

**Q1) Compare man-made electrical and bioelectrical systems. Give examples of both of them.**

Bioelectricity is recognized to follow the identical laws of electricity in the atmosphere, conducting wires, semiconductors, etc. Despite this, there are some important distinctions among bioelectrical systems and man-made electrical systems.

### Man-made systems:

- Charge carriers are electrons within a conductor.
- Current flow within (insulated) conductors.
- Have a localized energy source such as battery, that drives currents through the interior of conductor such as wire, with currents outside the wire being due to leakage or other imperfections.

**Example:** Household electric bulb that runs off AC, so it is glowing and fading at 60 times per second, which hardly makes any difference to the light emitted since the change between on and off is occurring at rate imperceptible to the naked eye.

### Bioelectrical systems:

- Charge carriers are ions within an electrolyte
- Current flow inside and outside of (particularly insulated) cell membranes.
- Derive their electrical energy from the ionic concentration differences across cell membranes. Thus the energy resources intrinsically are distributed in space along the membrane and involve a flow of current across membrane. Thus current necessarily flows both inside and outside electrically active cells.

**Example:** Heart, as an electricity generating power station. The blood vessels act like electric wires, while blood represents the charges or electrons flowing in each wire. Then, voltage is analogous to the pressure in blood vessels, and vascular resistance will be  $\propto$  Ohms that is proportional to blood flow.

Q2) Derive Nernst-Planck equation in 2D.

④ Ions respond to electric field

$$\vec{J}_e = -u_p \cdot \frac{\vec{z}_p}{|z_p|} \cdot c_p \cdot \vec{\nabla} \Phi$$

$$E = -\vec{\nabla} \cdot \vec{\Phi}$$

$$\vec{J}_D = -D_p \cdot \vec{\nabla} C \quad \{ \text{flux due to diffusion} \}$$

$$\vec{J}_e = -u_p \cdot \frac{\vec{z}_p}{|z_p|} \cdot c_p \cdot \vec{\nabla} \Phi \quad \{ \text{flux due to electric field} \}$$

$$\text{Total flux: } \vec{J}_p = \vec{J}_D + \vec{J}_e$$

$$= -D_p \cdot \vec{\nabla} \cdot C_p - u_p \cdot \frac{\vec{z}_p}{|z_p|} \cdot c_p \cdot \vec{\nabla} \Phi$$

$$\xrightarrow{\text{diffusion}} D_p = \frac{u_p \cdot R \cdot T}{|z_p| \cdot F} \Rightarrow u_p = \frac{D_p \cdot |z_p| \cdot F}{R \cdot T}$$

$$\vec{J}_p = -D_p \left( \vec{\nabla} C_p + \frac{\vec{z}_p \cdot (c_p \cdot F) \cdot \vec{\nabla} \Phi}{R \cdot T} \right) \quad \left[ \begin{array}{l} \text{Nernst-Planck} \\ \text{Equation} \end{array} \right]$$

$$\vec{J}_p = F \cdot z_p \cdot \vec{J}_p = -D_p \cdot F \cdot z_p \left( \underbrace{\vec{\nabla} C_p}_{\substack{\downarrow \text{ion flux} \\ \text{density}}} + \underbrace{\frac{\vec{z}_p \cdot (c_p \cdot F) \cdot \vec{\nabla} \Phi}{R \cdot T}}_{\substack{\text{concentration} \\ \text{difference can} \\ \text{generate electrical} \\ \text{current}}} \right)$$

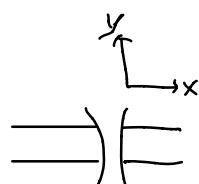
drift due to  
 electrical field  
 on the membrane

