ASSIGNMENT-2

UNNATI GUPTA

Download all python codes from

```
https://github.com/unnatigupta2320/Assignment−2/
    tree/master/codes
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and latex-tikz codes from

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https://github.com/unnatigupta2320/Assignment−2/
    tree/master
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1 Question No. 2.36

Construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.

2 Solution

1) Let us assume vertices of given quadrilateral $MIST$ as **M**,**I**,**S** and **T**.
2) Let us generalize the given data:

$$\angle M = 75° = \theta \tag{2.0.1}$$

$$\angle I = 105° = \alpha \tag{2.0.2}$$

$$\angle S = 120° = \gamma \tag{2.0.3}$$

$$\|\mathbf{I} - \mathbf{M}\| = 3.5 = a, \tag{2.0.4}$$

$$\|\mathbf{S} - \mathbf{I}\| = 6.5 = b, \tag{2.0.5}$$

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} \tag{2.0.6}$$

3) Also, Let us assume the other two sides as

$$\|\mathbf{S} - \mathbf{T}\| = c \tag{2.0.7}$$

$$\|\mathbf{M} - \mathbf{T}\| = \|\mathbf{T}\| = d (\because \mathbf{M} = 0) \tag{2.0.8}$$

4) Finding out that quadrilateral is possible or not:-
   - For this quadrilateral $MIST$ we have,

$$\angle M + \angle I = 75° + 105° = 180°, \tag{2.0.9}$$

$$\implies MT \parallel IS (\because \text{MI being the transversal})$$

   - As, sum of adjacent angle on same side is 180° only when lines are **parallel**.Also,

$$\implies \angle S + \angle T = 180° \tag{2.0.10}$$

$$\implies \angle T = 60° \tag{2.0.11}$$

$$\text{Let } \angle T = 60° = \beta \tag{2.0.12}$$

   - Now sum of all the angles given and (2.0.12) is 360°.So construction of given quadrilateral is **possible**.

Lemma 2.1. *The coordinate of S and T can be written as follows:*

$$\implies \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2.0.13}$$

$$\implies \mathbf{T} = d \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}; \tag{2.0.14}$$

where,If we consider,

$$b \times \frac{\cos\gamma}{\cos\beta} = P \tag{2.0.15}$$

then $d =$

$$\frac{P[b\cos^2\theta + a\cos\theta + \sin^2\theta] - b^2 - ab\cos\theta}{P - \sin^2\theta + b\cos^2\theta} \tag{2.0.16}$$

Proof. • For finding coordinates of S:-
The vector equation of line is given by:

$$\mathbf{S} = \mathbf{I} + \lambda m \tag{2.0.17}$$

$$\|\mathbf{S} - \mathbf{I}\| = |\lambda| \times \|\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}\| \tag{2.0.18}$$

$$\implies \|\mathbf{S} - \mathbf{I}\| = |\lambda| \tag{2.0.19}$$

Now using (2.0.5) we get:

$$\implies b = |\lambda| \tag{2.0.20}$$

$$\implies \mathbf{S} = \begin{pmatrix} a \\ 0 \end{pmatrix} + b \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2.0.21}$$

- For finding coordinates of T:-

