Assignment-4

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_4/tree/master/codes

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_4

1 Question No. 2.17

Find the perpendicular distance of the following lines from the origin and angle between the perpendicular and positive x-axis.

a. $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{x} = -8$

b. $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 2$

c. $\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4$

2 Solution

1) All the given data can be tabularised in table 2.1:

	Line_1	Line_2	Line_3
c_1,c_2,c_3	8	-2	-4
$\mathbf{n}_1$,$\mathbf{n}_2$,$\mathbf{n}_3$	$\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

TABLE 2.1: Given Data

2) For finding the perpendicular distance of lines from origin:-
   - The formula for calculating perpendicular distance between the point and a given line is :
$$d = \frac{\left|c - \mathbf{n}^T \mathbf{A}\right|}{\|\mathbf{n}\|} \tag{2.0.1}$$
     where,$\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$ is the normal vector of line.
   - If we have to find distance from origin, then above formula get reduced to:
$$d = \frac{|c|}{\|\mathbf{n}\|} \tag{2.0.2}$$

3) For finding angle between the perpendicular and positive x-axis:-
   - If the normal vector $\mathbf{n}_{\text{L}}$ of a line is:-
$$\mathbf{n}_{\text{L}} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \tag{2.0.3}$$
   - Then the normal vector $\mathbf{n}_{\text{P}}$ of a line perpendicular to it is:
$$\mathbf{n}_{\text{p}} = \begin{pmatrix} -n_2 \\ n_1 \end{pmatrix} \tag{2.0.4}$$
   - Also, slope of perpendicular line,m_{P} can be written as:
$$m_{\text{p}} = \frac{-(-n_2)}{n_1} \tag{2.0.5}$$
   - If a line with slope m_{p} makes an angle θ with positive x-axis,then:
$$\implies \tan\theta = m_{\text{p}} \tag{2.0.6}$$
$$\implies \theta = \tan^{-1}(m_{\text{p}}) \tag{2.0.7}$$

4) For $Line_1$, $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{x} = -8$
   - We have:
$$c_1 = 8, \mathbf{n}_1 = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \tag{2.0.8}$$
   - Using (2.0.2) we get:
$$d_1 = \frac{|c_1|}{\|\mathbf{n}_1\|} \tag{2.0.9}$$
$$d_1 = \frac{8}{\sqrt{1+3}} \tag{2.0.10}$$
$$d_1 = \frac{8}{2} = 4 \text{ units} \tag{2.0.11}$$
   - Using (2.0.4),the normal vector $\mathbf{n}_{\text{p1}}$ of the line perpendicular to given line is:
$$\mathbf{n}_{\text{p1}} = \begin{pmatrix} \sqrt{3}, 1 \end{pmatrix} \tag{2.0.12}$$
   - Using (2.0.5) the slope of this perpendicular

line is given by:

$$m_{\mathrm{p}1} = \frac{-\sqrt{3}}{1} \quad (2.0.13)$$
$$m_{\mathrm{p}1} = -\sqrt{3} \quad (2.0.14)$$

- Using (2.0.7)the angle which perpendicular is making with positive x-axis is:

$$\theta_1 = \tan^{-1}(m_{\mathrm{p}_1}) \quad (2.0.15)$$
$$\theta_1 = \tan^{-1}(-\sqrt{3}) \quad (2.0.16)$$
$$\theta_1 = -60^\circ \quad (2.0.17)$$

5) For $Line_2$, $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 2$
   - We have:

$$c_2 = -2, \mathbf{n}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.18)$$

   - Using (2.0.2) we get:

$$d_2 = \frac{|c_2|}{\|\mathbf{n}_2\|} \quad (2.0.19)$$
$$d_2 = \frac{2}{\sqrt{1}} \quad (2.0.20)$$
$$d_2 = 2 \text{ units} \quad (2.0.21)$$

   - Using (2.0.4),the normal vector $\mathbf{n}_{\mathrm{p}2}$ of the line perpendicular to given line is:

$$\mathbf{n}_{\mathrm{p}2} = \begin{pmatrix} -1, 0 \end{pmatrix} \quad (2.0.22)$$

   - Using (2.0.5) the slope of this perpendicular line is given by:

$$m_{\mathrm{p}2} = \frac{-1}{0} \quad (2.0.23)$$
$$\implies m_{\mathrm{p}2} = \infty \quad (2.0.24)$$

   - Using (2.0.7)the angle which perpendicular is making with positive x-axis is:

$$\theta_2 = \tan^{-1}(m_{\mathrm{p}_2}) \quad (2.0.25)$$
$$\theta_2 = \tan^{-1}(\infty) \quad (2.0.26)$$
$$\theta_2 = 90^\circ \quad (2.0.27)$$

6) For $Line_3$,$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4$
   - We have:

$$c_3 = -4\mathbf{n}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.28)$$

   - Using (2.0.2) we get:

$$d_3 = \frac{|c_3|}{\|\mathbf{n}_3\|} \quad (2.0.29)$$
$$d_3 = \frac{4}{\sqrt{1+1}} \quad (2.0.30)$$
$$d_3 = \frac{4}{\sqrt{2}} \quad (2.0.31)$$
$$d_3 = \frac{4}{1.41} \quad (2.0.32)$$
$$d_3 = 2.828 \text{ units} \quad (2.0.33)$$

