EE5340

INTRODUCTION TO QUANTUM COMPUTING AND PHYSICAL BASICS OF COMPUTING

Concluding Remarks

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Google's Claims & IBM's Rebuttal

"We argue that an ideal simulation of the same task can be performed on a classical system in 2.5 days and with far greater fidelity."

- 2.5 days: Conservative, worst-case estimate
  - Cost of classical simulation can further be reduced
- Quantum Supremacy:
  - "the point where quantum computers can do things that classical computers can't"

200 secs on Sycamore vs. **10K years on a classical computer**?

- Storage requirement for entire state vector prohibitive
- Hence, need to use an algorithm trading off space (storage) for time
  - Is this the best-case for classical processing?
    - Memory hierarchy?
    - Algorithmic optimizations?

  Can extend the range of quantum circuits that can be practically simulated with classical algorithms…

New classical simulation of 53-qbit Sycamore [2]

Performance Attributes	Content
Perfomance	1.2 Eflops (single-precision) 4.4 Eflops (mixed-precision)
Maximum problem Size	10×10 (qubits) ×(1 + 40 + 1) (depth)
Category of achievement	Peak performance, and time to solution
Type of method used	Simulating the RQC by contracting a tensor network
Results reported on basis	Whole application
Precision reported	Single precision, and mixed precision (single/half)
System scale	107,520 nodes (41,932,800 cores)
Measurement mechanism	Flops counts and timers

New classical simulation of 53-qbit Sycamore [2]

- 2021 Gordon Bell Price Winner

The Gordon Bell Prize is awarded each year to recognize **outstanding achievement in high-performance computing**. The purpose of the award is to track the progress over time of parallel computing, **with particular emphasis on rewarding innovation in applying high-performance computing to applications in science, engineering, and large-scale data analytics**.

- Sunway supercomputer
  - Using over 41.9 million Sunway cores
  - Physical footprint much larger than Sycamore

    Criteria 3: Fair resource allocation to classical machine, applies
- Numerous algorithmic tricks including
  - Customized tensor network contraction algorithm
  - Mixed precision with adaptive precision scaling
  - Customized parallelization
- **Google's 10K years became 304 seconds**

