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Assignment 1

1 Understand the classifiers

1.1 Previous Research

1. P = 10'000 and n = 100
   Training sample accuracy is 99% and the 50% accuracy for the test sample.

   Answer:
   A 99% accuracy on the training sample suggests that the model is performing very well on the training data 99% of the time the model has a correct prediction. As for the actual test sample only half of the time the model has a correct prediction. From the data set we can see that it is a classic case of the curse of dimensionality, having a high number of predictors compared to the number of observations. This suggests that there is a risk of overfitting. As the number of features is much more than the number of observations the model can become overly complex, capturing noise in the data rather than meaningful patterns as we can see in our example where the training sample accuracy is 99% compared to the actual test sample.

2. Training data error rate – 10%, Test data error rate – 15%, K=1 average error rate = 9%.

   Answer:
   I believe their decision to choose the KNN as their best model is a bad decision. With 100 observations and 10,000 features using a KNN model with K=1 will be less meaningful compared to a logistic regression, because the model will simply be assigned to the class of the single nearest neighbor risking overfitting the data, which may not be the best choice for a data with high dimensions. Using the KNN K= 1 model could essentially lead to overfitting due to the curse of dimensionality. Thus, it is more appropriate to choose the logistic model because it is more resistant to overfitting and can generalize better.

3. Income exceeds 10,000 Kenyan Shilling ⇒ Person adopts mobile money.

   Answer:
   The parameter value associated with the income variable in a training sample trained using a logistic regression will be (c) β = ∞. The income variable perfectly predicts whether a person adopts mobile money, indicating there is a perfect separation in the data. Therefore, the parameter value associated with the income variable will approach infinity, to achieve perfect accuracy on the separation by driving the probabilities for the incomes below 10,000 Kenyan Shilling to 0 and for the incomes above 10,000 Kenyan Shilling to 1. We can also see this behaviour from the log odds ratio formula:

   $$\log odds\ ratio = \ln\left(\frac{P(Y=1\,|X)}{1-P(Y=1|X)}\right) = \beta_0 + \beta_1 X$$

   - When $\beta_1 \to \infty$, for X > 10,000 the value $\beta_0 + \beta_1 X$ becomes very large, making P(Y = 1 | X) approach 1.

2 Linear Discriminant Analysis

1.
   Answer:
   Normal density function as a function of x, μ_1 and σ^2:
   $f_k(x) = \frac{1}{\sqrt{2\pi_k\sigma_k}} \exp\left(-\frac{1}{2\sigma_K^2}(x - \mu_k)^2\right)$.

2.

Answer:

From equation 12 and 13 in chapter 4 we have that:

$$p_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi\sigma}} \exp(-\frac{1}{2\sigma^2}(x - \mu_K)^2)}{\sum_{i=1}^{k} \pi_i \frac{1}{\sqrt{2\pi\sigma}} \exp(-\frac{1}{2\sigma^2}(x-\mu_i)^2)} \qquad (12)$$

$$\log(p_k(x)) = x\frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k) \qquad (13)$$

Using the equations we get the following:

$$\log\left(\frac{P_1(x)}{1-P_1(x)}\right) = \log\left(\frac{p_1(x)}{p_2(x)}\right).$$

$$\log\left(\frac{p_1(x)}{p_2(x)}\right) = \log(p_1(x)) - \log(p_2(x)) = x\,\frac{\mu_1}{\sigma^2} - \frac{\mu_1}{2\sigma^2} + \log(\pi_1) - \left(x\,\frac{\mu_2}{\sigma^2} - \frac{\mu_2^2}{2\sigma^2} + \log(1 - \pi_1)\right)$$

$$= x\,\frac{\mu_1}{\sigma^2} - \frac{\mu_1^2}{2\sigma^2} + \log(\pi_1) - \frac{x\mu_2}{\sigma^2} + \frac{\mu_2^2}{2\sigma^2} - \log(1 - \pi_1)$$

$$= \frac{x}{\sigma^2}(\mu_1 - \mu_2) - \frac{1}{2\sigma^2}(\mu_1^2 - \mu_2^2) + \log\left(\frac{\pi_1}{1-\pi_1}\right)$$

We can identify that:

- $c_1 = \frac{1}{\sigma^2}(\mu_1 - \mu_2)$
- $c_0 = -\frac{1}{2\sigma^2}(\mu_1^2 - \mu_2^2) + \log\left(\frac{\pi_1}{1-\pi_1}\right)$

3.

