ASSIGNMENT-6

Unnati Gupta

Download all python codes from

https://github.com/unnatigupta2320/Assignment_6/blob/master/codes.py

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_6

1 Question No-2.105 (quadforms)

Find the area enclosed by the parabola $4y = 3x^2$ and the line $\begin{pmatrix} -3 & 2 \end{pmatrix} \mathbf{x} = 12$.

2 Solution

1) Given equation of parabola is:

$$4y = 3x^2 \tag{2.0.1}$$
$$3x^2 - 4y = 0 \tag{2.0.2}$$

2) Comparing with the standard equation :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{2.0.3}$$
$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, f = 0 \tag{2.0.4}$$

3) We can find the eigen values corresponding to the $\mathbf{V}$,

$$|\mathbf{V} - \lambda \mathbf{I}| = \begin{vmatrix} 3-\lambda & 0 \\ 0 & -\lambda \end{vmatrix} = 0 \tag{2.0.5}$$
$$\implies (3-\lambda)(-\lambda) = 0 \tag{2.0.6}$$

$\therefore$ Eigen values are

$$\lambda_1 = 0, \lambda_2 = 3 \tag{2.0.7}$$

4) Calculating the eigen vectors corresponding to $\lambda_1 = 0, \lambda_2 = 3$ respectively.

$$\mathbf{V}\mathbf{x} = \lambda \mathbf{x} \tag{2.0.8}$$
$$\begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = 0 \implies \mathbf{p_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.9}$$
$$\begin{pmatrix} 0 & 0 \\ 0 & -3 \end{pmatrix} \mathbf{x} = 3\mathbf{x} \implies \mathbf{p_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.10}$$

5) By eigen decomposition on $\mathbf{V}$,

$$\mathbf{V} = \mathbf{P}\mathbf{D}\mathbf{P}^T \tag{2.0.11}$$

Where,

$$\mathbf{P} = \begin{pmatrix} \mathbf{p_1} & \mathbf{p_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.12}$$
$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix} \tag{2.0.13}$$

6) To find the vertex of the parabola ,

$$\begin{pmatrix} \mathbf{u^T} + \eta \mathbf{p_1^T} \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -\mathbf{f} \\ \eta \mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{2.0.14}$$
$$\text{where, } \eta = \mathbf{u^T p_1} = 2 \tag{2.0.15}$$
$$\implies \begin{pmatrix} 0 & 4 \\ 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.16}$$

7) The above equation implies that,

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.17}$$

8) To find the point of intersection:
   - Let $\mathbf{K}$ and $\mathbf{L}$ are point of intersection.
   - The point of contacts are given using:

$$\begin{pmatrix} \mathbf{u^T} + \kappa \mathbf{n^T} \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -\mathbf{f} \\ \kappa \mathbf{n} - \mathbf{u} \end{pmatrix} \tag{2.0.18}$$
$$\text{where, } \kappa = \frac{\mathbf{p_1^T u}}{\mathbf{p_1^T n}} = 1 \tag{2.0.19}$$

   - Putting values from (2.0.4) in above equation, we get:

$$\implies \mathbf{K} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{2.0.20}$$
$$\implies \mathbf{L} = \begin{pmatrix} 4 \\ 12 \end{pmatrix} \tag{2.0.21}$$

9) For finding area enclosed by parabola and line:

Area required,

$$A = \text{Area under line} - \text{Area under parabola} \tag{2.0.22}$$

$$A = Ar(KLMNK) - Ar(KCLMCNK) \tag{2.0.23}$$

$$A = A_1 - A_2 \tag{2.0.24}$$

10) For finding area under the line that is A_1- **KLMN** is a trapezium. So its area can be given as:

$$A_1 = MN \times \frac{(KN + ML)}{2} \tag{2.0.25}$$

$$A_1 = 6 \times \frac{(3 + 12)}{2} \tag{2.0.26}$$

$$A_1 = 45 \text{ units} \tag{2.0.27}$$

11) For finding area under the parabola that is A_2-

$$A_2 = \int_{-2}^{4} y dx \tag{2.0.28}$$

$$A_2 = \int_{-2}^{4} \frac{3}{4} x^2 dx \tag{2.0.29}$$

$$A_2 = \frac{3}{4} \int_{-2}^{4} x^2 dx \tag{2.0.30}$$

$$A_2 = \frac{3}{4 \times 3} \left(4^3 - (-2)^3\right) \tag{2.0.31}$$

$$A_2 = \frac{1}{4} (64 + 8) \tag{2.0.32}$$

$$A_2 = \frac{72}{4} \tag{2.0.33}$$

$$A_2 = 18 \text{ units} \tag{2.0.34}$$

12) Putting (2.0.27) and (2.0.34) in (2.0.24) we get required area A as:

$$A = A_1 - A_2 \tag{2.0.35}$$

$$A = 45 - 18 \tag{2.0.36}$$

$$A = 27 \text{ units} \tag{2.0.37}$$

So, the required area A is **27 units**

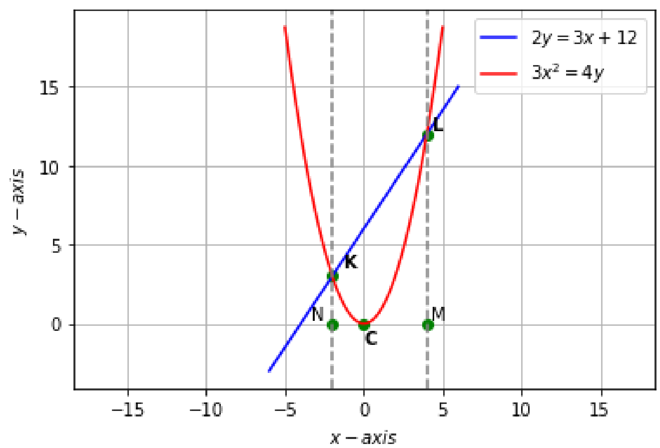

Fig. 2.1: Plot of the parabola and line