PRACTICAL - 1.

1) $$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$\lim_{x\to a}\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}}$$

$$=\frac{2}{3\sqrt{3}}$$

2) $$\lim_{x\to\pi/6}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$$

By Subsituting $x-\frac{\pi}{6}=h$

$x=h+\pi/6$ where $h\to 0$

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$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

Using
$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \frac{1}{2} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{2}\right)}{\pi - 6h - \pi}$$

$$\lim_{h\to 0} \frac{\cos \cdot \frac{\sqrt{3}}{2} h - \sin\frac{h}{2} - \sin\frac{3h}{2} - \frac{\cos\sqrt{3}h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{-\sin\frac{4h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{\sin \cancel{4}h}{3\,\cancel{12}h}$$

$$\frac{1}{3}\lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$$

4) $\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing Numerator & denominator

Sin+Sin

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2+3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2+3}}{\sqrt{x^2+5}+\sqrt{x^2+3}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\frac{8(\sqrt{x^2+3}+\sqrt{x^2+1})}{2(\sqrt{x^2+5}+\sqrt{x^2+3})}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2(1+\frac{3}{x^2})}+\sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+\frac{5}{x^2})}+\sqrt{x^2(1-\frac{3}{x^2})}}$$

After applying the limit we get,

$= 4.$

5) $f(x)=\frac{\sin 2(\frac{\pi}{2})}{\sqrt{1-\cos 2(\pi/2)}}$ $\quad\therefore f(\pi/2)=0$

f at $x=\pi/2$ define.

(ii) $\lim_{x\to\pi/2^+} f(x)=\lim_{x\to\pi/2^+}\frac{\cos x}{\pi-2x}$.

By Subsituting Method

$x-\frac{\pi}{2}=h$

$x=h+\frac{\pi}{2}$

where $h\to 0$.

$$\lim_{h\to 0}\frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi-2\left(h+\frac{\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos(h+\frac{\pi}{2})}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h\cdot \cos\frac{\pi}{2} - \sin h\cdot \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h\cdot 0 - \sin h}{-2h}$$

$$\lim_{h\to 0} \frac{-\sin h}{-2h}$$

$$\frac{1}{2}\lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{2}$$

b) $\lim_{x\to\pi/2} f(x) = \lim_{x\to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$$\lim_{x\to\pi/2^-} \frac{2\sin x\cdot\cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x\to\pi/2^-} \frac{2\cancel{\sin x}\cos x}{\sqrt{2}\cancel{\sin x}}$$

$$\lim_{x\to\pi/2^-} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}}\lim_{x\to\pi/2^-} \cos x$$

$\therefore$ LHS $\neq$ RHS

$\therefore$ f is not continous at $x = \pi/2$

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(ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \le x < 6$ $\quad$ at $x=3$ & $x=6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$

at $x=3$

(i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define.

(ii) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is defined at $x=3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ LHS = RHS

f is continuous at $x=3$

for $x = 6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{\cancel{27}\,3}{\cancel{9}\,1} = 3$

2. $\lim_{x \to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x \to 6^+} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x \to 6^+} (x-3) = 6-3 = 3$

$\lim_{x \to 6^-} x+3 = 3+6 = 9$

$\therefore$ LHS $\neq$ RHS

function is not continous

6.

1) $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x < 0 \\ = K & x = 0 \end{cases}$ at $x = 0$

$\Rightarrow$ f is continous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = K$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = K$

$2\lim_{x \to 0} \frac{\sin^2 x}{x^2} = K$

$2\lim_{x \to 0} \left(\frac{\sin \cdot 2x}{x}\right)^2 = K$

$2(2)^2 = K$

$\boxed{K = 8}$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$

$= K$ $\quad x = 0$ $\Big\}$ at $x = 0$

$\Rightarrow f(x) = (\sec^2 x)^{\cot^2 x}$

$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$

we know that

$\lim_{x \to 0} (1 + px)^{\frac{1}{px}} = e$

$\therefore \quad = e$

$\therefore \quad K = e$

(iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$

$= K$ $\quad x = \pi/3$ $\Big\}$ at $x = \frac{\pi}{3}$

$x - \frac{\pi}{3} h$

$x = h + \frac{\pi}{3}$

where $h \to 0$

$f(\pi/3 + h) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$

$\lim_{h \to 0} \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

using $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\lim_{h \to 0} \dfrac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{\pi - \pi - 3h}$

$\lim_{h \to 0} \dfrac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \cdot \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{-3h}$

$$\lim_{h\to 0} \frac{\left(\frac{\sqrt{3} - \sqrt{3}\times\sqrt{3}\cdot\tan h) - (\sqrt{3}+\tan h)}{1-\tan \pi/3 \cdot \tan h}\right)}{-3h}$$

$$\lim_{h\to 0} \frac{\left(\frac{\sqrt{3} - \sqrt{3}\times\sqrt{3}\cdot\tan h) - (\sqrt{3}+\tan h)}{1-\tan \pi/3 \cdot \tan h}\right)}{-3h}$$

$$\lim_{h\to 0} \frac{\left(\frac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{1-\sqrt{3}\tan h}\right)}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\tan h)} \qquad \frac{\tan h}{h} = 1$$

$$= \frac{4}{3} \frac{1}{(1-\sqrt{3}(0))}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right)$$

$$= 4$$

Q7.

