Prueba:

$$\|\tilde{y_k} - \tilde{s_k}\| \leq \bar{c}\epsilon_k \|\tilde{s_k}\|, \quad \|\tilde{s_k} - \tilde{y_k}\| \leq \bar{c}\epsilon_k \|\tilde{s_k}\|$$

$$(1 - \bar{c}\epsilon_k)\|\tilde{s_k}\| \leq \|\tilde{y_k}\| \leq (1 + \bar{c}\epsilon_k)\|\tilde{s_k}\| \tag{1}$$

$$(1 - \bar{c}\epsilon_k)^2\|\tilde{s_k}\|^2 - 2\tilde{y_k}^T\tilde{s_k} + \|\tilde{s_k}\|^2 \leq \|\tilde{y_k}\|^2 - 2\tilde{y_k}^T\tilde{s_k} + \|\tilde{s_k}\|^2 \leq \bar{c}^2\epsilon_k^2\|\tilde{s_k}\|^2,$$

$$2\tilde{y_k}^T\tilde{s_k} \geq (1 - 2\bar{c}\epsilon_k + \bar{c}^2\epsilon_k^2 + 1 - \bar{c}^2\epsilon_k^2)\|\tilde{s_k}\|^2 = 2(1 - \bar{c}\epsilon_k)\|\tilde{s_k}\|^2$$

$$\tilde{m_k} = \frac{\tilde{y_k}^T\tilde{s_k}}{\|\tilde{s_k}\|^2} \geq 1 - \bar{c}\epsilon_k \tag{2}$$

$$\tilde{M_k} = \frac{\|\tilde{y_k}\|^2}{\tilde{y_k}^T\tilde{s_k}} \leq \frac{1 + \bar{c}\epsilon_k}{1 - \bar{c}\epsilon_k} \tag{3}$$

$$\tilde{M_k} \leq 1 + \frac{2\bar{c}}{1 - \bar{c}\epsilon_k}\epsilon_k \leq 1 + \bar{c}\epsilon_k \tag{4}$$

$$\frac{-x}{1-x} - \ln(1-x) = h\left(\frac{1}{1-x}\right) \leq 0$$

$$\ln(1 - \bar{c}\epsilon_k) \geq \frac{-\bar{c}\epsilon_k}{1 - \bar{c}\epsilon_k} \geq -2\bar{c}\epsilon_k$$

$$\ln(\tilde{m_k}) \geq \ln(1 - \bar{c}\epsilon_k) \geq -2\bar{c}\epsilon_k \geq -2c\epsilon_k \tag{5}$$

$$0 < \psi(\tilde{B_{k+1}}) \leq \psi(\tilde{B_k}) + 3c\epsilon_k + \ln(cos^2\tilde{\theta_k}) + \left[1 - \frac{\tilde{q_k}}{cos^2\tilde{\theta_k}} + \ln\left(\frac{\tilde{q_k}}{cos^2\tilde{\theta_k}}\right)\right] \tag{6}$$

$$\sum_{j=0}^{\infty}\left(\ln\left(\frac{1}{cos^2\tilde{\theta_j}}\right) - \left[1 - \frac{\tilde{q_j}}{cos^2\tilde{\theta_j}} + \ln\left(\frac{\tilde{q_j}}{cos^2\tilde{\theta_j}}\right)\right]\right) \leq \psi(\tilde{B_0}) + 3c\sum_{j=0}^{\infty}\epsilon_j < +\infty$$

$$\lim_{j\to\infty}\ln\left(\frac{1}{cos^2\tilde{\theta_j}}\right) = 0, \qquad \lim_{j\to\infty}\left(1 - \frac{\tilde{q_j}}{cos^2\tilde{\theta_j}} + \ln\left(\frac{\tilde{q_j}}{cos^2\tilde{\theta_j}}\right)\right) = 0$$

$$\lim_{j\to\infty} cos\tilde{\theta_j} = 1, \qquad \lim_{j\to\infty}\tilde{q_j} = 1 \tag{7}$$

lo del final

$$
\begin{aligned}
\frac{\|G_*^{-1/2}(B_k - G_*)s_k\|^2}{\|G_*^{1/2}s_k\|} = \frac{\|(\tilde{B}_k - I)\tilde{s}_k\|^2}{\|\tilde{s}_k\|^2} \\
&= \frac{\|\tilde{B}_k\tilde{s}_k\|^2 - 2\tilde{s}_k^T\tilde{B}_k\tilde{s}_k + \tilde{s}_k^T\tilde{s}_k}{\tilde{s}_k^T\tilde{s}_k} \\
&= \frac{\tilde{q}_k^2}{cos\tilde{\theta}_k^2} - 2\tilde{q}_k + 1.
\end{aligned}
$$

$$
\lim_{k\to\infty} \frac{\|(B_k - G_*)s_k\|}{\|s_k\|} = 0
$$