Epidemiological model on macro level

Modelling and Simulation

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Contents

1 Introduction

The first aim of this project is to determine the possibilities for determining the value of the effectiveness of various restrictive measures taken by the government of the Czech Republic for the period from September 1, 2020 till the last day of the project - December 7, 2020.

The second aim is to create a predictive model for determining the number of persons who have illness in the same time, persons who have been ill or otherwise have immunity

Used model contains different scenarios of quarantine precautions (using different types of lockdown). Based on simulations of this scenarios, influence of particular scenario is shown. As an experiment, theoretical scenarios from the article and current lockdown type in Czech Republic are analyzed.

1.1 Contributors

This project is solved by team of two students: Abramov Mikhail and Pavel Yadlouski.

1.2 Model validation

Results of theoretical scenarios simulation are compared with reference results from the article. The article by itself was subjected to critical analysis and minor formulas adjustments. Experiment with lockdown type in Czech Republic is compared with reality:)

2 Topic analysis

As epidemic situation in the world become worth with time, there is need to take appropriate precautions based on mathematical models and simulation. Epidemiological model can be described as by stochastic as by deterministic model. Stochastic model can describe epidemics on micro level. For example on micro level time period between visitors came to the market is stochastic.

But on macro level with large populations epidemics is described using deterministic model. In this model each individual of the population is assigned to different subgroup. And each subgroup represents a specific stage of the epidemic. The transition rates from one class to another are mathematically expressed as derivatives, this way model is defined by differential equations. In any time of the simulation following equation should valid:

$$P = \sum_{n=1}^{n} S_n^t \tag{1}$$

where P is size of initial and S_n^t is size of the subgroup S_n in the time t.

One of the base mathematical model for simulation of expansion of the epidemic is SIR[1] model. SIR model is the simplest compartment methods that can be extended using different methodologies. This method compares three values:

1. **S** – the number of **s**usceptible individuals
2. **I** – the number of **i**nfectious individuals.
3. **R** – the number of **r**emoved (and immune) or deceased individuals

[1] https://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology

2.1 Sources

The necessary information to research this topic was found in scientific articles from IRCACS-International Research Center for Applied Complexity Sciences in Colombia written by Danny Ibarra-Vega[2].

2.2 Approaches

For more complex view of system behavior, a mathematical model has been adopted with the Systems Dynamics(SD) methodology[2]. The core of this methodology is that SD models solve the problem of simultaneity (mutual causation) by updating all variables in small time increments with positive and negative feedbacks and time delays structuring the interactions and control.

Used SD model extends basic SIR model with separating the number of recovered and deceased individuals into two variables and addition of auxiliary and state variables that represent hospital capacity, contacts, contacts with infected. As a result, there is a model of 4 stock variables and 4 corresponding differential equations (2).

3 Concept model

In the model, we proceed from the assumption that immunity is stable and guarantees the absence of recurrent disease for the duration of research period.