Answer:

Both Logistic regression and Linear Discriminant analysis models result in a log – odds ratio with respect to the features/X. Therefore, both models produce linear decision boundaries. The primary difference between the two models is that the coefficients of the Logistic regression model β_0 and β_1 are estimated using maximum likelihood, whereas c_0 and c_1 – coefficients of the LDA are computed using the estimated mean and variance from a normal distribution.

4.

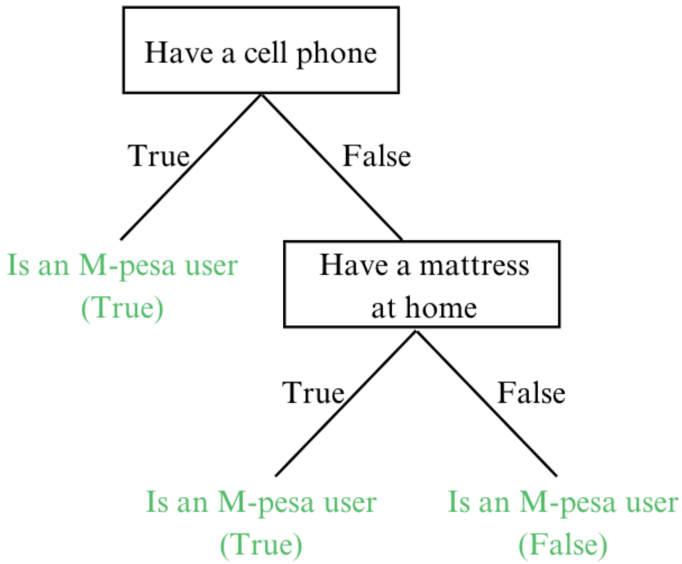
Answer:

Although LDA and logistic regression differ only in their fitting procedures, the predictions of the two models do not always give the same results. LDA assumes that the data comes from a Gaussian distribution with the same covariance matrix for each class, which can lead to better performance than logistic regression when this assumption is roughly true. On the other hand, logistic regression may perform better if the Gaussian assumptions of LDA are not satisfied.

3 Tree Based Methods

1.

Answer:



D*(Large Household size) = 0.918
D*(Have a cell phone) = 0.541
D*(Have a mattress at home) = 0.809
(Calculations done by hand can be found in the appendix)

Thus, “Have a cell phone is chosen as the root node, because it has the lowest entropy).
“Have a cell phone” can then be divided into:

- False – 4 samples
- True – 2 samples
  - This is a leaf node, as it correctly predicts all the train data.

Now considering the 4 samples where “Have a cell phone” is False, we found the following entropies:
D*(Have a mattress at home) = 0
D*(Large Household size) = 0.6885

The “Have a mattress at home” variable completely predicts all the sample data, thus is a leaf node.
*All the calculations can be found in the appendix.

2.

Answer:
The prediction generated by the tree will be: M-pesa user = True

3.

