Structure-aware Diversity Pursuit as AI Safety strategy against Homogenization

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Abstract

Generative AI models reproduce the biases in the training data and can further amplify them through mode collapse. We refer to the resulting harmful loss of diversity as homogenization. Our position is that homogenization should be a primary concern in AI safety. We introduce *xeno-reproduction* as the strategy that mitigates homogenization. For auto-regressive LLMs, we formalize xeno-reproduction as a structure-aware diversity pursuit. Our contribution is foundational, intended to open an essential line of research and invite collaboration to advance diversity.

1. Introduction

> *But even if we are not here next year, our DMs, our selfies, our late-night voice notes, they'll be. Our memory is the archive now.*
>
> *@bundleof_styx*
>
> *July 28, 2025 on Reels*

In this epigraph, trans intellectual *bundleof_styx* laments the recent transphobic turn in the United States, a shift that threatens the survival of her community. The stories in the margins have historically been excluded from *the archive* (Spivak, 1988), so their memory faded with them. Today, however, the internet allows (and forces) the recording of many more stories. These are still very subtle *traces* against the dominant narratives (Hussain, 2024). **How should technology respond to the faint echoes of the *minoritized*?**

AI safety recognizes that AI systems can amplify *biases* leading to concrete harm (Bengio et al., 2025). However, AI safety usually differentiates and prioritizes future catastrophic risk over present social harm (Morozov, 2024; Harding & Kirk-Giannini, 2025). In this paper, we respond to traces from the margins by foregrounding them within AI safety through *diversity*.

The harms from biases in *Machine Learning* (ML) systems are many, including representational (Katzman et al., 2023), allocational (Shelby et al., 2023), and narrative[2] (Coeckelbergh, 2023) harms. Concerning *Generative Artificial Intelligence* (GenAI), we particularly emphasize that biases result in **homogenization**, a *harmful loss of diversity in generated outputs* (Rudko & Bashirpour Bonab, 2025; Agarwal et al., 2025; Hussain, 2024; Sourati et al., 2025; Moon et al., 2025). Borrowing terminology from *critical theory* (Hester, 2018), we refer to the strategy that addresses homogenization as[3] **xeno-reproduction**.

Our main standpoint is that diversity is always relative to a context. We take the first steps to operationalize this principle by offering an abstract framework that aims to encapsulate some nuances of context. Our framework can be thought of as **structure-aware**, as it offers a vocabulary of *structures*, *systems*, and *compliances*. Given that an LLM defines a probability distribution over all possible trajectories, we **enhance our structural account with string statistics**. This allows us to further introduce the notions of *cores*, *orientations*, *deviances*, and *dynamics*. Finally, our formalism enables us to formalize xeno-reproduction.

Our contributions:

- We motivate the formalization of xeno-reproduction as a core AI safety strategy. (Section 2)
- We provide an expressive theoretical framework that allows us to jointly reason about the structures and the statistics of strings. (Section 3)
- We formalize homogenization (Section 4) and xeno-reproduction (Section 5).
- We provide initial theoretical results and touchpoints from our framework. (Section 6)

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[2] Narrative harms can also be considered as *aspirational* (Fazelpour & Magnani, 2025), *imaginative* (Gillespie, 2024), and *epistemic* (Barry & Stephenson, 2025) harms, or *hermeneutic* (Goetze, 2018) injustices.

[3] While homogenization reproduces "the same" and narrows *futurability* (Berardi, 2017), xeno-reproduction reproduces "the strange" and widens possibilities.

Our position is that AI safety should center homogenization in its research and mitigation agenda, and that structure-aware diversity pursuit is a key part of the strategy to address homogenization in LLMs. The goal of this paper is not to present a complete and empirically validated algorithm, but rather to offer a conceptual vocabulary and formal scaffolding to guide future research on diversity in LLMs.

2. Background

A case *against* homogenization is a case *for* diversity. Roughly, we can think of the **diversity of a *community* as the *average rarity* of its *members*** (Leinster, 2024). For a community of LLM outputs, a string is *rare* if neither it nor any *similar* strings are *generated* often. However, people tend to disagree on what kind of similarities and differences are meaningful (Vrijenhoek et al., 2024). Embracing *ambiguity* (Reinhardt, 2020) for us amounts to attending to *context*. This section situates diversity in the contexts meaningful to us, guiding our *desiderata* for xeno-reproduction.

2.1. Why is diversity lost?

The initial driver of diversity loss is the way our data is collected (Guo et al., 2024). The archive does not fully or accurately represent reality. Minoritized populations are often underrepresented or **misrepresented** in the existing corpora of data (Bengio et al., 2025).

Even if our training data perfectly reflected the world, generative models (Huang & Huang, 2025) generally do not capture the complete diversity of the training data. This phenomenon has been referred to as **mode collapse** (Jiang et al., 2025), a failure of distributional faithfulness that negatively impacts diversity. It was initially introduced in the context of GANs (Huang & Huang, 2025). For LLMs, the terminology has been somewhat loose (Schaeffer et al., 2025). *Generalized* mode collapse encompasses mode dropping (Huang et al., 2024; Yazici et al., 2020), no-breadth scenarios (Kalavasis et al., 2025b), coverage collapse (Schaeffer et al., 2025), overgeneralization (Li & Farnia, 2023), mode interpolation (Aithal et al., 2024), degeneration (Finlayson et al., 2023), and catastrophic forgetting (Cobbinah et al., 2025; Thanh-Tung & Tran, 2020).

2.2. Why is diversity important?

There are always rare events of interest[4] in the long tails of reality's distribution. For example, we want to understand, model, and prepare for extreme catastrophes (Gu et al., 2025), such as unexpected natural disasters. Similarly, we want to reproduce those rare bursts of genius that generate novel, paradigm-shifting innovations in our research work (Uzzi et al., 2013; Hofstra et al., 2020; Wu et al., 2019). We find examples of this in all domains (Stanley & Lehman, 2015), including: web server computing (Dean & Barroso, 2013), market research (Von Hippel, 1989), autonomous vehicles (Putra et al., 2024), cybersecurity (Edwards et al., 2016), and ecology (Leitão et al., 2016). How do we guide our GenAI models to reproduce the realities found in these long tails?

