Math. 304 Number Theory (Spring 2019)
Preparation and Assignment No. 1

February 25, 2019

1. Prove:

(a) Let a and b be natural numbers and n an integer. Show that the Diophantine equation $ax + by = n$ is soluble in integers x, y if and only if d divides n, where d denotes the greatest common divisor of a and b.

(b) Let $a_1, a_2, \ldots, a_m$ be natural numbers. Define a greatest common divisor of $a_1, a_2, \ldots, a_m$ and show that it exists uniquely for any natural number m.

(c) Show that the Diophantine equation $a_1x_1 + a_2x_2 + \ldots + a_mx_m = n$ is soluble in integers if and only if d divides n, where d denotes the greatest common divisor of $a_1, a_2, \ldots, a_m$.

2. Find integers x, y such that $95x + 432y = 1$.

3. Find integers x, y, z such that $35x + 55y + 77z = 1$.

4. Prove that $1 + \frac{1}{2} + \cdots + \frac{1}{n}$ is not an integer for $n > 1$.

5. Show that there are infinitely many primes of the form $4n + 3$.

6. Show that there are infinitely many primes of the form $4n + 1$.

7. Show that the series $1 + \frac{1}{2} + \cdots + \frac{1}{n} + \cdots$ diverges.

8. Show that the series $1 + \frac{1}{2} + \cdots + \frac{1}{p} + \cdots$, where p runs over all primes, diverges.

The End