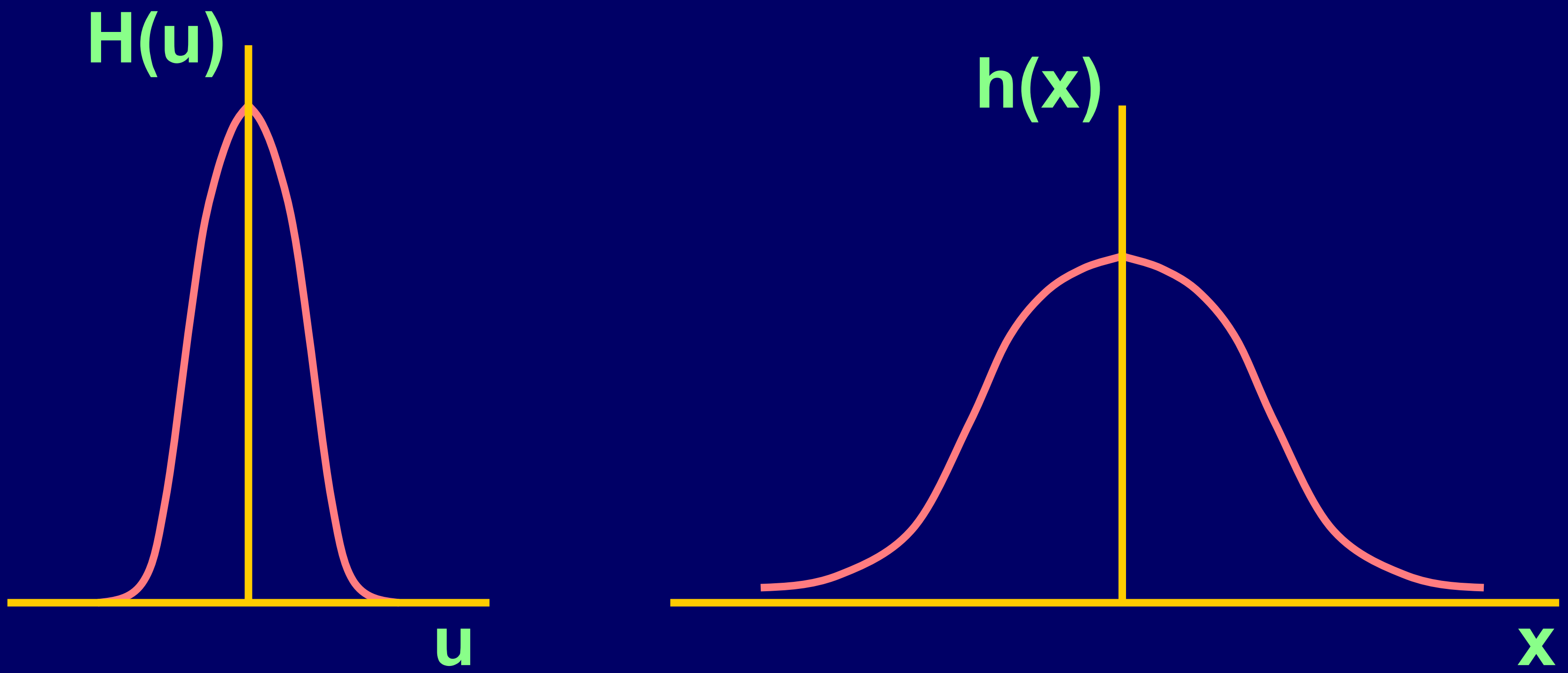
$$h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2 x^2}$$

1. **Both of these are Gaussian and real**
2. **Behave reciprocally with respect to one another**

   - when H(u) has broad profile (large σ), h(x) has a narrow profile and vice versa.

   - when $\sigma \rightarrow \infty$, H(u) tends to be a constant function and h(x) tends towards an impulse.

Lowpass Filter



LPF

All the values are +ve in both the domains.

$\Rightarrow$ Spatial domain filter mask will have all +ve coefficients.

Narrower the frequency domain filter, more it will attenuate the low frequencies, resulting in increased blurring.

$\Rightarrow$ This means in spatial domain a wider filter $\rightarrow$ a larger mask

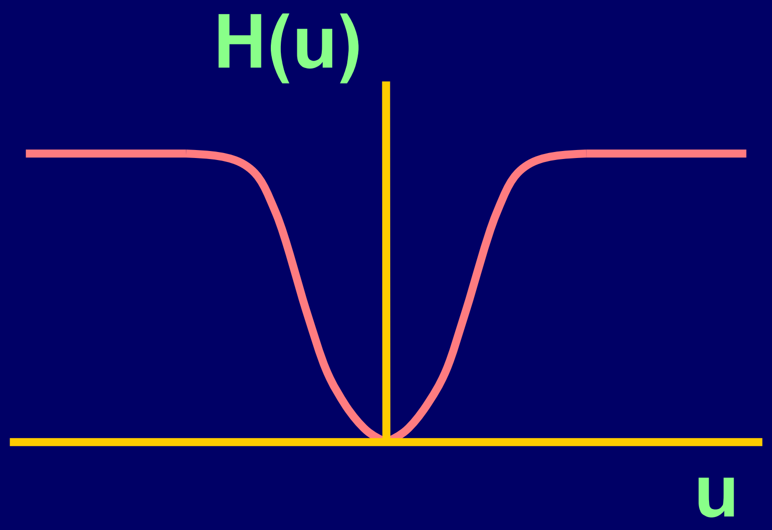
A Highpass filter in frequency domain can be constructed from Gaussian as

