

ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

<https://github.com/unnatigupta2320/Assignment-2/tree/master/codes>

and latex-tikz codes from

<https://github.com/unnatigupta2320/Assignment-2/tree/master>

1 QUESTION No. 2.36

Construct a quadrilateral MIST where $MI = 3.5$, $IS = 6.5$, $\angle M = 75^\circ$, $\angle I = 105^\circ$ and $\angle S = 120^\circ$.

2 SOLUTION

1) Let us assume vertices of given quadrilateral MIST as **M, I, S** and **T**.

2) Let us generalize the given data:

$$\angle M = 75^\circ = \theta \quad (2.0.1)$$

$$\angle I = 105^\circ = \alpha \quad (2.0.2)$$

$$\angle S = 120^\circ = \gamma \quad (2.0.3)$$

$$\|\mathbf{I} - \mathbf{M}\| = 3.5 = a, \quad (2.0.4)$$

$$\|\mathbf{S} - \mathbf{I}\| = 6.5 = b, \quad (2.0.5)$$

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \quad (2.0.6)$$

3) Also, Let us assume the other two sides as

$$\|\mathbf{S} - \mathbf{T}\| = c \quad (2.0.7)$$

$$\|\mathbf{M} - \mathbf{T}\| = d \quad (2.0.8)$$

4) Finding out that quadrilateral is possible or not:-

• For this quadrilateral MIST we have,

$$\angle M + \angle I = 75^\circ + 105^\circ = 180^\circ, \quad (2.0.9)$$

$$\Rightarrow MT \parallel IS (\because MI \text{ being the transversal})$$

• As, sum of adjacent angle on same side is 180° only when lines are **parallel**. Also,

$$\Rightarrow \angle S + \angle T = 180^\circ \quad (2.0.10)$$

$$\Rightarrow \angle T = 60^\circ \quad (2.0.11)$$

$$\text{Let } \angle T = 60^\circ = \beta \quad (2.0.12)$$

• Now sum of all the angles given and (2.0.12) is 360° . So construction of given quadrilateral is **possible**.

Lemma 2.1. The coordinate of S and T can be written as follows:

$$\Rightarrow \mathbf{S} = (\mathbf{I}) + b \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow \mathbf{T} = b \times \frac{c^T d}{c^T b} \times \frac{\cos \gamma}{\cos \beta} \times \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix} \quad (2.0.14)$$

Proof. • For finding coordinates of S:-
The vector equation of line is given by:

$$\mathbf{S} = \mathbf{I} + \lambda m \quad (2.0.15)$$

$$\|\mathbf{S} - \mathbf{I}\| = |\lambda| \times \left\| \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \right\| \quad (2.0.16)$$

$$\Rightarrow \|\mathbf{S} - \mathbf{I}\| = |\lambda| \quad (2.0.17)$$

Now using (2.0.5) and putting its value in above equation, we get

$$\Rightarrow |\lambda| = b \quad (2.0.18)$$

• For finding coordinates of T:-

The vector equation of line is given by:

$$\mathbf{T} = \mathbf{M} + \mu m = \mu m (\because \mathbf{M} = 0) \quad (2.0.19)$$

$$\mathbf{T} = \mu \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix} \quad (2.0.20)$$

Using inner products of vectors we get,

$$\frac{(\mathbf{S} - \mathbf{T})^T (\mathbf{S} - \mathbf{I})}{\|\mathbf{S} - \mathbf{T}\| \times \|\mathbf{S} - \mathbf{I}\|} = \cos \gamma \quad (2.0.21)$$

$$\frac{(S - T)^\top(M - T)}{\|S - T\| \times \|M - T\|} = \cos \beta \quad (2.0.22)$$

Now, dividing (2.0.21) and (2.0.22) we get:

$$\frac{(S - T)^\top(S - I)}{(S - T)^\top(M - T)} \times \frac{\|M - T\|}{\|S - I\|} = \frac{\cos \gamma}{\cos \beta} \quad (2.0.23)$$

$$\frac{(S - T)^\top(S - I)}{(S - T)^\top(M - T)} \times \frac{\cos \beta}{\cos \gamma} = \frac{\|S - I\|}{\|M - T\|} \quad (2.0.24)$$

