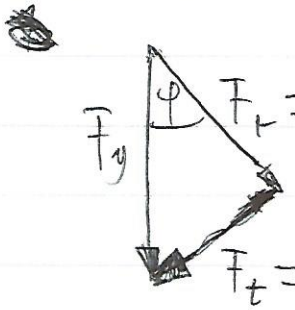
6.1.1



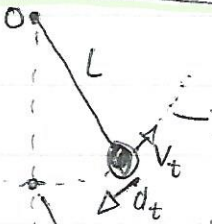
$F_r = F_y \cos(\varphi(t)) = mg\cos(\varphi(t))$

$F_t = mg\sin(\varphi(t))$

↳ auch Rückstellkraft genannt, zeigt immer zur Ruheposition

~~Equation of motion:~~

Translatorischer Ansatz



↳ motionpath, we can express motion in 1D

↳ equilibrium point?

↳ Ruheposition

Equation of motion: (Newton II)

$ma_t(t) = \sum F_t = F_t = -mg\sin(\varphi(t))$

↳ sum of all forces in tangential direction

↳ tangential acceleration

$\rightarrow v_t = \omega r \quad | \frac{d}{dt}$

$a_t = \dot{\omega} r$

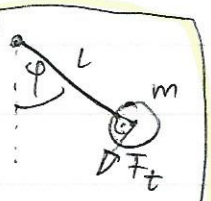
↳ $\dot{\omega} = \ddot{\varphi} \rightarrow a_t = \ddot{\varphi} r = \ddot{\varphi} L$

↳ seillänge

$m\ddot{\varphi}(t)L = -mg\sin(\varphi(t))$

$\ddot{\varphi}(t) = -\frac{g}{L}\sin(\varphi(t))$

Rotatorischer Ansatz:



$\Theta_o \cdot \ddot{\varphi}(t) = \sum T_o$

↳ Drehpunkt

$(\Theta_m + mL^2)\cdot\ddot{\varphi}(t) = F_t \cdot L$

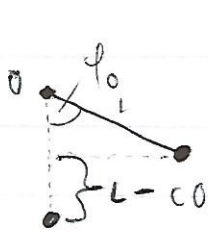
↳ Null-Punktmasse

↳ Steiner

$mL^2\ddot{\varphi}(t) = mg\sin(\varphi(t))\cdot L$

$\ddot{\varphi}(t) = \frac{g}{L}\sin(\varphi(t))$

6.1.2



$L - \cos(\varphi_0) l = h = L(1-\cos\varphi_0)$

~~Energieerhaltung:~~ ~~Falsch Ansatz~~

~~$E_{pot} = E_{kin}$~~ ← das wäre Fallgeschwindigkeit v

~~$mgh = \frac{1}{2} m v^2$~~

~~$g(L(1-\cos\varphi) = \frac{1}{2} v^2$~~

~~$\Theta_0 g l = \frac{1}{2} \Theta_0 \omega^2$~~

$E_{pot} = E_{kin\_rot}$

$mgh = \frac{1}{2} \Theta_0 \omega^2$ Θ_0 weil Punktmasse

$mgL(1-\cos\varphi_0) = \frac{1}{2}(\Theta_m + ml^2)\omega^2$

$2mgL(1-\cos\varphi_0) = ml^2\omega^2$

$\omega^2 = \frac{2g}{l}(1-\cos\varphi_0) \rightarrow \omega = \sqrt{\frac{2g}{l}(1-\cos\varphi_0)}$ $\omega = \dot{\varphi}$

~~$v = \omega r = \omega L = \sqrt{\frac{2g}{l}(1-\cos\varphi_0)} \cdot l$~~

6.1.3

3) $\ddot{\varphi}(t) = \frac{g}{l}\sin(\varphi(t))$

$\sin(\varphi) \approx \varphi$

$\ddot{\varphi} = \frac{g}{l}\varphi$

$\dot{\varphi} = \int \frac{g}{l}\varphi\, dt = \frac{g}{l}\int \varphi(t)\, dt = \frac{g}{l}\varphi \cdot t + c_1$

$\varphi = \int \frac{g}{l}\varphi t\, dt = \frac{g}{l}\frac{1}{2}\varphi t^2 + c_1 t + c_2$

$\ddot{\varphi}(t) = -\frac{g}{l}\sin(\varphi(t))$

with $\sin(\varphi) \approx \varphi$

$\ddot{\varphi} = -\frac{g}{l}\varphi \rightarrow \varphi = \frac{l}{g}\ddot{\varphi}$

Beweis:

$\frac{d^2}{d^2t}\left(\varphi_0 \sin\left(\sqrt{\frac{g}{l}}t\right)\right) = \frac{g}{l}\varphi$

$\varphi(t) = \varphi_0 \sin(kt)$, $k = \sqrt{\frac{g}{l}} = \omega = \dot{\varphi}$

$\frac{d}{dt}\left(\varphi(t) = \varphi_0 \sin(kt)\right) = \dot{\varphi}(t) = \varphi_0 k \cos(kt)$

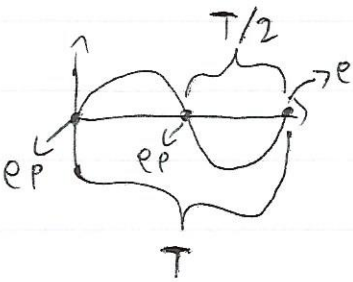
$\frac{d}{dt}\left(\dot{\varphi}(t) = \varphi_0 k \cos(kt)\right) = \ddot{\varphi}(t) = \varphi_0 k^2 \cdot -\sin(kt)$

$\ddot{\varphi}(t) = -\varphi_0\left(\sqrt{\frac{g}{l}}\right)^2 \sin\left(\sqrt{\frac{g}{l}}t\right) = -\varphi_0 \frac{g}{l}\sin\left(\sqrt{\frac{g}{l}}t\right)$

$\omega = \sqrt{\frac{g}{l}} = \dot{\varphi}$

↳ Eigenkreisfrequenz

4)



$T = 2\pi\frac{1}{\sqrt{g/l}} = \frac{2\pi\sqrt{l}}{\sqrt{g}}$

ep

→ every second through equilibrium point means we want:

$\frac{T}{2} = 1\,s \rightarrow T = 2\,s$econds

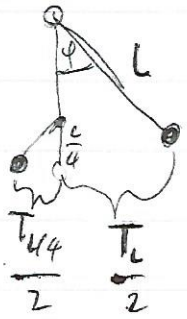
→ l

$\left(\frac{T\sqrt{g}}{2\pi}\right)^2 = l = \frac{T^2 g}{4\pi^2}$

$\rightarrow l(T=2s) = \frac{4\,s^2 \cdot 9{,}81\,\frac{m}{s^2}}{4\pi^2} = 0{,}994\,m = 99{,}4\,cm$

6.1.5

a)



$$T = \frac{T_L}{2} + \frac{T_{L/4}}{2}$$

$$T_L = \frac{2\pi\sqrt{L}}{\sqrt{g}}$$

$$T_{L/4} = \frac{2\pi\sqrt{\frac{L}{4}}}{\sqrt{g}} = \frac{2\pi\frac{\sqrt{L}}{2}}{\sqrt{g}} = \frac{\pi\sqrt{L}}{\sqrt{g}}$$

$$= \frac{2\pi\sqrt{L}}{2\sqrt{g}} + \frac{\pi\sqrt{L}}{2\sqrt{g}} = \frac{3\pi\sqrt{L}}{2\sqrt{g}}$$

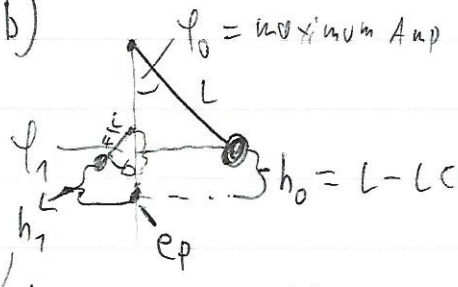
b)

$$T = \frac{\pi\sqrt{L}}{\sqrt{g}\cdot 2} + \frac{\pi\sqrt{L}}{2\sqrt{g}} = \frac{3\pi\sqrt{L}}{2\sqrt{g}}$$

$\hookrightarrow \frac{T_L}{2}$ $\hookrightarrow \frac{T_{L/4}}{2}$

6.1.5°

b)



$h_0 = L - L\cos(\varphi_0) = L(1-\cos\varphi_0)$

$\hookrightarrow h_1 = \frac{L}{4}(1-\cos\varphi_1)$

$E_{pot1} = E_{pot0}$

$mgh_1 = mgh_0$

~~$\frac{L}{4}(1-\cos\varphi_1) = L(1-\cos\varphi_0) \quad |\cdot\frac{4}{L} \quad 1-1 \quad |\cdot(-1) \quad |\arccos(\ldots)$~~

