$$\frac{-x}{1-x} - \ln(1-x) = h\left(\frac{1}{1-x}\right) \leq 0$$

$$\ln(1-\bar{c}\epsilon_k) \geq \frac{-\bar{c}\epsilon_k}{1-\bar{c}\epsilon_k} \geq -2\bar{c}\epsilon_k$$

$$\ln(\tilde{m_k}) \geq \ln(1-\bar{c}\epsilon_k) \geq -2\bar{c}\epsilon_k \geq -2c\epsilon_k \tag{1}$$

$$0 < \psi(\tilde{B_{k+1}}) \leq \psi(\tilde{B_k}) + 3c\epsilon_k + \ln(cos^2\tilde{\theta_k}) + \left[1 - \frac{\tilde{q_k}}{cos^2\tilde{\theta_k}} + \ln\left(\frac{\tilde{q_k}}{cos^2\tilde{\theta_k}}\right)\right] \tag{2}$$

$$\sum_{j=0}^{\infty}\left(\ln\left(\frac{1}{cos^2\tilde{\theta_j}}\right) - \left[1 - \frac{\tilde{q_j}}{cos^2\tilde{\theta_j}} + \ln\left(\frac{\tilde{q_j}}{cos^2\tilde{\theta_j}}\right)\right]\right) \leq \psi(\tilde{B_0}) + 3c\sum_{j=0}^{\infty}\epsilon_j < +\infty$$

$$\lim_{j\to\infty} \ln\left(\frac{1}{cos^2\tilde{\theta_j}}\right) = 0, \qquad \lim_{j\to\infty}\left(1 - \frac{\tilde{q_j}}{cos^2\tilde{\theta_j}} + \ln\left(\frac{\tilde{q_j}}{cos^2\tilde{\theta_j}}\right)\right) = 0$$

$$\lim_{j\to\infty} cos\tilde{\theta_j} = 1, \qquad \lim_{j\to\infty} \tilde{q_j} = 1 \tag{3}$$