Optimization, Robotics & Biomechanics (ORB)
Institute of Computer Engineering (ziti)
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Robotics 1 (WS 2018/2019)

Repetition Sheet 1

Mechanical basics

- Describe the meaning of 'degree of freedom' with your own words.
- How many variables are needed to fully describe a body / a point mass in space / in the plane?
- Formulate Newton's 3 Laws. Formulate Newton's law for rotations (also called Euler's equations).
- How do you compute the kinetic energy of translation and rotation of a body?
- 4) Deduce the formula for the kinetic energy of translation from the work that an external force exerts on the body.
- How do you compute the potential energy of a body?
- Explain the terms momentum and angular momentum.
- Describe the passive elements spring and damper in your own words and with an equation (for the linear case).
- What are stick friction (static friction) and slip friction (kinetic friction)? What do they depend on?
- What is a friction cone?
- Describe elastic and inelastic impacts mathematically.
- Explain the conservation laws for momentum and energy in your own words and with the corresponding equations.
- Can you give examples for energy loss and energy gain (input) in a system?
- What is a moment of inertia? Explain in your own words and give the mathematical definition. What is the moment of inertia of a thin stick about its center of mass (about an exit perpendicular to the stick)?
- What is the parallel axis theorem (Huygens -Steiner) used for? State the equation.

Kinematics of multibody systems

- What is a (rigid) multibody system?
- What is a kinematic chain? What types of chains are there? What is a kinematic tree? What is a kinematic loop?
- What is the difference between 'Kinematics' and 'Kinetics?
- Describe the configuration, work and task-space of a robot.
- How are degrees of freedom defined in the context of multibody systems? How can they be computed?
- Give the definition of Grüblers formula. Distinguish between 2D and 3D case. What is the special case, where Grüblers formula is wrong?
- Which types of joints do you know?

d)

Beschleunigungsarbeit(t)

$$W = F \cdot s(t)$$
$$= ma \cdot \frac{1}{2} a t^2$$
$$= \frac{1}{2} m v^2$$

= kin Energie zum Zeitpunkt t, also Kin Energie nachdem weg S abgearbeitet wurde

NR

$F = ma = const \rightarrow a = const$

$a = const$

$v = \int a \, dt = at$

$s = \frac{1}{2} a t^2$

- Which types of coordinate systems are there for modeling a multi-body-system?
- How are right and left handed coordinate systems defined?
- Give four different representations that describe rotations in 3D.
- b) Give the mathematical formulation for Rodrigues rotation formula. What is the geometrical interpretation?
- c) What does the rotation matrix for an elementary rotation about the x axis (the y axis, the z axis) look like for a given angle α? How do you compute the rotation matrix for rotations about different axis and angles?
- Explain the relationship between rotation matrices and Euler angles. Write down the general form of a rotation matrix with entries depending only on Euler angles (you can choose one type).
- Describe the degenerated cases where the Euler angles cannot be uniquely recovered from a given rotation matrix.
- d) What are homogeneous coordinates and what is a homogeneous transformation? Why is this formulation useful?
- ~~What are forward and inverse kinematics?~~
- e) Describe the Denavit Hartenberg convention.
- What is the 'operational space'?
- What is the difference between 'kinematically redundant' and 'intrinsically redundant'?
- Which relationship is described by differential kinematics?
- f) What is a 'geometrical' Jacobian? How is it calculated? What is it used for?
- g) How can the geometrical Jacobian be calculated in a kinematic chain with prismatic and revolute joints?
- h) Give the derivative of a rotation matrix. Interpret the skew-symmetric matrix that is involved in the formulation.
- What is a kinematic singularity? How is it connected to the Jacobian? Name two different types of singularities.
- What is manipulability? Explain in your own word. Which ways are there to picture or describe manipulability?
- What is dynamic manipulability?
- How can redundancy and manipulability be analyzed using the Jacobian?
- How are 'inverse differential kinematics' calculated? What happens in the case of kinematic singularities / redundancy?

~~Dynamics of multibody systems~~

kommt später!

