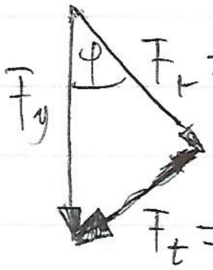
6.1.1



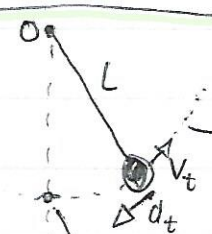
$F_r = F_y\cos(\varphi(t)) = mg\cos(\varphi(t))$

$F_t = mg\sin(\varphi(t))$

↳ auch Rückstellkraft genannt, zeigt immer zur Ruheposition

~~Equation of motion:~~

Traslatorischer Ansatz



→ motonpath, we can express motion in 1D

↳ equilibrium point?

↳ Ruheposition

Equation of motion: (Newton II)

$$m a_t(t) = \sum F_t = F_t = -mg\sin(\varphi(t))$$

↳ sum of all Forces in tangential direction

↳ tangential accelaration

$\rightarrow v_t = \omega r \quad \Big| \frac{d}{dt}$

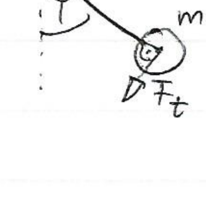
$a_t = \dot{\omega} r$

↳ $\dot{\omega} = \ddot{\varphi} \rightarrow a_t = \ddot{\varphi} r = \ddot{\varphi} L$ ↳ seillänge

$$\cancel{m}\ddot{\varphi}(t) L = -\cancel{m} g\sin(\varphi(t))$$

$$\ddot{\varphi}(t) = -\frac{g}{L}\sin(\varphi(t))$$

Rotatorischer Ansatz:



$$\Theta_0 \cdot \ddot{\varphi}(t) = \sum T_0$$

↳ Drehpunkt

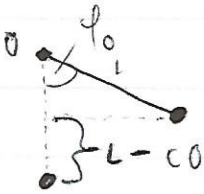
$$(\Theta_m + \underbrace{mL^2}_{\text{Steiner}})\cdot\ddot{\varphi}(t) = F_t \cdot L$$

↳ Null, Punktmasse

$$\cancel{m}L^{\cancel{2}}\ddot{\varphi}(t) = \cancel{m} g\sin(\varphi(t))\cdot\cancel{L}$$

$$\ddot{\varphi}(t) = \frac{g}{L}\sin(\varphi(t))$$

6.1.2



$$L - \cos(\varphi_0) L = h = L(1-\cos\varphi_0)$$

~~Energieerhaltung:~~ Falsch Ansatz

$$\cancel{E_{pot} = E_{kin}}$$ ← das wäre Fallgeschwindigkeit !

$$\cancel{mgh = \frac{1}{2} m v^2}$$

$$\cancel{gL(1-\cos\varphi) = \frac{1}{2} v^2}$$

$$\cancel{\Theta_0 g \varphi = \frac{1}{2}\Theta_0 \omega^2}$$

$$E_{pot} = E_{kin\_rot}$$

$$mgh = \frac{1}{2}\Theta_0 \omega^2$$

Θ_0 weil Punktmasse

$$mgL(1-\cos\varphi_0) = \frac{1}{2}(\cancel{\Theta_m} + mL^2)\,\omega^2$$

$$2\cancel{m}g\cancel{L}(1-\cos\varphi_0) = \cancel{m} L^{\cancel{2}} \omega^2$$

$\dot{\varphi}$

$$\omega^2 = \frac{2g}{L}(1-\cos\varphi_0) \rightarrow \omega = \sqrt{\frac{2g}{L}(1-\cos\varphi_0)}$$

$$v = \omega r = \omega L = \sqrt{\frac{2g}{L}(1-\cos\varphi_0)} \cdot L$$

6.1.3

3)

$$\ddot{\varphi}(t) = -\frac{g}{l}\sin(\varphi(t))$$

with $\sin(\varphi) \approx \varphi$

$$\ddot{\varphi} = -\frac{g}{l}\varphi \rightarrow \varphi = \frac{l}{g}\ddot{\varphi}$$

Beweis:

$$\frac{d^2}{d^2t}\left(\varphi_0 \sin\left(\sqrt{\tfrac{g}{l}}\,t\right)\right) = \frac{g}{l}\varphi$$

$$\varphi(t) = \varphi_0 \sin(kt) \quad , \quad k = \sqrt{\frac{g}{l}} = \omega = \dot{\varphi}$$

$$\frac{d}{dt}\left(\varphi(t) = \varphi_0 \sin(kt)\right) = \dot{\varphi}(t) = \varphi_0 k \cos(kt)$$