(i) $f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \cancel{=}\ 9 & x = 0 \end{cases}$ at $x = 0$

$$f(x) = \frac{1-\cos 3x}{x \tan x}$$

$$\lim_{x\to 0} = \frac{2\sin^2 3/2\, x}{x\tan x}$$

$$\lim_{x\to 0} \frac{\dfrac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{\dfrac{x \tan x}{x^2} \times x^2}$$

$$= \frac{2 \lim_{x\to 0} \left(\frac{3}{2}\right)^2}{1} = \frac{2\times 9}{4} = \frac{9}{2}$$

$\lim_{x\to 0} f(x) = \frac{9}{2} \neq f(0)$

$\therefore$ f is not continous at $x = 0$

Redefine function

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x \tan x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$

ii) $f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^\circ}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{cases}$ at $x=0$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \qquad \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x\to 0} \frac{3\cdot e^{3x}-1}{3x} \qquad \lim_{x\to 0} \sin\left(\frac{\pi x}{180}\right)$$

$$3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \qquad \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \log_e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

f is continous at $x = 0$

8. $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ $\quad x = 0$

is continous at $x = 0$

$\because$ given,

f is continous at $x = 0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiplying with numerator & denominator

$$= 1 + 2 \times \frac{1}{4} = 3/2 = f(0)$$

9. $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \pi/2$

$f(0)$ is continous at $x = \pi/2$

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}.$$

$$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1 + \sin x}{1 - \sin^2 x (\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1 + \sin x}{(1-\sin x)(1+\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$$

AK
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PRACTICAL - 2

Q1. (i) $\cot x$

$f(x) = \cot x$

$= \lim_{x\to a} \dfrac{f(x) - f(a)}{x-a}$

$= \lim_{x\to a} \dfrac{\cot x - \cot a}{x-a}$

$= \lim_{x\to a} \dfrac{1/\tan x - 1/\tan a}{x-a}$

$= \lim_{x\to 0} \dfrac{\tan a - \tan x}{(x-a)\tan x \tan a}$.

Put $x-a=h$ $\quad x=a+h$ $\quad$ as $x\to a,\ h\to 0$

$f(h) = \lim_{h\to 0} \dfrac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$

$= \lim_{h\to 0} \dfrac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$.

formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 - \tan A \cdot \tan B}$.

$\tan A - \tan B = \tan(A-B)(1+\tan A \cdot \tan B)$

$= \lim_{h\to 0} \dfrac{\tan(a-a-h) - (1+\tan a + \tan(a+h))}{h\times \tan(a+h)\tan a}$.

$= \lim_{h\to 0} -\dfrac{\tan h}{h} \times \dfrac{1+\tan a\tan(a+h)}{\tan(a+h)\tan a}$

$= \lim_{h\to 0} -\dfrac{\tan h}{h} \times \dfrac{1+\tan a\tan(a+h)}{\tan(a+h)\tan a}$

$= -1 \times \dfrac{1\times\tan^2 a}{\tan^2 a}$.

$= -\dfrac{\sec^2 a}{\tan^2 a} = \dfrac{-1}{\cos^2 a} \times \dfrac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$f'(a) = -\cos^2 a$

$\therefore$ f is differentiable $\forall\, a \in R$.

2. cosec x

$f(x) = \operatorname{cosec} x$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\sin x - 1/\sin a}{x - a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

Put $x - a = h$, $x = a + h$ as $x \to a$ $h \to 0$.

$$f(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

formula

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1/2 \times 2\cos\left(\frac{2a}{2}\right)}{\sin (a+0)}$$

$$= \frac{-\cos a}{\sin^2 a}$$

$$= -\cot a \ \operatorname{cosec} a.$$

Q2. $f(x) = 4x+1 \quad x \le 2$
$\quad\ = x^2+5 \quad x > 0$ } at $x=2$

then find f is differentiable or not.

LHS

$$f(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 4$$

$$f(2^-) = 4$$

RHS: $f(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$

$$\lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$\lim_{x\to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$f(2^+) = 4 \quad = 2+2 = 4$

$\therefore$ RHS = LHS

f is differentiable at $x = 2$

Q3.) $f(x) = 4x + 7 \quad x < 3$

$= x^2 + 3x + x^4 \quad x \geq 3$ } at $x = 3$ then, find f is differentiable or not

