ASSIGNMENT-9

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_9

and latex-tikz codes from

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1 Question No-2.63

If $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$, then show that $\mathbf{A}^3 - 23\mathbf{A} - 40\mathbf{I} = 0$.

2 Solution

Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$.

1) **The Characteristic equation** is given by:

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \tag{2.0.1}$$

$$\implies det \begin{pmatrix} 1-\lambda & 2 & 3 \\ 3 & -2-\lambda & 1 \\ 4 & 2 & 1-\lambda \end{pmatrix} = 0 \tag{2.0.2}$$

$$\implies (1-\lambda)\left((-2-\lambda)(1-\lambda) - 2\right) - 2\left(3(1-\lambda) - 4\right) + 3\left(6 + 4(2+\lambda)\right) = 0 \tag{2.0.3}$$

$$\implies (1-\lambda)\left(\lambda + \lambda^2 - 4\right) - 2(-1 - 3\lambda) + 3(14 + 4\lambda) = 0 \tag{2.0.4}$$

$$\implies -\lambda^3 + 5\lambda - 4 + 2 + 6\lambda + 42 + 12\lambda = 0 \tag{2.0.5}$$

$$\implies \lambda^3 - 23\lambda - 40 = 0 \tag{2.0.6}$$

The above equation is similar to equation to be proved.

2) According to **Cayley-Hamilton Theorem:** Every square matrix satisfies its own **characteristic equation**.

$$\therefore \mathbf{A}^3 - 23\mathbf{A} - 40\mathbf{I} = 0 \tag{2.0.7}$$

Hence Proved.