Diff Eq's - Gen Properties

Def: Any eq. of form $u_n = f(u_{n-1}, u_{n-2}, \dots, u_{n-m})$

E.g. ① $u_n = 2u_{n-1} + 2$ ⟷ $u_{n+7} = 2u_{n+6} + 2$

① $u_n = u_{n-1} + 1$

① $u_n = 2 \cdot u_{n-1}$ ⟷ $u_{n+1} = 2 \cdot u_n$

② $u_n = 3u_{n-1} + 2u_{n-2} + n^2$

① $u_n = K u_{n-1}(1 - u_{n-1})$ ⟷ $u_{n+1} = K u_n (1 - u_n)$

order

Order (of a diff eq.): diff b/w the larges & smallest indexes

Goal: To find a sequence $\{u_n\}$ and formula for u_n which "satisfies" the diff eq.

↳ in terms of, say, initial conditions

$u_{n+1} = 2u_n(1-u_n)$ eg of "logistic eq."

$u_{n+1} = 2\left(\frac{1}{2}\right)\left(1-\frac{1}{2}\right) = \frac{1}{2}$

$u_0 = \frac{1}{4}$

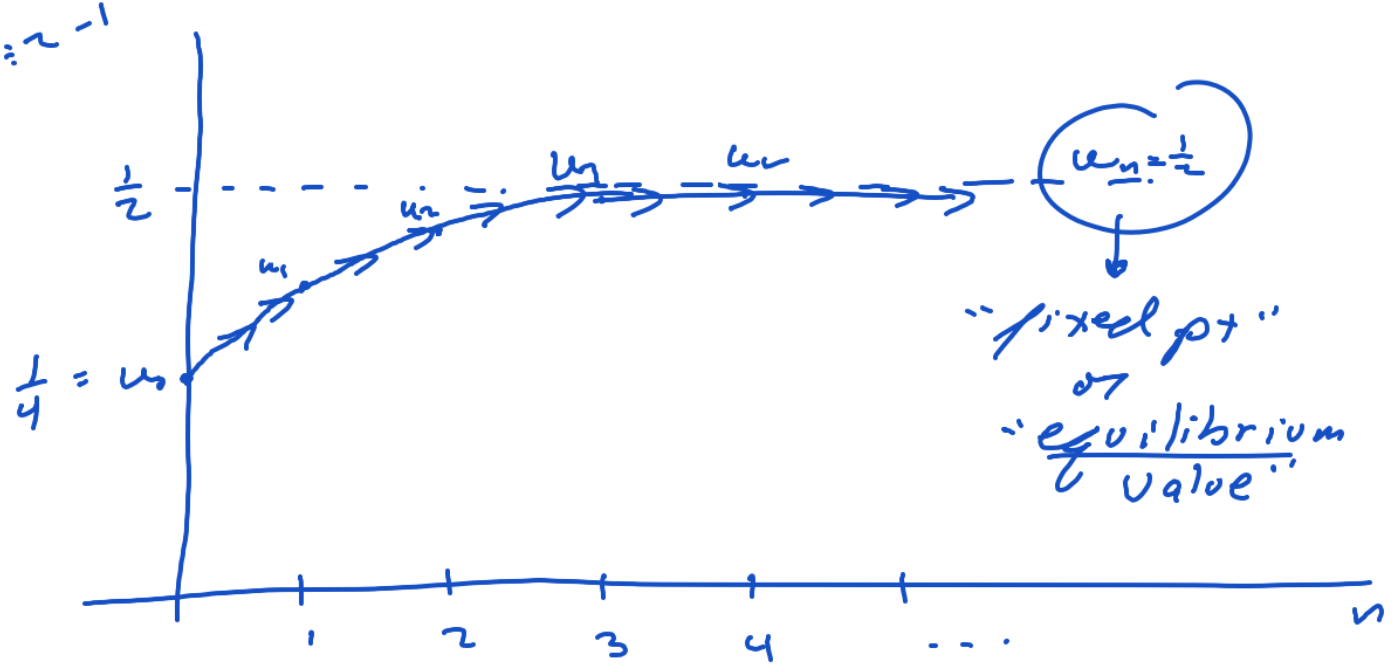
$u_1 = \frac{1}{2}\cdot\frac{3}{4} = \frac{3}{8}$ ← num = den + 2 − 1

