Epidemiological model on macro level

Modelling and Simulation

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Contents

1 Introduction

The first aim of this project is to determine the possibilities for determining the value of the effectivity of various restrictive measures taken by the government of the Czech Republic for the period from September 1, 2020 till the last day of the project - December 5, 2020.

The second aim is to create a predictive model for determining the number of persons who have illness in the same time, persons who have been ill or otherwise have immunity

Used model contains different scenarios of quarantine precautions (using different types of lockdown). Based on simulations of this scenarios, influence of particular scenario is shown. As an experiment, theoretical scenarios from the article and current lockdown type in Czech Republic are analyzed.

1.1 Contributors

This project is solved by team of two students: Abramov Mikhail and Pavel Yadlouski.

1.2 Model validation

Results of theoretical scenarios simulation are compared with reference results from the article. The article by itself was subjected to critical analysis and minor formulas adjustments. Experiment with lockdown type in Czech Republic is compared with reality based on statistics values.

2 Topic analysis

As epidemic situation in the world become worth with time, there is need to take appropriate precautions based on mathematical models and simulation. Epidemiological model can be described as by stochastic as by deterministic model. Stochastic model can describe epidemics on micro level. For example on micro level time period between visitors came to the market is stochastic.

But on macro level with large populations epidemics is described using deterministic model. In this model each individual of the population is assigned to different subgroup. And each subgroup represents a specific stage of the epidemic. The transition rates from one class to another are mathematically expressed as derivatives, this way model is defined by differential equations. In any time of the simulation following equation should valid:

$$P = \sum_{n=1}^{n} S_n^t \tag{1}$$

where P is size of initial and S_n^t is size of the subgroup S_n in the time t.

One of the base mathematical model for simulation of expansion of the epidemic is SIR[1] model. SIR model is the simplest compartment methods that can be extended using different methodologies. This method compares three values:

1. **S** – the number of **s**usceptible individuals
2. **I** – the number of **i**nfectious individuals.
3. **R** – the number of **r**emoved (and immune) or deceased individuals

[1] https://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology

2.1 Sources

The necessary information to research this topic was found in scientific articles from IRCACS-International Research Center for Applied Complexity Sciences in Colombia written by Danny Ibarra-Vega[2].

2.2 Approaches

For more complex view of system behavior, a mathematical model has been adopted with the Systems Dynamics(SD) methodology[2]. The core of this methodology is that SD models solve the problem of simultaneity (mutual causation) by updating all variables in small time increments with positive and negative feedbacks and time delays structuring the interactions and control.

Used SD model extends basic SIR model with separating the number of recovered and deceased individuals into two variables and addition of auxiliary and state variables that represent hospital capacity, contacts, contacts with infected. As a result, there is a model of 4 stock variables and 4 corresponding differential equations (2).

3 Concept model

In the model, we proceed from the assumption that immunity is stable and guarantees the absence of recurrent disease for the duration of research period.