Outliers (Bhandari et al., 2024) and anomalies (Ruef & Birkhead, 2024) are powerful (Beamish & Hasse, 2022; Cook et al., 2021). Each instance represents a possible real mechanism that we have not yet considered (Woodward, 2005; Rudman et al., 2023). Because we lack understanding, they often escape our systems of classification (Bowker & Star, 1999). Even experts can confuse (Sokol & Hüllermeier, 2025) aleatoric and epistemic uncertainty[5].

Some of the long tails of reality originate from structural inequity in society (Schwartz et al., 2022; Lopez, 2021). Without any intervention, GenAI is expected to worsen the lives of those minoritized (Hussain, 2024). The traces from the minoritized are not only faint but also often overlooked (Jasanoff, 2007; Mohamed et al., 2020) and even actively silenced (McQuillan, 2022). The result is that we do not even know what to look for, even when they are right *in front of* us (Gopinath, 2005). **Some of the most ethically important long-tail cases will be hard to detect**.

2.3. What is the risk of homogenization?

Narrative and storytelling are some of the oldest and most powerful technologies (Zurn et al., 2024). With phenomena like AI-induced psychosis (Preda, 2025), we are just beginning to grapple with the profound ways that LLMs can shape our minds and behavior. Over time, if LLMs deliver too little diversity (Bommasani et al., 2022), our ability to interpret our own experiences and entertain alternative possibilities will shrink (Gillespie, 2024). Eventually, homogenization leads to future *knowledge collapse* (Peterson, 2025), degradation of innovation, and erosion of the human experience (Han, 2024; Berardi, 2017; Preciado, 2013).

The last few years have made it clear that even "less advanced" technology, such as social networks, can have enormous negative impacts (Allcott et al., 2020). Algorithmic recommendations can also have a homogenizing effect, as they tend to standardize and narrow discourse (Putri et al., 2024). This fosters echo chambers and filter bubbles that amplify polarization and misinformation (Rodilosso, 2024). Tragically, in some cases, these dynamics have escalated

[4] For both positive and negative reasons.

[5] For instance, (He & Lab, 2025) recently showed how indeterminism in LLM inference (which can turn on-policy RL into off-policy RL (Yao et al., 2025)) can in fact be explained and reduced, so it is not truly stochastic.

into **real-world violence** (Facebook, 2021) and even genocide (Modok, 2023). This foreshadows the near-term existential risks of AI, especially as it becomes more powerful and more deeply integrated into our lives (Bucknall, 2022; Kasirzadeh, 2025; Kolt, 2024).

2.4. Why is diversity complex?

Diversity is complex (Mironov & Prokhorenkova, 2025) because it is always only meaningful in relation to a **context** (Peeperkorn et al., 2025). Indeed, all entropy is actually relative (Leinster, 2024). This suggests that **we need to be explicit about the context with a sufficient level of nuance**.

Most existing techniques to increase diversity in LLM outputs overlook context, and often fail in practice. For instance, increasing *temperature* increases *incoherence* more than *novelty* (Peeperkorn et al., 2024), limiting usefulness before hitting *text degeneration* (Lee et al., 2025). Despite hyperparameter tuning, *homogeneity bias* is persistent and particularly affects minoritized groups (Lee, 2025). In addition, advanced prompting techniques (which have been effective for reasoning tasks) do not help increase creativity in outputs (Morain & Ventura, 2025).

Not only do we lack reliable ways to increase the diversity of LLM output, but current practices are actively reducing it. Recent literature (Murthy et al., 2025; West & Potts, 2025; Meng et al., 2024) has shown that *alignment* degrades the capabilities of LLMs related to output diversity. The trade-offs introduced by alignment are only now coming into focus (Feng et al., 2025), but it is becoming increasingly clear there is a narrowing of the *generative horizon* (Feng et al., 2025).

2.5. Diverse how, anyway?

Recent work challenges the assumption that hallucinations are always *problematic* or *undesirable* (Yuan et al., 2025; Sun et al., 2025). Since diversity is *task-dependent* (Jain et al., 2025), **what counts as a hallucination is rather a *prescription***.

Indeed, many formalisms (Li et al., 2025) take a *normative* (Sui et al., 2024) approach to defining hallucinations, such as formulating the binary classification problem "*Is it Valid?*" (Kalai et al., 2025). However, we recognize that there are many ways for a model to hallucinate (Huang et al., 2025; Cossio, 2025), and we advocate for sufficiently expressive formalisms[6].

2.6. What do we want from the future?

From the foregoing discussion, we conclude that, to promote diversity, our desired strategy should guide our GenAI to:

- **Be queer**[7]: *Diverge* into the long tails of reality.
- **Center the subaltern**[8]: Take special *care* for the traces of the minoritized, which are rendered invisible by structural inequity and power.
- **Explore intentionally and explicitly**: Specify the *context* for diversity. Spell out if anything should be conserved or avoided during exploration.

3. Theoretical Framework

3.1. LLMs as trees of strings

Let $\{t_a, t_b, \ldots\}$ denote the finite token alphabet, with special tokens $\perp$ (start-of-sequence) and $\top$ (end-of-sequence). A **string** is a finite sequence of tokens beginning with $\perp$; a **trajectory** is a string ending with $\top$. We write prompts, continuations, and trajectories as:

$$\begin{aligned} x_p &= \perp t_1 \ldots t_p \\ x_{p+k} &= x_p t_{p+1} \ldots t_{p+k} \\ y = x_T &= x_{T-1} \top \end{aligned}$$

We denote the set of all strings by Str, the set of all trajectories by $\mathsf{Str}_\top \subset \mathsf{Str}$, and the sets of continuations and trajectories for a given prompt[9] x_p by $\mathsf{Str}(x_p)$ and $\mathsf{Str}_\top(x_p)$, respectively.

Any LLM induces a tree on Str: the root is $\perp$, each node is a string, the leaves are trajectories, and the edges connect strings to their next-token continuations with probability $p(t_{p+1}|x_p)$. Probabilities chain and decompose as $p(y|x_p) = p(y|x_{p+k})p(x_{p+k}|x_p)$. For any prompt x, we

[6] To paraphrase Eugenia Cheng (Cheng, 2022), abstraction is about making precise the different senses in which different things can be valid.

