Math. 304 Number Theory (Spring 2019)
Preparation No. 6

March 13, 2019

- 일반 강의로 들을 학생은 퀴즈를 제출하지 않고 나중에 과제물을 제출하면 됨.

- Study the following materials from the text:

(a) Section 4.3: (1) Gauss' lemma, (2) Computation of $(\frac{-1}{p}), (\frac{2}{p}), (\frac{3}{p}), (\frac{5}{p})$ via Gauss' lemma, (3) Quadratic reciprocity law (weak form), (4) Quadratic reciprocity law (1-dimensional proof).

- Quiz $\sharp 6$.

1. Let p be an odd prime, a an odd integer, and $k(\geq 3)$ an integer.

(a) Show: If we square the 2^{k-3} odd integers between 1 and 2^{k-2}, no two of the squares are congruent to modulo 2^k.

(b) Prove that $x^2 \equiv a \pmod{2^k}$ is soluble if and only if $a \equiv 1 \pmod 8$.

(c) Show: If $a \equiv 1 \pmod 8$ and s is a solution of $x^2 \equiv a \pmod{2^k}$, then $\pm s, \pm s + 2^{k-1}$ are all solutions of the congruence.

2. Use Gauss' lemma to compute $(\frac{11}{31})$.

3. Let p be an odd prime. In the text it is proved that $(\frac{2}{p}) = 1$ if $p \equiv 1, 7 \pmod 8$ and $(\frac{2}{p}) = -1$ if $p \equiv 3, 5 \pmod 8$. Do the similar job for $(\frac{3}{p})$.

4. Do Exercise (i) on page 33.

5. Do Exercise (ii) on page 33.

6. Determine whether the following congruences are soluble for integers x, y which are not congruent to zero (mod 17).

(a) $x^2 + 15y^2 \equiv 0 \pmod{17}$.

(b) $x^2 + 11y^2 \equiv 0 \pmod{17}$.

The End