

EconS 301: INTERMEDIATE MICROECONOMICS WITH CALCULUS

PRACTICE EXERCISES ON INTERMEDIATE MICROECONOMICS WITH CALCULUS

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This document contains a set of exercises (with answer keys) from the weekly review sessions. These sessions are organized by the teaching assistants of the EconS 301 course (Intermediate Microeconomics with Calculus) at Washington State University. I am thankful to the students taking this course at Washington State University in the 2009-2011 academic years, as well as the teaching assistants (many of them previous students of the class), for their useful comments and suggestions.

For more class materials (including lecture notes, class slides, past exams, etc.) please visit:
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EconS 301 – Intermediate Microeconomics

Review Session #1

1. Suppose the market demand curve for a product is given by $Q^d = 1000 - 10P$ and the market supply curve is given by $Q^s = -50 + 25P$.
 - a. What are the equilibrium price and quantity?
 - b. What is the Inverse Form of the demand curve?
 - c. At the market equilibrium, what is the price elasticity of demand?
 - d. Suppose the price in this market is \$25. What is the amount of excess demand?
2. Suppose demand for good A is given by $Q_A^d = 500 - 10P_A + 2P_B + 0.70I$ where P_A is the price of good A , P_B is the price of some other good B , and I is income. Assume that P_A is currently \$10, P_B is currently \$5, and I is currently \$100.
 - a. What is the elasticity of demand for good A with respect to the price of good A at the current situation?
 - b. What is the cross-price elasticity of the demand for good A with respect to the price of good B at the current situation?
 - c. What is the income elasticity of demand for good A at the current situation?
3. Consider two goods, A and B . For each of the following scenarios, develop the utility function $U(A,B)$ that matches the given information.
 - a. The consumer believes that good A and B are perfect substitutes with one unit of A equivalent to four units of B .
 - b. The consumer believes that good A and B are perfect compliments and always uses three units of B for every unit of A .
4. Consider the utility function $U(x, y) = 3x^2 + 5y$ with $MU_x = 6x$ and $MU_y = 5$.
 - a. Is the assumption that “more is better” satisfied for both goods?
 - b. What is the $MRS_{x,y}$ for this utility function?
 - c. Is the $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?
5. For the following sets of goods draw two indifference curves, U_1 and U_2 , with $U_2 > U_1$. Draw each graph placing the amount of the first good on the horizontal axis.
 - a. Hot dogs and chili (the consumer likes both and has a diminishing marginal rate of substitution of hot dogs for chili).
 - b. Sugar and Sweet'N Low (the consumer likes both and will accept an ounce of Sweet'N Low or an ounce of sugar with equal satisfaction).
 - c. Peanut butter and jelly (the consumer likes exactly two ounces of peanut butter for every ounce of jelly).
 - d. Nuts (which the consumer neither likes nor dislikes) and ice cream (which the consumer likes).
 - e. Apples (which the consumer likes) and liver (which the consumer dislikes).

Answers

Exercise 1.

- a. In equilibrium, we know the quantity supplied must equal the quantity demanded. So we simply set the supply equal to demand, and solve for price.

$$\begin{aligned}Q^d &= Q^s \\1000 - 10P &= -50 + 25P \\1050 &= 35P \\\left(\frac{1050}{35}\right) &= P \\30 &= P\end{aligned}$$

Now that we know the equilibrium price, we can plug it into either the demand or supply to solve for the equilibrium quantity.

Plugging price into Q^d yields:

$$\begin{aligned}Q^d &= 1000 - 10(30) \\Q^d &= 1000 - 300 \\Q^d &= 700\end{aligned}$$

Plugging price into Q^s yields:

$$\begin{aligned}Q^s &= -50 + 25(30) \\Q^s &= -50 + 750 \\Q^s &= 700\end{aligned}$$

Calculating the equilibrium quantity using both Q^d and Q^s is a simple way to check your algebra, as the quantity demanded should equal quantity supplied.

b.

The inverse demand curve can be useful when drawing the supply and demand graph.

$$\begin{aligned}Q^d &= 1000 - 10P \\10P &= 1000 - Q^d \\P &= 100 - .1Q^d\end{aligned}$$

- b. Price elasticity can be thought of as the percentage change in quantity demanded as a result of a one percent change in price. So we have,

$$\varepsilon_{Q,P} = \frac{\text{percentage change in quantity demanded}}{\text{percentage change in price}} = \frac{\Delta Q / Q}{\Delta P / P} = \frac{\Delta Q}{\Delta P} \frac{Q^{-1}}{P^{-1}} = \frac{\Delta Q}{\Delta P} \frac{P}{Q}$$

where $\frac{\Delta Q}{\Delta P} = \frac{\partial Q}{\partial P}$. Thus $\varepsilon_{Q,P} = \frac{\partial Q}{\partial P} \frac{P}{Q}$. In our exercise $\frac{\partial Q}{\partial P}$, the partial derivative of quantity with respect to price, is equal to -10.

$$\varepsilon_{Q,P} = \frac{\partial Q}{\partial P} \frac{P}{Q} = -10 \left(\frac{30}{700} \right) = -0.429$$

So given a one percent increase in the price, we will have a 0.429 percentage decrease in quantity demanded.

- c. Plugging the price of \$25 into the demand function:

$$\begin{aligned} Q^d &= 1000 - 10(25) \\ Q^d &= 1000 - 250 \\ Q^d &= 750 \end{aligned}$$

Plugging the price of \$25 into the supply function:

$$\begin{aligned} Q^s &= -50 + 25(25) \\ Q^s &= -50 + 625 \\ Q^s &= 575 \end{aligned}$$

So we have a demand of 750 units and a supply of 575 units. Thus, the excess demand is simply $Q^d - Q^s = 750 - 575 = 175$.

Exercise 2.

- a. We know the current market prices and income, so we can calculate the market quantity for good A.

$$Q_A^d = 500 - 10(10) + 2(5) + 0.70(100) = 480$$

Using that quantity, we can now calculate the price elasticity of demand for good A.

$$\varepsilon_{Q_A, P_A} = \frac{\partial Q_A}{\partial P_A} \frac{P_A}{Q_A}$$

$$\varepsilon_{Q_A, P_A} = -10 \left(\frac{10}{480} \right)$$

$$\varepsilon_{Q_A, P_A} = -0.208$$

- b. The cross-price elasticity determines how the quantity demanded of good A varies with the price of good B.

$$\varepsilon_{Q_A, P_B} = \frac{\partial Q_A}{\partial P_B} \frac{P_B}{Q_A}$$

$$\varepsilon_{Q_A, P_B} = 2 \left(\frac{5}{480} \right)$$

$$\varepsilon_{Q_A, P_B} = 0.021$$

Simply put, from the price elasticity of demand, we just replace P_A with P_B in both the partial derivative and the numerator of the second term.

- c. Similarly, income elasticity determines how the quantity demanded varies with income.

$$\varepsilon_{Q_A, I} = \frac{\partial Q_A}{\partial I} \frac{I}{Q_A}$$

$$\varepsilon_{Q_A, I} = 0.79 \left(\frac{100}{480} \right)$$

$$\varepsilon_{Q_A, I} = 0.146$$

Exercise 3.

- a. These types of questions are more intuitive than anything. We know that one unit of good A will provide the same level of utility as four units of good B. That is, good A provides four times the utility as good B. And we know the goods are perfect substitutes, so the *MRS* will be constant, and the utility function will be linear. So the utility function can be characterized by,

$$U(A, B) = 4A + B$$

- b. Recall the indifference curves for perfect compliments are straight vertical and horizontal lines at a right angle to each other (with the right angle closest to the origin). This is because a consumer does not derive any extra utility from additional units of one good without the other. In our exercise, one unit of good A is always used with three units of good B. The utility function will take the form,

$$U(A, B) = \min(3A, B)$$

The “min” simply means “take the minimum” of $3A$ and B .

For example, suppose we have 10 units of good B and 3 units of good A . Then we will have,

$$U(A, B) = \min(9, 10) = 9.$$

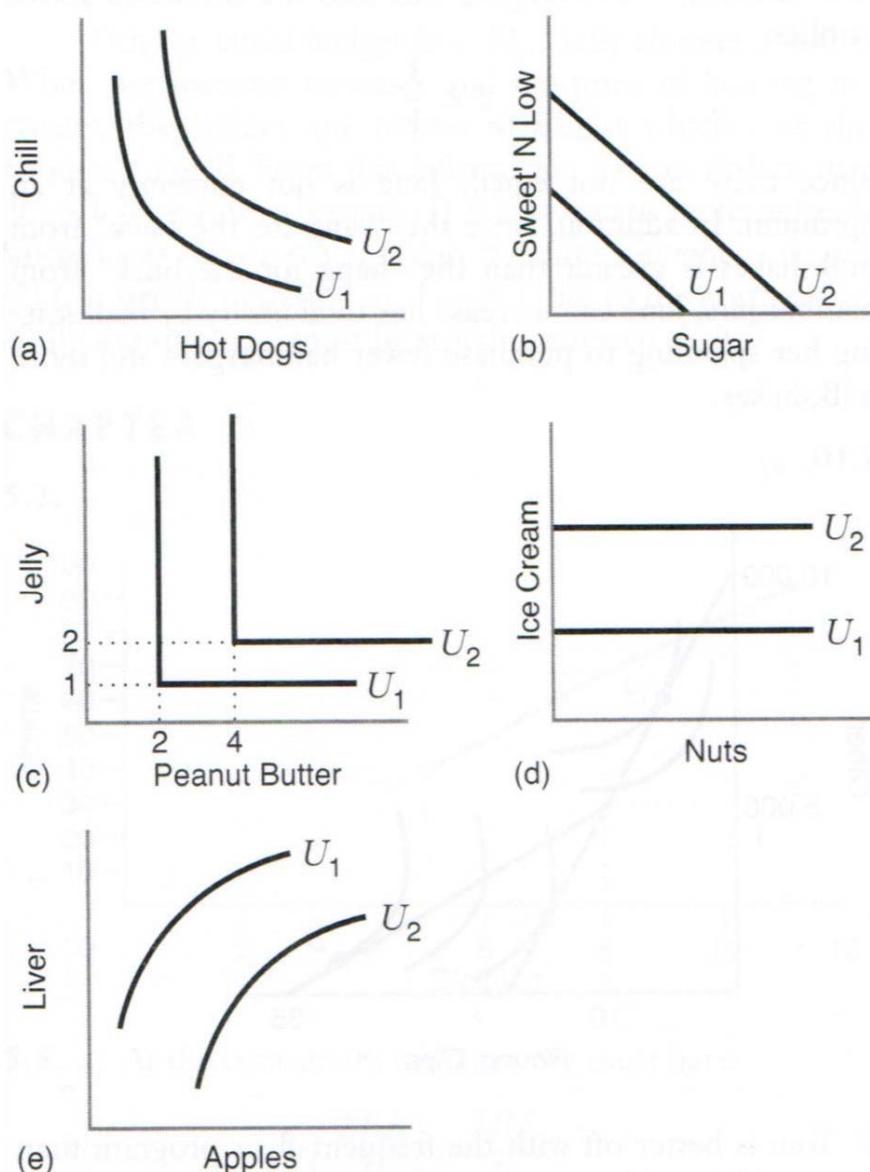
So we use 9 units of good B and 3 units of good A .

Exercise 4.

- a. It's worth noting that the marginal utility of a good is simply the partial derivative of utility function with respect to the good. The marginal utilities are given, and both are positive, so increasing consumption of either good will increase utility. Thus, more is always better.
- b.
$$MRS_{x,y} = \frac{\partial U / \partial x}{\partial U / \partial y} = \frac{MU_x}{MU_y} = \frac{6x}{5}$$
- c. The MRS is increase as the consumer substitutes towards more x and less y . This is simply because x appears in the numerator of the MRS .
- d. We know the utility function is not linear, so the indifference curves will not be straight lines. And we know that the $MRS_{x,y}$ is increasing, so the indifference curve will be bowed away from the origin, thus concave to the origin.

Exercise 5 (next page).

5.



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Review Session #2

1. Consider the utility function $U(x, y) = 3x^2 + 5y$ with $MU_x = 6x$ and $MU_y = 5$.
 - a) Is the assumption that ‘more is better’ satisfied for both goods?

Answer

Yes, since the marginal utility is greater than zero for both goods, increasing consumption of either good will increase total utility. Remember that marginal utility measures the amount of utility that you will receive from one extra unit of that good. Thus, if MU is positive, more of that good can only leave you better off.

- b) What is $MRS_{x,y}$ for this utility function? What does the $MRS_{x,y}$ tell us?

Answer

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{6x}{5}$$

The marginal rate of substitution tells us the tradeoff that this consumer is willing to make, while holding a constant level of utility. Also, the negative of the $MRS_{x,y}$ tells us the slope of the indifference curve.

- c) Is the $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?

Answer

$MRS_{x,y}$ is increasing as the consumer substitutes toward more x and less y since x appears in the numerator of $MRS_{x,y}$. That is, as consumption of good x increases, the numerator of this fraction gets larger, and therefore the $MRS_{x,y}$ gets larger as well.

- d) Will the indifference curves corresponding to this utility function be convex to the origin (bowed toward the origin), concave to the origin (bowed away), or straight lines? Explain.

Answer

The indifference curves will be concave to the origin since the marginal rate of substitution is increasing as the consumer substitutes x for y along an indifference curve. To further explain, as we move from left to right on the indifference curve the slope will get steeper because MRS is getting larger.

2. Olivia likes to eat both apples and bananas. At the grocery store, each apple costs \$0.20 and each banana cost \$0.25. Olivia's utility function for apples and bananas is given by $U(A, B) = 6\sqrt{AB}$ where $MU_A = 3\sqrt{\frac{B}{A}}$ and $MU_B = 3\sqrt{\frac{A}{B}}$. If Olivia has \$4 to spend on apples and bananas, how many of each should she buy to maximize her satisfaction?

Answer

Use the tangency condition to find the optimal amount of A relative to B .

$$MU_A/P_A = MU_B/P_B$$

$$\begin{aligned} \frac{3\sqrt{\frac{B}{A}}}{0.20} &= \frac{3\sqrt{\frac{A}{B}}}{0.25} \\ 15\sqrt{\frac{B}{A}} &= 12\sqrt{\frac{A}{B}} \\ \frac{225B}{A} &= \frac{144A}{B} \\ \frac{225B^2}{144} &= A^2 \\ A &= 1.25B \end{aligned}$$

Now plug this into the budget constraint to find the optimal amount of B to purchase.

$$\begin{aligned} 0.20(1.25B) + 0.25B &= 4 \\ 0.50B &= 4 \\ B &= 8 \end{aligned}$$

Finally, plug this result into the relationship between A and B above (that we found using the tangency condition) to determine the optimal amount of A ; $A = 1.25(8) = 10$. Therefore, she should buy 10 apples and 8 bananas to maximize her utility.

3. A consumer has income of \$180 per week and buys two goods, x and y . Initially, the prices are $(P_{x_1}, P_{y_1}) = (15, 10)$, and the consumer chooses basket 1 containing $(x_1, y_1) = (10, 3)$. Later, the prices change to $(P_{x_2}, P_{y_2}) = (12, 12)$. At these prices the consumer chooses basket 2 containing $(x_2, y_2) = (5, 10)$. The income is still \$180 per week. Do the consumer's choices in these two situations maximize utility?

Answer

We can use the theory of revealed preference to answer this question. At the initial prices the baskets cost:

$$\text{Basket 1 } 15(10) + 10(3) = 180$$

$$\text{Basket 2 } 15(5) + 10(10) = 175$$

The consumer chose basket 1 rather than basket 2 in this case. Since basket 1 is more expensive, the consumer must prefer basket 1 to basket 2. Notice that both baskets are affordable at the initial prices. This has to be true with each set of prices before we can make any inferences about preferences.

To be consistent to utility maximization, with the second set of prices basket 2 must cost less than basket 1 (since basket 2 was chosen over basket 1). Let's check:

$$\text{Basket 1 } 12(10) + 12(3) = 156$$

$$\text{Basket 2 } 12(5) + 12(10) = 180$$

Since basket 2 was chosen and basket 2 is more expensive, this would imply that basket 2 is preferred to basket 1. But this contradicts the initial situation where we discovered that basket 1 was preferred to basket 2. Therefore, these choices by the consumer do not satisfy utility maximization by the consumer.

4. Sally likes peanut butter and jelly together in her sandwiches. However, Sally is very particular about the proportions of peanut butter and jelly. Specifically, Sally likes 2 scoops of jelly with each 1 scoop of peanut butter. The cost of "scoops" of peanut butter and jelly are \$0.50 and \$0.20, respectively. Sally has \$9 each week to spend on peanut butter and jelly. (You can assume that Sally's mother provides the bread for the sandwiches.) If Sally is maximizing her utility subject to her budget constraint, how many scoops of peanut butter and jelly should she buy?

Answer

Notice that Sally only enjoys these goods if they are in the exact ratio she prefers. This means that peanut butter and jelly are perfect complements for Sally. Sally wants to consume at the "corner point" of her L-shaped indifference curve (least expensive point on any given utility curve). Thus, she will consume twice as much jelly as peanut butter. By consuming in this ratio, she will spend \$0.90 for each peanut butter/jelly bundle. She has a total of \$9 in income. Thus, she can afford 10 bundles:

$$\frac{\$9.00}{\$0.90} = 10.$$

Since each bundle contains two scoops of jelly, Sally should buy 20 scoops of jelly. Since each bundle contains one scoop of peanut butter, Sally should buy 10 scoops of peanut butter. At $PB = 10$ and $J = 20$, Sally is at the point where the corner of the L-shaped indifference curve just touches the budget constraint at that one point.

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Review Session #3

1. A consumer purchases two goods, food (F) and clothing (C). Her utility function is given by $U(F, C) = FC + F$. The marginal utilities are $MU_F = C + 1$ and $MU_C = F$. The price of food is P_F , the price of clothing is P_C , and the consumer's income is I .
 - a) What is the equation for the demand curve for clothing?

Answer

Setting up the tangency condition implies

$$\begin{aligned}\frac{MU_F}{P_F} &= \frac{MU_C}{P_C} \\ \frac{C+1}{P_F} &= \frac{F}{P_C} \\ FP_F &= P_C(C+1)\end{aligned}$$

Substituting this result into the budget line implies

$$\begin{aligned}P_C C + P_F F &= I \\ P_C C + P_C(C+1) &= I \\ P_C(2C+1) &= I \\ 2C+1 &= \frac{I}{P_C} \\ C &= \frac{I - P_C}{2P_C}\end{aligned}$$

- b) Is clothing a inferior good in this case?

Answer

Since the amount of clothing purchased will increase as income increases, as noted by the demand curve, clothing is a normal good, and not an inferior good.

2. Consider a consumer who purchases two goods, x and y . The consumer's utility function is $U(x, y) = xy$ with $MU_x = y$ and $MU_y = x$. In addition, the demand curve for y is given by $y = \frac{I}{2P_y}$. Assume initially that the consumer's income is \$160, the price of x is $P_x = \$8$, and the price of y is $P_y = \$1$.
- a) From the given information determine 1) the utility maximizing amount of x , 2) the utility maximizing amount of y , and 3) the total utility at the utility maximizing bundle.

Answer

Using the demand curve for y we can find the utility maximizing amount of y .

$$\begin{aligned}y &= \frac{I}{2P_y} \\y &= \frac{160}{2(1)} \\y &= 80\end{aligned}$$

Since each unit of y costs \$1, the consumer will spend $\$1(80) = \80 on y . This leaves \$80 to spend on x . Since x costs \$8, the utility maximizing amount of x is therefore 10 units. At this bundle, total utility is $U = xy = 10(80) = 800$.

- b) Now assume the price of y increases to \$2. Recompute the values from part a) at the new price.

Answer

Again, using the demand curve for y we can find the utility maximizing amount of y .

$$\begin{aligned}y &= \frac{I}{2P_y} \\y &= \frac{160}{2(2)} \\y &= 40\end{aligned}$$

Since each unit of y costs \$2, the consumer will spend $\$2(40) = \80 on y . This leaves \$80 to spend on x . Since x costs \$8, the utility maximizing amount of x is therefore 10 units. At this bundle, total utility is $U = xy = 10(40) = 400$.

- c) Determine the decomposition basket that identifies the substitution and income effects as the consumer moves from the optimal basket in part a) to the optimal basket in part b).

Answer

To determine the decomposition basket we note that this basket must satisfy two conditions. First, the decomposition basket must have total utility the same as at the *initial* prices, $U(x, y) = xy = 800$. Second, the decomposition basket must satisfy the tangency condition at the *new* prices. This implies

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

$$\begin{aligned}\frac{y}{8} &= \frac{x}{2} \\ y &= 4x\end{aligned}$$

These two conditions give two equations in two unknowns: $xy = 800$ and $y = 4x$.

Substituting the second equation into the first and solving implies $x = \sqrt{200} = 14.14$ and $y = 4\sqrt{200} = 56.56$.

- d) Identify the substitution and income effects as the consumer moves from the initial consumption basket to the final consumption basket.

Answer

Initially, the consumer chose to consume 80 units of y and at the final basket the consumer chose to consume 40 units of y . Therefore, the substitution effect is the initial amount less the amount in the decomposition basket, $80 - 56.56 = 23.44$ and the income effect is the amount in the decomposition basket less the final amount, $56.56 - 40 = 16.16$. Notice that these two amount sum to the total change of 40 units.

3. Karl's preferences over hamburgers (H) and beer (B) are described by the utility function: $U(H, B) = \min(2H, 3B)$. His monthly income is I dollars, and he only buys these two goods using his income. Denote the price of hamburgers by P_H and for beer P_B .

- a) Derive Karl's demand curve for beer as a function of the exogenous variables.

Answer

Recall that goods that are compliments have utility functions in the “min” functional form, where the indifference curves are L shaped. And, from the utility function, we know Karl's optimal bundle will always be such that $2H = 3B$. That is, he will always consume in the ratio of 1 hamburger for every 2/3rds beer, or any other combination following that ratio such that $2H = 3B$ is satisfied. If this were not true then he could decrease the consumption of one of the two goods, staying at the same level of utility and reducing expenditure. Also, at the optimal bundle, it must be true that $P_H H + P_B B = I$. Solving the first condition for H , that is $H = 3/2B$, and substituting into the second we get

$B(1.5P_H + P_B) = I$ which implies that the demand curve for beer is given by,

$$B = \frac{I}{(1.5P_H + P_B)}$$

- 4) Carina buys two goods, food F and clothing C , with the utility function $U = FC + F$. Her marginal utility of food is $MU_F = C + 1$ and her marginal utility of clothing is $MU_C = F$. She has an income of 20. The price of clothing is 4.
- a) Derive the equation representing Carina's demand for food, and draw this demand curve for prices of food ranging between 1 and 6.

Answer

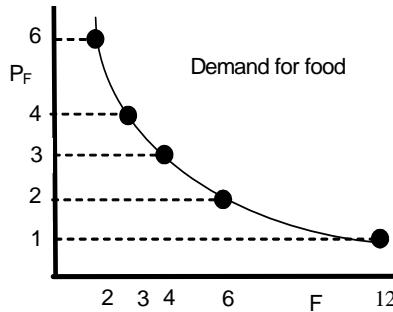
$$MU_F = C + 1 \quad MU_C = F$$

$$\text{Tangency: } MU_F/MU_C = P_F / P_C. \quad (C + 1)/F = P_F/4 \Rightarrow 4C + 4 = P_F F. \quad (\text{Eq 1})$$

$$\text{Budget Line: } P_F F + P_C C = I. \Rightarrow P_F F + 4C = 20. \quad (\text{Eq 2})$$

Substituting (Eq 1) into (Eq 2): $4C + 4 + 4C = 20$. Thus $C = 2$, independent of P_F .

From the budget line, we see that $P_F F + 4(2) = 20$, so **the demand for F is $F = 12/P_F$** .



- b) Calculate the income and substitution effect on Carina's consumption of food when the price of food rises from 1 to 4, and draw a graph illustrating these effects.

Answer

Initial Basket: From the demand for food in (a), $F = 12/1 = 12$, and $C = 2$.

Also, the initial level of utility is $U = FC + F = 12(2) + 12 = 36$.

Final Basket: From the demand for food in (a), we know that $F = 12/4 = 3$, and $C = 2$. And the final level of utility is, $U = 3(2) + 3 = 9$.

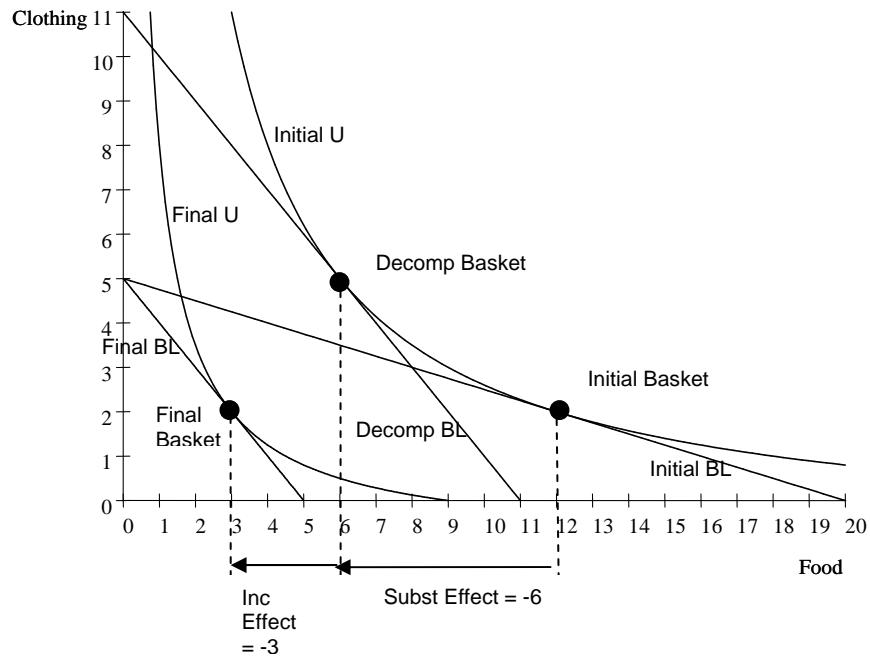
Decomposition Basket: Must be on initial indifference curve, with $U = FC + F = 36$ (Eq 3)

Tangency condition satisfied with final price: $MU_F/MU_C = P_F / P_C. \quad (C + 1)/F = 4/4 \Rightarrow C + 1 = F. \quad (\text{Eq 4})$

Eq 3 can be written as $F(C + 1) = 36$. Plugging Eq 4 into the rewritten Eq 3 we have $(C + 1)^2 = 36$, and thus, $C = 5$. Also, by Eq 4, $F = 6$. So the decomposition basket is $F = 6, C = 5$.

Income effect on F: $F_{\text{final basket}} - F_{\text{decomposition basket}} = 3 - 6 = -3$.

Substitution effect on F: $F_{\text{decomposition basket}} - F_{\text{initial basket}} = 6 - 12 = -6$.



- c) Determine the numerical size of the compensating variation (in monetary terms) associated with the increase in the price of food from 1 to 4.

Answer

$P_F F + P_C C = 4(6) + 4(5) = 44$. So she would need an additional income of 24 (plus her actual income of 20). The compensating variation associated with the increase in the price of food is -24.

EconS 301 – Intermediate Microeconomics

Review Session #4

1. Suppose a person's utility for leisure (L) and consumption (Y) can be expressed as $U = YL$ and this person has no non-labor income.
 - a) Assuming a wage rate of \$10 per hour, show what happens to the person's labor supply when the person wins a lottery prize of \$100 per day.

Answer

Plug in what we know to the utility function above, $U = YL$. Y should express total income earned and L should express all non-work time per day.

Rearranging yields $U = (Y^* + 10H)(24 - H) = 24Y^* + 240H - Y^*H - 10H^2$.

Maximizing utility with respect to H yields $dU/dH = 240 - Y^* - 20H = 0$. (Remember that where $U=0$ it is at its maximum.)

Before winning the lottery, $Y^* = 0$, so $H = 12$.

After winning the \$100 per day lottery, $Y^* = 100$, so $H = 7$.

Winning the lottery reduces this person's quantity of labor supplied by 5 hours when $w = \$10$. Intuitively this makes sense because the more wealth they have the less desirable work becomes. L becomes more attractive with greater wealth.

2. Suppose a person's utility for leisure (L) and consumption (Y) can be expressed as $U = Y + L0.5$. Show what happens to the person's labor supply curve when the income tax is cut from 70 % to 30 %. Denote hours worked as H and wage per hour as w.

Answer

Since $Y = \text{net income}$, $U = w(1 - t)H + (24 - H)0.5$. Note that $w(1-t)$ represents real wage.

Maximizing utility with respect to hours worked, H, yields $H = 24 - (2(1 - t)w) - 2$. Any decrease in t would increase the number of hours worked. Note: This person is a workaholic. Even at a net wage of \$1, this person only relaxes for 3/4 of an hour!

2. Suppose you work for a government agency that is considering removing certain agricultural subsidies. The removal of these subsidies will increase the price, thus lowering consumers' welfare. Because only aggregate market data is available, you are unable to measure the exact values for the compensated and equivalent variation by consumer. However, you are able to estimate the change in market consumer surplus. Assuming agricultural products are normal goods, how does your estimate of consumer surplus compare to the unknown EV and CV? Explain. Under what conditions will the three measures of welfare be close to one another?

Answer

For normal goods, the CS will be less than the CV and greater than the EV (in absolute value). Typically, these measures will be close for (1) small price changes, (2) small income effects/elasticity, and (3) small budget share. Remember that the CV and EV are used to measure the change of welfare (income) of the consumer after a price change and that typically the EV and the CV will not be close in magnitude. This is because most price changes have a nonzero income effect. Similarly, in the case of a quasi-linear utility function, the CV and EV will be the same because the income effect is zero.

