PORTFOLIO CONSTRUCTION USING NON-GAUSSIAN VAR ESTIMATION AND GSEV APPROACH

A Project Report
Submitted by

UNNIKRISHNAN NAMBIAR K

(Roll No. 17NA3FP14)

Under the guidance of

Dr SUJOY BHATTACHARYA

Vinod Gupta School of Management, IIT Kharagpur

In partial fulfilment of the requirements for the award of the degree of

BACHELOR OF TECHNOLOGY (HONOURS) AND MASTER OF TECHNOLOGY



VINOD GUPTA SCHOOL OF MANAGEMENT
INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
APRIL 2021

DECLARATION

I hereby declare that this report represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/ data/ fact/ source in my submission. I understand that any violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

Date: 21 April 2021 Unnikrishnan Nambiar K

Roll No. 17NA3FP14

Dept. of Ocean Engg. And Naval Arch.

Indian Institute of Technology Kharagpur

CERTIFICATE OF APPROVAL

This is to certify that the project report titled "PORTFOLIO CONSTRUCTION USING NON-GAUSSIAN VAR ESTIMATION AND GSEV APPROACH" submitted by UNNIKRISHNAN NAMBIAR K (17NA3FP14) to Indian Institute of Technology Kharagpur for the award of the degree of Bachelor of Technology (Honours) and Master of Technology is a bona fide record of research work carried out by him under my supervision. The contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Place: Kharagpur Dr. Sujoy Bhattacharya

Date: 21-04-2021. Associate Professor

Vinod Gupta School of Management

Indian Institute of Technology Kharagpur

West Bengal – 721302

ACKNOWLEDGEMENT

First & foremost, I would like to thank Prof. Sujoy Bhattacharya of Vinod Gupta School of

Management, IIT Kharagpur for giving me the opportunity to implement this relevant project

titled "PORTFOLIO CONSTRUCTION USING NON-GAUSSIAN VAR ESTIMATION AND

GSEV APPROACH". His guidance and virtual course lectures were crucial in helping me

complete my project on this fascinating subject. I would also like to thank Prof. Abhijeet

Chandra of Vinod Gupta School of Management, IIT Kharagpur for taking his valuable time

in grading this project. I'd also like to express my gratitude to all of the authors, journalists,

columnists, and scientists whose ideas and works were incorporated into my project. My

sincere gratitude is extended out to the University's entire staff and administration for providing

the infrastructure in the form of our online library and the virtual classes that they made so

easy. Finally, I would like to express my gratitude to my parents, seniors, mentors, and friends

at the university who have provided me with the support throughout the duration of this

project.

Unnikrishnan Nambiar K

Roll No. 17NA3FP14

Dept. of Ocean Engg. And Naval Arch.

Indian Institute of Technology Kharagpur

4

TABLE OF CONTENTS

ABSTRACT	6
INTRODUCTION	6
LITERATURE SURVEY	8
METHODOLOGY	10
THE GSEV APPROACH	
DATA	
RESULTS	13
DISCUSSION	17
BIBLIOGRAPHY	Error! Bookmark not defined.
ANNEXURE I: MODEL FIT STATISTICS	20

ABSTRACT

In this implementation, we have made use of a multivariate analysis of equity portfolio returns that accounts for some of the most studied and discussed traits of financial data: fat tails, positive and negative skew, stochastic clustering of volatility, and asymmetric dependence with the goal of increasing the model accuracy of quantitative portfolio risk measures. We have implemented the EGARCH model to account for volatility clustering and leverage effect, the skewed-t model to capture fat tails and asymmetric skew, and the skewed-t copula implemented in python to model asymmetric dependence. This model, known as the GSEV approach was further implemented on BRIC equity index data. We have further back tested this model, validating portfolios with lower downside risk than alternative approaches, namely the Markowitz framework. We further cement that investors can substantially curb the underestimation of risk by taking non-Gaussian models into account.

Keywords: GSEV, Stochastic volatility, Tail-risk, Skewed-t copula, Portfolio optimisation, BRIC Economies, Volatility Clustering, EGARCH processes.

INTRODUCTION

Since the advent of the Markowitz framework, experts have debated and shown that financial returns stray away from the idealised and ubiquitous normal distribution. True financial asset return distributions have fat tails compared to the prescribed Gaussian normal distribution (leptokurtosis) and more significant negative values than positive values (negative skew) [Mandelbrot, 1963].

