

Lecture #15. 곡선 이동

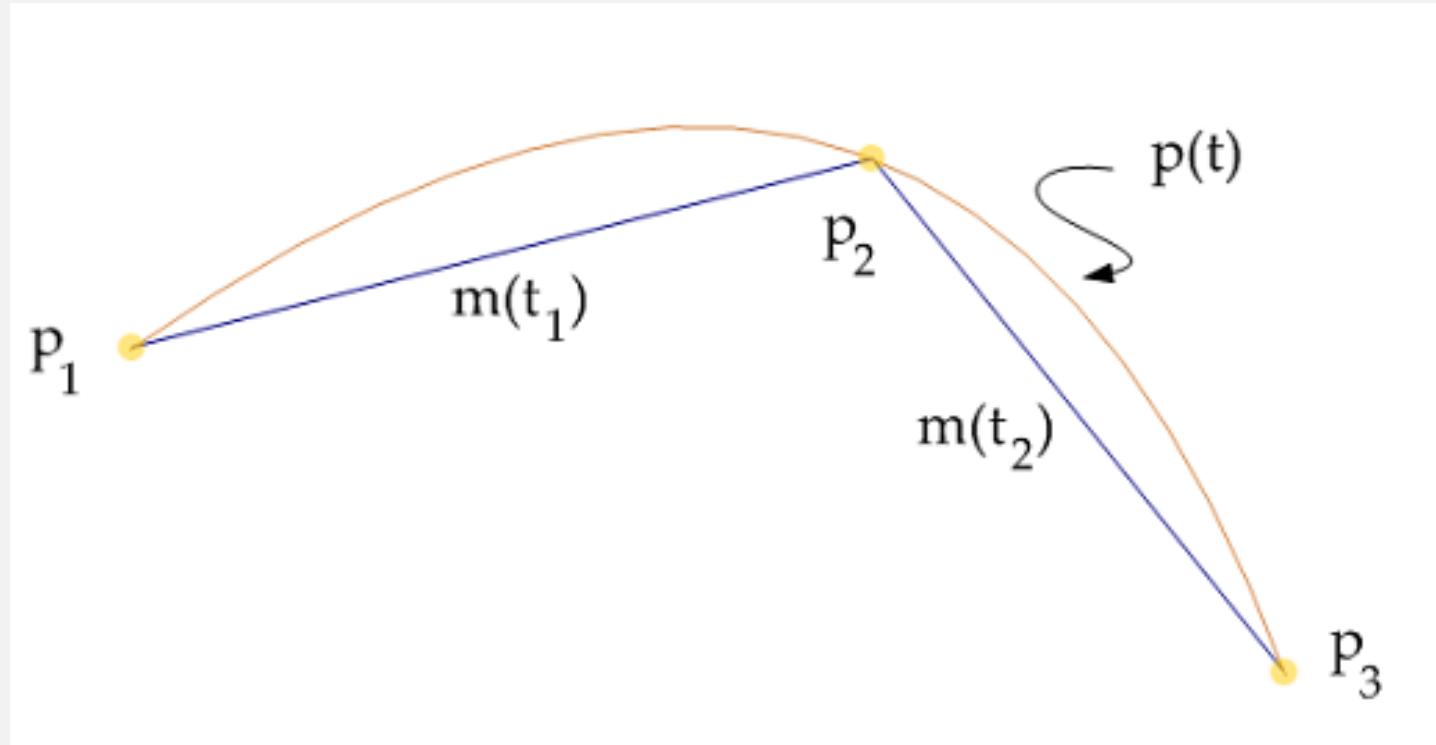
2D 게임 프로그래밍

이대현 교수

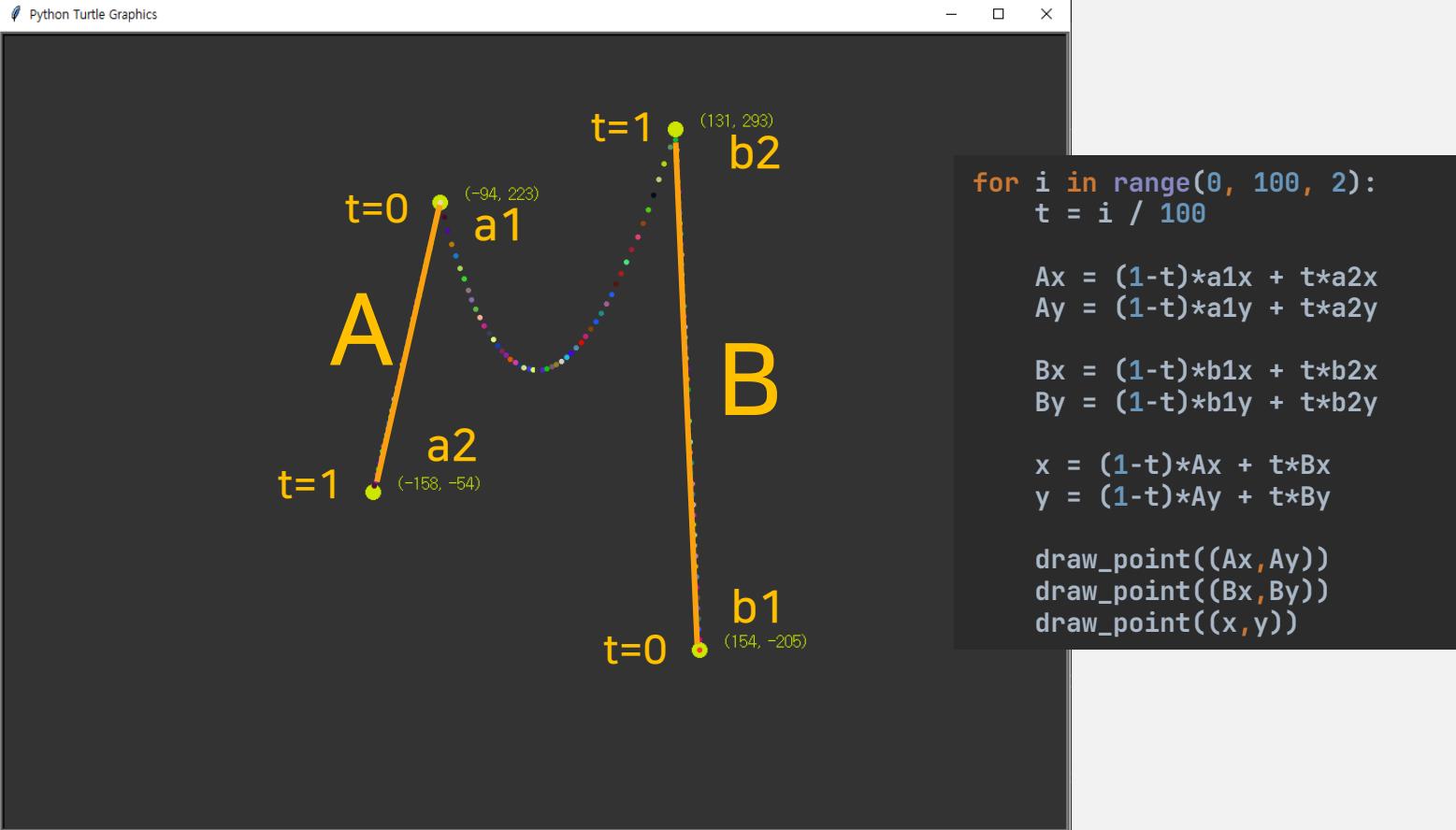
학습 내용

- 세 점 간의 부드러운 곡선 이동
- 네 점간의 부드러운 곡선 이동

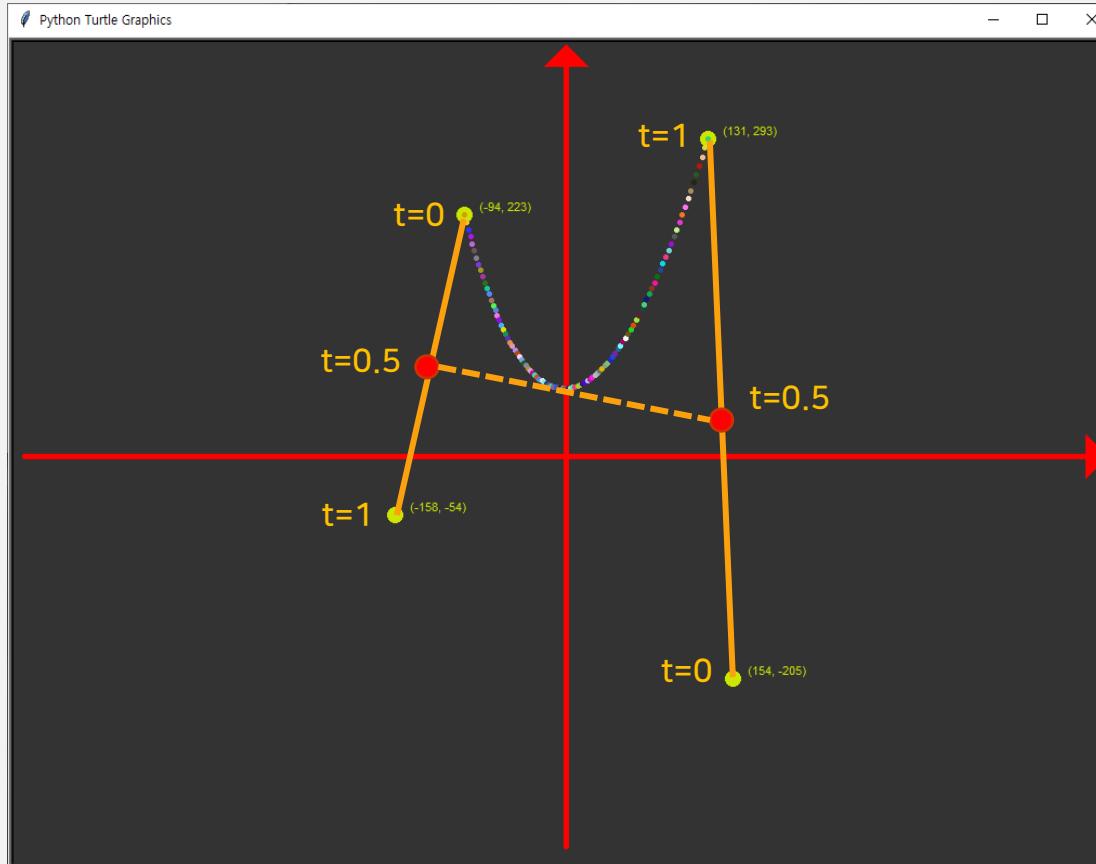
세 개의 점을 잇는 부드러운 곡선을 어떻게 그릴까?



