EE5340

INTRODUCTION TO QUANTUM COMPUTING AND PHYSICAL BASICS OF COMPUTING

Quantum Mechanics [Basics of the Basics]

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Basics

- States = vectors in a complex vector space
- Physical observables described by linear operators
  - Observables = what we can measure
  - Operators must be linear and Hermitian

Observables

- What we can measure
- Examples
  - Coordinates of a point
  - Energy of a system
  - ...
- "M(achine)" analogy

$$M|A >= |B >$$

Linear Operators $M|A>=|B>$

1. Should result in a unique output for every (input) vector in the space

2. For any complex z: $Mz|A>=z|B>$

3. $M(|A>+|B>)=M|A>+M|B>$

Linear Operators

$M|A> = |B>$

$$|A> = \sum_j \alpha_j |j> \qquad |B> = \sum_j \beta_j |j>$$

$$\sum_j M|j> \alpha_j = \sum_j \beta_j |j>$$

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$<k|j>=0$ if j and k not equal (1 otherwise)

$$\sum_j <k|M|j>\alpha_j = \beta_k \qquad \sum_j m_{kj}\alpha_j = \beta_k$$

m_{kj} matrix elements of M

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m_{kj} matrix elements of M

arranged by an NxN matrix

Eigenvectors and Eigenvalues

- A linear operator acting on a vector generally changes the direction of the vector
- Any linear operator preserves the direction of specific input vectors
  - Eigenvectors

$$M|\lambda >= \lambda|\lambda >$$

- λ: both a complex number and a ket
- Constant multiplier: eigenvalue (generally complex)

Linear Operators

Can also act on bra-vectors

$$< B| = [\beta_1^* \beta_2^* ... \beta_n^*]$$

$$< B|M$$

Hermitian Conjugation

Complex conjugation for operators

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`complex conjugate?`

$$\sum_i m^*_{ji}\alpha^*_i = \beta^*_j$$

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matrix form using kets?

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`complex conjugate?`

$$\sum_i m_{ji}^*\alpha_i^* = \beta_j^*$$

`matrix form using kets?`

$$<A|M^\dagger = [\alpha_1^*\ \alpha_2^*\ \alpha_3^*]\begin{bmatrix} m_{11}^* & m_{21}^* & m_{31}^* \\ m_{12}^* & m_{22}^* & m_{32}^* \\ m_{13}^* & m_{23}^* & m_{33}^* \end{bmatrix}$$

Hermitian Conjugation

changing an equation from ket-form to bra-form:

1. Interchange rows and column (i.e., transpose matrix)
2. Complex conjugate each matrix element

Hermitian Conjugation

$M^\dagger$ Hermitian conjugate (complex conjugate of transpose)

$$M|A>=|B>$$

$< A|M^\dagger =< B|$ where $M^\dagger = [M^T]^*$

Hermitian Operators

- Any measurement renders real numbers
- Observables are represented by linear operators
  - which are equal to their own Hermitian conjugates
  - called "Hermitian operators"
  - special properties

$$M^\dagger = M$$

$$m_{ji} = m_{ij}^*$$

Hermitian Operators

$M^\dagger = M$

- Have real eigenvalues

$L|\lambda >= \lambda|\lambda >$ `assume L is Hermitian`

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Cheatsheet: Hermitian Conjugation

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$<\lambda|L^\dagger=<\lambda|\lambda^*$

$<\lambda|L=<\lambda|\lambda^*$ as $L=L^\dagger$

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$<\lambda|L|\lambda>=\lambda^*<\lambda|\lambda>$

for both equations to be true $\lambda=\lambda^*$

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for both equations to be true $\lambda = \lambda^*$ QED

Hermitian Operators & Orthonormal Basis

- **Observables are represented by Hermitian operators**
- **Eigenvectors of a Hermitian operator form a complete set**
  - Linear combination of its eigenvectors can represent any vector that the operator can generate
- **The eigenvectors corresponding to different eigenvalues are orthogonal**
- **Even for equal eigenvalues, corresponding eigenvectors can be chosen orthogonal**
  - Different eigenvectors having the same eigenvalue: degeneracy

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`Eigenvectors of a Hermitian operator form an orthonormal basis!`