Their volatilities tend to cluster and are not constants as primitive models suggest. "Asset returns have asymmetric tail dependency, with correlations rising during periods of market stress [Karolyi and Stulz, 1996; Longin and Solnik, 2001; Alcock and Satchell, 2018]". This was pertinent in understanding the 2008 financial crisis where the tail risk of CDO portfolios were significantly underestimated [Watts, 2016].

Thus, the mean-variance model, according to experts, can lead to risk mismanagement and be harmful to investor health. Despite this, the mean-variance approach remains very much in use in the financial sector [Fabozzi, 2007] due to its simplicity.

Chen and Fan formalised a set of approaches in where a multivariate time series' conditional mean and conditional variance are parametrically defined, but the distribution of the standardised multivariate invention is specified semi-parametrically as a "parametric copula evaluated at non-parametric marginal densities". The family of such models were introduced as "semiparametric copula-based multivariate dynamic models (SCOMYD)".

The students-t copula was used in this implementation to model asymmetric dependency in larger dimensions. We have implemented and expanded on previous work by proposing a risk management and portfolio development approach that takes into account all the four characteristic traits mentioned and can be used even when there exist a large number of assets.

The idea of Value at Risk (VaR) is used to explain the above risk management concepts. By applying our proposed approach on BRIC equity index data, we compare it to existing frameworks. We show that, when compared to competing Markowitz model, the proposed method generates a substantial increase in investor utility certainty.

This structure of this report is as follows. We include a brief overview of the VaR estimation literature in Chapter 2 as well as a literature overview of alternative non-Gaussian approaches that have been used for portfolio optimization and construction. In Chapter 3, we develop a scalable model (GSEV) that mathematically elucidate the four stylized facts listed earlier, as well as discuss the dataset used in the implementation. In Chapter 4, we examine the empirical results of our evaluation methodologies, benchmark methods, and data, as well as the models' VaR forecasting efficiency. Chapter 5 wraps up the study and addresses the findings' practical consequences.

LITERATURE SURVEY

VaR estimation methods have been heavily criticised, despite their widespread usage in the financial sector. VaR has been argued to be incoherent as a risk indicator [Artzner, 1997]. Furthermore, VaR ignores the fact that, depending on market conditions, the same dollar loss will result in dramatically different economic outcomes [At-Sahalia, 2000].

Models that take the stochastic models of volatility into consideration, on the other hand, consistently outperform static models [Berkowitz and O'Brien, 2002; McAleer and Da Veiga, 2008; Skoglund et al., 2010; Moreira and Muir, 2017] . Models that incorporate non-Gaussian developments have also been shown to do better than pure parametric Gaussian models [McAleer and Da Veiga, 2008].

According to surveys, the most used approaches at commercial banks are relatively simple historical simulation and its alternative, the filtered historical simulation approach [Christoffersen, 2009; Pérignon and Smith, 2008; Gurrola-Perez and Murphy, 2015].

Several studies have attempted to improve investor utility by incorporating non-Gaussian characteristics into portfolio construction methods. According to Patton (2004), using skewed-t marginals and a rotated Gumbel copula to account for asymmetry and skewness results in a slight increase in realised usefulness over the usual 1/N strategy.

For investors with moderate to high levels of risk aversion, Jondeau and Rockinger (2005) combined a Taylor series approach with DCC-GARCH to find that the approach substantially outperforms mean–variance Markowitz portfolios.

It has been shown that full-scale optimisation based on S-shaped and bi-linear utility improves investor utility [Adler and Kritzman, 2007]. With the implementation of CRRA utility to model portfolio weights as a function of asset characteristics joint asset returns must be modelled [Aït-Sahalia and Brandt, 2001].

To account for asymmetric dependency, Gaussian marginals in conjunction with the Clayton copula were used and a large increase in average returns as well as stronger returns in bearish markets were found [Alcock and Hatherley, 2009].

Models for all four stylised facts had previously been developed, but they were focused on generalising the Archimedean copula method to higher dimensions (n > 2), which necessitated substantial and restrictive assumptions. [Viebig and Poddig, 2010].