$\varphi_1 = \varphi_0$

$\frac{L}{4}(1-\cos\varphi_1) = L(1-\cos\varphi_0)$

$1-\cos\varphi_1 = 4-\cos\varphi_0 \quad |+1$

$+\cos\varphi_1 = -3+\cos\varphi_0 \quad |\cdot(-1) \quad |\arccos$

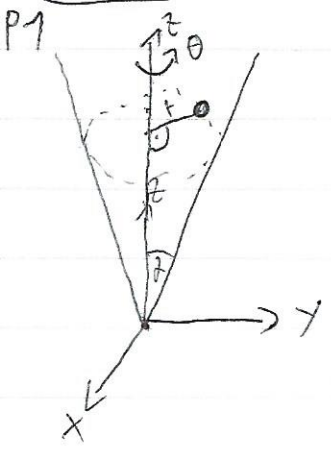
$\varphi_1 = \arccos(\cos(\varphi_0)-3)$

$z = f(r)$

6.3.1

$$q = \begin{bmatrix} r \\ \theta \end{bmatrix}$$

P1



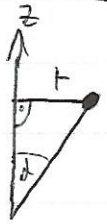
$$\mathcal{L} = T - U = f(q_{1\dots n}, \dot{q}_{1\dots n})$$

$U = mgz_B$

$T = \frac{m}{2}\dot{r}^2 + \frac{m}{2}\dot{z}_B^2 + \frac{1}{2}\left(I_{Rad}^{\nearrow 0, \text{weil Punktmasse}} + mr^2\right)\dot{\theta}^2$

$T = \frac{m}{2}\left(\dot{r}^2 + \dot{z}_B^2 + r^2\dot{\theta}^2\right) \quad (I)$

suche $z_B = f(\alpha)$:



$$\tan\alpha = \frac{r}{z_B} \rightarrow z_B = \frac{r}{\tan(\alpha)} \quad (II)$$

konstant $\forall$ $\rightarrow \tan(\alpha) = K$

II in I:

$$T = \frac{m}{2}\left(\dot{r}^2 + \frac{d}{dt}\left(\frac{r}{\tan(\alpha)}\right) + r^2\dot{\theta}^2\right) = \frac{m}{2}\left(\dot{r}^2 + \frac{1}{K}\dot{r} + r\dot{\theta}^2\right) = T$$

$\dot{K}$ = 0

NR $\rightarrow \frac{d}{dt}\left(\frac{r}{K}\right) = \frac{\dot{r}K - \dot{K}\cdot r}{K^2} = \frac{\dot{r}K}{K^2} = \frac{\dot{r}}{K}$

$$U = mgz_B = mg\frac{r}{K} = U$$

$\rightarrow$ Now I could just do $U = T \rightarrow E_{pot} = E_{kin}$, but we want 2 differential equations, one for r-movements, one for θ-movements. In the lecture, we didn't talk about how to use Lagrange to get movement equations.

euler Lagrange: $\rightarrow$ external torques & forces

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{q}_i}\right) - \frac{d\mathcal{L}}{dq_i} = Q_i = 0 \Rightarrow \frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{q}_i}\right) = \frac{d\mathcal{L}}{dq_i}$$

↳ doing this with $q_i = [r, \theta]$

6.3.1
P2

$$\mathcal{L} = \frac{m}{2}\left(\dot{r}^2 + \frac{1}{k}\dot{r} + r\dot{\theta}^2\right) - \frac{mg}{k}r$$

r - equation:

$$\frac{d\mathcal{L}}{dr} = \frac{1}{2}m\dot{\theta}^2 - \frac{mg}{k}$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{r}}\right) = \frac{d}{dt}\left(m\dot{r} + \frac{m}{2k}\right) = m\ddot{r}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{r}}\right) = \frac{d\mathcal{L}}{dr}$$

$$m\ddot{r} = \frac{1}{2}mr\dot{\theta}^2 - \frac{mg}{k}$$

$$\ddot{r} = \frac{1}{2}r\dot{\theta}^2 - \frac{g}{k}$$

θ - equation:

$$\frac{d\mathcal{L}}{d\theta} = 0$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{\theta}}\right) = \frac{d}{dt}\left(mr\dot{\theta}\right) = mr\ddot{\theta}$$

$$0 = mr\ddot{\theta}$$

6.3.2

P1

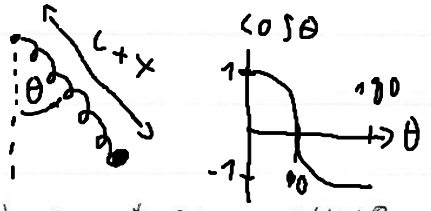
$$q = \begin{bmatrix} \theta \\ x \end{bmatrix}$$

kinetic Energy:

$$T = \frac{1}{2}\left(\cancel{I_{Ball}}^{0,\text{ weil Punktmasse}} + m(L+x)^2\right)\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2$$

