ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

```
https://github.com/unnatigupta2320/Assignment−2/
    tree/master/codes
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and latex-tikz codes from

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https://github.com/unnatigupta2320/Assignment−2/
    tree/master
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1 Question No. 2.36

Construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75^\circ, \angle I = 105^\circ$ and $\angle S = 120^\circ$.

2 Solution

1) Let us assume vertices of given quadrilateral $MIST$ as $\mathbf{M}$,$\mathbf{I}$,$\mathbf{S}$ and $\mathbf{T}$.
2) According to given data:

$$\angle M = 75^\circ, \angle I = 105^\circ, \angle S = 120^\circ \tag{2.0.1}$$

$$\|\mathbf{M} - \mathbf{I}\| = 3.5, \|\mathbf{I} - \mathbf{S}\| = 6.5 \tag{2.0.2}$$

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \tag{2.0.3}$$

3) For this quadrilateral $MIST$ we have,

$$\angle M + \angle I = 75^\circ + 105^\circ = 180^\circ, \tag{2.0.4}$$

$$\implies MT \parallel IS (\because \text{MI being the transversal})$$

4) As, sum of adjacent angle on same side is 180° only when lines are **parallel**.Now, ST is another transversal on these parallel lines then, we get:

$$\implies \angle S + \angle T = 180^\circ \tag{2.0.5}$$

$$\implies \angle T = 60^\circ \tag{2.0.6}$$

5) Now taking sum of all the angles given and (2.0.6) we get the sum as 360°.So construction of given quadrilateral is possible.
6) For finding coordinates of S:-

$$\mathbf{S} = \mathbf{I} + \lambda m \tag{2.0.7}$$

$$\|\mathbf{S} - \mathbf{I}\| = |\lambda| \times \|\begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix}\| \tag{2.0.8}$$

$$\implies \|\mathbf{S} - \mathbf{I}\| = |\lambda| \tag{2.0.9}$$

$$\implies |\lambda| = 6.5 \tag{2.0.10}$$

$$\implies \lambda = 6.5 \tag{2.0.11}$$

Now putting value of λ in (2.0.14) and solving we get,

$$\implies \mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \tag{2.0.12}$$

$$\implies \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix} \tag{2.0.13}$$

7) For finding coordinates of T:-

$$\mathbf{T} = \mathbf{M} + \mu m = \mu m (\because \mathbf{M} = 0) \tag{2.0.14}$$

Using inner products of vectors in quadrilateral $MIST$,

$$\frac{(S - T)^\intercal (S - I)}{\|S - T\| \times \|S - I\|} = \cos 120^\circ \tag{2.0.15}$$

$$\frac{(S - T)^\intercal (M - T)}{\|S - T\| \times \|M - T\|} = \cos 60^\circ \tag{2.0.16}$$

8) Now, dividing (2.0.15) and (2.0.16) then solving we get:

$$\frac{(S - T)^\intercal (S - I)}{(S - T)^\intercal (M - T)} \times \frac{\|M - T\|}{\|S - I\|} = \frac{\cos 120^\circ}{\cos 60^\circ} \tag{2.0.17}$$

$$\frac{(S - T)^\intercal (S - I)}{(S - T)^\intercal (M - T)} \times \frac{\cos 60^\circ}{\cos 120^\circ} = \frac{\|S - I\|}{\|M - T\|} \tag{2.0.18}$$

$$\frac{\mathbf{S}^T\mathbf{S} - \mathbf{S}^T\mathbf{I} - \mathbf{T}^T\mathbf{S} + \mathbf{T}^T\mathbf{I}}{\mathbf{S}^T\mathbf{M} - \mathbf{S}^T\mathbf{T} - \mathbf{T}^T\mathbf{M} + \mathbf{T}^T\mathbf{T}} \times (-1) = \frac{6.5}{\mu} \tag{2.0.19}$$

$$\frac{\|S\|^2 - \mathbf{S}^T\mathbf{I} - \mathbf{T}^T\mathbf{S} + \mathbf{T}^T\mathbf{I}}{\mathbf{S}^T\mathbf{T} - \|T\|^2} = \frac{6.5}{\mu} (\because \mathbf{M} = 0) \tag{2.0.20}$$

9) On solving (2.0.20) using (2.0.14) we have

$$\implies 6.491\mu^2 - 48.012\mu = 6.4935\mu^2 - 48.035\mu \tag{2.0.21}$$

$$\implies \mu = 9.35 \tag{2.0.22}$$

10) Now,putting value of μ in (2.0.14),we have $\mathbf{T}$ as

$$\implies \mathbf{T} = \mu m = 9.35\begin{pmatrix}\cos 75^\circ \\ \sin 75^\circ\end{pmatrix} \tag{2.0.23}$$

$$\implies \mathbf{T} = \begin{pmatrix}2.42 \\ 9.63\end{pmatrix} \tag{2.0.24}$$

11) Now,the vertices of given Quadrilateral MIST can be written as,

$$\mathbf{M} = \begin{pmatrix}0 \\ 0\end{pmatrix}, \mathbf{I} = \begin{pmatrix}3.5 \\ 0\end{pmatrix}, \mathbf{S} = \begin{pmatrix}5.18 \\ 6.27\end{pmatrix}, \mathbf{T} = \begin{pmatrix}2.42 \\ 9.63\end{pmatrix} \tag{2.0.25}$$

12) On constructing the quadrilateral $MIST$ we get:

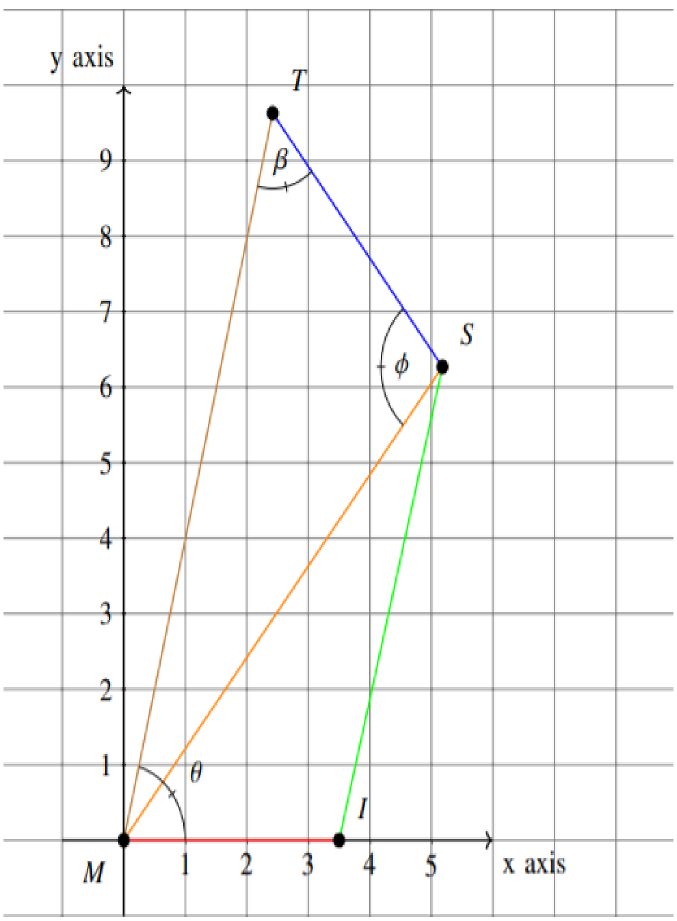

Fig. 2.1: Quadrilateral MIST