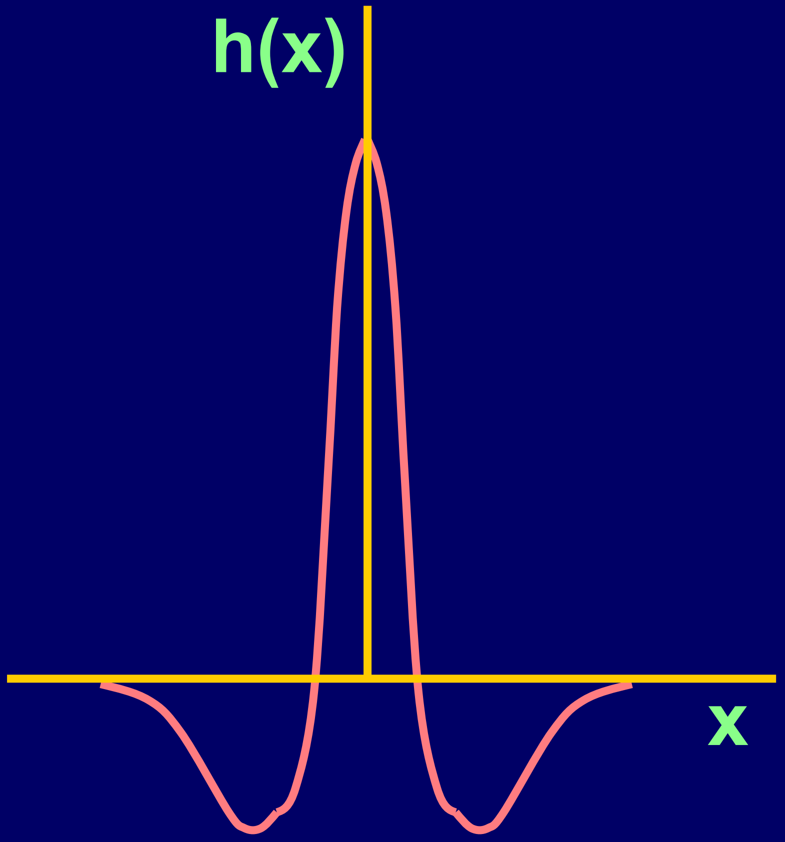
$$H(u) = A\left(1 - e^{-u^2/2\sigma^2}\right)$$

Corresponding spatial domain filter

$$h(x) = A\left[\left(\delta(x) - \sqrt{2\pi}\,\sigma e^{-2\pi^2\sigma^2x^2}\right]$$

Highpass Filter





Spatial filter has both +ve and –ve values. Once values turn negative, they don't turn +ve again

Smoothing Frequency Domain Filter

Edge and other sharp transitions lead to high frequency content in Fourier Transform

Smoothing (blurring) is obtained by attenuating specified range of high frequency components in the FT of a given image.

Basic model $\Rightarrow$

$$G(u,v) = H(u,v)\,F(u,v)$$

→ FT of image

Select a filter transfer function H(u,v) that yields G(u,v) by attenuating high frequency components of F(u,v)

Ideal LPF

$$H(u,v) = \begin{cases} 1 & D(u,v) \le D_o \\ 0 & D(u,v) > D_o \end{cases}$$

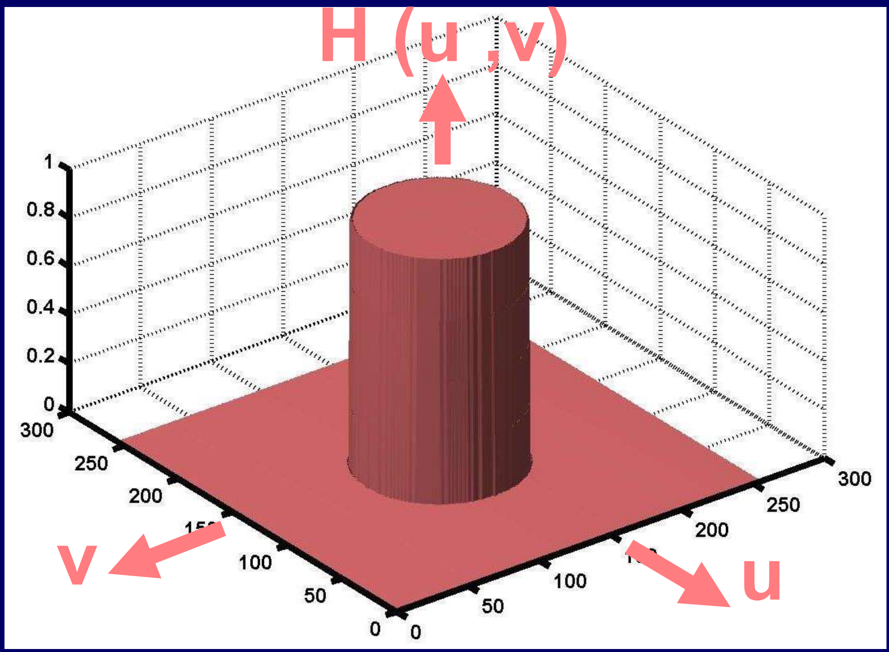
Cuts off all high frequency components of the FT that are at a distance greater than a specified distance D_o from the origin

For image size M x N $\Rightarrow$

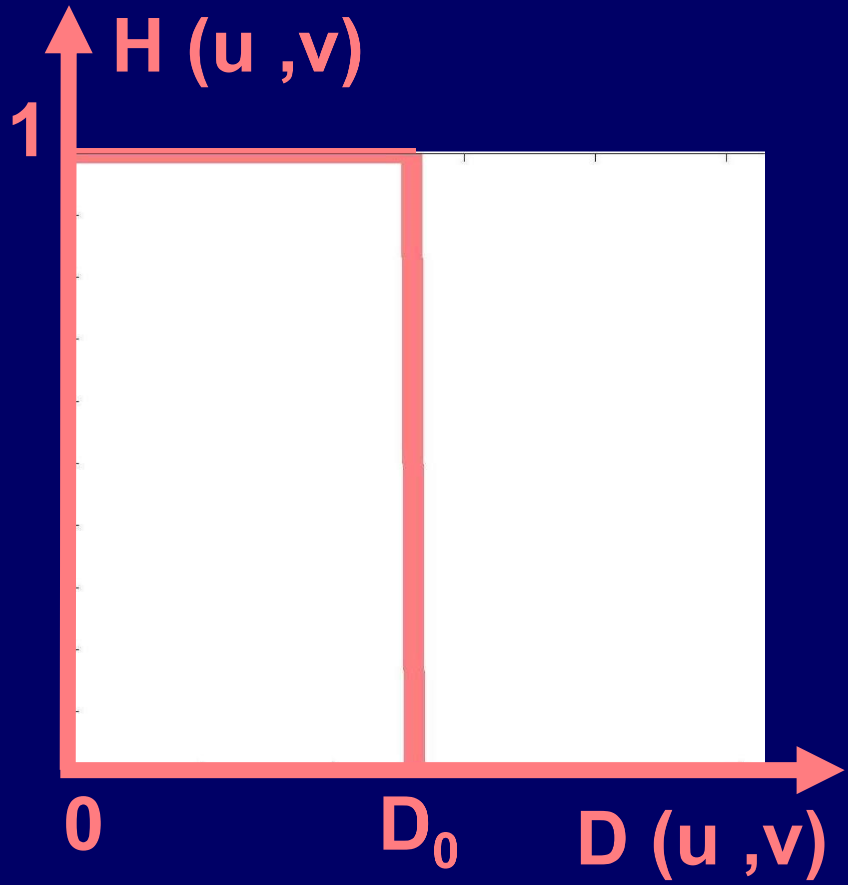
Center of frequency rectangle for centered FT is at (u,v) = (M/2 , N/2)

$$\Rightarrow D(u,v) = [(u-M/2)^2+(v-N/2)^2]^{1/2}$$

Ideal Lowpass Filter



Perspective plot



Cross section

Ideal $\Rightarrow$ All frequencies within circle of radius D0 are passed unattenuated.