At equilibrium, the electric field force is balanced by diffusion force

$$\vec{J}_p = 0$$

$$\vec{J}_p = 0 = -D_p \cdot F \cdot z_p \left( \underbrace{\vec{\nabla} C_p}_{=0} + \underbrace{\frac{\vec{z}_p \cdot (c_p \cdot F) \cdot \vec{\nabla} \Phi}{R \cdot T}}_{=0} \right)$$

$$\underbrace{\frac{\partial C_p}{\partial x} \hat{x} + \frac{\partial C_p}{\partial y} \hat{y}}_{=0} = -\hat{x} \underbrace{\frac{z_p c_p F}{R \cdot T} \cdot \frac{\partial \Phi}{\partial x}}_{=0} - \hat{y} \underbrace{\frac{z_p c_p F}{R \cdot T} \cdot \frac{\partial \Phi}{\partial y}}_{=0}$$



$$\frac{\partial C_P}{\partial x \cdot C_P} = \frac{z_p \cdot F}{R \cdot T} \cdot \frac{\partial \Phi}{\partial x} \quad \mid \quad \frac{\partial C_P}{\partial y \cdot C_P} = \frac{z_p \cdot F}{R \cdot T} \cdot \frac{\partial \Phi}{\partial y}$$

both gives

$$\frac{\partial C_P}{C_P} = \left( \frac{z_p \cdot F}{R \cdot T} \right) \frac{\partial \Phi}{\partial x}$$

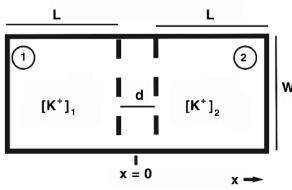
$$\int_e^i \frac{\partial C_P}{C_P} = - \left( \frac{z_p \cdot F}{R \cdot T} \right) \cdot \int_e^i \frac{\partial \Phi}{\partial x}$$

$$\ln([C_P]_i) - \ln([C_P]_e) = \left( \frac{z_p \cdot F}{R \cdot T} \right) (\Phi_i - \Phi_e)$$

Nernst-Planck  
Potential

$$V_m^{eq} = \Phi_i - \Phi_e = - \frac{R \cdot T}{z_p \cdot F} \cdot \ln \left( \frac{[C_P]_i}{[C_P]_e} \right) \text{ mV}$$

### Q3)



Exercises 9–15. Concentration cells.

**Exercises 9 to 15: Concentrations and Flux:** Two chambers numbered 1 and 2 have the following dimensions (see the figure). Dimension  $L$  is 100 microns, and  $W$  is 100 microns. Height  $H$  (out of surface) is 100 microns. The dotted lines show the edges of a boundary layer that has a thickness of 40 Å. (The dotted lines identify the position of a transition region, not a physical boundary.) The temperature is 300 degrees Kelvin, and the diffusion coefficient  $D_K$  is 1.96E-5 cm<sup>2</sup>/sec. Within either chamber the concentration and electric potential are uniform, though different between chambers. The  $K^+$  concentrations are  $K_1$  in chamber 1 and  $K_2$  in chamber 2. A linear change occurs in concentration and potential across the transition region. The potential

difference is the voltage  $V$ , where  $V = \phi_1 - \phi_2$ . Note the polarity, i.e., a positive value for  $V$  occurs when the potential in chamber 1 is the higher.

9. Concentration  $K_1$  is 0.22 mM and  $K_2$  is 0.02 mM. V is -0.04 Volts.

- a. At  $x = 0$ , what is  $I_K^D$ , the potassium current along  $x$  due to diffusion?

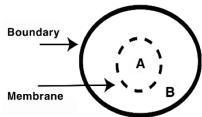
$$\begin{aligned} I_K^D &= \bar{J}_K^D \cdot A = -D_K \cdot F \cdot z_K \cdot (\nabla C_K) \cdot A \\ &= (-1.96 \times 10^{-5}) \left[ \frac{\text{cm}^2}{\text{s}} \right] (10^{-4}) \left[ \frac{\text{m}^2}{\text{cm}^2} \right] (96487) \left[ \frac{\text{C}}{\text{mol}} \right] \left( \frac{0.02 - 0.02}{40 \times 10^{-10}} \right) \left[ \frac{\text{Mol}}{\text{L} \times \text{m}} \right] (1 \times 10^3 \times 1000) \left[ \frac{\text{L}}{\text{m}^3} \right] (+) (100 \times 10^6)^2 \left[ \frac{\text{m}^2}{\text{m}^2} \right] \\ &= 945.5 \times 10^{-7} \text{ A} \\ &= 94.5 \text{ nA} \\ &\quad \left( \frac{0.02 - 0.02}{40 \times 10^{-10}} \right) \left[ \frac{\text{Mol}}{\text{L} \times \text{m}} \right] (1 \times 10^3 \times 1000) \left[ \frac{\text{L}}{\text{m}^3} \right] (+) (100 \times 10^6)^2 \left[ \frac{\text{m}^2}{\text{m}^2} \right] \end{aligned}$$