[7] We adopt *critical theory* language because technology is outpacing traditional concepts (Hadfield, 2023), and stale language fails to make the impacts of our theorizations explicit. A **theory with *teeth***, one that is attuned to real stakes (Saketopoulou, 2023), must remain *ground-bound* (Bettcher, 2025), foregrounding minoritized people rather than disembodied abstractions. Would it not be a bit silly/naive (at best) if we tried to "solve diversity" and did not engage (even if just in spirit) with the academic fields that explicitly study social bias? (e.g., Queer Theory, Postcolonial Studies, Black Studies, etc.).

[8] We characterize this desideratum as a type of *fairness* (Verma & Rubin, 2018). To increase diversity, we naturally seek to achieve structural *parity*. However, we also incorporate more *justice-oriented* notions of fairness (Rawls, 1971; Mittelstadt et al., 2023): **Interventions shall maximally benefit the least advantaged.**

[9] The case of "no prompt" corresponds to $x_p = \perp$.

have a *probability mass function*[10] on the trajectories for *any* particular prompt (Bradley & Vigneaux, 2025):

$$\sum_{y \in \mathsf{Str}_\top(x_p)} p(y|x_p) = 1 \tag{1}$$

3.2. Structure-awareness

We propose an abstract language that distinguishes among the different contexts in which we discuss diversity. We define **structure** as the *specification of a type of organization among the tokens of a string*.

For a string $x \in \mathsf{Str}$, the degree of **structure compliance** is $\alpha_i(x)$. *Ideal compliance* corresponds to $\alpha_i(x) = 1$, and *no compliance* corresponds to $\alpha_i(x) = 0$.

$$\alpha_i : \mathsf{Str} \to [0, 1] \tag{2}$$

We can consider many structures simultaneously. We call a **system** the collection of structures of interest. We define the **system compliance** as a *vector of compliances across particular structures*.

$$\Lambda_n(x) := (\alpha_1(x), \ldots, \alpha_n(x)) \tag{3}$$

To enable easy comparisons, we define operators[11] that aggregate compliance into scalar **system scores** and **difference scores**:

$$\|\Lambda_n(x)\|_\Lambda, \ \|\Lambda_n(x_r) - \Lambda_n(x_q)\|_\theta \ \in \ [0, 1] \tag{4}$$

3.3. Incorporating string statistics

For a given structure and an LLM, we can reason about its *expected structural compliance*. We call this the **structure core**:

$$\langle \alpha_i \rangle = \sum_{y \in \mathsf{Str}_\top} p(y) \alpha_i(y) \tag{5}$$

Similarly, we can reason about the *expected system compliance* as the **system core**:

$$\langle \Lambda_n \rangle = \sum_{y \in \mathsf{Str}_\top} p(y) \Lambda_n(y) \tag{6}$$

Leveraging these definitions, we can reason about the *deviation from the expected system compliance*. This would constitute a *set of deviations*, one for each structure. The **orientation** (Ahmed, 2006) of a given string relative to the given system core is:

$$\theta_n(x) = \Lambda_n(x) - \langle \Lambda_n \rangle \tag{7}$$

We can think of orientation as a characterization of *queerness* (Jedrusiak, 2024) for a string. If the system core tells us what is *normatively* complied with, orientations tell us in what ways a string is *non-normative*. Our framework is *expressive* because it allows us to think about **diversity per structure**.

To summarize *non-normativity* as a single number, we leverage Equation 4 to define the **deviance**:

$$\|\theta_n(x)\|_\theta = \partial_n(x) \in [0, 1] \tag{8}$$

3.4. What about prompting?

We can generalize our framework to account for prompts by making explicit the **conditioning on a given prompt** x_p:

$$\begin{aligned} \langle \alpha_i \rangle(x_p) &= \sum_{y \in \mathsf{Str}_\top(x_p)} p(y|x_p) \alpha_i(y) \\ \langle \Lambda_n \rangle(x_p) &= \sum_{y \in \mathsf{Str}_\top(x_p)} p(y|x_p) \Lambda_n(y) \\ \theta_n(x|x_p) &= \ \Lambda_n(x) \ - \ \langle \Lambda_n \rangle(x_p) \end{aligned} \tag{9}$$

The conditional probabilities under different prompts may differ substantially. Different prompts collapse to different modes (Zhang et al., 2025a). **We can think that a given prompt induces its *own normativity*.**

3.5. Dynamics of relative diversity

As noted in the last section, what is *non-normative* is *conditional on what came before*. Then, as a string is being completed, the set of possible trajectories is narrowed so the system core and orientations change. Trajectories that were essentially *unreachable* from the root of the tree may emerge as *attractors* once we condition on a specific *subtree*.

Given a trajectory $y = x_T$, for $k \in \{0, 1, \ldots, T\}$, we can define **states** for all the *intermediate continuations*:

$${}^x\phi_k {=} \langle \Lambda_n \rangle(x_k) \quad {}^y\phi_k {=} \theta_n(x_k|\bot) \quad {}^z\phi_k {=} \theta_n(y|x_k) \tag{10}$$

which form a discrete-time **dynamics**:

$$({}^x\phi_0, {}^y\phi_0, {}^z\phi_0) \to \ldots \to ({}^x\phi_T, {}^y\phi_T, {}^z\phi_T)$$

The state ${}^x\phi$ evolves from representing the expected system compliance of all possible continuations at ${}^x\phi_0 = \langle \Lambda_n \rangle(\bot)$, to the specific system compliance of a given trajectory at ${}^x\phi_T = \langle \Lambda_n \rangle(y) = \Lambda_n(y)$.

The state ${}^y\phi$ encodes how much the current path has *deviated* from normativity, evolving from a *zero deviance*[12] at ${}^y\phi_0 = \Lambda_n(\bot) - \langle \Lambda_n \rangle(\bot)$ to the full trajectory's orientation in the largest frame of reference at ${}^y\phi_T = \Lambda_n(y) - \langle \Lambda_n \rangle(\bot)$.

[10] We assume all *terminal* strings *finish* within a *finite context window*. Refer to (Bradley & Vigneaux, 2025) for full theoretical framework of LLMs are trees of strings.

