EE5340

INTRODUCTION TO QUANTUM COMPUTING AND PHYSICAL BASICS OF COMPUTING

Basics

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Quantum Bits: qbits

- Mathematical “objects”
- Have “state”:
  - Two observable (basis) states: |0> and |1>
  - |>: Dirac notation, standard notation for states in QM
  - vs. 2-state (0 or 1) classical bits
    - qbits can be in any state formed by linear combinations (superpositions) of basis states

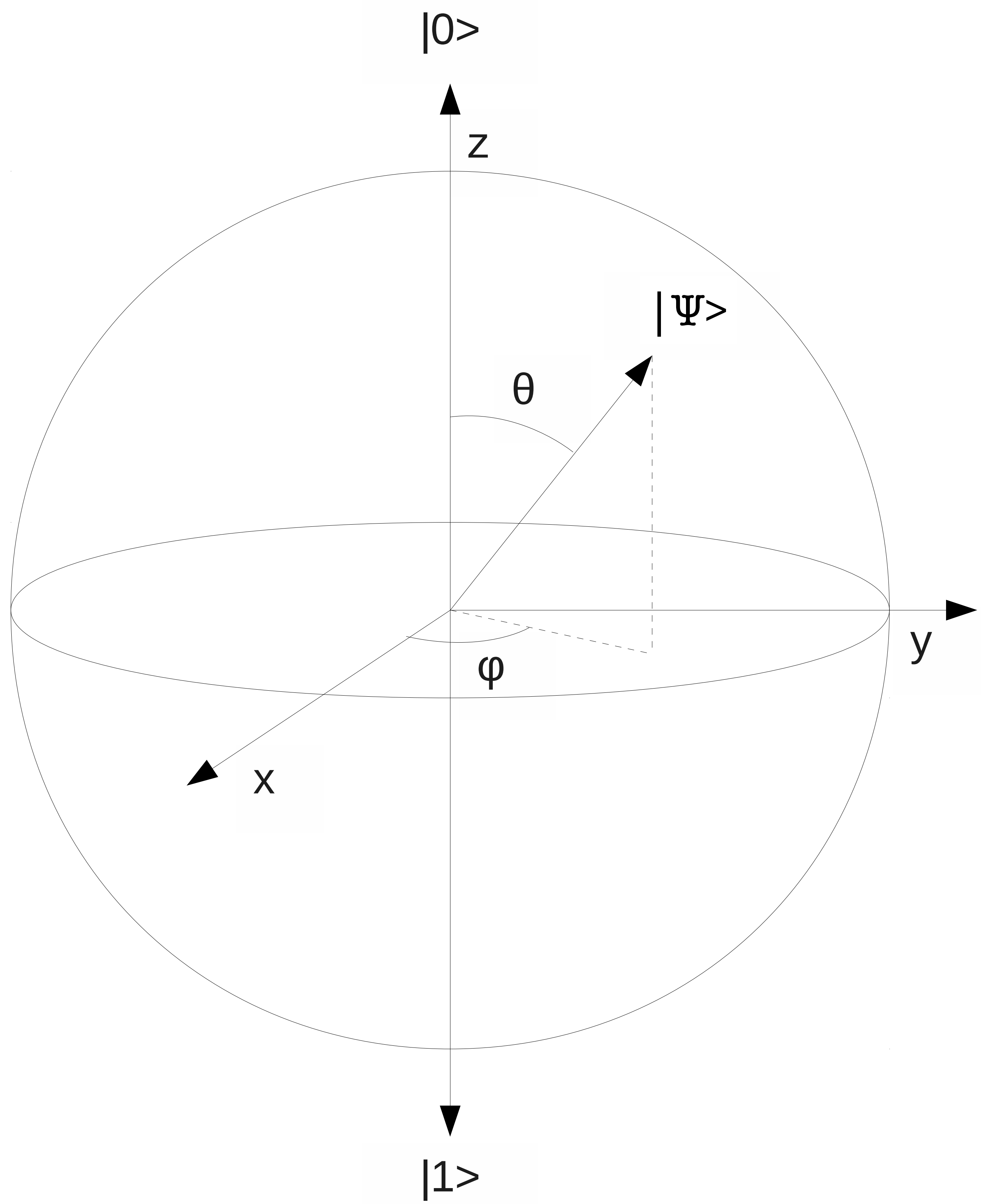
$$|\psi >= a|0> +b|1>= a\begin{bmatrix}1\\0\end{bmatrix} + b\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}a\\0\end{bmatrix} + \begin{bmatrix}0\\b\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$$

Quantum Bits: qbits

$$|\psi >= a|0> +b|1 >= a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

- State of a single qbit: $|\Psi> = a|0> + b|1>$
  - Amplitutes a and b are complex coefficients: $|a|^2 + |b|^2 = 1$
  - $|. >$ "Dirac-Ket" to denote a particular quantum state
  - $|a|^2$ Probability that the qbit will be observed in the state $|0>$
  - $|b|^2$ Probability that the qbit will be observed in the state $|1>$
- Without direct observation, the state of a single qbit spans $[a,b]^T$

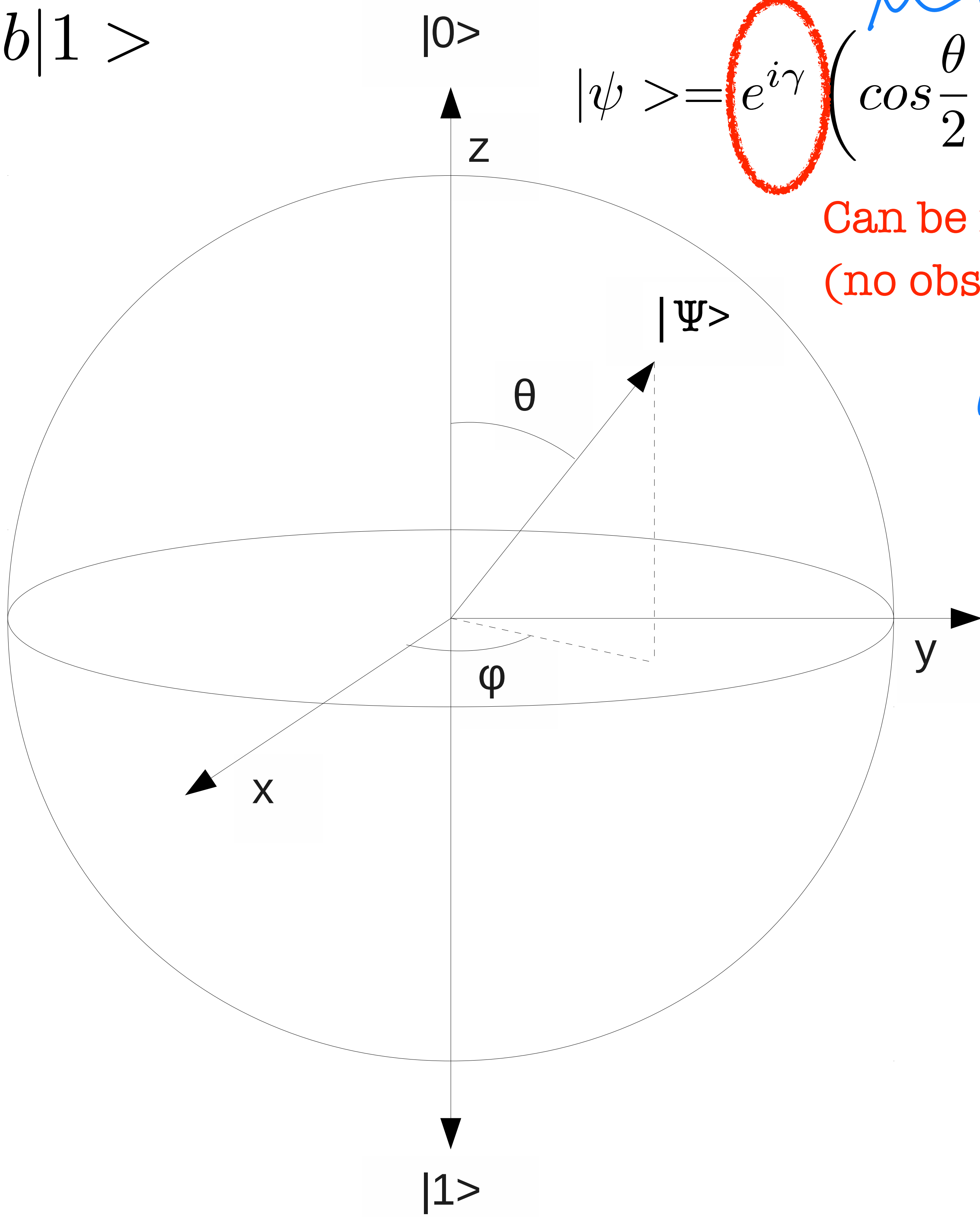
Bloch Sphere Representation



Bloch Sphere Representation

$$|\psi>= a|0> +b|1>$$

$$|a|^2 + |b|^2 = 1$$



$$|\psi>= e^{i\gamma}\left(\overbrace{cos\frac{\theta}{2}}^{a}|0> +\overbrace{e^{i\varphi}sin\frac{\theta}{2}}^{b}|1>\right)$$

Can be ignored

(no observable effects)

$$|0> = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$a = 1 = \cos\theta/2$

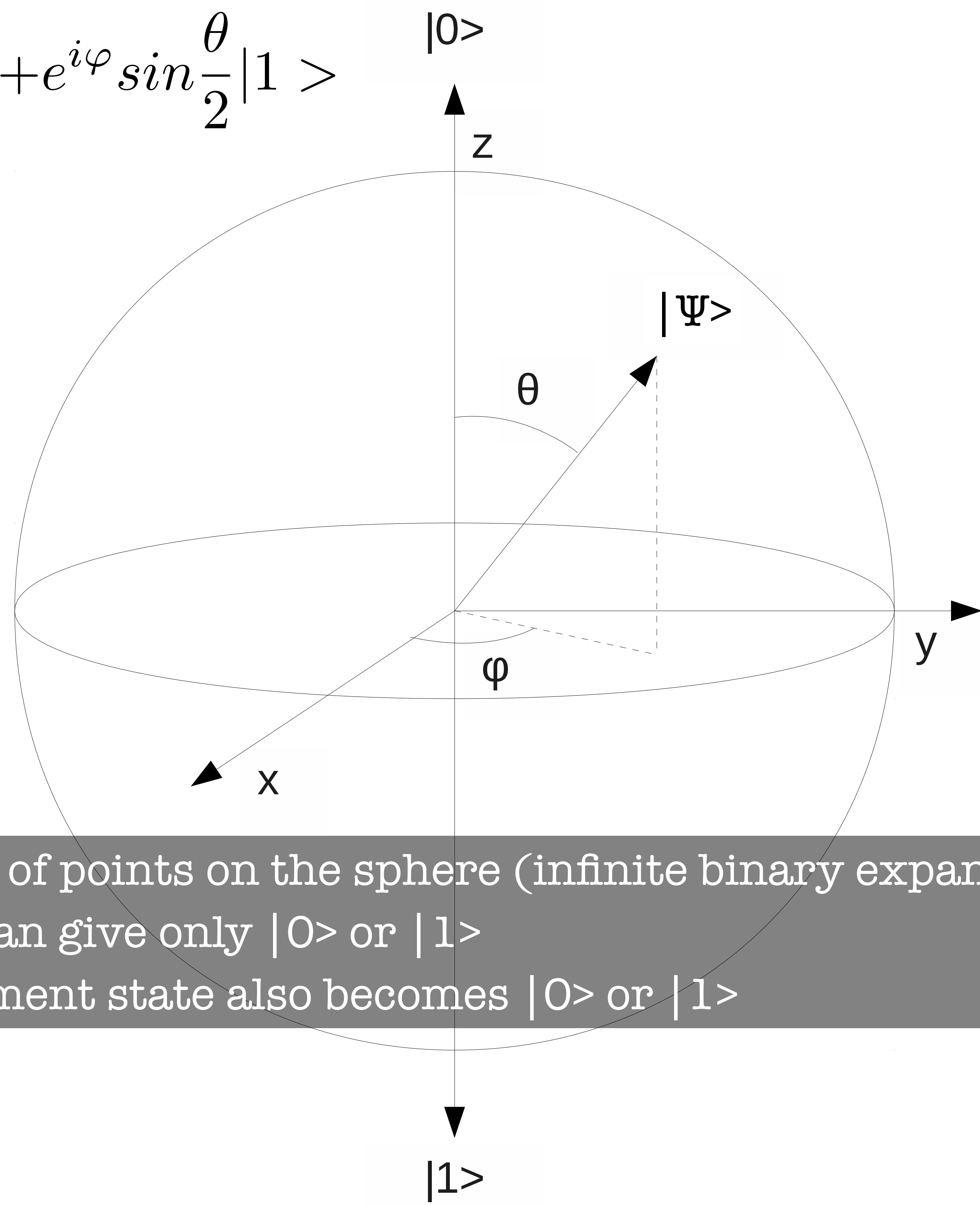
$b = 0$

$\theta = 0$

$\Rightarrow \sin\theta/2 = 0$

Bloch Sphere Representation

$$|\psi> = cos\frac{\theta}{2}|0> + e^{i\varphi} sin\frac{\theta}{2}|1>$$



- Infinite number of points on the sphere (infinite binary expansion for angles)
- Measurement can give only |0> or |1>
  - Post measurement state also becomes |0> or |1>

Measurement

$$|+> = \underbrace{\frac{1}{\sqrt{2}}}_{a}|0> + \underbrace{\frac{1}{\sqrt{2}}}_{b}|1>$$

- Example: measurement of |+> gives |0>
  - Then, post measurement state will be |0>
- A single measurement can only carry a single bit of information about the state of qbit
- Only if infinitely many qbits were prepared and measured, a and b could be determined

$$1/\sqrt{2} = \cos\theta/2 \Rightarrow \theta = \pi/2$$

$$1/\sqrt{2} = e^{i\varphi} \cdot \underbrace{\sin\theta/2}_{1/\sqrt{2}} \Rightarrow e^{i\varphi} = 1$$

$$\Rightarrow \varphi = 0$$

Tensor Products

$$|\psi_1 \psi_2\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \quad \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \\ b_1 \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} a_1 a_2 \\ a_1 b_2 \\ b_1 a_2 \\ b_1 b_2 \end{bmatrix}$$

Multiple qbits

- State of a quantum computer with 2 qbits?
  - 4-dimensional vector space: |00>, |01>, |10>, |11>
  - Without direct observation, the state of a 2-qbit system spans $[\alpha_{00}, \alpha_{01}, \alpha_{10}, \alpha_{11}]^T$

$$|\psi>= \alpha_{00}|00> + \alpha_{01}|01> + \alpha_{10}|10> + \alpha_{11}|11>$$

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1 \qquad \sum_{X \in \{0,1\}^2} |\alpha_X|^2 = 1$$

- The measurement result X = {00,01,10,11} occurs with probability $|\alpha_X|^2$
  - Post measurement state becomes |X>

- Measuring the first qbit is 0 with probability $|\alpha_{00}|^2 + |\alpha_{01}|^2$
  - Post measurement state becomes

$$|\psi'>= \frac{\alpha_{00}|00> + \alpha_{01}|01>}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}$$

$$|\psi_1\rangle|\psi_2\rangle = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \otimes \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{bmatrix} = \begin{bmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{bmatrix}$$

Post-measurement

$$|\psi_1\psi_2\rangle = \underbrace{\alpha_{00}'}_{\neq \alpha_{00}} |00\rangle + \underbrace{\alpha_{01}'}_{\neq \alpha_{01}} |01\rangle$$

EPR Pair or Bell State

$$\frac{|00>}{\sqrt{2}} + \frac{|11>}{\sqrt{2}}$$

- EPR: Fully entangled state
  - If one qbit is measured (collapses to "1" or "0")
    - The other qbit gets "destroyed"
    - The other qbit "collapses" to the same state

- Measuring the first qbit is 0 with probability $|\alpha_{00}|^2 + |\alpha_{01}|^2 = |\alpha_{00}|^2$
  - Post measurement state becomes

$$|\psi'>= \frac{\alpha_{00}|00> +\alpha_{01}|01>}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}} = \frac{\alpha_{00}|00>}{\sqrt{|\alpha_{00}|^2}} = \frac{\frac{1}{\sqrt{2}}|00>}{\sqrt{\frac{1}{2}}} = |00>$$

Multiple qbits

- n-qbit quantum system can represent 2^n bit strings
  - Distinguished by 2^n complex valued amplitudes

$$|\psi >= \sum_{i=0}^{2^{n-1}} c_i|x_i >$$

$$\sum_{i=0}^{2^{n-1}} |c_i|^2 = 1$$

Single qbit gates

- NOT gate

$$|\psi>= a|0> +b|1>\rightarrow b|0> +a|1>$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|\psi>= a|0> +b|1>= a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix}$$

Single qbit gates

- NOT gate

$$|\psi >= a|0 > +b|1 >\rightarrow b|0 > +a|1 >$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

