EC 215: INTERMEDIATE MICROECONOMIC THEORY FINAL EXAMINATION FOR FIRST SEMESTER, 2012-13 ACADEMIC YEAR

QUESTION ONE.

The city council of a small college town decides to regulate rents in order to reduce student living expenses. Suppose the average annual market-clearing rent for a two-bedroom apartment had been \$700 per month, and rents were expected to increase to \$900 within a year. The city council limits rents to their current \$700-per-month level.

a. Draw a supply and demand graph to illustrate what will happen to the rental price of an apartment after the imposition of rent controls.

Initially demand is D_1 and supply is S, so the equilibrium rent is \$700 and Q_1 apartments are rented. Without regulation, demand was expected to increase to D_2, which would have raised rent to \$900 and resulted in Q_2 apartment rentals. Under the city council regulation, however, the rental price stays at the old equilibrium level of \$700 per month. After demand increases to D_2, only Q_1 apartments will be supplied while Q_3 will be demanded. There will be a shortage of $Q_3 - Q_1$ apartments.

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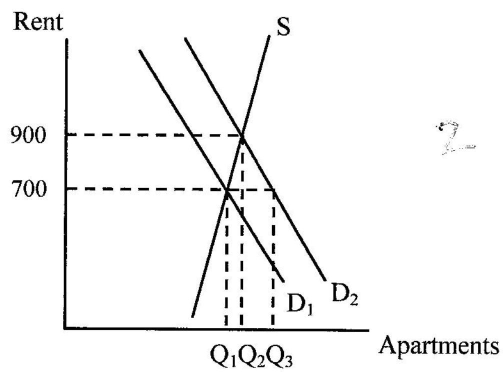

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b. Do you think this policy will benefit all students? Why or why not?

No. It will benefit those students who get an apartment, although these students may find that the cost of searching for an apartment is higher given the shortage of apartments. Those students who do not get an apartment may face higher costs as a result of having to live outside the college town. Their rent may be higher and their transportation costs will be higher, so they will be worse off as a result of the policy.

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QUESTION TWO.

Show that the two utility functions given below generate the identical demand functions for goods X and Y:

a. **U(X, Y) = log(X) + log(Y)**

b. **U(X, Y) = (XY)$^{0.5}$**

a). To find the demand functions for X and Y, corresponding to $U(X, Y) = \log(X) + \log(Y)$, we must maximize $U(X, Y)$ subject to the budget constraint. To do this, first write out the Lagrangian function, where λ is the Lagrange multiplier:

$$\Phi = \log(X) + \log(Y) - \lambda(P_X X + P_Y Y - I).$$

Differentiating with respect to X, Y and λ, and setting the derivatives equal to zero:

$$\frac{\partial\Phi}{\partial X} = \frac{1}{X} - \lambda P_X = 0$$

$$\frac{\partial\Phi}{\partial Y} = \frac{1}{Y} - \lambda P_Y = 0$$

$$\frac{\partial\Phi}{\partial\lambda} = I - P_X X - P_Y Y = 0.$$

The first two conditions imply that $P_X X = \frac{1}{\lambda}$ and $P_Y Y = \frac{1}{\lambda}$.

The third condition implies that $I - \frac{1}{\lambda} - \frac{1}{\lambda} = 0$, or $\lambda = \frac{2}{I}$.

Substituting this expression into $P_X X = \frac{1}{\lambda}$ and $P_Y Y = \frac{1}{\lambda}$ gives the demand functions:

$$X = \left(\frac{I}{2P_X}\right) \text{ and } Y = \left(\frac{I}{2P_Y}\right).$$

Notice that the demand for each good depends only on the price of that good and on income, not on the price of the other good. Also, the consumer spends exactly half her income on each good, regardless of the prices of the goods.

