ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

https://github.com/unnatigupta2320/Assignment−2/tree/master/codes

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment−2/tree/master

1 Question No. 2.36

Construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.

2 Solution

1) Let us assume vertices of given quadrilateral $MIST$ as **M**,**I**,**S** and **T**.
2) Let us generalize the given data:

$$\angle M = 75° = \theta \tag{2.0.1}$$
$$\angle I = 105° = \alpha \tag{2.0.2}$$
$$\angle S = 120° = \gamma \tag{2.0.3}$$
$$\|\mathbf{I} - \mathbf{M}\| = 3.5 = a, \tag{2.0.4}$$
$$\|\mathbf{S} - \mathbf{I}\| = 6.5 = b, \tag{2.0.5}$$
$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \tag{2.0.6}$$

3) Also, Let us assume the other two sides as

$$\|\mathbf{S} - \mathbf{T}\| = c \tag{2.0.7}$$
$$\|\mathbf{M} - \mathbf{T}\| = \|\mathbf{T}\| = d (\because \mathbf{M} = 0) \tag{2.0.8}$$

4) Finding out that quadrilateral is possible or not:-
   - For this quadrilateral $MIST$ we have,

$$\angle M + \angle I = 75° + 105° = 180°, \tag{2.0.9}$$
$$\implies MT \parallel IS (\because \text{MI being the transversal})$$

   - As,sum of adjacent angle on same side is 180° only when lines are **parallel**.
   - If we consider ∠ MSI and ∠ TMS ,then as alternate pair of angles are equal,we get:

$$\angle MSI = \angle TMS = \omega \tag{2.0.10}$$

   - Also, ST being another transversal, we get:

$$\implies \angle S + \angle T = 180° \tag{2.0.11}$$
$$\implies \angle T = 60° \tag{2.0.12}$$
$$\text{Let } \angle T = 60° = \beta \tag{2.0.13}$$

   - Now sum of all the angles given and (2.0.13) is 360°.So construction of given quadrilateral is **possible**.
5) For finding value of c:
   - Applying sine formula in △ MIS using (2.0.10) we have

$$\frac{MS}{\sin I} = \frac{MI}{\sin \omega} \tag{2.0.14}$$
$$\therefore MS = \frac{a \times \sin I}{\sin \omega} \tag{2.0.15}$$

   - Applying sine formula in △ MTS ,we have:

$$\frac{MS}{\sin T} = \frac{ST}{\sin \omega} \tag{2.0.16}$$

   - Putting value of (2.0.15) in(2.0.16) we get :

$$c = \frac{a \times \sin I}{\sin T} \tag{2.0.17}$$

Lemma 2.1. *The coordinate of S and T can be written as follows:*

$$\implies \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \tag{2.0.18}$$
$$\implies \mathbf{T} = d \begin{pmatrix} \cos M \\ \sin M \end{pmatrix}; \tag{2.0.19}$$

where the value of d is:

$$d = \frac{a}{2} \times \frac{\sin I}{\sin T} + b + a \cos M \tag{2.0.20}$$

Proof.
- For finding coordinates of S:-

The vector equation of line is given by:

$$\mathbf{S} = \mathbf{I} + b\mathbf{m} \tag{2.0.21}$$

$$\|\mathbf{S}-\mathbf{I}\| = b \times \|\begin{pmatrix}\cos M\\ \sin M\end{pmatrix}\| \tag{2.0.22}$$

$$\implies \mathbf{S} = \begin{pmatrix}a\\0\end{pmatrix} + b\begin{pmatrix}\cos M\\ \sin M\end{pmatrix} \tag{2.0.23}$$

- For finding coordinates of T:-

The vector equation of line is given by:

$$\mathbf{T} = \mathbf{M} + d\mathbf{m} = d\mathbf{m}(\because \mathbf{M} = 0) \tag{2.0.24}$$

$$\|\mathbf{T}\| = d \times \|\begin{pmatrix}\cos M\\ \sin M\end{pmatrix}\| \tag{2.0.25}$$

$$\implies \mathbf{T} = d\begin{pmatrix}\cos M\\ \sin M\end{pmatrix} \tag{2.0.26}$$

Using inner products of vectors in quadrilateral $MIST$ we get,

$$\frac{(\mathbf{S}-\mathbf{T})^{\top}(\mathbf{M}-\mathbf{T})}{\|\mathbf{S}-\mathbf{T}\| \times \|\mathbf{M}-\mathbf{T}\|} = \cos T \tag{2.0.27}$$

$$\frac{\mathbf{S}^{\top}\mathbf{M} - \mathbf{S}^{\top}\mathbf{T} - \mathbf{T}^{\top}\mathbf{M} + \mathbf{T}^{\top}\mathbf{T}}{\|\mathbf{S}-\mathbf{T}\| \times \|\mathbf{M}-\mathbf{T}\|} = \cos T \tag{2.0.28}$$

$$\frac{-\mathbf{S}^{\top}\mathbf{T} + \mathbf{T}^{\top}\mathbf{T}}{\|\mathbf{S}-\mathbf{T}\| \times \|\mathbf{M}-\mathbf{T}\|} = \cos T(\because \mathbf{M} = 0) \tag{2.0.29}$$

$$\frac{-\mathbf{S}^{\top}\mathbf{T} + \mathbf{T}^{\top}\mathbf{T}}{c \times d} = \cos T \tag{2.0.30}$$