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$$X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix}$$

- Quantum gates on single qbits can be described by 2x2 matrices
  - Normalization condition should be preserved after the gate operation:
    - $|a|^2 + |b|^2 = 1$ and $|a'|^2 + |b'|^2 = 1$ for $U[a\ b]^T = [a'\ b']^T$
  - Formally: $U^\dagger U = I$

Single qbit gates

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"Unitarity constraint" is the only constraint on q-gates

➡Any unitary matrix can specify a q-gate

➡There exists many single qbit q-gates

Single qbit gates

- Quantum gates on single qbits can be described by 2x2 matrices
  - Normalization condition should be preserved after the gate has acted
  - $|a|^2 + |b|^2 = 1$ and $|a'|^2 + |b'|^2 = 1$ for $G[a\ b]^T = [a'\ b']^T$
  - Formally: $U^\dagger U = I$

"Unitarity constraint" is the only constraint on q-gates

➡Any unitary matrix can specify a q-gate

➡There exists many single qbit q-gates

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Single qbit gates: Hadamard gate

- |0> becomes |+>, |1> becomes |->

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|+ >= H|0 >= \frac{1}{\sqrt{2}}|0 > + \frac{1}{\sqrt{2}}|1 >= \frac{1}{\sqrt{2}}(|0 > +|1 >)$$

$$|- >= H|1 >= \frac{1}{\sqrt{2}}|0 > - \frac{1}{\sqrt{2}}|1 >= \frac{1}{\sqrt{2}}(|0 > -|1 >)$$

Single qbit gates

- Gate operations correspond to rotations and reflections

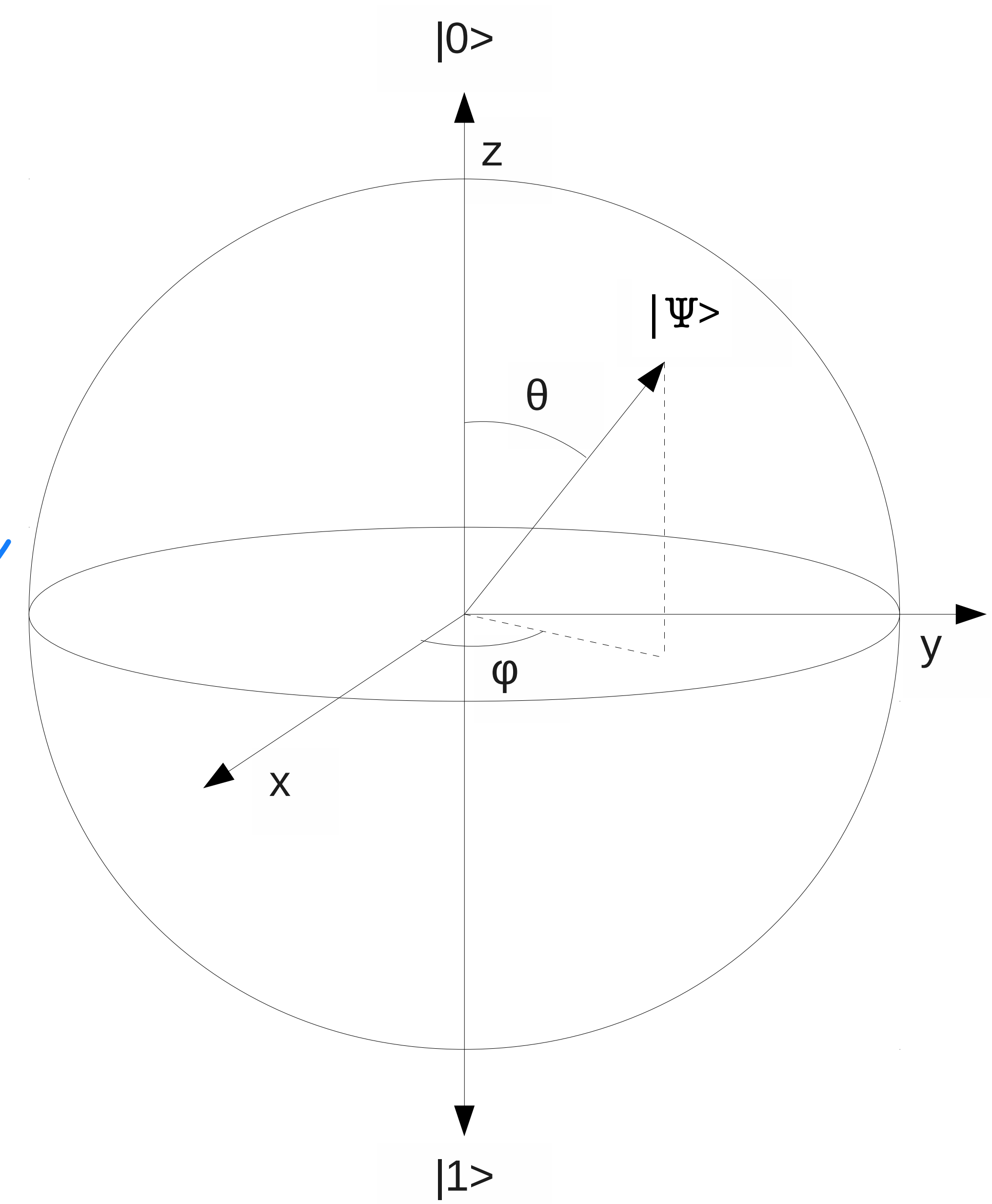
$$|-\rangle = \underbrace{1/\sqrt{2}}_{a}\,|0\rangle - \underbrace{1/\sqrt{2}}_{b}\,|1\rangle$$

$$= \cos\theta/2 \qquad = e^{i\varphi}\sin\theta/2$$

$$\Rightarrow \theta = \pi/2 \qquad \Rightarrow e^{i\varphi} = -1$$

$$\Rightarrow \varphi = \pi$$

$$H|1\rangle = |-\rangle$$



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

Single qbit gates

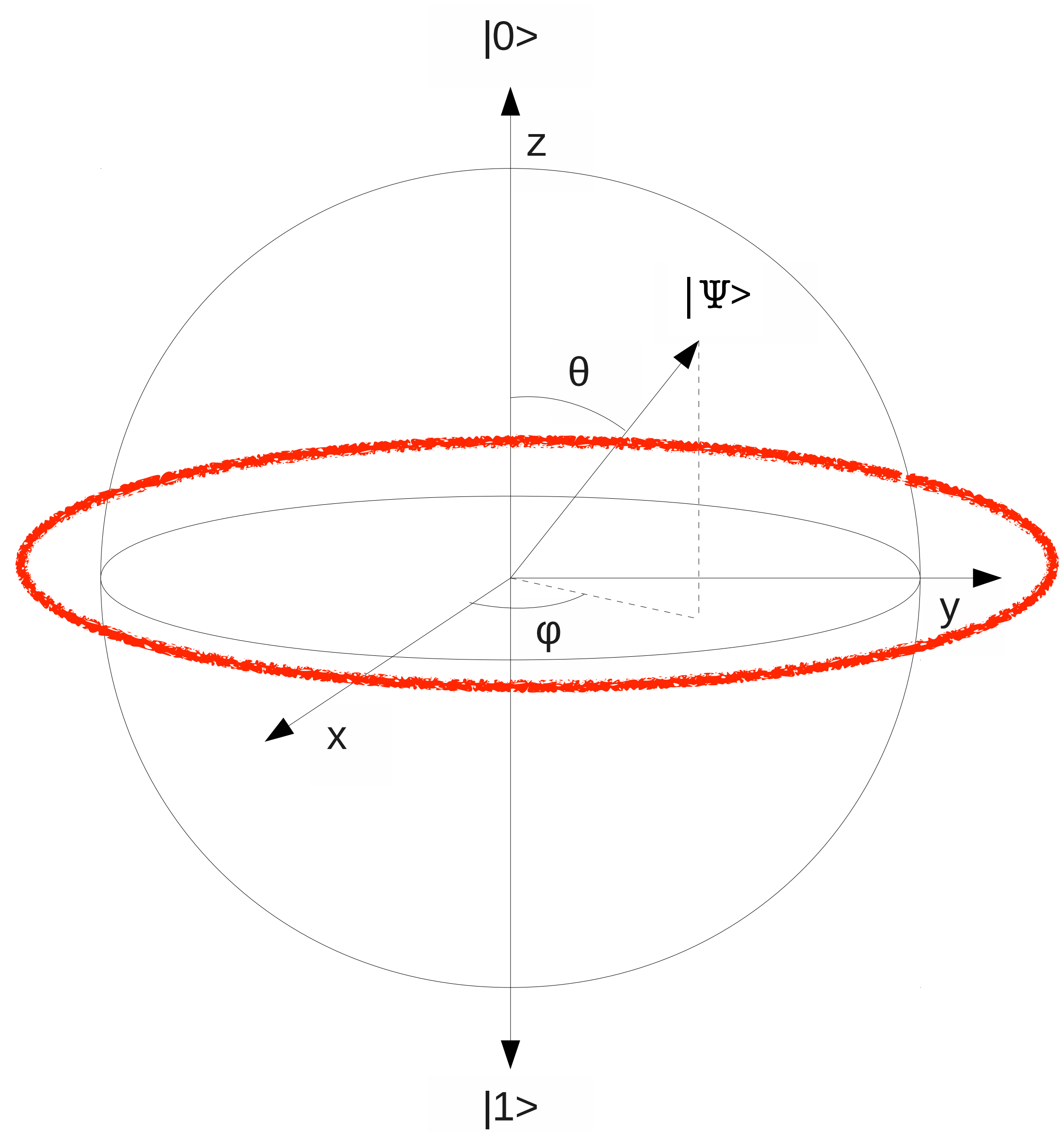
- Gate operations correspond to rotations and reflections

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|+ >= H|0 >= \frac{1}{\sqrt{2}}|0 > +\frac{1}{\sqrt{2}}|1 >= \frac{1}{\sqrt{2}}(|0 > +|1 >)$$

$$|- >= H|1 >= \frac{1}{\sqrt{2}}|0 > -\frac{1}{\sqrt{2}}|1 >= \frac{1}{\sqrt{2}}(|0 > -|1 >)$$

$$|\psi >= cos\frac{\theta}{2}|0 > +e^{i\varphi}sin\frac{\theta}{2}|1 >$$



Single qbit gates

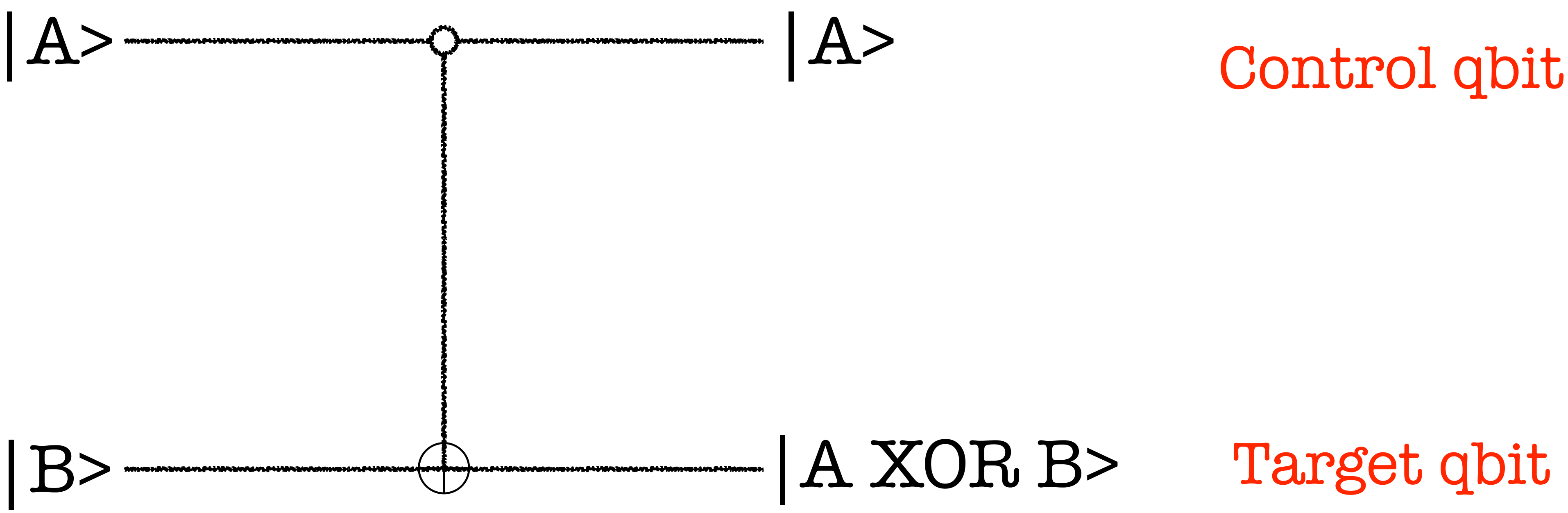
$$a|0> +b|1> \longrightarrow \boxed{\mathbf{X}} \longrightarrow b|0> +a|1>$$

$$a|0> +b|1> \longrightarrow \boxed{\mathbf{Z}} \longrightarrow a|0> -b|1>$$

$$a|0> +b|1> \longrightarrow \boxed{\mathbf{H}} \longrightarrow a\frac{|0> +|1>}{\sqrt{2}} + b\frac{|0> -|1>}{\sqrt{2}}$$

Multi qbit gates

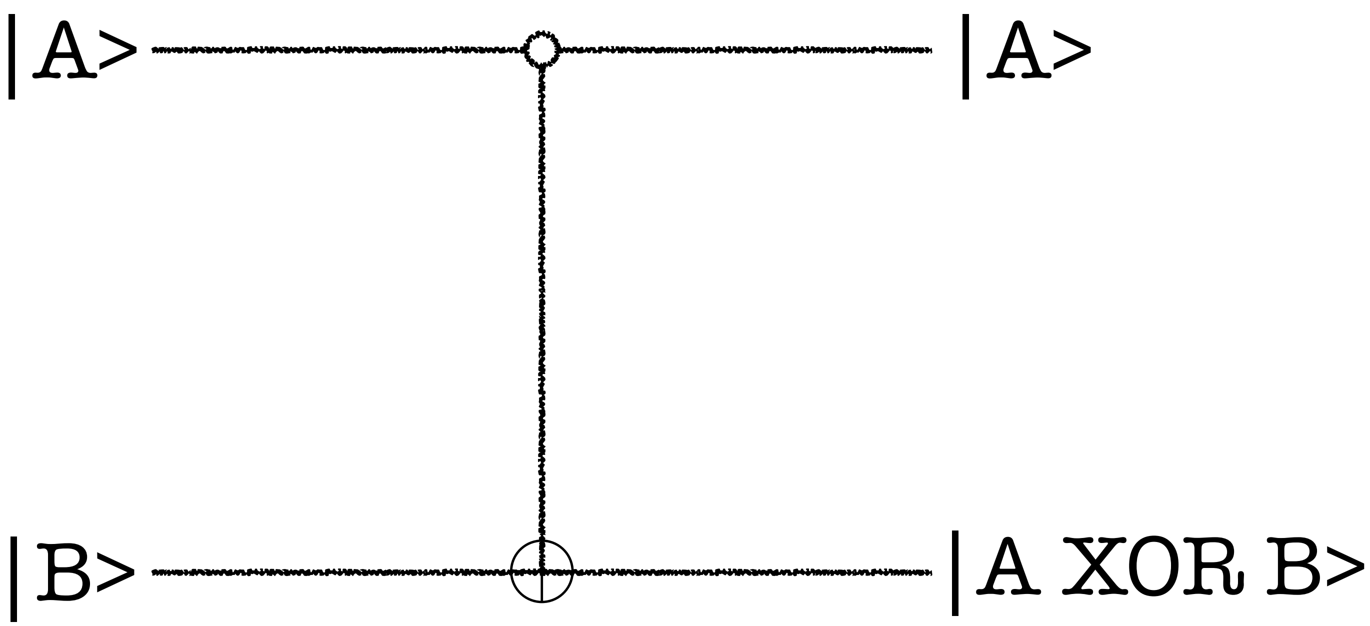
- Controlled NOT (CNOT)



A	B	A	A XOR B
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Multi qbit gates

- Controlled NOT (CNOT)



$$U_{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$U^{\dagger}_{CNOT} U_{CNOT} = I$$

- State |AB> with the first qbit corresponding to control:

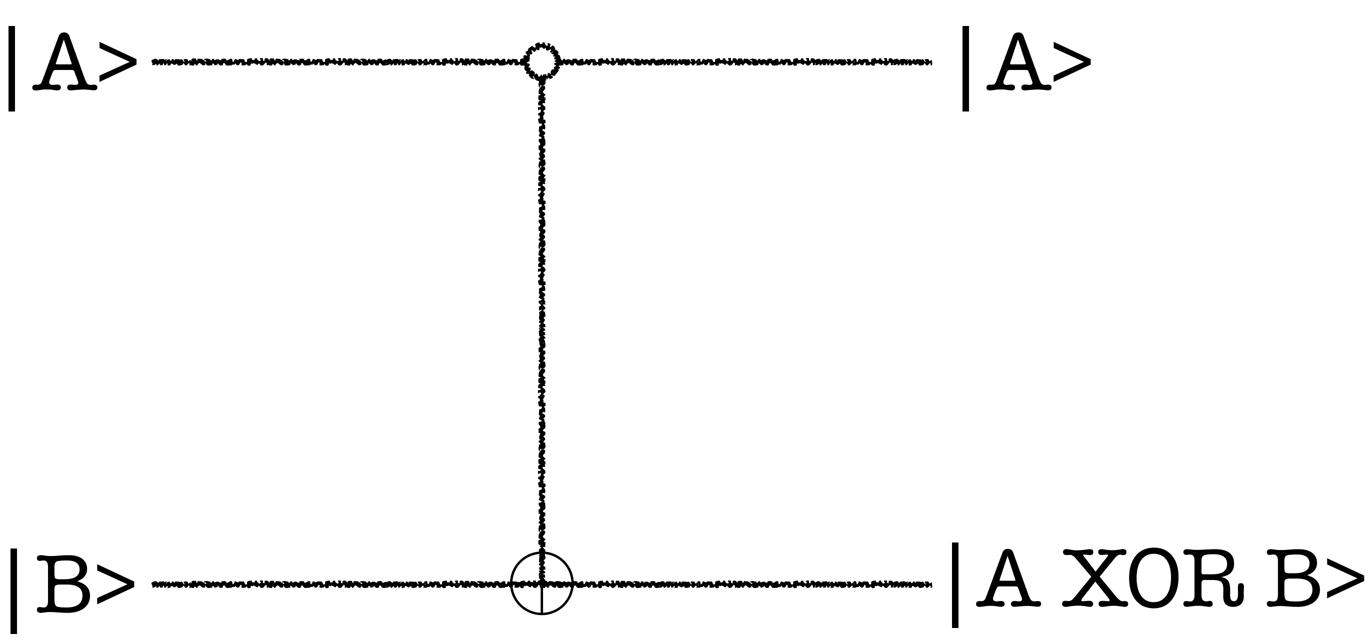
$$\begin{aligned} |00 > &\to |00 > \\ |01 > &\to |01 > \\ |10 > &\to |11 > \\ |11 > &\to |10 > \end{aligned}$$

$$\underbrace{|A, B >}_{|A> \otimes |B>} \to |A, A \oplus B >$$

tensor product

Multi qbit gates

- Controlled NOT (CNOT)



$$U_{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\equiv I$

$\equiv X$

$$U^{\dagger}_{CNOT} U_{CNOT} = I$$

- State |AB> with the first qbit corresponding to control:

$$|00> \to |00>$$
$$|01> \to |01>$$
$$|10> \to |11>$$
$$|11> \to |10>$$

I

X

$$\underbrace{|A, B>}_{|A> \otimes |B>} \to |A, A \oplus B>$$

tensor product

Tensor Products

Axiom:

$$(G_1 \otimes G_2)(|\psi_1\rangle \otimes |\psi_2\rangle) =$$

$$G_1|\psi_1\rangle \otimes G_2|\psi_2\rangle$$

Tensor Products

Axiom:

$$(G_1 \otimes G_2)(|\psi_1\rangle \otimes |\psi_2\rangle) =$$

$$\underbrace{G_1}_{\underbrace{}_{I}}|\psi_1\rangle \otimes \underbrace{G_2}_{I \text{ or } X \text{ (conditional)}}|\psi_2\rangle$$

Multi qbit gates

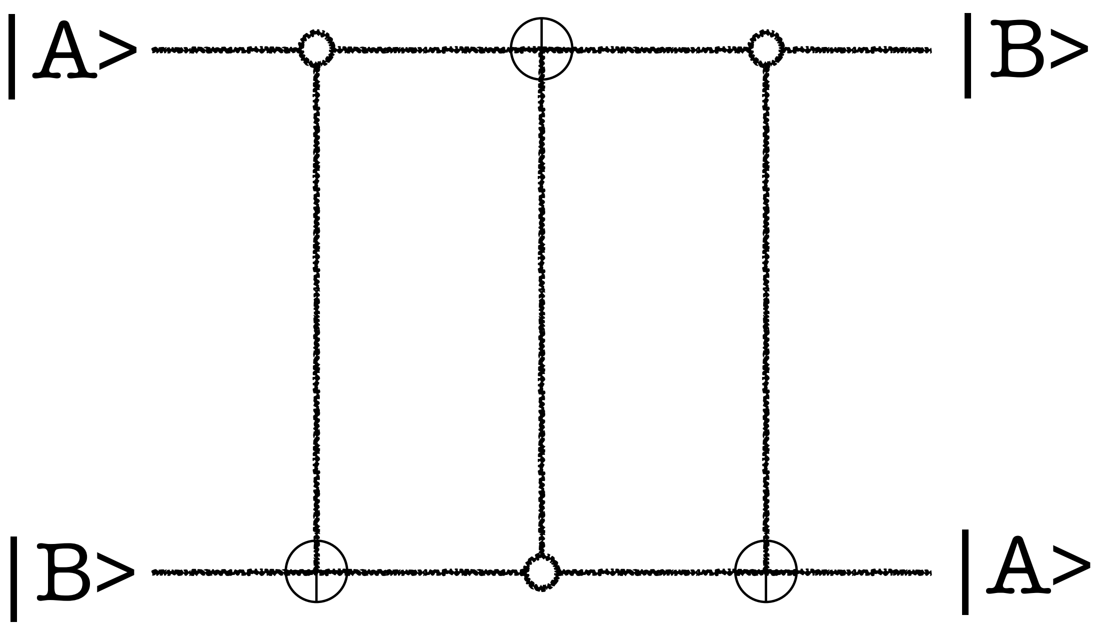
- The inverse of a unitary matrix always exists

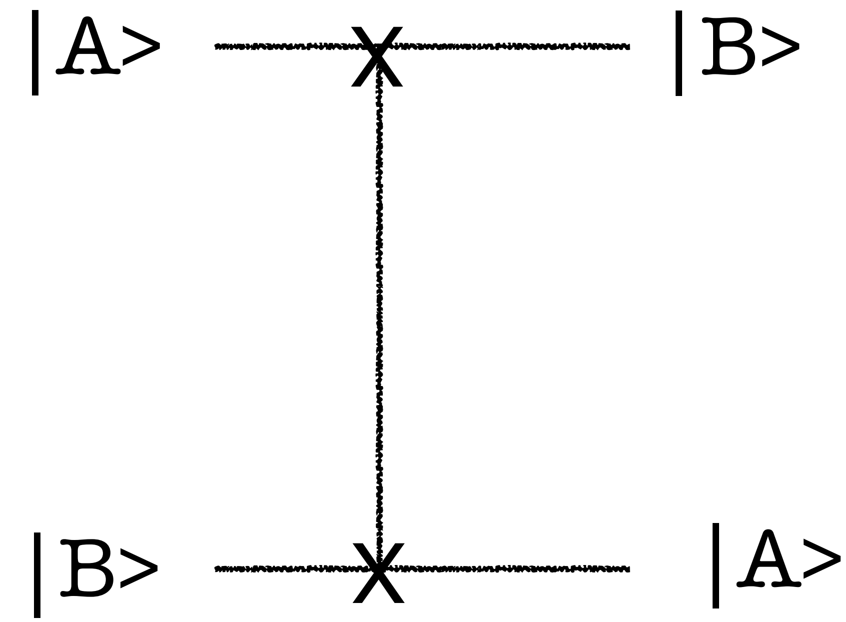
  ➡Unitary matrices are always invertible

  ➡Unitary quantum gates are always **reversible**
- **Any multiple qbit logic gate can be composed from CNOT and single qbit gates**

  [Quantum parallel of universality]

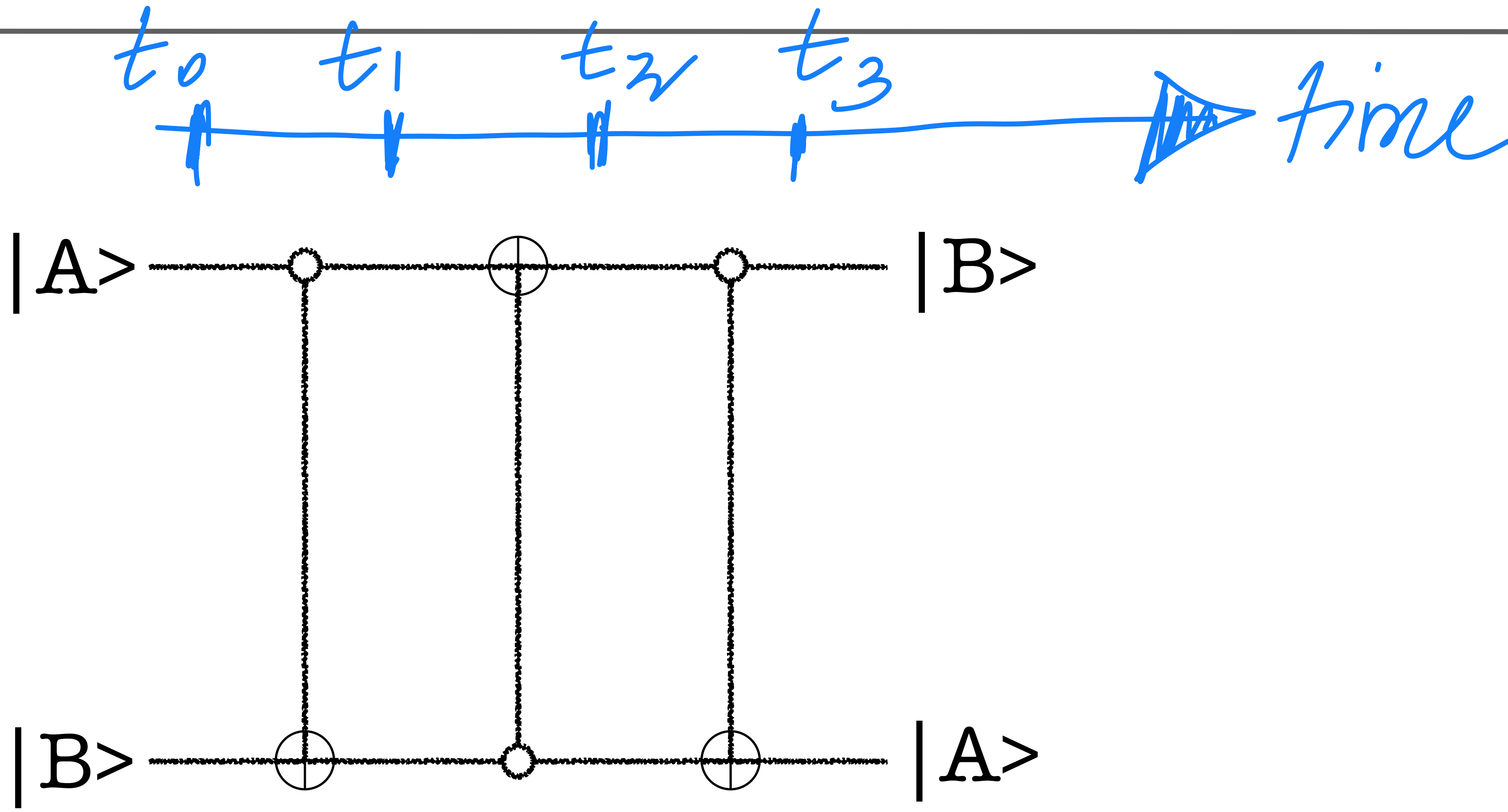
Example: 2 qbit swap

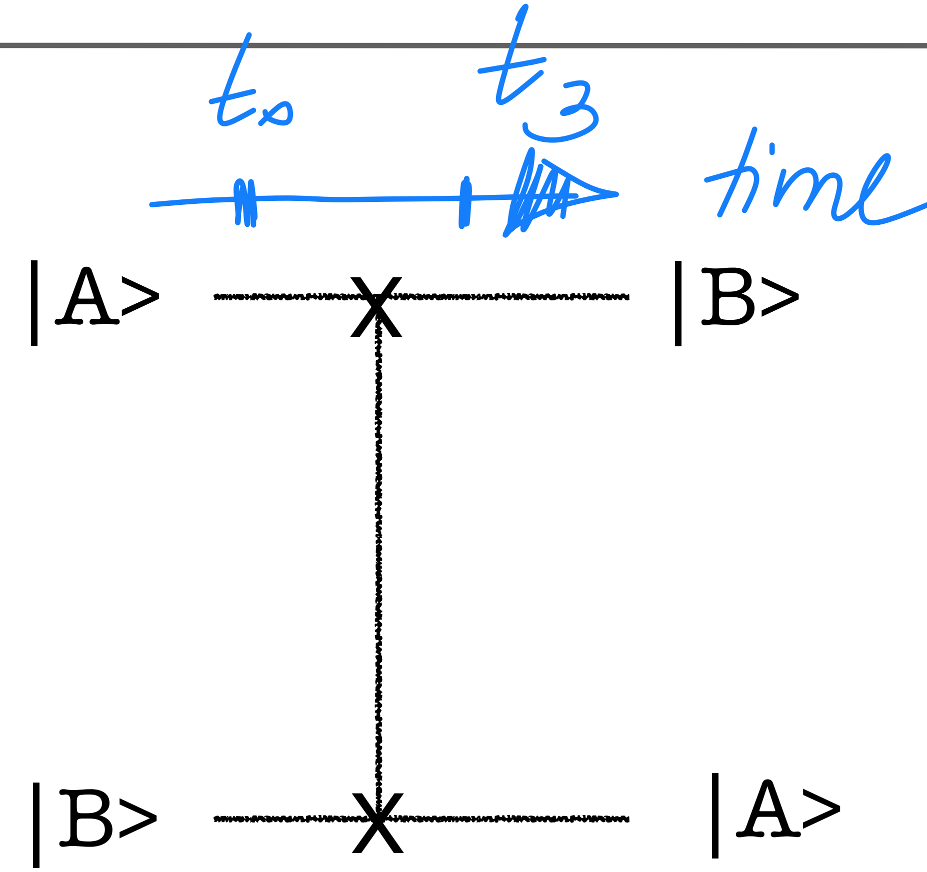




$$
\begin{aligned}
|A, B> &\to |A, A \oplus B> \\
\to |A \oplus (A \oplus B), A \oplus B> = |B, A \oplus B> &\\
\to |B, (A \oplus B) \oplus B> = |B, A> &
\end{aligned}
$$

Example: 2 qbit swap





@ t_0 @ t_1

$$|A, B> \rightarrow |A, A \oplus B>$$

@ t_2

$$\rightarrow |A \oplus (A \oplus B), A \oplus B> = |B, A \oplus B>$$

@ t_3 $$\rightarrow |B, (A \oplus B) \oplus B> = |B, A>$$

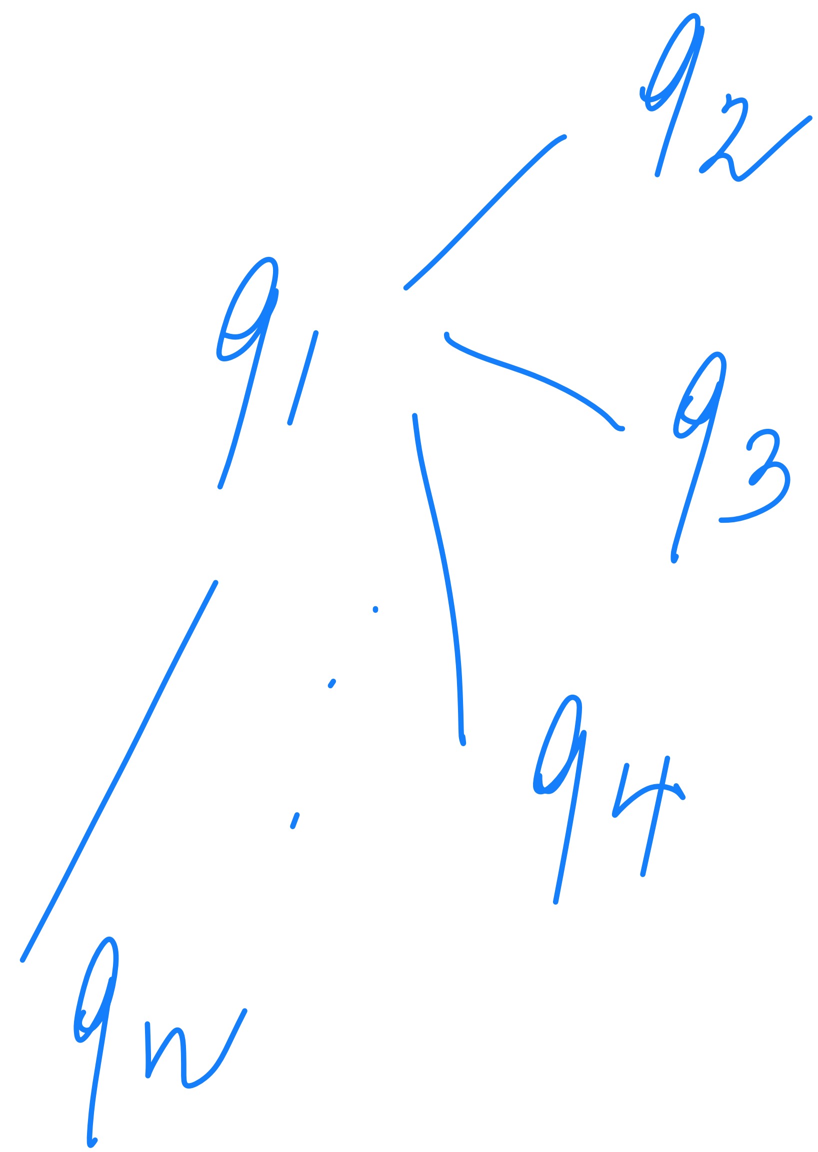
$$A \oplus (A \oplus B) = ?$$

A	$A \oplus A$
0	0
1	$(1+1) \bmod 2 = 0$

$$\Rightarrow A \oplus A \oplus B = B$$

Swap gate ?