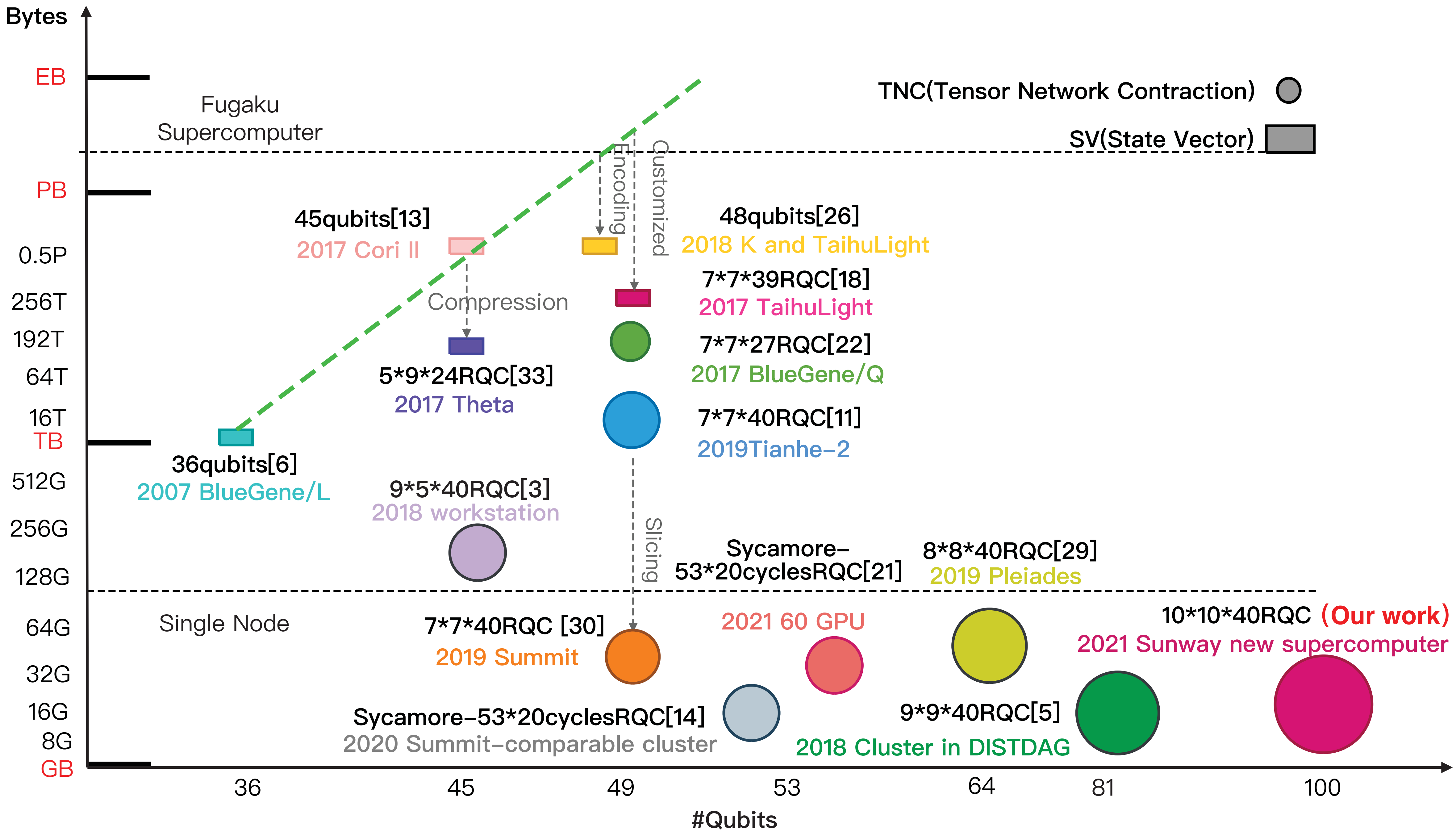
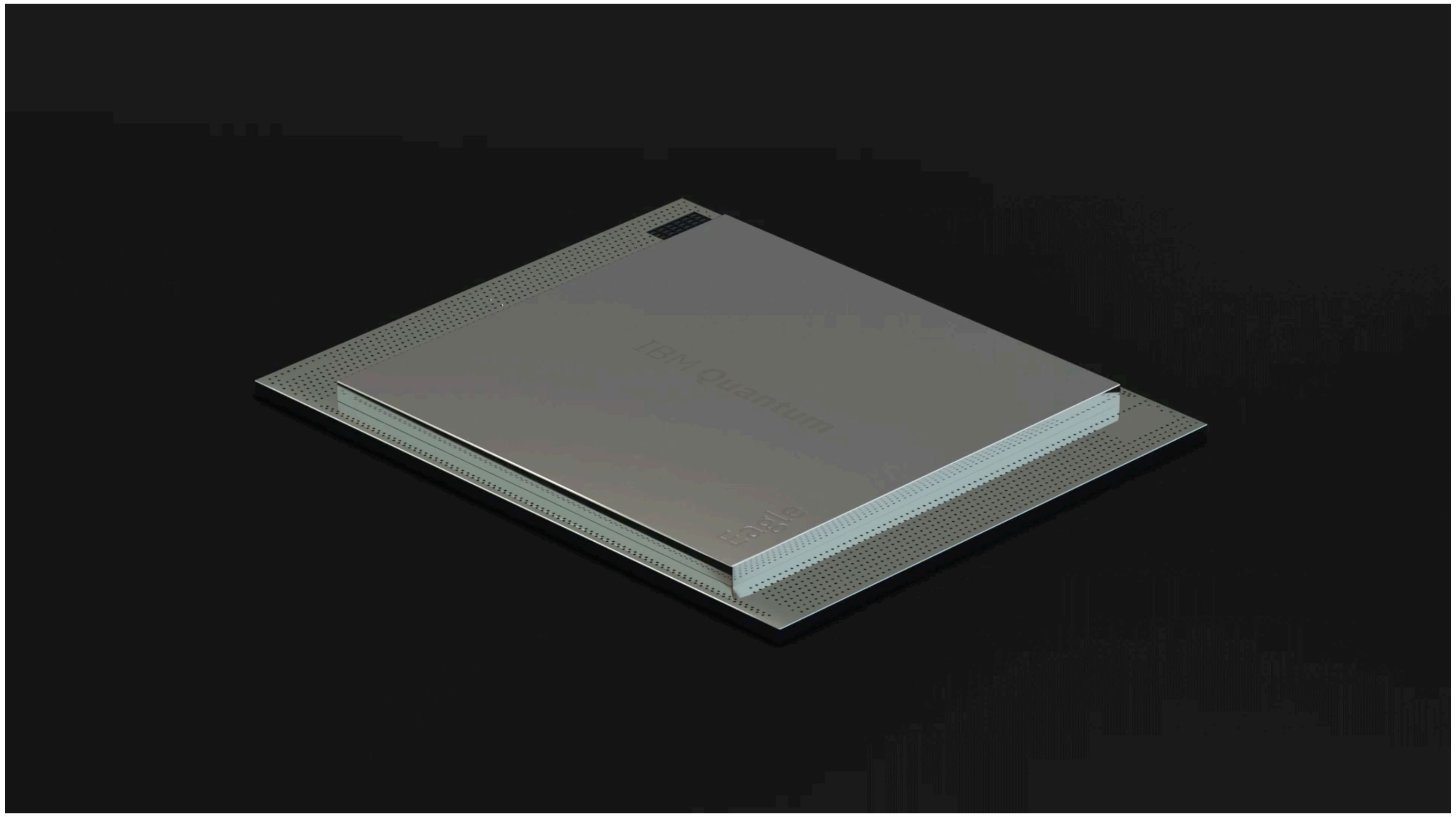