3. Ed's utility from vacations (V) and meals (M) is given by the function $U(V,M) = V^2M$. Last year, the price of vacations was \$200 and the price of meals was \$50. This year, the price of meals rose to \$75, the price of vacations remained the same. Both years, Ed had an income of \$1500.
 - a) Calculate the change in consumer surplus from meals resulting from the change in meal prices. (Note: we need to compare his optimal consumption baskets before and after the price change to be able to see the change in CS)

Answer

Ed's optimization problem is

$$\begin{aligned} \text{Max } & V^2M \\ \text{subject to } & p_M M + 200V = 1500 \end{aligned}$$

where p_M is the price of meals and the price of vacations is represented by the constant 200. Using the Lagrangian (solve utility function for V and then plug V into the budget constraint and solve for M), we derive the demand for meals:

$$M^* = 500/p_M$$

The change in consumer surplus is found from the integral (space under curve):

$$\Delta CS = \int_{50}^{75} \frac{500}{p_M} dp_M = -500 \ln(p_M) \Big|_{50}^{75} = -500(\ln 75 - \ln 50) = -202.7$$

So the change in consumer surplus is -\$202.7.

- b) What is the compensating variation for the price change in meals?

Answer

Recall the CV is simply the difference in the consumer's income and the income necessary to purchase the decomposition basket at the new prices. In this case, the CV is the amount of money needed to offset a consumer's harm from a price increase (that is, CV will be additive here because the consumer will need more money to be equally happy after the price increase as she was before). So we first need to find his initial optimum basket.

$$\max L = V^2 M + \lambda(y - p_M M - p_V V)$$

$$\left. \begin{aligned} \frac{\delta L}{\delta M} &= V^2 - \lambda p_M = 0 \\ \frac{\delta L}{\delta V} &= 2VM - \lambda p_V = 0 \end{aligned} \right\} \frac{V^2}{2VM} = \frac{V}{2M} = \frac{p_M}{p_V}$$

$V = \frac{2p_M M}{p_V}$ into the budget constraint

$$1500 = p_M M - p_V \left(\frac{2p_M M}{p_V} \right) \text{ solving for } M,$$

$$\frac{1500}{3p_M} = \frac{500}{p_M} = M \text{ plug back into } V \text{ and solve,}$$

$$V = \frac{2p_M \left(\frac{500}{p_M} \right)}{p_V} = \frac{1000}{p_V}$$

Ed's utility before the price change is based on his optimal consumption bundle where $M_1 = 500/50 = 10$ and $V_1 = 1000/200 = 5$. Thus, his initial utility is $U(5,10) = (5)(2)(10) = 250$. In order to find the decomposition basket, we need to use the MRS and the initial level of utility. From

above, we know the MRS is $\frac{V}{2M} = \frac{p_M}{p_V}$.

Using this and plugging in the new prices we can solve for V in terms of M,

$$V = \frac{2(75)M}{200} = \frac{150M}{200} = \frac{3M}{4}.$$

Plug this result into the utility function when initial utility is 250 and solving for M, $250 = (3M/4)^2 M \Rightarrow M = 7.63$. Solve for $V = 5.72$. This is our decomposition basket. The expenditure required to purchase this bundle is:

$$75M + 200V = 75(7.63) + 200(5.72) = 1717.25$$

(we need total expenditure because CV and EV are measures of income before and after price change)

Thus the CV is $\$1,500 - \$1,717.25 = \$217.25$.

- c) Calculate the equivalent variation for the price change in meals.

Answer

The EV is similar to the CV, except that we need to find the consumption basket that would put him on his new utility level holding prices constant (sort of the opposite of the decomposition basket where the new prices are used holding the initial utility constant). The EV is the amount of money Ed will pay to prevent the price increase. First we need to find his optimal consumption basket at new prices so we can find the new level of utility. To do this, simply use the demand equations we derived in part (b) and plug in the new prices. From above, $V = 5$ and $M = 500/75 = 6.67$. His utility from this bundle is $U = (5)^2(6.67) = 166.75$. Now use the MRS and

new prices to solve for V in terms of M, $V = \frac{2(50)M}{200} = \frac{100M}{200} = \frac{M}{2}$. Plug this result into the utility function with $U=166.75$ and we can solve for M,

$$166.75 = \left(\frac{M}{2}\right)^2 M = \frac{M^3}{4}$$

$$667 = M^3$$

$$8.74 = M$$

$$V = \frac{8.74}{2} = 4.37$$

The expenditure of this bundle is:

$$50(8.74) + 200(4.37) = 1311.$$

Ed would pay up to $1500 - 1311 = \$189$ to avoid the price change. This is the EV.

4. Linda consumes two goods, X and Y . Her utility function is $U = XY$, with $MU_X = Y$ and $MU_Y = X$. Initially, $P_X = \$18$ and $P_Y = \$2$. Linda's income is \$288. Then the price of X falls to \$8. [The following questions ask you to calculate a mathematical example of the income and substitution effects of a price decrease for good X .]
- a) Complete the following table.

Basket	X	Y	$U = XY$	$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$	Expenditure $P_X X + P_Y Y$
A					
B	12	48			
C					

Answer

For bundle A,

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

$$\frac{Y}{X} = \frac{18}{2} = \frac{9}{1}$$

$$9X = Y$$

$$P_X X + P_Y Y = I$$

$$18X + 2Y = 288$$

$$18X + 18X = 288$$

$$36X = 288$$

$$X = 8$$

$$Y = 72$$

For bundle C,

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

$$\frac{Y}{X} = \frac{8}{2} = \frac{4}{1}$$

$$4X = Y$$

$$P_X X + P_Y Y = I$$

$$8X + 2Y = 288$$

$$8X + 8X = 288$$

$$16X = 288$$

$$X = 18$$

$$Y = 72$$

Basket	X	Y	$U = XY$	$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$	Expenditure $P_X X + P_Y Y$
A	8	72	$8 * 72 = 576$	$\frac{Y}{X} = \frac{72}{8} = \frac{9}{1} = \frac{18}{2}$	$18 * 8 + 2 * 72 = 288$
B	12	48	$12 * 48 = 576$	$\frac{Y}{X} = \frac{48}{12} = \frac{4}{1} = \frac{8}{2}$	$8 * 12 + 2 * 48 = 192$
C	18	72	$18 * 72 = 1296$	$\frac{Y}{X} = \frac{72}{18} = \frac{4}{1} = \frac{8}{2}$	$8 * 18 + 2 * 72 = 288$

- b) The movement from point A to point B illustrates which effect, the income effect or the substitution effect? Explain.

Answer

The movement from point A to point B illustrates the substitution effect because the consumer moves along the same indifference curve (notice that total utility remains constant) to the new tangency point between the original indifference curve and the new budget constraint (where $P_X = \$8$). That is, we are looking at the change in optimal choice induced solely by the change in the price relationship between x and y and not any change due to a change in income.

- c) The movement from point B to point C illustrates which effect, the income effect or the substitution effect? Explain.

Answer

The movement from point B to point C illustrates the income effect because the consumer moves to a higher indifference curve. Point C represents the new tangency point between the new budget constraint and the new indifference curve. Here, we look solely at how the income change affects the optimal choice for this consumer. (Notice that the slope of the budget constraint does not change between points B and C but that the utility does.)

- d) Is good X a normal, inferior, or Giffen good? Explain.

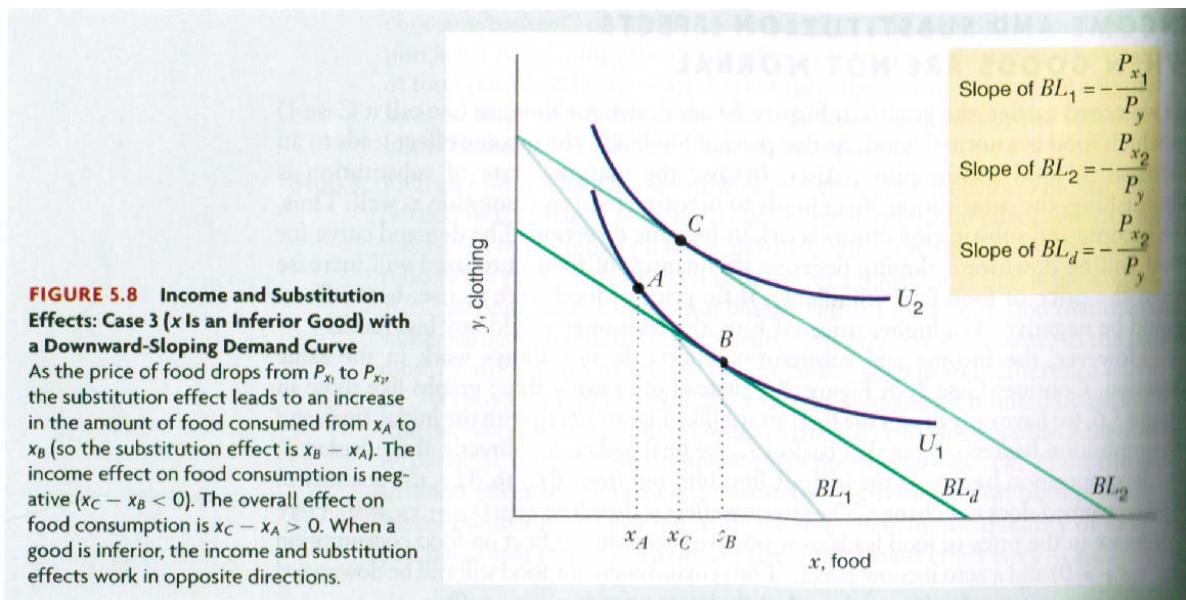
Answer

Good X is a normal good because as “income” increases from point B to point C, Linda consumes more X. Remember that the gap between B and C is solely measuring the effect of increased income (income effect) so we can use it to make conclusions on whether a good is giffen or normal. Note that the change between bundle A and bundle B could not be used to check this.

5. If x is an inferior good and the price of x rises
- The substitution effect will induce the consumer to purchase more x and the income effect will induce the consumer to purchase more x .
 - The substitution effect will induce the consumer to purchase more x and the income effect will induce the consumer to purchase less x .
 - The substitution effect will induce the consumer to purchase less x and the income effect will induce the consumer to purchase more x .
 - The substitution effect will induce the consumer to purchase less x and the income effect will induce the consumer to purchase less x .

Answer

Recall that with inferior goods, the income and substitution effects move in opposite directions. So immediately you can eliminate choices A and D. Now consider the example given in figure 5.8.

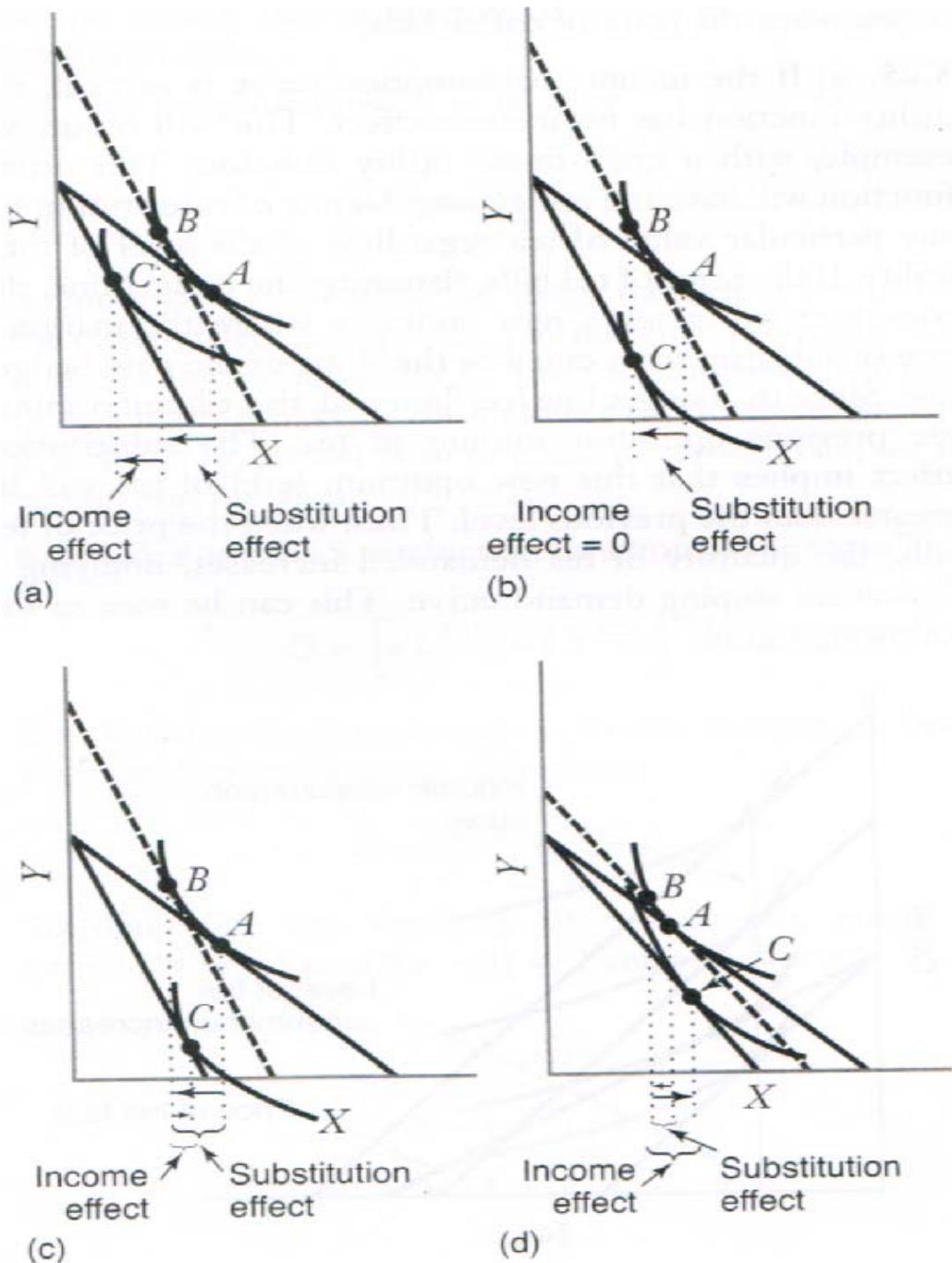


In this figure, we have a price decrease and we can see that the substitution effect (bundle A to B) causes the consumer to purchase more. Similarly, we can see the income effect (bundle B to C) causes the consumer to purchase less. Again this is for a price decrease. So for a price increase, you simply change the direction of the effects. So in our case, the SE results in less x and the IE results in more x , thus the answer is choice C.

6. Rich purchases two goods, food and clothing. He has a diminishing marginal rate of substitution of food for clothing. Let x denote the amount of food consumed and y the amount of clothing. Suppose the price of food increase from P_{x_1} to P_{x_2} . On a clearly labeled graph, illustrate the income and substitution effects of the price change on the consumption of food. Do so for each of the following cases:
- Food is a normal good.
 - The income elasticity of demand for food is zero.

- c. Food is an inferior good, but not a Giffen good.
- d. Food is a Giffen good.

Answer



7. Suppose that Bart and Homer are the only people in Springfield who drink 7-UP. Moreover, their inverse demand curves for 7-UP are, respectively, $P=10-4Q_B$ and $P=25-2Q_H$, and, of course, neither one can consume negative amounts. Write down the market demand curve for 7-UP in Springfield, as a function of all possible prices.

Answer

Recall that the market demand is simply the horizontal summation (adding in x and not in y) of all the individual demand curves. That is, you sum the quantities demanded. We have the inverse demand equations, so we need to solve each for their respective quantities,

$$P = 10 - 4Q_B$$

$$4Q_B = 10 - P$$

$$Q_B = \begin{cases} 2.5 - .25P & \text{when } P < 10 \\ 0 & \text{when } P \geq 10 \end{cases}$$

and

$$P = 25 - 2Q_H$$

$$2Q_H = 25 - P$$

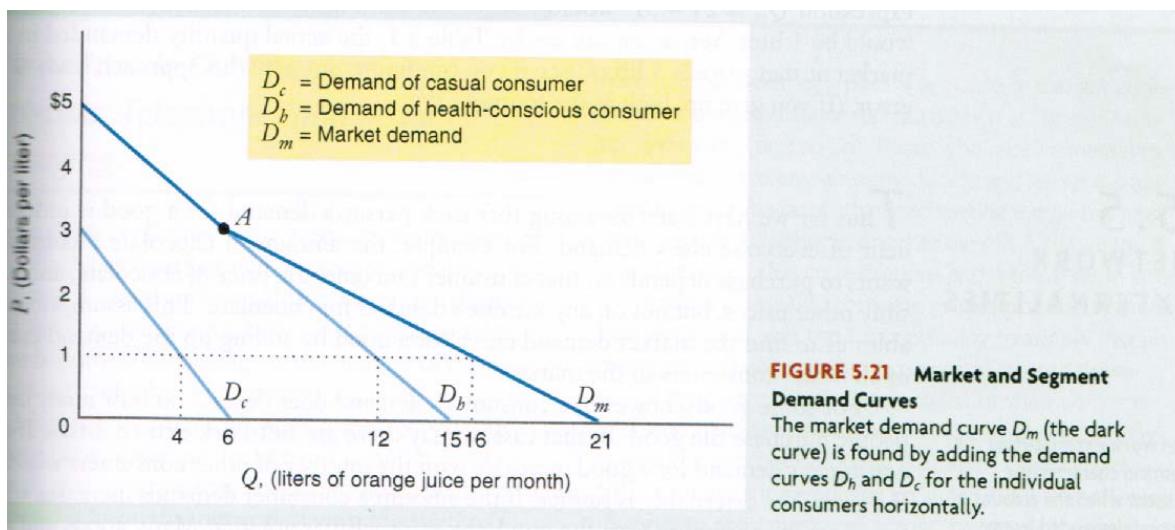
$$Q_H = \begin{cases} 12.5 - .5P & \text{when } P < 25 \\ 0 & \text{when } P \geq 25 \end{cases}$$

now sum Q_H and Q_B ,

$$Q_{\text{market}} = Q_B + Q_H = 2.5 - .25P + 12.5 - .5P = 15 - .75P$$

$$Q_{\text{market}} = \begin{cases} 15 - .75P & \text{when } P < 10 \\ 12.5 - .5P & \text{when } 10 \leq P < 25 \\ 0 & \text{when } P \geq 25 \end{cases}$$

For a graphical representation refer to Figure 5.21. Notice the “kink” once one consumer’s maximum price is reached.



EconS 301 – Intermediate Microeconomics Review Session #5

Exercise 1

You might think that when a production function has a diminishing marginal rate of technical substitution of labor for capital, it cannot have increasing marginal products of capital and labor. Show that this is not true, using the production function $Q = K^2 L^2$, with the corresponding marginal products $MP_k = 2KL^2$ and $MP_L = 2K^2L$.

Answer

Immediately we can see that the marginal products for capital and labor are increasing, since they are both positive. Now we simply need to find the $MRTS_{L,K}$. Recall that the $MRTS_{L,K}$ is simply the slope of the isoquant, and is equal to the ratio of marginal products. It is analogous to the MRS in consumer theory. So we have,

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{2K^2L}{2KL^2} = \frac{K}{L}.$$

Note that labor is in the denominator, thus as labor increases, the $MRTS_{L,K}$ is decreasing or diminishing. That is, as we move along the isoquant, increasing L , the slope is getting “less steep”. Graphically, isoquants that exhibit diminishing marginal rates of technical substitution are convex to the origin (bowed toward the origin).

Thus, we have shown a production function with increasing marginal products of labor and capital can have a diminishing marginal rate of technical substitution.

Exercise 2

A firm produces quantity Q of breakfast cereal using labor L and material M with the production function $Q = 50(ML)^{\frac{1}{2}} + M + L$. The marginal product functions for this production function are

$$MP_L = 25\left(\frac{M}{L}\right)^{\frac{1}{2}} + 1$$

$$MP_M = 25\left(\frac{L}{M}\right)^{\frac{1}{2}} + 1$$

- a) Are the returns to scale increasing, constant, or decreasing for this production function?

Answer

To determine the nature of returns to scale, increase all inputs by some factor λ and determine if output goes up by a factor more than, less than, or equal to λ .

$$\begin{aligned}
Q_\lambda &= 50(\lambda M \lambda L)^{\frac{1}{2}} + \lambda M + \lambda L \\
&= 50\lambda^{\frac{1}{2}}\lambda^{\frac{1}{2}}(ML)^{\frac{1}{2}} + \lambda M + \lambda L \\
&= 50\lambda(ML)^{\frac{1}{2}} + \lambda M + \lambda L \\
&= \lambda \left[50(ML)^{\frac{1}{2}} + M + L \right] \\
&= \lambda Q
\end{aligned}$$

Thus, by increasing all inputs by a factor λ , output goes up by a factor of λ . Since output goes up by the same factor as the inputs, this production function exhibits constant returns to scale.

- b) Is the marginal product of labor ever diminishing for this production function? If so, when? Is it ever negative, and if so, when?

Answer

Recall $MP_L = 25\left(\frac{M}{L}\right)^{\frac{1}{2}} + 1$. Suppose $M > 0$. Holding M constant, increasing L will decrease the MP_L .

The marginal product of labor is decreasing for all levels of labor. The MP_L , however, will never be negative since both components of the equation will always be greater or equal to zero. In fact, for this production function, $MP_L \geq 1$.

Exercise 3

A firm's production function is $Q = 5L^{\frac{2}{3}}K^{\frac{1}{3}}$ with $MP_K = \left(\frac{5}{3}\right)L^{\frac{2}{3}}K^{-\frac{2}{3}}$ and $MP_L = \left(\frac{10}{3}\right)L^{-\frac{1}{3}}K^{\frac{1}{3}}$.

- a) Does this production function exhibit constant, increasing, or decreasing returns to scale?

Answer

$$\begin{aligned}
Q_\lambda &= 5(\lambda L)^{\frac{2}{3}}(\lambda K)^{\frac{1}{3}} \\
&= 5\lambda^{\frac{2}{3}}\lambda^{\frac{1}{3}}L^{\frac{2}{3}}K^{\frac{1}{3}} \\
&= 5\lambda^{\frac{2}{3}}K^{\frac{1}{3}} \\
&= \lambda \left[5L^{\frac{2}{3}}K^{\frac{1}{3}} \right] \\
&= \lambda Q
\end{aligned}$$

Thus, we have constant returns to scale.

b) What is the marginal rate of technical substitution of L for K for this production function?

Answer

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{\left(\frac{10}{3}\right)L^{-\frac{1}{3}}K^{\frac{1}{3}}}{\left(\frac{5}{3}\right)L^{\frac{2}{3}}K^{-\frac{2}{3}}} = \frac{2K}{L}$$

c) What is the elasticity of substitution for this production function?

Answer

The formula for the elasticity of substitution is given by,

$$\sigma = \frac{\text{percentage change in capital to labor ratio}}{\text{percentage change in } MRTS_{L,K}} = \left(\frac{\Delta \left(\frac{K}{L} \right)}{\Delta MRTS_{L,K}} \right) \left(\frac{MRTS_{L,K}}{\frac{K}{L}} \right) = \frac{d \left(\frac{K}{L} \right)}{d MRTS_{L,K}} \left(\frac{MRTS_{L,K}}{\frac{K}{L}} \right)$$

So from part (b) we know,

$$MRTS_{L,K} = \frac{2K}{L} \rightarrow \frac{MRTS_{L,K}}{\left(\frac{K}{L} \right)} = 2 \text{ and } \frac{K}{L} = \frac{MRTS_{L,K}}{2}. \text{ So, } \frac{d \left(\frac{K}{L} \right)}{d MRTS_{L,K}} = \frac{1}{2}.$$

$$\text{Thus, } \sigma = \frac{d \left(\frac{K}{L} \right)}{d MRTS_{L,K}} \left(\frac{MRTS_{L,K}}{\frac{K}{L}} \right) = \left(\frac{1}{2} \right) 2 = 1.$$

Exercise 4

Suppose a firm's production function initially took the form $Q = 500(L + 3K)$. However, as a result of a manufacturing innovation, its production function is now $Q = 1000(0.5L + 10K)$.

a) Show that the innovation has resulted in technological progress in the sense defined in the text.

Answer

We simply need to show that given a fixed combination of inputs, the quantity produced will increase as a result of the innovation. Assume a fixed level of labor and capital at 2 and 2 units respectively. Now we just calculate the quantities produced before the innovation and after the innovation using these input levels. Before the innovation, $Q = 500(L + 3K) = 500(2 + 3(6)) = 4000$. And after the innovation, $Q = 1000(0.5L + 10K) = 1000(0.5(2) + 10(2)) = 21000$. So we obviously have more output after the innovation, thus we have technological progress.

- b) Is the technological progress neutral, labor saving, or capital saving?

Answer

We simply need to look at the marginal products of labor and capital before and after the innovation.

Before the innovation we have $MP_L = 500$ and $MP_K = 1500$. After the innovation we have $MP_L = 500$ and $MP_K = 10000$. So, obviously the marginal product of capital increased relative to the marginal product of labor, which remained constant. Thus, we have labor saving technology (ie they will use less labor and more capital).

MULTIPLE CHOICE EXERCISES

1. Identify the truthfulness of the following statements.
 - I. When the marginal product of labor is falling, the average product of labor is falling.
 - II. When the marginal product curve lies above the average product curve, then average product is rising.
 - a. Both I and II are true.
 - b. Both I and II are false.
 - c. I is true; II is false.
 - d. I is false; II is true.

Answer

Although both statements sound similar, they are actually different. The first statement is false, since the marginal product curve can lie above the average product curve and still be decreasing. And since the marginal product curve is above the average product curve, the average product curve will be increasing. Thus the first statement is false. The second statement is true, since when the marginal product curve crosses the average product curve at its highest point (maximum), thus since the marginal product curve lies below it, then it must be decreasing from its maximum. Thus the correct choice is D. Refer to page 191 in Besanko for a graph.

2. Which one of these is false when compared to the relationship between marginal and average product
 - a. When average product is increasing in labor, marginal product is greater than average product. That is, if AP_L increases in L, then $MP_L > AP_L$.
 - b. When average product is decreasing in labor, marginal product is less than average product. That is, if AP_L decreases in L, then $MP_L < AP_L$.
 - c. The relationship between MP_L and AP_L is not the same as the relationship between the marginal of anything and the average of anything.
 - d. When average product neither increases nor decreases in labor because we are at a point at which AP_L is at a maximum, then marginal product is equal to average product.

Answer

The only statement that is false is C, since the concept of average and marginal is mathematical and doesn't depend on what you are comparing.

3. The $MRTS_{L,K} =$

- a. $\frac{MP_K}{MP_L}$
- b. $-\frac{\Delta L}{\Delta K}$
- c. $\frac{MP_L}{MP_K}$
- d. $-\frac{MP_K}{MP_L}$

Answer

The correct answer is C, since from our previous problems we know $MRTS_{L,K} = \frac{MP_L}{MP_K}$

4. The production function $Q(L, K, M) = 25K^{0.5}L^{0.5}M^{0.5}$ exhibits

- a. decreasing returns to scale.
- b. constant returns to scale.
- c. increasing returns to scale.
- d. either decreasing or constant returns to scale, but more information is needed to determine which one.

Answer

$$\begin{aligned} Q_\lambda &= 25(\lambda K)^{\frac{1}{2}}(\lambda L)^{\frac{1}{2}}(\lambda M)^{\frac{1}{2}} \\ &= 25\lambda^{\frac{1}{2}}\lambda^{\frac{1}{2}}\lambda^{\frac{1}{2}}K^{\frac{1}{2}}L^{\frac{1}{2}}M^{\frac{1}{2}} \\ &= 25\lambda^{\frac{3}{2}}K^{\frac{1}{2}}L^{\frac{1}{2}}M^{\frac{1}{2}} \\ &= \lambda^{\frac{3}{2}}[25K^{\frac{1}{2}}L^{\frac{1}{2}}M^{\frac{1}{2}}] \\ &= \lambda^{\frac{3}{2}}Q \end{aligned}$$

Thus we have increasing returns to scale, so the answer is C.

EconS 301

Review Session #6 – Chapter 8: Cost Curves

- 8.12.** Consider a production function with two inputs, labor and capital, given by $Q = (\sqrt{L} + \sqrt{K})^2$. The marginal products associated with this production function are as follows:

$$MP_L = \left[L^{\frac{1}{2}} + K^{\frac{1}{2}} \right] L^{-\frac{1}{2}}$$

$$MP_K = \left[L^{\frac{1}{2}} + K^{\frac{1}{2}} \right] K^{-\frac{1}{2}}$$

Let $w = 2$ and $r = 1$

- a) Suppose the firm is required to produce Q units of output. Show how the cost-minimizing quantity of labor depends on the quantity Q . Show how the cost-minimizing quantity of capital depends on quantity Q .

Starting with the tangency condition we have

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\frac{\left[L^{1/2} + K^{1/2} \right] L^{-1/2}}{\left[L^{1/2} + K^{1/2} \right] K^{-1/2}} = \frac{2}{1}$$

$$\frac{K}{L} = 4$$

$$K = 4L$$

Plugging this into the total cost function yields

$$Q = \left[L^{1/2} + (4L)^{1/2} \right]^2$$

$$Q = \left[3L^{1/2} \right]^2$$

$$Q = 9L$$

$$L = \frac{Q}{9}$$

Inserting this back into the solution for K above gives

$$K = \frac{4Q}{9}$$

- b) Find the equation of the firm's long-run total cost curve.

$$TC = 2\left(\frac{Q}{9}\right) + \frac{4Q}{9}$$

$$TC = \frac{2Q}{3}$$

- c) Find the equation of the firm's long-run average cost curve.

$$AC = \frac{TC}{Q} = \left(\frac{2Q}{3} \right) / Q$$

$$AC = \frac{2}{3}$$

- d) Find the solution to the firm's short-run cost-minimization problem when capital is fixed at a quantity of 9 units (i.e. $\bar{K} = 9$).