Another interesting model was the Taylor Series approach to account for higher moments, which showed that unless sophisticated shrinkage estimators of higher moments were used, the system did not increase investor welfare [Martellini and Ziemann, 2010].

Furthermore, during studies based on the 2008 financial crisis, the implementation of the truncated Lévy-flight distribution to catch skew and heavy-tails found that the strategy outperformed mean–variance slightly [Xiong and Idzorek, 2011].

METHODOLOGY

THE GSEV APPROACH

In this section, we demonstrate the advantages of a risk management approach that can accommodate fat tails, bidirectional skew, stochastic volatility clustering, and asymmetric tail dependence. This model builds on previous work that employs GARCH models and copulas to model the same [Nystrom and Skoglund, 2002; Ghorbel and Trabelsi, 2009; Viebig and Poddig, 2010].

Copulas are useful because they allow you to distinguish the dependency structure from the univariate densities when building portfolios. The estimation of constant and dynamic conditional correlation (CCC and DCC) multivariate GARCH models [Bollerslev, 1990; Engle, 2002] is another way to model this.

For addressing stochastic volatility and the leverage effect, we use a univariate exponential GARCH (EGARCH) procedure, the non-centred student's t distribution to model the heavy-tails in the GARCH residuals, and the skewed-t copula to allow for asymmetric tail dependence in our approach. This is represented as the "GSEV" approach.

The fitting of the GARCH processes, the estimation of the univariate marginals, and the modelling of dependency structure are the three parts of the estimation problem. Filtering with a GARCH method produces an iid series, which makes fitting a parametric distribution easier. The estimation process is also simplified and accelerated with minimal efficiency loss.

With Student-t innovations, we use Nelson (1991)'s exponentially weighted GARCH model. We suit piecewise distributions to the univariate GARCH residuals, as McNeil and Frey (2000) and Nystrom and Skoglund (2002) did. We used maximum likelihood estimation to suit non centered-t distributions and a Gaussian kernel.

The skewed-t copula is used to model the dependency structure of the GARCH residuals. We are interested in the skewed-t copula since it can handle tail dependence, asymmetric

dependence between upper and lower tails, and heterogeneous dependence throughout asset pairs. It is also scalable in large dimensions.

The following measures are used to approximate and simulate the model:

- 1. Estimate an AR (1)-EGARCH (1,1) model for each asset *i*, and obtain parameter estimates, conditional variances, and residuals.
- 2. Calculate the standardised residuals, $\epsilon_{i,t} = \epsilon_{i,t}/\sigma_{i,t}$.
- 3. Fit the univariate distributions to, $\epsilon_{i,t}$.
- 4. Fit univariate skewed-t distributions to each vector of residuals from step 2 by MLE.
- 5. Estimate the covariance matrix Σ given by:

$$\left(\frac{v-2}{v}\right)\left(\operatorname{cov}(X) - \frac{\hat{\beta}\hat{\beta}'(2v^2)}{(v-2)^2(v-4)}\right)$$

where v and β are parameters of the skewed-t distributions estimated in Step 4. Cov(X) is the sample covariance matrix.

6. Draw *N* independent d-dimensional vectors from the multivariate Gaussian distribution defined by:

$$Z \sim N(0, \Sigma^{^{\wedge}})$$

7. Draw *N* independent random numbers from the inverse gamma distribution defined by:

$$W \sim IG\left(\frac{v}{2}, \frac{v}{2}\right)$$

8. Substitute *Z* and *W* into the following function to yield multivariate skewed t variables:

$$X = \mu + \beta W + \sqrt{W}Z$$

- 9. Approximate the cumulative distribution functions for the univariate marginals by numerical integration.
- 10. Transform X, based on Step 8, into uniformly distributed variables using the univariate cumulative distribution functions.
- 11. Convert Usk to innovations, Isk, using the inverse cumulative distribution functions for the univariate piecewise distributions developed in Step 3.
- 12. Simulate from an AR (1)-EGARCH (1,1) model for each asset i using innovations, I, and parameters θiE .

This algorithm is rapid and can be applied to problems with large dimensions. Fit statistics for the stochastic models are shown in Annexure 1.

DATA

Using a dataset of major stock index prices from BRIC economies, we compared our proposed model to portfolio construction approaches typically used by managers or that are well-known in the literature. Our data set encompass the period of 2014-2017 and is comprised of Brazilian (BVSP), Russian (MOEX), Indian (NIFTY), and Chinese (SSE) Indices. All our return series are daily.