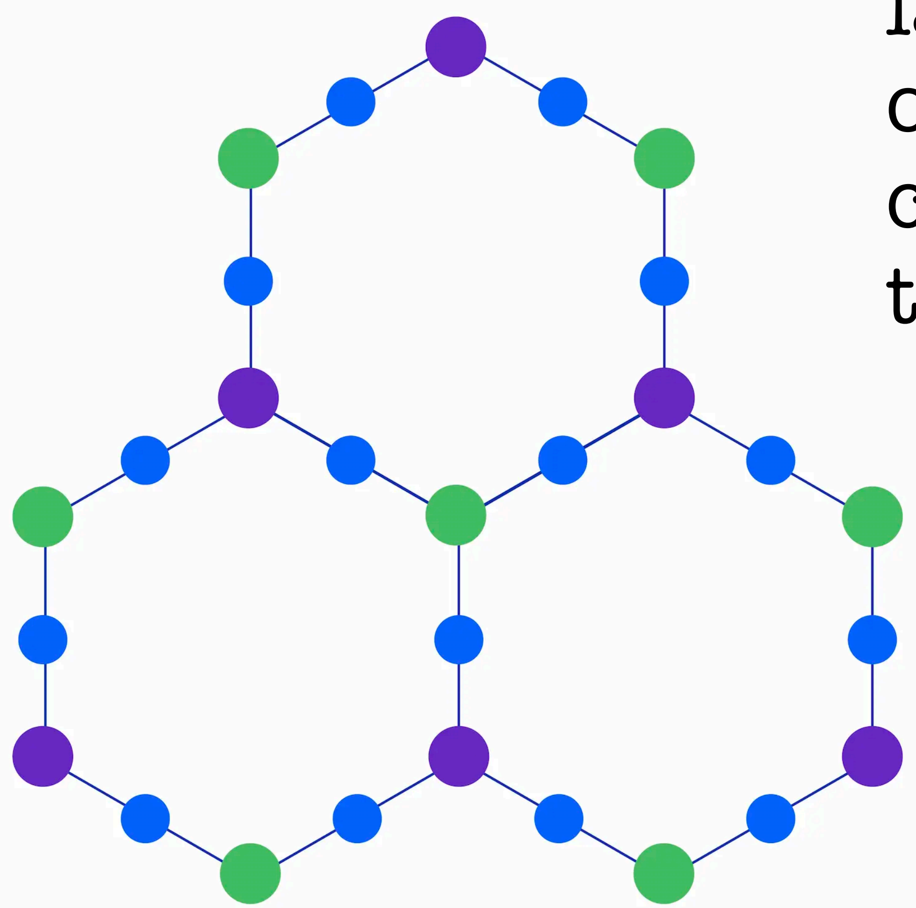