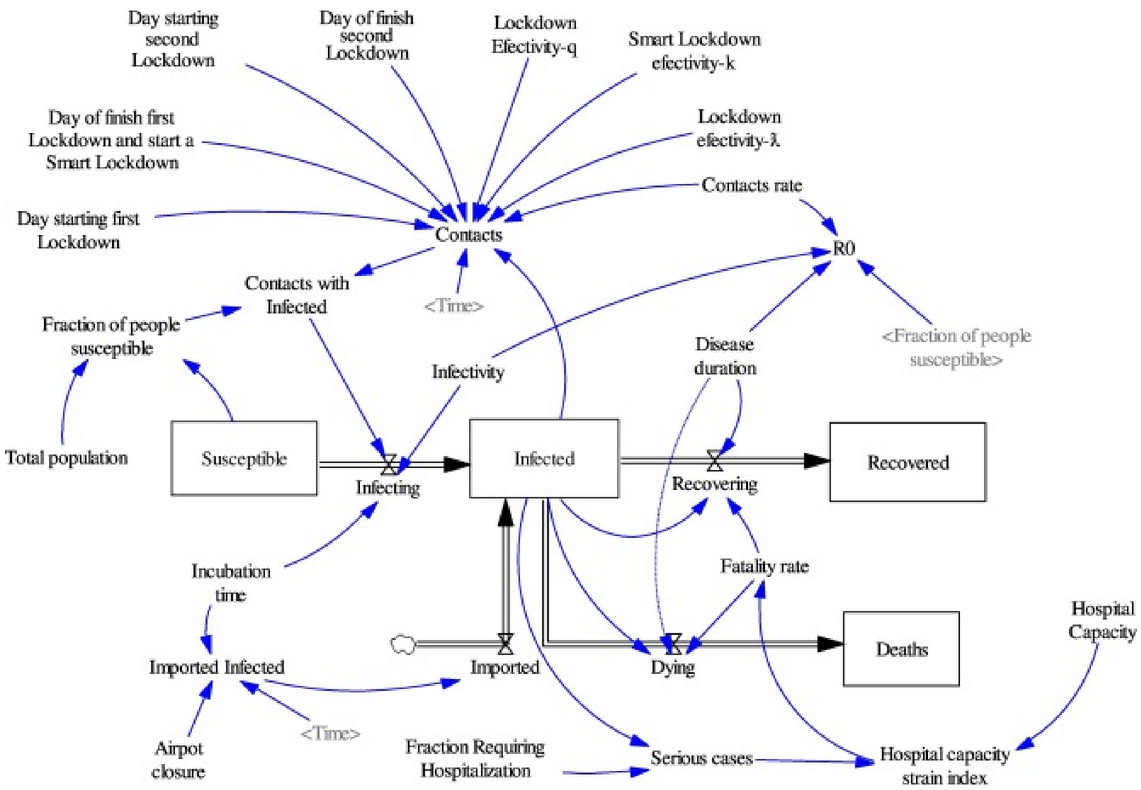


Figure 1: Stock and flows diagram

[2]https://en.wikipedia.org/wiki/System_dynamics

The diagram 1 from the reference article shows how each stock variable (susceptible, infectious, removed, deaths) is connected and influenced by other stock and auxiliary variable. Also Auxiliary variables are constructed from bibliographic references or some estimated.

Name	Initial value	Units	Reference
Susceptible	100,000	People	Assumed
Incubation time	5	Days	Wu et al. (2020)
Disease duration	14	Days	Wu et al. (2020)
Fraction requiring hospitalization	13	%	WHO report 73 (2020), Li et al. (2020)
Infectivity	0.025	Dimensionless	Estimated with RO
Contacts rate	70	Contacts/person	Assumed
Hospital capacity	1000	Beds	Assumed
Fatality rate	3	%	WHO report 73 (2020), Wu et al. (2020)

Table 1: Initial conditions [2]

Meaning and notation of individual variable is explained in the table 2

Type of variable	Parameter	Notation
Auxiliary	Contacts rate	μ
Auxiliary	Fatality rate	Fr
Auxiliary	Hospital capacity strain index	HiC
Parameter	Incubation time	it
Parameter	Disease duration	Dd
Parameter	Fraction requiring hospitalization	Fh
Parameter	Infectivity	β
Parameter	Hospital capacity	HC
Parameter	Lockdown effectivity	λ
Parameter	Smart lockdown effectivity	k
Parameter	Post lockdown effectivity	q
Parameter	Serious cases	SC
Parameter	Hospital capacity	HC
Stock	Susceptible	S
Stock	Infected	I
Stock	Recovered	R
Stock	Deaths	D

Table 2: Notation and variables [2]

Mathematical model of epidemic is

$$\frac{dS}{dt} = -\frac{\beta Ci}{it}$$

$$\frac{dI}{dt} = \frac{\beta C}{it} - \frac{I}{Dd}$$

$$\frac{dR}{dt} = \frac{I}{Dd} * (1 - Fr)$$

$$\frac{dD}{dt} = \frac{I}{Dd} * (Fr)$$

During implementation of the model from the article, we found out that some variables in auxiliary equations are not defined in the article. For example variable F is not present in the article. Using addition source article from Harvard University [1] we determined that variable F is Si divided on the total number of population P, otherwise we decided not to use additional variable for population and decided to use initial value of Susceptible as total population.

