

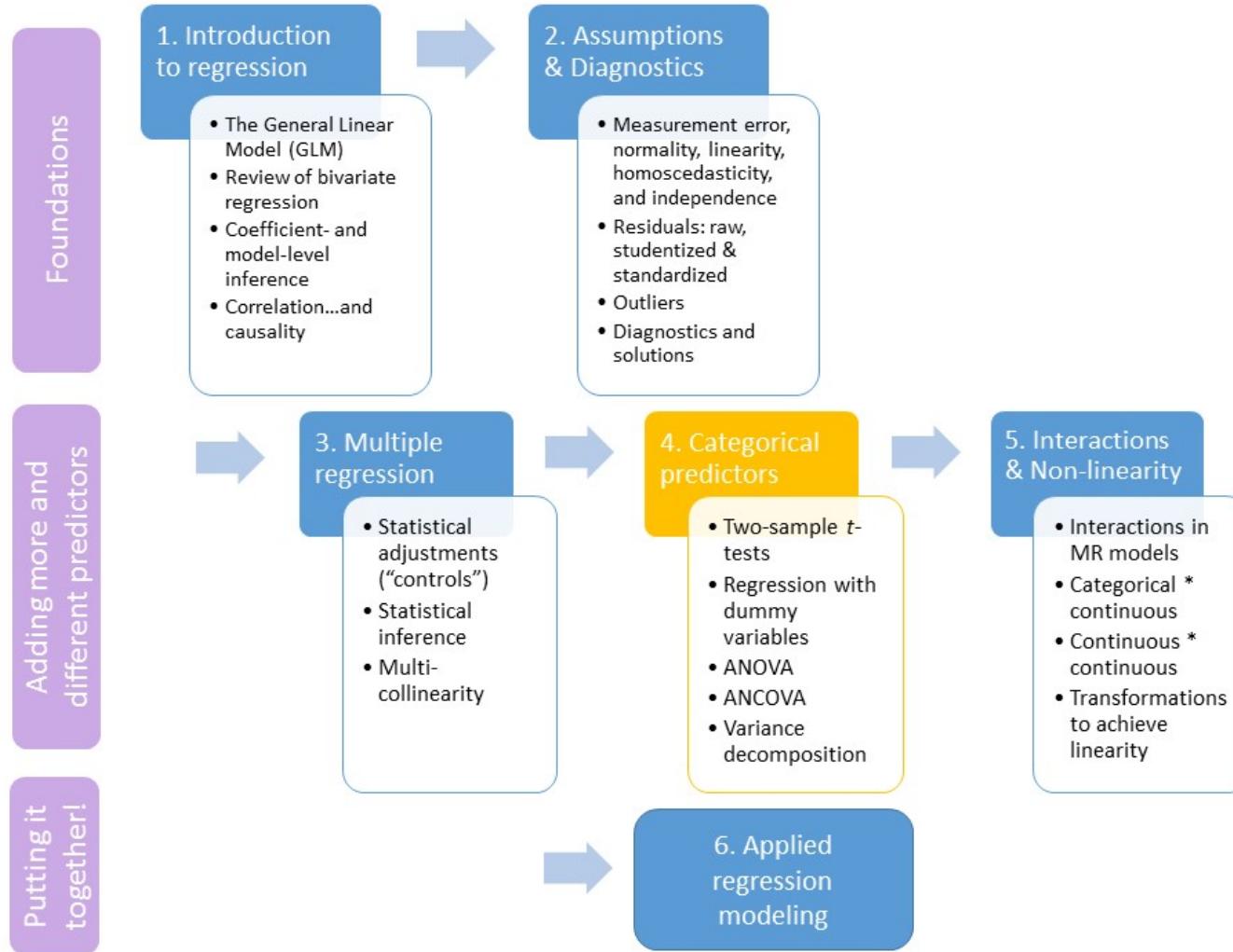
# Categorical predictors and ANOVA/ANCOVA

EDUC 643: Unit 4

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# Roadmap



# Unit goals

- Describe the relationship between dichotomous and polychotomous variables and convert variables between these forms, as necessary
- Conduct a two-sample  $t$ -test
- Describe the relationship between a two-sample  $t$ -test and regressing a continuous outcome on a dichotomous predictor
- Estimate a regression with one dummy variable as a predictor and interpret the results (including when the reference category changes)
- Estimate a multiple regression model with several continuous and dummy variables and interpret the results
- Estimate an ANOVA model and interpret the within- and between-group variance
  - Do the same for an ANCOVA model, adjusting for additional continuous predictors
- Describe the similarities and differences of Ordinary-Least Squares regression analysis and ANOVA/ANCOVA, and when one would prefer one approach to another
- Describe potential Type I error problems that arise from multiple group comparisons and potential solutions to these problems, including theory, pre-registration, ANOVA and *post-hoc* corrections
- Describe the relationship between different modeling approaches with the General Linear Model family

# Categorical variables

# Categorical variables

So far, we have only looked at General Linear Models (and their associated OLS regression estimating equations) involving **continuous predictors**. But what about **categorical predictors**?

What are categorical predictors?

- Categorical predictors are *predictors in statistical models whose values denote categories*. Of course, this begs the question...

# Categorical predictors

Important distinctions and conventions:

## Nominal predictors

- These have *unordered* values
- E.g., gender, religion, political party

## Ordinal predictors

- These have *ordered* values
- E.g., grade, developmental stage, education level (?)

Another important distinction: **dichotomies** (only 2 categories) vs. **polychotomies** (>2 categories)

# Our (new!) motivating question

A team of researchers based at the [University of Oregon](#) aimed to understand the effects of the COVID-19 pandemic on students' early literacy skills.<sup>1</sup>



Ann Swindells Professor in Special Education [Gina Biancarosa](#), former UO doctoral students David Fainstein, Chris Ives, and Dave Furjanic, along with CTL Research Manager Patrick Kennedy, used data from assessments of 471,456 students across 1,684 schools on the Dynamic Indicators of Basic Early Literacy Skills (DIBELS) to analyze the extent to which students' Oral Reading Fluency (ORF) scores differed across four waves of DIBELS assessment prior-to and during the pandemic.

Their study is forthcoming at the *Elementary School Journal*.

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[1] For various reasons, the pandemic is a "lousy natural experiment" for examining the effects of a particular policy response (e.g. virtual schooling). However, it is quite possible to seek to understand its global effects via just the type of analysis Furjanic et al. conducted.

# Our data

```
str(dibels)
```

```
## 'data.frame': 5396 obs. of 18 variables:  
## $ sch_deid : int 10001 10001 10001 10002 10002 10002 10003 10003 10003 ...  
## $ grade : int 1 2 3 1 2 3 1 2 3 1 ...  
## $ y1_boy_mean : num 30.6 71.3 102.9 34.1 79.5 ...  
## $ y1_moy_mean : num 51.8 105.7 132.6 62.3 118.1 ...  
## $ y2_boy_mean : num 26 71.1 90.5 32.6 68.1 ...  
## $ y2_moy_mean : num 46.5 97.8 111.2 50.9 98.6 ...  
## $ st : chr "AL" "AL" "AL" "AL" ...  
## $ school_magnet: chr "No" "No" "No" "No" ...  
## $ school_titlei: chr "Title I targeted assistance school" "Title I targete...  
## $ tr_ts : int 100 105 107 96 82 78 74 92 73 124 ...  
## $ school_enroll: int 312 312 312 256 256 256 239 239 239 418 ...  
## $ frpl_prop : num 0.442 0.442 0.442 0.449 0.449 ...  
## $ pre : num 41.2 88.5 117.8 48.2 98.8 ...  
## $ post : num 36.2 84.4 100.9 41.7 83.3 ...  
## $ asian_prop : num 0.06 0.1238 0.0841 0.1042 0.061 ...  
## $ black_prop : num 0.14 0.0857 0.1402 0.125 0.2439 ...  
## $ hisp_prop : num 0.09 0.0571 0.0561 0.0938 0.061 ...  
## $ white_prop : num 0.61 0.686 0.654 0.583 0.573 ...
```

# How similar is our data to Furjanic?

```
# How many unique schools are represented?  
length(unique(dibels$sch_deid))
```

```
## [1] 1527
```

```
# How many total students contribute test-scores?  
sum(dibels$tr_ts)
```

```
## [1] 396188
```

# Mean comparison

```
mean(dibels$pre)
```

```
## [1] 75.12461
```

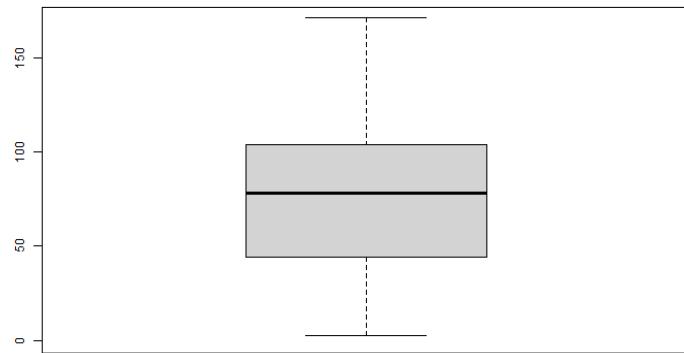
```
mean(dibels$post)
```

```
## [1] 70.67498
```

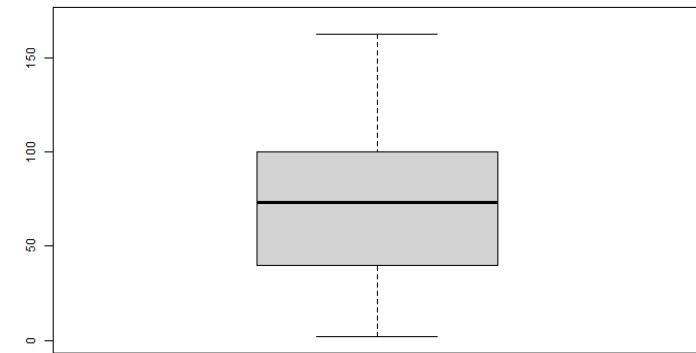
Means are 4.5 words per-minute apart.

# Understanding the distributions

Pre-pandemic

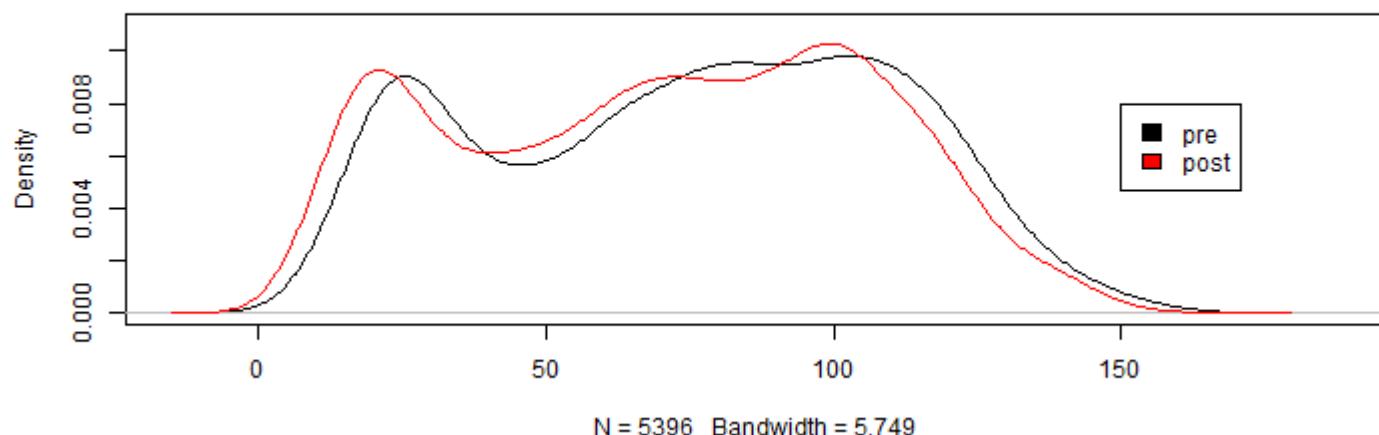


Post-pandemic onset



# Understanding the distributions

```
plot(density(dibels$pre), main=" ", sub=NULL, ylim=range(0,0.011))
lines(density(dibels$post), col="red")
legend(150, .008, legend=c("pre", "post"), fill=c("black", "red"))
```



But, as you may by now have anticipated, we are interested in knowing how likely we are to have gotten such a difference by idiosyncrasy of sampling from a population of school-grades in which there was no difference. Fortunately, we have just such a tool in our toolbox already. **What will the (asymptotic) distribution of means of repeatedly drawn samples from a given population be?**

# Two-sample $t$ -test

```
t.test(dibels$pre, dibels$post)
```

```
##  
##      Welch Two Sample t-test  
##  
## data:  dibels$pre and dibels$post  
## t = 6.4962, df = 10790, p-value = 8.602e-11  
## alternative hypothesis: true difference in means is not equal to 0  
## 95 percent confidence interval:  
##  3.106978 5.792286  
## sample estimates:  
## mean of x mean of y  
## 75.12461 70.67498
```

# I'm allowing in this  $t$ -test for the possibility that my sample in  
# each group is of different sizes and has different variance.  
# These assumptions affect the precision of my estimates. In some  
# settings, particularly experimental ones, I can impose stricter  
# assumptions and get more precise estimates.

