Solving Problems by Searching
Informed Search

COMP3411/9814: Artificial Intelligence

Overview

- Heuristics
- Informed Search Methods
  - Best-first search
  - Greedy best-first search
  - A* search
  - Iterative Deepening A* Search

Informed (Heuristic) Search

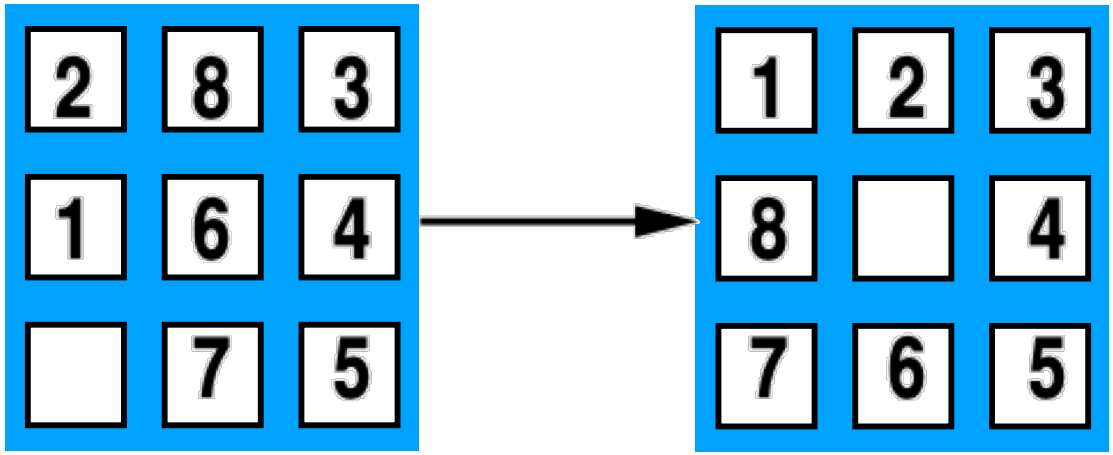
- Informed search strategy
  - uses problem-specific knowledge
  - finds solutions more efficiently than an uninformed strategy
- Uninformed search algorithms given no information about the problem other than its definition.
  - some can solve any solvable problem, none of them can do it efficiently
- Informed search algorithms, can do well given guidance on where to look for solutions.
- Implemented using a priority queue to store frontier nodes

Heuristics

- Heuristics are “rules of thumb” for deciding which, among several alternative courses of action, promises to be the most effective.
- Heuristic must be an underestimate of actual cost to get from current node to a goal
  - Called an admissible heuristic
- Denoted $h(n)$
  - $h(n) = 0$ when ever n is a goal node

Heuristics — Example

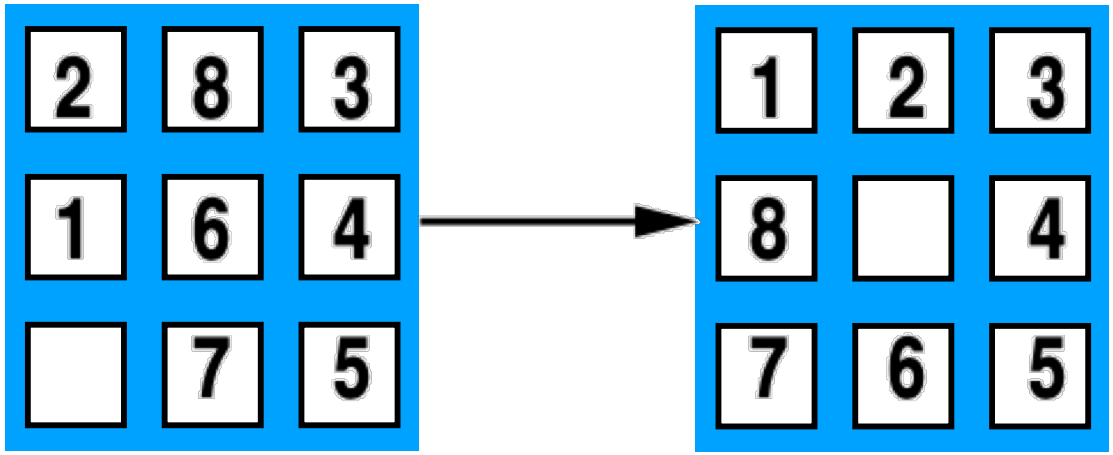
8-Puzzle — number of tiles out of place



$h(n) = 5$

Heuristics — Example

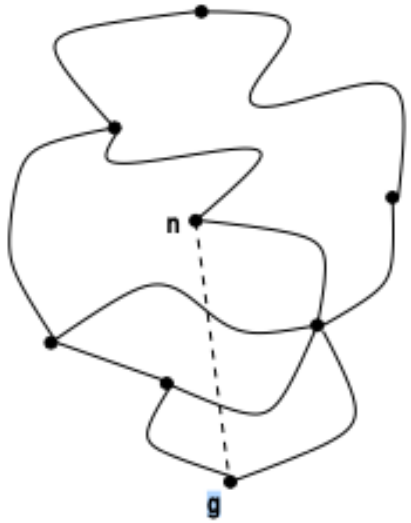
8-Puzzle — Manhattan distance (distance tile is out of place)



$$h(n) = 1 + 1 + 0 + 0 + 0 + 1 + 1 + 2 = 6$$

Heuristics — Example

Another common heuristic is the straight-line distance (“as the crow flies”) from node to goal



Therefore $h(n) =$ distance from n to g

Heuristic Search

- Don't ignore the goal when selecting paths.
- Extra knowledge (heuristics) can guide the search.
- $h(n)$ estimates the cost of shortest path from node n to a goal node.
- $h(n)$ must be efficient to compute.
- h can be extended to paths: $h(\langle n_0, \cdots, n_k \rangle) = h(n_k)$.
- $h(n)$ is an underestimate if there is no path from n to a goal with cost less than $h(n)$.
- An admissible heuristic is a non-negative function that is an underestimate of the actual cost of a path to a goal.