$u_2 = \frac{3}{4}\cdot\frac{5}{8} = \frac{15}{32}$

$u_3 = \frac{255}{512}$

$u_4 = \frac{65535}{131072}$

⋮



mathematical determ. of eq. val.

say D.Eq: $u_{n+1} = 2u_n(1-u_n)$

suppose u is a equil val.

then $u = 2u(1-u)$

$u = 2u - 2u^2$

$2u^2 - u = 0$

$u(2u-1) = 0$

$u = 0$ or $u = \frac{1}{2}$

let $u_0 = 0$

$u_1 = 2 \cdot 0(1-0) = 0$

$u_2 = 0$ $u_3 = 0$ ...

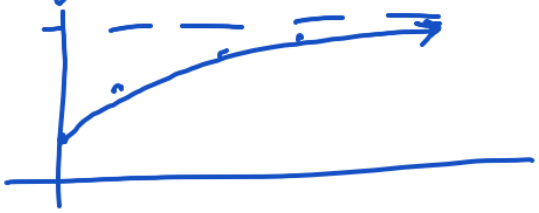
let $u_0 = \frac{1}{2}$

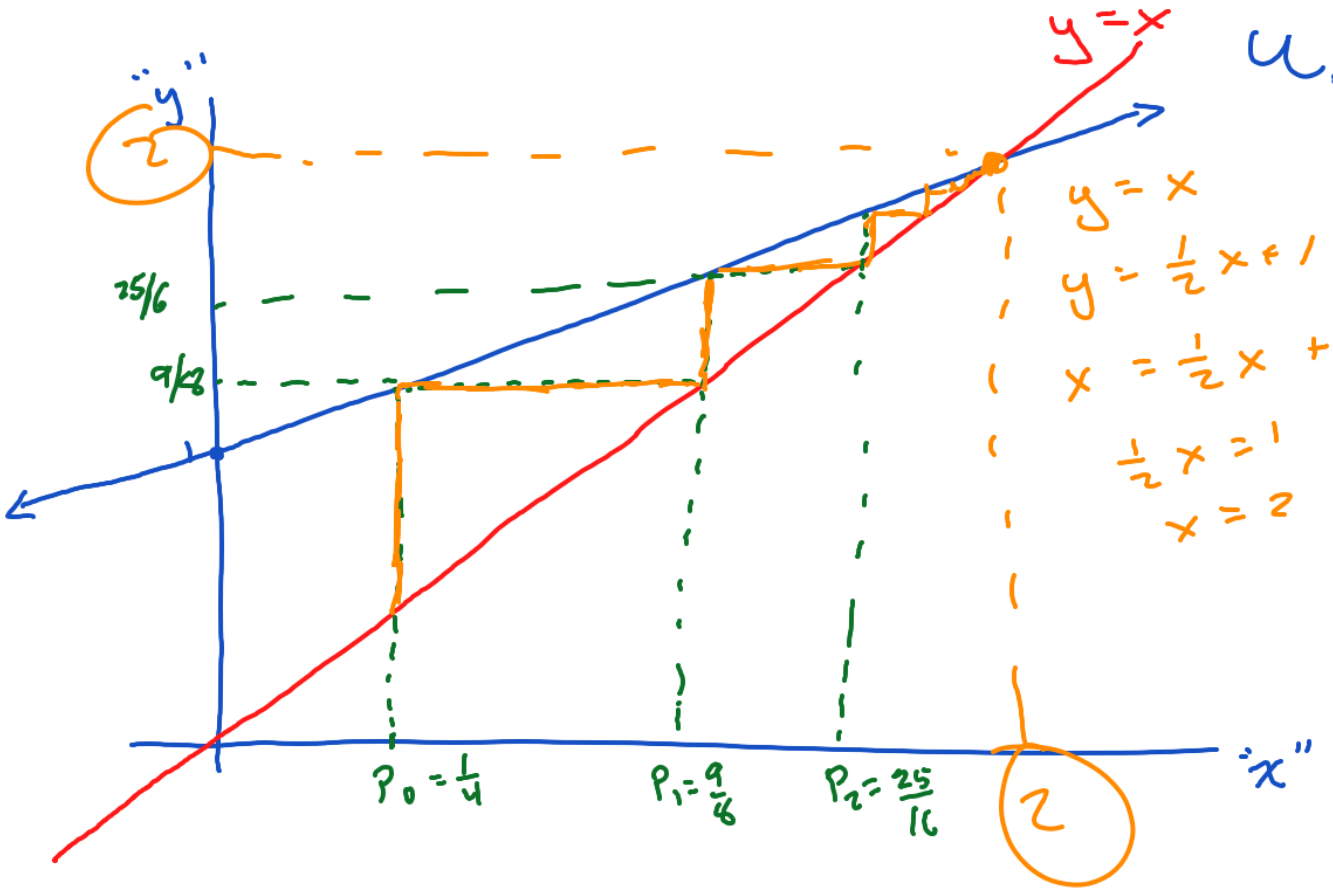
$u_1 = 2(\frac{1}{2})(1-\frac{1}{2}) = \frac{1}{2}$