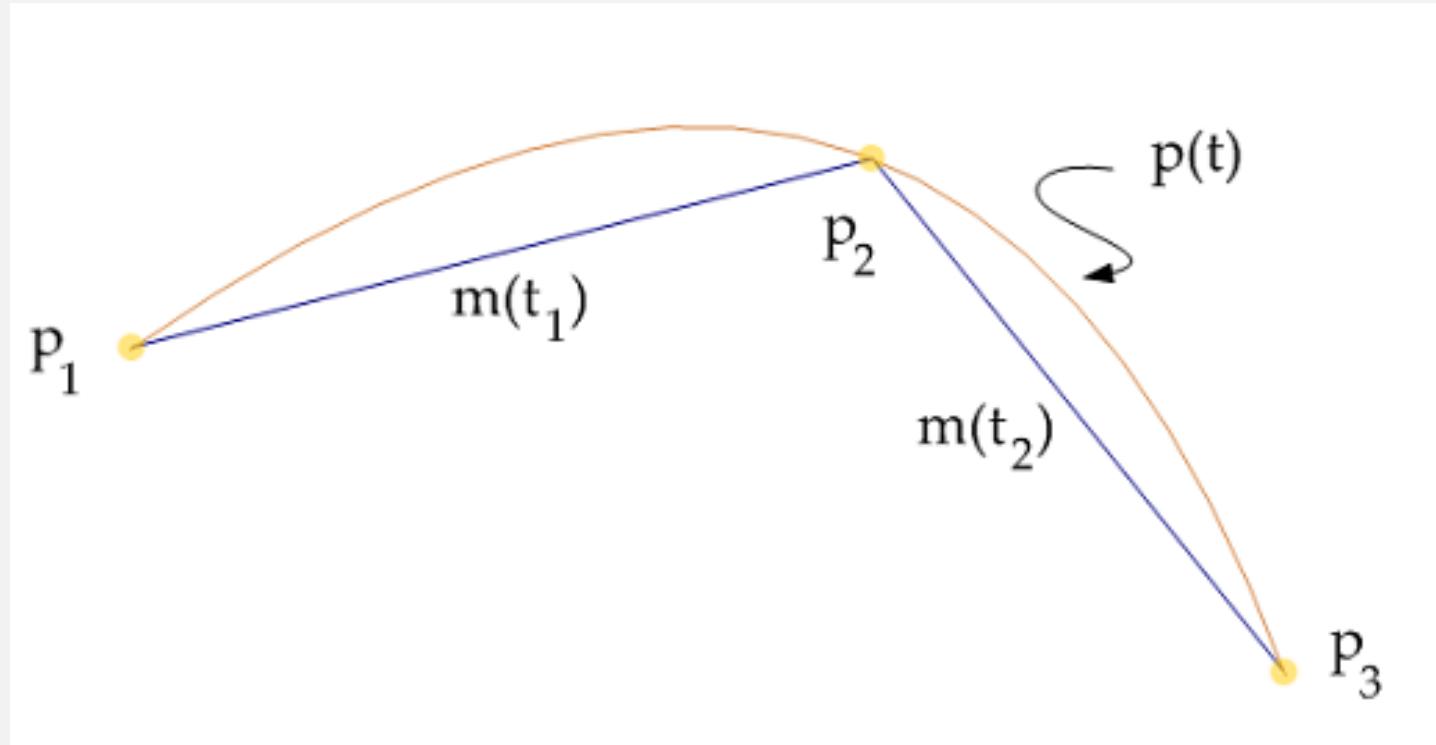
선분과 선분을 섞기



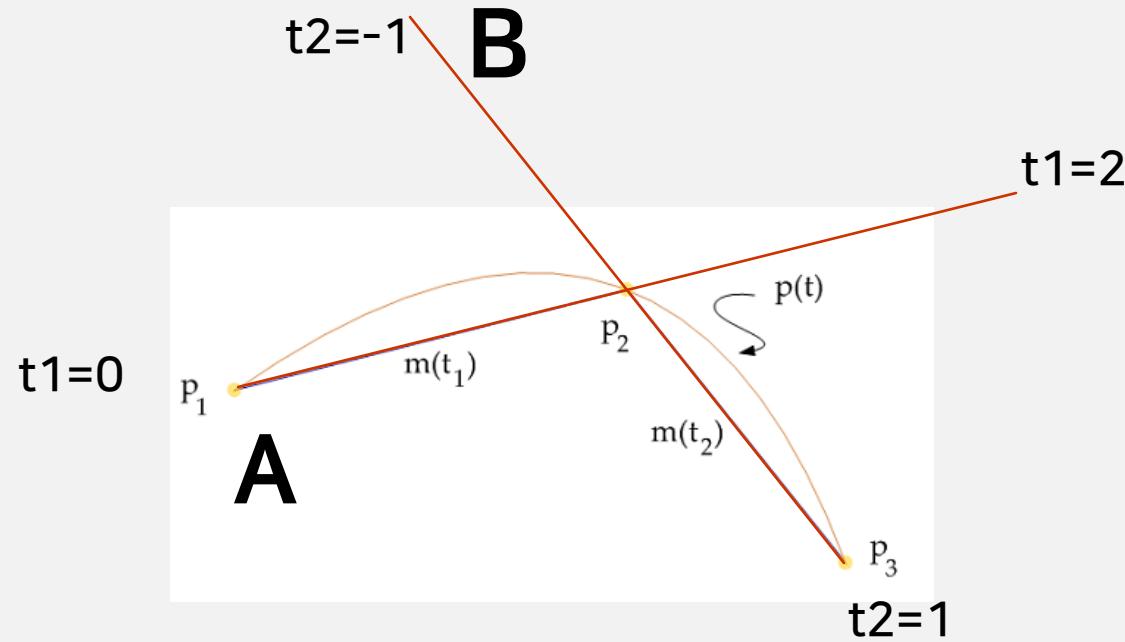
곡선은 두 선분의 중점을 지난다.