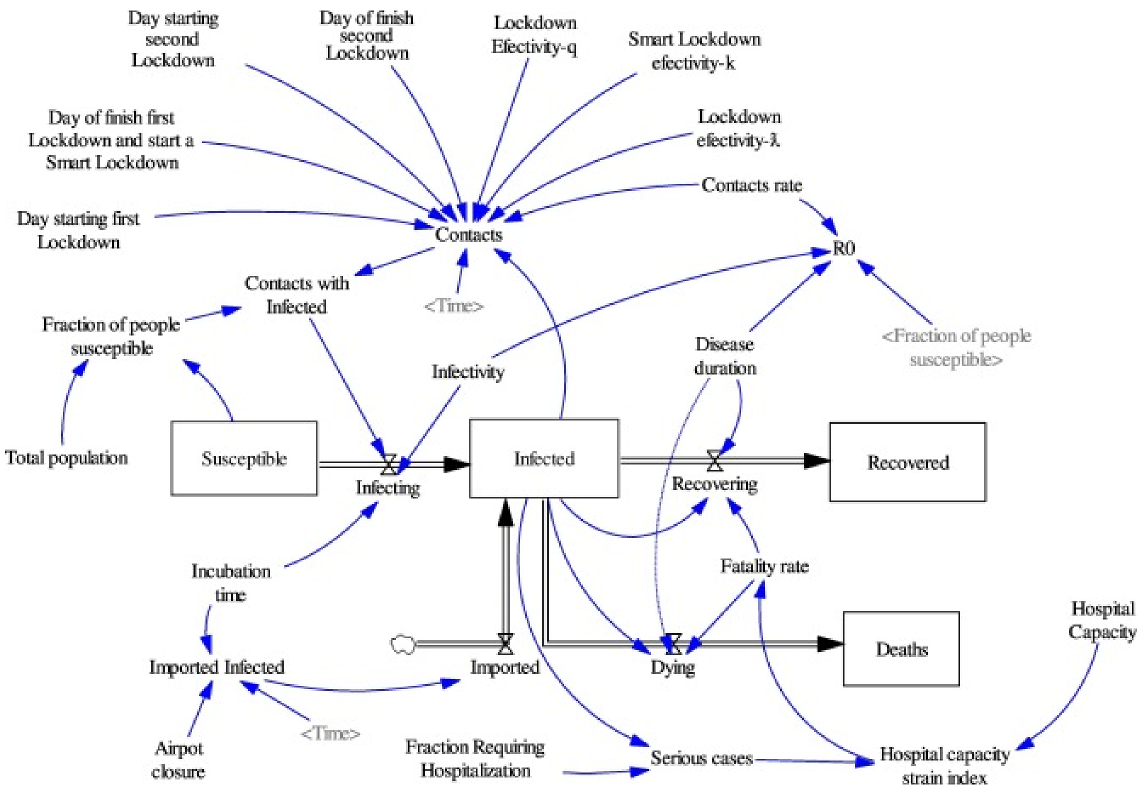


Figure 1: Stock and flows diagram

[2] https://en.wikipedia.org/wiki/System_dynamics

The diagram 1 from the reference article shows how each stock variable (susceptible, infectious, removed, deaths) is connected and influenced by other stock and auxiliary variable. Also Auxiliary variables are constructed from bibliographic references or some estimated.

Name	Initial value	Units	Reference
Susceptible	100,000	People	Assumed
Incubation time	5	Days	Wu et al. (2020)
Disease duration	14	Days	Wu et al. (2020)
Fraction requiring hospitalization	13	%	WHO report 73 (2020), Li et al. (2020)
Infectivity	0.025	Dimensionless	Estimated with RO
Contacts rate	70	Contacts/person	Assumed
Hospital capacity	1000	Beds	Assumed
Fatality rate	3	%	WHO report 73 (2020), Wu et al. (2020)

Table 1: Initial conditions [2]

Meaning and notation of individual variable is explained in the table 2

Type of variable	Parameter	Notation
Auxiliary	Contacts rate	μ
Auxiliary	Fatality rate	Fr
Auxiliary	Hospital capacity strain index	HiC
Parameter	Incubation time	it
Parameter	Disease duration	Dd
Parameter	Fraction requiring hospitalization	Fh
Parameter	Infectivity	β
Parameter	Hospital capacity	HC
Parameter	Lockdown effectivity	λ
Parameter	Smart lockdown effectivity	k
Parameter	Post lockdown effectivity	q
Parameter	Serious cases	SC
Parameter	Hospital capacity	HC
Stock	Susceptible	S
Stock	Infected	I
Stock	Recovered	R
Stock	Deaths	D

Table 2: Notation and variables [2]

Mathematical model of epidemic is

$$\frac{dS}{dt} = -\frac{\beta Ci}{it}$$

$$\frac{dI}{dt} = \frac{\beta C}{it} - \frac{I}{Dd}$$

$$\frac{dR}{dt} = \frac{I}{Dd} * (1 - Fr)$$

$$\frac{dD}{dt} = \frac{I}{Dd} * (Fr)$$

During implementation of the model from the article, we found out that some variables in auxiliary equations are not defined in the article. For example variable F is not present in the article. Using addition source article from Harvard University [1] we determined that variable F is Si divided on the total number of population P, otherwise we decided not to use additional variable for population and decided to use initial value of Susceptible as total population.