Figure 2: A summary of major classical RQC simulations. The x-axis denotes the number of qubits, while the y-axis shows the corresponding memory space required. The size of the circle/rectangular corresponds to the complexity (depth) of the circuit.

IBM's 127 qbit Eagle [3]

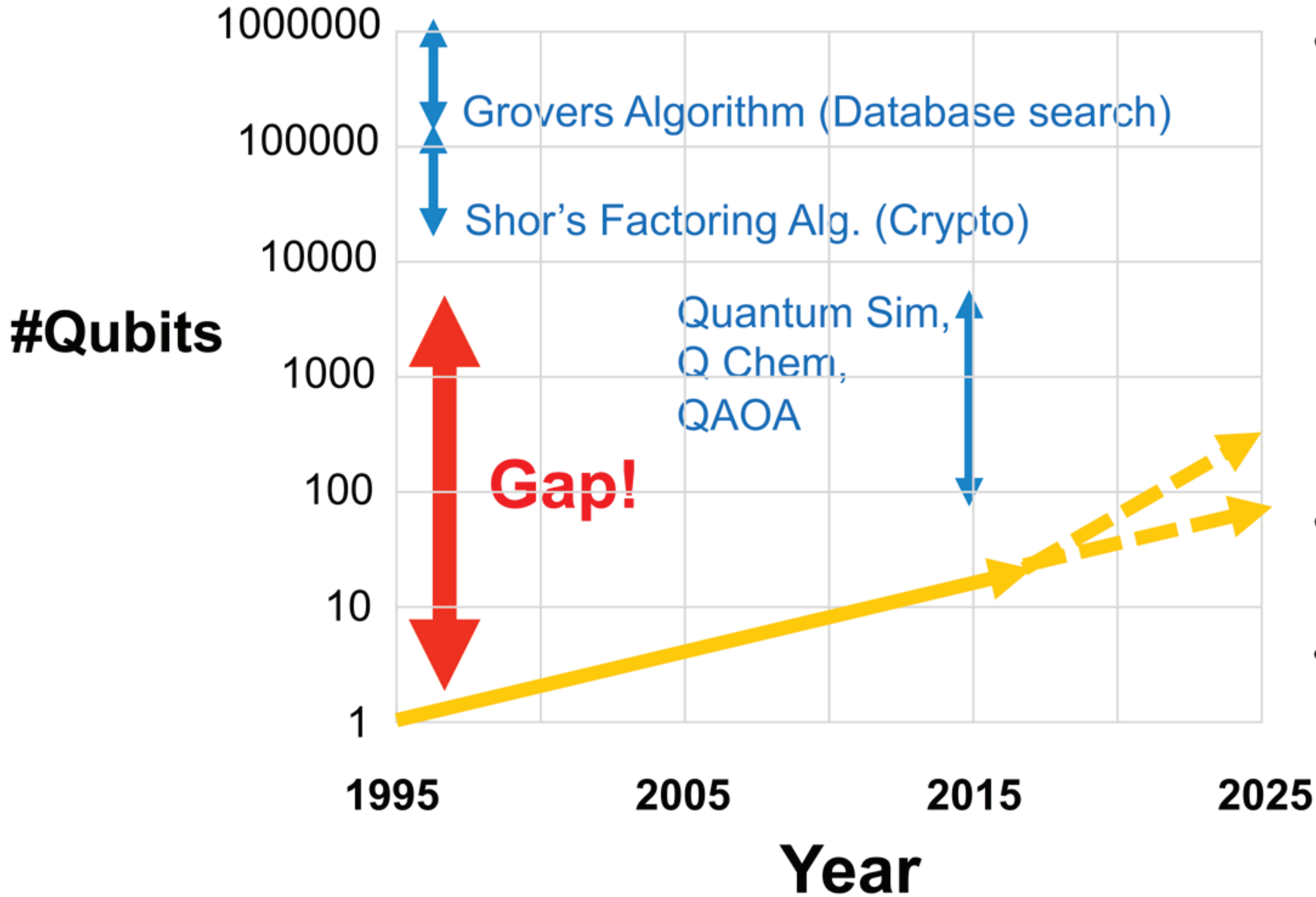


IBM Eagle [2]



Three unit cells of the heavy-hex lattice. Colors indicate the pattern of three distinct frequencies for control (dark blue) and two sets of target qubits (green and purple).