As,we have the coordinates of **S** and **T** as:

$$\mathbf{S} = \begin{pmatrix}a + b\cos M\\ b\sin M\end{pmatrix} \tag{2.0.31}$$

$$\mathbf{T} = \begin{pmatrix}d\cos M\\ d\sin M\end{pmatrix} \tag{2.0.32}$$

On, computing $\mathbf{S}^{\top}\mathbf{T}$,we get:

$$\mathbf{S}^{\top}\mathbf{T} = ad\cos M + bd \tag{2.0.33}$$

On, computing $\mathbf{T}^{\top}\mathbf{T}$,we get:

$$\mathbf{T}^{\top}\mathbf{T} = d^2\cos^2 M + d^2\sin^2 M = d^2 \tag{2.0.34}$$

Putting values of (2.0.33) and (2.0.34) in (2.0.30), we get:

$$\frac{d^2 - b \times d - ad\cos M}{c \times d} = \cos T \tag{2.0.35}$$

$$\frac{d - b - a\cos M}{c} = \cos T \tag{2.0.36}$$

Putting value of c from (2.0.17),we get:

$$d = \frac{a}{2} \times \frac{\sin I}{\sin T} + b + a\cos M \tag{2.0.37}$$

$$\therefore \mathbf{T} = d \times \begin{pmatrix}\cos M\\ \sin M\end{pmatrix} \tag{2.0.38}$$

□

6) Putting value of b=6.5 in (2.0.23) and using (2.0.1) we get,

$$\implies \mathbf{S} = \begin{pmatrix}a\\0\end{pmatrix} + b\begin{pmatrix}\cos M\\ \sin M\end{pmatrix} \tag{2.0.39}$$

$$\implies \mathbf{S} = \begin{pmatrix}3.5\\0\end{pmatrix} + 6.5\begin{pmatrix}\cos 75^\circ\\ \sin 75^\circ\end{pmatrix} \tag{2.0.40}$$

$$\implies \mathbf{S} = \begin{pmatrix}5.18\\6.27\end{pmatrix} \tag{2.0.41}$$

7) Using (2.0.37)and solving we get:

$$\implies d = \frac{a}{2} \times \frac{\sin I}{\sin T} + b + a\cos M \tag{2.0.42}$$

$$\implies d = \frac{3.5}{2} \times \frac{\sin 105^\circ}{\sin 60^\circ} + 6.5 + 3.5\cos 75^\circ \tag{2.0.43}$$

$$\implies d = 9.35 \tag{2.0.44}$$

8) Putting value of d and $\angle$ M in (2.0.26)we get:

$$\implies \mathbf{T} = 9.35\begin{pmatrix}\cos 75^\circ\\ \sin 75^\circ\end{pmatrix} \tag{2.0.45}$$

$$\implies \mathbf{T} = \begin{pmatrix}2.41\\9.03\end{pmatrix} \tag{2.0.46}$$

9) Now,the vertices of given Quadrilateral MIST can be written as,

$$\mathbf{M} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{I} = \begin{pmatrix}3.5\\0\end{pmatrix}, \mathbf{S} = \begin{pmatrix}5.18\\6.27\end{pmatrix}, \mathbf{T} = \begin{pmatrix}2.41\\9.03\end{pmatrix} \tag{2.0.47}$$

10) On constructing the quadrilateral $MIST$ we get:

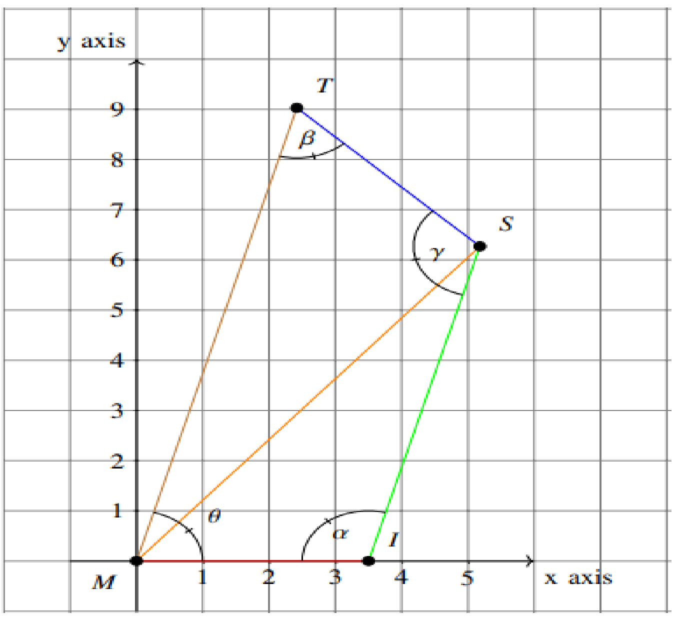

Fig. 2.1: Quadrilateral MIST