So, auxiliary equations that we used have following form

$$\begin{aligned} Ci &= C * F \\ F &= \frac{S}{S_{int}} \\ HiC &= \frac{SC}{HC} \\ SC &= I * Fh \\ Fr &= \begin{cases} 3\% \; if \; HiC < 5 \\ 7\% \; if \; 5 < HiC < 30 \\ 10\% \; if \; HiC > 30 \end{cases} \end{aligned}$$

Influence of lockdown scenarios is described in the following way

$$C = \begin{cases} I * \mu \; if \; t \le t_1 \\ I * \mu * \lambda \; if \; t_1 < t \le t_2 \\ I * \mu * k \; if \; t_2 < t \le t_3 \\ I * \mu * q \; if \; t_3 < t \le t_4 \end{cases}$$

There are 4 types of the lockdown:

1. without lockdown
2. standard lockdown
3. smart lockdown

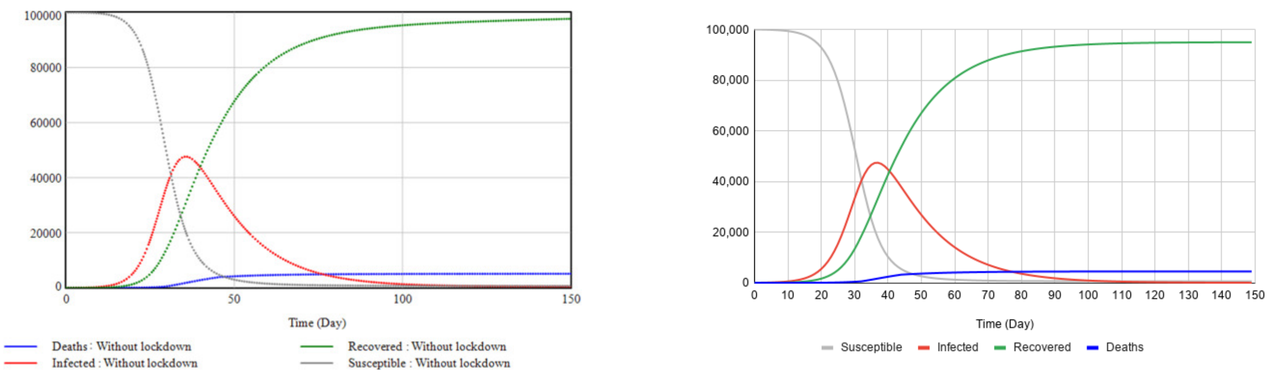
They are differ in effectivity of each one (parameters λ and k for standard and smart lockdown respectively). In addition, there is parameter q that shows post-lockdown effectivity. By effectivity we mean how much contact rate is reduced (in %).

4 Experiment

At first, we conducted experiments from the source article. This experiments simulate different quarantine scenarios[3].

4.1 First scenario

In this scenario there is no lockdown at all.



(a) Result from the article

(b) Our result

Figure 2: Results of the first scenario

4.2 Second scenario

The second scenario includes one standard lockdown for 60 days from the 25th day after first beginning of the simulation till 85th day (long).