The main aim behind considering the BRIC nations for this study is to analyse the characteristics of volatility clustering for the emerging markets. In the last few years, Brazil, Russia, China, India, and south Africa have shown consistent return thus including those countries for this study is relevant and suitable. Many institutional and private investors consider emerging countries as smart alternative investment avenues.

RESULTS AND ANALYSIS

The average overview statistics for the properties in the dataset are shown in Table 1. To search for normality, we used the Jarque–Bera tests on each index. For each asset, normality was rejected. We have also conducted the Augmented Dickey Fuller test for the indices and the results show that there was strong evidence to believe in the absence of unit roots. These are shown in Table 2. The absolute value of returns has pronounced autocorrelation, which is consistent with volatility clustering. The ACF and PACF plots are shown in Fig1.

Table 1. Dataset Summary Description

Tuble 1. Dataset Summary Description						
INDEX	BRAZIL (BVSP)	RUSSIA (MOEX)	INDIA (NIFTY)	CHINA (SSE)		
Annualized Return	0.009	0.017	0.009	0.005		
std	0.018	0.019	0.012	0.015		
min	-0.148	-0.091	-0.130	-0.103		
25%	-0.008	-0.009	-0.005	-0.006		
50%	0.001	0.000	0.001	0.001		
75%	0.010	0.012	0.007	0.007		
max	0.139	0.133	0.088	0.106		
Skewness	-0.50	0.26	-0.92	-0.63		
Kurtosis	11.68	3.12	14.64	7.60		
Jarque Bera	9912.37	719.42	15701.69	4275.23		
P (Normal)	0.00	0.00	0.00	0.00		

Table 2. Augmented Dickey Fuller Test

Tuble 2.114gmented Dieney 1 uner 1 est							
INDEX	BRAZIL (BVSP)	RUSSIA (MOEX)	INDIA (NIFTY)	CHINA(SSE)			
ADF Statistic	-10.01	-16.14	-8.76	-13.73			
p Value	0.00	0.00	0.00	0.00			
AIC Auto lag	17.00	5.00	23.00	7.00			
Critical 1%	-3.43	-3.43	-3.43	-3.43			
Critical5%	-2.86	-2.86	-2.86	-2.86			
Critical 10%	-2.57	-2.57	-2.57	-2.57			

Table 3. VaR forecast evaluation: at 99.5% confidence.

$\beta = 99.5\%$	EW Markowitz	GSEV
Violations	0.0431	0.0048
UC: p-value	0.00	0.90
Consecutive viol	0.19	0.06
SI: p-value	0.00	0.03
CC: p-value	0.00	0.12

