

ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

<https://github.com/unnatigupta2320/Assignment-2/tree/master/codes>

and latex-tikz codes from

<https://github.com/unnatigupta2320/Assignment-2/tree/master>

1 QUESTION No. 2.36

Construct a quadrilateral $MIST$ where $MI = 3.5$, $IS = 6.5$, $\angle M = 75^\circ$, $\angle I = 105^\circ$ and $\angle S = 120^\circ$.

2 SOLUTION

1) Let us assume vertices of given quadrilateral $MIST$ as $\mathbf{M}, \mathbf{I}, \mathbf{S}$ and \mathbf{T} .

2) Let us generalize the given data:

$$\angle M = 75^\circ = \theta \quad (2.0.1)$$

$$\angle I = 105^\circ = \alpha \quad (2.0.2)$$

$$\angle S = 120^\circ = \gamma \quad (2.0.3)$$

$$\|\mathbf{I} - \mathbf{M}\| = 3.5 = a, \quad (2.0.4)$$

$$\|\mathbf{S} - \mathbf{I}\| = 6.5 = b, \quad (2.0.5)$$

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \quad (2.0.6)$$

3) Also, Let us assume the other two sides as

$$\|\mathbf{S} - \mathbf{T}\| = c \quad (2.0.7)$$

$$\|\mathbf{M} - \mathbf{T}\| = d \quad (2.0.8)$$

4) Finding out that quadrilateral is possible or not:-

- For this quadrilateral $MIST$ we have,

$$\angle M + \angle I = 75^\circ + 105^\circ = 180^\circ, \quad (2.0.9)$$

$$\Rightarrow MT \parallel IS (\because MI \text{ being the transversal})$$

- As, sum of adjacent angle on same side is 180° only when lines are **parallel**. Also,

$$\Rightarrow \angle S + \angle T = 180^\circ \quad (2.0.10)$$

$$\Rightarrow \angle T = 60^\circ \quad (2.0.11)$$

$$\text{Let } \angle T = 60^\circ = \beta \quad (2.0.12)$$

- Now sum of all the angles given and (2.0.12) is 360° . So construction of given quadrilateral is **Possible**.

5) For finding coordinates of \mathbf{S} :-

- Using vector equation of line, we have:

$$\mathbf{S} = \mathbf{I} + \lambda \mathbf{m} \quad (2.0.13)$$

$$\|\mathbf{S} - \mathbf{I}\| = |\lambda| \times \left\| \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \right\| \quad (2.0.14)$$

$$\Rightarrow \|\mathbf{S} - \mathbf{I}\| = |\lambda| \quad (2.0.15)$$

- Now using (2.0.5) and putting its value in above equation, we get

$$\Rightarrow |\lambda| = b \quad (2.0.16)$$

$$\Rightarrow \lambda = 6.5 \quad (2.0.17)$$

- Now by putting value of λ in (2.0.13) and using (2.0.1) solving we get,

$$\Rightarrow \mathbf{S} = (\mathbf{I}) + b \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (2.0.18)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \quad (2.0.19)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix} \quad (2.0.20)$$

6) For finding coordinates of \mathbf{T} :-

- Again, using vector equation of line, we have:

$$\mathbf{T} = \mathbf{M} + \mu \mathbf{m} = \mu \mathbf{m} (\because \mathbf{M} = 0) \quad (2.0.21)$$

$$\mathbf{T} = \mu \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix} \quad (2.0.22)$$

- Using inner products of vectors we get,

$$\frac{(S - T)^\top(S - I)}{\|S - T\| \times \|S - I\|} = \cos \gamma \quad (2.0.23)$$

$$\frac{(S - T)^\top(M - T)}{\|S - T\| \times \|M - T\|} = \cos \beta \quad (2.0.24)$$

7) Now, dividing (2.0.23) and (2.0.24) we get:

$$\frac{(S - T)^\top(S - I)}{(S - T)^\top(M - T)} \times \frac{\|M - T\|}{\|S - I\|} = \frac{\cos \gamma}{\cos \beta} \quad (2.0.25)$$

$$\frac{(S - T)^\top(S - I)}{(S - T)^\top(M - T)} \times \frac{\cos \beta}{\cos \gamma} = \frac{\|S - I\|}{\|M - T\|} \quad (2.0.26)$$

8) Now using (2.0.5), (2.0.7) and (2.0.8) we get

$$\frac{c^T b}{c^T d} \times \frac{\cos \beta}{\cos \gamma} = \frac{b}{d} \quad (2.0.27)$$

- On solving we get:

$$\Rightarrow 6.491\mu^2 - 48.012\mu = 6.4935\mu^2 - 48.035\mu \quad (2.0.28)$$

$$\Rightarrow \mu = 9.35 \quad (2.0.29)$$

9) Now, putting value of μ in (2.0.22) we have \mathbf{T} as

$$\Rightarrow \mathbf{T} = |\mu| \begin{pmatrix} \cos \beta \\ \sin \beta \end{pmatrix} \quad (2.0.30)$$

$$\Rightarrow \mathbf{T} = 9.35 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \quad (2.0.31)$$

$$\Rightarrow \mathbf{T} = \begin{pmatrix} 2.42 \\ 9.63 \end{pmatrix} \quad (2.0.32)$$

10) Now, the vertices of given Quadrilateral MIST can be written as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 2.42 \\ 9.63 \end{pmatrix} \quad (2.0.33)$$

11) On constructing the quadrilateral *MIST* we get:

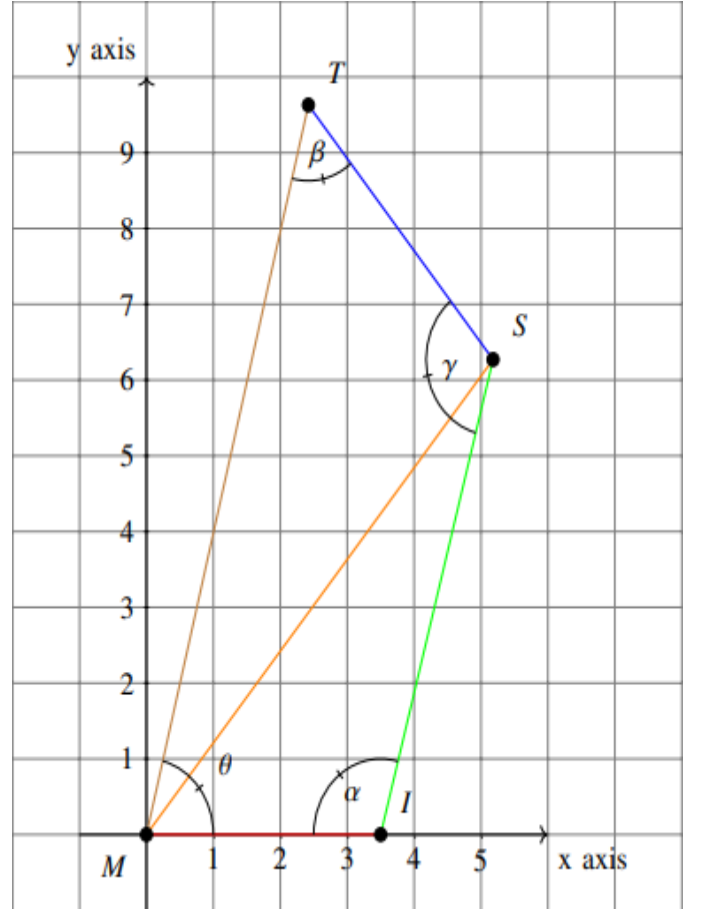


Fig. 2.1: Quadrilateral MIST