# Our old friend

We can now answer a lingering question from last term, and avoid having to make some torturous assumptions about what the "true" population mean is:

```
t.test(who$life_expectancy[who$status=="Developing"],  
       who$life_expectancy[who$status=="Developed"])  
  
##  
##      Welch Two Sample t-test  
##  
## data: who$life_expectancy[who$status == "Developing"] and who$life_expectancy[who$status == "Developed"]  
## t = -12.854, df = 103.88, p-value < 2.2e-16  
## alternative hypothesis: true difference in means is not equal to 0  
## 95 percent confidence interval:  
## -12.78858 -9.36995  
## sample estimates:  
## mean of x mean of y  
## 69.70199 80.78125
```

```
# The square braces [] allow me to subset my data  
# by the boolean operations within them
```

# Waves of data

Show 9 ▾ entries		Search:					
		sch_deid	grade	y1_boy_mean	y1_moy_mean	pre	post
1		10001	1	30.6	51.8	41.2	36.2
2		10001	2	71.3	105.7	88.5	84.4
3		10001	3	102.9	132.6	117.8	100.9
4		10002	1	34.1	62.3	48.2	41.7
5		10002	2	79.5	118.1	98.8	83.3
6		10002	3	95.7	127.4	111.5	105.8
7		10003	1	39.4	61.0	50.2	36.5
8		10003	2	86.3	122.3	104.3	88.6
9		10003	3	103.9	134.4	119.2	116.3

Showing 1 to 9 of 5,396 entries

Previous

1

2

3

4

5

...

600

Next

# Waves of data

Show 6 entries		Search:					
		sch_deid	grade	y1_boy_mean	y1_moy_mean	pre	post
1	10001	1		30.6		51.8	41.2
2	10001		2		71.3	105.7	88.5
3	10001		3	102.9		132.6	117.8
4	10002		1		34.1	62.3	48.2
5	10002		2	79.5		118.1	98.8
6	10002		3	95.7		127.4	105.8

Showing 1 to 6 of 5,396 entries

Previous 1 2 3 4 5 ... 900 Next

I actually have the same outcome stored across multiple variables. **What is it?** This is a classic example of a phenomenon you will come to know and hate: the curse of **wide** and **long** data structures.

# Wide and long data

Various types of analyses will necessitate different data structures:

## Long to wide

Country	Year	Mobile
A	1990	1
B	2000	80
A	2000	100
B	1990	2

Wide data

Country	1990	2000
A	1	100
B	2	80



## Wide to long

Wide data

Country	1990	2000
A	1	100
B	2	80



Tidy/long data

Country	Year	Mobile
A	1990	1
B	2000	80
A	2000	100
B	1990	2

Understanding exactly how to do this will take repeated time and practice, and you will nearly always need to look up and remind yourself how to do it. Bookmark and get familiar with this vignette: <https://tidyverse.org/articles/pivot.html>!

I'm going to show you the code to reshape your data (here from wide to long), but you don't need to be able to do this for assignments in this class. You should aim to understand substantively what is going on.

# Pivot longer

```
dibels_long <- dibels %>%
  pivot_longer(
    cols = c("y1_boy_mean", "y1_moy_mean",
            "y2_boy_mean", "y2_moy_mean"),
    names_to = "period",
    names_pattern = "(.*)_mean",
    values_to = "mean_orf")
dibels_long$period <- factor(dibels_long$period)
```

# Long DIBELS

Show 9 entries

Search:

	sch_deid	grade	period	mean_orf	pre	post
1	10001	1	y1_boy	30.6	41.2	36.2
2	10001	1	y1_moy	51.8	41.2	36.2
3	10001	1	y2_boy	26.0	41.2	36.2
4	10001	1	y2_moy	46.5	41.2	36.2
5	10001	2	y1_boy	71.3	88.5	84.4
6	10001	2	y1_moy	105.7	88.5	84.4
7	10001	2	y2_boy	71.1	88.5	84.4
8	10001	2	y2_moy	97.8	88.5	84.4
9	10001	3	y1_boy	102.9	117.8	100.9

Showing 1 to 9 of 21,584 entries

Previous

1

2

3

4

5

...

2,399

Next

# Dummy coding

The mean values for pre-pandemic and post-pandemic onset are no longer helpful:

```
dibels_long <- select(dibels_long, -c(pre, post))
```

But it will be helpful for us to be able to designate which observations refer to a time period before the pandemic, and which refer to a time period post-onset:

```
dibels_long <- mutate(dibels_long,
                      post = ifelse(period=="y1_boy" |
                                    period=="y1_moy", 0, 1))
```

# Dummy variables

Dummy (or indicator variables) distinguish between categories, but offer no meaningful quantitative information *on their own*.

By convention, the variable name corresponds to the category given by the value == 1, e.g.:

$post = 1$  if after pandemic onset

$post = 0$  if pre-pandemic

The category given the value 0 is called the **reference category**.

Good data management practice: call the categorical variable the value implied by its substantive meaning when equal to 1 (i.e., "post" rather than "pandemic"; "treat" rather than "condition") so that you are clear on what 0 and 1 represent.

# Polychotomies

In fact, dummy coding will prove essential for categorical variables with more than two categories as well, especially those that are nominal (i.e., unordered):

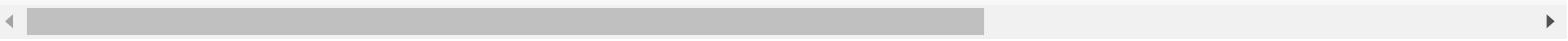
```
table(dibels_long$school_titlei)

##
##                                     Missing
##                                         752
## Not a Title I school
##                                         2752
## Title I schoolwide eligible-Title I targeted assistance program
##                                         1124
## Title I schoolwide eligible school-No program
##                                         120
## Title I schoolwide school
##                                         14712
## Title I targeted assistance eligible school-No program
##                                         284
## Title I targeted assistance school
##                                         1840
```

# Polychotomies

Here we have seven different levels of a school's Title I status. We can probably simplify these, but we need to be able to represent them using numerical values, when these levels don't inherently have a numerical structure. So... we use **dummy coding**. First, let's simplify the categories:

```
dibels_long <- dibels_long %>%
  mutate(title1 = case_when(school_titlei=="Missing" ~ "Missing",
                            school_titlei=="Not a Title I school" ~
                            school_titlei=="Title I schoolwide eligi-
                            school_titlei=="Title I schoolwide eligi-
                            school_titlei=="Title I schoolwide schoo-
                            school_titlei=="Title I targeted assista-
                            school_titlei=="Title I targeted assista
table(dibels_long$title1, exclude=NULL)
```



	Missing	Not Title I	Title I schoolwide	Title I targeted
##	752	2752	15956	2124

# Dummy coding

The most common process for representing categorical variables in regression is dummy coding.

- Dummy coding essentially creates a new (dummy-coded) variable for each level.

School Status	D1	D2	D3
Not Title I	0	0	0
Title I schoolwide	1	0	0
Title I targeted	0	1	0
Missing	0	0	1

- One group becomes the reference group (in this case "Not Title I").
- The dummy-coded variables are then coded "1" for their corresponding level, and 0 for all other levels.

# Dummy coding

In a sample dataset, we could conceive of the dummy coding scheme like this:

School	Title I status	D1 (Schoolwide)	D2 (Targeted)	...
10001	Not Title I	0	0	...
10002	Title I schoolwide	1	0	...
10003	Title I targeted	0	1	...
10004	Missing	0	0	...
10005	Title I schoolwide	1	0	

Since "Not Title I" is our reference, we don't create a new column for it (it's implied by 0s in all other groups).

Hence, for  $K$  categories in our original variable, we have  $K - 1$  dummy-coded variables.

# Dummies in R

From our polychotomous categorical variable, we can hand-create dummies:

```
dibels_long <- dibels_long %>%  
  mutate(title1_school = ifelse(title1=="Title I schoolwide", 1, 0)) %>%  
  mutate(title1_target = ifelse(title1=="Title I targeted", 1, 0)) %>%  
  mutate(title1_miss = ifelse(title1=="Missing", 1, 0))
```

But, R is actually really smart, so the most straightforward way is to turn our original variable into a factor and then let R automatically convert it into a series of dummies when we need them:

```
dibels_long$title1 <- factor(dibels_long$title1)
```

# Categorical predictors in regression

# Categorical predictors in regression

In our standard multiple regression model, we have noted that we've made several important assumptions about our outcome ( $Y_i$ ) and residuals ( $\varepsilon_i$ ):

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \dots + \beta_k X_{ki} + \varepsilon_i$$

but, we haven't made any particular assumptions about the form of the  $X$ s. In fact, regression models can easily accommodate categorical variables (both dichotomous and polychotomous)!

# Pre/post in regression

We can now estimate whether there was a difference in ORF scores pre- and post-pandemic onset in regression:

```
fit1 <- lm(mean_orf ~ post, data=dibels_long)
summary(fit1)

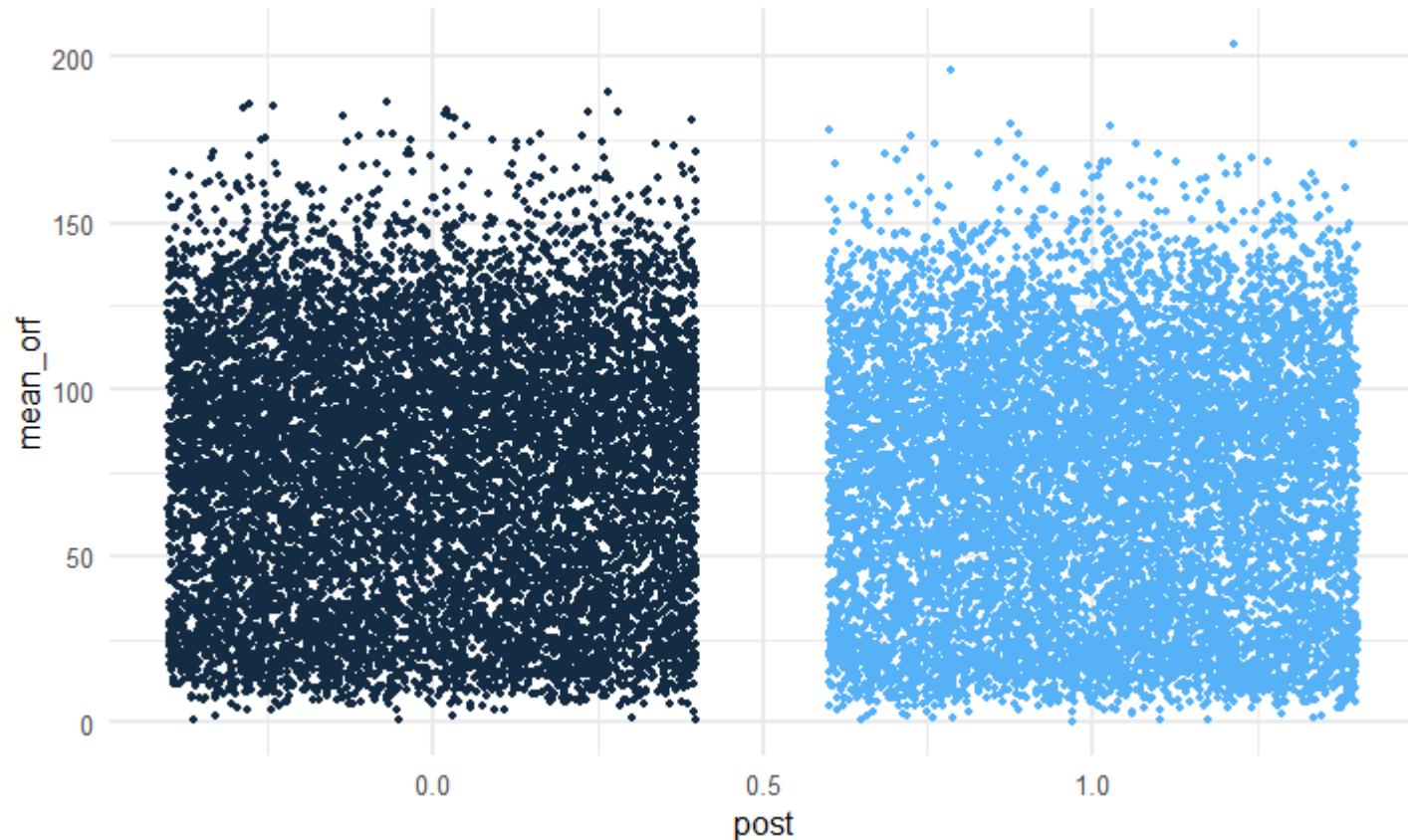
##
## Call:
## lm(formula = mean_orf ~ post, data = dibels_long)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -74.306 -33.112   1.504  28.995 133.325
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 75.1246    0.3665 204.958  <2e-16 ***
## post        -4.4496    0.5184  -8.584  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 38.08 on 21582 degrees of freedom
## Multiple R-squared:  0.002403   Adjusted R-squared:  0.002356
```