When $Q \leq 9$ the firm needs no labor. If $Q > 9$ the firm must hire labor. Setting $\bar{K} = 9$ and plugging in for capital in the production function yields

$$Q = [L^{1/2} + 9^{1/2}]^2$$

$$Q^{1/2} = L^{1/2} + 3$$

$$L^{1/2} = Q^{1/2} - 3$$

$$L = [Q^{1/2} - 3]^2$$

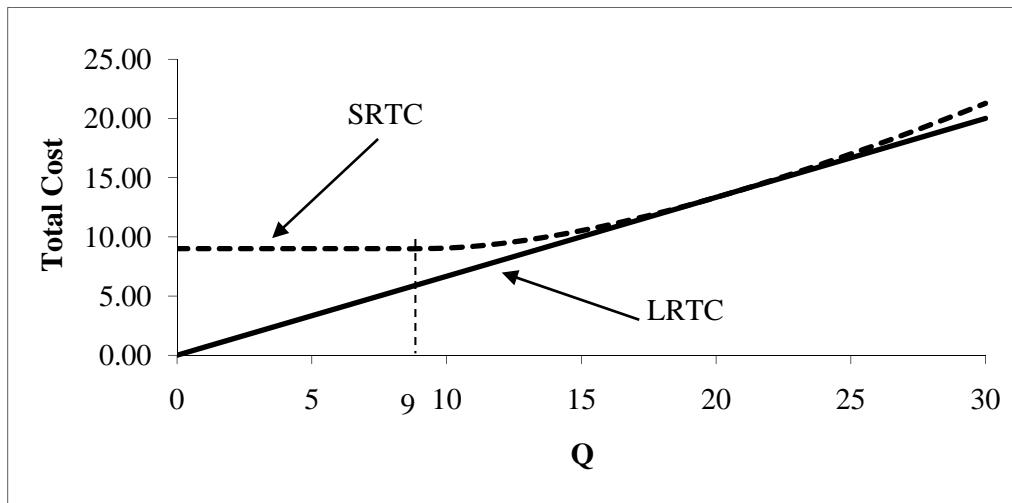
Thus,

$$L = \begin{cases} [Q^{1/2} - 3]^2 & \text{if } Q > 9 \\ 0 & \text{if } Q \leq 9 \end{cases}$$

- e) Find the short-run total cost curve, and graph it along with the long-run total cost curve.

$$TC = \begin{cases} 2(Q^{1/2} - 3)^2 + 9 & \text{when } Q > 9 \\ 9 & \text{when } Q \leq 9 \end{cases}$$

Graphically, short-run and long-run total cost are shown in the following figure.



- f) Find the associated short-run average cost curve.

$$AC = \frac{TC}{Q} = \begin{cases} \frac{2(Q^{1/2} - 3)^2 + 9}{Q} & \text{if } Q > 9 \\ \frac{9}{Q} & \text{if } Q \leq 9 \end{cases}$$

- 8.14.** A hat manufacturing firm has the following production function with capital and labor being the inputs: $Q = \min(4L, 7K)$ —that is it has a fixed-proportions production function. If w is the cost of a unit of labor and r is the cost of a unit of capital, derive the firm's long-run total cost curve and average cost curve in terms of the input prices and Q .

The fixed proportions production function implies that for the firm to be at a cost minimizing optimum, $4L = 7K$ and both of these equal Q . Therefore, $L = Q/4$ and $K = Q/7$. So the firm's total cost is $wL + rK = wQ/4 + rQ/7 = [\frac{w}{4} + \frac{r}{7}]Q$.

The average cost curve is $LRAC = TC/Q = \frac{w}{4} + \frac{r}{7}$. Note that this average cost curve is independent of Q and is simply a straight line.

- 8.19.** Consider a production function of three inputs, labor, capital, and materials, given by $Q = LKM$. The marginal products associated with this production function are as follows: $MP_L = KM$, $MP_K = LM$, and $MP_M = LK$. Let $w = 5$, $r = 1$, and $m = 2$, where m is the price per unit of materials.

- a) Suppose that the firm is required to produce Q units of output. Show how the cost-minimizing quantity of labor depends on the quantity Q . Show how the cost-minimizing quantity of capital depends on the quantity Q . Show how the cost-minimizing quantity of materials depends on the quantity Q .

Equating the bang for the buck between labor and capital implies

$$\begin{aligned} \frac{MP_L}{MP_K} &= \frac{w}{r} \\ \frac{KM}{LM} &= \frac{5}{1} \\ K &= 5L \end{aligned}$$

Equating the bang for the buck between labor and materials implies

$$\frac{MP_L}{MP_M} = \frac{w}{m}$$

$$\frac{KM}{KL} = \frac{5}{2}$$

$$M = \frac{5L}{2}$$

Plugging these into the production function yields

$$Q = L(5L) \left(\frac{5L}{2} \right)$$

$$Q = \frac{25L^3}{2}$$

$$L^3 = \frac{2Q}{25}$$

$$L = \left(\frac{2Q}{25} \right)^{1/3}$$

Substituting into the tangency condition results above implies

$$K = 5 \left(\frac{2Q}{25} \right)^{1/3}$$

and

$$M = \frac{5}{2} \left(\frac{2Q}{25} \right)^{1/3}$$

b) Find the equation of the firm's long-run total cost curve.

$$TC = 5 \left(\frac{2Q}{25} \right)^{1/3} + 5 \left(\frac{2Q}{25} \right)^{1/3} + 2 \left(\frac{5}{2} \right) \left(\frac{2Q}{25} \right)^{1/3}$$

$$TC = 15 \left(\frac{2Q}{25} \right)^{1/3}$$

c) Find the equation of the firm's long-run average cost curve.

$$AC = \frac{TC}{Q} = \frac{15}{Q} \left(\frac{2Q}{25} \right)^{1/3}$$

d) Suppose that the firm is required to produce Q units of output, but that its capital is fixed at a quantity of 50 units (i.e. $K = 50$). Show how the cost-minimizing quantity of labor depends on the quantity Q . Show how the cost-minimizing quantity of materials depends on the quantity Q .

Beginning with the tangency condition

$$\frac{MP_L}{MP_M} = \frac{w}{m}$$

$$\frac{KM}{KL} = \frac{5}{2}$$

$$M = \frac{5L}{2}$$

Setting $\bar{K} = 50$ and substituting into the production function yields

$$Q = L(50) \left(\frac{5L}{2} \right)$$

$$Q = 125L^2$$

$$L = \sqrt{\frac{Q}{125}}$$

Substituting this result into the tangency condition result above implies

$$M = \frac{5\sqrt{\frac{Q}{125}}}{2}$$

$$M = \sqrt{\frac{Q}{20}}$$

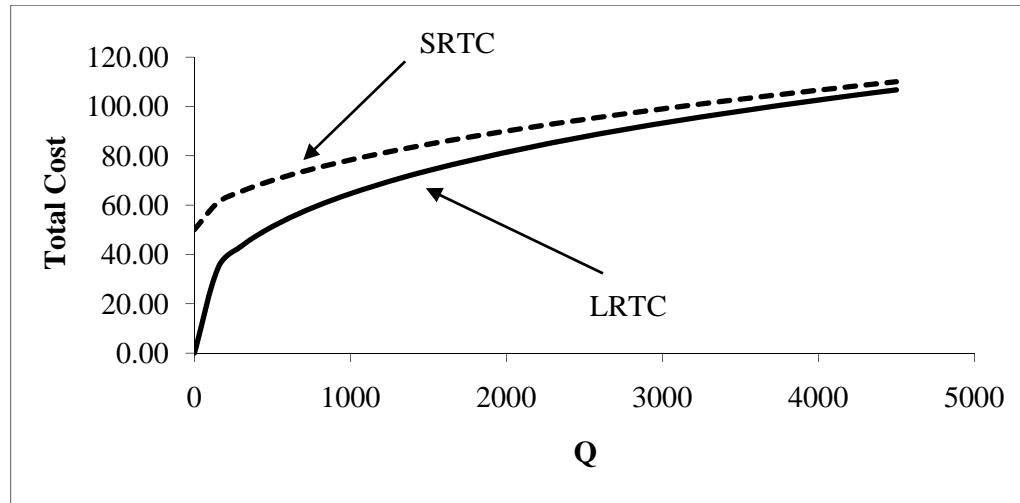
- e) Find the equation of the short-run total cost curve when capital is fixed at a quantity of 50 units (i.e. $\bar{K} = 50$) and graph it along with the long-run total cost curve.

In the short run,

$$TC = 5\sqrt{\frac{Q}{125}} + 50 + 2\sqrt{\frac{Q}{20}}$$

$$TC = 2\sqrt{\frac{Q}{5}} + 50$$

Graphically, short-run and long-run total cost curves are shown in the following figure.



f) Find the equation of the associated short-run average cost curve.

Short run average cost is given by

$$AC = \frac{TC}{Q} = \frac{2\sqrt{\frac{Q}{5}} + 50}{Q}$$

EconS 301 – Intermediate Microeconomics

Review Session #7 – Chapter 9: Perfectly Competitive Markets

1. The market for sweet potatoes consists of 1,000 identical firms. Each firm has a short-run total cost curve of $STC = 100 + 100q + 100q^2$, and a short-run marginal cost curve of $SMC=100+200q$ where q is output. Suppose that sunk costs are 75 and non-sunk costs are 25. What is the equation of an individual firm's short-run supply curve?
- a) $q = \frac{P}{200} - .5$ for $P \geq 100$, and $q=0$ otherwise.
 - b) $q = \frac{P}{100} - .5$ for $P \geq 200$, and $q=0$ otherwise.
 - c) $P = 100 + 200q$
 - d) $q = \frac{P}{200} - .5$ for $P \geq 200$, and $q=0$ otherwise.

Answer

Recall that the supply curve is simply the marginal cost curve above the minimum average variable cost. So we simply have to take the given marginal cost and solve for q , treating $MC=P$. Also, note that the supply will be zero for a certain range of prices. Thus, the answer is A.

2. A perfectly competitive firm's short-run supply curve is determined by the equation:
- a) $P = AC$ where $P \geq SMC$. Otherwise, supply is zero.
 - b) $P = AVC$ where $P \geq SMC$. Otherwise, supply is zero.
 - c) $P = SMC$ where $P \geq AC$. Otherwise, supply is zero.
 - d) $P = SMC$ where $P \geq AVC$ or $P \geq ANSC$ or $P \geq SAC$, depending on the level of sunk costs. Otherwise, supply is zero.

Answer

Again, recall that the supply curve is simply the marginal cost curve above the minimum average variable cost. Thus the answer is D.

3. Which of the following is *not* true in a long-run perfectly competitive equilibrium?
- a) $P = MC$, where P is market price and MC is the marginal cost of a firm.
 - b) $P = AC$, where P is market price and AC is the average cost of a firm.
 - c) $Q^d = nq$, where q is the supply of an individual firm, n is the number of firms in the industry, and Q^d is the market demand for a product.
 - d) Firms may earn negative profits.

Answer

Firms will not earn negative profits in a long-run equilibrium. They would simply exit the market. Thus the answer is D.

4. For an entire perfectly competitive industry, which of the following statements is *incorrect* in the long run?
- Economic profit for the industry equals zero.
 - Producer surplus equals economic rent.
 - Economic profit equals total revenues less total costs.
 - Producer surplus for the industry equals economic profit for the industry.

Answer

We know producer surplus equals economic profit, and we know in the long-run economic profit will be zero. We also know that economic rent is not zero, thus the answer is B.

5. In a perfectly competitive, increasing-cost industry in the long run, economic profit for the industry _____ and economic rent _____.
- can be positive; can be positive.
 - can be positive; equals zero.
 - equals zero; can be positive.
 - equals zero; equals zero.

Answer

We know in a perfectly competitive market, the economic profit is zero, so you can eliminate choices A and B. Also, we know that economic rent can be positive, thus the answer is C.

WRITTEN EXERCISES

6. In a certain market in the long-run, each firm and potential entrant has a long-run average cost curve $AC = 10Q^2 - 5Q + 20$ and long-run marginal cost curve $MC = 30Q^2 - 10Q + 20$ where Q is thousands of units per year. Market demand is given by $D(P) = 39,000 - 2,000P$.
- In equilibrium, how many units will each firm produce?

Answer

In the long-run equilibrium, each firm will produce where $P = AC = MC$. Thus,

$$\begin{aligned}10Q^2 - 5Q + 20 &= 30Q^2 - 10Q + 20 \\20Q^2 - 5Q &= 0 \\20Q - 5 &= 0 \\Q &= 0.25\end{aligned}$$

- b) What is the market equilibrium price?

Answer

Since each firm produces where $P = MC$, price will be

$$\begin{aligned}P &= 30Q^2 - 10Q + 20 \\P &= 30(0.25)^2 - 10(0.25) + 20 \\P &= 19.375\end{aligned}$$

- c) What is the equilibrium number of firms in the long-run?

Answer

Since total market demand is 250 and each firm is produce 0.25 units, the total number of firms in the market in equilibrium will be

$$\begin{aligned}N &= \frac{250}{0.25} \\N &= 1,000\end{aligned}$$

7. Suppose market demand is given by $D(P) = 25 - 0.25P$ and market supply is given by $S(P) = 0.2P - 2$.
- a) What are the market equilibrium price and quantity?

Answer

Setting market demand equal to market supply yields

$$\begin{aligned}25 - 0.25P &= 0.2P - 2 \\0.45P &= 27 \\P &= 60\end{aligned}$$

At $P = 60$, the equilibrium quantity sold will be

$$\begin{aligned}D(P) &= 25 - 0.25P \\D(60) &= 25 - 0.25(60) \\D(60) &= 10\end{aligned}$$

The equilibrium quantity is 10 units.

- b) What is producer surplus at the market equilibrium?

Answer

Producer surplus is given by area B in the figure above. Thus, producer surplus is $PS = 0.5(60 - 10)10 = 250$.

8. Suppose a firm's short-run total cost curve is given by

$$STC = 30Q^2 + 25Q + 15$$

with short-run marginal cost $SMC = 60Q + 25$.

- a) What is the equation for the firm's short-run supply curve?

Answer

First, we find the minimum of average variable cost by setting average variable cost equal to short-run marginal cost.

$$\begin{aligned}30Q + 25 &= 60Q + 25 \\ Q &= 0\end{aligned}$$

At $Q = 0$, average variable cost is $AVC = 30Q + 25 = 30(0) + 25 = 25$. The supply curve is the short-run marginal cost curve above the minimum point of average variable cost. Thus,

$$S(P) = \begin{cases} \frac{P - 25}{60} & P \geq 25 \\ 0 & P < 25 \end{cases}$$

EconS 301

Review Session #8 – Chapter 11: Monopoly and Monopsony

1. Which of the following describes a correct relation between price elasticity of demand and a monopolist's marginal revenue when inverse demand is linear, $P = a-bQ$?
 - a) Demand is elastic when $Q > a/2b$.
 - b) Demand is inelastic when $Q > a/b$.
 - c) Demand is unit elastic when $P = a/2b$.
 - d) Demand is elastic when $Q < a/2b$.

Answer

Recall that a monopolist maximizes profits when $MR=MC$. And recall that, given a linear demand, the marginal revenue will have a slope exactly twice as steep as the demand. Thus, we know that the marginal revenue is $MR = a-2bQ$. So, at a quantity of $Q = a/2b$, we will have $MR=0$. This point is also exactly in the middle of the demand curve, where the demand is unitary elastic. And we know the monopolist will have a $MR>0$, thus they will be operating at a quantity less than $Q = a/2b$, and the answer is D.

2. In order to calculate the Lerner Index for a particular firm, you need to know _____ and _____ for that firm.
 - a) marginal cost; marginal revenue
 - b) marginal cost; price
 - c) price; quantity
 - d) price; demand

Answer

The learner index is given by, $(P-MC)/P$. Thus, the answer is B.

3. A monopolist owns two plants in which to produce a product which has inverse demand $P = (770/3) - 3Q$. The monopolist has marginal cost curves of $MC_1 = 20+3Q_1$ and $MC_2 = 10+6Q_2$ in the two plants, respectively. Which of the following represents the optimal outputs in the two plants, Q_1 and Q_2 and the market price?
 - a) $Q_1 = 170/9; Q_2 = 100/9; P = 500/3$.
 - b) $Q_1 = 100/9; Q_2 = 170/9; P = 500/3$.
 - c) $Q_1 = 500/3; Q_2 = 170/9; P = 100/9$.
 - d) $Q_1 = 500/3; Q_2 = 100/9; P = 170/9$.

Answer

First we need to find the total marginal cost by summing the two inverse marginal cost curves over quantity,

$$MC_1 = 20 + 3Q_1 \Rightarrow Q_1 = \frac{MC_1 - 20}{3}$$

$$MC_2 = 10 + 6Q_2 \Rightarrow Q_2 = \frac{MC_2 - 10}{6}$$

$$Q_1 + Q_2 = \frac{MC_1 - 20}{3} + \frac{MC_2 - 10}{6} = \frac{3MC_T - 50}{6}$$

solving for MC_T ,

$$MC_T = 2Q_T + \frac{100}{6}$$

set up profit max condition $MC_T = MR$,

$$2Q_T + \frac{100}{6} = \frac{770}{3} - 6Q_T$$

$$Q_T = 30$$

$$P = \frac{770}{3} - 3(30) = \frac{500}{3}$$

$$MC_T = 2(30) + \frac{100}{6} = \frac{230}{3} \text{ into inverse MC curves,}$$

$$Q_1 = \frac{\left(\frac{230}{3}\right) - 20}{3} = \frac{170}{9}$$

$$Q_2 = \frac{\left(\frac{230}{3}\right) - 10}{6} = \frac{100}{9}$$

Thus, the answer is A.

4. The profit-maximizing monopsonist hires an optimal quantity of input (e.g. labor) so that
- the marginal expenditure on that input equals its marginal revenue product.
 - the average expenditure on that input equals its average revenue product.
 - the marginal expenditure on that input equals its average revenue product.
 - the average expenditure on that input equals its marginal revenue product.

Answer

We know the monopolist will use an input until $MC=MR$. Thus, the answer is A.

5. A monopsonist only uses labor to produce an output according to production function $Q = 2L$, where Q is output and L is labor. The output sells for a price of \$20 per unit. The supply curve for labor can be written $w = 4 + L$. What is the monopsonist's demand for labor in this market?
- $L = 12$.
 - $L = 18$.
 - $L = 22$.
 - $L = 24$.

Answer

The monopolist will use labor to the point where marginal expenditure is equal to marginal revenue product. Thus, we need to find these for labor.

$$ME_L = w + \left(\frac{\delta w}{\delta L} \right) L$$

$$ME_L = (4 + L) + L$$

$$ME_L = 4 + 2L$$

$$MRP_L = p \left(\frac{\delta Q}{\delta L} \right)$$

$$MRP_L = 20(2) = 40$$

$$ME_L = MRP_L$$

$$4 + 2L = 40$$

$$L = \frac{40 - 4}{2} = 18$$

Thus the answer is B.

WRITTEN EXERCISES

6. Assume that a monopolist sells a product with a total cost function

$$TC = 400 + Q^2$$

and a corresponding marginal cost function

$$MC = 2Q .$$

The market demand curve is given by the equation $P = 500 - Q$.

- a) Find the profit-maximizing output and price for this monopolist. Is the monopolist profitable?

Answer

To find the profit-maximizing price and quantity, set $MR = MC$.

$$MR = 500 - 2Q$$

$$MC = 2Q$$

$$2Q = 500 - 2Q$$

$$4Q = 500$$

$$Q = 125$$

Plug Q into the demand curve to find P .

$$\begin{aligned}P &= 500 - Q \\P &= 500 - 125 \\P &= 375\end{aligned}$$

Profit equals total revenue minus total cost.

$$\begin{aligned}\pi &= PQ - TC \\&= 125(375) - (400 + 125^2) \\&= 46,875 - 400 - 15,625 \\&= 30,852\end{aligned}$$

Yes, the monopolist is profitable.

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- b) Calculate the price elasticity of demand at the monopolist's profit-maximizing price. Also calculate the marginal cost at the monopolist's profit-maximizing output. Verify that the IEPR rule holds.

Answer

The price elasticity of demand at the profit-maximizing price is -3 .

$$\begin{aligned}\varepsilon_{Q,P} &= \frac{\Delta Q}{\Delta P} \frac{P}{Q} \\&= -1 \left(\frac{375}{125} \right) = -3\end{aligned}$$

The marginal cost when $Q = 125$ equals $2Q = 2(125) = 250$. Therefore, the IEPR rule holds.

$$\begin{aligned}IEPR &\Rightarrow \frac{P - MC}{P} = -\frac{1}{\varepsilon_{Q,P}} \\&\frac{375 - 250}{375} = -\frac{1}{-3} \\&\frac{1}{3} = \frac{1}{3}\end{aligned}$$

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7. Suppose a monopolist faces demand $Q^d = 200 - 5P$ and has a constant marginal cost of \$5.
- a) What price should the monopolist charge to maximize its profits?

Answer

To find the profit-maximizing price, set $MR = MC$.

$$\begin{aligned}Q &= 200 - 5P \\5P &= 200 - Q \\P &= 40 - 0.20Q \\MR &= 40 - 0.40Q\end{aligned}$$

$$\begin{aligned}40 - 0.40Q &= 5 \\Q &= 87.5\end{aligned}$$

At $Q = 87.5$, the monopolist will charge a price $P = 40 - 0.20(87.5) = 22.50$.

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- b) What is the Lerner Index of Market Power for this monopolist?

Answer

To calculate the Lerner Index, calculate

$$\begin{aligned}L &= \frac{P - MC}{P} \\L &= \frac{22.50 - 5}{22.5} \\L &= 0.78\end{aligned}$$

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EconS 301 – Intermediate Microeconomics

Review Session #9 – Chapter 12: Capturing Surplus

1. With second-degree price discrimination
 - a) The firm tries to price each unit at the consumer's reservation price.
 - b) The firm offers consumers a quantity discount.
 - c) The firm charges different consumer groups or market segments a different price.
 - d) A buyer can only purchase one product by agreeing to purchase some other product as well.

Answer: Recall second degree price discrimination is based on the quantity of a good purchased. Thus the answer is B.

2. Identify the truthfulness of the following statements.
 - I. If a seller engages in second-degree price discrimination, the seller captures more producer surplus than with uniform pricing.
 - II. The seller captures the maximum producer surplus by engaging in third-degree price discrimination.
 - a) Both I and II are true.
 - b) Both I and II are false.
 - c) I is true; II is false.
 - d) I is false; II is true.

Answer: All forms of price discrimination allow the producer to capture more surplus than uniform pricing, so statement one is true. And first degree price discrimination occurs when a producer captures ALL surplus from the consumers, so statement two is false. Thus the answer is C.

3. An expenditure schedule in which the average outlay changes with the number of units purchased is
 - a) Block tariff.
 - b) Nonlinear outlay schedule.
 - c) Average expenditure.
 - d) Usage charges.

Answer: The definition of a nonlinear outlay schedule is one in which the average outlay changes as the number of units purchased changes, thus the answer is B.

4. Which of the following is NOT a real-world example of third-degree price discrimination?
 - a) A railroad charges more to haul 100 tons of coal than it does to haul 100 tons of grain.
 - b) An airline charges a lower price for a coach ticket purchased four weeks in advance than for the same type of ticket purchased three days in advance.
 - c) A movie theater charges senior citizens a cheaper price for movie tickets than it charges non-senior citizens for the same movie ticket.

- d) Sam's Club® warehouses sell bulk quantities of macaroni and cheese for a cheaper per unit price than a grocery store, but the boxes are packaged together so that the customer must buy six boxes at a time.

Answer: Third degree price discrimination occurs when different groups of consumers are identified and charged a different, profit maximizing price, for each type of consumer. Thus choices a-c are all third degree price discrimination, while d is not. Thus, the answer is D.

5. Which of the following is a real-world example of tying?

- a) A movie theater charges senior citizens a cheaper price for movie tickets than it charges non-senior citizens for the same movie ticket.
- b) Sam's Club® warehouses sell bulk quantities of macaroni and cheese for a cheaper per unit price than a grocery store, but the boxes are packaged together so that the customer must buy six boxes at a time.
- c) An airline charges more for a first-class ticket than for a coach ticket.
- d) The manufacturer of an instant-prints camera is the only manufacturer of the film that the camera uses.

Answer: In the case of choice d, in order to use the camera, you must purchase the film manufactured by the same firm, thus “tying” the products. Hence, the answer is D.

6. You own a small bookstore. Your hired a marketing firm to calculate your own price elasticity of demand and your advertising elasticity of demand. Assume the firm has provided you with the relevant numbers regardless of minor adjustments in price or advertising budget. Your own price elasticity of demand is around -1.7 , and your advertising elasticity of demand is around 0.05 . How much should you mark-up your price over your marginal cost for your books?

- a) By a factor of 0.41.
- b) By a factor of 2.43.
- c) By a factor of 37 percent.
- d) By a factor of 70 percent.

Answer: Using the inverse elasticity pricing rule, we have

$$\begin{aligned}\frac{P - MC}{P} &= -\frac{1}{\varepsilon_{Q,p}} \\ \frac{P - MC}{P} &= -\frac{1}{-1.7} = \frac{10}{17} \\ P - MC &= \frac{10}{17}P \\ P \left(1 - \frac{10}{17}\right) &= MC \\ P = \frac{17}{7}MC &= 2.43MC\end{aligned}$$

Thus, books should be marked up 2.43 times the marginal cost. Hence, the answer is B.

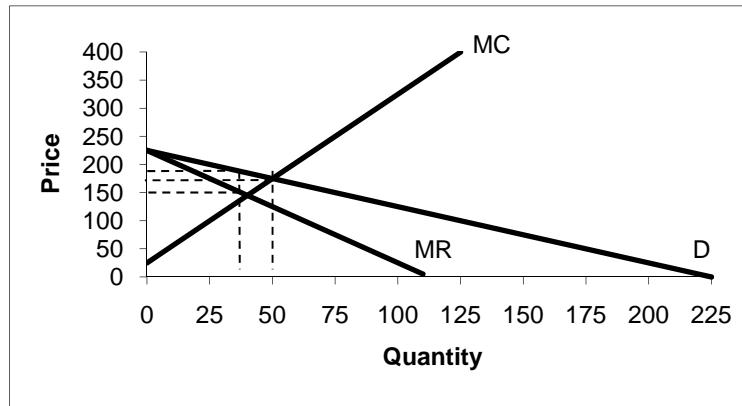
WRITTEN EXERCISES

1. Suppose a monopolist faces demand $P = 225 - Q$ and has marginal cost $MC = 25 + 3Q$. Complete the following table identifying consumer surplus, producer surplus, total surplus, and deadweight loss for two situations: (1) the monopoly charges a uniform price and (2) the monopoly engages in first-degree price discrimination.

	Uniform Price	First-degree Price Discrimination
Consumer Surplus		
Producer Surplus		
Total Surplus		
Deadweight Loss		

Answer

The following figure illustrates the two situations.



With uniform pricing the monopoly charges \$185 for each unit and sells 40 units. With first-degree price discrimination, the last unit is sold for \$175 and the monopoly sells 50 units. Here is the completed table.

	Uniform Price	First-degree Price Discrimination
Consumer Surplus	800	0
Producer Surplus	4,000	5,000
Total Surplus	4,800	5,000
Deadweight Loss	200	0

2. A monopolist faces demand from two different market segments. In the first segment, demand is given by $P_1 = 50 - 2Q_1$ and in the second segment demand is given by $P_2 = 30 - 0.5Q_2$. If the monopolist has a constant marginal cost $MC = 10$, what price should the monopolist charge each market segment and how many units will it sell to each segment if it engages in third-degree price discrimination?

Answer

To maximize profits from third-degree price discrimination, the monopolist should set $MR = MC$ in each market segment. In the first market segment this implies

$$50 - 4Q_1 = 10$$

$$Q_1 = 10$$

At this quantity, the monopoly will charge this segment a price $P_1 = 50 - 2(10) = 30$. In the second market segment we have

$$30 - Q_2 = 10$$

$$Q_2 = 20$$

At this quantity, the monopoly will charge this segment a price $P_2 = 30 - 0.5(20) = 20$.

3. Consider a simple bundling problem in which a producer sells two products to three potential customers. The customer's reservation prices for the two products and the firm's marginal costs are given in the following table.

Reservation Prices		
	Product A	Product B
Customer 1	50	40
Customer 2	75	30
Customer 3	100	10
Marginal Cost	10	5

- a) If the firm does not bundle the products, what price should it charge for Product A and for Product B to maximize profit? How much profit will the firm expect to earn?

Answer

If the firm does not bundle the products, then for Product A the firm should charge a price of \$75. At this price, the firm will sell Product A to Customer's 2 and 3 earning \$150 in revenue (with \$20 in cost). For Product B, the firm should charge a price of \$30. At this price the firm will sell Product B to Customer's 1 and 2 earning \$60 in revenue (with \$10 in cost). The firm's profit will be total revenue, \$210, less total cost, \$30, or \$180.

- b) If the firm can bundle the products, what price should it charge to maximize profit and how much profit can it expect to earn? How does this compare to result in part a)?

Answer

If the firm can bundle the products, then when determining the profit-maximizing price it looks at the reservation prices for the bundle. These are \$90, \$105, and \$110 for the three Customers. With these reservation prices, the firm will maximize profits by setting price at \$90 for the bundle. At this price, the firm will sell bundles to all three Customers earning revenue of \$270 and incurring cost of \$45. The firm can expect to earn a profit of \$225, or \$45 more than when they could not bundle.

4. Suppose you own your own business and you estimate your own price elasticity of demand at -2 , and that your advertising elasticity of demand is 0.2 .
 - a) How much should you mark-up your price over your marginal cost for your product to maximize profit?

Answer

To determine the optimal mark-up, use the inverse-elasticity pricing rule.

$$\begin{aligned}\frac{P - MC}{P} &= -\frac{1}{\varepsilon_{Q,P}} \\ \frac{P - MC}{P} &= -\frac{1}{-2} \\ 2P - 2MC &= P \\ P &= 2MC\end{aligned}$$

This implies that to maximize profit you should set your price at about double the marginal cost.

- b) What should your advertising-to-sales ratio be?

Answer

The optimal advertising-to-sales ratio should be

$$\begin{aligned}\frac{A}{PQ} &= -\frac{\varepsilon_{Q,A}}{\varepsilon_{Q,P}} \\ \frac{A}{PQ} &= -\frac{0.2}{-2} \\ \frac{A}{PQ} &= 0.10\end{aligned}$$

This implies your advertising expenses should be about 10% of your sales revenues.

5. A monopolist faces demand from two different market segments. In the first segment, demand is given by $P_1 = 100 - 5Q_1$ and in the second segment demand is given by $P_2 = 50 - 4Q_2$. If the monopolist has a constant marginal cost $MC = 10$, what price should the monopolist charge each

market segment and how many units will it sell to each segment if it engages in third-degree price discrimination?

Answer

To maximize profits from third-degree price discrimination, the monopolist should set $MR = MC$ in each market segment. In the first market segment this implies

$$\begin{aligned}10 &= 100 - 10Q_1 \\10Q_1 &= 90 \\Q_1 &= 9\end{aligned}$$

At this quantity, the monopoly will charge this segment a price $P_1 = 100 - 5Q_1 = 100 - 45 = 55$. In the second market segment we have

$$\begin{aligned}10 &= 50 - 8Q_2 \\8Q_2 &= 40 \\Q_2 &= 5\end{aligned}$$

At this quantity, the monopoly will charge this segment a price $P_2 = 50 - 4Q_2 = 50 - 20 = 30$.

