ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

```
https://github.com/unnatigupta2320/Assignment−2/
    tree/master/codes
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and latex-tikz codes from

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https://github.com/unnatigupta2320/Assignment−2/
    tree/master
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1 Question No. 2.36

Construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.

2 SOLUTION

1) Let us assume vertices of given quadrilateral $MIST$ as **M**,**I**,**S** and **T**.
2) Let us generalize the given data:

$$\angle M = 75° = \theta \tag{2.0.1}$$
$$\angle I = 105° = \alpha \tag{2.0.2}$$
$$\angle S = 120° = \gamma \tag{2.0.3}$$
$$\|\mathbf{I} - \mathbf{M}\| = 3.5 = a, \tag{2.0.4}$$
$$\|\mathbf{S} - \mathbf{I}\| = 6.5 = b, \tag{2.0.5}$$
$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \tag{2.0.6}$$

3) Also, Let us assume the other two sides as

$$\|\mathbf{S} - \mathbf{T}\| = c \tag{2.0.7}$$
$$\|\mathbf{M} - \mathbf{T}\| = \|\mathbf{T}\| = d (\because \mathbf{M} = 0) \tag{2.0.8}$$

4) Finding out that quadrilateral is possible or not:-
   - For this quadrilateral $MIST$ we have,

$$\angle M + \angle I = 75° + 105° = 180°, \tag{2.0.9}$$
$$\implies MT \parallel IS (\because \text{MI being the transversal})$$

   - As,sum of adjacent angle on same side is 180° only when lines are **parallel**.
   - If we consider $\angle$ MSI and $\angle$ TMS ,then as alternate pair of angles are equal,we get:

$$\angle MSI = \angle TMS = \omega \tag{2.0.10}$$

   - Also, ST being another transversal, we get:

$$\implies \angle S + \angle T = 180° \tag{2.0.11}$$
$$\implies \angle T = 60° \tag{2.0.12}$$
$$\text{Let } \angle T = 60° = \beta \tag{2.0.13}$$

   - Now sum of all the angles given and (2.0.13) is 360°.So construction of given quadrilateral is **possible**.
5) For finding value of c:
   - Applying sine formula in $\triangle$ MIS using (2.0.10) we have

$$\frac{MS}{\sin I} = \frac{MI}{\sin \omega} \tag{2.0.14}$$
$$\therefore MS = \frac{a \times \sin \alpha}{\sin \omega} \tag{2.0.15}$$

   - Applying sine formula in $\triangle$ MTS ,we have:

$$\frac{MS}{\sin T} = \frac{ST}{\sin \omega} \tag{2.0.16}$$

   - Putting value of (2.0.15) in(2.0.16) we get :

$$c = \frac{a \times \sin I}{\sin T} \tag{2.0.17}$$

Lemma 2.1. *The coordinate of S and T can be written as follows:*

$$\implies \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \tag{2.0.18}$$
$$\implies \mathbf{T} = d \begin{pmatrix} \cos M \\ \sin M \end{pmatrix}; \tag{2.0.19}$$

where the value of d is:

$$d = \frac{a}{2} \times \frac{\sin I}{\sin T} + b + a \cos M \tag{2.0.20}$$

Proof.