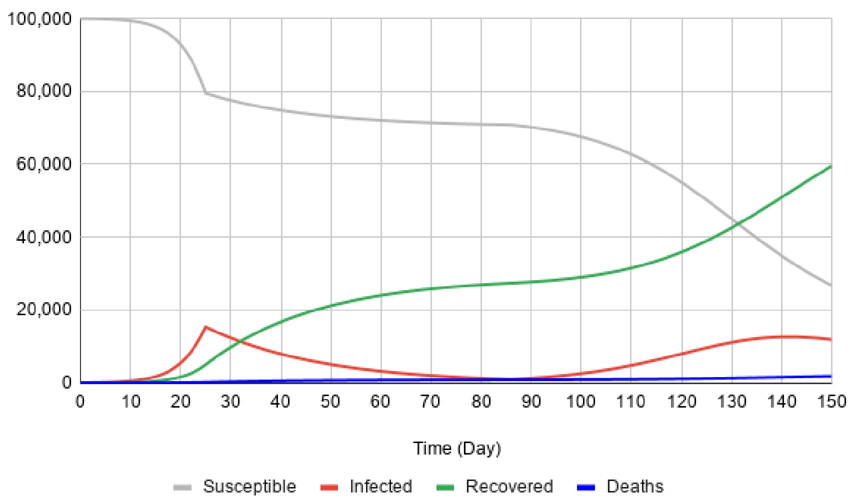


Figure 3: Results of the second scenario

[3]In the source article there is diagram only for the first scenario (4.1)

4.3 Third scenario

In the third theoretical scenario there are two standard and one smart lockdown. Each lasts for 30 days (short). Smart lockdown acts between two standard lockdown

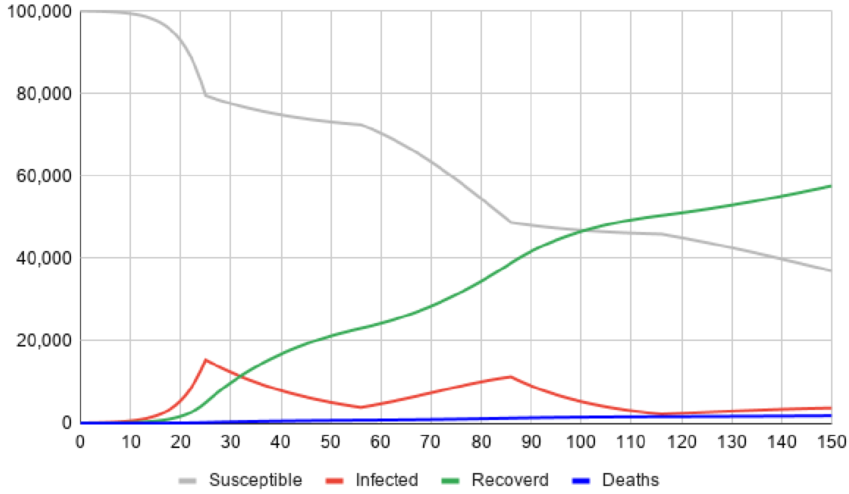


Figure 4: Results of the third scenario

4.4 Fourth scenario

In the last theoretical scenario there is only two lockdown: one standard and one smart. Each lasts for 40 days (medium).