$\Rightarrow f(3^+) = \lim_{x \to 3^+} \dfrac{f(x) - f(3)}{x - 3}$

$= \lim_{x \to 3^+} \dfrac{x^2 + 3x + 1 - (3^2 + 3 - 3 + 1)}{x - 3}$

$= \lim_{x \to 3^+} \dfrac{x^2 + 3x + 1 - 19}{x - 3}$

$= \lim_{x \to 3^+} \dfrac{x^2 + 3x - 18}{x - 3}$

$= \lim_{x \to 3^+} \dfrac{x^2 + 6x - 3x - 18}{x - 3}$

$= \lim_{x \to 3^+} \dfrac{x(x+6) - 3(x+6)}{x - 3}$

$= \lim_{x \to 3^+} \dfrac{(x+6)(x-3)}{(x-3)}$

$= 3 + 6$

$= 9.$

$f(3^+) = 9$

LHS $= Df(3^-)$

$= \lim_{x \to 3^-} \dfrac{f(x) - f(3)}{x - 3}$

$\lim_{x \to 3^-} \dfrac{4x + 7 - 19}{x - 3} = \lim_{x \to 3^-} \dfrac{4x - 12}{x - 3}$

$$\lim_{x\to 3^-} \frac{4(x-3)}{x-3}$$

$f(3^+) = 4$

RHS / LHS

f is not defined at $x = 3$

Qs 24. $f(x) = 8x - 5$ $\quad x \le 2$ $\quad$ at $x = 2$

$= 3x^2 - 4x + 7$ $\quad x < 2$

f is differentiable or not.

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHL $f(2^+) = \lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x\to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$

$= \lim_{x\to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$

$= \lim_{x\to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$

$= \lim_{x\to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$

$= \lim_{x\to 2^+} \frac{(3x+2)(x-2)}{x - 2}$

$= 3 \times 2 + 2 = 8$

$f(2^+) = 8$

LHS $f(2^-) = \lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x\to 2^-} \frac{8x - 5 - 11}{x - 2}$

$= \lim_{x\to 2^-} \frac{8(x-2)}{(x-2)}$

$f(2^-) = 8$

LHS = RHS

∴ f is differentiable at $x = 2$

PRACTICAL - 03.

Q1. Find the intervals in which function is increasing or decreasing.

1) $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

$\therefore$ f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \qquad \sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x) < 0$.

$3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$.

$+ \quad - \quad +$

$-\sqrt{5/3} \qquad +\sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2. $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x - 2) > 0$

$x - 2 > 0$

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$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

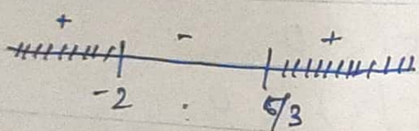
$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$



$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

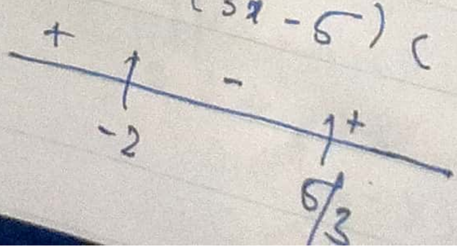
$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$



$f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore$ $3(x^2 - 9) > 0$

$\therefore$ $(x-3)(x+3) > 0$

$+ \quad - \quad +$

$-3 \qquad 3$

$\therefore$ $x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore$ $3x^2 - 27 < 0$

$\therefore$ $3(x^2 - 9) < 0$

$\therefore$ $(x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \qquad 3$

$\therefore$ $x \in (-3, 3)$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore$ $6x^2 - 18x - 24 > 0$

$\therefore$ $6(x^2 - 3x - 4) > 0$

$\therefore$ $x^2 - 4x + x - 4 > 0$

$\therefore$ $x(x-4) + (x-4) > 0$

$\therefore$ $(x-4)(x+1) > 0$

$+ \quad - \quad +$

$-1 \qquad 4$

$\therefore$ $x \in (-\infty, -1) \cup (4, \infty)$

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No.

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$

$+$

$-1 \qquad 4$

$\therefore x \in (-1, 4)$

Q2.

1)

$y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\because f''(x) > 0$

$\therefore x \in (1/2, \infty)$

2)

$y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$

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$\therefore \quad 12x^2 - 36x + 24 > 0$

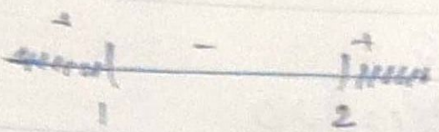
~~f is concave~~

$\therefore \quad 12(x^2 - 3x + 2) > 0$

$\therefore \quad x^2 - 2x - x + 2 > 0$

$\therefore \quad x(x-2) - 1(x-2) > 0$

$(x-2)(x-1) > 0$



$x \in (-\infty, 1) \cup (2, \infty)$

$y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore \quad 6x > 0$

$x > 0$

$x \in (0, \infty)$

$y = 69 - 24x - 9x^2 + 2x^3$

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x < -1/6$

$\because f''(x) \not> 0$

There exist no interval

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PRACTICAL - 04

Q1. Find Maximum & minimum values of following

i) $f(x) = x^2 + \frac{16}{x^2}$

$\Rightarrow f'(x) = 2x - 32/x^3$

Now Consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$\boxed{x = \pm 2}$

$f'(x) = 2 + 96/x^4$

$f''(x) = 2 + \frac{96}{24}$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum values at $x = 2$

$\therefore f(2) = 2^2 + 16/22$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$, $x = -2$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

Consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore 15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30$

$= 30 > 0$

$\therefore f$ has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = \cancel{-30(-1) + 60(-1)^3}$ $\quad 3 - 5(-1)^3 + 3(-1)^5$

$= \cancel{30 - 60}$ $\quad = 3 + 5 - 3$

$= \cancel{-30 < 0}$ $\quad = 5$

$\therefore f$ has maximum value 5 at $x = -1$ & has the minimum value 1 at $x = 1$

iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 > 0$

$\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= \cancel{9 - 12}$

$= -3$

$\therefore$ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$.

