Monopoly

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Monopoly

- We'll again use a partial equilibrium setup
- Consumer i has quasilinear preferences with standard assumptions:
$$u^i\left(m^i, x^i\right) = m^i + \phi^i(x^i)$$
- A single firm produces the good x using the convex cost function
$$c(q)$$

The firm's problem

- The firm still chooses q, but not does not take price as given. Instead, takes into account the affect of q on the equilibrium price.
- Can write price as a function of $p(q)$. What is $p(q)$?

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- The firm still chooses q, but not does not take price as given. Instead, takes into account the affect of q on the equilibrium price.
- Can write price as a function of $p(q)$. What is $p(q)$?
- $p(q)$ is the inverse demand function- what prices induces an aggregate demand of q.

The firm's problem (2)

- Firm's problem:

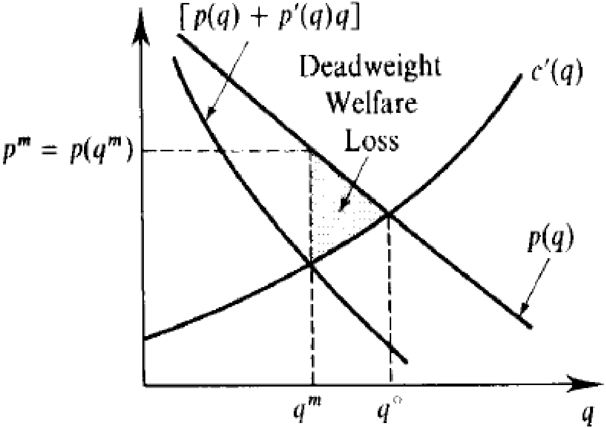
$$\max_q p(q)q - c(q)$$

- Yields FOC:

$$p'\left(q^m\right) q^m + p\left(q^m\right) = c'\left(q^m\right)$$

- LHS= marginal revenue, takes into account not just additional revenue from selling good, but also implied reduction in price.
- Relationship between q^m and $q^\star$
- $p'\left(q^m\right) q^m < 0$ given no Giffen goods with quasilinear utility. Therefore $q^m < q^\star$

Graphically



Crucially— no supply curve once we have market power!

Incidence

- We have DWL
$$\int_{q^m}^{q^o} \left[p(s) - c'(s)\right] ds$$
- Producer surplus (monopoly profits)
$$\int_0^{q^m} \left[p^m - c'(s)\right] ds$$
- Consumer surplus
$$\int_0^{q^m} [p(s) - p^m] \, ds$$

An Example

- Suppose aggregate demand is given by $p(q) = 100 - q$
- Firms cost curve: $c(q) = \frac{q^2}{2}$
- If the firm is a price taker, what is equilibrium quantity?
- If firm is price taker, what are consumer and producer surplus?
- If the firm acts as a monopolist, what is equilibrium quantity?
- If firm acts as a monopolist, what are consumer and producer surplus?

Price ceiling

- What if we mandate a price ceiling such that $p(q^m) < \bar{p}$?
- Graphically
- Price control in this case *increases* quantity and eliminates DWL (opposite of ECON 101 intuition)).

More on marginal revenue

- We have

$$MR = \frac{\partial p\left(q^m\right)}{\partial q} q^m + p\left(q^m\right)$$

- Therefore

$$MR = p\left(q^m\right)\left(1 + \frac{\partial p\left(q^m\right)}{\partial q} \frac{q^m)}{p\left(q^m\right)}\right)$$

- What is $\frac{\partial p(q^m)}{\partial q} \frac{q^m)}{p(q^m)}$?