- Describe the problems of forward and inverse dynamics.
- Which information / parameters of the segments are required to set up a dynamic multibody system model?
- What is an inertia matrix? Name some properties.
- Name two different methods for setting up equations of motion of multibody systems.
- Write down the Lagrange equations of the second kind (i.e for generalized coordinates that are a minimal description).
- What is a free body diagram? Can you draw it for a simple example, e.g. a double pendulum? How are actuators (in the joints) considered in a free body diagram?

b)

Schiefsymmetrisch

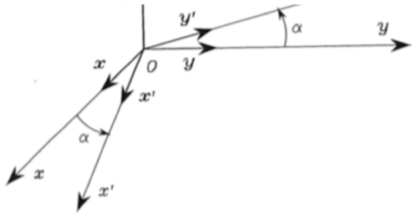
$$[a]_\times = \begin{pmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{pmatrix}$$

$$R = \exp([a]_\times) = I + \sin(\|a\|)\left[\frac{a}{\|a\|}\right]_\times + (1-\cos(\|a\|))\left[\frac{a}{\|a\|}\right]_\times^2$$

Ihre Hauptanwendung liegt darin, dass das Ergebnis einer Drehung um die Achse a mit Winkel $\|a\|$ als Matrix beschreibt.

c)

$$\boldsymbol{R}_z(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Corresponding rotation matrices for rotations β about the y axis and γ about the x axis:

$$\boldsymbol{R}_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \quad \boldsymbol{R}_x(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}$$

wir können um andere Achsen drehen, mit Routrigues oder indem wir elementar-Rotations-Matrizen kombinieren/ multiplizieren

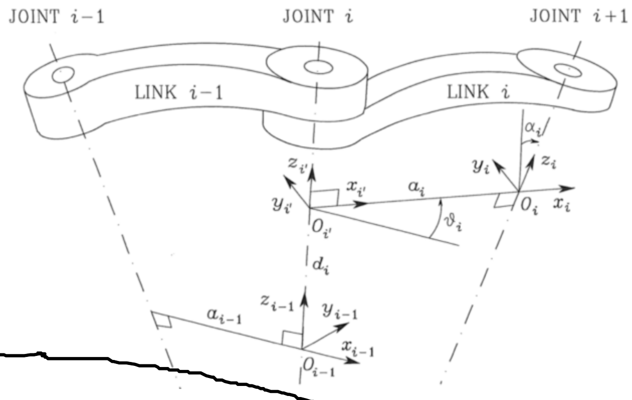
d)

$$\widetilde{\boldsymbol{p}} = \begin{bmatrix} \boldsymbol{p} \\ 1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{A}_1^0 = \begin{bmatrix} \boldsymbol{R}_1^0 & \boldsymbol{o}_1^0 \\ \boldsymbol{0}^T & 1 \end{bmatrix}$$

Bei den homogenen Koordinaten werden inhomogene Koordinaten in eine höheren Dimension projiziert, um so Transformationen linear abbilden zu können.
So kann mit einer homogenen 4x4 Matrix sowohl eine 3D-Rotation als auch eine 3D-Translation durchgeführt werden werden. Das nennt man dann eine homogene transformation.

$${}^{n-1}T_n = \mathrm{Rot}(z_{n-1}, \theta_n) \cdot \mathrm{Trans}(z_{n-1}, d_n) \cdot \mathrm{Trans}(x_n, a_n) \cdot \mathrm{Rot}(x_n, \alpha_n)$$

e)



f)

J beschreibt den Zusammenhang zwischen Geschwindigkeiten in den Joints und Geschwindigkeiten im End-Effektor.

Mithilfe der Jacobi kann auch die Mobilität eines Roboters untersucht und Singularitäten erkannt werden, die bei bestimmten konfis auftreten.

Herleitung: (translatorisch)

$$P_{tip} = f(q(t)) \quad \Big| \frac{d}{dt}$$

$$V_{tip} = \frac{df}{dq} \cdot \dot{q}$$

$$\Rightarrow J = \frac{df}{dq} \rightarrow A_n^0 = \text{Forwardkin-Matrix}$$

g)

$$\begin{bmatrix} \dot{p}_e \\ \omega_e \end{bmatrix} = \begin{bmatrix} J_{p1} & \dots & J_{pn} \\ J_{o1} & \dots & J_{pn} \end{bmatrix}$$

$$\begin{bmatrix} J_{pi} \\ J_{oi} \end{bmatrix} = \begin{cases} \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} & \text{— prismatic joint} \\ \begin{bmatrix} z_{i-1} \times (p_e - p_{i-1}) \\ z_{i-1} \end{bmatrix} & \text{— rot joint} \end{cases}$$

$$z_{i-1} = R_{i-1}^0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

h)