Consider

$f'(x) = 0$.

$\therefore\ 6x^2 - 6x - 12 = 0$

$\therefore\ 6(x^2 - x - 2) = 0$

$\therefore\ x^2 - x - 2 = 0$

$\therefore\ x^2 + x - 2x - 2 = 0$

$\therefore\ x(x+1) - 2(x+1) = 0$

$\therefore\ (x-2)(x+1) = 0$

$\therefore\ x = 2$ or $x = -1$

$\therefore\ f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore\ f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$\therefore\ f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore\ f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has maximum value 8 at $x = -1$ and f has minimum value -19 at $x = 2$.

22) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= 0.1712$

$\therefore$ The root of equation is 0.1712.

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(ii) $f(x) = x^3 - 4x - 9$ [2, 3]

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

let $x_0 = 3$ be the initial approximation

∴ By newton's law

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 0.5961/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$ $[1, 2]$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1 - 1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

let $x_0 = 2$ be initial approximation. By Newton's

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_2) / f'(x_2)$

$= 2 - 2.2 / 5.2$

$= 2 - 0.4230 = 1.577$.

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= \cancel{8 - 7.2 - 20 + 17 = -2.2}$

$= 0.6785$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -\underline{8.2164}$

$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$

$= 1.577 + 0.6755 / 8.2164$.

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 1$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$.

$f'(x_2) = 3(1.659)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.9731 - 10$

$= -7.7143$

$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618).$$

$$= 4.5892 - 4.9708 - 16.618 + 17$$

$$= 0.0004.$$

$$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$$

$$= -7.6977.$$

$$x_4 = x_3 - f(x_3)/f'(x_3)$$

$$= 1.618 + \frac{0.0004}{7.6977}$$

$$= \underline{1.6618}.$$

PRACTICAL - 05

INTEGRATION

Q1.1) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$.

$= \int \frac{1}{\sqrt{(x+1)^2-4}} \, dx$

put $x+1=t$

$dx = \frac{1}{t} dt$ where $t=1$ $t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} \, dt$.

using

$\int \frac{1}{\sqrt{x^2-a^2}} \, dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= (|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|) + C$

2). $\int (4e^{3x}+1) \, dx$

$\int 4e^{3x} dx + \int 1 \, dx$

$= 4\int e^{3x} dx + \int 1 \, dx$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + C$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\, dx$

$= \int 2x^2 dx - \int 3\sin(x)\, dx + \int 5x^{1/2} dx.$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dy$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}}\, dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{3^{1/2}} + \frac{4}{x^{1/2}}\, dx.$

$= \int x^{5/2} dx + 3x^{1/2}\, \cancel{dx} + \frac{4}{x^{1/2}}\, dy$

$= \frac{x^{5/2+1}}{\cancel{5/2+1}}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)\, dt$

Put $y = 2t^4$

$dy = 2\times4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times4t^3}\, dy$

Subsitute t^4 with $\frac{4}{2}$

$$= \int \frac{4/2 \times \sin(u)\, du}{8}$$

$$= \int \frac{4 \times \sin(4u)}{2} \Big/ 8 \; du$$

$$= \int \frac{4 \times \sin(\cancel{4}) \, du}{16.}$$

$$= \frac{1}{16} \int 4 \times \sin(u)\, du$$

$\#\ \int u\, dv = uv - \int du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1\, du \qquad v = -\cos(u)$

$$= \frac{1}{16} \left(4 \times (-\cos u)\right) - \int \cdot \cos(u)\, du.$$

$$= \frac{1}{16} \times \left(4 \times (-\cos(u)) + \sin(u)\right)$$

$\cancel{= \frac{1}{16} \times (4 \times}$ Return the subsitute

$$= 1/16 \times \left(2t^4 \times (-\cos(2t^4)) + \sin(2t^4)\right)$$

$$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$$

6) $\int \sqrt{x}\,(x^2 - 1)\, dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

No.

1.

2.

3

4

$= \int x^{1/2} \times x^2 - x^{1/2}\, dx$

$= \int x^{5/2} - x^{1/2}\, dx$

$= \int x^{5/2}\, dx - \int x^{1/2}\, dx$

$= I_1 \cdot \dfrac{x^{5/2+1}}{5/2+1} = \dfrac{x^{7/2}}{7/2} = \dfrac{2x^{7/2}}{7} = \dfrac{2\sqrt{x}^7}{7} = \dfrac{2x^3\sqrt{x}}{7}$

$= I_2 = \dfrac{x^{1/2+1}}{1/2+1} = \dfrac{x^{3/2}}{3/2} = \dfrac{2x^{3/2}}{3/2} = \dfrac{2\sqrt{x}^3}{3}$

