

# ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

<https://github.com/unnatigupta2320/Assignment-2/tree/master/codes>

and latex-tikz codes from

<https://github.com/unnatigupta2320/Assignment-2/tree/master>

## 1 QUESTION No. 2.36

Construct a quadrilateral  $MIST$  where  $MI = 3.5$ ,  $IS = 6.5$ ,  $\angle M = 75^\circ$ ,  $\angle I = 105^\circ$  and  $\angle S = 120^\circ$ .

## 2 SOLUTION

- 1) Let us assume vertices of given quadrilateral  $MIST$  as  $\mathbf{M}, \mathbf{I}, \mathbf{S}$  and  $\mathbf{T}$ .
- 2) Let us generalize the given data:

$$\angle M = 75^\circ = \theta \quad (2.0.1)$$

$$\angle I = 105^\circ = \alpha \quad (2.0.2)$$

$$\angle S = 120^\circ = \gamma \quad (2.0.3)$$

$$\|\mathbf{I} - \mathbf{M}\| = 3.5 = a, \quad (2.0.4)$$

$$\|\mathbf{S} - \mathbf{I}\| = 6.5 = b, \quad (2.0.5)$$

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \quad (2.0.6)$$

- 3) Also, Let us assume the other two sides as

$$\|\mathbf{S} - \mathbf{T}\| = c \quad (2.0.7)$$

$$\|\mathbf{M} - \mathbf{T}\| = \|\mathbf{T}\| = d (\because \mathbf{M} = 0) \quad (2.0.8)$$

- 4) Finding out that quadrilateral is possible or not:-

- For this quadrilateral  $MIST$  we have,

$$\angle M + \angle I = 75^\circ + 105^\circ = 180^\circ, \quad (2.0.9)$$

$$\Rightarrow MT \parallel IS (\because MI \text{ being the transversal})$$

- As, sum of adjacent angle on same side is  $180^\circ$  only when lines are **parallel**.

- If we consider  $\angle MSI$  and  $\angle TMS$ , then as alternate pair of angles are equal, we get:

$$\angle MSI = \angle TMS = \omega \quad (2.0.10)$$

- Also,  $ST$  being another transversal, we get:

$$\Rightarrow \angle S + \angle T = 180^\circ \quad (2.0.11)$$

$$\Rightarrow \angle T = 60^\circ \quad (2.0.12)$$

$$\text{Let } \angle T = 60^\circ = \beta \quad (2.0.13)$$

- Now sum of all the angles given and (2.0.13) is  $360^\circ$ . So construction of given quadrilateral is **possible**.

- 5) For finding value of  $c$ :

- Applying sine formula in  $\triangle MIS$  using (2.0.10) we have

$$\frac{MS}{\sin I} = \frac{MI}{\sin \omega} \quad (2.0.14)$$

$$\therefore MS = \frac{a \times \sin I}{\sin \omega} \quad (2.0.15)$$

- Applying sine formula in  $\triangle MTS$ , we have:

$$\frac{MS}{\sin T} = \frac{ST}{\sin \omega} \quad (2.0.16)$$

- Putting value of (2.0.15) in (2.0.16) we get :

$$c = \frac{a \times \sin I}{\sin T} \quad (2.0.17)$$

**Lemma 2.1.** The coordinate of  $S$  and  $T$  can be written as follows:

$$\Rightarrow \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.18)$$

$$\Rightarrow \mathbf{T} = d \begin{pmatrix} \cos M \\ \sin M \end{pmatrix}; \quad (2.0.19)$$

where the value of  $d$  is:

$$d = \frac{a}{2} \times \frac{\sin I}{\sin T} + b + a \cos M \quad (2.0.20)$$

*Proof.* • For finding coordinates of  $S$ :-

The vector equation of line is given by:

$$\mathbf{S} = \mathbf{I} + \lambda \mathbf{m} \quad (2.0.21)$$

$$\|\mathbf{S} - \mathbf{I}\| = |\lambda| \times \left\| \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \right\| \quad (2.0.22)$$

$$\Rightarrow \|\mathbf{S} - \mathbf{I}\| = |\lambda| \quad (2.0.23)$$

Now using (2.0.5) we get:

$$\Rightarrow |\lambda| = b \quad (2.0.24)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.25)$$

- For finding coordinates of T:-

The vector equation of line is given by:

$$\mathbf{T} = \mathbf{M} + \mu \mathbf{m} = \mu \mathbf{m} (\because \mathbf{M} = 0) \quad (2.0.26)$$

$$\|\mathbf{T}\| = |\mu| \times \left\| \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \right\| \quad (2.0.27)$$

Now,using (2.0.8),we get:

$$|\mu| = d \quad (2.0.28)$$

$$\Rightarrow \mathbf{T} = d \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.29)$$