Current State: Algorithms to Machines Gap



Next Steps in Quantum Computing: Computer Science's Role, Martonosi et al., CCF Report, November 2018

Quantum Optimization

Combinatorial Optimization Problems (COP)

- Minimize a cost (objective) function over a finite and discrete search space
- For most of these problems of practical interest
  - time required to find a solution increases much faster than a polynomial function of the problem size (i.e., the number of objects in the search space)
  - hence they belong to the complexity class NP
    - to be more specific, NP-complete
      - the subset of the hardest problems in NP
      - If we can solve any of these efficiently – with the time required growing as a polynomial function of the problem size – then we can solve all problems in NP efficiently
- An optimization problem can always be converted to a decision problem:
  - Is there a solution having cost less than some value?
  - By solving a sequence of these decision problems, we may obtain the minimal cost solution to the optimization problem
    - but the number of such queries may not always grow as a polynomial function of the problem size…
    - If the decision version of a problem is NP-complete, the problem is NP-hard

Ising Model mathematical model

- Introduced in 1920s to understand magnetic materials
- Represents a magnetic material as a collection of molecules
  - Each molecule has a spin
    - either aligned or anti-aligned with an applied magnetic field
  - Spins interact with each other in a pairwise fashion
- The energy (Hamiltonian) of a collection of N spins $s = [s_1, s_2, \ldots, s_N]$

$$E(s) = \sum_{<i,j>} J_{ij} s_i s_j + \sum_i h_i s_i$$

s_i Spin of molecule i (i=1, 2, ..., N), can either be +1 or -1

J_{ij} Interaction field between neighboring spins i and j

h_i Strength of applied field to molecule i

real valued

Ising Model cont.

$$E(s) = \sum_{<i,j>} J_{ij} s_i s_j + \sum_i h_i s_i$$

stable

- To establish equilibrium, the system tends to converge to lower energy states (i.e., s configurations) minimizing E(s):

if $J_{ij} > 0$ $\quad s_i s_j = -1 \quad J_{ij} s_i s_j$ becomes -

if $J_{ij} < 0$ $\quad s_i s_j = 1 \quad J_{ij} s_i s_j$ becomes -

- Inline with physical intuition, the collection of spins can be thought of residing in a finite-dimensional lattice, which can be represented by a generic graph...
- Let G = (V, E) be an undirected graph
  - V is the set of vertices
    - Each vertex $i \in V$ is a spin, and is characterized by s_i and h_i
  - E is the set of edges
    - Each edge is labeled by $< i, j > \in E$
    - The coupling strength (weight) along each edge is captured by J_{ij}

Complexity

- In its most generic form, finding the minimal energy, i.e., ground, state of the Ising model is proven to be NP-hard.
- Therefore, any physical system that complies with the Ising model and is trying to reach equilibrium, by construction, is solving an NP-hard problem, where the ground state of the system encodes the solution to an NP-hard optimization problem
- **NP-complete or NP-hard combinatorial optimization problems can be reduced to the ground-state finding problem of the Ising model in polynomial time**

$$E(s) = \sum_{<i,j>} J_{ij} s_i s_j + \sum_i h_i s_i$$

Ising Formulation of COPs

- Including all of Karp's 21 NP-complete problems, Ising formulations for many important NP-complete and NP-hard problems have been demonstrated
- These essentially boil down to finding proper J_{ij} and h_i assignments
- **As finding the exact solution (i.e., the minimal energy state) of the Ising model is proven to be NP-hard, no simulation or software-based implementation tailored to conventional von Neumann architectures (be it a CPU or a GPU) can tackle large problems at scale, by definition**

$$E(s) = \sum_{<i,j>} J_{ij} s_i s_j + \sum_i h_i s_i$$

Ising Formulation of COPs

$$E(s) = \sum_{<i,j>} J_{ij} s_i s_j + \sum_i h_i s_i$$

- As an example, the proof that Ising model is NP-hard can also be used to convert it to Quadratic Unconstrained Binary Optimization (QUBO)