# Pre/post in regression

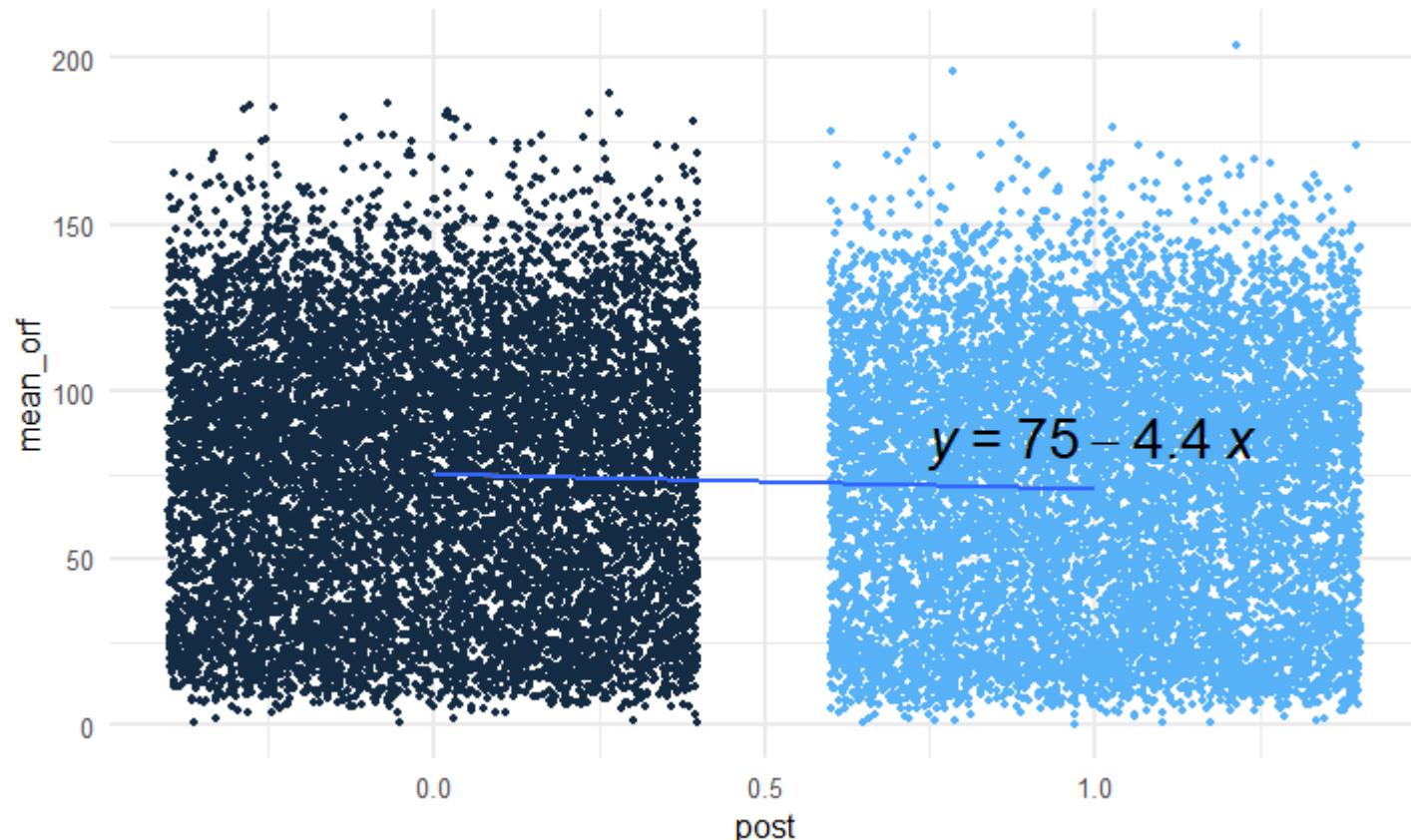
```
##  
## Call:  
## lm(formula = mean_orf ~ post, data = dibels_long)  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max  
## -74.306 -33.112    1.504  28.995 133.325  
##  
## Coefficients:  
##                 Estimate Std. Error t value Pr(>|t|)  
## (Intercept)  75.1246     0.3665 204.958 <2e-16 ***  
## post        -4.4496     0.5184 -8.584 <2e-16 ***  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 38.08 on 21582 degrees of freedom  
## Multiple R-squared:  0.003403, Adjusted R-squared:  0.003356  
## F-statistic: 73.69 on 1 and 21582 DF, p-value: < 2.2e-16
```

Our point estimate is identical to our original two-sample *t*-test, though our inference has changed slightly.

# Regression w. categorical predictors



# Regression w. categorical predictors



**y-intercept:** estimated value of  $Y$  when dichotomous predictor=0

**slope:** estimated difference in  $Y$  between categories of predictor

# Reference category

What happens if we change the reference category?

```
# Create a new dummy variable called "pre", coded  
# as one if test is prior to pandemic onset  
dibels_long <- mutate(dibels_long,  
                      pre = ifelse(period=="y1_boy" |  
                                   period=="y1_moy", 1, 0))  
  
# Fit the model  
fit2 <- lm(mean_orf ~ pre, data=dibels_long)
```

# Reference category

What happens if we change the reference category?

```
...
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|) 
## (Intercept) 70.6750   0.3665 192.819 <2e-16 ***
## pre         4.4496   0.5184   8.584 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 38.08 on 21582 degrees of freedom
## Multiple R-squared:  0.003403, Adjusted R-squared:  0.003356 
## F-statistic: 73.69 on 1 and 21582 DF,  p-value: < 2.2e-16
...
```

- Sign of slope is reversed
- Y-intercept is value of new reference category
- SE and inference remain exact same
- Full model statistics are the same

# What about the waves?

Up until now, we've focused on a simple comparison of pre- and post-pandemic onset scores. But this glosses over the facts that:

- Students typically improve substantially over the course of the year (we're lumping these time points together)
- We aren't able to capture the dynamic ways in which performance may have evolved over the early parts of the pandemic

We can use our multiple wave collection (now captured in our categorical polychotomous variable *period*) to address this.

```
table(dibels_long$period, exclude=NULL)  
  
##  
## y1_boy y1_moy y2_boy y2_moy  
##   5396    5396    5396    5396
```

# Categorical predictors

## Nominal predictors

- These have *unordered* values
- E.g., gender, religion, political party, state of residence
- **NEVER include a nominal predictor directly in a regression model**
  - You end up with the problem of "country-ness" as a predictor

## Ordinal predictors

- These have *ordered* values
- E.g., grade, developmental stage, education level (?)
- **CAN include an ordinal predictor directly in regression, but make sure this is what you want!**
  - Should you convert a political view scale (1=progressive, 2=liberal, 3=moderate, 4=conservative, 5=right-wing) to a series of dummies?
  - What about education (1=HS dropout, 2=HS grad, 3=some college, 4=college grad)?

# Polychotomies in regression

In a regression model, categorical predictors are typically entered in their dummy-coded format:

$$Y = \beta_0 + \beta_1 D_2 + \beta_2 D_3 + \beta_3 D_3 + \dots + \varepsilon$$

In our four-wave ORF regression, we can think of the equation like this:<sup>1</sup>

$$MEAN\_ORF_j = \beta_0 + \beta_1 Y1\_MOY_j + \beta_2 Y2\_BOY_j + \beta_3 Y2\_MOY_j + \varepsilon_j$$

where did *Y1\_BOY* (year 1, beginning of year) go?

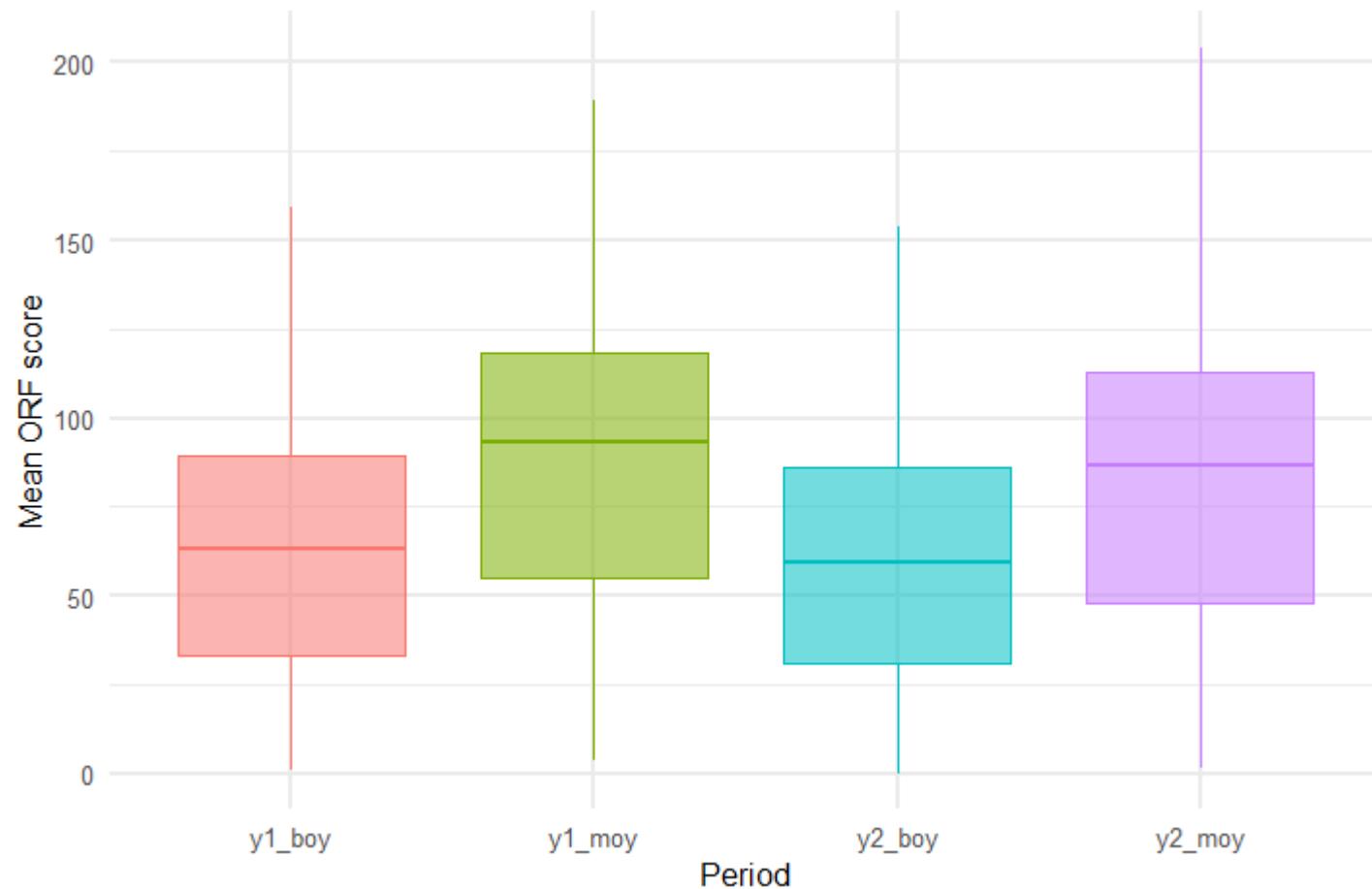
How should we interpret each of the coefficients?

