

Assignment-4

UNNATI GUPTA

Download all python codes from

https://github.com/unnatigupta2320/Assignment_4/tree/master/codes

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_4

1 QUESTION No. 2.17

Find the perpendicular distance of the following lines from the origin and angle between the perpendicular and positive x-axis.

- $(1 - \sqrt{3})x = -8$
- $(0 \ 1)x = 2$
- $(1 - 1)x = 4$

2 SOLUTION

- All the given data can be tabularised in table 2.1:

	Line ₁	Line ₂	Line ₃
c_1, c_2, c_3	8	-2	-4
$\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$	$\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

TABLE 2.1: Given Data

- For finding the perpendicular distance of lines from origin:-

- The formula for calculating perpendicular distance between the point and a given line is :

$$d = \frac{|c - \mathbf{n}^T \mathbf{A}|}{\|\mathbf{n}\|} \quad (2.0.1)$$

- If we have to find distance from origin, then above formula get reduced to:

$$d = \frac{|c|}{\|\mathbf{n}\|} \quad (2.0.2)$$

- For finding angle between the perpendicular and positive x-axis:-

- If the normal vector of line is \mathbf{n} , then:

$$\mathbf{n} = \begin{pmatrix} 1 \\ a_n \end{pmatrix} \text{ or,} \quad (2.0.3)$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ \tan \theta \end{pmatrix} \quad (2.0.4)$$

- where θ = the angle which perpendicular makes with positive x-axis,

$$\Rightarrow \tan \theta = a_n \quad (2.0.5)$$

$$\Rightarrow \theta = \tan^{-1}(a_n) \quad (2.0.6)$$

- For *Line*₁, $(1 - \sqrt{3})x = -8$

- We have:

$$c_1 = 8 \quad (2.0.7)$$

$$\mathbf{n}_1 = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \quad (2.0.8)$$

- Using (2.0.2) we get:

$$d_1 = \frac{|c_1|}{\|\mathbf{n}_1\|} \quad (2.0.9)$$

$$d_1 = \frac{8}{\sqrt{1+3}} \quad (2.0.10)$$

$$d_1 = \frac{8}{2} = 4 \text{ units} \quad (2.0.11)$$

- On comparing (2.0.3) and (2.0.8) we get:

$$a_1 = -\sqrt{3} \quad (2.0.12)$$

- Putting (2.0.12) in (2.0.6) the angle which perpendicular is making with positive x-axis is:

$$\theta_1 = \tan^{-1}(a_1) \quad (2.0.13)$$

$$\theta_1 = \tan^{-1}(-\sqrt{3}) \quad (2.0.14)$$

$$\theta_1 = -60^\circ \quad (2.0.15)$$

- For *Line*₂, $(0 \ 1)x = 2$

- We have:

$$c_2 = -2 \quad (2.0.16)$$

$$\mathbf{n}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.17)$$

- Using (2.0.2) we get:

$$d_2 = \frac{|c_2|}{\|\mathbf{n}_2\|} \quad (2.0.18)$$

$$d_2 = \frac{2}{\sqrt{1}} \quad (2.0.19)$$

$$d_2 = 2 \text{ units} \quad (2.0.20)$$

- Here, the line₂ is parallel to x-axis.
So, the angle it's perpendicular is making with positive x-axis is:

$$\therefore \theta_2 = 90^\circ \quad (2.0.21)$$

6) For Line₃, $(1 \ -1)\mathbf{x} = 4$

- We have:

$$c_3 = -4 \quad (2.0.22)$$

$$\mathbf{n}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.23)$$

- Using (2.0.2) we get:

$$d_3 = \frac{|c_3|}{\|\mathbf{n}_3\|} \quad (2.0.24)$$

$$d_3 = \frac{4}{\sqrt{1+1}} \quad (2.0.25)$$

$$d_3 = \frac{4}{\sqrt{2}} \quad (2.0.26)$$

$$d_3 = \frac{4}{1.41} \quad (2.0.27)$$

$$d_3 = 2.828 \text{ units} \quad (2.0.28)$$

- On comparing (2.0.3) and (2.0.23) we get:

$$a_3 = -1 \quad (2.0.29)$$

- Putting (2.0.29) in (2.0.6) the angle which perpendicular is making with positive x-axis is:

$$\theta_3 = \tan^{-1}(a_3) \quad (2.0.30)$$

$$\theta_3 = \tan^{-1}(-1) \quad (2.0.31)$$

$$\theta_1 = -45^\circ \quad (2.0.32)$$

7) If, \mathbf{d}_n = Perpendicular distance of line from the origin

\mathbf{m}_{pn} = Slope of the perpendicular to the line
 θ_n = Angle of perpendicular with positive x-axis.