Answer:
The prediction generated by the tree will be: M-pesa user = True

Eco482 A1

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1 Algorithmic implementation

[11]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from sklearn import preprocessing
from sklearn.model_selection import GridSearchCV
from sklearn.metrics import confusion_matrix, ConfusionMatrixDisplay
from sklearn.metrics import roc_auc_score, roc_curve
from sklearn.model_selection import train_test_split, cross_val_score

from sklearn.ensemble import RandomForestClassifier
from sklearn.linear_model import LogisticRegression
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.neighbors import KNeighborsClassifier
```

1.1

[2]:
```
df = pd.read_csv("mobile_money.csv")


df = df.apply(lambda x: x.replace({'yes': True, 'no': False}) if x.dtype ==␣
 ↪'object' else x)

param = ['mpesa_user','cellphone', 'totexppc', 'wkexppc', 'wealth', 'size',␣
 ↪'education_years',
                'pos', 'user_neg', 'ag', 'user_sick', 'sendd', 'recdd',␣
 ↪'bank_acct',
                'mattress', 'sacco', 'merry', 'occ_farmer', 'occ_public',␣
 ↪'occ_prof',
                 'occ_help', 'occ_bus', 'occ_sales', 'occ_ind', 'occ_other',␣
 ↪'occ_ue']

features_name = ['cellphone', 'totexppc', 'wkexppc', 'wealth', 'size',␣
 ↪'education_years',
```

```
                 'pos', 'user_neg', 'ag', 'user_sick', 'sendd', 'recdd',␣
 ↪'bank_acct',
                 'mattress', 'sacco', 'merry', 'occ_farmer', 'occ_public',␣
 ↪'occ_prof',
                  'occ_help', 'occ_bus', 'occ_sales', 'occ_ind', 'occ_other',␣
 ↪'occ_ue']
df = df[param].dropna()
outcome = df['mpesa_user']
df_feat = df[features_name]

df
```

[2]:

```
      mpesa_user  cellphone  totexppc  wkexppc    wealth  size  \
0           True        1.0  150808.0  68640.0  202600.0   1.0
1          False        0.0   33866.4  15246.4   13300.0   5.0
2           True        1.0   36072.8  15194.4  149700.0   5.0
3           True        1.0   21632.0   9412.0   20000.0   5.0
4           True        1.0   29848.0  15236.0   31000.0   5.0
…            …         …        …        …        …     …
2277        True        1.0  150200.0  66456.0   89100.0   5.0
2278        True        1.0   41142.5  30030.0   55450.0   8.0
2279       False        1.0   14102.0  11934.0   57130.0  10.0
2280        True        0.0   21236.5  11381.5  133500.0   8.0
2281       False        1.0   25375.5  19383.0   62500.0   8.0

      education_years  pos  user_neg  ag  …  merry  occ_farmer  occ_public  \
0                 3.0    0         0   0  …  False           0           0
1                 8.0    0         0   0  …   True           0           0
2                 0.0    0         0   0  …   True           0           0
3                12.0    0         1   0  …  False           0           0
4                 0.0    0         1   0  …  False           0           0
…                 …  …        …  ..  …     …      …          …
2277              8.0    0         1   0  …   True           0           0
2278             12.0    0         1   1  …   True           1           0
2279              7.0    0         0   0  …   True           0           0
2280              0.0    0         1   0  …  False           0           0
2281              3.0    0         0   0  …   True           0           0

     occ_prof occ_help occ_bus occ_sales  occ_ind  occ_other  occ_ue
0           0        0       0         0        0          1       0
1           0        0       1         0        0          0       0
2           1        0       0         0        0          0       0
3           1        0       0         0        0          0       0
4           0        0       0         1        0          0       0
…        …       …       …        …       …         …      …
2277        1        0       0         0        0          0       0
2278        0        0       0         0        0          0       0
```

```
2279        1        0        0        0        0        0        0
2280        0        0        0        0        0        1        0
2281        0        1        0        0        0        0        0

[2261 rows x 26 columns]
```

1.2

[3]:
```
outcome.describe()
```

[3]:
```
count      2261
unique        2
top        True
freq       1667
Name: mpesa_user, dtype: object
```

1.3

[20]:
```
grouped_stats = df.groupby('mpesa_user')[features_name].describe()
grouped_stats.T.head(50)
for var in features_name: print(df.groupby('mpesa_user')[var].describe())
```

