

Factorize the characteristic equation for coupling patches of waves

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Make output pretty.

```
1 on div; off allfac; on revpri;
```

Introduce variable $\xi = x/D$ to be the subgrid variable in each element.

```
2 depend xi,x; let df(xi,x)=>1/dd;
```

Define operator `cis` to represent $e^{i\theta}$. Then define the exponential structure, `ee`, we expect from element to element, index j , and its reciprocal, `ff`.

```
3 operator cis;  
4 let {cis(0)=>1, cis(~a)^2=>cis(2*a),  
5     cis(~a)*cis(~b)=>cis(a+b),  
6     df(cis(~a),~b)=>i*cis(a)*df(a,b)  
7     };  
8 ee:=cis(11*t/dd+k*j);  
9 ff:=cis(-11*t/dd-k*j);
```

Define subgrid fields for even elements, say those labelled 0, and odd elements, labelled 1 (or vice versa).

```
10 u0:= ( a0*cos(11*xi)+b0*sin(11*xi))*ee;
```

```

11 h0:=i*(-a0*sin(ll*xi)+b0*cos(ll*xi))*ee;
12 h1:= ( a1*cos(ll*xi)+b1*sin(ll*xi))*ee;
13 u1:=i*(-a1*sin(ll*xi)+b1*cos(ll*xi))*ee;

```

Check that the above subgrid fields satisfy the PDE in each element. Should get four zeros here.

```

14 pde:={df(u0,t)+df(h0,x)
15       ,df(h0,t)+df(u0,x)
16       ,df(u1,t)+df(h1,x)
17       ,df(h1,t)+df(u1,x)};

```

Code the coupling relations.

```

18 chl:=((1-gam/2)*(sub(xi==+1,h0)-sub(xi=-1,h0))
19        -gam/2*(sub({j=j+2,xi=-1},h0)-sub({j=j-2,xi=+1},h0))
20        )*ff;
21 chr:=(1/2*(sub({xi=-1,j=j+1},h0)+sub({xi=1,j=j-1},h0))
22        +sub(xi=0,h1)
23        )*ff;
24 cul:=(1/2*(sub({xi=-1,j=j+1},u1)+sub({xi=1,j=j-1},u1))
25        +sub(xi=0,u0)
26        )*ff;
27 cur:=((1-gam/2)*(sub(xi==+1,u1)-sub(xi=-1,u1))
28        -gam/2*(sub({j=j+2,xi=-1},u1)-sub({j=j-2,xi=+1},u1))
29        )*ff;

```

Form the matrix that multiplies each of the unknown coefficients.

```

30 a:=mat((df(chl,a0),df(chl,b0),df(chl,a1),df(chl,b1))
31         ,(df(chr,a0),df(chr,b0),df(chr,a1),df(chr,b1))
32         ,(df(cul,a0),df(cul,b0),df(cul,a1),df(cul,b1))
33         ,(df(cur,a0),df(cur,b0),df(cur,a1),df(cur,b1))
34         );

```

Get non-trivial solutions only when the determinant is zero, so find determi-

nant and factorize.

```
35 chareqn:=(det(a) where cis(~q)=>cos(q)+i*sin(q));
36 chareqn:=trigsimp(chareqn,expand);
37 chareqn:=factorize(chareqn);
```

Finished.

```
38 end;
```

The output finishes with the following which seems different to what Meng generated. Check.

```
39 chareqn := {{4,1},
40             {sin(11) + sin(k)*gam,1},
41             { - sin(11) + sin(k)*gam,1},
42             {1 + sin(k),1},
43             { - 1 + sin(k),1},
44             {1 + sin(11),1},
45             { - 1 + sin(11),1}}
```