n-qbit system

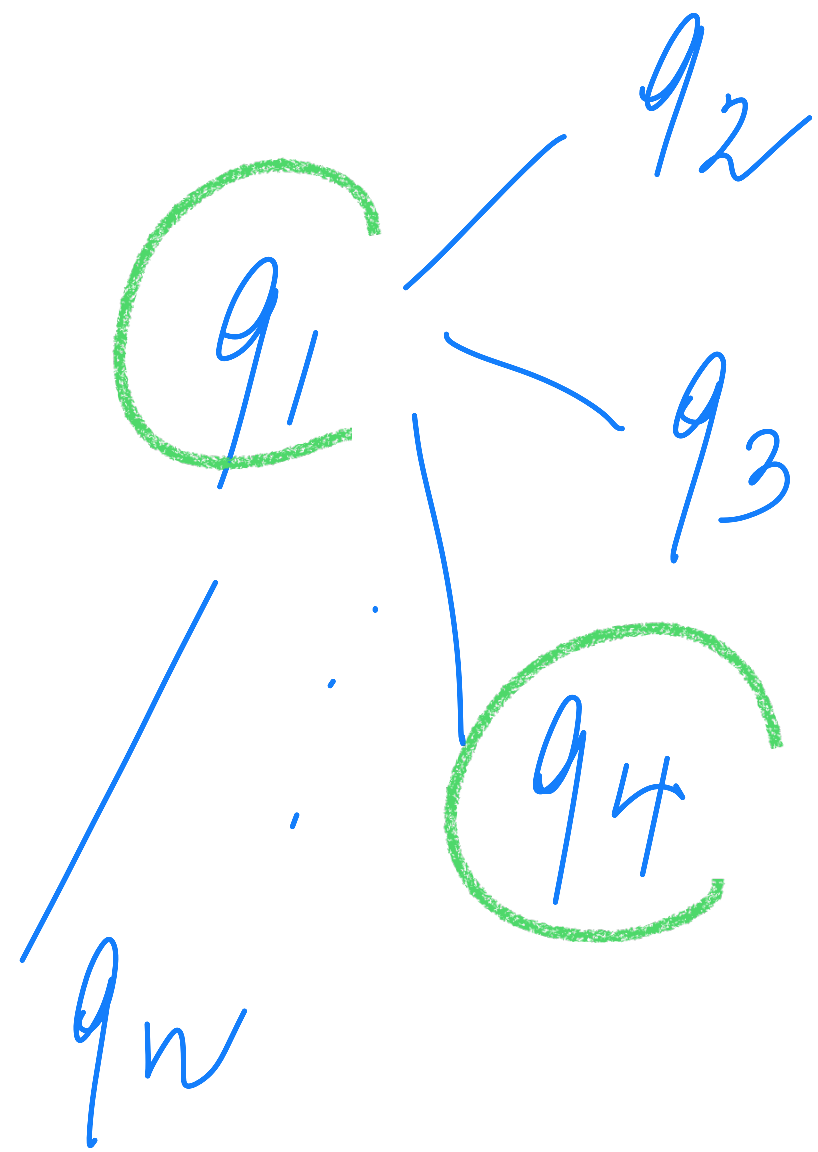


vs. $q_1 \quad q_2 \quad q_3 \quad q_4$

$q_5 \quad \cdots$

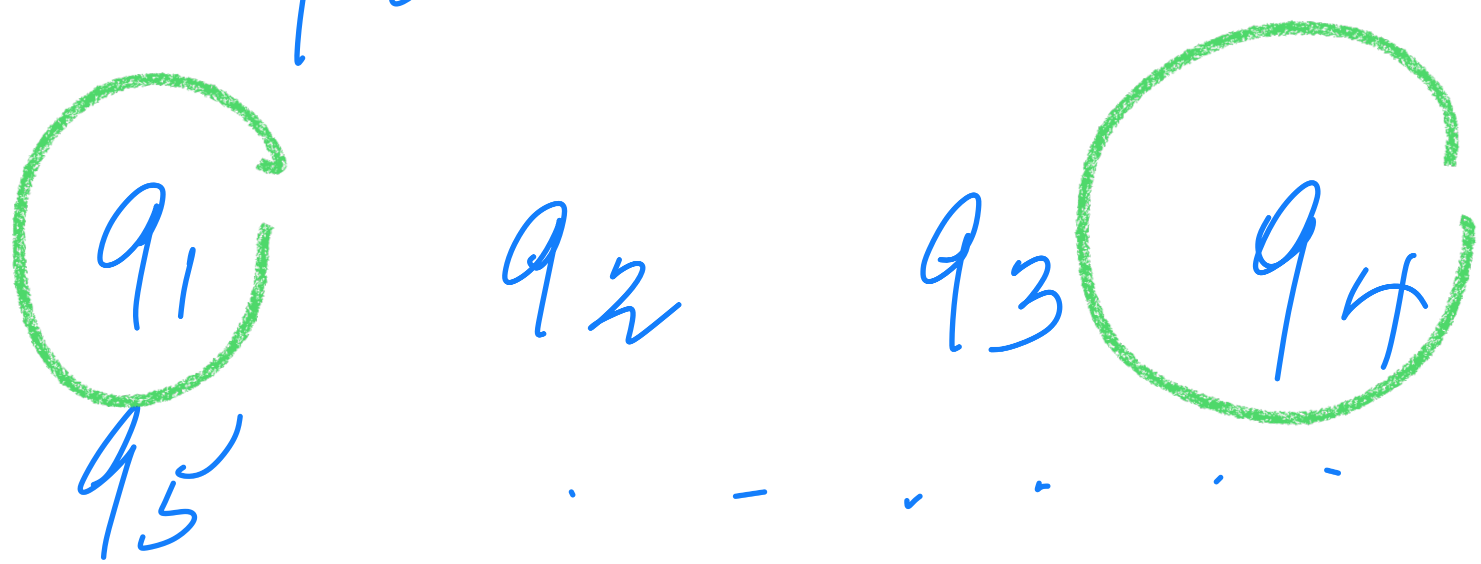
Swap gate ?

n-qbit system



CNOT(q1, q4)

vs.



n-bit U-gate

$$|\psi > \rightarrow |\psi' >$$

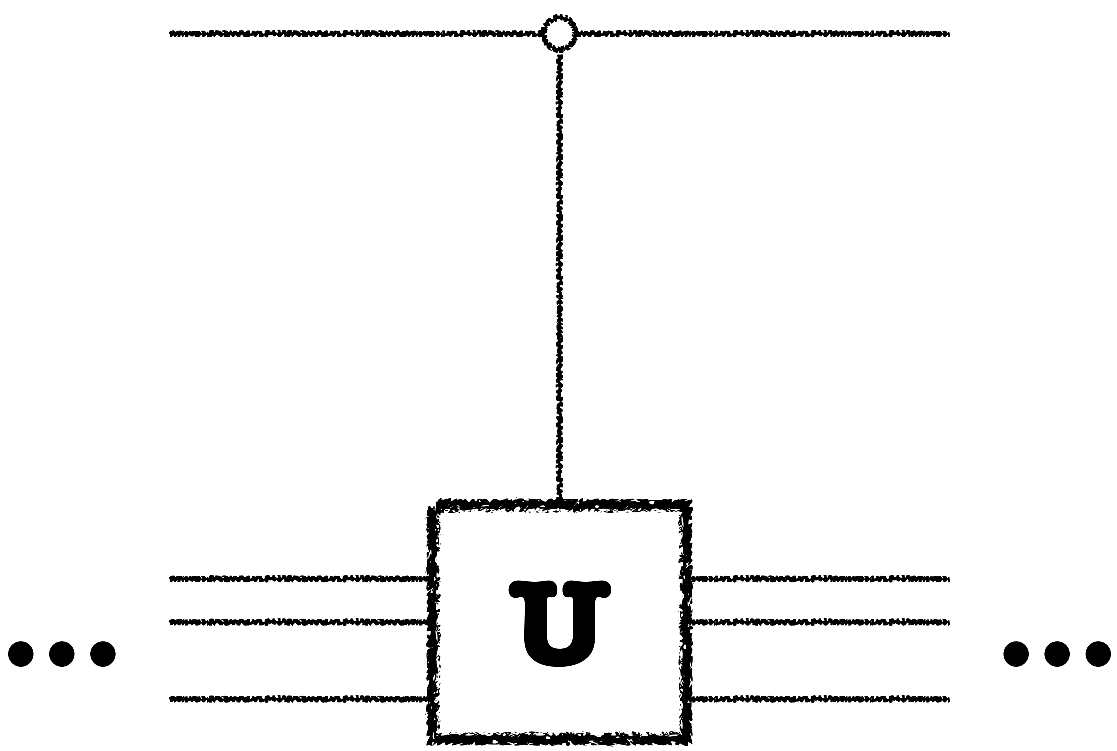
$$[c_0, c_1, c_2, c_3, ..., c_{2^n-1}]^T \rightarrow [c'_0, c'_1, c'_2, c'_3, ..., c'_{2^n-1}]^T$$

$\sum_{i=0}^{2^n-1} |c'_i|^2 = 1$ should be preserved

$$U|\psi >= U \begin{bmatrix} c_0 \\ c_1 \\ ... \\ c_{2^n-1} \end{bmatrix} = \begin{bmatrix} c'_0 \\ c'_1 \\ ... \\ c'_{2^n-1} \end{bmatrix} = |\psi' >$$