The vector equation of line is given by:

$$\mathbf{T} = \mathbf{M} + \mu m = \mu m(\because \mathbf{M} = 0) \tag{2.0.22}$$

$$\|\mathbf{T}\| = |\mu| \times \|\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}\| \tag{2.0.23}$$

Now,using (2.0.8),we get:

$$\implies d = |\mu| \tag{2.0.24}$$

$$\mathbf{T} = d\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{2.0.25}$$

Using inner products of vectors in quadrilateral $MIST$ we get,

$$\frac{(S-T)^{\top}(S-I)}{\|S-T\| \times \|S-I\|} = \cos\gamma \tag{2.0.26}$$

$$\frac{(S-T)^{\top}(M-T)}{\|S-T\| \times \|M-T\|} = \cos\beta \tag{2.0.27}$$

Now, dividing (2.0.26) and (2.0.27) we get:

$$\frac{(S-T)^{\top}(S-I)}{(S-T)^{\top}(M-T)} \times \frac{\|M-T\|}{\|S-I\|} = \frac{\cos\gamma}{\cos\beta} \tag{2.0.28}$$

$$\frac{S^{\top}T - S^{\top}I - T^{\top}S + T^{\top}I}{S^{\top}M - S^{\top}T - T^{\top}M + T^{\top}T} \times \frac{\cos\beta}{\cos\gamma} = \frac{\|S-I\|}{\|M-T\|} \tag{2.0.29}$$

$$\frac{S^{\top}T - S^{\top}I - T^{\top}S + T^{\top}I}{-S^{\top}T + T^{\top}T} = \frac{b}{d} \times \frac{\cos\gamma}{\cos\beta} \tag{2.0.30}$$

$$\frac{c^T b}{c^T d} \times \frac{\cos\beta}{\cos\gamma} = \frac{b}{d} \tag{2.0.31}$$

$$\text{Let } b \times \frac{\cos\gamma}{\cos\beta} = P \tag{2.0.32}$$

Now solving (2.0.30) we get, d=

$$\frac{P[b\cos^2\theta + a\cos\theta + \sin^2\theta] - b^2 - ab\cos\theta}{P - \sin^2\theta + b\cos^2\theta} \tag{2.0.33}$$

$$\text{and } \mathbf{T} = d \times \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{2.0.34}$$

□

5) Putting value of λ=6.5 in (2.0.13) and using (2.0.1) we get,

$$\implies \mathbf{S} = \begin{pmatrix}a\\ 0\end{pmatrix} + b\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{2.0.35}$$

$$\implies \mathbf{S} = \begin{pmatrix}3.5\\ 0\end{pmatrix} + 6.5\begin{pmatrix}\cos 75^\circ\\ \sin 75^\circ\end{pmatrix} \tag{2.0.36}$$

$$\implies \mathbf{S} = \begin{pmatrix}5.18\\ 6.27\end{pmatrix} \tag{2.0.37}$$

6) Using (2.0.33)and solving we get:

$$d = \frac{-6.5[0.435 + 0.905 + .933] - 42.25 - 5.888}{[-6.5 - 0.933 + 0.435]} \tag{2.0.38}$$

$$\implies d = 9.35 \tag{2.0.39}$$

7) Putting value of d and θ in (2.0.25)we get:

$$\implies \mathbf{T} = 9.35\begin{pmatrix}\cos 75^\circ\\ \sin 75^\circ\end{pmatrix} \tag{2.0.40}$$

$$\implies \mathbf{T} = \begin{pmatrix}2.42\\ 9.63\end{pmatrix} \tag{2.0.41}$$

8) Now,the vertices of given Quadrilateral MIST can be written as,

$$\mathbf{M} = \begin{pmatrix}0\\ 0\end{pmatrix}, \mathbf{I} = \begin{pmatrix}3.5\\ 0\end{pmatrix}, \mathbf{S} = \begin{pmatrix}5.18\\ 6.27\end{pmatrix}, \mathbf{T} = \begin{pmatrix}2.42\\ 9.63\end{pmatrix} \tag{2.0.42}$$

9) On constructing the quadrilateral $MIST$ we get:

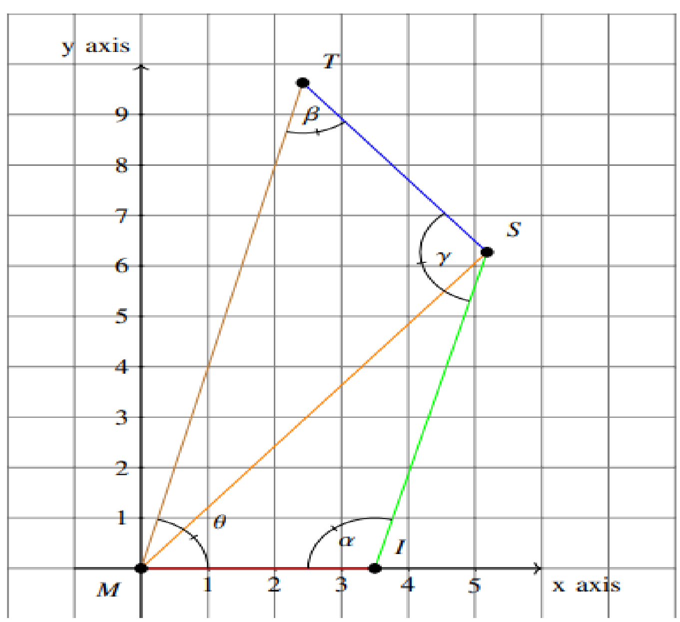

Fig. 2.1: Quadrilateral MIST