Integration I *2025*

UNSW Competitive Programming and Mathematics Society

Warm up

1. Evaluate $\displaystyle\int \frac{dx}{e^x + e^{-x}}$

2. Evaluate $\displaystyle\int \frac{dx}{\sqrt{x}(x+1)}$

3. Evaluate $\displaystyle\int \frac{\cos 2x}{\cos x}\, dx$

4. Evaluate $\displaystyle\int \frac{\ln x}{x(\ln x + 1)}\, dx$

5. Evaluate $\displaystyle\int \frac{dx}{\sqrt{e^x - 1}}$

6. Evaluate $\displaystyle\int \sqrt{x^2 + x + 1}\, dx$

7. Evaluate $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x \cos^3 \sqrt{x}}}\, dx$

8. Evaluate $\displaystyle\int \sqrt{\frac{x}{x^3 + 1}}\, dx$

9. Evaluate $\displaystyle\int_0^1 \frac{(x - x^2)^4}{1 + x^2}\, dx$, hence find a lower bound on π.

10. Evaluate $\displaystyle\int \left(\sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \csc^2 x\right)\, dx$

11. Without expanding, evaluate $\displaystyle\int (x-1)^2 (x-3)^2 (x-2)\, dx$

Trig

1. Evaluate $\displaystyle\int \left(\frac{1}{1 + \sin x} + \frac{1}{1 + \cos x} + \frac{1}{1 + \tan x} + \frac{1}{1 + \sec x} + \frac{1}{1 + \csc x} + \frac{1}{1 + \cot x} \right) dx$

2. Evaluate $\displaystyle\int \frac{x + \pi}{\sqrt{x^2 + \pi^2}}\, dx$

3. Evaluate $\displaystyle\int \frac{dx}{(x^2 + 1)^{\frac{3}{2}}}\, dx$

4. Evaluate $\displaystyle\int \frac{A \sin x + B \cos x}{C \sin x + D \cos x}$

5. Evaluate $\displaystyle\int \frac{\sin 2x}{2025 e^x + \sin x + \cos x}\, dx$

6. Evaluate $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{\cos^3 x - \cos x}{\cos^3 x - 1}}\, dx$

7. Evaluate $\displaystyle\int \sqrt{1 + \cos x}\, \cot x\, dx$

8. Evaluate $\displaystyle\int \frac{dx}{\sin^6 x + \cos^6 x}$

9. Evaluate $\displaystyle\int \tan(x) \tan(2x) \tan(3x)\, dx$

10. Evaluate $\displaystyle\int \frac{\sin 2x}{\sin^4 x + \cos^4 x}\, dx$ in two different ways, to show that $\tan^{-1}(\tan^2 x) + \frac{\pi}{4} = \cot^{-1}(\cos 2x)$.

11. *Evaluate* $\displaystyle\int_{-\frac{\pi}{4}}^{0} (1 + \tan x)(1 + \tan^2 x)(1 + \tan^4 x)(1 + \tan^8 x)\ldots\, dx$

12. **Evaluate** $\displaystyle\int \frac{\tan^2(x/2)\sqrt{\sin x \tan x}}{\sqrt{\cos^2 x + \cos x + 1}}\, dx$

13. *Evaluate* $\displaystyle\int \frac{\sqrt{2 - x - x^2}}{x^2}\, dx$

Other u subs and misc

1. Evaluate $\displaystyle\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$

2. Evaluate $\displaystyle\int \frac{dx}{x^2(x^3-1)^{\frac{3}{2}}}$

3. Evaluate $\displaystyle\int (1+\ln x)x^{2x}\,dx$

4. Evaluate $\displaystyle\int \frac{(x-1)^{\ln(x+1)}}{(x+1)^{\ln(x-1)}}\,dx$

5. Evaluate $\displaystyle\int \left(\frac{x^4}{x^6+1}\right)^2 dx$

6. *Evaluate* $\displaystyle\int \left[\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x\right]\ln x\,dx$

7. Evaluate $\displaystyle\int_0^1 (1-x^2)^{30}x^3\,dx$

8. *Evaluate* $\displaystyle\int \frac{x^2-1}{x^2+1}\frac{dx}{\sqrt{1+x^4}}$

By parts

1. Evaluate $\displaystyle\int_0^\infty \tan^{-1}\left(\frac{1}{x^2}\right)dx$

2. *Evaluate* $\displaystyle\int \exp\left(\frac{2}{1-\sqrt{x}}\right)dx$

3. Evaluate $\displaystyle\int (\sin^{-1}x - \cos^{-1}x)^2\,dx$

4. *Evaluate* $\displaystyle\int \frac{\ln x - 1}{(x+\ln x)^2}\,dx$

5. Evaluate $\displaystyle\int_0^\infty \ln\left(\frac{x^2+a^2}{x^2+b^2}\right)dx$

6. Evaluate $\displaystyle\int \frac{\sin^{-1}x}{x^3}\,dx$

7. Evaluate $\displaystyle\int \sin(\sqrt[3]{x})\,dx$

8. **Evaluate** $\displaystyle\int \left(\frac{x}{x\sin x + \cos x}\right)^2 dx$

9. *Evaluate* $\displaystyle\int \left(\frac{1}{\ln x} - \frac{1}{\ln^2 x}\right)dx$

10. **Evaluate** $\displaystyle\int \frac{(2-x^2)e^x}{(1-x)\sqrt{1-x^2}}\,dx$

Definite integral tricks

1. Evaluate $\displaystyle\int_0^\infty \frac{dx}{1+x^3}$ without partial fractions

2. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi\tan x}{2+2\tan x}\right)dx$

3. *Evaluate* $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x\ln(\sin x)\,dx$

4. *Evaluate* $\displaystyle\int_0^\pi \frac{2x^3 - 3\pi x^2}{(1+\sin x)^2}\,dx$

5. Evaluate $\displaystyle\int_0^\infty \frac{\ln x}{1+x^2}$

6. *Evaluate* $\displaystyle\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x}\,dx$

7. Evaluate $\displaystyle\int_0^{2\pi} \frac{x}{\sqrt{2-\sin x}}\,dx$

8. *Evaluate* $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1+\sqrt{\sin 2x}}}\,dx$

9. *Evaluate* $\displaystyle\int_0^4 \frac{\ln x}{\sqrt{4x-x^2}}\,dx$

10. *Evaluate* $\displaystyle\int_0^\infty \frac{\ln x}{x^2+2x+4}\,dx$

11. **Evaluate** $\displaystyle\int_0^1 \frac{\tan^{-1}x}{x^2+x+2}\,dx$

Finally, and this is NOT a Kings rule problem, evaluate

$$\int_0^{\frac{\pi}{2}} \frac{x\cos x}{2+\sqrt{1-\sin x}+\sqrt{1+\sin x}}\,dx.$$