ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

and latex-tikz codes from

1 QUESTION No. 2.36

Construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.

2 SOLUTION

1) Let us assume vertices of given quadrilateral $MIST$ as **M,I,S** and **T**.

2) Let us generalize the given data:

$$\angle M = 75° = \theta \qquad (2.0.1)$$

$$\angle I = 105° = \alpha \qquad (2.0.2)$$

$$\angle S = 120° = \gamma \qquad (2.0.3)$$

$$\|\mathbf{I} - \mathbf{M}\| = 3.5 = a, \qquad (2.0.4)$$

$$\|\mathbf{S} - \mathbf{I}\| = 6.5 = b, \qquad (2.0.5)$$

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \qquad (2.0.6)$$

3) Also, Let us assume the other two sides as

$$\|\mathbf{S} - \mathbf{T}\| = c \qquad (2.0.7)$$

$$\|\mathbf{M} - \mathbf{T}\| = d \qquad (2.0.8)$$

4) Finding out that quadrilateral is possible or not:-
   - For this quadrilateral $MIST$ we have,

$$\angle M + \angle I = 75° + 105° = 180°, \qquad (2.0.9)$$

$$\implies MT \parallel IS \,(\because \text{MI being the transversal})$$

- As, sum of adjacent angle on same side is $180°$ only when lines are **parallel**.Also,

$$\implies \angle S + \angle T = 180° \qquad (2.0.10)$$

$$\implies \angle T = 60° \qquad (2.0.11)$$

$$\text{Let } \angle T = 60° = \beta \qquad (2.0.12)$$

- Now sum of all the angles given and (2.0.12) is $360°$.So construction of given quadrilateral is **possible**.

Lemma 2.1. *The coordinate of S and T can be written as follows:*

$$\implies \mathbf{S} = \left(I \right) + b \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \qquad (2.0.13)$$

$$\implies \mathbf{T} = d \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} \qquad (2.0.14)$$

Proof. • For finding coordinates of S:-
The vector equation of line is given by:

$$\mathbf{S} = \mathbf{I} + \lambda m \qquad (2.0.15)$$

$$\|\mathbf{S} - \mathbf{I}\| = |\lambda| \times \|\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}\| \qquad (2.0.16)$$

$$\implies \|\mathbf{S} - \mathbf{I}\| = |\lambda| \qquad (2.0.17)$$

Now using (2.0.5) and putting its value in above equation,we get

$$\implies |\lambda| = b \qquad (2.0.18)$$

- For finding coordinates of T:-
The vector equation of line is given by:

$$\mathbf{T} = \mathbf{M} + \mu m = \mu m (\because \mathbf{M} = 0) \qquad (2.0.19)$$

$$\mathbf{T} = \mu \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} \qquad (2.0.20)$$

Using inner products of vectors we get,

$$\frac{(S - T)^\top (S - I)}{\|S - T\| \times \|S - I\|} = \cos\gamma \qquad (2.0.21)$$

$$\frac{(S - T)^{\top}(M - T)}{\|S - T\| \times \|M - T\|} = \cos\beta \qquad (2.0.22)$$

Now, dividing (2.0.21) and (2.0.22) we get:

$$\frac{(S - T)^{\top}(S - I)}{(S - T)^{\top}(M - T)} \times \frac{\|M - T\|}{\|S - I\|} = \frac{\cos\gamma}{\cos\beta} \qquad (2.0.23)$$

$$\frac{(S - T)^{\top}(S - I)}{(S - T)^{\top}(M - T)} \times \frac{\cos\beta}{\cos\gamma} = \frac{\|S - I\|}{\|M - T\|} \qquad (2.0.24)$$

Now using (2.0.5), (2.0.7) and (2.0.8) we get

$$\frac{c^T b}{c^T d} \times \frac{\cos\beta}{\cos\gamma} = \frac{b}{d} \qquad (2.0.25)$$

□

5) Putting value of $\lambda = 6.5$ in (2.0.13) and using (2.0.1) we get,

$$\implies S = I + b\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix} \qquad (2.0.26)$$

$$\implies S = \begin{pmatrix}3.5\\0\end{pmatrix} + 6.5\begin{pmatrix}\cos 75°\\\sin 75°\end{pmatrix} \qquad (2.0.27)$$

$$\implies S = \begin{pmatrix}5.18\\6.27\end{pmatrix} \qquad (2.0.28)$$

6) Using (2.0.25) and solving we get:

$$\implies 6.491\mu^2 - 48.012\mu = 6.4935\mu^2 - 48.035\mu \qquad (2.0.29)$$

$$\implies \mu = 9.35 \qquad (2.0.30)$$

7) Putting value of μ and β, we get:

$$\implies T = 9.35\begin{pmatrix}\cos 75°\\\sin 75°\end{pmatrix} \qquad (2.0.31)$$

$$\implies T = \begin{pmatrix}2.42\\9.63\end{pmatrix} \qquad (2.0.32)$$

8) Now, the vertices of given Quadrilateral MIST can be written as,

$$M = \begin{pmatrix}0\\0\end{pmatrix}, I = \begin{pmatrix}3.5\\0\end{pmatrix}, S = \begin{pmatrix}5.18\\6.27\end{pmatrix}, T = \begin{pmatrix}2.42\\9.63\end{pmatrix} \qquad (2.0.33)$$
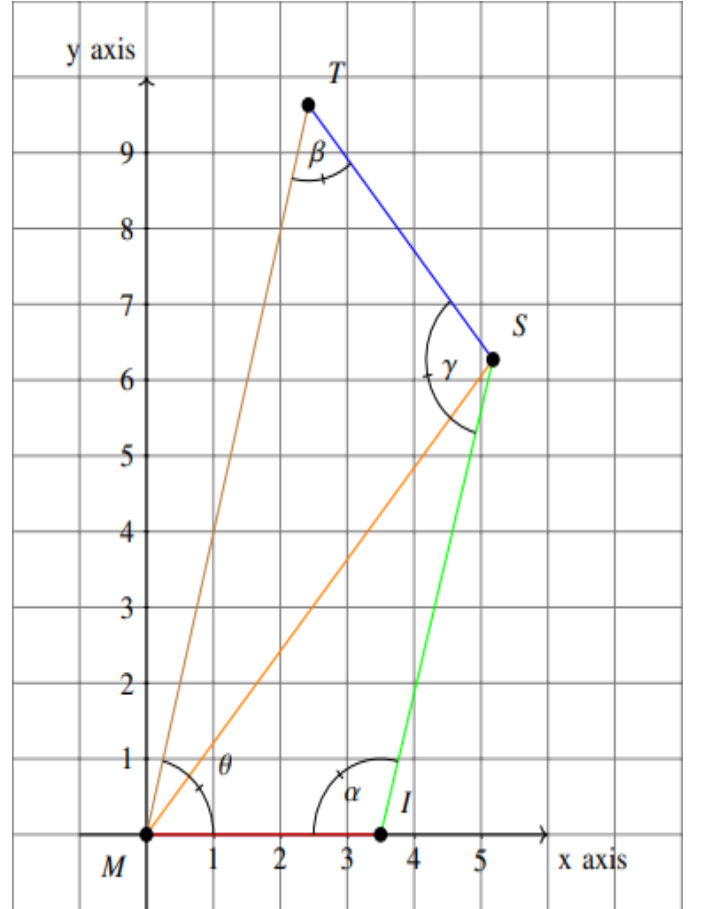
9) On constructing the quadrilateral $MIST$ we get:

Fig. 2.1: Quadrilateral MIST