Frequencies outside this circle are completely attenuated.

Cutoff frequency $\Rightarrow$

Point of transition between H(u,v) = 1 and H(u,v) = 0.

Such sharp cutoff frequencies can not be realized using electronic components.

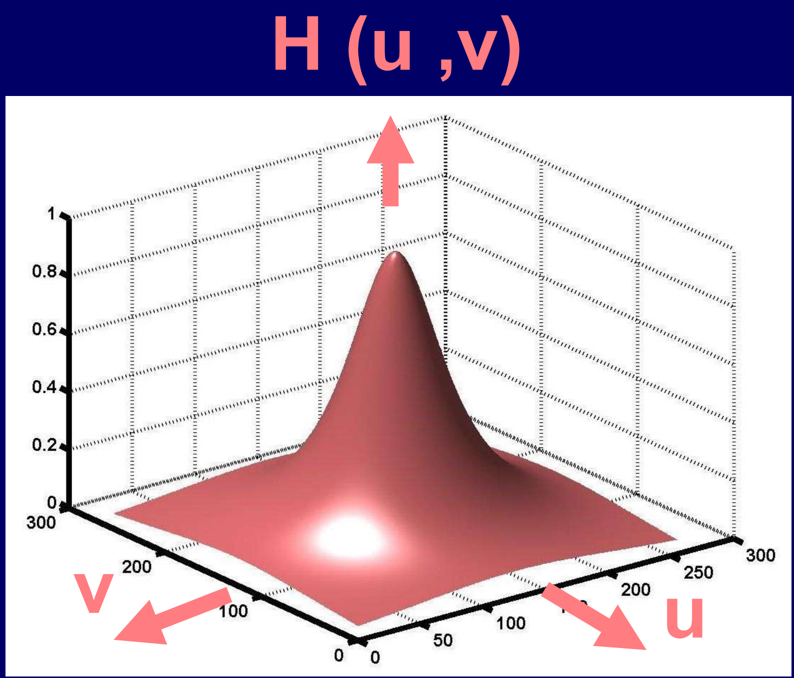
Butterworth LPF

$$H(u,v) = \frac{1}{1 + \left[\dfrac{D(u,v)}{D_0}\right]^{2n}}$$

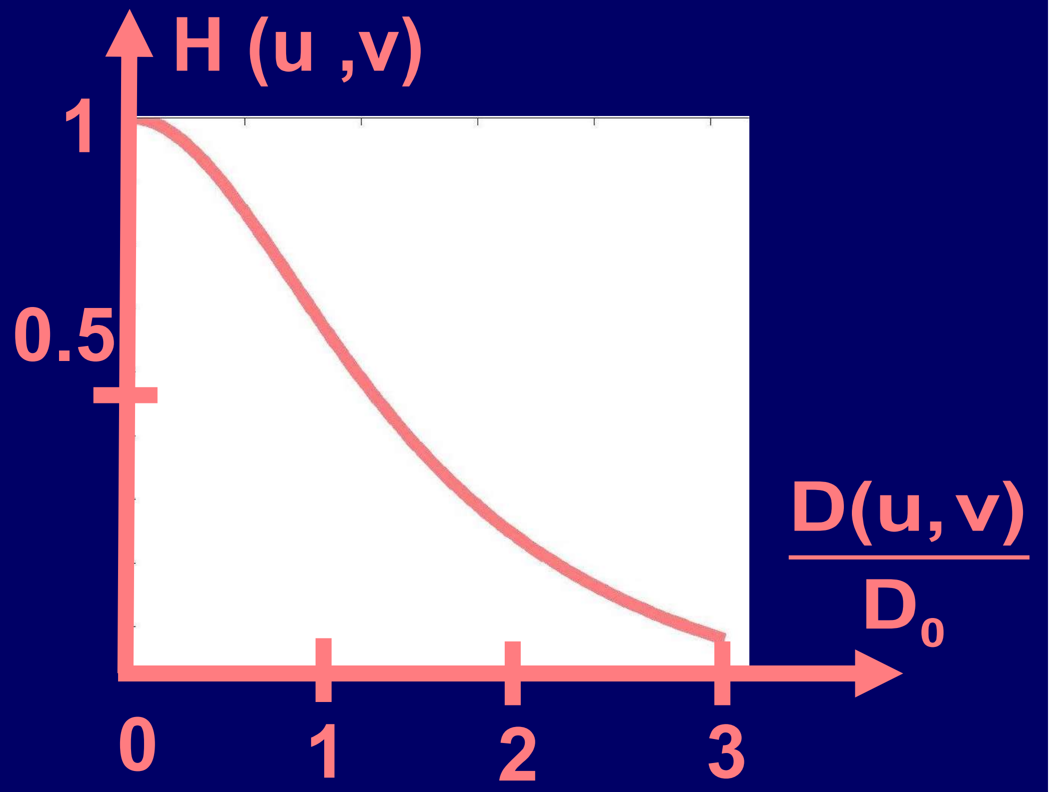
$\Rightarrow$ **Order n**

Butterworth Lowpass Filter

