

ASSIGNMENT 7

Unnati Gupta

Download all python codes from

https://github.com/unnatigupta2320/Assignment_7/blob/master/codes.py

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_7

1 QUESTION No 2.74(D)

In each of the following find the equation for the ellipse that satisfies the given conditions:

- a. Conjugate axis length= 8, foci= $\begin{pmatrix} \pm 5 \\ 0 \end{pmatrix}$

2 SOLUTION

Given that,

$$\text{Conjugate axis length} = 2b = 8 \quad (2.0.1)$$

$$\text{foci} = \begin{pmatrix} \pm 5 \\ 0 \end{pmatrix} \quad (2.0.2)$$

Lemma 2.1. *The standard equation of an ellipse is given by:*

$$\frac{\mathbf{y}^T D \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.3)$$

$$\text{where, } D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad (2.0.4)$$

Also, the length of semi major axis, a is

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \quad (2.0.5)$$

and the length of semi minor axis, b is

$$b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \quad (2.0.6)$$

- 1) Using (2.0.1) length of conjugate axis is:

$$2b = 8 \quad (2.0.7)$$

$$\Rightarrow b = 4 \quad (2.0.8)$$

$$\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = 4 \quad (2.0.9)$$

$$\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} = 16 \quad (2.0.10)$$

$$\Rightarrow \lambda_2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{16} \quad (2.0.11)$$

- 2) Also, we know that,

$$c^2 = a^2 - b^2 \quad (2.0.12)$$

$$\Rightarrow 25 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} \quad (2.0.13)$$

- 3) Putting (2.0.10) in above equation we get:

$$25 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} - 16 \quad (2.0.14)$$

$$41 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} \quad (2.0.15)$$

$$\Rightarrow \lambda_1 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{41} \quad (2.0.16)$$

- 4) Using lemma (2.1), the standard equation of ellipse is given by :

$$\frac{\mathbf{y}^T D \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.17)$$

- 5) Putting (2.0.11) and (2.0.16) in above equation we get:

$$\Rightarrow \frac{\mathbf{y}^T \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.18)$$

$$\Rightarrow \mathbf{y}^T \begin{pmatrix} \frac{1}{41} & 0 \\ 0 & \frac{1}{16} \end{pmatrix} \mathbf{y} = 1 \quad (2.0.19)$$

- 6) The Plot of ellipse is:

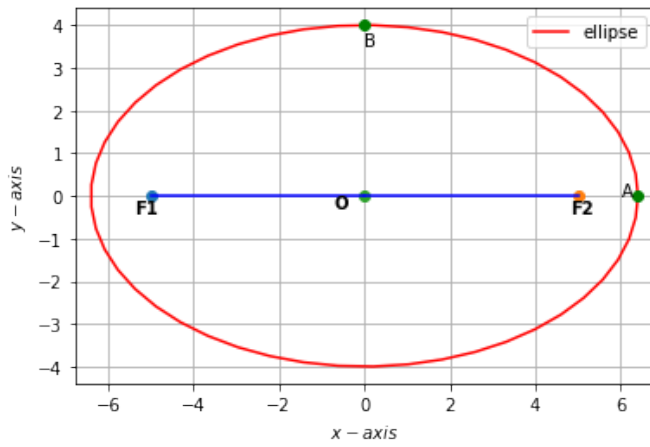


Fig. 2.1: Ellipse $\frac{x^2}{41} + \frac{y^2}{16} = 1$