$$U^{-1}|\psi' >= |\psi >$$

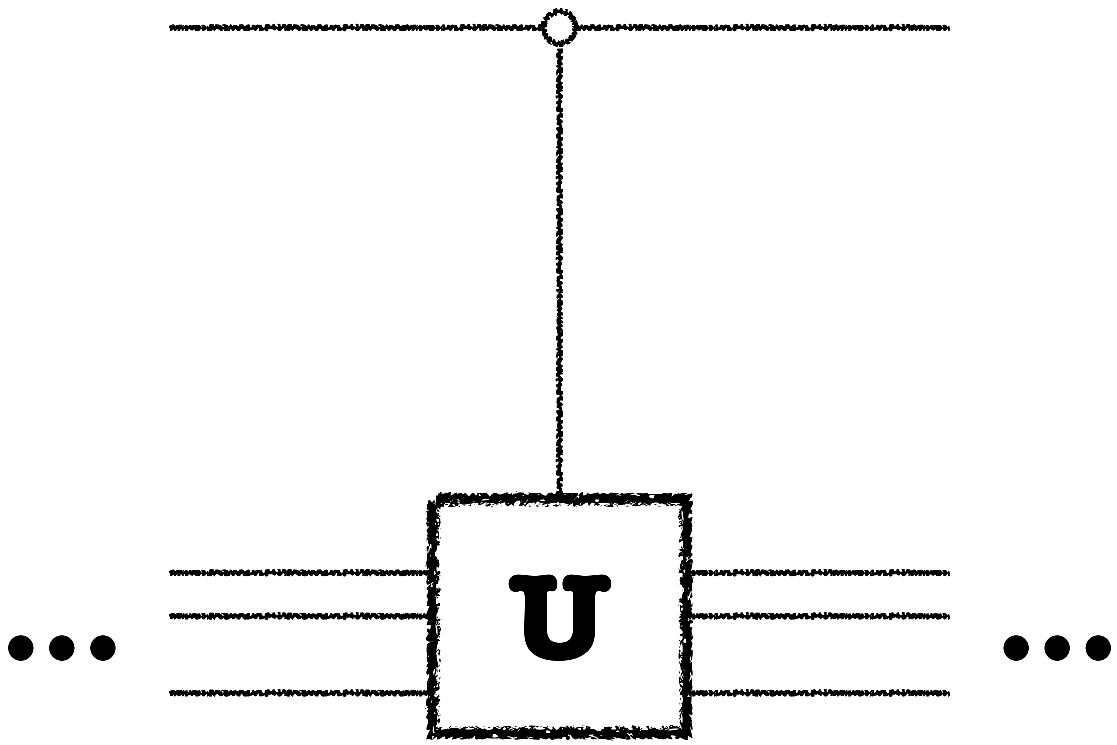
n-bit **controlled** U-gate

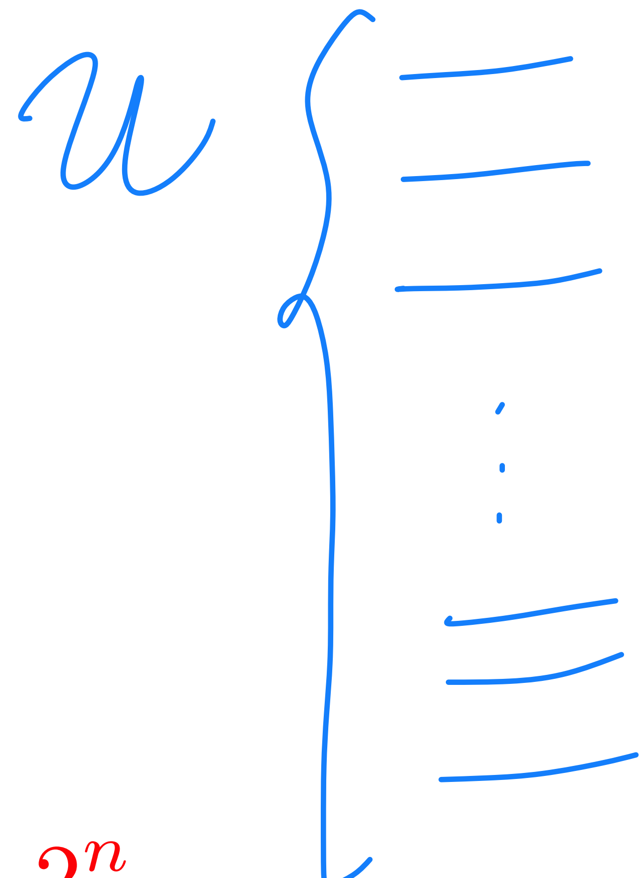


U gate: n-qbit gate
Corresponding square matrix U is $2^n \times 2^n$

n-bit **controlled** U-gate

Algorithm

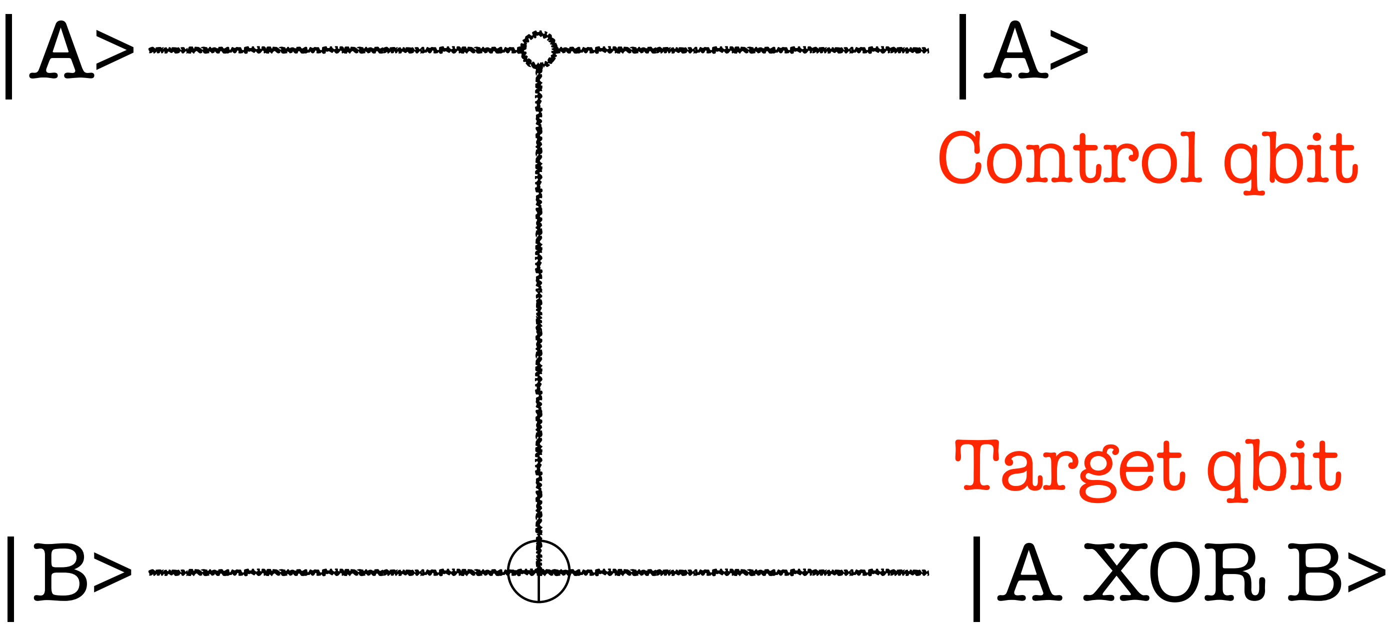


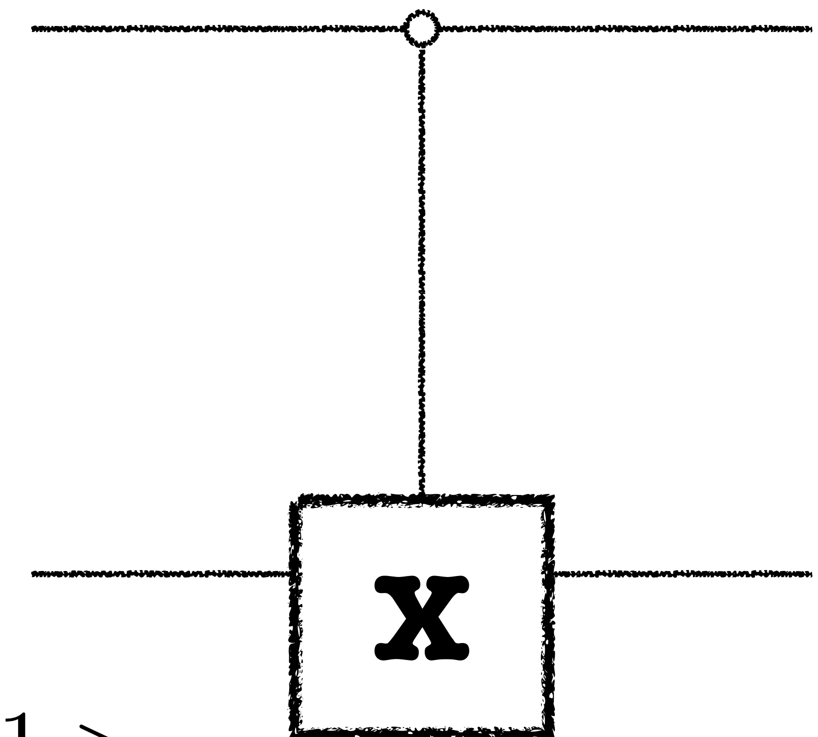


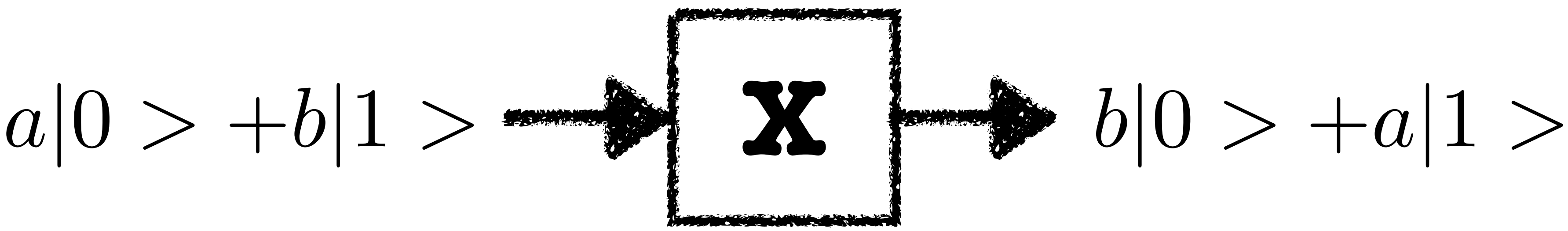
U gate: n-qbit gate

Corresponding square matrix U is $2^n \times 2^n$

CNOT







$$
\begin{aligned}
|00> &\rightarrow |00> \\
|01> &\rightarrow |01> \\
|10> &\rightarrow |11> \\
|11> &\rightarrow |10>
\end{aligned}
$$

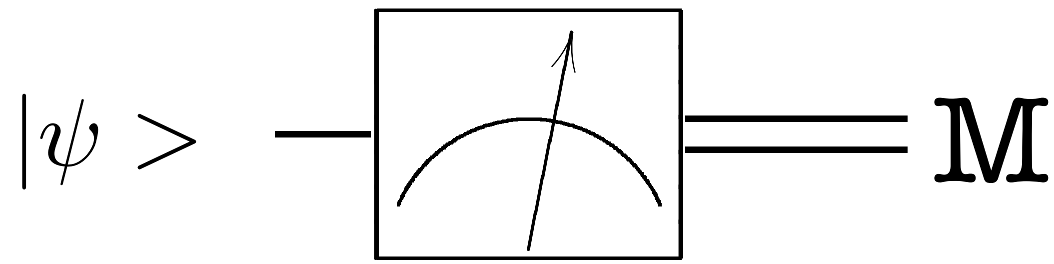
x

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$U_{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

x

Measurement “gate”



Measurement

- Many choices exist to express the basis states of a qbit , such as |0> and |1>
- Any arbitrary state can be re-expressed in terms of states |+> and |->

$$|\psi> = a|0> + b|1> = a\frac{|+> + |->}{\sqrt{2}} + b\frac{|+> - |->}{\sqrt{2}}$$

$$|\psi> = \frac{a+b}{\sqrt{2}}|+> + \frac{a-b}{\sqrt{2}}|->$$

- Measurement in basis |+> and |->:
  - |+> with probability

$$\frac{|a+b|^2}{2}$$

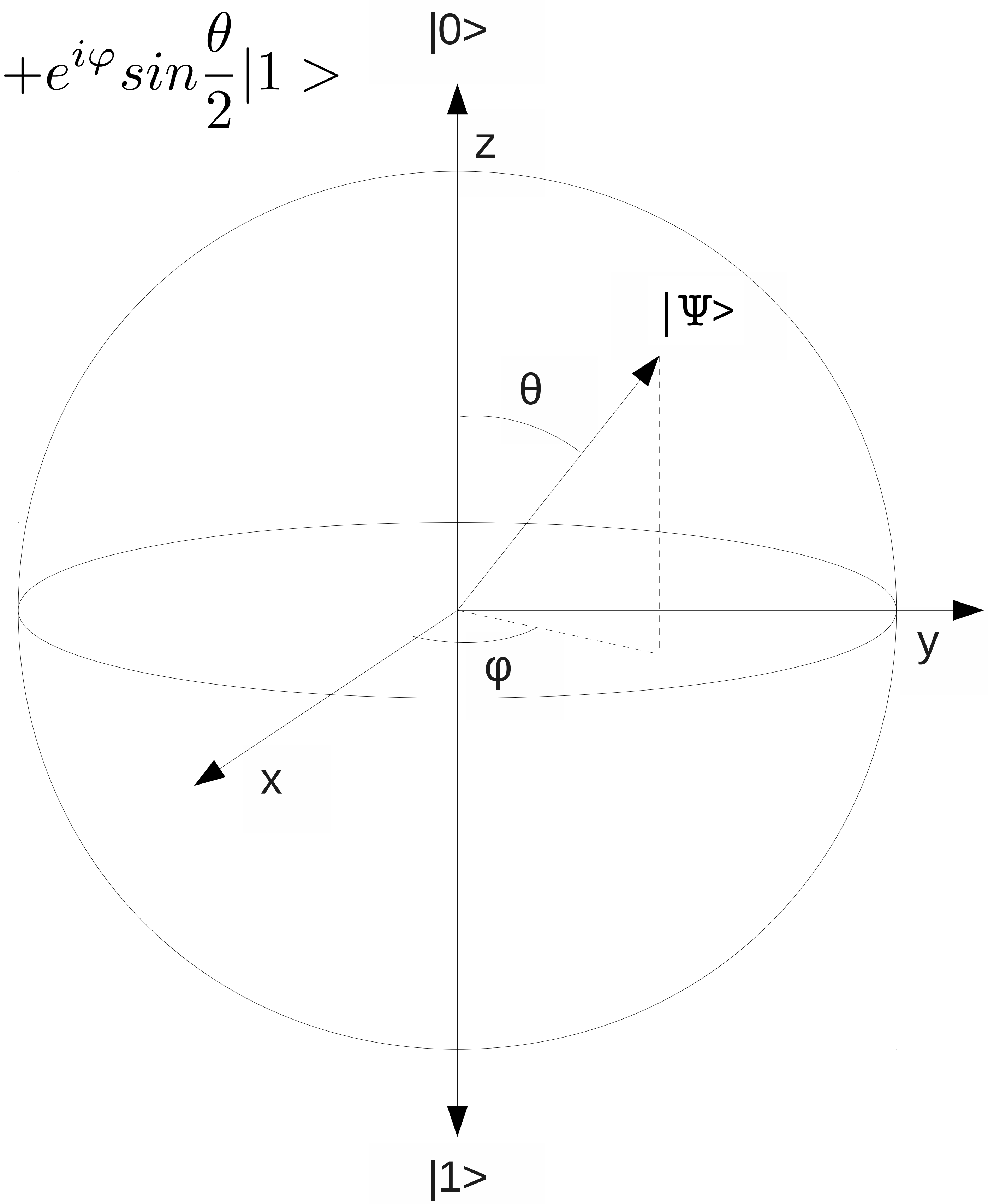
  - |-> with probability

$$\frac{|a-b|^2}{2}$$

- For any basis |x> and |y>, |Ψ> can be expressed as a linear combination: a'|x> + b'|y>
  - The only constraint is for |x> and |y> to be orthonormal such that $|a'|^2+|b'|^2=1$
  - The measurement renders |x> with probability $|a'|^2$;|y> ,with $|b'|^2$

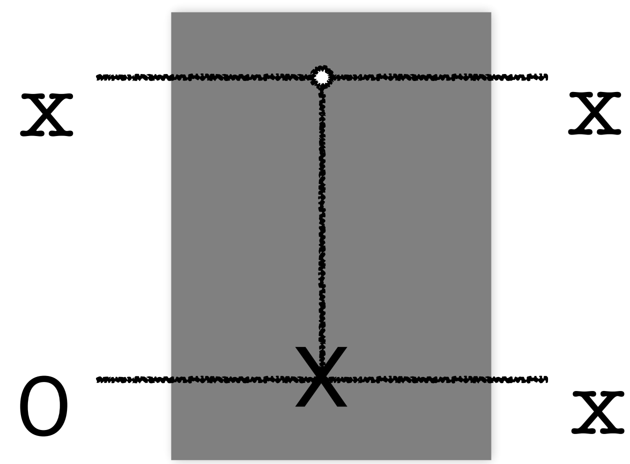
Bloch Sphere Representation

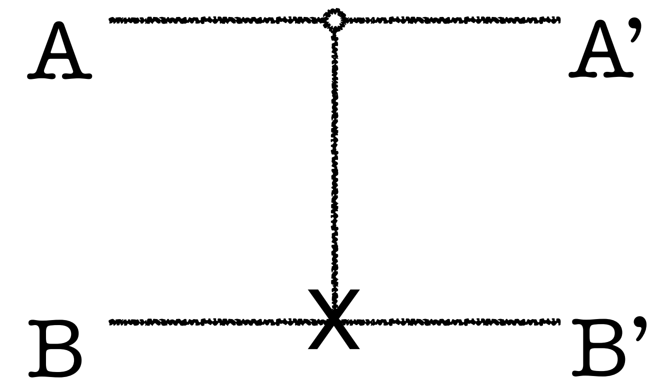
$$|\psi> = cos\frac{\theta}{2}|0> + e^{i\varphi}sin\frac{\theta}{2}|1>$$



Copy Circuit?

- Copy classical bit of unknown state
  - A, B = x, 0 (initial state)
  - A, B → A, A⊕B (CNOT)
    - A' = A = x
    - B' = A⊕B = x





A	B	A'	B'
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

B' = A XOR B

Copy Circuit?

$|\psi\rangle$?

- Copy classical bit of unknown state
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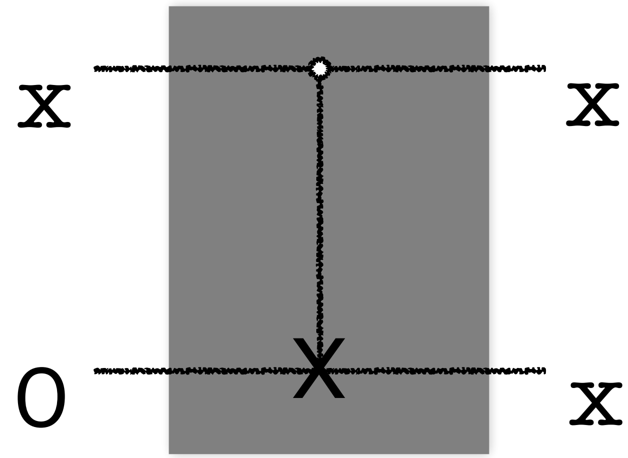
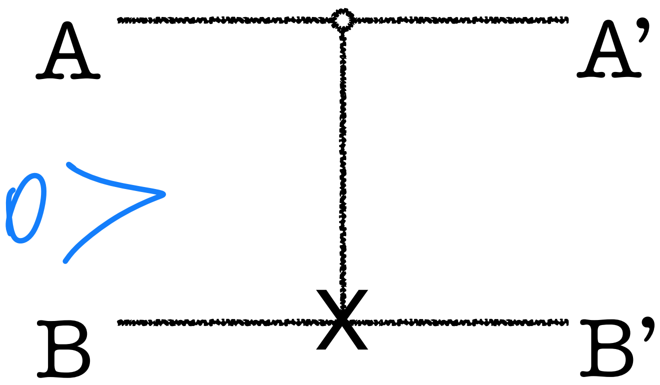


$|0\rangle$



A	B	A'	B'
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

B' = A XOR B

- Copy quantum bit of unknown state?
  - |Ψ> = a|0> + b|1>
  - Initial state: [a|0> + b|1>] |0> = a|00> + b|10>
  - Output state: a|00> + b|11>
    - Does the output state correspond to |Ψ> |Ψ>?
      - Only if |Ψ> = 0 or |Ψ> = 1
        - Quantum circuits can copy classic information
      - |Ψ> |Ψ> = a^2|00> + b^2|11> + ab|01> + ab|10>

$$|\psi >= a|0 > +b|1 >= a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

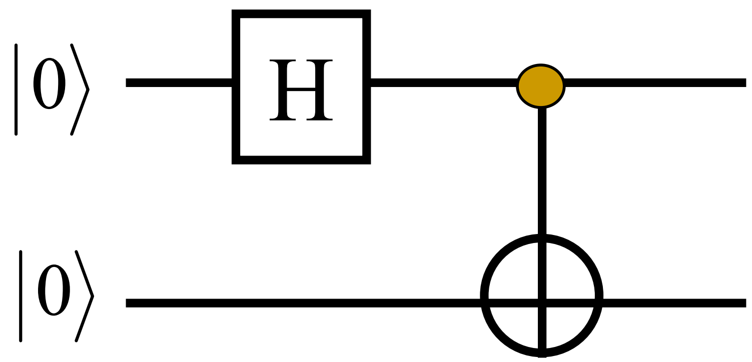
$$|\psi\rangle|\psi\rangle = \begin{bmatrix} a\begin{bmatrix} a \\ b \end{bmatrix} \\ b\begin{bmatrix} a \\ b \end{bmatrix} \end{bmatrix} = \begin{bmatrix} aa \\ ab \\ ab \\ bb \end{bmatrix}$$

$$|\psi\psi\rangle = a^2|00\rangle + ab|01\rangle + ab|10\rangle + b^2|11\rangle$$

vs. $a|00\rangle + b|11\rangle$ [CNOT output]