n =1

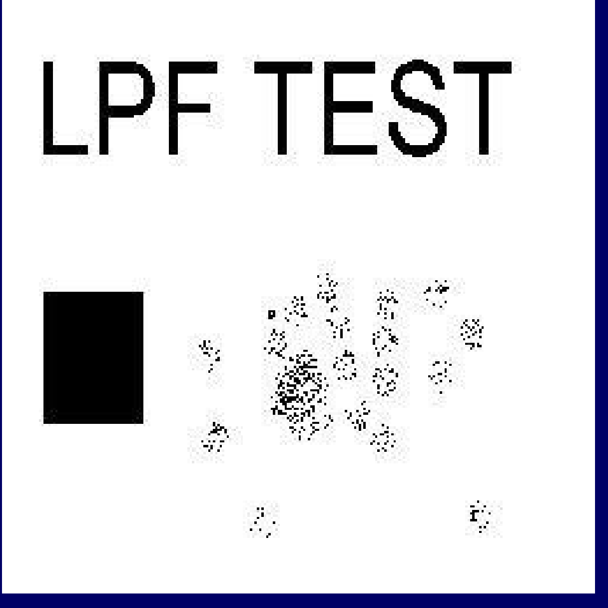


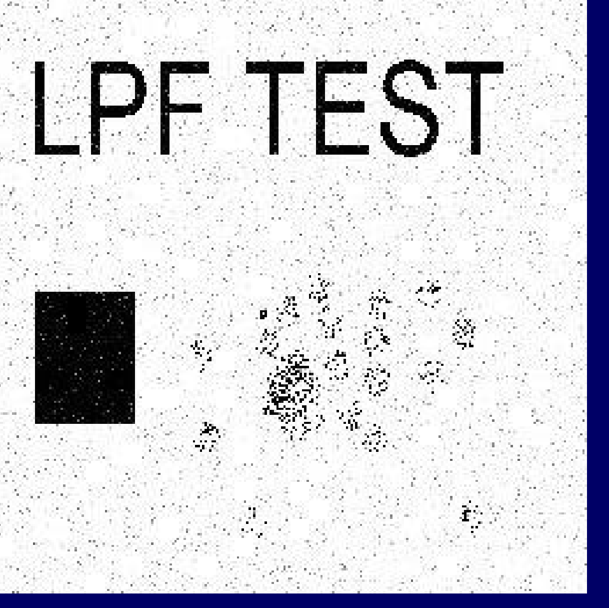
Perspective plot



Cross section

Lowpass Filter Result



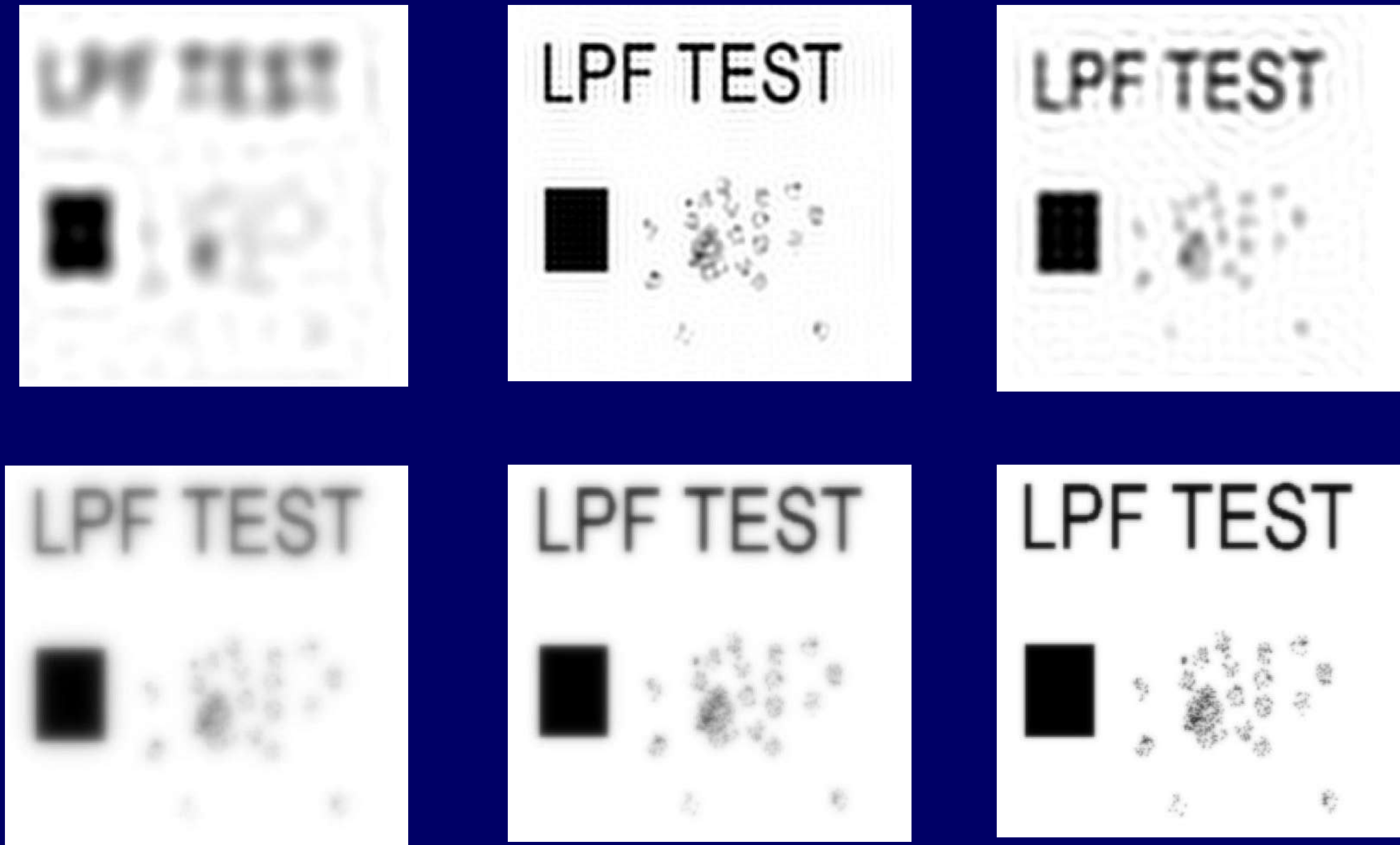


Ideal & Butterworth Lowpass Filter

Without Noise

Ideal

Butter



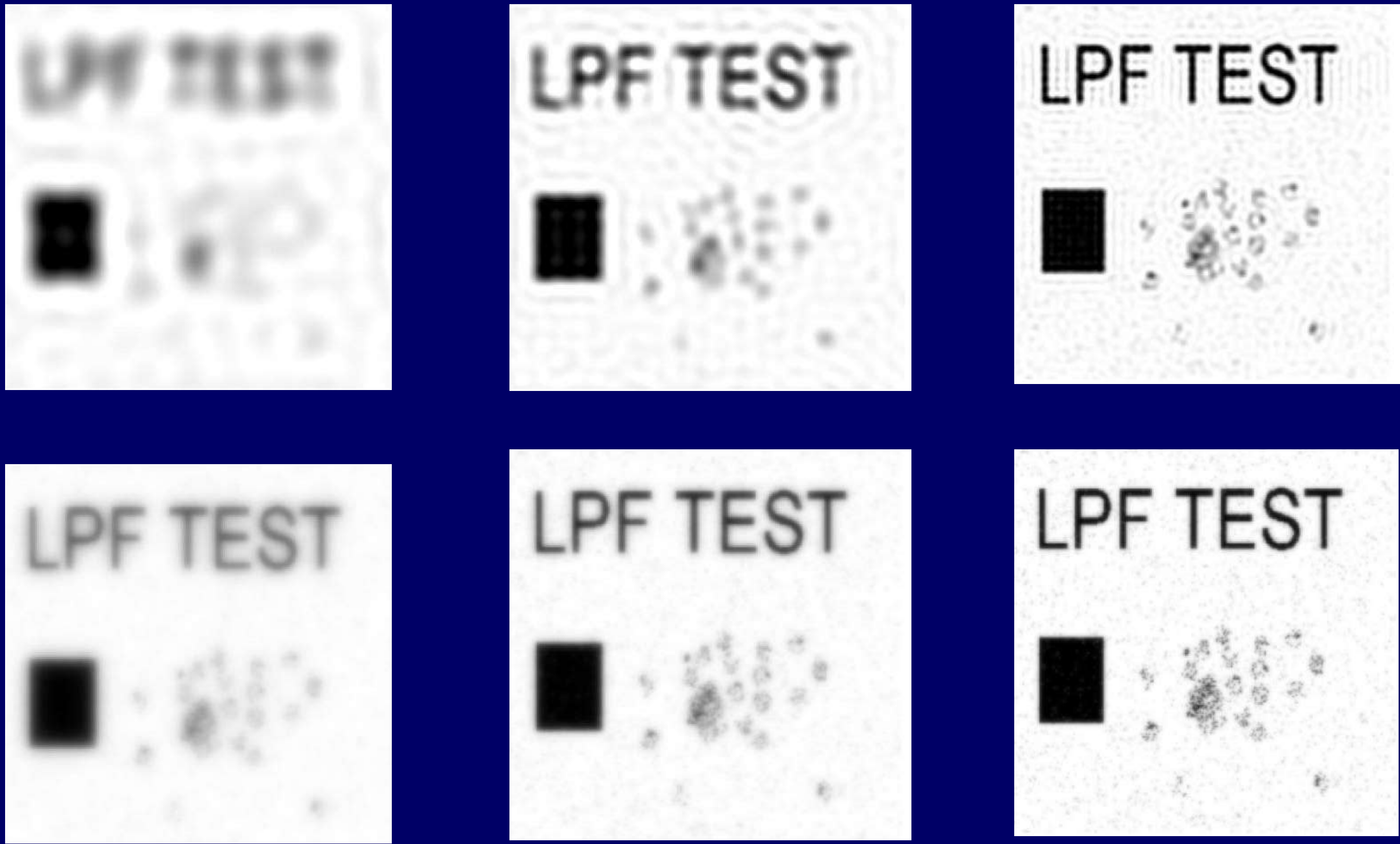
10 20 40

Ideal & Butterworth Lowpass Filter

With Noise

Ideal

Butter



10 20 40

Butterworth LPF

Blurring & Ringing