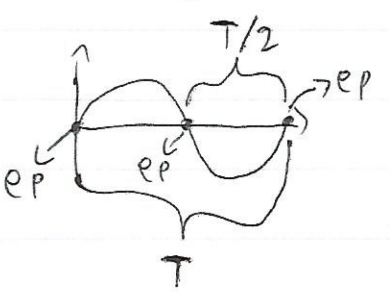
$$\frac{d}{dt}\left(\dot{\varphi}(t) = \varphi_0 k \cos(kt)\right) = \ddot{\varphi}(t) = \varphi_0 k^2 \cdot -\sin(kt)$$

$$\ddot{\varphi}(t) = -\varphi_0\left(\sqrt{\frac{g}{l}}\right)^2 \sin\left(\sqrt{\frac{g}{l}}\,t\right) = -\varphi_0 \frac{g}{l}\sin\left(\sqrt{\frac{g}{l}}\,t\right)$$

$$\omega = \sqrt{\frac{g}{l}} = \dot{\varphi}$$

↳ Eigenkreisfrequenz

4)



$$T = 2\pi \frac{1}{\sqrt{g/l}} = \frac{2\pi\sqrt{l}}{\sqrt{g}}$$

→ every second throug $\overbrace{\text{equilibrium point}}^{ep}$ means we want:

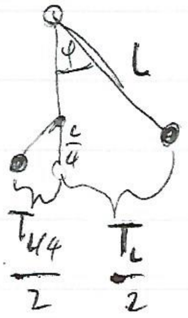
$$\frac{T}{2} = 1s \rightarrow T = 2\,\text{seconds}$$

$$\left(\frac{T\sqrt{g}}{2\pi}\right)^2 = l = \frac{T^2 g}{4\pi^2}$$

$$\rightarrow l(T=2s) = \frac{4s^2 \cdot 9{,}81\,\frac{m}{s^2}}{4\pi^2} = 0{,}994\,m = 99{,}4\,cm$$

(6-7.5)

a)



$$T = \frac{T_L}{2} + \frac{T_{L/4}}{2}$$

$$T_L = \frac{2\pi\sqrt{L}}{\sqrt{g}}$$

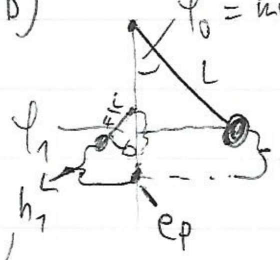
$$T_{L/4} = \frac{2\pi\frac{\sqrt{L}}{\sqrt{4}}}{\sqrt{g}} = \frac{2\pi\frac{\sqrt{L}}{2}}{\sqrt{g}} = \frac{\pi\sqrt{L}}{\sqrt{g}}$$

$$\cancel{T = \frac{2\pi\sqrt{L}}{2\sqrt{g}} + \frac{\pi\sqrt{L}}{\sqrt{g}} = \frac{3\pi\sqrt{L}}{2\sqrt{g}}}$$

$$T = \underbrace{\frac{2\pi\sqrt{L}}{\sqrt{g}\cdot 2}}_{\to \frac{T_L}{2}} + \underbrace{\frac{\pi\sqrt{L}}{2\sqrt{g}}}_{\to \frac{T_{L/4}}{2}} = \frac{3\pi\sqrt{L}}{2\sqrt{g}}$$

6.1.5

b)



$h_0 = L - L\cos(\varphi_0) = L(1-\cos\varphi_0)$

$\hookrightarrow h_1 = \frac{L}{4}(1-\cos\varphi_1)$

$E_{pot1} = E_{pot0}$

$\cancel{mg}h_1 = \cancel{mg}h_0$

$\cancel{L(1-\cos\varphi_1) = L(1-\cos\varphi_0) \quad |:L \quad |-1 \quad |\cdot(-1) \quad |\operatorname{acos}(..)}$