```
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.427609  0.495149  0.0  0.0  0.0  1.0  1.0
True        1667.0  0.922615  0.267281  0.0  1.0  1.0  1.0  1.0
             count          mean            std     min        25%        50%  \
mpesa_user
False        594.0  54237.974805   93427.263844   480.0  18554.200  31136.065
True        1667.0  84476.756585  102949.985330  2306.0  34411.905  57204.000

                75%        max
mpesa_user
False       53756.5  1576484.0
True        98570.0  1870776.0
             count          mean           std     min        25%        50%  \
mpesa_user
False        594.0  28578.544063  28099.286689     0.0  13000.000  20297.335
True        1667.0  35493.869666  27961.784440  1397.5  18412.625  27726.400

                  75%       max
mpesa_user
False       32844.500  237484.0
True        42734.095  263380.0
             count          mean           std  min      25%      50%  \
mpesa_user
False        594.0  76478.690236  2.970897e+05  0.0   7062.5  20250.0
```

```
True        1667.0  214923.106779  1.460980e+06  0.0  24950.0  54000.0

                 75%         max
mpesa_user
False        50225.0   4753200.0
True        112600.0  47200000.0
             count      mean       std  min  25%  50%  75%   max
mpesa_user
False        594.0  4.126263  2.449328  1.0  2.0  4.0  6.0  12.0
True        1667.0  4.262747  2.176337  1.0  3.0  4.0  6.0  13.0
             count      mean       std  min  25%  50%   75%   max
mpesa_user
False        594.0  6.489899  4.624234  0.0  2.0  7.0  10.0  19.0
True        1667.0  8.373725  5.287427  0.0  5.0  9.0  12.0  19.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.048822  0.215676  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.072585  0.259533  0.0  0.0  0.0  0.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.000000  0.000000  0.0  0.0  0.0  0.0  0.0
True        1667.0  0.529694  0.499267  0.0  0.0  1.0  1.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.139731  0.347000  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.108578  0.311203  0.0  0.0  0.0  0.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.000000  0.000000  0.0  0.0  0.0  0.0  0.0
True        1667.0  0.389922  0.487879  0.0  0.0  0.0  1.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.198653  0.399323  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.615477  0.486628  0.0  0.0  1.0  1.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.173401  0.378912  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.518296  0.499815  0.0  0.0  1.0  1.0  1.0
           count unique    top  freq
mpesa_user
False        594      2  False   432
True        1667      2   True  1188
           count unique   top  freq
mpesa_user
False        594      2  True   496
True        1667      2  True  1145
           count unique    top  freq
mpesa_user
```

```
False        594      2  False   527
True        1667      2  False  1289
           count unique    top freq
mpesa_user
False        594      2  False  356
True        1667      2  False  851
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.367003  0.482394  0.0  0.0  0.0  1.0  1.0
True        1667.0  0.136773  0.343710  0.0  0.0  0.0  0.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.005051  0.070947  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.051590  0.221264  0.0  0.0  0.0  0.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.132997  0.339857  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.240552  0.427547  0.0  0.0  0.0  0.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.085859  0.280391  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.144571  0.351773  0.0  0.0  0.0  0.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.171717  0.377452  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.162567  0.369081  0.0  0.0  0.0  0.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.074074  0.262112  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.112777  0.316415  0.0  0.0  0.0  0.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.031987  0.176112  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.026995  0.162116  0.0  0.0  0.0  0.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.042088  0.200958  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.044991  0.207347  0.0  0.0  0.0  0.0  1.0
             count      mean       std  min  25%  50%  75%  max
mpesa_user
False        594.0  0.089226  0.285309  0.0  0.0  0.0  0.0  1.0
True        1667.0  0.079184  0.270107  0.0  0.0  0.0  0.0  1.0
```

1.4

[5]:
```
#Split the data set into training and test 80-20

x_train, x_test, y_train, y_test = train_test_split(df_feat, outcome,
                                                    test_size = 0.2,␣
 ↪random_state = 21)