For BLPF $\Rightarrow$ order 1 $\Rightarrow$ no ringing

Ringing appears in case of BPLF of higher order.

Gaussian LPF

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

Allowing $\sigma = D_0$

$\Rightarrow$ $H(u,v) = e^{-D^2(u,v)/2D_0^2}$

$\Rightarrow$ IFT is also Gaussian

$\Rightarrow$ Leads to no ringing

Sharpening Frequency Domain filters

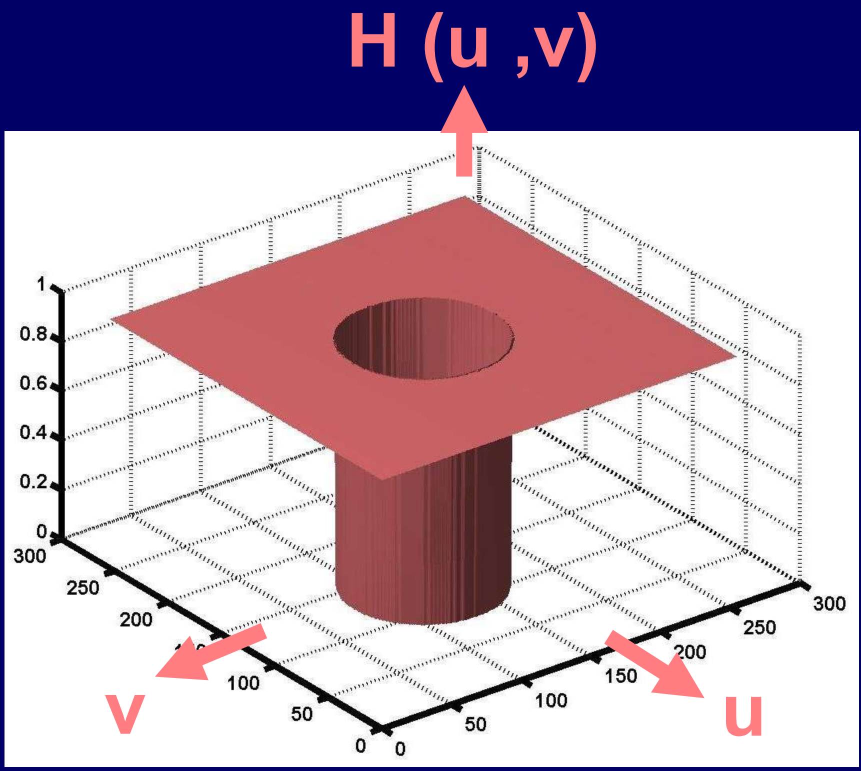
Sharpening obtained by Highpass filtering

