ASSIGNMENT 7

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1 QUESTION No 2.74(D)

In each of the following find the equation for the ellipse that satisfies the given conditions:

a. Conjugate axis length= 8, foci= $\begin{pmatrix} \pm 5 \\ 0 \end{pmatrix}$

2 SOLUTION

1) Given that,

$$\text{Conjugate axis length } = 2b = 8 \quad (2.0.1)$$

$$\text{foci} = \begin{pmatrix} \pm 5 \\ 0 \end{pmatrix} \quad (2.0.2)$$

2) We know that length of semi major axis, a is

$$a = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} \quad (2.0.3)$$

3) Also, length of semi minor axis, b is

$$b = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} \quad (2.0.4)$$

4) Using (2.0.1) length of conjugate axis is:

$$2b = 8 \quad (2.0.5)$$

$$\implies b = 4 \quad (2.0.6)$$

$$\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} = 4 \quad (2.0.7)$$

$$\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2} = 16 \quad (2.0.8)$$

$$\implies \lambda_2 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{16} \quad (2.0.9)$$

5) Also, we know that,

$$c^2 = a^2 - b^2 \quad (2.0.10)$$

$$\implies 25 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2} \quad (2.0.11)$$

6) Putting (2.0.8) in above equation we get:

$$25 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1} - 16 \quad (2.0.12)$$

$$41 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1} \quad (2.0.13)$$

$$\implies \lambda_1 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{41} \quad (2.0.14)$$

7) The standard equation of ellipse is given by :

$$\frac{\mathbf{y}^\top D y}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \quad (2.0.15)$$

8) Using (2.0.9) and (2.0.14) we get:

$$\implies \frac{\mathbf{y}^\top \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \quad (2.0.16)$$

$$\implies \mathbf{y}^\top \begin{pmatrix} \frac{1}{41} & 0 \\ 0 & \frac{1}{16} \end{pmatrix} = 1 \quad (2.0.17)$$
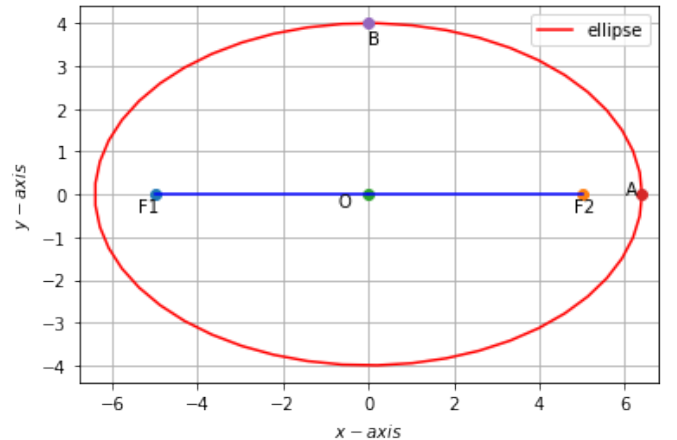
9) The Plot of ellipse is:

Fig. 2.1: Ellipse $\frac{x^2}{41} + \frac{y^2}{16} = 1$