6. Which of the following are examples of first-degree, second-degree, and third-degree price discrimination?
- a) The local movie theater offers discounts to all senior citizens.

Answer: Third degree—the firm is charging a different price to different market segments.

- b) Baseball tickets are \$7 each for individual games. Season tickets are \$200 for a 40-game season.

Answer: Second degree—the firm is offering quantity discounts.

- c) On Saturday afternoon, there is an auction of furniture, dishes, etc. from an estate.

Answer: First degree—each consumer is paying near his/her maximum willingness to pay.

- d) Two people are sitting at a dinner table on a 3-day cruise to the Bahamas. The first man says what a great deal he got for the trip because the ship had empty cabins. He paid \$300. The second man is embarrassed to say that he booked his trip six months prior and paid \$500 for a similar cabin.

Answer: Third degree—the firm is charging a different price to different market segments.

EconS 301 – Intermediate Microeconomics
Review Session #10 – Chapter 13: Market Structure and Competition

Exercise 13.2. A homogeneous products duopoly faces a market demand function given by $P = 300 - 3Q$, where $Q = Q_1 + Q_2$. Both firms have constant marginal cost $MC = 100$.

- a) What is Firm 1's profit-maximizing quantity, given that Firm 2 produces an output of 50 units per year? What is Firm 1's profit-maximizing quantity when Firm 2 produces 20 units per year?

With two firms, demand is given by $P = 300 - 3Q_1 - 3Q_2$. If $Q_2 = 50$, then

$$P = 300 - 3Q_1 - 150 \text{ or } P = 150 - 3Q_1. \text{ Setting } MR = MC \text{ implies}$$

$$\begin{aligned} 150 - 6Q_1 &= 100 \\ Q_1 &= 8.33 \end{aligned}$$

If $Q_2 = 20$, then $P = 240 - 3Q_1$. Setting $MR = MC$ implies

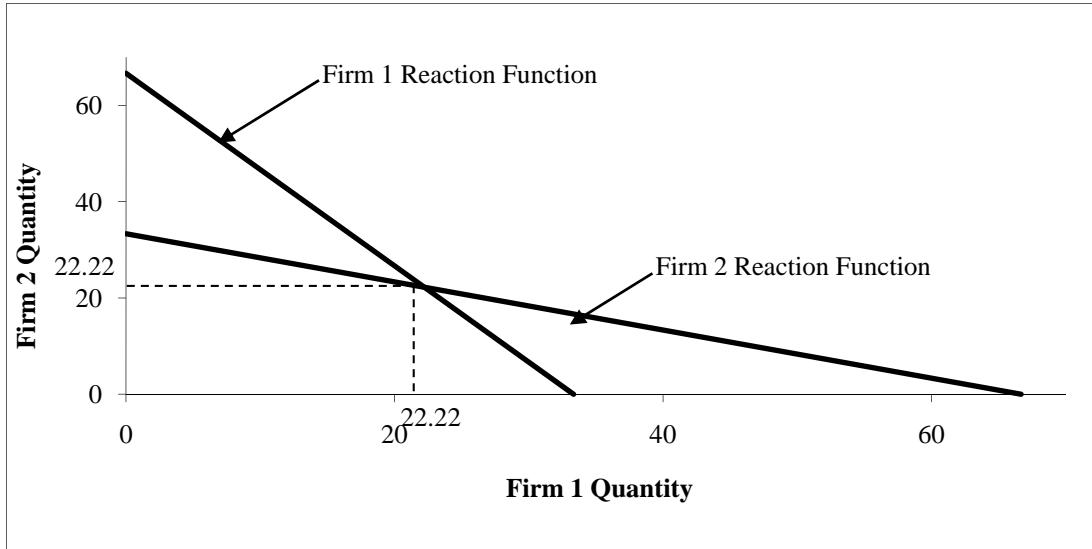
$$\begin{aligned} 240 - 6Q_1 &= 100 \\ Q_1 &= 23.33 \end{aligned}$$

- b) Derive the equation of each firm's reaction curve and then graph these curves.

For Firm 1, $P = (300 - 3Q_2) - 3Q_1$. Setting $MR = MC$ implies

$$\begin{aligned} (300 - 3Q_2) - 6Q_1 &= 100 \\ Q_1 &= 33.33 - 0.5Q_2 \end{aligned}$$

Since the marginal costs are the same for both firms, symmetry implies $Q_2 = 33.33 - 0.5Q_1$. Graphically, these reaction functions appear as



c) What is the Cournot equilibrium quantity per firm and price in this market?

Because of symmetry, in equilibrium both firms will choose the same level of output. Thus, we can set $Q_1 = Q_2$ and solve

$$\begin{aligned} Q_2 &= 33.33 - 0.5Q_2 \\ Q_2 &= 22.22 \end{aligned}$$

Since both firms will choose the same level of output, both firms will produce 22.22 units. Price can be found by substituting the quantity for each firm into market demand. This implies price will be $P = 300 - 3(44.44) = 166.67$.

d) What would the equilibrium price in this market be if it were perfectly competitive?

If this market were perfectly competitive, then equilibrium would occur at the point where $P = MC = 100$.

$$\begin{aligned} 300 - 3Q &= 100 \\ 200 &= 3Q \\ Q &= \frac{200}{3} \end{aligned}$$

e) What would the equilibrium price in this market be if the two firms colluded to set the monopoly price?

If the firms colluded to set the monopoly price, then

$$300 - 6Q = 100$$

$$Q = 33.33$$

At this quantity, market price will be $P = 300 - 3(20\%) = 200$.

f) What is the Bertrand equilibrium price in this market?

If the firms acted as Bertrand oligopolists, the equilibrium would coincide with the perfectly competitive equilibrium of $P = 100$.

g) What are the Cournot equilibrium quantities and industry price when one firm has a marginal cost of 100 but the other firm has a marginal cost of 90?

Suppose Firm 1 has $MC = 100$ and Firm 2 has $MC = 90$. For Firm 1, $P = (300 - 3Q_2) - 3Q_1$. Setting $MR = MC$ implies

$$(300 - 3Q_2) - 6Q_1 = 100$$

$$Q_1 = 33.33 - 0.5Q_2$$

For Firm 2, $P = (300 - 3Q_1) - 3Q_2$. Setting $MR = MC$ implies

$$(300 - 3Q_1) - 6Q_2 = 90$$

$$Q_2 = 35 - 0.5Q_1$$

Solving these two reaction functions simultaneously yields $Q_1 = 21.11$ and $Q_2 = 24.44$. With these quantities, market price will be $P = 163.36$.

Exercise 13.3. Zack and Andon compete in the peanut market. Zack is very efficient at producing nuts with a low marginal cost $C_z=1$; Andon, however, has a constant marginal cost $C_a=10$. If the market demand for nuts is $P = 100 - Q$, find the Cournot equilibrium price and the quantity and profit level for each competitor.

For Zack, $MR_Z = MC_Z$ implies $100 - 2q_Z - q_A = 1$, so Zack's reaction function is $q_Z = \frac{1}{2}*(99 - q_A)$. Similarly, $MR_A = MC_A$ implies $100 - 2q_A - q_Z = 10$ so Andon's reaction function is $q_A = \frac{1}{2}*(90 - q_Z)$. Solving these two equations in two unknowns yields $q_Z = 36$ and $q_A = 27$. The market price is $P = 100 - (36 + 27) = 37$. Zack earns $\pi_Z = (37 - 1)*36 = 1296$ and Andon earns $\pi_A = (36 - 10)*27 = 702$.

Exercise 13.6. Suppose that demand for cruise ship vacations is given by $P = 1200 - 5Q$, where Q is the total number of passengers when the market price is P

a) The market initially consists of only three sellers, Alpha Travel, Beta Worldwide, and Chi Cruiseline. Each seller has the same marginal cost of \$300 per passenger. Find the symmetric Cournot equilibrium price and output for each seller.

Consider first the problem of Alpha Travel. It produces until $MR_A = MC_A$ or $1200 - 5(Q_B + Q_C) - 10Q_A = 300$. Thus its reaction function is

$$Q_A = 90 - 0.5(Q_B + Q_C).$$

Symmetry implies that in equilibrium $Q_A = Q_B = Q_C$, so we can solve to find that $Q_i = 45$ for each firm. Thus the equilibrium price is $P = 525$.

b) Now suppose that Beta Worldwide and Chi Cruiseline announce their intention to merge into a single firm. They claim that their merger will allow them to achieve cost savings so that their marginal cost is less than \$300 per passenger. Supposing that the merged firm, BetaChi, has a marginal cost of $c < \$300$, while Alpha Travel's marginal cost remains \$300, for what values of c would the merger raise consumer surplus relative to part (a).

Reconsidering Alpha's profit-maximization problem, we now have that $MR_A = MC_A$ or $1200 - 5Q_{BC} - 10Q_A = 300$. Thus its reaction function is

$$Q_A = 90 - 0.5Q_{BC}.$$

The merged firm will produce until $MR_{BC} = MC_{BC}$ or $1200 - 5Q_A - 10Q_{BC} = c$, so its reaction function is

$$Q_{BC} = 120 - 0.1c - 0.5Q_A$$

Solving these two equations as a function of c yields $Q_{BC} = 100 - (2/15)*c$ and $Q_A = 40 + (1/15)*c$. Total output is then $Q = 140 - (1/15)*c$, and so the market price is $P = 500 + \frac{1}{3}*c$. Put simply, the merger raises consumer surplus only if the price falls; thus consumer surplus rises only when $500 + \frac{1}{3}*c < 525$, or $c < 75$.

Exercise 13.12. Consider an oligopoly in which firms choose quantities. The inverse market demand curve is given $P = 280 - 2(X + Y)$, where X is the quantity of Firm 1, and Y is the quantity of Firm 2. Each firm has a marginal cost equal to 40.

a) What is the Cournot equilibrium outputs for each firm? What is the market price at the Cournot equilibrium? What is the profit for each firm?

The table below summarizes the answer to this problem. The solution details follow.

	Firm 1 output	Firm 2 output	Market Price	Firm 1 Profit	Firm 2 Profit
Cournot	40	40	120	3,200	3,200
Stackelberg with Firm 1 as leader	60	30	100	3,600	1,800

a) Firm 1's marginal revenue is $MR = 280 - 2Y - 4X$. Equating MR to MC gives us:

$$280 - 2Y - 4X = 40,$$

$$240 - 2Y = 4X$$

$$X = 60 - 0.5Y$$

This is Firm 1's reaction function. Firm 2 is identical, and so Firm 2's reaction function will be a mirror image of Firm 1's

$$Y = 60 - 0.5X$$

If we solve these reaction functions simultaneously, we find $X = Y = 40$. At this output, the corresponding market price is $P = 280 - 2(40 + 40) = 120$. Each firm's profit is thus: $120*40 - 40*40 = 3,200$.

b) What is the Stackelberg equilibrium, when Firm 1 acts as the leader? What is the market price at the Stackelberg equilibrium? What is the profit for each firm?

To find the Stackelberg equilibrium in which Firm 1 is the leader, we start by writing the expression for Firm 1's total revenue:

$$TR = (280 - 2Y - 2X)X$$

In place of Y , we substitute in Firm 2's reaction function: $Y = 60 - 0.5X$

$$TR = [280 - 2(60 - 0.5X) - 2X]X = (160 - X)X$$

Firm 1's marginal revenue is therefore $MR = 160 - 2X$. Equating marginal revenue to marginal cost gives us:

$$160 - 2X = 40, \text{ or } X = 60.$$

To find Firm 2's output, we plug $X = 60$ back into Firm 2's reaction function:

$$Y = 60 - 0.5(60) = 30.$$

The market price is found by plugging $X = 60$ and $Y = 30$ back into the demand curve: $P = 280 - 2(60 + 30) = 100$.

Thus, at the Stackelberg equilibrium, Firm 1's profit is: $100*60 - 40*60 = 3,600$. Firm 2's profit is $100*30 - 40*30 = 1,800$.

Exercise 13.18. Apple's iPod has been the portable MP3-player of choice among many gadget enthusiasts. Suppose that Apple has a constant marginal cost of 4 and that market demand is given by $Q = 200 - 2P$.

a) If Apple is a monopolist, find its optimal price and output. What are its profits?

A monopolist sets $MR = MC$ (don't forget to invert the demand curve first!) so $100 - Q^m = 4$. Thus, $Q^m = 96$ and $P^m = 52$. As a monopolist, Apple's profits are $\pi = (52 - 4)*96 = 4608$.

- b) Now suppose there is a competitive fringe of 12 price-taking firms, each of whom has a total cost function $TC(q) = 3q^2 + 20q$ with corresponding marginal cost curve $MC = 6q + 20$. Find the supply function of the fringe (hint: a competitive firm supplies along its marginal cost curve above its shutdown price).

Each fringe firm maximizes profits by setting $P = MC = 6q + 20$, so we can derive a single firm's supply curve as $q = (P - 20)/6$, so long as $P > 20$. With the fringe comprising 12 firms, total supply is $Q_{\text{fringe}} = 12q$, or

$$Q_{\text{fringe}} = \begin{cases} 0 & P \leq 20 \\ 2P - 40 & P > 20 \end{cases}$$

- c) If Apple operates as the dominant firm facing competition from the fringe in this market, now what is its optimal output? How many units will fringe providers sell? What is the market price, and how much profit does Apple earn?

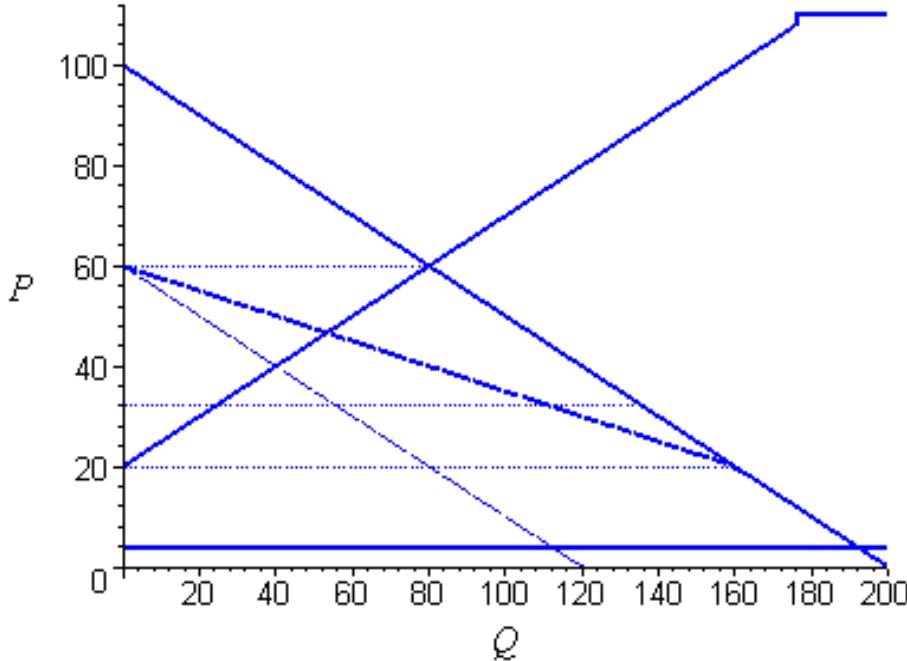
First, find Apple's (denoted DF for "dominant firm") residual demand, for $P > 20$: $Q_{DF} = Q_{mkt} - Q_{\text{fringe}} = 200 - 2P - (2P - 40) = 240 - 4P$. Inverting, this is $P = 60 - 0.25Q$. So Apple sets $MR = MC$ or

$$60 - 0.5Q_{DF} = 4$$

$$Q_{DF} = 112$$

From the residual demand, Apple's price is $P = 60 - 0.25*112 = 32$. At this price, the fringe supplies $Q_{\text{fringe}} = 2P - 40 = 2*32 - 40 = 24$. Apple's profits are $\pi = (32 - 4)*112 = 3136$.

- d) Graph your answer from part c



Exercise 13.23. Two firms, Alpha and Bravo, compete in the European chewing gum industry. The products of the two firms are differentiated, and each month the two firms set their prices. The demand functions facing each firm are:

$$Q_A = 150 - 10P_A + 9P_B$$

$$Q_B = 150 - 10P_B + 9P_A$$

where the subscript A denotes the firm Alpha and the subscript B denotes the firm Bravo. Each firm has a constant marginal cost of \$7 per unit.

- a) Find the equation of the reaction function for each firm.

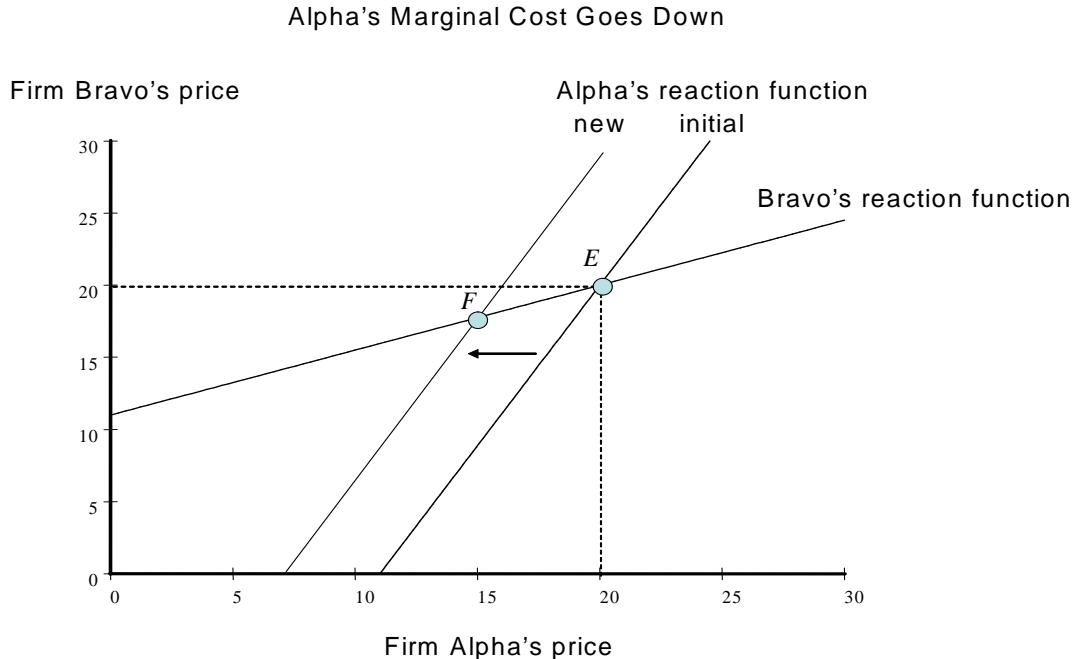
We will first solve for Alpha's reaction function. We begin by solving Alpha's demand function for P_A in terms of Q_A and P_B : $P_A = 15 - (1/10)Q_A + (9/10)P_B$. The corresponding marginal revenue equation is: $MR_A = 14 - (2/10)Q_A + (9/10)P_B$. Equating marginal revenue to marginal cost and solving for Q_A gives us Alpha's profit-maximizing quantity as a function of Bravo's price: $MR_A = MC_A \Rightarrow 15 - (1/10)Q_A + (9/10)P_B = 7$, which gives us: $Q_A = 40 + (9/2)P_B$. Now, substitute this expression for Q_A back into the expression for the demand curve with P_A on the left-hand side and Q_A on the right-hand side: $P_A = 15 - (1/10)[40 + (9/2)P_B] + (9/10)P_B \Rightarrow P_A = 11 + (9/20)P_B$. This is Alpha's reaction function.

- b) Find the Bertrand equilibrium price of each firm.

We can find Bravo's reaction function by following steps identical to those followed to derive Alpha's reaction function. Following these steps gives us: $P_B = 11 + (9/20)P_A$. We now have two equations (the two reaction functions) in two unknowns, P_A and P_B . Solving these equations gives us the Bertrand equilibrium prices: $P_A = P_B = 20$.

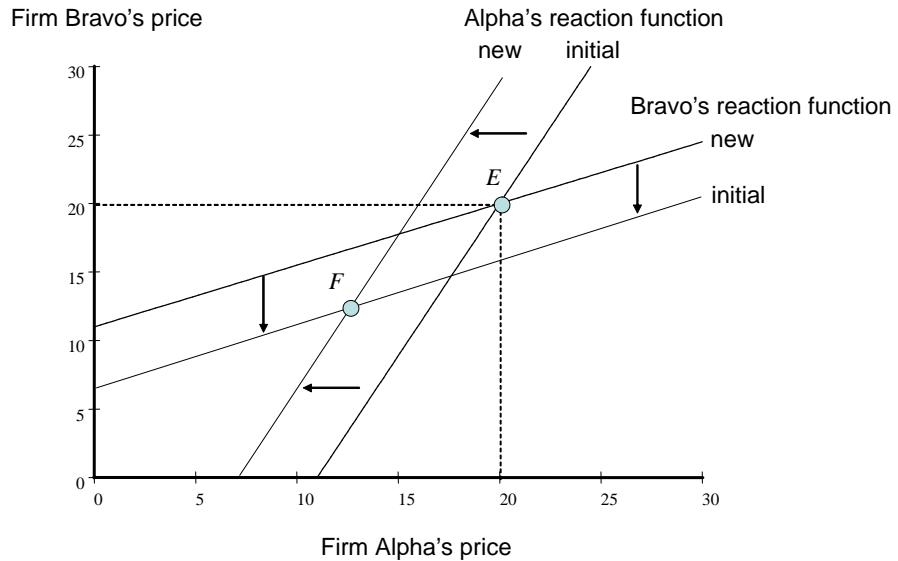
- c) Sketch how each firm's reaction function is affected by each of the following changes:

- i) Alpha's marginal cost goes down (with Bravo's marginal cost remaining the same).



ii) Alpha and Bravo's marginal cost goes down by the same amount.

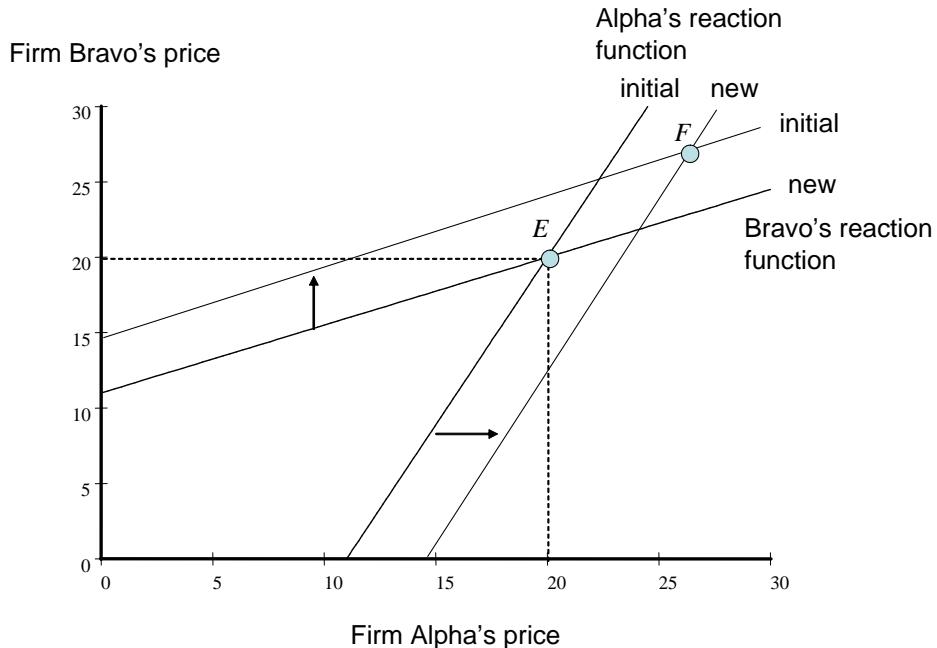
Alpha and Bravo's Marginal Cost Go Down by the Same Amount



3

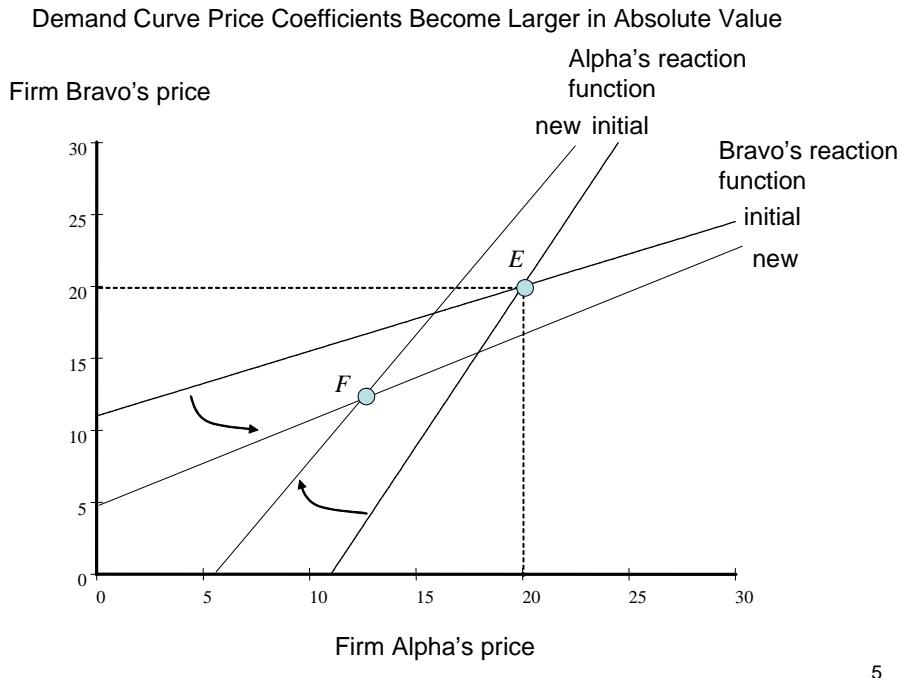
iii) Demand conditions change so that the “150” term in the demand function now becomes larger than 150.

Demand Curve Intercept Goes Up for Each Firm



4

- iv) The “10” and “9” terms in each demand function now become larger (e.g., they become “50” and “49” respectively).



- d) Explain in words how the Bertrand equilibrium price of each firm is affected by each of the following changes:

- i) Alpha’s marginal cost goes down (with Bravo’s marginal cost remaining the same).

Each firm’s equilibrium price goes down.

- ii) Alpha and Bravo’s marginal cost goes down by the same amount.

Each firm’s equilibrium price goes down.

- iii) Demand conditions change so that the “150” term in the demand function now becomes larger than 150.

Each firm’s equilibrium price goes up.

- iv) The “10” and “9” terms in each demand function now become larger (e.g., they become “50” and “49” respectively).

Each firm’s equilibrium price goes down.

Note: The graphs above can be used to explain these changes.

Exercise 13.31. The Thai food restaurant business in Evanston, Illinois, is monopolistically competitive. Suppose that each existing potential restaurant has a total cost function given by $TC = 10Q + 40,000$, where Q is the number of patrons per month and TC is the total cost per month. The fixed cost of \$40,000 includes fixed operating expenses (such as salary of the chef), the lease on the building space where the restaurant is located, and interest expenses on the bank loan needed to start the business in the first place.

Currently, there are 10 Thai restaurants in Evanston. Each restaurant faces a demand function given by $Q = \frac{4,000,000}{N} P^{-5} \bar{P}^4$, where P is the price of a typical entrée at the restaurant, \bar{P} is the price of a typical entrée averaged over all other Thai restaurants in Evanston, and N is the total number of restaurants. Each restaurant takes the prices of other Thai restaurants as given when choosing its own price.

a) What is the own-price elasticity of demand facing a typical restaurant?

Each firm faces an own price elasticity of demand of -5.

b) For a typical restaurant, what is the profit-maximizing price of a typical entrée?

Because the demand function exhibits constant elasticity, we can directly solve for a firm's profit-maximizing price by using the inverse elasticity pricing rule:

$$(P - MC)/P = -1/\varepsilon_{Q,P}$$

From the equation of the total cost function, we see that each firm faces a marginal cost of \$10 per patron. This implies

$$(P - 10)/P = -1/(-5), \text{ or } P = \$12.50.$$

c) At the profit-maximizing price, how many patrons does a typical restaurant serve per month? Given this number of patrons, what is the average total cost of a typical restaurant?

Each firm will charge a price of \$12.50, so therefore each restaurant attracts $(4,000,000/10)(12.50)^{-5}(12.50)^4 = 32,000$ patrons per month. Given this number of patrons each restaurant has an average total cost given by $10 + 40,000/32,000 = \$11.25$.

d) What is the long-run equilibrium number of Thai restaurants in the Evanston market?

To find the long-run equilibrium number of firms, we note that when there is an arbitrary number of firms N , the number of patrons going to each restaurant is $(4,000,000/N)(12.50)^{-5}(12.50)^4 = 320,000/N$. Given this number of patrons, each firm's average total cost is $10 + 40,000/(320,000/N) = 10 + 0.125N$. Because the demand function is constant elasticity, each firm will charge a price of \$12.50 no matter how many firms are in the market. In a long-run equilibrium, the price of 12.50 must equal each firm's average total cost. Thus: $10 + 0.125N = 12.50$, or 20.

EconS 301 – Intermediate Microeconomics

Review Session #11 – Chapter 14: Strategy and Game Theory

- 1) Asahi and Kirin are the two largest sellers of beer in Japan. These two firms compete head to head in dry beer category in Japan. The following table shows the profit (in millions of yen) that each firm earns when it charges different prices for its beer:

		Kirin			
		¥630	¥660	¥690	¥720
Asahi	¥630	180, 180	184, 178	185, 175	186, 173
	¥660	178, 184	183, 183	192, 182	194, 180
	¥690	175, 185	182, 192	191, 191	198, 190
	¥720	173, 186	180, 194	190, 198	196, 196

- Does Asahi have a dominant strategy? Does Kirin?
- Both Asahi and Kirin have a dominated strategy: Find and identify it.
- Assume that Asahi and Kirin will not play the dominated strategy you identified in part (b) (i.e., cross out the dominated strategy for each firm in the table.) Having eliminated the dominated strategy, show that Asahi and Kirin now have another dominated strategy.
- Assume that Asahi and Kirin will not play the dominated strategy you identified in part (c). Having eliminated this dominated strategy, determine whether A and Kirin now have a dominant strategy.
- What is the Nash equilibrium in this game?