How should we interpret the intercept?

---

[1] Following convention, I'm subscripting this equation with "*j*" because our data does not represent individual observations (which we typically subscript with "*i*"), but mean values, aggregate at the school-grade level.

# Why does this work?



# Interpreting coefficients

...

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	62.3436	0.4921	126.701	< 2e-16 ***
periody1_moy	25.5620	0.6959	36.734	< 2e-16 ***
periody2_boy	-2.7914	0.6959	-4.011	6.06e-05 ***
periody2_moy	19.4541	0.6959	27.957	< 2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 36.15 on 21580 degrees of freedom

...

On average, when measured in Fall 2019, grades in schools had a mean ORF score of 62.3

# Interpreting coefficients

...

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	62.3436	0.4921	126.701	< 2e-16 ***
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---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 36.15 on 21580 degrees of freedom

...

On average, when measured in Fall 2019, grades in schools had a mean ORF score of 62.3

On average, when measured in Winter 2020, grades in schools had a mean ORF score of 87.9 (62.3 + 25.6)

# Interpreting coefficients

...

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	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	62.3436	0.4921	126.701	< 2e-16 ***
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---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 36.15 on 21580 degrees of freedom

...

On average, when measured in Fall 2019, grades in schools had a mean ORF score of 62.3

On average, when measured in Winter 2020, grades in schools had a mean ORF score of 87.9 (62.3 + 25.6)

On average, when measured in Fall 2020, grades in schools had a mean ORF score of 59.6 (62.3 + (-2.8))

# Interpreting coefficients

...

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	62.3436	0.4921	126.701	< 2e-16 ***
periody1_moy	25.5620	0.6959	36.734	< 2e-16 ***
periody2_boy	-2.7914	0.6959	-4.011	6.06e-05 ***
periody2_moy	19.4541	0.6959	27.957	< 2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 36.15 on 21580 degrees of freedom

...

On average, when measured in Fall 2019, grades in schools had a mean ORF score of 62.3

On average, when measured in Winter 2020, grades in schools had a mean ORF score of 87.9 (62.3 + 25.6)

On average, when measured in Fall 2020, grades in schools had a mean ORF score of 59.6 (62.3 + (-2.8))

On average, when measured in Winter 2021, grades in schools had a mean ORF score of 81.79 (62.3 + 19.5)

# Interpreting coefficient significance

Coefficient significance tests still test the null hypothesis  $\beta_k = 0$ , but **we are testing against the reference group** implicit in our intercept.

...

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	62.3436	0.4921	126.701	< 2e-16 ***
periody1_moy	25.5620	0.6959	36.734	< 2e-16 ***
periody2_boy	-2.7914	0.6959	-4.011	6.06e-05 ***
periody2_moy	19.4541	0.6959	27.957	< 2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

...

Which DIBELS test wave differs significantly from our reference group: "Y1\_BOY"?

So, this is just a comparison of means, or a series of independent-sample *t*-tests!

# Changing reference category

If we change the model's reference category with a polychotomous variable, we *will* change the parameter estimates and associated tests. Each refers to the estimated mean difference for that group and the reference category. There can be significant variation from one group (e.g., time period) to another, but not all groups are different from each other.

```
# I can specify directly in my call which group to serve as reference
summary(lm(mean_orf ~ relevel(period, ref="y2_boy"), data=dibels_long))

...
## Coefficients:
##                                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)                  59.5522   0.4921 121.028 < 2e-16 *
## relevel(period, ref = "y2_boy")y1_boy    2.7914   0.6959   4.011 6.06e-05 *
## relevel(period, ref = "y2_boy")y1_moy   28.3534   0.6959   40.745 < 2e-16 *
## relevel(period, ref = "y2_boy")y2_moy   22.2455   0.6959   31.968 < 2e-16 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 36.15 on 21580 degrees of freedom
## Multiple R-squared:  0.1021,    Adjusted R-squared:  0.1019
## F-statistic: 817.7 on 3 and 21580 DF,  p-value: < 2.2e-16
...
```

# Prediction with categorical variables

Using the coefficients from our output, we have the following fitted regression equation:

$$MEAN_{j} \hat{ORF}_j = 62.3 + 25.6(Y1\_MOY_j) + (-2.8)(Y2\_BOY)_j + 19.6(Y2\_MOY_j)$$

What is the predicted ORF for school grades in the middle of the 2020–21 school year?

$$MEAN_{j} \hat{ORF}_j = 62.3 + 25.6(0) + (-2.8)(0) + 19.6(1) = 62.3 + 19.6 = 81.9$$

For dummy coded variables, we just add the appropriate effects for the group we are interested in, or omit them if they are in our reference group.

# So many tests

Reference group	Y1_BOY	Y1_MOY	Y2_BOY	Y2_MOY
Y1_BOY		1	2	3
Y1_MOY	4		5	6
Y2_BOY	7	8		9
Y2_MOY	10	11	12	

**DANGER:** we're back in the land of multiple hypothesis testing, and we may be inadvertently committing **Type I error!**

# Dangers of multiple hypothesis tests

If your goal is to find a "statistically significant" result, you will detect such a relationship 1 out of 20 times (on average).

Imagine rolling a die. What is the probability you roll a 1?  $1/6 = 0.167$

Now, roll it twice, what is the probability at least one of your rolls is a 1?

$$1 - (5/6 * 5/6) = 0.306$$

If you conduct enough tests, you'll detect a relationship eventually.

# Multiple tests in the wild

- How many tests?

$$\frac{(n \text{ categories})(n \text{ categories} - 1)}{2}$$

- ~ 80 counties, so 3160 tests

Table 1.4.2<sup>(1/2)</sup> Comparing countries' and economies' performance in mathematics

Mean score	Comparison country/economy	Countries and economies whose mean score is not statistically significantly different from the comparison country's/economy's score
591	B-S-J-Z (China)	
569	Singapore	
558	Macao (China)	Hong Kong (China) <sup>1</sup>
551	Hong Kong (China) <sup>1</sup>	Macao (China)
531	Chinese Taipei	Japan, Korea
527	Japan	Chinese Taipei, Korea, Estonia
526	Korea	Chinese Taipei, Japan, Estonia, Netherlands <sup>1</sup>
523	Estonia	Japan, Korea, Netherlands <sup>1</sup>
519	Netherlands <sup>1</sup>	Korea, Estonia, Poland, Switzerland
516	Poland	Netherlands <sup>1</sup> , Switzerland, Canada
515	Switzerland	Netherlands <sup>1</sup> , Poland, Canada, Denmark
512	Canada	Poland, Switzerland, Denmark, Slovenia, Belgium, Finland
509	Denmark	Switzerland, Canada, Slovenia, Belgium, Finland
509	Slovenia	Canada, Denmark, Belgium, Finland
508	Belgium	Canada, Denmark, Slovenia, Finland, Sweden, United Kingdom
507	Finland	Canada, Denmark, Slovenia, Belgium, Sweden, United Kingdom
502	Sweden	Belgium, Finland, United Kingdom, Norway, Germany, Ireland, Czech Republic, Austria, Latvia
502	United Kingdom	Belgium, Finland, Sweden, Norway, Germany, Ireland, Czech Republic, Austria, Latvia, France
501	Norway	Sweden, United Kingdom, Germany, Ireland, Czech Republic, Austria, Latvia, France, Iceland
500	Germany	Sweden, United Kingdom, Norway, Ireland, Czech Republic, Austria, Latvia, France, Iceland, New Zealand
500	Ireland	Sweden, United Kingdom, Norway, Germany, Ireland, Austria, Latvia, France, Iceland, New Zealand, Portugal <sup>1</sup>
499	Czech Republic	Sweden, United Kingdom, Norway, Germany, Ireland, Austria, Latvia, France, Iceland, New Zealand, Portugal <sup>1</sup>
499	Austria	Sweden, United Kingdom, Norway, Germany, Ireland, Czech Republic, Latvia, France, Iceland, New Zealand, Portugal <sup>1</sup>
496	Latvia	Sweden, United Kingdom, Norway, Germany, Ireland, Czech Republic, Austria, France, Iceland, New Zealand, Portugal <sup>1</sup> , Australia
495	France	United Kingdom, Norway, Germany, Ireland, Czech Republic, Austria, Latvia, Iceland, New Zealand, Portugal <sup>1</sup> , Australia
494	New Zealand	Norway, Germany, Ireland, Czech Republic, Austria, Latvia, France, Iceland, Portugal <sup>1</sup> , Australia
492	Portugal <sup>1</sup>	Czech Republic, Austria, Latvia, France, Iceland, New Zealand, Australia, Russia, Italy, Slovak Republic
491	Australia	Latvia, France, Iceland, New Zealand, Portugal <sup>1</sup> , Russia, Italy, Slovak Republic
488	Russia	Portugal <sup>1</sup> , Australia, Italy, Slovak Republic, Luxembourg, Spain, Lithuania, Hungary
487	Italy	Portugal <sup>1</sup> , Australia, Russia, Slovak Republic, Luxembourg, Spain, Lithuania, Hungary, United States <sup>1</sup>
486	Slovak Republic	Portugal <sup>1</sup> , Australia, Russia, Italy, Luxembourg, Spain, Lithuania, Hungary, United States <sup>1</sup>
483	Luxembourg	Russia, Italy, Slovak Republic, Spain, Lithuania, Hungary, United States <sup>1</sup>
481	Spain	Russia, Italy, Slovak Republic, Luxembourg, Lithuania, Hungary, United States <sup>1</sup>
481	Lithuania	Russia, Italy, Slovak Republic, Luxembourg, Spain, Hungary, United States <sup>1</sup>
481	Hungary	Russia, Italy, Slovak Republic, Luxembourg, Spain, Lithuania, United States <sup>1</sup>
478	United States <sup>1</sup>	Italy, Slovak Republic, Luxembourg, Spain, Lithuania, Hungary, Belarus, Malta
472	Belarus	United States <sup>1</sup> , Malta
472	Malta	United States <sup>1</sup> , Belarus
464	Croatia	Israel
463	Israel	Croatia
454	Turkey	Ukraine, Greece, Cyprus, Serbia
453	Ukraine	Turkey, Greece, Cyprus, Serbia
451	Greece	Turkey, Ukraine, Cyprus, Serbia
451	Cyprus	Turkey, Ukraine, Greece, Serbia
448	Serbia	Turkey, Ukraine, Greece, Cyprus, Malaysia
440	Malaysia	Serbia, Albania, Bulgaria, United Arab Emirates, Romania
437	Albania	Malaysia, Bulgaria, United Arab Emirates, Romania
436	Bulgaria	Malaysia, Albania, United Arab Emirates, Brunei Darussalam, Romania, Montenegro
435	United Arab Emirates	Malaysia, Albania, Bulgaria, Romania
430	Brunei Darussalam	Bulgaria, Romania, Montenegro
430	Romania	Malaysia, Albania, Bulgaria, United Arab Emirates, Brunei Darussalam, Montenegro, Kazakhstan, Moldova, Baku (Azerbaijan), Thailand
430	Montenegro	Bulgaria, Brunei Darussalam, Romania

# One fix

Instead of using  $\alpha = 0.05$  for each individual test, use  $\alpha = 0.05$  for the **family of tests** when we examine multiple contrasts to test a single hypothesis.