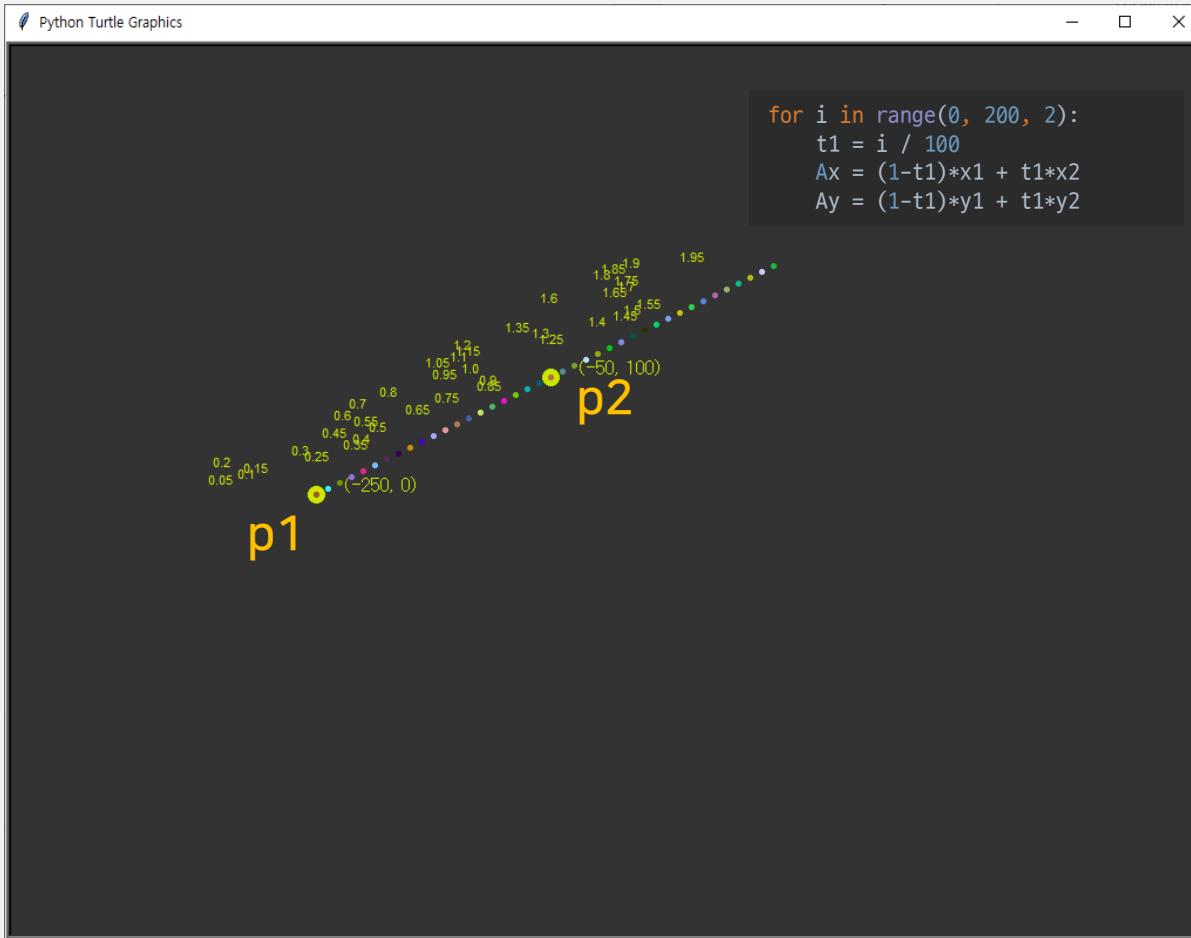
세 개의 점을 잇는 부드러운 곡선을 어떻게 그릴까?