$$H_{hpf}(u,v) = 1 - H_{lpf}(u,v)$$

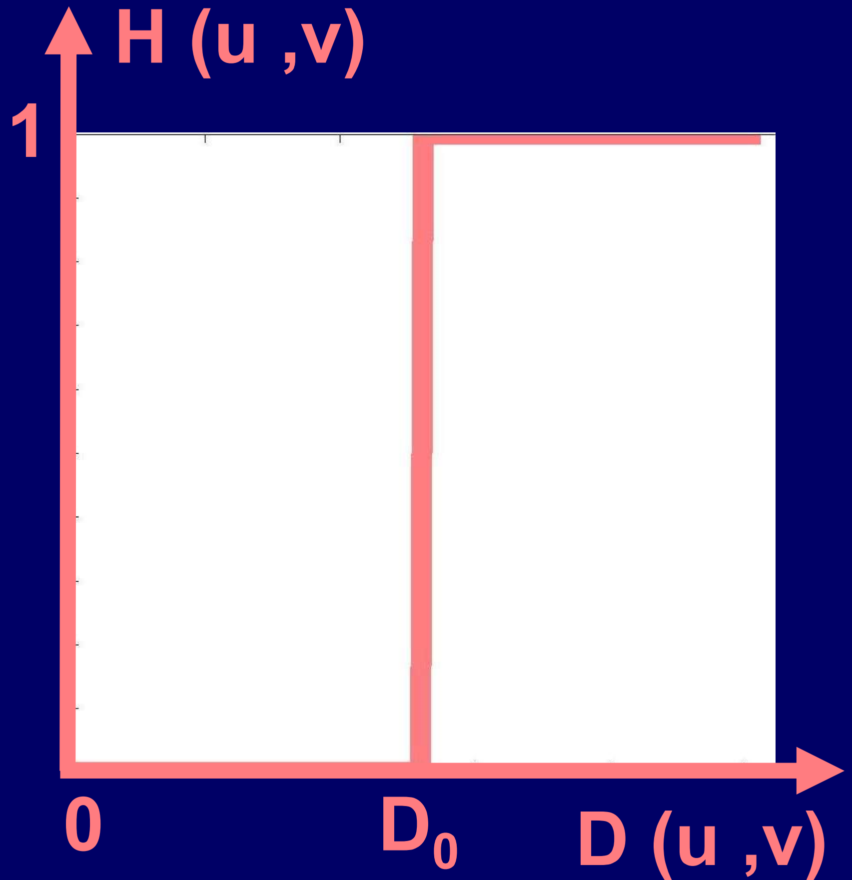
Ideal HPF

$$H(u,v) = \begin{cases} 0 & \text{if} \quad D(u,v) \le D_o \\ 1 & \text{If} \quad D(u,v) > D_o \end{cases}$$

Ideal Highpass Filter



Perspective plot



Cross section

Butterworth HPF

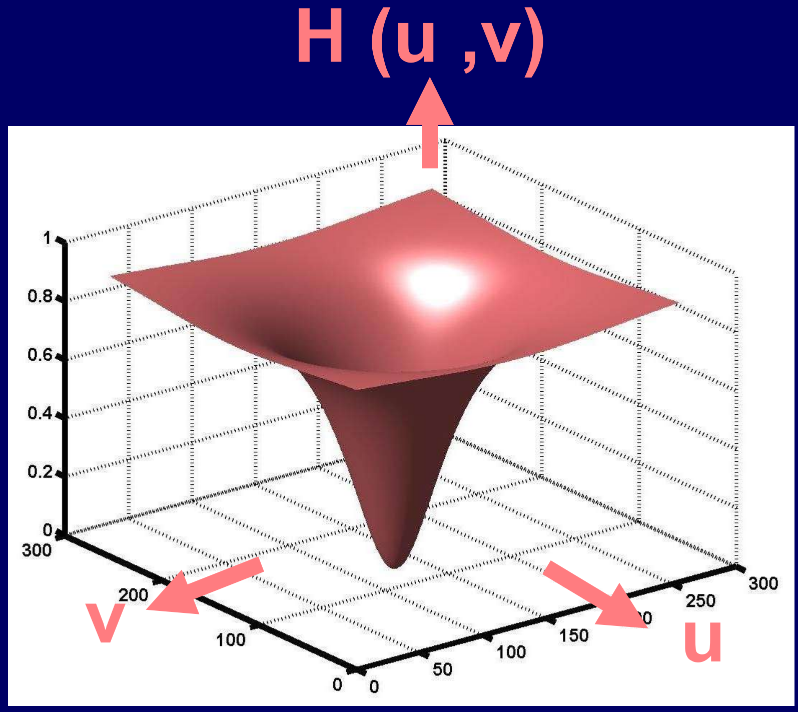
$$H(u,v) = \frac{1}{1 + \left[\dfrac{D_0}{D(u,v)}\right]^{2n}}$$

Gaussian HPF

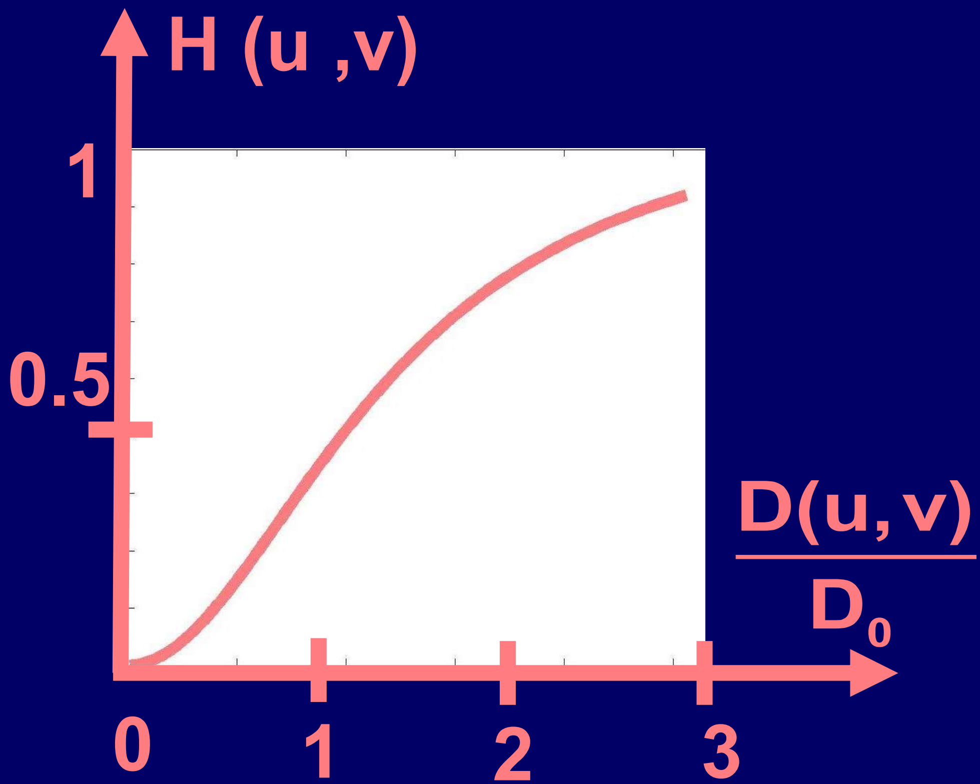
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