b). To find the demand functions for X and Y, corresponding to $U(X,Y) = (XY)^{0.5} = (X^{0.5})(Y^{0.5})$, first write out the Lagrangian function:

$$\Phi = (X)^{0.5}(Y)^{0.5} - \lambda(P_X X + P_Y Y - I)$$

Differentiating with respect to X, Y, λ and setting the derivatives equal to zero:

$$\frac{\partial\Phi}{\partial X} = 0.5X^{-0.5}Y^{0.5} - \lambda P_X = 0$$

$$\frac{\partial\Phi}{\partial Y} = 0.5X^{0.5}Y^{-0.5} - \lambda P_Y = 0$$

$$\frac{\partial\Phi}{\partial Y} = I - P_X X - P_Y Y = 0$$

Take the first two conditions, move the terms involving λ to the right hand sides, and then divide the first condition by the second. After some algebra, you'll find $\frac{Y}{X} = \frac{P_X}{P_Y}$, or $P_Y Y = P_X X$. Substitute for $P_Y Y$ in the third condition, which yields $I = 2P_X X$. Therefore, $X = \left(\frac{I}{2P_X}\right)$ and $Y = \left(\frac{I}{2P_Y}\right)$, which are the same demand functions we found for the other utility function.

$$0.5X^{-0.5}Y^{0.5} = \lambda P_X$$

$$0.5X^{0.5}Y^{-0.5} = \lambda P_Y$$

$$\frac{0.5Y^{\frac{1}{2}}}{X^{\frac{1}{2}}P_X} = \lambda = \frac{0.5X^{\frac{1}{2}}}{Y^{\frac{1}{2}}P_Y}$$

$$0.5Y^{\frac{1}{2}}Y^{\frac{1}{2}}P_Y = 0.5X^{\frac{1}{2}}X^{\frac{1}{2}}P_X$$

$$0.5YP_Y = 0.5XP_X$$

$$YP_Y = XP_X$$

Substitute $P_Y Y$ and XP_X in the 3rd equation

Thus $I - P_X X - XP_X = 0$ $\therefore$ $X = \frac{I}{2P_X}$

$I = 2P_X X$

Similarly $I = P_X X + P_Y Y = 2P_Y Y$

$$Y = \frac{I}{2P_Y}$$

QUESTION THREE

You manage a plant that mass-produces engines by teams of workers using assembly machines. The technology is summarized by the production function

$$q = 5\,KL$$

where q is the number of engines per week, K is the number of assembly machines, and L is the number of labor teams. Each assembly machine rents for r = $10,000 per week, and each team costs w = $5000 per week. Engine costs are given by the cost of labor teams and machines, plus $2000 per engine for raw materials. Your plant has a fixed installation of 5 assembly machines as part of its design.

a). What is the cost function for your plant — namely, how much would it cost to produce q engines?

The short-run production function is $q = 5(5)L = 25L$, because K is fixed at 5. This implies that for any level of output q, the number of labor teams hired will be $L = \dfrac{q}{25}$

The total cost function is thus given by the sum of the costs of capital, labor, and raw materials:

$$TC(q) = rK + wL + 2000q = (10{,}000)(5) + (5{,}000)\left(\frac{q}{25}\right) + 2{,}000\,q$$

$$TC(q) = 50{,}000 + 2200q.$$

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b). What are average and marginal costs for producing q engines?

The average cost function is then given by:

$$AC(q) = \frac{TC(q)}{q} = \frac{50{,}000 + 2200q}{q}.$$

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and the marginal cost function is given by:

$$MC(q) = \frac{dTC}{dq} = 2200.$$

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Marginal costs are constant at $2200 per engine.

c). How do average costs vary with output?

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Average costs will decrease as quantity increases because the average fixed cost of capital decreases.

TC = 50,000 + 2200q

AC = 50,000/q + 2200

= AFC ↓ + const

QUESTION FOUR.

Suppose you are the manager of a watchmaking firm operating in a competitive market. Your cost of production is given by $C = 200 + 2q^2$, where q is the level of output and C is total cost. (The marginal cost of production is 4q; the fixed cost is \$200.)

2 **a). If the price of watches is \$100, how many watches should you produce to maximize profit?**

MC = 4q

Profits are maximized where price equals marginal cost. Therefore,

FC = 200

$100 = 4q$

$100 = 4q$, or $q = 25$. Will produce 25 watches

2 **b). What will the profit level be?**

Profit is equal to total revenue minus total cost: $\pi = Pq - (200 + 2q^2)$. Thus,

$$\pi = (100)(25) - (200 + 2(25)^2) = \$1050.$$

c). At what minimum price will the firm produce a positive output?