- b. At  $x = 0$ , what is  $I_K^E$ , the potassium current along  $x$  due to the electric field?

$$\begin{aligned} I_K^E &= J_K^E \cdot A = -D_K \cdot F \cdot z_K \cdot \left( \frac{z_K \cdot F \cdot C_K}{R \cdot T} \right) \cdot (\nabla C_K) \cdot A \\ &= (-1.96 \times 10^{-5}) \left[ \frac{\text{cm}^2}{\text{s}} \right] (96487) \left[ \frac{\text{C}}{\text{mol}} \right] (+) \left( \frac{(+) (96487) (0.02 \times 10^{-3})}{(8.314) (300)} \right) \left( \frac{0.04}{40 \times 10^{-10}} \right) \left( 100 \times 10^6 \right)^2 \times \left( \frac{1 \text{ M}}{100 \text{ cm}} \right)^2 \\ &= -8.78 \times 10^{-5} \text{ A} \\ &= -87.8 \text{ nA} \end{aligned}$$

12. Concentration  $K_1$  is 0.4 mM and  $K_2$  is 0.02 mM. Voltage  $V$  is 0. What is  $j_K^D$  (the particle flux) along the  $x$  axis?

$$\begin{aligned} \bar{j}_K^D &= -D_K \cdot \nabla C_K \\ &= (-1.96 \times 10^{-5}) \left[ \frac{\text{cm}^2}{\text{s}} \right] (1 \times 10^{-3}) \left[ \frac{\text{m}^2}{\text{cm}^2} \right] \left( \frac{0.02 - 0.4}{40 \times 10^{-10}} \right) \left[ \frac{\text{Mol}}{\text{L} \times \text{m}} \right] (1 \times 10^3 \times 1000) \left[ \frac{\text{L}}{\text{m}^3} \right] \\ &= -0.186 \times 10^{-4} \frac{\text{mol}}{\text{s} \times \text{cm}^2} \end{aligned}$$



Exercises 16-20: The membrane is indicated by the dashed line.  
No flow occurs across the outer boundary.

16. Only sodium may cross the membrane separating the two compartments. The temperature is 24°C. What is the equilibrium potential?

$$V_m^{eq} = \frac{RT}{4F} \ln\left(\frac{(C_p)_B}{(C_p)_A}\right) \rightarrow mV$$

Exercises 16-21: Concentrations (mM) <sup>a</sup>		
Ion	A	B
K <sup>+</sup>	280	10
Na <sup>+</sup>	61	485
Cl <sup>-</sup>	51	485

<sup>a</sup> mM means millimoles per liter.

$$T = 273 + 24 = 297 \text{ K} \quad F \rightarrow \text{faraday constant}$$

$$R = 8.314 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1} \quad 2 \rightarrow \text{valence of Na}^+(+1)$$

$$E_{Na} = V_m^{eq} = \left[ \frac{8.314 \times 297}{4 \times 96485} \right] \ln\left(\frac{485}{61}\right) = 0.053 \frac{\text{J}}{\text{C}} = 53 \text{ mV}$$

19. The transmembrane potential is 1 millivolt, and the temperature is 18°C. How many ion species (Na<sup>+</sup>, K<sup>+</sup>, or Cl<sup>-</sup>) are NOT in equilibrium? (Allow a small tolerance for round-off.)

+ Three ion species are not in equilibrium at this transmembrane potential

$$\text{for } Na^+ : \quad V = \frac{RT}{nF} \ln\left(\frac{485}{61}\right)$$

$$\ln\left(\frac{1}{10}\right) \quad \frac{1}{n} \ln\left(\frac{485}{61}\right) = n^F$$

$$0.001 = \frac{8.314 \times 291}{n \times 96485} \ln\left(\frac{485}{61}\right)$$

$$n = 51.98 \text{ moles} \rightarrow \text{Number of ions} = 51.98 \times 6.023 \times 10^{23}$$

$$V_m^{Na} = 52 \text{ mV}$$

$$\text{for } K^+ : \quad V = \frac{RT}{nF} \ln\left(\frac{10}{280}\right)$$

$$= 3.13 \times 10^{25} \text{ ions}$$

$$0.001 = \frac{8.314 \times 291}{n \times 96485} \ln\left(\frac{10}{280}\right)$$