[11] While system compliance is formulated as a *vector*, this generalizes to other structures with appropriate operators. See Appendix B.

[12] A zero deviance is when an orientation has a deviation value of zero for all structures.

The state ${}^{z}\phi$ evolves from representing how *deviant* the trajectory is in the largest frame of reference at ${}^{z}\phi_0 = {}^{y}\phi = \Lambda_n(y) - \langle\Lambda_n\rangle(\perp)$, to a *zero deviance* at ${}^{z}\phi_T = 0 \quad .$

3.6. Normative orders

We notice that our framework allows us to define interesting *preorders*. For a fixed system, LLM and prompt, we can rank strings by how deviant they are, and also rank structures by how often strings comply with them:

$$\begin{aligned} x_a \preceq_{\partial_n} x_b \quad &\iff \quad \partial_n(x_a) \leq \partial_n(x_b) \\ \alpha_i \preceq_{\langle\cdot\rangle} \alpha_j \quad &\iff \quad \langle\alpha_i\rangle \leq \langle\alpha_j\rangle \end{aligned} \tag{11}$$

4. Homogenization

We can consider the *expected deviance* and the *deviance variance*:

$$\begin{aligned} \mathbb{E}_{y\sim p(\cdot|x_p)}[\partial_n] &= \sum_{y\in\mathsf{Str}_\top(x_p)} p(y|x_p)\; \partial_n(y|x_p) \\ \mathrm{Var}_{y\sim p(\cdot|x_p)}[\partial_n] &= (\mathbb{E}[\partial_n^2] - \mathbb{E}[\partial_n]^2)_{y\sim p(\cdot|x_p)} \end{aligned} \tag{12}$$

Then, we can see homogenization as **minimizing all deviance**:

$$\mathbb{E}_{y\sim p(\cdot|x_p)}[\partial_n] \;\mapsto\; 0 \qquad \mathrm{Var}_{y\sim p(\cdot|x_p)}[\partial_n] \;\mapsto\; 0 \tag{13}$$

Given a system core $\langle\Lambda_n\rangle$, we can normalize its structures as $\langle\overline{\alpha}_i\rangle := \frac{\langle\alpha_i\rangle(x_p)}{\sum_j^n \langle\alpha\rangle_j(x_p)}$. Then, we can compute the *core entropy*:

$$H(\langle\Lambda_n\rangle) = -\sum_{i=1}^{n} \langle\overline{\alpha}_i\rangle \log(\langle\overline{\alpha}_i\rangle) \tag{14}$$

Then, we can also think of homogenization as **making the system core more *uneven***. When the core has low entropy, fewer structures dominate:

$$H(\langle\Lambda_n\rangle) \;\mapsto\; 0 \tag{15}$$

5. Xeno-reproduction

To satisfy our desiderata, we propose a **structure-aware diversity pursuit**. We conceptualize this fundamentally as a *non-objective search* (Lehman & Stanley, 2011), *optionally* augmented with *fairness-oriented biases* and *explicit constraints*.

We present two complementary formulations. The *distribution-level formulation* accounts for how interventions shape the entire probability landscape. The *trajectory-level formulation* reinterprets distribution-level scores as reward signals for individual output trajectories. Both formulations share the same underlying values but differ in their computational affordances.

5.1. Distribution-level formulation

We *score* interventions through the *intervention* variable w that encompasses any[13] mechanism affecting the effective distribution of trajectories. We write w_0 for the *unintervened conditions* (the baseline).

5.1.1. Scoring diversity

We would like to evaluate how much more *diversity-seeking* our choice of w is compared to the baseline.

On the one hand, we can think of promoting diversity as inducing a new core that is different from the old one:

$$\mathsf{score}_{\mathsf{explore}}(w) = \|\langle\Lambda_n\rangle(w) - \langle\Lambda_n\rangle(w_0)\|_\theta \tag{16}$$

On the other hand, the new core should not be excessively *dominant*. We can think of promoting diversity as guiding output strings to *diverge* from any system core, and also be deviant *in their own way*:

$$\mathsf{score}_{\mathsf{diverge}}(w) = \mathbb{E}[\partial_n](w) + \mathrm{Var}[\partial_n](w) \tag{17}$$

Our **diversity score** ρ_d would then be the sum:

$$\rho_d(w) = \mathsf{score}_{\mathsf{explore}}(w) + \mathsf{score}_{\mathsf{diverge}}(w) \tag{18}$$

5.1.2. Scoring fairness

We would like to evaluate how much our choice of w inverts the normative ordering of the structure cores induced by w_0. To do so, we can leverage the *relative-order* sign:

$$s_{i,j}(w) = \mathrm{sign}(\langle\alpha_i\rangle(w) - \langle\alpha_j\rangle(w)) \tag{19}$$

We can score the *invertedness* of the *normative order* (Equation 11) as:

$$\mathsf{score}_{\mathsf{inverted}}(w) = \binom{n}{2}^{-1} \sum_{1\leq i<j\leq n} \mathbf{1}[s_{i,j}(w) \neq s_{i,j}(w_0)] \tag{20}$$

We also would like to evaluate how *even* the system core is:

$$\mathsf{score}_{\mathsf{even}}(w) = H(\langle\Lambda_n\rangle(w)) \tag{21}$$

Our **fairness score** ρ_f would then be the sum:

$$\rho_f(w) = \mathsf{score}_{\mathsf{inverted}}(w) + \mathsf{score}_{\mathsf{even}}(w) \tag{22}$$

5.1.3. Scoring adherence to constraints

To be explicit and intentional, we need to consider *constraints* (Eguchi, 2024). We can define systems that prescribe the structures that we would like to *target*, *conserve*

[13] We consider anything that depends on $p(y|x_p, w)$ to be parameterized by w as well. For instance, $\langle\Lambda_n\rangle$ would be parametrized as $\langle\Lambda_n\rangle(x_p, w)$, but for readability we just write $\langle\Lambda_n\rangle(w)$, folding the prompting into the interventional variable.

