Math. 304 Number Theory (Spring 2019)
Preparation No. 5

March 11, 2019

- 일반 강의로 들을 학생은 퀴즈를 제출하지 않고 나중에 과제물을 제출하면 됨.

- Study the following materials from the text:

(a) Section 4.1: (1) Reduction of general quadratic congruence to congruence $x^2 \equiv a (\mathrm{mod}\ p)$, (2) Definition of a is quadratic residue (resp. non-residue) (mod n), (3) Definition of Legendre symbol $(\frac{a}{p})$ where p is a prime and $(a, p) = 1$.

(b) Section 4.2: (1) Euler's criterion that $(\frac{a}{p}) \equiv a^{\frac{p-1}{2}}$. (2) Proof of Euler's criterion.

- Quiz ♯5.

1. Do Exercise (vi) on page 25.

2. Do Exercise (vii) on page 26.

3. Do Exercise $(viii)$ on page 26.

4. Let n be a natural number. Show that $\sum_{d|n} \phi(d) = n$.

5. Let j be an integer with $j \geq 3$.

(i) Show that the order of 5 (mod 2^j) is 2^{j-2}.

(ii) Show that every odd integer a is congruent (mod 2^j) to just one integer of the form $(-1)^l 5^m$, where $l = 0, 1$ and $m = 0, 1, \ldots, 2^{j-2} - 1$.

6. Use definition to compute the Legendre symbols $(\frac{a}{13})$ where $a = 1, 2, \ldots, 12$.

The End