Answer

- Neither player has a dominant strategy in this game.
- In this game, Asahi has a dominated strategy, ¥720 is dominated by ¥690, and Kirin has a dominated strategy, ¥720 is dominated by ¥690. Assuming neither player will play these dominated strategies we can remove them from the game. The reduced game is

		Kirin		
		¥630	¥660	¥690
Asahi	¥630	180, 180	184, 178	185, 175
	¥660	178, 184	183, 183	192, 182
	¥690	175, 185	182, 192	191, 191

- Now that we have eliminated a dominated strategy from the original game, both players now have a dominated strategy in the reduced game. Asahi has a dominated strategy, ¥690 is dominated by ¥660, and Kirin has a dominated strategy, ¥690 is dominated by ¥660. Assuming neither player will play these dominated strategies we can remove them from the game. The reduced game is

		<i>Kirin</i>	
		¥630	¥660
<i>Asahi</i>	¥630	180, 180	184, 178
	¥660	178, 184	183, 183

- d) Now that we have eliminated another dominated strategy from the original game, both players have a dominant strategy to choose ¥630.
- e) Based on the analysis above, the Nash equilibrium in this game has both players choosing ¥630.
- 2) Consider the following game, where $x > 0$:

		<i>Firm 2</i>	
		High Price	Low Price
<i>Firm 1</i>	High Price	140, 140	20, 160
	Low Price	$90 + x, 90 - x$	50, 50

- a) For what values of x do both firms have a dominant strategy? What is the Nash equilibrium (or equilibria) in these cases?
- b) For what values of x does only one firm have a dominant strategy? What is the Nash equilibrium (or equilibria) in these cases?
- c) Are there any values of x such that neither firm has a dominant strategy? Ignoring mixed strategies, is there a Nash equilibrium in such cases?

Answer

- a) For $x > 50$, both firms have a dominant strategy. The unique NE is (*Low, Low*).
- b) For $40 < x < 50$ only Firm 2 has a dominant strategy. The unique NE in this case is still (*Low, Low*).
- c) For $x < 40$, there are zero dominant strategies, and no NE exists.
- 3) In a World Series game, Kerry Wood is pitching and Alex Rodriguez is batting. The count on Rodriguez is 3 balls and 2 strikes. Wood has to decide whether to throw a fastball or a curveball. Rodriguez has to decide whether to swing or not swing. If Wood throws a fastball and Rodriguez doesn't swing, the pitch will almost certainly be a strike, and Rodriguez will be out. If Rodriguez does swing, however, there is a strong likelihood that he will get a hit. If Wood throws a curve and Rodriguez swings, there is a strong likelihood that Rodriguez will strike out. But if Wood throws a curve and Rodriguez doesn't swing, there is a good chance that it will be ball four and Rodriguez will walk (assume that a walk is as good as a hit in this instance).

The following table shows the payoffs from each pair of choices that the two players can make:

		Alex Rodriguez	
		Swing	Do Not Swing
Kerry Wood	Fastball	-100, 100	100, -100
	Curveball	100, -100	-100, 100

- a) Is there a Nash equilibrium in pure strategies game?
- b) Is there a mixed strategy Nash equilibrium in game? If so, what is it?

Answer

- a) There are no Nash equilibria in pure strategies in this game.
- b) This game does have a Nash equilibrium in mixed strategies. To find the mixed strategy equilibrium, we need to find the probabilities with which, for example, Rodriguez plays "Swing" and "Do Not Swing" so that Wood's payoff from playing "Fast Ball" and "Curve Ball" are the same. Letting the probability of "Swing" be P and "Do Not Swing" be $1 - P$, Wood's expected payoff if he chooses "Fast Ball" is $-100P + 100(1 - P)$. His expected payoff if he chooses "Curve Ball" is $100P - 100(1 - P)$. Equating these yields:

$$\begin{aligned} -100P + 100(1 - P) &= 100P - 100(1 - P) \\ 100 - 200P &= 200P - 100 \\ P &= 0.50 \end{aligned}$$

Thus, in equilibrium Rodriguez should play "Swing" with a 50% probability and "Do Not Swing" with a 50% probability. Since the problem is symmetric, Wood should play "Fast Ball" with a 50% probability and "Curve Ball" with a 50% probability.

- 4) In the mid-1990s, Value Jet wanted to enter market serving routes that would compete head-to-head with Delta Airlines in Atlanta. Value Jet knew that Delta might respond in one of two ways: Delta could price war or it could be "accommodating," keep price at a high level. Value Jet had to decide whether it would enter on a small scale or on a large scale. Annual profits (in millions of dollars) associated with strategy are summarized in the following table (where the first number is the payoff to Value Jet and the second the payoff to Delta):

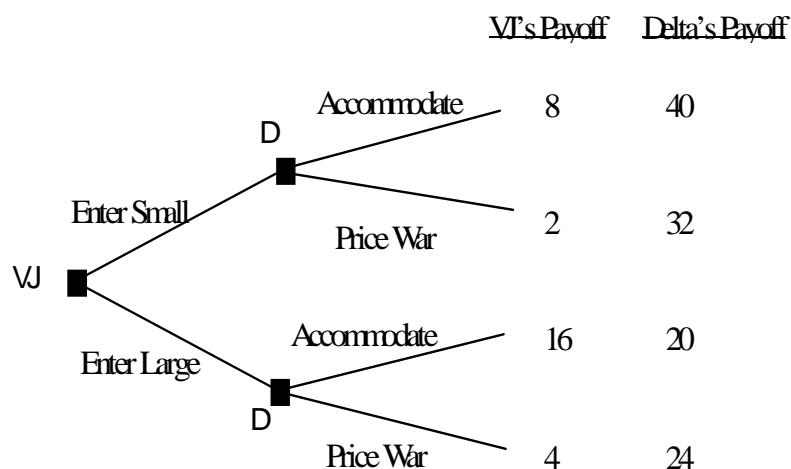
		Delta	
		Accommodate	Price Low (Price High)
Value Jet	Enter on Small Scale	8, 40	2, 32
	Enter on Large Scale	16, 20	4, 24

- a) If Value Jet and Delta choose their strategies simultaneously, what strategies would the two firms choose at the Nash equilibrium, and what would be the payoff for Value Jet? Explain.
- b) As it turned out, Value Jet decided to move first, entering on a small scale. It communicated this information by issuing a public statement announcing that it had limited aspirations in this marketplace and had no plans to grow beyond its initial small size. Analyze the sequential game in which Value Jet chooses "small" or large in the first stage and then Delta accommodates or starts a price war in the second stage. Did Value Jet enhance its profit by moving first and entering on a small scale? If so, how much more did it earn with this strategy? If not, explain why not? (Hint: Draw the game tree)

Answer

a) Value Jet has a dominant strategy to enter large. Given that, Delta would respond by launching a price war. Thus there is a unique pure strategy Nash equilibrium in which Value Jet enters large and Delta starts a price war. Value Jet's payoff is \$4 zillion.

b)



The game tree above models the two stages of the game. The payoffs are the same as in the matrix in (a). If VJ (Value Jet) builds low, D (Delta) will accommodate (preferring 40 over 32). Thus, D will get 8 if it enters small. If VJ (Value Jet) builds large, D (Delta) will start a price war (preferring 24 over 20). Thus, D will get 4 if it enters small. Value Jet's optimal strategy is to build small and for Delta to accommodate. Value Jet will then receive \$8 zillion. Value Jet has increased its profit from \$4 zillion in (a) to \$8 zillion in (b), so by moving first, gains an extra \$4 zillion in profit.

- 5) Besanko, Inc. and Braeutigam, Ltd. compete in the high-grade carbon fiber market. Both firms sell identical grades of carbon fiber, a commodity product that will sell at a common market price. The challenge for each firm is to decide upon a capacity expansion strategy. The following problem pertains to this choice.
- Suppose it is well known that long-run market demand in this industry will be robust. In light of that, the payoffs associated with various capacity expansion strategies that Besanko and Braeutigam might pursue are shown in the following table. What are the Nash equilibrium capacity choices for each firm if both firms make their capacity choices simultaneously?
 - Again, suppose that the table gives the payoffs to each firm under various capacity scenarios, but now suppose that Besanko can commit in advance to a capacity strategy. That is, it can choose no expansion, modest expansion, or major expansion. Braeutigam observes this choice and makes a choice of its own (no expansion or modest expansion). What is the equilibrium this sequential-move capacity game?

		Braeutigam	
		No Expansion	Modest Expansion
Besanko	No Expansion	\$1,013, \$1,013	\$844, \$1,125
	Modest Expansion	\$1,125, \$844	\$900, \$900
	Major Expansion	\$1,013, \$506	\$675, \$450

Answer

- As we see below (where squares represent Besanko's best response and circles represent Braeutigam's), the Nash equilibrium is for each player to choose MODEST EXPANSION.

		Braeutigam	
		No Expansion	Modest Expansion
Besanko	No Expansion	\$ 1,013	\$ 844
	Modest Expansion	\$ 1,125	\$ 900
	Major Expansion	\$ 1,013	\$ 675

\$ 506 \$ 450

b) If Besanko moves first, then he will commit to a MAJOR EXPANSION, and Braeutigam will choose NO EXPANSION. Here's why:

- If Besanko chooses NO EXPANSION, Braeutigam's best response is MODEST EXPANSION. Besanko's payoff is \$844.
- If Besanko chooses MODEST EXPANSION, Braeutigam's best response is MODEST EXPANSION. Besanko's payoff is \$900.
- If Besanko chooses MAJOR EXPANSION, Braeutigam's best response is NO EXPANSION. Besanko's payoff is \$1,013.

Besanko does best when he choose MAJOR EXPANSION, putting Braeutigam in a position in which it is optimal for him to choose NO EXPANSION.

EconS 301 – Intermediate Microeconomics

Review Session #12 – Chapter 17: Externalities and Public Goods

- 1) Why is it generally not socially efficient to set an emission standard allowing zero pollution?

Answer

If the government were to set an emissions standard requiring zero pollution, this standard would probably not be socially efficient. By setting the standard at zero, the government could reduce pollution by preventing polluting industries from producing goods that society values. By setting the standard at zero, however, the government will also eliminate the benefits to society from production of these goods. In general, the social benefits from producing will likely exceed the social costs up to some non-zero level of production (pollution) implying the socially efficient level of production is non-zero.

- 2) A competitive refining industry produces one unit of waste for each unit of refined product. The industry disposes of the waste by releasing it into the atmosphere. The inverse demand curve for the refined product (which is also the marginal benefit curve) is $P^d = 24 - Q$, where Q is the quantity consumed when the price consumers pay is P^d . The inverse supply curve (also the marginal private cost curve) for refining is $MPC = 2 + Q$, where MPC is the marginal private cost when the industry produces Q units. The marginal external cost is $MEC = 0.5Q$, where MEC is the marginal external cost when the industry releases Q units of waste.

- a) What are the equilibrium price and quantity for the refined product when there is no correction for the externality?
- b) How much quantity should the market supply at the social optimum?
- c) How large is the deadweight loss from the externality?
- d) Suppose the government imposes an emission fee of \$T per unit of emission. How large should this emission fee be if the market is to produce the economically (socially) efficient amount of the refined product?

Answer

- a) If there is no correction for the externality, the equilibrium will occur at the point where the marginal benefit curve, $P^d = 24 - Q$, intersects the marginal private cost curve, $MPC = 2 + Q$. This occurs at

$$\begin{aligned}24 - Q &= 2 + Q \\Q &= 11\end{aligned}$$

At $Q = 11$, price is $P = 13$.

- b) At the social optimum marginal benefit, $P^d = 24 - Q$, will equal marginal social cost, $MSC = MPC + MEC$. This occurs where

$$24 - Q = (2 + Q) + 0.5Q$$

$$Q = 8.80$$

Thus, the social optimum is to produce $Q = 8.80$.

- c) At the uncorrected equilibrium, the marginal social cost is $MSC = 2 + 1.5(11) = 18.5$. Thus, the deadweight loss will be $0.5(11 - 8.80)(18.5 - 13) = 6.05$.
 - d) The emissions fee of T should be set to shift the MPC curve so that it intersects the marginal benefit curve at $Q = 8.80$, the socially optimal quantity. At $Q = 8.80$ the marginal benefit is $P = 15.2$ and the marginal private cost is $MPC = 2 + 8.80 = 10.80$. Therefore, the optimal tax is $T = 15.2 - 10.8 = 4.4$.
- 3)** Amityville has a competitive chocolate industry with the (inverse) supply curve $P^s = 440 + Q$. While the market demand for chocolate is $P^d = 1200 - Q$, there are external benefits that the citizens of Amityville derive from having a chocolate odor wafting through the town. The marginal external benefit schedule is $MEB = 6 - 0.05Q$.
- a) Without government intervention, what would be the equilibrium amount of chocolate produced? What is the socially optimum amount of chocolate production?
 - b) If the government of Amityville used a subsidy of S per unit to encourage the optimal amount of chocolate production, what level should that subsidy be?

Answer

- a) The equilibrium level of output occurs where $P^d = P^s$, or $1200 - Q = 440 + Q$. Equilibrium output is then $Q = 380$. Taking into account the positive externality, the social optimal amount of production sets $P^d + MEB = P^s$, or $1200 - Q^* + 60 - 0.05Q^* = Q^* + 440$, yielding $Q^* = 400$.
 - b) With a subsidy of S , equilibrium occurs where $P^d + S = P^s$ or $1200 - Q + S = 440 + Q$. To get $Q = Q^* = 400$ the subsidy must satisfy $1200 - 400 + S = 440 + 400$ or $S = 40$.
- 4)** There are three consumers of a public good. The demand for the consumer are as follows:

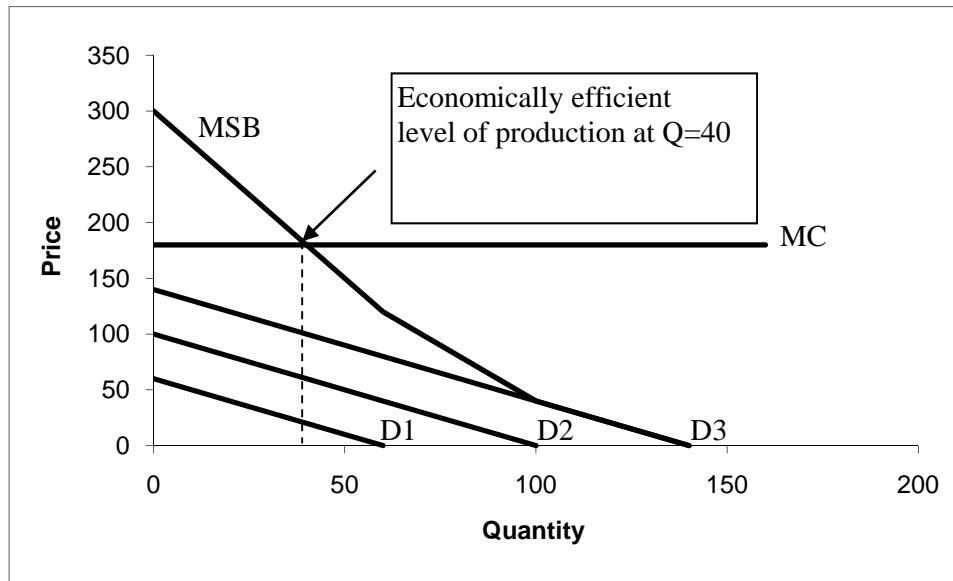
$$\text{Consumer 1: } P_1 = 60 - Q$$

$$\text{Consumer 2: } P_2 = 100 - Q$$

$$\text{Consumer 3: } P_3 = 140 - Q$$

Where Q measures the number of units of the good and P is the price in dollars. The marginal cost of the public good is \$180. What is the economically efficient level of the production of the good? Illustrate your answer clearly on a graph.

Answer



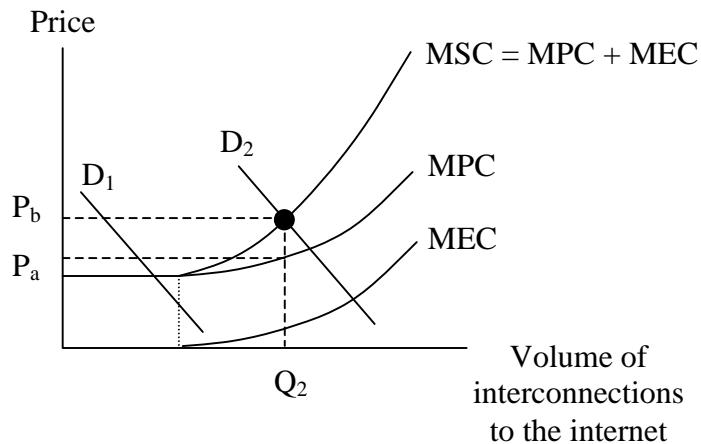
The economically efficient level of output occurs where $MSB = MC$. Since this occurs where all three consumers are in the market we have

$$\begin{aligned}
 (60 - Q) + (100 - Q) + (140 - Q) &= 180 \\
 3Q &= 120 \\
 Q &= 40
 \end{aligned}$$

- 5) Some observers have argued that the Internet is overused in times of network congestion.
- Do you think the Internet serves as common property? Are people ever denied access to the Internet?
 - Draw a graph illustrating why the amount of traffic is higher than the efficient level during a period of peak demand when there is congestion. Let your graph reflect the following characteristics of the Internet:
 - At low traffic levels, there is no congestion, with marginal private cost equal to marginal external cost.
 - However, at higher usage levels, marginal external costs are positive, and the marginal external cost increases as traffic grows.
 - On your graph explain how a tax might be used to improve economic efficiency in the use of the Internet during a period of congestion.
 - As an alternative to a tax, one could simply deny access to additional users once the economically efficient volume of traffic is on the Internet. Why might an optimal tax be more efficient than denying access?

Answer

- a) The Internet can be viewed as common property because virtually anyone has access to it. In practice, people are sometimes denied access, particularly when the congestion is great and consumers cannot connect to it.
- b) The graph might be very similar to Figure 17.5.



When the demand for connections to the internet is D_1 , there is no congestion. However, when the demand is high at D_2 , congestion creates a positive marginal external cost.

- c) When the demand is large, a tax equal to $(P_b - P_a)$ would lead users to demand the efficient number of connections Q_2 .
- d) A tax would ensure that users who value connections the most would be able to connect. If access is denied to some users, some users with a higher value for an interconnection might be unable to connect, while other users with a lower value for a connection might be able to go online. This would not be economically efficient because the scarce resource (connections) would not necessarily be allocated to consumers who value connection the most.

EconS 301 – Intermediate Microeconomics

Review Session #13 – Chapter 14: Strategy and Game Theory

- 1) Asahi and Kirin are the two largest sellers of beer in Japan. These two firms compete head to head in dry beer category in Japan. The following table shows the profit (in millions of yen) that each firm earns when it charges different prices for its beer:

		Kirin			
		¥630	¥660	¥690	¥720
Asahi	¥630	180, 180	184, 178	185, 175	186, 173
	¥660	178, 184	183, 183	192, 182	194, 180
	¥690	175, 185	182, 192	191, 191	198, 190
	¥720	173, 186	180, 194	190, 198	196, 196

- Does Asahi have a dominant strategy? Does Kirin?
- Both Asahi and Kirin have a dominated strategy: Find and identify it.
- Assume that Asahi and Kirin will not play the dominated strategy you identified in part (b) (i.e., cross out the dominated strategy for each firm in the table.) Having eliminated the dominated strategy, show that Asahi and Kirin now have another dominated strategy.
- Assume that Asahi and Kirin will not play the dominated strategy you identified in part (c). Having eliminated this dominated strategy, determine whether A and Kirin now have a dominant strategy.
- What is the Nash equilibrium in this game?

Answer

- Neither player has a dominant strategy in this game.
- In this game, Asahi has a dominated strategy, ¥720 is dominated by ¥690, and Kirin has a dominated strategy, ¥720 is dominated by ¥690. Assuming neither player will play these dominated strategies we can remove them from the game. The reduced game is

		Kirin		
		¥630	¥660	¥690
Asahi	¥630	180, 180	184, 178	185, 175
	¥660	178, 184	183, 183	192, 182
	¥690	175, 185	182, 192	191, 191

- Now that we have eliminated a dominated strategy from the original game, both players now have a dominated strategy in the reduced game. Asahi has a dominated strategy, ¥690 is dominated by ¥660, and Kirin has a dominated strategy, ¥690 is dominated by ¥660. Assuming neither player will play these dominated strategies we can remove them from the game. The reduced game is

		<i>Kirin</i>	
		¥630	¥660
<i>Asahi</i>	¥630	180, 180	184, 178
	¥660	178, 184	183, 183

- d) Now that we have eliminated another dominated strategy from the original game, both players have a dominant strategy to choose ¥630.
- e) Based on the analysis above, the Nash equilibrium in this game has both players choosing ¥630.
- 2) Consider the following game, where $x > 0$:

		<i>Firm 2</i>	
		High Price	Low Price
<i>Firm 1</i>	High Price	140, 140	20, 160
	Low Price	$90 + x, 90 - x$	50, 50

- a) For what values of x do both firms have a dominant strategy? What is the Nash equilibrium (or equilibria) in these cases?
- b) For what values of x does only one firm have a dominant strategy? What is the Nash equilibrium (or equilibria) in these cases?
- c) Are there any values of x such that neither firm has a dominant strategy? Ignoring mixed strategies, is there a Nash equilibrium in such cases?

Answer

- a) For $x > 50$, both firms have a dominant strategy. The unique NE is (*Low, Low*).
- b) For $40 < x < 50$ only Firm 2 has a dominant strategy. The unique NE in this case is still (*Low, Low*).
- c) For $x < 40$, there are zero dominant strategies, and no NE exists.
- 3) In a World Series game, Kerry Wood is pitching and Alex Rodriguez is batting. The count on Rodriguez is 3 balls and 2 strikes. Wood has to decide whether to throw a fastball or a curveball. Rodriguez has to decide whether to swing or not swing. If Wood throws a fastball and Rodriguez doesn't swing, the pitch will almost certainly be a strike, and Rodriguez will be out. If Rodriguez does swing, however, there is a strong likelihood that he will get a hit. If Wood throws a curve and Rodriguez swings, there is a strong likelihood that Rodriguez will strike out. But if Wood throws a curve and Rodriguez doesn't swing, there is a good chance that it will be ball four and Rodriguez will walk (assume that a walk is as good as a hit in this instance).

The following table shows the payoffs from each pair of choices that the two players can make:

		Alex Rodriguez	
		Swing	Do Not Swing
Kerry Wood	Fastball	-100, 100	100, -100
	Curveball	100, -100	-100, 100

- a) Is there a Nash equilibrium in pure strategies game?
- b) Is there a mixed strategy Nash equilibrium in game? If so, what is it?

Answer

- a) There are no Nash equilibria in pure strategies in this game.
- b) This game does have a Nash equilibrium in mixed strategies. To find the mixed strategy equilibrium, we need to find the probabilities with which, for example, Rodriguez plays "Swing" and "Do Not Swing" so that Wood's payoff from playing "Fast Ball" and "Curve Ball" are the same. Letting the probability of "Swing" be P and "Do Not Swing" be $1 - P$, Wood's expected payoff if he chooses "Fast Ball" is $-100P + 100(1 - P)$. His expected payoff if he chooses "Curve Ball" is $100P - 100(1 - P)$. Equating these yields:

$$\begin{aligned} -100P + 100(1 - P) &= 100P - 100(1 - P) \\ 100 - 200P &= 200P - 100 \\ P &= 0.50 \end{aligned}$$

Thus, in equilibrium Rodriguez should play "Swing" with a 50% probability and "Do Not Swing" with a 50% probability. Since the problem is symmetric, Wood should play "Fast Ball" with a 50% probability and "Curve Ball" with a 50% probability.

- 4) In the mid-1990s, Value Jet wanted to enter market serving routes that would compete head-to-head with Delta Airlines in Atlanta. Value Jet knew that Delta might respond in one of two ways: Delta could price war or it could be "accommodating," keep price at a high level. Value Jet had to decide whether it would enter on a small scale or on a large scale. Annual profits (in millions of dollars) associated with strategy are summarized in the following table (where the first number is the payoff to Value Jet and the second the payoff to Delta):

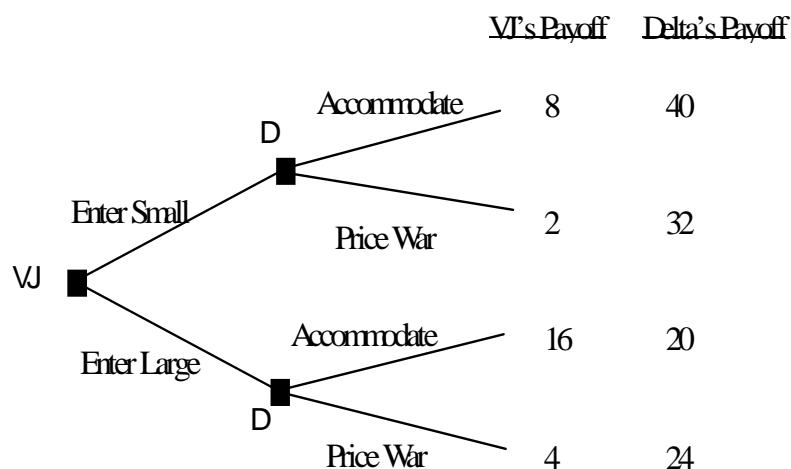
		Delta	
		Accommodate	Price Low (Price High)
Value Jet	Enter on Small Scale	8, 40	2, 32
	Enter on Large Scale	16, 20	4, 24

- a) If Value Jet and Delta choose their strategies simultaneously, what strategies would the two firms choose at the Nash equilibrium, and what would be the payoff for Value Jet? Explain.
- b) As it turned out, Value Jet decided to move first, entering on a small scale. It communicated this information by issuing a public statement announcing that it had limited aspirations in this marketplace and had no plans to grow beyond its initial small size. Analyze the sequential game in which Value Jet chooses "small" or large in the first stage and then Delta accommodates or starts a price war in the second stage. Did Value Jet enhance its profit by moving first and entering on a small scale? If so, how much more did it earn with this strategy? If not, explain why not? (Hint: Draw the game tree)

Answer

a) Value Jet has a dominant strategy to enter large. Given that, Delta would respond by launching a price war. Thus there is a unique pure strategy Nash equilibrium in which Value Jet enters large and Delta starts a price war. Value Jet's payoff is \$4 zillion.

b)



The game tree above models the two stages of the game. The payoffs are the same as in the matrix in (a). If VJ (Value Jet) builds low, D (Delta) will accommodate (preferring 40 over 32). Thus, D will get 8 if it enters small. If VJ (Value Jet) builds large, D (Delta) will start a price war (preferring 24 over 20). Thus, D will get 4 if it enters small. Value Jet's optimal strategy is to build small and for Delta to accommodate. Value Jet will then receive \$8 zillion. Value Jet has increased its profit from \$4 zillion in (a) to \$8 zillion in (b), so by moving first, gains an extra \$4 zillion in profit.

- 5) Besanko, Inc. and Braeutigam, Ltd. compete in the high-grade carbon fiber market. Both firms sell identical grades of carbon fiber, a commodity product that will sell at a common market price. The challenge for each firm is to decide upon a capacity expansion strategy. The following problem pertains to this choice.
- Suppose it is well known that long-run market demand in this industry will be robust. In light of that, the payoffs associated with various capacity expansion strategies that Besanko and Braeutigam might pursue are shown in the following table. What are the Nash equilibrium capacity choices for each firm if both firms make their capacity choices simultaneously?
 - Again, suppose that the table gives the payoffs to each firm under various capacity scenarios, but now suppose that Besanko can commit in advance to a capacity strategy. That is, it can choose no expansion, modest expansion, or major expansion. Braeutigam observes this choice and makes a choice of its own (no expansion or modest expansion). What is the equilibrium this sequential-move capacity game?

		Braeutigam	
		No Expansion	Modest Expansion
Besanko	No Expansion	\$1,013, \$1,013	\$844, \$1,125
	Modest Expansion	\$1,125, \$844	\$900, \$900
	Major Expansion	\$1,013, \$506	\$675, \$450

Answer

- As we see below (where squares represent Besanko's best response and circles represent Braeutigam's), the Nash equilibrium is for each player to choose MODEST EXPANSION.

		Braeutigam	
		No Expansion	Modest Expansion
Besanko	No Expansion	\$ 1,013	\$ 844
	Modest Expansion	\$ 1,125	\$ 900
	Major Expansion	\$ 1,013	\$ 675

\$ 506 \$ 450

b) If Besanko moves first, then he will commit to a MAJOR EXPANSION, and Braeutigam will choose NO EXPANSION. Here's why:

- If Besanko chooses NO EXPANSION, Braeutigam's best response is MODEST EXPANSION. Besanko's payoff is \$844.
- If Besanko chooses MODEST EXPANSION, Braeutigam's best response is MODEST EXPANSION. Besanko's payoff is \$900.
- If Besanko chooses MAJOR EXPANSION, Braeutigam's best response is NO EXPANSION. Besanko's payoff is \$1,013.

Besanko does best when he choose MAJOR EXPANSION, putting Braeutigam in a position in which it is optimal for him to choose NO EXPANSION.

EconS 301 – Intermediate Microeconomics

Review Session #14 – Chapter 17: Externalities and Public Goods

- 1) Why is it generally not socially efficient to set an emission standard allowing zero pollution?

Answer

If the government were to set an emissions standard requiring zero pollution, this standard would probably not be socially efficient. By setting the standard at zero, the government could reduce pollution by preventing polluting industries from producing goods that society values. By setting the standard at zero, however, the government will also eliminate the benefits to society from production of these goods. In general, the social benefits from producing will likely exceed the social costs up to some non-zero level of production (pollution) implying the socially efficient level of production is non-zero.

- 2) A competitive refining industry produces one unit of waste for each unit of refined product. The industry disposes of the waste by releasing it into the atmosphere. The inverse demand curve for the refined product (which is also the marginal benefit curve) is $P^d = 24 - Q$, where Q is the quantity consumed when the price consumers pay is P^d . The inverse supply curve (also the marginal private cost curve) for refining is $MPC = 2 + Q$, where MPC is the marginal private cost when the industry produces Q units. The marginal external cost is $MEC = 0.5Q$, where MEC is the marginal external cost when the industry releases Q units of waste.