Bell States

$$|00\rangle \Rightarrow$$



$$\Rightarrow \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

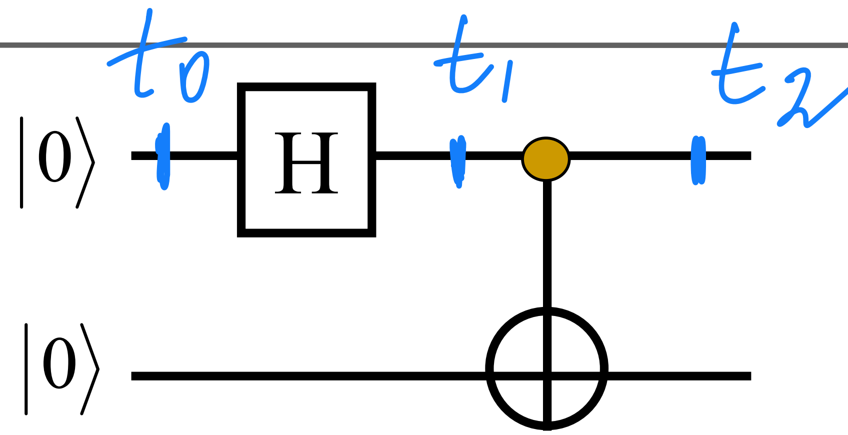
$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|00> \rightarrow |00>$$
$$|01> \rightarrow |01>$$
$$|10> \rightarrow |11>$$
$$|11> \rightarrow |10>$$

Bell States

$\longrightarrow$ time

$$|00\rangle \Rightarrow$$



$$\Rightarrow \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|00> \rightarrow |00>$$
$$|01> \rightarrow |01>$$
$$|10> \rightarrow |11>$$
$$|11> \rightarrow |10>$$

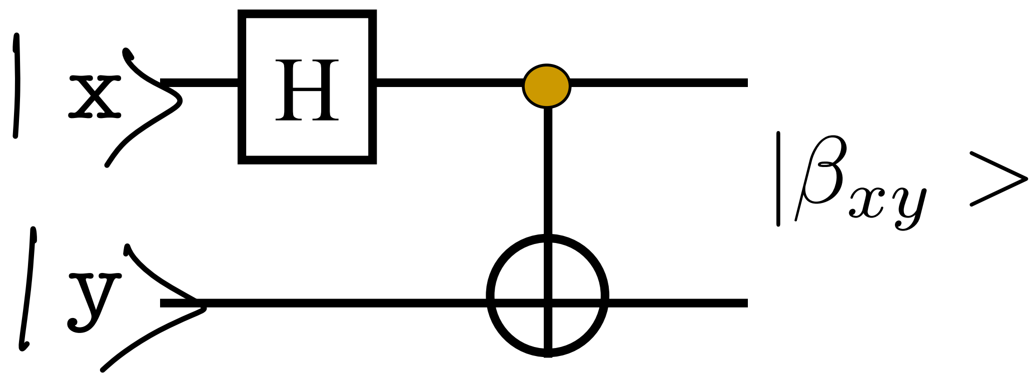
@ t_0: $|00>$

@ t_1: $\left(1/\sqrt{2}\,|0> + 1/\sqrt{2}\,|1>\right)|0>$

$1/\sqrt{2}\left(|00> + |10>\right)$

@ t_2: $1/\sqrt{2}\left(|00> + |11>\right)$

Bell States



$$|\beta_{xy}> = \frac{|0,y> + (-1)^x|1,\bar{y}>}{\sqrt{2}}$$

In	Out
$\lvert 00>$	$(\lvert 00> + \lvert 11>)/\sqrt{2} \equiv \beta_{00}$
$\lvert 01>$	$(\lvert 01> + \lvert 10>)/\sqrt{2} \equiv \beta_{01}$
$\lvert 10>$	$(\lvert 00> - \lvert 11>)/\sqrt{2} \equiv \beta_{10}$
$\lvert 11>$	$(\lvert 01> - \lvert 10>)/\sqrt{2} \equiv \beta_{11}$

Quantum Teleportation

- Alice and Bob live far apart
- When they were together, they generated an EPR pair
- Then, each took a qbit of the EPR pair, and left…
- Now, Alice is trying to deliver an unknown qbit of information to Bob
  - Alice does not know the state of the qbit $|\psi >$
  - She can only send classical information to Bob

Quantum Teleportation

- Alice and Bob live far apart
- When they were together, they generated an EPR pair
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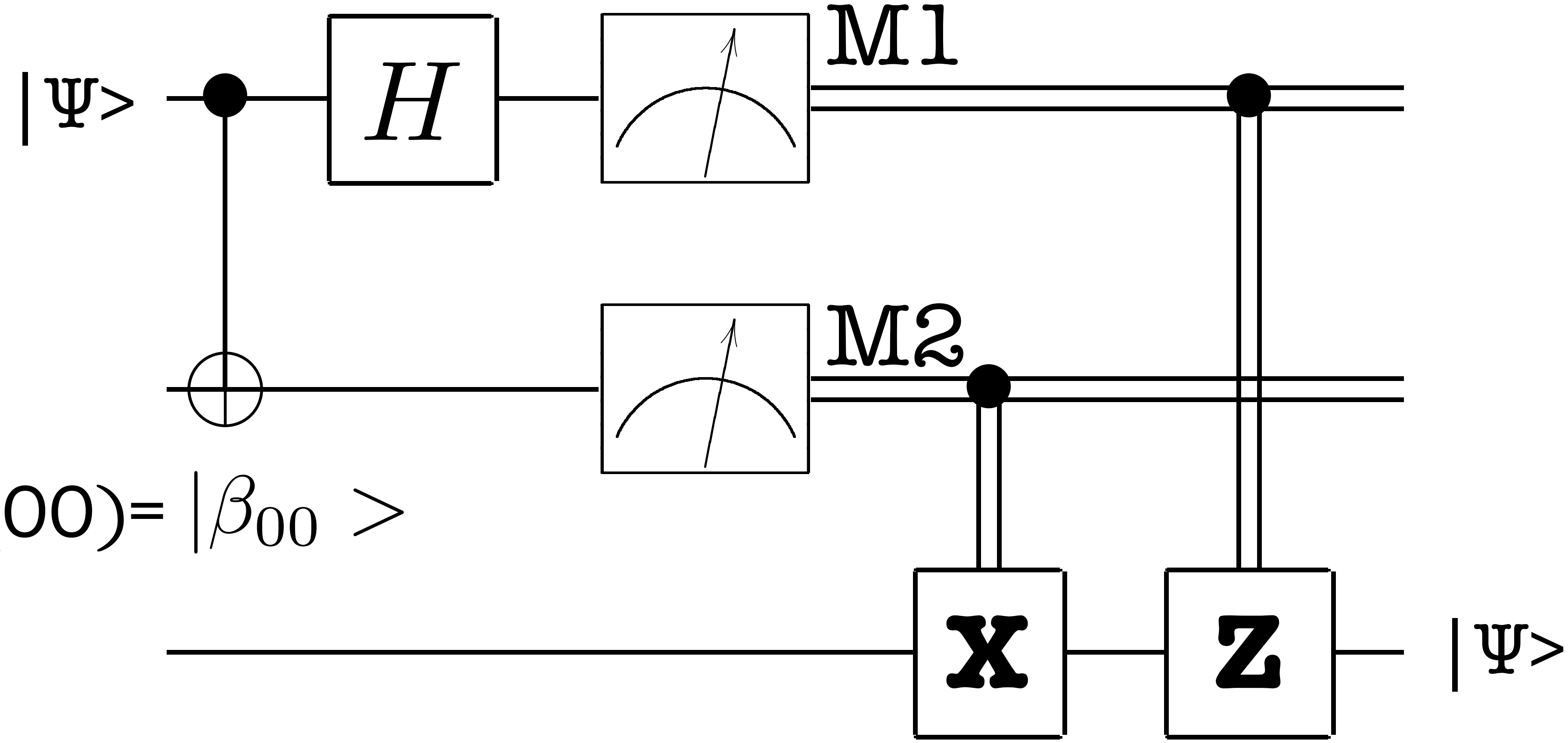
- Now, Alice is trying to deliver an unknown qbit of information to Bob
  - Alice does not know the state of the qbit $|\psi >$
  - She can only send classical information to Bob
- Complications
  - Alice cannot measure the state as she has only one copy...
  - Even if she knew the state
    - Describing it would take infinite # bits of classical information
    - Sending the state classically would take forever

Quantum Teleportation

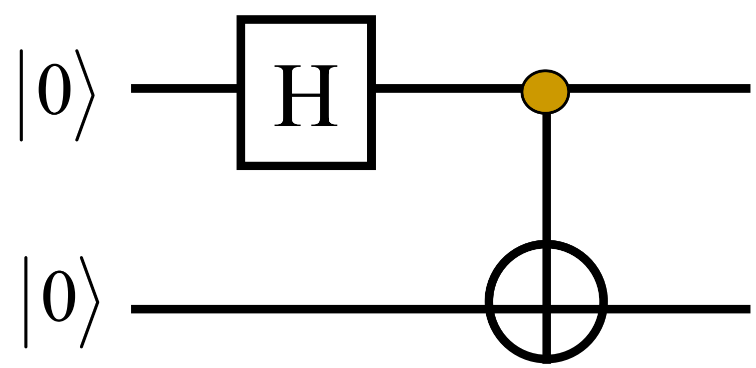
- Alice can use the entangled EPR pair instead:
  - Alice interacts the qbit with her half of the EPR pair
  - Next, she measures both qbits
    - She obtains one of possible classical results of 00, 01, 10, 11
    - She sends the classical result to Bob
    - Depending on Alice's classical message, Bob performs one of four operations on his half of the EPR pair
    - Bob can recover the original state of the qbit!

Quantum Teleportation



EPR (00):

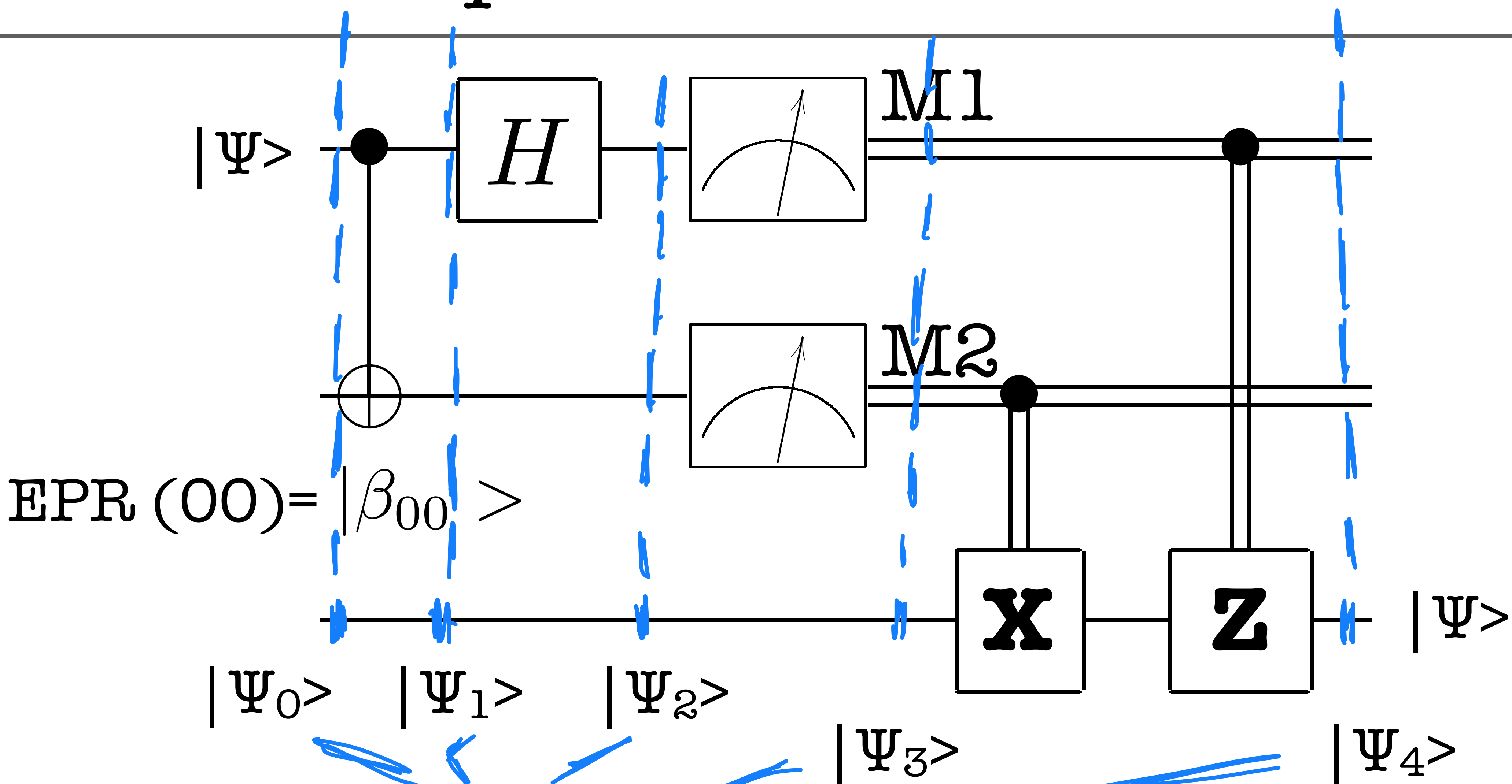
$$|00\rangle$$



$|\beta_{00}>$

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Quantum Teleportation

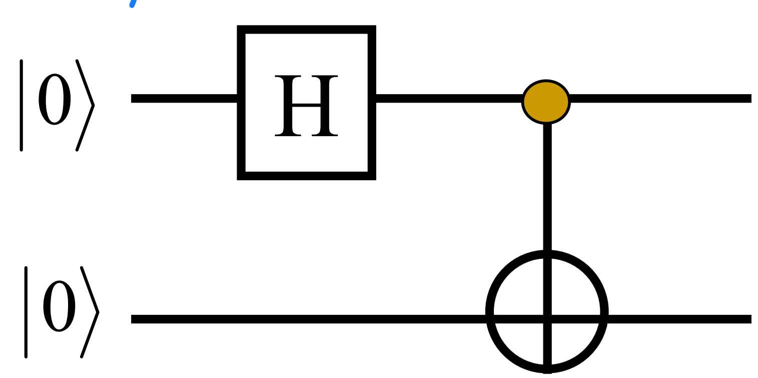


3qbit state

time

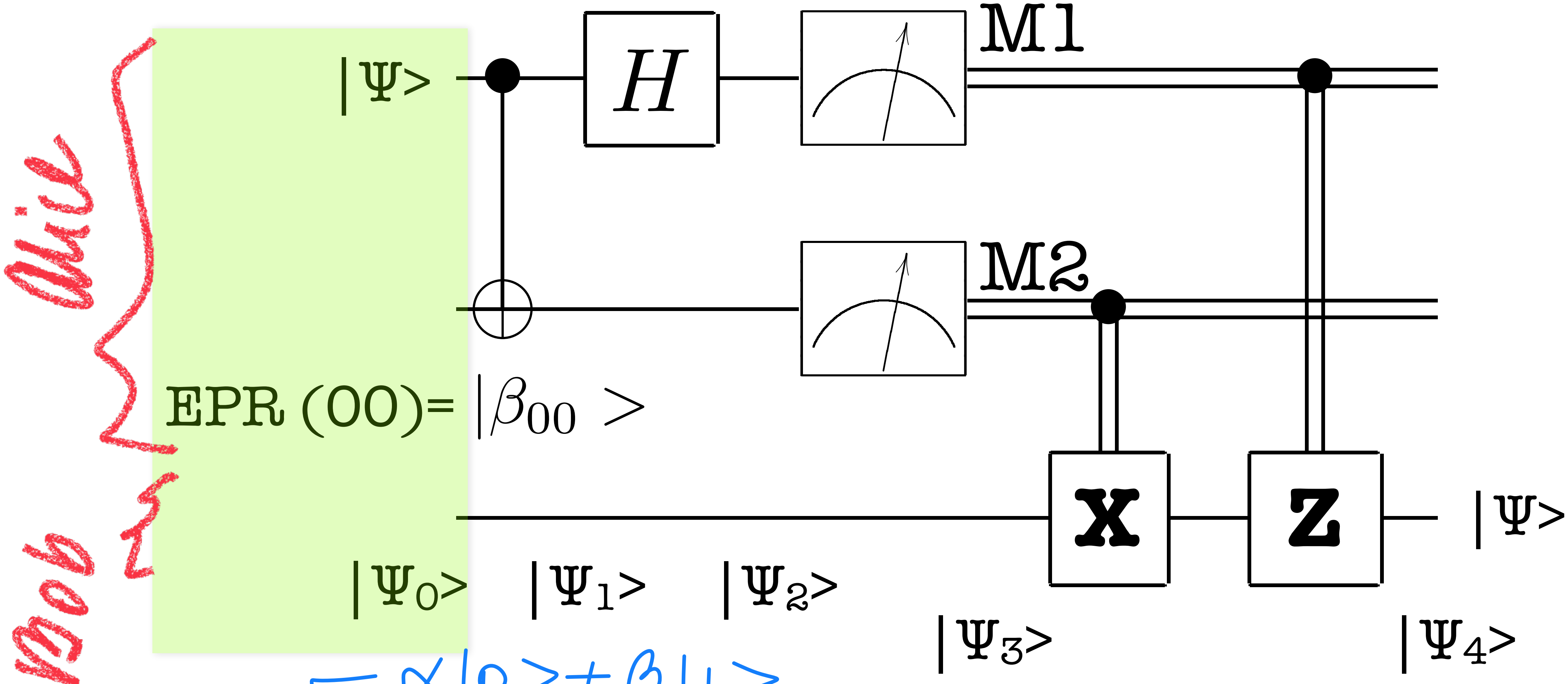
EPR (00):

$$|00\rangle \rightarrow$$



$$\rightarrow \quad |\beta_{00}> = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Quantum Teleportation