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- What is $\frac{\partial p(q^m)}{\partial q}\frac{q^m)}{p(q^m)}$?
- Inverse elasticity of demand

$$\frac{\partial p\left(q^m\right)}{\partial q}\frac{q^m)}{p\left(q^m\right)} \equiv \frac{1}{\epsilon}\left(q^m\right)$$

Markup

- Then

$$MR = p(q^m)\left(1 + \frac{1}{\epsilon}(q^m)\right) = MC$$

- Then

$$p(q^m) = MC\left(\frac{\epsilon}{\epsilon + 1}\right)$$

- We'll call $\left(\frac{\epsilon}{\epsilon+1}\right)$ the markup
- How much the price exceeds the firm's marginal cost.
- Note in perfect competition there is no markup- we have price equals marginal cost.
- If we have a measure of marginal cost, the markup can be used as a measure of how much prices are distorted due to market power.
- Markup disappears for perfectly elastic demand.
- Would a monopolist ever produce on the inelastic part of the demand curve ($\epsilon > -1$)?

Collusion

- Note that the monopolists profit is higher than the price takers profit.
- Similarly, if firm's collude, the sum of firm's profit is higher than the sum of firm's profit if they firms act independently.
- Therefore, if all firms can collude, they can be better off.
- Why don't we see more collusion? 1. Often times illegal. 2. Strong incentives to deviate from collusion.

Cartel's problem

- Suppose there are J firms who choose to collude.
- Firm j has cost function $c^j(q^j)$.
- Cartel's problem:

$$\max_{q^1,q^2,\ldots q^J} p(\sum_j q^j)\sum_j q^j - \sum_j c^j(q^j)$$

- For each firm j:

$$p'\left(\sum_{k\neq j} q^{km} + q^{jm}\right)\sum_j q^{jm} + p\left(\sum_{k\neq j} q^{km} + q^{jm}\right) = c^{j\prime}\left(q^{jm}\right)$$

- Consider now the problem of an individual firm, given that all the other firms are playing the collusive outcome

Deviation from Collusion

- Individual Firm's problem:

$$\max_{q^j} p(\sum_{k \neq j} q^{km} + q^j)q^j - c^j(q^j)$$

- Yields FOC:

$$p' \left(\sum_{k \neq j} q^{km} + \tilde{q}^j \right) \tilde{q}^j + p \left(\sum_{k \neq j} q^{km} + \tilde{q}^j \right) = c^{j\prime} \left(\tilde{q}^j \right)$$

- Only care about own marginal revenue, not everyone else. So $\tilde{q}^j > q^{jm}$.

Deviation from Collusion (2)

- Big incentives to deviate
- Easier to enforce with smaller number of firms
- Cost of enforcing collusion is an additional efficiency cost of collusion.

Price Discrimination

- Suppose the monopolist could charge different prices to different consumers
- Also could charge different prices for different units charged by the same consumer.
- Suppose monopolist can charge whatever price it wants for each unit sold to each consumer and for each unit purchased by each consumer. This is first-degree price discrimination.
- Would choose price for each q exactly equal to each individual consumers willingness to pay for that unit. This is given by inverse demand curve $p(q)$.
- In this case, marginal revenue for each unit q is exactly given by the inverse demand curve $p(q)$.

Perfect Price Discrimination

- Firm's FOC (MR=MC):

$$p\left(q^d\right) = c'\left(q^d\right)$$

- Therefore with perfect price discrimination, we again have $q^d = q^\star$. However, the full surplus is captured as firm profit.

Examples of price discrimination

- Financial aid
- Movies (student discounts)
- Airlines
- Bulk discounts
- Coupons
- Haggling

An Example

- Suppose aggregate demand is given by $p(q) = 100 - q$
- Firms cost curve: $c(q) = \frac{q^2}{2}$
- If the firm can perfectly price discriminate, what is equilibrium quantity?
- If the firm can perfectly price discriminate, what is equilibrium quantity?
- If the firm can perfectly price discriminate, what are consumer and producer surplus?

A quick aside: LR EQM

- So far we've considered settings where the number of firms J is given as a model primitive. What pins down J in the long run?
- Firms will enter as long as profits positive and leave if profits are negative. There in a long run equilibrium, we must have profits=0.

LR EQM

- Take consumer behavior as given, summarized by aggregated demand function $x(p)$
- Suppose all firms (and potential firms) have cost function $c(q)$
- Firms that don't enter earn profit of 0.
- Partial eqm setting with 1 produced good.

