Assignment-4

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_4/tree/master/CODES

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_4

1 Question No. 2.17

Find the perpendicular distance of the following lines from the origin and angle between the perpendicular and positive x-axis.

a. $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{x} = -8$
b. $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 2$
c. $\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4$

2 Solution

1) For finding the perpendicular distance of lines from origin:
   - Let a point $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$.
   - The perpendicular distance of a point $\mathbf{A}$ from a line ax+by+c=0 is given by:-

$$d = \frac{|a(x_1) + b(y_1) + c|}{||\mathbf{n}||} \tag{2.0.1}$$

$$\text{where, } \mathbf{n} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.2}$$

   - If the point is $\mathbf{O}= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, then

$$d = \frac{|a(0) + b(0) + c|}{||\mathbf{n}||} \tag{2.0.3}$$

$$d = \frac{|c|}{||\mathbf{n}||} \tag{2.0.4}$$

2) For finding angle between the perpendicular and positive x-axis:-
   - We have:

$$ax + by + c = 0 \tag{2.0.5}$$

$$by = -ax - c \tag{2.0.6}$$

$$y = \frac{-a}{b} - \frac{c}{b} \tag{2.0.7}$$

   - So,the slope,m_l can be written as:

$$m_l = \frac{-a}{b} \tag{2.0.8}$$

   - Let slope of the perpendicular line = m_p,then we know that:

$$m_l \times m_p = -1 \tag{2.0.9}$$

$$m_p = \frac{-1}{m_l} \tag{2.0.10}$$

   - Using (2.0.8) in above equation we get:

$$m_p = \frac{b}{a} \tag{2.0.11}$$

$$m_p = \tan\theta \tag{2.0.12}$$

$$\theta = \tan^{-1}(m_p) \tag{2.0.13}$$

3) For line, $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{x}$ =−8
   - We have:

$$c_1 = 8, \mathbf{n}_1 = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \tag{2.0.14}$$

   - Using (2.0.4) we get:

$$d_1 = \frac{|c|}{||\mathbf{n}||} \tag{2.0.15}$$

$$d_1 = \frac{8}{\sqrt{1+3}} \tag{2.0.16}$$

$$d_1 = \frac{8}{2} = 4 \text{ units} \tag{2.0.17}$$

   - Using (2.0.11)the slope of the line perpendicular to given line is:

$$m_{p1} = \frac{b}{a} \tag{2.0.18}$$

$$m_{p1} = \frac{-\sqrt{3}}{1} = -\sqrt{3} \tag{2.0.19}$$

- Also,Using (2.0.13) the angle between Perpendicular and Positive x-axis is:

$$\theta_1 = \tan^{-1}(m_2) \quad (2.0.20)$$
$$\theta_1 = \tan^{-1}(-\sqrt{3})(\because m_2 = -\sqrt{3}) \quad (2.0.21)$$
$$\theta_1 = -60^\circ \quad (2.0.22)$$

4) For line, $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 2$
   - We have:

$$c_2 = 2, \mathbf{n}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.23)$$

   - Using (2.0.4) we get:

$$d_2 = \frac{|c|}{||\mathbf{n}||} \quad (2.0.24)$$
$$d_2 = \frac{2}{\sqrt{1}} \quad (2.0.25)$$
$$d_2 = 2 \text{ units} \quad (2.0.26)$$

   - Using (2.0.11), the slope of the line perpendicular to given line is:

$$m_{p2} = \frac{b}{a} \quad (2.0.27)$$
$$\text{But as } a = 0 \implies m_{p2} = \infty \quad (2.0.28)$$

   - The angle between Perpendicular and Positive x-axis is:

$$\implies \theta_2 = \tan^{-1}(\infty) \quad (2.0.29)$$
$$\implies \theta_2 = 90^\circ \quad (2.0.30)$$

5) For line,$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4$
   - We have:

$$c_3 = 4, \mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.31)$$

   - Using (2.0.4) we get:

$$d_3 = \frac{|c|}{||\mathbf{n}||} \quad (2.0.32)$$
$$d_3 = \frac{4}{\sqrt{1+1}} \quad (2.0.33)$$
$$d_3 = \frac{4}{\sqrt{2}} \quad (2.0.34)$$
$$d_3 = \frac{4}{1.41} \quad (2.0.35)$$
$$d_3 = 2.828 \text{ units} \quad (2.0.36)$$