$$n = - \text{ moles} \rightarrow \text{Number of ions} = \times 6.023 \times 10^{23}$$

$$V_m^{K^+} = -93.55 \text{ mV}$$

$$\text{for } Cl^- : \quad V = \frac{RT}{nF} \ln\left(\frac{51}{51}\right)$$

$$= \times 10^{25} \text{ ions}$$

$$0.001 = \frac{8.314 \times 291}{n \times 96485} \ln\left(\frac{485}{51}\right)$$

$$n = - \text{ moles} \rightarrow \text{Number of ions} = \times 6.023 \times 10^{23}$$

$$V_m^{Cl^-} = -56.48 \text{ mV}$$

$$= \times 10^{25} \text{ ions}$$

28. A cardiac cell has length, width, and depth of 82, 12, and 5  $\mu\text{m}$ , respectively. At rest the membrane resistivity ( $\Omega\text{cm}^2$ ) is 20,000 and the capacitance is 1 microFarad per  $\text{cm}^2$ . If the cell membrane functions as a passive, parallel resistor-capacitor combination, what is the time constant of this cell?

+ A cardiac cell is represented by the shape of the brick

$$l = 82 \mu\text{m}$$

$$w = 12 \mu\text{m}$$

$$d = 5 \mu\text{m}$$

$$\text{Resistivity} = \rho = 20,000 \Omega/\text{cm}^2$$

$$\text{Capacitance} = C = 1 \text{ NF/cm}^2 \cdot A = 1 \times 10^{-2} \frac{\text{F}}{\text{m}^2} \cdot A = 4.1 \times 10^{-14} \text{ F} = 4.1 \times 10^{-12} \text{ F}$$

$\text{(cross-sectional area} = A = l \times d = 4.1 \times 10^{-12} \text{ m}^2\text{)} = 4.1 \times 10^{-10} \text{ m}^2 = 4.1 \times 10^{-6} \text{ cm}^2$

Time constant  $\tau = R \cdot C$

$$R = \frac{20,000}{4.1 \times 10^{-6}} \Omega = 4.88 \times 10^9 \Omega$$

$$\tau = R \cdot C = 4.88 \times 10^9 \times 4.1 \times 10^{-12}$$

$$= 20,008 \text{ msec}$$

30. What are suitable units for each of the following:

a.  $J$ , the current flux?  $\left[ \frac{\text{Coulombs}}{\text{sec} \cdot \text{cm}^2} \right]$

b.  $j$ , the particle flux?  $\left[ \frac{\text{moles}}{\text{sec} \cdot \text{cm}^2} \right]$

c.  $C$ , the capacitance?  $\left[ \frac{\text{coulombs}}{\text{volts}} \right]$

d.  $D$ , the diffusion coefficient?  $\left[ \frac{\text{m}^2}{\text{s}} \right]$

e.  $\phi$ , the electric potential? Volts

f.  $\mu$ , the mobility of an ion in response to an electric field?  $\left( \frac{\text{m}}{\text{sec}} \right) \left( \frac{\text{Volts}}{\text{m}} \right) = \left( \frac{\text{m}^2}{\text{Volts} \cdot \text{sec}} \right)$

**Exercises 34-45:** Short Nerve Segment. These exercises involve a short segment of an unmyelinated nerve. Each nerve segment has a circular cross-section with diameter  $d$  and length  $L$ . The temperature is 300 degrees Kelvin, and the membrane capacitance is 1.2 microFarads per cm<sup>2</sup>. The transmembrane potential (inside potential minus outside) is clamped to -40 millivolts. There is no variation of membrane potential along the axial coordinate. Use the parallel-conductance model for analysis, as needed. Membrane conductances are:  $g_K = 0.375$ ,  $g_{Na} = 0.01$  mS/cm<sup>2</sup>,  $g_L = 0.57$  mS/cm<sup>2</sup>. The membrane conductances are assumed to be constants, in these exercises. If more values are given than are needed to find the answer, a part of the exercise is selecting the relevant information.