Definition

A symmetric long run competitive equilibrium is a triple $(p^\star, q^\star, J^\star)$ such that

- Profit maximization: $q^\star$ solves

$$\max_{q>0} p^\star q - c(q)$$

- Market clearing

$$x(p^\star) = q^\star J^\star$$

- Free entry

$$p^\star q^\star - c(q^\star) = 0$$

Intuition

- The new condition: free entry

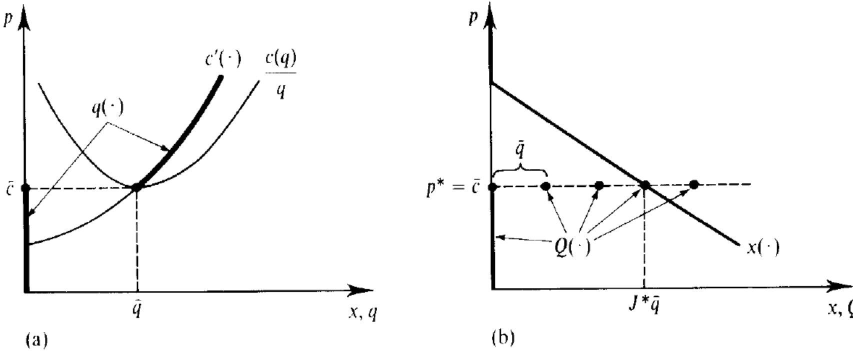
$$p^\star q^\star - c\left(q^\star\right) = 0$$

- Can write as price=average cost.

$$p^\star = \frac{c\left(q^\star\right)}{q^\star}$$

- What if firms have DRS? (only true at $q^\star = 0$ no LR eqm)
- What if firms have CRS (always true. Number of firms is indeterminte).
- Must have strictly positive minimum of AC. (Why? Need MC=AC and need upward slopping MC)
- Usually justified by fixed cost initially.

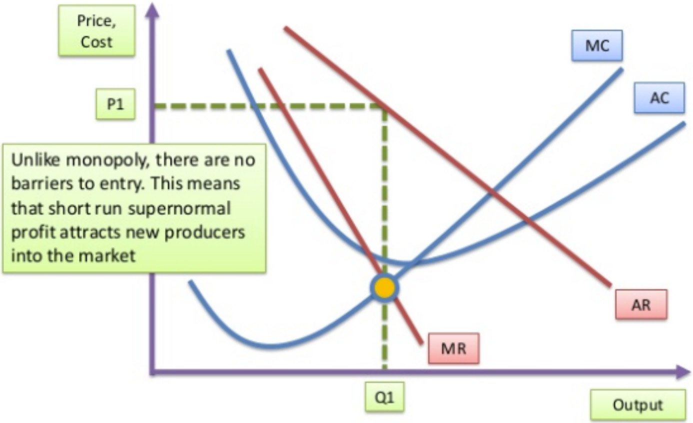
Graphically



Monopolistic Competition

- Firms set price but compete with other monopolistically competitive firms.
- Drives profits to 0.
- Graph.

Graphically



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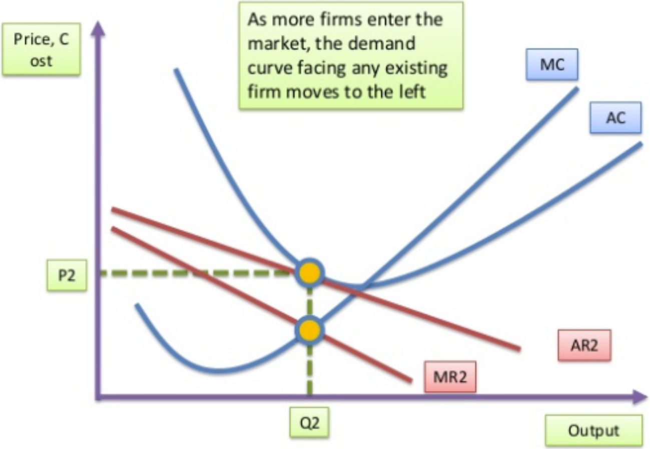