#Standardizing the data set
#stage 1
scaler = preprocessing.StandardScaler()
#stage2
scaler.fit(x_train)
#stage 3
x_train_scaled = scaler.transform(x_train)
x_test_scaled = scaler.transform(x_test)

#Logistic regression
log_reg = LogisticRegression()
log_reg.fit(x_train_scaled, y_train)

#Random Forest  Classifier
rforest = RandomForestClassifier()
rforest.fit(x_train_scaled, y_train)

#LDA
lda = LinearDiscriminantAnalysis()
lda.fit(x_train_scaled, y_train)
```

[5]:
```
LinearDiscriminantAnalysis()
```

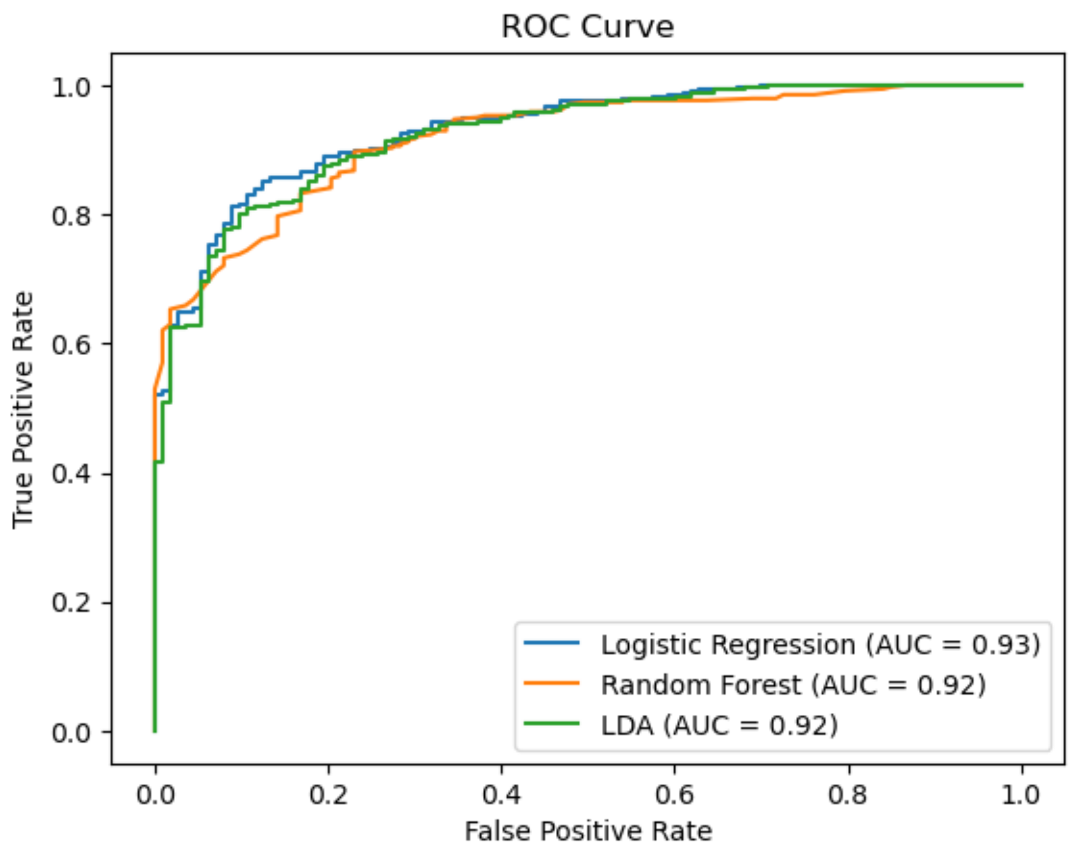
1.5

The best classifier is the Logistic Regression, as can be seen in the following.

[6]:
```
#Getting accuracies for the classifiers
accuracy = dict()

log_score = log_reg.score(x_test_scaled, y_test)

rforest_score = rforest.score(x_test_scaled, y_test)

lda_score = lda.score(x_test_scaled, y_test)

accuracy['Logistic regression accuracy'] = log_score
accuracy['Random forest accuracy'] = rforest_score
accuracy['LDA accuracy'] = lda_score
```