and *avoid*. We would like to score how much our choice of w affects the adherence to those constraints. Our **constraint score** ρ_c would be:

$$\begin{aligned}\rho_c(w) = \|\langle\Lambda_{\text{target}}\rangle(w)\|_\Lambda - \|\langle\Lambda_{\text{avoid}}\rangle(w)\|_\Lambda \\ - \|\langle\Lambda_{\text{conserve}}\rangle(w) - \langle\Lambda_{\text{conserve}}\rangle(w_0)\|_\theta\end{aligned} \tag{23}$$

5.1.4. Xeno-reproduction as search over interventions

The **intervention score** ρ_χ is a λ-weighted sum:

$$\rho_\chi(w) = \lambda_d \rho_d(w) + \lambda_f \rho_f(w) + \lambda_c \rho_c(w) \tag{24}$$

We formulate *xeno-reproduction* as the ***search over interventions***:

$$w \sim \pi(w) \;\propto\; e^{\rho_\chi(w)} \tag{25}$$

By sampling the intervention variable and applying it, we generate trajectories:

$$\mathbb{E}_{w\sim\pi(w)}[p(y|w)] = \int \pi(w)\, p(y|w)\, dw \tag{26}$$

5.2. Trajectory-level formulation

The trajectory-level formulation offers a complementary perspective that assigns *rewards* to individual outputs:

$$\begin{aligned}
r_d(y|x_p) &= \partial_n(y|x_p) \\
r_f(y|x_p) &= \sum_i^n \mathsf{v}_i \alpha_i(y) \qquad \mathsf{v}_i \propto (\langle\alpha_i\rangle(x_p))^{-1} \\
r_c(y|x_p) &= \sum_{t\in\text{target}} \alpha_t(y) - \sum_{a\in\text{avoid}} \alpha_a(y) - \sum_{c\in\text{conserve}} |\alpha_c(y) - \langle\alpha_c\rangle(x_p)|
\end{aligned} \tag{27}$$

The **stay reward** is:

$$r_\chi(y|x_p) = \lambda_d r_d(y|x_p) + \lambda_f r_f(y|x_p) + \lambda_c r_c(y|x_p) \tag{28}$$

We formulate *xeno-reproduction* as the ***search over trajectories***:

$$p(y|x_p, w) \propto p(y|x_p w_0)\, e^{r_\chi(y|x_p)} \tag{29}$$

The trajectory-level reward provides a sample-based *approximation* to the distribution-level strategy, enabling more tractable implementations.

6. Theoretical Results

Our framework opens several avenues for theoretical investigation. In this section, we highlight an initial result that reveals a fundamental tension in diversity-seeking interventions.

Theorem 6.1 (*Informal*, Diversity-Fairness Trade-off)**.** *The intervention that maximizes diversity is not the one that maximally uplifts underrepresented structures. No single intervention optimally serves both.*

See Appendix A for the formal statement and proof. This trade-off establishes that the choice of weights (λ_d, λ_f) in the combined objective ρ_χ encodes a value judgment about the relative priority of diversity versus fairness. Our framework makes this tension explicit.

Beyond this result, our structure-aware language admits natural generalizations and connects to existing theory. Appendix B develops generalized versions of cores and deviances, showing how different parameter choices reflect different viewpoints on diversity. Appendix C shows that hallucination frameworks and language generation theory can be recast within our vocabulary, suggesting a potential for *theoretical unification*.

7. Related Work

Xeno-reproduction immediately steps into conversation with ***Active Divergence*** (Berns et al., 2023; Broad et al., 2021; Berns, 2025; Berns & Colton, 2020; Tahiroglu & Wyse, 2024; Esling et al., 2022; Cole et al., 2025), as they both aim to *disorient* (Ahmed, 2006). Whereas Active Divergence focuses on maximizing raw *novelty* in artistic contexts, xeno-reproduction addresses homogenization and emphasizes context through *structures*. While Active Divergence work overlaps with *Computational Creativity*, xeno-reproduction is oriented towards AI safety.

Xeno-reproduction will seek the help of *Interpretability* to understand how structures relate to the models' internals. At a more foundational layer, they also come together to understand **Representation Bias**[14].

Reinforcement Learning (RL) and xeno-reproduction both leverage exploration. To improve LLM reasoning, exploration is leveraged during training (Song et al., 2025) and prompting (Yao et al., 2023). The ideas in search algorithms, such as AlphaSAGE (Chen et al., 2025) and **Quality-Diversity** (Pugh et al., 2016), are promising directions for xeno-reproduction.

8. Limitations and Future Directions

As we mentioned earlier, diversity is complex. Our framework is not complete; it is a starting point. Significant collaboration will be required to address homogenization effectively[15]. We have several notes outlining directions to extend this line of work to overcome current limitations.

[14] *Representation Bias* is the phenomenon when signals end up being represented more strongly, more reliably, or more prominently in the internal representations than others, even when, from a functional or computational perspective, those features are equally relevant. (Lampinen et al., 2024; 2025)

[15] We are very interested in collaborating with other researchers concerned about diversity. Don't hesitate to reach out.

Specification of structures. This paper has raised many questions about structures. The choice of structures to consider is always *opinionated*. However, we can still ask meaningful questions about the *structure between structures* and the *substructures* within a structure. We need a taxonomy of the types of structure we could consider, specifying how compliance could be estimated. Moreover, we hope to align our framework with emerging research in *computational learning theory* and *language generation* that formalizes the trade-offs associated with hallucinations [16] (Kalavasis et al., 2025b).

Computational tractability. Calculating the system core exactly requires summing over $y \in \mathsf{Str}_{\top}(x_p)$, which is intractable. To address this, we need to develop tractable, efficient approximation methods, possibly leveraging smart sampling (Macar et al., 2025), the structures of interest, or carefully designed prompting (Zhang et al., 2025a).

Operationalizing the xeno-reproduction. Our formalization of the xeno-reproduction strategy is one of many possible ones. We want to invite more researchers to reflect on the desiderata for diversity (against homogenization) and to propose their own formulations of xeno-reproduction. In particular, we are interested in formulations that operationalize it in a tractable and readily applicable way.

Connecting to evaluations. We would also like to understand how the current diversity evaluations (Jiang et al., 2025; Zhang et al., 2025b) are re-conceptualized from the perspective of cores and orientations.