- a) What are the equilibrium price and quantity for the refined product when there is no correction for the externality?
- b) How much quantity should the market supply at the social optimum?
- c) How large is the deadweight loss from the externality?
- d) Suppose the government imposes an emission fee of \$T per unit of emission. How large should this emission fee be if the market is to produce the economically (socially) efficient amount of the refined product?

Answer

- a) If there is no correction for the externality, the equilibrium will occur at the point where the marginal benefit curve, $P^d = 24 - Q$, intersects the marginal private cost curve, $MPC = 2 + Q$. This occurs at

$$24 - Q = 2 + Q$$

$$Q = 11$$

At $Q = 11$, price is $P = 13$.

- b) At the social optimum marginal benefit, $P^d = 24 - Q$, will equal marginal social cost, $MSC = MPC + MEC$. This occurs where

$$24 - Q = (2 + Q) + 0.5Q$$

$$Q = 8.80$$

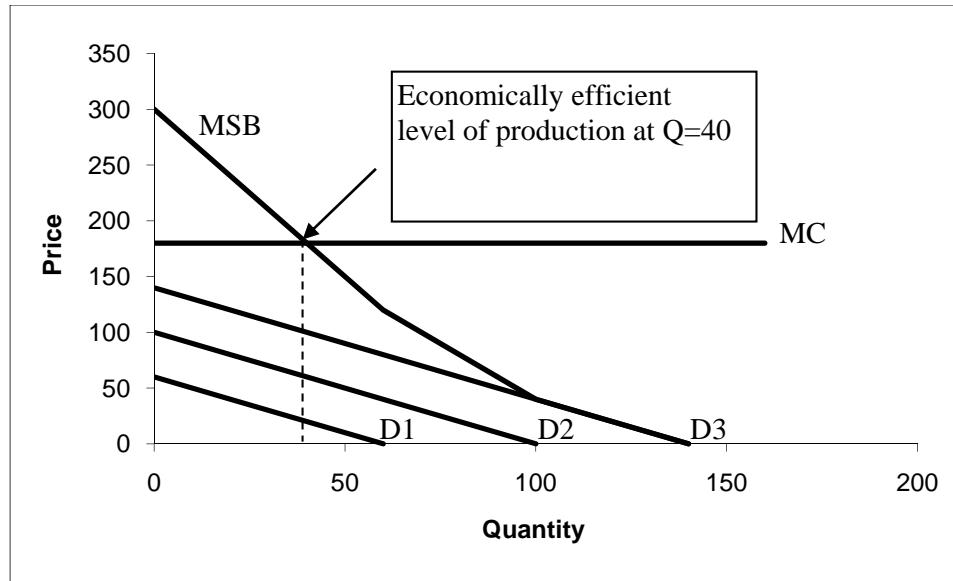
Thus, the social optimum is to produce $Q = 8.80$.

- c) At the uncorrected equilibrium, the marginal social cost is $MSC = 2 + 1.5(11) = 18.5$. Thus, the deadweight loss will be $0.5(11 - 8.80)(18.5 - 13) = 6.05$.
 - d) The emissions fee of $\$T$ should be set to shift the MPC curve so that it intersects the marginal benefit curve at $Q = 8.80$, the socially optimal quantity. At $Q = 8.80$ the marginal benefit is $P = 15.2$ and the marginal private cost is $MPC = 2 + 8.80 = 10.80$. Therefore, the optimal tax is $T = 15.2 - 10.8 = 4.4$.
- 3)** Amityville has a competitive chocolate industry with the (inverse) supply curve $P^s = 440 + Q$. While the market demand for chocolate is $P^d = 1200 - Q$, there are external benefits that the citizens of Amityville derive from having a chocolate odor wafting through the town. The marginal external benefit schedule is $MEB = 60 - 0.05Q$.
- a) Without government intervention, what would be the equilibrium amount of chocolate produced? What is the socially optimum amount of chocolate production?
 - b) If the government of Amityville used a subsidy of $\$S$ per unit to encourage the optimal amount of chocolate production, what level should that subsidy be?

Answer

- a) The equilibrium level of output occurs where $P^d = P^s$, or $1200 - Q = 440 + Q$. Equilibrium output is then $Q = 380$. Taking into account the positive externality, the social optimal amount of production sets $P^d + MEB = P^s$, or $1200 - Q^* + 60 - 0.05Q^* = Q^* + 440$, yielding $Q^* = 400$.
 - b) With a subsidy of $\$S$, equilibrium occurs where $P^d + S = P^s$ or $1200 - Q + S = 440 + Q$. To get $Q = Q^* = 400$ the subsidy must satisfy $1200 - 400 + S = 440 + 400$ or $S = 40$.
- 4)** There are three consumers of a public good. The demand for the consumer are as follows:
 Consumer 1: $P_1 = 60 - Q$
 Consumer 2: $P_2 = 100 - Q$
 Consumer 3: $P_3 = 140 - Q$
 Where Q measures the number of units of the good and P is the price in dollars. The marginal cost of the public good is \$180. What is the economically efficient level of the production of the good? Illustrate your answer clearly on a graph.

Answer



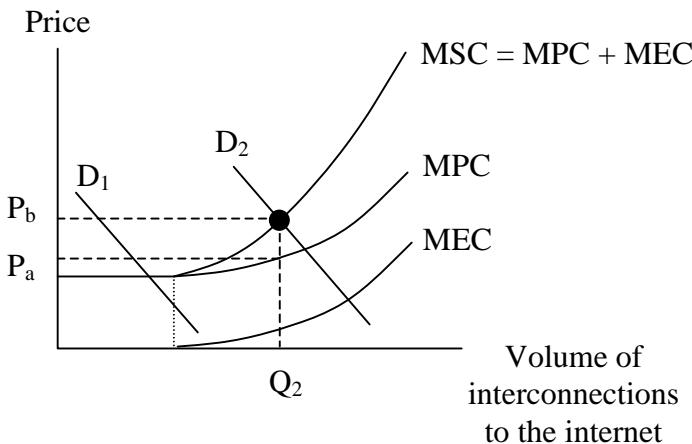
The economically efficient level of output occurs where $MSB = MC$. Since this occurs where all three consumers are in the market we have

$$\begin{aligned}
 (60 - Q) + (100 - Q) + (140 - Q) &= 180 \\
 3Q &= 120 \\
 Q &= 40
 \end{aligned}$$

- 5) Some observers have argued that the Internet is overused in times of network congestion.
- Do you think the Internet serves as common property? Are people ever denied access to the Internet?
 - Draw a graph illustrating why the amount of traffic is higher than the efficient level during a period of peak demand when there is congestion. Let your graph reflect the following characteristics of the Internet:
 - At low traffic levels, there is no congestion, with marginal private cost equal to marginal external cost.
 - However, at higher usage levels, marginal external costs are positive, and the marginal external cost increases as traffic grows.
 - On your graph explain how a tax might be used to improve economic efficiency in the use of the Internet during a period of congestion.
 - As an alternative to a tax, one could simply deny access to additional users once the economically efficient volume of traffic is on the Internet. Why might an optimal tax be more efficient than denying access?

Answer

- a) The Internet can be viewed as common property because virtually anyone has access to it. In practice, people are sometimes denied access, particularly when the congestion is great and consumers cannot connect to it.
- b) The graph might be very similar to Figure 17.5.



When the demand for connections to the internet is D_1 , there is no congestion. However, when the demand is high at D_2 , congestion creates a positive marginal external cost.

- c) When the demand is large, a tax equal to $(P_b - P_a)$ would lead users to demand the efficient number of connections Q_2 .
- d) A tax would ensure that users who value connections the most would be able to connect. If access is denied to some users, some users with a higher value for an interconnection might be unable to connect, while other users with a lower value for a connection might be able to go online. This would not be economically efficient because the scarce resource (connections) would not necessarily be allocated to consumers who value connection the most.

EconS 301 - REVIEW EXERCISES FOR MIDTERM #1

Chapter 2 Review Questions

2.12. Consider a linear demand curve, $Q = 350 - 7P$

a) Derive the inverse demand curve corresponding to this demand curve.

$$Q = 350 - 7P$$

$7P = 350 - Q$ (Remember, the inverse demand curve is the demand curve solved for P)

$$P = 50 - \frac{1}{7}Q$$

b) What is the choke price?

The choke price occurs at the point where $Q = 0$. Setting $Q = 0$ in the inverse demand equation above yields $P = 50$. That is, the choke price shows us the price at which consumers will not demand any quantity of the good.

c) What is the price elasticity of demand at $P = 50$?

At $P = 50$, the choke price, the elasticity will approach negative infinity. Since elasticity equals the percentage change in Quantity over the percentage change in price, at $Q=0$, the elasticity will approach negative infinity.

2.16. Consider the following demand and supply relationships in the market for golf balls: $Q^d = 90 - 2P - 2T$ and $Q^s = -9 + 5P - 2.5R$, where T is the price of titanium, a metal used to make golf clubs, and R is the price of rubber.

a) If $R = 2$ and $T = 10$, calculate the equilibrium price and quantity of golf balls.

Substituting the values of R and T , we get

$$\text{Demand : } Q^d = 70 - 2P$$

$$\text{Supply : } Q^s = -14 + 5P$$

In equilibrium, $70 - 2P = -14 + 5P$, which implies that $P = 12$. Substituting this value back, $Q = 46$.

b) At the equilibrium values, calculate the price elasticity of demand and the price elasticity of supply.

Elasticity of Demand = $-2(12/46)$, or -0.52 . Elasticity of Supply = $5(12/46) = 1.30$.

Notice that -2 is simply the $(\Delta Q/\Delta P)$, or derivative with respect to P of the demand function.

- c) At the equilibrium values, calculate the cross-price elasticity of demand for golf balls with respect to the price of titanium. What does the sign of this elasticity tell you about whether golf balls and titanium are substitutes or complements?

$\varepsilon_{golf,titanium} = -2(\frac{10}{46}) = -0.43$. The negative sign indicates that titanium and golf balls are complements, i.e., when the price of titanium goes up the demand for golf balls decreases. Remember that people like to consume complements together so the increase in the price of one is essentially like an increase in the price of the other, and therefore, the demand will decrease.

- 2.20.** Suppose that the market for air travel between Chicago and Dallas is served by just two airlines, United and American. An economist has studied this market and has estimated that the demand curves for round-trip tickets for each airline are as follows:

$$Q_U^d = 10,000 - 100P_U + 99P_A \text{ (United's demand)}$$

$$Q_A^d = 10,000 - 100P_A + 99P_U \text{ (American's demand)}$$

Where P_U is the price charged by United, and P_A is the price charged by American.

- a) Suppose that both American and United charge a price of \$300 each for a round-trip ticket between Chicago and Dallas. What is the price elasticity of demand for United flights between Chicago and Dallas?

$$Q_U^d = 10000 - 100(300) + 99(300)$$

$$Q_U^d = 9700$$

Using $P_U = 300$ and $Q_U^d = 9700$ gives

$$\varepsilon_{Q,P} = -100 \left(\frac{300}{9700} \right) = -3.09 \text{ (Simply plug numbers into the price elasticity of demand equation)}$$

- b) What is the market-level price elasticity of demand for air travel between Chicago and Dallas when both airlines charge a price of \$300? (Hint: Because United and American are the only two airlines serving the Chicago-Dallas market, what is the equation for the total demand for air travel between Chicago and Dallas, assuming that the airlines charge the same price?)

Market demand is given by $Q^d = Q_U^d + Q_A^d$. Assuming the airlines charge the same price we have...

$$Q^d = 10000 - 100P_U + 99P_A + 10000 - 100P_A + 99P_U$$

$$Q^d = 20000 - 100P + 99P - 100P + 99P \quad (\text{P}_A \text{ and } P_U \text{ simply become } P)$$

$$Q^d = 20000 - 2P$$

When $P = 300$, $Q^d = 19400$. This implies an elasticity equal to

$$\varepsilon_{Q,P} = -2 \left(\frac{300}{19400} \right) = -.0309$$

Chapter 3 Review Questions

3.4. Consider the utility function $U(x,y) = y\sqrt{x}$ with the marginal utilities $MU_x = y/(2\sqrt{x})$ and $MU_y = \sqrt{x}$.

a) Does the consumer believe that more is better for each good?

Since U increases whenever x or y increases, more of each good is better. This is also confirmed by noting that MU_x and MU_y are both positive for any positive values of x and y .

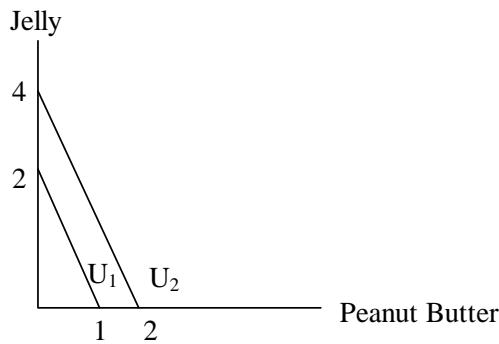
b) Do the consumer's preferences exhibit a diminishing marginal utility of x ? Is the marginal utility of y diminishing?

b) Since $MU_x = \frac{y}{2\sqrt{x}}$, as x increases (holding y constant), MU_x falls. Therefore the marginal utility of x is diminishing. However, $MU_y = \sqrt{x}$. As y increases, MU_y does not change. Therefore the preferences exhibit a constant, not diminishing, marginal utility of y .

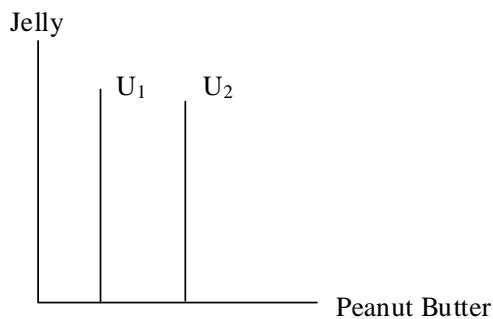
3.13. Draw indifference curves to represent the following types of consumer preferences.

a) I like both peanut butter and jelly, and always get the same additional satisfaction from an ounce of peanut butter as I do from 2 ounces of jelly.

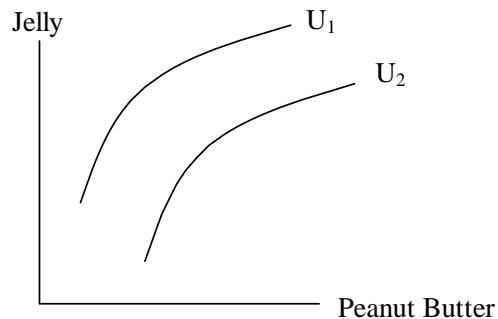
In the following pictures, $U_2 > U_1$.



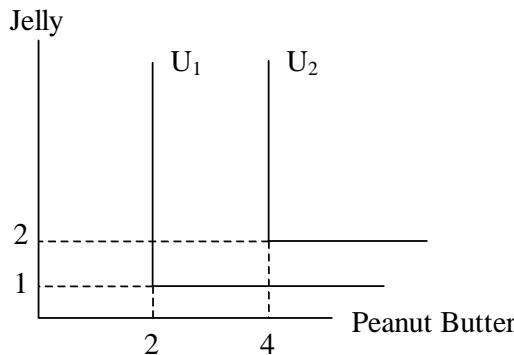
b) I like peanut butter, but neither like nor dislike jelly.



c) I like peanut butter, but dislike jelly.



d) I like peanut butter and jelly, but I only want 2 ounces of peanut butter for every ounce of jelly.



3.15. Consider the utility function $U(x,y) = 3x + y$, with $MU_x = 3$ and $MU_y = 1$.

a) Is the assumption that more is better satisfied for both goods?

2. Yes, the “more is better” assumption is satisfied for both goods since both marginal utilities are always positive. As you add more of either x or y , the total utility increases.

b) Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more of x ? Explain.

The marginal utility of x remains constant at 3 for all values of x . The MU_x equation is simply a constant of 3, so the change in x or y doesn't change MU_x .

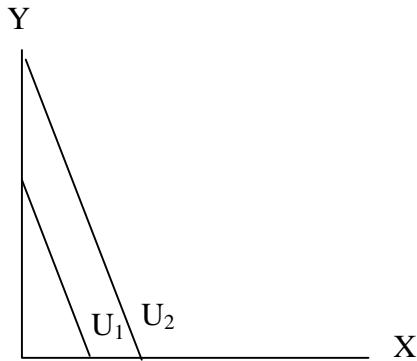
c) What is $MRS_{x,y}$?

b) $MRS_{x,y} = 3$ (because $MU_x/MU_y = 3/1 = 3$)

d) Is the MRS diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?

The $MRS_{x,y}$ remains constant moving along the indifference curve (look at above equation).

e) On a graph with x on the horizontal axis and y on the vertical axis, draw a typical indifference curve (it need not be exactly to scale, but it needs to reflect accurately whether there is a diminishing $MRS_{x,y}$). Also indicate on your graph whether the indifference curve will intersect either or both axis. Label the curve U_1 .



- f) On the same graph draw a second indifference curve U₂, with U₂>U₁.

(See above graph)

3.21. Suppose a consumer's preferences for two goods can be represented by the Cobb-Douglas utility function $U = Ax^\alpha y^\beta$, where A, α , and β are positive constants. The marginal utilities are $MU_x = \alpha Ax^{\alpha-1}y^\beta$ and $MU_y = \beta Ax^\alpha y^{\beta-1}$. Answer all parts of Problem 3.15 for this utility function.

a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) Since we do not know the value of α , only that it is positive, we need to specify three possible cases:

When $\alpha < 1$, the marginal utility of x diminishes as x increases.

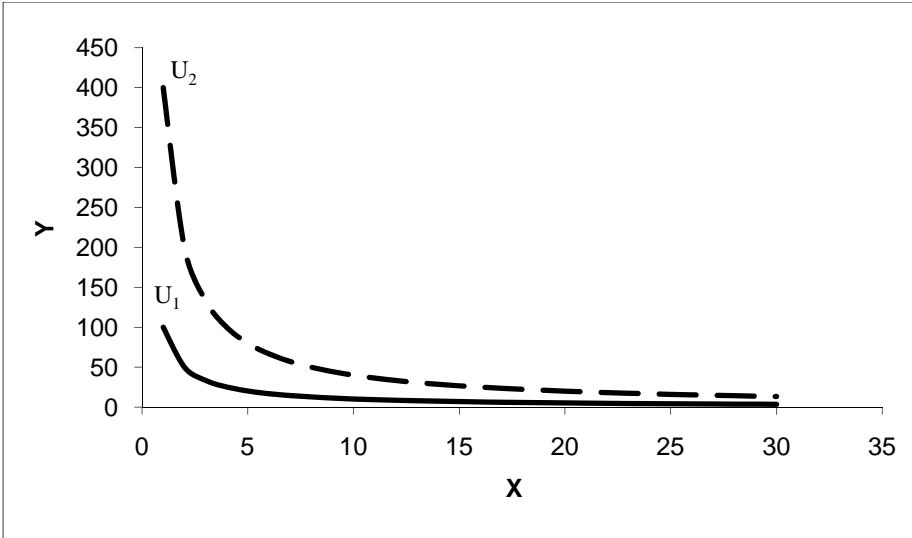
When $\alpha = 1$, the marginal utility of x remains constant as x increases.

When $\alpha > 1$, the marginal utility of x increases as x increases.

c) $MRS_{x,y} = \frac{\alpha Ax^{\alpha-1}y^\beta}{\beta Ax^\alpha y^{\beta-1}} = \frac{\alpha y}{\beta x}$

d) As the consumer substitutes x for y , the $MRS_{x,y}$ will diminish because x is in the denominator and drives down the entire fraction.

e & f) The graph below depicts indifference curves for the case where $A = 1$ and $\alpha = \beta = 0.5$. Thus $U(x, y) = x^{0.5}y^{0.5}$. Regardless, the indifference curves will never intersect either axis.



3.24. One type of Cobb-Douglas utility function is given by $U(x,y) = x^\alpha y^{1-\alpha}$, where $MU_x = \alpha x^{\alpha-1} y^{1-\alpha}$ and $MU_y = (1-\alpha)x^\alpha y^{-\alpha}$. Suppose that you are told that a person with Cobb-Douglas preferences has an $MRS_{x,y}$ of 4 when $x = 4$ and $y = 8$. What is the numerical value of α ?

First, the expression for $MRS_{x,y}$ is

$$\begin{aligned}
 MRS_{x,y} &= \frac{MU_x}{MU_y} \\
 c) \quad &= \frac{\alpha x^{\alpha-1} y^{1-\alpha}}{(1-\alpha)x^\alpha y^{-\alpha}} \text{ (plugging in the MU's and then rearranging)} \\
 &= \frac{\alpha}{1-\alpha} \frac{y}{x}
 \end{aligned}$$

Since we know that $MRS_{x,y} = 4$ when $x = 4$ and $y = 8$,

$$\begin{aligned}
 d) \quad 4 &= \frac{\alpha}{1-\alpha} \frac{8}{4} \\
 &= \frac{\alpha}{1-\alpha} \quad (\text{we set MRS equal to 4 and then plug in } x, y \text{ values and solve for } \alpha) \\
 2 - 2\alpha &= \alpha \\
 \alpha &= \frac{2}{3}
 \end{aligned}$$

a)

Chapter 4 Review Questions

4.2. The utility function that Ann receives by consuming food F and clothing C is given by $U(F,C) = FC + F$. The marginal utilities of food and clothing are $MU_F = C + 1$ and $MU_C = F$. Food costs \$1 a unit, and clothing costs \$2 a unit. Ann's income is \$22.

a) Ann is currently spending all of her income. She is buying 8 units of food. How many units of clothing is she consuming?

3. If Ann is spending all of her income then...

$$F + 2C = 22$$

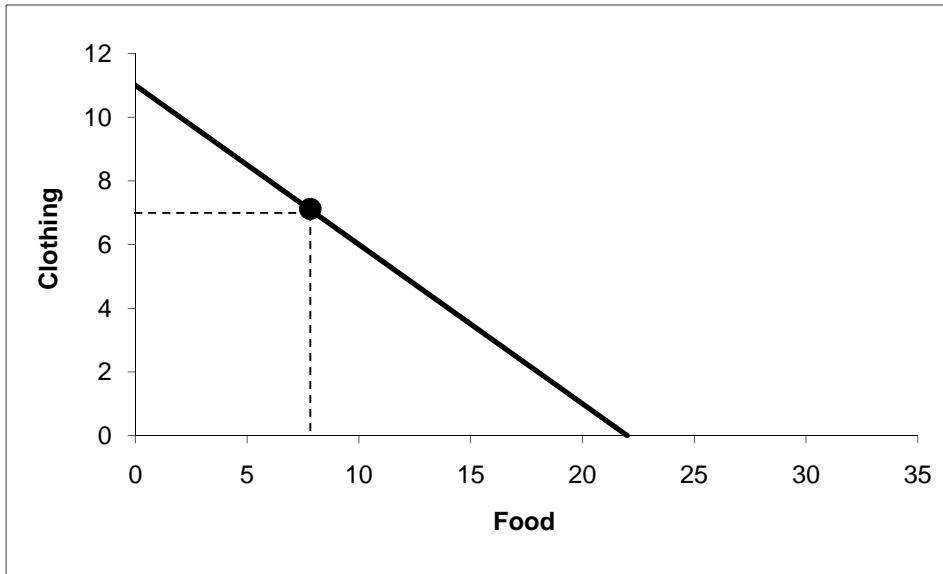
$$8 + 2C = 22 \quad (\text{Simply plugging in what we know to the Budget Line})$$

$$2C = 14$$

$$C = 7$$

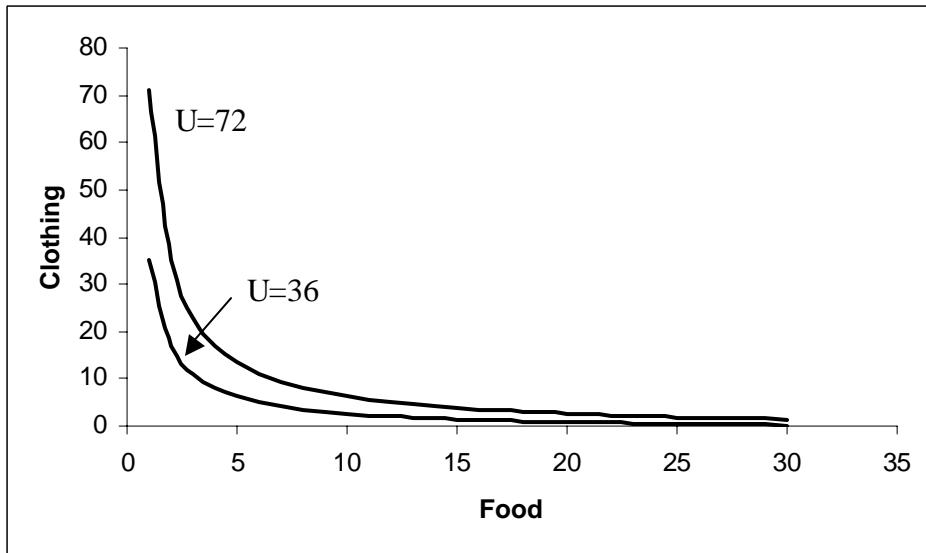
b) Graph her budget line. Place the number of units of clothing on the vertical axis and the number of units of food on the horizontal axis. Plot her current consumption basket.

Remember: Use the Budget Line and set P_y to zero to find horizontal intercept and P_x equal to zero to find the vertical intercept.

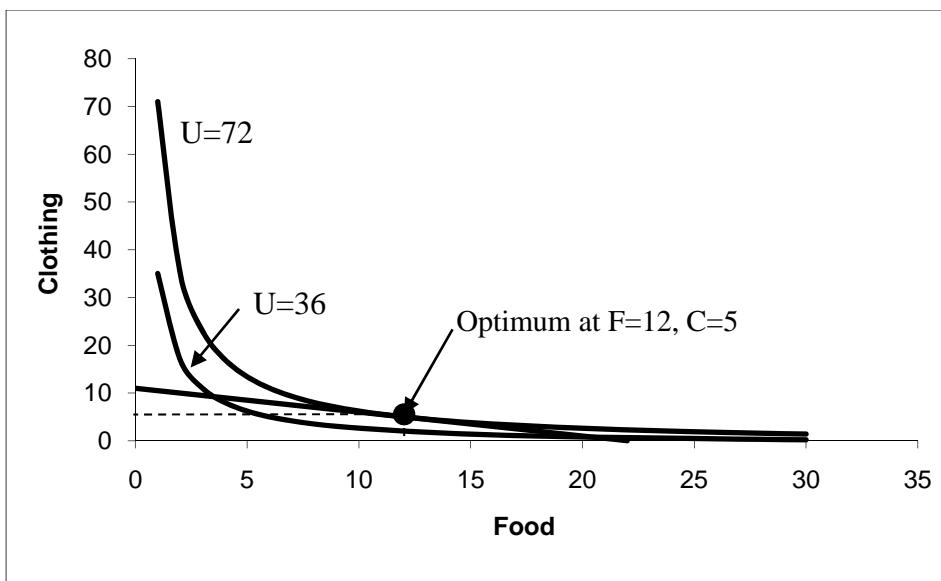


c) Draw the indifference curve associated with a utility level of 36 and the indifference curve associated with a utility of 72. Are the indifference curves bowed in toward the origin?

Yes, the indifference curves are convex, i.e., bowed in toward the origin. Also, note that they intersect the F -axis.



- d) Using a graph (and no algebra), find the utility-maximizing choice of food and clothing. That is, find the intersection of the budget line and the indifference curves.



- e) Using algebra, find the utility maximizing choice of food and clothing.

The tangency condition requires that

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C} \text{ (where slope of BL equals slope of IC)}$$

Plugging in the known information yields

$$\begin{aligned}\frac{C+1}{F} &= \frac{1}{2} \\ 2C + 2 &= F\end{aligned}$$

Substituting this result into the budget line, $F + 2C = 22$ results in

$$\begin{aligned}(2C + 2) + 2C &= 22 \\ 4C &= 20 \\ C &= 5\end{aligned}$$

Finally, plugging this result back into the tangency condition implies that $F = 2(5) + 2 = 12$. At the optimum the consumer chooses 5 units of clothing and 12 units of food.

f) What is the marginal rate of substitution of food for clothing when utility is maximized? Show this graphically and algebraically.

$$MRS_{F,C} = \frac{C+1}{F} = \frac{5+1}{12} = \frac{1}{2} \text{ The marginal rate of substitution is equal to the price ratio.}$$

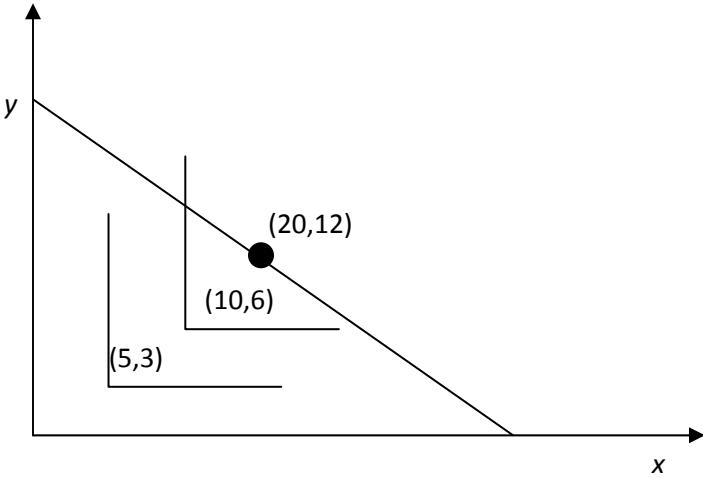
g) Does Ann have a diminishing marginal rate of substitution of food for clothing? Show this graphically and algebraically.

Yes, the indifference curves do exhibit diminishing $MRS_{F,C}$. We can see this in the graph in part c) because the indifference curves are bowed in toward the origin. Algebraically, $MRS_{F,C} = \frac{C+1}{F}$. As F increases and C decreases along an isoquant, $MRS_{F,C}$ diminishes.

