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| **Set:**  Intersection:A∩B. A, B are disjoint if A∩B =∅. A∪B=B↔A⊆B A∩B=B↔A⊇B. Difference:𝐀\𝐁 ≝ 𝐀 ∩ 𝐁𝒄  Symmetric Difference(xor): 𝐀 ⊕ 𝐁 ≝ (𝐀\𝐁) ∪ (𝐁\𝐀) Complementation A∪(A𝒄 )= 𝒰. A∩ (A𝒄 )= Ø  Commutativity A∪B=B∪A , A ∩ B = B ∩ A Identity A∪Ø=A , A∩𝒰=A  Associativity (A∪B)∪C=A∪(B∪C)  Distribution:A∪(B∩C)= (A∪B) ∩ (A∪C ), A∩ (B∪C)= (A∩B) ∪ (A∩C )  Idempotence ：A ∩ A = A A∪A = A  De Morgan laws (A∪B)c = A c ∩Bc , (A∩B)c = A c∪Bc  Double Complementation (A c)c=A  Annihilation A∩ Ø =A A∪𝒰 = 𝒰  Complement 补足  If A = B, then dual(A) = dual(B)。eg: A = (S∪T)∩ 𝒰, dual(A) = (S∩T)∪Ø.( ∩与∪,Ø与𝒰交换)  Eg: if Σ ={a,b,c} Σ \*Σ ={aa,bb,cc,ab,ba,ac,ca,bc,cb} 是有顺序的！  **Binary Relation**  Ø: 所有元素都没关系 𝒰：每个元素都与其他任意一个元素有关系  Reflexivity (R) all x ∈ S: (x,x) ∈ R. Antireflexivity (AR): all x ∈ S: (x,x) ∉ R  Symmetry (S): all x,y ∈ S: if (x,y) ∈ R then(y,x) ∈ R Antisymmetry (AS) : all x,y ∈ S: if (x,y) ∈ R and (y,x) ∈ R, then x=y  Transitivity (T): all x,y,z ∈ S, if (x,y) and (y,z) ∈ R, then (x,z) ∈ R.  R AR S AS T  = √ X. √ √. √  ≤. √. X. X √. √  <. X. √. X. √. √  Ø X. √. √ √. √  𝒰 √. X. √. X. √  1. √. X. X. X. √  = mod3√. X. √. X. √  做题方法：1.画两个椭圆 2.画点矩阵（eg: card(A) = m,card(B) = n how many relations on A?2|mXn| (R)or (AR)?2m^2-m(S) 2(m(m+1)/2),function?nm）  Equivalence relations:(R) (S) (T) Class:[s] = {t:t∈ S and sRt}. sRt iff [s] = [t] Partitions:~ ⊆ SxS, s ~t:s and t in same Si .  Eg:m~n iff m2=n2(mod5) S = {1…7}.Find all the equivalence classes: [1]={1,4,6}=[4]=[6] [5]={5} [3] ={2,3,7}=[2]=[7]  证明两个R相等方法：1.列举每个R的元素 2.证明两个R互为子集  找Equivalence Class 个数：eg：Equivalence relation R ⊆ Σ\*X Σ\*，Σ ={a,b} , L={w∈ Σ \*:3|len(w)}, class:空集,[a],[aa].分类依据，w%3为0，1，2  Partial order: (R)(AS)(T)(eg:(Z,≤),(Pow(X),⊆ )for some set X, (N,|), not posets(Z,<)(Z,|)  Hasse diagram  Pow(X) ordered by ⊆ : glb(A,B) = A ∩ B lub(A,B) = A∪B  Total order: (L) Linearity(any elements are comparable)  Topological sort:  Lexicographic order：按字典排序Lenlex  Filing order  **Function:**  1.二元关系 2.每个input都有唯一1个output  F1:N→Z，given by f(x)=x2中Dom(f)is N(x ∈ N),Codom(f)is Z, i mage is x2  F2:N→N，given by f(x)=x2中Dom(f)is N(x ∈ N),Codom(f)is N, lm(f) image is x2  f1 f2 are different function  g o f = g(f(x)) f的output必须在g的input里。