- Consider constant vector $\boldsymbol{p}'$ and vector $\boldsymbol{p}(t) = \boldsymbol{R}(t)\boldsymbol{p}'$.
- Derivative

$$\dot{\boldsymbol{p}}(t) = \dot{\boldsymbol{R}}(t)\boldsymbol{p}'$$

$$\dot{\boldsymbol{p}}(t) = \boldsymbol{S}(t)\boldsymbol{R}(t)\boldsymbol{p}'.$$

- With angular velocity $\boldsymbol{\omega}(t)$ of the frame of $\boldsymbol{R}(t)$

$$\dot{\boldsymbol{p}}(t) = \boxed{\boldsymbol{\omega}(t) \times} \boldsymbol{R}(t)\boldsymbol{p}'$$

- Therefore

$$\boldsymbol{S} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

with

$$\boldsymbol{\omega}(t) = [\omega_x \quad \omega_y \quad \omega_z]^T$$

- We can write

$$\dot{\boldsymbol{R}} = \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R}.$$

Dynamics of multibody systems

- Describe the problems of forward and inverse dynamics.
- Which information / parameters of the segments are required to set up a dynamic multibody system model?
- What is an inertia matrix? Name some properties.
- Name two different methods for setting up equations of motion of multibody systems.
- a) Write down the Lagrange equations of the second kind (i.e for generalized coordinates that are a minimal description).
- What is a free body diagram? Can you draw it for a simple example, e.g. a double pendulum? How are actuators (in the joints) considered in a free body diagram?
- How do you set up equations of motion with the Newton-Euler method?
- Explain the recursive Newton-Euler method.
- b) Can you set up the equations of motion of a simple pendulum with point mass at the end? How do these equations change if there is no point mass but the pendulum consists of a stick with homogeneous mass distribution?

a)

- **Lagrange function:**

$$\mathcal{L} = \mathcal{T} - \mathcal{U}$$

Kinetic Energy — Potential Energie

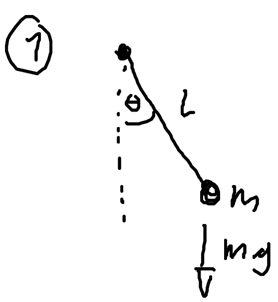
- **(Euler-)Lagrange equation** for every component (generalized coordinate)

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = \xi_i \qquad i = 1, \ldots, n$$

ξ_i is the generalized force corresponding to coordinate q_i

b)

1



$$I\ddot{\theta} = mg\cdot\sin\theta\cdot l$$

$$I = \cancel{I_0}^{0} + ml^2 = ml^2$$

$$\cancel{m}\, l^{\cancel{2}}\ddot{\theta} = \cancel{m} g \sin\theta \cancel{l}$$

$$l\ddot{\theta} = g\sin\theta \rightarrow \ddot{\theta}(t) = \frac{g}{l}\sin\theta(t)$$

2

$$I = \frac{1}{3}ml^2$$

$$\frac{1}{3}\cancel{m}\, l^{\cancel{2}}\ddot{\theta} = \cancel{m} g \sin\theta \cancel{l}$$

$$\ddot{\theta} = 3\frac{g}{l}\sin\theta$$

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Repetition Sheet 2

Trajectory planning

- Explain why a *trajectory* is a combination of a *path* and *time scaling*.
- Compare trajectory planning for Cartesian space and joint space. What are the respective strengths / weaknesses?
- Formulate PTP interpolation with trapezoidal velocity profile.
- Formulate PTP interpolation with sinusoidal velocity profile. pffffffff....
- What is the difference between synchronous and asynchronous PTP trajectories?
- How is a cubic spline defined? Name all necessary constraints.

Motion planning

- What is the difference between trajectory planning and motion planning?
- Give the formal definition for motion planning.
- Name some properties of motion planners regarding querrying, completeness and computational complexity.
- Compare types of motion planning methods and compare their advantages and disadvantages. What scenario is suited for which planning method?
- Construct one example each for *complete path planners* and *grid planners*.
- How can grid maps efficiently be stored and traversed?
- Why is it sometimes necessary to sample control space instead of configuration-space?
- Explain how a grid method can be applied to robot arm motion planning.
- Simulating collisions is expensive. How can distances between objects be approximated?
- Formulate the A^* algorithm.
- How does the A^* algorithm compare to other graph-based algorithms?
- Formulate the RRT algorithm. How can random samples be chosen? How can *nearest nodes* be defined?
- What is the difference between RRT and RRT* algorithms (and what is its effect)?