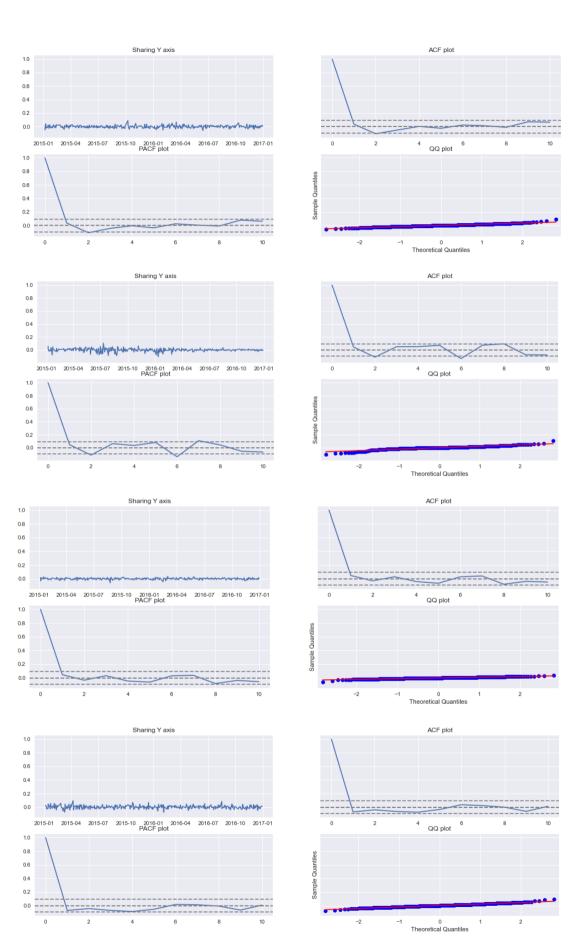


Fig.1. ACF, PACF plots BVSP, SSE, NIFTY, MOEX

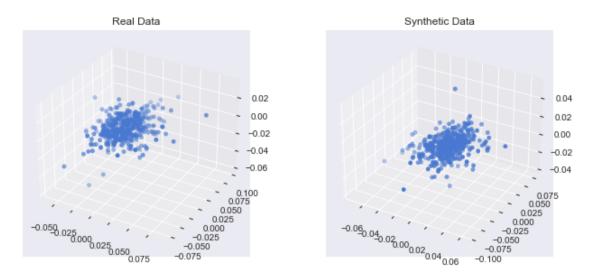


Fig.2.3D comparison plot of real and synthetic data from GSEV based on BSVP, MOEX, NIFTY data



Fig.3. Back test for EW Model based on Markowitz portfolio.

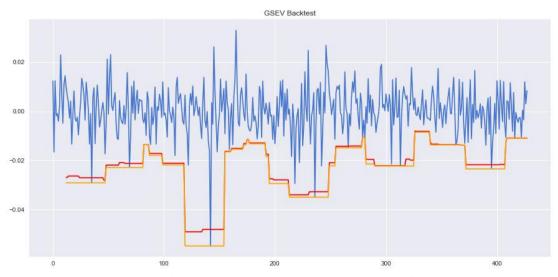


Fig.4. Back test for Minimum Variance Model based on GSEV portfolio.

It is apparent from Table 2 that there exists significant autocorrelation and the absence of unit roots. Fig 2 shows the synthetic data generated from the GSEV approach and the real data in a 3D plot. Hence, we can utilise this to predict better using the GSEV algorithm. The portfolio performance of the models is summarised in Figures 3 and 4. The out-of-sample CVaRs of the Markowitz EW model are significantly higher than the target. The GSEV approach, on the other hand, produces CVaRs that are similar to the target level producing the smallest risk forecast error.

The model also consistently produces a lower maximum drawdown, as measured by the peak to trough return, a widely used metric by practitioners. Since the copula imposes a consistent dependency structure across asset pairs, the approach's output is likely to deteriorate as the number of assets and complexity grows. Table 3 shows the relative success of the two models.

CONCLUSION AND DISCUSSION

This implementation was aimed to strengthen Gaussian approaches that systematically underestimate tail risk, in line with previous work in the field. Any strategy that quantifies tail risk using the Gaussian distribution poses a major risk to investor health. Conditional volatility approaches tend to be an important component of a sensible VaR estimation model since they reduce the proportion of consecutive breaches.

The GSEV approach produces more reliable VaR forecasts than the GGEV approach, which employs a symmetrical dependency structure, suggesting the significance of accounting for asymmetric dependence. We believe that these findings are applicable to other risk control procedures, but we'll leave that to further study.

We conclude that the GSEV method outperforms mean—variance in our out-of-sample portfolio rebalancing study. In comparison to the mean—variance method, historical simulation produces a small increase in economic value. The added benefit of the GGEV and GSEV approaches is important. The use of a conditional volatility approach in combination with a heavy-tailed distribution is a popular unifying factor across both of these approaches.

The GSEV approach has the potential to provide significant value addition while minimising risk. The GSEV method also has the lowest maximum drawdown (MDD) values and the most reliable portfolio VaR estimates out of study. This demonstrates how important it is to account for asymmetric dependency.

This study has a number of consequences for fund managers. Managers can use a conditional volatility model to quantify fat tail risk and optimise investment portfolios if asset return volatility is stochastic and clustered. Exponentially weighted covariance estimators, GARCH, stochastic volatility models, and realised volatility using HF data are some of the options suggested. In the case of HF results, noise from market microstructures must be taken into account. [Ait-Sahalia, 2010].

Investors can use a heavy-tailed distribution in the form of skewed-t distributions or other models if conditional returns are non-Gaussian. Finally, when calculating VaR and evaluating

portfolio weights, investors should consider asymmetric dependency. The skewed-t copula seems to be a solid and underplayed choice. The dispersion of our multivariate approach's risk estimates is a potential field for future research. This method is time-consuming to implement because it necessitates the calculation of a large number of parameters. In this context, it would be fascinating to put At-Sahalia and Brandt's more frugal portfolio selection strategy into practise (2001).

