ASSIGNMENT-9

Unnati Gupta

Download all python codes from

https://github.com/unnatigupta2320/Assignment_9

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_9

1 Question No-2.63

If $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$, then show that $\mathbf{A}^3 - 23\mathbf{A} - 40\mathbf{I} = 0$.

2 Solution

Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$.

1) **The Characteristic equation** is given by:

$$\implies |\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{2.0.1}$$

$$\implies \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & -2-\lambda & 1 \\ 4 & 2 & 1-\lambda \end{vmatrix} = 0 \tag{2.0.2}$$

$$\implies (1-\lambda)\left((-2-\lambda)(1-\lambda) - 2\right) - 2\left(3(1-\lambda) - 4\right) + 3\left(6 + 4(2+\lambda)\right) = 0 \tag{2.0.3}$$

$$\implies \lambda^3 - 23\lambda - 40 = 0 \tag{2.0.4}$$

The above equation is similar to equation to be proved.

2) According to **Cayley-Hamilton Theorem:** Every square matrix satisfies its own **characteristic equation**.

$$\therefore \mathbf{A}^3 - 23\mathbf{A} - 40\mathbf{I} = 0 \tag{2.0.5}$$

Hence Proved.