Example Heuristic Functions

- If the nodes are points on a Euclidean plane and the cost is the distance, $h(n)$ can be the straight-line distance from n to the closest goal.
- If the nodes are locations and cost is time, we can use the distance to a goal divided by the maximum speed.
- If goal is to collect a bunch of coins and not run out of fuel, cost is an estimate of how many steps to collect rest of the coins, refuel when necessary, and return to goal.
- A heuristic function can be found by simplifying calculation of true cost

Search Strategies

General Search algorithm:

- add initial state to queue

- repeat:
  - take node from front of queue
  - test if it is a goal state; if so, terminate
  - “expand” it, i.e. generate successor nodes and add them to the queue

Search strategies are distinguished by the order in which new nodes are added to the queue of nodes awaiting expansion.

Search Strategies

- BFS and DFS treat all new nodes the same way:
  - BFS add all new nodes to the back of the queue
  - DFS add all new nodes to the front of the queue

- Best First Search uses an evaluation function $f()$ to order the nodes in the queue
  - Similar to uniform cost search

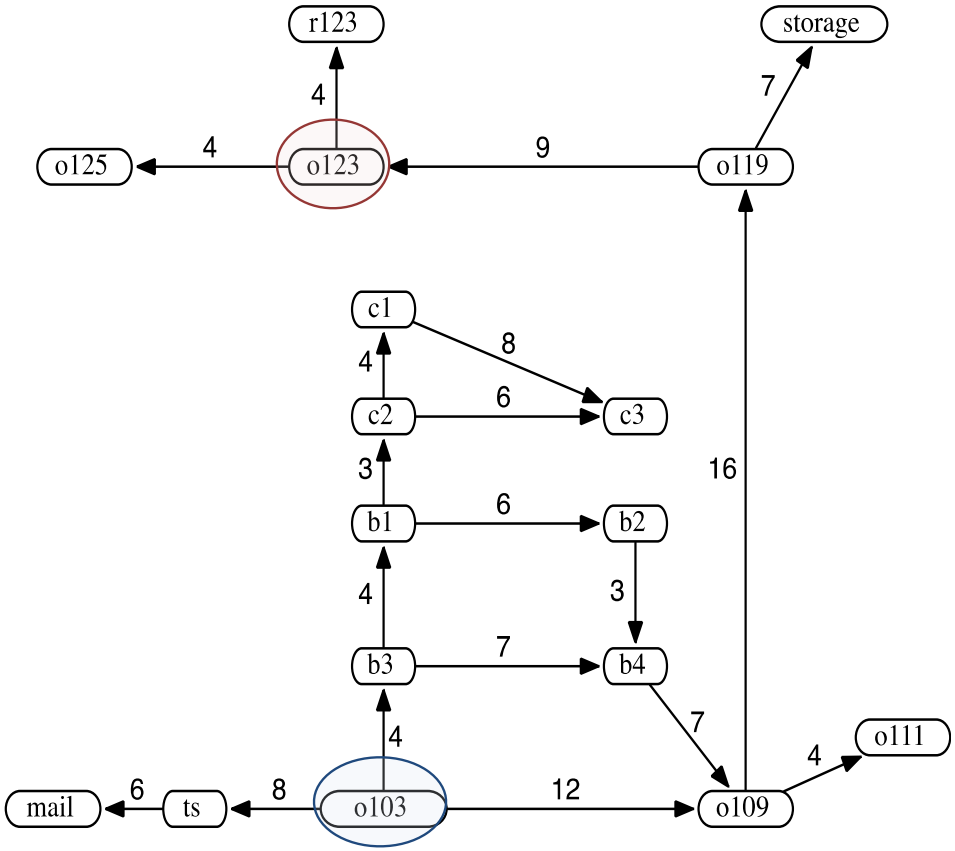
- Informed or Heuristic:
  - Greedy Search $f(n) = h(n)$ (estimates cost from node n to goal)
  - A* Search $f(n) = g(n) + h(n)$ (cost from start to n plus estimated cost to goal)

Delivery Robot Heuristic Function

Use straight-line distance as heuristic, and assume these values:

$$
\begin{array}{lll}
h(mail) = 26 & h(ts) = 23 & h(o103) = 21 \\
h(o109) = 24 & h(o111) = 27 & h(o119) = 11 \\
h(o123) = 4 & h(o125) = 6 & h(r123) = 0 \\
h(b1) = 13 & h(b2) = 15 & h(b3) = 17 \\
h(b4) = 18 & h(c1) = 6 & h(c2) = 10 \\
h(c3) = 12 & h(storage) = 12 &
\end{array}
$$

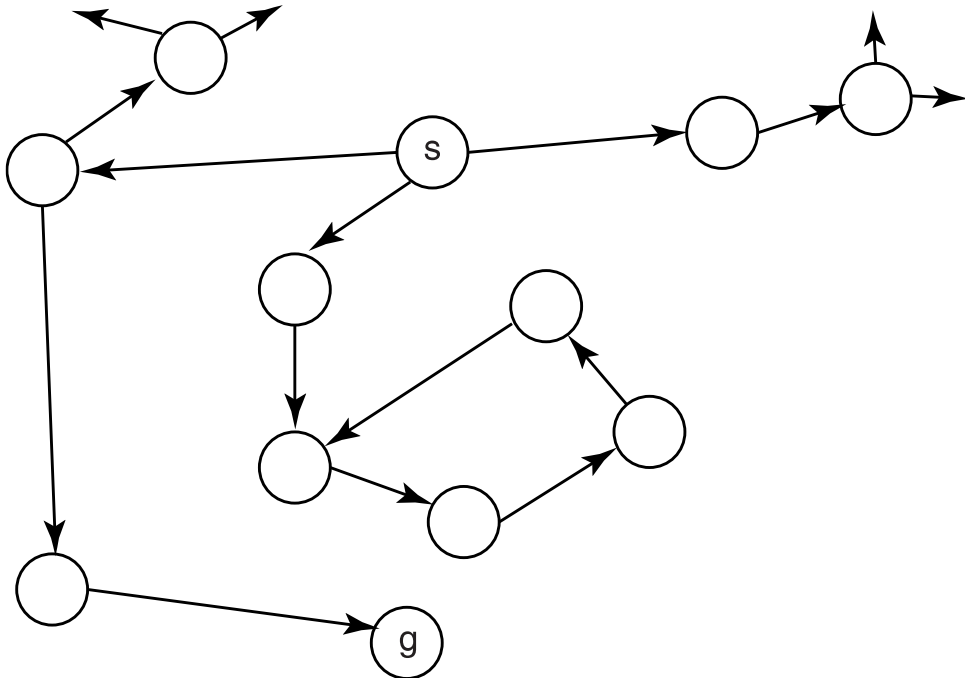
Heuristic function can be extended to paths by making heuristic value of path equal to heuristic value of node at the end of the path: $h(\langle n_0, \cdots, n_k \rangle) = h(n_k)$.