REFERENCES

- Adler, T., Kritzman, M., 2007. Mean-variance versus full-scale optimisation: In and out of sample. J. Asset Manage.
 7, 302-311.
- Aït-Sahalia, Y., Brandt, M.W., 2001. Variable selection for portfolio choice. J. Finance 56, 1297–1351.
- Ait-Sahalia, Y., Fan, J., Xiu, D., 2010. High-frequency covariance estimates with noisy and asynchronous financial data. J. Amer. Statist. Assoc. 492, 1504–1517.
- Aït-Sahalia, Y., Lo, A., 2000. Nonparametric risk management and implied risk aversion. J. Econometrics 94, 9-51.
- Allen D, Lizeri C., Satchel S., (2020) A comparison of non-Gaussian VaR estimation and portfolio construction techniques, 58 J. Empir. Financ. 58, 356–368.
- Alcock, J., Hatherley, A., 2009. Asymmetric dependence between domestic equity indices and its effect on portfolio construction. Aust. Actuar. J. 15, 143–180.
- Artzner, P., Delbaen, J.M., Eber, F., Heath, D., 1997. Thinking coherently. Risk 10, 68–71.
- Artzner, P., Delbaen, J.M., Eber, F., Heath, D., 1999. Coherent measures of risk. Math. Finance 9, 203-228.
- Berkowitz, J., O'Brien, J., 2002. How accurate are value-at-risk models at commercial banks? J. Finance 57, 1093–1111.
- Bollerslev, T., 1990. Modeling the coherence in short-run nominal exchange-rates A multivariate generalized ARCH model. Rev. Econ. Stat. 72, 498–505.
- Christoffersen, P.F., 2009. Value-At-Risk Models. Springer, Heidelberg, Berlin.
- Chen, X., Fan, Y., 2006. Estimation and model selection of semiparametric copula-based multivariate dynamic models under copula misspecification. J. Econometrics 135, 125–154.
- Fabozzi, F.J., Focardi, S., Jonas, C., 2007. Trends in quantitative equity management: Survey results. Quant. Finance 7, 115–122.
- Karolyi, G.A., Stulz, R.M., 1996. Why do markets move together? An investigation of US-Japan stock return comovements. J. Finance 51, 951–986.
- Mandelbrot, B., 1963. The variation of certain speculative prices. J. Bus. 36, 394–419.
- Markowitz, H.M., 1952. Portfolio selection. J. Finance 7, 77–91.
- Markowitz, H.M., 1959. Portfolio Selection: Efficient Diversification of Investments. John Wiley and Sons, Inc.
- Martellini, L., Ziemann, V., 2010. Improved estimates of higher-order comoments and implications for portfolio
- McAleer, M., Da Veiga, B., 2008. Single-index and portfolio models for forecasting value-at-risk thresholds. J. Forecasting 27, 217-23.
- Nystrom, K., Skoglund, J., 2002. A framework for scenario based risk management. SAS Institute and the University of Umea Working Paper.
- Viebig, J., Poddig, T., 2010. Modelling extreme returns and asymmetric dependence structures of hedge fund strategies using extreme value theory and copula theory. J. Risk 13, 23–55.
- West, K.D., Edison, H.J., Cho, D., 1993. A utility-based comparison of some models of exchange-rate volatility. J. Int. Econ. 35, 23–45.
- Xiong, J.X., Idzorek, T.M., 2011. The impact of skewness and fat tails on the asset allocation decision. Financ. Anal. J. 67, 23–35.