   - Using (2.0.11) the slope of the line perpendicular to given line is:

$$m_{p3} = \frac{b}{a} \quad (2.0.37)$$
$$m_{p3} = \frac{1}{-1} = -1 \quad (2.0.38)$$

   - Also,Using (2.0.13) the angle between perpendicular and positive x-axis is:

$$\theta_3 = \tan^{-1}(m_2) \quad (2.0.39)$$
$$\theta_3 = \tan^{-1}(-1)(\because m_2 = -1) \quad (2.0.40)$$
$$\theta_3 = -45^\circ \quad (2.0.41)$$

6) If,
   $\mathbf{d_n}$=Perpendicular distance of line from origin

   $\mathbf{m_{pn}}$=Slope of the perpendicular to the line

   θ_n= Angle of perpendicular with positive x-axis.

7) All the given and calculated data can be tabularised in table as 2.1:

	Line1	Line2	Line3
c_1,c_2,c_3	8	2	4
$\mathbf{n}_1$,$\mathbf{n}_2$,$\mathbf{n}_3$	$\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
d_1,d_2,d_3	4	2	2.828
$\mathbf{m_{p1}}$,$\mathbf{m_{p2}}$,$\mathbf{m_{p3}}$	$-\sqrt{3}$	∞	-1
θ_1,θ_2,θ_3	-60°	90°	-45°

TABLE 2.1: Data:-Given and Calculated

a. Plot of Line1 :-

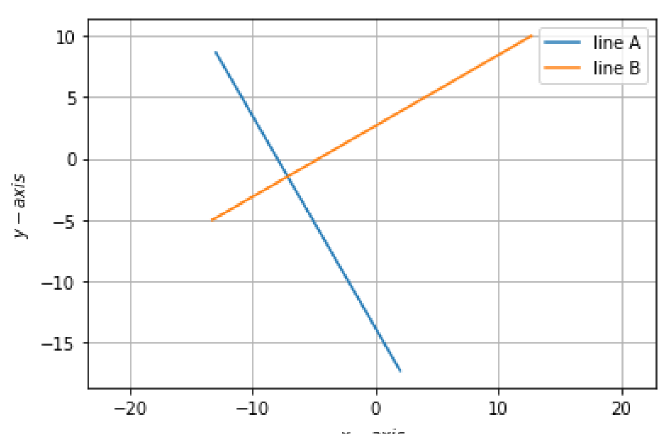

Fig. 2.1: Line1 and it's perpendicular

b. Plot of Line2 :-

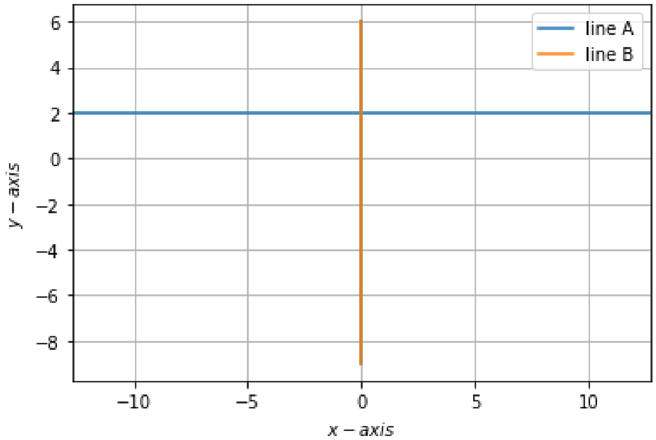

Fig. 2.2: Line2 and it's perpendicular

c. Plot of Line3 :-

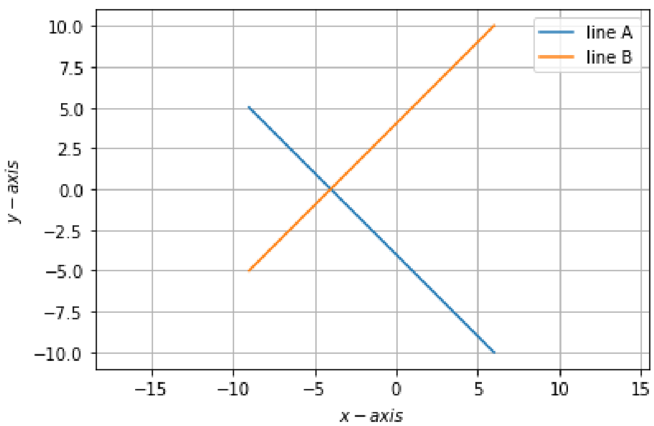

Fig. 2.3: Line3 and it's perpendicular