Assignment-4

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_4/tree/master/codes

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_4

1 Question No. 2.17

Find the perpendicular distance of the following lines from the origin and angle between the perpendicular and positive x-axis.

a. $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = -8$
b. $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 2$
c. $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 4$

2 Solution

1) All the given data can be tabularised in table 2.1:

	Line_1	Line_2	Line_3
c_1,c_2,c_3	8	-2	-4
$\mathbf{n}_1$,$\mathbf{n}_2$,$\mathbf{n}_3$	$\begin{pmatrix}1\\-\sqrt{3}\end{pmatrix}$	$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}1\\-1\end{pmatrix}$

TABLE 2.1: Given Data

2) For finding the perpendicular distance of lines from origin:-
   - The formula for calculating perpendicular distance between the point and a given line is :
$$d = \frac{\left|c - \mathbf{n}^T\mathbf{A}\right|}{\|\mathbf{n}\|} \tag{2.0.1}$$
     where,$\mathbf{n} = \begin{pmatrix}x_L\\y_L\end{pmatrix}$ is the normal vector of line.
   - If we have to find distance from origin, then above formula get reduced to:
$$d = \frac{|c|}{\|\mathbf{n}\|} \tag{2.0.2}$$

3) For finding angle between the perpendicular and positive x-axis:-
   - If the normal vector $\mathbf{n}_L$ of a line is:-
$$\mathbf{n}_L = \begin{pmatrix}x_L\\y_L\end{pmatrix} \tag{2.0.3}$$
   - Then the normal vector $\mathbf{n}_P$ of a line perpendicular to it is:
$$\mathbf{n}_p = \begin{pmatrix}-y_L\\x_L\end{pmatrix} \tag{2.0.4}$$
   - Also, slope of perpendicular line,m_P can be written as:
$$m_p = \frac{-(-y_L)}{x_L} \tag{2.0.5}$$
$$m_p = \frac{y_L}{x_L} \tag{2.0.6}$$
   - If a line with slope m_p makes an angle θ with positive x-axis,then:
$$\implies \tan\theta = m_p \tag{2.0.7}$$
$$\implies \theta = \tan^{-1}(m_p) \tag{2.0.8}$$

4) For $Line_1$, $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = -8$
   - We have:
$$c_1 = 8, \mathbf{n}_1 = \begin{pmatrix}1\\-\sqrt{3}\end{pmatrix} \tag{2.0.9}$$
   - Using (2.0.2) we get:
$$d_1 = \frac{|c_1|}{\|\mathbf{n}_1\|} \tag{2.0.10}$$
$$d_1 = \frac{8}{\sqrt{1+3}} \tag{2.0.11}$$
$$d_1 = \frac{8}{2} = 4 \text{ units} \tag{2.0.12}$$
   - Using (2.0.4),the normal vector $\mathbf{n}_{p1}$ of the line perpendicular to given line is:
$$\mathbf{n}_{p1} = \begin{pmatrix}\sqrt{3}\\1\end{pmatrix} \tag{2.0.13}$$
   - Using (2.0.6) the slope of this perpendicular

line is given by:

$$m_{p1} = \frac{-\sqrt{3}}{1} \tag{2.0.14}$$

$$m_{p1} = -\sqrt{3} \tag{2.0.15}$$

- Using (2.0.8)the angle which perpendicular is making with positive x-axis is:

$$\theta_1 = \tan^{-1}(m_{p_1}) \tag{2.0.16}$$

$$\theta_1 = \tan^{-1}(-\sqrt{3}) \tag{2.0.17}$$

$$\theta_1 = -60^\circ \tag{2.0.18}$$

5) For $Line_2$, $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 2$
   - We have:

$$c_2 = -2, \mathbf{n}_2 = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.19}$$

   - Using (2.0.2) we get:

$$d_2 = \frac{|c_2|}{||\mathbf{n}_2||} \tag{2.0.20}$$

$$d_2 = \frac{2}{\sqrt{1}} \tag{2.0.21}$$

$$d_2 = 2 \text{ units} \tag{2.0.22}$$

   - Using (2.0.4),the normal vector $\mathbf{n}_{p2}$ of the line perpendicular to given line is:

$$\mathbf{n}_{p2} = \begin{pmatrix}-1\\0\end{pmatrix} \tag{2.0.23}$$

   - Using (2.0.6) the slope of this perpendicular line is given by:

$$m_{p2} = \frac{-1}{0} \tag{2.0.24}$$

$$\implies m_{p2} = \infty \tag{2.0.25}$$

   - Using (2.0.8)the angle which perpendicular is making with positive x-axis is:

$$\theta_2 = \tan^{-1}(m_{p_2}) \tag{2.0.26}$$

$$\theta_2 = \tan^{-1}(\infty) \tag{2.0.27}$$

$$\theta_2 = 90^\circ \tag{2.0.28}$$

6) For $Line_3$,$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 4$
   - We have:

$$c_3 = -4, \mathbf{n}_3 = \begin{pmatrix}1\\-1\end{pmatrix} \tag{2.0.29}$$

   - Using (2.0.2) we get:

