Assignment-4

UNNATI GUPTA

Download all python codes from

https://github.com/unnatigupta2320/Assignment_4/tree/master/codes

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_4

1 Question No. 2.17

Find the perpendicular distance of the following lines from the origin and angle between the perpendicular and positive x-axis.

a. $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{x} = -8$
b. $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 2$
c. $\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4$

2 Solution

1) All the given data can be tabularised in table 2.1:

	Line_1	Line_2	Line_3
c_1, c_2, c_3	8	-2	-4
$\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$	$\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

TABLE 2.1: Given Data

2) For finding the perpendicular distance of lines from origin:-
   - The formula for calculating perpendicular distance between the point and a given line is :

$$d = \frac{\left|c - \mathbf{n}^T \mathbf{A}\right|}{\|\mathbf{n}\|} \tag{2.0.1}$$

   where, $\mathbf{n} = \begin{pmatrix} x_L \\ y_L \end{pmatrix}$ is the normal vector of line.
   - If we have to find distance from origin, then above formula get reduced to:

$$d = \frac{|c|}{\|\mathbf{n}\|} \tag{2.0.2}$$

3) For finding angle between the perpendicular and positive x-axis:-
   - Using inner products of vectors the angle θ between two lines is given by:

$$\cos\theta = \frac{\mathbf{n}_p{}^T \mathbf{n}_x}{\|\mathbf{n}_p\| \times \|\mathbf{n}_x\|} \tag{2.0.3}$$

$$\implies \theta = \cos^{-1} \frac{\mathbf{n}_p{}^T \mathbf{n}_x}{\|\mathbf{n}_p\| \times \|\mathbf{n}_x\|} \tag{2.0.4}$$

a. $Line_1$, $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{x} = -8$
   - We have:

$$c_1 = 8, \mathbf{n}_1 = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \tag{2.0.5}$$

   - Using (2.0.2) we get:

$$d_1 = \frac{|c_1|}{\|\mathbf{n}_1\|} \tag{2.0.6}$$

$$d_1 = \frac{8}{\sqrt{1+3}} \tag{2.0.7}$$

$$d_1 = \frac{8}{2} = 4 \text{ units} \tag{2.0.8}$$

   - The normal vector $\mathbf{n}_{p1}$ of the line perpendicular to given line is:

$$\mathbf{n}_{p1} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \tag{2.0.9}$$

   - Using (2.0.4) the angle which perpendicular is making with positive x-axis is:

$$\cos\theta_1 = \frac{\mathbf{n}_{p1}{}^T \mathbf{n}_x}{\|\mathbf{n}_{p1}\| \times \|\mathbf{n}_x\|} \tag{2.0.10}$$

$$\cos\theta_1 = \frac{\begin{pmatrix} \sqrt{3} & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix}}{\sqrt{4} \times \sqrt{1}} \tag{2.0.11}$$

$$\cos\theta_1 = \frac{-1}{2} \tag{2.0.12}$$

$$\implies \theta_1 = \cos^{-1}(-0.5) \tag{2.0.13}$$

$$\therefore \theta_1 = 120^\circ \tag{2.0.14}$$

b. *Line*$_2$, $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 2$

- We have:

$$c_2 = -2, \mathbf{n}_2 = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.15}$$

- Using (2.0.2) we get:

$$d_2 = \frac{|c_2|}{||\mathbf{n}_2||} \tag{2.0.16}$$

$$d_2 = \frac{2}{\sqrt{1}} \tag{2.0.17}$$

$$d_2 = 2 \text{ units} \tag{2.0.18}$$

- The normal vector $\mathbf{n}_{\text{p2}}$ of the line perpendicular to given line is:

$$\mathbf{n}_{\text{p2}} = \begin{pmatrix}-1\\0\end{pmatrix} \tag{2.0.19}$$

- Using (2.0.4) the angle which perpendicular is making with positive x-axis is:

$$\cos\theta_2 = \frac{\mathbf{n}_{\text{p2}}{}^T\mathbf{n}_{\text{x}}}{||\mathbf{n}_{\text{p2}}|| \times ||\mathbf{n}_{\text{x}}||} \tag{2.0.20}$$

$$\cos\theta_2 = \frac{\begin{pmatrix}-1 & 0\end{pmatrix}\begin{pmatrix}0\\-1\end{pmatrix}}{\sqrt{1} \times \sqrt{1}} \tag{2.0.21}$$

$$\cos\theta_2 = 0 \tag{2.0.22}$$

$$\implies \theta_2 = \cos^{-1}(0) \tag{2.0.23}$$

$$\therefore \theta_2 = 90^\circ \tag{2.0.24}$$

c. *Line*$_3$,$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 4$

- We have:

$$c_3 = -4, \mathbf{n}_3 = \begin{pmatrix}1\\-1\end{pmatrix} \tag{2.0.25}$$

- Using (2.0.2) we get:

$$d_3 = \frac{|c_3|}{||\mathbf{n}_3||} \tag{2.0.26}$$

$$d_3 = \frac{4}{\sqrt{1+1}} \tag{2.0.27}$$

$$d_3 = \frac{4}{\sqrt{2}} \tag{2.0.28}$$

$$d_3 = \frac{4}{1.41} \tag{2.0.29}$$

$$d_3 = 2.828 \text{ units} \tag{2.0.30}$$

- The normal vector $\mathbf{n}_{\text{p3}}$ of the line perpendicular to given line is:

$$\mathbf{n}_{\text{p3}} = \begin{pmatrix}1\\1\end{pmatrix} \tag{2.0.31}$$

- Using (2.0.4) the angle which perpendicular is making with positive x-axis is:

$$\cos\theta_3 = \frac{\mathbf{n}_{\text{p3}}{}^T\mathbf{n}_{\text{x}}}{||\mathbf{n}_{\text{p3}}|| \times ||\mathbf{n}_{\text{x}}||} \tag{2.0.32}$$

$$\cos\theta_3 = \frac{\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}0\\-1\end{pmatrix}}{\sqrt{2} \times \sqrt{1}} \tag{2.0.33}$$

$$\cos\theta_3 = \frac{-1}{\sqrt{2}} \tag{2.0.34}$$

$$\implies \theta_3 = \cos^{-1}\frac{-1}{\sqrt{2}} \tag{2.0.35}$$

$$\therefore \theta_3 = 135^\circ \tag{2.0.36}$$

4) If,
$\mathbf{d_n}$=Perpendicular distance of line from origin

$\mathbf{n_{pn}}$=Normal vector of the perpendicular to the line

θ_{n}= Angle of perpendicular with positive x-axis.

5) All the calculated data can be tabularised in table 2.2:

	Line$_1$	Line$_2$	Line$_2$
d$_1$,d$_2$,d$_3$	4	2	2.828
$\mathbf{n_{p1}},\mathbf{n_{p2}},\mathbf{n_{p3}}$	$\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$	$\begin{pmatrix}-1\\0\end{pmatrix}$	$\begin{pmatrix}1\\1\end{pmatrix}$
θ_1,θ_2,θ_3	120°	90°	135°

TABLE 2.2: Calculated Data

- The plots of all the three lines and there respective perpendiculars are as follows:-

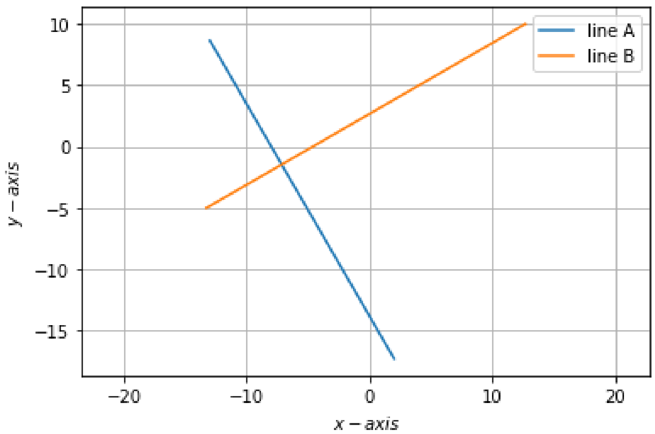

Fig. 2.1: Line_1 and it's perpendicular

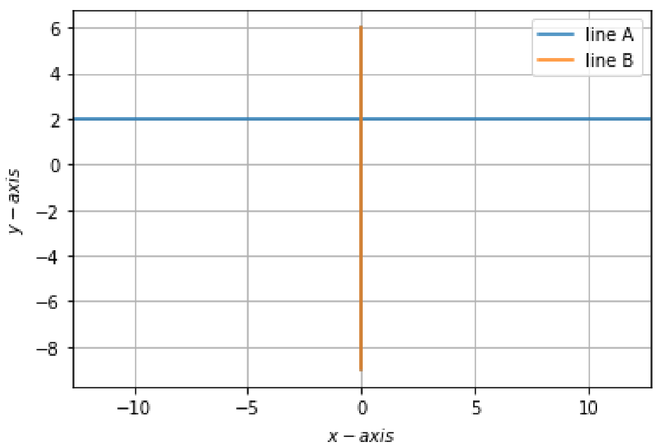

Fig. 2.2: Line_2 and it's perpendicular

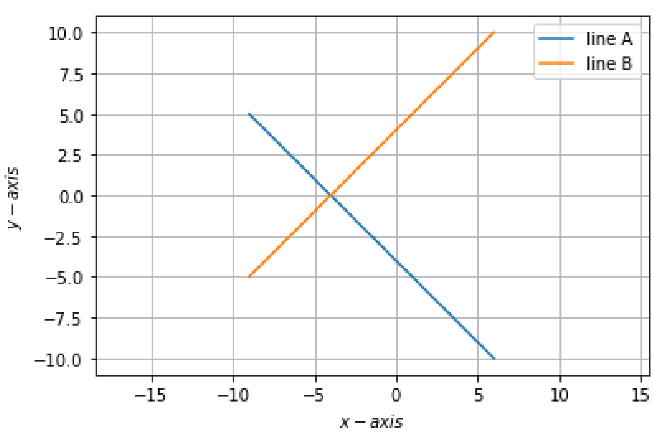

Fig. 2.3: Line_3 and it's perpendicular