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$$< \lambda_1|L = \lambda_1 < \lambda_1|$$ multiply by $|\lambda_2 >$

$$< \lambda_1|L|\lambda_2 >= \lambda_1 < \lambda_1|\lambda_2 >$$

subtract

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$$< \lambda_1|L|\lambda_2 >= \lambda_2 < \lambda_1|\lambda_2 >$$

$$0 = (\lambda_1 - \lambda_2) < \lambda_1|\lambda_2 >$$

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$$L|\lambda_2 >= \lambda_2|\lambda_2 >$$

$$0 = (\lambda_1 - \lambda_2) < \lambda_1|\lambda_2 >$$

for $\lambda_1 \neq \lambda_2$ inner product $< \lambda_1|\lambda_2 >$ must be 0

the two eigenvectors must be orthonormal

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$$|A >= \alpha|\lambda_1 > +\beta|\lambda_2 >$$

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$$L|A> = \alpha L|\lambda_1> + \beta L|\lambda_2>$$

$$L|A> = \alpha\lambda|\lambda_1> + \beta\lambda|\lambda_2>$$

$$L|A> = \lambda(\alpha|\lambda_1> + \beta|\lambda_2>) = \lambda|A>$$

Gram-Schmidt Procedure

- What if a linearly independent set of eigenvectors do not form an orthonormal set?
- Usually happens in a system with degenerate states
  - Distinct states with same eigenvalues
- It is possible to construct orthonormal vectors, example for two vectors:
  1. Divide first vector by its length, to render a unit vector parallel to the first
     - forms first vector in the orthonormal set
  2. Project the second into the direction of the first (inner product)
  3. Subtract the result from 2. from the second vector
  4. Normalize the outcome from 3. (divide by its own length)

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- Possible results of a measurement:
  - the eigenvalues of the operator representing the observable
  - State for which the measurement result is unambiguously a specific eigenvalue:
    - the corresponding eigenvector
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Principles

- **Observable or measurable quantities are represented by linear operators L**
  - Observable = measurable
  - Operator: states + (their) eigenvalues (i.e., possible measurement results)
- Example: (measurable) components of a spin σ_x, σ_y, σ_z
  - Each measurement renders +1 or -1
  - Result of a measurement is generally statistically uncertain
    - Except for particular states
    - E.g., measuring σ_z for $|u\rangle$ always renders +1; for $|d\rangle$, -1
  - Each observable is associated with a specific linear operator
    - in the 2D space of states describing spin

Principles

- **Possible results of a measurement:**
  - **the eigenvalues of the operator representing the observable**
  - State for which the measurement result is unambiguously a specific eigenvalue:
    - the corresponding eigenvector
  - If the system is in the eigenstate
    - the measurement result is guaranteed to be the corresponding eigenvalue

- The measurement result is **always** a real number from a set of possible results
  - Spin example: +1, -1
  - Each component of the spin operator must have eigenvalues equal to +1 and -1

Principles

- **Unambiguously distinguishable states are represented by orthogonal vectors**

- Two states are physically distinct if there is a measurement to tell them apart
- Spin example: |u> and |d> are distinguishable by measuring $\boldsymbol{\sigma}z$
  - Similarly for |l> and |r> $\boldsymbol{\sigma}x$, and |i> and |o> for $\boldsymbol{\sigma}y$
- Assume that we don't know the initial state, but it is either of |u> and |r>
  - No measurement can tell these possibilities apart
  - Even if $\boldsymbol{\sigma}z$ measurement renders +1 the initial state might have been |r>
  - |u> and |d> are physically distinguishable, |u> and |r> are not
- Inner product of two states can serve as a proxy for distinguishability ("overlap")
  - The principle requires physically distinct states to be represented by
    - orthogonal state vectors = vectors with no overlap
    - <u|d> = 0 where <u|r> =

Principles

- **If |A> is the state-vector of a system and the observable L is measured**
  - **the probability of observing the eigenvalue λ_i is**

$$P(\lambda_i) =< A|\lambda_i >< \lambda_i|A >$$

$$= [\alpha_1^* \ \alpha_2^* \ \alpha_3^*] \begin{bmatrix} \lambda_{i1} \\ \lambda_{i2} \\ \lambda_{i3} \end{bmatrix} [\lambda_{i1}^* \ \lambda_{i2}^* \ \lambda_{i3}^*] \begin{bmatrix} \alpha 1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = | < A|\lambda_i > |^2$$

- Probability = (overlap)^2

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- Possible results of a measurement:
  - the eigenvalues of the operator representing the observable.
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⇒ Operators that represent observables are Hermitian

- Measurement result must be a real number, hence eigenvalues of L
  - Eigenvectors (states) representing unambiguously distinguishable results
    - must have different eigenvalues
    - must be orthogonal