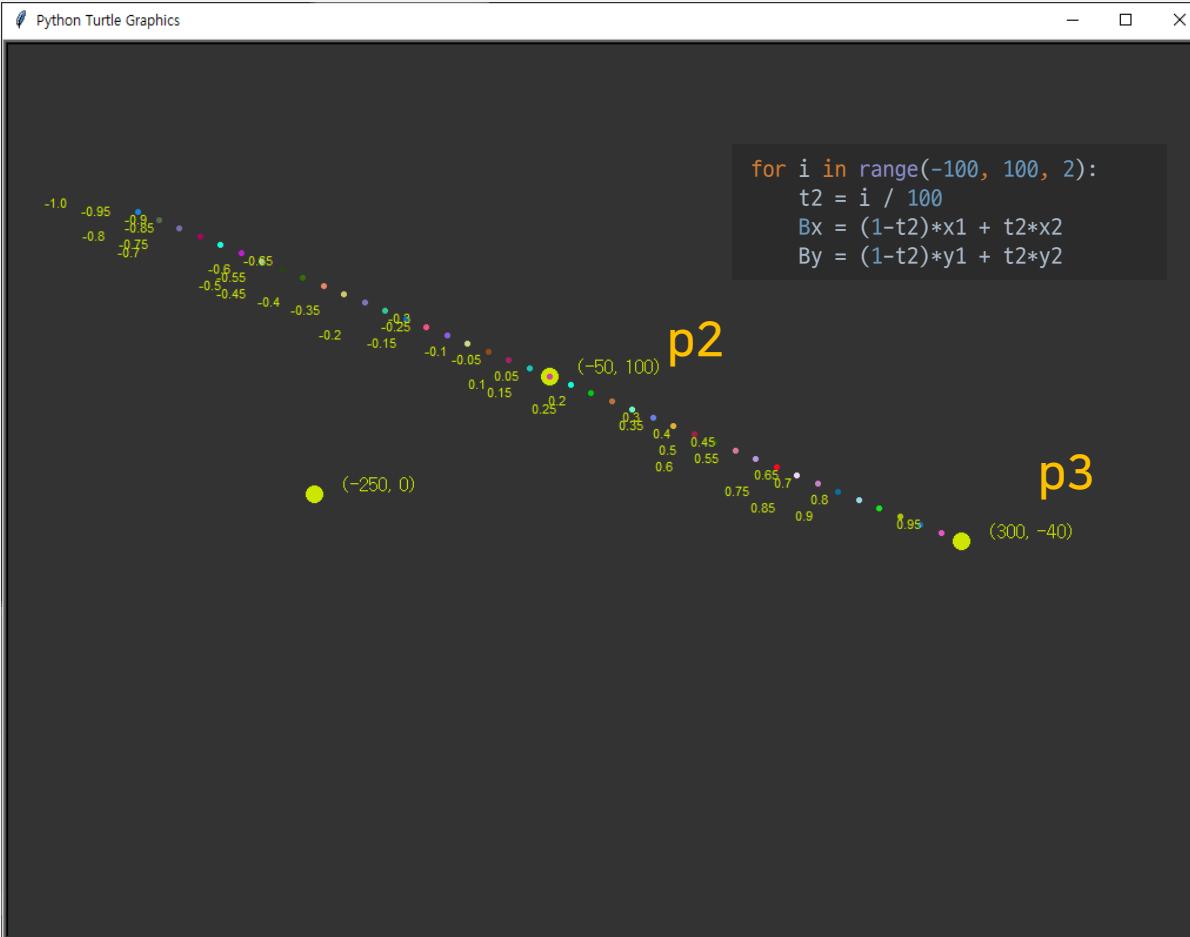
두개 직선 A, B로 확대

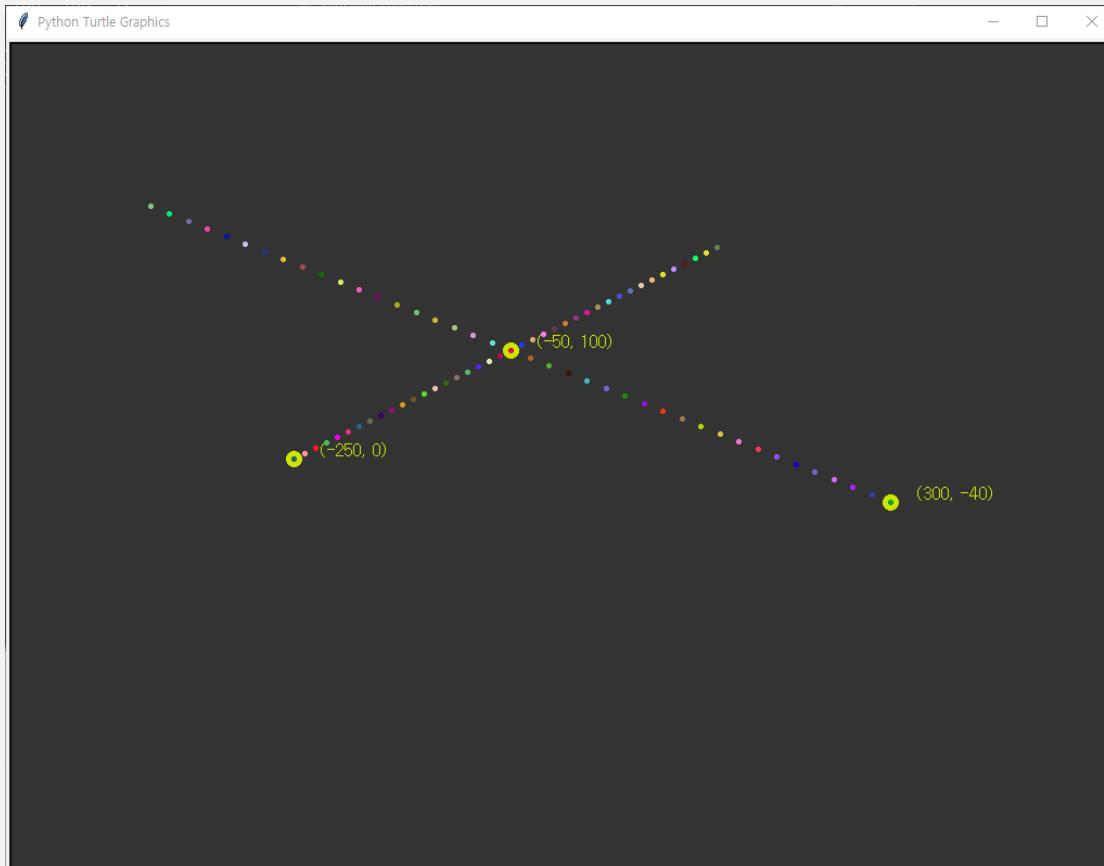


A 직선 그리기 ($t_1 = 0 \sim 2$)



B 직선 그리기 ($t2 = -1 \sim 1$)





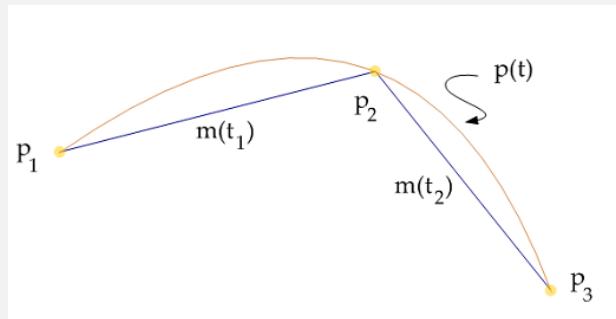
아이디어: 두개의 선분 $m(t_1)$ 과 $m(t_2)$ 를 $1-t:t$ 의 비율로 섞음.

