Assignment 3

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_3/tree/master/CODES

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_3

1 Question No. 2.59

Draw a line segment **AB** of length 8 units. Taking **A** as centre draw a circle of radius 4 units and taking **B** as centre draw a circle of radius 3 units. Construct tangents to each circle from the centre of other circle.

2 Solution

Given that, line segment AB=8 units.So,Let:

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix} b \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \tag{2.0.2}$$

Further the given data is tabularised in table 2.1:

	Symbols	Circle1	Circle2
Centre	$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 8 \\ 0 \end{pmatrix}$
Radius	r_1,r_2	4	3
Polar coordinates	$\mathbf{C}_1$,$\mathbf{C}_2$	$4\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$	$3\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$
Angle	θ	0-2π	0-2π

TABLE 2.1: Input values

1) For tangents at **Circle1** from **B**:-
   - Let BP and BQ be tangents to **Circle1** with radius 4 units.
   - We know a tangent is always perpendicular to the radius .

$$\therefore AP \perp BP, AQ \perp BQ. \tag{2.0.3}$$

   - Now,

$$\implies (\mathbf{A}-\mathbf{P})^T(\mathbf{B}-\mathbf{P}) = 0 \quad (\because AP \perp BP) \tag{2.0.4}$$

$$\implies \mathbf{P}^T(\mathbf{B}-\mathbf{P}) = 0 \quad (\because \mathbf{A} = 0) \tag{2.0.5}$$

$$\implies \mathbf{P}^T\mathbf{B} - \mathbf{P}^T\mathbf{P} = 0 \tag{2.0.6}$$

$$\implies \|\mathbf{P}\|^2 = \mathbf{P}^T\mathbf{B} \tag{2.0.7}$$

$$\implies \|\mathbf{P}\|^2 = \mathbf{B}^T\mathbf{P} \tag{2.0.8}$$

$$\implies \mathbf{B}^T\mathbf{P} = 16 \quad \left(\because \|\mathbf{P}\|^2 = 16\right) \tag{2.0.9}$$

$$\implies \begin{pmatrix} 8 & 0 \end{pmatrix}\mathbf{P} = 16 \quad \left(\because \mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}\right) \tag{2.0.10}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{P} = 2 \tag{2.0.11}$$

$$\implies \mathbf{P} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.12}$$

$$\implies \mathbf{P} = \mathbf{p} + \lambda\mathbf{m} \tag{2.0.13}$$

$$\text{where, } \mathbf{p} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2.0.14}$$

$$\text{and } \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.15}$$

   - We know,

$$\|\mathbf{p} + \lambda\mathbf{m}\|^2 = 16 \tag{2.0.16}$$

$$(\mathbf{p} + \lambda\mathbf{m})^T(\mathbf{p} + \lambda\mathbf{m}) = 16 \tag{2.0.17}$$

$$\lambda^2 = \frac{16 - \|\mathbf{p}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.18}$$

$$\lambda = \pm 3.46 \tag{2.0.19}$$

Substitute λ value in (2.0.13) we get the coordinates of **P** and **Q** as :

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3.46 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 2 \\ -3.46 \end{pmatrix} \tag{2.0.20}$$

2) For tangents at **Circle2** from **A**:-
   - In $\triangle$ ABR, using Pythagoras Theorem,we get:

$$\|\mathbf{A}-\mathbf{B}\|^2 = \|\mathbf{A}-\mathbf{R}\|^2 + \|\mathbf{B}-\mathbf{R}\|^2 \tag{2.0.21}$$
$$\|\mathbf{A}-\mathbf{R}\|^2 = \|\mathbf{A}-\mathbf{B}\|^2 - \|\mathbf{B}-\mathbf{R}\|^2 \tag{2.0.22}$$

