Random variables

Notable discrete distributions

Binomial distribution

Poisson distribution

Appears with the law of rare events. It can be seen as the limit of a binomial distribution B(n, p) where n $\rightarrow$ +∞, p $\rightarrow$ 0 and n*p = λ.

It also happens in queuing systems to model the number of arrivals to a service in a time interval. To have a Poisson distribution, the following conditions must be hold:

- The expected number of arrivals only depends on the length of the time interval
- The number of arrivals in different points of the interval are independent random variables
- The probability of two simultaneous arrivals is 0
  If this conditions holds, the distribution of the number of arrivals must be a Poisson distribution.
  X follows a Poisson distribution with parameter λ. We denote it X~P(λ) and its probability mass function is P(P(λ) = K) = $e^{-\lambda} * \frac{\lambda^K}{K!}$ for K = 0, 1, 2, 3...

E(X) = λ = Var(X)

Exercises

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P(being defective) = 0.05
a) probability of not having to inspect the whole batch:

P(NOT having to inspect the whole batch) $\rightarrow$ P(at most 2 defective components out of 30) $\rightarrow$ P(B(30, 0.05) $\leq$ 2) $\rightarrow$ P(B(30, 0.05) = 0) + P(B(30, 0.05) = 1) + P(B(30, 0.05) = 2) $\rightarrow$ **0.8121**.

b) the probability distribution of the random variable "number of components examined"

x	f(x)
30	0.8121
1000	1 - 0.8121 = 0.1879

c) expected number of components examined per batch
E(X) = 30*0.8121 + 1000*0.1879 = 212.263

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The number of breakdowns in an information system is a random variable X following a Poisson distribution with parameter 2 (λ = 2)

a) Expected number of breakdowns $\rightarrow$ E(X) = 2

b) What's the probability that the number of breakdowns is between 1.5 and 3.7

P(1.5 $\leq$ X $\leq$ 3.7) = P(X=2) + P (X=3) = $e^{-2} * \frac{2^2}{2!} + e^{-2} * \frac{2^3}{3!}$

c) P(-2.24 $\leq$ X $\leq$ 6.24) = $\sum_{k=0}^{6}$ P(X = K) = $\sum_{k=0}^{6} e^{-2} * \frac{2^k}{k!}$ = 0.9955

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P(0.3*t)

a) Two incoming messages in a period of 2 sec

Two messages in 10 seconds follows ~P(0.3*10)

P(Y = 2) = $e^{-3} * \frac{3^2}{2!}$ = 0.224

b) Number of messages in 5 messages between 2 and 4

It follows ~P(1.5)
P(2 $\leq$ Z $\leq$ 4) $\rightarrow$ P(Z = 2) + P(Z = 3) + P(Z = 4) = $e^{-1.5} * \frac{1.5^2}{2!} + e^{-1.5} * \frac{1.5^3}{3!} + e^{-1.5} * \frac{1.5^4}{4!}$

c) P more than 4 seconds pass between two messages. This is the same as say that in 4 seconds there was no messages.
It follows ~P(0.3*4)
P(P(1.2) = 0) = $e^{-1.2} * \frac{1.2^0}{0!}$ = 0.301