$$m(t_1) = (1 - t_1) p_1 + t_1 p_2$$

$$m(t_2) = (1 - t_2) p_2 + t_2 p_3$$

$$p(t) = (1 - t) m(t_1) + t m(t_2)$$

$$= (1 - t)((1 - t_1) p_1 + t_1 p_2) + t ((1 - t_2) p_2 + t_2 p_3)$$



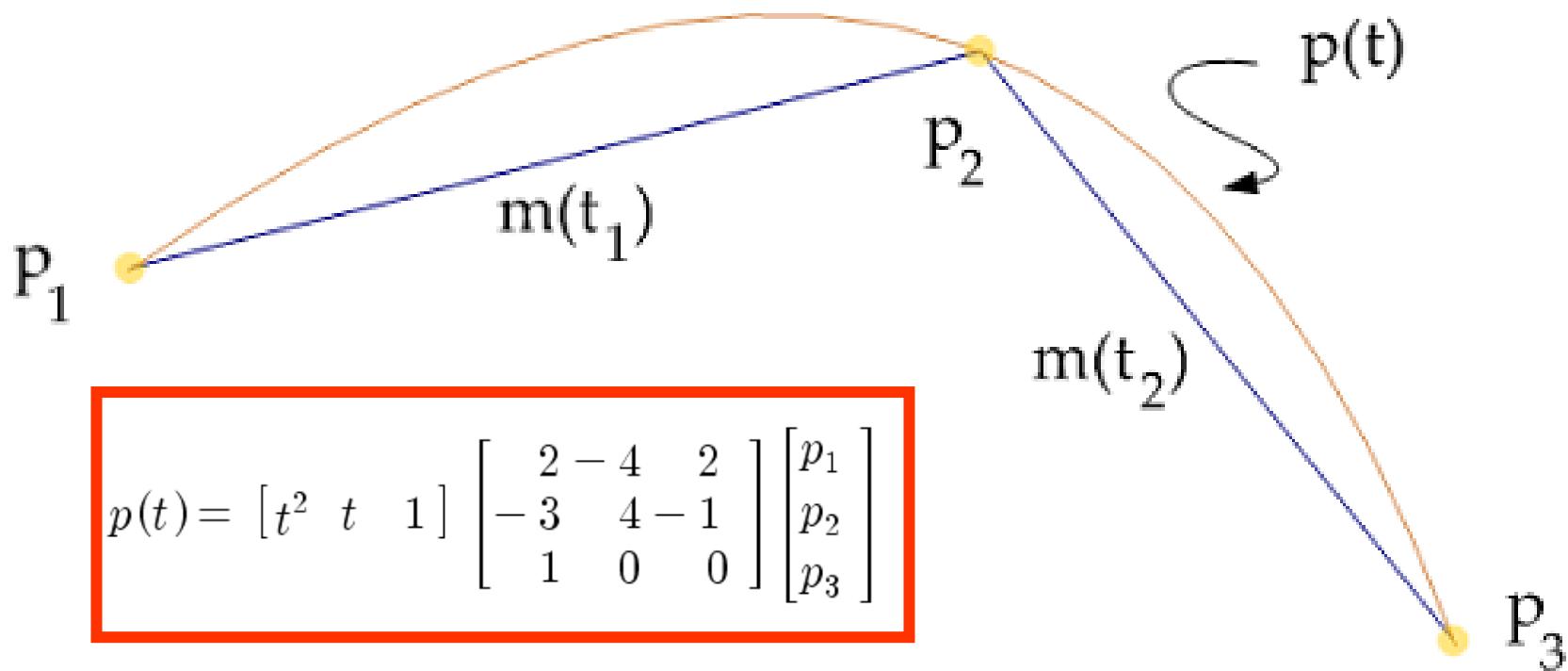
$m(t_1)$ 에서 $t_1 = 0$ 인 점 p_1 은 $p(t)$ 로 볼 때 $t = 0$ 인 점

