Assignment-4

UNNATI GUPTA

Download all python codes from

https://github.com/unnatigupta2320/Assignment_4/tree/master/codes

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_4

1 Question No. 2.17

Find the perpendicular distance of the following lines from the origin and angle between the perpendicular and positive x-axis.

a. $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = -8$
b. $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 2$
c. $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 4$

2 Solution

1) All the given data can be tabularised in table 2.1:

	Line_1	Line_2	Line_3
c_1,c_2,c_3	8	-2	-4
$\mathbf{n}_1$,$\mathbf{n}_2$,$\mathbf{n}_3$	$\begin{pmatrix}1\\-\sqrt{3}\end{pmatrix}$	$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}1\\-1\end{pmatrix}$

TABLE 2.1: Given Data

2) For finding the perpendicular distance of lines from origin:-
   - The formula for calculating perpendicular distance between the point and a given line is :
   $$d = \frac{\left|c - \mathbf{n}^T\mathbf{A}\right|}{\|\mathbf{n}\|} \tag{2.0.1}$$
   - If we have to find distance from origin, then above formula get reduced to:
   $$d = \frac{|c|}{\|\mathbf{n}\|} \tag{2.0.2}$$
3) For finding angle between the perpendicular and positive x-axis:-
   - If the normal vector of line is $\mathbf{n}$,then:
   $$\mathbf{n} = \begin{pmatrix}1\\a_{\text{n}}\end{pmatrix} \text{ or,} \tag{2.0.3}$$
   $$\mathbf{n} = \begin{pmatrix}1\\\tan\theta\end{pmatrix} \tag{2.0.4}$$
   - where θ = the angle which perpendicular makes with positive x-axis,
   $$\implies \tan\theta = a_{\text{n}} \tag{2.0.5}$$
   $$\implies \theta = \tan^{-1}(a_{\text{n}}) \tag{2.0.6}$$
4) For $Line_1$, $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = -8$
   - We have:
   $$c_1 = 8 \tag{2.0.7}$$
   $$\mathbf{n}_1 = \begin{pmatrix}1\\-\sqrt{3}\end{pmatrix} \tag{2.0.8}$$
   - Using (2.0.2) we get:
   $$d_1 = \frac{|c_1|}{\|\mathbf{n}_1\|} \tag{2.0.9}$$
   $$d_1 = \frac{8}{\sqrt{1+3}} \tag{2.0.10}$$
   $$d_1 = \frac{8}{2} = 4 \text{ units} \tag{2.0.11}$$
   - On comparing (2.0.3) and(2.0.8) we get:
   $$a_1 = -\sqrt{3} \tag{2.0.12}$$
   - Putting (2.0.12) in (2.0.6) the angle which perpendicular is making with positive x-axis is:
   $$\theta_1 = \tan^{-1}(a_1) \tag{2.0.13}$$
   $$\theta_1 = \tan^{-1}(-\sqrt{3}) \tag{2.0.14}$$
   $$\theta_1 = -60^\circ \tag{2.0.15}$$
5) For $Line_2$, $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 2$

- We have:

$$c_2 = -2 \tag{2.0.16}$$

$$\mathbf{n}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.17}$$

- Using (2.0.2) we get:

$$d_2 = \frac{|c_2|}{\|\mathbf{n}_2\|} \tag{2.0.18}$$

$$d_2 = \frac{2}{\sqrt{1}} \tag{2.0.19}$$

$$d_2 = 2 \text{ units} \tag{2.0.20}$$

- Here,the line$_2$ is parallel to x-axis. So,the angle it's perpendicular is making with positive x-axis is:

$$\therefore \theta_2 = 90^\circ \tag{2.0.21}$$

6) For $Line_3$,$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4$
   - We have:

$$c_3 = -4 \tag{2.0.22}$$

$$\mathbf{n}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.23}$$

   - Using (2.0.2) we get:

$$d_3 = \frac{|c_3|}{\|\mathbf{n}_3\|} \tag{2.0.24}$$

$$d_3 = \frac{4}{\sqrt{1+1}} \tag{2.0.25}$$

$$d_3 = \frac{4}{\sqrt{2}} \tag{2.0.26}$$

$$d_3 = \frac{4}{1.41} \tag{2.0.27}$$

$$d_3 = 2.828 \text{ units} \tag{2.0.28}$$

   - On comparing (2.0.3) and(2.0.23) we get:

$$a_3 = -1 \tag{2.0.29}$$

   - Putting (2.0.29) in (2.0.6) the angle which perpendicular is making with positive x-axis is:

$$\theta_3 = \tan^{-1}(a_3) \tag{2.0.30}$$

$$\theta_3 = \tan^{-1}(-1) \tag{2.0.31}$$

$$\theta_3 = -45^\circ \tag{2.0.32}$$

7) If, $\mathbf{d_n}$=Perpendicular distance of line from the origin
   θ_n= Angle of perpendicular with positive x-axis.
8) Then, All the calculated data can be tabularised in table 2.2:

	Line$_1$	Line$_2$	Line$_3$
d_1,d_2,d_3	4	2	2.828
θ_1,θ_2,θ_3	-60°	90°	-45°

TABLE 2.2: Calculated Data

9) The plots of all the three lines and there respective perpendiculars are as follows:-
   - Plot of Line$_1$:-

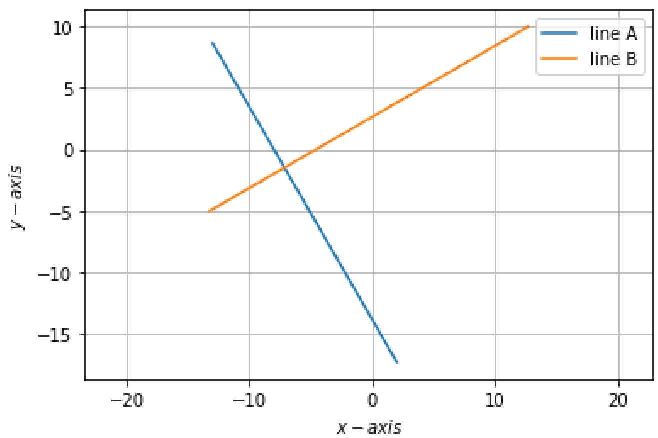

Fig. 2.1: Line$_1$ and it's perpendicular

   - Plot of Line$_2$:-

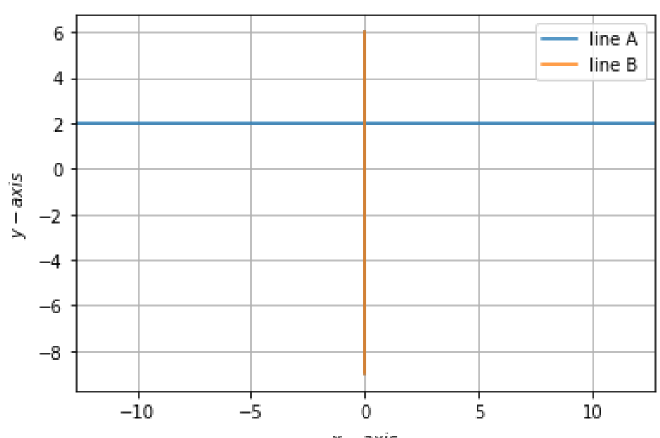

Fig. 2.2: Line$_2$ and it's perpendicular

   - Plot of Line$_3$:-

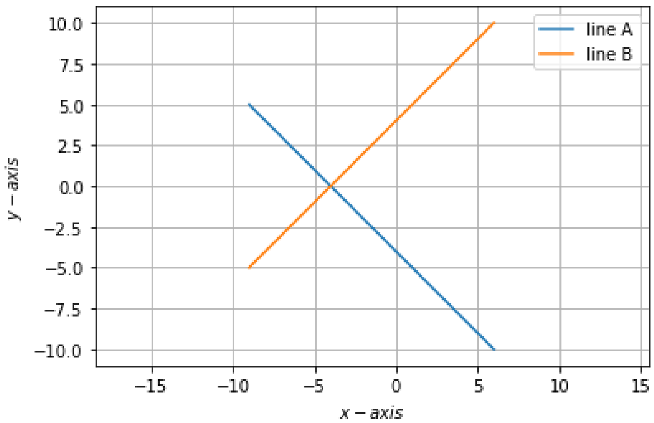

Fig. 2.3: Line_3 and it's perpendicular