An example: Dixit Stiglitz

- A representative consumer with preferences over a continuum of varieties (goods):

$$U = \left(\int_0^n q(\omega)^\rho \, d\omega \right)^{1/\rho}, \rho \in (0,1)$$

- n is the mass of varieties
- $q(\omega)$ is consumption of variety ω
- ρ is a measure of substitutability.
- Consumer has access to infinite varieities but each variety only forms an infinitesimal fraction of budget.
- Implications- no strategic concerns between firms and budget shares go to 0.
- Budget constraint:

$$\int_0^n p(\omega) \, q(\omega) \, d\omega = I$$

Consumers problem

- Consumer's problem given by the Lagrangian:

$$\max_{q} U^{\rho} - \lambda \left(p\left(\omega\right) q\left(\omega\right) d\omega - I \right)$$

- Yields an FOC for good ω.

$$\frac{\partial \mathcal{L}}{\partial q\left(\omega\right)} = \rho q\left(\omega\right)^{\rho-1} - \lambda p\left(\omega\right) = 0$$

- Rearrange to get Frisch demand:

$$q\left(\omega\right) = \left(\frac{\lambda p\left(\omega\right)}{\rho} \right)^{\frac{1}{\rho-1}}$$

Consumers problem (2)

- Ratio of Frisch demands

$$\frac{q(\omega_1)}{q(\omega_2)} = \left(\frac{p(\omega_1)}{p(\omega_2)}\right)^{\frac{1}{\rho-1}}$$

- Define $\sigma \equiv \frac{1}{1-\rho}$, then

$$q(\omega_1) = q(\omega_2)\left(\frac{p(\omega_1)}{p(\omega_2)}\right)^{-\sigma}$$

- Multiply both sides by $p(\omega_1)$:

$$p(\omega_1)\, q(\omega_1) = q(\omega_2)\, p(\omega_1)^{1-\sigma}\, p(\omega_2)^{\sigma}$$

- Integrate over ω_1

$$\int_0^n p(\omega_1)\, q(\omega_1)\, d\omega_1 = \int_0^n q(\omega_2)\, p(\omega_1)^{1-\sigma}\, p(\omega_2)^{\sigma}\, d\omega_1$$

Consumers problem (3)

- Integrate over ω_1

$$\int_0^n p(\omega_1)\, q(\omega_1)\, d\omega_1 = \int_0^n q(\omega_2)\, p(\omega_1)^{1-\sigma}\, p(\omega_2)^{\sigma}\, d\omega_1$$

- What is LHS?

Consumers problem (3)

- Integrate over ω_1

$$\int_0^n p(\omega_1)\, q(\omega_1)\, d\omega_1 = \int_0^n q(\omega_2)\, p(\omega_1)^{1-\sigma}\, p(\omega_2)^{\sigma}\, d\omega_1$$

- What is LHS? Then

$$I = q(\omega_2)\, p(\omega_2)^{\sigma} \int_0^n p(\omega_1)^{1-\sigma}\, d\omega_1$$

- Rearrange to get demand:

$$q(\omega_2) = \frac{I p(\omega_2)^{\sigma}}{\int_0^n p(\omega_1)^{1-\sigma}\, d\omega_1}$$

A price index

- Define $P \equiv \left(\int_0^n p\left(\omega_1\right)^{1-\sigma} d\omega_1 \right)^{\frac{1}{1-\sigma}}$
- Then Marshallian demand is

$$q\left(\omega_2\right) = \frac{Ip\left(\omega_2\right)^{\sigma}}{P^{1-\sigma}} = \left(\frac{p(\omega_2)}{P}\right)^{-\sigma} \frac{I}{P}$$

- How can we write U in terms of P and I (remember $\rho = \frac{\sigma-1}{\sigma}$

$$U = \left(\int_0^n \left(\frac{p(\omega)}{P}\right)^{-\sigma} \frac{I}{P} d\omega \right)^{1/\rho}$$

$$= P^{\sigma-1} I \underbrace{\left(\int_0^n p\left(\omega\right)^{1-\sigma} d\omega \right)^{\frac{\sigma}{\sigma-1}}}_{P^{-\sigma}}$$

$$= P^{\sigma-1} I P^{-\sigma} = \frac{I}{P}$$

- P is true cost of living index, in that $e(P,u) = Pu$.