Using inner products of vectors in quadrilateral *MIST* we get,

$$\frac{(\mathbf{S} - \mathbf{T})^\top (\mathbf{M} - \mathbf{T})}{\|\mathbf{S} - \mathbf{T}\| \times \|\mathbf{M} - \mathbf{T}\|} = \cos T \quad (2.0.30)$$

$$\frac{\mathbf{S}^\top \mathbf{M} - \mathbf{S}^\top \mathbf{T} - \mathbf{T}^\top \mathbf{M} + \mathbf{T}^\top \mathbf{T}}{\|\mathbf{S} - \mathbf{T}\| \times \|\mathbf{M} - \mathbf{T}\|} = \cos T \quad (2.0.31)$$

$$\frac{-\mathbf{S}^\top \mathbf{T} + \mathbf{T}^\top \mathbf{T}}{\|\mathbf{S} - \mathbf{T}\| \times \|\mathbf{M} - \mathbf{T}\|} = \cos T (\because \mathbf{M} = 0) \quad (2.0.32)$$

$$\frac{-\mathbf{S}^\top \mathbf{T} + \mathbf{T}^\top \mathbf{T}}{c \times d} = \cos T \quad (2.0.33)$$

As,we have the coordinates of  $\mathbf{S}$  and  $\mathbf{T}$  as:

$$\mathbf{S} = \begin{pmatrix} a + b \cos M \\ b \sin M \end{pmatrix} \quad (2.0.34)$$

$$\mathbf{T} = \begin{pmatrix} d \cos M \\ d \sin M \end{pmatrix} \quad (2.0.35)$$

On, computing  $\mathbf{S}^\top \mathbf{T}$ , we get:

$$\mathbf{S}^\top \mathbf{T} = ad \cos M + bd \quad (2.0.36)$$

On, computing  $\mathbf{T}^\top \mathbf{T}$ , we get:

$$\mathbf{T}^\top \mathbf{T} = d^2 \cos^2 M + d^2 \sin^2 M = d^2 \quad (2.0.37)$$

Putting values of (2.0.36) and (2.0.37) in (2.0.33), we get:

$$\frac{d^2 - b \times d - a \cos M}{c \times d} = \cos T \quad (2.0.38)$$

Putting value of c from (2.0.17),we get:

$$d = \frac{a}{2} \times \frac{\sin I}{\sin T} + b + a \cos M \quad (2.0.39)$$

$$\therefore \mathbf{T} = d \times \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.40)$$

□

- 6) Putting value of  $\lambda=6.5$  in (2.0.21) and using (2.0.1) we get,

$$\Rightarrow \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.41)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \quad (2.0.42)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix} \quad (2.0.43)$$

- 7) Using (2.0.39)and solving we get:

$$\Rightarrow d = \frac{a}{2} \times \frac{\sin I}{\sin T} + b + a \cos M \quad (2.0.44)$$

$$\Rightarrow d = \frac{3.5}{2} \times \frac{\sin 105^\circ}{\sin 60^\circ} + 6.5 + 3.5 \cos 75^\circ \quad (2.0.45)$$

$$\Rightarrow d = 9.35 \quad (2.0.46)$$

- 8) Putting value of  $d$  and  $\angle M$  in (2.0.29)we get:

$$\Rightarrow \mathbf{T} = 9.35 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \quad (2.0.47)$$

$$\Rightarrow \mathbf{T} = \begin{pmatrix} 2.41 \\ 9.03 \end{pmatrix} \quad (2.0.48)$$

- 9) Now,the vertices of given Quadrilateral *MIST* can be written as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.27 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 2.41 \\ 9.03 \end{pmatrix} \quad (2.0.49)$$

10) On constructing the quadrilateral  $MIST$  we get:

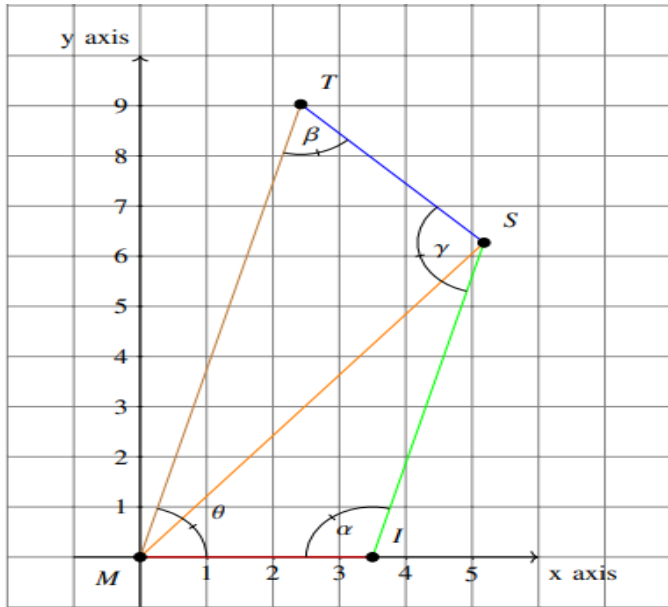


Fig. 2.1: Quadrilateral  $MIST$