$u_2 = \frac{1}{2}$ ...

Graphical Visualization of Diff. Eq.

① plot n vs. u_n ② cobwebs!





$$u_{n+1} = \boxed{f(u_n) = \frac{1}{2}u_n + 1}$$

linear fun.

$y = mx + b$

$$f\left(\frac{1}{4}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + 1$$

$$= \frac{1}{8} + 1$$

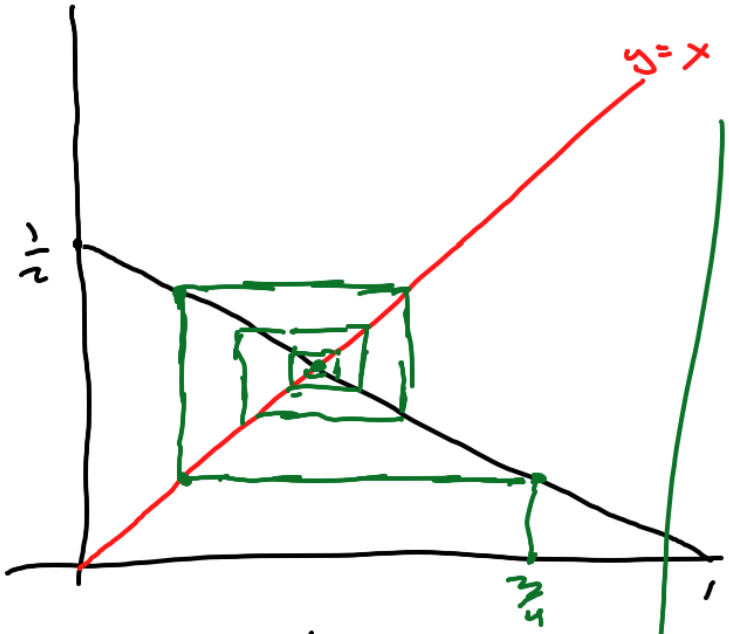
$$= \frac{9}{8}$$

$$f\left(\frac{9}{8}\right) = \left(\frac{1}{2}\right)\left(\frac{9}{8}\right) + 1$$