Butterworth Highpass Filter

n =1



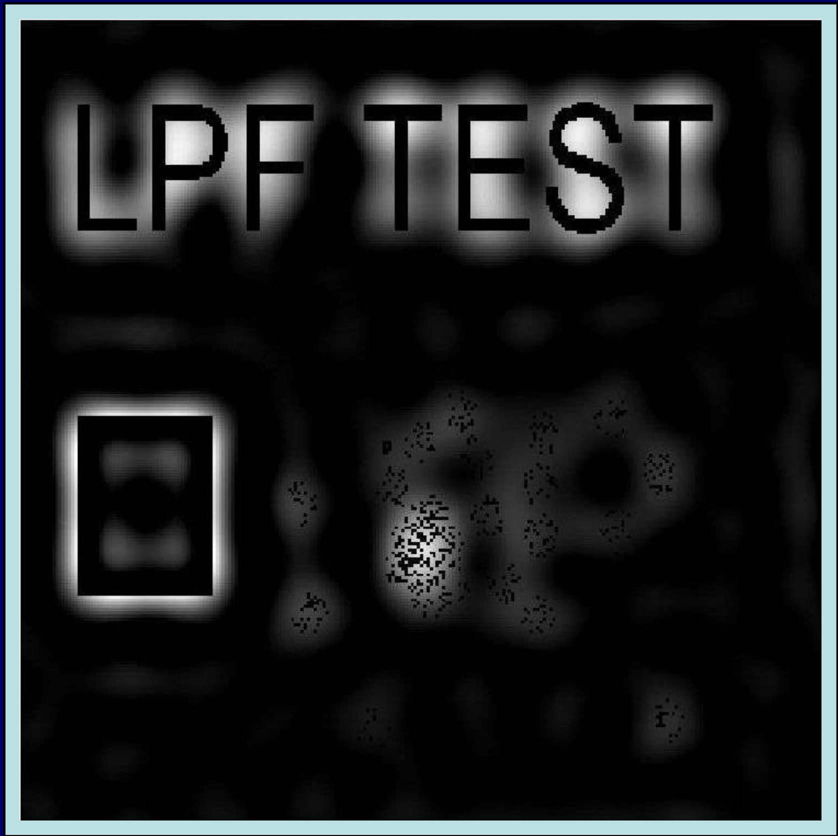
Perspective plot



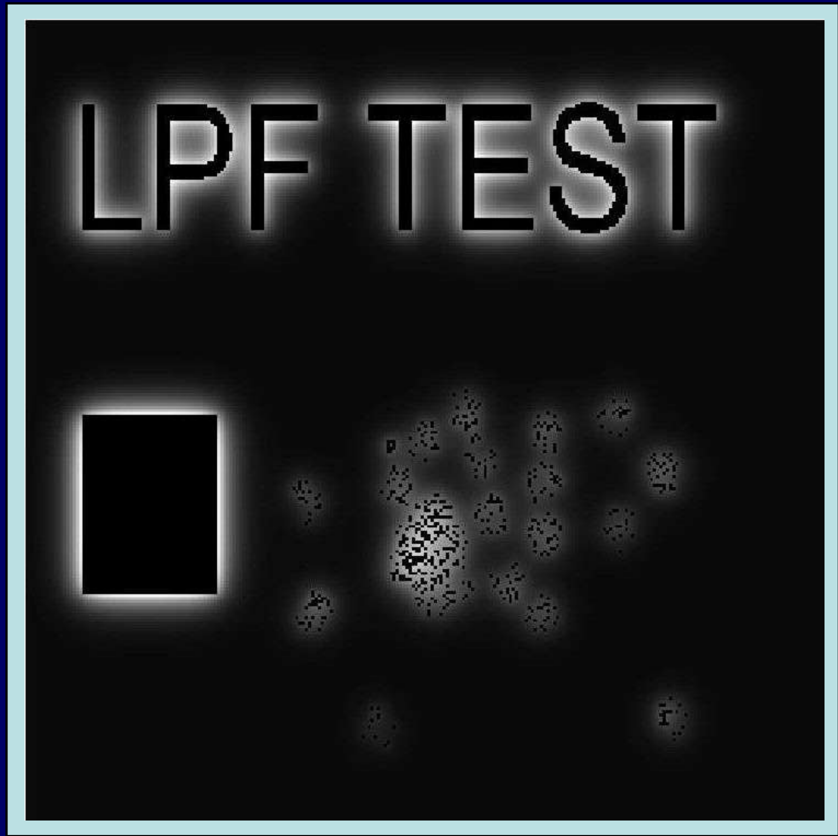
Cross section

Ideal & Butterworth Highpass Filter

10



Ideal

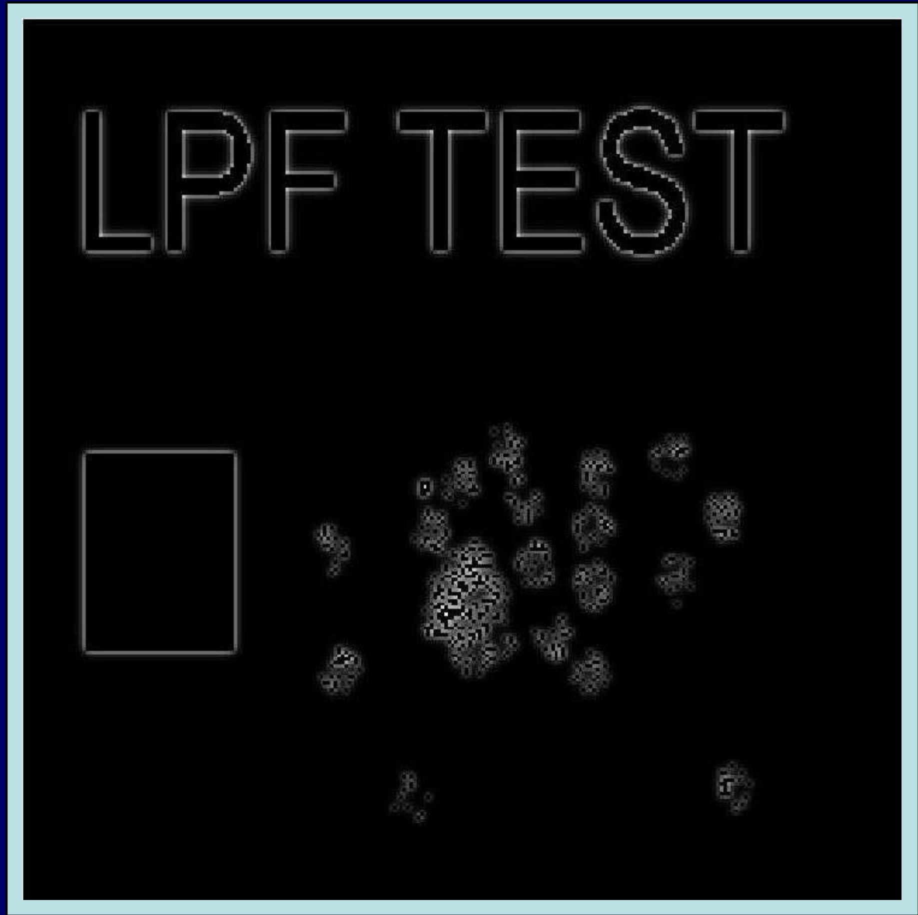


Butterworth

Ideal & Butterworth Highpass Filter

50

Ideal



Butterworth

Homographic Filter

$$f(x,y) = i(x,y)\,r(x,y)$$

$$\Rightarrow Z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

$$\Rightarrow Z(u,v) = F_i(u,v) + F_r(u,v)$$

Choose H(u,v) to work on Z(u,v)

$$S(u,v) = H(u,v)Z(u,v)$$

$$= H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$

$\Rightarrow$ IFT

$$s(x,y) = i'(x,y) + r'(x,y)$$

Finally,