$\cancel{\varphi_1 = \varphi_0}$

$\frac{L}{4}(1-\cos\varphi_1) = L(1-\cos\varphi_0)$

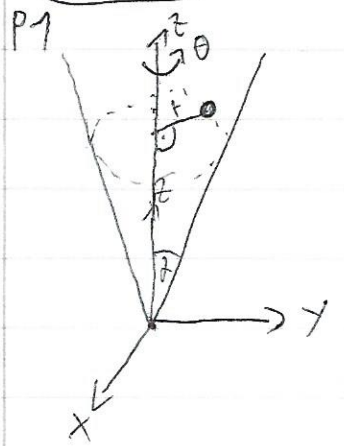
$1-\cos\varphi_1 = 4-\cos\varphi_0 \qquad |*1$

$+\cos\varphi_1 = -3+\cos\varphi_0 \qquad |\cdot(-1) \qquad |\operatorname{acos}$

$\varphi_1 = \operatorname{acos}(\cos(\varphi_0)-3)$

6.3.1

P1



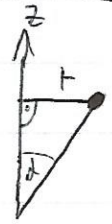
$$\mathcal{L} = T - U$$

$$U = m g z_B$$

$$T = \frac{m}{2}\dot{r}^2 + \frac{m}{2}\dot{z}_B^2 + \frac{1}{2}(\underbrace{I_{ball}}_{0,\ \text{weil Punktmasse}} + m r^2)\dot{\theta}^2$$

$$T = \frac{m}{2}\left(\dot{r}^2 + \dot{z}_B^2 + r^2\dot{\theta}^2\right) \quad (\mathrm{I})$$

suche $z_B = f(\alpha)$:



$$\tan\alpha = \frac{r}{z_B} \rightarrow z_B = \frac{r}{\underbrace{\tan(\alpha)}_{\text{konstant} \rightarrow \tan(\alpha) = K}} \quad (\mathrm{II})$$

II in I:

$$T = \frac{m}{2}\left(\dot{r}^2 + \frac{d}{dt}\left(\frac{r}{\cancel{\tan(\alpha)}}\right) + r^2\dot{\theta}^2\right) = \boxed{\frac{m}{2}\left(\dot{r}^2 + \frac{1}{K}\dot{r} + \dot{r}\dot{\theta}^2\right) = T}$$

$$\text{NR} \rightarrow \frac{d}{dt}\left(\frac{r}{K}\right) = \frac{\dot{r}K - \overset{0}{\cancel{\dot{K}}}\cdot r}{K^2} = \frac{\dot{r}\cancel{K}}{K^{\cancel{2}}} = \frac{\dot{r}}{K}$$

$$U = m g z_B = \boxed{m g \frac{r}{K} = U}$$

→ Now I could just do $U = T \rightarrow E_{pot} = E_{kin}$, but we want 2 differential equations, one for r-movement, one for θ-movements. ~~In the lecture, we didn't talk about how to use Lagrange to get movement equations.~~

euler Lagrange:

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{q}_i}\right) - \frac{d\mathcal{L}}{dq_i} = \underbrace{\xi_i}_{\text{external torques \& forces}} = \underline{0} \Rightarrow \frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{q}_i}\right) = \frac{d\mathcal{L}}{dq_i}$$

↳ doing this ~~for~~ with $q_i = [r, \theta]$

6.3.1

P2

$$\mathcal{L} = \frac{m}{2}\left(\dot{r}^2 + \frac{1}{k}\dot{r} + r\dot{\theta}^2\right) - \frac{mg}{k}r$$

r - equation:

$$\frac{d\mathcal{L}}{dr} = \frac{1}{2}m\dot{\theta}^2 - \frac{mg}{k}$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{r}}\right) = \frac{d}{dt}\left(m\dot{r} + \frac{m}{2k}\right) = m\ddot{r}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{r}}\right) = \frac{d\mathcal{L}}{dr}$$

$$\cancel{m}\ddot{r} = \frac{1}{2}\cancel{m}\dot{\theta}^2 - \frac{\cancel{m}g}{k}$$

$$\boxed{\ddot{r} = \frac{1}{2}\dot{\theta}^2 - \frac{g}{\tan\alpha}}$$

θ - equation:

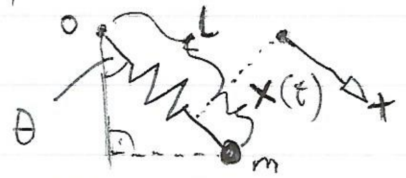
$$\frac{d\mathcal{L}}{d\theta} = 0$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{\theta}}\right) = \frac{d}{dt}\left(mr\dot{\theta}\right) = mr\ddot{\theta}$$

$$\boxed{0 = mr\ddot{\theta}}$$

6.3.2

P1



kinetic Energy:

$$T = \frac{1}{2}\left(\cancel{I_{Ball}}^{\,0,\ \text{weil Punktmasse}} + m(L+x)^2\right)\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2$$

$$\boxed{T = \frac{1}{2}m\left[(L+x)^2\dot{\theta}^2 + \dot{x}^2\right]}$$

potential energy

→ Spannkraft immer entgegengerichtet?

$$\boxed{U = \underbrace{\frac{1}{2}kx^2}_{\text{Spring}} + \underbrace{-mg(L+x)\cos(\theta)}_{\text{„height-energy"}}}$$

$$\mathcal{L} = 0 = T - U = \boxed{\frac{1}{2}m\left[\dot{x}^2 + (L+x)^2\dot{\theta}^2\right] - \frac{1}{2}kx^2 + mg(L+x)\cos\theta = \mathcal{L}}$$

$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{x}}\right) = \frac{d\mathcal{L}}{dx}$: equation of motion for x

$$\frac{d\mathcal{L}}{dx} = \cancel{\frac{1}{2}}m\cancel{2}(L+x)\cdot 1\cdot\dot{\theta}^2 - kx + mg\cos\theta = m(L+x)\dot{\theta}^2 - kx + mg\cos\theta$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{x}}\right) = \frac{d}{dt}\left(m\dot{x}\right) = m\ddot{x}$$

$$\Rightarrow \boxed{\cancel{m}\ddot{x} = \cancel{m}(L+x)\dot{\theta}^2 - kx + \cancel{m}g\cos\theta}$$

← Nicht m kürzen, war Vorsehen, geht nicht g → macht Sinn

$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{\theta}}\right) = \frac{d\mathcal{L}}{d\theta}$: Equation of motion for θ:

$$\frac{d\mathcal{L}}{d\theta} = -mg(L+x)\sin(\theta)\cdot 1$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{\theta}}\right) = \frac{d}{dt}\left(m(\underbrace{L+x}_{f(t)})^2\underbrace{\dot{\theta}}_{f(t)}\right) = 2m(L+x)\dot{\theta}\dot{x} + m(L+x)^2\ddot{\theta}$$

$$-\cancel{m}g\cancel{(L+x)}\sin\theta = 2\cancel{m}\cancel{(L+x)}\dot{\theta}\dot{x} + \cancel{m}(L+x)^{\cancel{2}}\ddot{\theta}$$

$$\boxed{-g\sin\theta = 2\dot{\theta}\dot{x} + (L+x)\ddot{\theta}}$$

$\Rightarrow \overset{2}{o} \; \overset{2}{x} \; \overset{2}{q}$

6.3.3

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{q}}\right) - \frac{d\mathcal{L}}{dq} = \tilde{\tau}$$

$$\tilde{\tau} = \begin{cases} x \to m\ddot{x} - m(L+x)\dot{\theta}^2 + kx - mg\cos\theta = \overset{M}{m\ddot{x}} \underbrace{- mL\dot{\theta}^2 - mx\dot{\theta}^2}_{C_{centripedal}} + kx \\ \theta \to 2\dot{\theta}\dot{x} + (L+x)\ddot{\theta} + g\sin(\theta) = \underset{coriolis}{2\dot{\theta}\dot{x}} + \underset{M}{L\ddot{\theta} + x\ddot{\theta}} + \underset{g}{g\sin(\theta)} \end{cases}$$

$$q = \begin{pmatrix} x \\ \theta \end{pmatrix} \qquad \ddot{q} = \begin{pmatrix} \ddot{x} \\ \ddot{\theta} \end{pmatrix}$$

$$M(q)\ddot{q} = \begin{bmatrix} m + \frac{kx}{\ddot{x}} & 0 \\ \cancel{L+x}\; 0 & L+x \end{bmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} m\ddot{x} + kx \\ L\ddot{\theta} + x\ddot{\theta} \end{pmatrix}$$

$$g = \begin{pmatrix} 0 \\ g\sin(\theta) \end{pmatrix}$$

$$C_{cent} = \begin{pmatrix} -mL\dot{\theta}^2 - mx\dot{\theta}^2 \\ 0 \end{pmatrix}$$

$$C_{cor} = \begin{pmatrix} 0 \\ 2\dot{\theta}\dot{x} \end{pmatrix}$$