Assignment-4

UNNATI GUPTA

Download all python codes from

https://github.com/unnatigupta2320/Assignment_4/tree/master/codes

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_4

1 Question No. 2.17

Find the perpendicular distance of the following lines from the origin and angle between the perpendicular and positive x-axis.

a. $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = -8$

b. $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 2$

c. $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 4$

2 Solution

1) All the given data can be tabularised in table 2.1:

	Line_1	Line_2	Line_3
c_1,c_2,c_3	8	-2	-4
$\mathbf{n}_1$,$\mathbf{n}_2$,$\mathbf{n}_3$	$\begin{pmatrix}1\\-\sqrt{3}\end{pmatrix}$	$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}1\\-1\end{pmatrix}$

TABLE 2.1: Given Data

2) For finding the perpendicular distance of lines from origin:-
   - The formula for calculating perpendicular distance between the point and a given line is :
   $$d = \frac{\left|c - \mathbf{n}^T\mathbf{A}\right|}{\|\mathbf{n}\|} \tag{2.0.1}$$
   where,$\mathbf{n}$ is the normal vector of line.
   - If we have to find distance from origin, then above formula get reduced to:
   $$d = \frac{|c|}{\|\mathbf{n}\|} \tag{2.0.2}$$

3) For finding angle between the perpendicular and positive x-axis:-
   - If $\mathbf{e}_1$=$\begin{pmatrix}1\\0\end{pmatrix}$ is the direction vector of x-axis then the angle θ between line and x-axis,is given using inner products of vectors as:
   $$\cos\theta = \frac{\mathbf{n}^T\mathbf{e}_1}{\|\mathbf{n}\| \times \|\mathbf{e}_1\|} \tag{2.0.3}$$
   $$\implies \theta = \cos^{-1}\frac{\mathbf{n}^T\mathbf{e}_1}{\|\mathbf{n}\| \times \|\mathbf{e}_1\|} \tag{2.0.4}$$

a. $Line_1$, $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x}$ =-8
   - We have:
   $$c_1 = 8, \mathbf{n}_1 = \begin{pmatrix}1\\-\sqrt{3}\end{pmatrix} \tag{2.0.5}$$
   - Using (2.0.2) we get:
   $$d = \frac{|c_1|}{\|\mathbf{n}_1\|} \tag{2.0.6}$$
   $$d = \frac{8}{\sqrt{1+3}} \tag{2.0.7}$$
   $$d = \frac{8}{2} = 4 \text{ units} \tag{2.0.8}$$
   - The direction vector $\mathbf{n}$ of the line perpendicular to given line is:
   $$\mathbf{n} = \begin{pmatrix}-1\\\sqrt{3}\end{pmatrix} \tag{2.0.9}$$
   - Using (2.0.4) the angle which perpendicular is making with positive x-axis is:
   $$\cos\theta = \frac{\mathbf{n}^T\mathbf{e}_1}{\|\mathbf{n}\| \times \|\mathbf{e}_1\|} \tag{2.0.10}$$
   $$\cos\theta = \frac{\begin{pmatrix}-1 & \sqrt{3}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}}{\sqrt{4} \times \sqrt{1}} \tag{2.0.11}$$
   $$\cos\theta = \frac{-1}{2} \tag{2.0.12}$$
   $$\implies \theta = \cos^{-1}(-0.5) \tag{2.0.13}$$
   $$\therefore \theta = 120^\circ \tag{2.0.14}$$

b. $Line_2$, $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 2$

- We have:

$$c_2 = -2, \mathbf{n}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.15)$$

- Using (2.0.2) we get:

$$d = \frac{|c_2|}{||\mathbf{n}_2||} \quad (2.0.16)$$
$$d = \frac{2}{\sqrt{1}} \quad (2.0.17)$$
$$d = 2 \text{ units} \quad (2.0.18)$$

- The direction vector $\mathbf{n}$ of the line perpendicular to given line is:

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.19)$$

- Using (2.0.4) the angle which perpendicular is making with positive x-axis is:

$$\cos\theta = \frac{\mathbf{n}^T \mathbf{e}_1}{||\mathbf{n}|| \times ||\mathbf{e}_1||} \quad (2.0.20)$$

$$\cos\theta = \frac{\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{\sqrt{1} \times \sqrt{1}} \quad (2.0.21)$$
$$\cos\theta = 0 \quad (2.0.22)$$
$$\implies \theta = \cos^{-1}(0) \quad (2.0.23)$$
$$\therefore \theta = 90^\circ \quad (2.0.24)$$

c. $Line_3$,$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4$

- We have:

$$c_3 = -4, \mathbf{n}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.25)$$