```python
accuracy_df = pd.DataFrame(accuracy, index=[0])
print(accuracy_df)


#Getting the AUC scores for the classifiers

log_auc = roc_auc_score(y_test, log_reg.predict_proba(x_test_scaled)[:, 1])
fpr_log, tpr_log, _ = roc_curve(y_test, log_reg.predict_proba(x_test_scaled)[:,␣
 ↪1])

rf_auc = roc_auc_score(y_test, rforest.predict_proba(x_test_scaled)[:, 1])
fpr_rf, tpr_rf, _ = roc_curve(y_test, rforest.predict_proba(x_test_scaled)[:,␣
 ↪1])

lda_auc = roc_auc_score(y_test, lda.predict_proba(x_test_scaled)[:, 1])
fpr_lda, tpr_lda, _ = roc_curve(y_test, lda.predict_proba(x_test_scaled)[:, 1])

#Plotting the curves
plt.figure()
plt.plot(fpr_log, tpr_log, label=f'Logistic Regression (AUC = {log_auc:.2f})')
plt.plot(fpr_rf, tpr_rf, label=f'Random Forest (AUC = {rf_auc:.2f})')
plt.plot(fpr_lda, tpr_lda, label=f'LDA (AUC = {lda_auc:.2f})')
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
plt.title('ROC Curve')
plt.legend(loc='lower right')
plt.show()
```

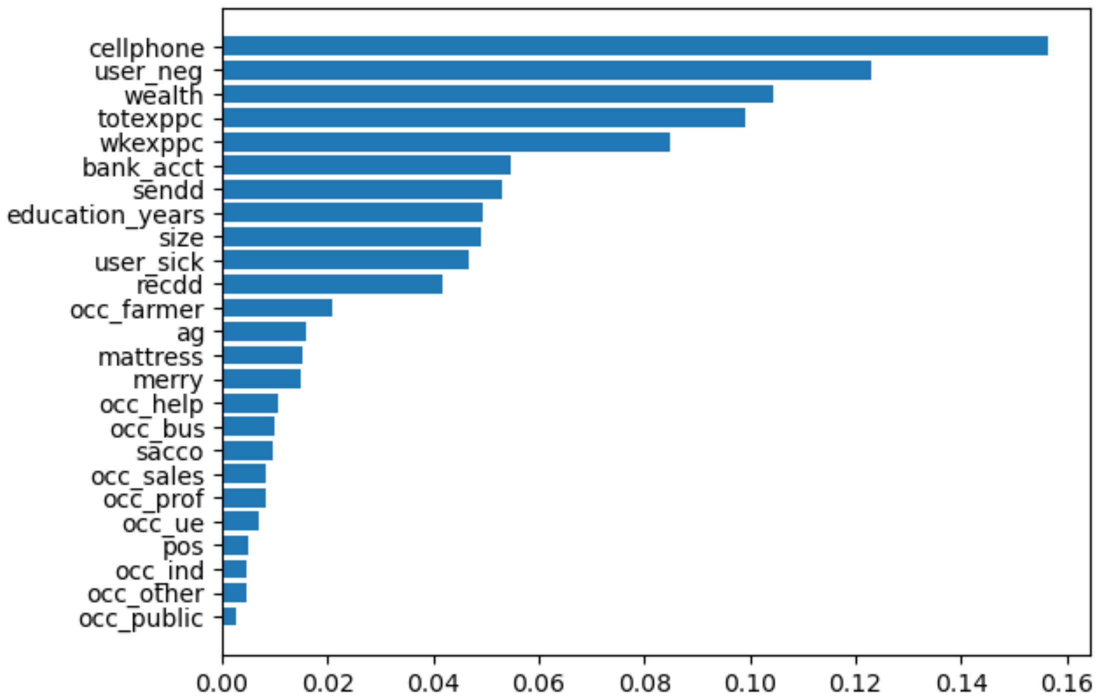
```
   Logistic regression accuracy  Random forest accuracy  LDA accuracy
0                      0.871965                0.863135       0.86755
```



1.6

[7]:
```
snames, simportances = zip(*sorted(zip(features_name, rforest.
 ↪feature_importances_),
                                   key=lambda pair: pair[1]))
plt.barh(snames, simportances)
```

[7]: <BarContainer object of 25 artists>



1.7

[12]:
```python
k_value = range(1, 11)
accuracies = []