Greedy Best-First Search

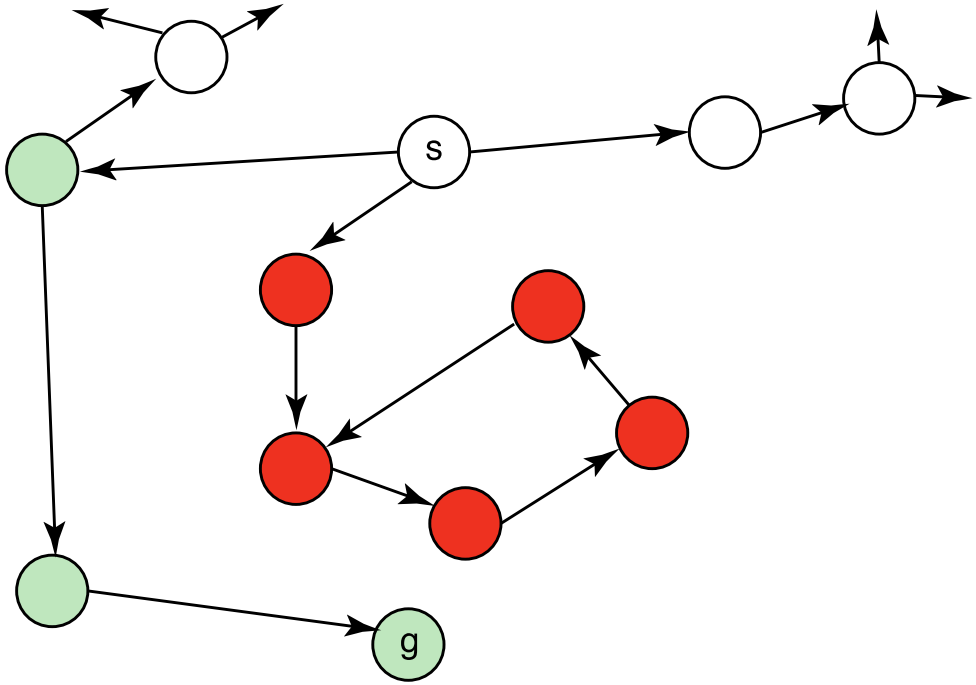
- Select node closest to goal according to heuristic function
- Evaluation function is exactly the heuristic function, i.e. $f(n) = h(n)$
  - $h(n) = 0$ if n is a goal state
  - greedy search minimises the estimated cost to the goal
    - expands node that is estimated to be closest to the goal.
- Frontier is a priority queue ordered by h.
- “Greedy” algorithm takes “best” node first.

Greedy Best-first Search Example



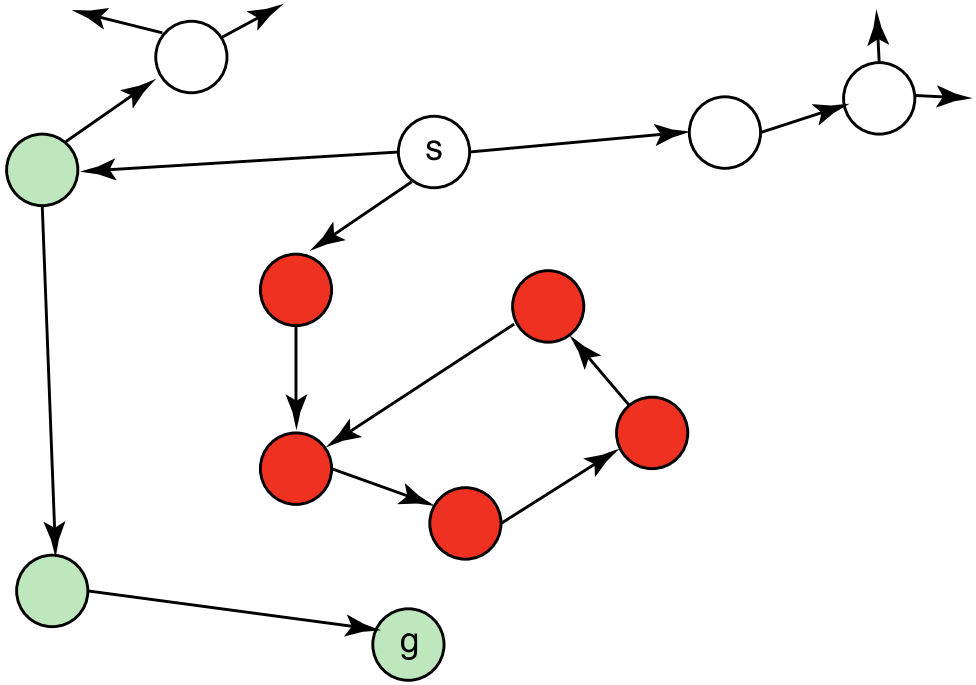
The graph is drawn to scale, where the cost of an arc is its length.
The aim is to find the shortest path from s to g.