$= \dfrac{2x^3\sqrt{x}}{7} + \dfrac{2\sqrt{x}^3}{3} + C$

क्ष) $\int \dfrac{\cos x}{3\sqrt{\sin(x)^2}}\, dx$

$= \int \dfrac{\cos x}{\sin x^{2/3}}\, dx$

Put $t = \sin(x)$

$t = \cos x$

$= \int \dfrac{\cancel{\cos x}}{\sin(x)^{3/2}} \times \dfrac{1}{\cancel{\cos(x)}}\, dt$

$= \dfrac{1}{\sin x^{3/2}}\, dt$

$= \dfrac{1}{t^{2/3}}\, dt$

$I = \int \dfrac{1}{t^{2/3}}\, dt = \dfrac{-1}{(2/3-1)\, t^{2/3-1}}$

$= \dfrac{-1}{-1/3\, t^{2/3-1}} = \dfrac{1}{1/3\, t^{-1/3}} = \dfrac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{t}$

$\int x^3 - 3x^2 + 1$

Put $x^3 - 3x^2 + 1 = dt$

$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x} dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 \cancel{8} 6x} dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)} dt$

$= \int \frac{1}{x^3 - 3x^2 + 1} \times 1/3 \, dt$

$= \int \frac{1}{3(x^3 - 3x^2 + 1)} dt = \int \frac{1}{3t} dt$

$= 1/3 \int 1/t \, dt \qquad \int 1/x \, dx = \ln(x)$

$= 1/3 \times \ln |t| + C$

$= 1/3 \times \ln (|x^3 - 3x^2 + 1|) + C.$

AB
04/01/2020

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PRACTICAL- 06.

Application of Integeration and numerical ...

Q1. Find the length of the following

1. $x = t\sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$

$x = \sin t$ $y = t\cos t$

$\frac{dx}{dt} = 1 - \cos t$ $\frac{dy}{dx} : \sin t$

$$L = \int_0^{2\pi} \sqrt{(1-\cos t + \sin t)^2 + (\sin t)^2}$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\ dt$$

$$= \int_0^{2\pi} \sqrt{2(1-\cos t)}\ dt$$

$$= \int_0^{2\pi} \sqrt{4\sin^2 t/2}\ dt$$

$$= 2\int_0^{2\pi} \sin t/2\ dt.$$

$$= 2\left[-\cos t/2\right]_0^{2\pi} = 2[-\cos 2\pi - \cos \theta]$$

$= \underline{\underline{4}}$

2. $y =$

3. y

2. $y = \sqrt{4-x^2}$, $x \in [-2, 2]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$y = \sqrt{4-x^2}$$

$$\frac{dy}{dx} = 2^2 \int_0^2 1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2 dx$$

$$= 2\int_0^2 \sqrt{\frac{1+x^2}{4-x^2}}\, dx.$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx.$$

$$= 4\left[\sin^{-1}(x/2)\right]_0^2$$

$$= 2\pi$$

3. $y = x^{3/2}$ in $[0, 4]$

$$f'(x) = 3/2\ x^{1/2}$$

$$[f(x)]^2 = 9/4\ x$$

$$L = \int_0^4 \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$$

$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$$

$$= \frac{1}{2}\int_0^4 \frac{(4+9x)^{1/2+1}}{\sqrt{1/2+1}}\, dx$$

$$= \frac{1}{27}\left[[4+0]^{3/2} - [4+36]^{3/2}\right]$$

$$= \frac{1}{27}\left[(4)^{3/2} - (40)^{3/2}\right]$$

4. $x = 3\sin t \qquad y = 3\cos t$

$\Rightarrow$

$$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dx} = -3\sin t$$

$$l = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9}\, dt \quad = 3\int_0^{2\pi} dt$$

$$= 3[t]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

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$x = \frac{1}{6} y^3 + \frac{1}{2y}$ $\qquad y \in [1,2]$

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4 - 1}{2y^2}$

$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$

$= \int_1^2 \sqrt{\frac{(y^2+1)}{(2y^2)}} \, dy$

$= \int_1^2 \frac{y^4+1}{2y^2} \, dy$

$= \int_1^2 y^2 \, dy + \frac{1}{2} \int_1^2 y^{-2} \, dy$

$= \frac{1}{2} \left[\frac{y^3}{3} - \frac{y^{-2}}{1} \right]_1^2$

$= \frac{1}{2} \left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1 \right]$

$= \frac{1}{2} \left[\frac{7}{3} - \frac{1}{2} \right] = \frac{17}{12}$

Q2. Using Simpson's Rule solve the following

1) $3\int_0^2 e^{x^2} \, dx$ with $h = 4$

$a = 0, \; b = 2, \; h = 4$

$4 = \frac{2-0}{4}$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

By Simpson's Rule

$$\int_0^2 e^{x^2}dx = \frac{0.5}{3}\left[(1+54.5931)+4(1.2890+9.4877) + 2(2.7182+54.5981)\right]$$

$$= \frac{0.5}{3}[55.5981+43.0868+114.63266]$$

$$= \underline{1.1279}$$

2) $\int_0^4 x^2 dx$

$$h = \frac{4-0}{4} = 1$$

3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$, $n=6$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$6\pi/18$
y	0	0.4166	0.58	0.70	0.8087	0.8[illegible]