Spin Operators

- Each component is represented by a linear operator, including **σ**z
- Eigenvectors of **σ**z are |u> and |d>, eigenvalues +1 and -1

$$\sigma_z|u>=|u>$$

$$\sigma_z|d>=-|d>$$

- States|u> and |d> are orthogonal to each other $<u|d>=0$

$$\begin{bmatrix}\sigma_{z,11} & \sigma_{z,12}\\ \sigma_{z,21} & \sigma_{z,22}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix} \qquad \begin{bmatrix}\sigma_{z,11} & \sigma_{z,12}\\ \sigma_{z,21} & \sigma_{z,22}\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=-\begin{bmatrix}0\\1\end{bmatrix}$$

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- There is only one matrix that satisfies these equations:

$$\sigma_z=\begin{bmatrix}\sigma_{z,11} & \sigma_{z,12}\\ \sigma_{z,21} & \sigma_{z,22}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

Spin Operators

- From experimental data: measurement of an observable
  - unambiguously renders +1 or -1 for (the so called) |u> and |d> states
- From principles
  - |u> and |d> are orthogonal, and are eigenvectors of $\boldsymbol{\sigma}$z
    - $\boldsymbol{\sigma}$z: linear operator representing this observable
  - Corresponding eigenvalues are measured values +1 and -1
- Similar derivation applies for $\boldsymbol{\sigma}$x and $\boldsymbol{\sigma}$y

Spin Operators

- Eigenvectors of **σ**x are |r> and |l>, eigenvalues +1 and -1
- |r> and |l> are linear super-positions of |u> and |d>

$$\sigma_x|r>=|r>$$

$$\sigma_x|l>=-|l>$$

$$|r>=\frac{1}{\sqrt{2}}|u>+\frac{1}{\sqrt{2}}|d>$$

$$|l>=\frac{1}{\sqrt{2}}|u>-\frac{1}{\sqrt{2}}|d>$$

$$\begin{bmatrix}\sigma_{x,11} & \sigma_{x,12}\\ \sigma_{x,21} & \sigma_{x,22}\end{bmatrix}\begin{bmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{bmatrix}=\begin{bmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{bmatrix}$$

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Spin Operators

- Eigenvectors of $\boldsymbol{\sigma}$y are |i> and |o>, eigenvalues +1 and -1

$$\sigma_y|i>=|i>$$

$$\sigma_y|o>=-|o>$$

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$$\begin{bmatrix}\sigma_{y,11} & \sigma_{y,12}\\ \sigma_{y,21} & \sigma_{y,22}\end{bmatrix}\begin{bmatrix}1/\sqrt{2}\\ i/\sqrt{2}\end{bmatrix}=\begin{bmatrix}1/\sqrt{2}\\ i/\sqrt{2}\end{bmatrix}$$

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Pauli Matrices

$$\sigma_z = \begin{bmatrix} \sigma_{z,11} & \sigma_{z,12} \\ \sigma_{z,21} & \sigma_{z,22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

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Putting It All Together

- Operators are used to calculate eigenvalues and eigenvectors
- Operators act on states, not on actual physical systems
- Operator acting on a state vector generates another state vector
- Measuring an observable is **not** the same as
  - operating with the corresponding operator L on the state
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- Example:

$$\sigma_z|u>=|u>$$

$$\sigma_z|d>=-|d>$$

If the system is prepared in state |d>

- measurement certainly renders -1
- post measurement state becomes -|d>
- |d> and -|d> are practically the same states

Putting It All Together

- Operators are used to calculate eigenvalues and eigenvectors
- Operators act on states, not on actual physical systems
- Operator acting on a state vector generates another state vector
- Measuring an observable is **not** the same as
  - operating with the corresponding operator L on the state
  - e.g., from initial state |A>, measurement cannot render L|A>
- Example:

$$\sigma_z|u>=|u>$$

$$\sigma_z|d>=-|d>$$

If the system is prepared in |r>, measurement renders, with equal probability,

- either +1 and a post measurement state |u>
- or -1 and a post measurement state -|d>

$$|r>=\frac{1}{\sqrt{2}}|u>+\frac{1}{\sqrt{2}}|d>$$

$$\sigma_z|r>=\frac{1}{\sqrt{2}}\sigma_z|u>+\frac{1}{\sqrt{2}}\sigma_z|d>$$

$$=\frac{1}{\sqrt{2}}|u>-\frac{1}{\sqrt{2}}|d>$$

Spin component along any axis?

$$\hat{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \qquad \sigma_n = \sigma\hat{n} = n_x\sigma_x + n_y\sigma_y + n_z\sigma_z$$

$$\sigma_n = \sigma\hat{n} = n_x \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + n_y \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} + n_z \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} n_z & (n_x - in_y) \\ (n_x + in_y) & -n_z \end{bmatrix}$$

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Given any state $|A >= \alpha_u|u > +\alpha_d|d >$

There exists a direction p such that $\sigma_p|A >= |A >$

How do states change with time?

