

ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

<https://github.com/unnatigupta2320/Assignment-2/tree/master/codes>

and latex-tikz codes from

<https://github.com/unnatigupta2320/Assignment-2/tree/master>

1 QUESTION No. 2.36

Construct a quadrilateral $MIST$ where $MI = 3.5$, $IS = 6.5$, $\angle M = 75^\circ$, $\angle I = 105^\circ$ and $\angle S = 120^\circ$.

2 SOLUTION

- 1) Let us assume vertices of given quadrilateral $MIST$ as $\mathbf{M}, \mathbf{I}, \mathbf{S}$ and \mathbf{T} .
- 2) Let us generalize the given data:

$$\angle M = 75^\circ = \theta \quad (2.0.1)$$

$$\angle I = 105^\circ = \alpha \quad (2.0.2)$$

$$\angle S = 120^\circ = \gamma \quad (2.0.3)$$

$$\|\mathbf{I} - \mathbf{M}\| = 3.5 = a, \quad (2.0.4)$$

$$\|\mathbf{S} - \mathbf{I}\| = 6.5 = b, \quad (2.0.5)$$

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \quad (2.0.6)$$

- 3) Also, Let us assume the other two sides as

$$\|\mathbf{S} - \mathbf{T}\| = c \quad (2.0.7)$$

$$\|\mathbf{M} - \mathbf{T}\| = \|\mathbf{T}\| = d (\because \mathbf{M} = \mathbf{0}) \quad (2.0.8)$$

- 4) Finding out that quadrilateral is possible or not:-

- For this quadrilateral $MIST$ we have,

$$\angle M + \angle I = 75^\circ + 105^\circ = 180^\circ, \quad (2.0.9)$$

$$\Rightarrow MT \parallel IS (\because MI \text{ being the transversal})$$

- As, sum of adjacent angle on same side is 180° only when lines are **parallel**. Also,

$$\Rightarrow \angle S + \angle T = 180^\circ \quad (2.0.10)$$

$$\Rightarrow \angle T = 60^\circ \quad (2.0.11)$$

$$\text{Let } \angle T = 60^\circ = \beta \quad (2.0.12)$$

- Now sum of all the angles given and (2.0.12) is 360° . So construction of given quadrilateral is **possible**.

Lemma 2.1. The coordinate of S and T can be written as follows:

$$\Rightarrow \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow \mathbf{T} = d \begin{pmatrix} \cos M \\ \sin M \end{pmatrix}; \quad (2.0.14)$$

where, If we consider,

$$b \times \frac{\cos S}{\cos T} = P \quad (2.0.15)$$

then $d =$

$$\frac{P[b \cos^2 M + a \cos M + \sin^2 M] - b^2 - ab \cos M}{P - \sin^2 M + b \cos^2 M} \quad (2.0.16)$$

Proof. • For finding coordinates of S :-
The vector equation of line is given by:

$$\mathbf{S} = \mathbf{I} + \lambda m \quad (2.0.17)$$

$$\|\mathbf{S} - \mathbf{I}\| = |\lambda| \times \left\| \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \right\| \quad (2.0.18)$$

$$\Rightarrow \|\mathbf{S} - \mathbf{I}\| = |\lambda| \quad (2.0.19)$$

Now using (2.0.5) we get:

$$\Rightarrow b = |\lambda| \quad (2.0.20)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.21)$$

- For finding coordinates of T :-

The vector equation of line is given by:

$$\mathbf{T} = \mathbf{M} + \mu \mathbf{m} = \mu \mathbf{m} (\because \mathbf{M} = 0) \quad (2.0.22)$$

$$\|\mathbf{T}\| = |\mu| \times \left\| \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \right\| \quad (2.0.23)$$

Now, using (2.0.8), we get:

$$\Rightarrow d = |\mu| \quad (2.0.24)$$

$$\mathbf{T} = d \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.25)$$

Using inner products of vectors in quadrilateral *MIST* we get,

$$\frac{(\mathbf{S} - \mathbf{T})^\top (\mathbf{S} - \mathbf{I})}{\|\mathbf{S} - \mathbf{T}\| \times \|\mathbf{S} - \mathbf{I}\|} = \cos S \quad (2.0.26)$$

$$\frac{(\mathbf{S} - \mathbf{T})^\top (\mathbf{M} - \mathbf{T})}{\|\mathbf{S} - \mathbf{T}\| \times \|\mathbf{M} - \mathbf{T}\|} = \cos T \quad (2.0.27)$$