$$g(x,y) = e^{s(x,y)} = e^{i'(x,y)}.e^{r'(x,y)}$$

$$= i_0(x,y).r_0(x,y)$$

$\Rightarrow$ illumination/reflectance components of output image

Illumination component $\Rightarrow$ slowly varying

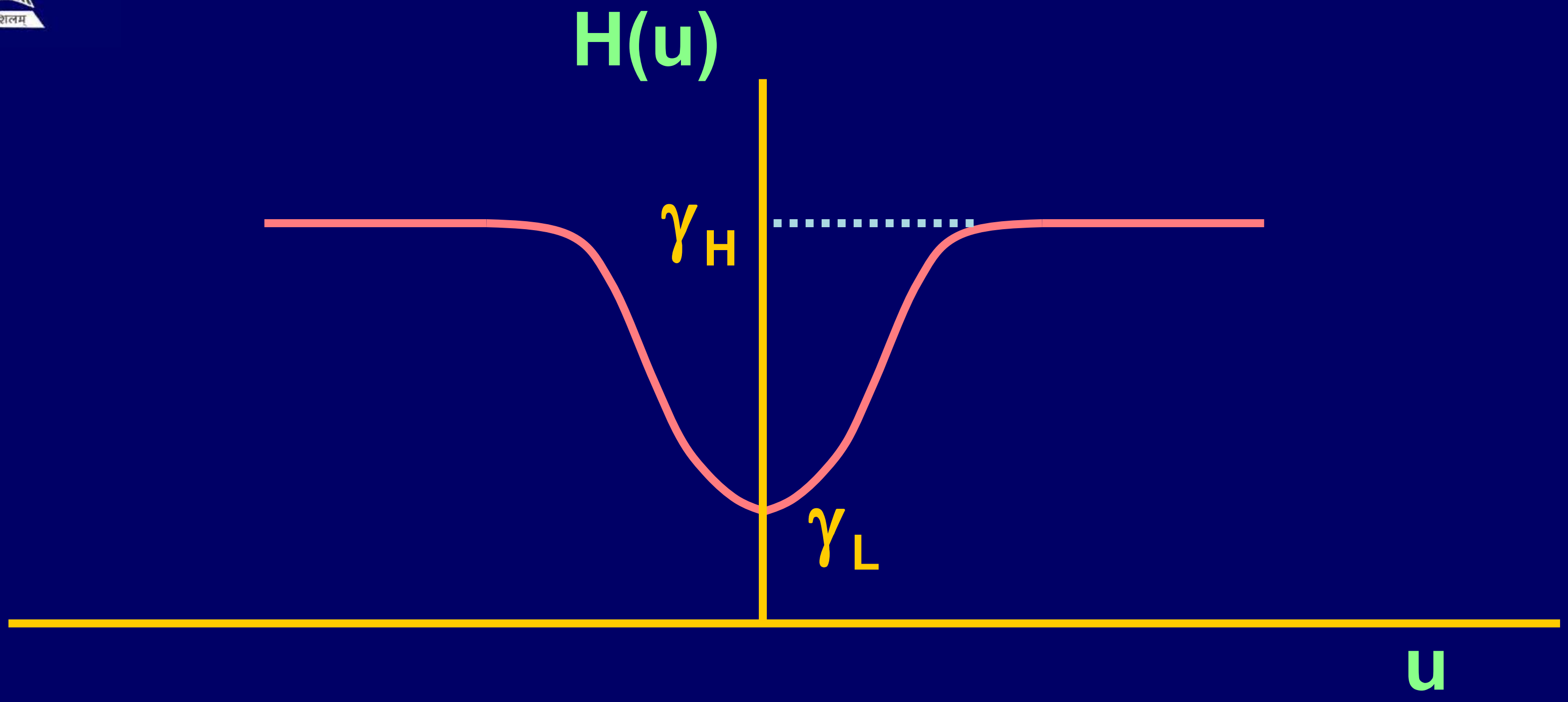
$\Rightarrow$ low freq. comp.

Reflectance $\Rightarrow$ changes abruptly (particularly at junctions)

$\Rightarrow$ high freq. comp.

Proper choice of H(u,v) can provide good control over illumination and reflectance components

Homomorphic Filter



$\gamma_H > 1$, $\gamma_L < 1$ — **Contribution due to Illumination is reduced, Reflectance is enhanced**

$$H(u,v) = (\gamma_H - \gamma_L)\left(1 - e^{-c\left(D^2(u,v)\right)/D_0^2}\right) + \gamma_L$$

Homomorphic Filter