Concentrations (mM)		
Ion	Intra	Extra
K+	280	10
Na+	50	437
Cl-	51	485

ward.

r 39. -0.960 microampères per square centimeter ( $\mu\text{A}/\text{cm}^2$ ). Note that the current is inward.

34. What is the resting potential?

The resting potential of a neuron or other excitable cell, relative to its surroundings when not stimulated or involved in passage of an impulse During resting, the membrane is negative on the inside. The resting membrane potential is maintained because of ionic gradient.

$$T = 300 \text{ K}$$

$$R = 8.314 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$$

$F$  = faraday constant

$z$  = +ve valence of

$$E_{Na} = V_m = \left[ \frac{8.314 \times 300}{1 \times 96,485} \text{ coul} \right] \ln \left( \frac{10}{50} \right) = 0.05614 \frac{\text{J}}{\text{C}} = 56.14 \text{ mV}$$

$$E_K = V_m = \left[ \frac{8.314 \times 300}{1 \times 96,485} \text{ coul} \right] \ln \left( \frac{10}{230} \right) = 0.05614 \frac{\text{J}}{\text{C}} = -86.14 \text{ mV}$$

$$E_{Cl} = V_m \left[ \frac{8.314 \times 300}{1 \times 96,485} G_{Cl} \right] \ln \left( \frac{195}{51} \right) = -0.09912 \frac{V}{C} = -99.12 \text{ mV}$$

$$g_K = 0.375 \text{ mS/cm}^2$$

$$g_{Na} = 0.01 \text{ mS/cm}^2$$

$$g_{Cl} = 0.77 \text{ mS/cm}^2$$

$$\begin{aligned} V_{rest} &= \frac{g_K E_K + g_{Cl} E_{Cl} + g_{Na} E_{Na}}{g_K + g_{Na} + g_{Cl}} \\ &= -67.99 \text{ mV} \end{aligned}$$

35. What is  $E_K$ , the equilibrium voltage for  $K^+$  ions?

$E_K$  is  $-86.14 \text{ mV}$  calculated in above.

39. What is  $J_{Na}$ , the current density for  $Na^+$  ions?

$$J_{Na} = I_{Na} = g_{Na} (V_m - E_{Na}) = 0.01 \frac{\text{mS}}{\text{cm}^2} \times (-67.99 - 27) = -0.960 \frac{\mu \text{A}}{\text{cm}^2}$$

current is inward.

Consider 6 and boundary conditions,

$$D.z'' - D\alpha z' + (k_1 + k_2) \cdot z = 0, \quad z(0) = C_0, \quad z(\infty) = 0$$

$$\text{Simplifying, } z'' - \frac{\alpha z'}{D} + \frac{(k_1 + k_2)}{D} \cdot z = 0 \rightarrow \text{ODE} \rightarrow \text{seek solution } z(z) = e^{mz}$$

$\downarrow$

$$m^2 - \alpha m + \frac{k_1 + k_2}{D} = 0$$

$$m = \frac{\alpha \pm \sqrt{\alpha^2 - 4 \frac{(k_1 + k_2)}{D}}}{2}, \quad (m = m_1(+), \text{ and } m_2(-))$$

where  $m$  is constant,  
may be complex.  
 $\backslash, -- \text{ constants } (m^{-2})$

The general solution of (8)

$$\text{Simplifying, } z(z) = P_1 e^{m_1 z} + P_2 e^{m_2 z} \quad \text{where } P_1, P_2 \text{ constants}$$

Now consider boundary conditions

$$z(0) = P_1 + P_2 = C_0 \rightarrow P_2 = C_0 - P_1$$

$$z(z+\omega) = P_1 e^{m_1 z} + P_2 e^{m_2 z} = 0 \rightarrow P_1 = 0, P_2 = C_0$$

$$\text{Therefore, } z = C_0 e^{m_2 z}$$

(5) can be rewritten as

$$DrR'' + (1 - D\alpha r)R' - k_2 r R = 0$$

$$P = Dr, \quad Q = (1 - D\alpha r), \quad S = -k_2 r, \quad \text{let } P = Dr = 0, \quad D \neq 0, \quad \text{we know } r \neq 0$$

$$\frac{Q}{P} = \frac{1 - D\alpha r}{Dr} \quad \text{and} \quad \frac{S}{P} = \frac{-k_2 r}{Dr} = -\frac{k_2}{D}$$