4.3. Consider a consumer with the utility function $U(x,y) = \min(3x, 5y)$, that is, the two goods are perfect complements in the ratio 3:5. The prices of the two goods are $P_x = \$5$ and $P_y = \$10$, and the consumer's income is \$220. Determine the optimum consumption basket.

This question cannot be solved using the usual tangency condition. However, you can see from the graph below that the optimum basket will necessarily lie on the "elbow" of some indifference curve, such as (5, 3), (10, 6) etc. If the consumer were at some other point, he could always move to such a point, keeping utility constant and decreasing his expenditure. The equation of all these "elbow" points is $3x = 5y$, or $y = 0.6x$. Therefore the optimum point must be such that $3x = 5y$.

The usual budget constraint must hold of course. That is, $5x + 10y = 220$. Combining these two conditions, that is, plugging y into the budget line, we get $(x, y) = (20, 12)$.

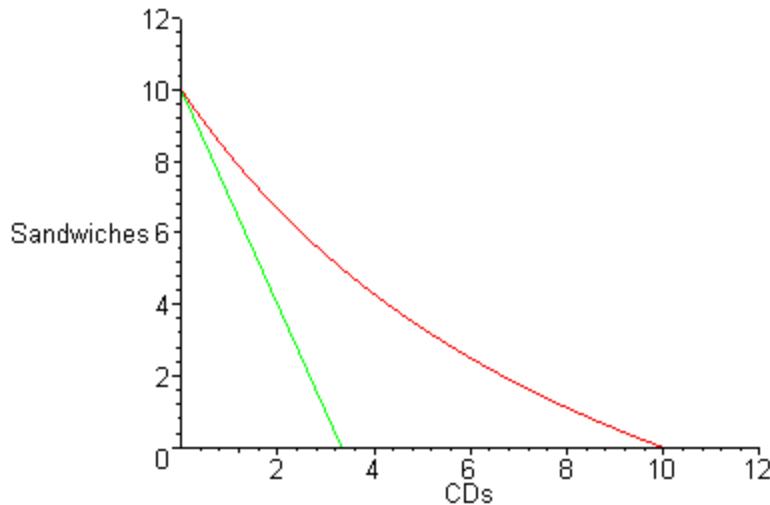


4.7. Helen's preferences over CDs (C) and sandwiches (S) are given by $U(S,C) = SC + 10(S + C)$, with $MU_C = S + 10$ and $MU_S = C + 10$. If the price of a CD is \$9 and the price of a sandwich is \$3, and Helen can spend a combined total of \$30 each day on these goods, find Helen's optimal consumption basket.

See the graph below. The fact that Helen's indifference curves touch the axes should immediately make you want to check for a corner point solution.

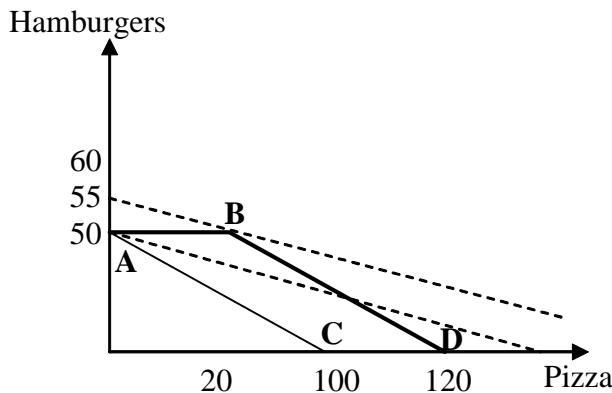
To see the corner point optimum algebraically, notice if there was an interior solution, the tangency condition implies $(S + 10)/(C + 10) = 3$, or $S = 3C + 20$. Combining this with the budget constraint, $9C + 3S = 30$, we find that the optimal number of CDs would be given by $18C = -30$ which implies a negative number of CDs. Since it's impossible to purchase a negative amount of something, our assumption that there was an interior solution must be false. Instead, the optimum will consist of $C = 0$ and Helen spending all her income on sandwiches: $S = 10$.

Graphically, the corner optimum is reflected in the fact that the slope of the budget line is steeper than that of the indifference curve, even when $C = 0$. Specifically, note that at $(C, S) = (0, 10)$ we have $P_C / P_S = 3 > MRS_{C,S} = 2$. Thus, even at the corner point, the marginal utility per dollar spent on CDs is lower than on sandwiches. However, since she is already at a corner point with $C = 0$, she cannot give up any more CDs. Therefore the best Helen can do is to spend all her income on sandwiches: $(C, S) = (0, 10)$. [Note: At the other corner with $S = 0$ and $C = 3.3$, $P_C / P_S = 3 > MRS_{C,S} = 0.75$. Thus, Helen would prefer to buy more sandwiches and less CDs, which is of course entirely feasible at this corner point. Thus the $S = 0$ corner cannot be an optimum.]



4.15. Paul consumes only two goods, pizza (P) and hamburgers (H) and considers them to be perfect substitutes, as shown by his utility function: $U(P, H) = P + 4H$. The price of pizza is \$3 and the price of hamburgers is \$6, and Paul's monthly income is \$300. Knowing that he likes pizza, Paul's grandmother gives him a birthday gift certificate of \$60 redeemable only at Pizza Hut. Though Paul is happy to get this gift, his grandmother did not realize that she could have made him exactly as happy by spending far less than she did. How much would she have needed to give him in cash to make him just as well off as with the gift certificate?

Paul's initial budget constraint is the line AC, allowing him to purchase at most 50 hamburgers or at most 100 pizzas. The \$60 cash certificate shifts out his budget constraint without changing the maximum number of hamburgers that he can buy. The new budget constraint is ABD and he can now buy a maximum of 120 pizzas.



Initially, Paul's optimal basket contains all hamburgers and no pizza, at point A where $(P, H) = (0, 50)$, because $MU_H/P_H = 4/6 > MU_P / P_P = 1/3$. His utility level at point A is $U(0, 50) = 200$. When he gets the gift certificate, Paul's optimal basket is at point B, spending

all of his regular income on hamburgers and the \$60 gift certificate on pizza. So point B is where $(P, H) = (20, 50)$ with a utility of $U(20, 50) = 220$.

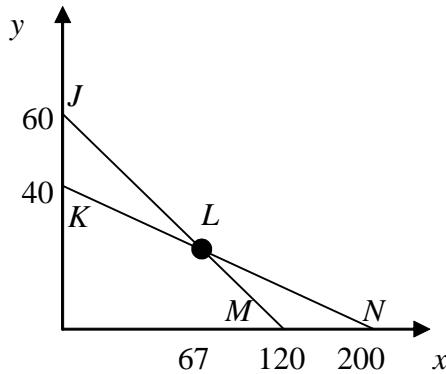
However, Paul could also achieve a utility of 220 by consuming $220/4 = 55$ hamburgers. To buy the extra 5 hamburgers he would require $5 \cdot 6 = \$30$. So, if he had received a cash gift of \$30 it would have made Paul exactly as well off as the \$60 gift certificate for pizzas.

4.22. Darrel has a monthly income of \$60. He spends this money making telephone calls home (measured in minutes of calls) and no other goods. He mobile phone company offers him two plans:

- Plan A: pay no monthly fee and make calls for \$0.50 per minute.
- Plan B: Pay a \$20 monthly fee and make calls for \$0.20 per minute.

Graph Darrell's budget constraint under each of the two plans. If Plan A is better for him, what is the set of baskets he may purchase if his behavior is consistent with utility maximization? What baskets might he purchase if Plan B is better for him?

Let x denote the number of phone calls, and y denote spending on other goods. Under Plan A, Darrell's budget line is JLM . Under Plan B, it is $JKLN$. These budget lines intersect at point L , or about $x = 67$.



If we know that Darrell chooses Plan A, his optimal bundle must lie on the line segment JL . No point between L and M would be optimal under this plan because then Darrell could have chosen a point under Plan B, between L and N , that would have given him more minutes, and left him with more money to buy other goods. However, we cannot exclude point L itself (Darrell could, for instance, have perfect complements preferences with an “elbow” at point L). Thus, if Darrell chooses Plan A his optimal basket could be anywhere between J and L , including either of these points.

Similarly, if he chose Plan B then his optimal basket must lie between L and N . Any point between L and K (but not including point L) would be dominated by a point under

Plan A between L and J . Thus, if Darrell chooses Plan B his optimal basket could be anywhere between L and N , including either of these points.

Chapter 5 Review Questions

- 5.7** David has a quasi-linear utility function of the form $U(x, y) = \sqrt{x} + y$, with associated marginal utility functions $MU_x = \frac{1}{2\sqrt{x}}$ and $MU_y = 1$.

a) Derive David's demand curve for x as a function of the prices, P_x and P_y . Verify that the demand for x is independent of the level of income at an interior optimum.

Denoting the level of income by I , the budget constraint implies that $p_x x + p_y y = I$ and the tangency condition is $\frac{1}{2\sqrt{x}} = \frac{p_x}{p_y}$, which means that $x = \frac{p_y^2}{4p_x^2}$. The demand for x does not depend on the level of income.

b) Derive David's demand curve for y . Is y a normal good? What happens to the demand for y as P_x increases?

From the budget constraint, the demand curve for y is, $y = \frac{I - p_x x}{p_y} = \frac{I}{p_y} - \frac{p_y}{4p_x} x$.

You can see that the demand for y increases with an increase in the level of income, indicating that y is a normal good. Moreover, when the price of x goes up, the demand for y increases as well.

- 5.19** Lou's preferences over pizza (x) and other goods (y) are given by $U(x, y) = xy$, with associated marginal utilities $MU_x = y$ and $MU_y = x$. His income is \$120.

a) Calculate his optimal basket when $P_x = 4$ and $P_y = 1$.

Using the tangency condition, $\frac{y}{x} = 4$, and the budget constraint, $4x + y = 120$,

$$y = 4x \text{ Insert into budget constraint}$$

$$4x + 4x = 120 \quad 4(15) + y = 120$$

$$8x = 120 \quad y = 60$$

$$x = 15$$

Lou's initial optimum is the basket $(x, y) = (15, 60)$ with a utility of 900.

b) Calculate his income and substitution effects of a decrease in the price of food to \$3.

First we need the decomposition basket. This would satisfy the new tangency condition, $\frac{y}{x} = 3$ and would give him as much utility as before, i.e. $xy = 900$.

$$\begin{aligned} y &= 3x \\ x(3x) &= 900 \quad 17.32y = 900 \\ x^2 &= 300 \quad y = 51.9 \\ x &= 17.32 \end{aligned}$$

This gives $(x, y) = (10\sqrt{3}, 30\sqrt{3})$ or approximately $(17.3, 51.9)$. Now we need the final basket, which satisfies the same tangency condition as the decomposition basket and also the new budget constraint: $3x + y = 120$.

$$\begin{aligned} \frac{y}{x} &= 3 \\ y &= 3x \\ 3x + 3x &= 120 \quad 3(20) + y = 120 \\ 6x &= 120 \quad y = 60 \\ x &= 20 \end{aligned}$$

Together, these conditions imply that $(x, y) = (20, 60)$. The substitution effect is therefore $17.3 - 15 = 2.3$, and the income effect is $20 - 17.3 = 2.7$.

c) Calculate the compensating variation of the price change.

The compensating variation is the amount of income Lou would be willing to give up after the price change to maintain the level of utility he had before the price change. This equals the difference between the consumer's actual income, \$120, and the income needed to buy the decomposition basket at the new prices. This latter income equals: $3*17.3 + 1*51.9 = 103.8$. The compensating variation thus equals $120 - 103.8 = \$16.2$.

d) Calculate the equivalent variation of the price change.

The equivalent variation is the amount of income that Lou would need to be given *before* the price change in order to leave him as well off as he would be after the price change. After the price change his utility level is $20(60)=1200$. Therefore the additional income should be such that it allows Lou to

purchase a bundle (x, y) satisfying the initial tangency condition, $\frac{y}{x} = 4$, and also such that $xy = 1200$.

This implies that $(x, y) = (10\sqrt{3}, 40\sqrt{3})$ or approximately $(17.3, 69.2)$. How much income would Lou need to purchase this bundle under the original prices? He would need $4(17.3) + 69.2 = 138.4$. That is he

would need to increase his income by $(138.4 - 120)$ dollars in order to be as well off as if the price of pizza were to decrease instead. Therefore his equivalent variation is \$18.4.

5.20 Carina buys two goods, food F and clothing C, with the Utility function $U=FC+F$. Her marginal utility of food is $MU_F = C+1$ and her marginal utility of clothing is $MU_C=F$. She has an income of \$20. The price of clothing is \$4.

a) Derive the equation representing Carina's demand for food, and draw this demand curve for prices of food between 1 and 6.

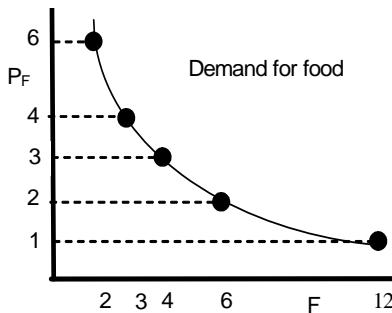
$$MU_F = C + 1 \quad MU_C = F$$

$$\text{Tangency: } MU_F/MU_C = P_F / P_C. \quad (C + 1)/F = P_F/4 \Rightarrow 4C + 4 = P_F F. \quad (\text{Eq 1})$$

$$\text{Budget Line: } P_F F + P_C C = I. \quad P_F F + 4C = 20. \quad (\text{Eq 2})$$

Substituting (Eq 1) into (Eq 2): $4C + 4 + 4C = 20$. Thus $C = 2$, independent of P_F .

From the budget line, we see that $P_F F + 4(2) = 20$, so **the demand for F is $F = 12/P_F$**



b) Calculate the income and substitution effects on Carina's consumption of food when the price rises from 1 to 4, and draw a graph illustrating these effects. Your graph does not need to be to scale, but it should be consistent with the data.

Initial Basket: From the demand for food in (a), $F = 12/1 = 12$, and $C = 2$.

Also, the initial level of utility is $U = FC + F = 12(2) + 12 = 36$.

Final Basket: From the demand for food in (a), we know that $F = 12/4 = 3$, and $C = 2$. (Also, $U = 3(2) + 3 = 9$.)

Decomposition Basket: Must be on initial indifference curve, with $U = FC + F = 36$ (Eq 5)

Tangency condition satisfied with final price: $MU_F/MU_C = P_F / P_C. \quad (C + 1)/F = 4/4 \Rightarrow C + 1 = F. \quad (\text{Eq 3})$

Eq 5 can be written as $F(C + 1) = 36$. Using Eq 3, $(C + 1)^2 = 36$, and thus, $C = 5$. Also, by Eq 3, $F = 6$.

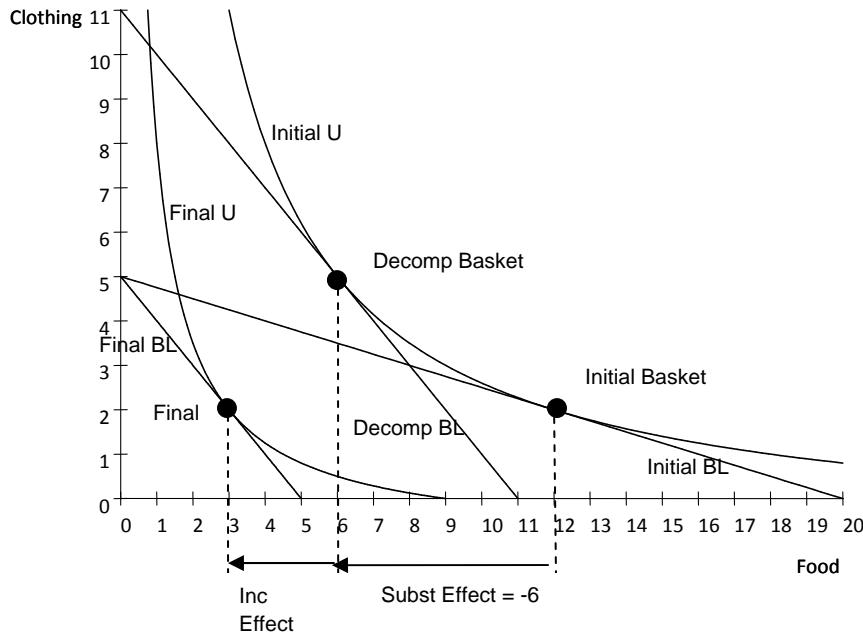
So the decomposition basket is $F = 6$, $C = 5$.

Income effect on F: $F_{\text{final basket}} - F_{\text{decomposition basket}} = 3 - 6 = -3$.

Substitution effect on F: $F_{\text{decomposition basket}} - F_{\text{initial basket}} = 6 - 12 = -6$.

c) Determine the numerical size of the compensating variation (in monetary terms) associated with the increase in the price of the good from 1 to 4.

$P_F + P_C C = 4(6) + 4(5) = 44$. So she would need an additional income of 24 (plus her actual income of 20). The compensating variation associated with the increase in the price of food is -24.



5.22 There are two consumers on the market: Jim and Donna. Jim's utility function is $U(x,y)=xy$, with associated marginal utilities $MU_x=y$ and $MU_y=x$. Donna's utility function is $U(x,y)=x^2y$, with associated marginal utility functions $MU_x=2xy$ and $MU_y=x^2$. Income of Jim of $I_J=100$ and income of Donna is $I_D=150$.

a) Find optimal baskets of Jim and Donna when the price of y is $P_y=1$ and price of x is P .

Jim's optimal basket is a solution to equations

$$MU_x / MU_y = P / P_y \text{ and } P x + P_y y = I_J.$$

Hence, we have $2xy / x^2 = P$ and $P x + y = 100$

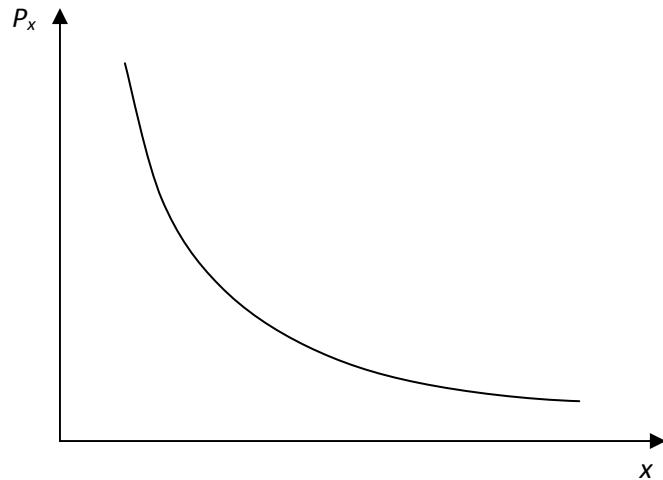
with solution $x = 200 / (3P)$ and $y = 100 / 3$.

Analogous system of equations for Donna is

$$y / x = P \text{ and } P x + y = 150 \text{ with solution } x = 75 / P \text{ and } y = 75.$$

b) On separate graphs plot Jim's and Donna's demand schedule for x for all values of P .

Approximate shape of the demand curve for Jim and Donna is depicted below.



c) Compute and plot aggregate demand when Jim and Donna are the only consumers.

Aggregate Demand:

$$D_x(P) = 200 / (3P) + 75 / P = 445 / (3P).$$

d) Plot aggregate demand when there is one more consumer that has identical utility function and income as Donna.

When there is one more consumer that has preferences identical to Donna's then her demand is also $75 / P$ and hence aggregate demand is

$$D_x(P) = 200 / (3P) + 75 / P + 75 / P = 650 / (3P).$$

Shape of the demand curve in this case is the same as in part b).

5.28 Consider Noah's preference for leisure (L) and other goods (Y), $U(L, Y) = \sqrt{L} + \sqrt{Y}$. The associated marginal utilities are $MU_L = \frac{1}{2\sqrt{L}}$ and $MU_Y = \frac{1}{2\sqrt{Y}}$. Suppose that $P_Y = \$1$. Is Noah's supply of labor backward bending?

If Noah's wage rate is w , then the income he earns from working is $(24 - L)w$. Since $P_Y = 1$, the number of units of other goods he purchases is $Y = (24 - L)w$. Also, the tangency condition gives us $\sqrt{\frac{Y}{L}} = w$.

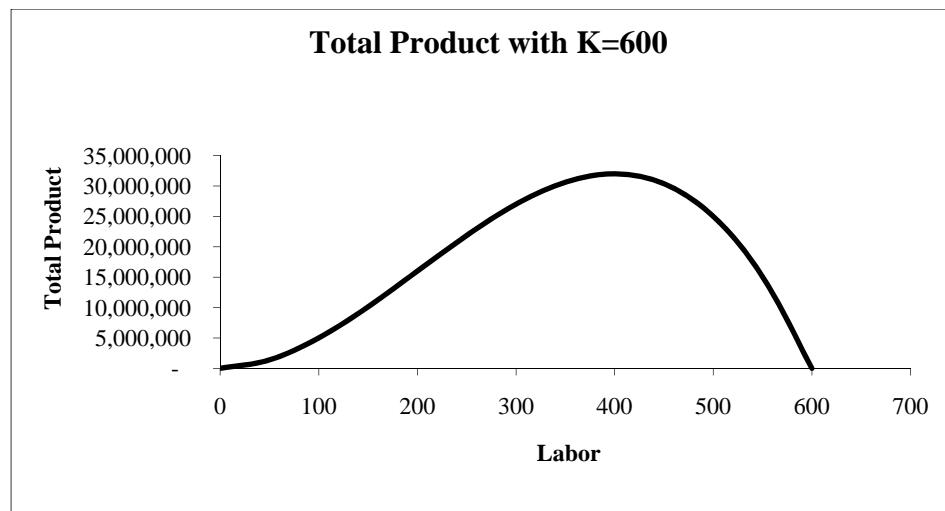
Combining the two conditions, $w^2 L = (24 - L)w$, or $L = \frac{24}{w+1}$. Clearly, the amount of leisure that

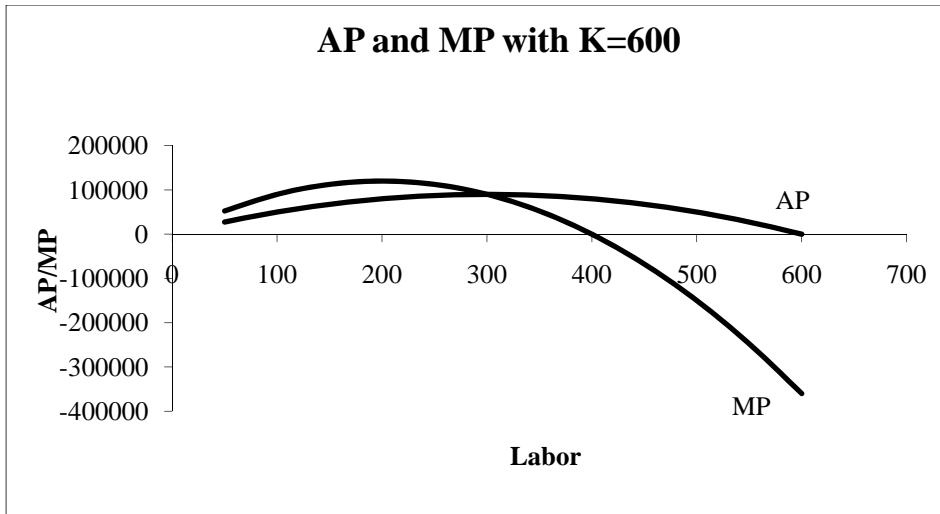
Noah consumes decreases with an increase in the wage rate, and this is true no matter what the wage rate is. Since the amount of labor that Noah supplies equals $(24 - L)$, we see that his supply of labor always increases with an increase in the wage rate. So, his labor supply curve is always positively sloped – that is, it is not backward bending.

Chapter 6 Review Questions

6.4 Suppose that the production function for DVDs is given by $Q=KL^2-L^3$, where Q is the number of disks produced per year, K is machine-hours of capital, and L is man hours of labor.

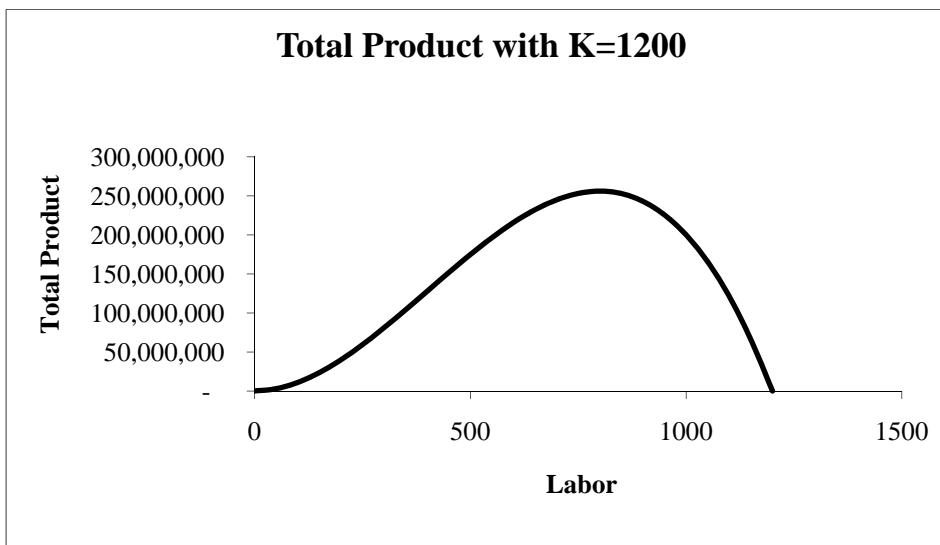
a) Suppose $K=600$. Find the total product function and graph it over the range $L=0$ and $L=500$. Then sketch the graphs of the average and marginal product functions. At what level of labor L does the average product curve appear to reach its maximum? At what level does the marginal product curve appear to reach its maximum?

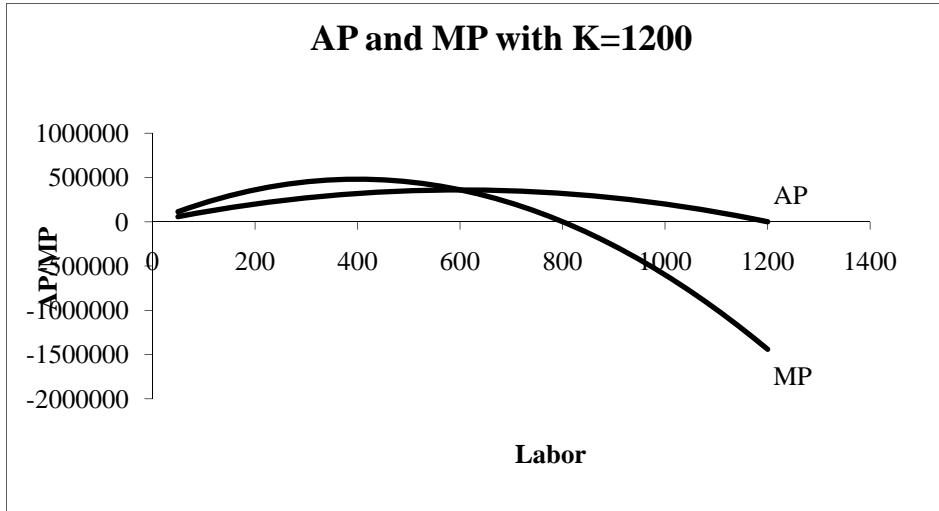




Based on the figure above it appears that the average product reaches its maximum at $L = 300$. The marginal product curve appears to reach its maximum at $L = 200$

b) Replicate the analysis in (a) for the case in which $K=1200$



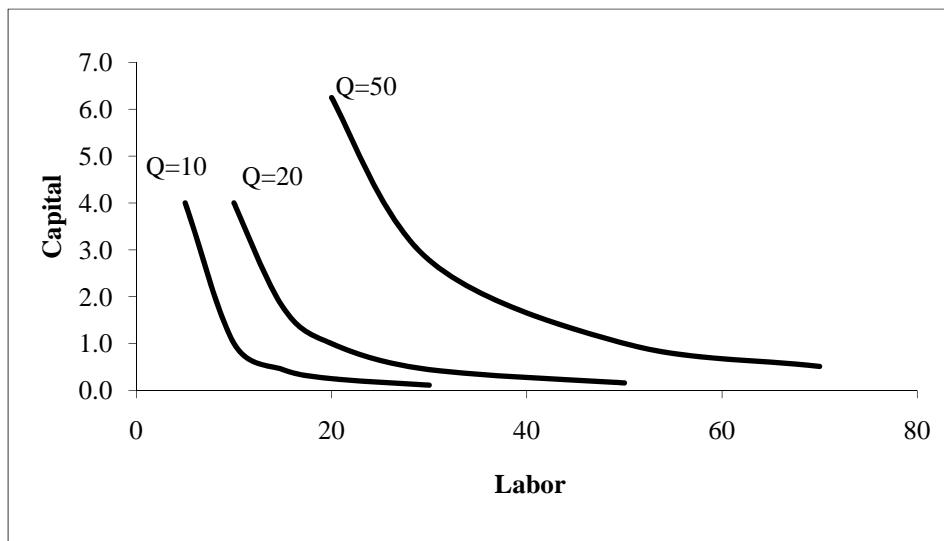


Based on the figure above it appears that the average product curve reaches its maximum at $L = 600$. The marginal product curve appears to reach its maximum at $L = 400$.

- c) When either $K=600$ or $K=1200$, does the total product function have a region of increasing marginal returns?

In both instances, for low values of L the total product curve increases at an increasing rate. For $K = 600$, this happens for $L < 200$. For $K = 1200$, it happens for $L < 400$.

- 6.9** Suppose the production function is given by the equation $Q = L\sqrt{K}$. Graph the isoquants corresponding to $Q=10$, $Q=20$, $Q=50$. Do these isoquants exhibit diminishing marginal rate of technical substitution?



Because these isoquants are convex to the origin they do exhibit diminishing marginal rate of technical substitution.

6.11 Suppose the production function is given by the following equation (where a and b are positive constants): $Q=aL+bK$. What is the marginal rate of technical substitution of labor for capital ($MRTS_{L,K}$) at any point along an isoquant?

For this production function $MP_L = a$ and $MP_K = b$. The $MRTS_{L,K}$ is therefore

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{a}{b}$$

6.12 You might think that when a production function has a diminishing marginal rate of technical substitution of labor for capital, it cannot have increasing marginal products of capital and labor. Show that this is not true, using the production function $Q=K^2L^2$, with the corresponding marginal products $MP_K=2KL^2$ and $MP_L=2K^2L$.