ANNEXURE I: MODEL FIT STATISTICS

BRAZIL (BVSP)

AR - EGARCH Model Results

Dep. Variable: BRAZIL (BVSP) R-squared: -0.000 Mean Model: AR Adj. R-squared: -0.000 Vol Model: EGARCH Log-Likelihood: 1138.88 Distribution: Standardized Skew Student's t AIC: -2265.77 Method: Maximum Likelihood BIC: -2241.41 No. Observations: 428 Date: Tue, Apr 20 2021 Df Residuals: 422 Time: 22:18:46 Df Model: 6 Mean Model coef std err t P> t 95.0% Conf. Int. Const 4.0389e-04 8.248e-04 0.490 0.624 [-1.213e-03,2.020e-03] Volatility Model coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.					
Vol Model: EGARCH Log-Likelihood: 1138.88 Distribution: Standardized Skew Student's t AIC: -2265.77 Method: Maximum Likelihood BIC: -2241.41 No. Observations: 428 Date: Tue, Apr 20 2021 Df Residuals: 422 Time: 22:18:46 Df Model: 6 Mean Model coef std err t P> t 95.0% Conf. Int. Const 4.0389e-04 8.248e-04 0.490 0.624 [-1.213e-03,2.020e-03] Volatility Model coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.	Dep. Variable:		BRAZIL (BVSP)	R-squared:	-0.000
Distribution: Standardized Skew Student's t AIC: -2265.77 Method: Maximum Likelihood BIC: -2241.41 No. Observations: 428 Date: Tue, Apr 20 2021 Df Residuals: 422 Time: 22:18:46 Df Model: 6 Mean Model coef std err t P> t 95.0% Conf. Int. Const 4.0389e-04 8.248e-04 0.490 0.624 [-1.213e-03,2.020e-03] Volatility Model coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.	Mean Model:		AR	Adj. R-squared:	-0.000
Method: Maximum Likelihood No. Observations: 428 No. Observations: 428 Pate: Tue, Apr 20 2021 Df Residuals: 422 Time: 22:18:46 Df Model: 6 Mean Model coef std err t P> t 95.0% Conf. Int. Const 4.0389e-04 8.248e-04 0.490 0.624 [-1.213e-03,2.020e-03] Volatility Model coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.	Vol Model:		EGARCH	Log-Likelihood:	1138.88
No. Observations: 428 Apr 20 2021 Df Residuals: 422 Time: 22:18:46 Df Model: 6 Mean Model	Distribution:	Standardized	Skew Student's t	AIC:	-2265.77
Date: Tue, Apr 20 2021 Df Residuals: 422 Time: 22:18:46 Df Model: 6 Mean Model coef std err t P> t 95.0% Conf. Int. Const 4.0389e-04 8.248e-04 0.490 0.624 [-1.213e-03,2.020e-03] Volatility Model coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.	Method:	Ma	aximum Likelihood	BIC:	-2241.41
Time:				No. Observations:	428
Time:	Date:		Tue, Apr 20 2021	Df Residuals:	422
coef std err t P> t 95.0% Conf. Int. Const 4.0389e-04 8.248e-04 0.490 0.624 [-1.213e-03,2.020e-03] Volatility Model coef std err t P> t 95.0% Conf. Int. Omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.	Time:				6
Const 4.0389e-04 8.248e-04 0.490 0.624 [-1.213e-03,2.020e-03] Volatility Model coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.			Mean Model		
Const 4.0389e-04 8.248e-04 0.490 0.624 [-1.213e-03,2.020e-03] Volatility Model coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.	=========				
Const 4.0389e-04 8.248e-04 0.490 0.624 [-1.213e-03,2.020e-03] Volatility Model coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.		coef std err	· t	P> t 95.0% Conf. Int.	
Volatility Model coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.			-		
Volatility Model coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.	Const 4.0	389e-04 8.248e-04	0.490	0.624 [-1.213e-03.2.020e-03]	
coef std err t P> t 95.0% Conf. Int. omega -0.3382 0.188 -1.795 7.271e-02 [-0.708,3.116e-02] alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.				[,,	
omega -0.3382					
omega -0.3382		coef std err	· t	P> t 95.0% Conf. Int.	
alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.					
alpha[1] 0.0842 3.931e-02 2.143 3.210e-02 [7.203e-03, 0.161] beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.	omega ·	-0.3382 0.188	-1.795 7.2	71e-02 [-0.708.3.116e-02]	
beta[1] 0.9581 2.313e-02 41.425 0.000 [0.913, 1.003] Distribution coef std err t P> t 95.0% Conf. Int.					
Distribution Coef std err t P> t 95.0% Conf. Int.					
coef std err t P> t 95.0% Conf. Int.	beca[1]			0.000 [0.515, 1.005]	
			D150.100010		
		coef std err	+	Ps + 95.0% Conf. Int.	
7 1367 2 278 3 132 1 7346-03 [2 671 11 602]				.,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	
	nu	7.1367 2.278	3.132 1.73	34e-03 [2.671, 11.602]	
lambda 0.0402 6.171e-02 0.652 0.515 [-8.075e-02, 0.161]					

