

ASSIGNMENT-6

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_6/blob/master/codes.py

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_6

1 QUESTION No-2.105 (QUADFORMS)

Find the area enclosed by the parabola $4y = 3x^2$ and the line $\begin{pmatrix} -3 & 2 \end{pmatrix} \mathbf{x} = 12$.

2 SOLUTION

1) Given equation of parabola is:

$$4y = 3x^2 \quad (2.0.1)$$

$$3x^2 - 4y = 0 \quad (2.0.2)$$

$$x^2 - \frac{4}{3}y = 0 \quad (2.0.3)$$

2) Comparing with the standard equation :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.4)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{2}{3} \end{pmatrix}, f = 0 \quad (2.0.5)$$

3) We can find the eigen values corresponding to the \mathbf{V} ,

$$|\mathbf{V} - \lambda \mathbf{I}| = \begin{vmatrix} 1 - \lambda & 0 \\ 0 & -\lambda \end{vmatrix} = 0 \quad (2.0.6)$$

$$\Rightarrow (1 - \lambda)(-\lambda) = 0 \quad (2.0.7)$$

\therefore Eigen values are

$$\lambda_1 = 0, \lambda_2 = 1 \quad (2.0.8)$$

4) Calculating the eigen vectors corresponding to $\lambda_1 = 0, \lambda_2 = 1$ respectively.

$$\mathbf{V} \mathbf{x} = \lambda \mathbf{x} \quad (2.0.9)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = 0 \Rightarrow \mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.10)$$

$$\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} = \mathbf{x} \Rightarrow \mathbf{p}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.11)$$

5) By eigen decomposition on \mathbf{V} ,

$$\mathbf{V} = \mathbf{P} \mathbf{D} \mathbf{P}^T \quad (2.0.12)$$

Where,

$$\mathbf{P} = (\mathbf{p}_1 \quad \mathbf{p}_2) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.0.13)$$

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.14)$$

6) To find the vertex of the parabola ,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -\mathbf{f} \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.0.15)$$

$$\text{where, } \eta = \mathbf{u}^T \mathbf{p}_1 = \frac{-2}{3} \quad (2.0.16)$$

$$\Rightarrow \begin{pmatrix} 0 & \frac{-4}{3} \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.17)$$

7) The above equation implies that,

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.18)$$

8) To find the point of intersection:

- Let \mathbf{K} and \mathbf{L} are point of intersection.
- The given line is:

$$\begin{pmatrix} -3 & 2 \end{pmatrix} \mathbf{x} = 12 \quad (2.0.19)$$

$$\text{or, } 2y = 3x + 12 \quad (2.0.20)$$

$$\therefore \mathbf{x} = \begin{pmatrix} x \\ \frac{12+3x}{2} \end{pmatrix} \text{ satisfies it.} \quad (2.0.21)$$

- From (2.0.4) we have:

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.22)$$

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 & -\frac{2}{3} \end{pmatrix} \mathbf{x} + 0 = 0 \quad (2.0.23)$$

- Putting (2.0.21) in above equation we get:

$$\begin{pmatrix} x & \frac{12+3x}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \frac{12+3x}{2} \end{pmatrix} + \begin{pmatrix} 0 & -\frac{4}{3} \end{pmatrix} \begin{pmatrix} x \\ \frac{12+3x}{2} \end{pmatrix} = 0 \quad (2.0.24)$$

$$\Rightarrow x \left(\frac{x}{\frac{12+3x}{2}} \right) + \left(0 \quad \frac{-4}{3} \right) \left(\frac{x}{\frac{12+3x}{2}} \right) = 0 \quad (2.0.25)$$

$$\Rightarrow x^2 + \frac{(-4)}{3} \left(\frac{12+3x}{2} \right) = 0 \quad (2.0.26)$$

$$\Rightarrow x^2 - 2x - 8 = 0 \quad (2.0.27)$$

$$\Rightarrow x = -2, 4 \quad (2.0.28)$$

- Putting values of x in (2.0.20), we get:

$$\text{When } x = -2 \Rightarrow y = 3 \quad (2.0.29)$$

$$\Rightarrow \mathbf{K} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \quad (2.0.30)$$

$$\text{When } x = 4, \Rightarrow y = 12 \quad (2.0.31)$$

$$\Rightarrow \mathbf{L} = \begin{pmatrix} 4 \\ 12 \end{pmatrix} \quad (2.0.32)$$

- 9) For finding area enclosed by parabola and line:

Area required,

$$A = \text{Area under line} - \text{Area under parabola} \quad (2.0.33)$$

$$A = Ar(KLMNK) - Ar(KCLMCNK) \quad (2.0.34)$$

$$A = A_1 - A_2 \quad (2.0.35)$$

- 10) For finding area under the line that is A_1 -
KLMN is a trapezium. So its area can be given as:

$$A_1 = MN \times \frac{(KN + ML)}{2} \quad (2.0.36)$$

$$A_1 = 6 \times \frac{(3 + 12)}{2} \quad (2.0.37)$$

$$A_1 = 45 \text{ units} \quad (2.0.38)$$

- 11) For finding area under the parabola that is A_2 -

$$A_2 = \int_{-2}^4 y dx \quad (2.0.39)$$

$$A_2 = \int_{-2}^4 \frac{3}{4} x^2 dx \quad (2.0.40)$$

$$A_2 = \frac{3}{4} \int_{-2}^4 x^2 dx \quad (2.0.41)$$

$$A_2 = \frac{3}{4 \times 3} (4^3 - (-2)^3) \quad (2.0.42)$$

$$A_2 = \frac{1}{4} (64 + 8) \quad (2.0.43)$$

$$A_2 = \frac{72}{4} \quad (2.0.44)$$

$$A_2 = 18 \text{ units} \quad (2.0.45)$$

- 12) Putting (2.0.38) and (2.0.45) in (2.0.35) we get
required area A as:

$$A = A_1 - A_2 \quad (2.0.46)$$

$$A = 45 - 18 \quad (2.0.47)$$

$$A = 27 \text{ units} \quad (2.0.48)$$

So, the required area A is **27 units**

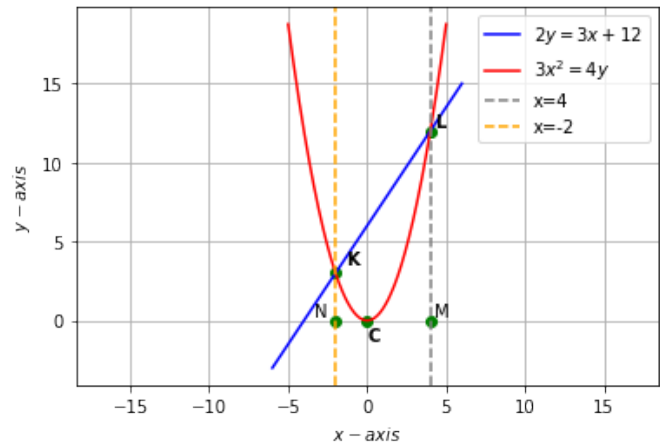


Fig. 2.1: Plot of the parabola and line