for k in k_value:
    knn = KNeighborsClassifier(n_neighbors=k)
    cv_scores = cross_val_score(knn, x_train_scaled, y_train, cv=5)
    accuracies.append(cv_scores.mean())

for k, acc in zip(k_value, accuracies):
    print(f"K={k}: Mean Accuracy={acc:.4f}")
```

```
K=1: Mean Accuracy=0.8081
K=2: Mean Accuracy=0.7799
K=3: Mean Accuracy=0.8346
K=4: Mean Accuracy=0.8258
K=5: Mean Accuracy=0.8490
K=6: Mean Accuracy=0.8440
K=7: Mean Accuracy=0.8523
K=8: Mean Accuracy=0.8468
K=9: Mean Accuracy=0.8518
K=10: Mean Accuracy=0.8507
```

1.8

```
[21]: log_regl = LogisticRegression(solver='liblinear')
param_grid = {'penalty':['l1', 'l2'],
              'C':[0.1, 1, 10, 100]}

grid_search = GridSearchCV(estimator = log_regl, param_grid = param_grid,
                           cv = 5)

grid_search.fit(x_train_scaled, y_train)

for mean_score, params in zip(grid_search.cv_results_['mean_test_score'],␣
 ↪grid_search.cv_results_['params']):
    print(f"C={params['C']}, Penalty={params['penalty']}: Mean CV␣
 ↪Score={mean_score:.4f}")
```