Greedy Best-first Search Example



Suppose the Euclidean straight-line distance to the goal g is used as the heuristic function.

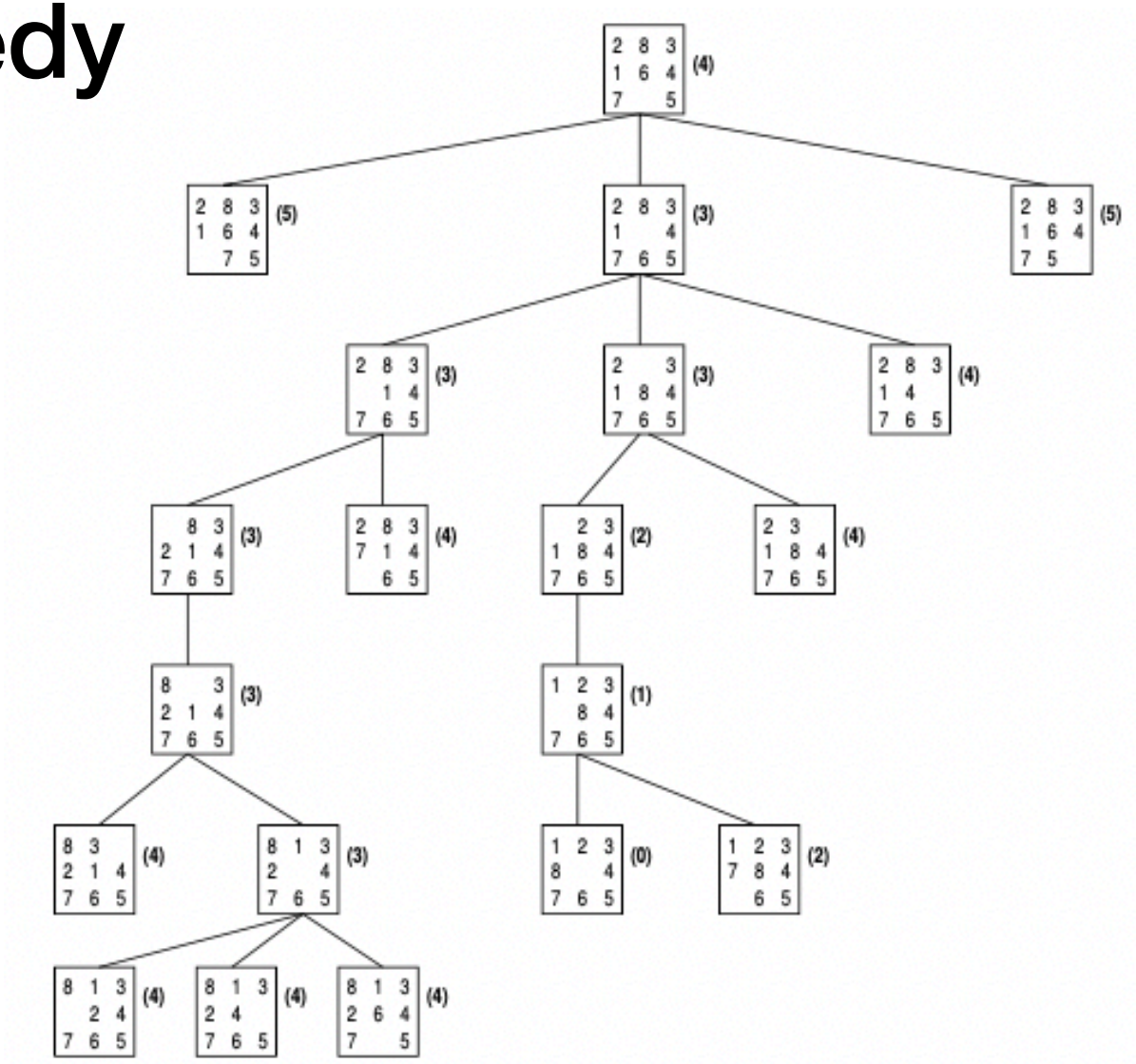
Greedy Best-first Search Example



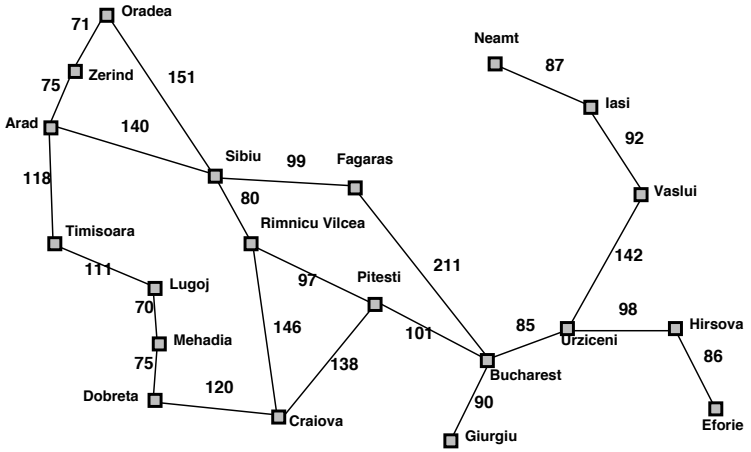
- A greedy depth-first search will select the node below s and never terminate
- Because all nodes below s look good, a greedy best-first search will cycle between them, never trying an alternate route from s.