So, auxiliary equations that we used have following form

$$\begin{aligned} Ci &= C * F \\ F &= \frac{S}{S_{int}} \\ HiC &= \frac{SC}{HC} \\ SC &= I * Fh \\ Fr &= \begin{cases} 3\% \; if \; HiC < 5 \\ 7\% \; if \; 5 < HiC < 30 \\ 10\% \; if \; HiC > 30 \end{cases} \end{aligned}$$

Influence of lockdown scenarios is described in the following way

$$C = \begin{cases} I * \mu \; if \; t \leq t_1 \\ I * \mu * \lambda \; if \; t_1 < t \leq t_2 \\ I * \mu * k \; if \; t_2 < t \leq t_3 \\ I * \mu * q \; if \; t_3 < t \leq t_4 \end{cases}$$

There are 4 types of the lockdown:

1. without lockdown
2. standard lockdown
3. smart lockdown

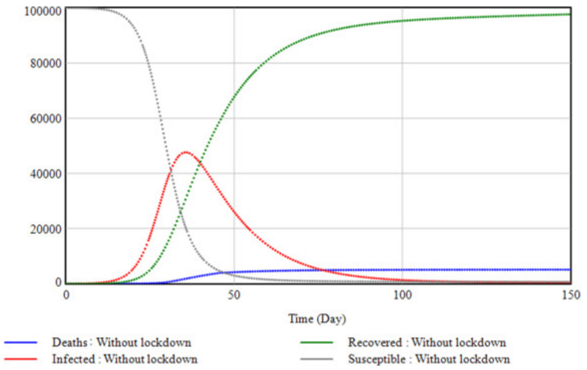
They are differ in effectivity of each one (parameters λ and k for standard and smart lockdown respectively). In addition, there is parameter q that shows post-lockdown effectivity. By effectivity we mean how much contact rate is reduced (in %).

4 Experiment

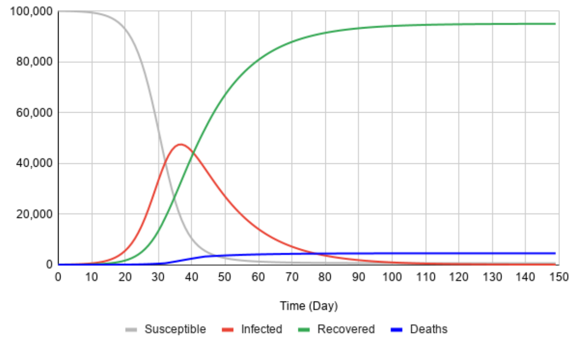
At first, we conducted experiments from the source article. This experiments simulate different quarantine scenarios[3].

4.1 First scenario

In this scenario there is no lockdown at all.



(a) Result from the article



(b) Our result

Figure 2: Results of the first scenario

4.2 Second scenario

The second scenario includes one standard lockdown for 60 days from the 25th day after first beginning of the simulation till 85th day (long).