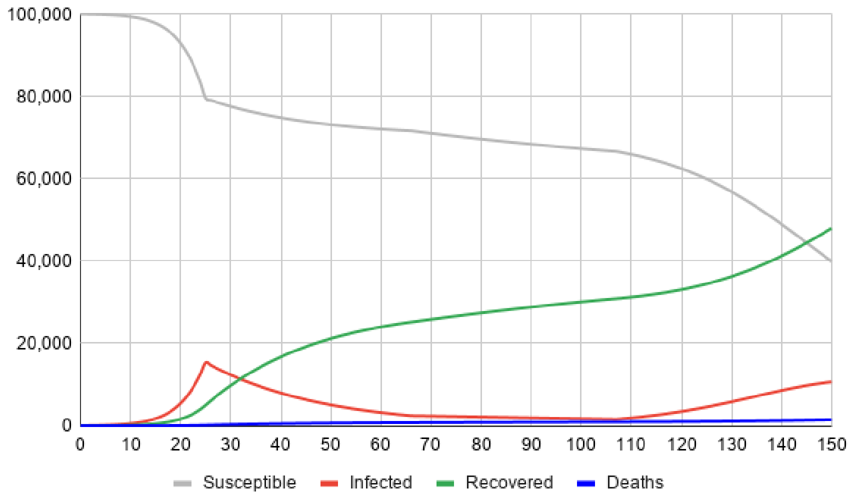


Figure 5: Results of the fourth scenario

4.5 Our experiment

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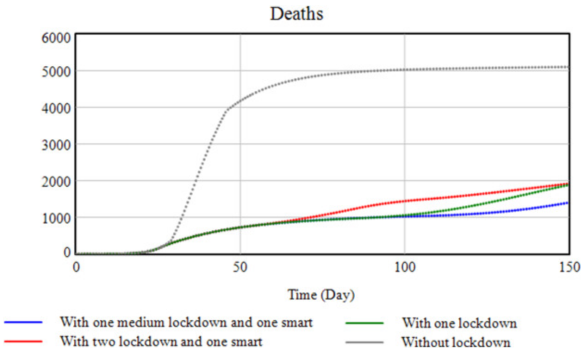
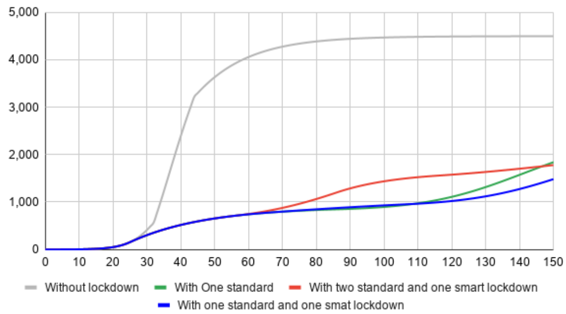
5 Conclusion

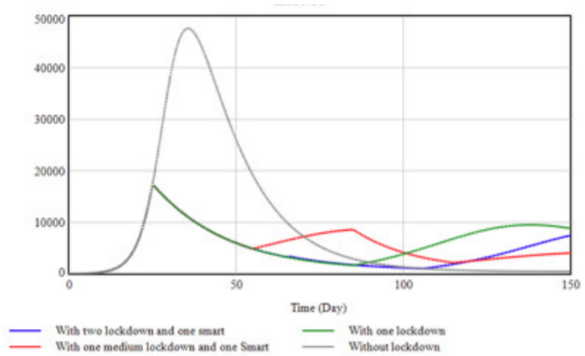


(a) Result from the article

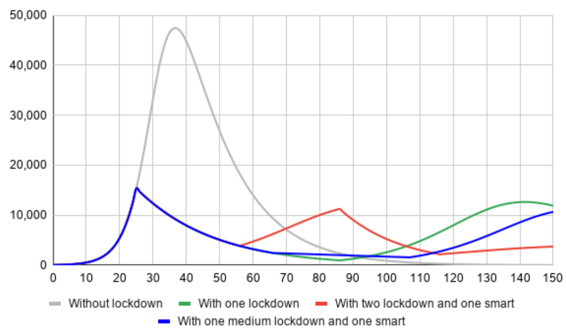


(b) Our result

Figure 6: Deaths



(a) Result from the article



(b) Our result

Figure 7: Infected

During implementation we met with serious inaccuracies that led to a difference in graphics

- **No initial values**
  In the source article there is no initial value of Infected variable

- **Logical mistake in question**
  The model assumes that recovered person can't be infected again. In source model question for Infected variable has following form:

$$\frac{dI}{dt} = \frac{\beta C}{it} - \frac{I}{Dd} * (1 - Fr)$$

The problem is that factor $(1 - Fr)$ includes only recovered people, but not dead people. Logically dead person can't be infected again too :) This mistake leads to violation of primary question (1)

After all the experiments, we came to the conclusion that the most effective scenario is the fourth (4.4) based on count of deaths.

5.1 Conclusion of our experiment

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[[TODO Misha napishy suda]]

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[[TODO Experiment with other country]]

References

[1] J. Fernandez-Villaverde and C. I. Jones. *Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities*. https://web.stanford.edu/~chadj/sird-paper.pdf, 2020. [Online; accessed November-2020].

[2] D. Ibarra-Vega. Lockdown, one, two, none, or smart. modeling containing covid-19 infection. a conceptual model. *Science of The Total Environment*, 730:138917, 2020.