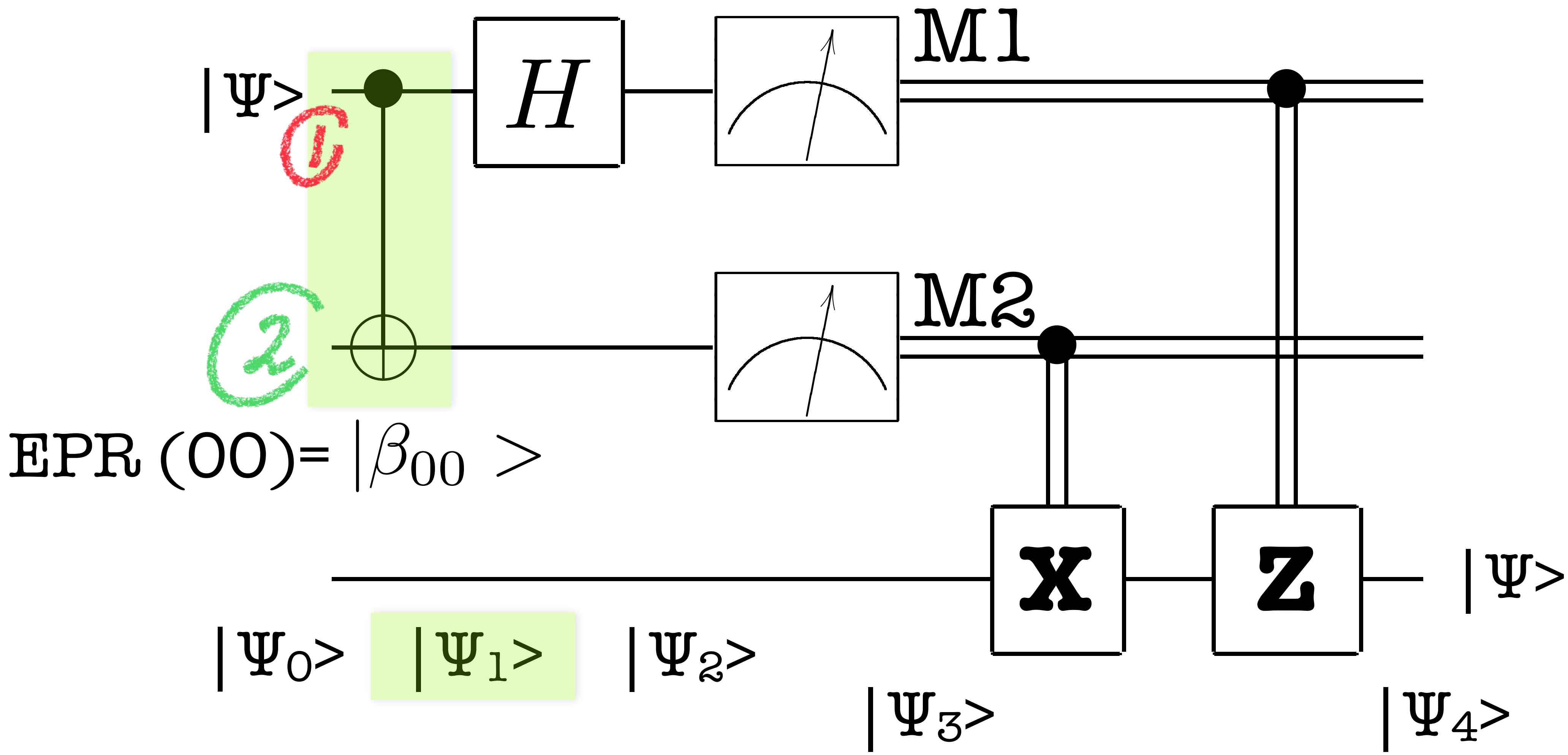


$\alpha|0>+\beta|1>$

$$|\psi_0>=|\psi>|\beta_{00}> \quad 1/\sqrt{2}\,(|00>+|11>)$$

$$|\psi_0>=\frac{1}{\sqrt{2}}[\alpha|0>(|00>+|11>)+\beta|1>(|00>+|11>)]$$

Quantum Teleportation



$$|\psi_0>=\frac{1}{\sqrt{2}}[\alpha|0>(|00>+|11>)+\beta|1>(|00>+|11>)]$$

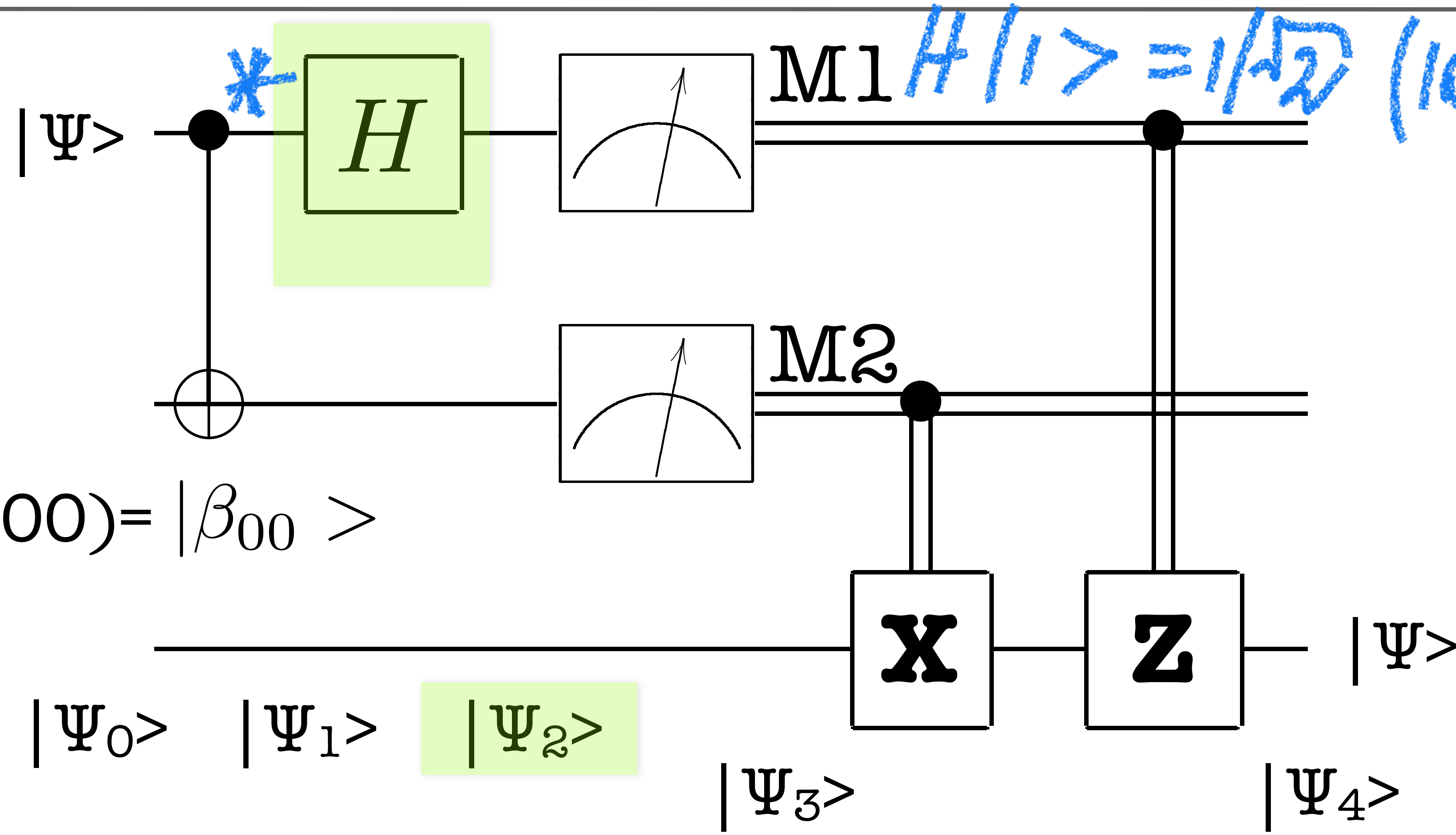
CNOT(1,2)

$$|\psi_1>=\frac{1}{\sqrt{2}}[\alpha|0>(|00>+|11>)+\beta|1>(|10>+|01>)]$$

Quantum Teleportation

$H|0\rangle = 1/\sqrt{2}\,(|0\rangle + |1\rangle)$

$H|1\rangle = 1/\sqrt{2}\,(|0\rangle - |1\rangle)$



$$|\psi_1 >= \frac{1}{\sqrt{2}}[\alpha|0 > (|00 > +|11 >) + \beta|1 > (|10 > +|01 >)]$$

H

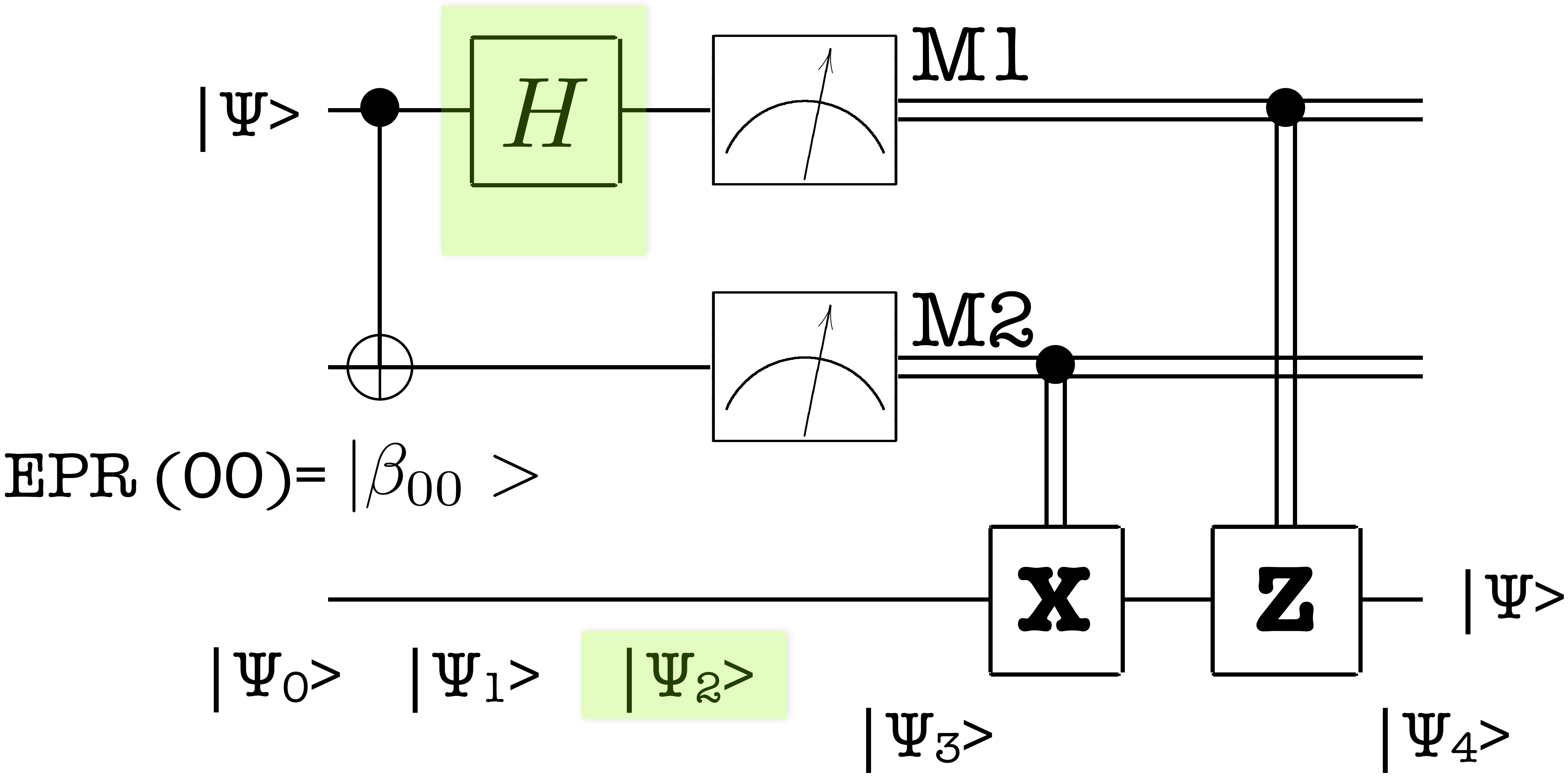
$$|\psi_2 >= \frac{1}{2}[\alpha(|0 > +|1 >)(|00 > +|11 >) + \beta(|0 > -|1 >)(|10 > +|01 >)]$$

$$|\psi_2\rangle =$$

$$\tfrac{1}{2}\left[\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle\right] =$$

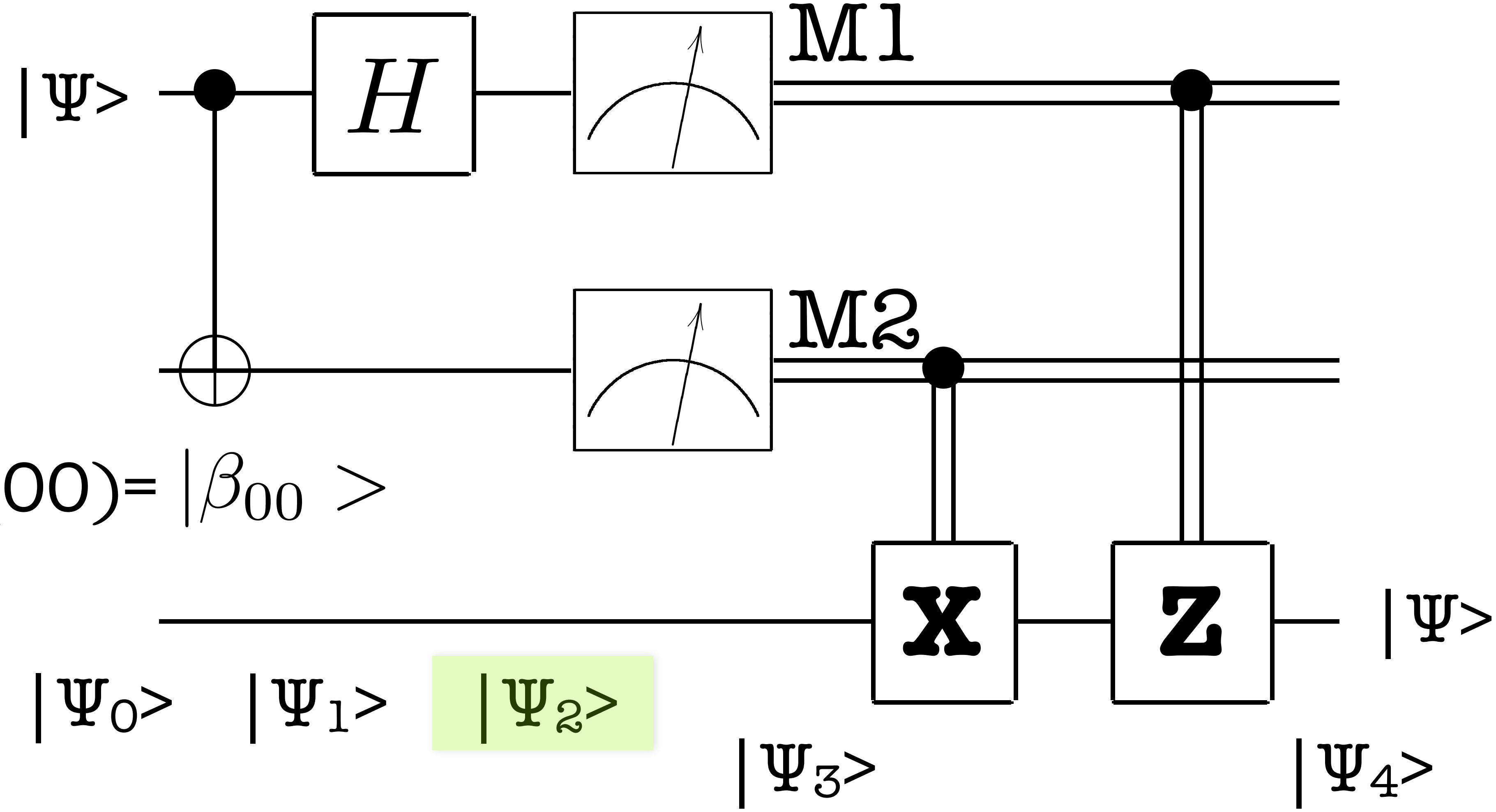
$$= \tfrac{1}{2}\left[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)\right]$$