Taste for variety

- Suppose all varieties have the same price p and therefore consumed in equal amounts q, such that $I = \int_0^n pq d\omega$.
- Write U as a function of n, the number of varieties. Use $I = npq$.

Taste for variety

- Suppose all varieties have the same price p and therefore consumed in equal amounts q, such that $I = \int_0^n pq d\omega$.
- Write U as a function of n, the number of varieties. Use $I = npq$.

$$U = \left(n \left(\frac{I}{np} \right)^{\rho} \right)^{1/\rho} = n^{\frac{1-\rho}{\rho}} \frac{I}{P} = n^{\frac{1}{\sigma-1}} \frac{I}{P}$$

- Increasing in n and moreso for lower values of σ.

Firms: Monopolistic competition

- Firms are monopolistically competitive. We'll also have increasing returns to scale. Why is this possible here but not in perfect competitition (Just need MR decreases faster than MC)?
- For simplicity, we'll write the amount of labor required to to produce q units of output. Similar to a cost function but in terms of labor:

$$l(q) = f + cq$$

- l is labor demanded
- f is fixed cost of production
- c is constant marginal cost. Implies AC decreasing in q
- 1 firm per variety.

Firms:Profits

- Profits:
$$\pi = pq - wcq - wf$$
- Firm chooses the quantity to maximize profits (can choose price too):
$$\frac{\partial \pi}{\partial q} = p + \frac{\partial p}{\partial q} q = wc$$
- Interpret this equation.
- Set prices
$$p\left(1 + \frac{1}{\varepsilon}\right) = wc$$
- $\frac{\partial q}{\partial p}\frac{p}{q} = \varepsilon$ is elasticity of demand with respect to price of of the variety.

Demand elasticity

- Recall that demand for a given variety is given by:

$$q = \left(\frac{p}{P}\right)^{-\sigma} \frac{I}{P}$$

- Solve for elasticity of demand. Note that change in price of variety has no effect on price index because there are a continuum of firms. (treat P as constant)

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- $$\frac{\partial q}{\partial p} = -\sigma p^{-\sigma-1} P^{\sigma-1} I$$

$$\frac{\partial q}{\partial p}\frac{p}{q} = -\sigma$$

- Constant elasticity of demand!

Firms:Pricing

- Recall
$$p\left(1+\frac{1}{\varepsilon}\right) = wc$$
- Therefore
$$p = wc\left(\frac{\sigma}{\sigma-1}\right)$$
- Optimal pricing is proportional mark-up.

Free -entry

- To find LR quantities, use 0 profit condition.
- $$\pi = pq - wcq - wf = 0$$
- Plugging in $p = wc\left(\frac{\sigma}{\sigma-1}\right)$
- $$wc\left(\frac{\sigma}{\sigma-1}\right)q - wcq = wf$$
- $$qc\left(\frac{\sigma}{\sigma-1} - 1\right) = f$$
- $$q = \frac{f}{c}(\sigma - 1)$$
- Intuition- higher f - fewer firms, more output per firm. Higher c - higher marginal cost, lower output per firm. Lower σ- less elastic demand, produce less because increasing q decreases price more.

Dixit Stiglitz

- Trade– traditional (Ricardian) models predict that firms should export goods they have comparative advantage in. In reality, lots of trade within industries. US both imports and exports cars.
- Further, lots of trading within countries with similar endowments.
- Melitz (2003) using Dixit Stiglitz says they are trading varieties within industry.
- Urban/economic geography/macro- provides a rational for agglomeration effects/ amplification effects because of IRS. Example, a small population increase leads to influx of firms, which because of love of variety, makes a city even more desirable. Can help to explain the existence of cities.