ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

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https://github.com/unnatigupta2320/Assignment−2/tree/master/codes
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and latex-tikz codes from

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https://github.com/unnatigupta2320/Assignment−2/tree/master
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1 Question No. 2.36

Construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.

2 Solution

1) Let us assume vertices of given quadrilateral $MIST$ as **M**,**I**,**S** and **T**.
2) According to given data:
$$\angle M = 75°, \angle I = 105°, \angle S = 120° \tag{2.0.1}$$
$$\|\mathbf{M} - \mathbf{I}\| = 3.5, \|\mathbf{I} - \mathbf{S}\| = 6.5 \tag{2.0.2}$$
$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \tag{2.0.3}$$
3) For this quadrilateral $MIST$ we have,
$$\angle M + \angle I = 75° + 105° = 180°, \tag{2.0.4}$$
$$\implies MT \parallel IS (\because \text{MI being the transversal})$$
4) As, sum of adjacent angle on same side is 180° only when lines are **parallel**.Now, ST is another transversal on these parallel lines then, we get:
$$\implies \angle S + \angle T = 180° \tag{2.0.5}$$
$$\implies \angle T = 60° \tag{2.0.6}$$
5) Now taking sum of all the angles given and (2.0.6) we get the sum as 360°.So construction of given quadrilateral is possible.
6) For finding coordinates of S:-
$$\mathbf{S} = \mathbf{I} + \lambda m \tag{2.0.7}$$
$$\|\mathbf{S} - \mathbf{I}\| = |\lambda| \times \| \begin{pmatrix} \cos 75° \\ \sin 75° \end{pmatrix} \| \tag{2.0.8}$$
$$\implies \|\mathbf{S} - \mathbf{I}\| = |\lambda| \tag{2.0.9}$$
$$\implies |\lambda| = 6.5 \tag{2.0.10}$$
$$\implies \lambda = 6.5 \tag{2.0.11}$$
Now putting value of λ in (2.0.7) and solving we get,
$$\implies \mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75° \\ \sin 75° \end{pmatrix} \tag{2.0.12}$$
$$\implies \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix} \tag{2.0.13}$$
7) For finding value of MT :-
In $\triangle MTS$, Let $\angle$ TSM = ϕ, $\angle$ TMS =ω, $\angle$ MTS =β, Now using sine formula in $\triangle MTS$ we have
$$\frac{\sin \phi}{MT} = \frac{\sin \omega}{ST} = \frac{\sin \beta}{SM} \tag{2.0.14}$$
$$\frac{\sin 95.47°}{MT} = \frac{\sin 60°}{8.14} \tag{2.0.15}$$
$$\implies MT = 9.35 \tag{2.0.16}$$
8) For finding coordinates of T:-
$$\mathbf{T} = \mathbf{M} + \mu m = \mu m (\because \mathbf{M} = 0) \tag{2.0.17}$$
Using inner products of vectors in quadrilateral $MIST$,
$$\frac{(S - T)^\top (S - I)}{\|S - T\| \times \|S - I\|} = \cos 120° \tag{2.0.18}$$
$$\frac{(S - T)^\top (M - T)}{\|S - T\| \times \|M - T\|} = \cos 60° \tag{2.0.19}$$
9) Now, dividing (2.0.18) and (2.0.19), we get:
$$\frac{(S - T)^\top (S - I)}{(S - T)^\top (M - T)} \times \frac{\|M - T\|}{\|S - I\|} = \frac{\cos 120°}{\cos 60°} \tag{2.0.20}$$

10) On solving (2.0.17) and (2.0.20) using (2.0.16), we have

$$\implies \mu = 9.35 \tag{2.0.21}$$

$$\implies \mathbf{T} = 9.35 \begin{pmatrix} \cos 75° \\ \sin 75° \end{pmatrix} \tag{2.0.22}$$

$$\implies \mathbf{T} = \begin{pmatrix} 2.42 \\ 9.63 \end{pmatrix} \tag{2.0.23}$$

11) Now,the vertices of given Quadrilateral MIST can be written as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 2.42 \\ 9.63 \end{pmatrix} \tag{2.0.24}$$

12) On constructing the given quadrilateral we, get:

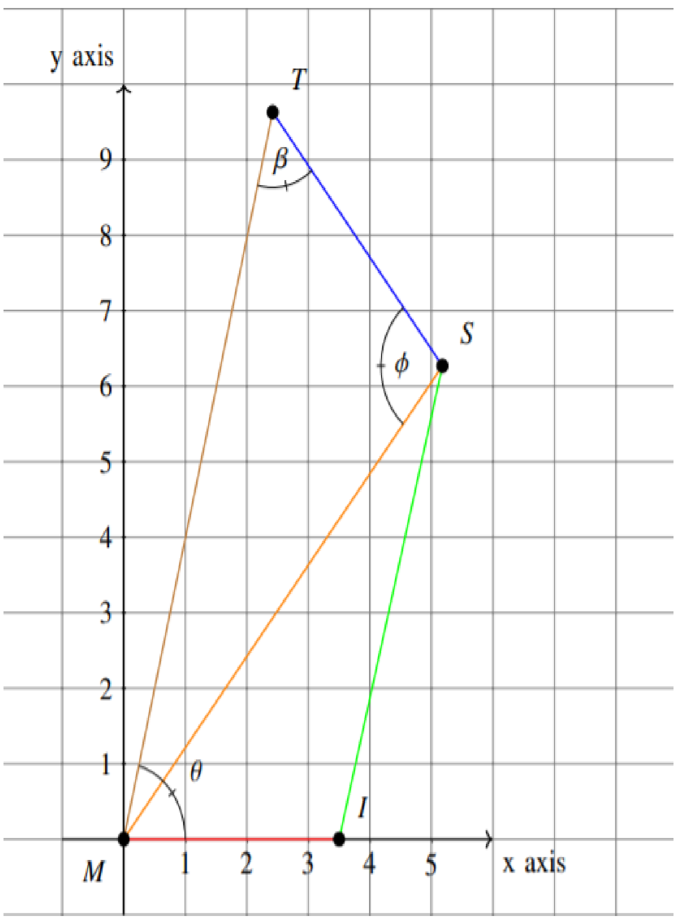

Fig. 2.1: Quadrilateral MIST