$m(t_1)$ 에서 $t_1 = 1$ 인 점 p_2 는 $p(t)$ 에서 $t = 1/2$ 인 중간점으로 간주

$m(t_2)$ 에서 $t_2 = 0$ 인 점 p_2 는 $t = 1/2$

따라서 $t_1 = 2t$, $t_2 = 2t - 1$.

카디널 스플라인(Cardinal Spline)



curve.py



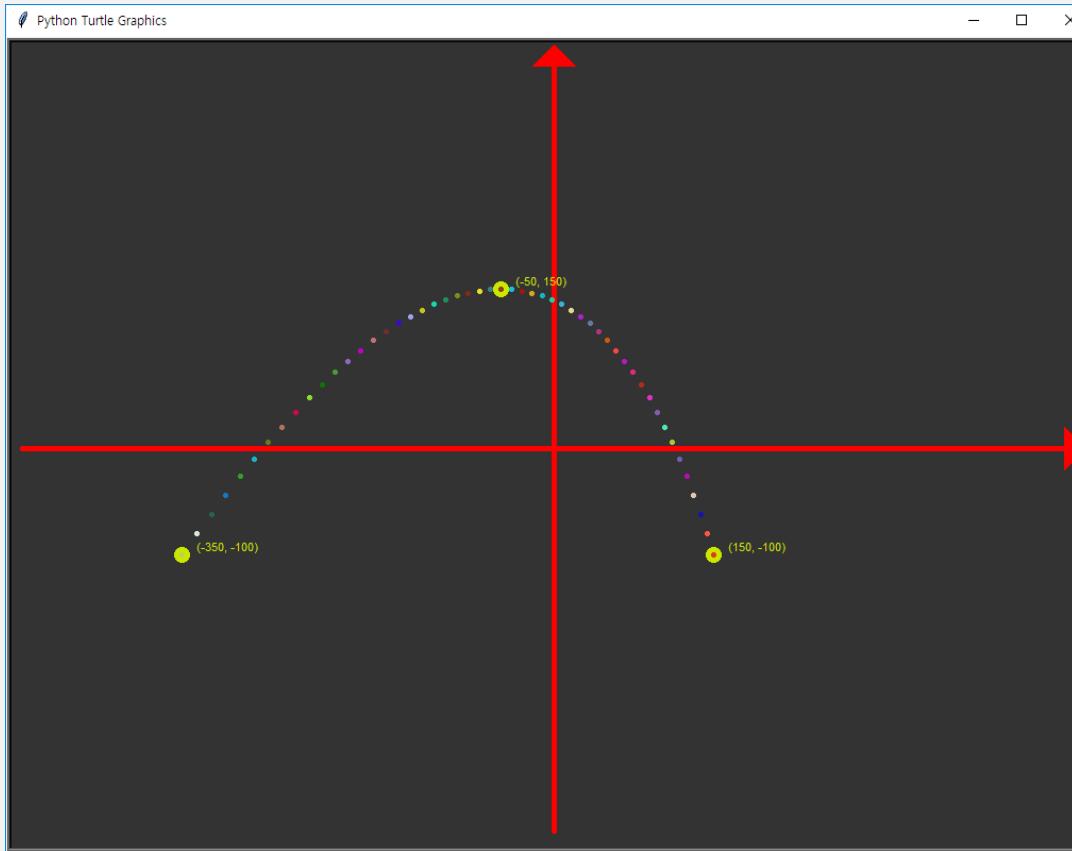
```
def draw_curve_3_points(p1, p2, p3):
    draw_big_point(p1)
    draw_big_point(p2)
    draw_big_point(p3)

    x1, y1 = p1; x2, y2 = p2; x3, y3 = p3

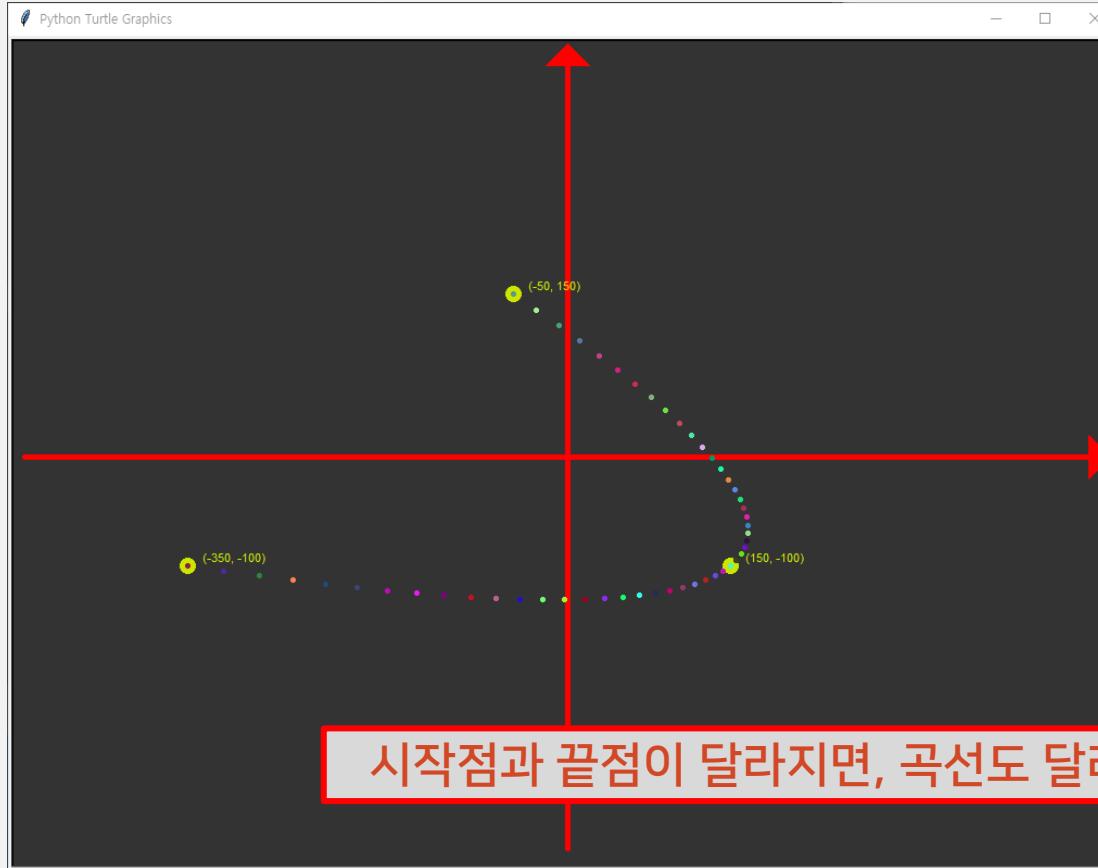
    for i in range(0, 100, 2):
        t = i / 100
        x = (2 * t ** 2 - 3 * t + 1) * x1 + (-4 * t ** 2 + 4 * t) * x2 + (2 * t ** 2 - t) * x3
        y = (2 * t ** 2 - 3 * t + 1) * y1 + (-4 * t ** 2 + 4 * t) * y2 + (2 * t ** 2 - t) * y3
        draw_point((x, y))

    draw_point(p3)
```

```
draw_curve_3_points((-350, -100), (-50, 150), (150, -100))
```