$$d_3 = \frac{|c_3|}{||\mathbf{n}_3||} \tag{2.0.30}$$

$$d_3 = \frac{4}{\sqrt{1+1}} \tag{2.0.31}$$

$$d_3 = \frac{4}{\sqrt{2}} \tag{2.0.32}$$

$$d_3 = \frac{4}{1.41} \tag{2.0.33}$$

$$d_3 = 2.828 \text{ units} \tag{2.0.34}$$

   - Using (2.0.4),the normal vector $\mathbf{n}_{p3}$ of the line perpendicular to given line is:

$$\mathbf{n}_{p3} = \begin{pmatrix}1\\1\end{pmatrix} \tag{2.0.35}$$

   - Using (2.0.6) the slope of this perpendicular line is given by:

$$m_{p3} = \frac{-1}{1} \tag{2.0.36}$$

$$m_{p3} = -1 \tag{2.0.37}$$

   - Using (2.0.8)the angle which perpendicular is making with positive x-axis is:

$$\theta_3 = \tan^{-1}(m_{p_3}) \tag{2.0.38}$$

$$\theta_3 = \tan^{-1}(-1) \tag{2.0.39}$$

$$\theta_3 = -45^\circ \tag{2.0.40}$$

7) If,
   $\mathbf{d_n}$=Perpendicular distance of line from origin

   $\mathbf{m_{pn}}$=Slope of the perpendicular to the line

   θ_n= Angle of perpendicular with positive x-axis.
8) All the calculated data can be tabularised in table 2.2:

	$Line_1$	$Line_2$	$Line_2$
d_1,d_2,d_3	4	2	2.828
$\mathbf{m_{p1}}$,$\mathbf{m_{p2}}$,$\mathbf{m_{p3}}$	$-\sqrt{3}$	∞	-1
θ_1,θ_2,θ_3	-60°	90°	-45°

TABLE 2.2: Calculated Data

- The plots of all the three lines and there respective perpendiculars are as follows:-

(A) Plot of Line_1 :-

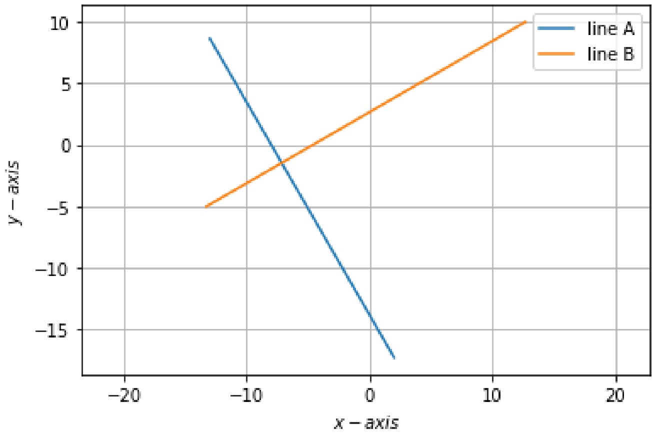

Fig. 2.1: Line_1 and it's perpendicular

(B) Plot of Line_2 :-

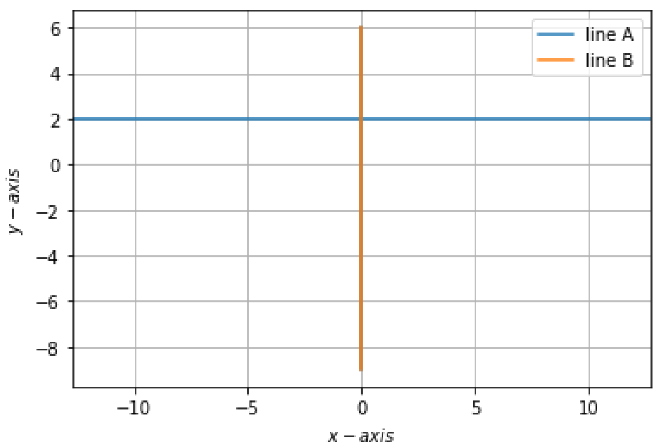

Fig. 2.2: Line_2 and it's perpendicular

(C) Plot of Line_3 :-

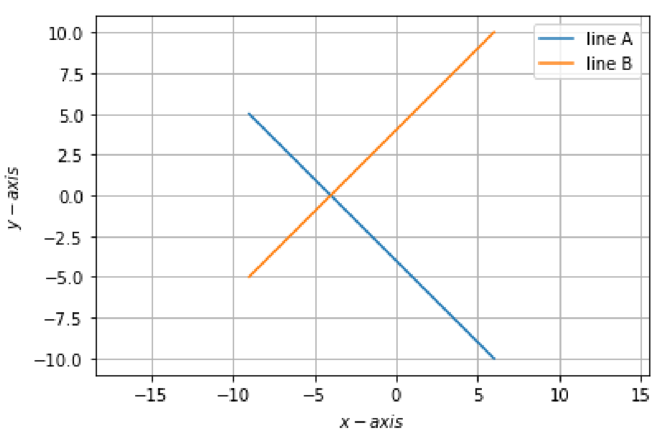

Fig. 2.3: Line_3 and it's perpendicular