f o f o f 可写成f3，eg:sin2(x)=(sin(x)2  Identity function on S: Ids(x)=x,x∈ S. eg:S→T, g o Ids=g, IdT o g = g  **Surjective(onto):**lm(f)=Codom(f) **/ Injective:**1对1(if same output →same input)**/ bijective**: surjective and injective  F2:N→N，given by f(x)=x2中Dom(f)is N(x ∈ N),Codom(f)is N, image is x2  f1 f2 are different function  A screenshot of a cell phone  Description automatically generated | **Recursion and Induction：**  **如何give a definition：**  **1. unwinding（硬找规律）:**   |  |  |  | | --- | --- | --- | | **f(0)=1, f(n) = 2f(n-1)**  **unwinding：** |  |  |   **(eg:，**  Big-Oh:Asymptotic Upper Bounds g(n) ≤ c · f(n) g ∈ O(f (n)) (R) (T)  Big-Omega: Asymptotic Lower Bounds g(n) ≥ c · f(n) g ∈ Ω(f (n)) (R) (T)（eg: g(n)=3n+1 →g(n)≥3n,for all n≥1,thus,,3n+1∈ Ω (n)  Big-Theta: tight bound c · f(n) ≤ g(n) ≤ d · f(n) g ∈ Θ(f ) (R) (S) (T)  If g(n) ∈ O(f(n)) thus f(n) ∈ Ω g(n)    O(log2n)= O(log3n)= ….O(log10n)=….. (回忆：2logn=n)  O(rn) ⊊O(sn) ⊊O(tn) for r<s<t. O(nk) ⊊ O(nl) ⊊O(nm) fro k<l<m  复杂度排序：    O(n1.5)=O(n√n) ≤n2/log(n)≤O(n2)  (1.1)logn=(2log(1.1))logn=(2logn)log(1.1)=2log(1.1)  General Result (From Lecture)  T(n) = T(n − 1) + bnk  solution T(n) = O(nk+1)  T(n)=cT(n−1)+bnk,c>1 solutionT(n)=O(cn )  (eg: T(n) = T(n-1) + 2n, thus c=1(so 用第一条), b=2,k=1, T(n) = O(n2) Thus T(n) = n2 + n + 1 is a valid formula for T(n))  Master Theorem  T(n)=aT(n/b)+f(n) where f(n)∈ Θ(nc(logn)k), Let d =logba. Then:  case1: if c<d ,then T(n) = O(nd)  case2: if c =d,then T(n) = O(nc(logn)k+1)  case3: if c>d then T(n) = O(f(n))  iff(n)∈O (h(n)) and g(n)∈O(k(n)) then f(n)+g(n)∈O(h(n)+k(n)),and f(n)g(n)∈O(h(n)k(n)) but f(n)/g(n)∉ O(h(n)/k(n))(eg: n ∈O( n2), n2 ∈O( n2), thus n2/n∉ O(n2/n2)=O(1))    1.结构化证明  setp 1 we can give a definition using the recursive nature of Σ \*  step 2 we first need the recursive definition of concatenation: 再定义一些相关概念  setp 3:Let f(n) be the proposition that 先写命题结论. we will show that f(n) holds for 写条件 by structural induction  Base cast : λ ∈ Σ \*,f(λ)=…… so f(λ)holds.  Inductive case:now suppose f(n)holds. if w ∈ Σ \* then aw in ∈ Σ \* for all a ∈ Σ ，把aw带入只有w的表达式中  2.数字证明  we will show that f(n) holds for 写条件 by induction  Base case：n=0, f(n) =  Inductive step:Now suppose f(n): 将f(n+1)带入f(n)的公式也成立  Graph:  n个点E 最多n(n-1)/2条(∈ O(n2))，in tree E = n-1(∈ O(n)) |