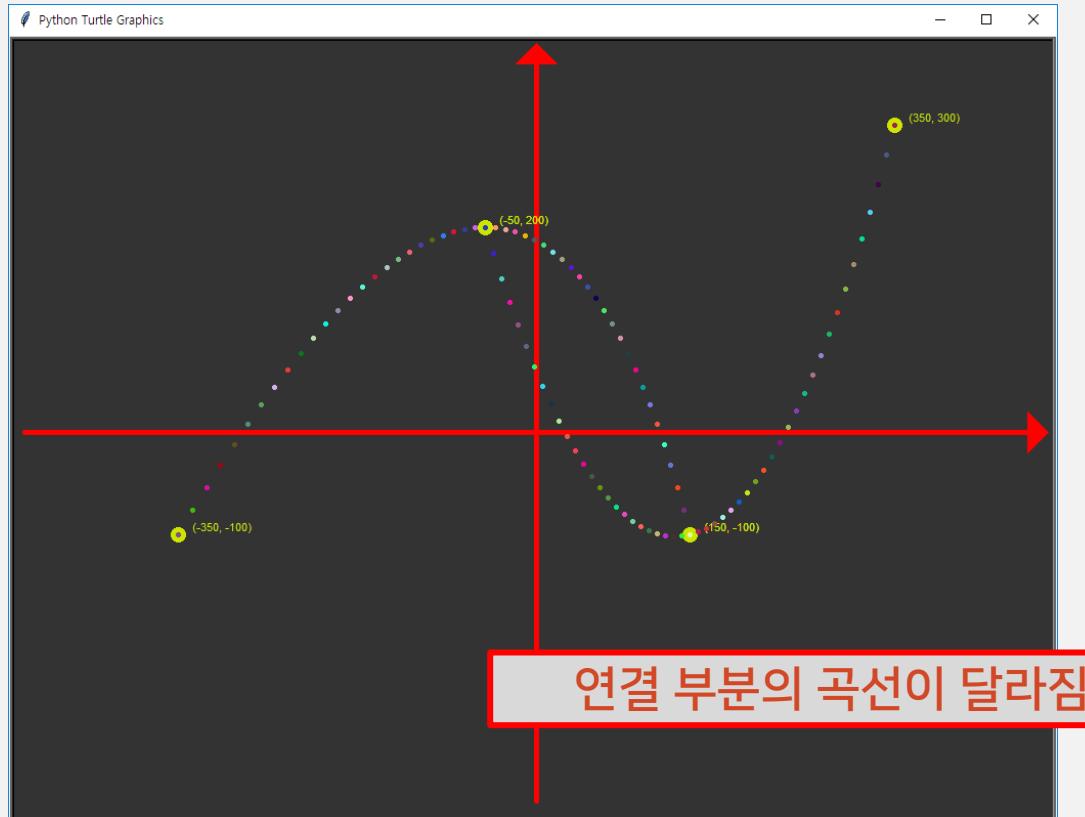


```
draw_curve_3_points((-50, 150), (150, -100), (-350, -100))
```