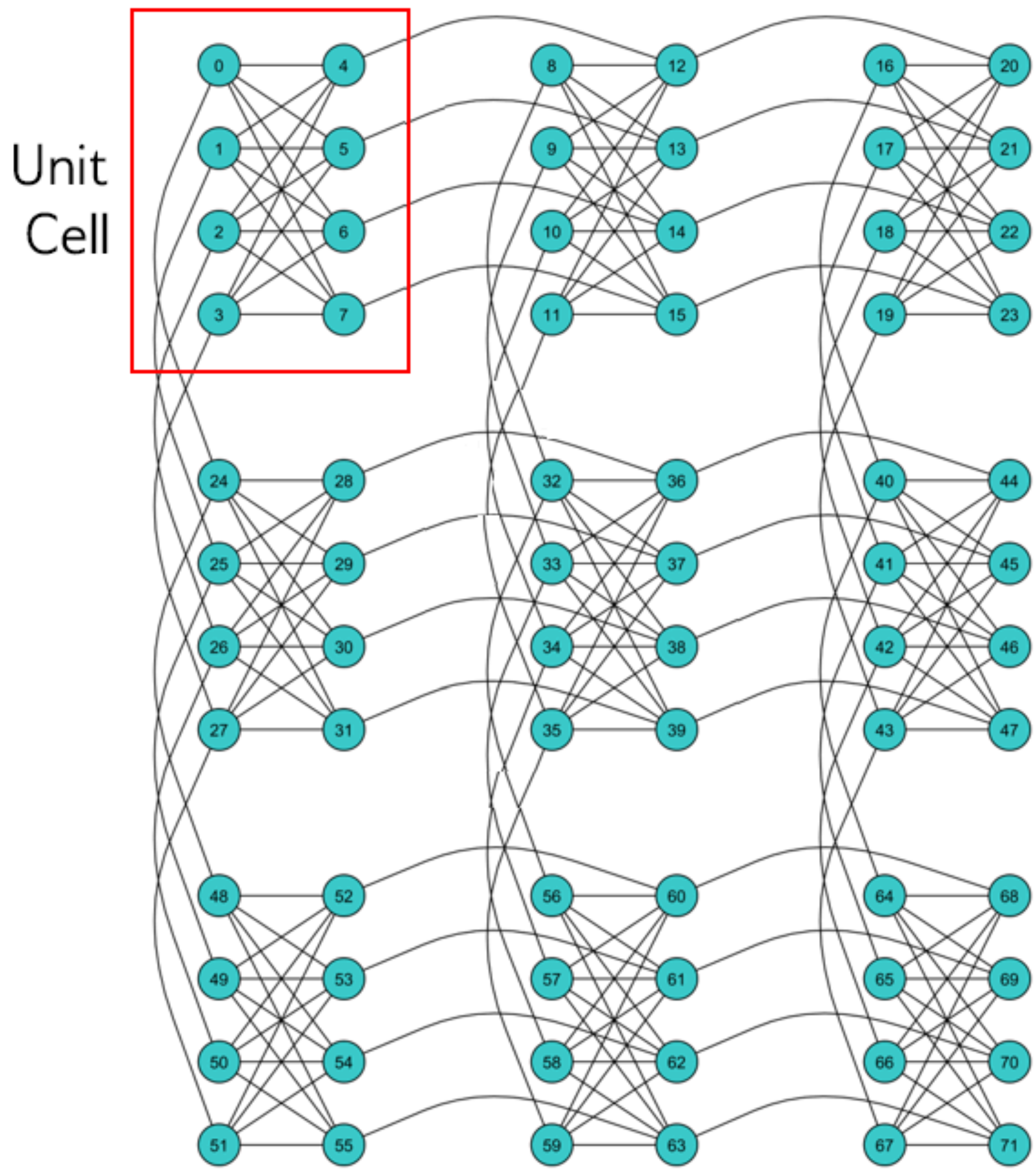
$s_i \in \{-1, +1\}$ become $y_i \in \{0, 1\}$ where $s_i = 2y_i - 1$

