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# **Robotics 1** (WS 2018/2019)

Exercise Sheet 6

Presentation during exercises in calendar week 50

## Exercise 6.1 – Single Pendulum

For this exercise we look at a pendulum that consists of a point mass $m$ on a string of length $l$ that swings frictionless with an angle amplitude of $\varphi(t)$.

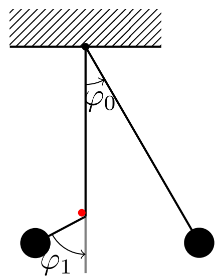


Figure 1: Single Pendulum with stopper

1. Split the gravitational force $F_g$ into a force keeping the string tight (radial to the motion) $F_r$ and one that accelerates the mass towards the equilibrium point (tangential to the motion) $F_t$. Express both as a function of $\varphi(t)$.

   Set up the equation of motion as discussed in the lecture earlier.

2. Assuming the pendulum starts with no initial velocity at an amplitude of $\varphi_0$. What is its velocity when reaching the equilibrium point?
   **Hint:** Use the conservation of energy.

3. The equation of motion is difficult to solve algebraigly. The *real pendulum* is reduced to the so called *simple pendulum* (German: *Mathematisches Pendel*). It is assumed that only small amplitudes $|\varphi|$ occure and therefore $\sin(\varphi) \approx \varphi$.

   Show, that the eq. 1 is a solution to the simple pendulum.

$$\varphi(t) = \varphi_o \sin(\sqrt{\frac{g}{l}} \cdot t) \tag{1}$$

4. Look at the periodicity of $\sin(\cdot)$. What is the period of motion $T$ for the pendulum?

   Assuming you want to build a clock with a pendulum that runs every second through the equilibrium point. How long is the string $l$?

5. Now we place a stopper (red) in such a way that for half a period the effective length of the string is reduced to $\frac{l}{4}$.

   a) What is the new period $T$ for this case?

   b) If $\varphi_0$ is the maximum amplitude for unrestrained case, what is the maximum amplitude $\varphi_1$ for the restrained case? Use the conservaton of energy.

**Exercise 6.2 – Double Pendulum with RBDL**

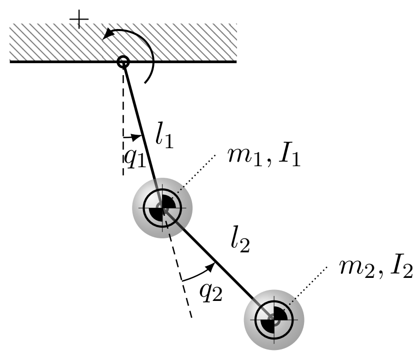


Figure 2: Double Pendulum

Given the double pendulum in Figure 2 the dynamic parameters are to be calculated from the the geometry of the system: Two massless rods of length $l_n$ and steel spheres with radii $r_n$ :

$$r_1 = 8cm \qquad \rho_1 = 7.874\frac{g}{cm^3} \qquad l_1 = 40cm$$

$$r_2 = 10cm \qquad \rho_2 = 7.874\frac{g}{cm^3} \qquad l_2 = 60cm$$

a) Download the code for this assignment, compile and run it. Look at the result with meshup. So far, made up values have been inside the program. Replace the formulae for the mass and inertia of a solid sphere with the correct ones and set the constants for the model as given in the table above. Look at the result.

b) Use the following initial values and discuss the result

   i) $\vec{q} = \left[\frac{\pi}{2}, 0\right], \dot{\vec{q}} = [0, 0]$

   ii) $\vec{q} = [\pi, \pi], \dot{\vec{q}} = [0, 0]$

   iii) $\vec{q} = [\pi, 0], \dot{\vec{q}} = [0, 0]$

   iv) $\vec{q} = [0, 0], \dot{\vec{q}} = [0, 0]$

   The last three initial poses are at an equilibrium. Some are stable some are not. Can you give a reason why?

c) Replace the RK4 integrator with the Euler and Heun integrator. How does this affect the stability?

**Exercise 6.3 – Euler-Lagrange formalism**

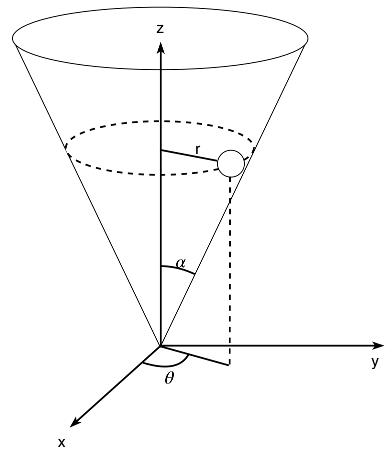


Figure 3: Ball in circular cone.

- Consider a ball rolling in a circular cone visualized in figure 3. The ball is modeled as a point mass with mass $m$. Friction is neglected completely. Find the equations of motion for $r$ and $\theta$ using the Euler-Lagrange formalism.

- Consider an elastic pendulum, modeled by a massless spring that lies in a straight line with spring constant $k$ and a mass $m$. The equilibrium length of the spring is $l$. Let the spring have length $l + x(t)$ and let its angle with the vertical be $\theta(t)$. Find the equations of motion for $x$ and $\theta$ using the Euler-Lagrange formalism.

- Split both resulting equations of motion into mass matrix component $M$, centripetal term $c_{\text{cent}}$, coriolis term $c_{\text{cor}}$ and gravitational term $g$, i.e.

$$\tau = M(q)\ddot{q} + \underbrace{c_{\text{cent}}(q,\dot{q}) + c_{\text{cor}}(q,\dot{q})}_{N(q,\dot{q})} + g(q)$$

  **Hint:** Quadric terms containing $\dot{q}_i^2$ are called *centripetal* terms, and quadric terms containing $\dot{q}_i\dot{q}_j, i \neq j$ are called *coriolis* terms.