To determine  $r=0$  is a regular or irregular singular point, look

$$(r-0) \cdot \frac{Q}{P} = r \cdot \frac{1 - D\alpha r}{Dr} = \frac{1}{D} - \alpha r \quad \text{and}$$

$$(r-0)^2 \frac{S}{P} = r^2 \frac{-k_2 r}{Dr} = -\frac{k_2 r^2}{D}$$

Both are analytical at  $r=0$ , thus  $r=0$  is regular singular point.  
If  $r=0$  is a regular singular point, we attempt to obtain a  
Frobenius series solution, which is series of form

$$R = \sum_{n=0}^{\infty} b_n r^{n+q}, \quad b_0 \neq 0$$

$\underline{r \rightarrow \infty}$

$n$  is real constant

$q = q_1, q_2, \dots$  multiple of  $b_0$

$b_n$  coefficient

→ Nernst-Planck Eq with initial & boundary conditions in 2D cylindrical space with radial symmetry (suppose that there is axisymmetry so that concentration does not vary with  $\theta$ )

$$\frac{\partial C}{\partial t} = D \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right) - D\alpha \left( \frac{\partial C}{\partial r} + \frac{\partial C}{\partial z} \right) \quad \textcircled{1}$$

$$C(r_1, z, 0) = 0$$

$$\alpha = \frac{z_i FFE}{RT} \text{ cm}^{-1}, z_i = \text{ion valence}$$

$$C(r_1, 0, t) = C_0$$

$$F = \text{faraday constant} = 96500 \frac{\text{coulombs}}{\text{mole}}$$

$$C(r, z \rightarrow \infty, t_m) = 0$$

$$E = \frac{N V}{l} \text{ V cm}^{-1} \quad \begin{matrix} \text{potential drop} \\ \text{across specimen} \end{matrix}$$

$$C(r, z, t) = R(r) Z(z) T(t) \quad \textcircled{2}$$

radical thick time effects

$$R = \text{gas constant} = 8.31 \frac{\text{J}}{\text{mole.K}}$$

T = Temperature  $\text{K}$

$$C(r, z, t) = \text{pore solution concentration}$$

$$D = \text{diffusion, migration coefficient}$$

$\text{cm}^2/\text{sec}$

Putting (2) to (1) :

$$\frac{D R'' + (r^{-1} - D\alpha) R'}{R} + \frac{DT'' - D\alpha Z'}{Z} = \frac{\dot{T}}{T} = -K_1 \quad \textcircled{3}$$

$$K_1 \rightarrow \text{separation constant} \rightarrow \text{sec}^{-1} \quad \dot{T} + K_1 T = 0 \quad \longrightarrow \quad \frac{\dot{T}}{T} = -K_1 \quad \textcircled{4}$$

$$\frac{D R'' + (r^{-1} - D\alpha) R'}{R} = -K_1 - \frac{DT'' - D\alpha Z'}{Z} = K_2$$

$$D R'' + (r^{-1} - D\alpha) R' - K_2 R = 0 \quad \textcircled{5}$$

$$D Z'' - D\alpha Z' + (K_1 + K_2) Z = 0 \quad \textcircled{6}$$

Now, considering <sup>initial and</sup> boundary conditions,

$$C(r, z, 0) = R(r) Z(z) T(0) = 0 \Rightarrow Z(z) = 0, T(0) \neq 0, R(r) \neq 0$$

$$C(r, 0, 0) = R(r) Z(0) T(0) = 0 \Rightarrow Z(0) = 0, T(0) \neq 0, R(r) \neq 0$$

and

$$C(r, z, t_m) = R(r) Z(z) T(t_m) = 0 \Rightarrow Z(z) = 0, T(t_m) \neq 0, R(r) \neq 0$$