8) Then, All the calculated data can be tabularised in table 2.2:

	Line ₁	Line ₂	Line ₃
d_1, d_2, d_3	4	2	2.828
$\mathbf{m}_{p1}, \mathbf{m}_{p2}, \mathbf{m}_{p3}$	$-\sqrt{3}$	∞	-1
$\theta_1, \theta_2, \theta_3$	-60°	90°	-45°

TABLE 2.2: Calculated Data

9) The plots of all the three lines and there respective perpendiculars are as follows:-

- Plot of Line₁ :-

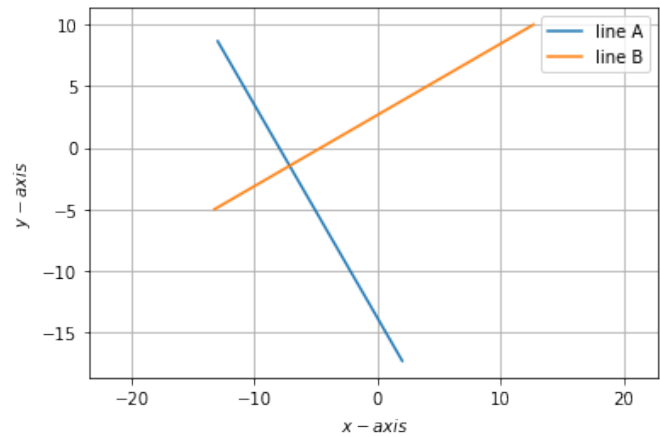


Fig. 2.1: Line₁ and it's perpendicular

- Plot of Line₂ :-

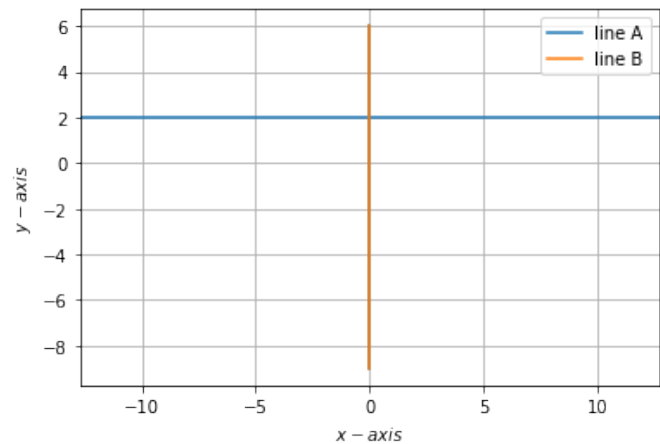


Fig. 2.2: Line₂ and it's perpendicular

- Plot of Line₃ :-

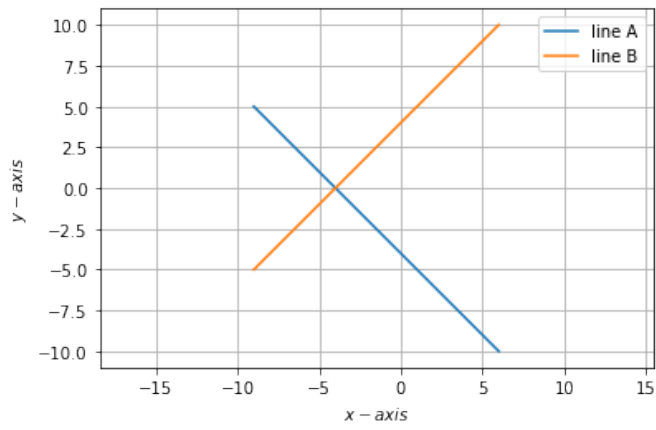


Fig. 2.3: Line₃ and it's perpendicular