- For finding coordinates of S:-

The vector equation of line is given by:

$$\mathbf{S} = \mathbf{I} + \lambda m \tag{2.0.21}$$

$$\|\mathbf{S} - \mathbf{I}\| = |\lambda| \times \|\begin{pmatrix} \cos M \\ \sin M \end{pmatrix}\| \tag{2.0.22}$$

$$\implies \|\mathbf{S} - \mathbf{I}\| = |\lambda| \tag{2.0.23}$$

Now using (2.0.5) we get:

$$\implies |\lambda| = b \tag{2.0.24}$$

$$\implies \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \tag{2.0.25}$$

- For finding coordinates of T:-

The vector equation of line is given by:

$$\mathbf{T} = \mathbf{M} + \mu m = \mu m (\because \mathbf{M} = 0) \tag{2.0.26}$$

$$\|\mathbf{T}\| = |\mu| \times \|\begin{pmatrix} \cos M \\ \sin M \end{pmatrix}\| \tag{2.0.27}$$

Now,using (2.0.8),we get:

$$|\mu| = d \tag{2.0.28}$$

$$\implies \mathbf{T} = d \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \tag{2.0.29}$$

Using inner products of vectors in quadrilateral $MIST$ we get,

$$\frac{(\mathbf{S} - \mathbf{T})^{\top}(\mathbf{M} - \mathbf{T})}{\|\mathbf{S} - \mathbf{T}\| \times \|\mathbf{M} - \mathbf{T}\|} = \cos T \tag{2.0.30}$$

$$\frac{\mathbf{S}^{\top}\mathbf{M} - \mathbf{S}^{\top}\mathbf{T} - \mathbf{T}^{\top}\mathbf{M} + \mathbf{T}^{\top}\mathbf{T}}{\|\mathbf{S} - \mathbf{T}\| \times \|\mathbf{M} - \mathbf{T}\|} = \cos T \tag{2.0.31}$$

$$\frac{-\mathbf{S}^{\top}\mathbf{T} + \mathbf{T}^{\top}\mathbf{T}}{\|\mathbf{S} - \mathbf{T}\| \times \|\mathbf{M} - \mathbf{T}\|} = \cos T (\because \mathbf{M} = 0) \tag{2.0.32}$$

$$\frac{-\mathbf{S}^{\top}\mathbf{T} + \mathbf{T}^{\top}\mathbf{T}}{c \times d} = \cos T \tag{2.0.33}$$

$$\frac{d^2 - b \times d - a \cos M}{c \times d} = \cos T \tag{2.0.34}$$

Putting value of c from (2.0.17),we get:

$$d = \frac{a}{2} \times \frac{\sin I}{\sin T} + b + a \cos M \tag{2.0.35}$$

$$\therefore \mathbf{T} = d \times \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \tag{2.0.36}$$

□

6) Putting value of λ=6.5 in (2.0.21) and using (2.0.1) we get,

$$\implies \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \tag{2.0.37}$$

$$\implies \mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \tag{2.0.38}$$

$$\implies \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix} \tag{2.0.39}$$

7) Using (2.0.35)and solving we get:

$$\implies d = \frac{a}{2} \times \frac{\sin I}{\sin T} + b + a \cos M \tag{2.0.40}$$

$$\implies d = \frac{3.5}{2} \times \frac{\sin 105}{\sin 60} + 6.5 + 3.5 \cos 75 \tag{2.0.41}$$

$$\implies d = 9.35 \tag{2.0.42}$$

8) Putting value of d and $\angle$ M in (2.0.29)we get:

$$\implies \mathbf{T} = 9.35 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \tag{2.0.43}$$

$$\implies \mathbf{T} = \begin{pmatrix} 2.41 \\ 9.03 \end{pmatrix} \tag{2.0.44}$$

9) Now,the vertices of given Quadrilateral MIST can be written as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 2.41 \\ 9.03 \end{pmatrix} \tag{2.0.45}$$

10) On constructing the quadrilateral $MIST$ we get:

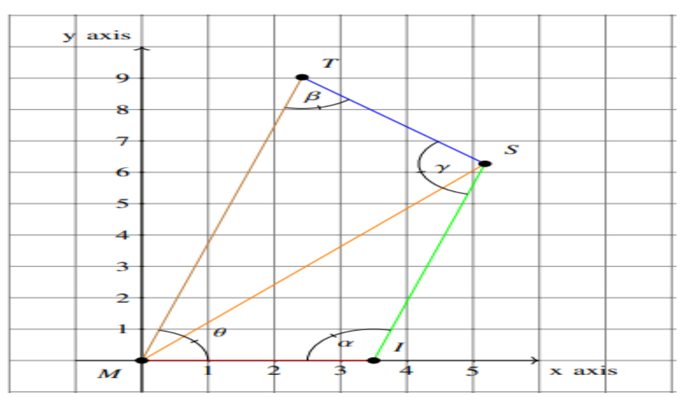

Fig. 2.1: Quadrilateral MIST