   - Using (2.0.4),the normal vector $\mathbf{n}_{\mathrm{p}3}$ of the line perpendicular to given line is:

$$\mathbf{n}_{\mathrm{p}3} = \begin{pmatrix} 1, 1 \end{pmatrix} \quad (2.0.34)$$

   - Using (2.0.5) the slope of this perpendicular line is given by:

$$m_{\mathrm{p}3} = \frac{-1}{1} \quad (2.0.35)$$
$$m_{\mathrm{p}3} = -1 \quad (2.0.36)$$

   - Using (2.0.7)the angle which perpendicular is making with positive x-axis is:

$$\theta_3 = \tan^{-1}(m_{\mathrm{p}_3}) \quad (2.0.37)$$
$$\theta_3 = \tan^{-1}(-1) \quad (2.0.38)$$
$$\theta_3 = -45^\circ \quad (2.0.39)$$

7) If,
   $\mathbf{d_n}$=Perpendicular distance of line from origin

   $\mathbf{m_{pn}}$=Slope of the perpendicular to the line

   θ_{n}= Angle of perpendicular with positive x-axis.

8) All the calculated data can be tabularised in table 2.2:

	Line_1	Line_2	Line_2
d_1,d_2,d_3	4	2	2.828
$\mathbf{m_{p1}}$,$\mathbf{m_{p2}}$,$\mathbf{m_{p3}}$	$-\sqrt{3}$	∞	-1
θ_1,θ_2,θ_3	-60°	90°	-45°

TABLE 2.2: Calculated Data

- The plots of all the three lines and there respective perpendiculars are as follows:-

(A) Plot of Line_1 :-

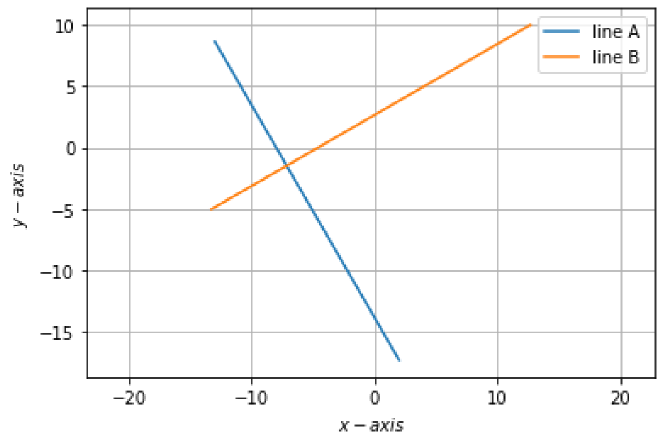

Fig. 2.1: Line_1 and it's perpendicular

(B) Plot of Line_2 :-

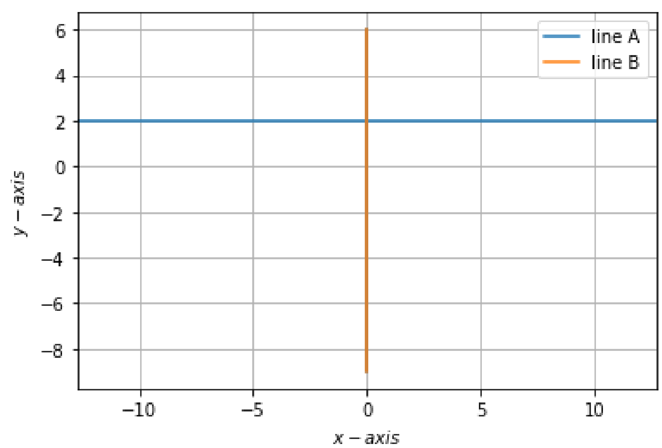

Fig. 2.2: Line_2 and it's perpendicular

(C) Plot of Line_3 :-

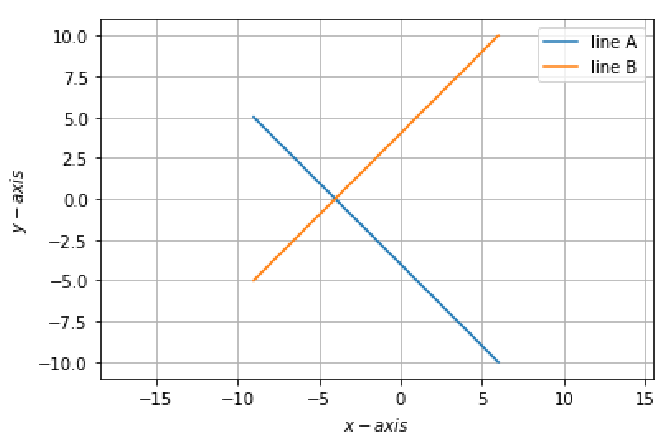

Fig. 2.3: Line_3 and it's perpendicular