## Bonferroni method

Take a given  $\alpha$ -threshold and "split it" across the entire family of tests.

Assuming  $\alpha = 0.05$ :

- For 2 tests, conduct each at 0.025 level;
- For 3 tests, conduct each at 0.0167 level;  
etc. ...

Use this new threshold to identify the critical  $t$ -statistic given the number of degrees of freedom. For the PISA example this would be  $p=0.05/3160=0.000016$

*Other approaches exist! Bonferroni is an extremely conservative one--beware!*

As tests increase, so do critical  $t$ -values:

# tests	# new $\alpha$	$t$ -statistic (df = $\infty$ )
1	0.0500	1.96
2	0.0250	2.24
3	0.0167	2.39
4	0.0125	2.50
5	0.0100	2.58
6	0.0083	2.64
10	0.0050	2.81
20	0.0025	3.02
50	0.0010	3.29
100	0.0005	3.48

# Bonferroni correction in R

```
pairwise.t.test(dibels_long$mean_orf, dibels_long$period,  
                 p.adjust.method = "bonferroni")
```

```
##  
##      Pairwise comparisons using t tests with pooled SD  
##  
## data:  dibels_long$mean_orf and dibels_long$period  
##  
##      y1_boy   y1_moy   y2_boy  
## y1_moy < 2e-16 -  
## y2_boy 0.00036 < 2e-16 -  
## y2_moy < 2e-16 < 2e-16 < 2e-16  
##  
## P value adjustment method: bonferroni
```

We can see that our inference has become slightly weaker for our  $Y1\_BOY$  vs.  $Y2\_BOY$  comparison, though still smaller than most traditional thresholds. Note that the others have become weaker too, but they were so small to begin with that we don't see this change.

# Another potential solution

Is there another way we could conceive of the assessment data collection wave variable that is *not* categorical?

```
dibels_long$time <- as.numeric(dibels_long$period)
summary(lm(mean_orf ~ time, dibels_long))

...
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 65.3975    0.6335 103.24  <2e-16 ***
## time        3.0009    0.2313   12.97  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 37.99 on 21582 degrees of freedom
## Multiple R-squared:  0.007738,   Adjusted R-squared:  0.007692
## F-statistic: 168.3 on 1 and 21582 DF,  p-value: < 2.2e-16
...
```

But perhaps, there are more satisfying ways to address this challenge!

# ANOVA

# ANOVA

- Analysis of variance (ANOVA) is a special case of the general linear model
- The primary goal of ANOVA is a comparison of means across different groups
  - $H_0 : \mu_1 = \mu_2 = \mu_3 \dots \mu_K$
- Although regression frameworks are more the norm across most disciplines, the ANOVA approach can be especially useful for:
  - Exploring and comparing the within- and between-group variation in the outcome
  - Simultaneously testing the main effects of categorical variables (*and avoiding some of the problems of multiple hypothesis testing*)

# Within and between



Can you describe the variability within- and between-test periods?

Check out the podcast [Within & Between](#) on quant methods and unpacking the hidden curriculum of academia hosted by Jessica Logan and Sara Hart (takes a developmental science perspective).

# Within and between

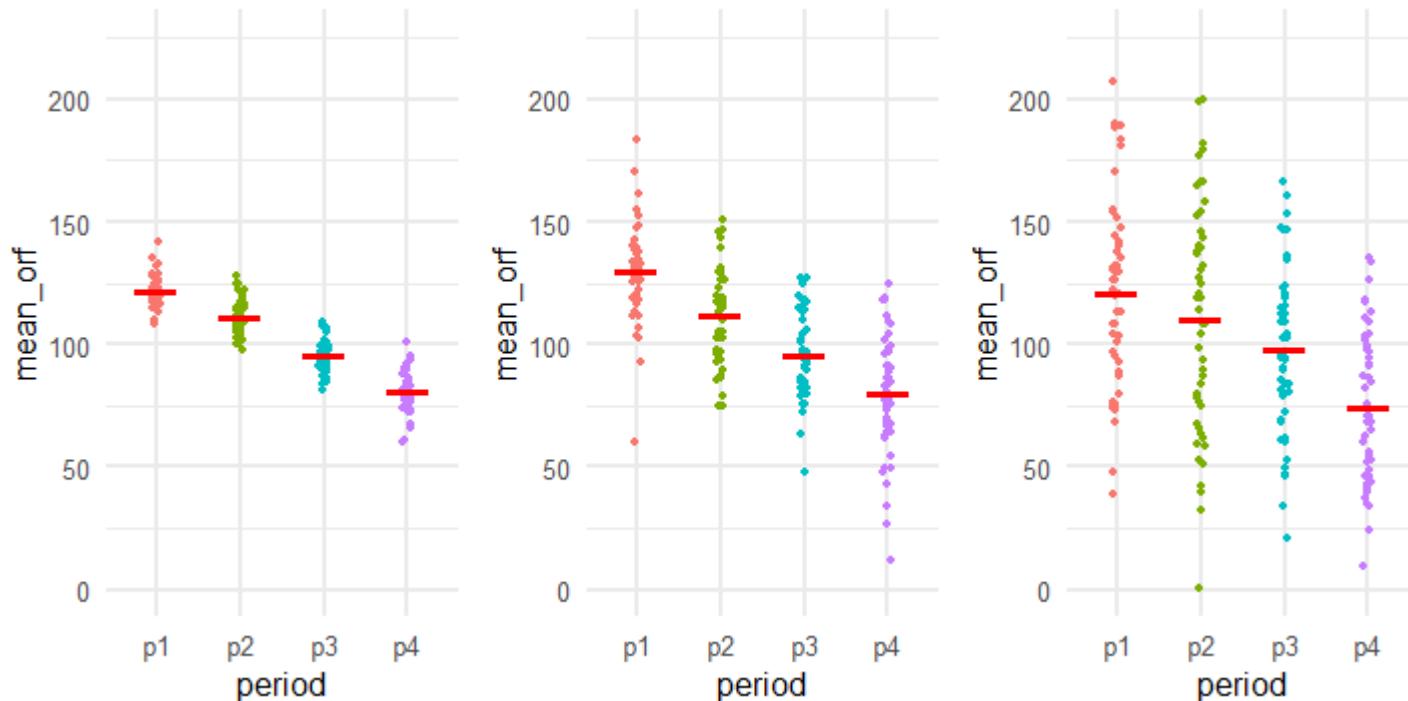


Questions we might want to answer about group differences:

1. Are observed differences between groups "real"?
2. In what context can we place these differences to evaluate their magnitude?

# Within and between

Let's imagine a slightly simpler example. Imagine three different data sets with a four-level categorical predictor and across each data set, the mean value of each category was the same.



How important are the differences in these group means across each dataset? Within-group variation provides important context for evaluating magnitude of between-group variation!

# Partitioning variance

In regression, we partition our total variance  $SS_{\text{total}}$  into our  $SS_{\text{model}}$  and  $SS_{\text{residual}}$ :

$SS_{\text{model}} = \text{Deviation of observed value from the predicted value } (Y_i - \hat{Y}_i)$

$SS_{\text{residual}} = \text{Deviation of predicted value from the grand mean } (\hat{Y} - \bar{Y}_i)$

In ANOVA, we apply a similar but slightly different conceptual process.

# Partitioning variance in ANOVA

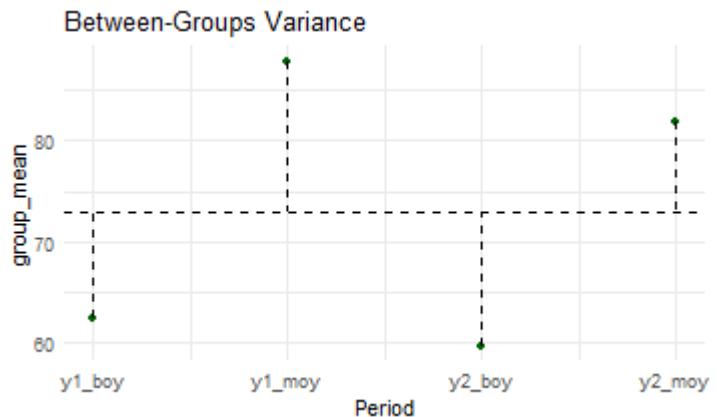
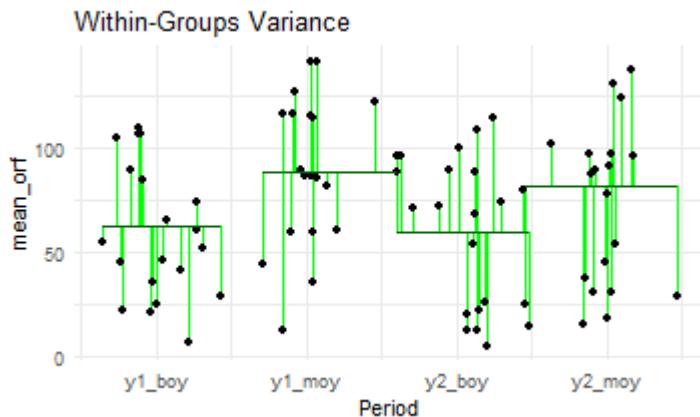
In ANOVA, we separate variance into between-group and within-group variance:

$SS_{\text{within}}$  = Deviation of observed value from its group mean ( $Y_{ik} - \bar{Y}_k$ )

$SS_{\text{between}}$  = Deviation of group mean from the grand mean ( $\bar{Y}_k - \bar{Y}$ )

$$SS_{\text{total}} = SS_{\text{within}} + SS_{\text{between}}$$

# Visualized variance partition



We can represent the residual variance around the group means (here on just a random selection of 20 observations from each period). Just like the error term in our regression model, it is all the remaining variance our predictor (*period*) can't explain.

In addition to each individual observation's deviation from its group mean, each group's mean also deviates from the grand mean of Oral Reading Fluency.

# ANOVA test statistic

When we conduct an ANOVA we are testing the significance of an  $F$ -statistic using the following formula:

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$$

The mean squares (MS) of between- and within-group variance is just the Sum of the Squares ( $SS$ ) for each group, divided by its degrees of freedom ( $df$ ):

$$MS_w = \frac{SS_w}{df_w}$$

$$df_w = N - G$$

$$MS_b = \frac{SS_b}{df_b}$$

$$df_b = G - 1$$

where  $w$  subscripts within,  $b$  subscripts between,  $N$  is the number of observations and  $G$  the number of groups.

# ANOVA significance test

The null hypothesis of an ANOVA is about the ratio of between- to within-group variance.

Essentially, when we state  $H_0 : \mu_1 = \mu_2 = \mu_3 \dots \mu_K$ , we are asking if the mean square variance of the group means around the grand mean is greater than the mean square variance of observations around their group mean. If the between-group variance were much larger than the within-group variance, then the  $F$ -statistic would exceed 1.

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}} = \frac{4.3}{1.5} = 2.87$$

If the between-group variance is equal to or smaller than the within-group variance, then our  $F$ -statistic will be  $\leq 1$ .

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}} = \frac{0.2}{1.5} = 0.13$$

# Calculating the $F$ -statistic

Let's find our  $F$ -statistic for our *period* variable.

Within-Group (Residual) Variance  $MS_{\text{Within}}$

```
# total n - number of groups (4)
nrow(dibels_long)
```

```
## [1] 21584
```

```
df_within <- 21584 - 4
sum((dibels_long$mean_orf - dibels_long$group_mean)^2) / df_within
```

```
## [1] 1306.462
```

# Calculating the $F$ -statistic

Let's find our  $F$ -statistic for our *period* variable.

Between-Group Variance  $MS_{\text{Between}}$

```
# number of groups (4) - 1
df_btwn <- 4-1

sum((mean(dibels_long$mean_orf) - dibels_long$group_mean)^2) / df_btwn

## [1] 1068298
```

# Calculating the $F$ -statistic

$$MS_{\text{Between}} = 1,068,298$$

$$MS_{\text{Within}} = 1,306$$

$$F = \frac{MS_{\text{Between}}}{MS_{\text{Within}}} = \frac{1,068,298}{1306} = 817.99$$

Our  $F$ -statistic is 818. Now that we see how it is calculated, let's fit an ANOVA in R to review the output and make an inference (note that we could now also consult an  $F$ -statistic lookup table to get the same info!).