A firm will produce in the short run if its revenues are greater than its total variable costs. The firm's short-run supply curve is its MC curve above minimum AVC. Here, $AVC = \frac{VC}{q} = \frac{2q^2}{q} = 2q$. Also, $MC = 4q$. So, MC is greater than AVC for any quantity greater than 0. This means that the firm produces in the short run as long as price is positive.

Produce when TR > TVC in the short-run

(2 Short-run supply = MC > min AVC VC = $2q^2$

AVC = $\frac{VC}{q} = \frac{2q^2}{q} = 2q$

-3 4q > 2q

So MC = 4q

AVC = 2q

∴ MC > AVC for any q > 0

The firm produces in the short-run as long as the price is positive

(5) P > 0 (P = 1)

15 pts

QUESTION FIVE.

A firm faces the following average revenue (demand) curve:

$$P = 120 - 0.02Q$$

where Q is weekly production and P is price, measured in cents per unit. The firm's cost function is given by C = 60Q + 25,000. Assume that the firm maximizes profits.

a). What is the level of production, price, and total profit per week?

The profit-maximizing output is found by setting marginal revenue equal to marginal cost. Given a linear demand curve in inverse form, $P = 120 - 0.02Q$, we know that the marginal revenue curve has the same intercept and twice the slope of the demand curve. Thus, the marginal revenue curve for the firm is $MR = 120 - 0.04Q$. Marginal cost is the slope of the total cost curve. The slope of $TC = 60Q + 25{,}000$ is 60, so MC equals 60. Setting $MR = MC$ to determine the profit-maximizing quantity:

$$120 - 0.04Q = 60, \text{ or}$$

$$Q = 1500.$$

Substituting the profit-maximizing quantity into the inverse demand function to determine the price:

$$P = 120 - (0.02)(1500) = 90 \text{ cents.}$$

Profit equals total revenue minus total cost:

$$\pi = (90)(1500) - (25{,}000 + (60)(1500)), \text{ so}$$

$$\pi = 20{,}000 \text{ cents per week, or } \$200 \text{ per week.}$$

b). If the government decides to levy a tax of 14 cents per unit on this product, what will be the new level of production, price, and profit?

Suppose initially that consumers must pay the tax to the government. Since the total price (including the tax) that consumers would be willing to pay remains unchanged, we know that the demand function is

$C = 60Q + 25,000$

$$P^* + t = 120 - 0.02Q, \text{ or}$$

$$P^* = 120 - 0.02Q - t,$$

where P^* is the price received by the suppliers and t is the tax per unit. Because the tax increases the price of each unit, total revenue for the monopolist decreases by tQ, and marginal revenue, the revenue on each additional unit, decreases by t:

$$MR = 120 - 0.04Q - t$$

where t = 14 cents. To determine the profit-maximizing level of output with the tax, equate marginal revenue with marginal cost:

(a)

$$120 - 0.04Q - 14 = 60, \text{ or}$$

$$Q = 1150 \text{ units.}$$

Substituting Q into the demand function to determine price:

(b)

$$P^* = 120 - (0.02)(1150) - 14 = 83 \text{ cents.}$$

Profit is total revenue minus total cost:

$$B = (83)(1150) - [(60)(1150) + 25,000] = 1450 \text{ cents, or}$$

(c)

$$\$14.50 \text{ per week.}$$

Note: The price facing the consumer after the imposition of the tax is 83 + 14 = 97 cents. Compared to the 90-cent price before the tax is imposed, consumers and the monopolist each pay 7 cents of the tax.

If the monopolist had to pay the tax instead of the consumer, we would arrive at the same result. The monopolist's cost function would then be

$$TC = 60Q + 25,000 + tQ = (60 + t)Q + 25,000.$$

The slope of the cost function is $(60 + t)$, so $MC = 60 + t$. We set this MC equal to the marginal revenue function from part (a):

$$120 - 0.04Q = 60 + 14, \text{ or}$$

$$Q = 1150.$$

Thus, it does not matter who sends the tax payment to the government. The burden of the tax is shared by consumers and the monopolist in exactly the same way.