Examples of Greedy Best-First Search

h is number of tiles out of place

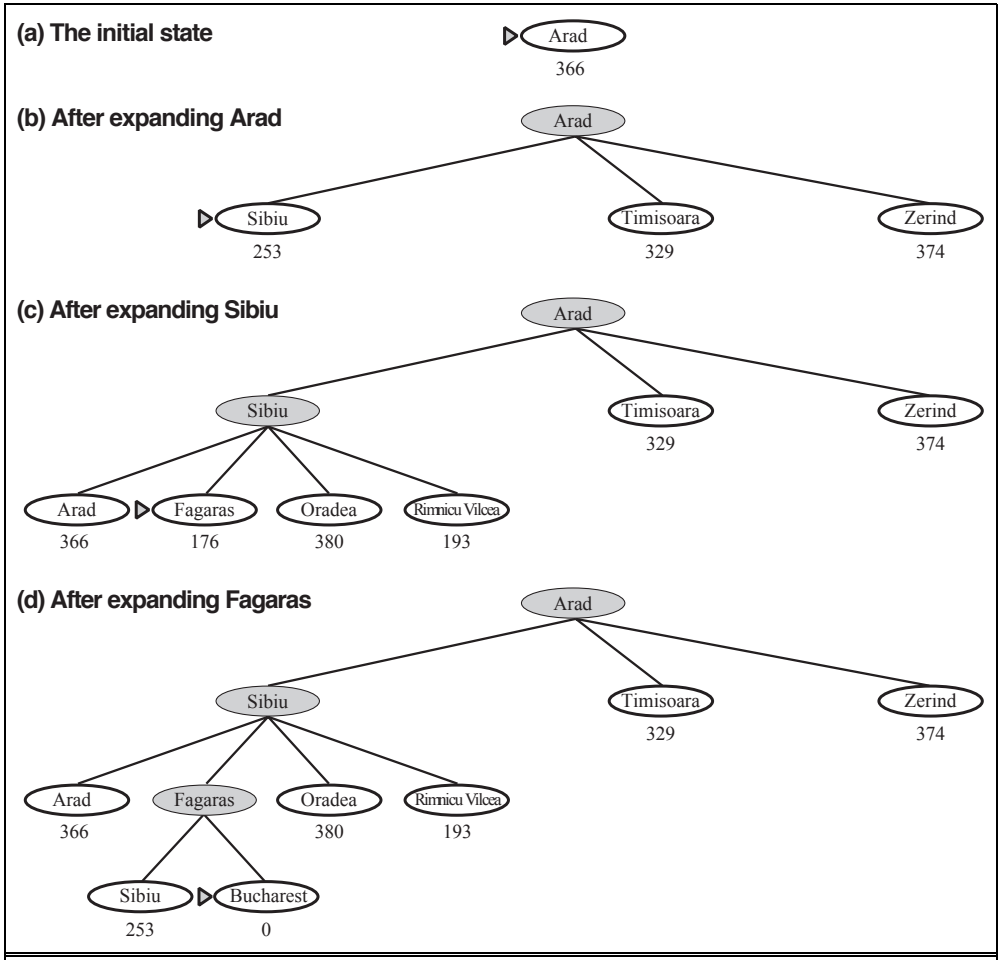


Examples of Greedy Best-First Search



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

- Stages in a greedy best-first tree search for route from Arad to Bucharest with the straight-line distance heuristic.
- Note that **straight-line distances are less than actual distances** in map.



Properties of Greedy Best-First Search

Complete: No. Can get stuck in loops.
(Complete in finite space with repeated-state checking)

Time: $O(b^m)$, where m is the maximum depth in search space.

Space: $O(bm)$ (retains all nodes in memory)

Optimal: No.

- Greedy Search has the same deficits as Depth-First Search.
- However, a good heuristic can reduce time and memory costs substantially.

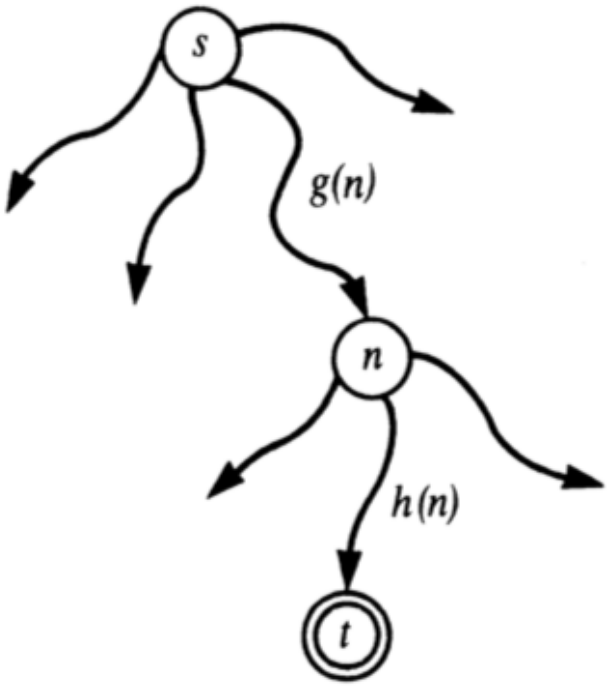
A* Search

- Use both cost of path generated and estimate to goal to order nodes on the frontier
  - $g(n)$ = cost of path from start to n
  - $h(n)$ = estimate from n to goal
- Order priority queue using function $f(n) = g(n) + h(n)$
- $f(n)$ is the estimated cost of the cheapest solution extending this path

A* Search

- Combines uniform-cost search and greedy search
- Greedy Search minimises $h(n)$
  - efficient but not optimal or complete
- Uniform Cost Search minimises $g(n)$
  - optimal and complete but not efficient