- 0 - 25 day – no lockdown
- 26 - 85 day – standard lockdown
- 86 - 150 – post-lockdown period

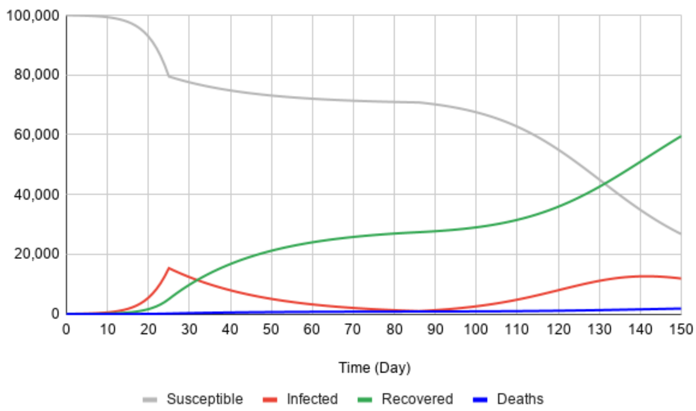


Figure 3: Results of the second scenario

[3]In the source article there is diagram only for the first scenario (4.1)

4.3 Third scenario

In the third theoretical scenario there are two standard and one smart lockdown. Each lasts for 30 days (short). Smart lockdown acts between two standard lockdown.

- 0 - 25 day – no lockdown
- 26 - 55 day – standard lockdown
- 56 - 85 day – smart lockdown
- 86 - 115 – standard lockdown
- 116 - 150 – post-lockdown period

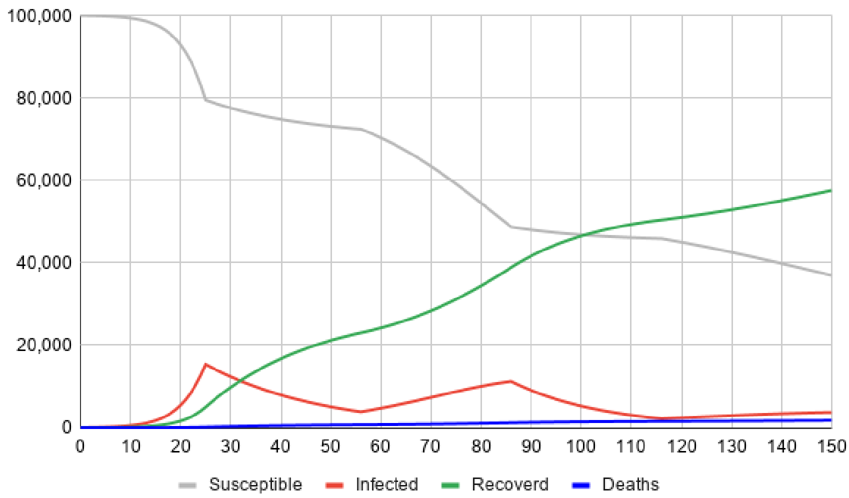


Figure 4: Results of the third scenario

4.4 Fourth scenario

In the last theoretical scenario there is only two lockdown: one standard and one smart. Each lasts for 40 days (medium).

- 0 - 25 day – no lockdown
- 26 - 65 day – standard lockdown
- 66 - 105 day – smart lockdown
- 106 - 150 – post-lockdown period

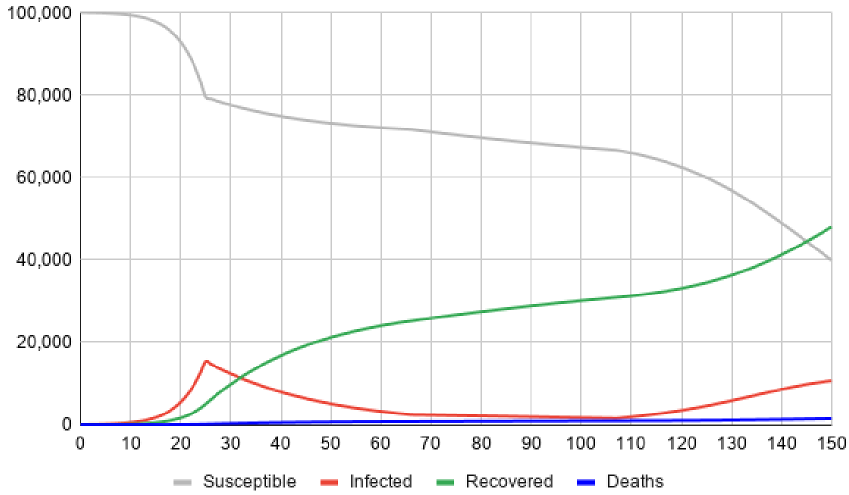


Figure 5: Results of the fourth scenario

4.5 Our experiment

The purpose of our experiment was to determine the level of measures effectivity taken by the government of the Czech Republic to counter covid19.