$$= \frac{9}{16} + \frac{16}{16} = \frac{25}{16}$$

$u_{n+1} = -k u_n + k$

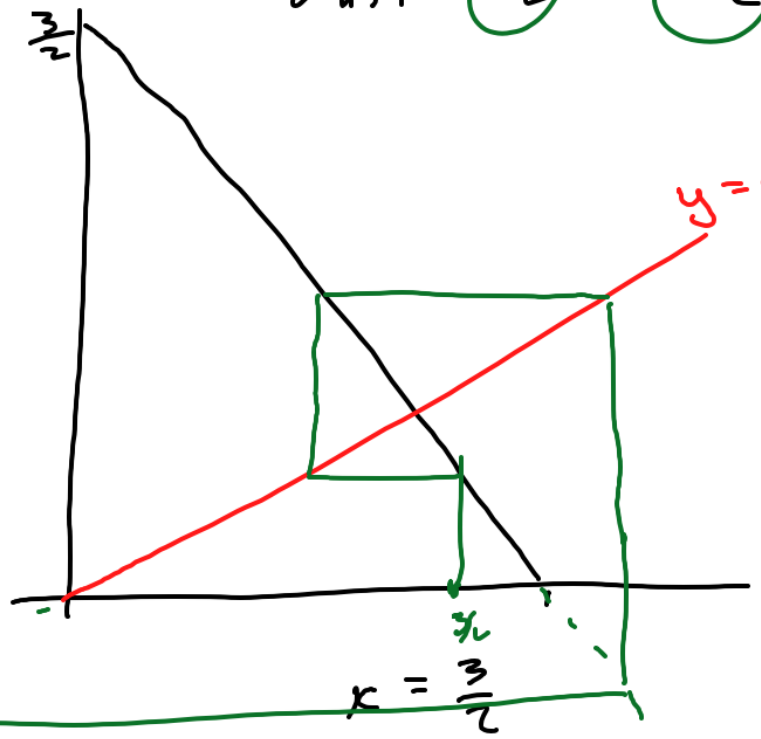
$u_{n+1} = -\frac{1}{2} u_n + \frac{1}{2}$



$k = \frac{1}{2}$

$u_0 = \frac{3}{4}$

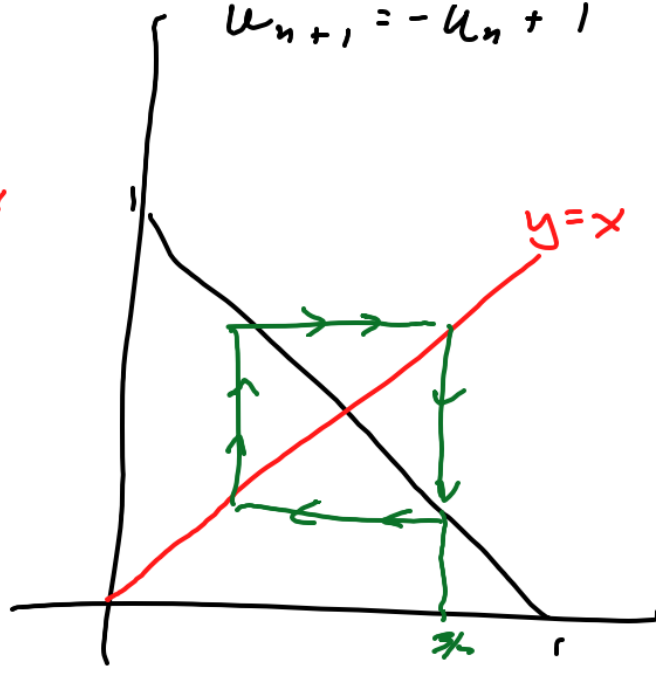
$u_{n+1} = -\frac{3}{2} u_n + \frac{3}{2}$



$k = \frac{3}{2}$

$u_0 = \frac{3}{4}$

$u_{n+1} = -u_n + 1$

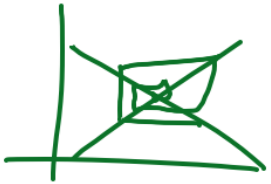


$k = 1$

$u_0 = \frac{3}{4}$

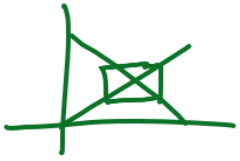
First-order Diff Eq $\boxed{u_{n+1} = -k u_n + a}$ has

(a) a stable fixed pt if $-1 < k < 1$



(b) a fixed pt that is not stable if $|k| > 1$

(c) a periodic fixed pt if $k = 1$



Challenge: write a C++ (SDL) app that draws cobweb for 1st ord Diff Eq

$u_{n+1} = -k u_n + a$