# ANOVA in R

Because ANOVA is just a particular method of analyzing variance in GLMs, we can wrap the `anova` command around our `lm` model fit.

```
fit3 <- lm(mean_orf ~ period, dibels_long)

anova(fit3)
```

```
## Analysis of Variance Table
##
## Response: mean_orf
##             Df  Sum Sq Mean Sq F value    Pr(>F)
## period      3 3204894 1068298   817.7 < 2.2e-16 ***
## Residuals 21580 28193461     1306
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We can see all the information we calculated earlier. With a  $p$ -value  $< 2.2 \times 10^{-16}$ , our  $F$ -statistic highly unlikely to be a product of a population in which the population means across the four waves of ORF administration were equal. Therefore we reject the null hypothesis and conclude that, on average in the population, the mean ORF scores differed significantly across the waves of assessment administration.

# Variance decomposition

- In addition to using ANOVA to "batch test" group differences, as we've seen it can be a useful tool to decompose the variance of your outcome into between and within group variation
- We can, in fact, extend this analysis to understand how much of the variation in an individual's outcome occurs across **different** groups.  
For example:
  - What proportion of the variation in child outcomes occurs within classrooms, compared to schools, compared to neighborhoods?
  - Are differences in school funding greater between schools, between districts or between states?
- You can explore more about these topics in our HLM sequence (EDLD 628/629)

# ANOVA vs. regression

- Both are implementations of the General Linear Model
- A regression with dummy indicator variables is statistically identical to ANOVA
- The  $F$ -test in a regression model represents a test of the model's variance against the residual
- In ANOVA, we can have one or more  $F$ -tests where we "batch test" a group of coefficients
  - This can help avoid Type I errors (rejecting the null when it is in fact true)
  - ANOVA doesn't tell you anything about the magnitude of the difference...which seems important?
- Learning regression is the more general approach, of which ANOVA is a special implementation; by learning regression you have a more flexible tool kit

# Presenting our results

```
modelsummary(list(fit1, fit3),
            stars=T,
            vcov = "robust",
            gofomit = "Adj. |AIC|BIC|Log|RMSE|RSE|Std.Err",
            coef_rename = c("post" = "Post-Pandemic Onset",
                            "periody1_moy" = "Winter 2020",
                            "periody2_boy" = "Fall 2020",
                            "periody2_moy" = "Winter 2021"))
```

**Table 1.** Estimates of grade-level average Oral Reading Fluency (ORF) score across waves of DIBELS administration, 2019–2021

	Model 1	Model 2
(Intercept)	75.125*** (0.369)	62.344*** (0.449)
Post-Pandemic Onset	-4.450*** (0.518)	
Winter 2020		25.562*** (0.696)
Fall 2020		-2.791*** (0.631)
Winter 2021		19.454*** (0.700)
Num.Obs.	21584	21584
R2	0.003	0.102

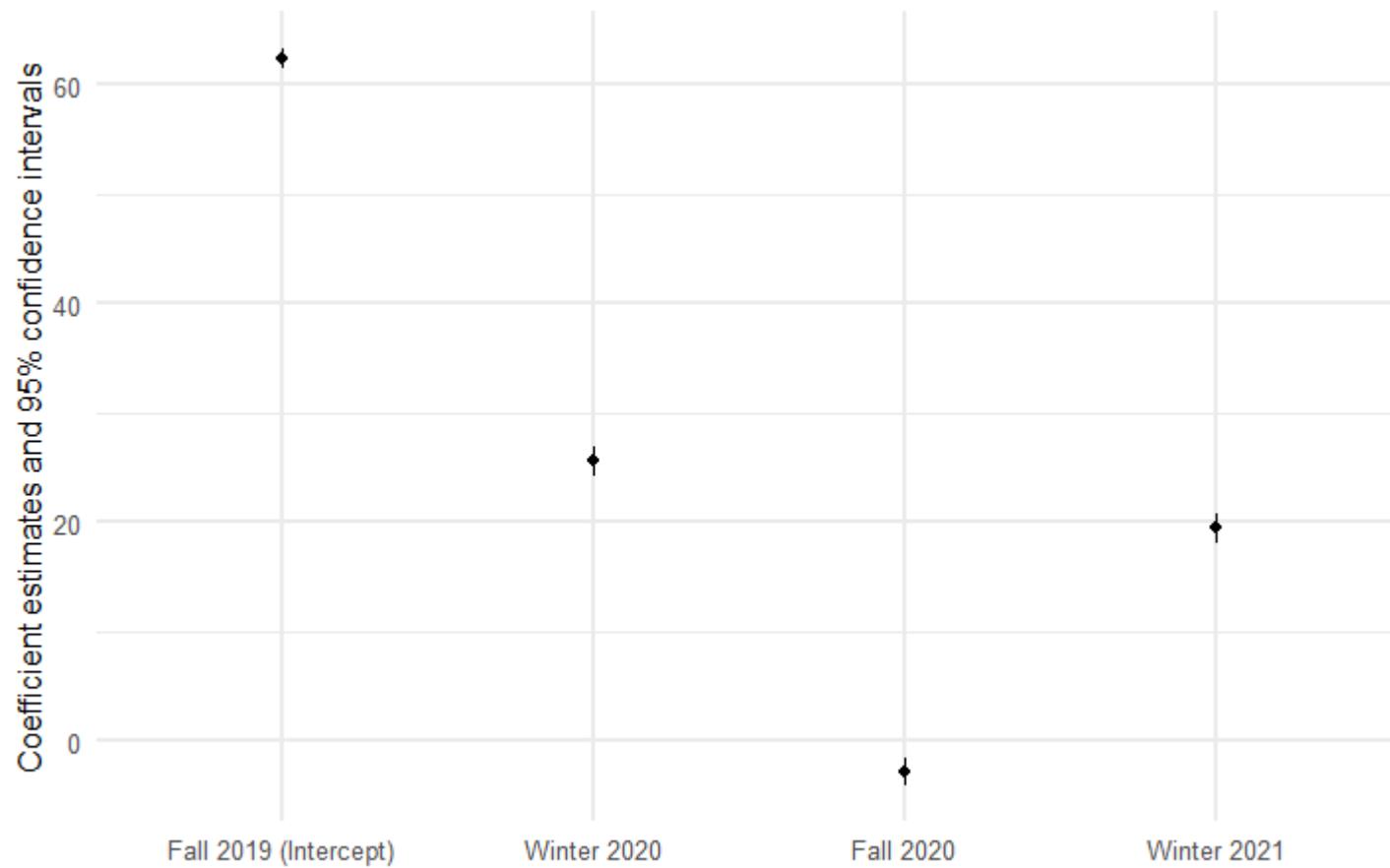
Cells report coefficients and heteroscedastic-robust standard errors in parentheses.  
Each observation is a school-grade-test value.

+ p < 0.1, \* p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001

# Visualizing results

```
coef_plot <- modelplot(fit3,
  coef_rename = c("(Intercept)" = "Fall 2019 (Intercept)",
    "periody1_moy" = "Winter 2020",
    "periody2_boy" = "Fall 2020",
    "periody2_moy" = "Winter 2021"),
  vcov = "robust") +
  coord_flip() +
  theme_minimal(base_size = 16)
```

# Visualizing results



# Multiple regression with categorical variables

# Mawwrr predictors

Now that we've learned the basic concept of multiple regression, it's a fairly simple task to add additional covariates (either continuous or categorical) to our equation.

**What theoretically justified covariates might be sensible to include? How would we make such a determination?**

Preliminarily, let's look at three: *GRADE* and *SCHOOL\_ENROLL*:

$$MEAN\_ORF_j = \beta_0 + \beta_1 Y1\_MOY_j + \beta_2 Y2\_BOY_j + \beta_3 Y2\_MOY_j + \beta_4 GRADE_j + \beta_5 SCHOOL\_ENROLL_j + \dots + \varepsilon_j$$

# Mawwrr predictors

$$MEAN\_ORF_j = \beta_0 + \beta_1 Y1\_MOY_j + \beta_2 Y2\_BOY_j + \beta_3 Y2\_MOY_j + \\ \beta_4 GRADE_j + \beta_5 SCHOOL\_ENROLL_j + \dots + \varepsilon_j$$

Before fitting any models, **can we interpret what each of these coefficients will now represent?**

To what do I need to attend when specifying the predictor *GRADE* in my model?

# MR with categoricals

```
fit4<-lm(mean_orf ~ period + as.factor(grade) +
           school_enroll, data=dibels_long)
summary(fit4)

...
## 
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)    
## (Intercept)            16.0605642  0.3715846 43.222 < 2e-16 ***
## periody1_moy          25.5620256  0.3324056 76.900 < 2e-16 ***
## periody2_boy          -2.7913676  0.3324056 -8.397 < 2e-16 ***
## periody2_moy          19.4541290  0.3324056 58.525 < 2e-16 ***
## as.factor(grade)2     38.9506245  0.3267840 119.194 < 2e-16 ***
## as.factor(grade)3     60.2892942  0.3572456 168.761 < 2e-16 ***
## as.factor(grade)4     81.9649741  0.3805173 215.404 < 2e-16 ***
## as.factor(grade)5     82.7401708  0.3913447 211.425 < 2e-16 ***
## school_enroll         0.0026358  0.0007061   3.733  0.00019 ***
## --- 
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 17.27 on 21575 degrees of freedom
## Multiple R-squared:  0.7952,    Adjusted R-squared:  0.7951 
## F statistic: 1.047e+04 on 8 and 21575 DF, p-value: < 2.2e-16
```

```

## 
## Call:
## lm(formula = mean_orf ~ period + as.factor(grade) + school_enroll,
##      data = dibels_long)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -98.502 -10.816  -0.482  10.385  86.022 
## 
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)    
## (Intercept)           16.0605642  0.3715846  43.222 < 2e-16 ***
## periody1_moy         25.5620256  0.3324056  76.900 < 2e-16 ***
## periody2_boy        -2.7913676  0.3324056  -8.397 < 2e-16 ***
## periody2_moy         19.4541290  0.3324056  58.525 < 2e-16 ***
## as.factor(grade)2    38.9506245  0.3267840 119.194 < 2e-16 ***
## as.factor(grade)3    60.2892942  0.3572456 168.761 < 2e-16 ***
## as.factor(grade)4    81.9649741  0.3805173 215.404 < 2e-16 ***
## as.factor(grade)5    82.7401708  0.3913447 211.425 < 2e-16 ***
## school_enroll       0.0026358  0.0007061   3.733  0.00019 ***
## ---                
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 
## 
## Residual standard error: 17.27 on 21575 degrees of freedom
## Multiple R-squared:  0.7952,    Adjusted R-squared:  0.7951 
## F-statistic: 1.047e+04 on 8 and 21575 DF,  p-value: < 2.2e-16

```

# Show what you know

Use the **adjusted means** to show your findings when your question predictor is categorical. Set all predictors to their sample means or to the value of the category and then compute the predicted value of your outcome at each level of your categorical question predictor:

```
mean(dibels_long$school_enroll)  
## [1] 337.3341  
  
prop.table(table(dibels_long$grade))  
  
