k-CANADIAN TRAVELLER PROBLEM ON AN OUTERPLANAR GRAPHS

Abstract

In the realm of modern science and information technologies, questions related to optimal traversal have become an integral part of numerous fields. In this context, the exploration of the traveling salesman problem on outerplanar graphs garners attention as a current and intricate challenge. Outerplanar graphs constitute an subset of graphs where vertices and edges are positioned in a manner that they do not intersect in a plane and all vertices belong to the outer face of the drawing. The traveling salesman problem on such graphs addresses issues of routing, optimization, and navigation, finding broad applications in domains spanning both mathematics and computer science, as well as practical scenarios such as route planning in communication networks and transportation systems. In this article, we will delve into the fundamental aspects of the traveling salesman problem on outerplanar graphs, scrutinizing existing approaches to its resolution and investigating their applicability across various contexts.

Keywords Outerplanar graph · Canadian Traveller Problem

Introduction

We work on a weighted outerplanar graph $G = (V, E, w)$, $s, t \in V$.

Set of blocked edges : $E_k \subset E$, $|E_k| \leq k$.

Graph G/E_K does not disconnected s from t.

Considering G with its outerplanar structure it admits an outer face adjacent to all vertexes. We distinguish two sides of the outer face, for example:

from s to t in clockwise direction: F_A, and from t to s as F_B.

$s, t \in F_A$, F_B and $F_A \cap F_B = \{s, t\}$, $F_A \cup F_B = \{V\}$

Jmps and traversing chords.

A **Jmp** is an edge connecting two vertexes of F_A or two vertexes of F_B which do not contains s or t.

A **travesing chord** is an edge connecting a vertexes of F_A with a vertexes of F_B. Travesing chords $\subseteq F_A \times F_B$.

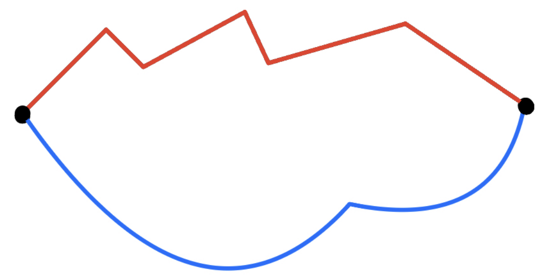

FIGURE 1. red - F_A; blue - F_B.

Property of outerplaner graph.

A travesing chord which does not contain s or t, say $(x, y) \in (F_A, F_B) \cap E$ let an $(s,t)-$ separator

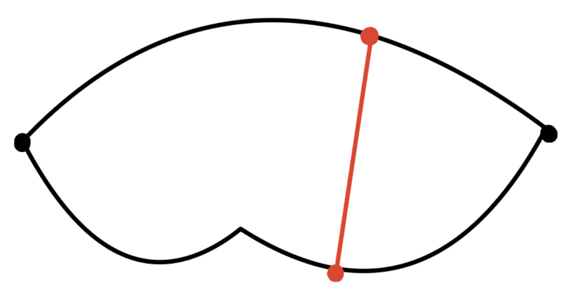

FIGURE 2. red - (x, y).

s and t are disconnected in $G/\{x, y\}$.

The algorithm

Current graph and shortest path.

Given some travel on a graph G, we denote:

- E'_k the set of blockages discovered ($E'_k \supset E_k$);
- P_{min} the shortest (s,t)-path of $G/E_k(G_{curr})$ and w_{min} it's weight (value w_{min} increasen);
- G_{curr} the current graph: $G_{curr} = G/E'_k$;
- For any pair (u,v) of vertexes, $P(u \to v)$ is shortest (u,v)-path on G_{curr}. Let $w(u \to v)$ be it's weight;
- T_{trav}: subgraph (not induced) of G containing the edges traversed by the traveller.

With own strategy, T_{trav} is a tree.
By def, $T_{trav} \cap E_k = \emptyset$.

With our strategy (described later), on edge will be traversed either once or twice (in both directions) during each *"exponential step"*.
$T_{trav} = T^1_{trav} \cup T^2_{trav}$ where T^1_{trav} is traversed once and T^2_{trav} is traversed twice.
We denote by $w_{trav}\ (u,v)$ the cost of the simple path between $u \in T_{trav}$ and $v \in T_{trav}$ on the tree.

Algorithm 1 Main Algorithm

Input
  G, s, t
Fix $w^*{:=}\ w_{min}$
while the traveller did not reach t **do**
  Step(G_{curr}, z, t, w^*) ▷ Where G_{curr} is updated when the blockage discovered. w^* indicate the budget allowed for the travel (we only consider (still-path of cost between w^* and $2w^*$)).
end while

Each exponential step: the idea is to consider paths from s to t with cost between $w*$ and $2w*$, where w^* is a bound equal to w_{min} at the beginning.

Algorithm 2 Step

Input

G, s, t, w^*

while the traveller did not reach t OR there are (s,t)-paths of cost $< 2w^*$ **do**

$P_{eff} \leftarrow P_{min}$

Traverse P_{eff} until blockage ▷ assume wlog that the traveller is blocked on F_A

E1

$(v,w) \leftarrow$ maximal jmp $s.t. v \in T_{trav}, w \in P_{eff}/T_{trav}, (v,w)$ open and $w(s,v)_{trav} + w(v,w) + w(w,t) < 2w^*$ ▷ cost of the shortest (s,t)-path apparently open passing through (v,w)

if (v,w) exists **then**

Backtrack on T_{trav} towards v

$P_{eff} \leftarrow T_{trav}(s,v) \cup (v,w) \cup P(w \to t)$

Traverse P_{eff} from v to t

if blocked **then**

Go back to E1

end if

end if

$u \leftarrow$ position of the traveller ▷ $u \in T_{trav}$

Backtrack on $T_{trav}(s,u)$ until finding a vertex $x \in T_{trav}(s,u)$ such that:

- either path $T_{trav}(s,x) \cup P(x \to t)$ has cost $< 2w^*$ Cax A

- or $x \in F_B$ Cax B

if x exists AND Cax A **then**

$y \leftarrow$ vertex s.t. (x, y) is the first edge on path $P(x \to t)$ ▷ (x,y) is necessarily a travesty chord

$G' \leftarrow G$ without vertex x and the source side of separator $\{x,y\}$ ▷ G' is an outerplanar subgraph of G

Step(G', y, t, w^*) ▷ The idea is to "resource" at vertex y. Induced, we know that all paths from s to t with cost $< 2w^*$ will pass through y.

end if

if x exists AND Cax B **then**

$z \leftarrow$ successor of x on P_{eff} ▷ (x,z) is a travesty chord

$G' \leftarrow G$ without the source side of separator $\{x,z\}$

Step(G', z, t, w^*)

end if

if x does not exist **then**

Terminate ▷ we go to the next step and we know that $w_{min} \geq 2w^*$

end if

end while