Assignment-4

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1 QUESTION No. 2.17

Find the perpendicular distance of the following lines from the origin and angle between the perpendicular and positive x-axis.

a. $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{x} = -8$
b. $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 2$
c. $\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4$

2 SOLUTION

1) For finding the perpendicular distance of lines from origin:

- Let a point $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$.
- The perpendicular distance of a point $\mathbf{A}$ from a line ax+by+c=0 is given by:-

$$d = \frac{|a(x_1) + b(y_1) + c|}{\|\mathbf{n}\|} \qquad (2.0.1)$$

$$\text{where, } \mathbf{n} = \begin{pmatrix} a \\ b \end{pmatrix} \qquad (2.0.2)$$

- If the point is $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, then

$$d = \frac{|a(0) + b(0) + c|}{\|\mathbf{n}\|} \qquad (2.0.3)$$

$$d = \frac{|c|}{\|\mathbf{n}\|} \qquad (2.0.4)$$

2) For finding angle between the perpendicular and positive x-axis:-

- We have:

$$ax + by + c = 0 \qquad (2.0.5)$$

$$by = -ax - c \qquad (2.0.6)$$

$$y = \frac{-a}{b} - \frac{c}{b} \qquad (2.0.7)$$

- So, the slope, m_1 can be written as:

$$m_1 = \frac{-a}{b} \qquad (2.0.8)$$

- Let slope of the perpendicular line = m_p, then we know that:

$$m_1 \times m_p = -1 \qquad (2.0.9)$$

$$m_p = \frac{-1}{m_1} \qquad (2.0.10)$$

- Using (2.0.8) in above equation we get:

$$m_p = \frac{b}{a} \qquad (2.0.11)$$

$$m_p = \tan\theta \qquad (2.0.12)$$

$$\theta = \tan^{-1}(m_p) \qquad (2.0.13)$$

3) For line, $\begin{pmatrix} 1 & -\sqrt{3} \end{pmatrix} \mathbf{x} = -8$
- We have:

$$c_1 = 8, \mathbf{n}_1 = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \qquad (2.0.14)$$

- Using (2.0.4) we get:

$$d_1 = \frac{|c|}{\|\mathbf{n}\|} \qquad (2.0.15)$$

$$d_1 = \frac{8}{\sqrt{1+3}} \qquad (2.0.16)$$

$$d_1 = \frac{8}{2} = 4 \text{ units} \qquad (2.0.17)$$

- Using (2.0.11) the slope of the line perpendicular to given line is:

$$m_{p1} = \frac{b}{a} \qquad (2.0.18)$$

$$m_{p1} = \frac{-\sqrt{3}}{1} = -\sqrt{3} \qquad (2.0.19)$$

- Also,Using (2.0.13) the angle between Perpendicular and Positive x-axis is:

$$\theta_1 = \tan^{-1}(m_2) \quad (2.0.20)$$

$$\theta_1 = \tan^{-1}(-\sqrt{3})(\because m_2 = -\sqrt{3}) \quad (2.0.21)$$

$$\theta_1 = -60° \quad (2.0.22)$$

4) For line, $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 2$

- We have:

$$c_2 = 2, \mathbf{n}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.23)$$

- Using (2.0.4) we get:

$$d_2 = \frac{|c|}{\|\mathbf{n}\|} \quad (2.0.24)$$

$$d_2 = \frac{2}{\sqrt{1}} \quad (2.0.25)$$

$$d_2 = 2 \text{ units} \quad (2.0.26)$$

- Using (2.0.11), the slope of the line perpendicular to given line is:

$$m_{p2} = \frac{b}{a} \quad (2.0.27)$$

But as $a = 0 \implies m_{p2} = \infty \quad (2.0.28)$

- The angle between Perpendicular and Positive x-axis is:

$$\implies \theta_2 = \tan^{-1}(\infty) \quad (2.0.29)$$

$$\implies \theta_2 = 90° \quad (2.0.30)$$

5) For line, $\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4$

- We have:

$$c_3 = 4, \mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.31)$$

- Using (2.0.4) we get:

$$d_3 = \frac{|c|}{\|\mathbf{n}\|} \quad (2.0.32)$$

$$d_3 = \frac{4}{\sqrt{1+1}} \quad (2.0.33)$$

$$d_3 = \frac{4}{\sqrt{2}} \quad (2.0.34)$$

$$d_3 = \frac{4}{1.41} \quad (2.0.35)$$

$$d_3 = 2.828 \text{ units} \quad (2.0.36)$$

- Using (2.0.11) the slope of the line perpendicular to given line is:

$$m_{p3} = \frac{b}{a} \quad (2.0.37)$$

$$m_{p3} = \frac{1}{-1} = -1 \quad (2.0.38)$$

- Also,Using (2.0.13) the angle between perpendicular and positive x-axis is:

$$\theta_3 = \tan^{-1}(m_2) \quad (2.0.39)$$

$$\theta_3 = \tan^{-1}(-1)(\because m_2 = -1) \quad (2.0.40)$$

$$\theta_3 = -45° \quad (2.0.41)$$

6) If,

d_n=Perpendicular distance of line from origin

m_{pn}=Slope of the perpendicular to the line

θ_n= Angle of perpendicular with positive x-axis.

7) All the given and calculated data can be tabularised in table as 2.1:

	Line1	Line2	Line3
c_1,c_2,c_3	8	2	4
$\mathbf{n}_1,\mathbf{n}_2,\mathbf{n}_3$	$\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
d_1,d_2,d_3	4	2	2.828
m_{p1},m_{p2},m_{p3}	$-\sqrt{3}$	∞	-1
$\theta_1,\theta_2,\theta_3$	$-60°$	$90°$	$-45°$

TABLE 2.1: Data:-Given and Calculated
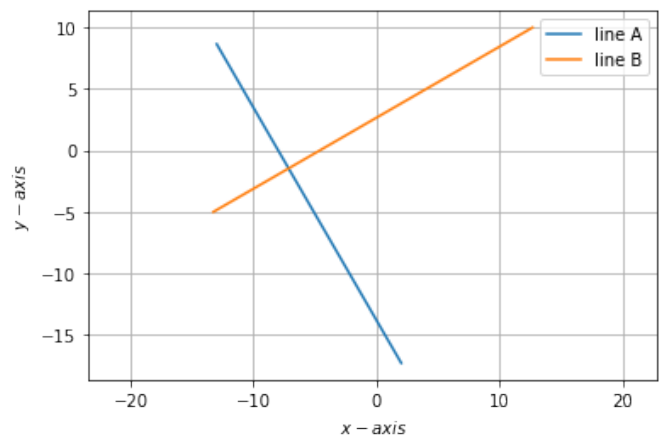
a. Plot of Line1 :-

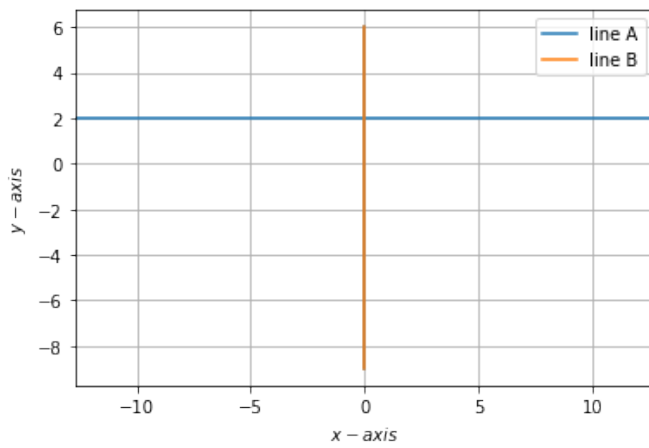Fig. 2.1: Line1 and it's perpendicular

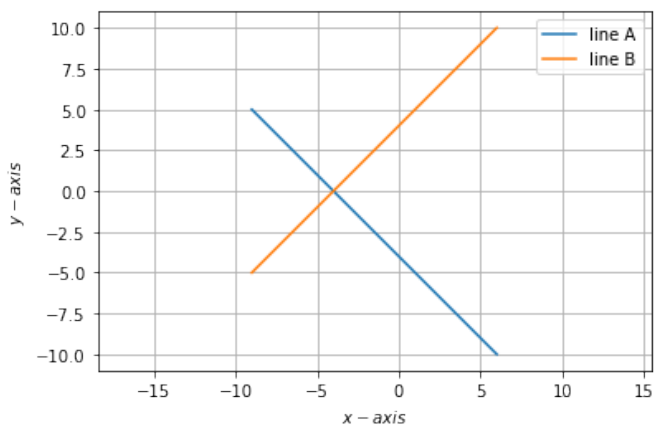b. Plot of Line2 :-

Fig. 2.2: Line2 and it's perpendicular

c. Plot of Line3 :-

Fig. 2.3: Line3 and it's perpendicular