

ASSIGNMENT-9

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Download all python codes from

https://github.com/unnatigupta2320/Assignment_9/codes.py

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_9

1 QUESTION No-2.63

If $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$, then show that
 $\mathbf{A}^3 - 23\mathbf{A} - 40\mathbf{I} = 0$.

2 SOLUTION

Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$.

1) **The Characteristic equation** is given by:

$$\Rightarrow |\mathbf{A} - \lambda\mathbf{I}| = 0 \quad (2.0.1)$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & -2-\lambda & 1 \\ 4 & 2 & 1-\lambda \end{vmatrix} = 0 \quad (2.0.2)$$

$$\begin{aligned} \Rightarrow (1-\lambda)((-2-\lambda)(1-\lambda)-2) \\ -2(3(1-\lambda)-4)+3(6+4(2+\lambda)) &= 0 \\ (2.0.3) \end{aligned}$$

$$\Rightarrow \lambda^3 - 23\lambda - 40 = 0 \quad (2.0.4)$$

The above equation is similar to equation to be proved.

2) According to **Cayley-Hamilton Theorem**:

Every square matrix satisfies its own **characteristic equation**.

$$\therefore \mathbf{A}^3 - 23\mathbf{A} - 40\mathbf{I} = 0 \quad (2.0.5)$$

Hence Proved.