Heuristic Function

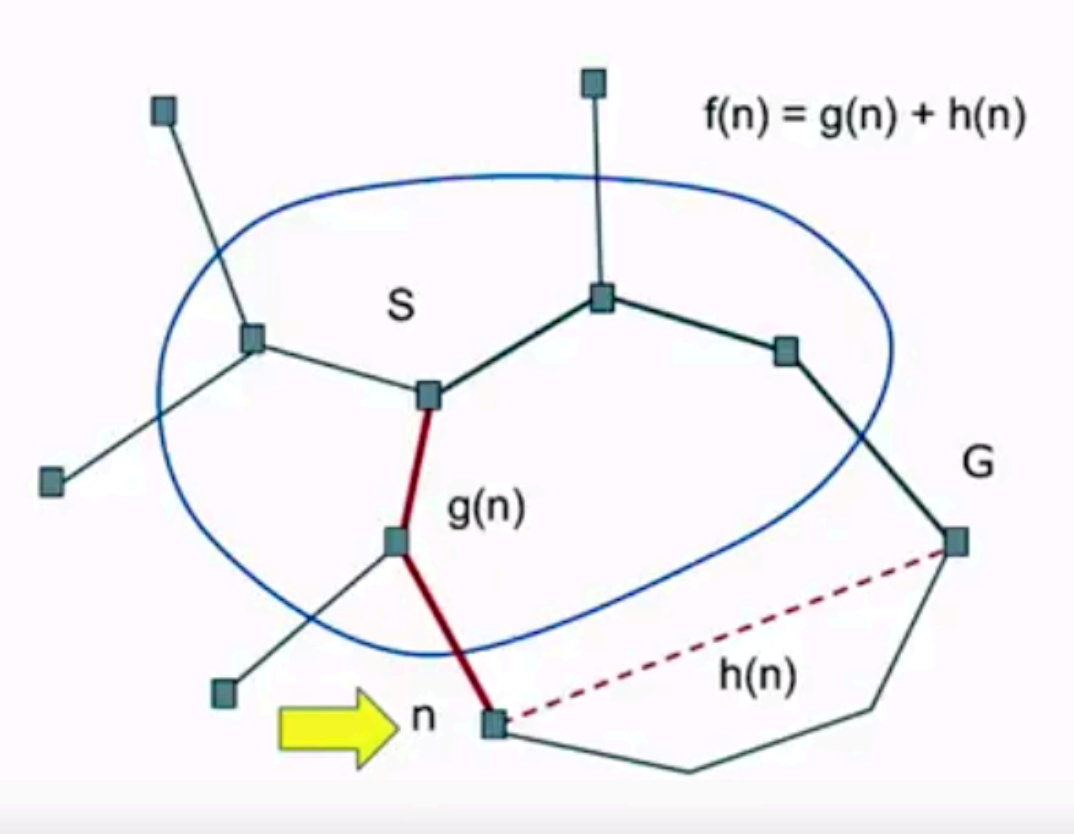


Heuristic estimate $f(n)$ of the cost of the cheapest paths from s to t via n: $f(n) = g(n) + h(n)$

g(n) is an estimate of the cost of an optimal path from s to n

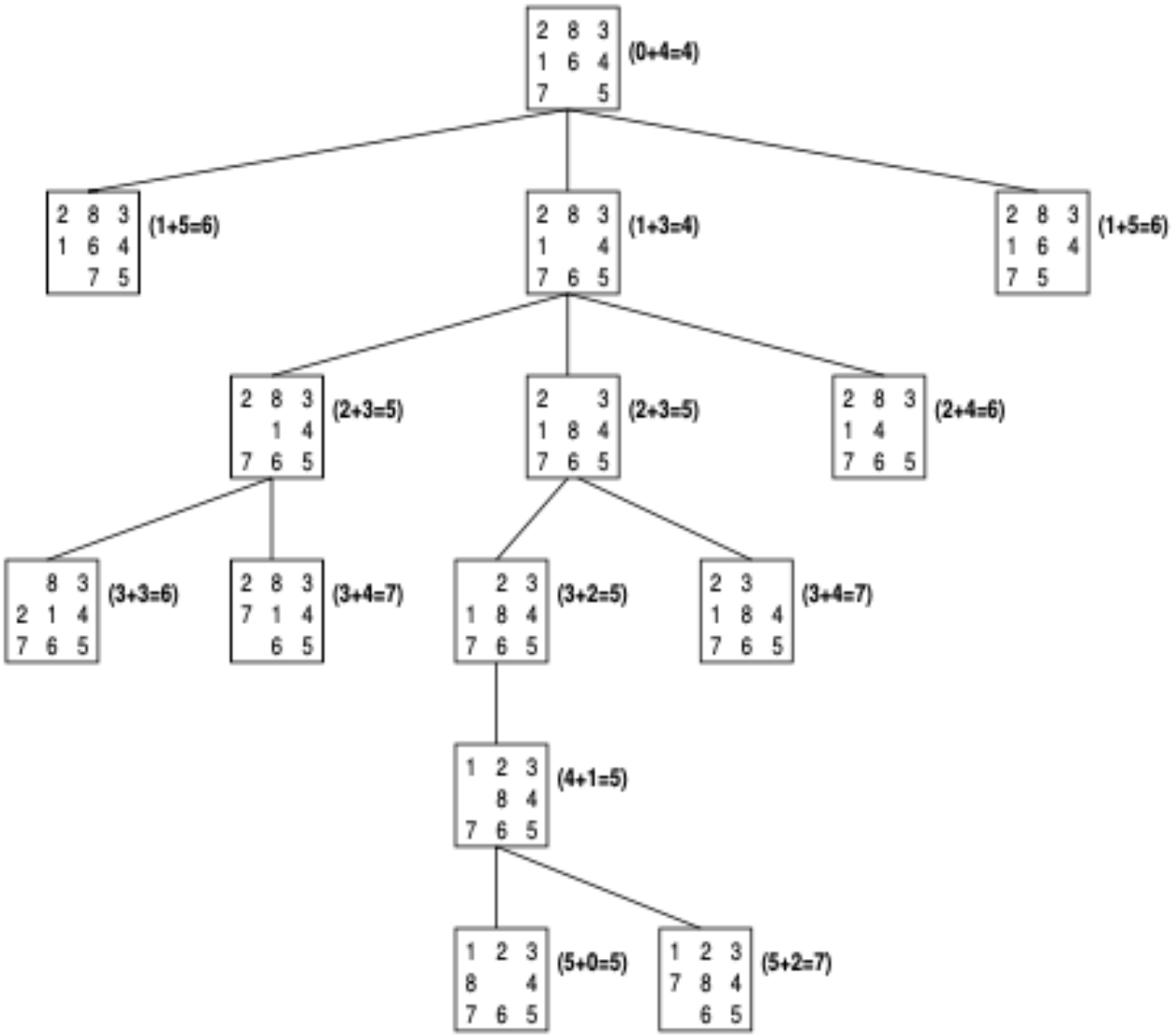
h(n) is an estimate of the cost of an optimal path from n to t.