Now, dividing (2.0.26) and (2.0.27) we get:

$$\frac{(\mathbf{S} - \mathbf{T})^\top (\mathbf{S} - \mathbf{I})}{(\mathbf{S} - \mathbf{T})^\top (\mathbf{M} - \mathbf{T})} \times \frac{\|\mathbf{M} - \mathbf{T}\|}{\|\mathbf{S} - \mathbf{I}\|} = \frac{\cos S}{\cos T} \quad (2.0.28)$$

$$\frac{\mathbf{S}^\top \mathbf{T} - \mathbf{S}^\top \mathbf{I} - \mathbf{T}^\top \mathbf{S} + \mathbf{T}^\top \mathbf{I}}{\mathbf{S}^\top \mathbf{M} - \mathbf{S}^\top \mathbf{T} - \mathbf{T}^\top \mathbf{M} + \mathbf{T}^\top \mathbf{T}} \times \frac{\cos T}{\cos S} = \frac{\|\mathbf{S} - \mathbf{I}\|}{\|\mathbf{M} - \mathbf{T}\|} \quad (2.0.29)$$

$$\frac{\mathbf{S}^\top \mathbf{T} - \mathbf{S}^\top \mathbf{I} - \mathbf{T}^\top \mathbf{S} + \mathbf{T}^\top \mathbf{I}}{-\mathbf{S}^\top \mathbf{T} + \mathbf{T}^\top \mathbf{T}} = \frac{b}{d} \times \frac{\cos S}{\cos T} \quad (2.0.30)$$

$$\frac{c^T b}{c^T d} \times \frac{\cos T}{\cos S} = \frac{b}{d} \quad (2.0.31)$$

$$\text{Let } b \times \frac{\cos S}{\cos T} = P \quad (2.0.32)$$

Now solving (2.0.30) we get, d=

$$\frac{P[b \cos^2 M + a \cos M + \sin^2 M] - b^2 - ab \cos M}{P - \sin^2 M + b \cos^2 M} \quad (2.0.33)$$

$$\text{and } \mathbf{T} = d \times \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.34)$$

□

(2.0.1) we get,

$$\Rightarrow \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.35)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \quad (2.0.36)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix} \quad (2.0.37)$$

6) Using (2.0.33) and solving we get:

$$d = \frac{-6.5[0.435 + 0.905 + .933] - 42.25 - 5.888}{[-6.5 - 0.933 + 0.435]} \quad (2.0.38)$$

$$\Rightarrow d = 9.35 \quad (2.0.39)$$

7) Putting value of d and M in (2.0.25) we get:

$$\Rightarrow \mathbf{T} = 9.35 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \quad (2.0.40)$$

$$\Rightarrow \mathbf{T} = \begin{pmatrix} 2.42 \\ 9.63 \end{pmatrix} \quad (2.0.41)$$

8) Now, the vertices of given Quadrilateral *MIST* can be written as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 2.42 \\ 9.63 \end{pmatrix} \quad (2.0.42)$$

9) On constructing the quadrilateral *MIST* we get:

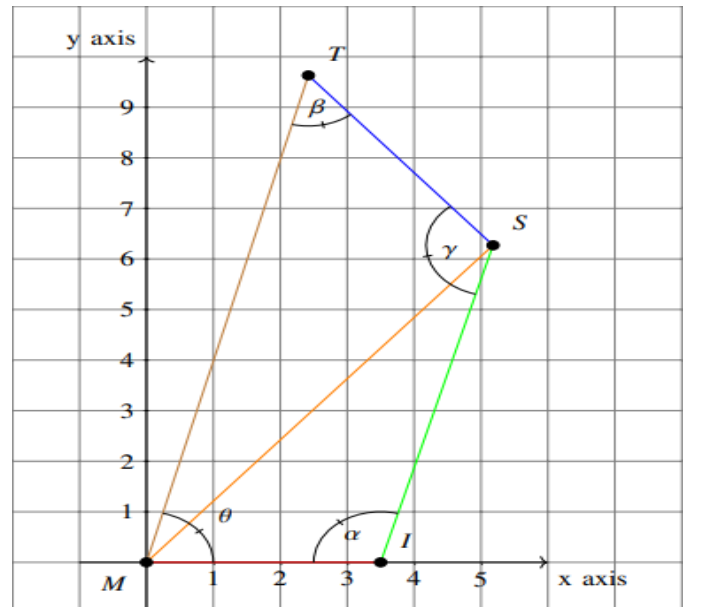


Fig. 2.1: Quadrilateral *MIST*

5) Putting value of $\lambda=6.5$ in (2.0.13) and using