ASSIGNMENT-13

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Download all python codes from

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https://github.com/unnatigupta2320/
    Assignment_13
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and latex-tikz codes from

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1 Question No-6.20

An unbiased dice is thrown twice. Let the event A be 'odd number on the first throw' and B be 'Odd number on second throw'. Check the independence of event A and B.

2 Solution

Lemma 2.1. *Two events are independent if knowing one event occurred doesn't change the probability of the other event.*
∴ A and B are said to be independent if and only if:-

$$\Pr(AB) = \Pr(A)\Pr(B) \tag{2.0.1}$$

1) Let X_0 and X_1 be the random variables representing the numbers we get when a dice is thrown for first and second time respectively.

$$X_0 \epsilon \{1, 2, 3, 4, 5, 6\} \tag{2.0.2}$$
$$X_1 \epsilon \{1, 2, 3, 4, 5, 6\} \tag{2.0.3}$$

2) Also, the probability

$$\Pr(X = i) = \begin{cases} \frac{1}{6} & 1 \le i \le 6 \\ 0 & otherwise \end{cases} \tag{2.0.4}$$

3) According to question,the events are:-

Events	**Description**
A	Odd number on first throw
B	Odd number on second throw
AB	Odd Numbers appears on both throws

4) For the event **A**:-
   - The probability of odd number on first throw is-

$$\Pr(A) = \sum_{i=1,3,5} \Pr(X_0 = i) \tag{2.0.5}$$

   - So,

$$\Pr(A) = \Pr(X_0 = 1) + \Pr(X_0 = 3) + \Pr(X_0 = 5) \tag{2.0.6}$$

$$\implies \Pr(A) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \tag{2.0.7}$$
$$\implies \Pr(A) = \frac{3}{6} \tag{2.0.8}$$
$$\implies \Pr(A) = \frac{1}{2} \tag{2.0.9}$$

5) For the event **B**:-
   - The probability of odd number on second throw is-

$$\Pr(B) = \sum_{i=1,3,5} \Pr(X_1 = i) \tag{2.0.10}$$

   - So,

$$\Pr(B) = \Pr(X_1 = 1) + \Pr(X_1 = 3) + \Pr(X_1 = 5) \tag{2.0.11}$$

$$\implies \Pr(B) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \tag{2.0.12}$$
$$\implies \Pr(B) = \frac{3}{6} \tag{2.0.13}$$
$$\implies \Pr(B) = \frac{1}{2} \tag{2.0.14}$$

6) For the event **AB**:-
   - The probability that odd numbers appears on

both throw is-

$$\implies \Pr(AB) = \sum_{i=1,3,5} (\Pr(X_0 = i)) \sum_{i=1,3,5} (\Pr(X_1 = i)) \tag{2.0.15}$$

$$\implies \Pr(AB) = \left(\frac{3}{6}\right)\left(\frac{3}{6}\right) \tag{2.0.16}$$

$$\implies \Pr(AB) = \frac{9}{36} \tag{2.0.17}$$

$$\implies \Pr(AB) = \frac{1}{4} \tag{2.0.18}$$

7) Now to check whether the events are **independent**, we use Lemma (2.1).

$$\implies \Pr(AB) = \Pr(A)\Pr(B) \tag{2.0.19}$$

8) Putting values from (2.0.9) and (2.0.14) we get,

$$\implies \Pr(AB) = \frac{1}{2} \times \frac{1}{2} \tag{2.0.20}$$

$$\implies \Pr(AB) = \frac{1}{4} \tag{2.0.21}$$

This is equal to value in equation (2.0.18). Hence, the events are **independent**.