Data provided by the information portal of the Ministry of Health[4] were used as reference materials to compare model with real situation.

Name	Initial value	Units	Reference
Susceptible	10,680,000	People	Statistics
Infected	4,917	People	Statistics
Recovered	19,783	People	Statistics
Deaths	428	People	Statistics
Incubation time	5	Days	Wu et al. (2020)
Disease duration	14	Days	Wu et al. (2020)
Fraction requiring hospitalization	13	%	WHO report 73 (2020), Li et al. (2020)
Infectivity	0.025	Dimensionless	Estimated with RO
Contacts rate	29	Contacts/person	Assumed
Hospital capacity	74760	Beds	Statistics
Fatality rate	1.7	%	Assumed

Table 3: Initial conditions

[4] https://onemocneni-aktualne.mzcr.cz/covid-19

The reasons for the change in the movement of the model are the measures introduced by the Government. In the model, we abstract from weather factors, because it is already obvious that in the warm summer time the virus slows down its spread. We also assume that every measure to reduce the spread of Coronavirus cannot have negative effectivity. And also we abstract from the differences in social groups and population heterogeneity.

Investigated measures and model time intervals:

- 0 - 9 day – no measures, setting the basic parameters of the model: Fatality rate, Contacts rate
- 10 - 20 day – indoor mandatory masks [5]
- 21 - 33 day – distance education in universities (First prague) [6]
- 34 - 37 day – distance education in all universities [7]
- 38 - 40 day – sports and recreation facilities are closed. [8]
- 41 - 49 day – distance education in schools [9]
- 50 - 56 day – outdoor mandatory masks, shops and restaurants are closed [10] [11]
- 57 - 77 day – imposed curfew [12]
- 78 - 92 day – PES4 [13]
- 93 - XX day – PES3 [14]

[5] https://www.mzcr.cz/tiskove-centrum-mz/od-ctvrtka-budou-povinne-rousky-ve-vnitrnich-prostorach-budov-v-cele-cr/

[6] https://www.idnes.cz/zpravy/domaci/koronavirus-covid-19-aktualizace-semafor-praha-cesko.A200921_181218_domaci_lesa

[7] https://apps.odok.cz/djv-agenda?date=2020-09-30

[8] https://www.lidovky.cz/domov/zive-vlada-projednala-nova-covidova-opatreni-kabinet-informuje-jake-restrikce-cekaji-cesko.A201008_110950_ln_domov_sei

[9] https://koronavirus.mzcr.cz/vlada-od-stredy-zprisni-preventivni-opatreni-posle-ochranne-pomucky-invalidnim-duchodcum

[10] https://www.kurzy.cz/zpravy/562948-od-stredy-21-rijna-do-odvolani-rousky-vsude-v-kontaktu-s-dalsi-osobou-i-venku/

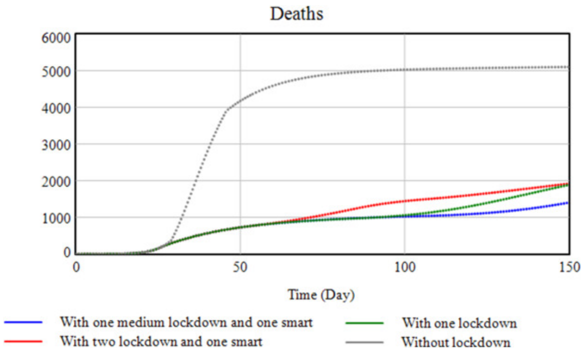
[11] https://prachaticky.denik.cz/zpravy_region/zavreni-obchodu-i-sluzeb-obchodnici-se-obavaji-financnich-problemu-20201021.html