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{\pi}{4} \times 12.1163$$

$$= \underline{0.7049}$$

PRACTICAL NO : 07
Differential equation

Q. Solve the following

1. $x \frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$; $Q(x) = \frac{e^x}{x}$

$IF = e^{\int_0^{1/x} dx} = x$

$y(IF) = \int Q(x)(IF)\,dx + C$

$= \int \frac{e^x}{x} \cdot x\,dx + C$

$= \int e^x\,dx + C$

$= e^x + C$

2. $e^x \frac{dy}{dx} + 2e^{xy} = 1$

$\frac{dy}{dx} + \frac{2e^x}{e^x} y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$, $Q(x) = e^{-x}$

$IF = e^{2x}$

$$Y(IF) = \int Q(x)(IF)\,dx + C$$
$$= \int e^{-x} e^{2x}\,dx + C$$
$$= \int e^{x}\,dx + C$$
$$Y \cdot e^{2x} = e^{x} + C$$

3. $x\frac{dy}{dx} = \cos x - 2y$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$$
$$P(x) = 2(x) \quad ; \quad Q(x) = \frac{\cos x}{x^2}$$
$$IF = e^{\int P(x)dx}$$
$$= e^{\int 2x\,dx}$$
$$= x^2$$
$$Y(IF) = \int Q(x)(IF)\,dx + C$$
$$= \int \frac{\cos x}{x^2} - x^2\,dx + C$$
$$= \int \cos x + C$$

4. $x\frac{dy}{dx} + 3y = \cancel{\frac{\sin x}{x^2}}$

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$$
$$P(x) = 3/x \quad ; \quad Q(x) = \sin x / x^3$$
$$P(x) = \int 3/x\,dx$$

5) e^2

6.

$IF = e^{\int P(x)\,dx} = x^3$

$= x^3$

$y(IF) = \int Q(x)(IF)\,dx + C$

$= \int \frac{\sin x}{x^3} \cdot x^3\,dx$

$= \int \sin x + C$

$= \underline{\underline{-\cos x}}$

5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2$; $Q(x) = \frac{2x}{e^{2x}} = 2xe^{-2x}$

$IF = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + C$

$= \int 2e^{-2x} \cdot e^{2x}\,dx$

$ye^{2x} = \underline{\underline{x^2 + C}}$

6. $\sec^2 x \cdot \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$

$\int \frac{\sec^2 x\,dx}{\tan} = -\int \frac{\sec^2 y}{\tan y}\,dy$

$\log|\tan x| = -\log|-\tan y| + C$

$\log|\tan x + \tan y| = C$

$\tan x + \tan y = e^C$

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7. $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$

$x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + c$

$\tan(x-y+1) = x + c$

8. $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x+3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$= \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$= \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$\int \frac{v+2}{v+1} \, dv = -3\,dx$$

$$\int \frac{v+1}{v} \, dx + \int \frac{1}{v+1} \, dv - 3\,dx$$

$$v + \log(v) = 3x + C$$

$$3x + 3y + \log|2x+3y+1| = 3x + c$$

$$3y + x - \log|2x+3y+1| + C$$

AV
11/01/2020

PRACTICAL - 08

Euler's formula

1) $\frac{dy}{dx} = y + e^x - 2$

$f(x,y) = y + e^x - 2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.187	3.57435
2	1	3.5743	4.2925	5.3615

$$\boxed{y_{n+1} = y_n + h f(x_n, y_n)}$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

$\therefore$ By Euler's formula,

$y(2) = 9.2831$

2) $\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

Using Euler's iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

$\therefore$ By Euler's formula

$$y(1) = 1.2942$$

3) $\dfrac{dy}{dx} = \sqrt{\dfrac{x}{y}}$ $\quad y(0) = 1 \quad x_0 = 0, \; h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

3) $\frac{dy}{dx} = 3x^2 + 1$ $y_0 = 2,\ x_0 = 1,\ h = 0$

for $h = 0.5$

Using Euler's iteration formula

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{x+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

By Euler's formula,

$y(2) = 28.5$

for $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.9048
4	2	8.9048		
5				

$\therefore$ By Euler's formula

$y(2) = 8.9048$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$

$y_0 = 1, \; x_0 = 1, \; h = 0.2$

Using Euler's iteration formula,

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

$\therefore$ By Euler's formula;

$y(1.2) = 1.6$

21/01/2020

039

PRACTICAL - 09

limits & partial order derivatives

1. $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

$\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

at $(-4,-1)$ Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(-4)^3 - 3(-4) + (-4)^2 - 1}{-4(-1)+5}$

$= \frac{-61}{9}$

2. $\lim_{(x,y)\to(-2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

at $(-2,0)$ Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(0+1)(4+0-8)}{2}$

$= \frac{-4}{2}$

$= -2$

S8:

