ASSIGNMENT-10

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_10/codes

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_10

1 Question No. 2.58

Solve x+2y ≤ 10,x+y ≥ 1,x-y ≤ 0,x,y ≥ 0

2 Solution

The given system of inequality can be written in matrix form as

$$\begin{pmatrix} -1 & -2 \\ 1 & 1 \\ -1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -10 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.1}$$

which can be further simplified into

$$\begin{pmatrix} -1 & -2 \\ 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -10 \\ 1 \\ 0 \end{pmatrix} \tag{2.0.2}$$

Let the surplus vector be

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq 0 \tag{2.0.3}$$

1)

$$\begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -10 \\ 1 \end{pmatrix} \tag{2.0.4}$$

$$\implies \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -10 \\ 1 \end{pmatrix} + \mathbf{u} \tag{2.0.5}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -10 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}^{-1} \mathbf{u} \tag{2.0.6}$$

$$\implies \mathbf{x} = \begin{pmatrix} -12 \\ 11 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix} \mathbf{u} \tag{2.0.7}$$

2)

$$\begin{pmatrix} -1 & -2 \\ -1 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -10 \\ 0 \end{pmatrix} \tag{2.0.8}$$

$$\implies \begin{pmatrix} -1 & -2 \\ -1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -10 \\ 0 \end{pmatrix} + \mathbf{u} \tag{2.0.9}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -1 & -2 \\ -1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -10 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & -2 \\ -1 & 1 \end{pmatrix}^{-1} \mathbf{u} \tag{2.0.10}$$

$$\implies \mathbf{x} = \begin{pmatrix} \frac{10}{3} \\ \frac{10}{3} \end{pmatrix} + \begin{pmatrix} \frac{-1}{3} & \frac{-2}{3} \\ \frac{-1}{3} & \frac{1}{3} \end{pmatrix} \mathbf{u} \tag{2.0.11}$$

Now,solution region which is common to regions of eq. (2.0.7) and eq. (2.0.11),is given by

$$\boxed{\mathbf{x} = \begin{pmatrix} \frac{10}{3} \\ \frac{10}{3} \end{pmatrix} + \begin{pmatrix} \frac{-4}{3} & \frac{-8}{3} \\ \frac{2}{3} & \frac{4}{3} \end{pmatrix} \mathbf{u}} \tag{2.0.12}$$

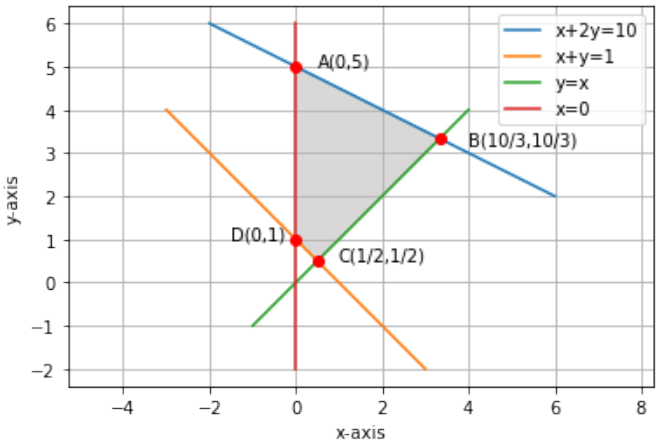

Fig. 2.1: Graphical Solution

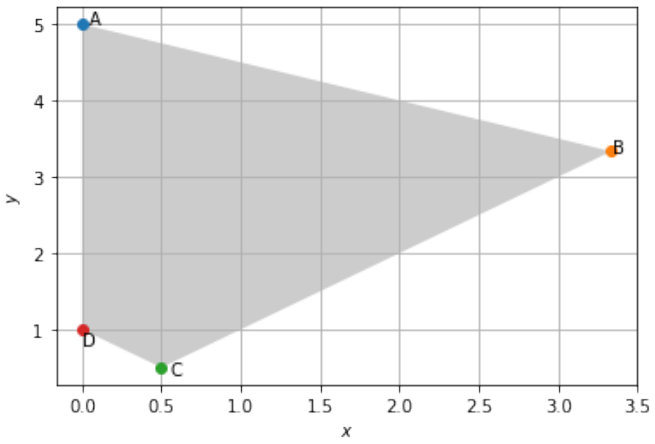

Fig. 2.2: Magnified Solution region