[12] https://www.denik.cz/z_domova/cesko-narizeni-aktualne-omezeni-20201028.html

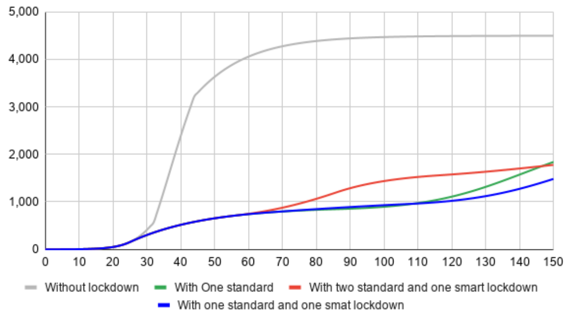
[13] https://onemocneni-aktualne.mzcr.cz/pes

[14] https://www.vlada.cz/cz/media-centrum/aktualne/do-tretiho-stupne-pes-prejde-cesko-ve-ctvrtek--rozhodla-vlada-185260/

5 Conclusion

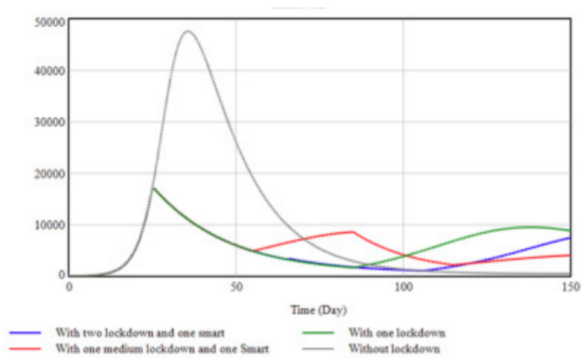


(a) Result from the article

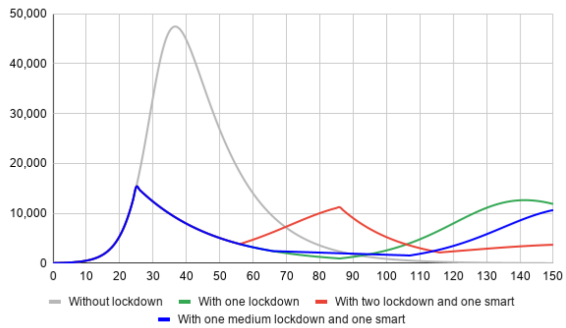


(b) Our result

Figure 6: Deaths



(a) Result from the article



(b) Our result

Figure 7: Infected

During implementation we met with serious inaccuracies that led to a difference in graphics

- **No initial values**
  In the source article there is no initial value of Infected variable

- **Logical mistake in question**
  The model assumes that recovered person can't be infected again. In source model question for Infected variable has following form:

$$\frac{dI}{dt} = \frac{\beta C}{it} - \frac{I}{Dd} * (1 - Fr)$$

  The problem is that factor $(1 - Fr)$ includes only recovered people, but not dead people. Logically dead person can't be infected again too :) This mistake leads to violation of primary question (1)