##  
##      1          2          3          4          5  
## 0.2657524 0.2524092 0.1864344 0.1540030 0.1414010
```

# Show what you know

Use the **adjusted means** to show your findings when your question predictor is categorical. Set all predictors to their sample means or to the value of the category and then compute the predicted value of your outcome at each level of your categorical question predictor:

For Y1\_MOY:

$$MEAN_{ORF_j} = 16.06 + 25.56(1) + (-2.79)(0) + 19.45(0) + 38.95(0.25) + 60.29(0.19) + 81.96(0.15) + 82.74(0.14) + 0.003(337.33) = 87.70$$

For Y2\_BOY:

$$MEAN_{ORF_j} = 16.06 + 25.56(0) + (-2.79)(1) + 19.45(0) + 38.95(0.25) + 60.29(0.19) + 81.96(0.15) + 82.74(0.14) + 0.003(337.33) = 59.35$$

For Y2\_MOY:

$$MEAN_{ORF_j} = 16.06 + 25.56(0) + (-2.79)(0) + 19.45(1) + 38.95(0.25) + 60.29(0.19) + 81.96(0.15) + 82.74(0.14) + 0.003(337.33) = 81.59$$

For Y1\_BOY:

$$MEAN_{ORF_j} = 16.06 + 25.56(0) + (-2.79)(0) + 19.45(0) + 38.95(0.25) + 60.29(0.19) + 81.96(0.15) + 82.74(0.14) + 0.003(337.33) = 62.14$$

# Present in simple tabular format

**Table 1.**

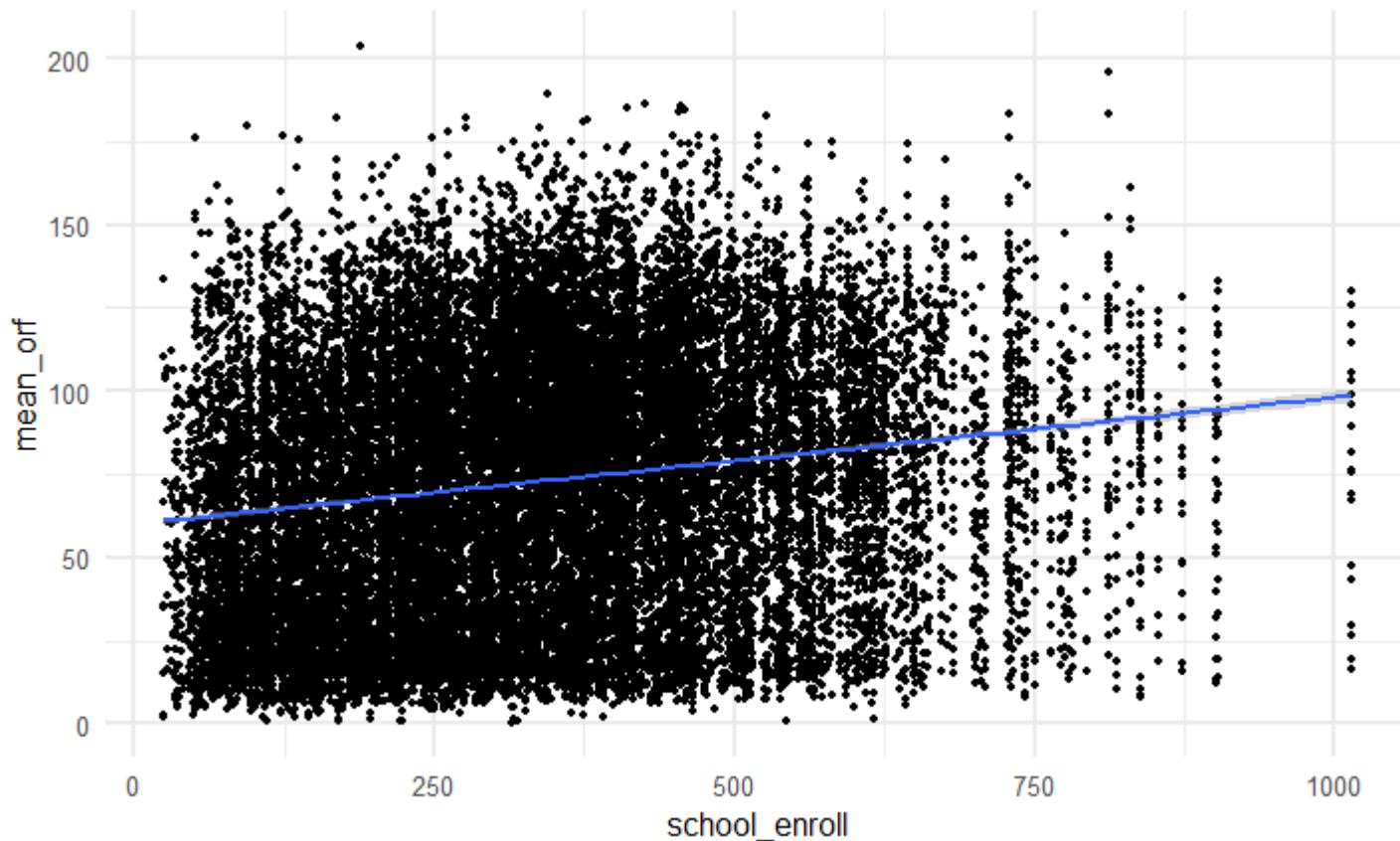
Mean Oral Reading Fluency score across different administrations of the DIBELS 8

Wave	Unadjusted	Adjusted
Fall 2019	62.3	62.1
Winter 2020	87.9	87.7
Fall 2020	59.6	62.1
Winter 2021	81.8	81.6

\*Adjusted mean = adjusting for grade and school size.

# An aside

If our question predictor were continuous, and we wanted to adjust for a categorical, how might we do so? With just one other predictor?



# An aside

If our question predictor were continuous, and we wanted to adjust for a categorical, how might we do so? With just one other predictor?



What multiple regression assumption is being relaxed here?

# An aside

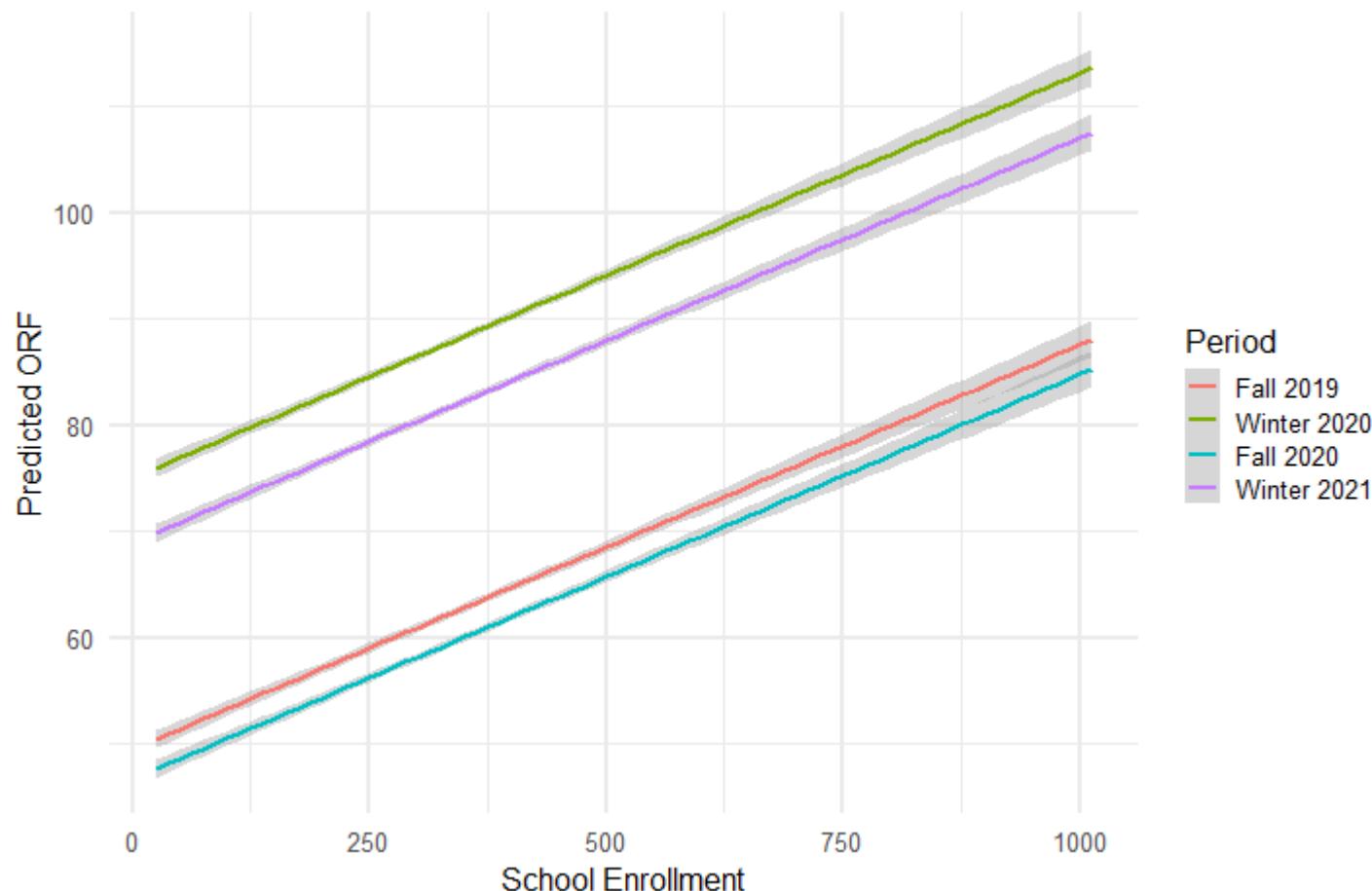
If our question predictor were continuous, and we wanted to adjust for a categorical, how might we do so? With multiple predictors?

```
df3 <- margins::margins(fit4,
                         at = list(period=c("y1_boy", "y1_moy",
                                            "y2_boy", "y2_moy")))

# Use prototypical values in resulting dataset to show results
proto <- ggplot(data=df3, aes(x=school_enroll, y=fitted,color=period)) +
  geom_smooth(method='lm') +
  xlab("School Enrollment") + ylab("Predicted ORF") +
  scale_color_discrete(name = "Period",
                        breaks=c("y1_boy", "y1_moy",
                                "y2_boy", "y2_moy"),
                        labels=c("Fall 2019", "Winter 2020",
                                "Fall 2020", "Winter 2021")) +
  theme_minimal(base_size=16)
```

# An aside

If our question predictor were continuous, and we wanted to adjust for a categorical, how might we do so? With multiple predictors?



# Alongside previous results

```
modelsummary(list(fit3, fit4),
            stars=T,
            vcov = "robust",
            gofomit = "Adj. |AIC|BIC|Log|RMSE|RSE|Std.Err",
            coef_rename = c("periody1_moy" = "Winter 2020",
                            "periody2_boy" = "Fall 2020",
                            "periody2_moy" = "Winter 2021",
                            "as.factor(grade)2" = "2nd Grade",
                            "as.factor(grade)3" = "3rd Grade",
                            "as.factor(grade)4" = "4th Grade",
                            "as.factor(grade)5" = "5th Grade",
                            "school_enroll" = "School Enrollment (#)"))
```

# Alongside previous results

	Model 1	Model 2
(Intercept)	62.344*** (0.449)	16.061*** (0.326)
Winter 2020	25.562*** (0.696)	25.562*** (0.332)
Fall 2020	-2.791*** (0.631)	-2.791*** (0.311)
Winter 2021	19.454*** (0.700)	19.454*** (0.330)
2nd Grade		38.951*** (0.297)
3rd Grade		60.289*** (0.345)
4th Grade		81.965*** (0.406)

# Alongside previous results: alternative

	Model 1	Model 2
(Intercept)	62.344*** (0.449)	16.061*** (0.326)
Winter 2020	25.562*** (0.696)	25.562*** (0.332)
Fall 2020	-2.791*** (0.631)	-2.791*** (0.311)
Winter 2021	19.454*** (0.700)	19.454*** (0.330)
Covariates?	No	Yes
Num.Obs.	21584	21584
R2	0.102	0.795
+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001		

# Putting into words

# ANCOVA

# ANCOVA

- **Analysis of covariance (ANCOVA)** is an extension of ANOVA and multiple regression
- It is also a part of the broader family of General Linear Models
- The model relates categorical predictors to a continuous outcome, adjusting for the effects of other covariates
  - Note: you may see in some (older) sources the statement that ANCOVA models can only adjust for the effects of other categorical covariates. This is not true as long as you are careful to specify your categorical covariates as dummy indicators
- The null hypothesis is still the same as ANOVA ( $\mu_1 = \mu_2 = \mu_K$ ).

# ANCOVA results

We can examine whether there are differences in the ORF scores by when students sat for the test, while adjusting for students' grade level and their school's size