Investigation of dynamics. Tracking how cores and orientations evolve could help us understand how LLMs explore solutions and deal with ambiguity. Certain words in a sentence may act as "branching points" where the dynamics bifurcate dramatically. Identifying these could reveal where diversity is most at stake during generation. Eventually, we could apply this to real-time *Chain-of-Thought monitoring* (Korbak et al., 2025).

Ethical Analysis. Our framework raises unresolved tensions. *Who should define the structures of interest?* Community participation is needed so that the right type of diversity is considered. *Is it always beneficial to make the traces more visible?* Minoritized populations sometimes prefer *opacity* as protection. Consent-based approaches are needed to ensure our methods do not cause harm.

9. Alternative views

Skepticism of technical solutions to diversity. Some authors point out (Wachter et al., 2021; Davis & Williams, 2025; Green & Viljoen, 2020) that technical interventions might not be appropriate for what (at its core) is a social justice and inequity problem. Better interventions could alternatively focus on institutional change, community participation, or even stopping AI development altogether (Goldfarb, 2024) to protect the types of diversity that we care about. We recognize that xeno-reproduction could fall into the *solutionism trap* (Selbst et al., 2019). We still believe that technical solutions are worth considering alongside other interventions.

Diversity can be *risky*. The type of open-ended search promoted by xeno-reproduction comes with risks. Some authors (Sheth et al., 2025) have raised concerns about *unpredictability*, *uncontrollability*, and *misalignment*. However, we remain hopeful that we can promote diversity responsibly. The open-endedness afforded by diversity could ultimately make AI safety *antifragile* (Hughes et al., 2024; Taleb, 2013).

10. Conclusion

This paper presents a case for diversity and identifies xeno-reproduction as an strategy that intentionally promotes it. This paper also presents an expressive framework for accounting for the structures of strings and their corresponding statistics. This is just an initial step towards scholarships that seriously theorize diversity and foreground its impact on people at the margins.

Call to action

In this paper, we call for AI Safety:

- To integrate homogenization into threat models and evaluations, expand theoretical and empirical work on diversity, and propose serious interventions.
- To be explicit on what context diversity is being defined in, and attempt to give sufficient nuance in conceptualizations.
- To be sincerely committed to *pluralism*, and engage with perspectives from *critical theory* such as Queer theory, Black studies, and Postcolonial studies.

Impact Statement

This paper introduces abstractions and a formal framework to center diversity in AI Safety. However, there are important risks. **The same methods that aim to amplify diversity could be used to squash, exploit, and control it.** Additionally, any formalization of diversity also risks reproducing the exclusions we aim to address.

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[16] See Appendix C for discussion.

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Appendix A. Trade-off between diversity and fairness

In this appendix, we show the fundamental tension between diversity and fairness by proving the existence of a *Pareto* trade-off between them. A Pareto trade-off says that among *efficient solutions*, improvement on one objective *necessarily worsens* the other (Ehrgott, 2005). To establish such a trade-off, it suffices to exhibit two efficient solutions, each of which is better than the other on a different objective. This shows that no single solution can *dominate* both.

Taking into account ρ_d and ρ_f defined as in Equation 18 and Equation 22:

Definition A.1 (Pareto Dominance)**.** An intervention w *Pareto-dominates* intervention w' if $\rho_d(w) \geq \rho_d(w')$ and $\rho_f(w) \geq \rho_f(w')$ with at least one strict inequality.

Theorem A.2 (Trade-off Between Diversity and Fairness)**.** *Let $n \geq 2$ and let w_0 induce a non-uniform baseline core with $\langle \alpha_n \rangle(w_0) < 1/n < \langle \alpha_1 \rangle(w_0)$. Then there exist interventions w_d, w_f such that neither Pareto-dominates the other:*

$$\rho_d(w_d) > \rho_d(w_f) \quad \textit{and} \quad \rho_f(w_d) < \rho_f(w_f) \tag{A.1}$$

This demonstrates the existence of a fundamental trade-off *between the two objectives.*

Proof. We construct two interventions that exhibit opposite strengths. For simplicity, we assume ***deterministic*** **generation**; adding stochasticity would only increase $\mathsf{score}_{\mathsf{diverge}}$ and strengthen the trade-off. We also say w_0 induces a non-uniform system core $\langle \Lambda_n \rangle(w_0) = (\mu_1, \ldots, 0, \mu_n)$ and assume $\| \cdot \|_\theta$ is the Euclidean norm $\| \cdot \|_2$

Let w_d induce $\langle \Lambda_n \rangle(w_d) = (0, \ldots, 0, 1)$. Then:

$$\begin{aligned}
\mathsf{score}_{\mathsf{explore}}(w_d) &= \sqrt{(1-\mu_n)^2 + \sum_{i=1}^{n-1} (\mu_i)^2} > 0 \\
\mathsf{score}_{\mathsf{diverge}}(w_d) &= 0 \qquad \text{(deterministic)} \\
\mathsf{score}_{\mathsf{even}}(w_d) &= H(0, \ldots, 0, 1) = 0 \\
\mathsf{score}_{\mathsf{inverted}}(w_d) &= 1 \quad \text{(all pairwise orderings change)}
\end{aligned}$$

Thus:

$$\rho_d(w_d) = \mathsf{score}_{\mathsf{explore}}(w_d) > 0 \qquad \rho_f(w_d) = 1$$

Let w_f induce $\langle \Lambda_n \rangle(w_f) = (1/n, \ldots, 1/n)$. Then:

$$\begin{aligned}
\mathsf{score}_{\mathsf{explore}}(w_f) &= \sqrt{\sum_{i=1}^{n} (1/n - \mu_i)^2} > 0 \\
\mathsf{score}_{\mathsf{diverge}}(w_f) &= 0 \quad \text{(deterministic)} \\
\mathsf{score}_{\mathsf{even}}(w_f) &= H(1/n, \ldots, 1/n) = \log n \\
\mathsf{score}_{\mathsf{inverted}}(w_f) &= 1 \quad \text{(all orderings flatten)}
\end{aligned}$$

Thus:

$$\rho_d(w_f) = \mathsf{score}_{\mathsf{explore}}(w_f) > 0 \qquad \rho_f(w_f) = 1 + \log n$$

