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Exercise Sheet 7

Presentation during exercises in calendar week 27

One-legged Hopping Robot

The purpose of this sheet will be to get you started on the modeling of contacts and multi-phase optimal control problems with respect to DAE-systems. This will be done based on a simple hopping robot (refer to Figure 1).

The hopping robot consists of two bodies (in the following reference frames) *Body* and *Leg*. It has two degrees of freedom. q_0 is the position of the reference frame *Body* with respect to the Z-coordinate. q_1 represents the translation between the reference frame *Body* and the reference frame *Leg*. Both reference frames are mechanically connected with a spring with constant k. The rest length of this spring is adjusted such that it occurs at $q_1 = 0$ (refer to Figure 1). In parallel to the spring force, a linear force u_0 is applied between the reference frames *Body* and *Leg* (a typical *PEA*, parallel elastic actuation arrangement).

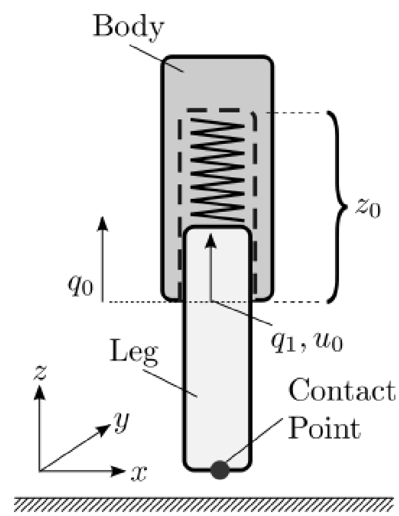

Figure 1: Simple Hopper

The robot is exposed to the conservative gravity field and the dynamic action of the *PEA* system. During the ground contact phase additional ground contact reactions *GCR* and ground collision impulsion at the time of the touch down are present in the dynamic modeling of the hopper. Consequently, the model has three distinct configurations: *flight*, *collision* and *ground-contact* (refer to Figure 2).

The different states are described by the following characteristics:

Flight-Phase No contact - $g(q(t)) > 0$ the lowest point of the leg should be above ground level. *GCR* are not present. The dynamics of the system are described by the *ODE*,

$$M(q)\,\ddot{q} + \mathrm{NLE}(q, \dot{q}) = \tau(t) + f_{\text{spring}}(q)\,. \tag{1}$$

Collision Ground impact occurs when the lowest point of the leg reaches ground-level, $g(q(t)) > 0$. In the following, it is assumed that the collision is modeled perfectly inelastic.

From collision dynamics the simplified description holds,

$$M(q)\,\dot{q}^{+} - J(q)^{T}\,\Lambda = M(q)\,\dot{q}^{-}, \tag{2a}$$

$$J(q)\,\dot{q}^{+} = 0. \tag{2b}$$

The velocities $\dot{q}^{-}$ and $\dot{q}^{+}$ in equation (2a) and (2b) describe the velocity state of the system before (-) and after (+) the collision, respectively. The resulting discontinuities of the velocity vector $\dot{q}$ can be accounted for in MUSCOD-II with a particular pseudo-integrator which is called with `ind_strans` (normal integrator `ind_rkf45`) at the corresponding phase configuration in the config section of the DAT-file.

Contact The lowest point of the leg remains at ground level $g(q(t)) = 0$. The contact is uni-lateral. This means that it only holds if and only if $f_{\text{Contact}} \geq 0$. The vertical contact force f_{Contact} is in the following expressed with the parameter λ. The resulting *DAE* follows to,

$$M(q)\,\ddot{q} + \text{NLE}(\dot{q}, q) - J^{T}\lambda = \tau(t) + f_{\text{spring}}(q), \tag{3a}$$

$$g(q(t)) = 0, \tag{3b}$$

$$\lambda > 0. \tag{3c}$$

All different model states are effectively described with RBDL (please refer to the documentation - section *External Contacts*).

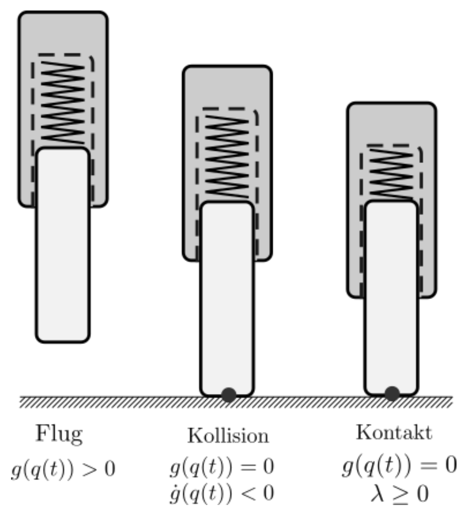

Figure 2: Overview of the model phases

Exercise 7.1 – Formulation of the Model

The formulation of *DAE* based models is problematic in MUSCOD-II. Therefore, in the following we are going to apply an index reduction to the equations (3a) and (3b), respectively.

- Apply an index reduction to index-1 by derivating the algebraic consistency equation (3b). What additional constraints on the initial values need to be respected, such that resulting index-1 *DAE* system is consistent to the original index-3 *DAE* system?

- Write the index-1 *DAE* system as a set of linear equations following,

$$Ax = b, \tag{4}$$

$$x = \left[\begin{array}{cc} \ddot{q} & \lambda \end{array} \right]^T \tag{5}$$

  Which form does the matrix A and the vector b have?

- (Bonus) What are the conditions to the matrix A such that A is regular and (4) yields a unique solution? **HINT:** The proof is the same as for quadratic programs. RBDL contains implementations to solve (2) and (3) with good efficiency. However, these methods tend to fail when A is not regular.

Exercise 7.2 – Stage-Definition and Optimization

The provided template code consists of the library `./SRC/hoppingrobot.cc`, a dat-file template `./DAT/hoppingrobot.dat` as well as the build system and the lua model of the hopping robot `hoppingrobot.lua` for RBDL and visualization of the results in MeshUp .

In the following, the problem formulation should be configured such that, an optimal control trajectory is determined for:

1. Minimum actuation force of the *PEA* system,
2. Minimum impact of the leg on the ground,

of a periodic hopping motion. The following hints should help you:

- Configure the build system such that `make` builds a distinct target (x2) for each problem formulation.
- Configure the integrators in the section `libind` of each DAT-file.
- Configure in each library file:
  - The right hand side functions for all model stages `ffcn_flight`, `ffcn_touchdown`, `ffcn_contact`, as well as the kinematics update, `update_generalized_variables`.
  - The decoupled interior point constraints `rdfcn_flight_i`, `rdfcn_touchdown_s`, `rdfcn_contact_i` and `rdfcn_contact_e`
  - The coupled interior point constraints `rcfcn_periodic_s` and `rcfcn_periodic_e`.
  - The objectives for actuation and impulsion, `lfcn` and `mfcn`, respectively.
  - The entry point `def_model`, objectives for each model stage as well as the constraint configurations.