ASSIGNMENT 7

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_7/blob/master/codes.py

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_7

1 Question No 2.74(d)

In each of the following find the equation for the ellipse that satisfies the given conditions:

a. Conjugate axis length= 8, foci= $\begin{pmatrix}\pm5\\0\end{pmatrix}$

2 Solution

Given that,

$$\text{Conjugate axis length } = 2b = 8 \tag{2.0.1}$$

$$\text{foci} = \begin{pmatrix}\pm c\\0\end{pmatrix} = \begin{pmatrix}\pm5\\0\end{pmatrix} \tag{2.0.2}$$

Here c= distance of foci of ellipse from origin

Lemma 2.1. *The standard equation of an ellipse is given by:*

$$\frac{\mathbf{y}^\top D\mathbf{y}}{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f} = 1 \tag{2.0.3}$$

$$\text{where, } D = \begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix} \tag{2.0.4}$$

Also,the length of semi major axis, a is

$$a = \sqrt{\frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1}} \tag{2.0.5}$$

and the length of semi minor axis, b is

$$b = \sqrt{\frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_2}} \tag{2.0.6}$$

1) Using (2.0.1) length of conjugate axis is:

$$2b = 8 \tag{2.0.7}$$

$$\implies b = 4 \tag{2.0.8}$$

$$\sqrt{\frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_2}} = 4 \tag{2.0.9}$$

$$\frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_2} = 16 \tag{2.0.10}$$

$$\implies \lambda_2 = \frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{16} \tag{2.0.11}$$

2) Also, we know that,

$$c^2 = a^2 - b^2 \tag{2.0.12}$$

$$\implies 25 = \frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1} - \frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_2} \tag{2.0.13}$$

3) Putting (2.0.10) in above equation we get:

$$25 = \frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1} - 16 \tag{2.0.14}$$

$$41 = \frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1} \tag{2.0.15}$$

$$\implies \lambda_1 = \frac{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f}{41} \tag{2.0.16}$$

4) Using lemma (2.1),the standard equation of ellipse is given by :

$$\frac{\mathbf{y}^\top D\mathbf{y}}{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f} = 1 \tag{2.0.17}$$

5) Putting (2.0.11) and (2.0.16) in above equation we get:

$$\implies \frac{\mathbf{y}^\top\begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix}\mathbf{y}}{\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u}-f} = 1 \tag{2.0.18}$$

$$\implies \mathbf{y}^\top\begin{pmatrix}\frac{1}{41} & 0\\0 & \frac{1}{16}\end{pmatrix}\mathbf{y} = 1 \tag{2.0.19}$$

6) The Plot of ellipse is:

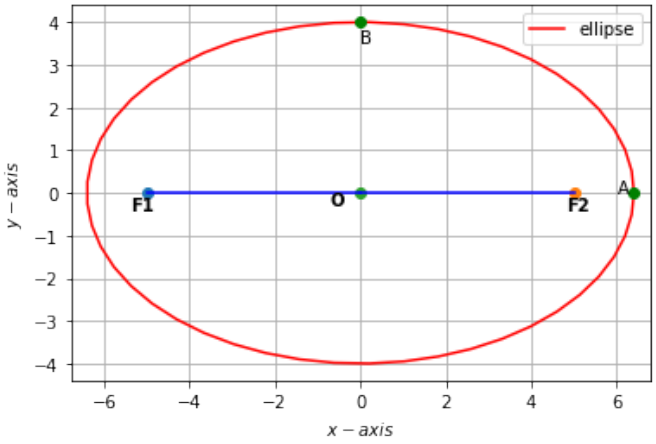

Fig. 2.1: Ellipse $\frac{x^2}{41} + \frac{y^2}{16} = 1$