We now check if $\mathsf{score}_{\mathsf{explore}}(w_d) > \mathsf{score}_{\mathsf{explore}}(w_f)$. We assume the inequality and verify if it holds:

$$\begin{aligned}
\text{assuming :} \quad & \mathsf{score}_{\mathsf{explore}}(w_d) > \mathsf{score}_{\mathsf{explore}}(w_f) \\
\text{squaring :} \quad & (1-\mu_n)^2 + \sum_{i=1}^{n-1} (\mu_i)^2 > \sum_{i=1}^{n} (1/n - \mu_i)^2 \\
\text{rearranging :} \quad & \sum_{i=1}^{n-1} \mu_i > (n-1)\left(\mu_n - \tfrac{1}{2}\right)
\end{aligned}$$

Since $\mu_n < 1/n \leq 1/2$ for $n \geq 2$, the right-hand side is negative. Since $\mu_1 > 1/n > 0$ and $\mu_i \in [0, 1]$, the left-hand side is positive. Thus, $\mathsf{score}_{\mathsf{explore}}(w_d) > \mathsf{score}_{\mathsf{explore}}(w_f)$ holds. Since $\mathsf{score}_{\mathsf{diverge}}(w_0) = \mathsf{score}_{\mathsf{diverge}}(w_f) = 0$, we conclude $\rho_d(w_d) > \rho_d(w_f)$.

By noting $1 < 1 + \log n$ for $n \geq 2$, we also conclude $\rho_f(w_d) < \rho_f(w_f)$.

Since $\rho_d(w_d) > \rho_d(w_f)$ and $\rho_f(w_d) < \rho_f(w_f)$, neither intervention dominates the other. □

Corollary A.3 (Weight choice encodes value judgment)**.** *The choice of weights (λ_d, λ_f) in the combined objective $\rho_\chi = \lambda_d \rho_d + \lambda_f \rho_f$ reflects an irreducible value judgment about the relative priority of diversity versus fairness.*

Appendix B. Implementing generalized diversities

Our structure-aware language is intentionally *abstract* so it **admits multiple implementations**, not only the one we presented in the main paper. In this appendix, we think through two alternative choices:

1. Generalization of the structure core through the *escort power mean*
2. Reinterpretation of the deviance as *relative entropy*

Our goal with this appendix is to **inspire reflection** on diversity *beyond* what was explicitly presented in our framework.

B.1. Generalizing the *structure core*

Inspired by *value measures* (Leinster, 2024) and *escort distributions* (Bercher, 2011), we generalize the structure core as the ***escort power mean***:

$$\langle\alpha_{i(q,r)}\rangle(x_p) = \left(\frac{\sum_{y\in\mathsf{Str}_\top(x_p)} p(y|x_p)^r \alpha_i(y)^q}{\sum_{y\in\mathsf{Str}_\top(x_p)} p(y|x_p)^r}\right)^{1/q} \tag{B.1}$$

We simplify by considering the *escort distribution*:

$$p_{(r)}(y|x_p) = \frac{p(y|x_p)^r}{\sum_{y\in\mathsf{Str}_\top(x_p)} p(y|x_p)^r} \tag{B.2}$$

Then, the ***generalized structure core*** is:

$$\langle\alpha_{i(q,r)}\rangle(x_p) = \left(\mathbb{E}_{y\sim p_{(r)}(\cdot|x_p)}[\alpha_i(y)^q]\right)^{1/q} \tag{B.3}$$

When $q = 1$ and $r = 1$, the generalized structure core *recovers* our original structure core in Equation 5 and Equation 9. Different values for q, r give us alternative interesting cores. For instance:

$$\begin{aligned}
\langle\alpha_{i(1,0)}\rangle(x_p) &= \frac{1}{|\mathsf{Str}_\top(x_p)|}\sum_{y\in\mathsf{Str}_\top(x_p)}\alpha_i(y)\\
\langle\alpha_{i(1,\infty)}\rangle(x_p) &= \alpha_i(\arg\max_y p(y|x_p))\\
\langle\alpha_{i(\infty,1)}\rangle(x_p) &= \max_{y\in\mathrm{supp}(p(\cdot|x_p))}\alpha_i(y)\\
\langle\alpha_{i(-\infty,\infty)}\rangle(x_p) &= \min_{y\in\mathrm{modes}(p(\cdot|x_p))}\alpha_i(y)
\end{aligned}$$

For a given structure α_i, we can think of q selecting whether large or small compliance values dominate, and r selecting whether the *large body* or *long-tails* of $p(\cdot|x_p)$ dominate. **By parameterizing, we make *transparent* how we weigh rarity, signal strength, and balance**. Since different parameters reflect different *viewpoints* (Leinster, 2024), we shall always consider a full *diversity profile* before drawing conclusions about how our interventions impact diversity.

B.2. Reinterpreting *deviance*

We can think of a ***generalized orientation*** as:

$$\theta_{n,k}(y|x_p) = \mathsf{orient}\left(\Lambda_n(y), \langle\Lambda_n\rangle(x_p)\right) \tag{B.4}$$

with $\mathsf{orient} : [0,1]^n \times [0,1]^n \to [0,1]^k$.

Then, the ***generalized deviance*** is:

$$\begin{gathered}
\partial_{n,k}(y|x_p) = \|\theta_{n,k}(y|x_p)\|_{\mathsf{orient}}\\
\|\cdot\|_{\mathsf{orient}} : [0,1]^k \to \mathbb{R}^+
\end{gathered} \tag{B.5}$$

If we choose $\mathsf{orient}(\Lambda_x, \Lambda_y) = \Lambda_x - \Lambda_y$ and $\|\cdot\|_{\mathsf{orient}} = \|\cdot\|_\theta$, we *recover* our original deviance in Equation 8 and Equation 9.