QUESTION SIX.

Consider two firms facing the demand curve $P = 50 - 5Q$, where $Q = Q_1 + Q_2$. The firms' cost functions are $C_1(Q_1) = 20 + 10Q_1$ and $C_2(Q_2) = 10 + 12Q_2$.

q_1 π_1
q_2 π_2

a) What is each firm's equilibrium output and profit if they behave noncooperatively? Use the Cournot model.

In the Cournot model, Firm 1 takes Firm 2's output as given and maximizes profits. Firm 1's profit function is

$$\pi_1 = (50 - 5Q_1 - 5Q_2)Q_1 - (20 + 10Q_1), \text{ or}$$

$$\pi_1 = 40Q_1 - 5Q_1^2 - 5Q_1Q_2 - 20.$$

Setting the derivative of the profit function with respect to Q_1 to zero, we find Firm 1's reaction function:

$$\frac{\partial \pi_1}{\partial Q_1} = 40 - 10Q_1 - 5Q_2 = 0, \text{ or } Q_1 = 4 - \left(\frac{Q_2}{2}\right).$$

Similarly, Firm 2's reaction function is

$\pi_2 = (50 - 5q_1 - 5q_2)q_2 - (10 + 12q_2)$

$$Q_2 = 3.8 - \left(\frac{Q_1}{2}\right).$$

To find the Cournot equilibrium, we substitute Firm 2's reaction function into Firm 1's reaction function:

$$Q_1 = 4 - \left(\frac{1}{2}\right)\left(3.8 - \frac{Q_1}{2}\right), \ or\ Q_1 = 2.8.$$

Substituting this value for Q_1 into the reaction function for Firm 2, we find

$$Q_2 = 2.4.$$

Substituting the values for Q_1 and Q_2 into the demand function to determine the equilibrium price:

$$P = 50 - 5(2.8 + 2.4) = \$24.$$

The profits for Firms 1 and 2 are equal to

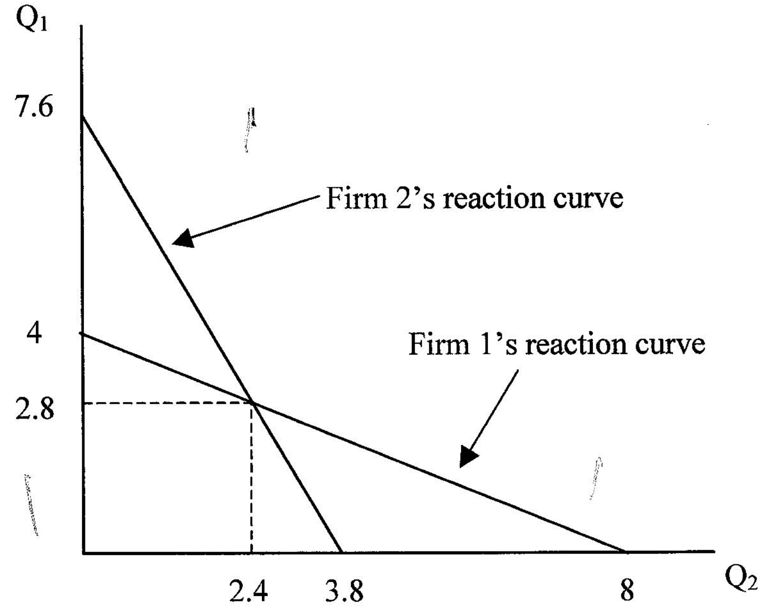
$$\pi_1 = (24)(2.8) - (20 + (10)(2.8)) = \$19.20, \text{ and}$$

$$\pi_2 = (24)(2.4) - (10 + (12)(2.4)) = \$18.80.$$

$P = 24$

b). Draw the firms' reaction curves and show the equilibrium.

The firms' reaction curves and the Cournot equilibrium are shown below.



END OF FINAL EXAMINATION