   - As, $\mathbf{A} = 0$, above equation get reduced to:

$$\|\mathbf{R}\|^2 = \|\mathbf{A}-\mathbf{B}\|^2 - \|\mathbf{B}-\mathbf{R}\|^2 \tag{2.0.23}$$
$$\|\mathbf{R}\|^2 = 8^2 - 3^2 = 55 \tag{2.0.24}$$

3) Let AR and AS be tangents to **Circle2** with radius 3 units.
   - We know a tangent is always perpendicular to the radius .

$$\therefore AR \perp BR, AS \perp BS. \tag{2.0.25}$$

   - Now,

$$\implies (\mathbf{A}-\mathbf{R})^T(\mathbf{B}-\mathbf{R}) = 0 \quad (\because AR \perp BR) \tag{2.0.26}$$
$$\implies \mathbf{R}^T(\mathbf{B}-\mathbf{R}) = 0 \quad (\because \mathbf{A} = 0) \tag{2.0.27}$$
$$\implies \mathbf{R}^T\mathbf{B} - \mathbf{R}^T\mathbf{R} = 0 \tag{2.0.28}$$
$$\implies \|\mathbf{R}\|^2 = \mathbf{R}^T\mathbf{B} \tag{2.0.29}$$
$$\implies \|\mathbf{R}\|^2 = \mathbf{B}^T\mathbf{R} \tag{2.0.30}$$

   - Now, we have $\|\mathbf{R}\|^2 = 55$, putting the value in above equation we get:

$$\implies \mathbf{B}^T\mathbf{R} = 55 \tag{2.0.31}$$
$$\implies \begin{pmatrix} 8 & 0 \end{pmatrix}\mathbf{R} = 55 \quad \left(\because \mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}\right) \tag{2.0.32}$$
$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{R} = \frac{55}{8} \tag{2.0.33}$$
$$\implies \mathbf{R} = \begin{pmatrix} \frac{55}{8} \\ 0 \end{pmatrix} + \mu\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.34}$$
$$\implies \mathbf{R} = \mathbf{r} + \mu\mathbf{m} \tag{2.0.35}$$
$$\text{where, } \mathbf{r} = \begin{pmatrix} \frac{55}{8} \\ 0 \end{pmatrix} \tag{2.0.36}$$
$$\text{and } \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.37}$$

   - We know,

$$\|\mathbf{r} + \mu\mathbf{m}\|^2 = 55 \tag{2.0.38}$$
$$(\mathbf{r} + \mu\mathbf{m})^T(\mathbf{r} + \mu\mathbf{m}) = 55 \tag{2.0.39}$$
$$\mu^2 = \frac{55 - \|\mathbf{r}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.40}$$
$$\mu = \pm 2.78 \tag{2.0.41}$$

Substitute μ value in (2.0.35) we get the coordinates of **R** and **S** as :

$$\mathbf{R} = \begin{pmatrix} 6.875 \\ 2.78 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 6.875 \\ -2.78 \end{pmatrix} \tag{2.0.42}$$

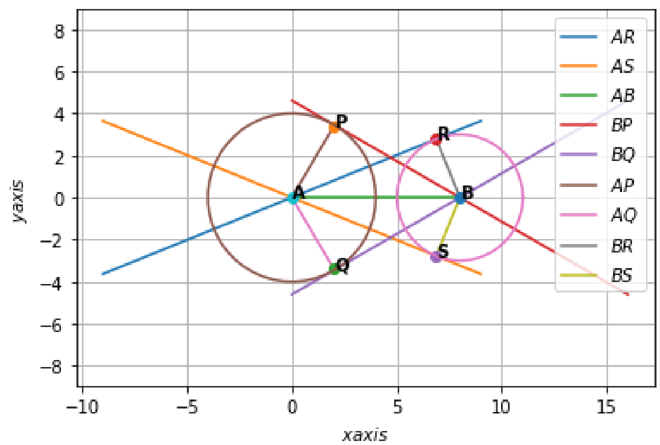

Fig. 2.1: Tangents from centres of Circle