For **relative entropy**, we consider the **Rényi entropy** defined (Leinster, 2024) as:

$$H_q(\mathbf{p}\,\|\,\mathbf{r}) = \frac{1}{q-1}\log\sum_{i\in\mathrm{supp}(\mathbf{p})} p_i^q\, r_i^{1-q} \tag{B.6}$$

Then, we can think of a *dummy* $\mathsf{orient}()$ that just stores Λ_x, Λ_y and a $\|\cdot\|_{\mathsf{orient}}$ operator that computes the *relative entropy* between them. For a given *normalized* core $\overline{\Lambda}_n = \{\langle\overline{\alpha}\rangle_1, ...\}$ and *normalized* system $\overline{\Lambda}_n = \{\overline{\alpha}_1, ...\}$, we define two *Hill number* (Leinster, 2024) deviances: the ***excess* deviance** and ***deficit* deviance**:

$$\partial_q^+(y, x_p) = e^{H_q(\overline{\Lambda}_n(y)\,\|\,\langle\overline{\Lambda}_n\rangle(x_p))} \tag{B.7}$$

$$\partial_q^-(y, x_p) = e^{H_q(\langle\overline{\Lambda}_n\rangle(x_p)\,\|\,\overline{\Lambda}_n(y))} \tag{B.8}$$

We could read ∂_q^+ as the *effective* **over-compliance** and ∂_q^- as the *effective* **under-compliance** with respect to the *normative compliance*.

For instance, as $q \to \infty$, we interpret:

- ∂_∞^+ as the largest *excess of compliance*

$$\partial_\infty^+ = \max_i \frac{\overline{\alpha}_i(y)}{\langle\overline{\alpha}_i\rangle(x_p)}$$

- ∂_∞^- as the largest *deficit of compliance*

$$\partial_\infty^- = \max_i \frac{\langle\overline{\alpha}_i\rangle(x_p)}{\overline{\alpha}_i(y)}$$

All of this to say, there are **multiple ways we can reason about *structures* and *statistics* jointly**. We encourage readers to develop alternative and competing formalisms that share our conceptual backbone: *structures* that make *context* explicit, *cores* that encode the normativity that *homogenization* push us toward, and *orientations* that capture perspectives of *non-normativity*. Above all, **we ask everyone to think deeper about diversity**.

Appendix C. Theoretical touchpoints

In this appendix, we explore how our theoretical framework connects to other frameworks. To that purpose, we consider an *unprompted* scenario of a *singleton* system with *binary* compliance for its single structure:

$$\Lambda_*(x) := (\alpha_*(x)) \qquad \alpha_*(x) \in \{0, 1\}$$

Then, the structure core represents the probability of compliance being exactly 1:

$$\mu := \langle \alpha_* \rangle = \sum_{c \in \{0,1\}} c \, \Pr(\alpha = c) = \Pr(\alpha = 1)$$

Our singleton deviance is expressed as:

$$\partial_*(x) = \|\alpha_*(x) - \mu|\|_\theta$$

C.1. Expected deviance and Gini-Simpson index

To calculate the *expected deviance*, we consider two choices for $\| \cdot \|_\theta$: absolute value and the squared ℓ_2 norm. For each, we find connections between $\mathbb{E}[\partial_*]$ and the *Gini-Simpson index* for a binary variable:

$$\mathbb{E}[|\alpha_* - \mu|] = 2\mu(1 - \mu) = \mathrm{GS}$$

$$\mathbb{E}[\|\alpha_* - \mu\|_2^2] = \mathrm{Var}[\alpha_*] = \mu(1 - \mu) = \frac{\mathrm{GS}}{2}$$

If we interpret GS as the *degree of mixing* in outcomes, then increasing the expected deviance drives *heterogeneity* rather than *concentration*.

C.2. Is-It-Valid classification for Hallucinations

To reason about hallucinations, authors in (Kalai et al., 2025) partition the space of *plausible* outputs into disjoint sets of *valid outputs* V and *errors* E. In their framework, a model *hallucinates* when it cannot solve the binary discrimination problem *Is-It-Valid?* (IIV). Their framework can be interpreted through our structure-aware language:

$$\alpha_{\mathrm{IIV}}(x) = \mathbf{1}[x \in V]$$

We can connect their generative hallucination rate given by $\mathrm{err} = \Pr_{x \sim \hat{p}}[x \in E] = \hat{p}(E)$ to the system core of a singleton IIV system:

$$\langle \alpha_{\mathrm{IIV}} \rangle = 1 - \mathrm{err}$$

The paper (Kalai et al., 2025) points out that future work should "consider degrees of hallucination". Our structure-aware framework provides the language to reason about these desired **graded notions of hallucination**: We can score a string under multiple structures, with scores encoding real-valued nuance *beyond the binary*.

C.3. Language Generation in the Limit

Recent work (Kleinberg & Mullainathan, 2024; Kalavasis et al., 2025a) studies language generation where a generator G, given strings from an unknown target language K, must output strings that are both **novel** and **valid**. We can reinterpret some of their framework as a special case of our structure-aware formulation.

Given a language collection $\mathcal{L} = \{L_1, L_2, \ldots\}$, we can define *membership structures* with corresponding cores that represent the probability of generating a string valid for each corresponding language:

$$\alpha_{L_i}(x) = \mathbf{1}[x \in L_i] \qquad \langle \alpha_{L_i} \rangle = \Pr[y \in L_i]$$

The literature is currently (Kalavasis et al., 2025b) exploring the trade-offs between *consistency* and *breadth*. An LLM generates strings *consistent* with our target language K if:

$$\langle \alpha_K \rangle = 1 \qquad \text{when} \qquad \mathbb{E}[\partial_K]_{y \sim p_{\mathsf{LLM}}} \to 0$$

An LLM generation has *breadth* when all strings of our target language $K \in \mathcal{L}$ can be generated:

$$\forall\, y \in K: \; p_{\mathsf{LLM}}(y) > 0 \iff K \subseteq \mathrm{supp}(p_{\mathsf{LLM}})$$

Our structure-aware framework gives us insight that homogenization is *relative to a system*. Indeed, pushing for consistency shall not imply that we push for homogenization in every context. Generally, for $\Lambda_K \neq \Lambda_m$:

$$\mathbb{E}[\partial_K] \to 0 \quad \neq \quad \mathbb{E}[\partial_m] \to 0$$

Thinking explicitly through structures and systems allows us to formulate *interesting* questions (for instance, is $\Lambda_K = \Lambda_{\mathrm{IIV}}$?) that will help us make connections between all these theoretical efforts. We present these touchpoints as **starting points for deeper exploration**.