1) $f(x,y) = xye^{x^2} + y^2$

$\therefore f_x = \frac{\delta f}{\delta x}(x,y)$

$= \frac{\delta(xye^{x^2}+y^2)}{\delta x}$

$= ye^{x^2+y^2}(2x)$

$\therefore f(x) = 2xye^{x^2+y^2}$

$f_y = \frac{\delta f(x,y)}{\delta y}$

$= \frac{\delta(xye^{x^2}+y^2)}{\delta x}$

$= xe^{x^2+y^2}(2y)$

$\therefore f_y = 2yxe^{x^2+y^2}$

2) $f(x,y) = e^x \cos y$

$f_x = \frac{\delta}{\delta x} f(x,y)$

$f_x = e^x \cos y$

$f_y = \frac{\delta f(x,y)}{\delta y}$

$= \frac{\delta(e^x \cos y)}{\delta y}$

$f_y = -e^x \sin y$

3. $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$fx = \frac{\delta f(x,y)}{\delta y}$

$= \frac{\delta f(x^3y^2 - 3x^2y + y^3 + 1)}{\delta x}$

$= 3x^2y^2 - 6xy$

$fy = \frac{\delta f(x,y)}{\delta y}$

$= \frac{\delta f(x^3y^2 - 3x^2y + y^3 + 1)}{\delta y}$

$= 2x^3y - 3x^2 + 3y^2$

Q3.

1. $f(x,y) = \frac{2x}{1+y^2}$

$fx = \frac{\delta f(x,y)}{\delta x}$

$= \frac{1+y^2 \frac{\delta(2x)}{\delta x} - 2x \frac{\delta(1+y^2)}{\delta x}}{(1+y^2)^2}$

$= \frac{2+2y^2}{(1+y)^2}$

$=$

$\frac{2}{1+y^2}$

~~At~~ $f(0,0)$

$\frac{2}{1+0} \cdot 2$

$$fy = \frac{\delta f}{\delta y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\delta}{\delta x}(2x) - 2x \frac{\delta}{\delta x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2 (0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= \underline{\underline{0}}$$

2a) ✓

1) $f(x,y) = \frac{y^2 xy}{x^2}$

$$fx = \frac{x^2 \cancel{\frac{\delta f}{\delta x}(y^2 - xy)} - (y^2 - xy)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x2y - 2x(y^2 - xy)}{x^4}$$

$f_y = 2y - \frac{x}{x^2}$

$f_{xx} = \frac{\delta}{\delta x}\left(-x^2y - \frac{2x(y^2 - xy)}{x^4}\right)$

$= \frac{x^4\,(\delta/\delta x\,(-x^2y - 2xy^2 + 2x^2y)) - (-x^2y - 2xy + 2x^2y)\,\delta/\delta x\,(x^4)}{(x^4)^2}$

$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^8}$ — ①

$f_{yy} = \frac{\delta}{\delta y}\left(2y - \frac{x}{x^2}\right)$

$= \frac{2-0}{x^2} = \frac{2}{x^2}$ — ②

$f_{xy} = \frac{\delta}{\delta y}\left(\frac{-x^2y - 2xy^2 + 2x^3y}{x^4}\right)$

$= \frac{x^2 - 4xy + 2x^2}{x^4}$ — ③

$f_{yx} = \frac{\delta}{\delta x}\left(2y - \frac{x}{x^2}\right)$

$= \frac{x^2\frac{\delta}{\delta x}(xy - x) - (xy - x)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$

$= \frac{-x^2 - 4xy - 2x^2}{x^4}$

from ③ & ④

$f_{xy} = f_{yx}$.

2. $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{\delta}{\delta x}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 3x^2 + 6xy^2 - \frac{2x}{x+1}$

$f_y = \frac{\delta}{\delta y}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 6x^2y$

$f_{xx} = 6x + 6y^2 \left(\frac{x^2+1 \frac{\delta}{\delta x}(2x) - 2x\left(\frac{\delta}{\delta x}(x^2+1)\right)}{(x^2+1)^2} \right)$

$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2} \right)$ — ①

$f_{yy} = \frac{\delta}{\delta y}(6x^2y)$

$= 6x^2$ — ②

$f_{xy} = \frac{\delta}{\delta y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0$

$= 12xy$ — ③

$f_{yx} = \frac{\delta}{\delta x}(6x^2y)$

$= 12xy$ — ④

from ③ & ④

$f_{xy} = f_{yx}$

3. $f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$f_{xx} = \frac{\delta}{\delta x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)(y) + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$ — ①.

$f_{yy} = \frac{\delta}{\delta y}(x\cos(xy) + e^{x+y})$

$= -x\cdot\sin(xy)(x) + e^{x+y}(1)$

$= -x\cdot\sin(xy) + e^{x+y}$

$f_{xy} = \frac{\delta}{\delta y}(y\cos(xy)) + e^{x+y}(1)$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ — ③

$f_{yx} = \frac{\delta}{\delta x}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — ④

from 3 & 4

$f_{xy} \neq f_{yx}$

$f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$$fx = \frac{1}{2\sqrt{x^2+y^2}}(2x)$$

$$= \frac{x}{\sqrt{x^2+y^2}}$$

$$fy = \frac{1}{2\sqrt{x^2+y^2}}(2y)$$

$$= \frac{y}{\sqrt{x^2+y^2}}$$

fx at $(1,1) = \frac{1}{\sqrt{2}}$

fy at $(1,1) = \frac{1}{\sqrt{2}}$

$$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)(y-b)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$$

$$= \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$$

$$= \frac{x+y}{\sqrt{2}}$$

3. $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x}$ $f_y = \frac{1}{y}$

f_x at $(1,1) = 1$ f_y at $(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a)$ $f_y(a,b)(y-$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$.