$$E(y) = \sum_{i,j} y_i Q_{ij} y_j$$

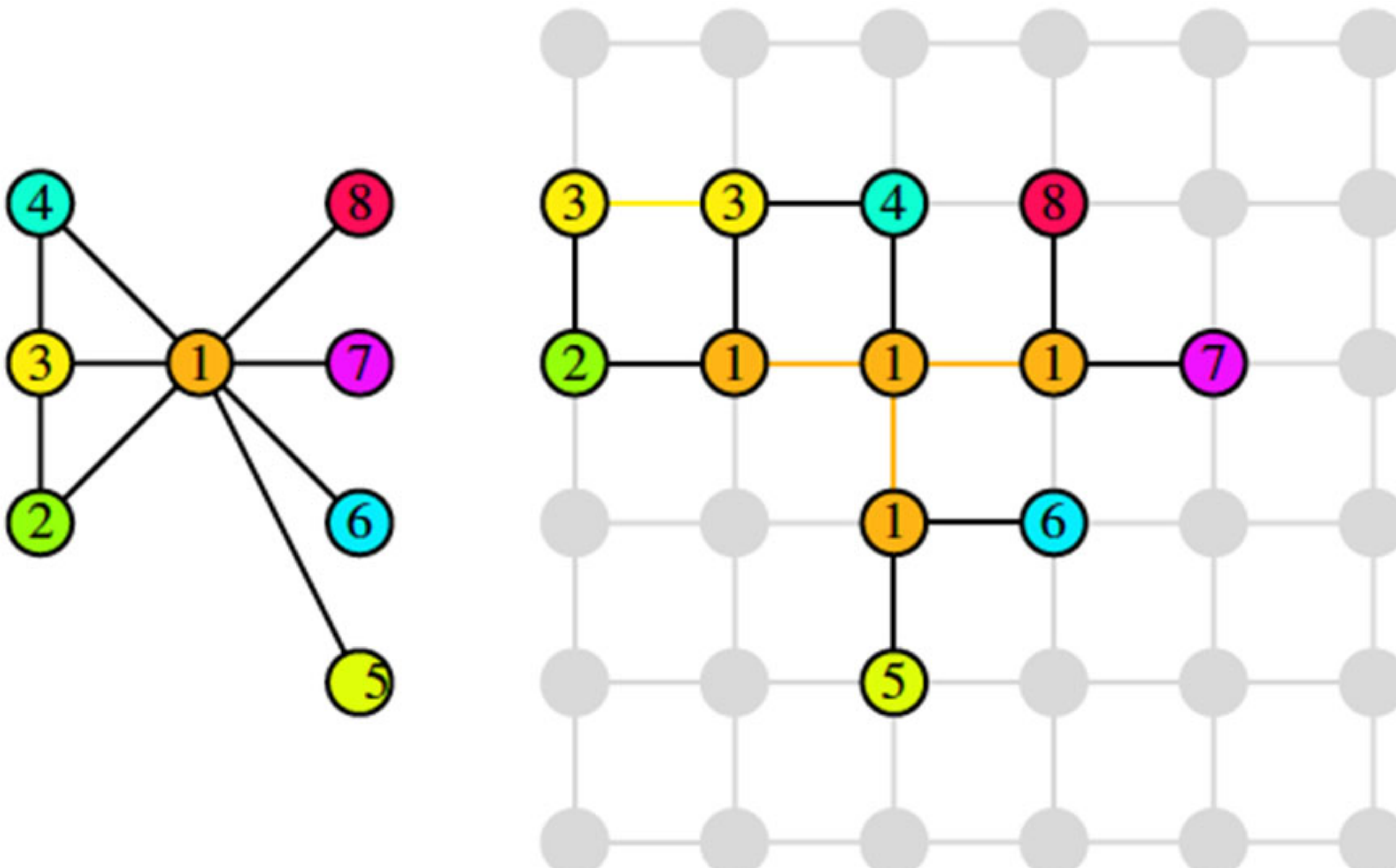
Ising Machines

- Rich design space
  - featuring both exotic and conventional physical substrates
- In theory, an ideal Ising machine would purely rely on the dynamical evolution (according to the Ising model) of its underlying physical system in solving a COP
  - Exotic designs come closer
    - Strictly speaking, none achieves this ideal
    - Practical limitations
      - Topology constraints
      - Commercial Quantum Ising Machines from DWave
        - Quantum Noise
        - Operation at cyrogenic temperatures

DWave Chimera



Graph Embedding



Baselines for Comparison?

- Non-quantum Ising machines?
- Approximation algorithms?

