ASSIGNMENT-13

Unnati Gupta

Download all python codes from

https://github.com/unnatigupta2320/Assignment_13

and latex-tikz codes from

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1 Question No-6.20

An unbiased dice is thrown twice. Let the event A be "odd number on the first throw" and B be "Odd number on second throw". Check the independence of event A and B.

2 Solution

Lemma 2.1. *Two events are independent if knowing one event occurred doesn't change the probability of the other event.*
∴ A and B are said to be independent if:

$$P(A \cap B) = P(A)P(B) \tag{2.0.1}$$

1) According to given data we have,

Events	Description
A	Odd number on first throw
B	Odd number on second throw
$A \cap B$	Odd Numbers appears on both throw

2) When a die is thrown twice the possible outcomes are:

$$\begin{aligned} S = \{&(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\ &(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\ &(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\ &(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ &(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\ &(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} \end{aligned} \tag{2.0.2}$$

3) For the event **A** : Odd number on first throw
   - The Sample space is:

$$\begin{aligned} S_A = \\ \{&(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\ &(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\ &(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\} \end{aligned} \tag{2.0.3}$$

   - So, the probability of odd number on first throw is-

$$\implies Pr(A) = \frac{18}{36} \tag{2.0.4}$$

$$\implies Pr(A) = \frac{1}{2} \tag{2.0.5}$$

4) For the event **B**: Odd number on second throw
   - The Sample space is:

$$\begin{aligned} S_B = \\ \{&(1,1),(2,1),(3,1),(4,1),(5,1),(6,1),\\ &(1,3),(2,3),(3,3),(4,3),(5,3),(6,3),\\ &(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\} \end{aligned} \tag{2.0.6}$$

   - So, the probability of odd number on second throw is-

$$\implies Pr(B) = \frac{18}{36} \tag{2.0.7}$$

$$\implies Pr(B) = \frac{1}{2} \tag{2.0.8}$$

5) For,$A \cap B$: Odd Numbers appears on both throw
   - The Sample space is:

$$\begin{aligned} S = \{&(1,1),(1,3),(1,5),\\ &(3,1),(3,3),(3,5),\\ &(5,1),(5,3),(5,5)\} \end{aligned} \tag{2.0.9}$$

   - So, the probability that odd numbers appears

on both throw is-

$$\implies Pr(A \cap B) = \frac{9}{36} \tag{2.0.10}$$

$$\implies Pr(A \cap B) = \frac{1}{4} \tag{2.0.11}$$

6) Now to check whether the events are **independent**, we use Lemma (2.1).

$$\implies Pr(A \cap B) = Pr(A)Pr(B) \tag{2.0.12}$$

7) Putting values from (2.0.5) and (2.0.8) we get,

$$\implies Pr(A \cap B) = \frac{1}{2} \times \frac{1}{2} \tag{2.0.13}$$

$$\implies Pr(A \cap B) = \frac{1}{4} \tag{2.0.14}$$

This is equal to value in equation (2.0.11).
Hence, the events are **independent**.