ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

https://github.com/unnatigupta2320/Assignment−2/tree/master/codes

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment−2/tree/master

1 Question No. 2.36

Construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.

2 Solution

1) Let us assume vertices of given quadrilateral $MIST$ as **M**,**I**,**S** and **T**.
2) Let us generalize the given data:

$$\angle M = 75° = \theta \tag{2.0.1}$$
$$\angle I = 105° = \alpha \tag{2.0.2}$$
$$\angle S = 120° = \gamma \tag{2.0.3}$$
$$\|\mathbf{I} - \mathbf{M}\| = 3.5 = a, \tag{2.0.4}$$
$$\|\mathbf{S} - \mathbf{I}\| = 6.5 = b, \tag{2.0.5}$$
$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \tag{2.0.6}$$

3) Also, Let us assume the other two sides as

$$\|\mathbf{S} - \mathbf{T}\| = c \tag{2.0.7}$$
$$\|\mathbf{M} - \mathbf{T}\| = d \tag{2.0.8}$$

4) Finding out that quadrilateral is possible or not:-
   - For this quadrilateral $MIST$ we have,

$$\angle M + \angle I = 75° + 105° = 180°, \tag{2.0.9}$$
$$\implies MT \parallel IS (\because \text{MI being the transversal})$$

   - As, sum of adjacent angle on same side is 180° only when lines are **parallel**.Also,

$$\implies \angle S + \angle T = 180° \tag{2.0.10}$$
$$\implies \angle T = 60° \tag{2.0.11}$$
$$\text{Let } \angle T = 60° = \beta \tag{2.0.12}$$

   - Now sum of all the angles given and (2.0.12) is 360°.So construction of given quadrilateral is **possible**.

5) For finding coordinates of S:-

   Proof. The vector equation of line is given by:

$$\mathbf{S} = \mathbf{I} + \lambda m \tag{2.0.13}$$

$$\|\mathbf{S} - \mathbf{I}\| = |\lambda| \times \| \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \| \tag{2.0.14}$$
$$\implies \|\mathbf{S} - \mathbf{I}\| = |\lambda| \tag{2.0.15}$$

   Now using (2.0.5) and putting its value in above equation,we get

$$\implies |\lambda| = b \tag{2.0.16}$$

□

   Lemma 2.1. *The coordinate of S can be written as:*

$$\implies \mathbf{S} = \begin{pmatrix} I \end{pmatrix} + \lambda \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2.0.17}$$

6) Putting value of λ=6.5 in (2.0.17) and using (2.0.1) we get,

$$\implies \mathbf{S} = \mathbf{I} + b \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2.0.18}$$
$$\implies \mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75° \\ \sin 75° \end{pmatrix} \tag{2.0.19}$$
$$\implies \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix} \tag{2.0.20}$$

7) For finding coordinates of T:-

   Proof. The vector equation of line is given by:

$$\mathbf{T} = \mathbf{M} + \mu m = \mu m (\because \mathbf{M} = 0) \tag{2.0.21}$$

$$\mathbf{T} = \mu \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} \tag{2.0.22}$$

Using inner products of vectors we get,

$$\frac{(S-T)^{\top}(S-I)}{\|S-T\| \times \|S-I\|} = \cos\gamma \tag{2.0.23}$$

$$\frac{(S-T)^{\top}(M-T)}{\|S-T\| \times \|M-T\|} = \cos\beta \tag{2.0.24}$$

Now, dividing (2.0.23) and (2.0.24) we get:

$$\frac{(S-T)^{\top}(S-I)}{(S-T)^{\top}(M-T)} \times \frac{\|M-T\|}{\|S-I\|} = \frac{\cos\gamma}{\cos\beta} \tag{2.0.25}$$

$$\frac{(S-T)^{\top}(S-I)}{(S-T)^{\top}(M-T)} \times \frac{\cos\beta}{\cos\gamma} = \frac{\|S-I\|}{\|M-T\|} \tag{2.0.26}$$

Now using (2.0.5), (2.0.7) and (2.0.8) we get

$$\frac{c^T b}{c^T d} \times \frac{\cos\beta}{\cos\gamma} = \frac{b}{d} \tag{2.0.27}$$

□

Lemma 2.2. *The coordinates of T can be written as:*

$$\implies \mathbf{T} = |\mu| \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} \tag{2.0.28}$$

8) Using (2.0.27)and solving we get:

$$\implies 6.491\mu^2 - 48.012\mu = 6.4935\mu^2 - 48.035\mu \tag{2.0.29}$$

$$\implies \mu = 9.35 \tag{2.0.30}$$

9) Putting value of μ and β,we get:

$$\implies \mathbf{T} = 9.35 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \tag{2.0.31}$$

$$\implies \mathbf{T} = \begin{pmatrix} 2.42 \\ 9.63 \end{pmatrix} \tag{2.0.32}$$

10) Now,the vertices of given Quadrilateral MIST can be written as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 2.42 \\ 9.63 \end{pmatrix} \tag{2.0.33}$$

11) On constructing the quadrilateral $MIST$ we get:

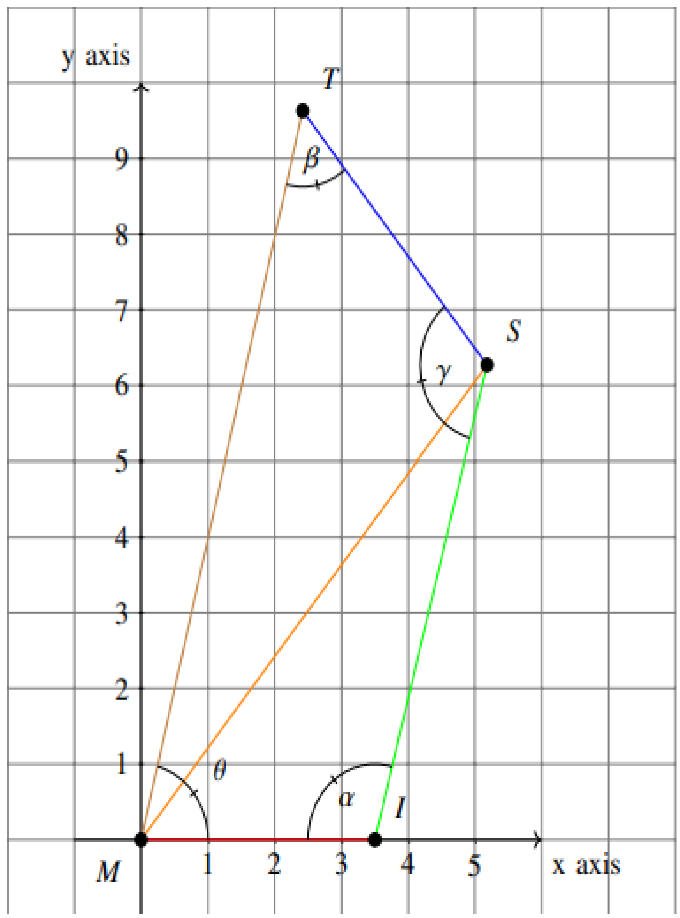

Fig. 2.1: Quadrilateral MIST