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PRACTICAL-10

Q1. Find the directional derivative of the following function at given point q in the direction of given vector.

1) $f(x,y) = x+2y-3$ $\quad a=(1,-1)$ $\quad u = 3i - j$

Here, $u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f = \left(1+\frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$

$= f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1, \frac{-h}{\sqrt{10}}\right)$

$= \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{-4 + \sqrt{10} + 4}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

ii) $f(x) = y^2 - 4x + 1 \quad a = (3,4) \quad u = \hat{i} + 5\hat{j}$

here $u = i + 5j$ is not a unit vector

$$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$$

$$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$\frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

find gradient vector.

i) $f(x, y) = x^y + y^x \Rightarrow a = (1, 1)$

$f(x) = y \cdot x^{y-1} + y^x \log y$

$fy = x^y \log x + xy^{x-1}$

$\nabla f(x, y) = (fx, fy)$

$= \left(yx^{y-1} + y^2 \log y,\ x^y \log x + xy^{x-1}\right)$

$f(1,1) = (1+0, 1+0)$

$= (1, 1)$

ii) $f(x, y) = (-\tan^{-1} x) \cdot y^2 \quad a = (1, -1)$

$fx = \frac{1}{1+x^2} \cdot y^2$

$fy = 2y \tan^{-1} x$

$\nabla f(x, y) = (fx, fy)$

$= \left(\frac{y^2}{1+x^2},\ 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, \; xz - e^{x+y+z}, \; xy - e^{x+y+z}$

$f(1,-1,0) = (-1)(0) - e^{1+(-1)+0}, \; (1)(0) - e^{1+(-1)+0}, \; (1),(-1) \ldots$

$= (0 - e^0, \; 0 - e^0, \; -1 - e^0)$

$= (-1, -1, -2)$

Q3. Find the equation of tangent & normal to each of the following using curves at given point.

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1,0)$:

$\therefore x_0 = 1, \; y_0 = 0$

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eq^n of tangent

$f_x (x-x_0) + f_y (y-y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \;\; 2(1) + e^0 \, 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ → It is the required eq^n of tangent

eq^n of normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1, 0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore d = -1$

i) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

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$= 2y + 3$

$(x_0, y_0) = (2, -2) \therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

eq^n of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ → It is required eq^n of tangent

eq^n of normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore \boxed{d = 6}$

Q4. find the eq^n of tangent and normal

i) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0)$ $\therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(\;\;"\;\;) = 2(0) + 3 = 3$

$f_z(\;"\;) = -2(1) + 2 = 0$

eq^n of tangent

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x - 2) + 3(y - 1) + 0(z - 0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0$ → This is required eq^n of tangent

Eq^n of normal at $(4, 3, -1)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x - 2}{4} = \frac{y - 1}{3} = \frac{z + 11}{0}$$

Q5. Find the local maxima & minima

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ —①

$f_y = 0$

$2y - 3x - 4 = 0$
$2y - 3x = 4$
$x = 0$

Subsitute value of x in eqⁿ 1
$2(0) - y = -2$
$+ y = \cancel{-}2$

$\therefore$ Critical points are $(0,2)$ $\quad \therefore y = 2$

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here $r > 0$
$= rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$

$\therefore$ f has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$
$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$
$0 + 4 - 0 + 0 - 8$
$= \underline{-4}$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$
$f_x = 8x^3 + 6xy$
$f_y = 3x^2 - 2y$
$f_x = 0$
$\therefore 8x^3 + 6xy = 0$
$2x(4x^2 + 3y) = 0$
$4x^2 + 3y = 0$ —— (1)

$f_y = 0$
$3x^2 - 2y = 0$ —— (2)

Multiply eqn (1) with 3
(2) with 4

$4x^2+3(0)=0$

$4x^2=0$

$x=0$

Critical point is $(0,0)$

$r=f_{xx}=24x^2+6x$

$t=f_{yy}=0-2=-2$

$s=f_{xy}=6x-0=6x=6(0)=0$

r at $(0,0)$

$=24(0)+6(0)=0$

$\therefore r=0$

$rt-s^2=0(-2)-(5)^2$

$=0-0$

$=0$

$r=0$ & $rt-s^2=0$

nothing to say)

$f(x,y)$ at $(0,0)$

$2(0)^4+3(0)^2(0)-0$

$=0+0-0$

$=0$

iii) $f(x,y)=x^2-y^2+2x+8y-70$

$f_x=2x+2$

$f_y=-2y+8$

$f_x=0 \quad \therefore 2x+2=0$

$x=\frac{-2}{2} \quad \therefore x=-1$

$f_y=0 \quad -2y+8=0$

$y=\frac{-8}{-2}=4$

$\therefore y=4$

Critical point is $(-1,4)$

Scanned with CamScanner

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$-t > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x, y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 37 - 70$

$= 33$

AK 01/02/2020