Math. 304 Number Theory (Spring 2019)
Preparation and Assignment No. 2

March 4, 2019

1. Let n be a natural number and a an integer relatively prime to n. Prove:

(a) If $\{a_1, a_2, \cdots, a_n\}$ is a complete set of residues modulo n, so is $\{aa_1, aa_2, \cdots, aa_n\}$.

(b) If $\{a_1, a_2, \cdots, a_{\phi(n)}\}$ is a reduced set of residues modulo n, so is $\{aa_1, aa_2, \cdots, a_{\phi(n)}\}$.

2. Solve the linear congruence $3x \equiv 9 (\text{mod } 24)$.

3. Solve the simultaneous linear congruences $x \equiv 1 (\text{mod } 9), x \equiv 5 (\text{mod } 7), x \equiv 3 (\text{mod } 5)$.

4. Use congruence technique to show that the following Diophantine equations are not soluble in integers.

(i) $x^2 + y^2 = 103$.

(ii) $x^2 + y^2 + z^2 = 95$.

5. Show that $x = 6$ is the only solution to the equation $x^3 = (x-1)^3 + (x-2)^3 + (x-3)^3$ in positive integers.

6. Find the last digit of 7^{139} and 13^{2018}.

7. Show that the Diophantine equation $x^2 + 1 = 23y$ has no integer solutions.

8. Let a be an odd integer. Show that if $k \geq 3$, $a^{2^{k-2}} \equiv 1 (\text{mod } 2^k)$.

The End