Quantum Teleportation



$$|\psi_2> = \frac{1}{2}[|00>(\alpha|0>+\beta|1>)+|01>(\alpha|1>+\beta|0>)$$
$$+|10>(\alpha|0>-\beta|1>)+|11>(\alpha|1>-\beta|0>)]$$

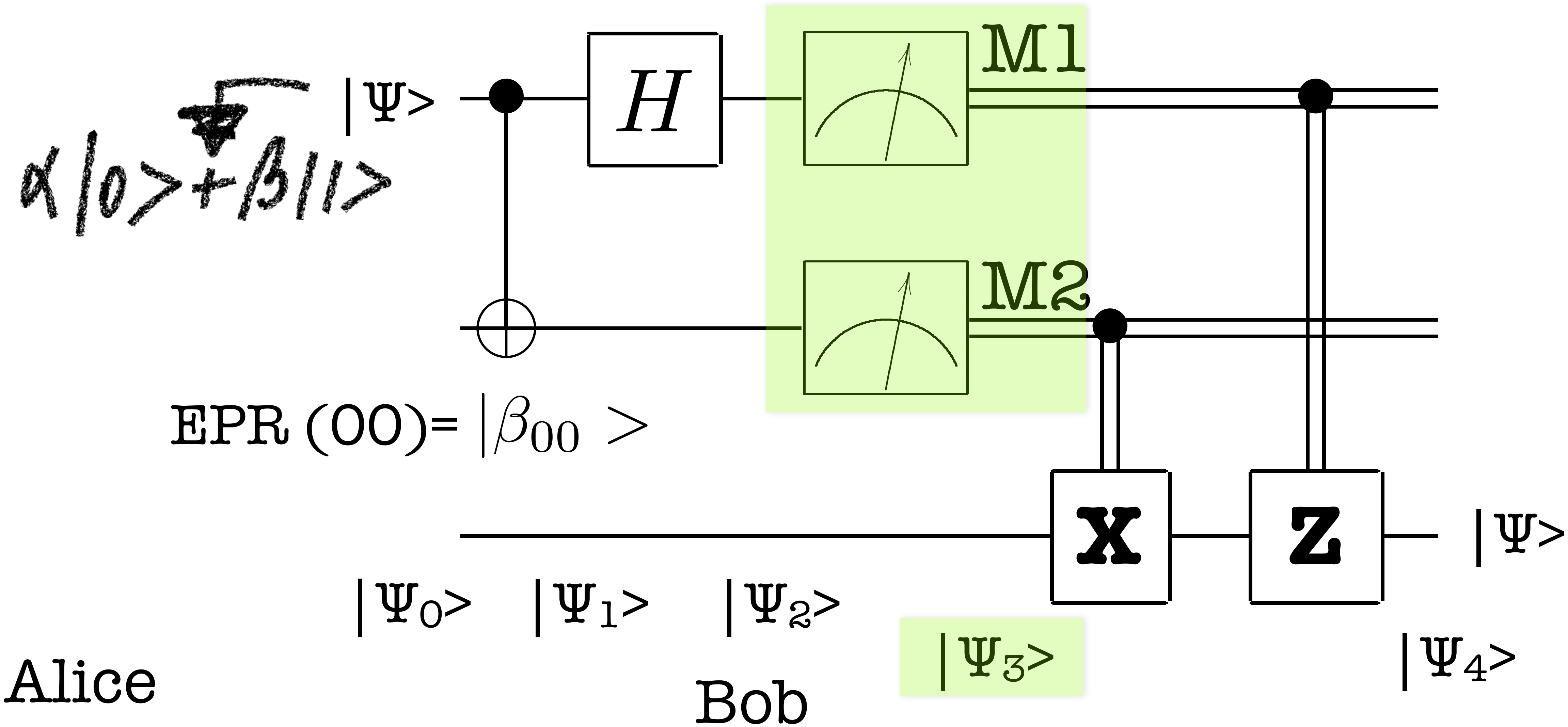
Quantum Teleportation



Alice Bob

$$|\psi_2 >= \frac{1}{2}[|00 > (\alpha|0 > +\beta|1 >) + |01 > (\alpha|1 > +\beta|0 >)$$
$$+|10 > (\alpha|0 > -\beta|1 >) + |11 > (\alpha|1 > -\beta|0 >)]$$

Quantum Teleportation



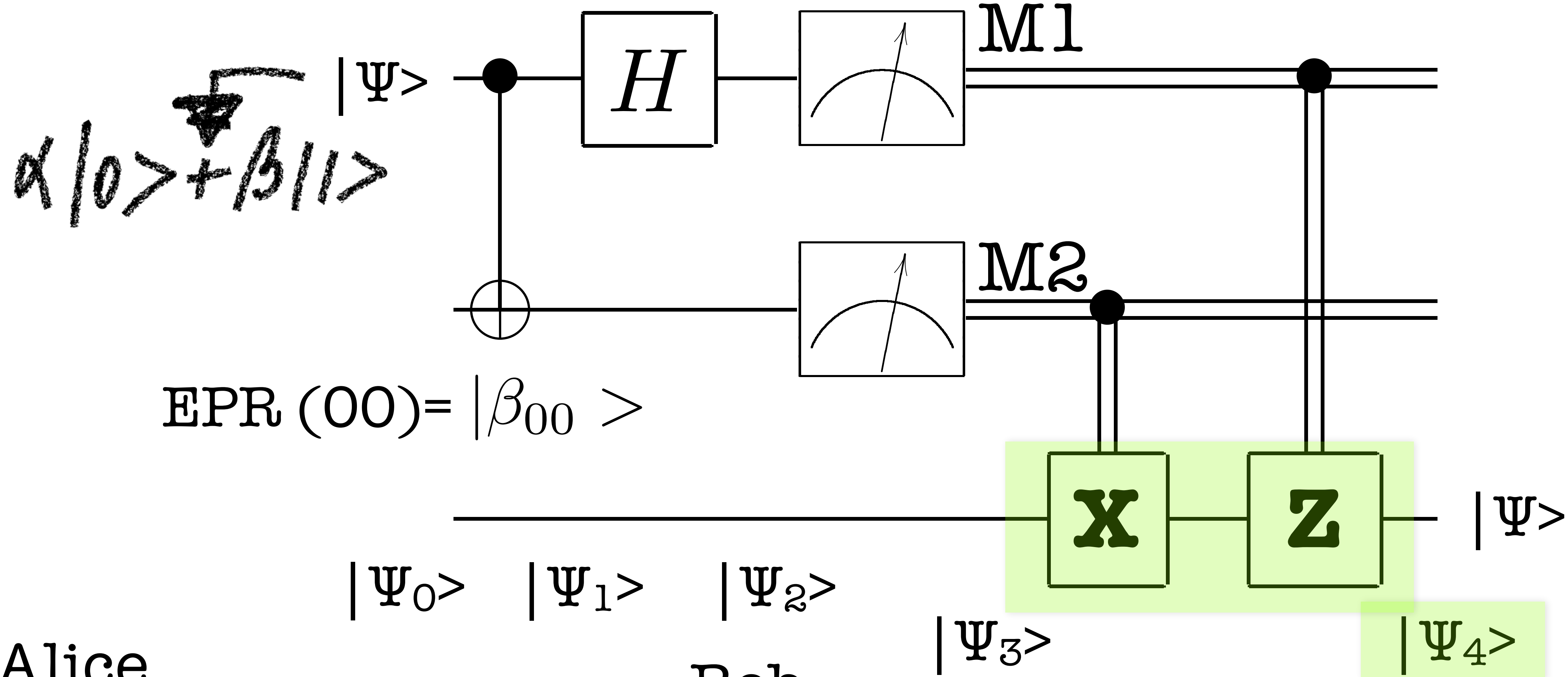
$$00 \rightarrow |\psi_3(00)\rangle \equiv [\alpha|0\rangle + \beta|1\rangle]$$

$$01 \rightarrow |\psi_3(01)\rangle \equiv [\alpha|1\rangle + \beta|0\rangle]$$

$$10 \rightarrow |\psi_3(10)\rangle \equiv [\alpha|0\rangle - \beta|1\rangle]$$

$$11 \rightarrow |\psi_3(11)\rangle \equiv [\alpha|1\rangle - \beta|0\rangle]$$

Quantum Teleportation



Alice

Bob

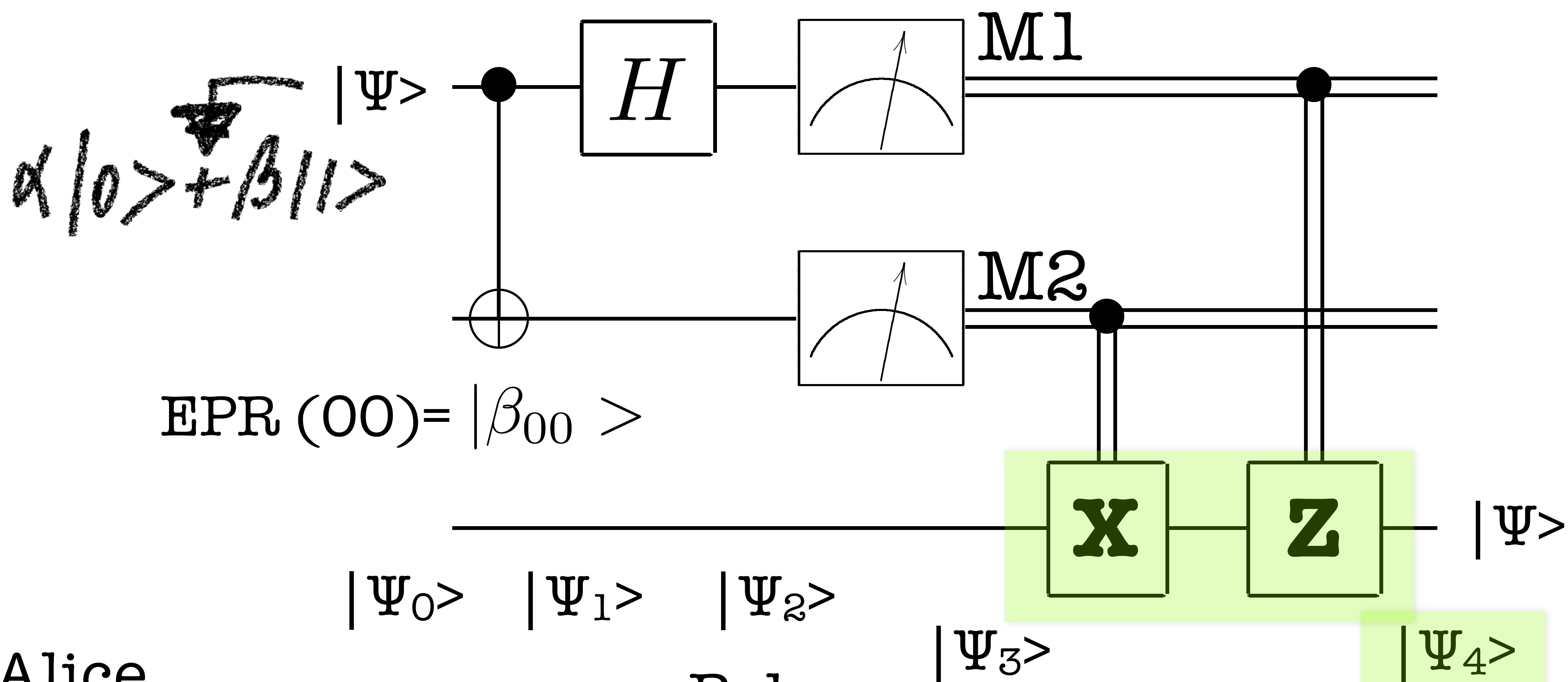
$00 \rightarrow |\psi_3(00)> \equiv [\alpha|0> +\beta|1>]$ Intact

$01 \rightarrow |\psi_3(01)> \equiv [\alpha|1> +\beta|0>]$ X

$10 \rightarrow |\psi_3(10)> \equiv [\alpha|0> -\beta|1>]$ Z

$11 \rightarrow |\psi_3(11)> \equiv [\alpha|1> -\beta|0>]$ XZ

Quantum Teleportation



Alice

Bob

$00 \rightarrow |\psi_3(00)> \equiv [\alpha|0> +\beta|1>]$ Intact

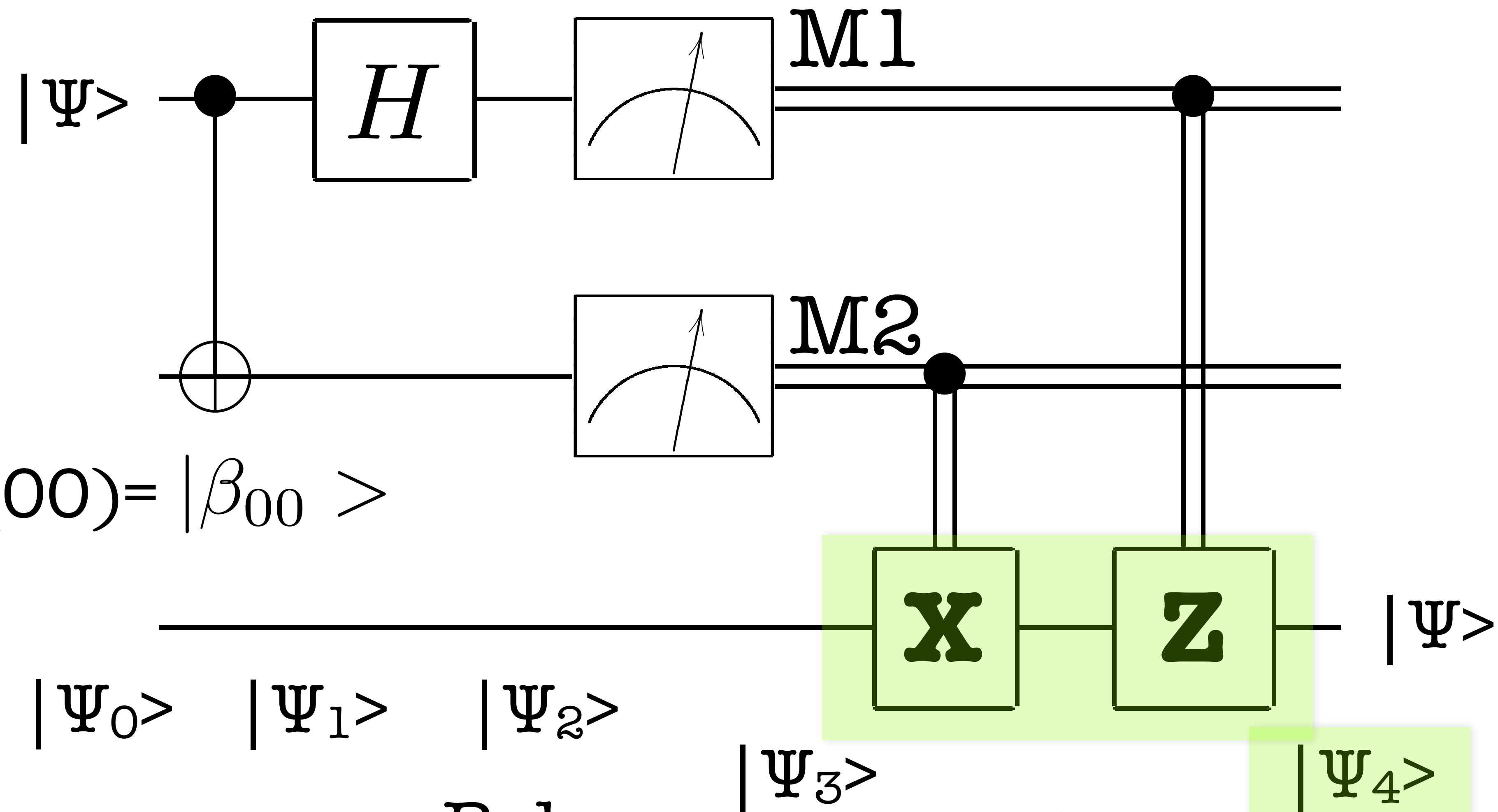
$01 \rightarrow |\psi_3(01)> \equiv [\alpha|1> +\beta|0>]$ X

$10 \rightarrow |\psi_3(10)> \equiv [\alpha|0> -\beta|1>]$ Z

$11 \rightarrow |\psi_3(11)> \equiv [\alpha|1> -\beta|0>]$ XZ

time

Quantum Teleportation



Alice

Bob

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$$01 \rightarrow |\psi_3(01)> \equiv [\alpha|1> + \beta|0>]$$ X

$$10 \rightarrow |\psi_3(10)> \equiv [\alpha|0> - \beta|1>]$$ Z

$$11 \rightarrow |\psi_3(11)> \equiv [\alpha|1> - \beta|0>]$$ ZX

$$Z^{M1}X^{M2}$$

matrix form

Bibliography

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- Metodi et al., Quantum Computing for Computer Architects

EE5340

INTRODUCTION TO QUANTUM COMPUTING AND PHYSICAL BASICS OF COMPUTING

Basics

Ulya Karpuzcu