After integrating (5),  $T(+)=\alpha e^{-K_1 t}$  where  $\alpha=\text{constant}$ ,  $\alpha=T(0) \neq 0$

$$T(+) = T(0) \cdot e^{-K_1 t} \quad \textcircled{7}$$

$$R' = \sum_{n=0}^{\infty} (n+q) b_n r^{n+q-1}$$

$$R'' = \sum_{n=0}^{\infty} (n+q)(n+q-1) b_n r^{n+q-2}$$

$$\text{④ } D \sum_{n=0}^{\infty} (n+q)(n+q-1) b_n r^{n+q-1} + \sum_{n=0}^{\infty} (n+q) b_n r^{n+q-1} - D \alpha \sum_{n=1}^{\infty} (n+q-1) b_{n-1} r^{n+q-1} - K_2 \sum_{n=2}^{\infty} b_{n-2} r^{n+q-1} = 0$$

combine these series into one summation, from  $n=2$ , and write terms corr.  $\underset{n=1}{\cancel{n=0}}$

$$R(r) = \sum_{n=0}^{\infty} b_n r^{n+1}, n=0, 1, 2$$

$$= b_0 + \sum_{n=1}^{\infty} b_n r^{n+1}, n=1, 2, \dots$$

$$(r, t) = C_0 T(t) e^{m_2 r} e^{-K_1 t} \sum_{n=0}^{\infty} b_n r^{n+1}$$

$T(t) \rightarrow \text{time function at } t=0$   
 $m_2 = \frac{\alpha - \sqrt{\alpha^2 - 4 \frac{K_1 + K_2}{D}}}{2}$

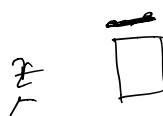
$K_1, K_2 \rightarrow \text{separation constants}$   
 $b_n \rightarrow \text{coefficients}$   
 $n \rightarrow \text{exponent of } r.$

$$\vec{J}_P = -D_P \cdot F \cdot z_P \left[ \vec{\nabla} C_P + \frac{z_P (P \cdot F) \vec{P} \Phi}{R \cdot T} \right]$$

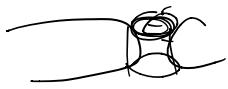
At equilibrium, the electric field force is balanced by diffusion force

$$\vec{J}_P = 0$$

$$\vec{J}_P = 0 = -D_P \cdot F \cdot z_P \left[ \vec{\nabla} C_P + \frac{z_P (P \cdot F) \vec{P} \Phi}{R \cdot T} \right]$$



$$\frac{\partial \overset{\circ}{C_P}}{\partial r} + \frac{\partial \overset{\circ}{C_P}}{\partial z} \cdot \hat{z} = - \hat{r} \frac{\overset{\circ}{Z_P C_P F}}{R \cdot \tau} \cdot \frac{\partial \Phi}{\partial r} - \hat{z} \frac{\overset{\circ}{Z_P C_P F}}{R \cdot \tau} \cdot \boxed{\frac{\partial \Phi}{\partial z}}$$



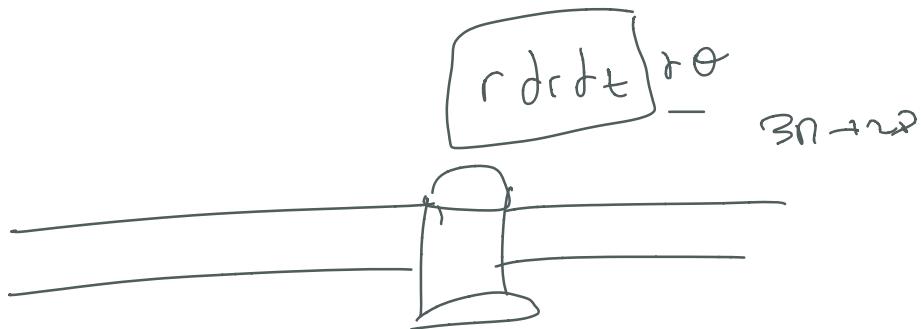
$$\frac{1}{C_P} \frac{d C_P}{d z} = - \left( \frac{Z_P F}{R \cdot \tau} \right) \frac{\partial \Phi}{\partial z}$$

; (intracellular)

$$\int_e^i \frac{d C_P}{C_P} = - \left( \frac{Z_P F}{R \cdot \tau} \right) \int_e^i d \Phi$$

$$\ln(C_P)_i - \ln(C_P)_e = \left( \frac{Z_P F}{R \cdot \tau} \right) \underbrace{(\Phi_i - \Phi_e)}_{= V_m}$$

$$V_m = \frac{-R \cdot \tau}{Z_P F} \cdot \ln \left( \frac{C_P)_i}{C_P)_e} \right)$$



... 0 ...

431 71

✓

←→