```
anova(fit4)
```

```
## Analysis of Variance Table
##
## Response: mean_orf
##                               Df  Sum Sq Mean Sq   F value    Pr(>F)
## period                  3 3204894 1068298 3583.554 < 2.2e-16 ***
## as.factor(grade)        4 21757555 5439389 18246.165 < 2.2e-16 ***
## school_enroll           1     4154     4154    13.933 0.0001899 ***
## Residuals                21575  6431752      298
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- As before, we reject the null and conclude that there is a difference, on average in the population, between waves of the ORF administration, *adjusting for the effects of students' grade and school size*
- However, our  $F$ -statistic is now **MUCH** bigger
- We've dramatically shrunk the RSS (28,193,461 vs. 6,431,752)

# ANOVA v. ANCOVA

Let's contrast an ANOVA with an ANCOVA test:

```
anova(fit3, fit4)

## Analysis of Variance Table
##
## Model 1: mean_orf ~ period
## Model 2: mean_orf ~ period + as.factor(grade) + school_enroll
##   Res.Df      RSS Df Sum of Sq    F    Pr(>F)
## 1  21580 28193461
## 2  21575  6431752  5  21761709 14600 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- In our basic ANOVA, our residual SS was 28,193,461 (it still is now). Now we see the residual SS for our ANCOVA fit is 6,431,752, or meaningfully (and statistically significantly smaller)
- Our variance has been "reorganized" with the addition of `school_enroll`.
- We can compare the two model fits with a new  $F$ -statistic that assesses whether one explains more of the variance ("is a better fit") than the other. It is.

# ANCOVA v. MR

We can also compare our ANCOVA output to our regression output and see our dummy-coded, "unbatched" analysis:

```
...
## 
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)    
## (Intercept)           16.0605642  0.3715846 43.222 < 2e-16 ***
## periody1_moy        25.5620256  0.3324056 76.900 < 2e-16 ***
## periody2_boy        -2.7913676  0.3324056 -8.397 < 2e-16 ***
## periody2_moy        19.4541290  0.3324056 58.525 < 2e-16 ***
## as.factor(grade)2   38.9506245  0.3267840 119.194 < 2e-16 ***
## as.factor(grade)3   60.2892942  0.3572456 168.761 < 2e-16 ***
## as.factor(grade)4   81.9649741  0.3805173 215.404 < 2e-16 ***
## as.factor(grade)5   82.7401708  0.3913447 211.425 < 2e-16 ***
## school_enroll       0.0026358  0.0007061   3.733  0.00019 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 17.27 on 21575 degrees of freedom
## Multiple R-squared:  0.7952,    Adjusted R-squared:  0.7951 
## F-statistic: 1.047e+04 on 8 and 21575 DF,  p-value: < 2.2e-16
...
```

# Common statistical tests are linear models

## Common statistical tests are linear models

Last updated: 02 April, 2019

See worked examples and more details at the accompanying notebook: <https://lindeloev.github.io/tests-as-linear>

Common name	Built-in function in R	Equivalent linear model in R	Exact?	The linear model in words	Icon	
<b>Simple regression: <math>\text{Im}(y \sim 1 + x)</math></b>	<b>y is independent of x</b> P: One-sample t-test N: Wilcoxon signed-rank	<code>t.test(y)</code> <code>wilcox.test(y)</code>	<code>lm(y ~ 1)</code> <code>lm(signed_rank(y) ~ 1)</code>	✓ <a href="#">for N &gt; 14</a>	One number (intercept, i.e., the mean) predicts <b>y</b> . - (Same, but it predicts the <i>signed rank</i> of <b>y</b> .)	
	P: Paired-sample t-test N: Wilcoxon matched pairs	<code>t.test(y1, y2, paired=TRUE)</code> <code>wilcox.test(y1, y2, paired=TRUE)</code>	<code>lm(y2 - y1 ~ 1)</code> <code>lm(signed_rank(y2 - y1) ~ 1)</code>	✓ <a href="#">for N &gt; 14</a>	One intercept predicts the pairwise <b>y<sub>2</sub>-y<sub>1</sub></b> differences. - (Same, but it predicts the <i>signed rank</i> of <b>y<sub>2</sub>-y<sub>1</sub></b> .)	
	<b>y ~ continuous x</b> P: Pearson correlation N: Spearman correlation	<code>cor.test(x, y, method='Pearson')</code> <code>cor.test(x, y, method='Spearman')</code>	<code>lm(y ~ 1 + x)</code> <code>lm(rank(y) ~ 1 + rank(x))</code>	✓ <a href="#">for N &gt; 10</a>	One intercept plus <b>x</b> multiplied by a number (slope) predicts <b>y</b> . - (Same, but with <i>ranked x</i> and <b>y</b> )	
<b>Multiple regression: <math>\text{Im}(y \sim 1 + x_1 + x_2 + \dots)</math></b>	<b>y ~ discrete x</b> P: Two-sample t-test P: Welch's t-test N: Mann-Whitney U	<code>t.test(y1, y2, var.equal=TRUE)</code> <code>t.test(y1, y2, var.equal=FALSE)</code> <code>wilcox.test(y1, y2)</code>	<code>lm(y ~ 1 + G<sub>2</sub>)<sup>A</sup></code> <code>glm(y ~ 1 + G<sub>2</sub>, weights=...<sup>B</sup>)<sup>A</sup></code> <code>lm(signed_rank(y) ~ 1 + G<sub>2</sub>)<sup>A</sup></code>	✓ ✓ <a href="#">for N &gt; 11</a>	An intercept for <b>group 1</b> (plus a difference if <b>group 2</b> ) predicts <b>y</b> . - (Same, but with one variance <i>per group</i> instead of one common.) - (Same, but it predicts the <i>signed rank</i> of <b>y</b> .)	
	P: One-way ANOVA N: Kruskal-Wallis	<code>aov(y ~ group)</code> <code>kruskal.test(y ~ group)</code>	<code>lm(y ~ 1 + G<sub>2</sub> + G<sub>3</sub> + ... + G<sub>N</sub>)<sup>A</sup></code> <code>lm(rank(y) ~ 1 + G<sub>2</sub> + G<sub>3</sub> + ... + G<sub>N</sub>)<sup>A</sup></code>	✓ <a href="#">for N &gt; 11</a>	An intercept for <b>group 1</b> (plus a difference if group ≠ 1) predicts <b>y</b> . - (Same, but it predicts the <i>rank</i> of <b>y</b> .)	
	P: One-way ANCOVA	<code>aov(y ~ group + x)</code>	<code>lm(y ~ 1 + G<sub>2</sub> + G<sub>3</sub> + ... + G<sub>N</sub> + x)<sup>A</sup></code>	✓	- (Same, but plus a slope on <b>x</b> .) <i>Note: this is discrete AND continuous. ANCOVAs are ANOVAs with a continuous x.</i>	
	P: Two-way ANOVA	<code>aov(y ~ group * sex)</code>	<code>lm(y ~ 1 + G<sub>2</sub> + G<sub>3</sub> + ... + G<sub>N</sub> + S<sub>2</sub> + S<sub>3</sub> + ... + S<sub>K</sub> + G<sub>2</sub>*S<sub>2</sub>+G<sub>3</sub>*S<sub>3</sub>+...+G<sub>N</sub>*S<sub>K</sub>)</code>	✓	Interaction term: changing <b>sex</b> changes the <b>y ~ group</b> parameters. <i>Note: G<sub>2..K</sub> is an indicator (0 or 1) for each non-intercept levels of the <b>group</b> variable. Similarly for S<sub>2..K</sub> for sex. The first line (with G) is main effect of group, the second (with S) for sex and the third is the <b>group × sex</b> interaction. For two levels (e.g. male/female), line 2 would just be "S<sub>2</sub>" and line 3 would be S<sub>2</sub> multiplied with each G<sub>i</sub>.</i>	[Coming]
<b>Counts ~ discrete x</b> N: Chi-square test	<code>chisq.test(groupXsex_table)</code>	<b>Equivalent log-linear model</b> <code>glm(y ~ 1 + G<sub>2</sub> + G<sub>3</sub> + ... + G<sub>N</sub> + S<sub>2</sub> + S<sub>3</sub> + ... + S<sub>K</sub> + G<sub>2</sub>*S<sub>2</sub>+G<sub>3</sub>*S<sub>3</sub>+...+G<sub>N</sub>*S<sub>K</sub>, family=...)<sup>A</sup></code>	✓	Interaction term: (Same as Two-way ANOVA.) <i>Note: Run glm using the following arguments: glm(model, family=poisson()) As linear-model, the Chi-square test is log(y) = log(N) + log(α) + log(β) + log(αβ) where α and β are proportions. See more info in the accompanying notebook.</i>	Same as Two-way ANOVA	
N: Goodness of fit	<code>chisq.test(y)</code>	<code>glm(y ~ 1 + G<sub>2</sub> + G<sub>3</sub> + ... + G<sub>N</sub>, family=...)<sup>A</sup></code>	✓	(Same as One-way ANOVA and see Chi-Square note.)	1W-ANOVA	

List of common parametric (P) non-parametric (N) tests and equivalent linear models. The notation  $y \sim 1 + x$  is R shorthand for  $y = 1 \cdot b + a \cdot x$  which most of us learned in school. Models in similar colors are highly similar, but really, notice how similar they *all* are across colors! For non-parametric models, the linear models are reasonable approximations for non-small sample sizes (see "Exact" column and click links to see simulations). Other less accurate approximations exist, e.g., Wilcoxon for the sign test and Goodness-of-fit for the binomial test. The signed rank function is `signed_rank = function(x) sign(x) * rank(abs(x))`. The variables G<sub>i</sub> and S<sub>i</sub> are "[dummy coded indicator variables](#) (either 0 or 1) exploiting the fact that when  $\Delta x = 1$  between categories the difference equals the slope. Subscripts (e.g., G<sub>2</sub> or y<sub>i</sub>) indicate different columns in data. lm requires long-format data for all non-continuous models. All of this is exposed in greater detail and worked examples at <https://lindeloev.github.io/tests-as-linear>.

<sup>A</sup> See the note to the two-way ANOVA for explanation of the notation.

<sup>B</sup> Same model, but with one variance per group: `glm(value ~ 1 + G2, weights = varIdent(form = ~1|group), method="ML")`.



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<https://lindeloev.net>

# It was the GLM the whole time...



4:54 AM - Oct 27, 2015 · Twitter Web App

254 Retweets 40 Quote Tweets 1,836 Likes

# Synthesis and wrap-up

# Putting categorical predictors together

1. Regression models can easily include dichotomous and polychotomous predictors
  - Can be used for either nominal or ordinal predictors with sensible planning around dummy variables and the omitted reference category
2. All assumptions are about Y at particular values of X (or Xs)—no assumptions about the distribution of the predictors
3. The same toolkit we've developed for continuous predictors can be used for dichotomous and polychotomous predictors (including hypothesis tests, correlations and plots)
4. Be aware that when you introduce many categorical predictors you are implicitly engaging in multiple hypothesis testing
  - ANOVA/ANCOVA can help you address this, but be careful not to focus on just interpreting *p*-values
5. ANOVA/ANCOVA are just special cases of multiple regression
  - Can be useful to avoid problems of multiple hypothesis testing and understanding within- and between-variation
  - Can tell you little to nothing about the magnitude of group differences

# Goals for the unit

- Describe the relationship between dichotomous and polychotomous variables and convert variables between these forms, as necessary
- Conduct a two-sample  $t$ -test
- Describe the relationship between a two-sample  $t$ -test and regressing a continuous outcome on a dichotomous predictor
- Estimate a regression with one dummy variable as a predictor and interpret the results (including when the reference category changes)
- Estimate a multiple regression model with several continuous and dummy variables and interpret the results
- Estimate an ANOVA model and interpret the within- and between-group variance
  - Do the same for an ANCOVA model, adjusting for additional continuous predictors
- Describe the similarities and differences of Ordinary-Least Squares regression analysis and ANOVA/ANCOVA, and when one would prefer one approach to another
- Describe potential Type I error problems that arise from multiple group comparisons and potential solutions to these problems, including theory, pre-registration, ANOVA and *post-hoc* corrections
- Describe the relationship between different modeling approaches with the General Linear Model family

# To-Dos

## Reading:

- Finish by Feb. 16: LSWR Chapter 14 and 16.6

## Assignment 3:

- Due Feb. 14, 11:59pm (**note extension**)

## Assignment 4:

- Due Feb. 28, 11:59pm