- The state at time t: U(t) acting on system state at time 0
  - $|\psi(t)>$ is determined by $|\psi(0)>$
  - U: time development operator of the system

$$|\psi(t) >= U(t)|\psi(0) >$$

- State vector evolves in a deterministic manner
- Measurement is still of statistical nature
- Quantum evolution of states allows computation of probabilities of measurements

$|\psi(t)>= U(t)|\psi(0)>$

- U(t) is required to be a linear operator
  - State-space is a vector-space

Linear Operators $M|A>=|B>$

1. Should result in a unique output for every (input) vector in the space

2. For any complex z: $Mz|A>=z|B>$

3. $M(|A>+|B>)=M|A>+M|B>$

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$U^\dagger(t)U(t)$ acts as I between members of a basis set

Unitarity

$$U^\dagger(t)U(t) = I$$ if U is unitary

- Time evolution is unitary
- One more principle of QM
  - The evolution of state vectors with time is unitary

Principles

- Observable or measurable quantities are represented by linear operators L
- Possible results of a measurement:
  - the eigenvalues of the operator representing the observable.
  - State for which the measurement result is unambiguously a specific eigenvalue:
    - the corresponding eigenvector
  - If the system is in the eigenstate
    - the measurement result is guaranteed to be the corresponding eigenvalue
- Unambiguously distinguishable states are represented by orthogonal vectors.
- If |A> is the state-vector of a system and the observable L is measured
  - the probability of observing the eigenvalue λi is

$$P(\lambda_i) =< A|\lambda_i >< \lambda_i|A >$$

- Evolution of state-vectors with time is unitary

Hamiltonian

$$U^{\dagger}(\epsilon)U(\epsilon) = I$$ unitarity

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$$H^\dagger - H = 0$$ unitarity condition

H is Hermitian, hence is observable

H: Quantum Hamiltonian

Hamiltonian

$$|\psi(t)>= U(t)|\psi(0)> \qquad U(\epsilon) = I - i\epsilon H$$

$$|\psi(\epsilon)>= |\psi(0)> -i\epsilon H|\psi(0)>$$

$$\frac{|\psi(\epsilon)> -|\psi(0)>}{\epsilon} = -iH|\psi(0)>$$

$$\epsilon \to 0$$

$$\frac{\partial|\psi>}{\partial t} = -iH|\psi>$$

Hamiltonian

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$$\frac{|\psi(\epsilon)>-|\psi(0)>}{\epsilon}=-iH|\psi(0)>$$

$$\epsilon \to 0$$

$$\frac{\partial|\psi>}{\partial t}=-iH|\psi>$$ dimension fix $$\hbar\frac{\partial|\psi>}{\partial t}=-iH|\psi>$$

Expectation $< L >= \sum_i \lambda_i P(\lambda_i)$

$$|A >= \sum_i \alpha_i |\lambda_i >$$

$$< A|L|A >=?$$

$$L|A >= \sum_i \alpha_i L|\lambda_i >$$

Let L operate on both sides ...

$$L|A >= \sum_i \alpha_i \lambda_i |\lambda_i >$$

|λi> are L's eigenvectors ...

$$< A|L|A >= \sum_i (\alpha_i^* \alpha_i)\lambda_i$$

take inner product with <A|,
use orthonormality of eigenvectors ...