~~$\frac{1}{2}m\left[\dot{x}^2 + (L+x)^2\dot{\theta}^2\right]$~~

$$T = \frac{1}{2}m\left[(L+x)^2\dot{\theta}^2 + \dot{x}^2\right]$$



potential energy

~~Spannkraft innere entgegengerichtet~~ → z

$$U = \frac{1}{2}kx^2 + \cancel{mgh} - mgy(L+x)\cos(\theta)$$

Die Gleichung hab ich aus dem Internet, keine Ahnung wieso das jetzt die height energy sein soll...

$\frac{1}{2}kx^2$: Spring

„height-energy"

$$\mathcal{L} = 0 = T - U = \frac{1}{2}m\left[\dot{x}^2 + (L+x)^2\dot{\theta}^2\right] - \frac{1}{2}kx^2 + mgy(L+x)\cos\theta = \mathcal{L}$$

$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{x}}\right) = \frac{d\mathcal{L}}{dx}$: equation of motion for x

$$\frac{d\mathcal{L}}{dx} = \frac{1}{2}m\,2(L+x)\cdot 1\cdot\dot{\theta}^2 - kx + mg\cos\theta = m(L+x)\dot{\theta}^2 - kx + mg\cos\theta$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{x}}\right) = \frac{d}{dt}\left(m\dot{x}\right) = m\ddot{x}$$

$$\Rightarrow \cancel{m}\ddot{x} = m(L+x)\dot{\theta}^2 - kx + \cancel{m}g\cos\theta$$

← Nicht m kürzen, war vorsehen, geht nicht $\frac{g}{0}$ → macht Sinn

$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{\theta}}\right) = \frac{d\mathcal{L}}{d\theta}$: Equation of motion for θ:

$$\frac{d\mathcal{L}}{d\theta} = -mg(L+x)\sin(\theta)\cdot 1$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{\theta}}\right) = \frac{d}{dt}\left(\underbrace{m(L+x)^2}_{f(t)}\underbrace{\dot{\theta}}_{f(t)}\right) = 2m(L+x)\dot{\theta}\dot{x} + m(L+x)^2\ddot{\theta}$$

$$-\cancel{mg}(L+x)\sin\theta = 2\cancel{m}(L+x)\dot{\theta}\dot{x} + \cancel{m}(L+x)^2\ddot{\theta}$$

$$-g\sin\theta = 2\dot{\theta}\dot{x} + (L+x)\ddot{\theta}$$

$\Rightarrow \dot{x}^2 \cdot \dot{\theta}^2$

6.3.3

$$\frac{d}{dt}\left(\frac{dT}{d\dot{q}}\right) - \frac{dT}{dq} = \tilde{J}$$

$C_{centripedul}$

$$\tilde{J} = \begin{matrix} x \\ \theta \end{matrix} \begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \begin{matrix} m\ddot{x} - m(L+x)\dot{\theta}^2 + kx - mg\cos\theta = m\ddot{x} - mL\dot{\theta}^2 - mx\dot{\theta}^2 + kx \\ 2\dot{\theta}\dot{x} + (L+x)\ddot{\theta} + g\sin(\theta) = 2\dot{\theta}\dot{x} + L\ddot{\theta} + x\ddot{\theta} + g\sin(\theta) \end{matrix}$$

coriolis, M, g

$$q = \begin{pmatrix} x \\ \theta \end{pmatrix} \qquad \ddot{q} = \begin{pmatrix} \ddot{x} \\ \ddot{\theta} \end{pmatrix}$$

$$M(q)\ddot{q} = \begin{bmatrix} m + \frac{kx}{\ddot{x}} & 0 \\ \cdots & \cdots \\ 0 & L+x \end{bmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} m\ddot{x} + kx \\ L\ddot{\theta} + x\ddot{\theta} \end{pmatrix}$$

$$g = \begin{pmatrix} 0 \\ g\sin(\theta) \end{pmatrix}$$

$$C_{cent} = \begin{pmatrix} -mL\dot{\theta}^2 - mx\dot{\theta}^2 \\ 0 \end{pmatrix}$$

$$C_{cor} = \begin{pmatrix} 0 \\ 2\dot{\theta}\dot{x} \end{pmatrix}$$