```
C=0.1, Penalty=l1: Mean CV Score=0.8794
C=0.1, Penalty=l2: Mean CV Score=0.8739
C=1, Penalty=l1: Mean CV Score=0.8778
C=1, Penalty=l2: Mean CV Score=0.8766
C=10, Penalty=l1: Mean CV Score=0.8766
C=10, Penalty=l2: Mean CV Score=0.8766
C=100, Penalty=l1: Mean CV Score=0.8766
C=100, Penalty=l2: Mean CV Score=0.8766
```

1.9

In our result, it is observed that when C increases from 0.1 to 100 (meaning regularization becomes weaker), the Mean CV Score stabilizes at approximately 0.8766. This suggests that when C reaches a specific point, additional decrease in regularization does not enhance the model's effectiveness. The optimal result is achieved when C=0.1 with l1 regularization, indicating that a moderate amount of regularization assists the model in managing bias and variance, resulting in improved generalization. Nevertheless, as the C rises, there is a possibility of slight overfitting in the model, leading to a plateau in the score without notable improvements. Regularization helps make sure the model is less complex and reduces the chances of overfitting to noise in the training data, resulting in improved cross-validation performance with optimal regularization (moderate C).

1.10

The Logistic Regression is the best classifier for our data. It has the highest cross-validation accuracy of 0.8794 for C=0.1, L1 penalty and the highest AUC of 0.93 compared to the other classifiers. While KNN, Random Forest and LDA has good accuracy, their accuracy and AUC values are lower compared to the Logistic Regression's.

1.11

The main findings from the research on M-Pesa adoption include: 1.Cell phone possession was identified as the most important factor, with having a bank account, overall wealth, sending money transfers, and receiving money transfers following closely behind. 2.Logistic Regression stood out among the classifiers, delivering top performance with a 0.879 cross-validation accuracy and a 0.93 AUC, establishing it as the most dependable model for forecasting M-Pesa adoption. Additional classifiers such as Random Forest, K-Nearest Neighbors (KNN), and Linear Discriminant Analysis (LDA) had strong performances, but Logistic Regression consistently outshined them in terms of accuracy and AUC. 3.These results indicate that having access to communication technology and being financially included are essential for the adoption of M-Pesa in Kenya.

Appendix:

3. Tree-based methods

1)

Large household size	M-pesa user
False	True
False	False
False	True
True	False
True	True
True	False

Finding D for large household size.

$$D_{False} = -\left(\hat{P}_{F,T}\log_2(\hat{P}_{F,T}) + \hat{P}_{F,F}\log_2(\hat{P}_{F,F})\right)$$

$$= -\left(\frac{2}{3}\log_2\left(\frac{2}{3}\right) + \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right)$$

$$= -\left(0.667\times(-0.585) - 0.333\times 1.585\right)$$

$$= -(-0.390 - 0.528)$$

$$= 0.918$$

$$D_{True} = -\left(\hat{P}_{T,T}\log_2(\hat{P}_{T,T}) + \hat{P}_{T,F}\log_2(\hat{P}_{T,F})\right)$$

$$= -\left(\frac{1}{3}\log_2\left(\frac{1}{3}\right) + \frac{2}{3}\log_2\left(\frac{2}{3}\right)\right)$$

$$= 0.918$$

$$D^*_{household\ size} = \frac{3}{6}\times 0.918 + \frac{3}{6}\times 0.918 = 0.918$$

Finding D for having a cell phone

Have a cell phone	M-pesa user
True	True
False	False
True	True
False	False
False	True
False	False

$$D_{False} = -\left(\frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{3}{4}\log_2\left(\frac{3}{4}\right)\right)$$

$$= 0.811$$

$$D_{True} = -\left(\frac{2}{2}\log_2\left(\frac{2}{2}\right) + 0\log_2(0)\right) = 0$$

$$D^{*}_{\text{have a cellphone}} = \frac{4}{6} \times 0.811 + \frac{2}{6} \times 0 \approx 0.541$$

Finding D for having a mattress at home.

Have a mattress at home	M-pesa user
False	True
False	False
False	True
False	False
True	True
False	False

$$D_F = -\left(\frac{2}{5}\log_2\left(\frac{2}{5}\right) + \frac{3}{5}\log_2\left(\frac{3}{5}\right)\right)$$
$$= 0.971$$

$$D_T = -\left(1 \log(1) + 0 \log(0)\right)$$

$$= 0$$

$$D^{*}_{\text{have a mattress at home}} = \frac{5}{6} \times 0.971 + \frac{1}{6} \times 0$$

$$= 0.809$$

Thus "have a cell phone" will be the root of the tree

Have a cell phone → False (4 samples)
↘ True (2 samples)

→ leads to M-pesa user True for all 2 samples

→ Thus, a leaf node

Now consider entropy on 4 samples of "Have a cell phone = False"

Have a cell phone	Have a mattress at home	M-pesa user
True	False	True
False	False	False
True	False	True
False	False	False
False	True	True
False	False	False

Finding D for Have a mattress at home for the 4 samples when Have a cell phone is False

$$D_F = -\left(\frac{3}{3}\log_2\left(\frac{3}{3}\right) + \frac{0}{3}\log_2\left(\frac{0}{3}\right)\right)$$
$$= 0$$

$$D_T = -\left(1 \log_2(1) + 0 \log_2(0)\right)$$
$$= 0$$

$$D^* = \frac{3}{4} \times 0 + \frac{1}{4} \times 0 = 0 \implies \text{Leaf Node!}$$

Large household size	Have a cell phone	M-pesa user
False	True	True
False	False	False
False	True	True
True	False	False
True	False	True
True	False	False

$$D_F = 0$$

$$D_T = -\left(\frac{2}{3}\log_2\left(\frac{2}{3}\right) + \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right) = 0.918$$

$$D^* = \frac{1}{4} \times 0 + \frac{3}{4} \times 0.918$$

$$= 0.6885$$