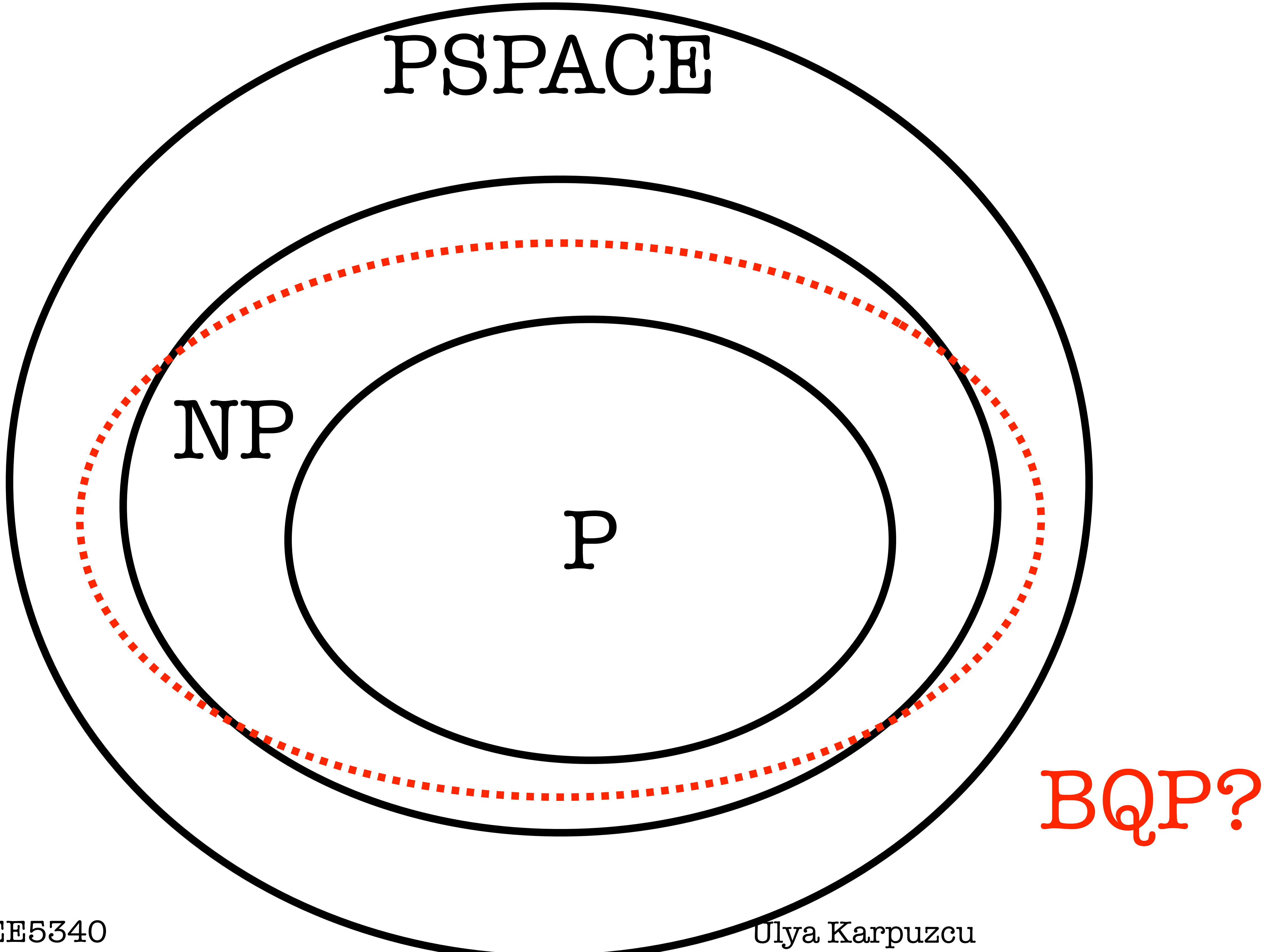
NISQ Era

- NISQ: Noisy-intermediate Scale Quantum Computer
- Quantum computers with 50-100 qubits
  - May be able to perform better than today's classical computers
  - Noise will limit the size of circuits that can be executed reliably
- Significant step toward more powerful future machines
  - Goal: Fully fault tolerant quantum computing
  - Premise: QEC
    - Principle: Encode quantum state in a very entangled state
    - Many additional physical qubits necessary…

NISQ usually refers to quantum computers with noisy gates, unprotected by error correction, as limited by

- Number of qubits
- Gate error rates, measurement and state preparation error rates
- Gate latency
- Connectivity

How powerful are quantum computers?



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[2] Closing the "quantum supremacy" gap: achieving real-time simulation of a random quantum circuit using a new Sunway supercomputer, SC'21

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Concluding Remarks

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