A* Search



- S = start
- G = Goal
- n = current node
- g(n) = actual cost from **S** to **n**
- h(n) = estimated distance from **n** to **G**

A* Search



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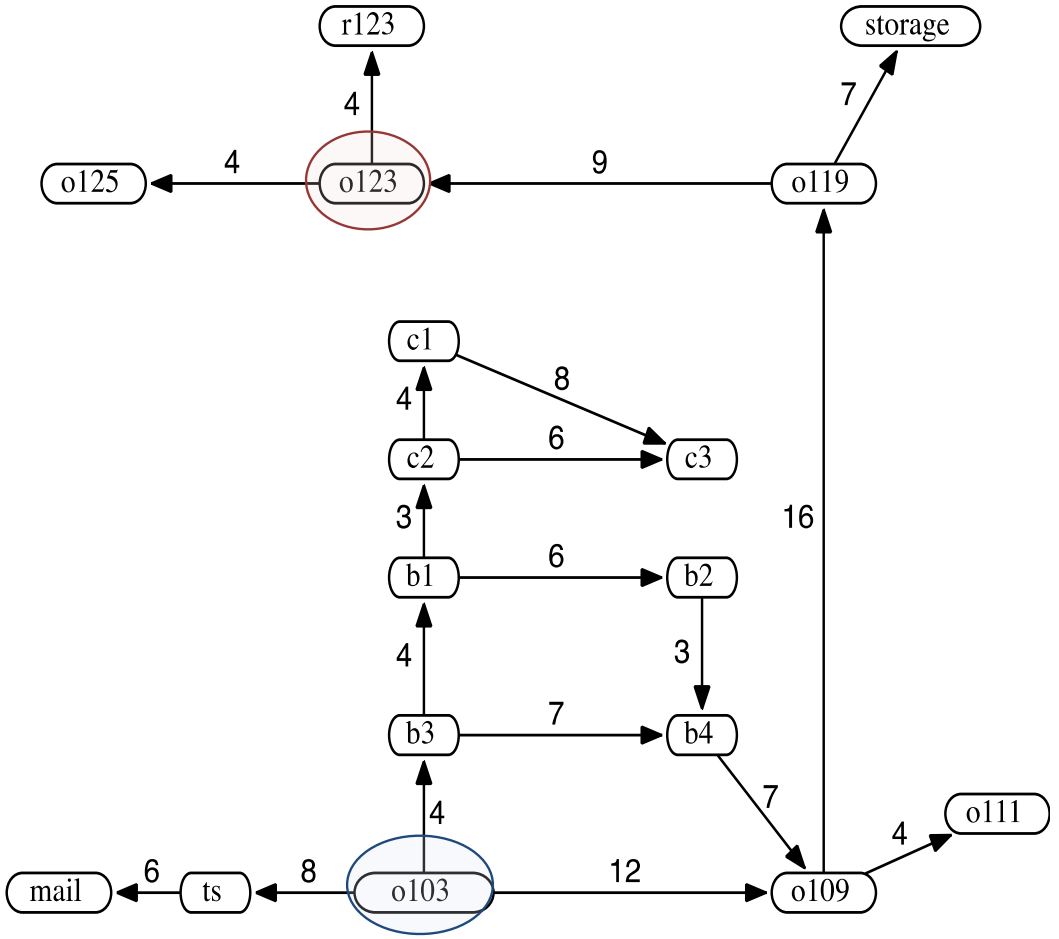
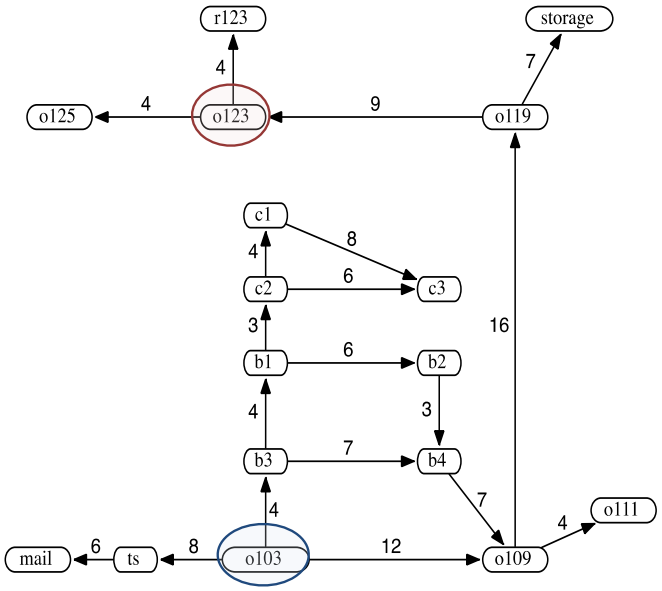


A* Search - the Delivery Robot



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$h(c3) = 12$ $h(storage) = 12$

1. **[o103$_{21}$]** $h(o103) = 21$
2. **[b3$_{21}$, ts$_{31}$, o109$_{36}$]** $f(\langle o103,b3\rangle) = g(\langle o103,b3\rangle) + h(b3) = 4 + 17 = 21$
3. **[b1$_{21}$, b4$_{29}$, ts$_{31}$, o109$_{36}$]**
4. **[c2$_{21}$, b2$_{29}$, b4$_{29}$, ts$_{31}$, o109$_{36}$]**
5. **[c1$_{21}$, b2$_{29}$, b4$_{29}$, c3$_{29}$, ts$_{31}$, o109$_{36}$]**
6. **[b2$_{29}$, b4$_{29}$, c3$_{29}$, ts$_{31}$, c3$_{35}$, o109$_{36}$]**
7. **[b4$_{29}$, ts$_{31}$, c3$_{35}$, o109$_{36}$]**
8. **[ts$_{31}$, c3$_{35}$, b4$_{35}$, o109$_{36}$, o109$_{42}$]**

.....

