Combinatorial Game Theory *2025*

UNSW Competitive Programming and Mathematics Society

Combinatorial Game Questions

1. Lawrence and Rita play a berry game, they line up raspberries (R) and blueberries (B) and begin eating them consecutively from the right end of the row. They can eat as many consecutive berries of the same type. If the table starts off with RBBRRRRBBBB, will the person who start first get the last berry

2. A-ob and Bliss play a game. Starting with 100 stones in a pile, they take turns removing stones from the pile. Each turn, the number of stones removed is between 1 and 10 inclusive. Whoever takes the last stone loses. Who has a winning strategy?

3. Alice and Bob have an unlimited supply of coins, and take turns placing these coins on a circular table with Alice going first. Coins placed must not overlap any other coin, and must completely lie on the table. Who has a winning strategy?

4. Anna and Ben play Hackenbush. They take turns chopping branches until a player cannot move and they are declared a loser. Anna cannot cut blue branches and Ben cannot cut red branches. They notice that whoever moves first will lose in a game with two trees of 3 red branches on a blue stem and one tree of 2 blue branches on a red stem. Is this still the case for a game with two trees of n red branches on a blue stem and one tree of $n-1$ blue branches on a red stem? [See Figure 2]

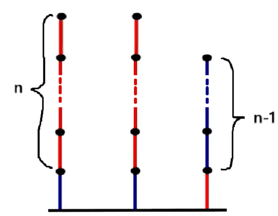

Figure 2: Is this position a first player loss?

5. Will and Emmy take turns drawing blue and red lines along a 5×5 square dot grid. Will tries to complete a closed loop of blue lines while Emmy tries to prevent Will from doing so. Can either of them guarantee the other fails?

6. Potato boy and Quint dingle are playing a game. Initially, there are 3 piles, each containing 2025 stones. The players take turns to make a move, with Potato boy going first. Each move consists of choosing one of the available piles, removing the unchosen pile(s) from the game, and then dividing the chosen pile into 2 or 3 non-empty piles. A player loses the game if they are unable to make a move. Which player has a winning strategy?

7. Alice and Bob are playing another game. Starting with n stones in a pile, Alice and Bob take turns in removing stones from the pile such that either:

   - One stone is removed
   - Half of the stones from the pile are removed (if there are an odd number of stones, the number of removed stones is rounded up).

   The winner is who takes the last stone. For what positive integers n does Alice have a winning strategy?

8. Yang and Zhangoose play a game. Initially, numbers $1 - 2025$ are written on a whiteboard. Each turn, the player removes a number on the whiteboard, and any of its factors that are on the whiteboard. Whoever removes the last numbers wins. Prove that Yang has a winning strategy if they go first.

9. Let n be a positive integer. Anna and Ben play a game in which they take turns choosing positive integers $k \leq n$. The rules of the game are:

   (i) A player cannot choose a number that has been chosen by either player on any previous turn

   (ii) A player cannot choose a number consecutive to any of those the player has already chosen on any previous turn

   (iii) The game is a draw if all numbers have been chosen; otherwise the player who cannot choose a number anymore loses the game.

   Anna takes the first turn. Determine the outcome of the game, assuming that both players use optimal strategies.