ASSIGNMENT-9

Unnati Gupta

Download all python codes from

https://github.com/unnatigupta2320/Assignment_9

and latex-tikz codes from

https://github.com/unnatigupta2320/Assignment_9

1 Question No-2.63

If $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$, then show that $\mathbf{A}^3 - 23\mathbf{A} - 40\mathbf{I} = 0$.

2 Solution

Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$.

1) Calculating $\mathbf{A}^3$:
   - We will firstly calculate $\mathbf{A}^2$:-

$$\mathbf{A}^2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix} \tag{2.0.1}$$

$$= \begin{pmatrix} 1+6+12 & 2-4+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9-2+1 \\ 4+6+4 & 8-4+2 & 12+2+1 \end{pmatrix} \tag{2.0.2}$$

$$\implies \mathbf{A}^2 = \begin{pmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{pmatrix} \tag{2.0.3}$$

   - Now, $\mathbf{A}^3$ can be given as :-

$$\mathbf{A}^3 = \mathbf{A}^2\mathbf{A} \tag{2.0.4}$$

$$\mathbf{A}^3 = \begin{pmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix} \tag{2.0.5}$$

$$= \begin{pmatrix} 19+12+32 & 38-8+16 & 57+4+8 \\ 1+36+32 & 2-24+16 & 3+12+8 \\ 14+18+60 & 28-12+30 & 42+6+15 \end{pmatrix} \tag{2.0.6}$$

$$\implies \mathbf{A}^3 = \begin{pmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{pmatrix} \tag{2.0.7}$$

2) Calculating $23\mathbf{A}$:-

$$23\mathbf{A} = 23\begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix} \tag{2.0.8}$$

$$\implies 23\mathbf{A} = \begin{pmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{pmatrix} \tag{2.0.9}$$

3) Calculating $40\mathbf{I}$:-

$$40\mathbf{I} = 40\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.10}$$

$$\implies 40\mathbf{I} = \begin{pmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{pmatrix} \tag{2.0.11}$$

4) Considering LHS of given equation we have:-

$$LHS = \mathbf{A}^3 - 23\mathbf{A} - 40\mathbf{I} \tag{2.0.12}$$

5) Putting values from (2.0.7), (2.0.9) and (2.0.11) we get:-

$$\implies \text{LHS} = \begin{pmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{pmatrix} - \begin{pmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{pmatrix} - \begin{pmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{pmatrix} \tag{2.0.13}$$

$$= \begin{pmatrix} 63-23-40 & 46-46 & 69-69 \\ 69-69 & -6+46-40 & 23-23 \\ 92-92 & 46-46 & 63-23-40 \end{pmatrix} \tag{2.0.14}$$

$$\implies \text{LHS} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.15}$$

$$\implies \text{LHS} = 0 = RHS \tag{2.0.16}$$

$$\implies \mathbf{A}^3 - 23\mathbf{A} - 40\mathbf{I} = 0 \tag{2.0.17}$$

Hence, proved.