First, note that $MRTS_{L,K} = L/K$, which diminishes as L increases and K falls as we move along an isoquant. So $MRTS_{L,K}$ is diminishing. However, the marginal product of capital MP_K is *increasing* (not diminishing) as K increases (remember, the amount of labor is held fixed when we measure MP_K .) Similarly, the marginal product of labor is *increasing* as L grows (again, because the amount of capital is held fixed when we measure MP_L). This exercise demonstrates that it is possible to have a diminishing marginal rate of technical substitution even though both of the marginal products are increasing.

6.17 What can you say about returns to scale of the linear production function $Q=aK+bL$, where a and b are positive constants.

$$\begin{aligned} Q &= aK + bL \\ &= \lambda aK + \lambda bL \\ &= \lambda[aK + bL] \\ &= \lambda[Q] \end{aligned}$$

Therefore a linear production function has constant returns to scale.

6.18 What can you say about the returns to scale of the Leontief production function $Q=\min(aK,bL)$, where a and b are positive constants?

A general fixed proportions production function is of the form $Q = \min(aK, bL)$.

$$\begin{aligned}
Q &= \min(aK, bL) \\
&= \min(\lambda aK, \lambda bL) \\
&= \lambda \min(aK, bL) \\
&= \lambda[Q]
\end{aligned}$$

Therefore the production function has constant returns to scale.

6.19 A firm produces a quantity Q of breakfast cereal using labor L and material M with the production function $Q = 50\sqrt{ML} + M + L$. The marginal product functions for this production function

$$MP_L = 25\sqrt{\frac{M}{L}} + 1$$

are

$$MP_M = 25\sqrt{\frac{L}{M}} + 1$$

a) Are the returns to scale increasing, constant, or decreasing for this production function?

To determine the nature of returns to scale, increase all inputs by some factor λ and determine if output goes up by a factor more than, less than, or the same as λ

$$\begin{aligned}
Q_\lambda &= 50\sqrt{\lambda M \lambda L} + \lambda M + \lambda L \\
Q_\lambda &= 50\lambda\sqrt{ML} + \lambda M + \lambda L \\
Q_\lambda &= \lambda \left[50\sqrt{ML} + M + L \right] \\
Q_\lambda &= \lambda Q
\end{aligned}$$

By increasing the inputs by a factor of λ output goes up by a factor of λ . Since output goes up by the same factor as the inputs, this production function exhibits constant returns to scale.

b) Is the marginal product of labor ever diminishing for this production function? If so, when? Is it ever negative, and if so, when?

The marginal product of labor is

$$MP_L = 25\sqrt{\frac{M}{L}} + 1$$

Suppose $M > 0$. Holding M fixed, increasing L will have the effect of decreasing MP_L . The marginal product of labor is decreasing for all levels of L . The MP_L , however, will never be negative since both components of the equation above will always be greater than or equal to zero. In fact, for this production function, $MP_L \geq 1$.

6.25 A firm's production function is initially $Q = \sqrt{KL}$, with $MP_K = 0.5\left(\frac{\sqrt{L}}{\sqrt{K}}\right)$ and $MP_L = 0.5\left(\frac{\sqrt{K}}{\sqrt{L}}\right)$. Over time, the production function changes to $Q = K\sqrt{L}$, with $MP_K = \sqrt{L}$ and $MP_L = 0.5\left(\frac{K}{\sqrt{L}}\right)$.

a) Verify that this change represents technological progress.

With any positive amounts of K and L , $\sqrt{KL} < K\sqrt{L}$ so more Q can be produced with the final production function. So there is indeed technological progress.

b) Is this change labor saving, capital saving, or neutral?

With the initial production function

With the final production function

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{0.5K}{L} .$$

For any ratio of capital to labor, $MRTS_{L,K}$ is lower with the second production function. Thus, the technological progress is labor-saving.

EconS 301 – Intermediate Microeconomics

Midterm #2 – Handout for the Review Session

Chapter 7

Exercise 7.8 The tangency condition implies...

$$10 = \frac{K}{L}$$

$$10L = K$$

Substituting into the production function yields

$$121,000 = LK$$

$$121,000 = L(10L)$$

$$121,000 = 10L^2$$

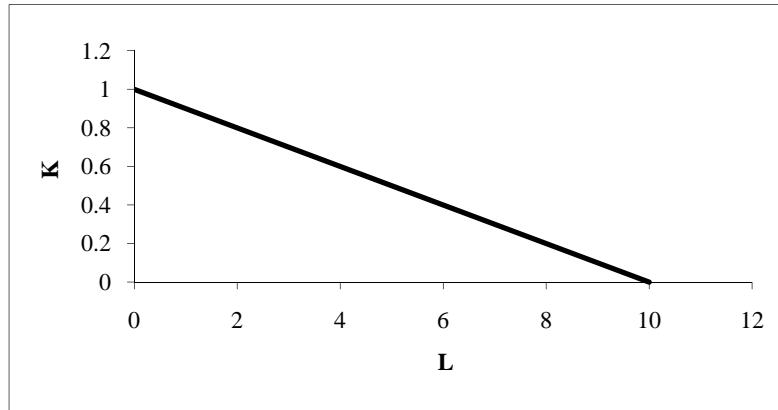
$$12,100 = L^2$$

$$110 = L$$

Since $K = 10L$, $K = 1,100$. The cost-minimizing quantities of labor and capital to produce 121,000 airframes are $K = 1,100$ and $L = 110$.

Exercise 7.9

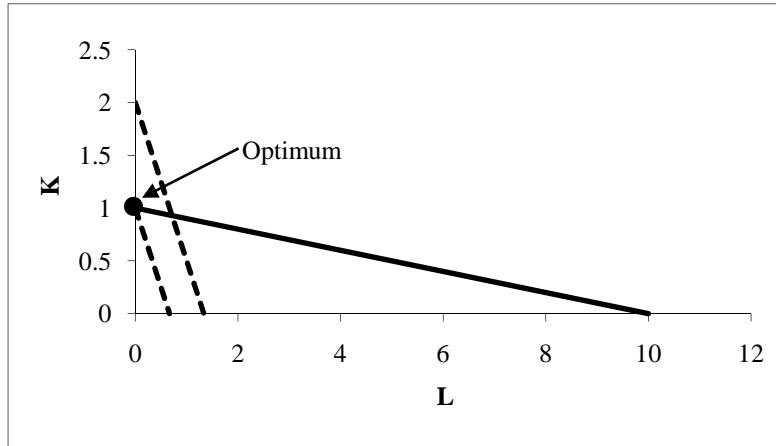
a)



K and L are perfect substitutes, meaning that the production function is linear and the isoquants are straight lines. We can write the production function as $Q = 10,000K + 1000L$, where Q is the number of workers for whom payroll is processed.

- b) If $r = 5$ and $w = 7.50$, the slope of a typical isocost line will be $-7.5/5.0 = -1.5$. This is steeper than the isoquant implying that the firm will employ only computer time (

K) to minimize cost. The cost minimizing combination is $K = 1$ and $L = 0$. This outcome can be seen in the graph below. The isocost lines are the dashed lines.



The total cost to process the payroll for 10,000 workers will be $TC = 5(1) + 7.5(0) = 5$.

- c) The firm will employ clerical time only if $MP_L / w > MP_K / r$. Thus we need $0.1 / 7.5 > 1/r$ or $r > 75$.

Exercise 7.22

Using the tangency condition, initially $\frac{K}{L} = 1$, implying that $K = L$. Since $KL = 100$, we get $K = L = 10$.

Under the new prices, the tangency condition implies that $K=4L$. This means that the optimal input combination is $(L, K) = (5, 20)$.

The percent change in price is $(4 - 1)*100 = 300\%$. While the percent change in the demand for labor is $[(5 - 10)/10]*100 = -50\%$. Therefore the price elasticity of demand over this range of prices is $-50/300 = -1/6$.

Chapter 8

Exercise 8.15 Since we can assume an interior solution, the tangency condition must hold. Therefore the optimal bundle must be such that $\frac{K}{L+1} = \frac{w}{r}$. This means $L+1 = \frac{rK}{w}$. Substituting this back into the production function, we see that $Q = \frac{rK^2}{w}$, so $K = \sqrt{\frac{Qw}{r}}$.

This implies that $L = \sqrt{\frac{Qr}{w}} - 1$. The total cost curve is then $TC = wL + rK = 2\sqrt{wrQ} - w$. If we substitute $2w$ and $2r$ in the place of w and r respectively, we get $TC_2 = 2\sqrt{(2w)(2r)Q} - (2w) = 4\sqrt{wrQ} - 2w = 2*TC$, so total cost does indeed double when input prices double.

Exercise 8.18

- a) Even if the firm hires zero units of labor, with K fixed at 16 it can still produce up to $Q = \sqrt{0} + \sqrt{16} = 4$ units of output. So for $Q \leq 4$, $L = 0$ is the cost-minimizing choice of labor and the short-run total cost function is just the cost of capital: $C = rK + wL = 2(16) + 1(0) = 32$.
- b) For $Q > 4$, the firm needs to hire positive amounts of labor, according to $Q = \sqrt{L} + \sqrt{16}$ or $L = (Q - 4)^2$. So for $Q > 4$, the short-run total cost function is $C(Q) = rK + wL = 2(16) + 1(Q - 4)^2 = 32 + (Q - 4)^2$.

Chapter 9

Exercise 9.8

- a) In order to maximize profit Ron should operate at the point where $P = MC$.

$$20 = 10 + 0.20Q$$

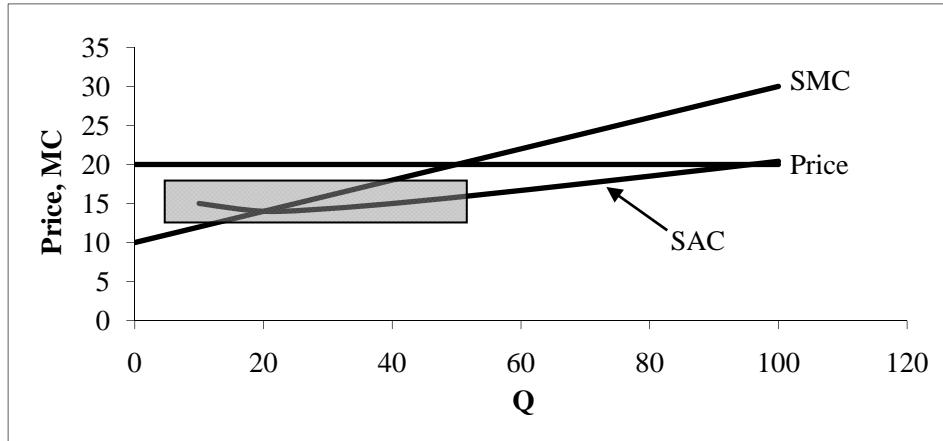
$$Q = 50$$

- b) Ron's profit is given by $\pi = TR - TC$.

$$\pi = 20(50) - (40 + 10(50) + 0.10(50)^2)$$

$$\pi = 210$$

- c) The firm's profit is equal to the shaded area in the graph below. It is a rectangle whose height is the market price and the average cost of the 50th unit, and whose width is the 50 units being produced.



- d) If all fixed costs are sunk, then $ANSC = AVC = (10Q + 0.1Q^2)/Q = 10 + 0.1Q$. So the first step is to find the minimum of $ANSC$ by setting $ANSC = SMC$, or $10 + 0.1Q = 10 + 0.2Q$ which occurs when $Q = 0$. The minimum level of $ANSC$ is thus 10. For prices below 10 the firm will not produce and for prices above 10, its supply curve is found by setting $P = SMC$:

$$P = 10 + .2Q$$

$$Q = 5P - 50$$

The firm's short-run supply curve is thus

$$s(P) = \begin{cases} 0 & \text{if } P < 10 \\ 5P - 50 & \text{if } P \geq 10 \end{cases}$$

- e) If all fixed costs are non-sunk, as in this case, then $ANSC = ATC = (40/Q) + 10 + 0.1Q$. The minimum point of $ANSC$ occurs where $ANSC = SMC$:

$$\frac{40}{Q} + 10 + .1Q = 10 + .2Q$$

$$Q = 20$$

The minimum level of $ANSC$ is thus 14. For prices below 14 the firm will not produce and for prices above 14, its supply curve is found by setting $P = SMC$ as before.

$$s(P) = \begin{cases} 0 & \text{if } P < 14 \\ 5P - 50 & \text{if } P \geq 14 \end{cases}$$

Exercise 9.13

Total industry supply is the sum of the supply curves of the individual firms. Since we have 100 type A firms, total supply from type A firms is $100s_A(P) = 200P$, and since we have 30 type B firms, total supply from type B firms is $30s_B(P) = 300P$. The short-run industry supply curve is thus $S(P) = 200P + 300P = 500P$. The short-run market equilibrium occurs at the price at which quantity supplied equals quantity demanded, or $5000 - 500P = 500P$, or $P = 5$. At this price, a type A firm supplies 10 units, while a type B firm supplies 50 units.

Exercise 9.23

In a long-run equilibrium all firms earn zero economic profit implying $P = AC$ and each firm produces where $P = MC$. Thus,

$$40 - 12Q + Q^2 = 40 - 6Q + \frac{1}{3}Q^2$$

$$Q = 9$$

So each individual firm produces $Q = 9$, and the long-run equilibrium price must be $P = 40 - 12(9) + 9^2 = 13$. Since $D(P) = 2200 - 100P$,

$$D(P) = 2200 - 100(13)$$

$$D(P) = 900$$

If each firm produces 9 units, the market will have 100 firms in equilibrium.

- a) Minimum efficient scale occurs at the point where average cost reaches a minimum. This point occurs where $MC = AC$.

$$2Q = \frac{144}{Q} + Q$$

$$Q = 12$$

At $Q = 12$,

$$AC = \frac{144}{Q} + Q$$

$$AC = 24$$

- b) In the long-run, the equilibrium price will be determined by the minimum level of average cost for firms with average CEOs. Thus, $P = 24$. At this price, firms having average CEOs will earn zero economic profit and firms with exceptional CEOs will earn positive economic profit.
- c) At the price, the firms with an average CEO will produce where $P = MC$

$$24 = 2Q$$

$$Q = 12$$

The firms with an exceptional CEO will also produce where $P = MC$

$$Q = 24$$

- d) At this price

$$D(P) = 7200 - 100P$$

$$D(P) = 4800$$

- e) Since there are 100 exceptional CEOs and assuming they are all employed, the total supply from exceptional CEO firms will be

$$S_E = 100(24)$$

$$S_E = 2400$$

This leaves $Q = 4800 - 2400 = 2400$ units to be supplied by firms with average CEOs. Thus,

$$N_A = \frac{2400}{12}$$

$$N_A = 200$$

- f) To calculate the exceptional CEO's economic rent we must compute the highest salary the firm would pay this CEO. This salary is the amount that would drive economic profit to zero. Call this amount S^* . Since the exceptional CEO firm is producing $Q = 24$, the firm's average cost is

$$AC = \frac{144}{24} + \frac{1}{2}(24)$$

$$AC = 18$$

Since $P = 24$, the exceptional CEO has produced a \$6 per unit cost advantage. This implies

$$\frac{S^* - 144}{24} = 6$$

$$S^* = 288$$

Economic rent is the difference between this salary, \$288,000, and the reservation wage of \$144,000. Thus, the exceptional CEO's economic rent is \$144,000.

- g) Firms that hire exceptional CEOs for \$144,000 will gain all of the CEO's economic rent and will therefore earn economic profit of \$144,000.
- h) In a long-run competitive equilibrium, exceptional CEO salaries should be bid up as other firms compete for the exceptional CEOs. This should bid up the salary of the CEOs until economic profits for firms with exceptional CEOs are driven to zero. Thus, exceptional CEO salaries should approach \$288,000 in a long-run equilibrium.

Chapter 11

Exercise 11.10

- a) If demand is given by $P = 300 - Q$ then $MR = 300 - 2Q$. To find the optimum set $MR = MC$.

$$300 - 2Q = Q$$

$$Q = 100$$

At $Q = 100$ price will be $P = 300 - 100 = 200$. At this price and quantity total revenue will be $TR = 200(100) = 20,000$ and total cost will be $TC = 1200 + .5(100)^2 = 6,200$. Therefore, the firm will earn a profit of $\pi = TR - TC = 13,800$.

- b) The price elasticity at the profit-maximizing price is

$$\varepsilon_{Q,P} = \frac{\Delta Q}{\Delta P} \frac{P}{Q}$$

With the demand curve $Q = 300 - P$, $\frac{\Delta Q}{\Delta P} = -1$. Therefore, at the profit-maximizing price

$$\varepsilon_{Q,P} = -1 \left(\frac{200}{100} \right)$$

$$\varepsilon_{Q,P} = -2$$

The marginal cost at the profit-maximizing output is $MC = Q = 100$. The inverse elasticity pricing rule states that at the profit-maximizing price

$$\frac{P - MC}{P} = -\frac{1}{\varepsilon_{Q,P}}$$

In this case we have

$$\begin{aligned}\frac{200 - 100}{200} &= -\frac{1}{-2} \\ \frac{1}{2} &= \frac{1}{2}\end{aligned}$$

Thus, the IEPR holds for this monopolist.

Exercise 11.15

- a) The monopolist will operate where $MR = MC$. With demand $P = a - bQ$, marginal revenue is given by $MR = a - 2bQ$. Setting this equal to marginal cost implies

$$\begin{aligned}a - 2bQ &= c + eQ \\ Q &= \frac{a - c}{2b + e}\end{aligned}$$

At this quantity price is

$$\begin{aligned}P &= a - b\left(\frac{a - c}{2b + e}\right) \\ P &= \frac{ab + ae + bc}{2b + e}\end{aligned}$$

- b) Since

$$Q = \frac{a - c}{2b + e}$$

increasing c or decreasing a will reduce the numerator of the expression, reducing Q .

- c) Since $e \geq 0$ and

$$P = \frac{ab + ae + bc}{2b + e}$$

increasing a will increase the numerator for this expression. This will therefore increase the equilibrium price.

Exercise 11.19

- a) Profit-maximizing firms generally allocate output among plants so as to keep marginal costs equal. But notice that $MC_2 < MC_1$ whenever $1 + 0.5Q_2 < 8$, or $Q_2 < 14$. So for small levels of output, specifically $Q < 14$, Gillette will only use the first plant. For $Q > 14$, the cost-minimizing approach will set $Q_2 = 14$ and $Q_1 = Q - 14$.

Suppose the monopolist's profit-maximizing quantity is $Q > 14$. Then the relevant $MC = 8$, and with $MR = 968 - 40Q$ we have

$$968 - 40Q = 8$$

$$Q = 24$$

Since we have found that $Q > 14$, we know this approach is valid. (You should verify that had we supposed the optimal output was $Q < 14$ and set $MR = MC_2 = 1 + 0.5Q$, we would have found $Q > 14$. So this approach would be invalid.) The allocation between plants will be $Q_2 = 14$ and $Q_1 = 10$. With a total quantity $Q = 24$, the firm will charge a price of $P = 968 - 20(24) = 488$. Therefore the price will be \$4.88 per blade.

- b) If $MC = 10$ at plant 1, by the logic in part (a) Gillette will only use plant 2 if $Q < 18$. It will produce all output above $Q = 18$ in plant 1 at $MC = 10$. Assuming $Q > 18$, setting $MR = MC$ implies

$$968 - 40Q = 10$$

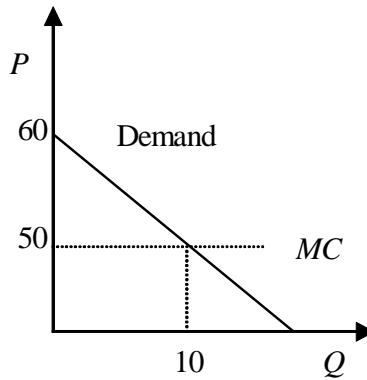
$$Q = 23.95$$

(So again, this approach is valid. You can verify that setting $MR = MC_2$ would again lead to $Q > 18$.) The firm will allocate production so that $Q_2 = 18$ and $Q_1 = 5.95$. At $Q = 23.95$, price will be \$4.89.

Chapter 12

Exercise 12.7

- a) Setting $MR = MC$, we have $60 - 2Z = 50$ or $Z = 5$, with $P = 55$. You earn $\pi = 55*5 - (K + 50*5) = 25 - K$, so profits are positive only if $K < 25$.
- b) $P = MC = 50$ implies the customer purchases $Z = 10$ units. See the graph below.



- c) At $P = 10$, the customer gets $CS = \frac{1}{2}*(60 - 50)*10 = 50$. Thus, the largest fixed-fee you could charge her, while ensuring that she is willing to participate in this market, is $F = CS = 50$.
- d) Now, your revenues are $R = 50 + 50*5 = 300$, so profits are $\pi = 300 - (K + 50*5) = 50 - K$. Now the firm can operate profitably so long as $K < 50$. By enabling the firm to

extract more surplus, (here, second-degree) price discrimination allows you to operate in a market where sunk fixed costs range as high as $K = 50$, whereas using standard monopoly pricing the firm wouldn't participate unless $K < 25$.

- e) For N customers, your profits are $\pi = N*300 - (K + 50*5*N) = 50*N - K$, so profits are positive only when $K < 50*N$.

Exercise 12.11

- a) With third-degree price discrimination the firm should set $MR = MC$ in each market to determine price and quantity. Thus, in Europe setting $MR = MC$

$$70 - 2Q_E = 10$$

$$Q_E = 30$$

At this quantity, price will be $P_E = 40$. Profit in Europe is then $\pi_E = (P_E - 10)Q_E = (40 - 10)30 = 900$. Setting $MR = MC$ in the US implies

$$110 - 2Q_U = 10$$

$$Q_U = 50$$

At this quantity price will be $P_U = 60$. Profit in the US will then be $\pi_U = (P_U - 10)Q_U = (60 - 10)50 = 2500$. Total profit will be $\pi = 3400$.

- b) If the firm can only sell the drug at one price, it will set the price to maximize total profit. The total demand the firm will face is $Q = Q_E + Q_U$. In this case

$$Q = 70 - P + 110 - P$$

$$Q = 180 - 2P$$

The inverse demand is then $P = 90 - 0.5Q$. Since $MC = 10$, setting $MR = MC$ implies

$$90 - Q = 10$$

$$Q = 80$$

At this quantity price will be $P = 50$. If the firm sets price at 50, the firm will sell $Q_E = 20$ and $Q_U = 60$. Profit will be $\pi = 50(80) - 10(80) = 3200$.

- c) The firm will sell the drug on both continents under either scenario. If the firm can price discriminate, total consumer surplus will be $0.5(70 - 40)30 + 0.5(110 - 60)50 = 1700$ and producer surplus (equal to profit) will be 3400. Thus, total surplus will be 5100. If the firm cannot price discriminate, consumer surplus will be $0.5(70 - 50)20 + 0.5(110 - 50)60 = 2000$ and producer surplus will be equal to profit of 3200. Thus, total surplus will be 5200.

Exercise 12.23

- a) You should sell two burgers for $P_B = 5$, and one order of fries for $P_F = 3$. Total surplus is then $PS + CS = (10 + 3) + (3 + 0) = 16$.

- b) In order for the profit-maximizing bundle price to be \$8, it must be true that $8 + x < 2*8$, i.e. that $x < 8$. In order for the profit-maximizing price of fries to be greater than \$3, it must be true that $x > 2*3$, or $x > 6$. Thus, we know that $6 \leq x < 8$.

Exercise 12.24

- a) Using the inverse elasticity price rule,

$$\begin{aligned}\frac{P - MC}{P} &= -\frac{1}{\varepsilon_{Q,P}} \\ \frac{P - MC}{P} &= -\frac{1}{-3} \\ \frac{P}{MC} &= 1.5\end{aligned}$$

The firm should set price at about 1.5 times marginal cost.

- b) The optimal advertising-to-sales ratio can be found by equating

$$\begin{aligned}\frac{A}{PQ} &= -\frac{\varepsilon_{Q,A}}{\varepsilon_{Q,P}} \\ \frac{A}{PQ} &= -\frac{0.5}{-3} \\ \frac{A}{PQ} &= 0.167\end{aligned}$$

Thus, advertising expense should be about 16 or 17 percent of sales revenue.

Chapter 13

Exercise 13.4

- a) The inverse market demand curve is $P = 100 - (Q/40) = 100 - (Q_1 + Q_2)/40$. Firm 1's reaction function is found by equating $MR_1 = MC_1$:

$$MR_1 = [100 - Q_2/40] - Q_1/20$$

$$MR_1 = MC_1 \Rightarrow [100 - Q_2/40] - Q_1/20 = 20$$

Solving this for Q_1 in terms of Q_2 gives us:

$$Q_1 = 1,600 - 0.5Q_2$$

Similarly, Firm 2's reaction function is found by equating $MR_2 = MC_2$:

$$MR_2 = [100 - Q_1/40] - Q_2/20$$

$$MR_2 = MC_2 \Rightarrow [100 - Q_1/40] - Q_2/20 = 40$$

Solving this for Q_2 in terms of Q_1 gives us:

$$Q_2 = 1,200 - 0.5Q_1$$

- b) The two reaction functions give us two equations in two unknowns. Using algebra we can solve these to get: $Q_1 = 1,333.33$ and $Q_2 = 533.33$.

We find the Cournot equilibrium price by plugging these quantities back into the inverse market demand curve:

$$P = 100 - (1333.33 + 533.33)/40 = 53.33$$

Exercise 13.12

The table below summarizes the answer to this problem. The solution details follow.

	Firm 1 output	Firm 2 output	Market Price	Firm 1 Profit	Firm 2 Profit
Cournot	40	40	120	3,200	3,200
Stackelberg with Firm 1 as leader	60	30	100	3,600	1,800

- a) Firm 1's marginal revenue is $MR = 280 - 2Y - 4X$. Equating MR to MC gives us:

$$280 - 2Y - 4X = 40,$$

$$240 - 2Y = 4X$$

$$X = 60 - 0.5Y$$

This is Firm 1's reaction function. Firm 2 is identical, and so Firm 2's reaction function will be a mirror image of Firm 1's

$$Y = 60 - 0.5X$$

If we solve these reaction functions simultaneously, we find $X = Y = 40$. At this output, the corresponding market price is $P = 280 - 2(40 + 40) = 120$. Each firm's profit is thus: $120*40 - 40*40 = 3,200$.

- b) To find the Stackelberg equilibrium in which Firm 1 is the leader, we start by writing the expression for Firm 1's total revenue:

$$TR = (280 - 2Y - 2X)X$$

In place of Y , we substitute in Firm 2's reaction function: $Y = 60 - 0.5X$

$$TR = [280 - 2(60 - 0.5X) - 2X]X = (160 - X)X$$

Firm 1's marginal revenue is therefore $MR = 160 - 2X$. Equating marginal revenue to marginal cost gives us:

$$160 - 2X = 40, \text{ or } X = 60.$$

To find Firm 2's output, we plug $X = 60$ back into Firm 2's reaction function:

$$Y = 60 - 0.5(60) = 30.$$

The market price is found by plugging $X = 60$ and $Y = 30$ back into the demand curve: $P = 280 - 2(60 + 30) = 100$.

Thus, at the Stackelberg equilibrium, Firm 1's profit is: $100*60 - 40*60 = 3,600$. Firm 2's profit is $100*30 - 40*30 = 1,800$.

Exercise 13.23

- a) We will first solve for Alpha's reaction function. We begin by solving Alpha's demand function for P_A in terms of Q_A and P_B : $P_A = 15 - (1/10)Q_A + (9/10)P_B$. The corresponding marginal revenue equation is: $MR_A = 14 - (2/10)Q_A + (9/10)P_B$. Equating marginal revenue to marginal cost and solving for Q_A gives us Alpha's profit-maximizing quantity as a function of Bravo's price: $MR_A = MC_A \Rightarrow 15 - (2/10)Q_A + (9/10)P_B = 7$, which gives us: $Q_A = 40 + (9/2)P_B$. Now, substitute this expression for Q_A back into the expression for the demand curve with P_A on the left-hand side and Q_A on the right-hand side: $P_A = 15 - (1/10)[40 + (9/2)P_B] + (9/10)P_B \Rightarrow P_A = 11 + (9/20)P_B$. This is Alpha's reaction function.
- b) We can find Bravo's reaction function by following steps identical to those followed to derive Alpha's reaction function. Following these steps gives us: $P_B = 11 + (9/20)P_A$. We now have two equations (the two reaction functions) in two unknowns, P_A and P_B . Solving these equations gives us the Bertrand equilibrium prices: $P_A = P_B = 20$.
- c) The following diagrams show how each change affects the reaction functions.

Exercise 13.28

- a) Monopoly price = \$6 per pair of shoes, $Q = 4$ million shoes are sold.
- b) Let's begin by deriving Firm 1's reaction function.

Step 1: Firm 1's demand curve can be written as:

$$P_1 = [10 - P_2] - Q_1$$

The corresponding MR curve is thus:

$$MR_1 = [10 - P_2] - 2Q_1$$

Step 2: Set MR_1 equal to MC and solve for Q_1 in terms of P_2

$$[10 - P_2] - 2Q_1 = 1$$

$$Q_1 = 4.5 - 0.5P_2$$

Step 3: Substitute back into the inverse demand curve to give us P_1 in terms of P_2 .

$$P_1 = [10 - P_2] - [4.5 - 0.5P_2] = 5.5 - 0.5P_2$$

Step 4: Firm 2's reaction function is easy. The firms are symmetric so it takes the same form as firm 1's reaction function:

$$P_2 = 5.5 - 0.5P_1$$

Step 5: Solve for the equilibrium

We have two equations in two unknowns. If we solve these reaction functions, we get $P_1 = P_2 = 3.67$.

Since $P_1 + P_2 =$ the total price of a pair of shoes $= 3.67 + 3.67 = 7.33 > 6$, a pair of shoes is now more expensive than when a single shoe monopolist controlled the market. Consumers are worse off after the breakup!

In effect what is going on here is this: after the breakup of the shoe monopoly, the right-shoe producer does not take into account the negative impact of his raising price on the demand for left-shoes. Ditto for the left-shoe producer. A monopolist seller of both right and left shoes, by contrast, would internalize this effect. The result: the independent firms raise price “too much” since they are “ignoring” part of the “cost” of raising price.