- Using (2.0.2) we get:

$$d = \frac{|c_3|}{||\mathbf{n}_3||} \quad (2.0.26)$$
$$d = \frac{4}{\sqrt{1+1}} \quad (2.0.27)$$
$$d = \frac{4}{\sqrt{2}} \quad (2.0.28)$$
$$d = \frac{4}{1.41} \quad (2.0.29)$$
$$d = 2.828 \text{ units} \quad (2.0.30)$$

- The direction vector $\mathbf{n}$ of the line perpendicular to given line is:

$$\mathbf{n} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (2.0.31)$$

- Using (2.0.4) the angle which perpendicular is making with positive x-axis is:

$$\cos\theta = \frac{\mathbf{n}^T \mathbf{e}_1}{||\mathbf{n}|| \times ||\mathbf{e}_1||} \quad (2.0.32)$$

$$\cos\theta = \frac{\begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{\sqrt{2} \times \sqrt{1}} \quad (2.0.33)$$
$$\cos\theta = \frac{-1}{\sqrt{2}} \quad (2.0.34)$$
$$\implies \theta = \cos^{-1}\frac{-1}{\sqrt{2}} \quad (2.0.35)$$
$$\therefore \theta = 135^\circ \quad (2.0.36)$$

4) If,
$\mathbf{d_n}$=Perpendicular distance of line from origin

θ_n= Angle of perpendicular with positive x-axis.
5) All the calculated data can be tabularised in table 2.2:

	$Line_1$	$Line_2$	$Line_2$
d_1,d_2,d_3	4	2	2.828
θ_1,θ_2,θ_3	120°	90°	135°

TABLE 2.2: Calculated Data

- The plots of all the three lines and there respective perpendiculars are as follows:-

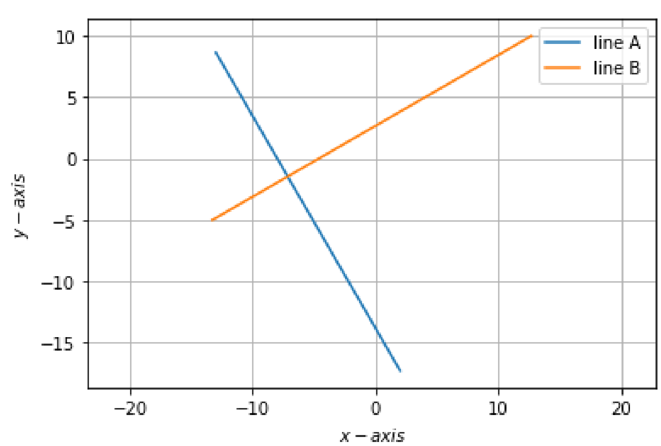

Fig. 2.1: $Line_1$ and it's perpendicular

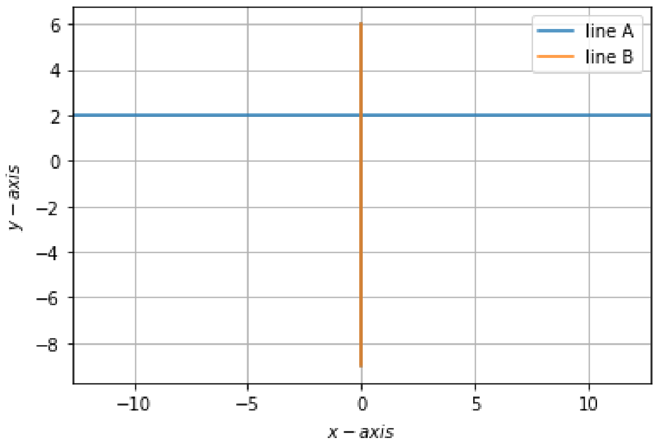

Fig. 2.2: Line_2 and it's perpendicular

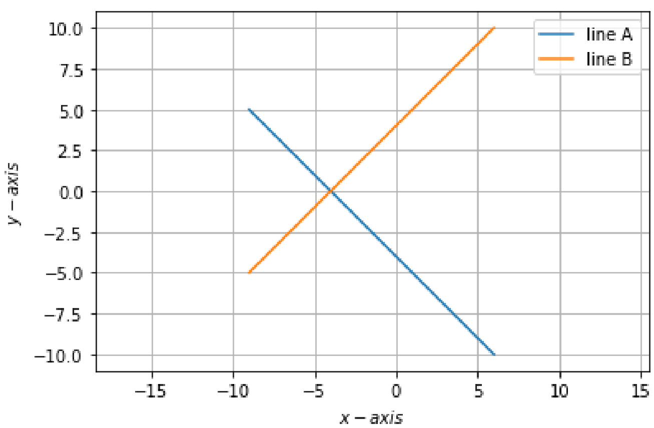

Fig. 2.3: Line_3 and it's perpendicular