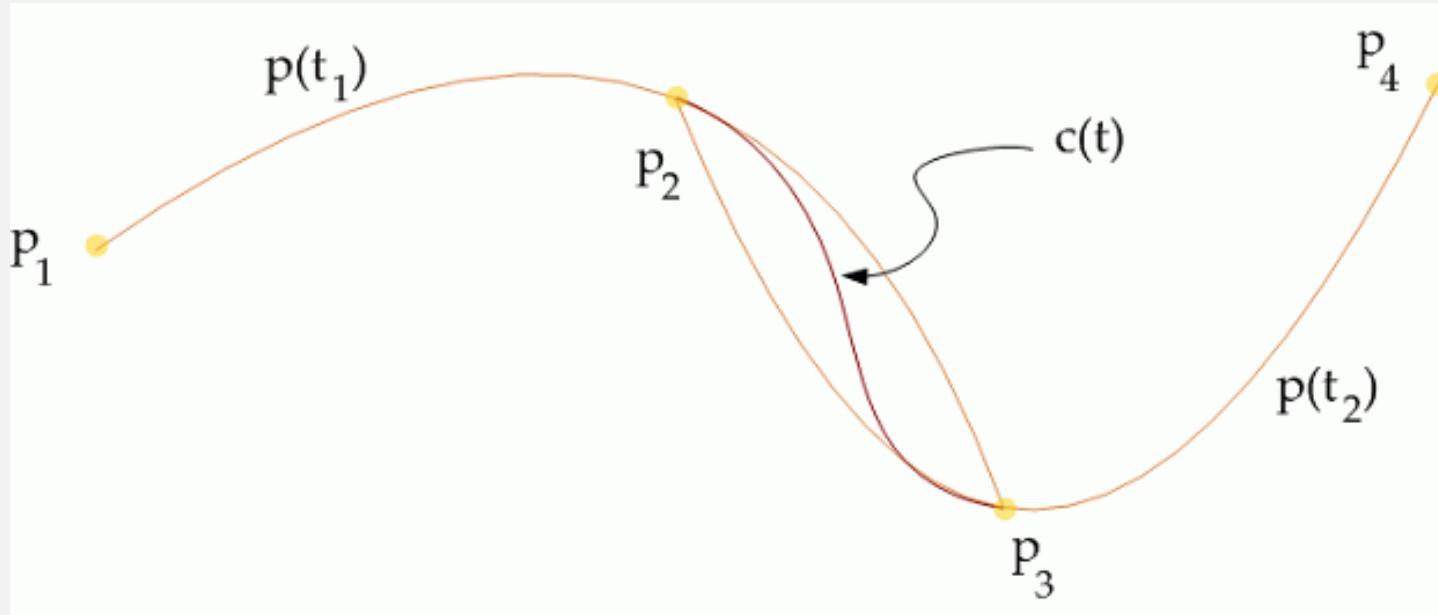
연속되는 점들 사이의 이동

```
draw_curve_3_points((-350, -100), (-50, 200), (150, -100))  
draw_curve_3_points((-50, 200), (150, -100), (350, 300))
```

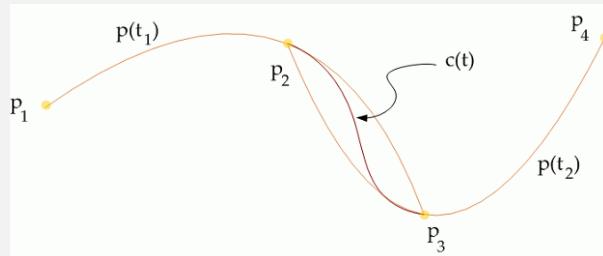


네 점을 연결하는 부드러운 곡선은?

아이디어: 두개의 곡선 $p(t_1)$ 과 $p(t_2)$ 를 $1-t:t$ 의 비율로 섞음.



$$\begin{aligned}
c(t) &= (1-t)p(t_1) + tp(t_2) \\
&= \frac{1}{2}((-t^3 + 2t^2 - t)p_1 + (3t^3 - 5t^2 + 2)p_2 + (-3t^3 + 4t^2 + t)p_3 + (t^3 - t^2)p_4) \\
&= [t^3 \quad t^2 \quad t \quad 1] \frac{1}{2} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} \quad (0 \leq t \leq 1) \\
&= T \ M_c \ P
\end{aligned}$$



곡선 $c(t)$ 는 p_2-p_3 구간에서만 정의됨.

점 네개가 주어질 때, 가운데 두점 사이에서만 정의됨.

curve.py



```
def draw_curve_4_points(p1, p2, p3, p4):
    draw_big_point(p1); draw_big_point(p2); draw_big_point(p3); draw_big_point(p4)

    # draw p1-p2
    for i in range(0, 50, 2):
        t = i / 100
        x = (2*t**2-3*t+1)*p1[0]+(-4*t**2+4*t)*p2[0]+(2*t**2-t)*p3[0]
        y = (2*t**2-3*t+1)*p1[1]+(-4*t**2+4*t)*p2[1]+(2*t**2-t)*p3[1]
        draw_point((x, y))
    draw_point(p2)

    # draw p2-p3
    for i in range(0, 100, 2):
        t = i / 100
        x = ((-t**3 + 2*t**2 - t)*p1[0] + (3*t**3 - 5*t**2 + 2)*p2[0] + (-3*t**3 + 4*t**2 + t)*p3[0] + (t**3 - t**2)*p4[0])/2
        y = ((-t**3 + 2*t**2 - t)*p1[1] + (3*t**3 - 5*t**2 + 2)*p2[1] + (-3*t**3 + 4*t**2 + t)*p3[1] + (t**3 - t**2)*p4[1])/2
        draw_point((x, y))
    draw_point(p3)

    # draw p3-p4
    for i in range(50, 100, 2):
        t = i / 100
        x = (2*t**2-3*t+1)*p2[0]+(-4*t**2+4*t)*p3[0]+(2*t**2-t)*p4[0]
        y = (2*t**2-3*t+1)*p2[1]+(-4*t**2+4*t)*p3[1]+(2*t**2-t)*p4[1]
        draw_point((x, y))
    draw_point(p4)
```

p1-p2구간은 p1,p2,p3로부터 앞의 50%를 계산

P2-p3구간은 p1, p2, p3, p4로부터 계산

p3-p4구간은 p2, p3, p4로부터 뒤의 50%를 계산

```
draw_curve_4_points((-350, -100), (-50, 200), (150, -100), (350, 300))
```