Covariance estimator: robust

RUSSIA (MOEX)

AR - EGARCH Model Results

Dep. Varia	able:		RUSSIA (MC	OEX) R-so	quared:	-0.000
Mean Model	l:			AR Adj.	. R-squared:	-0.000
Vol Model:	:		EG/	ARCH Log-	-Likelihood:	1043.99
Distributi	ion: Sta	ndardized S	kew Student'	s t AIC:	:	-2075.99
Method:		Max	cimum Likelih	nood BIC:	:	-2051.63
				No.	Observations:	428
Date:		Т	ue, Apr 20 2	2021 Df F	Residuals:	422
Time:			22:18	8:46 Df N	Model:	6
			Mean Model			
				.======		
			t		95.0% Conf. Int.	
Const		1.034e-03		3.601e-02	[1.415e-04,4.196e-03]	
	coef	std err	t	P> t	95.0% Conf. Int.	
omega	-0.0339	3.956e-04	-85.653	0.000	[-3.466e-02,-3.311e-02]	
alpha[1]	-0.0392	6.825e-03	-5.744	9.268e-09	[-5.257e-02,-2.582e-02]	
beta[1]	0.9958	1.272e-07	7.829e+06	0.000	[0.996, 0.996]	
		D	istribution			
=======	coef		t	P> t	95.0% Conf. Int.	
					[1.550, 15.397]	
lambda	0.0699	5.283e-02	1.323	0.186	[-3.364e-02, 0.173]	

Covariance estimator: robust

INDIA (NIFTY)

AR - EGARCH Model Results

Dep. Varia	able:		INDIA (NIFT)	 R-so 	quared:	-0.000
Mean Mode	1:			AR Adj.	. R-squared:	-0.000
Vol Model	:		EGAR	CH Log-	-Likelihood:	1362.65
Distribut:	ion: Sta	ndardized S	kew Student's	t AIC:	:	-2713.29
Method:		Max	imum Likelihoo	od BIC:	:	-2688.94
				No.	Observations:	428
Date:		Ti	ue, Apr 20 202	21 Df F	Residuals:	422
Time:			22:18:4	46 Df N	Model:	6
			Mean Model			
	coef	std err	t	P> t	95.0% Conf. Int.	
						-
Const	9.1317e-05			0.852	[-8.688e-04,1.051e-03]	
		Vol	atility Model			
					05 00 0 5 T ·	
					95.0% Conf. Int.	
					[-1.200, 0.237]	
					[-3.025e-02, 0.126]	
					[0.869, 1.026]	
			stribution		. , ,	
	coef	std err	t	P> t	95.0% Conf. Int.	
					[2.283, 10.258]	
lambda	-0.0947	6.165e-02	-1.536	0.125	[-0.215,2.616e-02]	

Covariance estimator: robust

CHINA (SSE)

AR - EGARCH Model Results

Dep. Vari	able:		CHINA (SSE) R-so	quared:		-0.000
Mean Mode	1:			AR Adj.	R-squared:		-0.000
Vol Model	:		EG	ARCH Log-	Likelihood:		1145.18
Distribut	ion: Sta	ndardized Sk	ew Student	's t AIC:			-2278.36
Method:		Maxi	mum Likeli	hood BIC:			-2254.01
					Observations	:	428
Date:		Tu		2021 Df F			422
Time:				8:46 Df M	Nodel:		6
			Mean Model				
	coef	std err	t	P> t	95.0%	Conf. Int.	
Const	3.7923e-04	6.346e-04	0.598	0.550	[-8.645e-04,	1.623e-03]	
		Volat	ility Mode			-	
		std err	t	P> t			
omega	-0.0124	4.513e-02			[-0.101,7.6	04e-02]	
beta[1]	0.1668 0.9963	5.554e-03	179.380	0.000	[0.985,	1.007]	
			tribution				
		std err	t	P> t			
nu	3.4496	0.615				4.6541	
	-0.0776						

Covariance estimator: robust