- Lowest-cost path is eventually found.
- Forced to try many different paths, because some temporarily seem to have the lowest cost.
- Still does better than lowest-cost-first search and greedy best-first search.

Optimality of A*

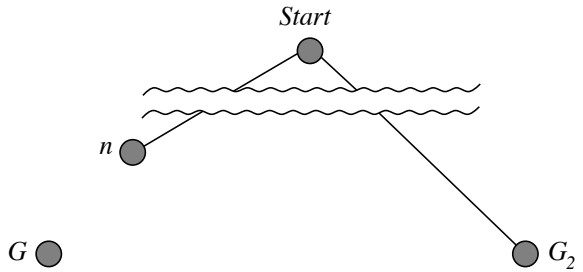
- Heuristic h is said to be admissible if

  $\forall n \ h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n to goal

- If h is admissible then $f(n)$ never overestimates the actual cost of the best solution through n.
  - $f(n) = g(n) + h(n)$, where $g(n)$ is the actual cost to n and $h(n)$ is an underestimate
- Example: h = *straight line distance* is admissible because the shortest path between any two points is a line.
- A^* is optimal if h is admissible.
- Admissible heuristics are by nature optimistic because they think the cost of solving the problem is less than it actually is.

Optimality of A* Search

Suppose a sub-optimal goal node G_2 has been generated and is in the queue. Let n be the last unexpanded node on a shortest path to an optimal goal node G.



$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&= g(G) && \text{since } G_2 \text{ is sub-optimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

Hence $f(G_2) > f(n)$, and A* will never select G_2 for expansion because queue is always ordered, e.g. $[\cdots, n, \cdots, G_2]$.

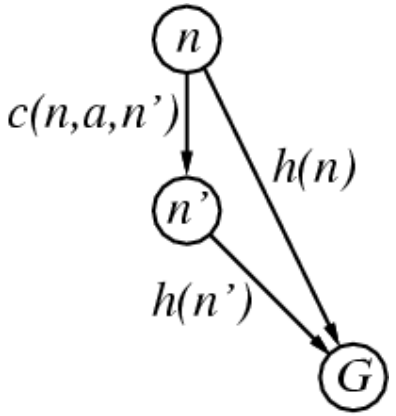
Consistent Heuristics

- A heuristic is consistent if $f(n)$ is nondecreasing along any path
- I.e. for every node n, every successor n' of n generated by any action a,

$$h(n) \leq c(n, a, n') + h(n')$$

If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$



Optimality of A* Search

Complete: Yes, unless infinitely many nodes with $f \leq$ cost of solution

Time: Exponential in *relative error in* $h \times$ *length of solution*

Space: Keeps all nodes is memory

Optimal: Yes (assuming h is admissible).

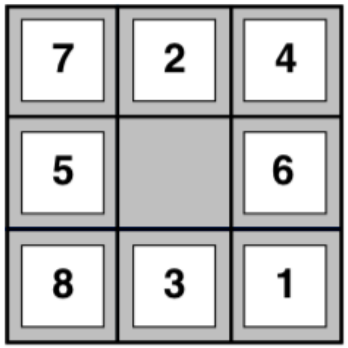
Iterative Deepening A* Search

- Iterative Deepening A* is a low-memory variant of A* that performs a series of depth-first searches but cuts off each search when the f exceeds current threshold, initially $f(start)$.
- The threshold is increased with each successive search.

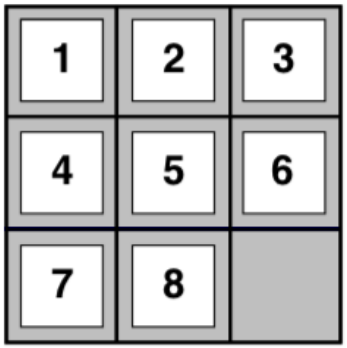
Examples of Admissible Heuristics

$h_1(n)$ = total number of misplaced tiles

$h_2(n)$ = total Manhattan distance = $\sum$ distance from goal position



Start State



Goal State

$h_1(Start) = 6$

$h_2(Start) = 4 + 0 + 3 + 3 + 1 + 0 + 2 + 1 = 14$

How to Find Heuristic Functions ?

- Admissible heuristics can often be derived from the exact solution cost of a simplified or “relaxed” version of the problem. (i.e. with some of the constraints weakened or removed)
  - If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution.
  - If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution.

Dominance

- if $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 dominates h_1 and is better for search. So the aim is to make the heuristic h as large as possible, but without exceeding h^*.

- typical search costs:

14-puzzle	IDS	$= 3{,}473{,}941$ nodes
	$A^*(h_1)$	$= 539$ nodes
	$A^*(h_2)$	$= 113$ nodes
24-puzzle	IDS	$\approx 54 \times 10^9$ nodes
	$A^*(h_1)$	$= 39{,}135$ nodes
	$A^*(h_2)$	$= 1{,}641$ nodes

Summary of Informed Search

- Heuristics can be applied to reduce search cost.
- Greedy Search tries to minimise cost from current node *n* to the goal.
- A* combines the advantages of Uniform-Cost Search and Greedy Search
- A* is complete, optimal and optimally efficient among all optimal search algorithms.
- Memory usage is still a concern for A*. IDA* is a low-memory variant.

Summary

- Informed search makes use of problem-specific knowledge to guide progress of search
- This can lead to a significant improvement in performance
- Much research has gone into admissible heuristics
  - Even on the automatic generation of admissible heuristics