Now using (2.0.5), (2.0.7) and (2.0.8) we get

$$\frac{c^T b}{c^T d} \times \frac{\cos \beta}{\cos \gamma} = \frac{b}{d} \quad (2.0.25)$$

$$d = b \times \frac{c^T d}{c^T b} \times \frac{\cos \gamma}{\cos \beta} \quad (2.0.26)$$

$$\text{which gives } \mathbf{T} = d \times \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix} \quad (2.0.27)$$

□

5) Putting value of $\lambda=6.5$ in (2.0.13) and using (2.0.1) we get,

$$\Rightarrow \mathbf{S} = \mathbf{I} + b \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (2.0.28)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \quad (2.0.29)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix} \quad (2.0.30)$$

6) Using (2.0.25) and solving we get:

$$\Rightarrow 6.491\mu^2 - 48.012\mu = 6.4935\mu^2 - 48.035\mu \quad (2.0.31)$$

$$\Rightarrow \mu = 9.35 \quad (2.0.32)$$

7) Putting value of μ and β , we get:

$$\Rightarrow \mathbf{T} = 9.35 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \quad (2.0.33)$$

$$\Rightarrow \mathbf{T} = \begin{pmatrix} 2.42 \\ 9.63 \end{pmatrix} \quad (2.0.34)$$

8) Now, the vertices of given Quadrilateral MIST can be written as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 2.42 \\ 9.63 \end{pmatrix} \quad (2.0.35)$$

9) On constructing the quadrilateral *MIST* we

get:

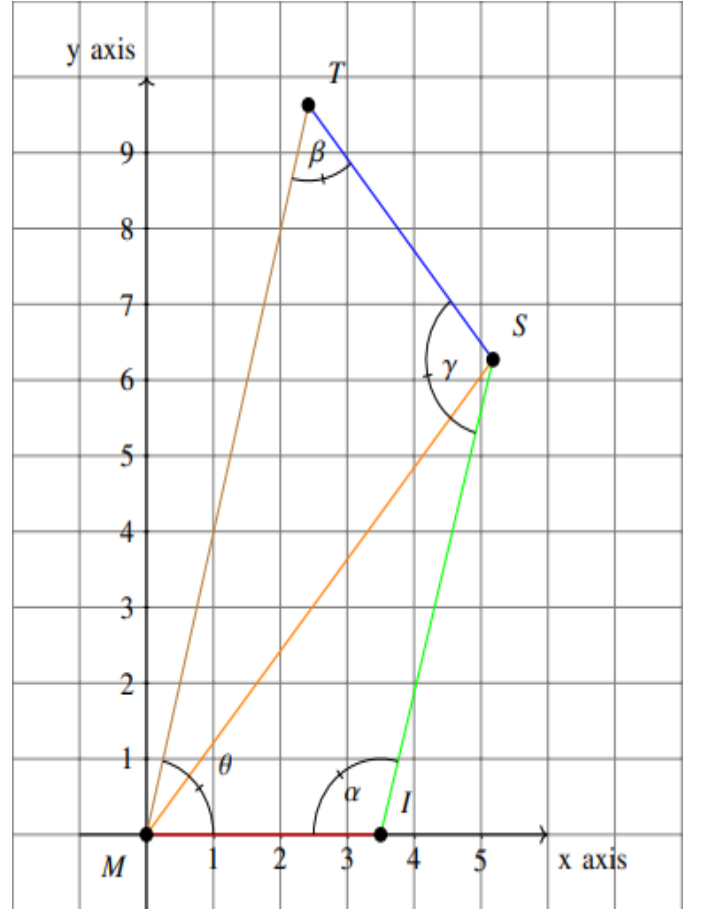


Fig. 2.1: Quadrilateral MIST