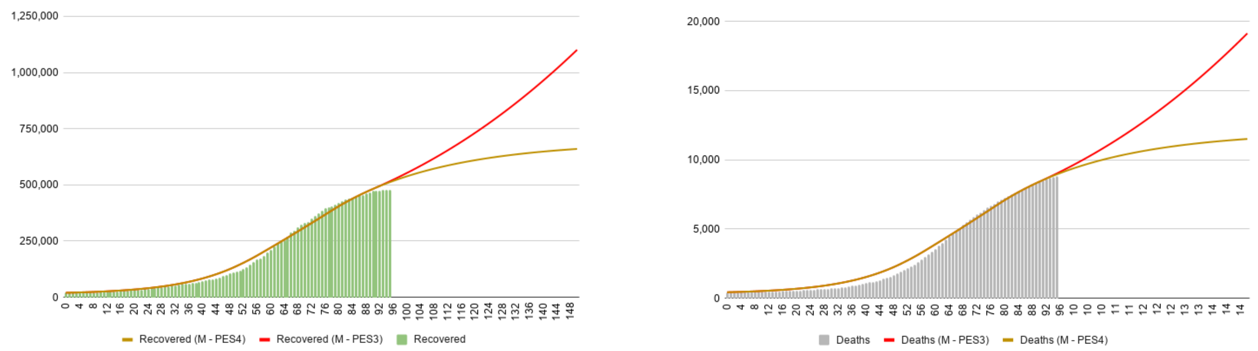
After all the experiments, we came to the conclusion that the most effective scenario is the fourth (4.4) based on count of deaths.

5.1 Conclusion of our experiment

According to the results obtained, the measures received the following percentage of effectivity (percentage decrease in the number of social contacts):

- 0 - 9 day – no measures – 0% – Total: 0%
- 10 - 20 day – indoor mandatory masks – 5% – Total: 5%
- 21 - 33 day – distance education in universities (first Prague) – 1% – Total: 6%
- 34 - 37 day – distance education in all universities – 1% – Total: 7%
- 38 - 40 day – sports and recreation facilities are closed – 1% – Total: 8%
- 41 - 49 day – distance education in schools – 3% – Total: 11%
- 50 - 56 day – outdoor mandatory masks, shops and restaurants are closed – 15% – Total: 26%
- 57 - 77 day – imposed curfew – 20% – Total: 46%
- 78 - 92 day – PES4 – 25% – Total: 71%
- 93 - XX day – PES3 – -35% – Total: 36%

Next, we will consider two scenarios with a third and fourth level PES.
We based our analysis on death and recovered values as on more statistically accurate.



(a) Recovered

(b) Deaths

Figure 8: Experiment results

Thus, we can say that measures by themselves cannot be effective. Measures can only be effective in combination. It is not enough to close universities and leave restaurants and bars open. The opposite is not enough too, when leaving schools and universities open.

Despite the fact that we did not have enough time to assess the decrease in the level of the PES system, we tried to predict two scenarios, from which it follows that it is necessary to maintain the fourth level of danger, which has been shown by the system itself[15].

[15] https://www.novinky.cz/koronavirus/clanek/on-line-index-rizika-stoupl-zpet-do-ctvrteho-stupne-zbyva-jediny-bod-40344376

We also came to the conclusion that the actual mortality differs from the expected one and is 1.7% instead of 3%, which may be due to the increase in testing coverage and the emergence of treatment methods.

We recommend a more systematic approach when introducing restrictive measures, because it is often not so easy to determine which measure really influenced the improvement of the situation. At the moment the PES system can solve this problem, since it consolidates a number of measures at each level within itself and their application becomes systemic.

References

[1] J. Fernandez-Villaverde and C. I. Jones. *Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities*. https://web.stanford.edu/~chadj/sird-paper.pdf, 2020. [Online; accessed December-2020].

[2] D. Ibarra-Vega. Lockdown, one, two, none, or smart. modeling containing covid-19 infection. a conceptual model. *Science of The Total Environment*, 730:138917, 2020. https://www-sciencedirect-com.ezproxy.lib.vutbr.cz/science/article/pii/S0048969720324347 [Online; accessed December-2020].

[3] P. Peringer, D. Leska, and D. Martinek. Simlib/c++ (simulation library for c++). http://www.fit.vutbr.cz/~peringer/SIMLIB, 19.10.2018. [Online; accessed December-2020].