$$< A| = \sum_i \alpha_i^* < \lambda_i|$$

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Expectation

$$< L >= \sum_i \lambda_i P(\lambda_i)$$

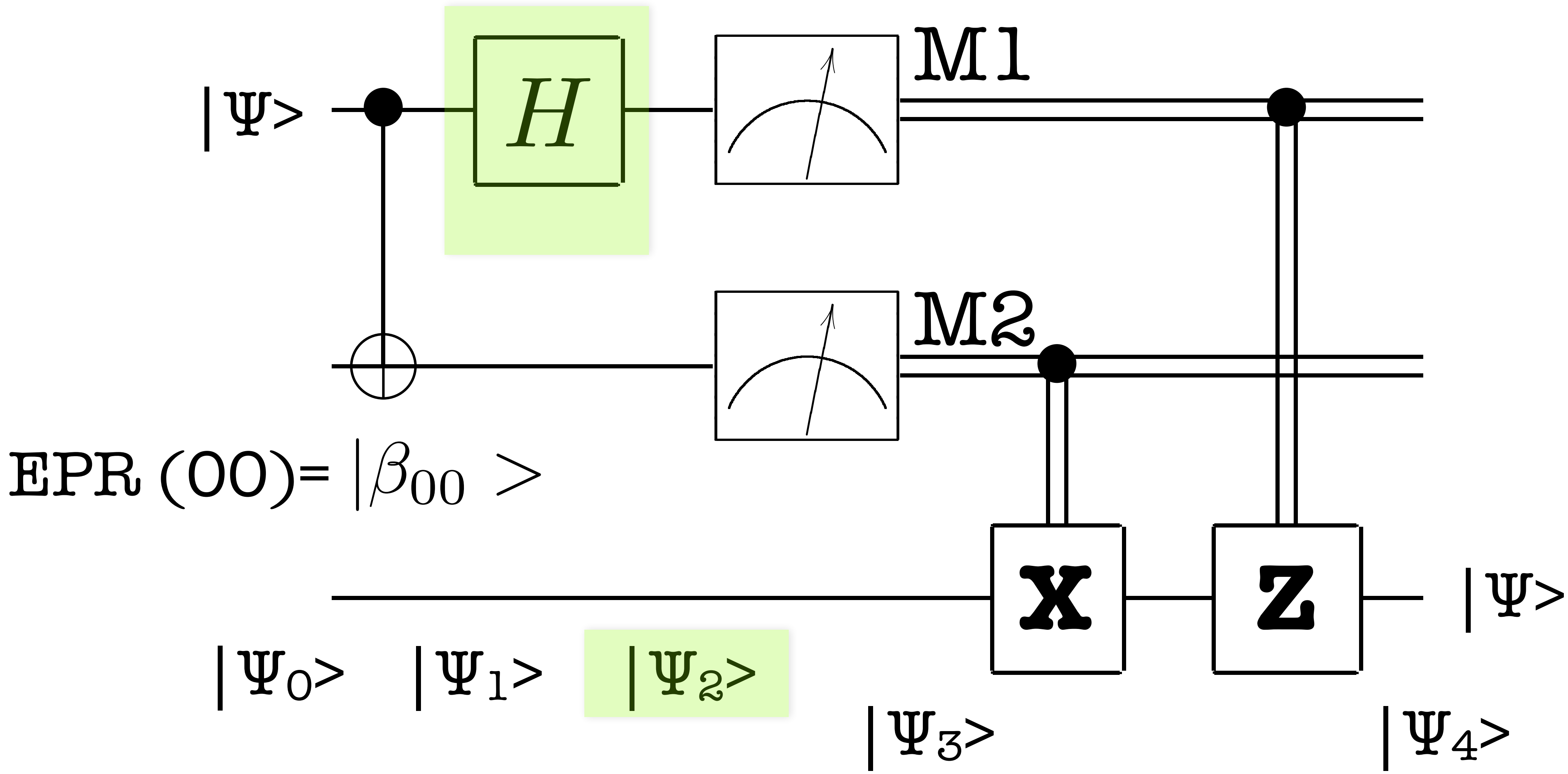
$$|A >= \sum_i \alpha_i |\lambda_i >$$

$$< A|L|A >=?$$

$$< L >$$

$$\sigma_z |r >= \frac{1}{\sqrt{2}} \sigma_z |u > + \frac{1}{\sqrt{2}} \sigma_z |d >$$

Quantum Teleportation



$$|\psi_2>= \frac{1}{2}[|00>(\alpha|0>+\beta|1>)+|01>(\alpha|1>+\beta|0>)$$
$$+|10>(\alpha|0>-\beta|1>)+|11>(\alpha|1>-\beta|0>)]$$

Bibliography

- Nielsen & Chuang, Quantum Computation & Quantum Information, Ch.2
- Susskind & Friedmann, Quantum Mechanics, Ch.3, 4

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INTRODUCTION TO QUANTUM COMPUTING AND PHYSICAL BASICS OF COMPUTING

Quantum Mechanics [Basics of the Basics]

Ulya Karpuzcu