ASSIGNMENT-14

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_14

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_14

1 Question No. 2.1(b)(Optimization)

Find the absolute maximum and absolute minimum value of $f(x) = (x-1)^2 + 3, x \in [-3, 1]$.

2 Solution

Lemma 2.1. *A function f(x) is said to be convex if following inequality is true for* $\lambda \in [0, 1]$:

$$\lambda f(x_1) + (1-\lambda) f(x_2) \geq f(\lambda x_1 + (1-\lambda)x_2) \quad (2.0.1)$$

Given :

$$(x-1)^2 + 3, x \in [-3, 1], x \in [-3, 1] \quad (2.0.2)$$

Checking convexity of $f(x)$:

$$\lambda\left((x_1-1)^2+3\right) + (1-\lambda)\left((x_2-1)^2+3\right) \geq ((\lambda x_1 + (1-\lambda)x_2) - 1)^2 + 3 \quad (2.0.3)$$

resulting in

$$x_1^2\left(\lambda - \lambda^2\right) + x_2^2\left(\lambda-\lambda^2\right) - 2x_1x_2\left(\lambda-\lambda^2\right) \geq 0 \quad (2.0.4)$$

$$\implies \left(\lambda-\lambda^2\right)\left(x_1^2 + x_2^2 - 2x_1x_2\right) \geq 0 \quad (2.0.5)$$

$$\implies \lambda(1-\lambda)(x_1-x_2)^2 \geq 0 \quad (2.0.6)$$

Hence,using lemma 2.1, given $f(x)$ is a convex function .

1) For Maxima :
   Using gradient ascent method,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \quad (2.0.7)$$

$$\implies x_{n+1} = x_n + \alpha(2x_n - 2) \quad (2.0.8)$$

Taking $x_0 = -3, \alpha = 0.001$ and precision= 0.00000001,values obtained using python are:

$$\boxed{\text{Maxima} = 18.999999999940298 \approx 19} \quad (2.0.9)$$

$$\boxed{\text{Maxima Point} = -2.9999900126845568 \approx -3} \quad (2.0.10)$$

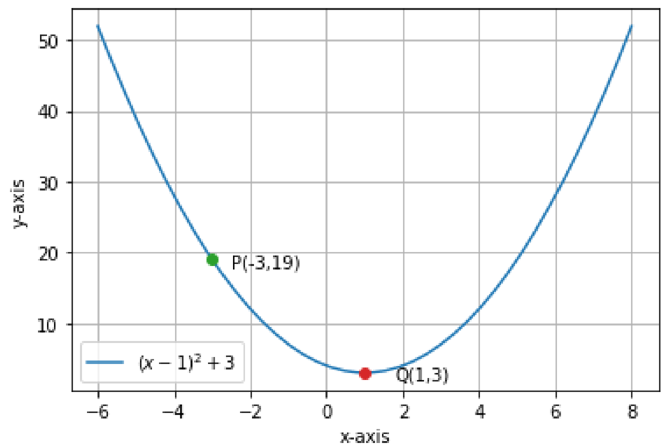

Fig. 2.1: $f(x) = (x-1)^2 + 3$

2) For Minima :

x	$f(x)$
-3	19
1	3

TABLE 2.1: Value of $f(x)$

Critical point is given by

$$\nabla f(x) = 0 \quad (2.0.11)$$

$$\implies x = 1 \quad (2.0.12)$$

and,end points are $x = -3$ and $x = 1$.
Using table 2.1,

$$\boxed{\text{Minima} = 3} \quad (2.0.13)$$

$$\boxed{\text{Minima Point} = 1} \quad (2.0.14)$$