

# Assignment 3

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Download all python codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-3/Codes>

and latex-tikz codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-3>

## 1 QUESTION No. 2.59

Draw a line segment **AB** of length 8 units. Taking **A** as centre draw a circle of radius 4 units and taking **B** as centre draw a circle of radius 3 units. Construct tangents to each circle from the centre of other circle.

## 2 SOLUTION

Given that, line segment  $AB=8$  units. So, Let:

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} = \begin{pmatrix} b \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \quad (2.0.2)$$

Further the given data is tabularised in table 2.1:

	Symbols	Circle1	Circle2
Centre	<b>O</b>	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 8 \\ 0 \end{pmatrix}$
Radius	$r_1, r_2$	4	3
Polar coordinates	<b>C<sub>1</sub>, C<sub>2</sub></b>	$4 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$	$3 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$
Angle	$\theta$	$0-2\pi$	$0-2\pi$

TABLE 2.1: Input values

1) For tangents at **Circle1** from **B**:-

- Let  $BP$  and  $BQ$  be tangents to **Circle1** with radius 4 units.
- We know a tangent is always perpendicular to the radius.

$$\therefore AP \perp BP, AQ \perp BQ. \quad (2.0.3)$$

• Now,

$$\Rightarrow (\mathbf{A} - \mathbf{P})^T (\mathbf{B} - \mathbf{P}) = 0 \quad (\because AP \perp BP) \quad (2.0.4)$$

$$\Rightarrow \mathbf{P}^T (\mathbf{B} - \mathbf{P}) = 0 \quad (\because \mathbf{A} = 0) \quad (2.0.5)$$

$$\Rightarrow \mathbf{P}^T \mathbf{B} - \mathbf{P}^T \mathbf{P} = 0 \quad (2.0.6)$$

$$\Rightarrow \|\mathbf{P}\|^2 = \mathbf{P}^T \mathbf{B} \quad (2.0.7)$$

$$\Rightarrow \|\mathbf{P}\|^2 = \mathbf{B}^T \mathbf{P} \quad (2.0.8)$$

$$\Rightarrow \mathbf{B}^T \mathbf{P} = 16 \quad (\because \|\mathbf{P}\|^2 = 16) \quad (2.0.9)$$

$$\Rightarrow \begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{P} = 16 \quad \left( \because \mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \right) \quad (2.0.10)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{P} = 2 \quad (2.0.11)$$

$$\Rightarrow \mathbf{P} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.12)$$

$$\Rightarrow \mathbf{P} = \mathbf{p} + \lambda \mathbf{m} \quad (2.0.13)$$

$$\text{where, } \mathbf{p} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$\text{and } \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.15)$$

• We know,

$$\|\mathbf{p} + \lambda \mathbf{m}\|^2 = 16 \quad (2.0.16)$$

$$(\mathbf{p} + \lambda \mathbf{m})^T (\mathbf{p} + \lambda \mathbf{m}) = 16 \quad (2.0.17)$$

$$\lambda^2 = \frac{16 - \|\mathbf{p}\|^2}{\|\mathbf{m}\|^2} \quad (2.0.18)$$

$$\lambda = \pm 3.46 \quad (2.0.19)$$

Substitute  $\lambda$  value in (2.0.13) we get the coordinates of **P** and **Q** as :

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3.46 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 2 \\ -3.46 \end{pmatrix} \quad (2.0.20)$$

2) For tangents at **Circle2** from **A**:-

- In  $\triangle ABR$ , using Pythagoras Theorem, we get:

$$\|A - B\|^2 = \|A - R\|^2 + \|B - R\|^2 \quad (2.0.21)$$

$$\|A - R\|^2 = \|A - B\|^2 - \|B - R\|^2 \quad (2.0.22)$$

- As,  $A = 0$ , we get:

$$\|R\|^2 = \|A - B\|^2 - \|B - R\|^2 \quad (2.0.23)$$

$$\|R\|^2 = 8^2 - 3^2 = 55 \quad (2.0.24)$$

3) Let  $AR$  and  $AS$  be tangents to **Circle2** with radius 3 units.

- We know a tangent is always perpendicular to the radius .

$$\therefore AR \perp BR, AS \perp BS. \quad (2.0.25)$$

- Now,

$$\Rightarrow (A - R)^T (B - R) = 0 \quad (\because AR \perp BR) \quad (2.0.26)$$

$$\Rightarrow R^T (B - R) = 0 \quad (\because A = 0) \quad (2.0.27)$$

$$\Rightarrow R^T B - R^T R = 0 \quad (2.0.28)$$

$$\Rightarrow \|R\|^2 = R^T B \quad (2.0.29)$$

$$\Rightarrow \|R\|^2 = B^T R \quad (2.0.30)$$

- Now, we have  $\|R\|^2 = 55$ , putting the value in above equation we get:

$$\Rightarrow B^T R = 55 \quad (2.0.31)$$

$$\Rightarrow \begin{pmatrix} 8 & 0 \end{pmatrix} R = 55 \quad \left( \because B = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \right) \quad (2.0.32)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \end{pmatrix} R = \frac{55}{8} \quad (2.0.33)$$

$$\Rightarrow R = \begin{pmatrix} \frac{55}{8} \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.34)$$

$$\Rightarrow R = r + \mu m \quad (2.0.35)$$

$$\text{where, } r = \begin{pmatrix} \frac{55}{8} \\ 0 \end{pmatrix} \quad (2.0.36)$$

$$\text{and } m = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.37)$$

- We know,

$$\|r + \mu m\|^2 = 55 \quad (2.0.38)$$

$$(r + \mu m)^T (r + \mu m) = 55 \quad (2.0.39)$$

$$\mu^2 = \frac{55 - \|r\|^2}{\|m\|^2} \quad (2.0.40)$$

$$\mu = \pm 2.78 \quad (2.0.41)$$

Substitute  $\mu$  value in (2.0.35) we get the coordinates of  $R$  and  $S$  as :

$$R = \begin{pmatrix} 6.875 \\ 2.78 \end{pmatrix}, S = \begin{pmatrix} 6.875 \\ -2.78 \end{pmatrix} \quad (2.0.42)$$

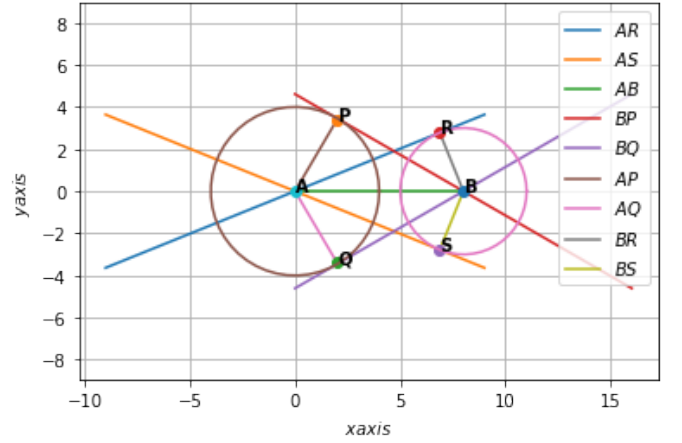


Fig. 2.1: Tangents from centres of Circle